The University of Adelaide

NUMERICAL SOLUTION OF PRANDTL'S LIFTING-LINE EQUATION

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This thesis is dedicated to
my mother and my late father,
whom I love very much.

The fear of the Lord is the beginning of knowledge :
but fools despise wisdom and instruction.

For the Lord giveth wisdom :
out of his mouth cometh knowledge and understanding.

Proverbs 1:7 ; 2:6

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Summary

Prandtl's lifting-line equation is an integro-differential equation which is used for calculating the span-wise distribution of circulation around wings which are three-dimensional quasi-planar, with negligible sweep and large aspect ratio, placed at a small angle of attack to a stream of incompressible fluid. Since Prandtl introduced this famous equation, there have been many who have tried to solve the equation. Stewartson (1960) has solved the equation analytically for a semi-infinite wing of constant chord, the final result being a function involving a double integral. To calculate the value of this function numerically, we need a computer routine which is, in general, very time consuming. In this thesis, we shall develop an efficient numerical method for solving Prandtl's lifting-line equation directly, using a non-uniform grid which is concentrated at the wing-tip. The results agree to nearly three significant figures with computation, based on Stewartson's double integral, for the case of semi-infinite wings of constant chord. Our method is useful for semi-infinite wings in a more general case, not only of constant chord. The same equation but with negative constant chord, which applies to a non-aerodynamic problem, e.g. a compliant layer problem, is also studied. However, our method is less accurate for this problem, because wave-like behaviour occurs.

Signed Statement

The contents of this thesis have not been submitted to any university for the purpose of obtaining any other degree or diploma. Also, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Boedi Koerniawan

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