A New Solution for an Edge Dislocation with Applications to the Stress and Fracture Analysis of Multilayered Media

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Abstract

The stress and fracture analysis of multilayered materials and structures containing crack-like defects is of interest in many research areas, such as composites, bio-mechanics, and geomechanics, and engineering applications, such as coatings, electronics, and adhesive joints. The main objective of this thesis is to further develop a general methodology and utilise it for the examination of fracture problems in multilayered materials. The general methodology is based upon the distributed dislocation technique and edge dislocation solutions obtained within the framework of plane theory of linear elasticity. This methodology has been shaped by the seminal contributions of many researchers over the past fifty years and currently represents a powerful tool for the analysis of crack problems.

New theoretical models and techniques are developed in the present thesis for a range of multi-disciplinary problems utilising the adopted methodology. The research gaps and objectives are formulated specifically for each problem and discussed in separate chapters of this thesis. The solution of each of these problems represents an original and substantial contribution towards the respective area of research. The significant outcomes of this thesis include: a new approach for the analysis of reinforced cracks in layered media, a new mechanism for height control of hydraulic fractures in layered hydrocarbon reservoirs, and a new predictive model for skier-triggered avalanches.

The original contributions of this thesis also include a new fundamental solution for an interfacial edge dislocation, which recovers all previously published solutions for edge dislocations in isotropic multilayered media. The obtained solution can be utilised to derive the governing integral equations for a wide variety of quasi-static crack problems in linearly elastic and isotropic multilayered materials, without any restrictions on the crack orientation or number of elastic layers. Therefore, the newly obtained solution further extends the general
methodology to effectively solve a wide class of fracture problems in multilayered materials and structures.

This thesis is presented in the form of a compendium of publications in high impact specialist journals. The main body of the thesis contains four articles which are united by the above mentioned theme and methodology. Three appendices are also included, which represent a compilation of the candidate’s publications on related topics. A complete publication list is provided in the forthcoming pages.
Declaration

I certify that this thesis contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this thesis will, in the future, be used in a submission in my name for any other degree or diploma in any other university or other tertiary institution without the prior approval of the University of Adelaide.

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Aditya Khanna                   Date
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I dedicate the thesis to my parents, Vijay and Renu Khanna and my sister, Maulshree Khanna. It is due to their continued support that I was able to pursue my interests freely.
List of publications

This thesis is comprised of published and submitted journal articles in accordance with the Academic Program Rules 2015 of The University of Adelaide. A complete list of articles written during the candidature is presented here. The main body of the thesis is based on the following journal articles:


The following journal articles are of closely related to the main topic of research and are included in the thesis as appendices:


The remaining journal articles authored by the candidate are not related to the main topic of the thesis. These articles are listed below but not included in the thesis.


The outcomes of the undertaken research were also presented at several international peer-reviewed conferences. The complete list of conference articles is provided below, however these articles are not included in the thesis.


14) A. Kotousov, **A. Khanna** and S. Bun, An analysis of elasto-plastic fracture criteria, Recent Advances in Structural Integrity Analysis - Proceedings of the International


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