

Modelling the Wave-Induced Collisions of Ice Floes

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Signed Statement

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint award of this degree.

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Abstract

The wave-induced collisions and rafting of ice floes are investigated experimentally and theoretically. Results from a series of wave basin experiments are presented. Ice floes are simulated experimentally using thin plastic disks. The first round of experiments focusses on measuring the oscillatory surge, heave, pitch and drift motions of solitary floes. The second and third rounds of experiments record the motions of two adjacent floes. Rafting is suppressed in the second round, and allowed in the third round. Collision and rafting regimes are identified, and collision behaviours are quantified over a range of incident wavelengths and wave amplitudes.

Two mathematical models are proposed to model the wave-induced motions of solitary floes. The first is based on slope-sliding theory, and the second is based on linear potential-flow theory. Both models are validated using results from the single-floe experiments. Model-data comparisons show that the slope-sliding model is valid in the long-wavelength regime, and potential-flow model is more accurate in shorter wavelengths.

A two-floe collision model is then developed to replicate the conditions of the two-floe experiments. Slope-sliding theory is used to model floe motions. A time-stepping algorithm is implemented to determine the occurrence of collision and rafting events. Predicted collision behaviours are compared with results from the two-floe experiments. Good agreement is attained in incident waves of intermediate to long wavelengths.