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Si Tran Nguyen Nguyen, Jinzhe Gong, Martin F. Lambert, Aaron C. Zecchin, Angus R. Simpson

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# Least squares deconvolution for leak detection with a pseudo random binary sequence excitation

Si Tran Nguyen Nguyen, Jinzhe Gong, Martin F. Lambert, Aaron C. Zecchin, Angus R. Simpson

*School of Civil, Environmental and Mining Engineering, University of Adelaide, SA 5005, Australia*

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## Abstract

Leak detection and localisation is critical for water distribution system pipelines. This paper examines the use of the time-domain impulse response function (IRF) for leak detection and localisation in a pressurised water pipeline with a pseudo random binary sequence (PRBS) signal excitation. Compared to the conventional step wave generated using a single fast operation of a valve closure, a PRBS signal offers advantageous correlation properties, in that the signal has very low autocorrelation for lags different from zero and low cross correlation with other signals including noise and other interference. These properties result in a significant improvement in the IRF signal to noise ratio (SNR), leading to more accurate leak localisation. In this paper, the estimation of the system IRF is formulated as an optimisation problem in which the  $l_2$  norm of the IRF is minimised to suppress the impact of noise and interference sources. Both numerical and experimental data are used to verify the proposed technique. The resultant estimated IRF provides not only accurate leak location estimation, but also good sensitivity to small leak sizes due to the improved SNR.

*Keywords:* Leak detection, linear system deconvolution, pseudo random binary sequence excitation, PRBS, water pipeline, hydraulic transient,

1 **1. Introduction**

2       Underground water distribution pipeline systems represent critical in-  
3 frastructure for modern cities. The maintenance of this infrastructure poses  
4 a major challenge as the pipelines are often buried underground. There has  
5 been growing attention to address this and many techniques for pipeline de-  
6 fect detection and condition assessment have been proposed [1, 2]. Typical  
7 defects of an aging pipeline include the leaks, blockages and internal and/or  
8 external corrosion in the pipes.

9       Leakage in water distribution systems can lead to significant economic  
10 cost due to water loss and associated additional energy consumption [3]. In  
11 addition, potential health risks to users due to pathogen intrusion during low  
12 pressure events [4]. Many leak detection techniques for water pipe systems  
13 have been developed and a selective literature review of leak detection tech-  
14 niques is presented in [5]. Hydraulic transient-based methods are relatively  
15 new techniques for leak detection and condition assessment of water pipeline  
16 systems. Since the original paper by Liggett et al. [6], researchers have con-  
17 tinued to examine the interaction of transient pressure waves with leaks and  
18 blockages. The developed approaches can be sub-divided into time-domain-  
19 based, the frequency-domain-based techniques and time-frequency domain  
20 based techniques.

21       The time-domain techniques for leak detection mainly include time-  
22 domain reflectometry (TDR) techniques, impulse response function (IRF)-  
23 based methods, and inverse transient analysis (ITA) methods. Silva et al.  
24 [7] discussed TDR techniques for leak detection in which a pulse time delay

25 was measured to compute the location of leak in the pipe. Later, Brunone  
26 and Ferrante [8], presented a method to detect a leak using a step transient  
27 pressure wave. The IRF-based methods were discussed in [9, 10], assuming  
28 the water pipe system was a linear time invariant (LTI) system. Thus, the  
29 measured output is the convolution of the input and the system impulse  
30 response. Given the measured input and output, one can estimate the IRF  
31 of a LTI pipe system, from which the leak locations can then be determined.  
32 With the ITA for leak detection [11, 12, 13, 14], the pressure responses at  
33 one or more locations are recorded during a transient event. A numerical  
34 pipeline model (with one or multiple leaks) is then iteratively calibrated  
35 to match the numerical pressure responses with the measured results. The  
36 success of ITA-based leak detection heavily relies on an accurate forward  
37 simulation, which is challenging for real pipes due to pipeline parameter  
38 uncertainties [13] and [15].

39 In the frequency-domain, the location and size of a leak can be inferred  
40 from the pipeline system frequency response diagram (FRD). The existence  
41 of a leak will introduce a sinusoidal pattern in the resonant or the anti-  
42 resonant peaks in an FRD, depending on the boundary condition of the  
43 pipeline system [16]. Early studies of FRD-based leak detection are re-  
44 ported in [17, 18]. Covas et al. [19] and Lee et al. [20] used the leak-induced  
45 sinusoidal pattern in the resonant peaks to gain an understanding of the  
46 system while Sattar et al. [21] proposed to use the leak-induced pattern in  
47 anti-resonant responses. Recently, Gong et al. [22] proposed a FRD-based  
48 leak detection technique that only uses the first three resonant peaks. How-  
49 ever, accurate extraction of the FRD of a pipeline is challenging, especially  
50 when the pipe is embedded in a complex network. Zecchin et al. [23] used a  
51 frequency-domain approach for estimation of pipe network parameters using

52 the maximum likelihood estimator. This was extended in [24] to the use of  
53 the expectation maximisation algorithm to deal with unmeasured boundary  
54 conditions.

55 Early research on leak detection based on Cepstrum has been shown  
56 to effectively locate a leak in a network [25, 26]. In addition, a wavelet  
57 transform was also used for leak detection research and reported in [27, 18,  
58 28, 29, 29, 30]. In [31], empirical mode decomposition was used for leak  
59 detection using transient step excitation to the system, and the extracted  
60 feature was then mapped to the time-domain to localise the leak. These  
61 techniques can be classified as the time-frequency based approach.

62 When compared to the FRD-based techniques, in which the system is  
63 typically required to enter a steady oscillatory condition for the extraction  
64 of the response of the whole system [32], the time-domain IRF technique  
65 relies on the analysis of only the primary wave reflections from leaks for  
66 detection and localisation. The application of some FRD-based leak de-  
67 tection methods is restricted to simple pipe system configurations, such as  
68 a reservoir-pipeline-reservoir or reservoir-pipeline-valve system, because the  
69 FRD of a more complex system is difficult to interpret and derived rela-  
70 tionships for the simple system will break down. The restriction on system  
71 complexity can be relaxed if the time-domain IRF method is to be used,  
72 since the particular segment of signal that contains the primary leak reflec-  
73 tions can be extracted for independent analysis, while the wave reflections  
74 due to boundaries and network connections can be truncated. Hence, the  
75 time-domain IRF approach is preferable to be used for leak detection for  
76 specific pipe sections in a complex network.

77 The time-domain leak detection using IRF provides a straightforward in-  
78 terpretation of where the anomalies (e.g. leaks or blockages) are located. De-

79 spite the merits, IRF-based leak detection techniques have not been widely  
80 adopted in the field. In early works (e.g., [33]), a typical simple step wave  
81 was generated as the excitation signal using a fast valve closure operation.  
82 However, the step signal is not robust to system noise and other interference  
83 sources, which can lead to high false alarm rates [34].

84 The use of pseudo random binary sequence (PRBS) for IRF estimation  
85 and then leak detection was first discussed in [9]. However, only numerical  
86 experimentation was used for verification and the technique used for the esti-  
87 mation of the IRF was not robust against the noise and interference sources:  
88 the IRF was determined based on the division of the output signal by the  
89 input in the frequency domain followed by an inverse Fourier transform,  
90 referred to as the spectral division technique. This approach whilst simple  
91 to implement, can potentially amplify the small noisy components at cer-  
92 tain frequencies in the denominator, especially for PRBS excitation signal,  
93 leading to an increase in the noise level in the IRF estimate and potential  
94 false detections. Further discussion of the PRBS spectral characteristic is  
95 discussed in section 2 illustrating the zero power frequencies of the PRBS  
96 at the clock frequency and its harmonics.

97 In the current research, a specific type of PRBS signal, the Inverse Re-  
98 peat Pseudo Random Binary Sequence (IRS), is used as the excitation. This  
99 type of signal is very robust to system noise and other interference sources  
100 due to its correlation property. Furthermore, the IRS is antisymmetric in  
101 each period, which helps to reduce the effect of nonlinear system responses  
102 to the determination of linear system response functions [35]. The IRF es-  
103 timate proposed in this paper is performed in the time domain by solving  
104 a least squares deconvolution problem. An optimisation problem is formu-  
105 lated seeking to minimise the least squares error and the  $l_2$  norm of the IRF

106 to allow suppression of the impact of noise and interference sources. The  
107 objective function is convex, leading to a unique solution that is indepen-  
108 dent of initialisations and a closed-form solution can be obtained. Numerical  
109 and experimental verifications are presented in this paper to illustrate the  
110 robustness and effectiveness of the proposed method against various sources  
111 of interference. A comparison of the new method is made with the conven-  
112 tional Cepstrum method [25] using both numerical and laboratory data.

## 113 **2. Pseudo Random Binary Sequence excitation and problem for-** 114 **mulation**

The PRBS signal is commonly used in the electrical and electronics fields for identifying an electrical systems properties [36]. The signal offers greater robustness against the effect of noise in the system compared to step excitations as commonly used in both electrical [37, 38] and hydraulics [22] applications. Figure 1 illustrates the (a) time and (b) frequency responses of the IRS (a specific type of PRBS) [36] which will be used throughout in this paper. The IRS signal used throughout in this paper is designed to have one period of 20.46 s and a 3 dB bandwidth of 50 Hz. The pressure signal generation is described in [32] by the movement of two solenoids which control the valve opening and closing. The solenoid movement is measured by a linear voltage displacement transducer (LVDT) during the signal generation, which is used to approximate the input signal to the system. Figure 2 illustrates an example of anomaly identification in pipelines based on transient analysis in the time domain. A reservoir-pipeline-valve system configuration is considered in Fig. 2 as it facilitates the numerical simulations to be discussed. An excitation pressure signal similar to that in

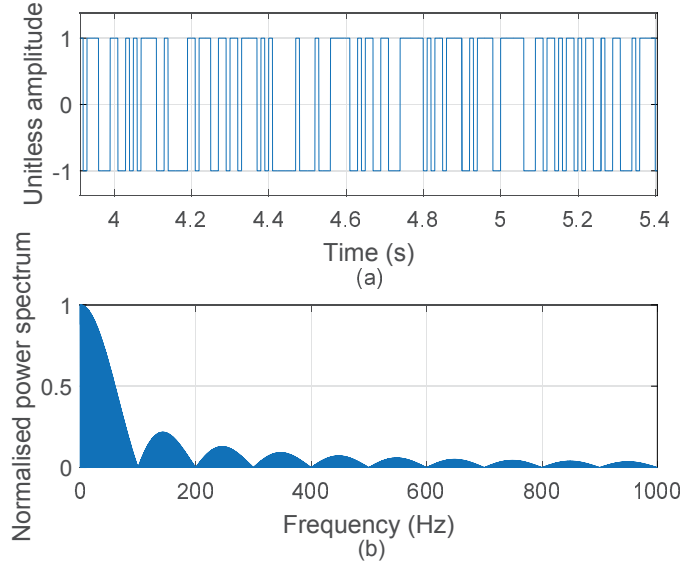


Figure 1: The time domain of the IRS (a) and corresponding frequency spectrum (b). One period of the IRS corresponds to 20.46 s.

Fig. 1(a), denoted as  $s(t)$ , is generated as an input to the water pipeline system (Fig. 2), which is an approximation of the IRS valve movement pattern. For simplicity, it is assumed that signal dissipation and dispersion are negligible when the transient signal  $s(t)$  travels in the pipeline towards the anomalies at distances  $d_1, d_2, \dots, d_N$  from the point of signal generation and measurement. As discussed, the anomalies within the pipe will cause reflections of the excitation wave  $s(t)$ , which will be measured by a pressure sensor positioned at the point of signal generation. The received signal at the sensor, denoted as  $r(t)$ , is the superposition of the excitation signal and the reflected signals caused by anomalies, which can be written as

$$r(t) = s(t) + \sum_{i=1, \dots, N} R_i s(t - \tau_i), \quad (1)$$

115 for  $R_i$  representing the reflection coefficient, indicating the ratio between



116 energy of the reflected wave at the anomalies and the incident wave's,  $\tau_i$   
 117 being the time taken for the transient pressure to travel to the anomalies  
 118 and back to be measured by the sensor;  $\tau_i = 2d_i/a$  where  $a$  is the transient  
 119 pressure wave speed. Note that higher order reflections are neglected (i.e.  
 120 reflections resulting from an already reflected wave), as they are typically  
 121 significantly smaller in magnitude, with reflection coefficients on the order  
 122 of  $R^3, R^5$ , etc., where  $R < 0.2$  are typical for pipeline anomaly detection  
 123 applications. The problem of detection and localisation of the anomalies in  
 124 the pipe is equivalent to estimating the reflection coefficient  $R_i$  and the time  
 125 delay  $\tau_i$  of the equation (1), whose concept is similar to the conventional  
 126 time-domain reflectometry based method [39].

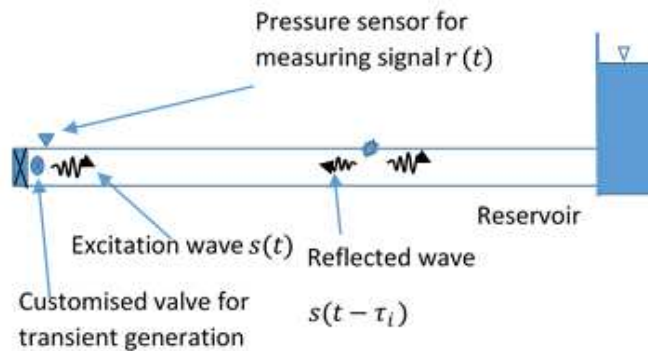


Figure 2: Transient-based strategy for anomaly identification in a pipeline.

### 127 3. Least squares deconvolution for water pipeline system identifi- 128 cation

A pipeline system can be considered as an approximate Linear Time Invariant (LTI) system for an appropriate magnitude level of transient excitation [22]. Therefore, if the input to the system is denoted as  $x(t)$  and

the output as  $y(t)$ , the relationship between the input and output signals is given by

$$y(t) = \int_{-\infty}^{+\infty} h(\tau)x(t - \tau)d\tau \quad (2)$$

129 where  $h(t)$  is the impulse response function (IRF) of the system - describ-  
130 ing the dynamic properties of the system in consideration. Eq. (2) rep-  
131 resents the convolution operation of the two signals  $h(t)$  and  $x(t)$  in the  
132 time domain. In the frequency domain, this relationship can be expressed  
133 as  $Y(f) = H(f)X(f)$ , for signals  $X(f)$ ,  $H(f)$ , and  $Y(f)$  representing the  
134 Fourier transforms of the time domain signal  $x(t)$ ,  $h(t)$ , and  $y(t)$ , respec-  
135 tively.

To compute a system transfer function in the frequency domain given the input and output signals, conventionally a simple division operation is applied, given by

$$H(f) = Y(f)/X(f) \quad (3)$$

136 for  $H(f)$  being the system transfer function, which is the Fourier transform  
137 of the IRF  $h(t)$ . An issue can be found using this spectral division approach  
138 if the input signal spectrum contains zero frequency components such as the  
139 IRS signal in Fig. 1(b). Thus, division by this signal in frequency domain  
140 will cause the inversion of zero value at certain frequencies. This is caused  
141 by the line spectrum characteristic of IRS, whose spectrum only has energy  
142 at some specific frequencies [examples of these zero energy frequencies can  
143 be observed at 100, 200 Hz, etc. (the clock frequency and its harmonics), in  
144 Fig. 1(b)].

To address this issue, a time-domain technique is proposed. Consider the discretised version of the signals in the convolution operation in Eq. (2),

in the discrete time domain it can be expressed as

$$y[n] = \sum_{k=1}^N h[k]x[n - k + 1]. \quad (4)$$

The expression in Eq. (4) is a linear operation which can be represented by a matrix multiplication, formulated as

$$\mathbf{y} = \mathbf{X}\mathbf{h}. \quad (5)$$

The signal  $\mathbf{y}$  is a  $N \times 1$  column vector representing the output signal  $y[n]$ ,  $\mathbf{X}$  being the  $N \times N$  convolution matrix constructed using the input signal  $x[n]$  and  $\mathbf{h}$  being the  $N \times 1$  column vector representing the impulse response function in discrete time domain. The convolution matrix  $\mathbf{X}$  and column vector  $\mathbf{y}, \mathbf{h}$  are constructed as follow:

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} \mathbf{x}[1] & 0 & 0 & \cdots & 0 \\ \mathbf{x}[2] & \mathbf{x}[1] & 0 & \cdots & 0 \\ \mathbf{x}[3] & \mathbf{x}[2] & \mathbf{x}[1] & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}[N] & \mathbf{x}[N-1] & \mathbf{x}[N-2] & \cdots & \mathbf{x}[1] \end{bmatrix} \\ \mathbf{y} &= [\mathbf{y}[1] \ \mathbf{y}[2] \ \mathbf{y}[3] \ \cdots \ \mathbf{y}[N]]^T \\ \mathbf{h} &= [\mathbf{h}[1] \ \mathbf{h}[2] \ \mathbf{h}[3] \ \cdots \ \mathbf{h}[N]]^T \end{aligned} \quad (6)$$

where superscript  $T$  represents the transpose operation. The impulse response  $\mathbf{h}$  will be computed by solving a linear equation system Eq. (5), given by

$$\mathbf{h} = \mathbf{X}^{-1}\mathbf{y}, \quad (7)$$

<sup>145</sup> for an invertible matrix  $\mathbf{X}$ , which is not the case in all scenarios.

To reduce the effect of noise sources in the pipeline system, a least squares optimisation for the IRF estimation is proposed to estimate the IRF  $\mathbf{h}$  given the input matrix  $\mathbf{X}$  and output vector  $\mathbf{y}$ , whose objective function is formulated as

$$f(\mathbf{h}) = \|\mathbf{y} - \mathbf{X}\mathbf{h}\|_2^2 + \lambda\|\mathbf{h}\|_2^2 \quad (8)$$

where  $\mathbf{h}, \mathbf{X}, \mathbf{y}$  represent the impulse response function, input matrix and output of the system, defined as in Eq. (6);  $\lambda$  determines the weighting ratio between the two terms in Eq. (8). Minimising  $f(\mathbf{h})$  in Eq. (8) effectively minimises the energy of the impulse response,  $\|\mathbf{h}\|_2^2$ , and simultaneously minimising the least squares error term, defined by  $\|\mathbf{y} - \mathbf{X}\mathbf{h}\|_2^2$ . Increasing the value  $\lambda$  effectively increases the weighting on regularisation  $\|\mathbf{h}\|_2^2$ , which suppresses the interference energy at the resultant IRF. This comes at the expense of a higher least squares error defined by  $\|\mathbf{y} - \mathbf{X}\mathbf{h}\|_2^2$ . If  $\lambda = 0$ , the problem becomes a conventional deconvolution problem as similar to that in Eq. (7). The second term of Eq. (8)  $\|\mathbf{h}\|_2^2$  should only have energy at the time points where the anomalies reflections occur which is usually small since the reflected energy is small compared to the incident wave. Rewriting the function in Eq. (8) gives us:

$$f(\mathbf{h}) = (\mathbf{y} - \mathbf{X}\mathbf{h})^T(\mathbf{y} - \mathbf{X}\mathbf{h}) + \lambda\mathbf{h}^T\mathbf{h} \quad (9)$$

The first derivative  $df/d\mathbf{h}$  is given as

$$df/d\mathbf{h} = -2\mathbf{y}^T\mathbf{X} + 2\mathbf{h}^T\mathbf{X}^T\mathbf{X} + 2\lambda\mathbf{h}^T \quad (10)$$

Equating  $df/d\mathbf{h} = 0$  yields

$$\mathbf{h} = (\mathbf{X}^T\mathbf{X} + \lambda\mathbf{I})^{-1}\mathbf{X}^T\mathbf{y} \quad (11)$$

146 The matrix  $\mathbf{I}$  represents the identity matrix. The expression in (11) gives  
 147 the closed form solution for the optimum of objective function defined in (8),  
 148 given the matrix  $\mathbf{X}$ , output  $\mathbf{y}$  and  $\lambda$ . It should be noted that this approach  
 149 can be used for non-invertible input matrix  $\mathbf{X}$ , in which case the solution  
 150 of the IRF cannot be obtained using Eq. (7). For example, quite often the  
 151 time interval of interest of the IRF (in the order of seconds, equivalent to the  
 152 time taken to for the wave to travel to the boundary and back and sensor)  
 153 is much smaller than the IRS period (20.46 s), thus the size of the vector  $\mathbf{h}$   
 154 is much smaller than that of  $\mathbf{y}$ . This will result in the convolution matrix  
 155  $\mathbf{X}$  being a non-square matrix and its inverse  $\mathbf{X}^{-1}$  does not exist.

#### 156 4. Experimental set-up and numerical verification

157 The following experimental configuration is used to test the performance  
 158 of the proposed approach.

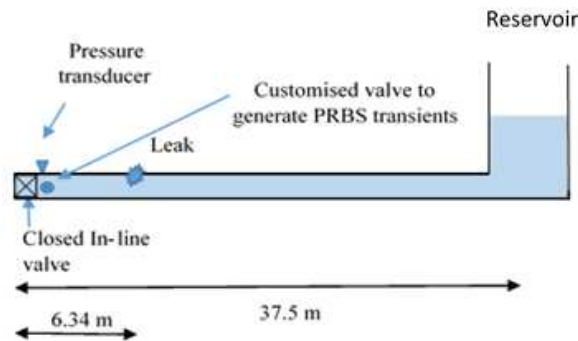


Figure 3: The experimental configuration of a valve-leak-reservoir system. The transient generator is located near the valve next to the pressure transducer, the leak is at 6.34 metres from the valve and the reservoir is at 37.5 metres.

159 The pipe system in Fig. 3 is located at the Robin Hydraulics Laboratory

160 at the University of Adelaide. It is bounded by a closed in-line valve at  
 161 one end and the reservoir at the other end with the pressure head of 38.5  
 162 metres. The pipe is made of copper with an internal diameter of 22.14  
 163 mm. A customised valve is connected to the pipe for IRS transient signals  
 164 to be generated, located 145 mm from the closed in-line valve. The head  
 165 response of the system is measured by a pressure transducer (Druck PDCR  
 166 810, Leicester, UK) mounted on the main pipe with a sampling rate of  
 167 5000 samples per second. A small orifice is used to simulate a leak in the  
 168 pipe. As discussed in [32], the IRS signal is generated by the movement of  
 169 two solenoids controlling the valves opening and closing and the solenoid  
 170 movement. The movement is measured by a linear voltage displacement  
 171 transducer (LVDT) to estimate the input signal to the system.

172 The configuration used in Fig. 3 is considered to avoid the issue of  
 173 directional wave propagation, referred to as the ambiguity issue of the  
 174 reflected waves from the anomalies arriving at the sensor from multiple  
 175 paths/directions. For such issue, pure time delay information cannot re-  
 176 solve the ambiguity. Whilst there exists techniques to resolve such issue  
 177 [40], it is beyond the scope of this paper.

178 The following steps are taken for leak detection using the IRS excitation  
 179 signal and least squares approach:

- 180 • Initialise the start time  $t_0$ . The IRS period in second given by  $T_P =$   
 181 20.46 s and the sampling frequency is given by  $F_s = 5000$  Hz. The  
 182 corresponding samples per period is given by  $[T_P F_s]$ ;  $[.]$  represents the  
 183 integer rounding operation.
- Let  $T[n], R[n]$  be the discrete time signals representing the solenoids  
 manoeuver and the measured head pressure at the transducer, respec-

tively, the input  $x[n]$  and output  $y[n]$  to the LTI system in consideration is given by:

$$\begin{cases} x[n] = T[n + n_0] \\ y[n] = R[n + n_0], \end{cases}$$

184 for  $n = 1, \dots, N_p; n_0 = [t_0 F_s]$ .

- 185 • The discrete signals  $x[n]$ ,  $y[n]$  will be used to construct the matrix
- 186  $\mathbf{X}$  and column vector  $\mathbf{y}$  as in (6), which will be then fed into the
- 187 optimisation problem defined in (8), the optimum is given by (11).

#### 188 4.1. Numerical verification

Using the method of characteristics (MOC) [41], numerical data is generated for the configuration in Fig. 3. The MOC, discussed in [41] offers a step-by-step method to solve the partial differential equations describing the relationship between the pressure head  $H$  and flow  $Q$  at a given time  $t$  and location  $x$ . The governing equations are given as:

$$\frac{\partial Q}{\partial x} + \frac{gA}{a^2} \frac{\partial H}{\partial t} = 0 \quad (12)$$

$$\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{fQ|Q|}{2gDA^2} = 0, \quad (13)$$

189 where  $a$  is the wave speed,  $A$  is the pipe cross-sectional area at the location

190  $x$  in consideration;  $D$  is the pipe diameter,  $f$  is the Darcy-Weisbach friction

191 factor. The MOC solves the differential equations (12), (13) in discrete the

192 time domain which gives a good approximation of the solutions, given a

193 sufficiently small time step along the characteristic lines [41].

194 The pressure head at the transducer of the configuration in Fig. 3 is

195 generated numerically as the customised valve's movement is controlled to

196 follow an IRS pattern. The associated change in flow/pressure following the

197 equations (12), (13) at the valve will result in an excitation wave (input to  
198 the system), which will approximate an IRS signal. The copper pipeline has  
199 internal diameter of 22.14 mm, an estimated wave speed of 1321 m/s and a  
200 simulated leak with diameter of 2 mm.

201 The input to the system is the approximation of the IRS signal as shown  
202 in Fig. 1. It should be noted the current version of the IRS generator  
203 used in the laboratory as described in [32] continuously generates a new  
204 period of IRS signal after it finishes the previous period. For consistency,  
205 the numerical data is designed to reflect this behaviour to be compared with  
206 the real data result in the next section.

207 One of the key challenges for this system is the long PRBS period of 20.46  
208 s with respect to the short pipeline in consideration for the given nominal  
209 wave speed of 1321 m/s (with a pipeline return time of 56 ms). Therefore,  
210 the output signal of one period long of data measured by the transducer will  
211 consist of the superposition of multiple components including the incident  
212 wave actively generated by the PRBS generator, and multiple reflections  
213 from the leak, reservoir, closed in-line valve, secondary reflections from leak  
214 and reservoir, etc. These multiple reflected waves will interfere with each  
215 other since they are not well separated in time. Setting the start time  $t_0$   
216 to be 5 s, Fig. 4 illustrates the normalised IRF estimate for the numerical  
217 experiment using the least squares approach. It should be noted that the  
218 illustrated IRF in Fig. 4 is normalised by the magnitude of the first sample,  
219 which indicates the incident wave.

220 In Fig. 4(a), the estimated IRF illustrates the reflections from the leak  
221 at 0.0098 s, and from the water reservoir at 0.057 s. Therefore, the leak  
222 location found will be  $(0.0098/0.057) \times 37.5 = 6.44$  metres from the in-line  
223 valve which is approximately the location of the known leak location of 6.34



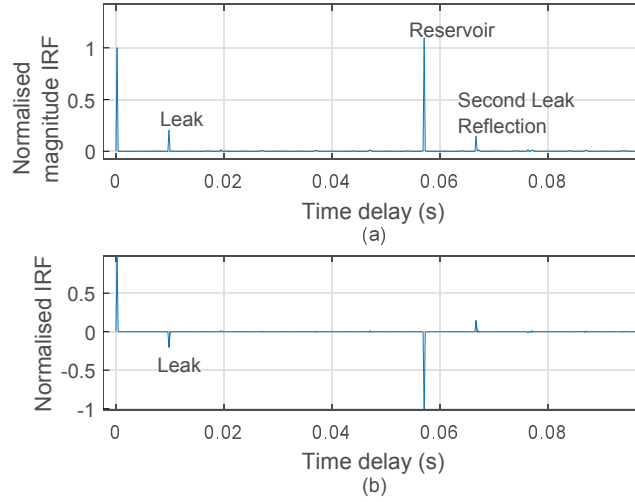


Figure 4: The normalised magnitude of the estimated IRF (a) and the normalised IRF including the signs at the reflections (b) from numerical data. The time delays of the reflections are found to be 0.0098 s (corresponding to the leak) and 0.057 s (corresponding to the pressurised reservoir).

224 metres (see Fig. 3). The error is due to the low sampling frequency of 5000  
 225 Hz, chosen to be consistent with experimental data to be presented in the  
 226 laboratory verification section.

227 In Fig. 4, the normalised magnitude plot is presented in Fig. 4(a)  
 228 to show the timings (location) that have energy, caused by the incidental  
 229 and reflected waves. The IRF result is from a numerical experiment without  
 230 interference, hence the IRF normalised magnitude plot shows no interference  
 231 from the noise component, i.e., the energy at the other times different to the  
 232 expected transient events is negligible. The bottom panel shows the actual  
 233 sign of the IRF; the negative sign reflection in Fig. 4(b) indicates a reduction  
 234 in transient pressure at the considered location, suggesting a leak instead  
 235 of a blockage which would have an increase in transient pressure and hence

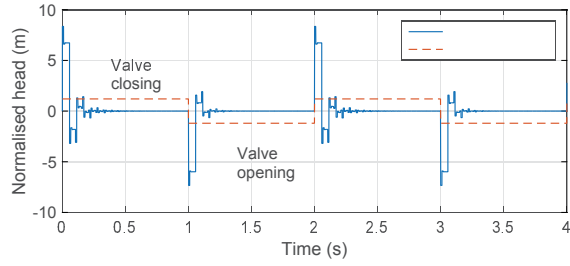
236 positive IRF reflection. The second leak reflection can be observed as the  
237 transient wave reflects from the leak for the second time, the time difference  
238 between these two leak reflections is approximately the time for the wave to  
239 travel to the pressurised reservoir and back to the pressure transducer.

#### 240 4.2. Comparison with the Cepstrum method

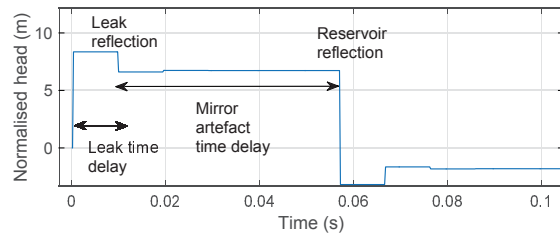
241 The MOC is again used to generate pressure head data at the sensor  
242 in the configuration 3 with the excitation signal similar to the Cepstrum  
243 method discussed in [25]. The normalised head is illustrated in Fig. 5 in  
244 blue solid trace whilst the solenoid valve movement is shown by the dashed  
245 trace.

246 If the normalised pressure in Fig. 5 is denoted as  $\mathbf{x}$ , the Cepstrum of  
247  $\mathbf{x}$  is given by  $DFT\{\log\{DFT\{\mathbf{x}\}\}\}$ ; where  $DFT\{.\}$  represents the discrete  
248 Fourier transform operation,  $\log\{.\}$  represents the computation of natural  
249 logarithm operation. The Cepstrum of the normalised head pressure in Fig.  
250 5(a) is illustrated in Fig. 5(c).

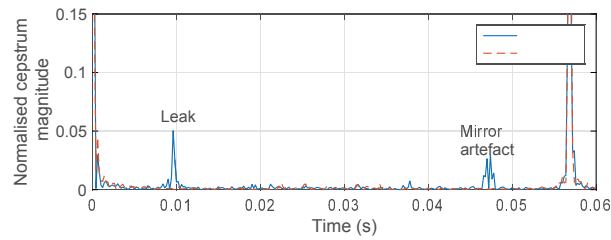
251 Figure 5(c) illustrates the Cepstrum of the normalised signal in Fig. 5.  
252 It should be noted that whilst a very good SNR can be achieved at the  
253 leak (at time 0.0096 s), there exists a strong artefact at 0.0472 s. It can  
254 be found that this artefact is related to the leak time as  $0.0472 = 0.0568 -$   
255  $.0096$ , where 0.0096, 0.0568 s are the leak time delay and the reservoir delay,  
256 respectively. Consider the zoomed in plot of the normalised pressure signal  
257  $\mathbf{x}$  in Fig. 5(b). It should be noted Cepstrum of the time domain signal  $\mathbf{x}$   
258 searches for the regularity in the signal, i.e., the regular change in pressure  
259 in the time domain trace. In Fig. 5(b), it can be observed that at least  
260 three significant components in Cepstrum domain are expected (or time  
261 intervals in the original time domain) including: 1) the leak time delay (the



(a) Time-domain trace



(b) Zoomed-in signal



(c) Cepstrum

Figure 5: (a) Normalised head numerically generated using the MOC technique of the configuration in Fig. 3 (blue solid trace) and the corresponding movement of the solenoid valve (red dashed trace); (b) the zoomed-in signal of that in (a) and (c) Cepstrum; the blue solid trace represents the Cepstrum with a leak whilst red dashed trace represents one without a leak.

262 time interval between the rise/drop in pressure due to the solenoid valve  
 263 closing/opening and the drop/rise in pressure due to the leak), 2) the time

264 interval between the leak reflection and the reservoir reflection and 3) the  
265 time interval between the solenoid valve closing/opening and the change in  
266 pressure due to the reservoir. It should be noted that a numerical experiment  
267 has been made to move the leak location to different position along the  
268 pipeline and found that the artefact location has also moved accordingly.

269 The mirror artefact observed in Fig. 5(c) corresponds to the time interval  
270 between the change in pressure due to the leak and the change in pressure  
271 due to the water reservoir, which is dependent on the location of the leak  
272 (unknown) and the location of the reservoir (known). This artefact is em-  
273 bedded in the time domain signal, however, not corresponding to a wave  
274 reflection due to any anomaly. Further verification regarding this artefact  
275 is made using the laboratory test in the next section (a similar artefact at  
276 approximately the same time can be observed). The proposed least squares  
277 deconvolution seeks the location of the reflected wave, thus suppressing the  
278 effect of this mirror artefact.

## 279 **5. Laboratory verification**

280 Laboratory data is used to test the proposed algorithm. The experiments  
281 were conducted in the Robin Hydraulics Laboratory, University of Adelaide,  
282 Australia for the configuration illustrated in Fig. 3. All the experiments  
283 conducted lasted for 10 minutes, with the first few seconds of data measured  
284 under the steady state condition (with the side discharge valve open). The  
285 transient event is then started by triggering the IRS excitation.

### 286 *5.1. Verification of the proposed method using laboratory data*

287 Laboratory tests for IRS excitation signal with a discharge orifice size  
288 of 2 mm, 1 mm and an irregular orifice to simulate the leak (irregular leaks

289 were simulated using a partially-opened side discharge valve). Discussion  
290 of the effect of irregular orifice to the system can also be found in [8, 42].  
291 Multiple tests on different days were also considered for the verification. The  
292 IRS reference signal, measured by the LVDT (measuring the dynamic valve  
293 opening), and the pressure head, measured by the transducer mounted on  
294 the main pipe are illustrated in Fig. 6. In this experiment the reference  
295 signal measured by the LVDT is used as an approximation of the system  
296 input signal and was used to construct the input matrix  $\mathbf{X}$  described in Eq.  
297 (6). Similarly, the output pressure measured by the transducer is heavily  
298 dependent on the device sensitivity to the frequency band of interest, its  
299 dynamic range and other practical instrumentation issues [43]. The pressure  
300 measurement at the transducer is used to construct the output column vector  
301  $\mathbf{y}$ . The IRF result, as determined by the proposed least squares approach,  
302 is shown in Fig. 7.

303 In Fig. 7, the leak location can be clearly observed at approximately  
304 0.0094 s together with those representing the incident wave at the beginning,  
305 the reflected wave caused by the reservoir at 0.0566 s and the second leak  
306 reflection at 0.066 s. The experimental IRF response is very similar to that  
307 of the numerical result suggesting the robustness against the aforementioned  
308 practical issues in application to a real pipe system using this method. The  
309 leak location can be computed as  $0.0094/T \times L = 6.23$  metres compared to  
310 the actual location of 6.34 metres, for  $L$  being the pipe length which is equal  
311 to 37.5 m,  $T$  being the time taken for the transient wave to travel two pipe  
312 lengths from the signal generator to the reservoir and back to the transducer,  
313 given as 0.0566 s [refer to Fig. 7(a) reflection time of pressurised reservoir].  
314 The sampling frequency for data acquisition for this experiment is 5000  
315 Hz. Thus for the wave speed computed as  $2L/T = 1325$  m/s, each sample

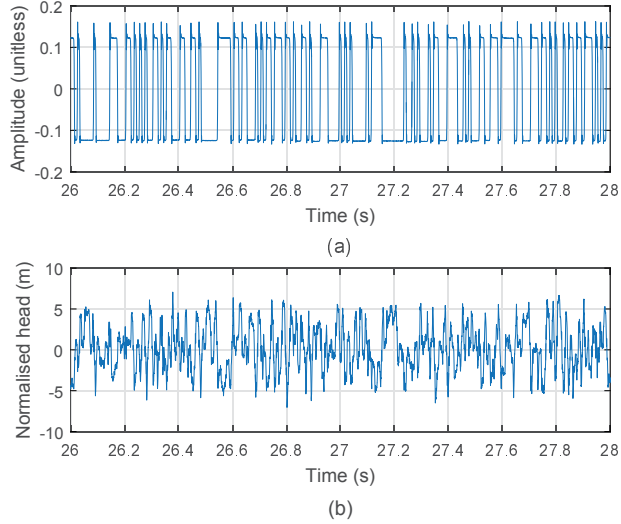


Figure 6: (a) The dimensionless IRS valve perturbation and (b) the pressure head measured by the pressure transducer (in a normalised form).

316 corresponds to a distance of  $1325/5000 = 0.26$  metres. This effectively  
 317 means that the leak location error computed by the proposed method is  
 318 due to the low sampling frequency and not the algorithm error, since the  
 319 next time sample of the IRF from the computed leak time corresponds to  
 320  $6.23 + 0.26 = 6.49$  metres which is greater than the actual leak location of  
 321 6.34 metres.

322 The leak locations computed based on least squares IRF estimation for  
 323 different datasets are given in Table 1 for  $T_0, T$  representing the timings  
 324 of the leak and reservoir reflections, respectively. The estimation error is  
 325 computed as  $|L_{est} - L_{actual}|/L$  for  $L_{est}, L_{actual}$  being the estimated and actual  
 326 distance from the in-line valve (transducer) to the leak, respectively. As  
 327 seen by the low error in Table 1, the proposed least squares IRF estimation  
 328 for IRS excitation has been shown effectively and accurately localise the

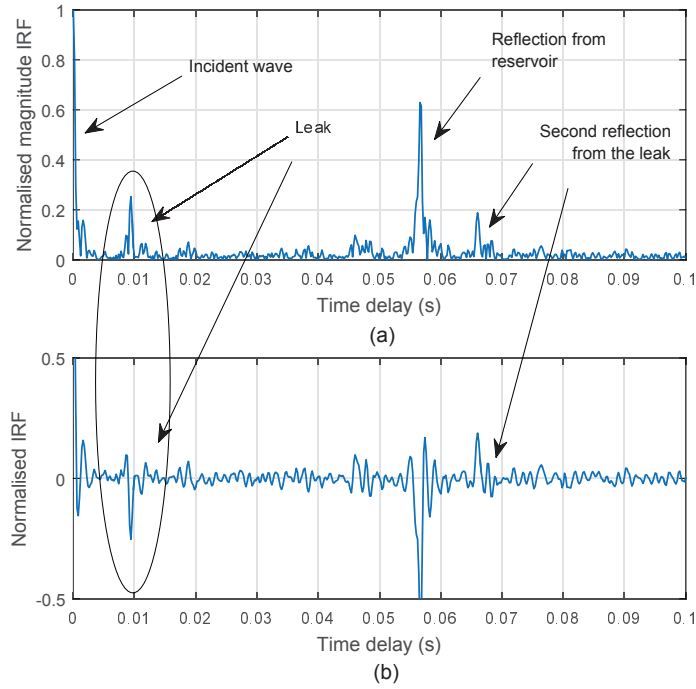


Figure 7: IRF estimate for laboratory data for 2 mm leak diameter: (a) normalised magnitude and (b) normalised amplitude.

329 location of leak in a pipeline system. Verifications were made using different  
 330 tests with the simulated leak orifice varying from a circular orifice of 2 mm  
 331 diameter (Test 1 repeat 1 and repeat 2 and Test 2), 1 mm diameter (Tests  
 332 3, 4) and an irregular orifice (Tests 5, 6). As seen in Table 1, the highest  
 333 estimation error is 0.37%. Compared to the existing literature research using  
 334 PRBS excitation for leak localisation in frequency domain [32] where an  
 335 estimation error was reported to be of approximately 2% for a similar scale  
 336 pipeline system, the proposed IRF estimation has significantly improved  
 337 the accuracy. It should be noted that the higher error is obtained with the  
 338 smaller leak diameter (1 mm) in which the reflection from leak is smaller

Data-set	$T_0; T$ (s)	Leak location (m)	Error (%)	Leak size diameter (mm)
Test 1- Repeat 1	0.0094;0.0568	6.206	0.35 %	2 mm
Test 1- Repeat 2	0.0094;0.0566	6.23	0.29 %	2 mm
Test 2- Repeat 1	0.0094;0.0566	6.23	0.29 %	2 mm
Test 3	0.0092;0.0566	6.1	0.37 %	1 mm
Test 4	0.0092;0.0566	6.1	0.37%	1 mm
Test 5	0.0094;0.0566	6.23	0.29 %	Irregular Orifice
Test 6	0.0094;0.0566	6.23	0.29 %	Irregular Orifice

Table 1: The tabulated results for leak localisation for various laboratory tests.

339 compared to the 2 mm diameter experiments.

### 340 5.2. Comparison with Cepstrum method using laboratory data

341 A comparison between the proposed approach and the Cepstrum method  
342 discussed in [25] is performed using laboratory data for the configuration in  
343 Fig. 3. The normalised pressure head is shown in Fig. 8 (a) with the  
344 rise in pressure at time 0 due to a valve closure. The drop in pressure at  
345 approximately 0.01 s is due to the leak. It should be noted that this pattern  
346 is similar to that observed in with the numerical counterpart in Fig. 5(b).  
347 One can argue that in an ideal environment such as that in Fig. 5(b), the  
348 leak can easily be observed and its time/location can be calculated from the  
349 time domain trace. In Fig. 8 (a), it can be observed that in a real scenario,

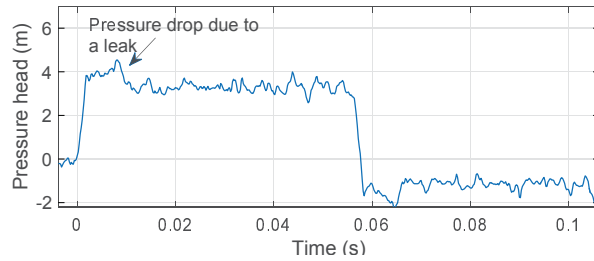


350 the non-ideal movement of the solenoid valve used to generate the transient  
351 pressure and the system noise interference can lead to false-detection or  
352 mis-identification of the anomalies if purely using the time-domain trace. For  
353 example, in Fig. 8 (a), there are locations where the pressure perturbation  
354 does not correspond to any features in the pipe (at approximately 0.05 s).

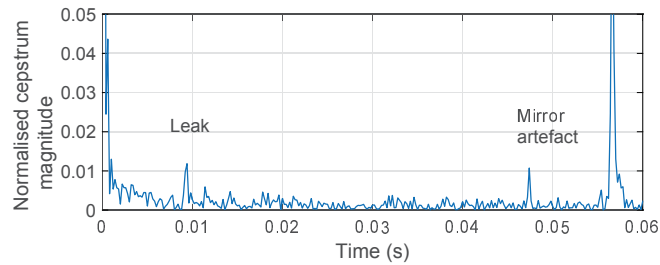
355 The Cepstrum of the signal in Fig. 8 (a) is shown in Fig. 8 (b) whilst  
356 the least squares approach result is shown in Fig. 8 (c). An improved SNR  
357 can be observed by both approaches. A similar artefact can be observed in  
358 Fig. 8 (b) compared to that in Fig. 5(c) obtained from the numerical data.  
359 In Fig. 8 (c), the artefact is suppressed using the proposed algorithm with  
360 IRS excitation.

## 361 **6. Application to a network configuration**

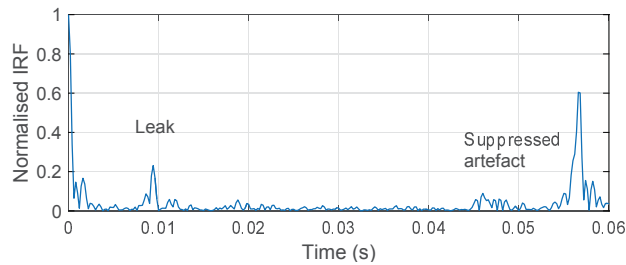
362 For completeness, the following pipe network is considered for compar-  
363 ison between the proposed approach and the Cepstrum method [25], the  
364 configuration is shown in Fig. 9. Three pipe sections of copper material  
365 and internal diameter of 12.6 mm, the lengths are 8.5, 9 and 11 metres for  
366 pipe sections 1, 2 and 3, respectively. The pressurised tanks each have a  
367 pressure of 19 metres. A 1 mm diameter leak is located at approximately  
368 9.5 metres from the pressure transducer on the pipe 3 section. An in-line  
369 valve was located at the boundary of pipe 1, next to a customised valve  
370 used to generate the transient pressure as seen in Fig. 9. The configuration  
371 and parameters are designed similar to that described in [25], except the  
372 use of the in-line valve instead of the inlet at the boundary of pipe 1. It is  
373 because the use of the reflection from the inlet would result in a destructive  
374 interference at the pressure transducer for the PRBS excitation.



(a) Normalised head pressure



(b) Cepstrum of the signal in (a)



(c) Least squares deconvolution using IRS

Figure 8: Normalised head pressure (a); Cepstrum of the laboratory data (b) and impulse response function using the proposed least squares approach (c) of system of a 22.14 mm internal diameter copper pipeline and a 2 mm diameter leak.

375 The method of characteristics (MOC) is again used to generate the tran-  
 376 sient pressure at the pressure transducer for the configuration in Fig. 9 with  
 377 the wave speed specified as 1447 m/s. The excitation to the system was

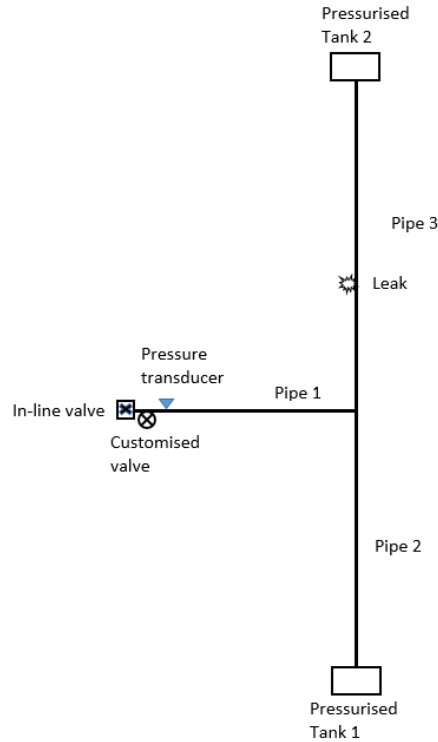
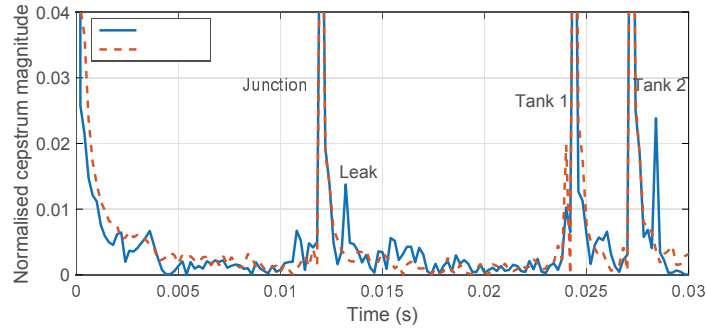


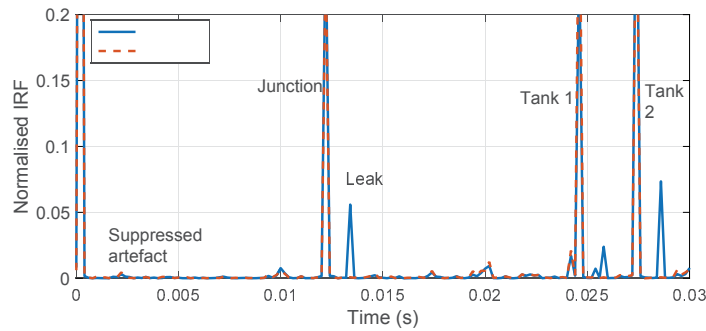
Figure 9: T network configuration.

378 generated similar to that described in [25] by a solenoid valve closing and  
 379 opening periodically with the time period of 1 s. Data was generated for  
 380 the structure with and without the leak. Both Cepstrum and least squares  
 381 results are shown in Fig. 10 where the dashed traces represent the results  
 382 without leak, solid traces are ones with leak.

383 From Fig. 10 (a) it could be seen that apart from the known features  
 384 such as reflections from the junction, leak and pressurised tank 1 and 2,  
 385 there exist other small artefacts in the Cepstrum results. Since this is a  
 386 result of a numerically generated signal in which an ideal pressure rise/drop  
 387 occurs at each time a change in impedance/flow occurs, the system can



(a) Cepstrum method [25]



(b) Proposed least squares deconvolution (IRS)

Figure 10: The Cepstrum of the laboratory data (a) and impulse response function using the proposed least squares approach (b); in both figures the red dashed traces represent the results without leak whilst blue solid traces are those with a leak.

388 be considered to be free from noise interference. Therefore, the artefacts  
 389 observed in the Cepstrum method are due to the interactions between the  
 390 pressure changes in the time domain trace and its regularity, similar to that  
 391 found for the single pipeline scenario, which becomes more complicated when  
 392 there are more features in the configuration. It should be noted that the  
 393 de-noising step described in [25] was omitted in the Cepstrum analysis since  
 394 there exists no noise in the numerical data. In Fig. 10 (b), the artefacts are

395 suppressed using the proposed approach. It is because the proposed least  
396 squares deconvolution with IRS excitation seeks for the reflected excitation  
397 wave which is different to the Cepstrum approach that searches for the  
398 regularity of change in the pressure trace.

## 399 **7. Conclusion**

400 This paper investigates the use of a new least squares deconvolution ap-  
401 proach for impulse response function estimation for leak localization in a  
402 pipeline system. The PRBS excitation has been shown to significantly im-  
403 prove the signal to noise ratio for the IRF estimation, hence significantly  
404 improving the localization accuracy even for small leak sizes compared to  
405 the existing frequency domain counterpart [32]. The proposed method pro-  
406 vides an elegant new way for detecting and localizing the leak in a water  
407 pipeline or simple network, which can be easily extended for other types of  
408 anomalies such as blockages and changes in pipe wall thickness a critical  
409 step leading to condition assessment of water pipes. Verification of the algo-  
410 rithms was made using both numerical and experimental data with various  
411 experimental datasets used to test the algorithm performance. The perfor-  
412 mance comparison has also been undertaken between the proposed approach  
413 and Cepstrum approach. A satisfactory suppression of the artefacts in the  
414 IRF could be obtained by using the proposed approach compared to Cep-  
415 strum. This was achieved by formulating the pipeline system deconvolution  
416 problem as a least squares optimisation problem with an  $l_2$  regularisation.

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