



**An efficient modelling technique for static and
dynamic response of thin-walled box girder
bridges**

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B.Sc. (Hons), M.Sc. (Civil and Structural Engineering)

Thesis submitted for the degree of

Doctor of Philosophy

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The University of Adelaide, Australia

December 2015

Dedication

This thesis is deeply dedicated:

To my lovely parents, Mrs Akhtar Rokhi and Mr Hossein Kashefi, for their continuous support, care and sacrifices

And

To the soul of my beloved grandfather Mr Ali Rokhi whose prayers and advice enlighten my life.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Acknowledgments

The successful completion of this thesis has not been possible without support, encouragement and assistance of several individuals for whom I would like to express my gratitude here.

Firstly, I would like to thank my supervisors, Associate Professor Abdul Hamid Sheikh, Professor Michael Griffith and Dr Mohamed Ali for all of their support, encouragement and tireless effort in helping me to achieve my goals. I am especially grateful of my main supervisor, Associate Professor Abdul Hamid Sheikh, whose encouragement, admirable attitude and humbleness in sharing and teaching his deep knowledge in the field has inspired me to a great extent and promoted me both technically in my profession and as a person. His unconditional support and dedication in responding my enquiries during the meetings or remotely using all possible means (e.g. mobile, email, skype, etc.) have escalated my respect and appreciation for him even more. I owe him my confidence in programming and improvement in my writing skills. I would like to sincerely thank him for encouraging my research and for allowing me to grow as a researcher as well as for having the ever interesting and inspiring conversations. Words cannot express the deepness of my gratitude to him.

I would like to extend my gratefulness to the School of Civil, Environmental and Mining Engineering staff and administration for their individual helps. Special thanks to Mr Stephen Carr and Mr Jeffery Newman for helping me in my programing, Professor Martin Lambert for his understanding and inspiring charismatic personality as well as his willingness to help and support every PhD student, and Ms Julie Ligertwood and Ms Josie Peluso for their support, kindness and encouragement.

I am also thankful to Dr Jose Pedro Blasques from National Laboratory for Sustainable Energy of Technical University of Denmark for his technical knowledge and constant willingness to help. His invaluable notes have really helped in different stages of this thesis.

I would like to acknowledge the Australian Government for supporting this research through Endeavor Postgraduate Award and making this amazing, fruitful and unforgettable experience possible.

Also I would like to thank the South Australian Government and Japanese Government for funding me through Mason Lakes Fellowship Program (MLFP) to implement the experimental part of my thesis in Bridge Laboratory of Nagoya University. My deepest appreciation goes to

my host supervisor in Nagoya University, Professor Kazuo Tateishi, for his invaluable guidance, unconditional support and indescribable kindness during the period of this fellowship. Also I am very much appreciative of Prof. Tateishi to provide me with the opportunity to work in his equipped laboratory together with his expert team. I am specifically grateful of having the pleasure of working with Dr Shimizu who has the attitude and substance of a genuine researcher and a real colleague. Without his persistence and help the on-time completion of the experiments would not have been possible.

My special appreciation and deepest gratitude are extended to my fellow friends whose technical and emotional support, encouragement and even the long depressed coffee hours with them kept me motivated during this PhD journey: Dr Masoumeh Zargar, Ms Bahar Forghani, Ms Melika Tavakolian, Dr Katelijjn Van Hende, Dr Veiko Palge, Mr Michael Di Matteo, Dr Moein Navvab Kashani, Dr Hamideh Elekaei, Mr David Peter Wright, Mr Exequiel Sepulveda Escobedo, Dr Christopher Stokes, Dr Xuyuan Li, Dr Jialu Ma (Sammy) and all my fellow postgraduate students for their friendship throughout my PhD.

Finally and most importantly, my immeasurable gratitude is extended to my family especially to my mother Mrs Akhtar Rokhi and my father Mr Hossein Kashefi for having faith in me and supporting me through their continuous encouragement and wisdom. Their prayers, strong beliefs and wisdom have always enlightened my path to success and inspired me. I owe my achievements so far in my life to them. Also I would like to express my deep appreciation to my sister Ms Keynoosh Kashefi for being the best possible role model in my life, my brother Mr Amir Kanan Kashefi who has always been my best friend and my niece Ms Nikta Adam for her enthusiasm and joyful conversations. Their love, friendship and support not only assisted in the successful completion of my PhD but made the experience even more enjoyable, fruitful and memorable.

Abstract

Highway bridges are crucial parts of the civil infrastructure which require special attention at the time of their analysis and design. Box girder deck systems are amongst the most popular types of highway bridge structures and the understanding of their behaviour under different loads plays an important role in their structural design. Therefore, an accurate assessment of the response of these bridges under these loads is of great importance.

Box girder bridges are essentially thin-walled beams having closed or a combination of closed and open cross-sections. The behaviour of these thin-walled structures under an arbitrary loading scenario is quite complex, primarily due to the cross-sectional warping (out-of-plane warping) and the distortion of the sections (in-plane warping). The accurate calculation of warping displacements has been the area of many research studies because the variation of warping displacements over a cross-section does not follow a standard pattern.

With the help of a detailed Finite Element Analysis (FEA), it is possible to obtain results which may be reasonably close to the exact three dimensional (3D) elasticity solutions of these thin-walled structures. However, this approach involves significant computational resources and efforts, especially for bridges having complex geometries. The use of this modelling approach is not feasible, particularly at the preliminary design stage when the analysis is typically performed many times and the design is being modified and improved iteratively.

On the other hand, a specific feature of typical box girders is that one of its dimensions (length) is very large compared to the other two dimensions. Utilising this trait, many researchers have tried to condense the 3D problem into a one-dimensional (1D) problem and treat these structures as beams. Although this approach makes the analysis highly efficient, the existing beam theories involve many approximations in order to account for out-of-plane warping and distortion of these structures. This can affect the accuracy of the solution significantly for thin-walled box girder bridges.

In the present thesis, a novel method is introduced which can offer a very accurate solution to the problem and at the same time the method is computationally efficient. The proposed technique splits the 3D elasticity problem into a two-dimensional (2D) cross-sectional problem and a 1D beam problem. The 2D beam cross-sectional problem is solved using a 2D finite element discretization where the effects of in-plane warping as well as out-of-plane warping are considered. The 2D finite element analysis generates the 'exact' constitutive matrix (or

stiffness matrix) for the beam cross-section which ensures proper coupling between the different modes of deformation. This cross-sectional stiffness matrix is then used in the 1D beam analysis based on a usual 1D beam finite element model. The stress resultants obtained from the 1D beam analysis and the results obtained from the 2D cross-sectional analysis are used to determine the warping displacements and finally recover the 3D stress and displacement fields of the thin-walled beams. The computational efficiency of this approach is significant in terms of prediction of the 3D response of these structures. In order to implement the method, computer programs were developed in FORTRAN specifically for the present purpose.

The major research contributions of the current study are presented in the form of three journal papers and one conference paper. Firstly, the mathematical formulation of the method is presented in details and its accuracy is examined by the analysis thin-walled girders having different cross-section configurations under various loading conditions. The results are then validated against those obtained by 3D FE models of these structures. In the second paper, the method is extended to dynamic analysis of box girder superstructures. Numerical examples of thin-walled box girders are solved by the proposed approach under dynamic loading (e.g. time varying and moving loads) to show its performance. The free vibration analyses of these structures are also carried out and the results are compared with the results obtained by 3D finite element analyses of these structures. Finally, the behaviour of straight and curved thin-walled box girders is investigated through experimental studies. Detailed 3D finite-element analysis of these girders is carried out and the results are compared with the experimental results. Also, the experimental results obtained for the straight specimen are used to validate the proposed analysis approach. Additionally, the vibration frequencies of the specimens are measured using the data obtained from their impact excitations.

The results obtained from the proposed method are found to have a very good correlation with the 3D FEM in all investigations. Considering the level of accuracy and efficiency required for the analysis of bridge super-structures, the proposed modelling approach seems to have a very good potential in its application for different problems. It is expected that this research will initiate further developments of this technique for its extension in the analysis of wide variety of bridge configurations (e.g. curved, composite) and for solving various problems (e.g. geometrical nonlinearity). **Keywords: finite element method, warping, cross-sectional analysis, static and dynamic response, experimental validations**

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