QUERIES

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QUERY: 91 Fisher (Design of Experiments, Section 50) shows how to assess the effects of quality and the quality and quantity interaction on the hypothesis of proportional response in the case of 4 qualities of nitrogen at 3 equally spaced intervals, the lowest of which is zero. How would this be done if the lowest level were not zero?

ANSWER: Suppose for any fertilizer we have

Quantity,	0	1	2	3
Yield		a	b	c

with totals A, B, C for all four fertilizers together. Then I imagine it will be agreed that the 2 d.f. denominated QUANTITY will have the sum of squares

$$\frac{1}{4}A^2 + \frac{1}{4}B^2 + \frac{1}{4}C^2 - \frac{1}{12}(A + B + C)^2$$

$$= \frac{1}{8}(C - A)^2 + \frac{1}{24}(A - 2B + C)^2$$

The remainder for QUALITY and INTERACTION will then have

$$S(a^2) - \frac{1}{4}A^2 + S(b^2) - \frac{1}{4}B^2 + S(c^2) - \frac{1}{4}C^2$$

which may, of course, be subdivided, as for example,

$$\frac{1}{2}S(c - a)^2 - \frac{1}{8}(C - A)^2$$

$$\frac{1}{6}S(a - 2b + c)^2 - \frac{1}{24}(A - 2B + C)^2$$

$$\frac{1}{3}S(a + b + c)^2 - \frac{1}{12}(A + B + C)^2$$

The question is what apportionment of this total is most proper for the separation of QUALITY and INTERACTION. I think opinions may legitimately differ. Evidently, the 3 d.f. having

$$\frac{1}{14} S(a + 2b + 3c)^2 - \frac{1}{56} (A + 2B + 3C)^2,$$

represent differences in quality as measured by linear response. A second orthogonal set having

$$\frac{1}{6} S(a - 2b + c)^2 - \frac{1}{24} (A - 2B + C)^2$$

represent differences in quadratic response, which may or may not be thought to be properly included in pure QUALITY. In any case, there remain three more d.f. having

$$\frac{1}{21} S(4a + b - 2c)^2 - \frac{1}{84} (4A + B - 2C)^2,$$

which seem to me to be properly described as INTERACTION or RESIDUE.

All the algebra needed is that for the identity

$$\frac{1}{2} (c - a)^2 + \frac{1}{3} (a + b + c)^2$$

$$\equiv \frac{1}{14} (a + 2b + 3c)^2 + \frac{1}{21} (4a + b - 2c)^2,$$

but the whole process will look more convincing with numerical data.

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