

Optimal allocation of vaccines in metapopulations

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*In memory of my grandfather,
Teo Koh Soon,
(20 January 1929 – 29 December 2014),
who during our holiday at Oxford back in 1997,
inspired me to the great heights of academia.*

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Signed Statement

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Abstract

Infectious diseases have had a devastating impact on society and the world's population throughout the years. For example, the Spanish flu in 1918 and most recently, the Ebola epidemic in West Africa. The introduction of vaccines has managed to keep some infectious diseases under control and in the case of smallpox, eradicated it from the world's population. Hence, vaccines are a very effective method to control the spread of an infectious disease and where possible, the world's population should be vaccinated against all possible diseases. However, the production of vaccines is expensive and if there is a novel strain of a disease, it is often unlikely that a vaccine has been developed to combat the outbreak immediately. Instead, a vaccine must be developed during an epidemic. Both of these situations therefore result in a limited supply of vaccines. Thus, it is of great interest and importance to public health officials to be able to know how best to allocate a limited supply of vaccines to a population. This is the main idea and theme of this thesis.

We investigate two possible questions that can arise from this problem. The first is to consider the case where there is a novel strain of a disease and vaccines are developed during an epidemic. For many public health officials in countries or cities around the world, the question is how best to allocate a limited supply of vaccines to the population to minimise the number of people that become infected, after the infection is already present in the population. That is, they are interested in

determining the optimal allocation of limited vaccines which take into account the changing dynamics of the epidemic. This naturally lends itself to dynamic programming, which determines the optimal actions whilst accounting for the dynamics of a process. Hence, we explore the use of dynamic programming techniques, backward dynamic programming and approximate dynamic programming, to attempt to solve this problem. We observe that backward dynamic programming does not scale well with the size of the population and so an alternative method is to consider approximate dynamic programming. The approximate dynamic programming algorithms we consider fall under the category of *lookup tables*. We find, through order calculations, that these methods do not work efficiently for our problem and so other types of algorithms need to be explored in order for approximate dynamic programming to be applied.

The second question of interest is to consider an epidemic occurring in some part of the world and government officials in a currently uninfected country have access to a limited supply of vaccines. Then, their interest lies in how best to distribute this supply of vaccines to the population to minimise the mean final epidemic size, that is, the number of people that become infected over the course of an epidemic, should an epidemic arise. Further, we also investigate whether vaccines should be withheld until the first onset of infection in the population or be distributed before infection is present in the population. Results from this investigation can help inform public health officials in countries where infection is not yet present make better informed decisions on how to best utilise their resources.

As we find that prophylactic vaccination (vaccinating before infection is present in the population) results in a smaller mean final epidemic size, the main body of work focuses on prophylactic vaccination. We develop a method to determine an approximately optimal prophylactic vaccination scheme. For a range of different examples and parameter values, we compare the performance of our approximate strategy with other strategies in the literature. Through this comparison, we find

that our approximate strategy performs better or just as well as the other strategies across a range of problems and parameter values, which gives us confidence in our approximate strategy. Then, we consider a real-world example involving Australia and compare the performance of our approximate strategy with the other strategies, as well as testing the robustness of the various strategies to a change in the underlying epidemic model. We observe through these comparisons that our approximate strategy performs well for intermediate population sizes and is also quite robust to changes in the underlying model.