University of Adelaide
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Discrete Morse Theory and L^2 Homology

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Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

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Abstract

A brief overview of Forman's discrete Morse theory is presented, from which analogues of the main results of classical Morse theory can be derived for combinatorial Morse functions, these being functions mapping the set of cells of a CW complex to the real numbers satisfying some combinatorial relations. The discrete analogue of the strong Morse inequality was proved by Forman for finite CW complexes using a Witten deformation technique.

This deformation argument is adapted to provide strong Morse inequalities for infinite CW complexes which have a finite cellular domain under the free cellular action of a discrete group. The inequalities derived are analogous of the L^2 Morse inequalities of Novikov and Shubin ([21]), and the asymptotic L^2 Morse inequalities of an inexact Morse 1-form as derived by Mathai and Shubin in [18].

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