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Nucleon mass and pion loops: Renormalization

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Using Dyson-Schwinger equations, the nucleon propagator is analyzed nonperturbatively in a field-theoretical model for the pion-nucleon interaction. Infinities are circumvented by using pion-nucleon form factors which define the physical scale. It is shown that the correct, finite, on-shell nucleon renormalization is important for the value of the mass shift and the propagator. For physically acceptable forms of the pion-nucleon form factor the rainbow approximation together with renormalization is inconsistent. Going beyond the rainbow approximation, the full pion-nucleon vertex is modeled by its bare part plus a one-loop correction including an effective Δ. It is found that a consistent value for the nucleon mass shift can be obtained as a consequence of a subtle interplay between wave function and vertex renormalization. Furthermore, the bare and renormalized pion-nucleon coupling constants are approximately equal, consistent with results from the cloudy bag model.

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I. INTRODUCTION

The interest in a consistent analysis of the effects of the pion cloud on nucleon properties is nurtured by two areas of recent research. For one, the extrapolation of nucleon lattice data, quenched or unquenched, to physical values of the quark (or pion) mass is mainly determined by pionic effects [1,2]. Furthermore, these effects will pose constraints on the development of covariant nucleon models such as those presented in Refs. [3,4]. The philosophy in superimposing the pion cloud onto a model of the nucleon is that one is building an effective theory in which the structure of the core (bare) nucleon is generated by nonperturbative QCD. The effect of the form factor, which is mathematically equivalent to a Pauli-Villars regulator in the heavy baryon limit [5], is to suppress emission and absorption of high momentum pions. This effectively yields a resummation of conventional, dimensionally regularized chiral perturbation theory which appears to have better convergence properties [6].

Besides suppressing unphysical contributions of high-energy pions in loop diagrams, the form-factor enhanced vertex defines an effective interaction between composite nucleons and pions which can be related to the axial form factor of the nucleon, see below. Thus the form factor serves a dual role in this context: on the one side it regulates otherwise divergent loop integrals and on the other side it describes the effective interaction between nucleons and pions and thus can be related to experiment.

The most basic observable of interest is the nucleon mass or, more precisely, the nucleon mass shift associated with the pion cloud. In a previous study [5], the connection of covariant Euclidean nucleon mass-shift calculations to extant results from the cloudy bag model (CBM) [7] and chiral perturbation theory (χPT) [8,9] was analyzed. A nonperturbative analysis using the Dyson-Schwinger (DS) equation in rainbow approximation (with no renormalization) suggested that almost all of the nucleon mass shift can be attributed to the covariant one-loop pion dressing. We will extend this analysis by supplementing the DS equation with correct on-shell renormalization conditions (Sec. II). To properly implement those, one has to forgo an angular approximation employed in Ref. [5]. In Sec. III we calculate a one-loop correction to the πNN vertex which is subsequently employed in the DS equation. In the last section, we summarize and present our conclusions.

II. THE MODEL

We consider a pseudovector Lagrangian for the πN interaction which reads in Euclidean space

$$\mathcal{L}_{\pi NN}=\frac{g}{2M_N}\ \bar{\Psi}\ i\gamma^\mu\gamma_5\ \boldsymbol{\tau}\Psi\cdot(\partial_\mu\boldsymbol{\pi}). \tag{1}$$

The DS equation for the nucleon propagator G is given by

$$G^{-1}(p)=G_0^{-1}(p)+i\not{p}\Sigma_V(p^2)+\Sigma_S(p^2), \tag{2}$$

$$i\not{p}A(p^2)+B(p^2)=Z_2\ (i\not{p}+Z_M M_N) + Z_1\int\frac{d^4k}{(2\pi)^4}\Gamma^0\ G(k)\ \Gamma\ D(p-k),$$

$$\Gamma^0=(\not{p}-\not{k})\gamma_5\ \boldsymbol{\tau}\ g. \tag{3}$$

Here, $i\not{p}\Sigma_V$ and Σ_S are the vector and scalar self-energies of the nucleon. Likewise, $i\not{p}A$ and B denote vector and scalar parts of the inverse nucleon propagator, respectively. Because of the small mass of the pion (which results in a negligible contribution of nucleon loops to the pion self-energy), we can approximate the full pion propagator $D(q)$ by the free scalar propagator, $D^0(q)=(q^2+m_\pi^2)^{-1}$. Γ^0 and Γ stand for the free and the full πNN vertices. The renormalization constants Z_1, Z_2, and Z_M refer to the πNN vertex, nucleon wave function and nucleon mass, respectively. The relation to bare quantities is given by

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$$g=\frac{Z_2}{Z_1}\,g_{\rm bare},\quad M_N=\frac{M_{\rm bare}}{Z_M}. \tag{4}$$

To account for the compositeness of the particles, we introduce a form factor at each πNN vertex into this idealized field-theoretic model,

$$\Gamma^{[0]}(p,k)\to\Gamma^{[0]}(p,k)u[(p-k)^2]. \tag{5}$$

In the case where both nucleons are on shell ($p^2=k^2=-M_N^2$), this form factor is most naturally related to the axial form factor of the nucleon. It is commonly parametrized as a dipole,

$$u(q^2)=\left(\frac{\lambda^2-m_\pi^2}{\lambda^2+q^2}\right)^2, \tag{6}$$

but we will also investigate(as in Ref. [5]) the exponential form

$$u_{\rm exp}(q^2)=\exp\left(-\frac{q^2+m_\pi^2}{\Lambda^2}\right). \tag{7}$$

From neutrino scattering experiments, the dipole width parameter (in the on-shell case) is determined as $\lambda=1.03\pm0.04$ GeV [10]. We note that because G parity forbids a three-pion vertex; the effect of the vertex renormalization on the *shape* (i.e., the q dependence) of this form factor can only come through recoil corrections. These are suppressed by powers of the pion mass divided by the baryon mass and vanish in the heavy baryon limit. We neglect this variation which would yield a much smaller variation in λ than the phenomenological variation considered below. The choice of λ sets the scale for our calculations, fixing (for example) the bare nucleon mass through Eqs. (9)–(12) below.

Even after the introduction of form factors, we assume that it is sensible to extract nonperturbative information from the DS Eq. (2). We note that now the loop integral in the equation is convergent and all renormalization constants will therefore be finite. A commonly used approximation to it is the rainbow approximation $\Gamma=\Gamma^0$. The full πNN vertex Γ fulfills its own DS equation which takes the symbolic form:

$$\Gamma=Z_1\;\Gamma^0+\Gamma^0\;(G^{[1]}G^{[2]})K_4. \tag{8}$$

Here, K_4 is the full, off-shell, amputated N-N scattering matrix. The rainbow approximation neglects contributions from the last term, and it is only consistent with Eq. (8) by setting $Z_1=1$, i.e., there is no vertex renormalization in the rainbow approximation.

We employ on-shell renormalization for the unknown nucleon propagator, G. This requires that G has a pole at $p^2=-M_N^2$ (the physical mass),

$$M_N[Z_2+\Sigma_V(-M_N^2)]=\Sigma_S(-M_N^2)+Z_2Z_MM_N, \tag{9}$$

$$M_{\rm bare}-M_N=M_N(Z_M-1)$$
$$=\frac{1}{Z_2}[M_N\Sigma_V(-M_N^2)-\Sigma_S(-M_N^2)]. \tag{10}$$

The renormalization constant Z_2 is determined by the condition that the residue at the pole be unity,

$$\left.\frac{\partial G^{-1}}{\partial(i\not p)}\right|_{i\not p=-M_N}=1\quad\rightsquigarrow\;, \tag{11}$$

$$-\Sigma_V(-M_N^2)+2M_N^2\left.\frac{\partial\Sigma_V}{\partial p^2}\right|_{p^2=-M_N^2}-2M_N\left.\frac{\partial\Sigma_S}{\partial p^2}\right|_{p^2=-M_N^2}$$
$$=Z_2-1. \tag{12}$$

Since the inverse bare propagator is $Z_2\,(i\not p+Z_MM_N)$, see Eq. (2), it has a zero at $p^2=-Z_M^2\,M_N^2$ and therefore the nucleon mass shift is given by $\delta M=M_N(1-Z_M)$. Mass renormalization ensures that the self-energy has cuts starting from the physical thresholds. Furthermore one can show that the spectral densities of the solution G multiplied by Z_2 are properly normalized (i.e., Z_2 can be interpreted as the probability of finding a ''bare'' nucleon inside the pion-dressed one). The last remark shows that finite renormalization is absolutely necessary for the correct probabilistic interpretation of results from our model DS equation. We emphasize once more that through the physicality conditions (10), (12), and (23), see below, the variation of the form factor cutoff Λ corresponds directly to a variation in the renormalization constants Z_1, Z_2, and Z_M. Equivalent constants appear in the (bare) Lagrangian of χPT and it can be shown that the expansion of the constants there corresponds to the expansion of the Λ-independent pieces of the Z_i in our picture [1,6].

In the exploratory study [5] the DS equation, Eq. (2), was solved in rainbow approximation after putting $Z_1=Z_2=Z_M=1$ and disregarding the above renormalization conditions. The mass of the dressed nucleon was then found as the solution of $-M_D^2\,A(-M_D^2)+B(-M_D^2)=0$, $M_D<M_N$. In this simplified scenario it was possible to employ a certain angular approximation to the loop integral in Eq. (2) to calculate all angular integrals analytically. However, if one incorporates the renormalization conditions (10) and (12), this angular approximation fails because it underestimates the slopes of Σ_V and Σ_S considerably (and these enter the expression for Z_2). As a result, we must resort to a numerical computation of one angular integral. Since we have to evaluate the renormalization conditions on the nucleon mass shell, it is necessary to continue the DS equation to complex momenta. This intricate procedure is outlined in the Appendix.

We illustrate the results for the mass shift in Fig. 1, employing (as in Ref. [5]) as the renormalized coupling constant $g=M_N/f_\pi$, i.e., $g_A=1$. We chose a dipole form factor with cutoffs λ in a range compatible with the measured axial form factor [11]. Indeed, the mass shift for the unrenormalized rainbow treatment and the one in the one-loop approxi-

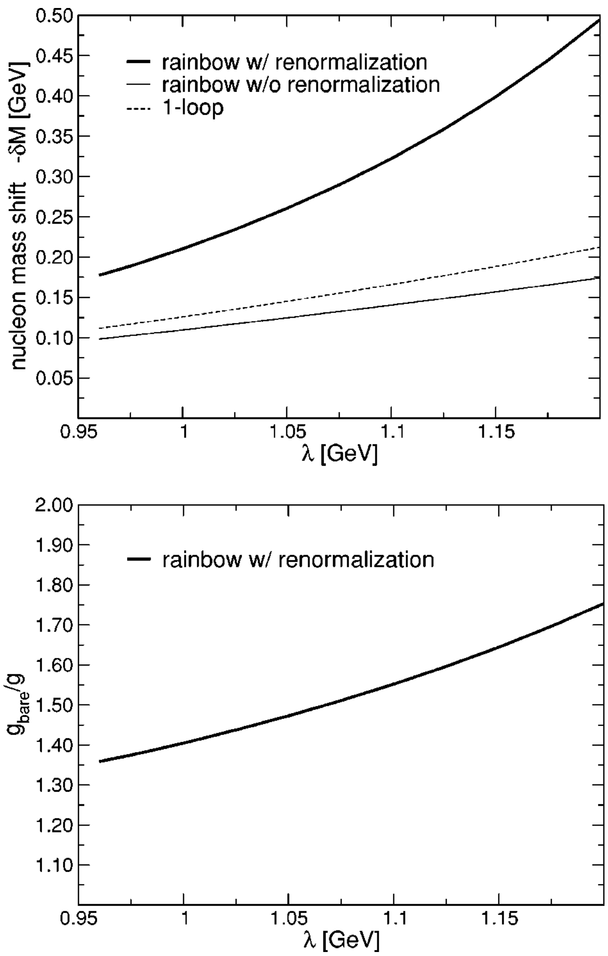

FIG. 1. Top panel: nucleon mass shift in the renormalized treatment (defined as $\delta M = M_N - M_{\rm bare}$) and in the unrenormalized treatment (defined as $\delta M = M_D - M_N$) of the rainbow approximation. Bottom panel: the ratio $g_{\rm bare}/g = Z_1/Z_2$. λ denotes the cutoff for the dipole-type form factor.

mation are very close. In contrast, the mass shift in the renormalized rainbow treatment is larger by a factor which rises from 1.8 ($\lambda = 0.96$ GeV) to 2.9 ($\lambda = 1.2$ GeV). The explanation for this peculiar behavior can be found in the bottom panel of Fig. 1 where the ratio of bare to renormalized coupling is plotted. This ratio equals $1/Z_2$ in the rainbow treatment and is considerably larger than 1. If one divides the DS equation, Eq. (2), by Z_2, one sees that the loop integral is proportional to the bare coupling and therefore drives the mass shift to larger absolute values.

A repetition of the calculations with the physical $g_A = 1.26$ only aggravates the difference between renormalized and unrenormalized results. Whereas the mass shift in the unrenormalized rainbow treatment scales with g_A^2 (as the one-loop result), the mass shift in the renormalized case shoots up to values between 400 MeV and 1.9 GeV (depending on λ), because of the nonlinear nature of the DS equation.

The question arises as to whether such a strong renormalization of the pion-nucleon coupling constant, as visible in the rainbow solutions, is physically reasonable. An analysis of pionic corrections to nucleons in the CBM to one-loop order [7,12] reveals that $Z_1 \approx Z_2$ (and therefore $g_{\rm bare} \approx g$) if one includes the Δ resonance in the analysis of the self-energies and the pion-nucleon vertex [13]. This is a strong indication that the rainbow approximation is unreliable for this problem: there $Z_1 = 1$ is set artificially and $Z_2 \sim 0.6-0.7$ for physical πNN form factors.

Therefore, we will model the full πNN vertex Γ by the bare one plus a one-loop correction that includes the Δ in an effective manner, i.e., as a spin-3/2 particle described by Rarita-Schwinger spinors. Ideally, we would like to employ self-consistent propagators of N and Δ in this study, but solving a coupled system of two-loop DS equations for N and Δ is beyond the scope of this study. Instead, we pursue a modest but nevertheless resource-consuming modification: the one-loop vertex correction to Γ will be calculated using free N and Δ propagators and the result will be inserted back into the DS Eq. (2). By that token, the problem becomes numerically tractable, and we expect the bulk of the effect on the solutions to be buried in a reasonable estimate for Z_1.

III. CORRECTION TO THE πNN VERTEX

We calculate the covariant one-loop correction to the πNN vertex by including the Δ as an effective degree of freedom. Pictorially the equation for the vertex is displayed in Fig. 2, and as mentioned above we take for the nucleon and Δ propagators free spin-1/2 and spin-3/2 propagators, respectively,

$$G^0(p^2) = \frac{-i\not{p} + M_N}{p^2 + M_N^2}, \tag{13}$$

$$G_\Delta^{0,\mu\nu}(p^2) = \frac{-i\not{p} + M_\Delta}{p^2 + M_\Delta^2}\ \mathbb{P}^{\mu\nu}, \tag{14}$$

$$\mathbb{P}^{\mu\nu} = \delta^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu + \frac{2}{3}\frac{p^\mu p^\nu}{M_\Delta^2} - \frac{i}{3}\frac{p^\mu\gamma^\nu - p^\nu\gamma^\mu}{M_\Delta}, \tag{15}$$

with $M_N = 0.94$ GeV and $M_\Delta = 1.23$ GeV the physical masses of both particles. For the $\pi N\Delta$ and $\pi\Delta\Delta$ interactions we employ tree level vertices derived from the simplest covariant interaction Lagrangians, i.e.,

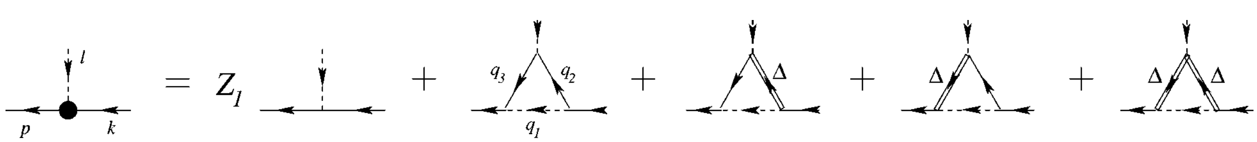

FIG. 2. One-loop corrections to the πNN vertex.

$$\mathcal{L}_{\pi N\Delta}=\frac{g_{\pi N\Delta}}{2M_N}\ \overline{\Psi}^{\mu}\ \boldsymbol{T}^{3/2}_{1/2}\Psi\cdot(\partial_\mu\boldsymbol{\pi})+\text{H.c.}$$

$$\rightsquigarrow\Gamma^0_{\pi N\Delta}=\frac{g_{\pi N\Delta}}{2M_N}\ iq^{\mu}\ \boldsymbol{T}^{3/2}_{1/2}, \tag{16}$$

$$\mathcal{L}_{\pi\Delta\Delta}=\frac{g_{\pi\Delta\Delta}}{2M_N}\overline{\Psi}^{\rho}\ i\gamma^{\mu}\gamma_5\ \boldsymbol{T}^{3/2}_{3/2}\ \Psi^{\rho}\cdot(\partial_\mu\boldsymbol{\pi})$$

$$\rightsquigarrow\Gamma^0_{\pi\Delta\Delta}=\frac{g_{\pi\Delta\Delta}}{2M_N}\ \not{q}\gamma_5\ \boldsymbol{T}^{3/2}_{3/2}. \tag{17}$$

Here q is the momentum of the incoming pion. For the isospin 1/2-3/2 transition matrix we adopt the convention

$$\boldsymbol{T}^{3/2}_{1/2}=C^{3/2M}_{1/2m,1k}. \tag{18}$$

As indices in the Clebsch-Gordan coefficient, $M,m,$and k stand for the isospin-z components of Δ, N, and $\boldsymbol{\pi}$. Furthermore, the isospin matrices $\boldsymbol{T}^{3/2}_{3/2}$ are just the ones for the four-dimensional SU(2) representation.

For the numerical values of the coupling constants $g_{\pi N\Delta}$ and $g_{\pi\Delta\Delta}$ we relate them to g via SU(6) quark model expressions. This of course leaves room to fine tuning which is not the main interest here. To apply the quark model, we note the nonrelativistic limit of both vertices,

$$\Gamma^0_{\pi N\Delta}\rightarrow i\frac{g_{\pi N\Delta}}{2M_N}\ \boldsymbol{T}^{3/2}_{1/2}\ (\boldsymbol{S}^{3/2}_{1/2}\cdot\boldsymbol{q}), \tag{19}$$

$$\Gamma^0_{\pi\Delta\Delta}\rightarrow-\frac{2i}{3}\ \frac{g_{\pi\Delta\Delta}}{2M_N}\ \boldsymbol{T}^{3/2}_{3/2}\ (\boldsymbol{S}^{3/2}_{3/2}\cdot\boldsymbol{q}). \tag{20}$$

The transition matrix $\boldsymbol{S}^{3/2}_{1/2}$ is identical to $\boldsymbol{T}^{3/2}_{1/2}$, and it just refers to the spin degrees of freedom. Comparing to the expressions in Ref. [7], we readily find

$$g_{\pi N\Delta}=\sqrt{\frac{72}{25}}\,g, \tag{21}$$

$$g_{\pi\Delta\Delta}=\frac{6}{5}\ g. \tag{22}$$

Here, we employ for the renormalized coupling constant the physical value $g=g_A M_N/f_\pi$ with $g_A=1.26$. Having fixed the strength of our interactions, we proceed now to the calculation of the renormalization constant Z_1. We choose to fix it at the virtual point where both nucleons and the pion are on shell, i.e.,

$$\overline{u}(k)\ \Gamma\ u(p)\overset{!}{=}\frac{g}{2M_N}\ \overline{u}(k)(\not{k}-\not{p})\gamma_5\ u(p)$$
$$\times[p^2=k^2=-M_N^2,\ (p-k)^2=-m_\pi^2]\ . \tag{23}$$

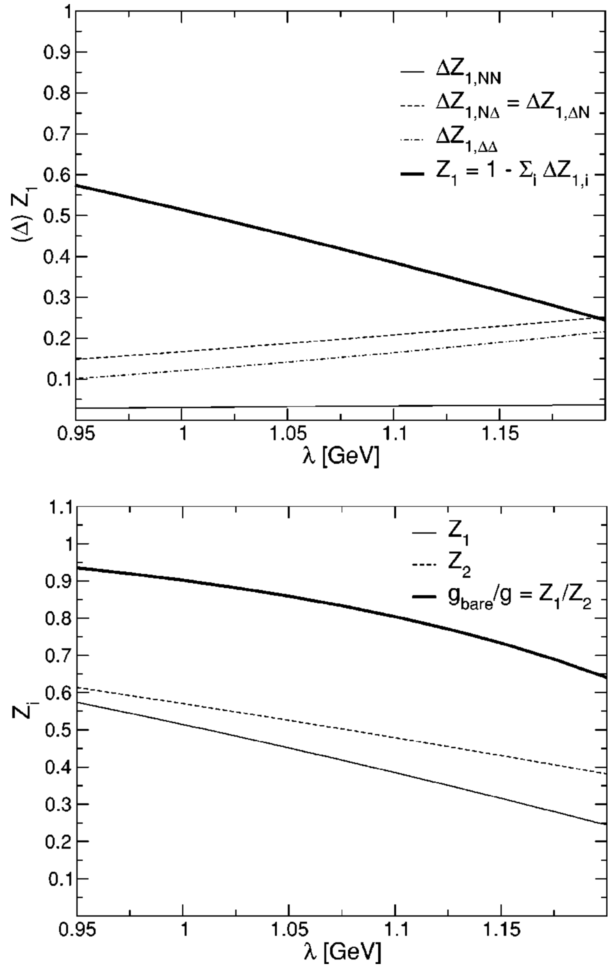

FIG. 3. Top panel: Vertex renormalization constant Z_1. The different contributions to $\Delta Z_1=1-Z_1$, stemming from the loop diagrams shown in Fig. 2, are labeled with $\Delta Z_{1,mn}$, with $m,n\in\{N,\Delta\}$ labeling the baryons in the loop. Bottom panel: Z_1, Z_2, and their ratio in the one-loop approximation. λ denotes the cutoff for the dipole-type form factor.

This has the advantage that the πNN form factor should reduce to 1 at this point.

The resulting Z_1 as a function of the dipole cutoff is plotted in Fig. 3. From the right panel one can see that the loop nucleons contribute less than 0.05 to $1-Z_1$. If this were the whole story, the rainbow treatment of the DS equation would seem to be justified, yielding mass shifts >500 MeV as demonstrated before. The bulk of the difference $1-Z_1$ comes from the two graphs with one intermediate Δ. The resulting bare coupling is somewhat lower than the renormalized one. If the Δ were also included in the nucleon self-energy, we would expect a somewhat larger bare coupling since its contribution lowers Z_2 additionally. Keeping that in mind, we expect the present results for the mass shift in the improved DS equation to be a lower bound.

To one-loop order, there are additional corrections to the πNN vertex. The simple interaction Lagrangian from Eq. (1) constitutes actually a first order expansion of a chirally covariant pion-nucleon Lagrangian of the form [14]

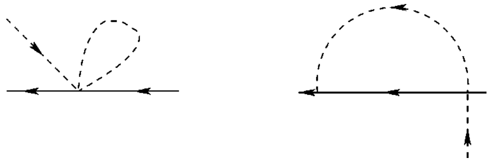

FIG. 4. Tadpole and Weinberg-Tomozawa corrections to the πNN vertex.

$$\mathcal{L}_{\rm cov}=\bar{\Psi}(-\tilde{D}_\mu\gamma^\mu+M_N)\Psi + \frac{g}{2M_N}\;\bar{\Psi}\;\;i\gamma^\mu\gamma_5\;\;\boldsymbol{\tau}\;\Psi\;\;(D_\mu\boldsymbol{\pi}) \qquad (24)$$

with the following definitions for the covariant derivatives:

$$\tilde{D}_\mu=\partial_\mu - i\left[\frac{\cos(\pi/f_\pi)-1}{2}\right]\;\boldsymbol{\tau}\cdot(\boldsymbol{e}_\pi\times\partial_\mu\boldsymbol{e}_\pi), \qquad (25)$$

$$D_\mu\boldsymbol{\pi}=(\partial_\mu\pi)\;\;\boldsymbol{e}_\pi+f_\pi\sin(\pi/f_\pi)\;\;\partial_\mu\boldsymbol{e}_\pi\;\;. \qquad (26)$$

Expanding $D_\mu\boldsymbol{\pi}$ to third order and $\tilde{D}_\mu$ to second order in $\boldsymbol{\pi}$, one finds a tadpole vertex correction and a loop correction with a Weinberg-Tomozawa (WT) term, see Fig. 4. These terms contribute to ΔZ_1 with different signs. Numerical evaluation shows that the positive WT term is somewhat larger than the absolute value of the tadpole. For the considered range of cutoff parameters, the sum of both terms constitutes at most a 10% correction to ΔZ_1 which we will neglect in the following. The smallness of this correction shows that indeed the one-loop effects with an intermediate delta are the main reason for $Z_1\sim Z_2$ to this order.

Our one-loop improved model for the vertex can now be inserted into the DS Eq. (2). Technically, we proceed by projecting the Dirac structure of the vertex onto basic covariant matrices,

$$\begin{aligned}\Gamma_{\rm Dirac}(p,k,l)&=(i\gamma_5)V_1(p^2,k^2,p\cdot k)+(\not{p}\gamma_5)V_2(p^2,k^2,p\cdot k)\\&+(\not{k}_{\rm T}\gamma_5)V_3(p^2,k^2,p\cdot k)\\&+(i\not{k}_{\rm T}\not{p}\gamma_5)V_4(p^2,k^2,p\cdot k),\end{aligned} \qquad (27)$$

$$k_{\rm T}=k-p\;\;\frac{k\cdot p}{p^2}, \qquad (28)$$

from which the scalar functions $V_{1\,\ldots\,4}$ can be readily traced out, using the program FORM [15]. The three remaining scalar integrals are evaluated numerically. We calculate the functions V_i on a three-dimensional grid in the variables p^2, k^2 and $\hat{p}\cdot\hat{k}$, needed for the solution of the DS equation.

The results for mass shift and the wave-function renormalization constant Z_2 are depicted in Fig. 5. In the physically interesting region $\lambda\in[0.95,1.05]$ GeV the mass shift stays rather flat, at a value around 200 MeV. For a harder form factor it actually *drops*, which can again be explained by looking at the ratio $g_{\rm bare}/g$: beyond $\lambda\sim1.1$ GeV $g_{\rm bare}/g$ becomes less than 1/2. As explained before, this ratio enters the pion loop integral in the DS equation through Z_1. Since

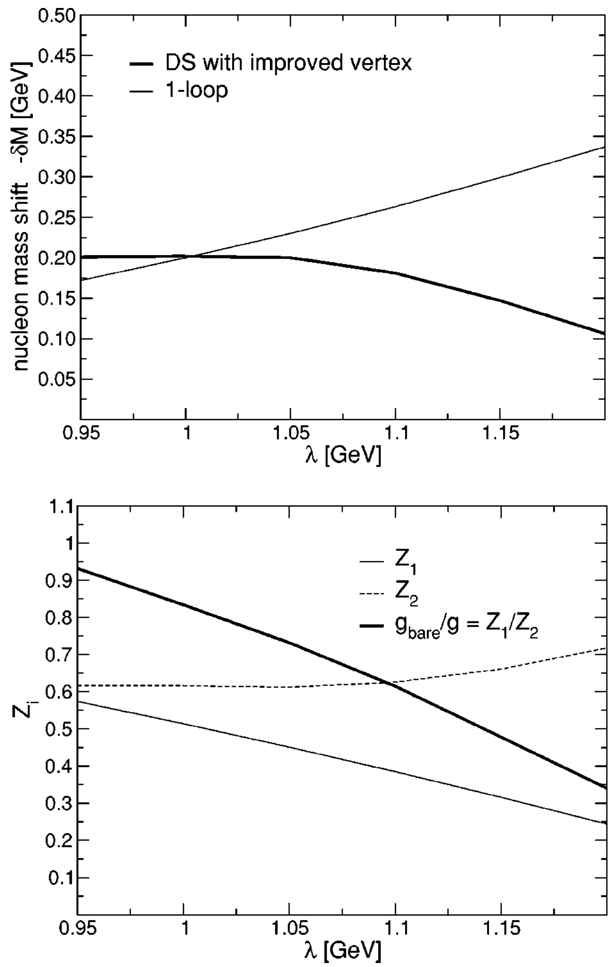

FIG. 5. Top panel: Mass shift for the vertex-improved DS solution in comparison to the one-loop approximation. Bottom panel: Z_1, Z_2, and their ratio in the DS solution. λ denotes the cutoff for the dipole-type form factor.

Z_1 itself is very low there, the one-loop treatment of the πNN vertex can be questioned.

The feature of a plateau in the mass shift for a certain range of cutoff parameters(which happens to coincide with the experimental results for the axial form factor) is actually a desired one in effective models with cutoff functions, since it indicates the relative independence of the results on the specific choice of cutoff. Whether that still holds after self-consistent inclusion of the Δ resonance is an open question. Certainly, by the arguments given above, its proper inclusion should lead to $g_{\rm bare}/g>1$ and a larger mass shift. Keeping this in mind, a value $-\delta M\approx-\delta M_{\text{one-loop}}\approx200$ MeV constitutes a lower bound. Although numbers might vary after the self-consistent inclusion of the delta, the conclusion that a properly renormalized rainbow approximation leads to unphysically large results will not change. We have demonstrated that this was a consequence of the assumption $Z_1=1$ which does not hold in the pion-nucleon system.

In Fig. 6 we show the results for an exponential πNN form factor. Since the exponential is an entire function, we can investigate also somewhat softer form factors without encountering analytical problems. One-loop mass shifts are

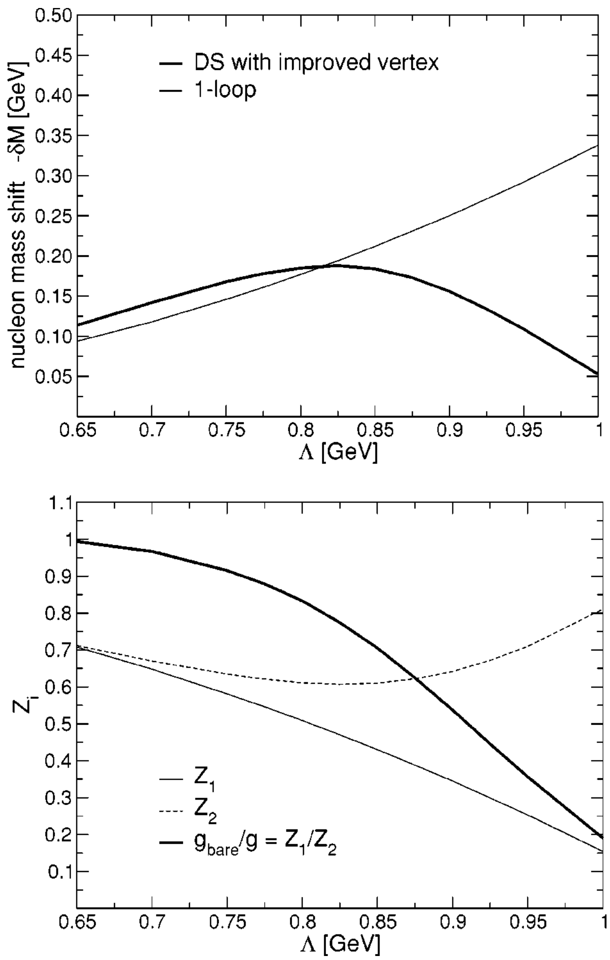

FIG. 6. Mass shift and renormalization constants arising from a pion-nucleon form factor of exponential type with cutoff Λ.

the same for dipole and exponential, if $\lambda \approx 1.2\Lambda$. The result for the self-consistent mass shift looks overall very similar to the previous case. Its curve flattens around $\Lambda = 0.8$ GeV at a value of approximately 190 MeV and then drops because of the rapidly decreasing bare coupling constant.

IV. SUMMARY AND CONCLUSIONS

We have investigated the covariant Dyson-Schwinger equation for the nucleon in a field-theoretic model for the pion-nucleon interaction with a pseudovector Lagrangian. Its treatment in the commonly used rainbow approximation, including proper renormalization, leads to a strong renormalization of the pion-nucleon coupling constant, $g_{\rm bare}/g > 1$, in contradiction to perturbative one-loop calculations. We were therefore led to calculate the one-loop perturbative correction to the vertex, including the Δ resonance, and to resolve the DS equation. For physically reasonable values for the cutoff in the dipole pion-nucleon form factor, $\lambda \approx 1$ GeV, we find $g_{\rm bare}/g \leqq$ and a rather stable value for the nucleon mass shift of around -200 MeV, consistent with one-loop results in both covariant treatments and semirelativistic approaches such as the cloudy bag model. The fully self-consistent inclusion of the Δ resonance is expected to raise the absolute value of the mass shift even further.

ACKNOWLEDGMENTS

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APPENDIX: SOLVING THE DYSON-SCHWINGER EQUATION FOR COMPLEX MOMENTA

Here we present a more detailed account of the procedure to solve the DS Eq. (2),

$$i\not{p}A(p^2)+B(p^2)=Z_2\;(i\not{p}+Z_M M)+i\not{p}\Sigma_V(p^2)+\Sigma_S(p^2), \tag{A1}$$

$$i\not{p}\Sigma_V(p^2)+\Sigma_S(p^2)=Z_1\int\frac{d^4k}{(2\pi)^4}\Gamma^0(p-k)\times\frac{1}{i\not{k}A(k^2)+B(k^2)}\;\Gamma(p,k)D^0(p-k). \tag{A2}$$

In order to evaluate the renormalization conditions (10), and (12), we will need (as shown below) the functions $A(p^2)$ and $B(p^2)$ for $p^2\in[-(M_N-m_\pi)^2,\infty)$, i.e., if $p=(\mathbf{0},p_4)$, p_4 may assume imaginary values.

We introduce the two component vectors $\Sigma(p^2)=[\Sigma_S(p^2),\Sigma_V(p^2)]$ and $S(p^2)=[A(p^2),B(p^2)]$. After applying suitable traces and doing the two trivial angular integrals we arrive at an equation of the form

$$\Sigma_i(p^2)=\frac{3}{(2\pi)^3}\int_0^\infty\frac{k^2\,dk^2}{k^2A^2(k^2)+B^2(k^2)}\times\int_{-1}^{1}\frac{\sqrt{1-z^2}\;dz}{p^2+k^2+m_\pi^2-2p_4|k|z}\,K_{ij}(p^2,k^2,z)S_j(k^2), \tag{A3}$$

where $z=\cos\psi$ refers to the angle between the Euclidean vectors p and k. If we want to evaluate the loop integral for $p^2<0$ (p_4 imaginary), we have to note that for $p^2+m_\pi^2<0$ poles from the pion propagator cross the real k_4 axis. These have to be avoided by a suitable deformation of the integration contour. This situation is depicted in Fig. 7. Alternatively, the original real path may be retained provided that a loop contour around the poles is added. This leads to

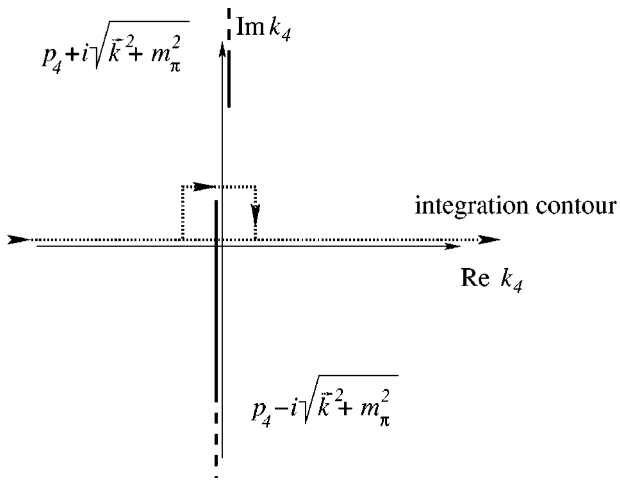

FIG. 7. The k_4 integration in the pion loop. Once the singularities have crossed the real axis, the proper integration contour consists of the real axis and a loop contour around the singularities that have crossed the axis.

$$\Sigma_i(p^2)=\frac{3}{(2\pi)^3}\int_0^\infty \frac{k^2\ dk^2}{k^2A^2(k^2)+B^2(k^2)}\times\int_{-1}^{1}\frac{\sqrt{1-z^2}\ dz}{p^2+k^2+m_\pi^2-2p_4k_4z}\ K_{ij}(p^2,k^2,z)S_j(k^2)+\frac{3}{(2\pi)^3}\ \theta(-p^2-m_\pi^2)\ \int_{\rm poles}\frac{d^3\boldsymbol{k}}{2\sqrt{\boldsymbol{k}^2+m_\pi^2}}\times\frac{K_{ij}(p^2,k^2,z)}{k^2A^2(k^2)+B^2(k^2)}\ S_j(k^2)|_{(p-k)^2=-m_\pi^2}. \tag{A4}$$

The loop contour in k_4 picks up residues $(-2\pi i)/(-2i\sqrt{\boldsymbol{k}^2+m_\pi^2})$ from the pion propagator, and the integrand for the remaining integral in three-space has to be evaluated at the pion poles (thus the value of z to be used in K_{ij} is determined). We convert the integral over $\boldsymbol{k}$ into an integral over the squared four-momentum k^2 using

$$d\boldsymbol{k}^2\frac{|\boldsymbol{k}|}{2\sqrt{\boldsymbol{k}^2+m_\pi^2}}=-dk^2\ \ k^2\ \ F(k^2,p^2), \tag{A5}$$

$$F(k^2,p^2)=\frac{\sqrt{(k^2+p^2+m_\pi^2)^2-4k^2p^2}}{4k^2p^2}. \tag{A6}$$

Hence, we arrive at the final expression for the self-energies, valid also for timelike momenta up to $-p^2=M_N^2$:

$$\Sigma_i(p^2)=\frac{3}{(2\pi)^3}\int_0^\infty \frac{k^2\ dk^2}{k^2A^2(k^2)+B^2(k^2)}\times\int_{-1}^{1}\frac{\sqrt{1-z^2}\ dz}{p^2+k^2+m_\pi^2-2p_4k_4z}\ K_{ij}(p^2,k^2,z)S_j(k^2)-\frac{3}{(2\pi)^2}\ \theta(-p^2-m_\pi^2)\times\int_{-(\sqrt{-p^2}-m_\pi)^2}^{-p^2-m_\pi^2}k^2F(k^2,p^2)dk^2\times\frac{K_{ij}(p^2,k^2,z)}{k^2A^2(k^2)+B^2(k^2)}\ S_j(k^2)|_{(p-k)^2=-m_\pi^2}. \tag{A7}$$

Now one sees clearly that in order to evaluate the renormalization conditions at $p^2=-M_N^2$, one needs to know the self-consistent solution to $A(p^2)$ and $B(p^2)$ for the interval $p^2\in[-(M_N-m_\pi)^2,\infty)$. The DS equation can now be solved iteratively, performing both z and k^2 integrations numerically. By virtue of the pion-nucleon form factors, the numerical treatment is not hampered by ultraviolet divergencies and stable results are achieved by using a mininum of 100 mesh points for each integral.

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