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Gluon propagator in the infrared region

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The gluon propagator is calculated in quenched QCD for two different lattice sizes (16^3×48 and 32^3×64) at β=6.0. The volume dependence of the propagator in the Landau gauge is studied. The smaller lattice is instrumental in revealing finite volume and anisotropic lattice artifacts. Methods for minimizing these artifacts are developed and applied to the larger lattice data. New structure seen in the infrared region survives these conservative cuts to the lattice data. This structure serves to rule out a number of models that have appeared in the literature. A fit to a simple analytical form capturing the momentum dependence of the nonperturbative gluon propagator is also reported. [S0556-2821(98)50215-9]

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I. INTRODUCTION

The infrared behavior of the nonperturbative gluon propagator has been the subject of extensive research and debate. Knowledge of this behavior is generally regarded as being central to understanding the mechanism of confinement in quantum chromodynamics (QCD). Moreover, it is of great importance in various model-based approaches. For example, some studies based on Dyson-Schwinger equations [1–3] have indicated that an infrared enhanced gluon propagator may be required for confinement; however, others [4–6] have pointed to the possibility of a dynamically generated gluon mass, or other forms leading to an infrared finite (or even vanishing) propagator (see [7] and references therein).

Computing the gluon propagator directly on the lattice should provide an opportunity to resolve these contradictory claims. However, previous lattice studies [8,9] have been unable to access the “deep” infrared region, where the most interesting nonperturbative behavior is expected. Significant finite volume effects introduced through zero momentum components prevent the study of the infrared behavior of the propagator on a small lattice.

The main aim of this study is to obtain a definite behavior for the gluon propagator for momenta smaller than 1 GeV, where the interesting physics is expected to reside. In the following, we will report such results for momenta as small as 0.4 GeV. These momenta are small enough to reveal new structure in the gluon propagator and to place strong constraints on its infrared behavior.

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II. THE GLUON PROPAGATOR ON THE LATTICE

The gauge links $U_\mu(x)$ may be expressed in terms of the continuum gluon fields as

$$U_\mu(x) = \mathcal{P} e^{ig_0 \int^{x+\hat{\mu}}_x A_\mu(z)dz} = e^{ig_0 A_\mu(x+\hat{\mu}/2)} + O(a^3),$$

(2.1)

where $\mathcal{P}$ denotes path ordering. From this, the dimensionless lattice gluon field $A_\mu^L(x)$ may be obtained via

$$A_\mu^L(x+\hat{\mu}/2) = \frac{1}{2ig_0} (U_\mu(x) - U_\mu^\dagger(x)) - \frac{1}{6ig_0} \text{Tr}(U_\mu(x) - U_\mu^\dagger(x)),$$

(2.2)

which is accurate to $O(a^2)$. Denoting the discrete momenta available on a finite, periodic volume by $\hat{q}$, the momentum space gluon field is

$$A_\mu(\hat{q}) = \sum_i e^{-iq\cdot x + i\hat{\mu}/2} A_\mu^L(x+\hat{\mu}/2)$$

$$= e^{-iq\cdot x + i\hat{\mu}/2} \left[ (U_\mu(\hat{q}) - U_\mu^\dagger(-\hat{q})) - \frac{1}{3} \text{Tr}(U_\mu(\hat{q}) - U_\mu^\dagger(-\hat{q})) \right].$$

(2.3)

where $U_\mu(\hat{q}) = \sum_i e^{-iq\cdot x} U_\mu(x)$ and $A_\mu(\hat{q}) = i^n A_\mu^n(\hat{q})$. The available momentum values $\hat{q}$ are given by

$$\hat{q}_\mu = \frac{2\pi n_\mu}{aL_\mu}, \quad n_\mu = 0, \ldots, L_\mu - 1$$

(2.4)
where $L_μ$ is the length of the box in the $μ$ direction. The factor $e^{-iqμa^2}$, which comes from the gauge fields being defined on the links rather than the sites of the lattice, is crucial to removing $O(a)$-errors and in particular obtaining the correct tensor structure for the gluon propagator [10].

The gluon propagator $D^{ab}_{μν}(q)$ is defined by

$$D^{ab}_{μν}(q) = \langle A^a_μ(q)A^b_ν(-q) \rangle. \quad (2.5)$$

We choose to study the gauge dependent propagator in the Landau gauge, which can be implemented numerically by maximizing $F[g] = \Sigma_μ x \ReTr U^*_μ(x)$, where

$$U^*_μ(x) = g(x)U_μ(x)g^{†}(x + \hat{μ}). \quad (2.6)$$

In the continuum limit, this is related to the fact that fields satisfying the Landau gauge condition correspond to stationary points of $F^{cont}[g] = \|A|^2 = \int d^4x \ReTr(A^a_μ)^2(x)$ [5].

The Landau gauge gluon propagator in the continuum has the form

$$D^{ab}(q) = \left( δ_{μν} - \frac{qμqν}{q^2} \right) δ^{ab}D(q^2). \quad (2.7)$$

The scalar function $D(q^2)$ can then be extracted using

$$D(q^2) = \frac{1}{3} \Sigma_μ \frac{1}{8} \Sigma_a D^{aa}_{μμ}(q). \quad (2.8)$$

For $a \rightarrow 0$, the lattice propagator is related to the continuum one by $a^2D(q^2) = D^{cont}(q^2) + O(a^2)$.

A well-known lattice artifact is that the tree level propagator of a massless boson field does not reproduce the expected continuum result of

$$D^{(0)}(q^2) = \frac{1}{q^2}, \quad (2.9)$$

but rather produces

$$a^2D^{(0)}(q^2) = \frac{a^2}{Σ_μ(2 \sin qμa/2)^2}. \quad (2.10)$$

In the following, we are particularly interested in the quantity $q^2D(q^2)$, which is expected to approach 1 up to logarithmic corrections as $q^2 \rightarrow ∞$. To ensure this result, we will work with the momentum variable defined as

$$q_μ = \frac{2}{a} \sin \frac{qμa}{2}. \quad (2.11)$$

The momenta $q$ and $q^*$ are often defined the other way around in the lattice literature. However, we feel it is more instructive to define $q$ as above, such that lattice results reproduce the continuum formula (2.7) and the tree level formula (2.9).
Since the low momentum region holds the greatest non-perturbative interest, it is instructive to stress that the low momentum component along one or more of the lattice axes is zero. Since the length of the lattice in the time direction is three times that of the spatial directions, we notice a clear difference between points which correspond to momenta directed along spatial axes from those purely in the time direction. These finite volume artifacts are clearly displayed at small momenta by the discrepancies between the filled squares and triangles denoting momenta directed along lattice axes lying below the majority of points that lie within a certain solid angle of the diagonal.

Some residual anisotropy remains at moderate momenta around $q^a \sim 1.5$ despite including the kinematical correction of Eq. (2.11). This anisotropy is clearly displayed in Fig. 2 by the filled squares and triangles denoting momenta directed along lattice axes lying below the majority of points from off-axis momenta for $q^a \sim 1.4$. Since tree-level and one-loop $O(4)$ breaking effects should be removed by the kinematical correction, the remaining anisotropy appears to have its origin in quantum effects beyond one loop.

In order to minimize lattice artifacts for large momentum components, we select momentum vectors lying within a cylinder directed along the diagonal $(t,x,y,z) = (1,1,1,1)$ of the lattice. This allows one to access the largest of momenta with the smallest of components. We found the selection of a cylinder with a radius of one spatial momentum unit ($\Delta q^a < 1 \times 2\pi / L$, where $L$ is the number of sites along a spatial axis) provides a reasonable number of points falling along a single curve for large momenta. The data surviving this cut are displayed in Fig. 3.

However, this cut does not address the large finite volume errors surviving in Fig. 3. To remove these problematic momenta, we consider a further cut designed to remove momentum vectors which have one or more vanishing components. This is implemented by keeping only momentum directions that lie within a certain solid angle of the diagonal. We found that a cone of angle $20^\circ$ measured from the diagonal at the origin was sufficient to provide a set of points lying along a smooth curve. The solid points in Fig. 3 represent these data. After these conservative cuts, there is relatively little structure left in the infrared region on our small lattice. Armed with this knowledge of how to obtain reliable lattice data, we now turn our attention to the gluon propagator data obtained from our larger lattice.

B. Large lattice analysis

Figure 4 displays the gluon propagator data for all momentum directions and values on the larger lattice. Again, only a $Z_3$ averaging has been performed. Examination of the infrared region indicates that finite volume artifacts are very small on the larger lattice. In particular, the agreement between purely spatial (filled squares) and time-like momentum vectors (filled triangles) at $q^a = 0.20$ appears to indicate that finite size effects are relatively small on this lattice. Such an observation is consistent with topological studies of the QCD vacuum which provide some insight into the typical scale of QCD vacuum fluctuations.
While the two points at $qa=0.20$ indicate that finite volume errors are small on this lattice, we also consider the additional angular cut, requiring that all points must lie within a cone of opening angle $20^\circ$ from the diagonal at the origin. The data surviving this cautiously conservative cut are illustrated by the solid points in Fig. 5. It is interesting to see that the turnover in $q^2a^2D(q^2)$ in the infrared region survives even this extreme cut on our larger lattice.

V. MODELING THE PROPAGATOR

The approach we have described here differs in several respects from that of [9]. In particular, the momentum vectors surviving our cuts are very different. In [9] only a small number of low-lying spatial momentum values are used along with all available time momentum values. In contrast, we have treated momenta in the spatial and time directions on an equal footing and selected momenta lying near the 4-dimensional diagonal, where lattice artifacts are expected to be minimal. In fact, nearly all the momenta used in [9] would be excluded by our cuts to the data.

In addition, we include the kinematical correction of Eq. (2.11). The authors of Ref. [9] do not include such a correction when performing their fits. Instead, their fits are constrained to the low-momentum region where such artifacts are hoped to be small.

We have considered a number of models for the propagator which have been suggested in the literature, as well as some simple analytical forms which are intended to capture the essence of the nonperturbative gluon propagator. A detailed analysis of these models is currently in progress [13].

The data for the fit are those obtained on the large lattice with the cylindrical cut. To balance the sensitivity of the fit over the available range of $qa$, we average adjacent lattice momenta lying within $\Delta qa<0.005$. In all fits the first point, at $qa\sim0.1$, is omitted as it may be sensitive to the finite volume of the lattice.

Our results so far indicate that the following analytical form:

$$D(q^2)=Z\left(\frac{A}{(q^2a^2)^{\alpha+\beta}(M^2)^{\gamma+\delta}}+\frac{1}{q^2a^2M^2}\right),$$

(5.1)

provides a satisfactory description of the data over a wide range of momenta. Our best fit to Eq. (5.1) is illustrated in Fig. 6. This fit yields $\chi^2/dof=3.5$, a somewhat high value. However, if the first four points are omitted, a more acceptable value of 1.6 is found.

We have studied the stability of the fits by varying the fitting range, and formula (5.1) turns out to give stable values for all the parameters over a wide region of fitting ranges. Our best estimates for the parameters, using all the available data, are

$$Z=1.214^{+5}_{-4.5}+70^{+9}_{-5.24} A=1.059^{+9}_{-4.5}+45^{+4}_{-2} \alpha=0.784^{+4}_{-1}+80^{+6}_{-20}$$

$$M=0.375^{+0.45}_{-0.2}+50^{+0.2}_{-2}+50^{+0.2}_{-2}$$.  

(5.2)
where the errors denote the uncertainties in the last digit(s) of the parameters. The first set of errors are statistical uncertainties in the fit to the full data set, and the second set of errors are due to fluctuations in the parameters as the fitting interval is varied. The estimate for $M$ corresponds to a physical value for the parameter in the region of 700 MeV.

VI. CONCLUSIONS

We have performed an accurate, nonperturbative study of the infrared behavior of the gluon propagator in the pure gauge theory on the lattice. We were able to isolate a set of data points for which systematic lattice errors seem to be negligible. Our data indicate an infrared behavior less singular than $1/q^2$. This can be inferred from our plots by noticing the clear turnover in the behavior of $q^2 a^2 D(q)$ around $q^2 = 1$ GeV$^2$. This is in agreement with suggestions from previous lattice results. In particular, there is evidence that the nonperturbative gluon propagator may be infrared finite. Our data appear to rule out models such as those proposed in [3], which lead to an infrared enhanced propagator.

Work in progress focuses on a detailed analysis of various analytic forms for the gluon propagator [13]. A stability analysis of the fit parameters is central to identifying the model best able to describe the gluon propagator in both the nonperturbative and the well known perturbative regimes. We are also exploring the possibility of extracting the value of $D(q=0)$ in the infinite volume limit, which one may be able to extrapolate from data at different volumes. A study of the effects of Gribov copies [14] and a complete analysis of the tensor structure of the gluon propagator are also issues under consideration.

Finally, one promising line of research appears to be the study of the gluon propagator using improved lattice actions [15]. These should yield a significant reduction of finite lattice spacing effects. Then, by performing simulations at larger values of the lattice spacing, one would be able to measure the propagator on larger physical volumes, thus gaining access to very low momentum values. For example, one may be able to use lattice spacings as large as 0.4 fm, so that a modest $16^4$ lattice would have a physical size of 6.4 fm.

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