



# The Structure of Clifford Algebra

A thesis presented

by

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## Abstract

This thesis presents an analysis of the general structure of the Clifford algebra while seeking to understand the geometrical content of an arbitrary Clifford element.

By extending the Lie algebra to the entire algebra using the anticommutator, the properties of any Clifford algebra may be concisely summarized in four identities. This formulation may be used to analyse the general structure of Clifford algebra but it is found that the symmetry properties are not consistent with the geometrical content. The extension of the graded Lie algebra to the entire algebra generates a geometrical algebra which simplifies the analysis of the structure of Clifford algebra. This produces a direct proof of the fundamental theorem which relates the Clifford and Grassmann algebras via the Pfaffian.

Using the Pfaffian an operator on the tensor algebra is defined to generate the explicit coset structure of the Clifford quotient of the tensor algebra. The associativity property of this tensor representation of Clifford algebra leads to a Pfaffian expansion analogous to the Laplace expansion of a determinant. Further, the explicit statement of the algebra norm provides a proof for the Hada mard theorem.

## **Signed Statement**

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University, and to the best of the candidate's knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

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