



**Necessary Conditions for  
the Variant Optimal Design of  
Linear Consecutive Systems**

by

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# Summary

A linear consecutive- $k$ -out-of- $n$ :F system is an ordered sequence of  $n$  components that fails if and only if at least  $k$  consecutive components fail. A linear consecutive- $k$ -out-of- $n$ :G system is an ordered sequence of  $n$  components that works if and only if at least  $k$  consecutive components work. This thesis establishes several sets of conditioning relating to the variant optimal design of such systems. A review of the existing research in the theory of variant optimal design of linear consecutive- $k$ -out-of- $n$  systems is included.

It is established that nonsingularity of the design is a necessary condition for the optimal design of a linear consecutive- $k$ -out-of- $n$ :F system with  $2k \leq n \leq 3k$ ,  $k > 1$ . The significance of this result is illustrated by the fact that designs satisfying it can be better than designs satisfying other known necessary conditions. Procedures to improve designs not satisfying this con-

dition are given. An equivalent necessary condition for the optimal design of a linear consecutive- $k$ -out-of- $n$ :G system with  $2k \leq n \leq 3k$ ,  $k > 1$  is also given, as well as procedures to improve designs not satisfying it.

The existing necessary conditions for the optimal design of systems with  $n \geq 2k$  provide comparisons between reliabilities of components restricted to positions from 1 to  $k$  and positions from  $n$  to  $(n - k + 1)$ . Necessary conditions for the optimal design of systems with  $n \geq 2k$  are given that involve components at some other positions, including the  $(k + 1)$ -th component. Procedures to improve designs not satisfying those conditions are also given.

A binary search method to obtain real optimal solutions is constructed for systems with  $n \in \{2k + 1, 2k + 2\}$ . A randomization method, based on all necessary conditions developed in this thesis, is constructed for sub-optimal solutions for systems with  $n \geq 2k$ .

This material is the content of the following papers (submitted for publication):

- “Variant optimal designs of linear consecutive- $k$ -out-of- $n$  systems”,

- “Optimal design of consecutive-k-out-of-n systems”, and
- “The  $(k+1)$ -th component of linear consecutive-k-out-of-n systems”.

# Signed Statement

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

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Signed

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