Topics in Equivariant Cohomology

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Thesis submitted for the degree of
Master of Philosophy
in
Pure Mathematics
at
The University of Adelaide
Faculty of Mathematical and Computer Sciences

School of Mathematical Sciences

February 1, 2017

Contents

Abstract

The equivariant cohomology of a manifold M acted upon by a compact Lie group G is defined to be the singular cohomology groups of the topological space

$$(M \times EG)/G.$$

It is well known that the equivariant cohomology of M is parametrised by the Cartan model of equivariant differential forms. However, this model has no obvious geometric interpretation – partly because the expression above is not a manifold in general. Work in the 70s by Segal, Bott and Dupont indicated that this space can be constructed as the geometric realisation of a simplicial manifold that is naturally built out of M and G. This simplicial manifold carries a complex of so-called simplicial differential forms which gives a much more natural geometric interpretation of differential forms on the topological space $(M \times EG)/G$.

This thesis provides a model for the equivariant cohomology of a manifold in terms of this complex of simplicial differential forms. Explicit chain maps are constructed, inducing isomorphisms on cohomology, between this complex of simplicial differential forms and the more standard models of equivariant cohomology, namely the Cartan and Weil models.

Signed Statement

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Acknowledgements

I must first sincerely thank my supervisor Danny Stevenson. Not only was Danny an expert resource for many of the technical questions I had along the way; the patience, time and care he put into reading this thesis and guiding my research cannot be overstated. It has truly been a privilege to have him as my supervisor and as a lecturer over the last few years. I have especially enjoyed his large collection of humorous anecdotes to reassure me when times got tough. Thank you for everything.

I would also like to thank my co-supervisor Michael Murray for being there whenever I needed him. Michael's keen eye has been invaluable for drafting this thesis and he has given me great insight and perspective at several points along the way. Thank you for all the kindness you have shown me.

It would be remiss of me to not thank my friends and family that helped me complete this dissertation. To Brett, Kate, Henry, Thomas, Tessa, Brock, Felix, James, Max, Pete and Trang: thanks for being such good company in the giant fish tank where it all began. To John, Serrin and Amelia: thanks for always being a shoulder to lean on regardless of how far apart we are. To my family, both in Adelaide and in Perth: thank you for all your love and support.

Last and certainly not least, I would like to express the utmost gratitude to my parents, Megan and Chris, for providing me with unwavering support and encouragement throughout my years of study and research. This thesis would not have been possible without them. Thank you for the abundance of love you have given me and the weekly hot dinners.