Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

Non-crossover dither creeping mutation genetic algorithm for pipe network optimization

Feifei Zheng[1], Aaron C. Zecchin[2], Angus R. Simpson[3] and Martin F. Lambert[4]

Abstract: A non-crossover dither creeping mutation-based genetic algorithm (CMBGA) for pipe network optimization has been developed and is analyzed. This CMBGA differs from the classic GA optimization in that it does not utilize the crossover operator, but instead it only uses selection and a proposed dither creeping mutation operator. The creeping mutation rate in the proposed dither creeping mutation operator is randomly generated in a range throughout a GA run rather than being set to a fixed value. In addition, the dither mutation rate is applied at an individual chromosome level rather than at the generation level. The dither creeping mutation probability is set to take values from a small range that is centered about 1/*ND* (where *ND*=number of decision variables of the optimization problem being considered). This is motivated by the fact that a mutation probability of approximately 1/*ND* has been previously demonstrated to be an effective value and is commonly used for the GA. Two case studies are used to investigate the effectiveness of the proposed CMBGA. An objective of this paper is to compare the performance of the proposed CMBGA with four other GA variants, and other published results. The results show that the proposed CMBGA exhibits considerable improvement over the considered GA variants, and comparable performance with respect to other

[1]Research Associate, School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia, 5005, Australia. fefei.zheng@adelaide.edu.au
[2]Lecturer, School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia, 5005, Australia azecchin@civeng.adelaide.edu.au
[3]Professor, School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia, 5005, Australia. angus.simpson@adelaide.edu.au
[4]Professor, School of Civil, Environmental and Mining Engineering, University of Adelaide, Adelaide, South Australia, 5005, Australia. mlambert@civeng.adelaide.edu.au

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

previously published results. A big advantage of CMBGA is its simplicity and that it requires the tuning of fewer parameters compared with other GA variants.

CE Database subject headings: Optimization; water distribution systems; genetic algorithms; dither creeping mutation.

INTRODUCTION

Evolutionary algorithms (EAs) have been used to optimize WDSs since the early 1990s. Amongst these EAs, GAs have gained popularity due to their ease of implementation and search ability (Simpson et al. 1994, Dandy et al. 1996; Vairavamoorthy and Ali 2000; Nicklow et al. 2010; Zheng et al. 2011). For GAs used by the water community, the crossover operator has been considered to be the dominant operator while mutation has been considered to be a second order operator. Thus high crossover probabilities and low mutation probabilities have been suggested for better performance of GAs for the optimization of WDSs (Simpson et al. 1994). A typical parameter combination for GA optimization of WDSs is a crossover probability of 0.9 and a mutation probability of 0.01 (Dandy et al. 1996).

In contrast, some other EAs such as Evolutionary Strategy (ES) (Rechenberg 1965) and Evolutionary Programming (EP) (Fogel et al. 1966) have concentrated on mutation as the main driving evolution operator. ES algorithms with adaptive mutation rates have been found to be effective when dealing with some optimization tasks such as mechanical design problems and Flow Shop Scheduling problems (Rechenberg 1965). Fogel and Atmar (1990) suggested that crossover has no general advantage over mutation based on

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

testing a number of optimization problems. As a result, mutation-based GAs have been proposed to solve some optimization problems (Falco et al. 2002).

Although the traditional crossover-based GA has been widely used in the pipe network optimization (Nicklow et al. 2010), little is known about the effectiveness of a GA driven by only mutation in terms of water network optimization. This paper aims to develop and investigate a non-crossover dither creeping mutation-based GA (CMBGA) to optimize the design of WDSs. In the proposed CMBGA, only the selection and dither creeping mutation operators are applied. The performance of the proposed CMBGA is assessed based on two benchmark WDS case studies. Results show that the proposed CMBGA outperforms the crossover based GA and exhibits a comparable performance, if not better than, other published algorithms. A big advantage of CMBGA is its simplicity and that it requires the tuning of fewer parameters compared with other GA variants. This provides one of the motivations for this technical note.

THE PROPOSED NON-CROSSOVER CREEPING MUTATION-BASED GA

The CMBGA proposed in this paper is differentiated by the fact that crossover is not used. Additionally, a dither creeping mutation operator is introduced into the CMBGA to replace the commonly used bitwise mutation operator. A flowchart of the proposed CMBGA applied to WDS optimization is illustrated in Figure 1 and the details of the proposed CMBGA are discussed in the following sections. In the proposed CMBGA, a maximum number of allowable evaluations is used as the stopping criterion. The CMBGA run is stopped when the criterion is satisfied.

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

Initialization

An initial random population of N integer coded solutions is generated by uniformly randomizing individuals within the search space as:

$$x_{i,0}^{j} = U_i^j(0, K-1) \quad j=1, 2, \ldots ND, i=1, 2, \ldots N \tag{1}$$

where $x_{i,0}^{j}$ represents the initial value of the j^{th} parameter in the i^{th} individual at the initial population, where an individual is given by $X_{i,0}=[x_{i,0}^{1}, x_{i,0}^{2}, \ldots\ldots x_{i,0}^{D}]^{\mathrm{T}}$. U_i^j represents a randomly generated integer variable within the range of 0 to K-1 for the j^{th} parameter in the i^{th} individual. The symbols N, ND and K are population size, number of decision variables and number of pipe diameter choices respectively. The details of integer coding used in this paper are given by Vairavamoorthy and Ali (2000) not hence are not repeated in this technical note.

Hydraulic analysis

For each network design, a minimum allowable pressure head for each demand node needs to be satisfied. A steady state hydraulic solver is used to compute the heads at each node for the given water demands. The actual pressure head for each node is compared with its corresponding minimum allowable pressure head, thereby computing the head deficit (if any). The head deficits for every node are cumulated and this value $P_{i,G}$ is recorded for its corresponding network design to be used in the selection phase.

Objective function calculation

The integer strings are decoded into the corresponding pipe diameters and hence N network designs are produced. The total material and construction cost for each network design is computed as:

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

$$f_i = \sum_{j=1}^{ND} L_j C_i^j \qquad (2)$$

where f_i is the objective function value for the individual i, L_j represents the length of the pipe j and C_i^j is the cost per unit for the pipe diameter of pipe j in the individual i.

Selection

Constraint tournament selection (Deb 2000) is used to determine the individuals that survive to the next generation (a noted advantage of this method is that it does not require a penalty multiplier parameter, which would need to be tuned). For two candidate solutions $X_{A,G}$ and $X_{B,G}$, the selection algorithm is given as:

$$X'_{G+1} = \begin{cases} \underset{X \in (X_{A,G}, X_{B,G})}{\text{argmin}} \; f(X), & \text{if } X_{A,G} \text{ and } X_{B,G} \text{ are both feasible solutions.} \\ \underset{X \in (X_{A,G}, X_{B,G})}{\text{argmin}} \; P(X), & \text{otherwise.} \end{cases} \qquad (3)$$

where X'_{G+1} is the individual at generation G+1 which is either $X_{A,G}$ or $X_{B,G}$, $f(X)$ is the objective function value for string X, and $P(X)$ is the cumulative head deficit for string X. If a vector X is a feasible solution, $P(X)=0$. As can be seen from Equation (3), the solution with a smaller value of objective function is selected between two feasible solutions. A feasible solution is selected ($P(X)=0$) when compared with an infeasible solution ($P(X)>0$);The solution with less head constraint violation is chosen between two infeasible solutions.

Dither creeping mutation

The dither creeping mutation proposed in this paper combines creeping mutation (Dandy et al. 1996) and the dither mutation strategy (Das et al. 2002). Within the proposed dither creeping mutation mechanism, each string, i=1,….,N, is first assigned a

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

probability (P_{dcm}^{i}), where $P_{dcm}^{i} \in [P_{dcm}^{\min}, P_{dcm}^{\max}]$ is a uniform random variable. Each integer of each string *i* is selected with a probability of P_{dcm}^{i} to be mutated. Then the selected integer has a probability P_d of being mutated to the adjacent integer value below in the pipe choice table and a probability 1-P_d of being mutated to the adjacent integer value above. For an integer that is already set to the smallest (largest) value, upward (or downward) mutation is allowed only. The dither creeping mutation algorithm used in the proposed CMBGA is given in Figure 2. For *K* pipe diameter choices, the integer numbers from 0 to *K*-1 are associated with each pipe diameter ordered from the smallest to the largest.

In Figure 2, P_{dcm}^{i} is the dither creeping mutation probability; $P_{dcm}^{\max}$ and $P_{dcm}^{\min}$ are the maximum and minimum allowable dither creeping mutation probabilities; $x_{i,G}^{j}$ is the i^{th} integer of the string; *Rand* is a uniformly distributed random variable between 0 and 1; and P_d is conditional probability of downward mutation.

The proposed dither creeping mutation used in this paper is novel in that the mutation probability used for each string is uniformly randomly generated rather than being set to a fixed value. Thus different strings in the proposed CMBGA will be subject to different creeping mutation probabilities at the same generation and the same string will be also subject to a variety of mutation probabilities at different stages. This differs significantly to the creeping mutation GA used by Dandy et al. (1996), for which, a fixed mutation probability was used throughout all the optimization and all the strings were subject to an identical mutation probability.

It is noted that the ES (Rechenberg 1965) and the creeping mutation-based GA (CMBGA) proposed here have the similar features in that both of them do not utilize

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

crossover operator. However, there exist some important differences between these two optimization algorithms. ES (such as (μ+λ) ES) normally selects the best μ individuals from the total (μ+λ) individuals to become parents for the next generation (Rechenberg 1965), where μ is the population size (parents) and λ is the number of offspring produced by the μ parents. In contrast all *N* individuals of the next generation are selected from the *N* parents utilizing constraint tournament selection strategy for the proposed CMBGA. A self-adaptive mutation strategy is normally used for ES (such as 1/5 success rule proposed by Rechenberg (1965)), while a dither creeping mutation strategy has been adopted in the proposed CMBGA.

CASE STUDIES

Two case studies from the literature are used to investigate the effectiveness of the proposed CMBGA. These include the New York Tunnels Problem (NYTP) (Dandy et al. 1996) and the Hanoi Problem (HP) (Fujiwara and Khang 1990). The CMBGA has been coded in C++ and combined with the EPANET2 hydraulic network solver.

In this study, the dither mutation rate takes values from a range that is centered about 1/*ND* (the inverse value of number of decision variables). This is motivated by the fact that a mutation probability of approximately 1/*ND* has been demonstrated to be an effective value and is normally used for the GA (Simpson et al. 1994). An interval is used to form the lower and upper bounds of the range of dither mutation rate. For example, for a WDS optimization problem with the 1/*ND*≈0.05, the range of $P_{dcm} \in [0.03, 0.07]$ is used for the proposed CMBGA. The number of decision variables, the range for the dither creeping mutation probability (P_{dcm}), the population size and the maximum number of allowable evaluations for each case study are given in Table 1. A sensitivity analysis

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

of the conditional probability of downward mutation (P_d) was conducted for each case study. The results show that P_d=0.5 exhibited consistently good performance for each case study and hence this value was used in the proposed GA method.

CMBGA PERFORMANCE DISCUSSION AND COMPARISON

Four other GA variants have been studied in this paper in order to enable the comparison with the proposed CMBGA. These include a crossover-based GA with bitwise mutation (SGA), a crossover-based GA with creeping mutation (CGA), a non-crossover GA with traditional bitwise mutation (NBGA), and a crossover dither creeping mutation GA (CDGA).

For each case study, each GA variant used the same population size and the same maximum allowable number of evaluations (outlined in Table 1). Integer coding and constraint tournament selection (tournament size=2) were used for all the GA variants. For each GA variant, an elite count of 5 was used. The elite count is the number of individuals with the best fitness values in the current generation that are guaranteed to survive to the next generation (Gibbs et al. 2008). The other parameter values for the two GA variants applied to each case study are given in Table 1. These parameter values have been fine-tuned by hand for each case study to give the best performance. For the crossover dither creeping mutation GA (CDGA), the crossover probability of 0.9 was used for each case study as a high crossover probability (typically 0.9) is normally used for a GA. The only difference between the proposed CMBGA and the CDGA is that no crossover (P_c=0) was used in the CMBGA, while a large crossover probability (P_c=0.9) was used in CDGA.

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

For each GA variant, a total of 1000 trial runs was performed and the results of CMBGA and the four other GA variants are given in Table 2. The current best known solutions for the NYTP and HP case studies were first reported by Maier et al. (2003) with a cost of $ 38.64 million and Reca and Martínez (2006) with a cost $6.081 million respectively.

As shown in Table 2, it is clearly seen that the proposed CMBGA consistently outperformed the other four GA variants in terms of solution quality and efficiency. In particular, the proposed CMBGA found the best known solution for the NYTP case study with a success rate of 62%, which is higher than those found by other four GA variants. For the HP case study, the proposed CMBGA found the best known solution in 82% of cases, which is significantly higher than the other four GA variants.

Interestingly, the proposed non-crossover dither creeping mutation GA performed slightly better than the crossover dither creeping mutation GA for the NYTP case study, while significantly better for the HP case study. This illustrates that the non-crossover dither creeping mutation GA is more effective than the crossover dither creeping mutation GA in exploring the search space for highly constrained problems (HP problem). As can be seen from Table 2, the non-crossover GA with traditional bitwise mutation (NBGA) performed the worst for each case study, showing that the non-crossover mutation strategy with a simple mutation approach (bitwise mutation) is not effective in comparison to using the proposed dither creeping mutation method.

As can be seen from Table 2, in terms of percent of trials that found the best solution for the NYTP case study, the hybrid discrete dynamically dimensioned search (HD-DDS, Tolson et al. 2009) performed better than the proposed CMBGA, however, the CMBGA

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

yielded a better solution quality than the max-min ant system (MMAS, Zecchin et al. 2007). It is noted that the control parameters of the HD-DDS and MMAS were reported by Tolson et al. (2009) and Zecchin et al. (2007) respectively and the results with the best parameter values were included in Table 2.

In comparing the algorithmic performance for the HP case study, it can be seen that the CMBGA achieved the highest percent of best solutions found with a value of 82%, which is significantly higher than the other algorithms. The proposed CMBGA produced the lowest average solution with a value of $6.112 million, which deviates only 0.51% from the best known solution. For the HP case study, the CDGA with other crossover probabilities including P_c=0.05, 0.1 and 0.6 were performed and the percentage of the best known solution found were 42%, 40% and 8% respectively. This compares to 82% of the CMBGA, indicating a significantly better performance of the CMBGA.

CONCLUSIONS

Within this paper, a dither creeping mutation based GA with no crossover (CMBGA) has been proposed. It differs significantly from the commonly used GA approach as no crossover operator is used. A big advantage of CMBGA is its simplicity and that it requires the tuning of fewer parameters compared with other GA variants. It should be noted that the proposed CMBGA is a variant of the GA with constraint tournament selection (but with the crossover probability set to be zero) and its effectiveness has been demonstrated for pipe network optimization in this paper.

The proposed CMBGA has been compared with the other four GA variants applied to the NYTP and HP case studies. The statistics results obtained show that the proposed CMBGA exhibits improvements in finding optimal solutions for the two case studies

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

compared with the other GA variants. In addition, the proposed CMBGA shows a comparable performance to the other EAs (MMAS and HD-DDS). An extension of the proposed method to deal with multi-objective optimization of pipe networks is one possible area of future work.

REFERENCES

Dandy, G. C., Simpson, A. R., and Murphy, L. J. (1996). “An improved genetic algorithm for pipe network optimization.” *Water Resources Research*, 32(2), 449-457.

Das, S., Konar, K., and Chakraborty, U. K. (2005). “Two improved differential evolution schemes for faster global search.” In *Proceedings of Conference on Genetic and Evolutionary Computation (GECCO 2005)*, 991-998.

Deb, K. (2000). "An efficient constraint handling method for genetic algorithms." *Computer Methods in Applied Mechanics and Engineering*, 186(2-4): 311-338.

Falco, I. D., Ciooppa, A. D., and Tarantino, D. (2002). “Mutation-based genetic algorithm: performance evaluation.” *Applied Soft Computing*, 1, 285-299.

Fogel, D. B., and Atmar, J. W. (1990). “Comparing genetic operators with Gaussian mutation in simulated evolutionary process using linear systems.” *Biological Cybernetics*, 63, 111-114.

Fujiwara, O., and Khang, D. B. (1990). “A two-phase decomposition method for optimal design of looped water distribution networks.” *Water Resources Res*earch, 23(6), 977-982.

Gibbs, M. S., Dandy, G. C., and Maier, H. R. (2008). "A genetic algorithm calibration method based on convergence due to genetic drift." *Inf. Sci.*, 178(14), 2857-2869.

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

Holland, J. H. (1975). "*Adaptation in Natural and Artificial Systems*." University of Michigan Press, Ann Arbor, Mich.

Maier, H. R., et al. (2003). "Ant colony optimization for the design of water distribution systems." *J. Water Resources Planning and Management*, 129(3), 200-209.

Nicklow, J., Reed, P., Savic, D., Dessalegne, T., Harrell, L., Chan-Hilton, A., Karamouz, M., Minsker, B., Ostfeld, A., Singh, A., and Zechman, E. (2010). "State of the art for genetic algorithms and beyond in water resources planning and management." *Journal of Water Resources Planning and Management*, 136(4), 412-432.

Rechenberg, I. (1965). "*Cybernetic solution path of an experimental problem*." Royal Aircraft Establishment, Library Translation Number 1122, Farnborough, England.

Reca, J., and Martínez, J. (2006) "Genetic Algorithms for the design of looped irrigation water distribution networks." *Water Resources Research*, 42, W05416, doi:10.1029/2005WR004383.

Simpson, A. R., Dandy, G. C., and Murphy, L. J. (1994). "Genetic algorithms compared to other techniques for pipe optimization." *J. Water Resources Planning and Management*, 120(4), 423-443.

Spears, W. M. (1993). "Crossover or mutation." *Proceedings of the Foundations of Genetic Algorithms*, Morgan Kaufmann, 2, 221-237.

Tolson, B. A., et al. (2009). "Hybrid discrete dynamically dimensioned search (HD-DDS) algorithm for water distribution system design optimization." *Water Resources Res*earch, 45, W12416, doi:10.1029/2008WR007673.

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

Vairavamoorthy, K., and Ali, M. (2000). “Optimal design of water distribution systems using genetic algorithms.” *Computer-Aided Civil and Infrastructure Engineering*, 15(5), 374-382.

Zecchin, A. C., et al. (2007). “Ant colony optimization applied to water distribution system design: Comparative study for five algorithms.” *J. Water Resources Planning and Management*, 133(1), 87-92.

Zheng, F., Simpson, A. R., and Zecchin, A. C. (2011). "Dynamically expanding choice-table approach to genetic algorithm optimization of water distribution systems." *Journal of Water Resources Planning and Management*, 137(6), 547-551.

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013;
posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

Figure captions list

Figure 1 Flowchart of the proposed CMBGA for pipe network optimization

Figure 2. Dither creeping mutation algorithm

Figure1

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013;
posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

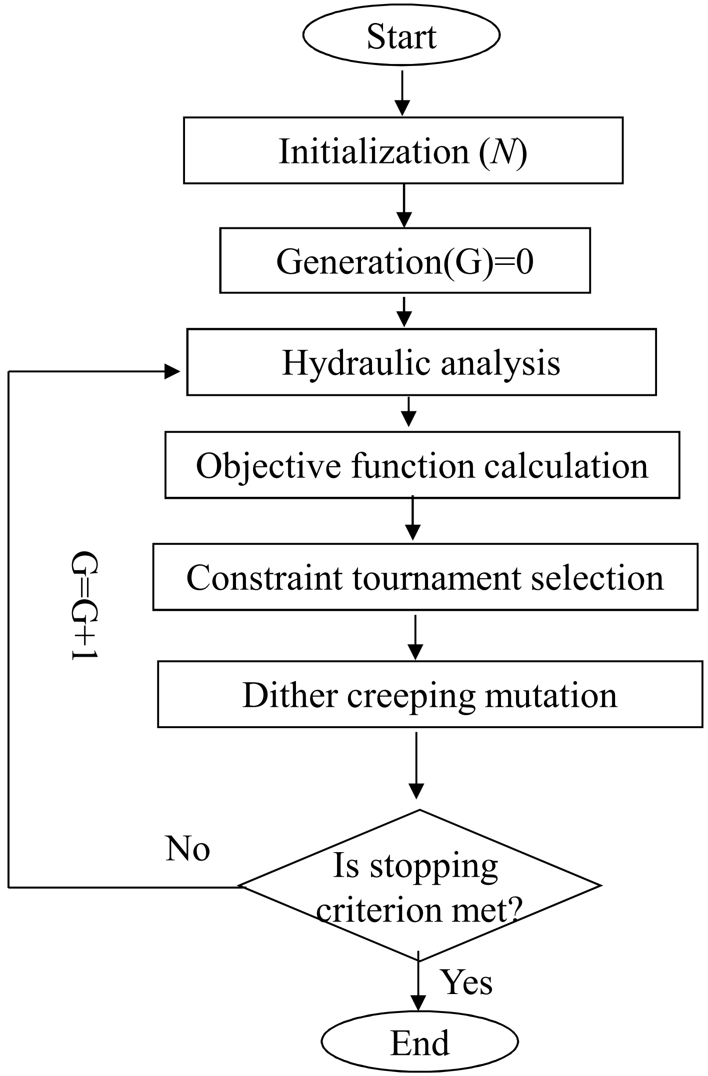

Figure 1 Flowchart of the proposed CMBGA for pipe network optimization

Figure2

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

FOR i=1, 2 ...., N

$$P_{dcm}^{i} = P_{dcm}^{\min} + rand \times (P_{dcm}^{\max} - P_{dcm}^{\min})$$

FOR j=1, 2 ...., ND

IF $Rand < P_{dcm}^{i}$

IF $Rand <= P_d$

$$x_{i,G}^{j} = \max[0, x_{i,G}^{j} - 1]$$

ELSE

$$x_{i,G}^{j} = \min[K - 1, x_{i,G}^{j} + 1]$$

END IF

END IF

END FOR

END FOR

Figure 2. Dither creeping mutation algorithm

Table 1 Parameter values for GA variants applied to each case study

Case study	No. of decision variables (*ND*)	SGA (*N*=100)	CGA (*N*=100)	NBGA (*N*=100)	CDGA (*N*=100)	CMBGA (*N*=100)	Maximum number of allowable evaluations
NYTP	21 (1/*ND*≈0.05)	P_c=0.5, P_m=0.03, two-point crossover	P_c=0.9, P_{cm}=0.04, two-point crossover	P_c=0.0, P_{cm}=0.04	P_c=0.9, P_{dcm} ∈[0.03, 0.07], two-point crossover	P_c=0.0, P_{dcm} ∈[0.03, 0.07],	50,000
HP	34 (1/*ND*≈0.03)	P_c=0.6, P_m=0.02, two-point crossover	P_c=0.6, P_{cm}=0.02, two-point crossover	P_c=0.0, P_{cm}=0.02	P_c=0.9, P_{dcm} ∈[0.01, 0.05] two-point crossover	P_c=0.0, P_{dcm} ∈[0.01, 0.05]	100,000

P_c: crossover probability. P_m: bitwise mutation probability. P_{cm}: traditional creeping mutation probability.

Journal of Water Resources Planning and Management. Submitted July 14, 2012; January 30, 2013; posted ahead of print February 1, 2013. doi:10.1061/(ASCE)WR.1943-5452.0000351

Table 2 Comparison of GA variants performance

Case studies	Algorithm	Number of different runs[4]	Best solution found ($ millions)	Percent of best solutions found	Average cost ($ millions)	No. of average evaluations[5]
NYTP	CMBGA[1]	1000	38.64	62%	38.82	42,385
	CDGA[1]	1000	38.64	56%	38.89	45,659
	CGA[1]	1000	38.64	50%	39.04	44,324
	SGA[1]	1000	38.64	45%	39.16	49,950
	NBGA[1]	1000	38.64	7%	40.07	49,875
	HD-DDS[2]	50	38.64	86%	38.64	47,000
	MMAS[3]	20	38.64	60%	38.84	30,700
HP	CMBGA[1]	1000	6.081	82%	6.112	70,423
	CDGA[1]	1000	6.081	12%	6.241	71,236
	CGA[1]	1000	6.081	2%	6.264	70,164
	SGA[1]	1000	6.099	0%	6.329	68,568
	NBGA[1]	1000	6.133	0%	6.259	73,695
	HD-DDS[2]	50	6.081	8%	6.252	100,000
	MMAS[3]	20	6.134	0%	6.386	85,600

[1]This study. [2]Tolson et al. (2009). [3]Zecchin et. al. (2007). [4]Based on different starting random number seeds. [5]The evaluations to find the first occurrence of the best solution.