

## Appendix

The decomposition of profile subspaces for the canonical form of the sum of profiles model of Chapter 5 is given by Algorithm 5.2.2. The MATLAB code below is an implementation of that algorithm.

### *Forward pass*

```
// The profile matrices should be in M
// The column positions of M_i should be in c where they
// specify the last column in each case with a zero entry
// to begin.
// The column sizes of m_i must be in q
// The number of profile designs is r
// Find the QR decomposition of M_1 to M_r

for i=1:r,[n t]=qr(m(:,c(i)+1:c(i+1)));...
m1(:,c(i)+1:c(i+1))=n(:,1:q(i));end
t=[]. n=[]

// m1 is the baseline matrix of QR decomp of M_1 to M_r
// m2 is the matrix of intersections currently being intersected
// again with those in m1
// m3 is the new matrix of intersections
// m3 is added to n the matrix of all intersections
// c1 contains [ 0 | last col of Q1 | last col of Q2 | .... ]
// c2 contains [ 0 | last col of int1 | last col of int2 | .... ]
// c3 contains [ 0 | last col of int1 | last col of int2 | .... ]
//   for the new intersections
// ord contains the order of the intersection for every
//   intersection in m
// cnti is a row vector in which the number of intersections at
//   each level is recorded
// cntr is the counter of all intersections
// cnt3 is the counter of all intersections at the current level
// z is an indicator matrix which contains the non-zero
//   intersections
// z1 is the indicator for m1
// z2 is the indicator for m2
// z3 is the indicator for m3
```

```
n=m1;m3=m1;c1=c;c3=c1;cntr=r;ord=J(1,r);
cnti(1)=r;z=I(r);z1=z;z3=z;cnt3=r+1;
```

```
// Find all the intersections of the subspaces of order 2,3,...,r
```

```
for k=1:r-1,cnti(k+1)=0;m2=m3;c2=c3;c3=[];c3(1,1)=0;z2=z3;...
cnt2=cnt3-1;cnt3=1;z3=J(r,1)*0;m3=J(p,1);
exec('split.mat');exec('remove.mat');end
c2=[],z2=[],cnt2=[],ord=[],s=[],d=[],t=[],a=[],qa=[]
```

Split.mat

```
for i=1:r, for j=1:cnt2;exec('svd.mat');end
```

Svd.mat

```
if abs(z2(i,j)-z1(i,i))>0.1,exec('checkz.mat') ; ...
if ind = 0,...
    C=m2(:,c2(j)+1:c2(j+1))*m1(:,c1(i)+1:c1(i+1));...
    [s d t]=svd(C);[rs cs]=size(d);exec('min.mat');...
    num1=0;exec('num1.mat');...
    if num1>0,exec('splitup.mat');...
        else num1=0;...
    end;...
    else C=[];...
end;...
end; ...
```

Checkz.mat

```
ind = 0;
for l=1:cnt3, j; z2(:,j)+z1(:,i); z3(:,l); ...
    if abs(norm(z2(:,j)+z1(:,i)-z3(:,l))) < 0.0000000001, ...
        ind = 1; ...
    else ind=ind; ...
    end; ...
end;
```

Min.mat

```
if rs>cs, min=cs; else min=rs; end
```

Num1.mat

```
for k1=1:min,k1;...
    if abs(d(k1,k1)-1) <= 0.0000001,...
        num1=num1+1;...
    else num1=num1;...
    end;
end
```

Splitup.mat

```
u = m2(:,c2(j)+1:c2(j+1))*s;c3(1,cnt3+1)=c3(1,cnt3)+num1;
m3=[m3 u(:,1:num1)];
cnti(k+1) = cnti(k+1)+1;z(:,cntr+1)=z2(:,j)+z1(:,i);
z3(:,cnt3+1)=z2(:,j)+z1(:,i);
n(:,c(cntr+1)+1:c(cntr+1)+num1)=u(:,1:num1);
cnt3=cnt3+1;cntr=cntr+1;c(cntr+1)=c(cntr)+num1;ord=[ord k+1];
```

Remove.mat

```
if cnti(k+1)>=1,m3=m3(:,2:c3(cnti(k+1)+1)+1);...
z3=z3(:,2:cnti(k+1)+1);...
else m3=m2,z3=z2,c3=c2,cnti=cnti(1:k),return;end
```

### *Backward pass*

```
// back movement to find the minimal spanning set
// Steps
//
// 1. c2 contains the number of columns in the highest intersection
//    collection.
// 2. m2 contains those columns
// 3. n contains all the columns in all non-trivial intersections
// 4. ord contains the order of the intersections
// 5. c contains the column mark for each of the intersections
// 6. cnti contains the number of intersections of order 1,2,...
// 7. z is an indicator matrix specifying the intersections
// 8. z2 gives the indicator for the highest order intersections
// Initial setting up
// 1. place z2 into zstar
// 2. place m2 into nstar
// At each stage
```

```

// 1. form the matrix of higher order intersections for which the
//    current intersection has a subset of unities in z
// 2. find the qr decomp of this matrix
// 3. find the intersection of the space with the space of higher
//    intersections
// 4. find the basis for the remainder
// 5. place the basis into m1
// 6. place the indicator into z1
// 7. continue through all such intersections

```

```

zstar=z3,nstar=m3,cstar=c3,[r3 cz3]=size(z3);cnts=cz3;...
r3=[],cz3=[];mark=cntr+1; [row col]=size(cnti);col=[]
for i=row-1:-1:1,...
    if cnti(i)<>0,exec('backinit.mat');exec('back.mat');end;...
end

```

Backinit.mat

```

mark = mark-cnti(i+1) ;
z1 = z(:,mark-cnti(i):mark-1) ;
m1 = n(:,c(mark-cnti(i))+1:c(mark)) ;
c1 = c(mark-cnti(i):mark)-c(mark-cnti(i))*J(1,cnti(i)+1) ;

```

Back.mat

```

for j=1:cnti(i), a=J(p,1)*0; dim=1;...
exec('backspan.mat');exec('backcol.mat');end

```

Backspan.mat

```

for l=1:cnts, exec('back1.mat'); exec('back2.mat'); end

```

Back1.mat

```

ind = 0 ;
for k = 1:r, ...
    if z1(k,j) = 1, if zstar(k,l) <> 1, ind=1; end; ...
end

```

Back2.mat

```

if ind=0,a=[a nstar(:,cstar(l)+1:cstar(l+1))];...

```

```
dim=dim+cstar(l+1)-cstar(l);end
```

```
Backcol.mat
```

```
if dim>1,exec('backcol0.mat');else exec('backcol2.mat');end
```

```
Backcol0.mat
```

```
a=a(:,2:dim);[qa ra]=qr(a);qa=qa(:,1:dim-1);...  
C=m1(:,c1(j)+1:c1(j+1))*qa;[s d t]=svd(C);[rs cs]=size(d);...  
exec('min.mat');num1=0;exec('num1.mat');...  
if num1<c1(j+1)-c1(j),exec('backcol1.mat');end
```

```
Backcol1.mat
```

```
zstar(:,cnts+1) = z1(:,j); cnts = cnts + 1;  
u = m1(:,c1(j)+1:c1(j+1))*s ;  
cstar(cnts+1)=cstar(cnts)+c1(j+1)-c1(j)-num1 ;  
nstar(:,cstar(cnts)+1:cstar(cnts+1))=u(:,num1+1:c1(j+1)-c1(j));
```

```
Backcol2.mat
```

```
zstar(:,cnts+1) = z1(:,j); cnts=cnts+1;  
cstar(cnts+1)=cstar(cnts)+c1(j+1)-c1(j) ;  
nstar(:,cstar(cnts)+1:cstar(cnts+1))=m1(:,c1(j)+1:c1(j+1)) ;
```

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