

**Geostatistical Modelling and Simulation of Karst Systems**

A thesis by

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# Contents

Contents.....	ii
List of Figures .....	iv
List of Tables.....	ix
List of Abbreviations.....	x
Declaration of Originality .....	xiv
Acknowledgements .....	xv
<b>Chapter 1: Introduction.....</b>	<b>1</b>
1.1 Overview .....	1
1.2 Research Objective.....	5
1.3 Thesis Outline .....	6
<b>Chapter 2: Literature Review .....</b>	<b>8</b>
2.1 Overview of Karst Aquifers .....	8
2.2 Geometry of Karst Networks .....	9
2.3 Existing Techniques for Modelling Meandering Channels and Karst Systems ...	10
2.3.1 Traditional geostatistical techniques .....	11
2.3.2 Multiple-point geostatistics .....	16
2.4 Current Approaches in Evaluating Geostatistical Methods .....	21
2.5 Gap Statement .....	23
2.5.1 Descriptive list of gaps .....	24
<b>Chapter 3: Methodology .....</b>	<b>26</b>
3.1 Introduction to Geostatistical Methods .....	26
3.2 Potentially Useful Methods .....	27
3.2.1 Two-point statistics .....	28
3.2.2 Multiple-point statistics.....	29
3.2.2.1 Gridded MPS method.....	31
3.2.2.2 Non-gridded MPS method .....	35
3.2.2.3 Modified non-gridded MPS method .....	38
3.3 Evaluation of Modified Non-gridded MPS .....	42
3.3.1 Measuring variability among realisations .....	43
3.3.2 Measuring variability between training image and realisations.....	45
<b>Chapter 4: Results.....</b>	<b>46</b>
4.1 Gridded MPS.....	46
4.1.1 Gridded MPS scanning algorithm .....	46
4.1.2 Gridded MPS simulation algorithm .....	50
4.2 Modified Non-gridded MPS.....	51
4.2.1 Modified non-gridded MPS pre-processing.....	51
4.2.2 Modified non-gridded MPS analysis.....	55
4.2.3 Modified non-gridded MPS simulation.....	57
4.3 Distance Measures.....	59
4.3.1 The within-realisation variability .....	60
4.3.2 The between-realisation variability .....	62
4.3.3 Sensitivity of non-gridded template size .....	64
4.4 Case Study (Demonstration Examples).....	68
4.4.1 Olwolgin Cave.....	69
4.4.1.1 Gridded MPS of Olwolgin Cave .....	70
4.4.1.2 Modified non-gridded MPS for Olwolgin Cave .....	72
4.4.2 Tank Cave .....	74

4.4.2.1 Gridded MPS of Tank Cave.....	75
4.4.2.2 Modified non-gridded MPS for Tank Cave.....	77
4.4.3 Distance measures.....	79
<b>Chapter 5: Conclusions and Recommendations for Future Work .....</b>	<b>87</b>
<b>5.1 Conclusion .....</b>	<b>87</b>
<b>5.2 Recommendations for Future Work .....</b>	<b>93</b>
Appendices.....	95
References.....	110

## List of Figures

Figure 1.1. Schematic diagram of a heterogeneous karst aquifer (adapted from Goldscheider & Drew 2007) .....	2
Figure 1.2. Turbulent water flow in a cave (photo: Wenger).....	2
Figure 1.3. Formation of sinkholes in Florida (photo: Yunji De Nies & Daisha Riley 2012).....	3
Figure 1.4. Surface stream sinking underground via a sinkhole (photo: Goldscheider) .....	4
Figure 2.1. Schematic model of karst aquifer illustrating various zones (Geyer 2008).....	9
Figure 2.2. 3D finite element model generated by coupling the karst conduit simulator with flow simulation (after Jaquet et al. 2004).....	12
Figure 2.3. 2D map of conduit network realisations; black lines: inlets, which are known as the starting points of the conduits; green points: outlet or karst spring (Borghi et al. 2012) .....	13
Figure 2.4. (a) 3D map of the non-connected conduits; (b) 3D map of a realisation of a karst network with connectivity using a modified diffusion-limited aggregation technique (Pardo-Igúzquiza et al. 2011).....	15
Figure 2.5. 3D map of karst network realisations generated by Henrion et al. (2008) ...	16
Figure 2.6. Top view of pattern simulation of large-scale passages; (a) original data, (b) simulated network, circle: conditioning data, lines: simulated connection (Erzeybek Balan 2012) .....	20
Figure 3.1. General geostatistical simulation workflow (Desbarats 1996) .....	27
Figure 3.2. A geological model using a training image (Honarkhah & Caers 2010).....	30
Figure 3.3. Example of gridded training image (Honarkhah & Caers 2010).....	31
Figure 3.4. Example of gridded template; a 3×3 template on an 11×11 grid (Arpat & Caers 2007).....	31
Figure 3.5. (a) Sampling of a gridded training image using a gridded template, (b) observed patterns (after Arpat & Caers 2007).....	32
Figure 3.6. General algorithm of distance-based pattern modelling .....	34
Figure 3.7. Generic template (adapted from Erzeybek Balan 2012).....	36

Figure 3.8. Example of a two-node template, a centre node (red circle) and two template nodes (blue circles) (adapted from Erzeybek Balan 2012) .....	37
Figure 3.9. Left: non-gridded training image; middle and right: two realisations generated with non-gridded MPS by Erzeybek Balan (2012) using a C++ code .....	39
Figure 3.10. General algorithm of the modified non-gridded MPS.....	41
Figure 3.11. Template used to scan the non-gridded training image by modified non-gridded MPS .....	42
Figure 3.12. General framework of evaluation of the modified non-gridded MPS.....	43
Figure 4.1. Conceptual model of a channel system 101×101×1 used as a gridded training image for the gridded MPS scanning analysis .....	47
Figure 4.2. Entropy plot for determining the optimum template size.....	48
Figure 4.3. MDS dimension selection; upper: scree plot; lower: correlation coefficient vs dimension .....	49
Figure 4.4. MDS result using the 15×15 pattern template.....	50
Figure 4.5. Two realisations using the gridded MPS simulation algorithm .....	51
Figure 4.6. (a) Conceptual model of a channel system with resolution 101×101×1, (b) digitised model used as a non-gridded training image for the modified non-gridded MPS scanning analysis (generated by Digitize it 2010) .....	52
Figure 4.7. Digitised model used as training data for the modified non-gridded MPS scanning algorithm; arrow indicates north.....	53
Figure 4.8. Frequency histogram of the observed patterns as obtained by scanning the training image with the template.....	55
Figure 4.9. Two realisations using the modified non-gridded MPS simulation algorithm; black points denote simulated nodes .....	57
Figure 4.10. Two realisations using the non-gridded MPS simulation algorithm; black points denote simulated nodes.....	58
Figure 4.11. Distances between training image and 50 realisations for gridded MPS; blue dots are the distance values between each realisation and the training image; red dots represent the average of the distances.....	60
Figure 4.12. Distances between training image and 50 realisations for modified non-gridded MPS; blue dots are the distance values between each realisation and training image; red dots represent the average of the distances.....	61

Figure 4.13. Distance plot between realisations and training image for gridded and modified non-gridded MPS methods; red dots represent the average of the distances .....	61
Figure 4.14. Distances between 50 realisations in gridded MPS; blue dots are the distance values between pairs of realisations; red dots represent the average of the distances.....	62
Figure 4.15. Distances between 50 realisations from modified non-gridded MPS; blue dots are the distance value between pairs of realisations; red dots represent the average of the distances .....	63
Figure 4.16. Distance plot between realisations for gridded and modified non-gridded MPS methods; red dots represent the average of the distances .....	63
Figure 4.17. Distance plot for the modified non-gridded MPS with a small template size (i.e., distances between training image and 50 realisations using a template with a smaller tolerance window and lag distance).....	64
Figure 4.18. Distance plot for the modified non-gridded MPS with a small template size (i.e., distances between 50 realisations using a template with a smaller tolerance window and lag distance).....	65
Figure 4.19. Distance plot for the modified non-gridded MPS with a large template (i.e., distances between training image and 50 realisations using a template with a larger tolerance window and lag distance) .....	65
Figure 4.20. Distance plot for the modified non-gridded MPS with a large template (i.e., distances between 50 realisations using a template with a larger tolerance window and lag distance) .....	66
Figure 4.21. Distance plot between realisations and training image for modified non-gridded MPS method using different template sizes .....	66
Figure 4.22. Distance plot between realisations for modified non-gridded MPS method using different template sizes .....	67
Figure 4.23. Olwolgin Cave downstream section .....	68
Figure 4.24. Map of Tank Cave, the longest phreatic cave in the Mt Gambier area, with spatial analysis of passage directions (Harris 2008) .....	69
Figure 4.25. Olwolgin Cave grey-scale intensity image .....	70
Figure 4.26. Olwolgin Cave digitised binary image at resolution of 170×100×1 (training image) .....	70

Figure 4.27. (a) Entropy plot; (b) profile log-likelihood; for optimum template size selection .....	71
Figure 4.28. Variance of entropy plot.....	71
Figure 4.29. Three realisations generated using gridded MPS simulation algorithm for Olwolgin Cave.....	72
Figure 4.30. Digitised model of Olwolgin Cave used as training data for the modified non-gridded MPS scanning algorithm; arrow indicates north.....	73
Figure 4.31. Conditioning data used for the modified non-gridded MPS simulation; arrow indicates north.....	73
Figure 4.32. Two different realisations of Olwolgin Cave generated by modified non-gridded MPS .....	74
Figure 4.33. Tank Cave binary image at resolution of 100×100×1 (training image).....	75
Figure 4.34. (a) Entropy plot; (b) profile log-likelihood; presenting the application of optimum template size selection.....	75
Figure 4.35. Variance of entropy plot.....	76
Figure 4.36. Three realisations resulted using gridded MPS simulation algorithm for Tank Cave .....	77
Figure 4.37. Digitised model of Tank Cave used as training data for the modified non-gridded MPS scanning algorithm; arrow indicates north .....	78
Figure 4.38. Conditioning data used for the modified non-gridded MPS simulation algorithm; arrow indicates north .....	78
Figure 4.39. Two different realisations of Tank Cave generated by modified non-gridded MPS .....	79
Figure 4.40. Distances between training image and 50 realisations for gridded MPS for Olwolgin Cave; blue dots are the distance values between each realisation and the training image; red dots represent the average of the distances .....	80
Figure 4.41. Distances between training image and 50 realisations for modified non-gridded MPS for Olwolgin Cave; blue dots are the distance values between each realisation and the training image; red dots represent the average of the distances .....	80
Figure 4.42. Distances between 50 realisations for gridded MPS for Olwolgin Cave; blue dots are the distance values between pairs of realisations; red dots represent the average of the distances.....	81

Figure 4.43. Distances between 50 realisations for modified non-gridded MPS for Olwolgin Cave; blue dots are the distance values between pairs of realisations; red dots represent the average of the distances .....	81
Figure 4.44. Distances between training image and 50 realisations for gridded MPS for Tank Cave; blue dots are the distance values between each realisation and the training image; red dots represent the average of the distances .....	82
Figure 4.45. Distances between training image and 50 realisations for modified non-gridded MPS for Tank Cave; blue dots are the distance values between each realisation and the training image; red dots represent the average of the distances .....	82
Figure 4.46. Distances between 50 realisations for gridded MPS for Tank Cave; blue dots are the distance values between pairs of realisations; red dots represent the average of the distances .....	83
Figure 4.47. Distances between 50 realisations for modified non-gridded MPS for Tank Cave; blue dots are the distance values between pairs of realisations; red dots represent the average of the distances .....	83
Figure 4.48. Distance plot between 50 realisations and training image for gridded and modified non-gridded MPS methods for Olwolgin Cave.....	84
Figure 4.49. Distance plot between 50 realisations for gridded and modified non-gridded MPS methods for Olwolgin Cave .....	84
Figure 4.50. Distance plot between 50 realisations and training image for gridded and modified non-gridded MPS methods for Tank Cave.....	85
Figure 4.51. Distance plot between realisations for gridded and modified non-gridded MPS methods for Tank Cave .....	85

## List of Tables

Table 4.1. Template properties .....	54
Table 4.2. First template properties .....	56
Table 4.3. Second template properties .....	56
Table 4.4. Third template properties .....	56

## List of Abbreviations

2D	two-dimensional
3D	three-dimensional
CPU	central processing unit
FILTERSIM	filter-based pattern simulation
FMA	fast marching algorithm
JSD	jensen shannon divergence
MDS	multi-dimensional scaling
MPS	multiple-point statistics
RAM	random-access memory
SGS	Sequential Gaussian Simulation
SIMPAT	Simulation of Patterns
SIS	Sequential Indicator Simulation
SNESIM	single normal equation simulation

## **Abstract**

Groundwater is a significant water resource and in many parts of the world it occurs in karst aquifers. The modelling of karst systems is a critical component of groundwater resource assessment and flow. Geostatistical techniques have shown useful applications in the area of groundwater research because of their ability to quantify spatial variability, uncertainty and risk. Traditional geostatistical methods, based on variogram models, use only two-point statistics and thus are not capable of modelling the complex and, high-connectivity structures of karst networks. This has led to an increasing focus on spatial multiple-point statistics (MPS) to model these complex systems.

In this approach, a training image is used instead of a variogram. Patterns are obtained by scanning and sampling the training image and during the simulation they are reproduced using MPS. There are two implementations of MPS: (i) gridded and (ii) non-gridded. In gridded MPS, the training image, templates and simulations are based on rigid grids, whereas the spatially flexible non-gridded approach does not depend on rigidly specified grids. The non-gridded approach is relatively new (Erzeybek Balan 2012), and applications, especially in hydrogeology are few; however, the method has been used to simulate paleokarsts in petroleum applications. Non-gridded MPS has potential to improve the modelling of karst systems by replacing the fixed gridding procedure, used in the original form of MPS, by a more flexible grid adapted to each specific application. However, there are some weaknesses in the non-gridded approach reported in the literature. For example, the proposed template cannot properly represent the tortuous nature of a network, and the variation of the passage widths is not taken into account. In the case of a simple channelised system with a constant width, sampling

the central line of the passages is sufficient; however, most karst systems have networks with significantly varying widths. In addition, the variability among the realisations generated by non-gridded MPS is relatively small, indicating that the realisations do not cover the full space of uncertainty. In practical applications, it is not possible to know the exact extent of the full space of uncertainty, but the observed variability of the geology and geomorphology of similar structures would tell us when the variability among the simulations is too small (or too large). A lack of significant variability among simulated realisations makes the method inapplicable. This thesis presents a modified non-gridded MPS method that increases the variability among realisations and adequately captures the tortuosities of karst networks. To do this, it includes the width and constructs an optimal template based on a representative variety of directions adapted to each network instead of considering only a few major directions using a generic template as applied by Erzeybek Balan (2012).

The performance of Erzeybek Balan's (2012) non-gridded MPS method has only been visually demonstrated, which is not a sufficiently robust measure of performance. In this thesis, a systematic measure is developed to evaluate the variability among the realisations. This provides an objective way of comparing an important feature of the simulations generated by gridded MPS and the proposed modified non-gridded MPS.

The research starts with an investigation and modification of non-gridded MPS. A widely used demonstration image, which is based on a channelised system, is used to compare the performances of the original non-gridded MPS (Erzeybek Balan 2012) and the modified version proposed in this thesis. A distance-based measure is used to evaluate and compare pattern reproduction and the variability of the realisations generated by the modified non-gridded MPS and standard gridded MPS methods. This

distance measure can be used to compare the multiple-point histograms of the realisations and training images. Gridded MPS and modified non-gridded MPS are then applied to two different karst systems—Olwolgin Cave and Tank Cave—and the realisations generated by each method are evaluated in terms of pattern reproduction and the extent of the uncertainty space.

The comparison examples demonstrate that the proposed modified non-gridded MPS generates a larger uncertainty space than that generated by gridded MPS. The results also confirm that modified non-gridded MPS performs significantly better than the original version of non-gridded MPS in terms of a larger (and more realistic) space of uncertainty and pattern reproduction when applied to a complex karst system.

## **Declaration of Originality**

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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