



A STUDY OF A NON-LINEAR DIFFUSION EQUATION

A Thesis

by

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SUMMARY

The absorption mechanism by which a plant removes ions from the soil can often be represented by a non-linear radial diffusion equation, in which the diffusion coefficient is dependent on concentration and the boundary condition applied at the absorbing surface is initially discontinuous. In the study of this equation numerical solutions are obtained using an IBM 1620 digital computer by setting up suitable implicit finite difference schemes. The initial discontinuity of the solution gives rise to difficulties in starting the computational procedure and an attempt is made to overcome these difficulties by utilizing suitable asymptotic approximations. An asymptotic approximation not only makes it possible to begin the numerical solution at a finite time, where the boundary condition is continuous, but provides a means of checking the numerical results and furthermore suggests a means of solving the problem without using finite differences. However, the simplest and most easily applied asymptotic approximation is useful only in a very small time range. Other asymptotic solutions are also developed and discussed, but it is found that their application would involve extensive computations.

To the best of my knowledge and belief this thesis contains no material which has been accepted for the award of another degree at any University and contains no material previously published or written by another person , except where due reference is made in the text of the thesis .

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