Electromagnetic radiation of baryons containing two heavy quarks

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The two heavy quarks in a baryon which contains two heavy quarks and a light one can constitute a scalar or axial vector diquark. We study the electromagnetic radiation of such baryons: (i) $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$, (ii) $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$, (iii) $\Xi^{**}_{(bc)_0}(1/2,l=1)\to\Xi_{(bc)_0}+\gamma$, (iv) $\Xi^{**}_{(bc)_0}(3/2,l=1)\to\Xi_{(bc)_0}+\gamma$, and (v) $\Xi^{**}_{(bc)_0}(3/2,l=2)\to\Xi_{(bc)_0}+\gamma$, where $\Xi_{(bc)_{0(1)}},\Xi^*_{(bc)_1}$ are *S*-wave bound states of a heavy scalar or axial vector diquark and a light quark, and $\Xi^{**}_{(bc)_0}(l\geqslant 1)$ are *P*- or *D*-wave bound states of a heavy scalar diquark and a light quark. The analysis indicates that these processes can be attributed to two categories and the physical mechanisms which are responsible for them are completely distinct. Measurements can provide good judgment for the diquark structure and a better understanding of the physical picture.

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I. INTRODUCTION

The lack of an effective way to properly handle nonperturbative QCD effects becomes a more and more intriguing problem when one needs to extract information from data. In other words, the hadronic matrix elements cannot be reliably estimated in the present theoretical framework. Thanks to the heavy quark effective theory (HQET) [1], an extra symmetry $\mathrm{SU}(2)_s\otimes\mathrm{SU}(2)_f$ greatly simplifies the picture in heavy flavor involved processes. Developments in this field enable us to more accurately evaluate hadronic transition matrix elements since the number of form factors is reduced in the heavy quark limit [2].

As many authors have suggested, a diquark structure may exist in baryons [3]. If it is real physics or at least a good approximation, we only need to deal with two-body problems instead of three-body problems. Consequently, the number of independent form factors can be remarkably reduced. Especially when the baryons contain two heavy quarks, it is reasonable to assume that the two heavy quarks constitute a color-antitriplet bosonlike diquark of spin 0 or 1 [4]. Based on this picture Savage and Wise studied the spectrum of baryons with two heavy quarks [5] and in the potential model, the spectra have been evaluated [6].

Although the diquark structure is very likely, the small color-antitriplet system is not point like in general. Consequently, we should replace the vertex gained from any fundamental theory such as the standard model by an effective vertex. A (or a few) reasonable form factor(s) will be involved in the effective vertex for compensating the nonpointlike spatial dispersion of the diquark. The form factor(s) can be derived in many ways, and one of them is the Bethe-Salpeter (BS) equation. With the effective vertex, we estimated the production and weak decay rates of such baryons [7] in our previous work based on the superflavor symmetry [8,9].

To further investigate the diquark structure and governing mechanisms inside the diquark, we will study the electromagnetic radiation of baryons with two heavy quarks in the present work. Since such processes are cleaner, we may expect to gain more exact knowledge from the data. In fact, similar electromagnetic radiation processes for baryons containing only one heavy quark have been discussed in the literature recently [10].

At the tree level, the γ emission is a pure electromagnetic process. In this work we study two cases which in fact are determined by completely different mechanisms. First, we consider (i) $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$ and (ii) $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$, where $\Xi_{(bc)_1}$ and $\Xi^*_{(bc)_1}$ are spin-1/2 and -3/2 baryons, respectively, which consist of a heavy axial vector diquark and a light quark in the *S*-wave bound state and $\Xi_{(bc)_0}$ is a spin-1/2 baryon which consists of a heavy scalar diquark and a light quark. Then we study (iii) $\Xi^{**}_{(bc)_0}(1/2,l=1)\to\Xi_{(bc)_0}+\gamma$, (iv) $\Xi^{**}_{(bc)_0}(3/2,l=1)\to\Xi_{(bc)_0}+\gamma$, and (v) $\Xi^{**}_{(bc)_0}(3/2,l=2)\to\Xi_{(bc)_0}+\gamma$ where $\Xi^{**}_{(bc)_0}(s,l\geqslant 1)$ are spin-1/2 ($s=1/2$) and -3/2 ($s=3/2$) baryons, respectively, composed of a heavy scalar diquark and a light quark in higher angular momentum states. It is noted that we study the $(bc)_{1(0)}$ diquark because only (bc) can constitute either spin-1 or -0 states with even parity (i.e., the orbital angular momentum between Q and Q' is set to be 0 in our discussion).

In the reactions (i) $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$ and (ii) $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$, the axial vector $(bc)_1$ transits into a scalar $(bc)_0$ by emitting a photon, whereas in the radiation (iii) $\Xi^{**}_{(bc)_0}(1/2,l=1)\to\Xi_{(bc)_0}+\gamma$, (iv) $\Xi^{**}_{(bc)_0}(3/2,l=1)\to\Xi_{(bc)_0}+\gamma$, and (v) $\Xi^{**}_{(bc)_0}(3/2,l=2)\to\Xi_{(bc)_0}+\gamma$, the diquark $(bc)_0$ remains in the spin-0 state, and the photon is radiated from the light quark hand. The latter three reactions are analogous to the radiation of an atom where the electron transits from a higher (angular and/or radial) excited state into a lower one and emits a photon. In our case, the light quark of $\Xi^{**}_{(bc)_0}(s,l\geqslant 1)$ in an angular momentum excited state transits into the ground state ($l=0$) $\Xi_{(bc)_0}$ and emits a photon. Analysis indicates that the possibility of radiating a photon from the spin-0 heavy diquark is very small, exactly as in the case of atoms.

Of course, in general, there may be processes like $\Xi^*_{(bc)_1}(3/2)\to\Xi_{(bc)_1}(1/2)+\gamma$. However, since the spin interaction between gluons and heavy diquarks decouples in the heavy quark limit, the mass splitting between $\Xi^*_{(bc)_1}$ and $\Xi_{(bc)_1}$ is 0. Consequently, in the heavy quark limit, a radiative transition between these two states is forbidden by the null phase space. So we do not discuss such processes in this work.

In the next section, we present our formulation for the two different radiation mechanisms and in Sec. III, we give the numerical results. The last section is devoted to a discussion and conclusion and finally in the Appendix, we give all the concerned expressions which are omitted in our context.

II. FORMULATION

In this section, we discuss the two different mechanisms, respectively.

A. Radiation from the heavy diquark hand

As discussed in the Introduction, for the radiation processes $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$ and $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$, the axial vector diquark transits into a scalar diquark by emitting a photon and the light quark remains as a spectator. In this case, all the nonperturbative effects can be attributed to a form factor at the leading order of expansion with respect to the heavy quark mass. To evaluate the transition matrix elements, we employ superflavor symmetry [8,9], which is applicable to this situation.

At the effective vertex $AS\gamma$, where A and S denote axial vector and scalar diquarks, respectively, and γ is the emitted photon, a form factor can be derived in terms of the BS equation [7]. The transition amplitude can be written as

$$T=\epsilon^*_\alpha\langle J^\alpha\rangle, \tag{1}$$

where ϵ^*_α is the polarization vector of the axial vector diquark, J^α is the effective current at the quark level, and $\langle J^\alpha\rangle$ is the corresponding transition amplitude.

For $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$,

$$\begin{aligned}\langle J^\alpha\rangle&=\langle\Xi_{(bc)_0}(v')|J^\alpha|\Xi_{(bc)_1}(v)\rangle\\&=\xi(v'\cdot v)if\epsilon^{\alpha\delta\rho\sigma}v_\rho v'_\sigma\overline{u'}(v')\gamma_5\gamma_\delta u(v),\end{aligned} \tag{2}$$

and for $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$,

$$\begin{aligned}\langle J^\alpha\rangle&=\langle\Xi_{(bc)_0}(v')|J^\alpha|\Xi^*_{(bc)_1}(v)\rangle\\&=\xi(v'\cdot v)if\epsilon^{\alpha\delta\rho\sigma}v_\rho v'_\sigma\overline{u'}(v')u_\delta(v),\end{aligned} \tag{3}$$

where $\xi(v\cdot v')$ is the Isgur-Wise function, v and v' are the four-velocities of the parent and daughter baryons, respectively, u is the four-component spinor for the parent or produced baryon $\Xi_{(bc)_{1(0)}}$, and u_δ is the Rarita-Schwinger spinor vector corresponding to $\Xi^*_{(bc)_1}$ with spin 3/2. The form factor is evaluated in the BS equation approach and all the details were given in our previous work [7].

Obviously, for expressions (2),(3), we have

$$k_\alpha\langle J^\alpha\rangle=0,$$

where $k_\alpha\equiv(m'v'-mv)_\alpha$ is the energy-momentum vector of the emitted photon. This equality guarantees the current conservation; it is the Ward identity which assures U(1) gauge invariance and the emitted photon is transverse.

Taking the amplitude squared, we have, for $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$,

$$\begin{aligned}\frac{1}{2}\sum_{all\ spins}|T|^2=\frac{e^2}{36}|\xi(v\cdot v')|^2&\\ \times\mathrm{Tr}[C^{\rho\sigma}C^{\rho'\sigma'}\epsilon_{\alpha\delta\rho\sigma}\epsilon_{\alpha'\delta'\rho'\sigma'}\overline{u'}&\\ \times\gamma_5\gamma^\delta u\bar{u}\gamma^{\delta'}\gamma_5u']\sum_\lambda\epsilon^\alpha_{(\lambda)}\epsilon^{*\alpha'}_{(\lambda)}&\end{aligned} \tag{4}$$

and, for $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$,

$$\begin{aligned}\frac{1}{4}\sum_{all\ spins}|T|^2=\frac{e^2}{36}|\xi(v\cdot v')|^2\mathrm{Tr}[C^{\rho\sigma}C^{\rho'\sigma'}\epsilon_{\alpha\delta\rho\sigma}\epsilon_{\alpha'\delta'\rho'\sigma'}&\\ \times\overline{u'}u^\delta\overline{u}^{\delta'}u']\sum_\lambda\epsilon^\alpha_{(\lambda)}\epsilon^{*\alpha'}_{(\lambda)},&\end{aligned} \tag{5}$$

where

$$C^{\rho\sigma}=fv^\rho v'^\sigma \tag{6}$$

and, numerically,

$$f\sim 1.$$

In our case, the photon emitted from the heavy diquark only carries very small momentum and energy; thus $v\cdot v'$ would be very close to unity, so

$$\xi(v\cdot v')\approx 1.$$

Then we can easily obtain the widths of these radiative decay processes as

$$\begin{aligned}\Gamma=\frac{1}{2M}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E}\frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}(2\pi)^4&\\ \times\delta^4(P-p-k)\frac{1}{2s+1}\sum_{all\ spins}|T|^2,&\end{aligned} \tag{7}$$

where P, p, and k are the four-momenta of the initial, final baryons, and emitted photon, respectively, and M is mass of of the initial baryon. Because it is a two-body final state case, the integration is very easy to carry out.

B. Radiation from the light quark hand

In this case, $\Xi^{**}_{(bc)_0}(s,l\geqslant 1)$ are composed of a scalar diquark and a light quark in a higher angular momentum state ($l\geqslant 1$); thus the radiation is realized via a process that the light quark transits from a higher angular momentum state into the ground state ($l=0$) via emitting a photon. This process is analogous to the photon radiation of atoms where the electron jumps from an excited state (radial or $l\geqslant 1$) into the ground state via emitting a photon.

In these processes, the heavy diquark acts as a spectator. Since the reaction happens on the light flavor side, HQET is not applicable in this case. Instead, we use the BS equation to calculate the transition matrix elements. For consistency, the wave functions of $\Xi^{**}_{(bc)_0}[1/2(3/2),l=1,2]$ and $\Xi_{(bc)_0}(1/2,l=0)$ are also obtained in terms of the BS equation. The wave functions are given in the following:

$$\kappa_P^{(1/2,0)}(p)=(\phi_1^{(10)}(p)+\phi_2^{(10)}(p)\not{p}_t)u(P) \quad \left(s=\frac{1}{2},l=0\right), \tag{8}$$

$$\kappa_P^{(1/2,1)}(p)=(\phi_1^{(11)}(p)+\phi_2^{(11)}(p)\not{p}_t)\gamma_5 u(P) \quad \left(s=\frac{1}{2},l=1\right), \tag{9}$$

$$\kappa_P^{(3/2,1)}(p)=(\phi_1^{(31)}(p)+\phi_2^{(31)}(p)\not{p}_t)p_{t\mu}u^{\mu}(P) \quad (s=3/2,l=1), \tag{10}$$

$$\kappa_P^{(3/2,2)}(p)=[\phi_1^{(32)}(p)+\phi_2^{(32)}(p)\not{p}_t]\gamma_5 p_{t\mu}u^{\mu}(P) \quad \left(s=\frac{3}{2},l=2\right), \tag{11}$$

where $u(P)$ is the spinor for the baryon of spin 1/2 and $u^{\mu}(P)$ is the Rarita-Schwinger spinor vector. Here we use the transverse momentum p_t which is defined as

$$p_t^{\mu}=p^{\mu}-p_l v^{\mu}, \tag{12}$$

and v^{μ} is the four-velocity of the concerned baryon; $p_l\equiv p\cdot v$ is the longitudinal momentum.

The vertex $\overline{q}q\gamma$ is the typical QED coupling. Taking the loop integration with the obtained BS wave functions we can have the transition amplitude squared as the following:

For $\Xi^{**}_{(bc)_0}(1/2,l=1)\to\Xi_{(bc)_0}+\gamma$,

$$\frac{1}{2}\sum_{all\ spins}|T|^2=\frac{e^2}{2}\sum_{all\ spins}|\overline{u}(v')G^{\mu}u(v)\epsilon_{\mu}^{(\lambda)*}|^2, \tag{13}$$

where

$$G^{\mu}\equiv G_1\gamma^{\mu}\gamma_5+G_2\gamma^{\mu}\not{v}_t'\gamma_5+G_3\not{v}_t'\gamma^{\mu}\gamma_5+G_4\times(-2\gamma^{\mu}\gamma_5+\not{v}\gamma^{\mu}\gamma_5)+G_5\not{v}_t'\gamma^{\mu}\not{v}_t'\gamma_5. \tag{14}$$

For $\Xi^{**}_{(bc)_0}(3/2,l=1)\to\Xi_{(bc)_0}+\gamma$,

$$\frac{1}{4}\sum_{all\ spins}|T|^2=\frac{e^2}{4}\sum_{all\ spins}|\overline{u}(v')H^{\mu}_{\nu}u^{\nu}(v)\epsilon_{\mu}^{(\lambda)*}|^2, \tag{15}$$

where

$$H^{\mu}_{\nu}\equiv H_1\gamma^{\mu}v'_{\nu}+H_3\gamma^{\mu}\not{v}_t'v'_{\nu}+2H_4 g^{\mu}_{\nu}+H_5\not{v}_t'\gamma^{\mu}v'_{\nu}+H_7\not{v}_t\gamma^{\mu}\not{v}_t'v'_{\nu}. \tag{16}$$

For $\Xi^{**}_{(bc)_0}(3/2,l=2)\to\Xi_{(bc)_0}+\gamma$,

$$\frac{1}{4}\sum_{all\ spins}|T|^2=\frac{e^2}{4}\sum_{all\ spins}|\overline{u}(v')F^{\mu}_{\nu}\gamma_5 u^{\nu}(v)\epsilon_{\mu}^{(\lambda)*}|^2, \tag{17}$$

where

$$F^{\mu}_{\nu}=F_1\gamma^{\mu}v'_{\nu}+F_3\gamma^{\mu}\not{v}_t'v'_{\nu}+2F_4 g^{\mu}_{\nu}+F_5\not{v}_t'\gamma^{\mu}v'_{\nu}+F_7\not{v}_t\gamma^{\mu}\not{v}_t'v'_{\nu}. \tag{18}$$

All the coefficients G_i, H_i, and F_i in Eqs. (14), (16), (18) are related to the integrals involving the BS wave functions $\tilde{\Phi}(p_t)^i$ and $\tilde{\Phi}(p_t')^f$ which correspond to the initial and final baryons, respectively. In [7] we set up the formalism for the BS approach where the kernel, which is controlled by nonperturbative QCD effects, is assumed to consist of both scalar confinement and one-gluon-exchange terms. Then the numerical solutions for the BS wave functions were found by solving the integral equations numerically in the so-called covariant instantaneous approximation, which guarantees the covariance of our formalism. The derivation is very tedious, so here we only give the explicit expressions in the Appendix. We would like to make a comment on the the integrals of the BS wave functions. Normally the initial and final states are not in the same frame; their wave functions have different arguments p_t and p_t', which correspond to the relative transverse momenta of the constituents in the initial and final baryons, respectively. Although they can be related if we consider the light quark (or heavy diquark) as a spectator, it is still hard work to do the integrations. Fortunately, the recoil is very small in our case; it is a very good approximation to keep the leading order in the expansion of $v\cdot v'-1$ through our calculations. Obviously $|p_t'|-|p_t|$ is proportional to $(v\cdot v'-1)^{\kappa}$ ($\kappa>0$). We can also expand the wave functions in this way. The wave functions and their derivatives can be obtained numerically by solving the BS equations.

Unlike the case of radiation from the heavy diquark hand, the gauge invariance is slightly broken. In general, with expressions (14), (16), (18), $k_{\alpha}\langle J^{\alpha}\rangle\neq 0$. This is because all coefficients are related to integrals over the BS wave functions, which are solved based on the kernel where a certain approximation is taken; hence the gauge invariance would be artificially violated [11]. This violation is unphysical; namely, if we could deal with nonperturbative QCD in a proper way, which is so far impossible, the U(1) gauge invariance would be perfectly retained. However, we will show that this deviation can only be manifest in the formulation, but almost not in the numerical results.

In this situation, the emitted photon is not fully transverse, as some longitudinal component is mixed in. Of course, this longitudinal component is not physical. This artificial U(1) gauge invariance violation is due to the approximation in the BS formalism. The crucial point is how much the unphysical part is in our BS approach, in other words, to what extent it influences the width evaluation. Probing the degree of U(1) gauge invariance violation is equivalent to checking the Ward-identity violation, which is directly related to the fraction of the unphysical longitudinal component of the photon polarization. The amplitude of transition is $\epsilon_\alpha \langle J^\alpha \rangle$ where $\langle J^\alpha \rangle$ is obtained in the BS formalism. Then we can replace the photon polarization vector ϵ_α by its four-momentum k_α. If the Ward identity is respected, $k_\alpha \langle J^\alpha \rangle$ should be zero exactly. Its deviation from zero indicates violation of the Ward identity and is what we want to know. $\langle J^\alpha \rangle$ includes the BS integrals which can only be computed numerically. In this way, we obtain the following equation which can be used to estimate the U(1) gauge violation:

$$k_\alpha \langle J^\alpha \rangle = a(m-m') + b\overline{m}(v-v')^2, \tag{19}$$

where m,v and m,v' are the masses and four-velocities of the initial and final baryons and $\overline{m}$ is $(m+m')/2$; a,b are two constants which are composed of complicated BS integrals and obtained in the above derivations. We numerically evaluate them and confirm that they are of order $O(1)$. It indicates that the gauge invariance violation is related to the differences of masses and four-velocities of the initial and final baryons. Since the recoil is very small, which is because the mass difference $(m-m')/m$ is small, we keep only the leading order in the expansion of $v\cdot v'-1$ through our calculations; the breakdown of the gauge invariance in the formulation has little effect on the numerical results of the decay widths. We will come to this point in the last section for further discussion.

The partial width is obtained in the same way as in Sec. II A.

III. NUMERICAL RESULTS

A. Radiation from the heavy diquark hand

Since there are yet no data for the masses of baryons containing two heavy quarks, we have to take the theoretically estimated values which are given in the literature. Here we use the results given by Ebert *et al.* [6] as $M_{\Xi^*_{(bc)}} = 7.02$ GeV and $M_{\Xi_{(bc)}} = 6.95$ GeV. We have

$$\Gamma(\Xi_{(bc)_1} \to \Xi_{(bc)_0} + \gamma) \sim 2.75\times 10^{-9} \text{ GeV},$$

$$\Gamma(\Xi^*_{(bc)_1} \to \Xi_{(bc)_0} + \gamma) \sim 7.48\times 10^{-9} \text{ GeV}.$$

Namely, the widths are of the order of eV's.

B. Radiation from the light quark hand

For consistency, we have also obtained the binding energies of the baryons concerned in terms of the BS equation. We have

$$E_{\Xi^{**}_{(bc)_0}(3/2,l=2)} = 1.39 \text{ GeV}, \quad E_{\Xi^{**}_{(bc)_0}(3/2,l=1)} = 0.69 \text{ GeV},$$

$$E_{\Xi^{**}_{(bc)_0}(1/2,l=1)} = 0.66 \text{ GeV}, \quad E_{\Xi_{(bc)_0}(1/2,l=0)} = 0.026 \text{ GeV}.$$

In this framework, we have

$$M_{\Xi^{**}_{(bc)_0}(s,l)} = m_1 + m_2 + E_{\Xi^{**}_{(bc)_0}(s,l)},$$

$$M_{\Xi_{(bc)_0}} = m_1 + m_2 + E_{\Xi_{(bc)_0}},$$

where m_1 and m_2 are the masses of the light quark and the heavy scalar diquark, respectively, and E is the binding energy. To evaluate the binding energies, we take the simplest potential form which contains only the Coulomb and linear confinement pieces as the BS kernel [7].

Numerically, we take

$$m_1 = 0.33 \text{ GeV (for } u \text{ and } d \text{ quark)},$$
$$0.5 \text{ GeV (for } s \text{ quark)},$$
$$m_2 = 6.52 \text{ GeV},$$

as inputs [3].

We use these values in the numerical evaluations and obtain

$$\Gamma(\Xi^{**}_{(bc)_0}(1/2,l=1) \to \Xi_{(bc)_0}(1/2) + \gamma) \sim 1.5\times 10^{-4} \text{ GeV},$$

$$\Gamma(\Xi^{**}_{(bc)_0}(3/2,l=1) \to \Xi_{(bc)_0}(1/2) + \gamma) \sim 3.7\times 10^{-5} \text{ GeV},$$

$$\Gamma(\Xi^{**}_{(bc)_0}(3/2,l=2) \to \Xi_{(bc)_0}(1/2) + \gamma) \sim 6.2\times 10^{-4} \text{ GeV}.$$

As discussed above, these partial widths are evaluated in terms of the BS equation. Indeed these reactions are governed by a mechanism different from that in Sec. III A, and the methods we use for evaluating the widths are distinct.

In this subsection, we obtain the masses of $\Xi^{**}_{(bc)_0}[1/2(3/2),l\geqslant 1]$ and $\Xi_{(bc)_0}(1/2)$ and the transition matrix element $\langle \Xi_{(bc)_0}(1/2)|J_\mu|\Xi^{**}_{(bc)_0}[1/2(3/2),l\geqslant 1]\rangle$ in the same framework, i.e., the BS equation. In fact, there is no any substantial difference from the values we take in Sec. III A for $\Xi_{(bc)_1} \to \Xi_{(bc)_0} + \gamma$ and $\Xi^*_{(bc)_1} \to \Xi_{(bc)_0} + \gamma$.

It is noted that the mass difference between the angular momentum excited state $\Xi^{**}_{(bc)_0}(3/2,l\geqslant 1)$ and the ground S

state $\Xi_{(bc)_0}$ is about 0.6–1.4 GeV. It is much larger than that between $\Xi^*_{(bc)_1}$ and $\Xi_{(bc)_0}$ (0.07 GeV). This is easy to understand: the former one is due to the orbital angular momentum excitation and the latter one is due to an energy splitting between axial vector and scalar diquarks, which is caused by the spin-spin interaction. Therefore for $\Xi^{**}_{(bc)_0}(s,l\geqslant 1)\rightarrow\Xi_{(bc)_0}+\gamma$ the threshold effects are not obvious and the widths are about four orders of magnitude larger than $\Gamma(\Xi_{(bc)_1}(\Xi^*_{(bc)_1})\rightarrow\Xi_{(bc)_0}(1/2)+\gamma)$. In other words, the remarkable width difference for the two processes is due to threshold effects while the matrix elements for both reactions are of the same order of magnitude.

IV. CONCLUSION AND DISCUSSION

HQET is proved to be effective in many processes where heavy flavors are involved. In most cases, the light flavors in the hadrons just behave as spectators for the reactions and these degrees of freedom are manifest in the hadronization processes, and therefore determine the form factors such as the Isgur-Wise function. However, in some cases, the light flavors may participate in reactions and sometimes can play a crucial role. As we know, when the quark level final state interaction is involved, W annihilation and especially Pauli interference can be very important in inclusive B meson decays [12,13]; then the contribution from the light flavor could be as important as that from the heavy one.

In this work, we chose two different kinds of processes where the heavy and light flavors are active, respectively. $\Xi_{(bc)_1}$ and $\Xi^*_{(bc)_1}$ consist of an axial vector diquark and a light quark. When they transit into $\Xi_{(bc)_0}$ by radiating a photon, the axial vector diquark turns into a scalar one, and the light quark serves as a spectator in this process. On the contrary, $\Xi^{**}_{(bc)_0}$ $[1/2(3/2),l\geqslant 1]$ consists of a scalar heavy diquark and a light quark at angular momentum excited states ($l=1,2$ in this work). Thus, when it transits into $\Xi_{(bc)_0}$, the heavy diquark stands as a spectator and the light quark jumps from a higher-excited state into the ground state while radiating a photon. For the former one, HQET definitely applies, and by superflavor symmetry, we can expect to obtain a more accurate result of the decay width. Once the doubly heavy baryon masses are measured, we can immediately have the final numbers with our formula for the partial width. As long as HQET works, the result should be close to the data. Of course, there is also an uncertain factor; it is the form factor at the effective vertex of $SA\gamma$. We obtain it in terms of the BS equation, where the potential kernel would bring up some uncertainty. However, in this case, the diquark is composed of two heavy quarks, so the nonrelativistic Cornell potential works well as understood. Moreover, careful studies indicate that for so small a recoil situation $(v\cdot v')\sim 1$, the form factor f is close to 1. Therefore, we can expect that the relative errors for the partial widths of $\Xi_{(bc)_1}\rightarrow\Xi_{(bc)_0}+\gamma$ and $\Xi^*_{(bc)_1}\rightarrow\Xi_{(bc)_0}+\gamma$ are quite small. The widths are of the order of eV's and similar to that for atomic radiation. The smallness is easy to understand. Let us use $\Xi^*_{(bc)_1}\rightarrow\Xi_{(bc)_0}+\gamma$ as an example. From Eq. (5), we have

$$\frac{1}{4}\sum_{all\ spins}|T|^2=\frac{4e^2}{27}f^2M_{1/2}M'_{3/2}(v\cdot v'-1)(1+v'\cdot v)^2, \tag{20}$$

where $M_{1/2}$ and $M'_{3/2}$ are the masses of $\Xi_{(bc)}(1/2)$ and $\Xi^*_{(bc)}(3/2)$, respectively. In this case, $v\cdot v'-1$ is close to zero and it is nothing but the threshold effect. With this expression, we can easily obtain the partial width of this radiative decay as

$$\Gamma=\frac{\alpha}{216}f^2\frac{(M'^2_{3/2}-M^2_{1/2})^3}{M'^5_{3/2}M^2_{1/2}}(M'_{3/2}+M_{1/2})^2. \tag{21}$$

It is noted that the width is proportional to $(M'^2_{3/2}-M^2_{1/2})^3/M'^5_{3/2}$; hence for a small difference between the masses of the parent and daughter baryons, the threshold effects are very obvious. One can expect these threshold effects to strongly suppress the width.

As a matter of fact, these radiative decay processes where the heavy axial vector diquark emits a photon and transits into a scalar one are analogous to the radiative decay $J/\psi\rightarrow\eta_c+\gamma$ whose partial width is about 1.13 keV [15]. But there are several suppression factors in the doubly heavy baryon case. First, in J/ψ, c and $\bar{c}$ reside in a color singlet, but in the diquark, b and c quarks are in a color $\bar{3}$ state; there should be a factor of 1/8 suppression for the diquark transition. From the formula (21), one has a factor $(M'^2_{3/2}-M^2_{1/2})/M'^5_{3/2}$, so totally there could be a suppression of about 5×10^{-3} compared to the J/ψ radiative decay. The net result is of eV order.

For $\Xi^{**}_{(bc)_0}$ $[1/2(3/2),l\geqslant 1]\rightarrow\Xi_{(bc)_0}+\gamma$, HQET does not apply and we need to employ the BS equation method to evaluate the transition matrix elements. In the calculations, the BS wave functions of the initial and final states are needed. Since in such radiative decays the recoil energy momentum of the final baryon is very small compared to the involved energy scales, we expect the theoretical predictions to be quite reliable.

It is noted that Eqs. (2), (3) are derived in terms of superflavor symmetry where the Ward identity holds and gauge invariance is assured. When we derive Eqs. (14), (16), (18), the subprocess of radiating a photon from the light quark hand is a typical QED vertex, which rigorously guarantees the Ward identity, but at the hadron level, as a result of a lack of knowledge about properly dealing with nonperturbative QCD effects, we adopt the BS equation's kernel motivated from the non-relativistic potential model. This leads to artificial violation of the U(1) gauge invariance. However, as shown in Eq. (19), the gauge invariance violation is related to $(m-m')/m$ or $(v-v')^2$, and both of them are very small in our case, so we can also expect that the violation degree of the U(1) gauge invariance little affects the decay-width evaluation, even though the formulation does not assure gauge invariance. This is similar to photon emission from an atom at rest.

The numerical results show that for $\Xi_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$ and $\Xi^*_{(bc)_1}\to\Xi_{(bc)_0}+\gamma$, the partial width is of the order of eV's, and for $\Xi^{**}_{(bc)_0}[3/2(1/2),l=1(2)]\to\Xi_{(bc)_0}(1/2)+\gamma$, it is of 10–100 keV. The difference is due to threshold effects.

In addtion to the study of the reaction mechanisms, this work also concerns elucidating the diquark structure in baryons. It is believed that the two heavy quarks inside a baryon can constitute a diquark of a scalar or axial vector which is a relatively stable physical subject [7]. Our calculations are based on such a physical picture and future experiments should test it. One point is definite: that the large difference of the decay widths corresponding to the emission from the light quark hand and the heavy diquark hand is not caused by different mechanisms, but threshold effects. As we discussed above, the mass differences between $\Xi_{(bc)_1}(\Xi^*_{(bc)_1})$ and $\Xi_{(bc)_0}$ are small because the light quark resides at the ground state $(l=0)$, whereas the mass difference between $\Xi^{**}_{(bc)_0}(s,l\geqslant 1)$ and $\Xi_{(bc)_0}$ is sizable. Expression (21) explicitly indicates this point. Indeed the diquark picture provides all information and the order of magnitude as long as the the phase space factors are reasonably removed.

A lack of data on baryons which consist of two heavy quarks so far makes drawing a definite conclusion difficult. But it is possible that data can be accumulated in the near future experiments. Once we have the data on the masses, we can reevaluate the numbers of decay widths easily. Then, comparing the calculated results with the data, we can determine the validity of the diquark structure and the reaction mechanisms. No doubt, the experiments for electromagnetic radiation are difficult, but as suggested [14], the radiative decay may be measurable soon, and the background in this case is clean. We believe that the results can enrich our knowledge on baryons, so it is worthy of careful investigations.

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APPENDIX

Here we present the explicit expressions of the form factors G_i, H_i and F_i in Eqs. (14),(16),(18).

$$G_i\equiv\int\frac{dp_l}{2\pi}g_i,\quad H_i\equiv\int\frac{dp_l}{2\pi}h_i,\quad F_i\equiv\int\frac{dp_l}{2\pi}f_i, \tag{A1}$$

$$g_1=\int\frac{d^3p_t}{(2\pi)^3}ac',$$

$$g_2=\frac{1}{\sqrt{(v\cdot v')^2-1}}\int\frac{d^3p_t}{(2\pi)^3}ad'|\vec{p}_t|\cos\theta,$$

$$g_3=\frac{1}{\sqrt{(v\cdot v')^2-1}}\int\frac{d^3p_t}{(2\pi)^3}bc'|\vec{p}_t|\cos\theta,$$

$$g_4=\frac{-1}{2}\int\frac{d^3p_t}{(2\pi)^3}bd'|\vec{p}_t|^2(1-\cos^2\theta),$$

$$g_5=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}bd'|\vec{p}_t|^2(1-3\cos^2\theta),$$

$$h_1=\frac{1}{\sqrt{(v\cdot v')^2-1}}\int\frac{d^3p_t}{(2\pi)^3}ac''|\vec{p}_t|\cos\theta,$$

$$h_2=\frac{-1}{2}\int\frac{d^3p_t}{(2\pi)^3}ad''|\vec{p}_t|^2(1-\cos^2\theta),$$

$$h_3=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}ad''|\vec{p}_t|^2(1-3\cos^2\theta),$$

$$h_4=\frac{-1}{2}\int\frac{d^3p_t}{(2\pi)^3}bc''|\vec{p}_t|^2(1-\cos^2\theta),$$

$$h_5=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}bc''|\vec{p}_t|^2(1-3\cos^2\theta),$$

$$h_6=\frac{-1}{2}\int\frac{d^3p_t}{(2\pi)^3}bd''(\lambda_2M_{(3/2,l=1)}+p_l)|\vec{p}_t|^2(1-\cos^2\theta),$$

$$h_7=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}bd''\times(\lambda_2M_{(3/2,l=1)}+p_l)|\vec{p}_t|^2(1-3\cos^2\theta),$$

$$f_1=\frac{1}{\sqrt{(v\cdot v')^2-1}}\int\frac{d^3p_t}{(2\pi)^3}ac''|\vec{p}_t|\cos\theta,$$

$$f_2=\frac{-1}{2}\int\frac{d^3p_t}{(2\pi)^3}ad|\vec{p}_t|^2(1-\cos^2\theta),$$

$$f_3=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}ad|\vec{p}_t|^2(1-3\cos^2\theta),$$

$$f_4=\frac{-1}{2}\int\frac{d^3p_t}{(2\pi)^3}bc|\vec{p}_t|^2(1-\cos^2\theta),$$

$$f_5=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}bc|\vec{p}_t|^2(1-3\cos^2\theta),$$

$$f_7=\frac{-1}{2}\frac{1}{(v\cdot v')^2-1}\int\frac{d^3p_t}{(2\pi)^3}bd(\lambda_2M_{(3/2,l=2)}+p_l)\times|\vec{p}_t|^2(1-3\cos^2\theta), \tag{A2}$$

where θ is the angle between p_t and v_t,

$$a=\left[\frac{2\omega'_{p_t}(\omega'_{p_t}-m_1-E_{(1/2,l=0)})}{(E_{(1/2,l=0)}-p'_l)+i\epsilon}\right.$$

$$\left.\times\frac{2m_1+p'_l}{(p'_l+m_1)^2-\omega'^2_{p_t}+i\epsilon}\right]\tilde{\Phi}_2^{(10)},$$

$$b=\left[\frac{2\omega'_{p_t}(\omega'_{p_t}-m_1-E_{(1/2,l=0)})}{(E_{(1/2,l=0)}-p'_l)+i\epsilon}\right.$$

$$\left.\times\frac{1}{(p'_l+m_1)^2-\omega'^2_{p_t}+i\epsilon}\right]\tilde{\Phi}_2^{(10)},$$

$$c=-i\left[\frac{2\omega_{p_t}(\omega_{p_t}-m_1-E_{(3/2,l=2)})}{(E_{(3/2,l=2)}-p_l)+i\epsilon}\right.$$

$$\left.\times\frac{-p_l}{(p_l+m_1)^2-\omega^2_{p_t}+i\epsilon}\right]$$

$$\times[2m_2(p_l-E_{(3/2,l=2)})]\tilde{\Phi}_2^{(32)},$$

$$d=-i\left[\frac{2\omega_{p_t}(\omega_{p_t}-m_1-E_{(3/2,l=2)})}{(E_{(3/2,l=2)}-p_l)+i\epsilon}\right.$$

$$\left.\times\frac{1}{(p_l+m_1)^2-\omega^2_{p_t}+i\epsilon}\right][2m_2(p_l-E_{(3/2,l=2)})]\tilde{\Phi}_2^{(32)},$$

$$c'=-i\left[\frac{2\omega_{p_t}(\omega_{p_t}-m_1-E_{(1/2,l=1)})}{(E_{(1/2,l=1)}-p_l)+i\epsilon}\right.$$

$$\left.\times\frac{-p_l}{(p_l+m_1)^2-\omega^2_{p_t}+i\epsilon}\right][2m_2(p_l-E_{(1/2,l=1)})]\tilde{\Phi}_2^{(11)},$$

$$d'=-i\left[\frac{2\omega_{p_t}(\omega_{p_t}-m_1-E_{(1/2,l=1)})}{(E_{(1/2,l=1)}-p_l)+i\epsilon}\right.$$

$$\left.\times\frac{1}{(p_l+m_1)^2-\omega^2_{p_t}+i\epsilon}\right][2m_2(p_l-E_{(1/2,l=1)})]\tilde{\Phi}_2^{(11)},$$

$$c''=-i\left[\frac{2\omega_{p_t}(\omega_{p_t}-m_1-E_{(3/2,l=1)})}{(E_{(3/2,l=1)}-p_l)+i\epsilon}\right.$$

$$\left.\times\frac{2m+p_l}{(p_l+m_1)^2-\omega^2_{p_t}+i\epsilon}\right][2m_2(p_l-E_{(3/2,l=1)}]\tilde{\Phi}_2^{(31)},$$

$$d''=-i\left[\frac{2\omega_{p_t}(\omega_{p_t}-m_1-E_{(3/2,l=1)})}{(E_{(3/2,l=1)}-p_l)+i\epsilon}\right.$$

$$\left.\times\frac{1}{(p_l+m_1)^2-\omega^2_{p_t}+i\epsilon}\right][2m_2(p_l-E_{(3/2,l=1)})]\tilde{\Phi}_2^{(31)},\tag{A3}$$

where $\tilde{\Phi}_i^{(s,l)}$ are the BS wave functions after integrating over p_l,

$$\tilde{\Phi}_i^{(s,l)}\equiv\int\frac{dp_l}{2\pi}\phi_i^{(s,l)}(p_l,p_t^2),$$

$\omega_{p_t}=\sqrt{|p_t|^2+m_1^2}$, and we have defined

$$\lambda_2=\frac{m_2}{m_1+m_2},$$

with m_1 being the light quark mass and m_2 the heavy diquark mass ($m_1\ll m_2$). $E_{(1/2,l)}$ and $E_{(3/2,l)}$ are binding energies in the corresponding baryons.

All the functions are obtained by carrying out the BS integrations which are very tedious, but straightforward (see Ref. [7]).

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