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Formulating a fiscal reaction function for ADAM

Resumé:

We attempt to clarify the formation of interest income and examine the role of a fiscal reaction function in ADAM in keeping the public debt from exploding when there is 1% increase in the public purchase of goods and services and the VAT rate. The paper briefly demonstrates the effectiveness of the proposed fiscal rule in stabilizing the public budget.

Dea

Keywords: fiscal reaction function, public budget, forward looking expectation

1. Introduction

This paper is a continuation of the previous paper "Collecting inspiration to develop a fiscal reaction rule for ADAM". In the simulations of the previous paper, the implicit interest rates of financial assets held by the public sector and the other sectors were not quite constant and not equal to the steady-state nominal GDP growth rate of the baseline.

In this paper, we construct a new baseline where the implicit interest rates are much closer to the steady state growth while keeping the net assets of the sectors proportional to GDP in the long run. With this baseline, we activate the backward and forward-looking fiscal reaction function introduced in the previous paper and analyze how the fiscal reaction function stabilize the public debt to GDP ratio in case of a permanent change in public purchases.

2. Constructing a new baseline

If the implicit interest rate of all financial assets were equal to the steady-state nominal GDP growth, which in a standard baseline equals 3.53% $(= 1.02 \cdot 1.015 - 1)$, the steady-state primary surplus would be zero in all sectors. Besides, the ratio of any sector's net asset to GDP, including public net debt, should be constant in steady state. In order to approach a steady state with 3.53% implicit interest rate and a constant ratio of the sectors' net asset to GDP, we have made four modifications in the *june14* ADAM model and in the $newdea$ data bank, which contains the baseline.

1) The ADAM interest income or outlay equations are in several cases specified as an exogenous interest rate of 3.53% multiplied on the average of the one period lagged and contemporaneous asset or liability. This formulation ensures that the contemporaneous flow into the asset affects the interest flow by half an annual interest rate. However, the precise formulation also makes the implicit interest rate in steady state deviate marginally from the growth rate of 3.53%.

To see that, consider the following interest revenue equation, where we use that the asset W grows by 3.53%:

$$Ti = 0.0353 \cdot (W + W_{-1})/2 = 0.0353 \cdot (1.0353 \cdot W_{-1} + W_{-1})/2 > 0.0353 \cdot W_{-1}$$

Thus, the interest revenue is higher than required to grow the asset by 3.53%. We might ignore this somewhat trivial technicality and apply a more flexible interpretation of equality between growth and interest rate. However, it is straightforward to bring interest and growth rate on an equal footing by re-writing the interest revenue equation as:

$$Ti = \left((0.0353/1.0353) \cdot W + 0.0353 \cdot W_{-1}\right)/2 = 0.0353 \cdot W_{-1}$$

The last equality sign holds when W grows by 3.53%, and the interest rate used for the contemporaneous W is just the prepaid interest rate equivalent to a normal interest rate of 3.53%.

Introducing this little re-specification in ADAM will e.g. change a public revenue equation like tii_z_os from (1.1) to (1.2)

$$Tii\_z\_os = iwdi \cdot (wnq\_os(-1) + wnq\_os)/2 + biwb \cdot wb\_z\_os(-1) + dttii\_z\_os \tag{1.1}$$

$$Tii\_z\_os = iwdi \cdot (wnq\_os(-1) + wnq\_os/(1 + iwdi))/2 + biwb \cdot wb\_z\_os(-1) + dttii\_z\_os \tag{1.2}$$

There are a number of equations of this type in ADAM and this re-specification is applied to all of them.

Moreover, the equations for public interest expenditure on domestic government krone bonds ($Tiid\_os\_z$), public interest expenditure on foreign loans ($Tiie\_os\_z$) and the equation for total mortgage interest expenditure of the financial sector ($Tiim\_cf\_x$) are changed in a similar way. The modified $Tiid\_os\_z$, $Tiie\_os\_z$ and $Tiim\_cf\_x$ relations are shown below as (1.3), (1.4) and (1.5) respectively:

$$Tiid\_os\_z = Tiid\_os\_z \cdot (1 - brwbd\_os\_z) + kiwbnd \cdot \frac{1}{2} \cdot iwbos \cdot [Tfbgd\_os\_z/(1 + iwbos) + Tfbgd\_os\_z(-1)]/ktfbgd\_os\_z \tag{1.3}$$

$$Tiie\_os\_z = Tiie\_os\_z(-1) \cdot (1 - brwbe\_os\_z) + kiwbnu \cdot \frac{1}{2} \cdot iwbos \cdot [Tfbge\_os\_z/(1 + iwbos) + Tfbge\_os\_z(-1)]/ktfbge\_os\_z \tag{1.4}$$

$$Tiim\_cf\_x = Tiim\_cf\_x(-1) \cdot (1 - brwbm\_cf\_z) + ktflm\_cf\_x \cdot \frac{1}{2} \cdot iwbz \cdot [(Tflm\_h\_cf + Tflm\_cr\_cf + Tflm\_ok\_cf + Tflm\_e\_cf + Tflm\_cf\_cf + \mathrm{brwbm\_cf\_z} * \mathrm{wbm\_cf\_z}(-1))/(1 + iwbz) + (Tflm\_h\_cf(-1) + Tflm\_cr\_cf(-1) + Tflm\_ok\_cf(-1) + Tflm\_e\_cf(-1) + Tflm\_cf\_cf(-1) + brwbm\_cf\_z(-1) \cdot Wbm\_cf\_z(-2))] \tag{1.5}$$

In ADAM, the interest flows $Tiid\_os\_z$ and $Tiim\_cf\_x$ are used to determine the implicit interest rate $biwb$ for all domestic bonds:

$$biwb = \frac{Tiid\_os\_z + \mathrm{Tiim\_cf\_x}}{Wbd\_os\_z(-1) + Wbm\_cf\_z(-1)}$$

With the interest flows in the nominator determined by the modified equations $biwb$ comes closer to $iwbos = iwbz = 0.0353$ in the standard baseline. We do not precisely get $iwbz$ to equal 3.53%, and this reflects that the stocks involved, i.e. Wbd_os_z and Wbm_cf_z, plus the related gross transactions, $Tfbgd\_os\_z$ and $Tflm\_h\_cf + Tflm\_cr\_cf + Tflm\_ok\_cf + Tflm\_e\_cf + Tflm\_cf\_cf + brwbm\_cf\_z \cdot wbm\_cf\_z(-1)$ are not yet growing by 3.53% by the end of the baseline scenario to 2100.

We do manage to get the total public net asset Wn_o to grow very close to 3.53% in the baseline before the end of the baseline. However, the dead-weight effect of some public assets not growing makes the growth rate of the residually determined domestic krone bond debt Wbd_os_z deviate from 3.53%. For a couple of public asset items it is easy to introduce the desired growth rate by determining the related transactions, i.e. $Tfnq\_os = 0.0353 \cdot wnq\_os(-1)$ and $Tfs\_x\_os = 0.0353 \cdot ws\_x\_os(-1)$, but we have not attempted to do more than that.

At the end of the baseline period, domestic government debt Wbd_os_z is still growing by more than 3.53% and the mortgage debt Wbm_cf_z is still growing by less. Consequently, the implicit rate for government bonds per se, $Tiid\_os\_z/Wbd\_os\_z(-1)$, is marginally higher than 3.53% while the implicit rate for mortgage bonds is marginally lower than 3.53%. Given equations (1.3), (1.4) and (1.5) to determine interest flows all implicit interest rates would be 3.53% when the exogenous interest rates are 3.53, provided all assets and liabilities were growing by 3.53%.

2) The second change is to split the public budget into interest income and primary surplus in another way in order to facilitate the calculations. In ADAM, the public primary surplus is calculated as the public budget surplus (Tfn_o) minus wealth-related income (Tin_o) reflecting the national accounts system and its formal definitions. However, Tin_o includes oil field-related income and income from publicly owned quasi companies neither of which relates to a financial wealth component. This makes the implicit interest rate deviate from 3.53% and the deviation makes the official primary surplus deviate from zero in steady state. To make a clean split, we modify our use of the public wealth income by excluding items without a wealth component. The modified public interest income is presented by equation (1.6).

$$Tin\_o^* = (Tin\_o - (Tirn\_o - Tirn\_ok) - (Tiuo\_z\_o - Tiuo\_z\_ok - Tiuo\_z\_oo) \qquad (1.6)$$

The corresponding modified primary surplus is equal to the public budget minus the modified interest income. In steady state, the implicit interest rate of the modified interest income should be 3.53% and the modified primary surplus should be zero.

3) Unfortunately, the modifications to interest income equations described in 1) and 2) do not suffice to obtain the a 3.53% implicit interest rate in the public sector. To achieve the goal, we add a relation, which sets the modified interest income of the public sector (Tin_o^*) equal to 3.53% of the lagged public net asset ($Wn\_o(-1)$). This is obtained by adjusting the interest flow between the public and financial sector and the instrument is the discount rate $iwdi$, which is determined from.

$$iwdi = [iwdea \cdot wn\_o(-1) - (Tin\_o^* - iwdi \cdot (wnq\_os(-1) + wnq\_os/(1 + iwdi))/2] /[(wnq\_os(-1) + wnq\_os/(1 + iwdi))/2] \qquad (1.7)$$

$iwdea$ is the desired implicit interest, i.e. 3.53%, and $iwdi \cdot (wnq\_os(-1) + wnq\_os/(1 + iwdi))/2$ is the discount-rate-related contribution to public interest revenues.

4) Moreover, we use the capital transfer from the public to the foreign sector (Tk_o_e) to help stabilize the public debt/GDP ratio in the baseline. Specifically, we introduce two auxiliary equations, (1.8) and (1.9). The first auxiliary equation is:

$$Tk\_o\_e = tssyd * Y \qquad (1.8)$$

Y is GDP, and $tssyd$ a factor, which makes the capital transfer proportional to GDP. The factor is endogenously modeled as displayed in (1.9).

$$tssyd = tssyd(-1) - 0.5 \cdot \left(ctssyd(-1) - \frac{Tfn\_o(-1) - tin\_o^*(-1) + tyd(-1)}{y(-1)}\right) \quad (1.9)$$

$ctssyd$ is the desired value of the factor, *tyd* is expenditure on unemployment benefits. The desired value of the total primary surplus $Tfn\_o - tin\_o^*$ is zero, so $ctssyd$ indicates steady state unemployment benefits as a GDP share.
Equations (1.8) and (1.9) are saying that if at time $t-1$ the primary surplus excluding unemployment benefits $(Tfn\_o - tin\_o^* + tyd)$ over GDP is larger than $ctssyd$, then $tssyd$ will increase at time t and public capital outlays will be higher. Thus, the two auxiliary equations can help us create a steady-state primary surplus of zero in the base line. Equation (1.9) is exogenized in the multiplier experiment.

5) In order to stabilize the private financial wealth to GDP ratio, we introduce two minor changes in the baseline of the data bank. Firstly, the adjustment term for the long-run private consumption equation is set to be constant to get a constant savings rate. Secondly, we conduct a temporary 1% rise in the adjustment term of the housing stock equation. The latter shock speeds up the transition to steady state where the ratio of the housing capital stock $(fkbh)$ relative to private consumption excluding housing $(fcpuxh)$ is constant.

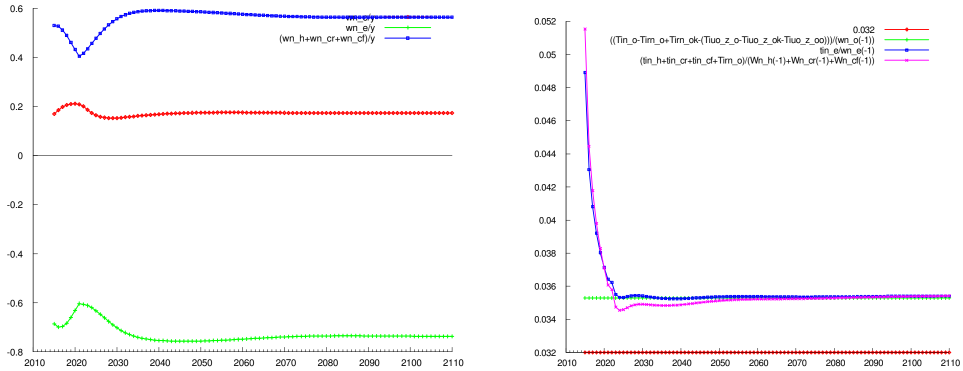
The private sector contains of three subsectors, households (h), financial corporations (cf) and non-financial corporations (cr). The net asset to GDP ratios, i.e. $wn\_h/y, wn\_cf/y, wn\_cr/y$, are not stable for the three subsectors and their implicit interest rates are not 3.53% in the baseline. To get the correct implicit interest rates and stable asset to GDP ratios, we use transfers between sectors.

We make a 1.07% of GDP capital transfer from the non-financial corporate sector to the foreign sector, a 0.94% of GDP capital transfer from the foreign sector to households, and to balance also the financial corporate sector, we reduce its capital transfer tkn_cf by 0.59% of GDP. Moreover an adjustment term in the dividend income of the non-financial corporate sector, Tiu_z_cr, is set to zero. This adjustment term affects the dividend income of the non-financial sector and the residually calculated dividend income of the financial corporate sector, so it is only a question of balancing both corporate sectors in ADAM, the combined corporate sector is not affected.

The mentioned changes in capital transfers and adjustment terms yield relatively constant ratios between net assets and GDP in.

All together, the five modifications help us produce a baseline with relatively constant ratios between net assets and GDP in all five sectors of the model and bring the corresponding implicit interest rates closer to 3.53%, particularly in the long term. This is illustrated in figure 1.1 below.

Figure 1.1 Financial wealth to GDP ratios and the implicit interest rates of the public, private and foreign sectors



3. The fiscal reaction function and multiplier analysis

In this section, we insert a fiscal reaction function in standard ADAM and using the baseline and minor model modifications explained in section 2 we investigate how a fiscal reaction function can stabilize the public debt to GDP ratio in case of a permanent shock to the public purchase of goods and services. More specifically, we employ a forward and backward looking fiscal reaction function, which resembles the MULTIMOD reaction function developed by IMF. In MULTIMOD's version of the reaction function, the tax rate instrument responds to deviations between the public debt and its target. But in ADAM, the tax rate instrument responds to deviations between the primary surplus and its target. The exact ADAM reaction function is given by:

$$\text{tsysp1}_t = \sum_{i=t-2}^{i=t+15} \frac{(\text{tsysp1}_i)}{17} + \gamma(P_{t-1} - P^*_{t-1}) \qquad (1.10)$$

Where, $tsysp1_t$ is the income tax rate at time t, γ is a negative parameter, P is the corrected primary surplus given by $(Tfn\_o - Tin\_o^*)/Y$ and P^* is the target for P. This fiscal reaction function incorporates forward looking expectations via the leaded tax rate. It has not been possible to solve the model with a leaded primary surplus in the reaction function (1.10). By induction, the tax rate at time t is a function of the primary surplus ratio in every time period. This should allow us to bring forward in time the reaction in the tax rate, which should moderate the pro-cyclicality of the fiscal reaction. However, given the crowding out time and the normal accelerator in the model etc. we should expect some volatility in the response of unemployment and other variables.

We now conduct a simulation experiment where the public purchase of goods and services is increased permanently by 1%. Figure 2.1 demonstrates how the public debt to GDP ratio responds presented as absolute deviations to the public debt ratio in the baseline scenario. Over the first decades we get fluctuations in the public debt ratio. The immediate fall in the debt ratio reflects the increase in income tax rate plus the reduction in unemployment benefits and the higher tax base. Thereafter the increased tax rate drives consumption and housing investments down resulting in higher unemployment and lower tax base. This makes the public debt ratio increase. However, the changes in the tax rates, cf. figure 2.3, and the crowding out of any unemployment effect eventually stabilize the public debt to GDP ratio.

For the same reason and over the same period we see fluctuations in the public budget balance (Tfn_o) and modified primary surplus($Tfn\_o - Tin\_o^*$). We note that the debt ratio does not necessarily return to its baseline but it should be parallel to its baseline scenario in steady state. The modified primary surplus does return to its baseline.

Fig 2.1 Public debt/GDP ratio

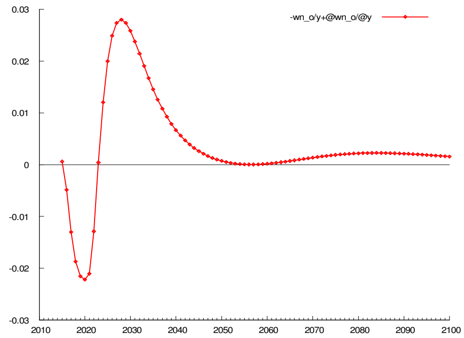


Fig 2.2 Public budget and primary surplus

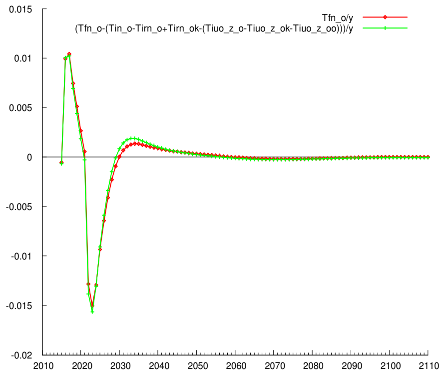


Figure 2.3 presents the calculated change in the two income tax rates presented as relative percentage deviations to the baseline. The changes in the income tax rate is calculated by the reaction function in (1.10), and we note that the long-term rate increase is close to 1%, e.g. from 0.15 to 0.1515. The immediate change in the tax rates is close to the long-term change, but the tax rates do fluctuate somewhat over the first 50 years.

Figure 2.4 shows the simulated deviations of the unemployment, employment and labor force from the baseline scenario. The extra domestic demand created by the higher public purchase increases employment temporarily. But the first round increase in employment is reversed by the rise in the income tax rate and the crowding out effect of higher wages. After some cycles, the labour market variables return to their base line, which contains the structural values of these variables. In other words, public purchase increases in ADAM do not have a permanent effect in the labor market – neither with nor without a fiscal reaction function.

Fig. 2.3 Income tax rates

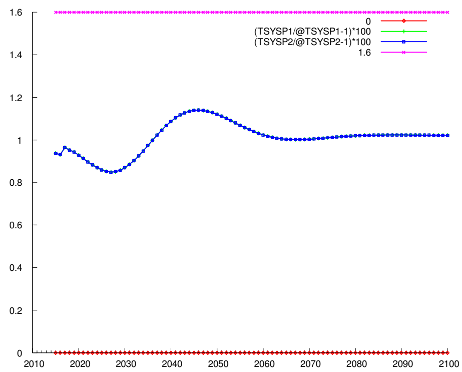
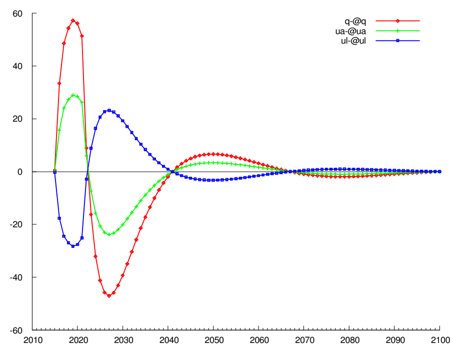


Fig. 2.4 Labor market



4. Conclusion

We investigate the role of a fiscal reaction function in stabilizing the public budget in ADAM. A forward-looking fiscal reaction rule is solved using the Fair-Taylor algorithm of the Gekko simulation software. The analysis seems to confirm that a tax reaction function can stabilize the public debt ratio in the long run.

The employed fiscal reaction rule is a simple ECM form with leads inspired by the MULTIMOD. The precise fiscal reaction function applied is not necessarily the best. It is natural to test the rule for alternative shocks and to try alternative specifications of the reaction function. To handle alternative specifications we may need a stronger algorithm for handling leads in Gekko.

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