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Kaon-soliton bound state approach to the pentaquark states

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We show that in hidden local symmetry theory with the vector manifestation (VM), a $K^+$ can be bound to skyrmion to give an exotic baryon that has the quantum numbers of the $\Theta^+$ pentaquark with spin 1/2 and even parity which is consistent with large $N_c$ counting. The vector meson $K^+$ subject to the VM in the chiral limit plays an essential role in inducing the binding.

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The discovery of a $\Theta^+$ baryon [1] with a mass of 1540 MeV and with a narrow width less than 25 MeV is one of the most exciting events in recent hadron physics. It is an interesting exotic state that cannot be simply made of three quarks as the other baryons. The existence of such an exotic state has been anticipated also in various models [2]. Among them, the most precise predictions for the mass and the width were made in the SU(3) chiral models [2]. Among them, the most precise predictions for such an exotic state has been anticipated also in various of three quarks as the other baryons. The existence of one of the most exciting events in recent hadron physics.

On the other hand, the large strange quark mass compared with the two nonstrange light-quark masses renders the SU(3) rigid rotator quantization problematic and, in particular, for the exotic state, it has been argued [4–6] that its excitation energy of order $N_c$ is inconsistent with the scale separation needed to justify collective coordinate quantization. Since the Skyrme model is considered to be a valid description of the baryons in the large-number-of-colors ($N_c$) limit, such an $N_c$ inconsistency could be a severe defect although the results are satisfactory phenomenologically. In Ref. [5], it has even been argued that the collective exotic states cannot be the genuine prediction at large $N_c$. This argument was given a further support by an exactly solvable model [7].

The objective of this paper is to show that the bound state approach first proposed by Callan and Klebanov [4,8] for the nonexotic $S = -1$ hyperons, in which the fluctuations in the strangeness direction are of order $N_c^0$, with a vibrational character and those in the light degrees of freedom are of order $1/N_c$ with a rotational character, can be suitably applied to hidden local symmetry Lagrangian [9,10] to obtain a bound or quasibound $K^+$ soliton having the quantum numbers of the exotic pentaquarks.

In fact, Itzhaki et al. [11,12] recently studied this $K^+$-soliton system using the usual three-flavor Skyrme Lagrangian consisting of the octet pseudo-Goldstone fields and arrived at the conclusion that with the parameters appropriate for the $S = -1$ hyperons, the repulsive Wess-Zumino (WZ) term for the $K^+$ channel prevented the binding and there can at best be a near-threshold $S = +1$ bound state—which can be quantized to $I = 0, J^P = 1/2^+$ pentaquark state—only when the SU(3) symmetry is strongly broken and the strength of the WZ term is reduced. However, in contrast to the $S = -1$ case where the bound state approach and the rigid rotator approach match consistently to each other with the bound state turning into the rotor zero mode, the absence of the $S = +1$ bound state in the SU(3) symmetric limit raises a serious question on the validity of the rigid rotor approach where $S = +1$ exotic states come out independent of the strange quark mass. Our study reaches a similar conclusion in the chiral limit.

In this work, we show that when vector mesons are incorporated into the chiral Lagrangian as was done in [13], a dramatic change can take place in the structure of the $S = +1$ skyrmions. In particular, we will see that the hidden local symmetry (HLS) theory [9] endowed with the vector manifestation (VM) as formulated recently by Harada and Yamawaki [10] can render under certain conditions that are not unreasonable the $K^+$ meson bound to an SU(2) skyrmion à la Callan and Klebanov to give a state with the quantum numbers of the $\Theta^+$. The crucial element in the treatment is that the vector mesons need to figure explicitly with the VM since as will be clarified below, the role of the vector mesons becomes more prominent for the $K^+$-nucleon interactions. There are qualitatively two important mechanisms at work in arriving at our result. First, the vector meson $K^+$ exerts a level
repulsion from above, thereby weakening the “pushing-up” effect of the repulsive WZ term for the $S = +1$ kaon state. Next, the explicit vector degrees of freedom with masses subject to the VM [10] soften the contact interaction terms between the pseudoscalars, reducing the strength of the WZ term. That vector mesons are important for bound pentaquarks for heavy flavors has been understood for some time [14]: This is intimately connected with heavy-quark symmetry. The significant new element in our theory is that the VM to which HLS theory flows as the spontaneously broken chiral symmetry is restored renders feasible a systematic chiral perturbation calculation with the vector mesons treated on the same footing as the pseudo-Goldstone bosons [10]. A striking case in support of this argument is the recent analysis of the chiral doublers in light-heavy-quark hadrons starting from the VM fixed point and taking into account small deviations from the VM in low order perturbation theory confirms that the VM is not too far from nature [15].

To bring out the above points in the simplest form, we consider the HLS Lagrangian for three flavors [9] with only the relevant degrees of freedom retained. The normal part of the Lagrangian will be taken in the form

\[
\mathcal{L} = \frac{f^2}{4} \left\{ \text{Tr} \left( \mathcal{D}_\mu \xi_L \mathcal{A}_L^{\dagger} \right)^2 - \text{Tr} \left( \mathcal{D}_\mu \xi_R \mathcal{A}_R^{\dagger} \right)^2 + \text{Tr} \left( \mathcal{M} \left( \xi_L \mathcal{A}_L^{\dagger} \xi_R^{\dagger} \right) + \xi_R^{\dagger} \mathcal{A}_L - 2 \right) \right\} + a \text{Tr} \left( \mathcal{D}_\mu \xi_L \mathcal{A}_L^{\dagger} + \mathcal{D}_\mu \xi_R \mathcal{A}_R^{\dagger} \right)
\]

\[
- \frac{1}{2g^2} \text{Tr} \left( F_{\mu\nu} \cdot F^{\mu\nu} \right),
\]

where $\xi_L^{\dagger} \xi_R = U \in \text{SU(3)}$ describes the pseudoscalar octets with masses $\mathcal{M} = \text{diag}(m_\pi^2, m_\pi^2, m_\pi^2)$, and the covariant derivatives are defined as $\mathcal{D}_\mu = \partial_\mu - iV_\mu$ with the vector meson nonets $V_\mu$ taken as hidden gauge fields (with the $\omega$ included as the isosinglet vector meson). $F_{\mu\nu} = \mathcal{D}_\mu V_\nu - \mathcal{D}_\nu V_\mu$ is the field strength tensor. The constant $a$ which will play a crucial role in our work represents the ratio $(f_\pi/f_\rho)^2$ where $f_\rho$ is the decay constant of the scalar field that gets “eaten up” to give the vector meson mass and $f_\pi$ is the pion decay constant, the empirical value of which is 93 MeV.

We shall take the anomalous part of the Lagrangian which plays a key role in our treatment in the form

\[
\mathcal{L}_{an} = \mathcal{L}_{\text{WZ}}^0 - 10C(\mathcal{L}_1 - \mathcal{L}_2),
\]

where $\mathcal{L}_{\text{WZ}}^0$ comes from the five-dimensional Wess-Zumino action that we shall refer to as “irreducible” WZ term and we have chosen only a special combination of two terms among four general homogeneous solutions of the anomaly equation of Ref. [16]. Other choices are found not to affect the essential feature of our results. Explicitly, they are

\[
\mathcal{L}_1 = \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( L_\mu R_\nu L_\rho R_\sigma - R_\mu L_\nu L_\rho R_\sigma \right),
\]

and

\[
\mathcal{L}_2 = \epsilon^{\mu\nu\rho\sigma} \text{Tr} \left( L_\mu R_\nu L_\rho R_\sigma \right)
\]

with $L_\mu (R_\mu) = \mathcal{D}_\mu \xi_L (R_\mu) \xi_L^{\dagger} (R_\mu)$ and $C = -iN_c/240\pi^2$. This particular combination of the homogeneous solutions makes the amplitude of the five-pseudoscalar process $K^+(k^+) + K^-(k^-) \rightarrow \pi^+(q^+) \pi^-(q^-) \pi^0 (q^0)$ entirely given by $\Gamma_{\text{WZ}}^0$ in the chiral limit to be consistent with QCD anomaly [17].

The basic premise of the Harada-Yamawaki approach is that the parameters of the Lagrangian are to be determined à la Harada-Yamawaki [10] by matching the effective theory (via, e.g., correlators) to QCD at a matching scale $\Lambda$ near the chiral scale $\sim 4\pi f_\pi \sim 1$ GeV. Physical quantities such as the masses and coupling constants of the fluctuating fields are obtained by doing loop calculations with the Lagrangian. This means that not all the parameters that figure for the soliton justified at the large $N_c$ limit are physical ones. The quantum corrections are generically suppressed by $1/N_c$ factors. Now what figures the most importantly for our problem is the parameter $a$ which ranges between 1 and 2 in nature. The vector meson mass arising from Higgsing in HLS theory is $m_v^2 = a g^2 f_\pi^2$.

At the VM fixed point, $a = 1$. The Harada-Yamawaki study shows that in the large $N_c$ limit, $a = \sim 1.3$ but at the scale corresponding to the on-shell vector meson, loop corrections make $a$ approach 2 at which point the vector meson dominance is exact [18]. On the other hand, there is a compelling indication that in the background of baryonic matter, $a$ is close to 1 with the vector dominance maximally violated [19] even though the gauge coupling $g$ departs from the VM fixed-point value. Following the modern literature [20], we call this point with $a = 1$ and $g \neq 0$ “Georgi vector limit” (GVL). This indicates that the range of mass parameters that would figure in the soliton and in fluctuations in the strangeness direction would be that which is closer to 1. It is intriguing that a detailed analysis [10,18] indicates that $a = 2$ is “accidental” and nature is closer to one—the (GVL)—with the vector dominance violated maximally although other parameters (such as $f_\pi$) are subject to significant quantum corrections. It is also intriguing that on a much more fundamental level, $a = 1$ represents the “theory-space locality” in the little Higgs mechanism for the $\pi^+ - \pi^0$ electromagnetic mass difference [20,21]. Thus our focus then will be the property of the soliton for $a$ in the vicinity of 1.

For comparison with the work of Izhaki et al. [11], we consider also the limit—which is artificial in the model—where $a \rightarrow \infty$ while keeping the Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) relation with the parameters $f_\pi$ and $g$ fixed finite. Then, the second term in the Lagrangian (1) containing $a$ gives constraints on the vector meson octets as $V_\mu = -\nu_\mu$ and the isoscalar $\omega$ as $\omega^\mu = -\frac{N_c}{2m_\pi} B_\mu$, where
where $K_{\mu\nu} = D_\mu K^* - D_\nu K^*$, and $q_{\mu\nu} = D_\mu q_\nu - D_\nu q_\mu$. Note that we have two kinds of covariant derivatives, one with the induced vector fields $v_\mu$, $D_\mu = (\partial_\mu + i q_\mu)$, and the other with the vector meson background $q_\mu$, $D_\mu = (\partial_\mu - i q_\mu)$.

The key point of our approach is that the presence of $K^*$ modifies especially the terms of first order in time derivative, which distinguishes $S = \pm 1$ fluctuations. It should be recalled that in the model with pseudoscalars only as in [8] such a term arises uniquely from the irreducible Wess-Zumino term. We shall refer to the terms that distinguish $S = \pm 1$ fluctuations other than the irreducible WZ term as “WZ-like terms.” In Eq. (7), we have many such WZ-like terms. They originate not only from the homogeneous Wess-Zumino term but also from the covariant derivatives with the static $\omega$ configuration as a gauge potential. The most striking result is that in addition to the irreducible WZ term, $(iN_c/4f)^2 B^{(0)}[(D_\mu K)^\dagger K - K^\dagger D_\mu K] \times 2$, of the Callan-Klebanov Lagrangian, there is an additional term with a factor $-2\bar{\omega}$ from the homogeneous part of the anomalous Lagrangian (2). However, this does not necessarily mean that the $S = \pm 1$ fluctuation will now feel an attractive interaction with the skyrmion. One can easily check that when the vector mesons are integrated out, the additional terms are exactly canceled by (i) the term with the nonvanishing time component of $q^{(0)}_{\mu = 0}$ (= the classical $\omega$ configuration) in the third line of Eq. (7), with the $\omega_0$ replaced by its infinite mass limit $\omega^\infty_0$, and (ii) the eighth line in Eq. (7), with $K^*$'s replaced by their infinite mass limit which can be read off from the fourth line as

$$K^{*\infty}_\mu = -\frac{1}{gf} \omega^{(0)}_\mu K.$$  

The sources of the various terms that cancel in the infinite mass limit are schematically illustrated in Fig. 1.

However, with finite vector meson masses as required in HLS theory, the cancellation becomes imperfect. The deviation of $\omega_0$ from its infinite mass limit $\omega^\infty_0$ is such that $<\omega_0/\omega^\infty_0> < 1$ independently of fluctuations in the strangeness direction. Thus, the incomplete cancellation effectively reduces the net strength of the Wess-Zumino attraction in the $S = -1$ channel, which was the motiva-
tion for introducing the vectors in Ref. [13] to solve the over-binding problem for the hyperons in the Callan-Klebanov model. What was not noticed in [13] was, however, that the $K_\mu^+$ solution depends strongly on its strangeness. In the case of the $S = -1$ fluctuation, the closer it is to $K_{\mu}^{\text{sol}}$, the more exact the cancellation among the WZ-like terms becomes. Thus, the net strength of the Wess-Zumino attraction becomes stronger approaching the irreducible strength. As for the $S = +1$ fluctuation, however, the opposite phenomenon takes place; the more $K_{\mu}^+$ deviates from its infinite mass limit, the less cancellation among the WZ-like terms occurs. The sum of the irreducible Wess-Zumino term and the WZ-like terms can then give an attraction although the fourth term in the Lagrangian prevents $K_{\mu}^+$ from deviating too much from $K_{\mu}^{\text{sol}}$.

Now, our task is to solve the equations of motion for $K$ and $K^*$ moving in the background potentials provided by the static soliton configuration sitting at the origin. This is straightforward though tedious. Through the background potentials, $K$ and $K^*$ are strongly coupled. Since the soliton solution is invariant only under the simultaneous rotations in the spatial and isospin spaces, the eigenstates are classified by their “grand spin” quantum number $\Lambda$ associated with the operator $\Lambda = L + S + I$, where $L$, $S$, and $I$ are the angular momentum, spin, and isospin operators.

To have an initial idea, we first take the bare Lagrangian obtained by Harada and Yamawaki [10] by matching the effective field theory correlators to the QCD ones at $\Lambda = 1.1$ GeV. The resulting parameters take the values: $f_\pi = 145$ MeV, $g = 3.69$, and $\alpha = 1.33$. These parameters reflect in some sense a “large” $N_c$ limit and hence differ from the physical values by the loop corrections down by $1/N_c$. We expect the soliton mass to be too big compared with the physical mass of the nucleon without $O(N_c^0)$ (e.g., Casimir) corrections (which of course should be calculated) but what is relevant for us is the effective mass of $K^+$ in the background of the skyrmion. The calculation shows that the $K^+$ is indeed bound. The binding energy turns out to be not appreciably big, around 3 MeV but the sign of the mass shift seems robust.

Since nature seems to indicate that $\alpha$ is close to 1, it would be interesting to be able to “dial” $\alpha$ toward 1 and see how the system evolves. This operation is rather intricate and nontrivial, however, because of various “consistency conditions” associated with the VM as can be seen in [10]. For instance, much of the low-energy hadron dynamics can be understood with $\alpha$ set to its fixed-point value 1 while the other parameters of the Lagrangian such as $g$ and $f_\pi$ depart from their fixed-point values. This comes about because there are subtle connections between various parameters controlled by the fact that the theory flows to the VM fixed point.

We have not done this “self-consistent” calculation yet. Since what we are mainly interested in is whether or not the $K^+$-soliton system can be bound, and under what conditions, we shall simply fix all the model parameters to the empirical values except for $\alpha$ which we vary: $N_c = 3, f_\pi = 93$ MeV, $m_\pi = 140$ MeV, $m_K = 495$ MeV, and $g = 5.85$. In interpreting our results, we should keep in mind that we do have at our disposal at least two constraints. One is that the (leading $N_c$) bare Lagrangian of Harada and Yamawaki mentioned above does give a bound $K^+$-soliton system. The other is that at $\alpha = 1$, chiral symmetry is realized with $f_\pi = f_{\pi}$ and $(0)A^\mu|\pi\rangle = ip^\mu f_\pi$ and $(0)V^\mu|\sigma\rangle = ip^\mu f_{\sigma}$, so it is likely that the coupling of the $K^+$-soliton bound state to the $K\pi$ continuum is zero in the spirit of the arguments presented in [22,23].

The solutions for the $K^+$-soliton system we have so obtained are summarized in Fig. 2. Plotted there are the eigenenergies vs $a$ of the bound states or the resonance states found in $S = \pm 1, \lambda^P = \frac{1}{2}, \frac{3}{2}$ channels. The widths of the resonance states in the kaon-soliton channel are indicated by error bars, which, however, may not be interpreted in the present form as the physical width of the $\Theta^+$. Among others, the system should be (collective) quantized for such an interpretation. We see in Fig. 2 that the $S = -1$ states depend little on $a$ for the relevant range $a \geq 1$ (or as long as the corresponding mass parameter $m_{K^*}$ is larger than $m_K$). This means that the structure of the $S < 0$ states, well described with [13] or without [8] vector mesons, will not be affected by the change in $a$.

There are two bound states stable against the change of $a$ which correspond, when quantized, to the normal $S = -1, -2$ hyperons with positive parity or $\Lambda(1405)$ with negative parity and one bound state or narrow-width

![FIG. 2 (color online). The eigenenergies of $S = \pm 1, \lambda^P = \frac{1}{2}, \frac{3}{2}$ states obtained for various $a$ values. The width of the resonance in the $K^+$-skyrmion channel is given by the error bar.](image-url)
resonance corresponding to $\Lambda(1520)$ with negative parity. On the other hand, the $S = +1$ state is extremely sensitive to the value of $a$. Around $a = 2$, the $S = +1$, $\lambda^P = \frac{3}{2}^+$ state could have a resonance state. The energy of the resonance state is above the kaon mass but below the mass of $K^*$. Except for those near $m_K$ threshold, the resonance states have much too large a width to be a candidate for the $\Theta^+$. If $a$ has the value around 1.3 as appropriate in the large $N_c$ limit [10] where $m_{K^*}$ is comparable to $m_K$, there can be a bound state or a sharp resonance with very narrow width in the $S = +1$, $\lambda^P = \frac{1}{2}^+$ channel.

It is noteworthy that there is no low-lying $S = +1$, $\lambda^P = \frac{3}{2}^+$ state in the model. Similar results are obtained for any values of $m_{K^*}$; that is, there are always stable $S = -1$ bound states below $m_K$ but for the $S = +1$ states the bound state or a resonance state with narrow width are possible only when there is a $K^*$ with $m_{K^*}$ close to or even less than $m_K$. This means that to have a bound state or a narrow resonance, a substantial modification of the vector meson mass in the presence of the soliton is required. Whether or not this can actually take place is not clear because of the “self-consistency” issue mentioned above. Since $m_{K^*}$ is nonzero except at the VM, there will be no bound or narrow-width pentaquark state in the chiral limit and hence our approach anchored on HLS/VM does not go over to the rigid rotor picture even in that limit.

In summary, when the vector mesons are incorporated à la Harada-Yamawaki HLS with the VM, a bound state or a sharp resonance can arise in the Callan-Klebanov picture of pentaquark baryons for the range of values for $a$ implied by a variety of considerations, $1 \lesssim a \lesssim 1.4$. When quantized, it has the quantum numbers $I = 0$, $J^P = \frac{1}{2}^+$ of the $\Theta^+$ pentaquark in question. Here, the vector mesons play a very important role through a simple mechanism of level repulsion which softens the WZ terms. We have achieved this result without affecting the successful description of the $S < 0$ hyperons [8,13].

Now the question is what our model can say about the chiral soliton structure of the putative $\Theta^+$ believed to be seen in the kaon-nucleon channel. Within the given scheme, the binding appears to be quite robust as long as the value of $a$ is near 1: It depends little on other parameters of the theory as long as they are reasonable for nonexotic baryons. However, whether the system is bound or not is extremely sensitive to the value of $a$. Suppose that $a$ is such that the system is bound. When the skyrmion is collective-quantized so that both $\Theta^+$ and nucleon have the proper quantum numbers, the bound $K^+$-soliton system will turn into a bound state lying below the $KN$ threshold. This is clearly not the $\Theta^+$ resonance one is talking about. To the best of our knowledge, such a bound state has not been seen in experiments. This however does not necessarily mean that it does not exist. It could be that the coupling to the kaon-nucleon continuum is much too weak to produce such a state. On the other hand, if the bound $K^+$-soliton complex could acquire additional mass by some—so far unknown—repulsion mechanism, such that its mass lies above the $KN$ threshold, it then could produce a narrow-width Feshbach-type resonance discussed by Jaffe and Jain [24]. One could think of the bound $K^+$ soliton as an analog of the bound diquark-antiquark state $[ud]^2\tilde{s}$ viewed as a Castillejo-Dalitz-Dyson pole discussed in [24] with the Pauli-blocking repulsion hypothesized in [25] playing the role of the “unknown mechanism” in our picture. At present, we have no idea how this repulsion can be implemented in our model.

A more plausible possibility is that $a$ lies near 1.4 at which a near-threshold resonance is formed. The width for that resonance will be dictated by the $O(N^3)$ equation of motion with higher order corrections strongly suppressed, so it can be tiny as indicated in Fig. 2. To check whether or not this description is viable will require the collective quantization. Even if this can provide a description of the system, it will however be unsatisfactory unless one can understand why $a$ is “fine-tuned” to a particular value. The answer may lie in understanding why nature favors $a$ near the Georgi vector limit value $a = 1$ [20].

Finally, if the notions of HLS with VM and the soliton structure for the pentaquarks are correct, since the HLS/VM theory moves towards the VM fixed point as matter density increases [26], in particular, as $a$ quickly approaches 1, the pentaquark would be definitely bound in dense medium if it were only a resonance in free space. The spectroscopy of the pentaquark multiplets is being worked out and will be a subject of a forthcoming report, together with details left out in this short article.

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