

Improving Partial Mutual Information Based Input Variable Selection for Data Driven Environmental and Water Resources Models

XUYUAN LI

B.E. (Hons)

Thesis submitted in fulfilment of the requirements for the degree of Doctor of Philosophy

The University of Adelaide Faculty of Engineering, Computer and Mathematical Sciences School of Civil, Environmental and Mining Engineering

Copyright[©] March 2015

Improving Partial Mutual Information Based Input Variable Selection for Data Driven Environmental and Water Resources Models

By:

Xuyuan Li, B.E. (Hons)

Supervised by:

Professor Holger R. Maier, B.E. (Hons), Ph.D., MIEAust, CPEng (NPER) Professor of Integrated Water Systems Engineering, Associate Editor, Water Resources Research, Member of Editorial Board, Environmental Modelling and Software, School of Civil, Environmental & Mining Engineering, The University of Adelaide

Dr. Aaron C. Zecchin, Ph.D., B.E. (Hons), B.Sc. (Math & Comp. Sci.), Senior lecturer, School of Civil, Environmental & Mining Engineering, The University of Adelaide

Thesis submitted in fulfillment of the requirements for the degree of **Doctor of Philosophy**

School of Civil, Environmental & Mining Engineering Faculty of Engineering, Computer and Mathematical Sciences The University of Adelaide North Terrace, Adelaide, SA 5005, Australia Phone: +61 8 8313 1575 Fax : +61 8 8303 4359 Email: <u>xli@civeng.adelaide.edu.au</u>, <u>xliadelaide@gmail.com</u> Copyright© Xuyuan Li, March, 2015.

II

TABLE OF CONTENT

LIST OF FIGURES
LIST OF TABLESXI
NOMENCLATURE & ABBREVIATIONSXIV
ABSTRACTXX
STATEMENT OF ORIGINALITYXXIV
ACKNOWLEDGEMENTSXXVI
Chapter 1 INTRODUCTION
1.1 Background1
1.1.1 ANNs in environmental and water resources modelling1
1.1.2 IVS1
1.1.3 PMI IVS
1.1.4 Bandwidth issue in PMI IVS6
1.1.5 Boundary issue in PMI IVS7
1.2 Objectives
1.3 Thesis overview15
Chapter 2 JOURNAL PAPER 1 - Selection of Smoothing Parameter
Estimators for General Regression Neural Networks - Applications to
Hydrological and Water Resources Modelling19
2.1 Introduction
2.2 GRNNs
2.3 Methodology
2.3.1 Procurement of input/output data with different degrees of
normality and non-linearity
2.3.2 Estimation of GRNN smoothing parameters using different
estimation methods
2.3.3 Development of benchmark MLP model
2.3.4 Model performance assessment
2.3.5 Test regime

TABLE OF CONTENT

2.4 Results and discussion
2.4.1 Synthetic case studies
2.4.2 Real case studies
2.5 Summary and conclusions
-
2.6 Acknowledgments
Chapter 3 JOURNAL PAPER 2 - Improved PMI-Based Input Variable
Selection Approach for Artificial Neural Network and Other Data Driven Environmental and Water Resource Models
3.1 Introduction
3.2 PMI IVS
3.3 Methodology
3.3.1 Generation of input/output data with different degrees of normality
3.3.2 Estimation of PDF and MI using different bandwidth estimators . 75
3.3.3 Performance assessment
3.3.4 Test regime
3.4 Results and discussion
3.4.1 Selection accuracy
3.4.2 Computational efficiency
3.4.3 Suggested rules and guidelines
3.5 Testing of proposed rules and guidelines
3.6 Summary and conclusions 102
3.7 Acknowledgments
Chapter 4 JOURNAL PAPER 3 - Improved Partial Mutual Information-
Based Input Variable Selection by Consideration of Boundary Issues
Associated With Bandwidth Estimation
4.1 Introduction
4.2 Background on PMI IVS and Boundary Issues

TABLE OF CONTENT

4.2.1 PMI IVS113
4.2.2 Boundary issues in PMI IVS115
4.2.3 Potential solutions to solve boundary issues in PMI IVS117
4.3 Methodology
4.3.1 Generate input/output data with different degrees of normality 122
4.3.2 Estimate MI using different boundary correctors and suggested
bandwidth estimators124
4.3.3 Estimate residuals using alternative approaches and suggested
bandwidth estimators
4.3.4 Test regime
4.3.5 Assess performance of IVS over 30 trials
4.4 Results and Discussion
4.4.1 Selection accuracy136
4.4.2 Computational efficiency147
4.4.3 Suggested rules and guidelines152
4.5 Validation on Murray Bridge and Kentucky River Basin case studies
4.5.1 Background
4.5.2 Experimental Procedure
4.5.2 Experimental Flocedule
4.5.2 Experimental Procedure 158 4.5.3 Results and discussion 158
4.5.3 Results and discussion158
4.5.3 Results and discussion1584.6 Summary and Conclusions160
4.5.3 Results and discussion1584.6 Summary and Conclusions1604.7 Acknowledgments163
4.5.3 Results and discussion1584.6 Summary and Conclusions1604.7 Acknowledgments163Chapter 5 CONCLUSIONS165
4.5.3 Results and discussion1584.6 Summary and Conclusions1604.7 Acknowledgments163Chapter 5 CONCLUSIONS1655.1 Thesis summary165
4.5.3 Results and discussion1584.6 Summary and Conclusions1604.7 Acknowledgments163Chapter 5 CONCLUSIONS1655.1 Thesis summary1655.2 Research contributions167

APPENDICES 192
APPENDIX-A Supplementary Material from Paper 1 (Chapter 2) 192
APPENDIX-B Supplementary Material from Paper 2 (Chapter 3) 220
B.1 Mathematical derivations
B.2 Supplementary figures and tables
APPENDIX-C Supplementary Material from Paper 3 (Chapter 4)
C.1 Mathematical explanation and derivations
C.2 Supplementary figures and tables
APPENDIX-D Copy of Publications
D.1 Copy of Paper 1 from Chapter 2 (as published)256
D.2 Copy of Paper 2 from Chapter 3 (as published)

LIST OF FIGURES

Figure 1.1 Framework of thesis
Figure 2.1 General architecture of a GRNN
Figure 2.2 Overview of proposed assessment approach
Figure 2.3 Predictive accuracy for the validation data of MLPs and GRNNs
for different synthetic data-generating models and distributions for which
optimal parameters have been obtained using different methods
Figure 2.4 Computational efficiency of MLPs and GRNNs for different
synthetic data-generating models and distributions for which optimal
parameters have been obtained using different methods
Figure 2.5 Suggested smoothing parameter estimators under different problem
situations
Figure 2.6 Predictive accuracy of MLPs and GRNNs with different smoothing
parameter estimators for the validation data for the real case studies
Figure 2.7 Predictive efficiency of MLPs and GRNNs with different
smoothing parameters for the validation data for the real case studies
Figure 3.1 Procedure of PMI IVS adopted in this study
Figure 3.2 Outline of the proposed experimental approach
Figure 3.3 Correct selection rate of EAR4 model with alternative bandwidth
estimators
Figure 3.4 Correct selection rate of TEAR10 model with alternative
bandwidth estimators
Figure 3.5 Correct selection rate of NL model with alternative bandwidth
estimators
Figure 3.6 KDE accuracy measured by K-S statistics for EAR4 & TEAR10
models
Figure 3.7 Residual accuracy measured by CE for EAR4 model
Figure 3.8 KDE accuracy measured by K-S statistics for NL model
Figure 3.9 Residual accuracy measured by CE for NL model
Figure 3.10 Computational efficiency of EAR4 model with different
bandwidth estimators
Figure 3.11 Suggested bandwidth estimators under different distribution
scenarios

Figure 3.12 The River Murray in South Australia (Maier and Dandy, 1996).95
Figure 3.13 Correct selection rate and efficiency of salinity forecast at Murray
Bridge with proposed and alternative bandwidth estimators98
Figure 3.14 The Kentucky River Basin in USA (Jain et al., 2004)99
Figure 3.15 Correct selection rate and efficiency of flow forecast at Kentucky
River Basin with proposed and alternative bandwidth estimators102
Figure 4.1 Graphical representation of the boundary issue in 2D (Hazelton and
Marshall, 2009)
Figure 4.2 Taxonomy of methods for dealing with boundary issues in mutual
information and residual estimation
Figure 4.3 Overview of the proposed analysis for the PMI IVS influenced by
bandwidth and boundary issues
Figure 4.4 Selection accuracy of the PMI with suggested settings for EAR4
models
Figure 4.5 Relative change of K-S and MI in-between M1 and B3 for EAR4
model
Figure 4.6 Accuracy of residual estimation with alternative estimators for
EAR4 model (3 cases)141
Figure 4.7 Selection accuracy of the PMI with suggested settings for TEAR10
models
Figure 4.8 Selection accuracy of the PMI with suggested settings for NL
models
Figure 4.9 Accuracy of residual estimation with alternative estimators for
TEAR10 model (3 cases)145
Figure 4.10 Accuracy of residual estimation with alternative estimators for NL
model (3 cases)146
Figure 4.11 Selection efficiency of the PMI IVS with tested methods for
EAR4 models
Figure 4.12 Suggested PMI IVS approaches under distinct scenarios153
Figure 4.13 Selection accuracy and efficiency of the PMI IVS with suggested
settings for Murray Bridge case
Figure 4.14 Selection accuracy and efficiency of the PMI IVS with suggested
settings for Kentucky River basin case

LIST OF TABLES

Table 1.1 Review of input variable selection methods for ANNs applied to
environmental and water resources problems (developed based on May, 2010)
Table 1.2 Bandwidth estimators applied within the statistics literature
Table 1.3 Boundary correctors proposed within the statistics literature12
Table 2.1 Details of the simulated input distributions for the time series
models (EAR4, TEAR10)
Table 2.2 Details of the simulated input distributions for the nonlinear model
(NL)
Table 2.3 Inputs and outputs used to forecast salinity at Murray Bridge 1, 5, &
14 days in advance
Table 2.4 Inputs and output used to model rainfall-runoff from the Kentucky
River basin
Table 2.5 Selected smoothing parameter estimators with different fitness
functions and assumptions of normality and error basis
Table 3.1 Details of the distributions used to generate values of the exogenous
input variables and the statistical properties of the generated data for all time
series models (EAR4, TEAR10)73
Table 3.2 Details of the distributions used to generate values of the input
variables and the statistical properties of the generated data for the non-linear
model (NL)
Table 3.3 GRNN bandwidth estimation techniques used for residual
estimation during the PMI IVS process (based on the guidelines from Li et al.
(2014b))
Table 3.4 Average ratio of different kernel bandwidths under different
distribution scenarios for EAR4 model
Table 3.5 Candidate inputs and output for the salinity case study
Table 3.6 Candidate inputs and output used for the rainfall-runoff case study
Table 4.1 Details of the distributions used to generate values of the exogenous
input variables and the statistical properties of the generated data for all time
series models (EAR4, TEAR10)122

Table 4.2 Details of the distributions used to generate values of the input
variables and the statistical properties of the generated data for the non-linear
model (NL)
Table 4.3 GRNN bandwidth estimation techniques used for residual
estimation during the PMI IVS128
Table 4.4 Different approaches used for PMI IVS by considering bandwidth
and boundary issues
Table 4.5 Candidate inputs and output used to forecast salinity at Murray
Bridge 14 days in advance
Table 4.6 Candidate inputs and outputs used to forecast flow at Kentucky
River Basin 1 day in advance

NOMENCLATURE & ABBREVIATIONS

Symbols

 $\widehat{m}_{v_i}(X_{i^*}^j)$: residual estimate of v_i based on X_{i^*}

 (X^{j}, y^{j}) : observed pairs of input and output data

 $\hat{\varphi}_4(g) = n^{-1} \sum_{i=1}^n \hat{L}^{(4)}(X^i; g)$: fourth order integrated squared density derivative

 $F_{emp}(X_i^j)$: empirical CDF of the input variable estimated by a histogram

 $F_{est}(X_i^j)$: estimated kernel based CDF of the input variable

 $I_{X_{i},y}$: mutual information

 $I_{v_i,u}$: partial mutual information

 $\widehat{ISB}(h)$: estimation of the integrated squared bias

 $K^{(n)}$: n^{th} derivative of kernel function K

 $\widetilde{R(f'')}$: approximation of the integrated squared second derivative of f

 $S_{x,i}^2$: sample variance of the input X_i

 $S_{xy,i}$: covariance between input X_i and output y

 S_y^2 : sample variance of output y

 X_{i^*} selected inputs

 $X^{(j)}$: *j-th* data point formed by the interpolated and original data points

 X_s : significant input set

 $\hat{f}(X, y)$: estimation of the joint probability density function between inputs X and output y

 p_{t-n} : exogenous input with lag n

 \hat{y} : estimation of the actual output y

 \bar{y} : sample mean of the observations

 e_1 : vector having 1 in the first entry and 0 elsewhere

 ε_t : introduced error term

 $\mu_2(K) = \int x^2 K(x) dx$: second moment of K

 $\mu_k(L) = \int u^k L(u) du$: *k-th* moment of *L*

 $\mu_n(K)$: n^{th} moment of kernel function K

 $\rho_{xy,i}$: correlation coefficient between input X_i and output y

 σ_i : sample standard deviation of the X_i^j

*: convolution operation

h: kernel bandwidth

K: kernel function

 $B(u; h_x)$: univariate boundary kernel with bandwidth h_x and variable

 $u = (X_i - X_i^j) / h_x$

B(u, v; H): bivariate boundary kernel with bandwidth matrix H and vector

(u, v) where $u = (X_i - X_i^j)/h_x$ and $v = (y - y^j)/h_y$

E[y|X]: conditional expectation of output y given input X

L: pilot kernel

O(h): bias of density function

P(t): lagged effective rainfall

Q(t-1): lagged runoff

 $R(K) = \int [K(x)]^2 dx$: integrated square of kernel function

a: left boundary of kernel density

c: right boundary of kernel density

e: number of effective inputs

f(X, y): joint probability density function between inputs X and output y

g: pilot bandwidth

k: kurtosis

k: order of pilot kernel *L*

m(x): regression function

m: number of inputs

n: number of observations

r: stage number of L

s: skewness

sup: supremum function

 $\boldsymbol{H} = h_i^2 \begin{bmatrix} S_{x,i}^2 & S_{xy,i} \\ S_{xy,i} & S_y^2 \end{bmatrix}$: bivariate bandwidth matrix $\boldsymbol{X} = [X_1 \dots X_m]^T$: input variables $\boldsymbol{h} = [h_1 \quad \cdots \quad h_n]^T$: kernel bandwidth vector

Abbreviation

ACF: auto-correlation function
AIC: Akaike information criterion
AMISE: asymptotic mean integrated squared error
ANNs: artificial neural networks
BCV: biased cross validation
BCVDPI: a combination of BCV and DPI
BE: backward elimination (pruning)
BJ: Box-Jenkins
BK: boundary kernel
BP: back-propagation algorithm
CE: coefficient of efficiency
CK: conventional kernel
CSR: correct selection rate
CT: computational time
DELSA: distributed evaluation of local sensitivity analysis
DPI: 2-stage direct plug-in
EAR4: exogenous auto-regressive time series model (with time order up to 4)
EMISE: exact mean integrated squared error
ES: exhaustive search
ETC: empirical translation correction
EVT1: extreme value type I distribution
EXP: exponential distribution
FS: forward selection
GAMMA: gamma distribution
GRNN: general regression neural network
GRR: Gaussian reference rule
GSS: golden section search
HS: heuristic search
ICAIVS: hybrid independent component analysis and input variable selection
filter
IIS: tree-based iterative input variable selection
IoAd: index of agreement

- IVS: input variable selection KDEs: kernel density estimates K-S: Kolmogorov-Smirnov statistic KT: kernel transformation L1UL: Lock 1 upper river level LBE: local bandwidth (enlarging) LBR: local bandwidth (reducing) LHOP: local high order polynomial LLM: local linear method LLP: local linear polynomial LOGN: log-normal distribution LOGPT3: log-Pearson type III distribution LOS: Loxton river salinity LQP: local quadratic polynomial LSCV: least squared cross validation MAE: mean absolute error MAS: Mannum river salinity MBS: Murray Bridge river salinity MCE: modified coefficient of efficiency MI: mutual information MIoAd: modified index of agreement MLPANNs: multi-layer perceptron artificial neural networks MLPs: multi-layer perceptrons MOS: Morgan river salinity MPI: modified persistence index MVC: multi-variable calibration MVCA: multi-variable calibration with mean absolute error as the objective function MVCS: multi-variable calibration with squared error as the objective function NL: nonlinear input-output function NORM: normal distribution NS: normal scale OM: optimisation method
- PA: pseudo-data approach

PACF: partial auto-correlation function

PC: partial correlation

PCA: principal component analysis

PDF: probability density function

PI: persistence index

PMI: partial mutual information

PNNs: probabilistic neural networks

PSO: particle swarm optimisation

PT3: Pearson type III distribution

RBFs: radial basis functions

RC: reflection correction

RE: residual estimation

RMSE: root mean squared error

RNNs: recurrent neural networks

RVSDEM: recursive variable selection embedded in dynamic emulation models

SCV: smoothed cross validation

SOM-GAGRNN: self-organising map genetic algorithm general regression neural network

SVC: single variable calibration

SVCA: single variable calibration with mean absolute error as the objective function

SVCS: single variable calibration with squared error as the objective function

SVO: single variable optimisation

SVR: single variable regression

TEAR10: threshold exogenous auto-regressive time series model (with time order up to 10)

WAS: Waikerie river salinity

ABSTRACT

Artificial neural networks (ANNs), as one of the most commonly used data driven models for environmental and water resources problems, have been applied successfully and extensively over the last two decades and are still gaining in popularity. Consideration of the methods used in the steps in the development of ANNs, which consist of data collection, data processing, input variable selection, data division, calibration and validation, are vitally important, as ANN model development is based on data, rather than understanding of the underlying physical processes.

Among these methods, input variable selection (IVS) plays a significant role, as the performance of the developed model can be compromised if inputs having a pronounced relationship with the modelled output are omitted. In contrast, calibration becomes extremely challenging and modelling validation, as well as knowledge extraction, are problematic if redundant or superfluous inputs are included. Given the facts explained above, various techniques have been developed for the sake of more accurate IVS.

Partial mutual information (PMI) is one of the most promising approaches to IVS, as it has a number of desirable properties, such as the ability to account for input relevance, the ability to cater to both linear and non-linear inputoutput relationships and the ability to check the redundancy of selected inputs. PMI is a stepwise input selection algorithm, which only selects one variable per iteration, as part of which the strength of the relationship between each potential input and the output is quantified using mutual information (MI) and input redundancy is accounted for by removing the influence of already selected inputs. This is achieved by developing models between the selected input and the output and assessing the strength of the relationship (in terms of MI) between the remaining potential inputs and the residuals of these models during the next iteration, which is referred to as PMI.

Although PMI IVS has already been applied successfully to a number of studies in hydrological and water resource modelling, present implementations predominantly depend on the assumption that the data used to develop the model follow a Gaussian distribution. This assumption has the

potential to affect two steps in the PMI process, including the estimation of MI/PMI and the estimation of the residuals. In terms of MI/PMI estimation, this requires kernel density estimates of the modelling data to be obtained for the estimation of marginal and joint probability density functions (PDFs), which rely on estimates of kernel bandwidths (or smoothing parameters) and in most studies, the Gaussian reference rule is used for this purpose, which only results in optimal bandwidth estimates if the modelling data follow a Gaussian distribution. However, this is unlikely to be the case when dealing with water resources and other environmental data. In terms of residual estimation (RE), this has generally been done using general regression neural networks (GRNNs), which also require estimates of kernel bandwidths to be obtained and therefore suffers from the same issues as MI/PMI estimation.

The purpose of this thesis is to assess the impact the assumption that the data follow a Gaussian distribution has on the performance of PMI IVS and the efficacy of potential methods for overcoming any problems associated with this assumption. In order to achieve this, a large number of numerical tests are conducted on synthetic data with different degrees of normality and nonlinearity, investigating the effectiveness of a range of options for (i) bandwidth estimation (caused by making Gaussian assumptions for non-Gaussian circumstances when adopting kernel based estimations in both MI/PMI and RE) and (ii) for dealing with boundary issues (caused by using a symmetrical kernel for bounded/unsymmetrical data when implementing kernel based estimations in both MI/PMI and RE), as well as methods for RE that do not require kernel density estimates. The results from these tests are used to develop preliminary guidelines for the selection of the most appropriate bandwidth and the most effective treatment of the boundary issue, which are then validated for two water resources case studies with different data properties and problem linearity, including forecasting of river salinity in the River Murray, Australia, and rainfall-runoff modelling in the Kentucky River, USA.

The major research contributions are presented in three journal publications. The motivations underlying these publications include: 1) the development and testing of rigorous and novel analytical procedures for assessing if, and to

XXI

what degree, the performances of residual and MI estimates are affected by bandwidth selection and boundary issues; 2) clear explanation of the inaccurate performance of conventional PMI IVS under the influence of bandwidth selection and boundary issues; 3) the development of effective preliminary guidelines based upon synthetic studies dealing with both bandwidth selection and boundary issues under different scenarios categorised by data normality and problem linearity; 4) the development of more robust and reliable PMI IVS software for realistic environmental and water resource problems. Overall, the research outcomes suggest that the performance of PMI IVS is significantly influenced by bandwidth selection and boundary issues and can be effectively improved by following the proposed empirical guidelines, although the findings of this work could be tested more broadly, including for data sets with a wider range of attributes, such as different degrees of noise, collinearity and interdependency, as well as incomplete information.

STATEMENT OF ORIGINALITY

I **Xuyuan Li** hereby certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

I give consent to this copy of my thesis when deposited in the University Library, being made available for loan and photocopying, subject to the provisions of the Copyright Act 1968.

I acknowledge that copyright of published works contained within this thesis resides with the copyright holder(s) of those works.

I also give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library catalogue and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

A list of works contained within this thesis is given in Section 5.3.

ACKNOWLEDGEMENTS

I would take this opportunity to gratefully acknowledge my principal supervisor Professor Holger R. Maier and co-supervisor Dr. Aaron C. Zecchin, who have supervised me with invaluable support, encouragement, and patience. I really appreciate the critical and rigorous attitude, timely and effective feedback, great foresight, and comfortable research environment provided by both of my supervisors. I have enjoyed studying under their supervision and learnt a lot from them during my candidature. Without their excellent supervision, the present thesis would hardly be possible.

I would like to sincerely thank Prof. A. Sharma, Dr. G.J. Bowden, Dr. R.J. May and Dr. G.B. Humphrey who brought me into the present research topic and kindly provided their suggestions and research materials.

I would express my appreciation to my fellow postgraduates within the School for sharing their wisdom, experiences, successes, and lessons with me. It was my pleasure to meet Dr. Xun Sun, Dr. Wenyan Wu, Dr. Feifei Zheng, Dr. Liang Huang, Dr. Jeffrey P. Newman, Dr. Christopher Stokes, and Dr. Tao Zhang and have had a number of in-depth academic discussions with them. Special thanks are also given to School Editor Barbara Brougham, Computer Technician Dr. Stephen Carr, all School Administrators, and the research scholarship (AGRS) provided by the University of Adelaide.

Last but not least, I would like to give an immense gratitude to my parents, Mr Xuelong Li and Ms Yongjing Liu, for their altruistic care and support from beginning to end (*ab ovo usque ad mala*).