

**Theoretical Study on Toughening
and Design against Fracture of
Functionally Graded Thermal Barrier
Coatings**

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Declaration

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Abstract

Functionally graded thermal barrier coatings (FG TBCs) are advanced multi-phase composites that are engineered to have a smooth spatial gradation of material constituents, which are normally ceramics and metal. The smooth gradation results in the reduction of thermal stresses, minimization or elimination of stress concentrations and singularities at interface corners, and increase in bonding strength. FG TBCs are able to survive very high temperatures and temperature gradients, which makes them very promising in many current and future applications including nuclear reactors, engines, turbines and leading edges of hypersonic vehicles.

FG TBCs inherit fatal weaknesses of ceramics, its brittleness, which often leads to fractures during temperature excursions. Despite of many studies on toughening of brittle ceramics conducted in the past three decades, there was not much work so far done on the toughening of FG TBCs.

The present project has two aims. The first aim is to develop a general micromechanical theory of the stress-induced transformation toughening of multi-phase composites and the second aim is to develop a theoretical model for FG TBCs toughened by transformable particles, which can be used in the design and fabrication of FG TBCs for applications where the high fracture resistance is mandatory.

A new theoretical model for transformation toughening in multi-phase composites is developed based on a combination of micromechanics and fracture mechanics approaches. According to the developed model, the effect of thermal residual stresses due to the mismatch in thermal expansion coefficients of constituent phases on toughening is found to be very strong.

A methodology of design of FG TBCs toughened by phase transformation of ZrO_2 is investigated by incorporating the developed micromechanics-based model for transformation toughening in multi-phase composites into the classical lamination theory (CLT). A new parameter such as an effective stress intensity factor is introduced for investigating the fracture behaviour and toughening effect in FG TBCs. As an example, Ni- ZrO_2 FG TBC systems subjected to a thermal shock conditions are analysed and general guidelines for the design of such system with improved fracture properties are developed.

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