A MATHEMATICAL MODEL
OF PLANT ROOT GROWTH

by

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CONTENTS

LIST OF TABLES AND FIGURES

LIST OF SYMBOLS AND THEIR UNITS

A	$\frac{q^*}{4\pi D C_0^*} = \frac{q}{4\pi D}$ [cm]
B	$\frac{V}{2D}$ [cm^{-1}]
C^*	Nutrient concentration [moles cm^{-3}]
C_0^*	Initial uniform value of C^*
C	$\frac{C^*}{C_0^*}$ [dimensionless]
D	Diffusivity [cm^2 sec^{-1}]
E	Fractional depletion of nutrient [dimensionless]
k	$\frac{M}{D}$ [cm^{-1}]. Constant of proportionality in boundary condition (equation (3.1))
L	Depletion of nutrient [moles cm^{-3}] (Chapter 2)
M	Uptake parameter [cm sec^{-1}]
$\underset{\sim}{n}$	Unit vector normal to root surface or root profile in direction of increasing C
Q	Strength of instantaneous sink [moles] (Chapter 2)
q^*	Strength of continuous sink [moles sec^{-1}]
q	$\frac{q^*}{C_0^*}$ [cm^3 sec^{-1}]
r	Polar distance [cm]
t	Time [sec]
V	Velocity of sink (in direction of negative z axis) [cm sec^{-1}]
x,y,z	Cartesian coordinates [cm]

α	Angle between isopycnal and circular arc
β	Angle between isopycnal and root profile
Γ	$C_r - kC$
Δ	$C_r^2 + \frac{C_\theta^2}{r^2} - k^2C^2$
ζ	$z + Vt$
θ	Polar angle, measured from $O\zeta$
λ	$\frac{4}{Ak}$ [dimensionless] (Chapter 4)
ρ	$\sqrt{x^2 + y^2 + \zeta^2}$

ISOPYCNAL: This term is used to describe a line or surface joining points of equal nutrient concentration. It corresponds to the isothermal of heat conduction. (It is taken from Physical Oceanography, where an isopycnal surface in the sea passes through points of equal salinity).

SUMMARY

In this thesis, the action of a growing plant root is represented by a point sink removing nutrient at a constant rate as it moves through a homogeneous soil at constant velocity. It is assumed that the nutrient diffuses through the soil towards the sink by a process of linear diffusion with constant diffusivity. In the region of varying nutrient concentration around such a moving sink, a "mathematical plant root" is defined by the condition that at each point of its surface, the rate of nutrient uptake is proportional to the nutrient concentration at the point.

This definition of a plant root is shown to lead to a first order differential equation, solutions of which have been found by numerical integration. It transpires that there are two bounding surfaces between which the root surface must lie, and that much of the root surface lies extremely close to one of these bounding surfaces.

This model of plant growth involves four parameters: the strength and velocity of the moving sink of nutrient, the diffusivity of the nutrient in the soil and the constant of proportionality in the uptake condition. Numerical values of these parameters have been provided by plant biologists.

Plant roots often consist of a mass of fine hairs, the envelope of which is referred to as the "root hair

envelope". Root surfaces have been obtained for various parameter combinations, and some of the results obtained do have shapes strongly suggestive of a root hair envelope.

The nutrient distribution around the moving sink is quasi steady for large time, and most of the numerical results described are concerned with the corresponding quasi steady root surfaces. There is also some discussion of the growth with time of the root surfaces associated with both stationary and moving sinks of nutrient.

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University, and, to the best of my knowledge and belief, contains no material previously published or written by another person, except when due reference is made in the text of the thesis.

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A C K N O W L E D G E M E N T

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