The strong isospin-breaking correction for the gluonic penguin contribution to ϵ'/ϵ at next-to-leading order in the chiral expansion

Carl E. Wolfe*
Nuclear Theory Center, Indiana University, Bloomington, Indiana 47408

Kim Maltman†
Department of Mathematics and Statistics, York University, 4700 Keele St., Toronto, Ontario, Canada M3J 1P3
and Special Research Centre for the Subatomic Structure of Matter, University of Adelaide, Australia 5005
(Received 27 July 2000; published 5 December 2000)

The strong isospin-breaking correction Ω_{st}, which appears in estimates of the standard model value for the direct CP-violating ratio ϵ'/ϵ, is evaluated to next-to-leading order (NLO) in the chiral expansion using chiral perturbation theory. The relevant linear combinations of the unknown NLO CP-odd weak low-energy constants (LEC's) which, in combination with one-loop and strong LEC contributions, are required for a complete determination at this order, are estimated using two different models. It is found that, to NLO, $\Omega_{st}=0.08\pm0.05$, significantly reduced from the ''standard'' value, 0.25 ± 0.08, employed in recent analyses. The potentially significant numerical impact of this decrease on standard model predictions for ϵ'/ϵ, associated with the decreased cancellation between gluonic penguin and electroweak penguin contributions, is also discussed.

DOI: 10.1103/PhysRevD.63.014008 PACS number(s): 13.25.Es, 11.30.Hv, 11.30.Rd, 14.40.Aq

I. INTRODUCTION

The recent improved experimental results for the ratio of direct to indirect CP-violation parameters, ϵ'/ϵ, obtained by both the KTEV and NA48 Collaborations [1,2], have spurred on continuing efforts to reduce the theoretical uncertainties in our expectations for the value of ϵ'/ϵ in the standard model. While short-distance effects are under control (the Wilson coefficients of the effective weak Hamiltonian being known to two-loop order) [3], there remains significant uncertainty in the theoretical calculation of the long-distance $K\to\pi\pi$ hadronic matrix elements. These have been estimated using a number of techniques and models in Refs. [4–10] (for recent reviews, see Refs. [11,12]).

An important ingredient in the calculation of the hadronic matrix elements is the inclusion of isospin-breaking (IB) effects. Strong isospin breaking arising from the up/down quark mass difference, $\delta m\equiv m_d-m_u$, induces a $\Delta I=3/2$ contribution to the $K\to\pi\pi$ matrix elements of the gluonic penguin operator, Q_6,[1] which, in the isospin limit, is pure $\Delta I=1/2$. This ''leakage'' of octet ($\Delta I=1/2$) strength into the $\Delta I=3/2$ component of the $K^0\to\pi\pi$ decay amplitudes has the effect of reducing the magnitude of the Q_6 contribution to ϵ'/ϵ which one obtains in the isospin-conserving (IC) limit. This is conventionally represented by a multiplicative factor $1-\Omega_{st}$ applied to the $\Delta I=1/2$ matrix element. Explicitly, one writes [3]

$$\frac{\epsilon'}{\epsilon}\sim[P^{(1/2)}-P^{(3/2)}] \tag{1}$$

with

$$P^{(1/2)}=\sum y_i\langle Q_i\rangle_0(1-\Omega_{st}),$$

$$P^{(3/2)}=\frac{1}{\omega}\sum y_i\langle Q_i\rangle_2, \tag{2}$$

where $\omega\simeq1/22$, the y_i are the parts of the Wilson coefficients associated with the top quark [and hence, through the corresponding Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, with direct CP violation], and the subscripts 0 and 2 denote the isospin of the $\pi\pi$ final state. Note that we have dropped overall factors in Eq. (1) in order to highlight the dependence on Ω_{st}. In typical analyses one finds a significant cancellation between the $\Delta I=1/2$ and $\Delta I=3/2$ contributions, and therefore a nontrivial sensitivity to Ω_{st}.

The IB correction, Ω_{st}, is obtained as follows. Writing the isospin decomposition of the $K^0\to\pi\pi$ decay amplitudes as

$$A_{00}=\sqrt{\frac{1}{3}}A_0e^{i\phi_0}-\sqrt{\frac{2}{3}}A_2e^{i\phi_2},$$

$$A_{+-}=\sqrt{\frac{1}{3}}A_0e^{i\phi_0}+\frac{1}{\sqrt{6}}A_2e^{i\phi_2}, \tag{3}$$

where A_0 and A_2 are, respectively, the (in general complex-valued) $\Delta I=1/2$ and $\Delta I=3/2$ amplitudes, and $\phi_i=\delta_i^{\pi\pi}$ are the usual $\pi-\pi$ scattering phases, Ω_{st} is given by the ratio

$$\Omega_{st}=\frac{\mathrm{Im}\,\delta A_2}{\omega\,\mathrm{Im}\,A_0} \tag{4}$$

*Email address: wolfe@niobe.iucf.indiana.edu
†Email address: maltman@fewbody.phys.yorku.ca

[1]We employ throughout the paper the notation of Ref. [3] for the four-quark operators of the effective weak Hamiltonian.

where δA_2 is the octet leakage contribution to the $\Delta I=3/2$ amplitude, and $\omega=(\mathrm{Re}A_2/\mathrm{Re}A_0)\simeq 1/22.2$ (reflecting the $\Delta I=1/2$ rule enhancement).[2]

The possibility that the octet leakage contribution to the matrix element of Q_6 induced by $\delta m\neq 0$ and electric charge differences could have a significant impact on estimates of ϵ'/ϵ was first discussed in Refs. [13,14]. At leading order (LO) in the chiral expansion, the leakage contribution to A_2 is saturated by π^0-η mixing and the kinematic effect produced by the K^0-$K^{\pm}$ mass splitting and the momentum-dependence of the LO weak vertices. This leads to the well-known LO value $\Omega_{st}=0.13$. The difference between this LO value and the conventional value employed in recent analyses of ϵ'/ϵ, $\Omega_{st}=0.25\pm 0.08$ [14], results from an estimate of those next-to-leading order (NLO) effects mediated by the η' through the $K\to\pi\eta'$ transition and π-η' mixing, which effects would be expected to be dominant in the large-N_c limit. In the framework of the conventional low-energy effective theory employed in this paper (that involving only the π, K and η degrees of freedom, in which the η' and other higher resonances have been integrated out), such effects correspond to contributions to the NLO weak low-energy constants (LEC's) (to be discussed below), although use of phenomenological values for the octet-singlet mixing angle also effectively incorporates NLO η'-mediated contributions proportional to the renormalized strong LEC, L_7^r, of Gasser and Leutwyler [15]. As has been recently pointed out, however, other NLO contributions might also be important. An example is that discussed in Ref. [16]. If one considers the effect of NLO strong dressing on the external legs in $K\to\pi\pi$, there is a large NLO η'-induced contribution (proportional to L_7^r) associated with treating π^0-η mixing at NLO. This mixing contribution, however, always occurs in the fixed combination $3L_7^r+L_8^r$ (see, for example, the expressions for the angles, $\hat{\theta}_1$ and $\hat{\theta}_2$, describing NLO mixing, given in Ref. [17]). As pointed out in Ref. [16], there is a strong numerical cancellation between the η'-induced L_7^r and the scalar-resonance-induced L_8^r contributions, clearly demonstrating the importance of including NLO contributions other than those induced by the η'. The effect of this cancellation was found, in Ref. [16], to lower Ω_{st} to 0.16 ± 0.03 (thus increasing the standard model prediction of ϵ'/ϵ by about 21% [16]). The possibility of additional non-η'-induced NLO contributions to Ω_{st} was also recently explored in Ref. [18]. The focus of this work was on IB contributions associated with NLO weak LEC's, and the numerical results suggest the possibility of very large corrections to Ω_{st} associated with scalar meson exchange. However, as we discuss below, the results of Ref. [18] suffer from important technical shortcomings which make them numerically unreliable. The significance of the possible reduction in the value of Ω_{st} suggested by Refs. [16,18] is obvious from Eqs. (1) and (2).

[2]We follow standard phase conventions in which A_0 and A_2 are real in the absence of CP-violation.

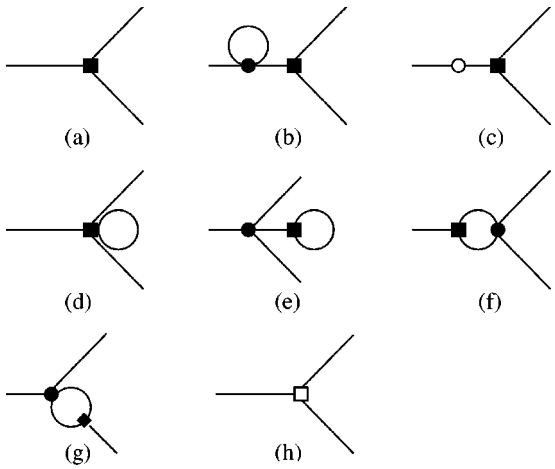

FIG. 1. Feynman diagrams for $K\to\pi\pi$ up to $\mathcal{O}(p^4)$ in the chiral expansion. Closed circles represent $\mathcal{O}(p^2)$ strong vertices, open circles $\mathcal{O}(p^4)$ strong vertices, closed boxes $\mathcal{O}(p^2)$ weak vertices, and open boxes $\mathcal{O}(p^4)$ weak vertices. No one-line weak tadpoles occur because, in the weak effective Lagrangian employed, they have already been rotated away. (b) and (c) should be understood to represent collectively the strong dressing on all the external lines.

Chiral perturbation theory (ChPT) provides a natural framework for the calculation of Ω_{st} since it ensures that all contributions of a given chiral order may be obtained in a computationally straightforward manner. The complete set of NLO contributions is a sum of NLO strong LEC, one-loop, and NLO weak LEC contributions, each of which is separately renormalization-scale-dependent, divergent, and therefore unphysical. The sum of these contributions is, however, necessarily finite and scale-independent. The cancellation of both divergences and scale-dependence in the final result provides a highly nontrivial check of the explicit calculations. The set of Feynman graphs to be evaluated is shown in Fig. 1, where Fig. 1(a) represents the LO, Fig. 1(c) the NLO strong LEC, Figs. 1(b) and 1(d)–1(g) the one-loop, and Fig. 1(h) the NLO weak LEC contributions, respectively. In Ref. [16] only the NLO contributions associated with external line dressing [Figs. 1(b,c)] were considered, while Ref. [18] examined only the contribution of Fig. 1(h).

In Refs. [19,20] it was shown that, in the CP-even sector, the inclusion of one-loop isospin-breaking corrections in $K\to\pi\pi$ decay amplitudes acts to *decrease* the magnitude of the octet leakage contribution to the $\Delta I=3/2$ amplitude. This decrease ensures that the $\Delta I=1/2$ rule enhancement obtained in an IC analysis is accurate to better than 10%. In the present paper we obtain a complete NLO $[\mathcal{O}(p^2\delta m)]$ determination of Ω_{st} in ChPT, and show that the same qualitative situation holds in the CP-odd sector. This means, according to Eq. (4), that one should expect a further reduction of Ω_{st} beyond that associated with the NLO strong mixing effects studied in Ref. [16]. The absence of reliable values for the CP-odd weak LEC's is *the* crucial stumbling block on the way to an accurate numerical result for Ω_{st} at NLO, the NLO strong LEC and loop corrections being, as we will see below, well-determined. In the present work we appeal to two models (the weak deformation model, and the chiral quark model) of the contribution of the gluonic penguin operator, Q_6, to the CP-odd weak LEC's in order to probe the prob-

able scale of the model-dependence in our estimates of Ω_{st}. It is important to emphasize that, while the estimates of the weak LEC contributions to Ω_{st} are model-dependent, the one-loop and strong LEC contributions (discussed below) are, though scale-dependent, model-independent. As we will see, the combination of these model-independent NLO contributions is, at typical hadronic scales, rather large, and negative, suggesting a significant reduction of Ω_{st} as compared to the conventional value.

The rest of the paper is organized as follows: In the next section we briefly review the chiral Lagrangian approach to the calculation of nonleptonic kaon decay amplitudes and discuss the models employed for the relevant NLO weak LEC combinations. In Sec. III we present our numerical results for Ω_{st}. The impact of our findings on theoretical estimates for the value of ϵ'/ϵ in the standard model are discussed and conclusions are presented in Sec. IV.

II. *CP*-ODD $K^0 \to \pi\pi$ DECAY AMPLITUDES IN CHIRAL PERTURBATION THEORY

The diagrams which have to be calculated to obtain the $K \to \pi\pi$ decay amplitudes to NLO are, as noted above, those given in Fig. 1. We now briefly review the ingredients needed for these calculations, referring the reader to Ref. [19–21] for the technical details. The low-energy representation of the nonleptonic weak interactions is obtained from the effective chiral Lagrangian, $\mathcal{L}_W$, which was written to LO in Ref. [22], and up to NLO in Ref. [23] (or, in equivalent reduced forms, in Refs. [24,25]). (In the CP-odd case, the gluonic penguin operator, which is the focus of the present work, and which, together with the electroweak penguin operator, dominates ϵ'/ϵ in the standard model, is pure octet; we, therefore, need only the octet components of $\mathcal{L}_W$.) We work with a form of the effective weak chiral Lagrangian in which the weak mass term appearing at LO [22] has been rotated away [23]. The LO (second order in the chiral counting) part of the octet Lagrangian, $\mathcal{L}^{(2)}_{W(8)}$, is thus given, in the absence of external fields, by [22]

$$\mathcal{L}^{(2)}_{W(8)} = c_2^{\pm} \mathrm{Tr}[\lambda^{\pm} \partial_\mu U^\dagger \partial^\mu U] \tag{5}$$

where $U = \exp(i\lambda \cdot \pi/F)$, with π^a the usual octet of pseudoscalar fields, and F is the pion decay constant in the chiral limit. The superscripts $\pm$ label the CP-even and odd cases respectively, with $\lambda^+ = \lambda_6$ and $\lambda^- = \lambda_7$. c_2^- thus represents the LO CP-odd octet weak coupling strength.

The NLO octet weak effective chiral Lagrangian is similarly given by either [24]

$$\mathcal{L}^{(4)}_{W(8)} = \frac{c_2^{\pm}}{F_\pi^2} \sum_{i=1}^{37} N_i^{\pm} O_i(\lambda^{\pm}), \tag{6}$$

or [23,25]

$$\mathcal{L}^{(4)}_{W(8)} = \sum_{i=1}^{48} E_i^{\pm} \tilde{O}_i(\lambda^{\pm}) \tag{7}$$

where the operators which contribute to the $K \to \pi\pi$ amplitudes correspond to $i = \{5,6,\ldots,13\}$ and $i = \{1,\ldots,5, 10,\ldots,15,32,\ldots,40\}$ in Eqs. (6) and (7) respectively. In quoting our results below, we will employ the notation of Eq. (6), and hence work with the operator basis given by

$$\begin{aligned}
O_5(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm}\{S, L_\mu L^\mu\}] \\
O_6(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} L_\mu]\mathrm{Tr}[S L^\mu] \\
O_7(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} S]\mathrm{Tr}[L_\mu L^\mu] \\
O_8(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} L_\mu L^\mu]\mathrm{Tr}[S] \\
O_9(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm}\{P, L_\mu L^\mu\}] \\
O_{10}(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} S^2] \\
O_{11}(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} S]\mathrm{Tr}[S] \\
O_{12}(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} P^2] \\
O_{13}(\lambda^{\pm}) &= \mathrm{Tr}[\lambda^{\pm} P]\mathrm{Tr}[P]
\end{aligned} \tag{8}$$

where $L_\mu = iU^\dagger \partial_\mu U$, $S = \chi^\dagger U + U^\dagger \chi$ and $P = i(\chi^\dagger U - U^\dagger \chi)$, with $\chi = 2B_0 M_q$ (where M_q is the quark mass matrix). Note that in Eq. (6) the weak LEC's are expressed as products of factors c_2^-/F_π^2 and N_i^-. The latter will, henceforth, be referred to as reduced CP-odd NLO weak LEC's. (As we will see below, this reduced form has certain advantages for estimates of Ω_{st}.)

The remaining ingredient needed in order to calculate the diagrams of Fig. 1 is the strong chiral Lagrangian. We use the standard form of Gasser and Leutwyler given by $\mathcal{L}_S = \mathcal{L}_S^{(2)} + \mathcal{L}_S^{(4)} + \cdots$, where the superscripts indicate the chiral order and, in the absence of external fields, one has [15]

$$\mathcal{L}_S^{(2)} = \frac{F^2}{4}\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger] + \frac{F^2}{4}\mathrm{Tr}[\chi U^\dagger + U\chi^\dagger], \tag{9}$$

$$\begin{aligned}
\mathcal{L}_S^{(4)} = {} & L_1(\mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger])^2 \\
& + L_2 \mathrm{Tr}[\partial_\mu U \partial_\nu U^\dagger]\,\mathrm{Tr}[\partial^\mu U \partial^\nu U^\dagger] \\
& + L_3 \mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger \partial_\nu U \partial^\nu U^\dagger] \\
& + L_4 \mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger]\,\mathrm{Tr}[\chi U^\dagger + U\chi^\dagger] \\
& + L_5 \mathrm{Tr}[\partial_\mu U \partial^\mu U^\dagger (\chi U^\dagger + U\chi^\dagger)] \\
& + L_6(\mathrm{Tr}[\chi U^\dagger + U\chi^\dagger])^2 \\
& + L_7(\mathrm{Tr}[\chi U^\dagger - U\chi^\dagger])^2 + L_8 \mathrm{Tr}[\chi U^\dagger \chi U^\dagger \\
& + U\chi^\dagger U\chi^\dagger] + H_2 \mathrm{Tr}[\chi\chi^\dagger],
\end{aligned} \tag{10}$$

where $\{L_i\}$, F and B_0 are the usual strong LEC's, in the notation of Ref. [15]. Recall that, when using dimensional regularization, the NLO LEC's, $\{L_i\}$, are formally divergent and have a Laurent expansion in $d-4$ (where d is the space-time dimension) of the form

$$L_i = \Gamma_i \lambda + L_i^r + L_i^{(-1)}(d-4) + \ldots \tag{11}$$

where

$$\lambda = \frac{1}{32\pi^2}\left[\left(\frac{2}{d-4}\right) + \gamma_E - 1 - \ln(4\pi)\right]. \tag{12}$$

The $\{L_i^r\}$ are the usual scale-dependent renormalized versions of the L_i [15], for which we employ the values found in Ref. [26], while the $\{\Gamma_i\}$ are constant coefficients (frequently called scaling coefficients) which govern the scale dependence of the L_i^r. The $\{L_i^{(-1)}\}$ contribute first to physical observables at next-to-next-to-leading (sixth) order (NNLO), through one-loop graphs involving a single NLO vertex proportional to L_i and, as such, are on a similar footing as the LEC's present in $\mathcal{L}_S^{(6)}$. [The NLO weak LEC's, $\{N_i^{\pm}\}$, in Eq. (6), of course, have a similar expansion.]

The *formal* difference between the $K^0 \to \pi\pi$ vertices extracted from Eq. (5) using λ^+ and λ^- is the switch $c_2^+ \to i c_2^-$. This is also, therefore, the only difference between the corresponding LO CP-even and CP-odd decay amplitudes. Since the NLO strong LEC and one-loop contributions to these amplitudes involve a single LO weak vertex, one readily sees, from Fig. 1, that the substitution $c_2^+ \to i c_2^-$ also converts the CP-even version of these contributions into the corresponding CP-odd version. The substitution $c_2^+ N_i^+ \to i c_2^- N_i^-$, similarly accomplishes the CP-even $\to$ CP-odd conversion of the weak LEC contributions, Fig. 1(h). If one considers the *ratio* of the NLO contributions to the LO contributions, therefore, the only difference between CP-even and CP-odd cases for the $K^0 \to \pi\pi$ amplitudes is the difference in the numerical values and physical interpretation of the renormalized reduced weak LEC's, N_i^r. Thus, for example, the contributions to Im δA_2 arising from the diagrams of Figs. 1(b)–1(g) are immediately obtained from the first row of Table II of Ref. [19] by multiplying the entries by $i c_2^-$. The formal contributions from the CP-odd NLO weak LEC's, $[A_0]_{WLEC}$ and $[\delta A_2]_{WLEC}$, arising from Fig. 1(h) are given in the Appendix in terms of the N_i^- of Eq. (6). Unlike the case of the CP-even sector, where linear combinations of octet NLO weak LEC's corresponding to isospin-conserving contributions were fit to the available $K \to \pi\pi$ and $K \to \pi\pi\pi$ data in Ref. [27], both the CP-odd isospin-breaking *and* isospin-conserving NLO weak LEC combinations are unknown. As there is not sufficient data available to perform fits analogous to those in the CP-even sector, it is necessary to resort to models to estimate the weak LEC contributions.

In general the numerical value of the isospin-conserving combination of NLO weak LEC's, which appears in $[A_0]_{WLEC}$ [Eq. (A1) of the Appendix], can be determined from the expressions for the hadronic matrix elements at NLO (for which many calculations exist). However the IB combination which appears in $[\delta A_2]_{WLEC}$ cannot be so determined. In what follows we consider three models which could be used to estimate the NLO weak LEC contribution to Ω_{st}. These are the weak deformation model (WDM) of Ref. [24] from which direct NLO weak LEC estimates are available, the chiral quark model (χQM) as implemented by the Trieste group in Refs. [8–10] for which all the necessary ingredients required to calculate the NLO weak LEC contributions to Im A_0 and Im δA_2 are readily available, and the ''scalar saturation model'' of Ref. [18], which combines the factorization approximation with the assumption of scalar meson exchange saturation of the relevant strong LEC's.

TABLE I. Model estimates of the NLO weak LEC combinations appearing in Eq. (A1) for the WDM and χQM as described in the text (with $r_\pi = m_\pi^2/m_K^2$).

	χQM ($\times 10^{-3}$)	WDM ($\times 10^{-3}$)
$\tilde{K}_1^- - r_\pi \tilde{K}_2^-$	-4.024	-0.673
$\tilde{J}_3^- - r_\pi \tilde{J}_4^-$	-3.571	0.673

The weak deformation model of Ref. [24] proceeds from the observation that the LO weak chiral Lagrangian of Eq. (5) can be generated from the LO strong chiral Lagrangian of Eq. (9) by a simple ''topological deformation.'' The model hypothesis is that this same deformation can be used to generate the entire $\Delta S = 1$ chiral Lagrangian. The WDM thus provides no information about the LO weak LEC values, $c^{\pm}$, but gives explicit expressions for the reduced NLO weak LEC's, $N_i^{\pm}$, in terms of the NLO strong LEC's, L_i. For the NLO weak LEC's relevant to $K \to \pi\pi$ we have [24]

$$[N_5^-]_{WDM} = -\frac{3}{2}[N_6^-]_{WDM} = -[N_7^-]_{WDM} = -L_5$$

$$[N_8^-]_{WDM} = 4L_4 + 2L_5. \tag{13}$$

All other NLO weak LEC's appearing in Eq. (6) vanish in the WDM. Since, in these relations, the divergent parts of the $\{L_i\}$ do not generate the correct divergent parts of the $\{N_i^-\}$, one must interpret Eq. (13) as applying to the *renormalized* versions of the LEC's. Moreover, since the scaling of the weak LEC's is not correctly given by that of the strong LEC's, Eq. (13) can be taken to hold only at a single scale, which is assumed to be a typical hadronic scale, μ_h. Assuming resonance dominance suggests $\mu_h \sim m_\rho$. Note also that, because of the scale dependence of the weak LEC's, even though, in the WDM, the remaining N_i^r vanish at the assumed matching scale μ_h, they are nonzero at other scales. Numerical values for the IC and IB combinations of NLO weak LEC's relevant to $K^0 \to \pi\pi$ in the WDM can be found in the last column of Table I, where we have used conventional values for the renormalized strong LEC's, L_4^r and L_5^r at $\mu = m_\rho$ taken from Ref. [26].

In another approach, the hadronic matrix elements of the four-quark operators relevant for nonleptonic kaon decay have been estimated in the χQM by the Trieste group [8–10]. In particular, expressions for the gluonic penguin contribution to the LO weak LEC's were obtained in the second of Refs. [10], and preliminary estimates of the contribution to NLO weak LEC's were given in Ref. [8]. As pointed out by the authors of Ref. [8], however, the latter expressions contain errors and are not to be used [28]. We have used the updated results of Ref. [9], which give the

contributions of Q_6 to A_0 to NLO, to extract the CP-odd weak LEC combination which enters $\mathrm{Im}\,A_0$. To obtain the IB combination of NLO weak LEC's entering δA_2, we have calculated the matrix element $\langle \pi^+\pi^0|Q_6|K^+\rangle = (\sqrt{3}/2)\,\delta A_2$ in the χQM using the formalism described in Ref. [9], together with the corrected results for the basic ingredients required to compute such matrix elements in the model given there. The model parameters which enter these estimates are the constituent quark mass, M, the vacuum quark condensate, and the gluon condensate. These parameters were constrained to reproduce the $\Delta I = 1/2$ Rule in the CP-even sector in Ref. [9] using a matching scale $\Lambda_\chi = 0.8$ GeV (the scale at which the scale-dependence of the chiral loops and that of the short-distance expressions is roughly matched in the model). The fitted parameters have the values $M = 0.2 \pm 0.02$ GeV, $\langle \bar{q}q\rangle = (-0.240^{+30}_{-10}\ \mathrm{GeV})^3$, and $\langle \alpha_s GG/\pi\rangle = (0.334 \pm 0.004\ \mathrm{GeV})^4$. The resulting values for the NLO weak LEC combinations are presented in the first column of Table I. Numerical estimates for the NLO weak LEC contributions to $\mathrm{Im}\,A_0$ and $\mathrm{Im}\,\delta A_2$ are presented in the next section.

The third approach to estimating the CP-odd weak LEC's is that of Ref. [18]. In this reference, one begins with the ''factorization approximation'' for Q_6,[3] in which the low-energy representation of Q_6 is assumed to be given by the product of the low-energy representations of the scalar densities of which the unrenormalized operator is formally a product. Since the low-energy representation of each such density is obtained by taking the derivative of $\mathcal{L}_S[\chi,\chi^\dagger]$, with respect to the appropriate component of χ or $\chi^\dagger$ (treated here as external sources) [15], the model version of the low-energy representation of Q_6 becomes the product of two such derivatives. This product can be organized by chiral order. The LO term (second order in chiral counting), arises from the product of the derivatives of $\mathcal{L}_S^{(2)}$ and $\mathcal{L}_S^{(4)}$, and leads to the conventional factorization approximation for c_2^-,

$$c_2^- = \frac{G_f}{\sqrt{2}} V_{us}V_{ud}\,\mathrm{Im}\,C_6(16B_0^2F_\pi^2L_5) \tag{14}$$

where C_6 is the Wilson coefficient accompanying Q_6 in the expressions for the effective weak Hamiltonian.[4] At NLO, having dropped the seagull contributions associated with the second derivative of $\mathcal{L}_S^{(8)}$ with respect to the sources, one is left with two types of terms, those involving a product of two derivatives of $\mathcal{L}_S^{(4)}$, and those involving a product of one derivative of $\mathcal{L}_S^{(2)}$ and one of $\mathcal{L}_S^{(6)}$. Let us denote the resulting contributions to the factorization approximation for the (nonreduced) NLO weak LEC's, E_i^-, (as employed in Ref. [18]) by $[E_i^-]_1$ and $[E_i^-]_2$, respectively. The authors of Ref. [18] then employ a model in which the relevant *renormalized* fourth order and sixth order strong LEC's are assumed to be saturated by scalar resonance exchange. There is, however, a problem with the approach of Ref. [18], associated with the $[E_i^-]_1$ contributions. To understand the origin of this problem, consider, for example, the results of Eq. (17) of Ref. [18], translated into our notation:

$$[E_1^-]_1 = [E_3^-]_1 = [-E_5^-]_1 = \frac{G_f}{\sqrt{2}} V_{us}V_{ud}\,\mathrm{Im}\,C_6(32B_0^2)L_8^2. \tag{15}$$

As noted above, L_8 has a Laurent expansion of the form given by Eq. (11). Note, first, that this means that the contributions $[E_i^-]_1$ of Eq. (15) begin at $O[1/(d-4)^2]$, in contrast to the actual E_i^-, whose Laurent expansions begin at $O[1/(d-4)]$. While one might plausibly ignore this discrepancy, arguing that only the finite parts of the expressions at some hadronic scale are to be used in any case, a related problem remains, even for the finite parts. Explicitly, the fact that both factors of L_8 in Eq. (15) contain a $1/(d-4)$ term means that the finite part of L_8^2 is *not* $[L_8^r]^2$, as assumed in Ref. [18], but rather $[L_8^r(\mu)]^2 + 5/(384\pi^2)L_8^{(-1)}(\mu)$, where the explicit value of Γ_8, given in Ref. [15] has been used. Since, as explained above, the $L_i^{(-1)}$ are on the same footing as the sixth order strong LEC's which enter $[E_i^-]_2$, the model expressions of Ref. [18] for the sum $[E_i^-]_1 + [E_i^-]_2$ are numerically incomplete. The model, moreover, provides no means of estimating the $L_i^{(-1)}$, making it impossible to correct this defect. In view of this problem, we conclude that, at present, it is not possible to estimate the NLO weak CP-odd LEC's using the factorization approximation.

Given the problem just discussed with the numerical estimates of Ref. [18], we restrict our attention to the WDM and χQM in obtaining estimates for the NLO weak LEC contributions to $\mathrm{Im}\,A_0$ and $\mathrm{Im}\,\delta A_2$. The resulting values for Ω_{st} will be given in the next section. Since the χQM is a microscopic model, and the WDM is not, we will take the value of Ω_{st} obtained using the former model as our central value, and use the deviation from this central value of the result obtained using the WDM as a *minimal* measure of the theo-

[3]The factorization approximation becomes exact in the limit of large N_c. In this limit, taking Q_6 for example, if one renormalizes the two densities of which Q_6 is a formal product, then one will also have renormalized the four-quark operator Q_6. For two such renormalized densities, $J(x)$ and $J'(y)$, at *different* points x,y, one can straightforwardly construct the low-energy representation of the product $J(x)J'(y)$ using standard methods. In general, of course, the low-energy representation of such a product is *not* simply the product of the low-energy representations of the individual densities, but also contains seagull terms. It turns out that, for Q_6, the NLO part of this representation, which is the part investigated by the authors of Ref. [18], does indeed contains seagulls. Since these diverge as $x \to y$, it is necessary to interpret the factorization approximation as corresponding to an approximate low-energy representation of Q_6 obtained by dropping the seagull terms in the low-energy representation of $lim_{x\to y} J(x)J'(y)$, i.e., to one obtained by taking simply the product of the low-energy representations of the two densities.

[4]In ChPT, c_2^- is finite, and scale-independent, whereas L_5 is divergent. To make sense of this relation one, therefore, usually assumes that L_5 is to be replaced by its renormalized value, $L_5^r(\mu)$, evaluated at some typical hadronic scale $\mu = \mu_h$.

retical uncertainty in our prediction for Ω_{st} associated with the model dependence of the weak NLO LEC's.

III. NUMERICAL RESULTS

The isospin-breaking correction to the gluonic penguin operator, Q_6, evaluated to $\mathcal{O}(p^2\delta m)$, can be written in terms of its LO [$\mathcal{O}(\delta m)$] value and NLO corrections as

$$\Omega_{st}^{(2)}[1+R_2-R_0] \tag{16}$$

with

$$\Omega_{st}^{(2)}=\frac{\sqrt{2}}{6\omega}\frac{B_0\delta m}{(\overline{m}_K^2-m_\pi^2)} \tag{17}$$

where $\overline{m}_K=(m_{K^0}+m_{K^+})/2$. [Note that the result of Eq. (17) is unambiguous and independent of the LO weak coupling c_2^-.] The NLO corrections, R_i, are given by

$$R_0=\frac{\mathrm{Im}\,A_0^{(NLO,ND)}}{\mathrm{Im}\,A_0^{(LO)}}, \quad R_2=\frac{\mathrm{Im}\,\delta A_2^{(NLO,ND)}}{\mathrm{Im}\,\delta A_2^{(LO)}} \tag{18}$$

where the superscript (NLO,ND) indicates the sum of non-dispersion NLO contributions (involving NLO weak and strong LEC's and the nondispersive parts of loop graphs). Note that, in Eq. (18), R_2 arises from IB effects, whereas R_0 is purely isospin-conserving. (The IB correction to R_0 would generate a contribution to Ω_{st} of $\mathcal{O}([\delta m]^2)$, and thus is beyond the scope of the present work.) To separate the model-independent contributions associated with strong LEC and loop effects [Figs. 1(b)–1(g)] from those of the model-dependent NLO weak LEC terms [Fig. 1(h)], it is convenient to further expand R_i as

$$R_i=R_i^{(non-WLEC)}+R_i^{(WLEC)} \tag{19}$$

where the superscripts indicate NLO weak LEC (WLEC) and one-loop-plus-strong-LEC (non-WLEC) contributions respectively. We begin our discussion with the non-WLEC contributions. These are model-independent and unambiguous, albeit renormalization-scale-dependent, since the NLO contributions concerned all involve exactly one weak $\mathcal{O}(p^2)$ vertex. The resulting overall factor of c_2^- in the non-WLEC part of the numerator of Eq. (18) therefore cancels with the corresponding factor in the denominator. This cancellation removes all of the short-distance uncertainties (Wilson coefficients, CKM matrix elements, etc.) contained in c_2^-. In addition, because the LO coupling strength cancels and the strong vertices (if any) are identical for the CP-even and CP-odd cases, diagram-by-diagram, the non-WLEC contributions to Ω_{st} are ''universal,'' that is, they are the same for CP-even and CP-odd cases. To evaluate these contributions we use as numerical input the values $m_\pi=135$ MeV, $\overline{m}_K=495$ MeV, $m_\eta=549$ MeV, and

$$B_0\delta m=\left(\frac{m_d-m_u}{m_d+m_u}\right)m_\pi^2=5552\pm 674 \ \ \mathrm{MeV}^2 \tag{20}$$

TABLE II. The NLO non-WLEC contributions to R_0 and R_2 at the renormalization scale μ.

μ	$R_0^{(non-WLEC)}$	$R_2^{(non-WLEC)}$
m_η	-0.01690	-0.2359
m_ρ	0.4203	-0.3147

(as determined by Leutwyler in Ref. [29]). With these values, we have the usual result $\Omega_{st}^{(2)}=0.128\approx 0.13$. The NLO non-WLEC contributions to R_0 and R_2 are given in Table II. The results are presented at two different renormalization scales, $\mu=m_\eta$ and $\mu=m_\rho$, in order to display explicitly the scale dependence of the non-WLEC contributions. When using the WDM estimate for the NLO weak LEC's, we employ $\mu=m_\rho$ (consistent with the expectations of resonance saturation), and when using the χQM estimates, $\mu=0.8$ GeV (the matching scale employed in Ref. [9] in obtaining fits for the χQM parameters).

It is immediately apparent that the non-WLEC contributions to R_0 and R_2 both act to reduce Ω_{st} as compared to its LO value. The R_2 non-WLEC contribution is weakly scale dependent and, being ''universal,'' follows immediately from the corresponding CP-even results of Ref. [19]. Although for scales $\mu\sim m_\rho$ $R_0^{(non-WLEC)}$ is positive, and hence acts to lower Ω_{st}, the scale dependence, in this case, is significantly stronger. The increase in the magnitude of $\mathrm{Im}\,A_0$ associated with the loop contributions is what one would expect given the attractive final state interactions (FSI) in the $I=0$ channel, and is analogous to the A_0 FSI enhancement discussed previously for the CP-even case [30,31,27]. The effect of FSI on the CP-odd amplitudes has also been recently discussed in Refs. [32–34]. In Refs. [32,34] it is argued that the value of $\mathrm{Im}\,A_0$ obtained from approaches which do not generate the final state $\pi\pi$ $I=0,2$ phases for the $I=0,2$ $K\to\pi\pi$ amplitudes should be enhanced by FSI by a factor of ~ 1.55, while the value of $\mathrm{Im}\,A_2$ should be suppressed by the weakly repulsive $I=2$ FSI by a factor of ~ 0.92. The numerical values of the enhancement/suppression are obtained using the Omnes representation for the amplitude, and correspond to the subtraction point $s=0$, for which the ChPT representation of the amplitude is presumed to be accurate. It should be borne in mind that the $I=0,2$ FSI, corresponding to Fig. 1(f), are already correctly included in our calculations, up to NLO in the chiral expansion. That the $I=0$ FSI, for example, produce a significant enhancement of $\mathrm{Im}\,A_0$, follows from the known FSI enhancement of A_0 in the CP-even case [27] and the ''universality'' of the one-loop contributions. One should also note that the Omnes function part of the representation of the amplitudes does not incorporate all of the NLO effects; some NLO effects remain in the polynomial prefactor. Thus the question of interest to us, namely whether the *complete* set of NLO contributions raises or lowers $\mathrm{Im}\,A_0$ *relative to its LO value* (i.e., whether R_0 is positive or negative) is not determined solely by the character of the loop contributions; it is perfectly possible, in principle, for the NLO LEC contributions to be sufficiently negative that the full NLO determina-

TABLE III. The NLO weak counterterm (WLEC) contributions to the correction factors as estimated in the WDM and χQM.

	χQM	WDM
R_0^{WLEC}	-0.231	-0.0331
R_2^{WLEC}	0.205	-0.0331

tion of R_0 is negative, even in the presence of the attractive FSI phases. For the models we have considered, this is not the case, and the combination of non-WLEC and WLEC contributions to R_0 is positive, leading to a suppression of Ω_{st} below its LO value. It is important to note that, so long as one adheres to the convention of incorporating the effect of the $I=2$ leakage contribution by means of a multiplicative correction factor applied to the contribution to the $I=0$ amplitude, there is an amplification effect at work in the gluonic penguin contribution to ϵ'/ϵ associated with the NLO contributions to $\mathrm{Im}A_0$: the more NLO effects *increase* the isospin-conserving contribution to $\mathrm{Im}A_0$, the more they simultaneously *decrease* Ω_{st}. Since the contribution to ϵ'/ϵ, including IB, is proportional to the product of the isospin-conserving contribution and the factor $1-\Omega_{st}$, both effects serve to enhance the Q_6 contribution to ϵ'/ϵ.

The NLO weak LEC contributions to R_0 and R_2 are estimated using the models described in the previous section. The numerical results are displayed in Table III. We note first that the contributions to $R_0^{(WLEC)}$ and $R_2^{(WLEC)}$ in the WDM are identical, and hence cancel in the difference, R_2-R_0, entering Eq. (16). In the χQM, the WLEC contributions to Ω_{st} are positive. Indeed the results of Tables II and III show significant cancellation between the WLEC and non-WLEC contributions in the χQM.

The total NLO correction factor, $1+R_2-R_0$ which multiplies $\Omega_{st}^{(2)}$, resulting from the combination of WLEC and non-WLEC contributions, is

$$1+R_2-R_0=\begin{cases}0.64 & (\chi\mathrm{QM})\\ 0.27 & (\mathrm{WDM}).\end{cases} \tag{21}$$

Taking the χQM result as a central value, and the deviation of the WDM result from this central value as a minimal measure of the model-dependence of our result, we find the IB correction to the gluonic penguin contribution to ϵ'/ϵ to be

$$\Omega_{st}=0.08\pm0.05\pm0.01 \tag{22}$$

where the first error represents the uncertainty associated with the model dependence of the NLO weak LEC's, and the second the uncertainty in the input value of $B_0\delta m$. The central value in Eq. (22) is significantly lower than both the conventionally-employed value, 0.25 ± 0.08, and the result of Ref. [16], 0.16 ± 0.03. That the relative uncertainty increases from about 30% to 62% is a reflection of the uncertain state of our knowledge of the NLO weak LEC's. We emphasize, however, that, regardless of the actual value of the weak NLO LEC contributions, the model-independent one-loop and strong LEC contributions, which are unambiguous, produce a significant reduction of Ω_{st} for any plausible choice of hadronic scale. As such, the inclusion of the loop contributions is crucial to any attempt to evaluate Ω_{st} beyond LO. For the models considered for the weak NLO LEC's, the net effect is to drive Ω_{st} significantly below its LO value.

TABLE IV. The dependence of ϵ'/ϵ on Ω_{st} in the standard model assuming, for illustrative purposes, the central values for $\Lambda_{\overline{MS}}$, $m_s(m_c)$, m_t and $\mathrm{Im}\,\lambda_t$ as given in Refs. [12]. The units of ϵ'/ϵ are 10^{-4}. The values of ϵ'/ϵ corresponding to $\Omega_{st}=0.25$ are taken from Table III of the second of Refs. [12] and the range of values for B_6, B_8 is the same as covered by that table.

B_6	B_8	ϵ'/ϵ $(\Omega_{st}=0.25)$	ϵ'/ϵ $(\Omega_{st}=0.08\pm0.05)$
1.0	0.6	8.4	11.2 ± 0.8
1.0	0.8	7.0	9.8 ± 0.8
1.0	1.0	5.5	8.2 ± 0.8
1.3	0.6	12.8	16.6 ± 1.2
1.3	0.8	11.3	15.1 ± 1.1
1.3	1.0	9.9	13.7 ± 1.1

IV. DISCUSSION AND CONCLUSIONS

As noted above, the significant cancellation between gluonic penguin and electroweak penguin contributions means that predictions for the value of ϵ'/ϵ in the standard model can depend rather sensitively on Ω_{st}. The exact degree of sensitivity, of course, depends on the relative size of these two dominant contributions, on which there is, as of yet, no clear theoretical concensus. In order to illustrate the impact of the decrease of Ω_{st} from the conventional central value, 0.25, to 0.08 ± 0.05, let us use the rough approximation to Eq. (1) discussed in Ref. [5]

$$\frac{\epsilon'}{\epsilon}\propto[B_6(1-\Omega_{st})-0.4B_8] \tag{23}$$

(where we have dropped an overall constant multiplicative factor irrelevant to the present discussion). Maintaining the constraint, $B_6>B_8$, imposed by the Munich group [5,12], and using the values $B_6=1.0\pm0.3$ and $B_8=0.8\pm0.2$ employed by them, we find the results shown in Table IV. The range of values for B_6, B_8 covered in the Table IV is the same as that in Table III of the second of Refs. [12], from which the values for ϵ'/ϵ corresponding to $\Omega_{st}=0.25$ have also been taken. All results correspond to central values of the input parameters $\Lambda_{\overline{MS}}^{(4)}$, $m_s(m_c)$, m_t and $\mathrm{Im}\,\lambda_t$. From Table IV we see that the decrease in Ω_{st} corresponds to an increase in ϵ'/ϵ of between 21% and 63% ($40\pm11\%$ for the central B_6, B_8 values). The increase in the magnitude of ϵ'/ϵ is between 2×10^{-4} and 5×10^{-4}, to be compared to the current experimental world average $(19.3\pm2.4)\times10^{-4}$. The magnitude of the increase will, of course, be even larger for models with larger values of B_6.

It is useful to comment in more detail on the application of the corrections discussed above to microscopic models such as the χQM and the extended NJL model [7]. Such

models allow one, in principle, to compute the corrections corresponding to the NLO weak LEC's self-consistently within the model, as was, for example, done by the Trieste group [9] for $\mathrm{Im}A_0$. As pointed out in Ref. [35] (in the context of the χQM), however, modifying the model predictions for ϵ'/ϵ obtained using the conventional value of Ω_{st} is more complicated than simply re-scaling the gluonic penguin contribution to take into account the new value of the factor $1-\Omega_{st}$. The reason is that the conventional value of Ω_{st} (assumed to be the same for the CP-even and CP-odd cases) enters also the determination of the CP-even amplitude A_2 in the model; a change in Ω_{st} in the CP-even sector would thus necessitate a re-fitting of the parameters of the model. In fact, in Ref. [35] it was noted that the re-fitting of parameters necessitated by a shift in Ω_{st} would almost entirely compensate for the effect of the shifted value of Ω_{st} in the model determination of ϵ'/ϵ. One should, however, bear in mind the caveat that this observation is based on the implicit assumption that Ω_{st} is the same in the CP-even and CP-odd sectors. Although this is true for the non-WLEC contributions, there is no reason to expect it to be true for the NLO weak LEC contributions. In fact, since these contributions correspond to the hadronization of very different effective operators, it would be rather surprising to find them taking on the same values. Fortunately one does not need to speculate idly on this question: in models such as the χQM it is possible to simply compute the NLO terms corresponding to the weak NLO LEC contributions. Having fitted the model parameters in the CP-even sector, one would then obtain, self-consistently, a determination of $[\Omega_{st}]_{WLEC}$ for both the CP-even and CP-odd cases. In order to make sure the determination of the strong IB correction to ϵ'/ϵ is under control, it is important to separately determine the NLO weak LEC contributions to the leakage amplitudes in the CP-even and CP-odd sectors.

To summarize, we have presented a complete NLO calculation of the isospin-breaking correction to the gluonic penguin operator contribution to $\epsilon'/\epsilon,\Omega_{st}$. It is found that model-independent NLO one-loop and strong LEC contributions are of the opposite sign to the LO contribution, and numerically large, for typical hadronic scales. Combined with model estimates for the NLO weak LEC contributions, we find a significant reduction of Ω_{st} as compared to the ''standard'' value of 0.25 ± 0.08. Our final result is

$$\Omega_{st}=0.08\pm0.05 \tag{24}$$

(where the uncertainties associated with model-dependence and $B_0\delta m$ have been added in quadrature). We recommend that this central value, together with, to be conservative, even larger errors, be employed in future estimates of ϵ'/ϵ in the standard model.

ACKNOWLEDGMENTS

We thank S. Bertolini and J. Eeg for clarifying the current state of the χQM calculations and pointing out the existence of the errors in the earlier LEC estimates. C.E.W. acknowledges the support of the United States Department of Energy under grant #DE-FG0287ER-40365. K.M. acknowledges the ongoing support of the Natural Sciences and Engineering Research Council of Canada, and the hospitality and support of the Special Research Center for the Subatomic Structure of Matter at the University of Adelaide.

APPENDIX: NLO WEAK LEC CONTRIBUTIONS TO $K^0\to\pi\pi$

The octet NLO weak LEC contributions to the $I=0$ and $I=2$ CP-odd $K^0\to\pi\pi$ amplitudes are given by

$$[A_0]_{WLEC}=-\frac{c_2^-}{F^2}\left(\frac{2\sqrt{6}}{F^3}\right)(m_K^2-m_\pi^2)(m_K^2\tilde{K}_1^- - m_\pi^2\tilde{K}_2^-)$$

$$[\delta A_2]_{WLEC}=\frac{c_2^-}{F^2}\left(\frac{2B_0(m_d-m_u)}{\sqrt{3}F^3}\right)(m_K^2\tilde{J}_3^- - m_\pi^2\tilde{J}_4^-) \tag{A1}$$

where the $\tilde{K}_i^-$ are isospin-conserving NLO weak LEC combinations (the CP-even analog of $\tilde{K}_1^-$ is discussed in Refs. [27,24]), and the $\tilde{J}_i^-$ are isospin-breaking LEC combinations whose CP-even analogs are discussed in Refs. [19,20]. In the notation of Ref. [24] these are given by

$$\tilde{K}_1^- = [N_5^{-,r} - 2N_7^{-,r} + 2N_8^{-,r} + N_9^{-,r}]$$

$$\tilde{K}_2^- = [-2N_5^{-,r} - 4N_7^{-,r} - N_8^{-,r} + 2N_{10}^{-,r} + 4N_{11}^{-,r} + 2N_{12}^{-,r}]$$

$$\tilde{J}_3^- = [N_5^{-,r} + 6N_6^{-,r} - 2N_8^{-,r} - N_9^{-,r} - 4N_{10}^{-,r} - 8N_{12}^{-,r} - 12N_{13}^{-,r}]$$

$$\tilde{J}_4^- = [2N_5^{-,r} + 6N_6^{-,r} + N_8^{-,r} - 2N_{10}^{-,r} - 10N_{12}^{-,r} - 12N_{13}^{-,r}]. \tag{A2}$$

[1] KTeV Collaboration, A. Alavi-Harati *et al.*, Phys. Rev. Lett. **83**, 22 (1999).

[2] NA48 Collaboration, V. Fanti *et al.*, Phys. Lett. B **465**, 335 (1999).

[3] A. Buras, M. Jamin, and M. Lautenbacher, Nucl. Phys. **B408**, 209 (1993); M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, *ibid.* **B415**, 403 (1994).

[4] W. Bardeen, A. Buras, and J. Gerard, Nucl. Phys. **B293**, 787 (1987); J. Heinrich, E. Paschos, J. Schwartz, and Y. Wu, Phys. Lett. B **279**, 140 (1992); T. Hambye, Acta Phys. Pol. B **28**, 2479 (1997); T. Hambye, G. Kohler, and P. Soldan, Eur. J. Phys. C **10**, 271 (1999); T. Hambye and P. Soldan, hep-ph/9908232; T. Hambye, G. Kohler, E. Paschos, and P. Soldan, Nucl. Phys. **B564**, 391 (2000).

[5] A. Buras, M. Jamin, and M. Lautenbacher, Phys. Lett. B **389**, 749 (1996); A. Buras and M. Lautenbacher, *ibid.* **318**, 212 (1993).
[6] M. Ciuchini, E. Franco, G. Martinelli, and L. Reina, Phys. Lett. B **301**, 263 (1993); M. Ciuchini, E. Franco, G. Martinelli, L. Reina, and L. Silvestrini, Z. Phys. C **68**, 239 (1995); R. Gupta, Nucl. Phys. B (Proc. Suppl.) **63**, 278 (1998); G. Kilcup, R. Gupta, and S. Sharpe, Phys. Rev. D **57**, 1654 (1998); L. Conti *et al.*, Phys. Lett. B **421**, 273 (1998); G. Martinelli, Nucl. Phys. B (Proc. Suppl.) **73**, 58 (1999).
[7] J. Bijnens and J. Prades, J. High Energy Phys. **01**, 023 (1999); **06**, 035 (2000).
[8] S. Bertolini, J. Eeg, and M. Fabbrichesi, Nucl. Phys. **B449**, 197 (1995).
[9] S. Bertolini, J. Eeg, M. Fabbrichesi, and E. Lashin, Nucl. Phys. **B514**, 63 (1998).
[10] M. Fabbrichesi and E. Lashin, Phys. Lett. B **387**, 609 (1996); V. Antonelli, S. Bertolini, J. Eeg, M. Fabbrichesi, and E. Lashin, Nucl. Phys. **B469**, 143 (1996); S. Bertolini, J. Eeg, M. Fabbrichesi, and E. Lashin, *ibid.* **B514**, 93 (1998).
[11] S. Bertolini, M. Fabbrichesi, and J. Eeg, Rev. Mod. Phys. **72**, 65 (2000); S. Bosch, A. Buras, M. Gorbahn, S. Jäger, M. Jamin, M. Lautenbacher, and L. Silvestrini, Nucl. Phys. **B565**, 3 (2000).
[12] M. Jamin, hep-ph/9911390; A. Buras, hep-ph/9908395.
[13] J. Donoghue, E. Golowich, B. Holstein, and J. Trampetic, Phys. Lett. B **179**, 361 (1986).
[14] A. Buras and J. Gerard, Phys. Lett. B **192**, 156 (1987); H.Y. Cheng, *ibid.* **201**, 155 (1988); M. Lusignoli, Nucl. Phys. **B325**, 33 (1989).
[15] J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) **158**, 142 (1984); Nucl. Phys. **B250**, 465 (1985).
[16] G. Ecker, G. Muller, H. Neufeld, and A. Pich, Phys. Lett. B **477**, 88 (2000).
[17] K. Maltman, Phys. Lett. B **351**, 56 (1995).
[18] S. Gardner and G. Valencia, Phys. Lett. B **466**, 355 (1999).
[19] C. Wolfe and K. Maltman, Phys. Lett. B **482**, 77 (2000).
[20] C. Wolfe, Ph.D. thesis, York University, Toronto, 1999.
[21] C. Wolfe and K. Maltman (in preparation).
[22] J. Cronin, Phys. Rev. **161**, 1483 (1967).
[23] J. Kambor, J. Missimer, and D. Wyler, Nucl. Phys. **B346**, 17 (1990).
[24] G. Ecker, J. Kambor, and D. Wyler, Nucl. Phys. **B394**, 101 (1993).
[25] J. Bijnens, E. Pallante, and J. Prades, Nucl. Phys. **B521**, 305 (1998).
[26] G. Ecker, Prog. Part. Nucl. Phys. **35**, 1 (1995).
[27] J. Kambor, J. Missimer, and D. Wyler, Phys. Lett. B **261**, 496 (1991).
[28] S. Bertolini and J. Eeg (private communications).
[29] H. Leutwyler, Phys. Lett. B **374**, 163 (1996); **378**, 313 (1996).
[30] W.A. Bardeen, A.J. Buras, and J.-M. Gerard, Phys. Lett. B **192**, 138 (1987); T.N. Truong, *ibid.* **207**, 495 (1988).
[31] A.A. Bel'kov, G. Bohm, D. Ebert, and A.V. Lanyov, Phys. Lett. B **220**, 459 (1989); N. Isgur, K. Maltman, J. Weinstein, and T. Barnes, Phys. Rev. Lett. **64**, 161 (1990).
[32] E. Pallante and A. Pich, Phys. Rev. Lett. **84**, 2568 (2000).
[33] A. Buras, M. Ciuchini, E. Franco, G. Isidori, G. Martinelli, and L. Silvestrini, Phys. Lett. B **480**, 80 (2000).
[34] E. Pallante and A. Pich, hep-ph/0007208.
[35] S. Bertolini, J. Eeg, and M. Fabbrichesi, hep-ph/0002234.