Thin-film flow in helical channels

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Abstract

In this thesis, we study fluid flows in helical channels. The primary motivating application for this work is the segregation of particles of different weights/densities in spiral particle separators, devices used in the mining and mineral processing industries to separate ores and clean coal. These devices feature very shallow flows, and so we use the thin-film approximation which enables significant analytic progress. It is most convenient to use a non-orthogonal, helicoidal coordinate system which allows a natural representation of helical channels with arbitrary cross-sectional profile, and arbitrary centreline slope and radius. We begin by studying particle-free flow in channels with rectangular cross-section. On taking the thin-film limit of the Navier-Stokes equations, we obtain a system of equations which has an analytic solution. This solution is investigated to determine the effects of changing the slope and curvature of the channel centreline, and the fluid flux down the channel. We then consider particle-free flow in helical channels with shallow, but otherwise arbitrary cross-section, and investigate the effect of changing the cross-sectional shape of the channel, guided in part by questions raised from studying rectangular channels. Except in a special case, this model must be solved numerically. Finally, we consider monodisperse particle-laden flow, using the diffusive-flux model proposed by Leighton and Acrivos (1987). We present the thin-film particle-laden flow model for shallow channels of arbitrary geometry and, assuming the particles are uniformly distributed in the vertical direction, solve the resulting system of equations numerically. We conclude by outlining future research directions.

Signed Statement

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