Abstract

High frequency combustion instabilities in liquid propellant rocket engines are spontaneously occurring pressure fluctuations that are coupled with unsteady combustion processes. Under the right conditions the unsteady fluctuations can grow to a point where they affect the operation of the combustion chamber. The cause of combustion instabilities, including which processes are responsible and under what conditions they arise, are not yet fully understood. The ability to predict and prevent combustion instabilities during the design of new combustion chambers, through better understanding, would dramatically reduce the uncertainty and risk in the development of new engines.

An experimental combustor, designated BKH, is used to conduct high frequency combustion instability experiments. BKH operates with liquid oxygen and gaseous hydrogen propellants at supercritical conditions analogous to real rocket engines. The chamber features an acoustic excitation system that imposes an acoustic disturbance representative of a high frequency instability upon a cluster of five coaxial injection elements in the center of the chamber. The response of the elements to the imposed acoustic disturbance is observed using high speed optical diagnostics.

The main aim of this project is to develop methods for predicting the flame response to high frequency acoustic forcing representative of combustion instability phenomena. BKH is employed as an experimental and numerical test case for investigating the flame response. Modelling and complementary data analysis methods are developed and applied to model the chamber flow field, identify and predict the excited acoustic disturbance, identify the flame response using optical data, and to predict the flame response numerically.

The BKH experiments are first characterised by modelling the chamber numerically and determining the local acoustic disturbance acting upon the flame. A steady state chamber model with supercritical oxygen-hydrogen combustion was computed using a specialised CFD code. The model results indicate the secondary injection in BKH has a strong influence on the resulting flame distribution.

A method for reconstructing the acoustic field from dynamic pressure sensor data was developed to determine the local acoustic disturbance acting upon the combustion zone over a range of excitation frequencies. A low-order acoustic modelling approach is also shown to predict the resonant mode frequencies and the evolution of the acoustic field.

The flame response to the imposed acoustic disturbance is identified by analysing optical data from BKH experiments and unsteady CFD modelling. Multi-variable dynamic mode decomposition (DMD) analysis is used to isolate the flame response to the imposed acoustic disturbance in shadowgraph and OH* imaging data. Wave-like structures propagating along the surface of the liquid oxygen (LOx) jet and a phase difference of 45° between acoustic pressure and observed intensity fluctuations were identified.

An unsteady model of an injection element subjected to representative acoustic forcing is used to predict the flame response for a range of excitation amplitudes. Velocity ratio fluctuations caused by acoustic coupling with the oxidiser post in a pressure antinode are identified. The trend of exponential decay of the length of the LOx core with increasing transverse acoustic amplitude excitation is reproduced numerically and the flattening and flapping motion of the flame was further investigated using the numerical results.
Declaration of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Scott K. Beinke
20th April 2017
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### Subscripts

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<td>Gas</td>
</tr>
<tr>
<td>$L$</td>
<td>Liquid</td>
</tr>
<tr>
<td>$R$</td>
<td>Real part</td>
</tr>
<tr>
<td>$I$</td>
<td>Imaginary part</td>
</tr>
<tr>
<td>$lam$</td>
<td>Laminar</td>
</tr>
<tr>
<td>$T$</td>
<td>Turbulent</td>
</tr>
<tr>
<td>$BPRMS$</td>
<td>Band-passed RMS result</td>
</tr>
<tr>
<td>$RMS$</td>
<td>Root Mean Squared result</td>
</tr>
</tbody>
</table>
Superscripts

\( a' \) Perturbation or oscillating value
\( \hat{a} \) Complex valued property
\( \bar{a} \) Mean property
\( a'' \) Forward reaction coefficient
\( a^b \) Backward reaction coefficient

Acronyms

AFRL Air Force Research Laboratory
AVBP LES solver jointly developed by Cerfacs, IFPEN, and EM2C
AVSP Acoustic solver developed by Cerfacs
BKD Combustion chamber (German: Brennkammer) ‘D’, operated by DLR
BKH Combustion chamber (German: Brennkammer) ‘H’, operated by DLR
BPRMS Band-passed Root Mean Squared
CAA Computational Aero Acoustics
CEA Chemical Equilibrium Analysis
CFD Computational Fluid Dynamics
CVRC Continuously Variable Research Combustor, operated by Purdue University
CRC Common Research Chamber, operated by DLR
DLR German Aerospace Center (Deutsches Zentrum für Luft und Raumfahrt)
DMD Dynamic Mode Decomposition
DNS Direct Numerical Simulation
EM2C Energetics and combustion lab at CentraleSupélec
FEM Finite Element Methods
FFT Fast Fourier Transform
FTF Flame Transfer Function
HF High Frequency
IFPEN French public-sector research, innovation and training center
JAXA Japanese Aerospace Exploration Agency
LEE Linearised Euler Equations
LES Large Eddy Simulation
LE-7A Mitsubishi LE-7(A) rocket engine
LF Low Frequency
LOx Liquid Oxygen
MIC Multiple-Injector Combustor, operated by ONERA
ONERA French national aerospace research center
P European Test Facility for Cryogenic Rocket Propulsion
PCCDYN Dynamic pressure sensors installed in BKH
POD Proper Orthogonal Decomposition
RANS Reynolds Averaged Navier Stokes
RMS Root Mean Squared
ROF Ratio of Oxidizer to Fuel mass flow rate
SSME Space Shuttle Main Engine
TAU CFD solver developed by the DLR
TIC Transverse Instability Combustor, operated by Purdue University
TUM Technical University of Munich (Technische Universität München)
URANS Unsteady Reynolds Averaged Navier Stokes
VHAM Very High Amplitude Modulator, operated by ONERA