The University of Adelaide

Elder Conservatorium of Music
Faculty of Arts

The infinite lattice:

a compositional exploration of symmetrical modes, modes of limited transposition and other musical parameters generated by low prime numbers.

Portfolio of Compositions and Exegesis

Submitted in fulfillment of the requirements for the degree of

Doctor of Philosophy

Dylan Joshua Crismani

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4. *K* for two pianos, two pianists, and two hands. Performance by Dylan Crismani on two just intoned pianos.
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Abstract

This doctoral research project at the Elder Conservatorium of Music, University of Adelaide, has addressed how symmetrical modes (in just intonation), and modes of limited transposition can be applied in conventional ensembles, and how low prime numbers can generate various aspects of a musical composition. The format of the submission is a portfolio of original compositions (chapter three) supported by an explanatory exegesis (chapters one, two, and four). The folio and exegesis investigate two key research areas. The first research area is concerned with how conventional instrumentalists, playing conventional instruments can play music written according to just intonation theory with limited alterations to musical notation, and with no specialised training in just intonation. This doctoral thesis exposes new methodologies, which address how consistent results in the first research area can be achieved. The second research area is concerned with how prime numbers can generate many aspects of a musical composition. The geometric symmetries of just intonation Tonnetzen are exposed, and discussed in relation to Un-transposable Harmonic Modes, which are also exposed for the first time. This thesis posits, and demonstrates that composing music in just intonation for non-just intonation specialists is a credible, and realistic pursuit provided the composer has sufficient knowledge of composition, just intonation theory, and adequate time to commit to the composition process. This thesis also aims to show that just intonation Tonnetzen, which are frequently represented as squares, are actually rectangular. New methods for generating tuning models, derived from geometric operations, are also discussed.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide.

I give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines.

I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

Acknowledgements

The exegesis and folio of music that follow herein would not have been possible without the help, support, and guidance of many exceptional individuals. First and foremost I wish to thank all the performers who brought my music to life both on the concert platform and in private. These very special, talented and devoted individuals include The Basel Chamber Orchestra cond. Franck Ollu, Gabriella Smart, Lisa Moore, Sonya Lifschitz, The Australian String Quartet (Dale Barltrop, Francesca Hiew, Stephen King, and Blair Harris), Angelina Zucco, Madeline Procopio, Thomas Marlin, Hilary Kleinig, Alex Hanysz, and The Elder Conservatorium Symphony Orchestra cond. Luke Dollman. Secondly I wish to thank my supervisors Professor Charles Bodman Rae and Stephen Whittington for their unwavering commitment to my development as a student of music, and to music itself. I wish to thank Matthew Sorrell for helping me to understand simple mathematics. I wish to thank my parents, Christopher Crismani, Michelle Barth, David Gee, and Shaun Byrne for their support in a number of ways. I wish to thank my siblings Wayne Crismani, and Marta Cifuentes-Ochoa for leading by example. I also wish to acknowledge the support I received from my friends. Particularly praiseworthy among my friends are Benjamin Robert Ford Smith, and John Polglase who were often guiding this project in one way or another, Iran Sanadzadeh, who always had the time to encourage me even while fighting her own doctoral battles, and Brett Walter for his interest, help and support regarding various aspects of the project. I wish to express my sincere gratitude to the University of Adelaide, Faculty of the Arts for providing me with a scholarship, to assist with living expenses, for the duration of the degree, and the Australian Government for providing the Research Training Program Scholarship. Finally, I wish to humbly acknowledge the previous work of other devoted scholars, composers, music theorists, and geometers on whose shoulders this thesis stands.

Track listing for CDs

CD 1

1. *Remembering is a Form of Forgetting* for symphony orchestra. Performance by Elder Conservatorium Symphony Orchestra, conductor Luke Dollman. 8’53”
2. *The Un-tempered Pianos* for two pianos and one pianist. Performance by Dylan Crismani on two just intoned pianos. 27’36”
3. *The Un-tempered Pianos* for two pianos and one pianist. Performance by Gabriella Smart on two just intoned pianos. 21’36”
4. *K* for two pianos, two pianists, and two hands. Performance by Dylan Crismani on two just intoned pianos. 13’42”
5. *K* for two pianos, two pianists, and two hands. Performance by Lisa Moore, and Sonya Lifschitz on two 12-TET pianos. 8’57”
6. *The tale of Kiều* for scordatura string orchestra. Performance by Basel Chamber Orchestra. 19’57”

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Glossary of Musical Terminology

Conventional instrument – The instruments belonging to a standard western orchestra, and piano.

Conventional instrumentalist – An instrumentalist with conservatory level training, who may be a student, or a professional level performer, but is not specialised in the field of just intonation.

Critical Band – The band of audio frequencies within which a second tone will interfere with the perception of the first tone by auditory masking.[1] *Critical bands* are used to quantify the ability of the human ear to distinguish between individual frequency tones.[2]

Integer ratio/co-prime ratio/ratio – These terms have been used interchangeably throughout the paper.

Maximal Un-transposable Harmonic Mode (MUHM) – The UHM that exists within a given tuning model and contains the maximum possible number of pitches in that particular system.

Mode of Synthesised Inversional Symmetry – A mode that is initially conceived as a set of pitches (set one) whose structure as a group may be symmetric, or asymmetric. Following the initial conception, the identity isometry is reflected into a new dimension, creating a second set of pitches whose structure is identical to the structure of set one. When combined, the two sets will form an inversionally symmetrical mode, whose symmetry has been intentionally synthesized by the theorist, or composer. This concept is exemplified in the tuning model of Tenney's *Chromatic Canon*.

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[1] Fletcher, Harvey. "Auditory Patterns." *Reviews of Modern Physics*. Vol. 12. New York, New York. January 1940.
[2] *Critical Bands in Human Hearing*. Web. Accessed 1 June 2018. <https://community.plm.automation.siemens.com/t5/Testing-Knowledge-Base/Critical-Bands-in-Human-Hearing/ta-p/416798>

Otonality – A tonality expressed by the over numbers of ratios having a Numerary Nexus; in conventional music theory, "major tonality".[3]

Tonnetz/lattice/matrix/tuning model/tuning scheme – These terms are used interchangeably throughout the paper.

Un-transposable Harmonic Mode (UHM) – An inversionally symmetrical mode, that is derived from the pure intervals found in the harmonic series, and cannot be transposed in the tuning model within which it was conceived.

Utonality – One of those tonalities expressed by the under numbers of ratios having a Numerary Nexus – in current musical theory, "minor" tonality.[4]

12-TET – This acronym is used throughout in place of twelve-tone equal temperament.

[3] Partch, Harry. *Genesis of a Music: An Account of a Creative Work, Its Roots and Its Fulfillments*. Second ed. New York: Da Capo Press, 1974. Page 72.
[4] Partch, Harry. *Genesis of a Music: An Account of a Creative Work, Its Roots and Its Fulfillments*. Second ed. New York: Da Capo Press, 1974. Page 75.

Glossary of Geometric Terminology

Coplanar – Geometric objects lying in a common plane are said to be coplanar.[5]

Geometric series – A type of infinite series where there is a constant ratio between the terms of the sequence.[6]

Glide reflection – A product of a reflection in a line and translation along the same line.[7]

Identity – An identity is a mathematical relationship equating one quantity to another (which may initially appear to be different).[8]

Invariant – Unchanged by specified mathematical or physical operations or transformations.[9]

Isometry – A mapping of a metric space onto another or onto itself so that the distance between any two points in the original space is the same as the distance between their images in the second space. Rotation and translation are isometries of the plane.[10]

Rational number – A rational number is a number that can be expressed as a fraction p/q where p and q are integers and $q \neq 0$. A rational number p/q is said

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[5] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Coplanar.html>
[6] Geometric series. Chegg Study. Web. Accessed 1 June 2018. <https://www.chegg.com/homework-help/definitions/geometric-series-29>
[7] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Glide.html>
[8] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Identity.html>
[9] Merriam-Webster Dictionary. Web. Accessed 12 October 2018. <https://www.merriam-webster.com/dictionary/invariant>
[10] Merriam-Webster Dictionary. Web. Accessed 12 October 2018. <https://www.merriam-webster.com/dictionary/isometry>

to have numerator *p* and denominator *q*. Real numbers that are not rational are called irrational numbers.[11]

Reciprocal – Two numbers are reciprocals if and only if their product is 1. To put it another way, a number and its reciprocal are inversely related. Therefore, the larger a (positive) number, the smaller its reciprocal.[12]

Reflection – The operation of exchanging all points of a mathematical object with their mirror images (i.e., reflections in a mirror).[13]

Rigid Motion – Any transformation, such as a translation or rotation, of a set such that the distance between points is preserved[14]. There are four basic rigid motions: Reflection, Glide reflection, rotation, and translation.

Rotation – The turning of an object or coordinate system by an angle about a fixed point. A rotation is an orientation-preserving orthogonal transformation.[15]

(Helical) screw (motion) – A translation along a straight line *L* and a rotation about *L* such that the angle of rotation is proportional to the translation at each instant. Also known as a twist.

Subordinate symmetry – A comparatively small symmetrical structure, existing within a larger overall symmetrical structure.

Transformation – The term used for four specific ways to manipulate the shape of a point, a line, or a shape.[16]

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[11] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/RationalNumber.html>
[12] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Reciprocal.html>
[13] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Reflection.html>
[14] Dictionary.com. Web. Accessed 12 October 2018. <https://www.dictionary.com/browse/rigid-motion>
[15] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Rotation.html>

Translation – A transformation consisting of a constant offset with no rotation or distortion.[17]

[16] *Math Warehouse.* Web. Accessed 02 June 2018. <http://www.mathwarehouse.com/transformations/>
[17] Weisstein, Eric W. *MathWorld* – A Wolfram Web Resource. Web. Accessed 02 June 2018. <http://mathworld.wolfram.com/Translation.html>

Introduction

The primary research area that this project is concerned with is just intonation, and the composition of a creative portfolio of music written in just intonation specifically to be performed by performers who are not required to possess any practical or theoretical knowledge of just intonation theory.

This central idea is perhaps best exemplified through the twentieth century godfather of just intonation, Harry Partch. A large portion of the instrumental music Partch wrote in just intonation requires the performers to have little to no knowledge of just intonation theory, they simply need to learn how to play his instruments. Much of this music, for example *Delusion of the Fury,*[1] is written as tablature, circumventing the need for all parties to have knowledge of this specialised musical area. The main aim was to work with this central idea of performers not needing to understand just intonation theory in order to play just intoned music, transferring the idea from a set of custom-built instruments, to conventional instruments.

During the course of this research project, two theories were developed which build on present theories of just intonation, making this project partially theory-building in nature. The first theory is, perhaps, a rather obvious exposition of the specific kind of geometric symmetry, which subsists in a just intonation Tonnetz, but is yet to be discussed at length in any published form. The second theory builds conceptually on the first theory. The second theory is a model of modes which exist in certain just intonation Tonnetzen and exhibit inversional symmetry.

In addition to attempting to find practical solutions to the performance of just intoned music, and developing theories about Tonnetzen and modes, this project also examines the ways in which prime numbers can influence the various temporal elements of a musical composition. This project was undertaken in order to address the following explicit aims:

[1] Partch, Harry. *Delusion of the Fury: A Ritual of Dream and Delusion.* Mainz, Schott music. 1965-66, rev. 1967.

1. To investigate a practical approach to how conventional instruments and instrumentalists can produce just intonation consistently throughout a piece of music in a chamber/orchestral context, and to apply it in compositions;
2. To investigate a number of tunings and evaluate the possibilities and limitations of using just intonation in a chamber and orchestral context;
3. To develop a methodology for the composition of music in just intonation for conventional instrumentalists and instruments;
4. To investigate how music can be composed in just intonation with limited alterations to standard notation;
5. To investigate and apply ways in which prime numbers can generate rhythm, form and temporal aspects of a piece of music;
6. To identify and analyse symmetrical modes, and modes with limited transpositions and apply them in compositions.

In order to address the research aims, the project was interrogated through the following key research questions:

1. What are the limitations and benefits of various approaches, and of using conventional instruments and instrumentalists, when writing chamber and orchestral works in just intonation?
2. What are the possibilities and limitations of the just intonation tuning models developed for this project in the context of a chamber/orchestral setting.
3. What processes and framework should a composer follow in order to compose music in just intonation for conventional instrumentalists, and instruments?
4. How can just intonation be achieved with limited alterations to musical notation in a chamber and orchestral context?
5. How can prime numbers generate aspects of a piece of music such as the parameters of pitch, rhythm, form and temporality?
6. How can symmetrical modes, and modes with limited transpositions, or no transpositions be found, analyzed, applied and truncated in just intonation?

The exegesis and folio have been structured according to IMRaD, Introduction, Methodology, Results, and Discussion. The introduction provides a project summary, and an explanation of the conceptual framework upon which the portfolio of creative works is based. A review of relevant repertoire and literature is intertwined with the conceptual framework. The methodology section presents the practical scaffolding for each piece in the folio which turns the abstract theoretical concepts into tangible results. The results section consists of the creative portfolio of musical compositions. The original compositions can be considered as the instructions for setting up experiments which are designed to test aspects of the hypothesis. The performance of these original compositions can be considered as the conducting of experiments. Following the experiments reflection on the entire process takes place in the final chapter which examines how the project has addressed the research aims. Chapter four also addresses the challenges presented in each creative scenario, along with the solutions that were formulated in order to overcome them. The nature of this project was experimental and further toward the quantitative end of the research spectrum than the qualitative end; therefore instead of presenting the creative portfolio and exegesis as two separate parts, which appears to be common in the field of music composition, they are presented here as a single, integrated thesis.

Chapter One: Conceptual Framework, and Repertoire and Literature Review

1.1 Why just intonation?

The theoretical concept of just intonation underpins most aspects of this project, although at times just intonation is approached somewhat transgressively. Purity of intonation and beat-less intervals were not my main goals as a composer. At the beginning of this project I possessed little knowledge on the topic of just intonation, I only knew that it was a way that composers could access new sounds, and harmonies. This was what seduced me about it. As I read and learned more on the topic, I learned that many composers claimed to use just intonation as a way to achieve a certain purity that 12-TET[2] does not offer. Ben Johnston writes the following:

> Just intonation is simply the easiest way to tune musical intervals by ear. It results in greatly heightened purity and clarity of sound for two reasons: it eliminates acoustic beats to the maximum possible, and second, it exploits resonance by utilizing harmonically simple combinations of pitches.[3]

La Monte Young appears to have similar motivations for using just intonation. In the performance notes of Chronos Kristalla, Young prescribes the following for the performance:

> I have chosen to use only one natural harmonic on each of the sixteen strings of the quartet. This way each harmonic can be tuned perfectly to a frequency standard.[4]

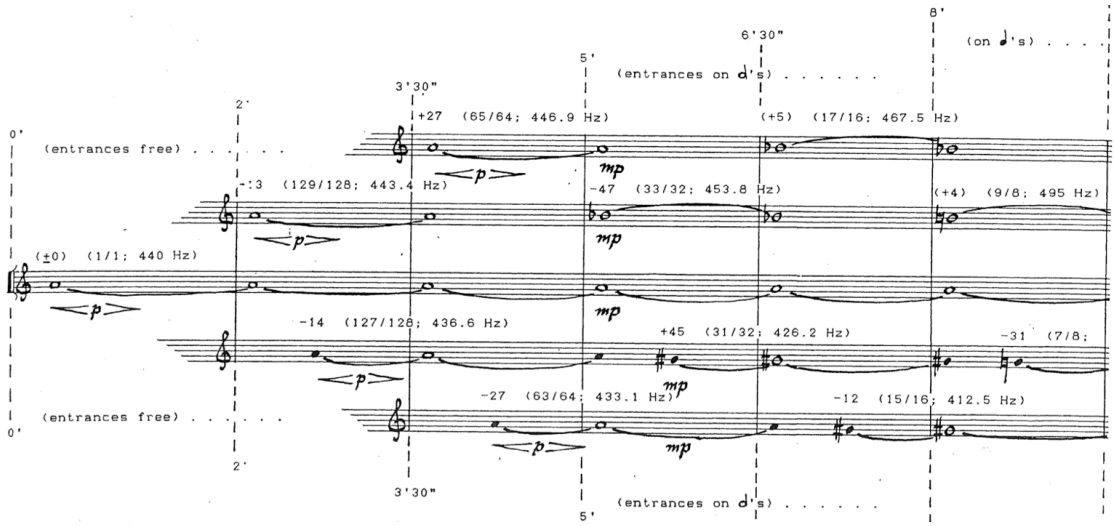
It appears that Young's primary goal, in this piece of music, is to achieve the most pure sounding intervals possible. I acknowledge that this is a valid approach for others, and does indeed produce elegant results, but my personal musical goals are different. After some time spent absorbing the theory of just intonation, and a few experiments conducted on my pianos, I discovered that just intonation could be used as a tool to achieve new sonic textures, not simply beat-less purity. The harmony in my imagination, which materializes in this project, consists of multiple different tunings of the same note played simultaneously, or in quick succession of one another. This concept, although aesthetically quite different from, can be compared to the concept of sruti in Indian classical music[5] or to

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[2] 12-TET is the acronym for twelve-tone equal temperament.
[3] Fonville, John. "Ben Johnston's Extended Just Intonation: A Guide for Interpreters." *Perspectives of New Music* 29.2 (1991): 106-37. Web.
[4] Young, La Monte. *Chronos Kristalla (Time Crystals) from The Magic Chord x 4*. Unpublished. 1990.
[5] Sinha, Purnima. *An approach to the study of Indian music*. Calcutta. Indian Publications Monograph Series No. 11. 1970. Pp. 18-22.

critical band theory. Two or more tones will interact with each other in various ways, depending on the degree of frequency separation, throughout different stages of the critical band the ear perceives roughness, and beating, and below certain frequencies it will perceive intertones[6]. Tenney has tackled this concept already, most notably, in his work *Critical Band*[7]. *Critical Band* unfolds as a drone in unison, with the opening four bars exploring various points within a single critical band, or five different tunings of the A natural, this is illustrated in example 1.1.

Example 1.1, bars 1-6 of James Tenney's *Critical Band.*



Bar 5 explores the relationship between two kinds of G#, and two kinds of Bb. This is also shown in example 1.1. Tenney is clearly interested in symmetry, and the way in which Tenney explores symmetry in *Critical Band* warrants discussion. Tenney could have explored critical bands and symmetry by using a set of integer ratios, and their reciprocals, as he does in *Chromatic Canon.*[8] Instead Tenney achieves symmetry, and exploration of the critical band through Hertz values. Each new note, above the fundamental is paired with a note below the fundamental that is exactly the same distance from the fundamental in Hertz. This is illustrated in table 1.1.

[6] Campbell, Murray, and Clive Greated. 'Hearing Musical Sounds' in *The Musician's Guide to Acoustics*. Oxford University Press, 1994. Pp. 57-61.
[7] Tenney, James. *Critical Band*. Canadian Music Centre. 1988.
[8] Tenney, James. *Chromatic Canon* for two pianos. Baltimore. Sonic Art Editions, 1983.

Table 1.1, The pitch-pairing scheme of *Critical Band*.

Note name	Represented by integer ratio	Appears in bar no(s).	Distance from fundamental (in Hertz)
A	2:1 (no corresponding pair)	13	+440
G#	15:8 (no corresponding pair)	12 – 13	+385
G	7:4	11 – 13	+330
F	13:8 (no corresponding pair)		+275
E	3:2	9 – 13	+220
D#	11:8	8	+165
C#	5:4	7 – 13	+110
B	9:8	6 – 9	+55
Bb	17:16	5 – 6	+27.5
Bb	33:32	4 – 5	+13.8
A	65:64	3 – 4	+6.9
A	129:128	2 – 3	+3.4
A	**1:1**	**1 - 13**	**0**
A	127:128	2 – 4	-3.4
A	63:64	3 – 5	-6.9
G#	31:32	4 – 6	-13.8
G#	15:16	5 – 7	-27.5
G	7:8	6 – 10	-55
E	4:3	7 – 13	-110
C#	5:8	9 – 11	-165
A	1:2	11 – 13	-220
A	1:4	13	-330

As Table 1.1 shows, *Critical Band* is mostly symmetrical, in the sense of vertical harmony. The way in which individual pitches are introduced according to their distance in hertz from the fundamental demonstrates the psycho-acoustical phenomenon of beating. The introduction of 129:128, and 127:128 cause 3.4 beats per second to emerge, 65: 64 and 64:63 cause 6.9 beat per second to emerge, and so on, until the 'beating' appears to cease, and a roughness in the musical texture is perceived instead.

Tenney's approach to exploring critical bands is surely elegant, but it differs somewhat from the approach that has been taken in this research project. The approach of this project differs as it involves the combination of modality with relatively large just intonation lattices. One way of perceiving a large lattice is that it is a harmonic fabric where multiple different tunings of the same limited number of pitches exist. Modes can then be constructed which utilize, for example, the five pitches C#, D#, F#, G#, A# of which there are two or three

tunings of each. If the various tunings of each note in this mode are always sounded simultaneously the ear really only perceives a pentatonic scale, but with a different sonic texture than if only one version of each pitch were sounded. Absolute precision and purity of tuning and intervals are, for the purposes of this project, not as critical as simply achieving these different sonic textures. A certain level of precision is of course desirable; aleatory music is not what this project aims to achieve but it also does not demand the level of purity and precision as La Monte Young seeks in Chronos Kristalla[9]. The middle path is sought.

1.2 Discourse on standard musicians, ensembles, and notation.
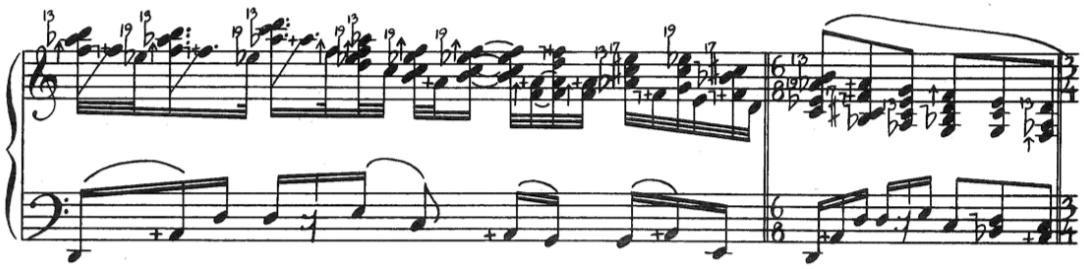
One of the main ideas behind this project was to address a perceived gap in the repertoire of just intoned music, written for specifically for instrumentalists who are not just intonation specialists. There exists a reasonably large body of work in just intonation written for highly specialized musicians, for example the string quartets of Ben Johnston, but little music exists that is accessible for performers who aren't willing to sacrifice many years to specializing in just intonation.
Ben Johnston's music is, from one perspective, more accessible for performers than Partch's music as Johnston scores his output for conventional instruments and musical ensembles. Despite this, his music is still often littered with ratios, his adaptions of accidentals, and in the case of string quartets nos. 2[10] and 3[11] a system of lines, reminiscent of a Pollock painting, connecting 3:2, 4:3, 5:4, 6:5, 8:5, 5:3, and 9:8 between instrumental parts on the full score. Johnston's contributions to music notation, theory, and composition are prolific and exceptional, no derision is intended here. The solid lines described, however, appear slightly excessive and seem to be more of a hindrance than a help to the analyst, but perhaps of some use to performers. The fact that these lines disappear in Johnston's subsequent string quartets indicates that maybe Johnston would, or did at some point concede that they are unnecessary.
Johnston's new system of musical notation for just intonation, outlined by

[9] Young, La Monte. *Chronos Kristalla (Time Crystals) from The Magic Chord x 4*. Unpublished. 1990.
[10] Johnston, Ben. String Quartet No. 2. Baltimore. Smith publications, 1964.
[11] Johnston, Ben. String Quartet No. 3. Baltimore. Smith publications, 1966.

Johnston[12] and Fonville[13] is innovative and provides the performer with precise mathematical detail about the tuning of musical pitches. There can be no doubt that Johnston's music requires specialist training. Firstly, the performers must learn the theory of just intonation. They must then learn Johnston's system of notation, which is by no means easy, even for those with a firm grasp of just intonation theory. Thirdly, once just intonation theory, and Johnston's system of notation are absorbed, the performers must then actually find these new pitches on their instruments, using their ears. Performing Johnston's music demands a significant time commitment from performers.

The one exception to this rule is of course Johnston's Suite for Microtonal Piano.[14] Once the piano is retuned there is absolutely no need for the performer to have any knowledge of just intonation theory, or Johnston's adapted notation. Despite this, Johnston persists in littering the score with ratios and his adapted style of notation, example 1.2 illustrates this.

Example 1.2, bars 52-53 of *Blues.* Excerpt from Ben Johnston's Suite for Microtonal Piano.



Information regarding the tuning of the piano is absolutely essential: this is provided in the performance notes. The addition of ratios in the musical score, however, is entirely unnecessary. The information is presumably meaningless to the performer, and could promptly be deduced from the tuning chart provided in the performance notes if a music analyst required that information.

[12] Johnston, Ben. *Maximum Clarity and Other Writings on Music.* Urbana and Chicago. University of Illinois Press, 2006.
[13] Fonville, John. "Ben Johnston's Extended Just Intonation: A Guide for Interpreters." *Perspectives of New Music* 29.2 (1991): 106-37. Web.
[14] Johnston, Ben. Suite for Microtonal Piano. Smith publications. Baltimore, USA. 1977.

Harry Partch takes various stances on the topic of notation, devoting two chapters to it in *Genesis of a Music.*[15] His approaches to notation include the use of tablature, systems of colours and shapes both on the instrument itself and on the five-line stave, and a system of notation written entirely in ratios.
Partch and Johnston, for various reasons, represent the less accessible end of the just intonation spectrum. At the other end of the micro-tonality accessibility spectrum lie composers such as Ligeti, Tenney, Sciarrino, and Andriessen. Of these composers, Tenney is the only one who actually appears to subscribe to the theory of just intonation. Ligeti's Hamburg Concerto[16] is an interesting middle ground between 12-TET, and just intonation. As a tempered keyboard instrument does not accompany the orchestra, it is unlikely that they are playing in 12-TET as each instrument has varying degrees of control over the intonation of individual pitches. Ligeti does alter standard notation slightly, but at this point in the 21st century his alterations lie towards the conservative end of the spectrum. Three types of arrow are used to indicate that certain pitches are natural harmonics and will sound flatter than usual, this is illustrated in example 1.3.

Example 1.3, excerpt from the performance notes of Ligeti's Hamburg Concerto

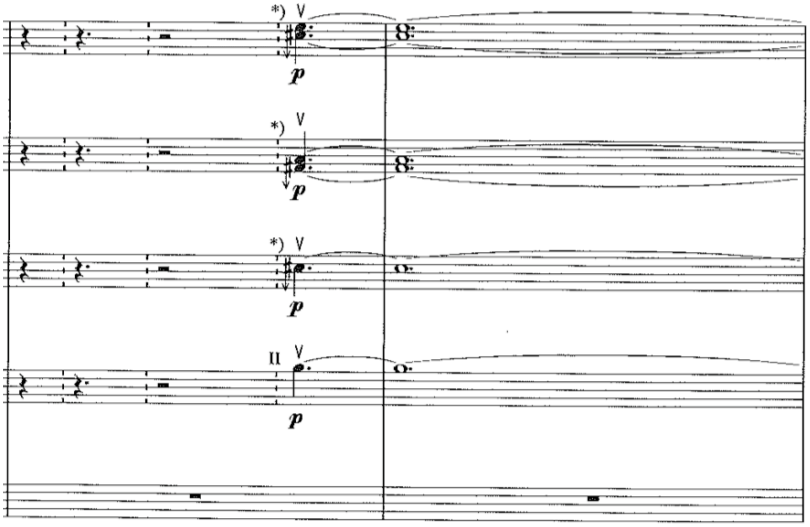
The lowering of the pitches is indicated approximately,
since there is no measurement for 14, 31 or 49 cent.
↓ just a bit lower (15%: 5th harmonic), ↓ lower (30%: 7th harmonic),
𝄲 quarter-tone lower (50%: 11th harmonic).
The four natural horns always produce non-tempered natural harmonics;
therefore the right hand must not correct the pitches.
Exception: the last horn chord in movement VI (at MM) is to be played stopped and sounds a semitone higher.

In other instances the strings are called upon to flatten certain notes which are not to be taken as harmonics. The string players are given ample time to prepare for such altered pitches, such as the passage shown in example 1.4.

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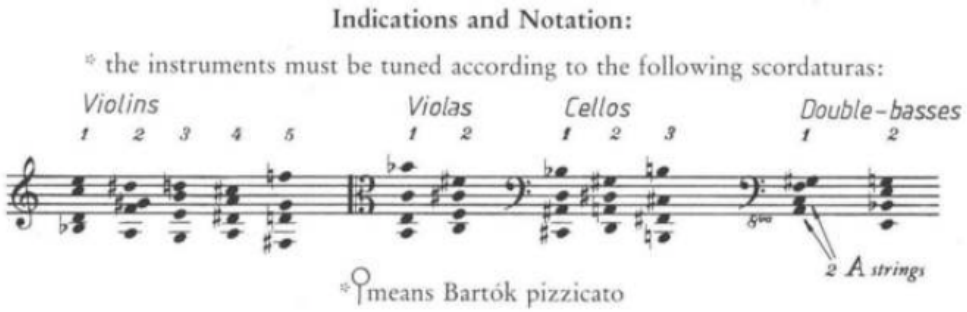
[15] Partch, Harry. *Genesis of a Music: An Account of a Creative Work, Its Roots and Its Fulfillments*. Second ed. New York. Da Capo Press, 1974. Pp 195-259.
[16] Ligeti, Gyorgy. *Hamburg Concerto,* for solo horn and chamber orchestra. Mainz. Schott music. 1998-99.

Example 1.4, bars 11-12 from Ligeti's Hamburg Concerto.



Notice that this is a long sustained passage in a section marked adagio, not a passage of semi-quavers set at a fast tempo with arrows and ratios attached to every note head, such as may be found in a Johnston string quartet. Additionally, these pitches are not to be as strictly tuned as the string writing of Johnston. Andriessen's Symphony for open strings[17] is similar to the Hamburg Concerto in that it is a middle ground between 12-TET and just intonation. Andriessen specifies a scordatura in the performance notes, which is shown in example 1.5, but doesn't provide precise ratios or Hertz values.

Example 1.5, excerpt from the performance notes of Louis Andriessen's Symphony for open strings.

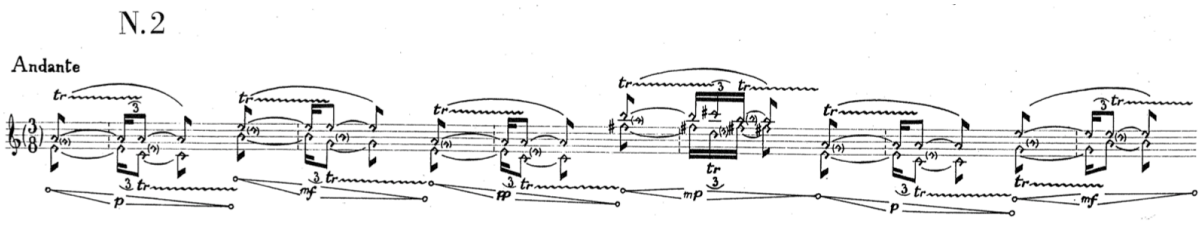


[17] Andriessen, Louis. Symfonie voor losse snaren (Symphony for open strings). Amsterdam. Donemus, 1978.

Should the performers tune their strings to a nearby piano? Should they use electronic tuners to find some exact Hertz value for each string? Should they use Pythagorean tuning? Andriessen doesn't specify. Each performance, therefore, will presumably be slightly different depending on how performers choose to tune their instruments. Andriessen tackles an interesting problem in this piece regarding coordination of the music. Due to the limitation imposed by the open string element of the piece, performers must work as a team to produce a single melodic line; a single player alone cannot produce a melody. This hocketing technique that Andriessen employs can be compared to a number of world musics such as Javanese and Balinese Gamelan music, and also to vocal music of the 13th and 14th centuries.

Sciarrino's Six Caprices for violin solo[18] are written entirely in natural harmonics, making it a piece that should, by default, be in just intonation, whether Sciarrino subscribes to the theory or not. Perhaps because of this lack of subscription to just intonation theory, the notation is completely void of exotic accidentals, ratios, or cent deviations. In fact the score is often void of the true sounding pitches, Sciarrino chooses to notate the score as a tablature simply showing the violinist where to place their fingers, this is illustrated in example 1.6.

Example 1.6, excerpt of the beginning the Caprice No.2 from Salvatore Sciarrino's Six Caprices for violin solo.



Treating notation as a tablature in the case of prescriptive scores, in the way that Sciarrino demonstrates in example 1.6, shall be employed in this project for reasons which Partch has already succinctly explained using rather colourful metaphors in reference to his chromelodeon notation:

[18] Sciarrino, Salvatore. *Six Capricci* per vioino. Milano. Casa Ricordi, 1976.

> . . . when the player sees an "F#," in the lower part of the treble clef, for example, he plays it automatically. However, he will not *hear* n "F#," but rather a low "E*b*" (11/7). At first the sounds are strange, but, as suggested above, the player's eyes and fingers are too well correlated to let that strangeness disturb him once the new psychological process has begun [...] The familiar notation is a track over which to run a newly conceived train. The designers of new trains – engines and coaches – might wish they had rails closer together or farther apart, or a track of an entirely different nature, but the wish is futile because the new trains, if they are to run in the service of public transportation, must use the system of tracks available.[19]

In some cases it is foreseen that the additional inclusion of sounding pitches in the score, despite perhaps not being entirely accurate, may aid performers in ensuring the tablature has been correctly written, and may help to uncover editorial errors.

1.3 Discourse on the symmetry of a Tonnetz and Untransposable Harmonic Modes (UHMs)[20]

In a letter to John Chalmers, written on April 26, 1975 Erv Wilson writes the following:

> The basic structure of "moments-of-symmetry" is embarrassingly simple. Unfortunately, what is simple is not always obvious, or vice versa. And we use certain devices repeatedly, without identifying them. Or we neglect to use them when we might well have chosen to do so, had we only recognized them.[21]

Wilson's sentiment resonates with the observations that are about to be outlined here. Although it appears both simple and obvious, and presumably numerous other music theorists have noticed the same things, no discourse has been uncovered, within the scope of this research project, on the physical, and musical occurrences, which are about to observed and commented upon here.

There are geometric symmetries which exist in a just intonation Tonnetz that could be used to aid the analysis, composition, and discussion of music in just

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[19] Partch, Harry. *Genesis of a Music: An Account of a Creative Work, Its Roots and Its Fulfillments.* Second ed. New York: Da Capo Press, 1974. Pp. 215-216.
[20] A glossary of terminology has been included at the beginning of this thesis, containing definitions of the geometry specific terms used in the discourse on symmetry.
[21] 'Erv Wilson's letter to Chalmers' in *The Wilson Archives*. Web. Accessed 12 December 2017. <http://anaphoria.com/mos.pdf>

intonation. James Tenney discusses the idea of geometric symmetry in just intonation in the article On Crystal Growth in Harmonic Space (Tenney, 1993-1998), and he clearly explores these ideas in Chromatic Canon for two pianos (Tenney, 1983). Similarly, and much more extensively, Tymoczko posits a theory of harmony based on geometry in *A Geometry of Music*.[22] Tymoczko is aware of, but apparently uninterested in, just intonation. He does not discuss the geometry specific to harmonic ratios and just intonation Tonnetz. His discourse centres around the geometry of chords and pitches in 12-TET. The discourse that follows herein makes explicit the specific type of symmetry which subsists in just intonation.

Wilson is interested in both just intonation, and the geometric patterns which are a result of it. His variable tuning models which are available in the Wilson archives[23], and on Wilsonic[24] demonstrate certain elements of the geometric symmetry which are about to be discussed, yet he provides no discourse on the geometric symmetry of a Tonnetz.

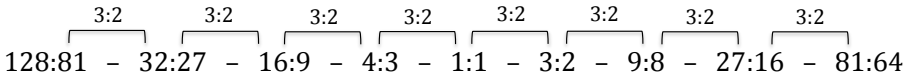
Symmetries of a line in just intonation

The two rigid motions of a line, that is translation and reflection[25] are both possible in just intonation if a single axis of a Tonnetz is considered in isolation. Example 1.7 shows a geometric series, or a series of perfect fifths as they would appear in a just intonation matrix. The operations discussed in this section are not only limited to a series of fifths, they can be applied to any series of a given interval. A series of fifths has been used simply because 3:2 is the first generative ratio encountered in just intonation.[26]

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[22] Tymoczko, Dmitri. *A Geometry of Music*: *Harmony and Counterpoint in the Extended Common Practice.* New York. Oxford University Press, 2011.
[23] *The Wilson Archives*. Web. Accessed on 10 May 2018. <http://www.anaphoria.com/wilson.html>
[24] Sattelite, Marcus (2014). Wilsonic (Version 1.7.91) [Mobile application software]. Retrieved from http://itunes.apple.com
[25] The musical terms transposition and inversion correspond to the geometric operations of translation and reflection.
[26] Doty, David B. *The Just Intonation Primer: An introduction to the theory and practice of Just Intonation.* Second ed. San Francisco. The Just Intonation Network, 1994. P. 33.

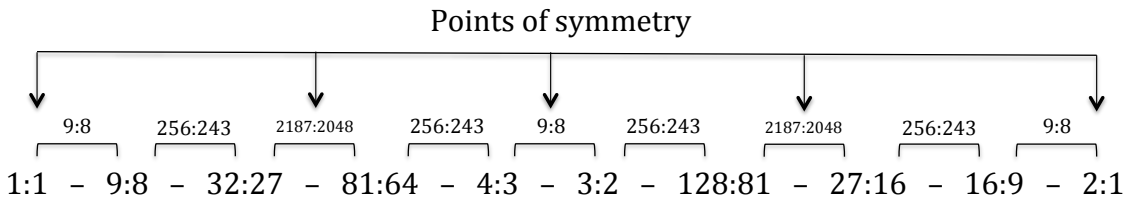
Example 1.7, a series of perfect fifths.



The interval, or quotient between adjacent harmonic ratios shown in example 1.7 is 3:2. Consider the scenario where 1:1 shifts two places to the right. Rigidity implies that if 1:1 moves two places to the right then all other points on the line must also move two places to the right. When this operation is conducted, all relationships between adjacent ratios in the line will continue to be 3:2, assuming the scale is open-ended, therefore translation is possible. Next consider a scenario where some chosen point is fixed.[27] If a mirror were placed on the fixed point, and only one half of the line were visible, the reflection would map a row of notes which had inverse relationships to the portion of the line that were visible. For example, if 1:1 were taken as the centre point, 3:2 would be reflected as 2:3, 9:8 would be reflected as 8:9 and so on. Once the rules of just intonation are taken into account[28] it can be seen that the rigid motion of reflection is possible in just intonation.

It should come as no surprise to the reader that a geometric series is symmetrical. What is interesting is that if the pitches from a series of fifths are re-arranged into ascending order to form a musical scale, or mode, some symmetry persists. This is shown in example 1.8.

Example 1.8, the mode generated by the series of fifths from example 1.7.



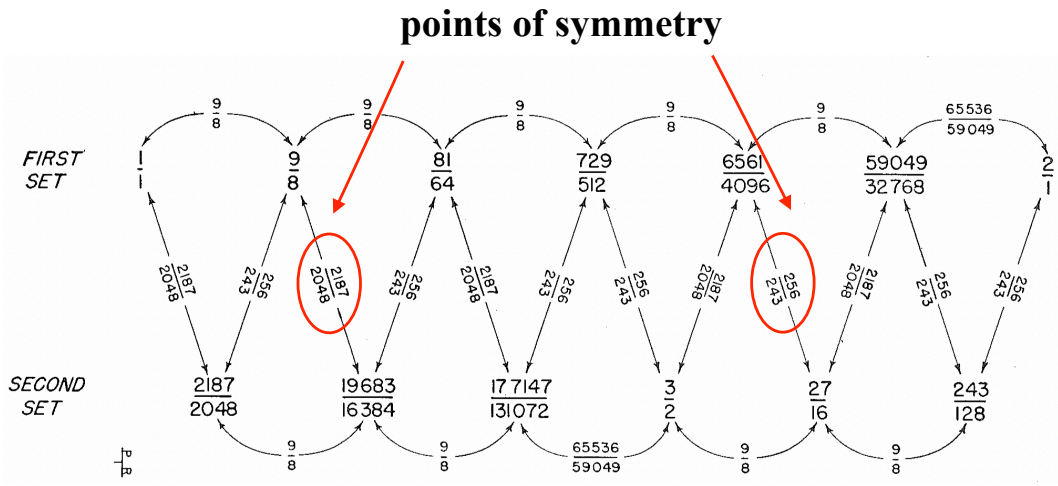
[27] A point can either be a pitch, or the interval between two pitches.

[28] What is meant by 'the rules of just intonation' is that harmonic ratios are frequently represented as fractions with values between one and two, as this is the way to neatly frame an interval inside a single octave. Ratios yielding values below one, or above two are altered in order to keep them within the desired range, unless explicit discussion of intervals greater than an octave is taking place.

The Harmonic ratio of 2:1 has been included in example 1.8 in order to close the mode and as a result complete the symmetrical pattern which is shown by the points of symmetry. Once the harmonic ratios are rearranged into a mode, the symmetry of the line changes. The ordering of pitches shown in example 1.8 is much less symmetric than the ordering of pitches in example 1.7. In example 1.7, any point on the line is a possible point of reflection, whereas in example 1.8 the possible points of symmetry are limited to four points out of eighteen.

This type of symmetrical musical scale, derived from a series of fifths can be traced back to Pythagoras, and Ling Lun. Ling Lun's scale is shown in example 1.9.

Example 1.9, Ling Lun's Scale, excerpt taken from Partch's *Genesis of a Music*.[29]



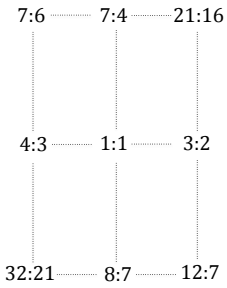
The points of symmetry, which have been marked, intercept 256:243 between 6561:4096 and 27:16, and 2187:2048 between 9:8 and 19683:16384. Although probably aware of its existence Ling Lun omits the Pythagorean comma, instead choosing to simply close the scale with 2:1, creating an inversionally symmetrical mode.

[29] Partch, Harry. *Genesis of a Music: An Account of a Creative Work, Its Roots and Its Fulfillments*. Second ed. New York. Da Capo Press, 1974. P. 401.

Symmetries of the plane in just intonation

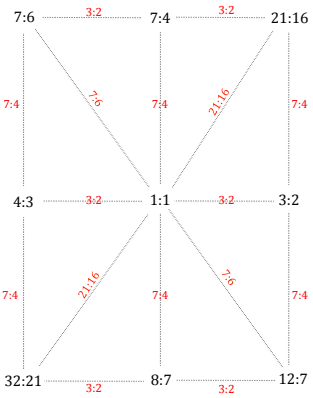
The symmetries of the plane, that is translation, reflection, rotation and glide reflection are also possible in just intonation. Consider the Tonnetz shown in example 1.10.

Example 1.10, a seven-limit just intonation Tonnetz.

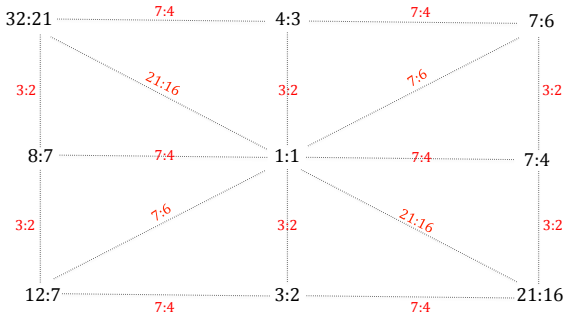


Example 1.10 shows a matrix where harmonic ratios are related by intervals of fifths on the *x*-axis and intervals of sevenths on the *y*-axis. The group of nine pitches shown in example 1.10 can be moved to some other arbitrary co-ordinates through a translation, assuming the system is open-ended. The relationships connecting each note to each other note in the lattice will remain invariant if each pitch shifts according to a set distance. This is given, as the Tonnetz is made up of coplanar lines whose symmetries have already been shown in example 1.7. The three parallel lines of fifths will reflect across the vertical line, and the three parallel lines of sevenths will likewise reflect across the horizontal line. Reflection across the line 7:6, 1:1, 12:7 yields variant results; likewise, reflection across the line 32:21, 1:1, 21:16 also yields variant results. This indicates that the shape in question does not meet the criteria of a square as rotation by 90 or 270 degrees produces results which are variant from the identity isometry. This is illustrated in examples 1.11 and 1.12.

Example 1.11, identity isometry of example 1.10, showing the intervals between the harmonic ratios shown in example 1.10.

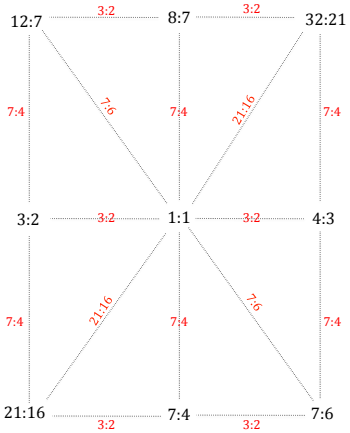


Example 1.12, rotation of the identity isometry by 90, or 270 degrees.



Example 1.12 shows that the results of this operation are variant, as 7:4 and 3:2 are not the inversions of each other, nor are 21:16, and 7:6. Rotation by 180 degrees produces results which are the same as that of the identity. This is shown in example 1.13.

Example 1.13, rotation of the identity isometry by 180 degrees.

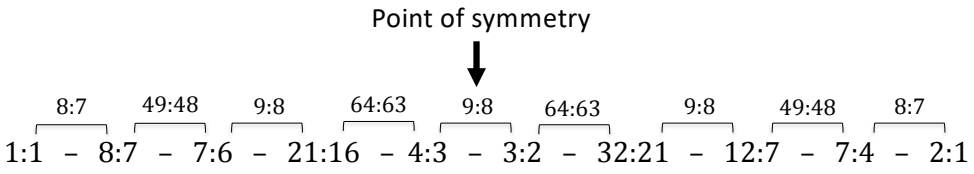


The variant results produced by a 90 or 270 degree rotation, combined with the invariant results produced by a 180 degree rotation indicate that the Tonnetz is not square shaped, but rectangular. Although it seems obvious, it needs to be stated that a square is not possible in a just intonation tonnetz because new dimensions are generated through introducing different prime numbers, whose values are not equal to one another.

As reflection and translation are both possible in just intonation, a glide reflection is also possible, as a glide reflection is simply a reflection followed by a translation. Glide reflection is only possible, however, if the reflection does not take place through the diagonal lines 7:6, 1:1, 12:7, and 32:21, 1:1, 21:16 as it has already been established that these lines produce results which are variant from that of the identity.

Symmetry persists when the harmonic ratios in the Tonnetz shown in example 1.10 are arranged in ascending order to form a mode. This is shown in example 1.14.

Example 1.14, the harmonic ratios from example 1.10 rearranged in the form of a mode.



As with example 1.8, example 1.14 shows that once a symmetrically shaped Tonnetz is arranged into an ascending order some symmetry persists, although it is different to the symmetry of the lattice shown in example 1.10. Again, the result of this particular reordering produces a mode that has fewer symmetries than the symmetries apparent in original layout of the Tonnetz.

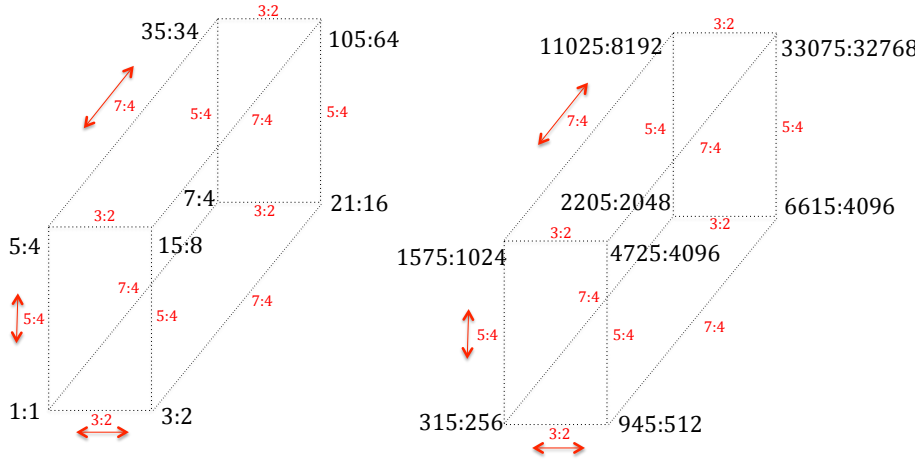
Symmetry in just intonation in three-dimensions

As the most symmetric shape available in a two-dimensional just intonation matrix is a rectangle, so the most symmetric shape in three-dimensions will be the rectangular prism, or cuboid. It should be noted that triangular based prisms,

pyramids, and octahedra do exist in just intonation Tonnetzen but will not be discussed here.

The rectangular prism has three planes *x, y, z* of reflection, along with three axes *x, y, z* of rotation. Translation, reflection, rotation, glide reflection, and helical screw motion are all possible. An example translation in three-dimensions is shown in example 1.15.

Example 1.15, translation of a 7-limit lattice.

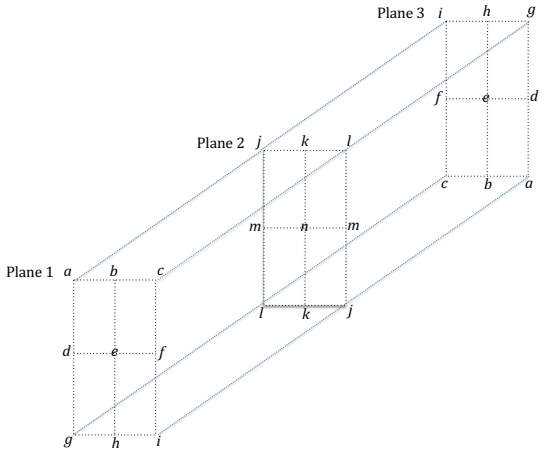


Example 1.15 shows that translation in three dimensions yields invariant results: all adjacent points on the lattice continue to be related by ratios of 3:2, 5:4, and 7:4. The translation conducted on the identity lattice was move east two spaces on the *x*-axis, move north one space on the *y*-axis, and upward one space on the *z*-axis. Each plane of symmetry will divide the rectangular prism into congruent halves. Reflection across lines on the plane have already been shown in the previous section, so reflections across the *x, y*, and *z* planes are given. Glide reflections are also given, as translation and reflection have been proven. 180-degree rotations in two-dimensions have also been proven in the previous section. 180-degree rotations are possible on any of the three axes in three dimensions; again 90, and 270 degree rotations yield results which are variant. Helical motion, that is a combination of a rotation with a parallel translation, is also possible in a three-dimensional just intonation Tonnetz.

When a three dimensional lattice is arranged into a mode, the pairing of pitches occurs in the following way: pitches on plane one reflect through plane two into plane three whilst undergoing a 180 degree rotation, while the pitches on plane

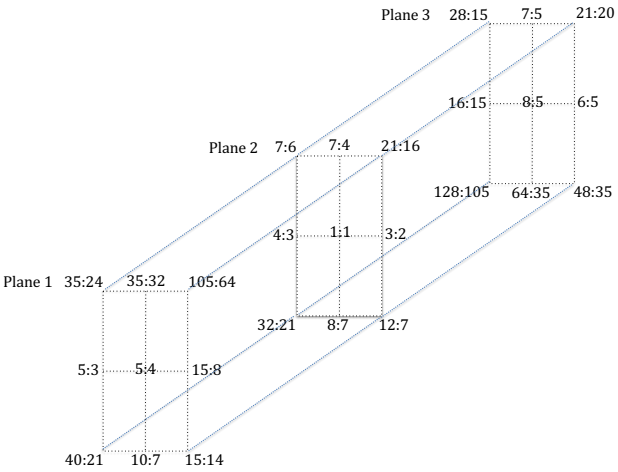
two are paired according to a 180 rotation. This is illustrated in example 1.16, using alphabetical pairs.

Example 1.16, plane one reflecting through plane two, and undergoing a 180 degree roation to form plane three.



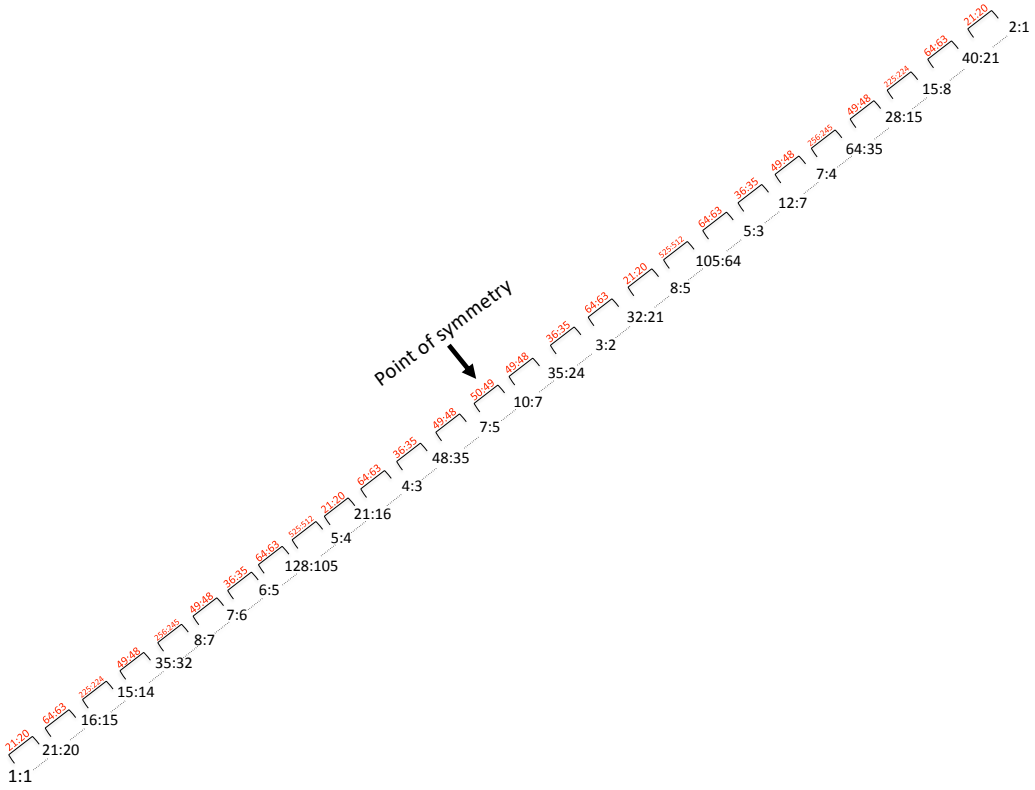
Example 1.17 is a different representation of Example 1.16, where harmonic ratios have been used to represent the various points on the lattice.

Example 1.17, the three-dimensional lattice shown in Example 1.16, with harmonic ratios instead of letters.



When the pitches in example 1.17 are arranged in ascending order the result is another symmetrical mode. The mode is shown in example 1.18.

Example 1.18, the harmonic ratios from example 1.17 re-arranged in ascending order as a mode.



The mode shown in example 1.18 is again, as with the one and two-dimensional modes, less symmetric than the arrangement of pitches as they lie in the lattice.

Un-transposable Harmonic Modes (UHMs)

The symmetries which occur in a just intonation Tonnetz result in Un-transposable Harmonic Modes, which from this point will be referred to as UHMs. UHMs are a new and original contribution to knowledge that have been uncovered through this doctoral project. A UHM is an inversionally symmetrical mode that is derived from the pure intervals found in the harmonic series, and cannot be transposed in the tuning model within which it was conceived;[30] this is the definition that has been developed for this discussion. UHMs are conceptually derived from Messiaen's modes of limited transposition, which were first discussed in *The Technique of My Musical Language* in 1944, but are actually quite different. Messiaen writes the following of his own modes:

[30] In this case the tuning model, although it could technically be continued infinitely in all directions, is closed.

> The series is closed, it is mathematically impossible to find others, at least in our tempered system of 12 semitones.[31]

Messiaen acknowledges that another system of tuning could potentially contain more modes of limited transposition but is content to leave them for someone else to decipher. UHMs pick up where Messiaen left off, in an alternative system of tuning, where the mathematical restrictions imposed by 12-TET are replaced by a new set of restrictions and the discovery of new modes is limited by a new set of rules. As UHMs are conceptually connected to Olivier Messiaen's modes of limited transposition it is logical that they should share some features. Firstly, as with Messiaen's modes, in each set of UHMs there is a maximal UHM from which the other UHMs in the set are derived through truncation. The maximal UHM can be defined as the UHM that exists within a given tuning model and contains the maximum possible number of pitches in that particular system, or a section of a system. If the tuning system is irregularly shaped, or is multi-dimensional, then it will likely contain more than one maximal UHM.[32] All other UHMs in that tuning system would be considered truncations of the maximal UHM. The second feature that UHMs share with Messiaen's modes of limited transposition is that they both exhibit inversional symmetry. Inversional symmetry, according to Cohn, can be considered a property of any pitch-set which […] pairs its pitches symmetrically about a pitch axis.[33] When truncated according to specific techniques, discussed in the next section of this paper, the maximal UHMs also yield inversionally symmetrical derivative UHMs, or truncations. Despite the conceptual connection to Messiaen's modes, UHMs do present slight differences. The first difference between UHMs and Messiaen's synthetic modes is that UHMs do not have a limited number of transpositions. UHMs theoretically have no transpositions. It should be acknowledged here that as just intonation is an open system, any given system in just intonation could be extended indefinitely in all directions, in which case it could be argued that UHMs could be transposed indefinitely. Conversely, it could also be argued that if the system in question

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[31] Messiaen, Olivier. *Technique de mon langage musical*. Paris. Alphonse Leduc, 1944. P. 58.

[32] This is my own definition of a maximal UHM.

[33] Cohn, Richard. "Inversional Symmetry and Transpositional Combination in Bartók." *Music Theory Spectrum*, vol. 10, (1988): page 21. Web.

were to be extended indefinitely then any previous UHMs would become invalid (as they would no longer meet the criteria for what constitutes a UHM, according to the definition of a UHM), and a new set of UHMs would become the UHMs of that new system. The second difference, which is closely linked to the first difference is that UHMs are conceived in just intonation whereas Messiaen's modes, were conceived in 12-TET.

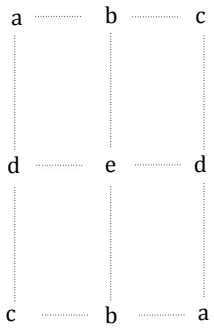
Inversion versus equidistance

Inversion in music, and mathematics implies the reversal of some relationship. Equidistance implies that a certain distance is traversed, in equal portions. In the case of UHM pitch-pairs, before moving any further ahead, the difference between these two things must be illuminated. Pitch pairs will always be equidistant from the point, line, or axis of symmetry, thus making the two notes which form a pitch pair the intervallic inversion of one another. Pitch pairs are not always mathematically inverse however. The factor affecting mathematical inversion of pitch pairs is the location, within the tuning model, of the fundamental ratio 1:1, as it is the placement of this ratio that will determine the value of all other ratios.

This concept may be best elucidated through two examples, the tuning models of Terry Riley's *Harp of New Albion*[34], and La Monte Young's *The Well Tuned Piano*[35]. These two tuning models are reasonably similar in that they are both two-dimensional systems, and they both contain a nine-pitch segment where eight pitches pair symmetrically around a central pitch as illustrated in example 1.19.

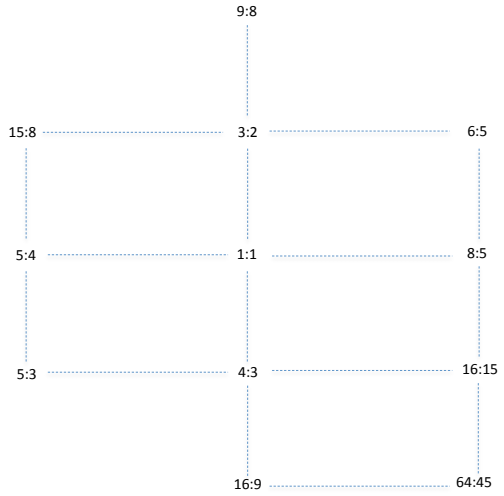
[34] Riley, Terry. *The Harp of New Albion*. Perf. Terry Riley. Munich. Celestial Harmonies, 1992.
[35] Young, La Monte. "The Well Tuned Piano." *YouTube*. YouTube, 23 June 2016. www.youtube.com/watch?v=9JV-66bGp7Y. Accessed 01 Mar. 2017.

Example 1.19, eight pitches, represented by algebraic (not musical) letter names paired around a central pitch.



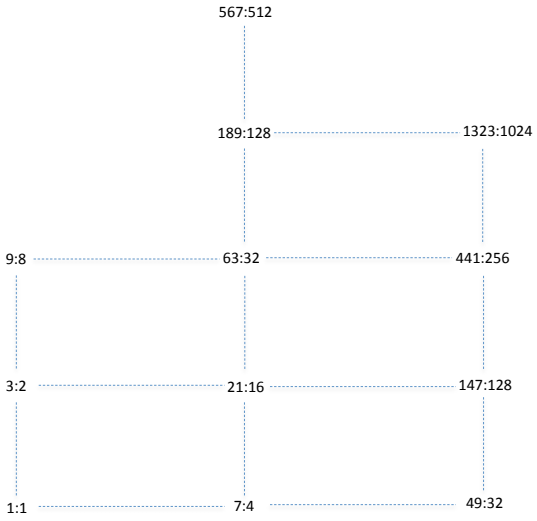
The main difference between the two tuning models is that Young's model is exclusively otonal, where Riley's model combines otonality and utonality. It is this difference that influences the placement of the fundamental, 1:1. Riley places the fundamental in the centre of the Tonnetz. This is shown in example 1.20.

Example 1.20, Terry Riley's tuning scheme for *The Harp of New Albion.*



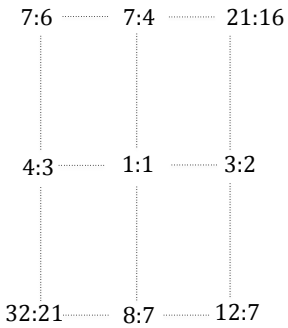
Riley's placement of the fundamental causes pitch-pairs to be intervallically inverse, mathematically inverse, and equidistant. Young, however, places the fundamental on one of the corners of the Tonnetz; this is shown in example 1.21.

Example 1.21, La Monte Young's tuning scheme for *The Well Tuned Piano.*



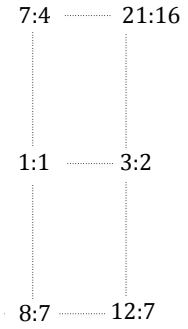
Young's placement of the fundamental causes pitch-pairs to be intervallically inverse, and equidistant, but not mathematically inverse. Of course, the central portion of his model could easily be translated into the following, different ratios shown in example 1.22.

Example 1.22, A alternative expression of the central portion of the tuning scheme for La Monte Young's *Well Tuned Piano.*



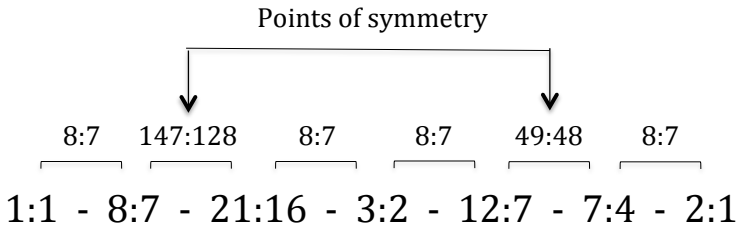
The translation, one space up on the *y*-axis, and one space to the right on the *x*-axis retains all relationships between pitches, and now causes all pitch-pair ratios to be mathematically inverse. In some cases, it is not possible to make pitch-pair ratios mathematically inverse. Take for example, the following two-dimensional lattice with the dimensions 2 x 3 shown in example 1.23.

Example 1.23, a two-dimensional lattice with dimensions 2 x 3.



This lattice contains three pitch-pairs which are equidistant from two points of symmetry, one point is the quotient of 7:4, and 12:7, the other point is the quotient of 21:16, and 8:7. The symmetry of the mode is shown in example 1.24.

Example 1.24, the mode resulting from the 2 x 3 lattice shown in example 1.23.



In such cases it is not possible to make the pitch-pair ratios mathematically inverse. This does not affect symmetry in any way, but it is a point worth observing especially in circumstances such as this one where correct understanding of these terms is pertinent to the theory that is being presented.

Truncating maximal UHMs

A MUHM is truncated by removing pairs of pitches which are equidistant from a point, line, or axis of symmetry. When a single pair is removed, this does not have any impact upon the symmetry of other pairs, as the symmetry of each pitch pair is completely independent of any other pitch pairs which may be included in the tuning model. Consequently, the removal of a single pair has little effect on the overall symmetrical structure of a mode. The following equation shows that $\boldsymbol{m}$ is equal to all possible combinations of equidistant pairs that can exist in any lattice.

$$\boldsymbol{m = 2^{k} - 1}$$

where

$$\boldsymbol{k = \left\lceil \frac{nd}{2} \right\rceil}$$

All the composer, theorist or analyst needs to do to use this equation is input two or more numbers (depending on how many dimensions there are) in place of *nd* in order to determine how many equidistant pitch pair combinations are available in a lattice. *nd* is equal to the product of the size of the lattice. For example, if the lattice forms a 3 x 3, 4 x 4, or 5 x 12 grid, then *nd* = 9, 16, or 60 respectively. If the lattice exists in three or more dimensions, then *nd* will be the product of the size of the three or more dimensions of the lattice.
For example, if one wanted to find all possible combinations of pitch-pairs from a 3 x 3 grid from example 1.22 then the equation would be used as follows.

$$\boldsymbol{m = 2^{\left\lceil \frac{9}{2} \right\rceil} - 1}$$

$$\boldsymbol{m = 2^{\lceil 5 \rceil} - 1}$$

$$\boldsymbol{m = 31}$$

The -1 at the end of the equation accounts for the fact that zero notes do not constitute a mode. This equation does however allow two-pitch, and even single-pitch combinations to constitute a mode. Some compositions such as *4'33*[36], Quattro Pezzi,[37] and 1960 #7[38] contain no pitches, a single pitch, and two pitches, respectively. These compositions are artistically valid, however, they can be considered the exception to the rule. Many composers would consider zero, single, and two note modes too restrictive to suit their needs. For this reason a second equation has been included in this paper which allows composers,

[36] Cage, John. *4'33"* for any instrument or combination of instruments. New York. Peters : Henmar, 1960.
[37] Scelsi, Giacinto. *Quattro Pezzi (su Una Nota Sola) Pour Orchestre (1959).* Paris. Salabert, 2001.
[38] Young, La Monte. *1960 #7*. Unpublished. 1960.

theorists or analysts to define the minimum number of pitches they wish for a mode to contain. The equation allowing this is as follows.

$$\boldsymbol{m = 2^k - f}$$

Where $\boldsymbol{f}$ is equal to the following.

$$\boldsymbol{f} = \sum_{t=0}^{\left\lfloor \frac{T+1}{2} \right\rfloor} \left(\frac{k!}{t!\,(k-t)!} \right)$$

T is equal to the T-pitch modes that are being removed. So if $\boldsymbol{m = 2^k - 1}$ removes all 0 note modes, then $\boldsymbol{m = 2^k - (1 + k)}$ will remove all one and two note modes also, and therefore $\boldsymbol{m = 2^k - \left(1 + k + \frac{k(k-1)}{2}\right)}$ will remove all three and four note modes. One must remember to take into account that when working with a lattice that has an odd number of pitches then one and two-pitch modes, and three and four-pitch modes, and so on are being removed simultaneously as the central pitch pairs with itself, and therefore frequently needs to be counted as two notes.

The equation does not take into account that certain transposable pitch-pairs, and sets of pitches typically exist within a lattice, as most lattices are composed of intersecting geometric series, as opposed to random collections of co-prime integer ratios. The feature that causes a set of pitches to be un-transposable is the most distantly related pitch-pair(s) in a lattice. Even within a small lattice, such as the one shown in example 1.10 the relationships of pitch pairs b-b, and d-d are transposable. The two pairs which ensure lack of transposition are a-a, and c-c, as these pairs are the most distantly related in the lattice, notice that they are also corner pairs. It is these distantly related, un-transposable pairs which need to be identified, and included in a mode if the composer wants to compose using un-transposable modes.

1.4 Discourse on the generative aspects of prime numbers on a piece of music

Prime numbers are the threads of cotton which, when intertwined, make up the fabric of just intonation. All musical pitches in just intonation are represented by some rational co-prime ratio, and tuning schemes are constructed from interlocked geometric series, which are the product of multiplication of these prime numbers by themselves, and each other. If prime numbers can so profoundly influence the realm of pitch, then surely can they also exert influence over other elements of music. Other composers and theorists have previously considered the idea of integer ratios being applied to various aspects of a musical composition. Cowell proposes the idea of expressing relationships in the harmonic series as polyrhythms in his 1930 work *New Musical Resources.*[39] Mathieu touches upon the same idea repeatedly in the opening chapters of *The Harmonic Experience.*[40] Johnston applies this concept in his string quartets, using number of complex polyrhythms, including 9:10, 5:4, 6:7, 5:7, 8:9, and 7:9 in his String Quartet no. 4[41] among others, which clearly correspond to the whole small integer ratios being used to generate the harmony in the piece,[42] this is illustrated in example 1.25.

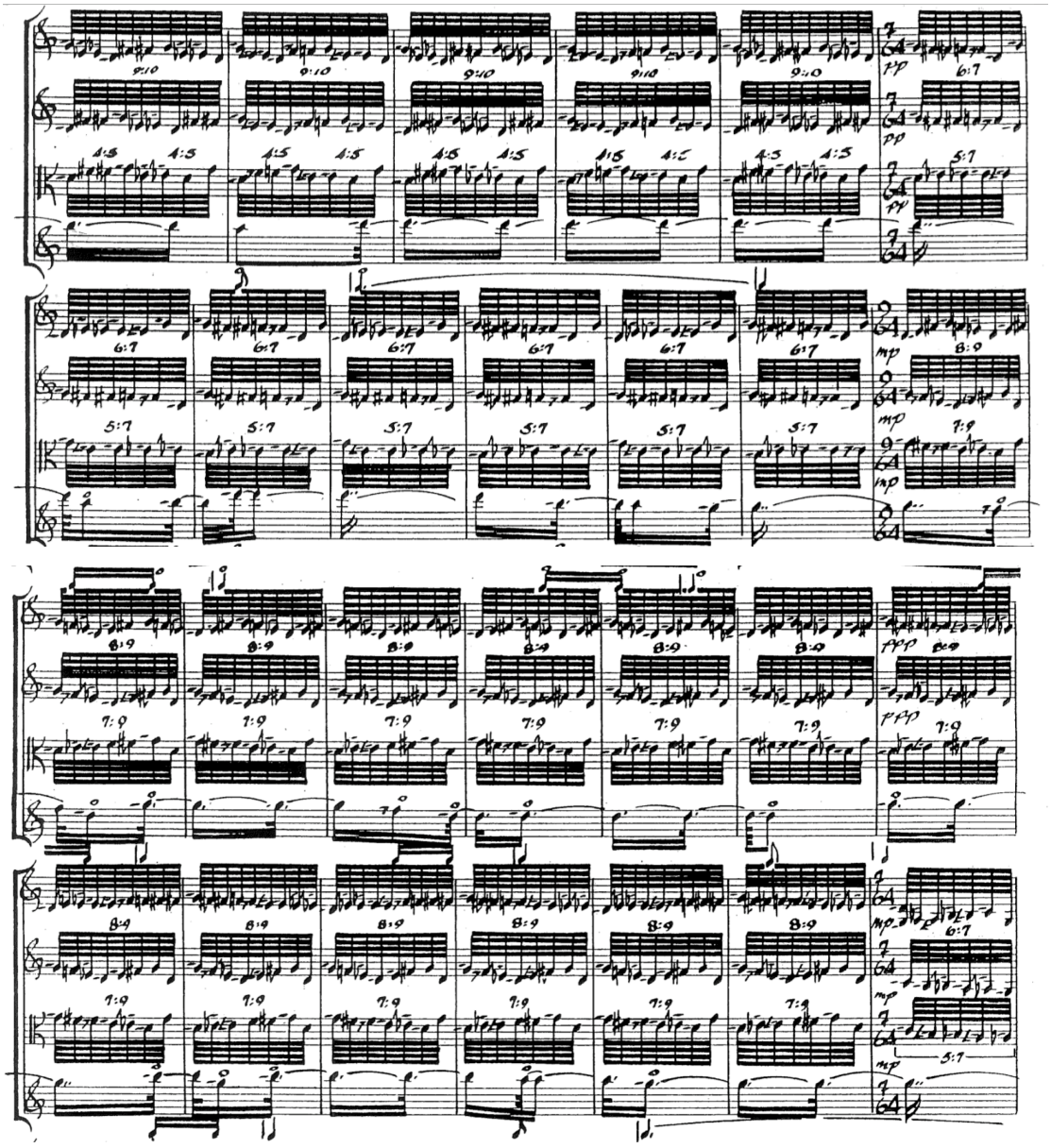
[39]Cowell, Henry. *New Musical Resources.* New York. Alfred A. Knopf, 1931. P. 51.

[40]Mathieu, W.A. *The Harmonic Experience: Tonal Harmony from its Natural Origins to its Modern Expression.* Rochester, Vermont: Inner Traditions International, 1997. Pages 1-31.

[41] Johnston, Ben. String Quartet No. 4. Baltimore. Smith publications, 1973.

[42] Shinn, Randall. "Ben Johnston's Fourth String Quartet." *Perspectives of New Music* 15.2 (1977): 145-73. Web.

Example 1.25, excerpt from Ben Johnston's String Quartet No. 4.



Other composers have linked some numerical concept, not necessarily prime numbers, to their music, such as Bartok[43] and Debussy[44]. Analysts of their music have argued that the golden ratio permeates through various aspects of their works. John Cage imagined and applied square root form in his music, [45] and this is consistent with 12-TET which is tuned according to $\sqrt[12]{2}$. Cage's early work *Six*

[43] Lendvai, Ernő. *Béla Bartók: An Analysis of His Music*. London. Kahn and Averill, 1971. P. 17.

[44] Howat, Roy. *Debussy in Proportion: A Musical Analysis.* Cambridge. Cambridge University Press, 1983. Pp. 1-11.

[45] Williams, Barry Michael. "The Early Percussion Music of John Cage 1935-1943." PhD Thesis. Michigan State University. 1990. P. 60.

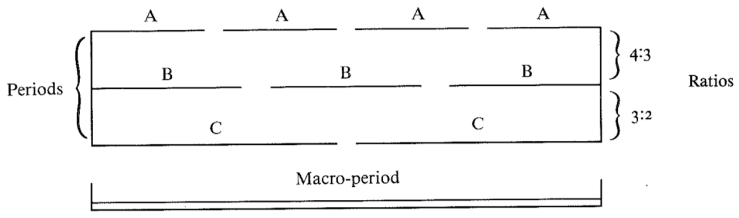
Short Inventions also makes use of a governing number, which influences the compositional procedures of the piece.[46] The numerical cohesiveness that Cage, Cowall et al. have posited resonates with integral serialist ideals and is exactly the type of organizational consistency that this project aims to implement in the creative works.

Co-prime ratios have a number of potential musical applications including harmonic ratios, polyrhythm, tempo relations, and macro-periodicity among others. In *African Polyphony and Polyrhythm*[47] Arom defines a macroperiod in the following way:

> A *macroperiod* is the cycle obtained when periods of different lengths are superposed, *and each individually is shorter*. This happens, for example, when two or more periods stand in a ratio of 2:3 and/or 3:4. The macroperiod then provides the only point at which *all the periods coincide*.

Arom also, conveniently provides the following diagram which illustrates this concept, shown here in example 1.26.

Example 1.26, diagram from African Polyphony and Polyrhythm showing a macroperiod.



1.5 Project scope

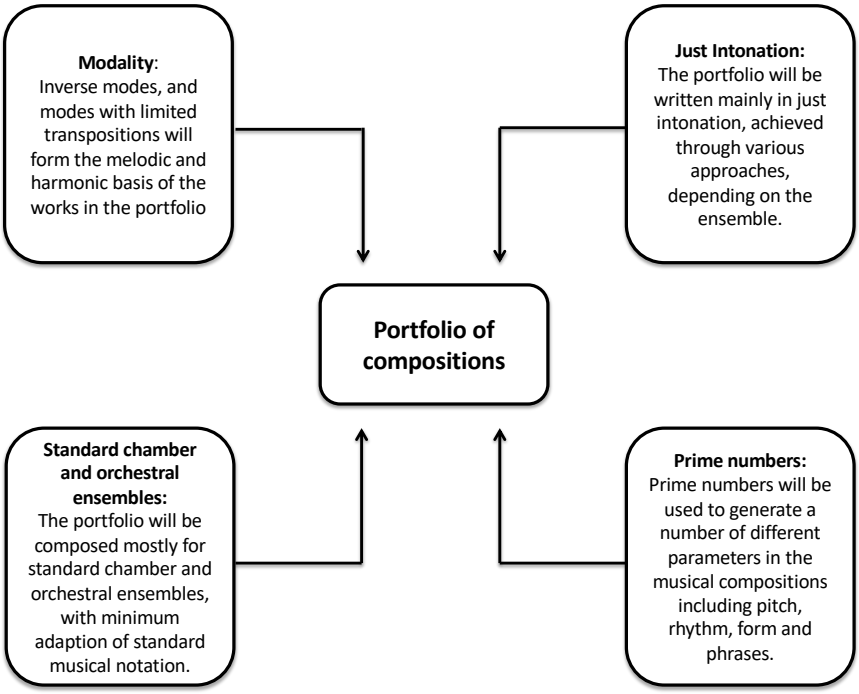
To recapitulate the conceptual goals of the project a diagram showing the four key sections, which have just been discussed, is shown in example 1.27.

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[46] Guessford, Jesse James. "The Compositional Procedures used in John Cage's *Six Short Inventions*, *First Construction* (in metal), and *Spontaneous Earth*." PhD Thesis. University of Illinois. 2004. P. 14.

[47] Arom, Simha. *African Polyrhythm and Polyphony: Musical structure and methodology.* Trans. by Martin Thom, Barbara Tuckett and Raymond Boyd. Cambridge. Cambridge University Press. 1991. P. 231.

Example 1.27, conceptual framework of the project.



Example 1.27 shows that the conceptual goal of the project is to generate a portfolio of music that is composed in specific microtonal tunings, but appears as relatively standard musical notation, and can be performed on conventional instruments, by conventional instrumentalists. Additionally, the portfolio of original compositions is conceived modally and achieves cohesion through the use of low prime numbers applied in various aspects of the scores.

Chapter two: Methodology

2.1 Methodology for *Remembering is a Form of Forgetting* for symphony orchestra.

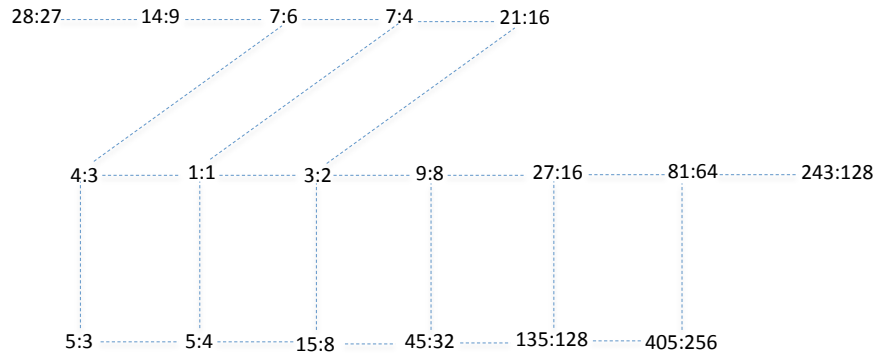
Premise

The first premise that underpins *Remembering is a Form of Forgetting* is that just intonation can be achieved by applying a principle where most instrumentalists in the orchestra are each assigned a small portion of the lattice, and by decreasing the number of pitches they need to tune, the likelihood of correct intonation increases. The second premise is that natural harmonics are to be used in place of training instrumentalists to play in just intonation, as natural harmonics, theoretically, are integer multiple of the fundamental. Where natural harmonics are not possible, techniques from *The Just Intonation Primer*[1] will be applied to increase the probability of pure intonation being achieved. It is understood that when using natural harmonics factors such as inharmonicity, bow pressure, and the temperature of wind and brass instruments are variable factors, and will affect the outcome of intonation. This piece of music aims to investigate ways in which an orchestral ensemble can realistically approach achieving a performance in just intonation, without rigorous specialist training.

Lattice

The seven-limit lattice for *Remembering is a Form of Forgetting,* mostly born of pragmatism, is shown in example 2.1.

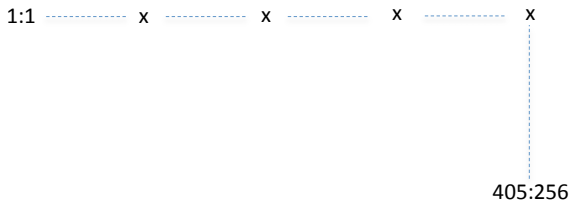
Example 2.1, lattice for *Remembering is a Form of Forgetting.*



[1] Doty, David B. *The Just Intonation Primer: An introduction to the theory and practice of Just Intonation.* Second ed. San Francisco. The Just Intonation Network, 1994. Pp. 60-72.

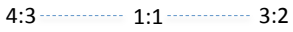
As previously stated, each instrument, or instrumental group has only been assigned a small portion of the lattice and in many cases pitch assignment is actually more limited than it need be, in order to create the greatest probability of just intonation occurring. Examples 2.2 through 2.8 show how the lattice has been assigned to each instrument of the orchestra. Bb and Eb clarinets have only been assigned the fundamental frequency (1:1), and the fifth harmonic above an E (81:64) fundamental, due to the inability of the clarinet to produce even numbered harmonics[2], this is shown in Example 2.2.

Example 2.2, portion of the lattice assigned to Bb and Eb clarinets.



Despite the flute, and horns having the capacity to play a number of harmonics, flutes one and two, and horns three and four have been restricted to the fundamental, and a fifth either side of it. The oboe has been restricted to these pitches as it is a double reed instrument and harmonics are theoretically more difficult to obtain on such an instrument. Example 2.3 shows the portion of the lattice assigned to flutes, oboes, and horns three and four.

Example 2.3, portion of the lattice assigned to flutes, oboes and horns three and four.



The orchestra tune to the cello's C string, so 1:1 will be in tune, 3:2 can be achieved through third, and sixth harmonics. 4:3 is one of a handful of intervals in this piece belonging to the sub-harmonic series and is therefore not available as a harmonic, although 4:3 is close to what an instrumentalist would play as a

[2] *Clarinet Acoustics: an introduction*. The University of New South Wales. Web. Accessed 17 April 2016. <http://newt.phys.unsw.edu.au/music/clarinet/>.

'normal' F. The cor Anglais, having similar design features to the oboe, has been assigned a similar amount of notes, this is shown in Example 2.4.

Example 2.4, portion of the lattice assigned to cor Anglais.



Trumpets in C, and horns one and two have been assigned only one note each, this is shown in Example 2.5.

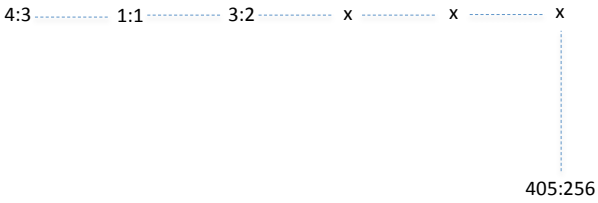
Example 2.5, portion of the lattice assigned to trumpets in C, and horns 1 & 2.



Trumpet one and horn one are assigned a 7:4 Bb, while trumpet two and horn two are assigned a 7:6 Eb. Provided that trumpet one and horn one are tuned correctly to the Cello's C string, and the instrumentalists can produce the seventh harmonic as it naturally occurs, i.e. without altering their embouchure, or hand position in the bell of the horn, then the 7:4 Bb should be possible. The same is true for the 7:6 Eb, although as previously stated, the tuning of 4:3 has not been specified and is not accessible as a harmonic, but a normal F should be within 2 cents of 4:3. So, if 4:3, or a note very close to 4:3 is taken as the fundamental tube length, then 7:6 should be easily accessible as the 7th harmonic.
Doty describes continuous pitch instruments as a mixed blessing, with a skilled player capable of producing a large number of pitches with a high level of accuracy, and an unskilled player possibly unable to produce a single correctly tuned pitch.[3] With the fear of a student ensemble's variable skill level sensibly instilled through Doty, trombones were assigned four pitches, as shown in example 2.6.

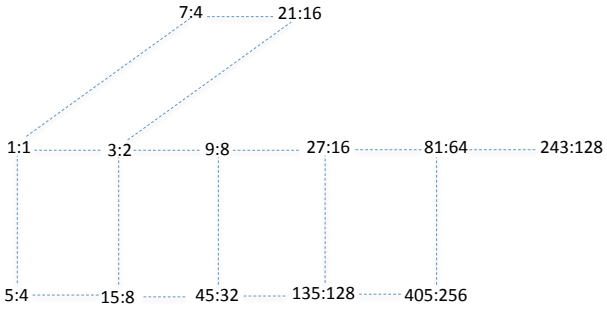
[3] Doty, David B. *The Just Intonation Primer: An introduction to the theory and practice of Just Intonation.* Second ed. San Francisco. The Just Intonation Network, 1994. Page 67.

Example 2.6, portion of the lattice assigned to trombones.



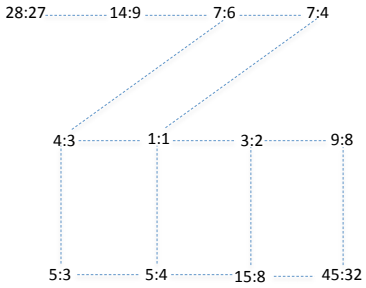
The string section, assigned only natural harmonics presented a versatile set of pitches. Example 2.7 shows the portion of the lattice assigned to the string section, and it omits 63:32, 189:128, and 567:512, which should all, theoretically, be easily accessible as the 7th harmonics of D, A, and E string respectively.

Example 2.7, portion of the lattice assigned to string section.



Cello solo, and contrabass solo parts were assigned a different set to the tutti string section, their portion of the lattice is shown in example 2.8. A number of sub-harmonics have been assigned to investigate and assess the use of altered notation, and how well soloists in an orchestral context could produce the desired intonation.

Example 2.8, portion of the lattice assigned to cello and contrabass soloists.



Bassoons have been assigned C 1:1, due to the instability of harmonics on a double reed instrument, and the tuba has been assigned C# 135:128. The best-

case scenario for this piece of music is that the lattice is achieved, and the relationship between all notes are small integer ratios. An alternative scenario, which would still produce results different from normal tuning, is that this piece of music would work in a similar way to *The Hamburg Concerto*[4] where Ligeti uses natural harmonics against a background of tuning which is unaltered. Finding a significant portion of the lattice through the use of natural harmonics is theoretically fine for string instruments, providing that the strings are tuned 1:1, 3:2, 9:8, 27:16, and 81:64. For wind, and brass instruments where the player tunes the fundamental below a given harmonic in real time, the use of harmonics is less reliable, as it depends upon the skill of the player to correctly tune the fundamental. Some of Doty's recommendations can counter this problem.

Doty's recommendations and their application

The Just Intonation Primer finishes with some of Doty's thoughts on obtaining satisfactory performances. Doty recommends the use of a fixed pitch anchoring instrument, the use of rehearsal tapes, and avoiding the use of cent deviation, ratios, or exotic accidentals in scores[5]. An anchoring instrument is not used in *Remembering is a Form of Forgetting*. Tuning tape tracks, and tuning with the aid of digital tuners are techniques that are to be used. Rehearsal tape tracks and tuning to a specific Hertz value with a digital tuner are especially important ways in which tuning a harmonic can be achieved for the brass and wind sections. Avoiding exotic accidentals, or convoluted notation is an explicit aim of this project. Despite this, it is necessary to indicate in some way to the soloists how sharp or flat pitches are to be. Table 2.1 shows how three simple symbols, a number seven; a dash; and an arrow, are used to indicate varying degrees of deviation from standard tuning.

[4] Ligeti, Gyorgy. *Hamburg Concerto,* for solo horn and chamber orchestra*.* Mainz. Schott Music, 1998-99.
[5] Doty, David B. *The Just Intonation Primer: An introduction to the theory and practice of Just Intonation.* Second ed. San Francisco. The Just Intonation Network, 1994. P. 71.

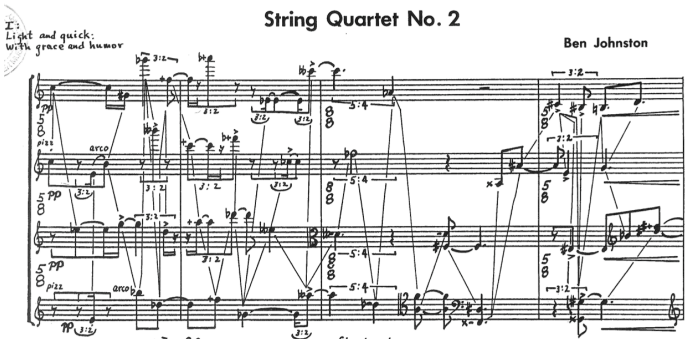
Table 2.1, the symbols appearing on the score in the solo parts, and what the symbols denote.

Ratio	Note name	Distance from fundamental (in cents)	Degree of difference from 12-TET (in semitones)	Degree of difference from 12-TET (in cents)	Symbol denoting change
1:1	C	0	n/a	0	n/a
28:27	Db	62	ca. 1/3 semitone flat	38 cents flat	7
9:8	D	203.9	Slightly sharp	3 cents sharp	↑
7:6	Eb	266	ca. 1/3 semitone flat	34 cents flat	7
5:4	E	386	Between 1/6 – 1/10 semitone flat	14 cents flat	-
4:3	F	498	Slightly flat	2 cents flat	↓
45:32	F#	590	1/10 semitone flat	10 cents flat	-
3:2	G	702	Slightly sharp	2 cents sharp	↑
14:9	Ab	764	ca. 1/3 semitone flat	36 cents flat	7
5:3	A	884	Between 1/6 – 1/10 semitone flat	16 cents flat	-
7:4	Bb	968	ca. 1/3 semitone flat	32 cents flat	7
15:8	B	1088	Between 1/6 – 1/10 semitone flat	12 cents flat	-
2:1	C	1200	n/a	0	n/a

Adaption of Johnston's notation and tuning technique

In string quartets numbers two[6] and three[7] Johnston connects intervals 3:2, 4:3, 5:4, 6:5, 8:5, 5:3, and 9:8 with solid lines. An example of how Johnston notates these connections is shown in example 2.9.

Example 2.9, the opening four measures of Ben Johnston's String Quartet no. 2.



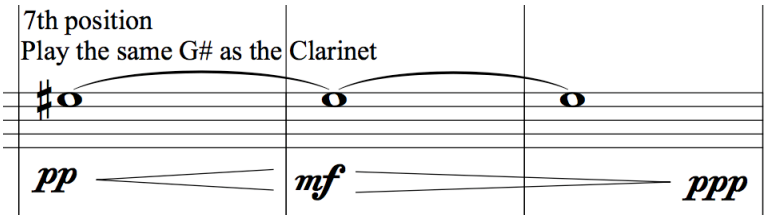
[6] Johnston, Ben. String Quartet No. 2. Baltimore. Smith publications, 1964.
[7] Johnston, Ben. String Quartet No. 3. Baltimore. Smith publications, 1966.

These solid lines are performance aids to be used to help performers find other player's notes, which they are to tune to. As discussed in the conceptual framework chapter, this style of notation is arguably untidy, and perhaps initially a little unnerving for performers. It also seems to suggest that performers must have the full score in front of them during rehearsal. As this research project is explicitly trying to avoid a style of notation that is convoluted, or messy, Johnston's notation style from these quartets cannot be directly adopted. One of Johnston's tuning concepts, however, can be retained. Johnston writes the following of his score in the performance notes.

> I wanted to write a piece in which the players would need to take much greater care than usual in locating the pitches. Each would be dependent upon making precisely the right interval with some other player's note.[8]

Johnston also states that a certain number of intervals need to be tuned by ear, to make the pure intonation system work. These intervals are the unison, octave, fifth, and perfect major third. The score for *Remembering is a Form of Forgetting* adapts Johnston's concept by using written word instructions in English in place of solid connecting lines. The written word instructions perform the same task, stipulating which instrument, and pitch, performers are to listen to, and tune to. Example 2.10 illustrates how Johnston's concept has been adapted in *Remembering is a Form of Forgetting*.

Example 2.10, *Remembering is a Form of Forgetting* excerpt of trombone part measures 49-51



To clarify the thought process behind this decision, example 2.10 shows that the trombone player is to listen for, and tune to the same G# that the clarinetist is playing. The logic behind this is that the clarinetist has much more limited control over the intonation of their G# than the trombonist, so the trombone should tune to the clarinet, not vice versa.

[8] Johnston, Ben. String Quartet No. 2. Baltimore. Smith publications, 1964. Performance notes.

2.2 Methodology for *The tale of Kiều* for scordatura string orchestra

Premise

The Tonnetz shown in example 2.11 appears, perhaps to some, so wildly unrealistic that it defies credibility. This may be true if a composer tried to apply this lattice without following the three tenets that underpin the score.

1. The string orchestra must follow the scordatura set forth in the opening pages of the score and in example 2.11, using digital tuners to tune string number IV of each instrument. The remaining strings may be tuned as open fifths above string IV.
2. The string players must be proficient enough in the production of natural harmonics to find up to and including the 8th harmonic of each string.
3. The composer must write the score out entirely in natural harmonics, after having composed the score as a set of sounding pitches.[9]

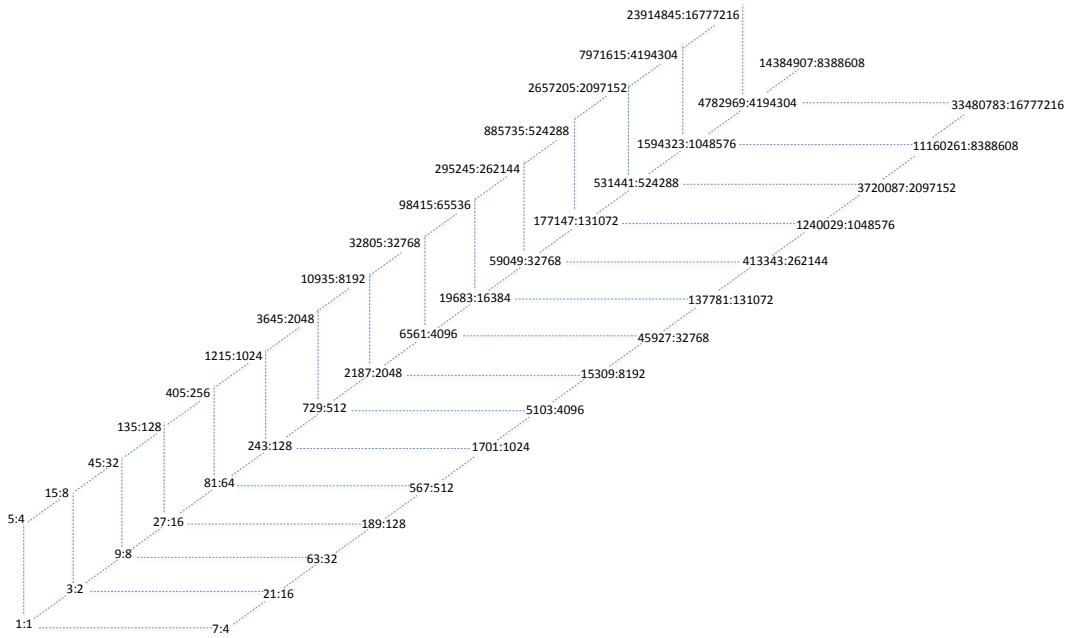
Provided these three reasonably simple rules are adhered to, the 46-pitches in the lattice shown in example 2.11 will become available for use.

Lattice and tuning of open strings

The lattice for *The tale of Kiều*, illustrated in example 2.11, shows a central Pythagorean spine of fifths starting at 1:1, and moving up to 14,384,907:8,388,608. This spine of fifths represents the open tuning of the orchestra. The outer two spines are simply the fifth and seventh harmonics of each open string.

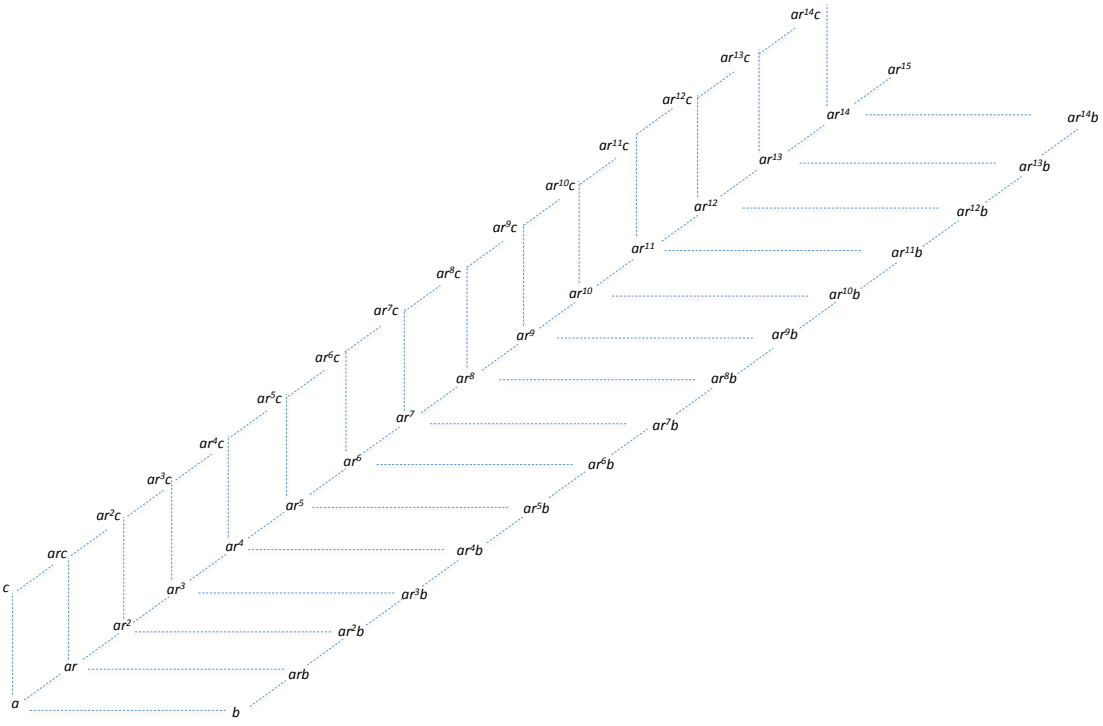
[9] The composer need not necessarily compose the left hand finger positions after the sounding pitches have been composed. The composer could write left hand finger positions followed by sounding pitches if they chose to do so.

Example 2.11, the lattice for *The tale of Kiều* for scordatura string orchestra.

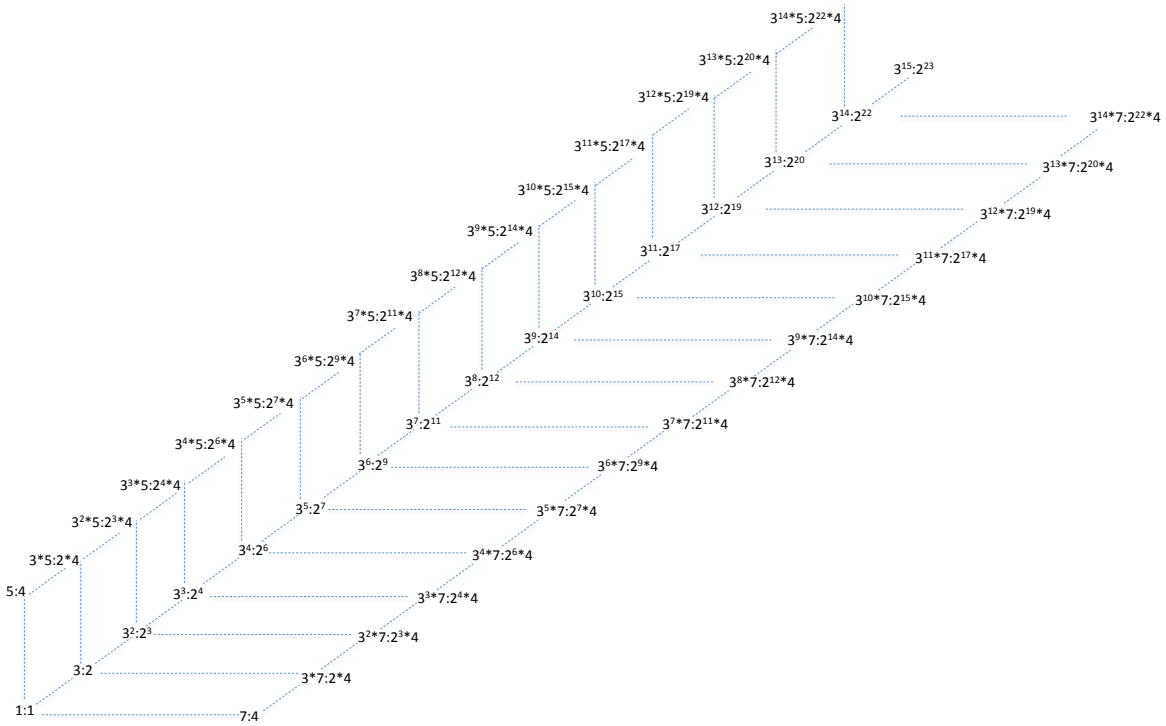


Simplified expressions of the lattice, including a purely algebraic representation, and a geometric series depiction are shown in figure x, and x, respectively.

Example 2.12, an algebraic representation of the lattice for *The tale of Kiều*.

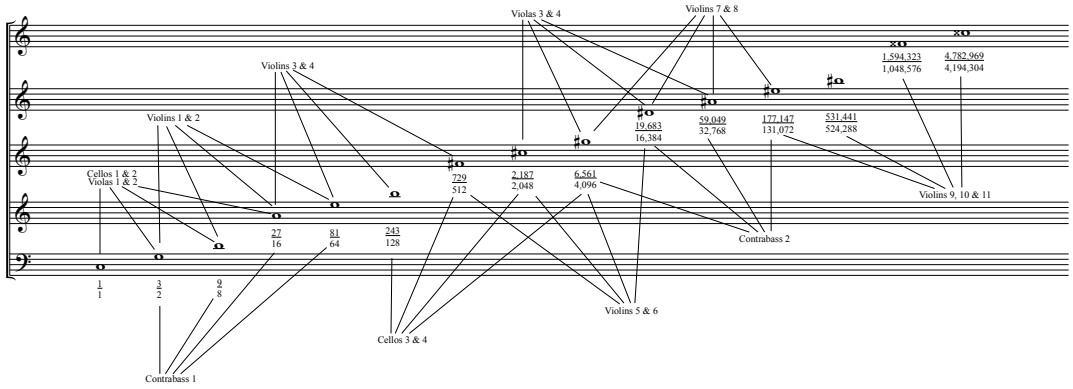


Example 2.13, the lattice for *The tale of Kiều* represented as three parallel geometric series'.
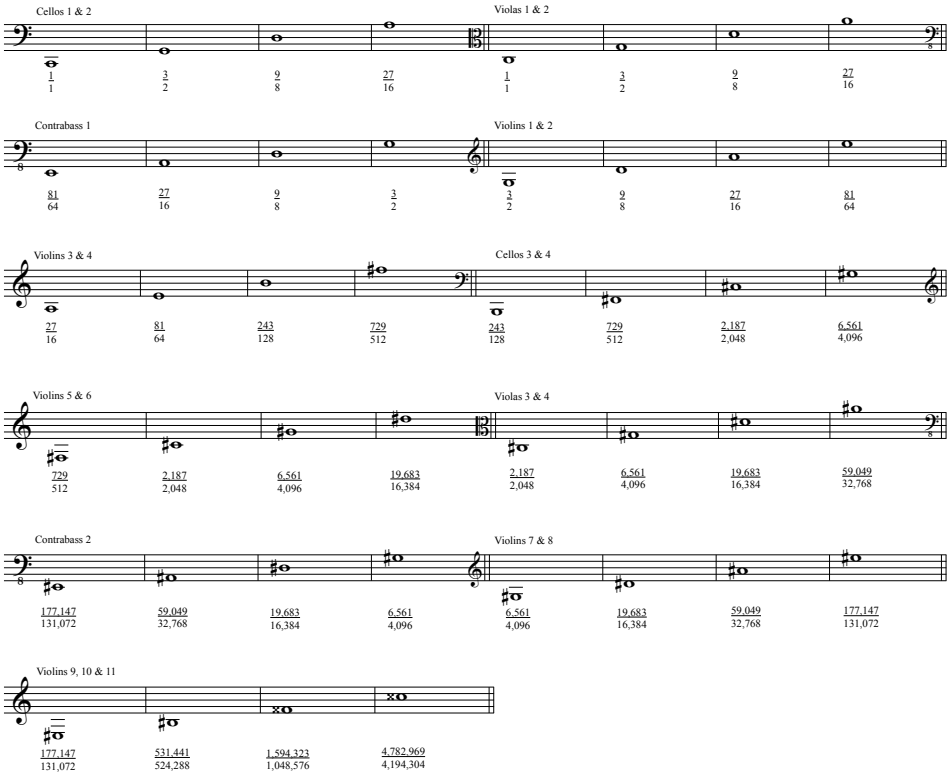


The open tuning of the orchestra can also be represented, conceptually, as notes on a musical staff as is shown example 2.14.

Example 2.14, the conceptual open tuning of the orchestra represented as pitches on a musical staff.



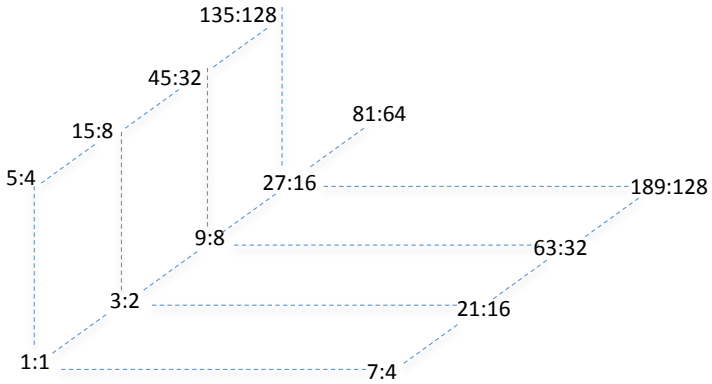
Example 2.14 is only a conceptual chart. It shows how the Pythagorean spine pieces together. It is inaccurate in terms of which octave specific pitches lie in. Example 2.15 provides the information regarding the specific, octave accurate, location of pitches on a musical staff.

Example 2.15, the octave accurate, open tuning of the orchestra represented as pitches on a musical staff.

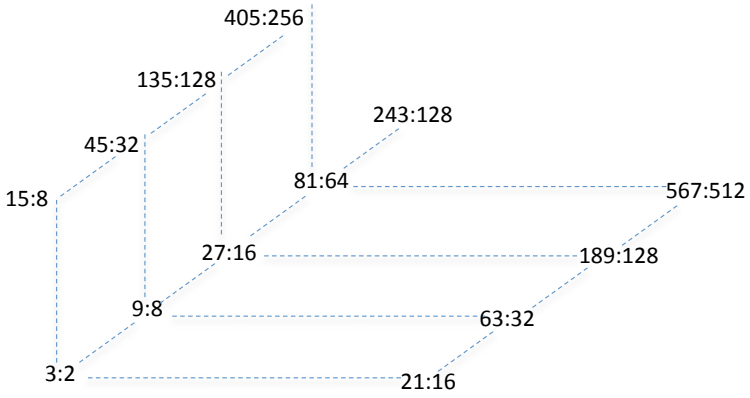


Once the orchestra has re-tuned according to the specific scordatura outlined here, each instrumental table will have access to 13 pitches. For clarity, the lattice has been broken into individual instrumental tables, and are illustrated in examples 2.16 through 2.23.

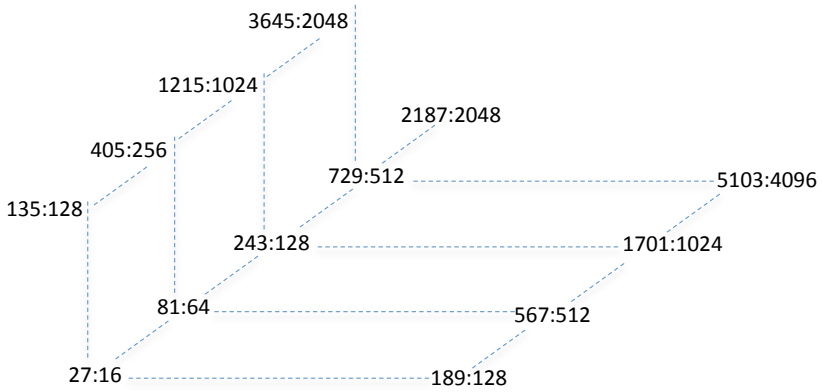
Example 2.16, portion of the lattice available to cellos 1, 2 and violas 1, and 2.



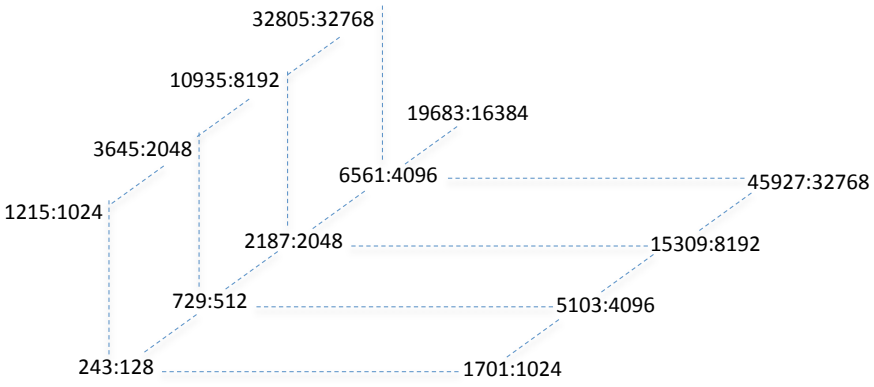
Example 2.17, portion of the lattice available to violins 1, 2, and contrabass 1.



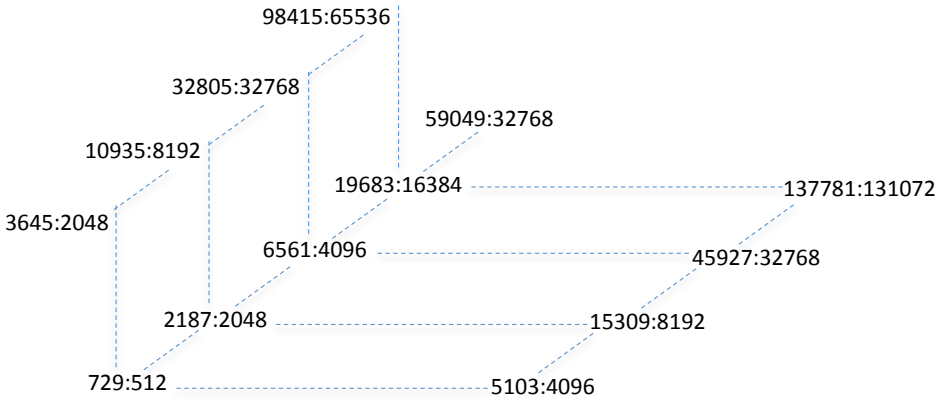
Example 2.18, portion of the lattice available to violins 3, and 4.



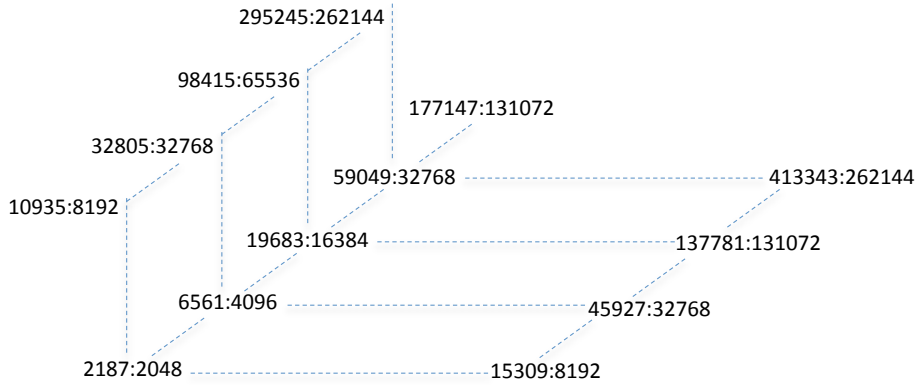
Example 2.19, portion of the lattice available to cellos 3, and 4.



Example 2.20, portion of the lattice available to violins 5, and 6.



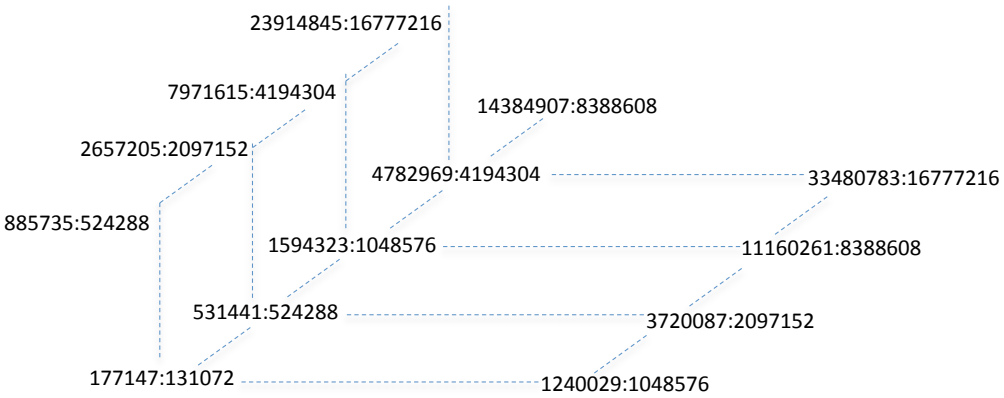
Example 2.21, portion of the lattice available to violas 3, and 4.



Example 2.22, portion of the lattice available to violins 7, 8, and contrabass 2.



Example 2.23, portion of the lattice available to violins 9, 10, 11, and 12.



Hertz conversion

In order to practically apply the integer-ratios shown in the lattices, they must be converted to hertz. Table 2.2 provides the instrumentalists with accurate values which they can use for tuning their open strings.

Table 2.2, absolute frequencies of the open tuning of the orchestra for *The tale of Kiều*, where the fundamental frequency is equal to 261.64Hz.

Instrument and string number	Value (Hz)	Instrument and string number	Value (Hz)	Instrument and string number	Value (Hz)
Cellos 1 & 2, IV	65.41	**Violins 3 & 4, IV**	220.75	**Contrabass 2, IV**	44.20
Cellos 1 & 2, III	98.12	**Violins 3 & 4, III**	331.13	**Contrabass 2, III**	58.93
Cellos 1 & 2, II	147.17	**Violins 3 & 4, II**	496.69	**Contrabass 2, II**	78.58
Cellos 1 & 2, I	220.75	**Violins 3 & 4, I**	745.03	**Contrabass 2, I**	104.77
Violas 1 & 2, IV	130.82	**Cellos 3 & 4, IV**	62.09	**Violins 7 & 8, IV**	209.54
Violas 1 & 2, III	196.24	**Cellos 3 & 4, III**	93.13	**Violins 7 & 8, III**	314.31
Violas 1 & 2, II	294.34	**Cellos 3 & 4, II**	139.69	**Violins 7 & 8, II**	471.47
Violas 1 & 2, I	441.5	**Cellos 3 & 4, I**	209.54	**Violins 7 & 8, I**	707.20
Contrabass 1, IV	41.39	**Violins 5 & 6, IV**	186.26	**Violins 9, 10, 11, IV**	176.80
Contrabass 1, III	55.19	**Violins 5 & 6, III**	279.39	**Violins 9, 10, 11, III**	265.20
Contrabass 1, II	73.59	**Violins 5 & 6, II**	419.08	**Violins 9, 10, 11, II**	397.80
Contrabass 1, I	98.12	**Violins 5 & 6, I**	628.62	**Violins 9, 10, 11, I**	596.70
Violins 1 & 2, IV	196.24	**Violas 3 & 4, IV**	139.69		
Violins 1 & 2, III	294.34	**Violas 3 & 4, III**	209.54		
Violins 1 & 2, II	441.5	**Violas 3 & 4, II**	314.31		
Violins 1 & 2, I	662.25	**Violas 3 & 4, I**	471.47		

Table 2.3 provides the composer with the values necessary for re-tuning a piano, or some digital interface so that they have an instrument for writing music on.

Table 2.3, absolute frequencies of all integer-ratios in the lattice for *The tale of Kiều*, where the fundamental frequency is equal to 261.64Hz.

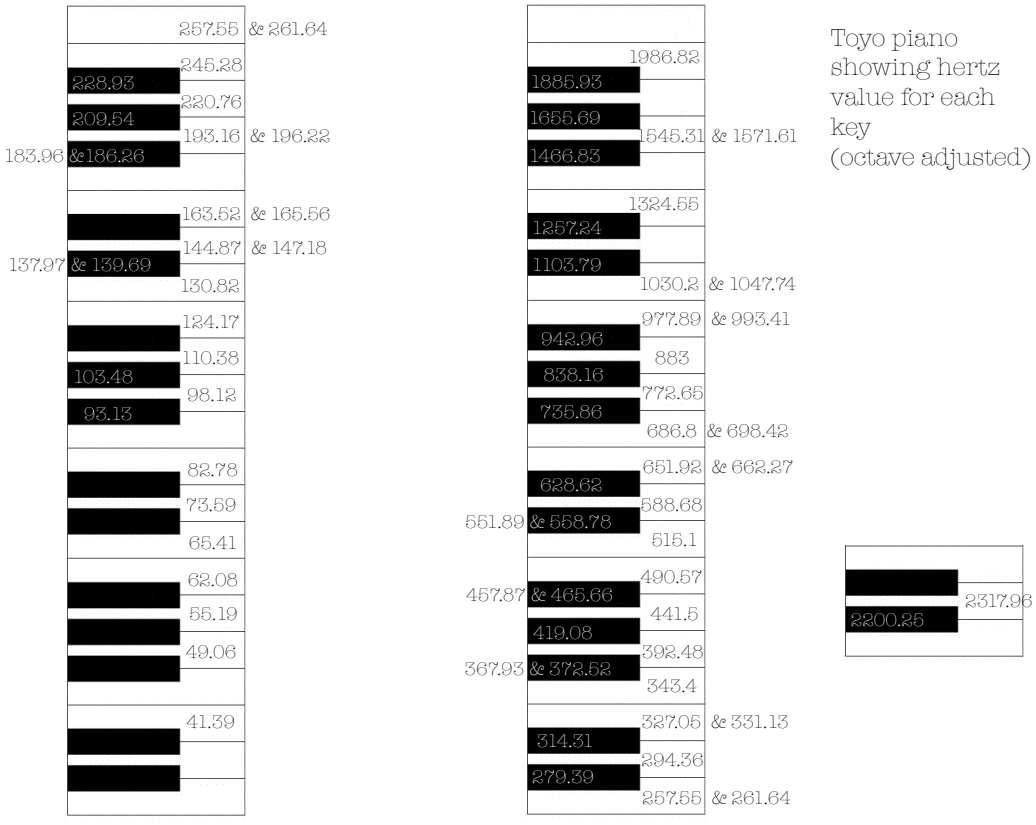
Integer ratio	**Value (Hz)**	**Integer ratio**	**Value (Hz)**	**Integer ratio**	**Value (Hz)**
1:1	261.64	**5:4**	327.05	**7:4**	457.87
3:2	392.46	**15:8**	490.58	**21:16**	343.40
9:8	294.35	**45:32**	367.93	**63:32**	515.10
27:16	441.52	**135:128**	275.95	**189:128**	386.33
81:64	331.14	**405:256**	413.92	**567:512**	289.75
243:128	496.71	**1215:1024**	310.44	**1701:1024**	434.62
729:512	372.53	**3645:2048**	465.66	**5103:4096**	325.96
2187:2048	279.40	**10935:8192**	349.25	**15309:8192**	488.95
6561:4096	419.10	**32805:32768**	261.94	**45927:32768**	366.71
19683:16384	314.32	**98415:65536**	392.90	**137781:131072**	275.03
59049:32768	471.48	**295245:262144**	294.68	**413343:262144**	412.55
177147:131072	353.61	**885735:524288**	442.02	**1240029:1048576**	309.41
531441:524288	265.21	**2657205:2097152**	331.51	**3720087:2097152**	464.12
1594323:1048576	397.81	**7971615:4194304**	497.27	**11160261:8388608**	348.09
4782969:4194304	298.36	**23914845:16777216**	372.95	**33480783:16777216**	522.13
14384907:8388608	448.66				

Now that the hertz values are available, they can be assigned to keys on the piano, or to a digital interface. Two pianos were used for the purposes of this project, and pitches were assigned as shown in examples 2.24 and 2.25.

Example 2.24, hertz assignment on Steinbach piano for *The tale of Kiều*.

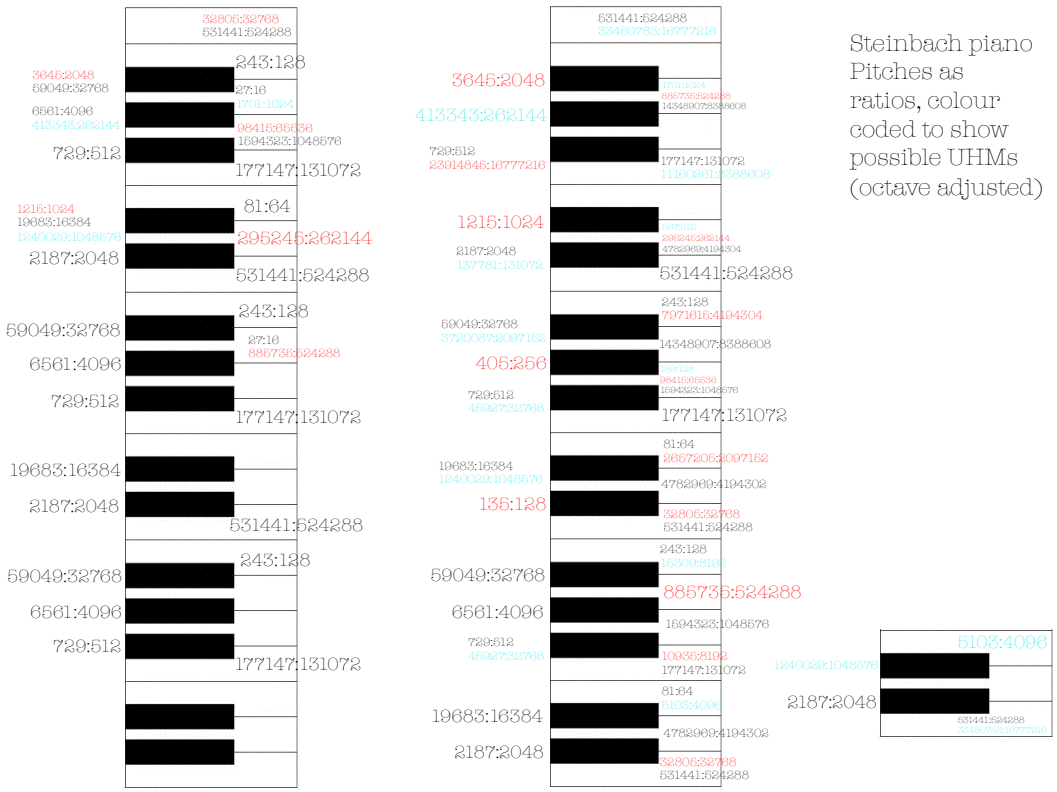


Example 2.25, hertz assignment on Toyo piano for *The tale of Kiều*.

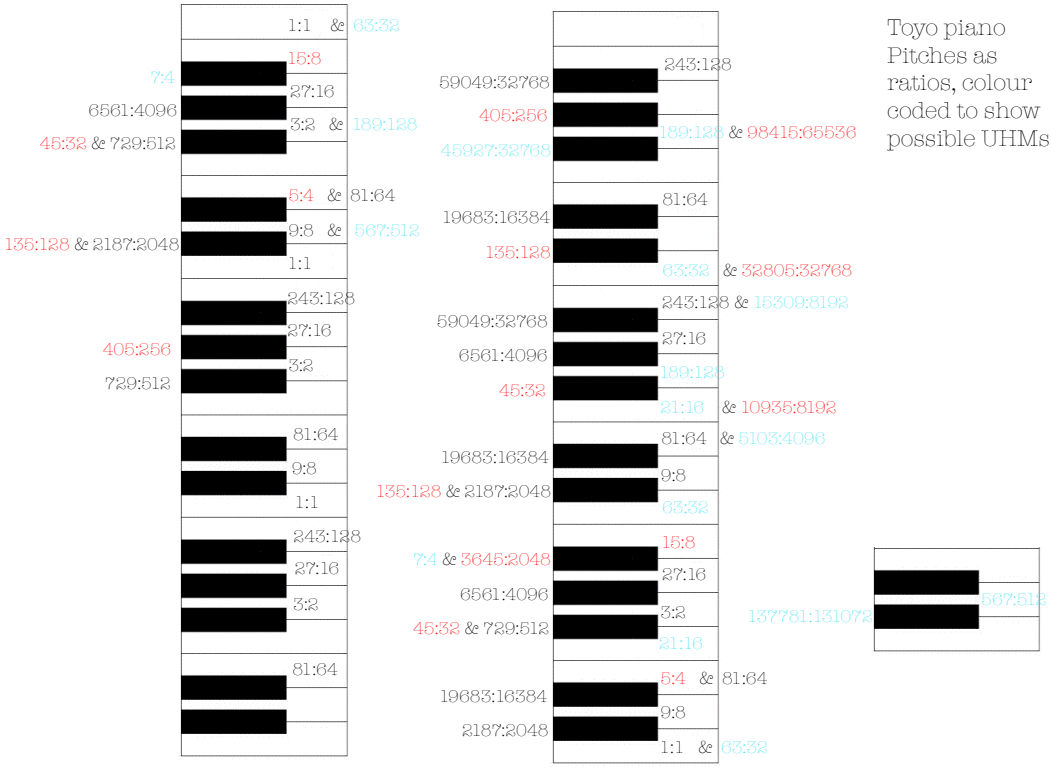


Once the hertz have been assigned to the chosen instrument then the process of musical composition can begin. If the composer plans to incorporate UHMs in the composition, then it is useful to make some map showing which integer-ratios are assigned to each key of the instrument. Again, this project applies the ratios to a piano, so copies of the piano charts are shown in example 2.26, and 2.27.

Example 2.26, integer-ratio assignment on Steinbach piano for *The tale of Kiều*.



Example 2.27, integer-ratio assignment on Toyo piano for *The tale of Kiều*.



If possible, practical, or necessary, colour coding ratios according to some system, such as prime factorization, can be a useful way to quickly identify which groups of UHMs an integer-ratio belongs to. Ratios in examples 2.26, and 2.27 have been colour coded according to the system outlined in table 2.4.

Table 2.4, colour coded integer-ratio identification chart.

Colour	Corresponding prime number	UHM group 1	UHM group 2
Black	3	5:4	7:4
Red	5	5:4	7:5
Blue	7	7:4	7:5

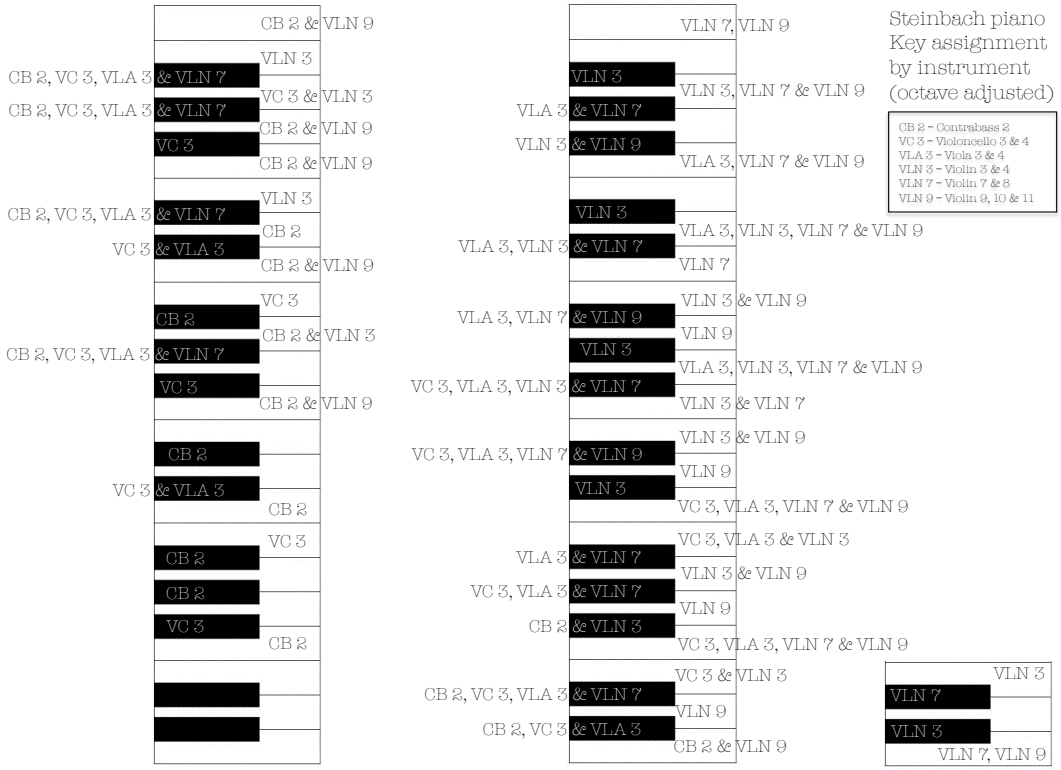
In the case of *The tale of Kiều* for scordatura string orchestra, where only three geometric series are used to generate the lattice, a third piano could have somewhat streamlined the compositional process, as each generating prime could then have been assigned to an individual piano. This would have caused

every two-piano combination to imply a set of UHMs. Despite only having access to two pianos for re-tuning, the colour coding chart, and system sufficed.

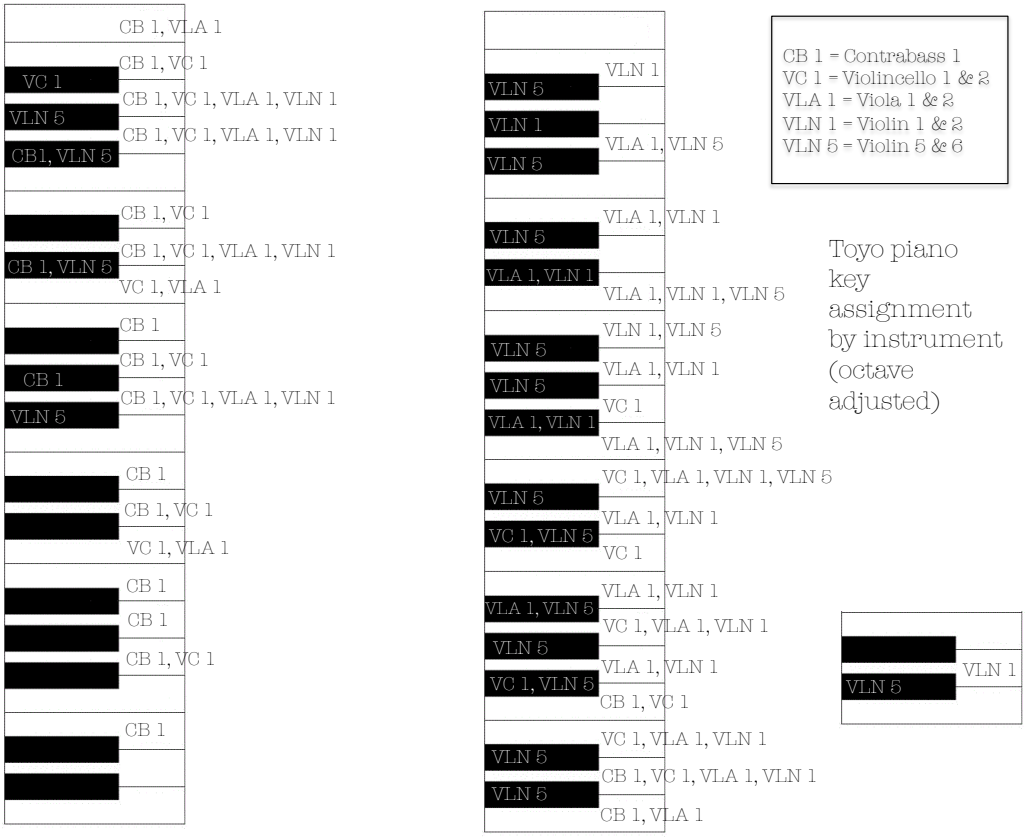
Orchestration (stage 1)

Once the music has been sketched using the methods outlined in previous sections, the composer can begin the process of orchestration. The first stage of this process is determining which notes can be played by which instrumentalists. This is of course only one approach. The composer could approach the composition with individual instrumental tables in mind at the very beginning of the composition. Whatever approach the composer takes, more charts, showing which instruments have been assigned to each key, are necessary. The piano charts illustrated in examples 2.28 and 2.29 allow the composer to know which notes on the piano keyboard belong to which instrumental tables.

Example 2.28, instrumental assignment on Steinbach piano for *The tale of Kiều*.



Example 2.29, instrumental assignment on Toyo piano *The tale of Kiều*.



Orchestration (stage 2)

The second stage of orchestration in *The tale of Kiều* for string orchestra involved transposing all sounding pitches up by an octave. This was necessary as the pianos used in this project were re-tuned an octave down from the true sounding pitches of the orchestra. The reasoning behind this is that re-tuning the extreme upper and lower ends of the piano is problematic, and most of this particular lattice lies in the upper octaves of the piano register.

Orchestration (stage 3)

Once the composer has composed the score as a set of sounding pitches and shifted them up by an octave, the entire composition must then be transposed to show the instrumentalists where to place their fingers. To complete this process a further set of transposing charts is necessary. The charts contained in Appendix C show sounding pitches, integer-ratios, and node points for each string of the orchestra (individual string charts by instrument).

2.3 Methodology: *13* for scordatura string quartet

Premise

The premise on which *13* is based is essentially the same as that of *The tale of Kiều* for scordatura string orchestra, albeit the particular scordatura is different, and there are fewer players. The three tenets are as follows.

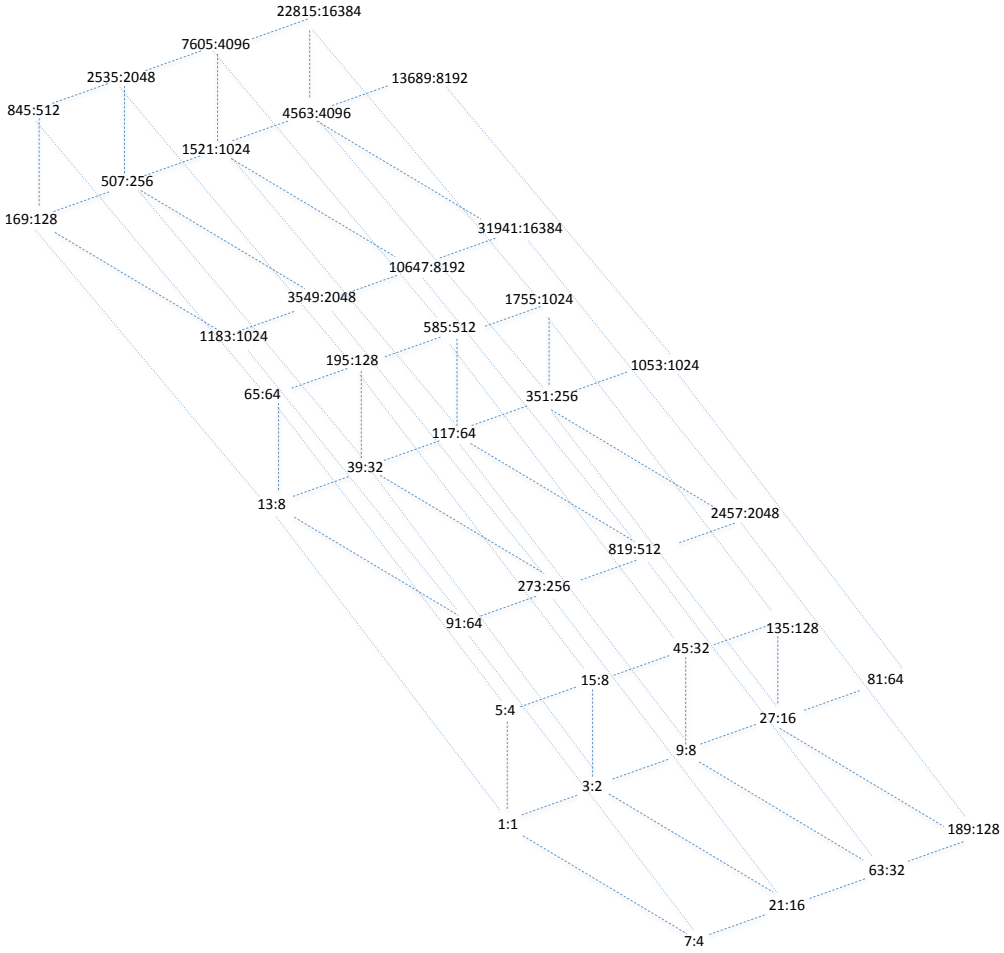
1. The string quartet must follow the scordatura set forth in the opening pages of the score and in example 2.32, using digital tuners to tune string number IV of each instrument. The remaining strings may be tuned as open fifths above string IV.
2. The string players must be proficient enough in the production of natural harmonics to find up to and including the 8^{th} harmonic of each string.
3. The composer must write the score out entirely in natural harmonics, after they have composed the score as a set of sounding pitches.

Provided these three rules are followed, the 39-pitch lattice shown in example 2.30 will become available for use.

Lattice

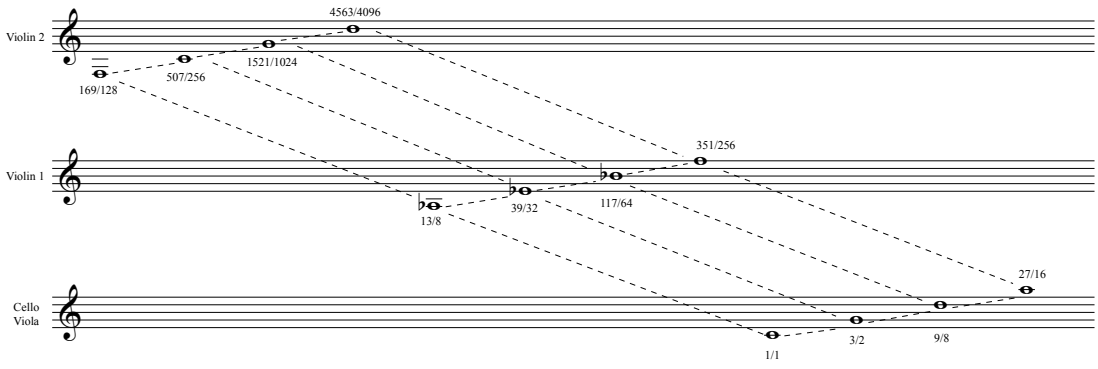
The lattice for *13* is made up of three structurally identical sets of pitches, separated by 13:8, and $(13:8)^2$.

Example 2.30, the lattice for *13* for scordatura string quartet.



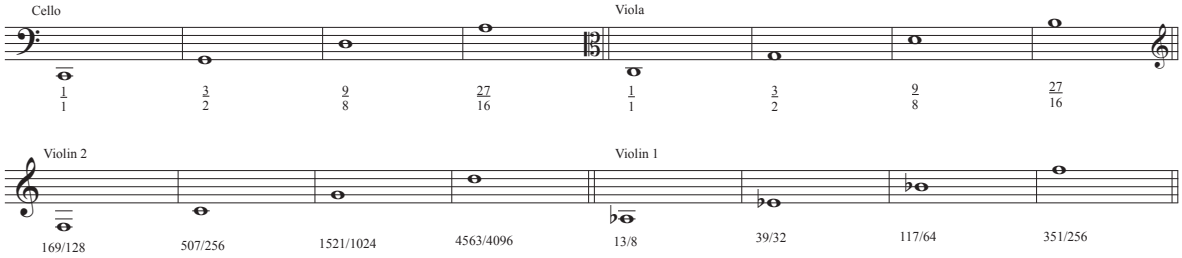
The open tuning of the string quartet is represented by the three lines 1:1 to 27:16, 13:8 to 351:256, and 169:128 to 4563:4096. Example 2.31 shows the open tuning conceptually as pitches on a musical staff.

Example 2.31, the conceptual open tuning of the quartet represented as pitches on a musical staff.



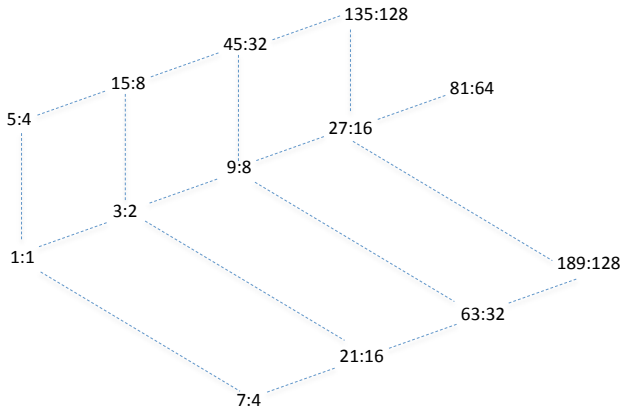
Example 2.31 shows how the lattice fits together on a musical staff, but is not octave accurate for cello and viola. Example 2.32 provides the information regarding the specific, octave accurate, location of pitches for all instruments.

Example 2.32, the octave accurate, open tuning of the quartet represented as pitches on a musical staff.

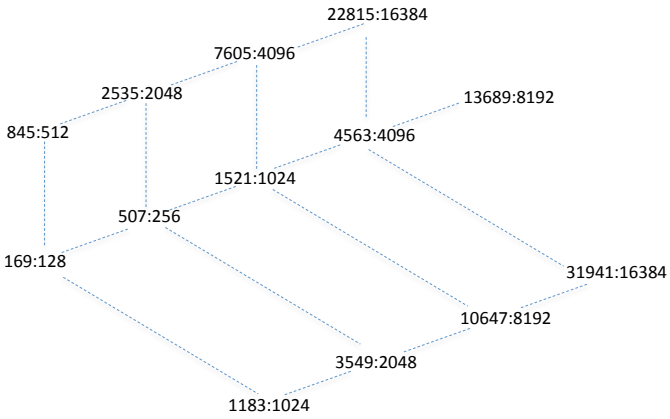


Once the string quartet has re-tuned to the specific scordatura outlined here, each instrument will have access to one of the structurally identical sets of 13 pitches. Examples 2.33 to 2.35 show how the lattice has been broken into three sets and assigned to the instruments of the string quartet.

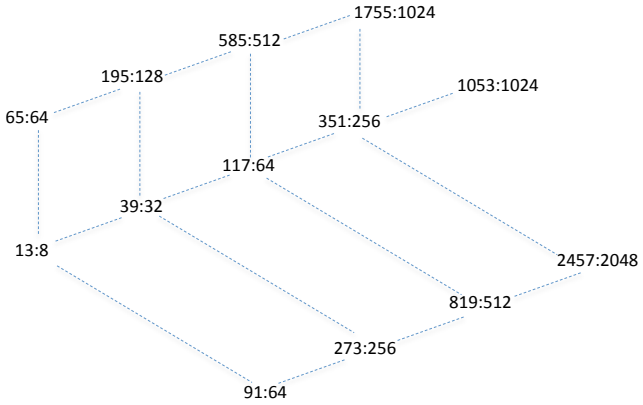
Example 2.33, portion of the lattice available to the viola and cello.



Example 2.34, portion of the lattice available to violin 2.



Example 2.35, portion of the lattice available to violin 1.



Hertz conversion

As with *The tale of Kiều*, in order to practically apply the integer ratios to the open strings of the quartet, and to a digital or acoustic instrument for the composer to use, hertz values must be calculated. These values are shown in tables 2.5, and 2.6.

Table 2.5, absolute frequencies of the open tuning of the orchestra, where the fundamental frequency is equal to 256 Hertz.

Instrument and string number	Value (Hz)
Cello, IV	64
Cello, III	96
Cello, II	144
Cello, I	216

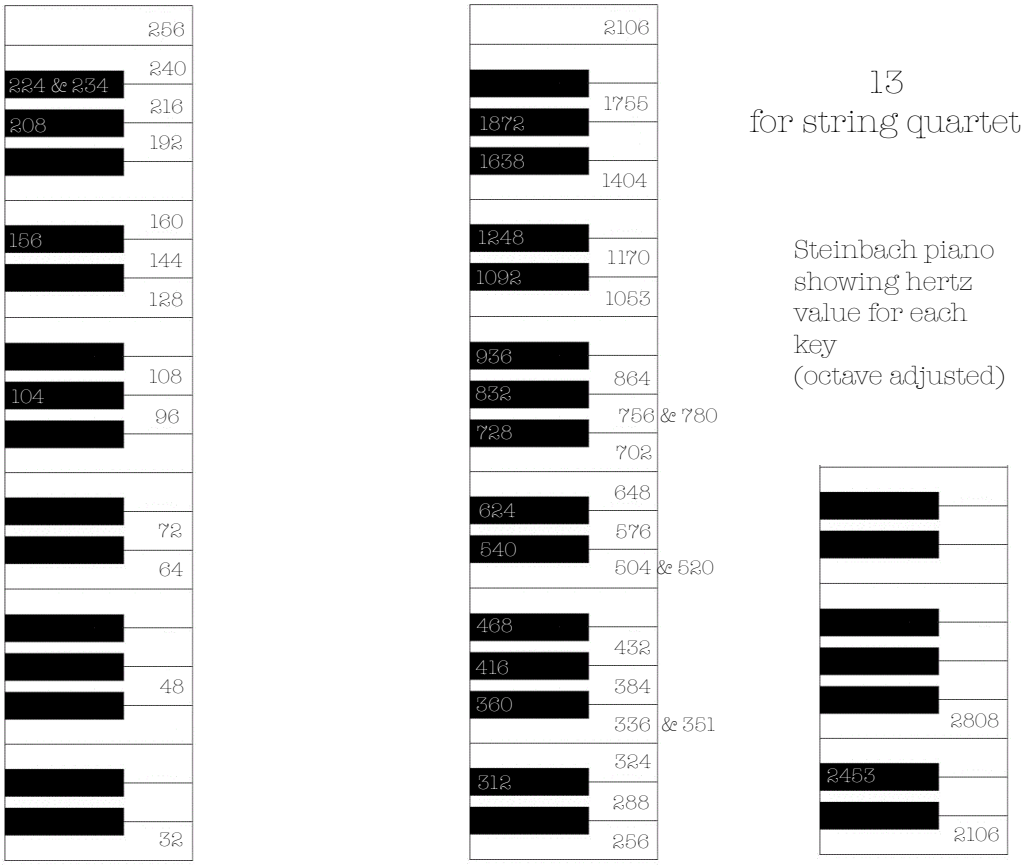
Viola, IV	128
Viola, III	192
Viola, II	288
Viola, I	432
Violin 2, IV	169
Violin 2, III	253.5
Violin 2, II	380.25
Violin 2, I	570.37
Violin 1, IV	208
Violin 1, III	312
Violin 1, II	468
Violin 1, I	624

Table 2.6, absolute frequencies of all integer-ratio in the lattice, where the fundamental frequency is equal to 256 Hertz.

Integer ratio	Value (Hz)	Integer ratio	Value (Hz)	Integer ratio	Value (Hz)
1:1	256	**13:8**	416	**169:128**	338
3:2	384	**39:32**	312	**507:256**	507
9:8	288	**117:64**	468	**1521:1024**	380.25
27:16	432	**351:256**	351	**4563:4096**	285.19
81:64	324	**1053:1024**	263.25	**13689:8192**	427.78
5:4	320	**65:64**	260	**845:512**	422.5
15:8	480	**195:128**	390	**2535:2048**	316.88
45:32	360	**585:512**	292.5	**7605:4096**	475.31
135:128	270	**1755:1024**	438.75	**22815:16384**	356.48
7:4	448	**91:64**	364	**1183:1024**	295.75
21:16	336	**273:256**	273	**3549:2048**	443.63
63:32	504	**819:512**	409.5	**10647:8192**	332.72
189:128	378	**2457:2048**	307.13	**31941:16384**	499.08

Hertz values were assigned to two pianos, as shown in examples 2.36, and 2.37.

Example 2.36, hertz assignment on Steinbach piano for *13*.



Example 2.37, hertz assignment on Toyo piano for *13*.



Again, as with *The tale of Kiều* for scordatura string orchestra, charts showing colour coded integer ratios mapped to two piano keyboards have been included in examples 2.38 and 2.39 to show how UHMs can be easily identified.

Example 2.38, integer ratio assignment on Steinbach piano for *13*.



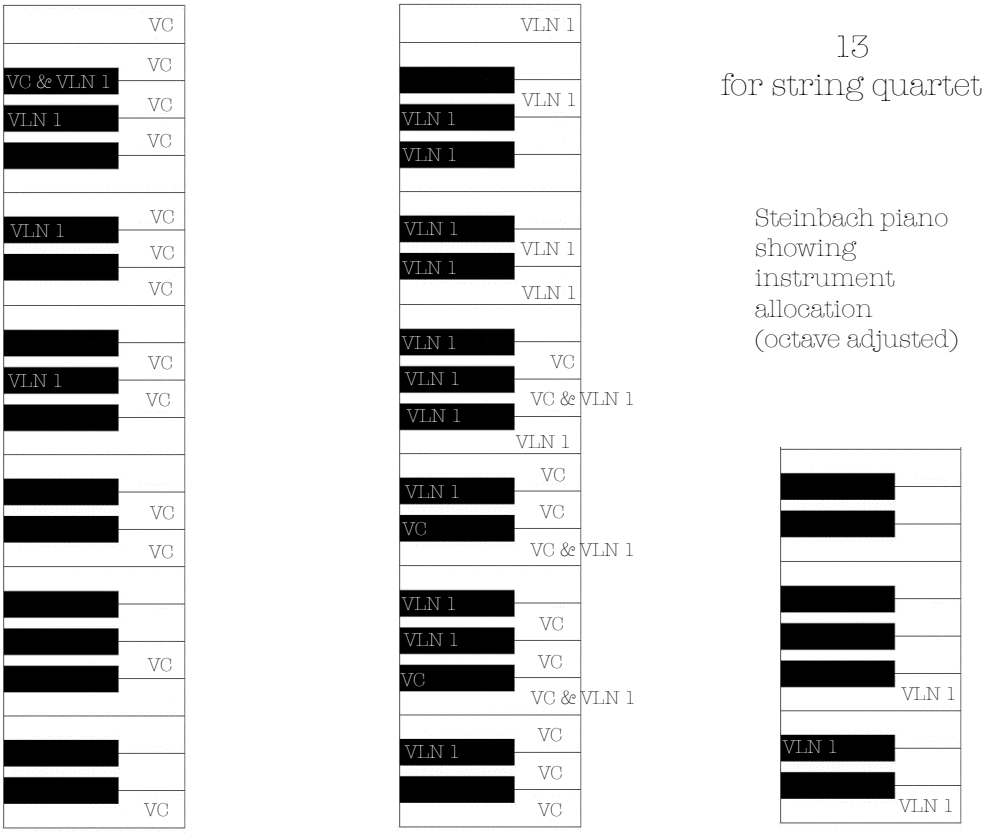
Example 2.39, integer ratio assignment on Toyo piano for *13*.



Orchestration (stage 1)

For the composer to orchestrate music written on the re-tuned pianos, as with *The tale of Kiều*, more charts, showing instrumental assignment to the two piano keyboards are necessary. These charts are shown in examples 2.40, and 2.41.

Example 2.40, instrumental assignment on Steinbach piano for *13*.



Example 2.41, instrumental assignment on Toyo piano.


VLA & VLN 2
VLA
VLA & VLN 2
VLN 2
VLA
VLA & VLN 2
VLA
VLN 2
VLA
VLN 2
VLA & VLN 2
VLA & VLN 2
VLA &

Orchestration (stage 2)

For the same reasons outlined in section 2.2 of this paper, the tuning system for *13* is shifted down by an octave on the two pianos. Therefore, the second stage of orchestration for this piece, is to shift the score back up by an octave.

Orchestration (stage 3)

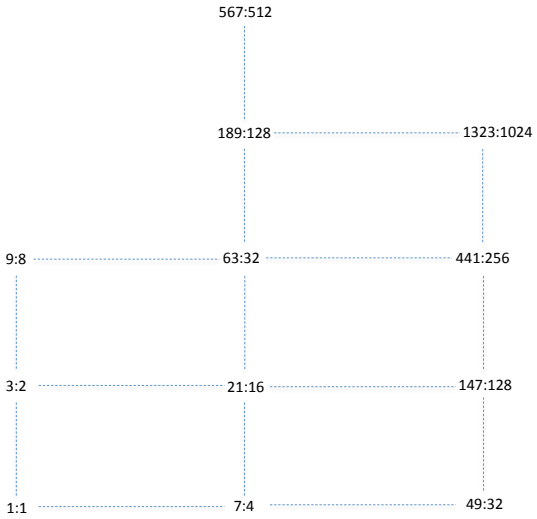
Once the composition is complete and the sounding pitches are placed in the correct octave, the score must be transposed to show players where to touch the strings. Transposition charts for *13* for scordatura string quartet are included in Appendix F.

2.4 Methodology: *The Un-tempered Pianos* for two pianos and one pianist and *K* for two pianos, two pianists, and two hands.

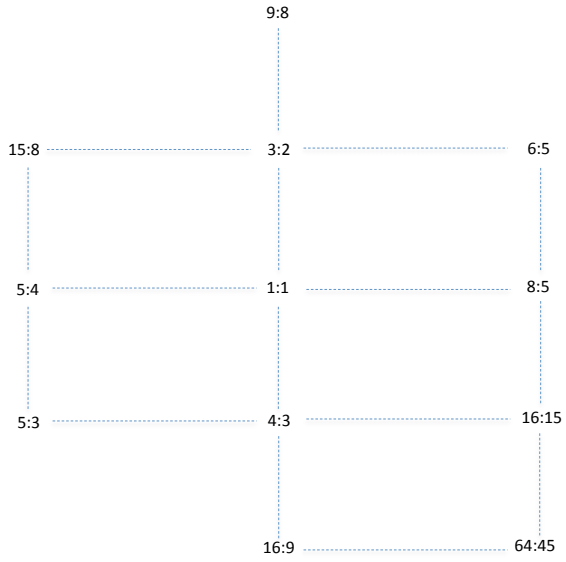
Lattice development

The lattice for *The Un-tempered Pianos* and *K* was originally derived from Young's *Well Tuned Piano* tuning model, which is shown in example 2.42, and Riley's *Harp of New Albion* tuning model, which is shown in example 2.43.

Example 2.42, the tuning model Young developed for *The Well Tuned Piano.*

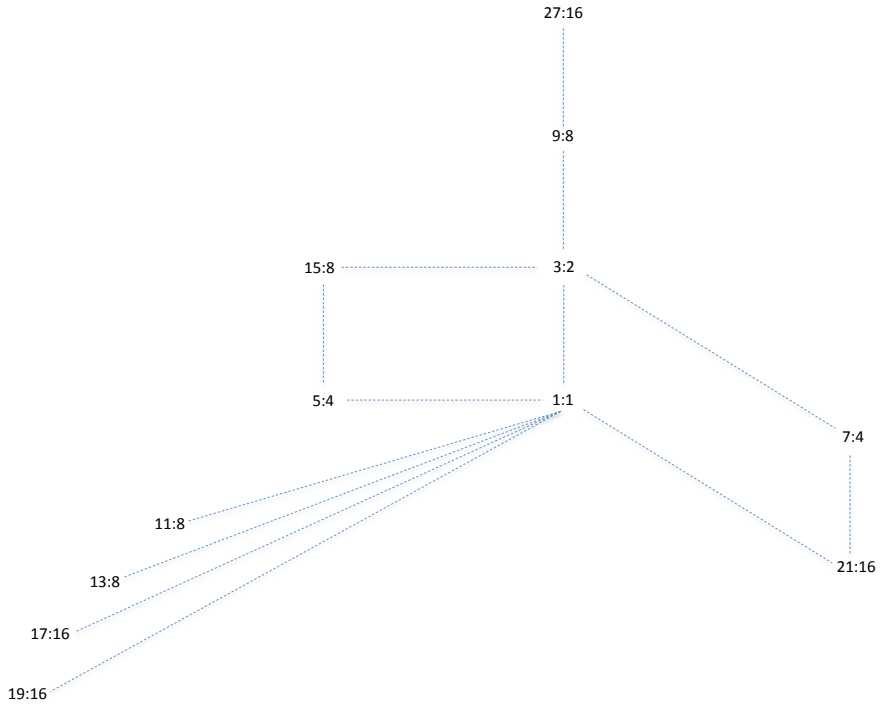


Example 2.43, the tuning model Riley developed for *The Harp of New Albion.*



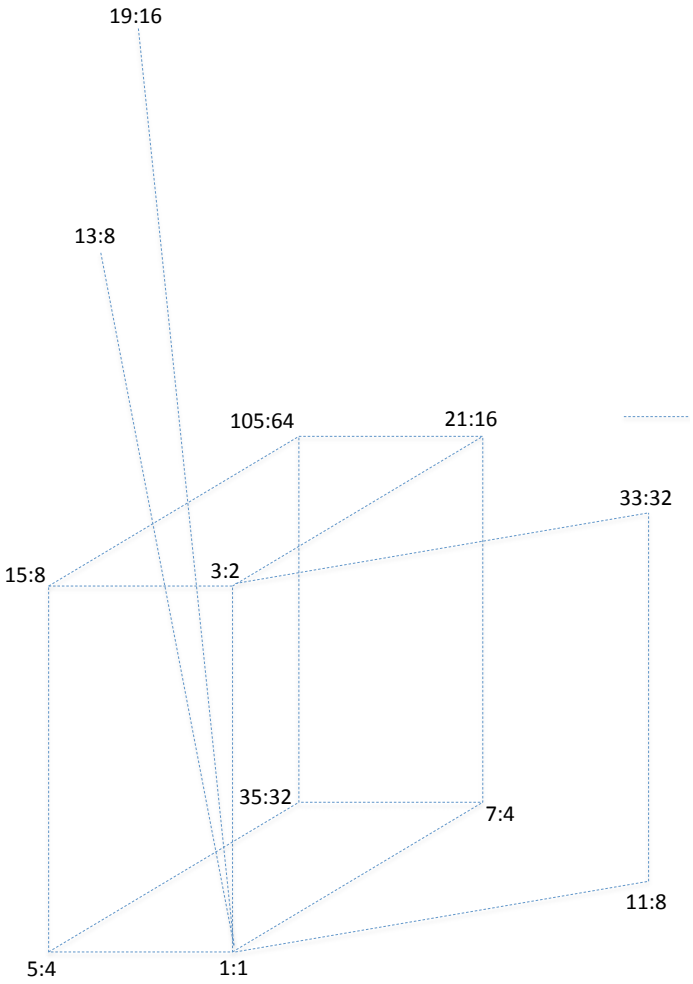
Young's system is a two dimensional lattice of pitches connected by fifths, and sevenths, while Riley's is a two dimensional lattice of pitches connected by thirds and fifths. The concept behind the original tuning model for *The Un-tempered Pianos* and *K* was to combine Young's and Riley's systems, and construct a system which would contain a certain number of pure thirds, fifths, and sevenths, whilst combining certain other higher prime-limit intervals for splashes of colour, in the way that Johnston does in *Suite* for microtonal piano, tuning model shown in Example 2.44.

Example 2.44, the tuning model Johnston developed for *Suite* for microtonal piano.



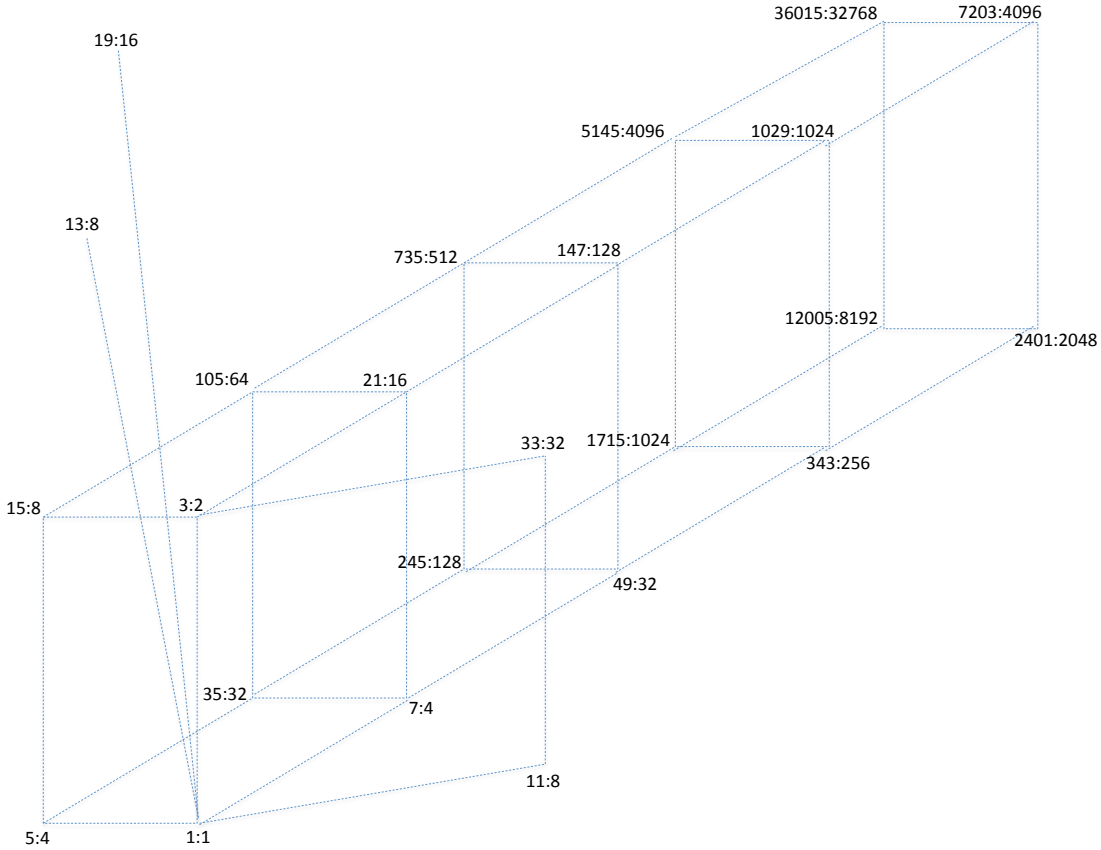
The initial inception of the lattice for *The Un-tempered Pianos* and *K* was for a single piano. This first draft of the lattice is shown in example 2.45.

Example 2.45, first draft of the lattice for *The Un-tempered Pianos* and *K.*



Over time the lattice was extended creating a spine of sevenths. This was done in order to expand the scope of symmetrical modes. To achieve this, a second piano was necessary. One piano retained the tuning shown in example 2.45, while the second piano was tuned according to the additional twelve integer-ratios shown in example 2.46.

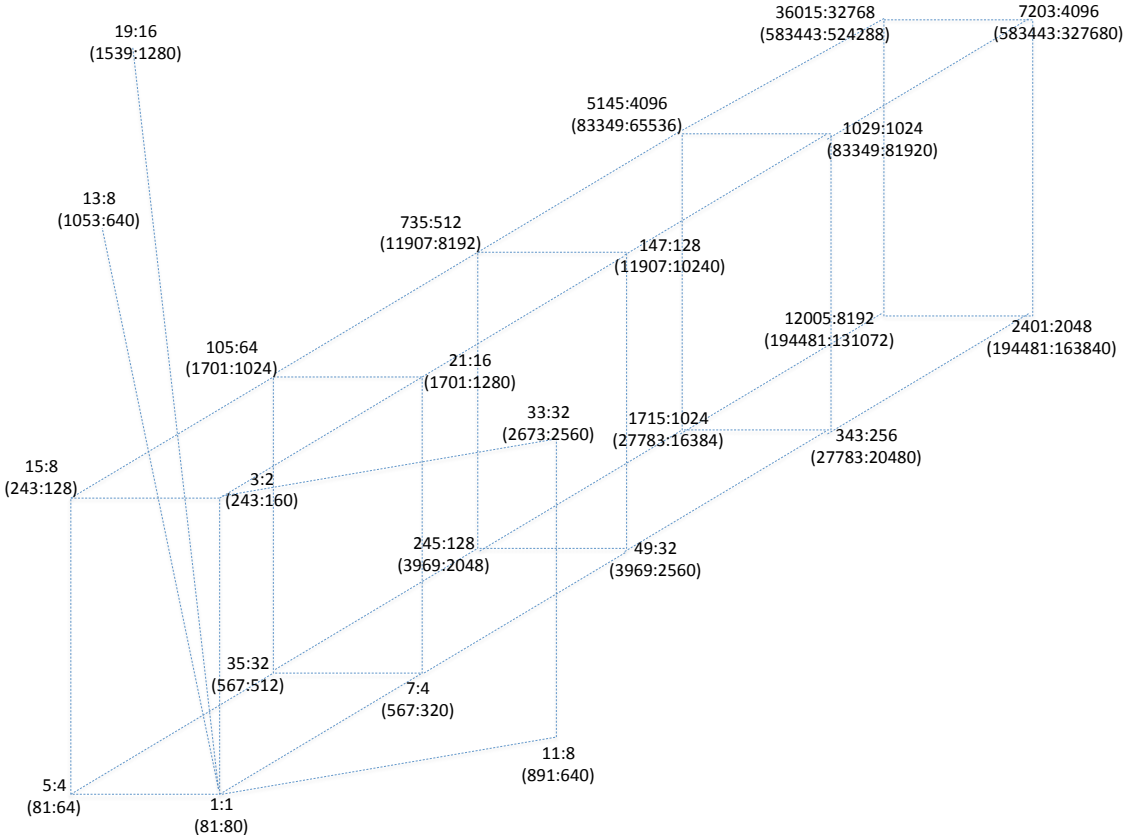
Example 2.46, second draft of the lattice for *The Un-tempered Pianos* and *K.*



Further experimentation altered the lattice for *The Un-tempered Pianos* and *K* again. Experimentation included sharpening each point on the lattice by a schisma, a Pythagorean comma, and a syntonic comma. The syntonic comma was eventually selected as the comma that would be included in the lattice due to the particular beating effect it caused in the middle and upper registers of the piano keyboard.

The lattice in example 2.47 shows that each pitch in the second draft of the lattice becomes inseparably paired to the pitch a syntonic comma sharper than itself in the final lattice. This is reflected in the piano tuning, whereby each hammer strikes two or three strings, and one string per key is not tuned to a unison, it is tuned a syntonic comma sharper than the other one or two strings. This specific technique is arguably what gives *The Un-tempered Pianos* and *K* their characteristic sound.

Example 2.47, the final incarnation of the lattice for *The Un-tempered Pianos* and *K.*



Tuning charts

Once the final lattice was determined, then the integer-ratios were converted to hertz and assembled in the charts shown in tables 2.7 through 2.10. These charts were used to retune the two pianos. Certain pitches in the lower register could not be sharpened by a syntonic comma, as there is only one string per key in the lowest register of most pianos. These lower strings were tuned according to the pitches shown in the second draft of the lattice.

Table 2.7, piano one hertz tuning chart for *The Un-tempered Pianos* and *K*.

Piano 1 tuning

1:1	33:32	35:32	19:16	5:4	21:16	11:8	3:2	13:8	105:64	7:4	15:8

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32	64	128	256	512	1024
C#	33	66	132	264	528	1056
D	35	70	140	280	560	1120
D#	38	76	152	304	608	1216
E	40	80	160	320	640	1280
F	42	84	168	336	672	1344
F#	44	88	176	352	704	1408
G	48	96	192	384	768	1536
G#	52	104	208	416	832	1664
A	52.5	105	210	420	840	1680
A#	56	112	224	448	896	1792
B	60	120	240	480	960	1920

Table 2.8, piano one hertz tuning chart for *The Un-tempered Pianos* and *K*, plus a syntonic comma.

Piano 1 tuning, plus 81:80

81:80	2673:2560	567:512	1539:1280	81:64	1701:1280	891:640	243:160	1053:640	1701:1024	567:320	243:128

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.4	64.8	129.6	259.2	518.4	1036.8
C#	33.41	66.82	133.64	267.28	534.56	1069.12
D	35.44	70.88	141.76	283.52	567.04	1134.08
D#	38.48	76.96	153.92	307.84	615.68	1231.36
E	40.5	81	162	324	648	1296
F	42.53	85.06	170.12	340.24	680.48	1360.96
F#	44.55	89.1	178.2	356.4	712.8	1425.6
G	48.6	97.2	194.4	388.8	777.6	1555.2
G#	52.65	105.3	210.6	421.2	842.4	1684.8
A	53.16	106.32	212.64	425.28	850.56	1701.12
A#	56.7	113.4	226.8	453.6	907.2	1814.4
B	60.75	121.5	243	486	972	1944

Table 2.9, piano two hertz tuning chart for *The Un-tempered Pianos* and *K.*

Piano 2 tuning

1029:1024	36015:32768	147:128	2401:2048	5145:4096	343:256	735:512	12005:8192	49:32	1715:1024	7203:4096	245:128

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.16	64.32	128.64	257.25	514.5	1029
C#	35.17	70.34	140.68	281.36	562.72	1125.44
D	36.75	73.5	147	294	588	1176
D#	37.52	75.04	150.08	300.16	600.32	1200.64
E	40.2	80.4	160.8	321.6	643.2	1286.4
F	42.88	85.76	171.52	343.04	686.08	1372.16
F#	45.94	91.88	183.76	367.52	735.04	1470.08
G	46.89	93.78	187.56	375.12	750.24	1500.48
G#	49	98	196	392	784	1568
A	53.95	107.18	214.36	428.72	857.44	1714
A#	56.29	112.58	225.16	450.32	900.64	1801.28
B	61.25	122.5	245	490	980	1960

Table 2.10, piano two hertz tuning chart for *The Un-tempered Pianos* and *K,* plus a syntonic comma.

Piano 2 tuning, plus 81:80

$\frac{83349}{81920}$	$\frac{583443}{524288}$	$\frac{11907}{10240}$	$\frac{194481}{163840}$	$\frac{83349}{65536}$	$\frac{27783}{20480}$	$\frac{11907}{8192}$	$\frac{194481}{131072}$	$\frac{3969}{2560}$	$\frac{27783}{16384}$	$\frac{583443}{327680}$	$\frac{3969}{2048}$

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.72	65.44	130.88	261.76	523.52	1047.04
C#	35.79	71.58	143.16	286.32	572.64	1145.28
D	37.4	74.8	149.6	299.2	598.4	1196.8
D#	38.17	76.34	152.68	305.36	610.72	1221.44
E	40.9	81.8	163.6	327.2	654.4	1308.8
F	43.63	87.26	174.52	349.04	698.08	1396.16
F#	46.74	93.48	186.96	373.92	747.84	1495.68
G	47.72	95.44	190.88	381.76	763.52	1527.04
G#	49.86	99.72	199.44	398.88	797.76	1595.52
A	54.53	109.06	218.12	436.24	872.48	1744.96
A#	57.26	114.52	229.04	458.08	916.16	1832.32
B	62.33	124.66	249.32	498.64	997.28	1994.56

Chapter three: Results (Musical scores)

3.1. *Remembering is a Form of Forgetting* for symphony orchestra

Remembering is a form of forgetting

for Symphony Orchestra

Inspired by the PhD thesis of Dr. Benjamin Robert Ford Smith:
Remembering is a form of forgetting: memory as a simulacrum

Dylan Crismani

2

Notes for the conductor and orchestra

Harmonics:

This piece is based on the harmonic series.
The notes in the harmonic series are not tempered. The interval relationships between notes in the harmonic series are theoretically pure. Therefore some of the notes in this piece need to be flattened as follows:

Bb = one third of a semitone flatter than a normal Bb.
Eb = one third of a semitone flatter than a normal Eb.
C# = one third of a semitone flatter than a normal C#.
G# = one third of a semitone flatter than a normal G#.

Woodwinds:

In addition to the small circles above note heads to indicate a harmonic I have also included a number. The number indicates which harmonic is to be played, where the fundamental pitch is 1.

e.g. The player reads a Bb. Above the Bb is the sign for a natural harmonic and the number 7. The fundamental pitch I am asking the player to take is a C and 7 indicates this.

Percussionists:

There is a repeated pattern throughout the piece; a drum roll on the timpani which lasts the duration of a semibreve or minim followed by the striking of a cymbal. The percussionist who has the duty of striking the cymbal should gather as many cymbals as are available. A different cymbal should be used for each strike, along with a variation of hard and soft mallets and striking the cymbal in different places. These decisions are to be made at the discretion of the percussionist.

It is not necessary to have dozens of cymbals, however, enough cymbals and mallets should be available for the percussionist to create slight timbral changes each time.

Strings:

8va signs only apply the the sounding pitch, not to the node points.
The string parts show two notes. The regular circle shaped note heads represent the desired sounding pitches. The diamond shaped noted head indicate where the player is to place their finger (node points) on the open string.

Tuning:

The orchestra should tune to C = 261 Hz or C = 256 Hz, not to A440 Hz

Notes for the soloists

♭7 This symbol indicates that the pitch is about a third of a semitone flat

↑ This symbol indicates that the pitch is slightly sharp

↓ This symbol indicates that the pitch is slightly flat

- This symbol indicates that the pitch is one sixth to one tenth of a semitone flat

These symbols will appear above or below the note which it is altering. Note that the symbol indicating one sixth to one tenth of a semitone flatness appears like a tenuto marking. It is not a tenuto marking. Tenuto is not used at all in this score.

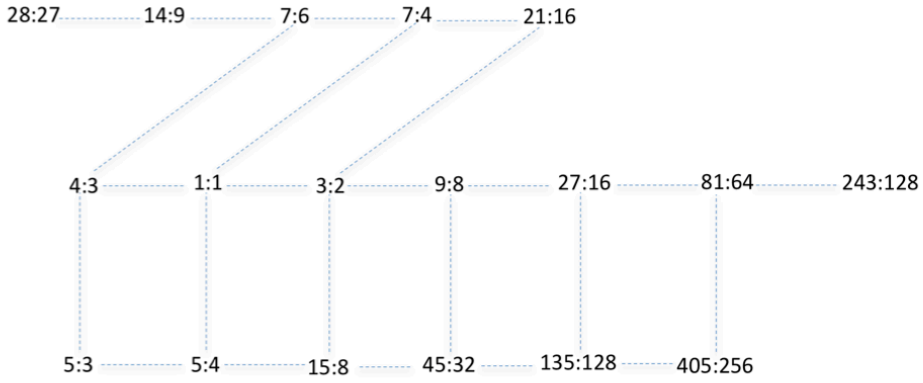
This piece does not rely heavily on the use of overly complex notation. The soloists are encouraged to use their ears, instead of their eyes, to guide them.

Performers should listen to recordings of the solo parts on youtube and aim to replicate the tuning they hear. Recordings can be accessed by searching the following words on google "Dylan Crismani Solo Cello / Solo CB "

These symbols have simply been included as reminders that may be useful during the performance of the piece.

4

Lattice


28:27
14:9
7:6
7:4
21:16
4:3
1:1
3:2
9:8
27:16
81:64
243:128
5:3
5:4
15:8
45:32
135:128
405:256

Orchestra

2 Flutes
2 Oboes
1 English Horn
1 Clarinet in Eb
2 Clarinets in Bb
2 Bassoons
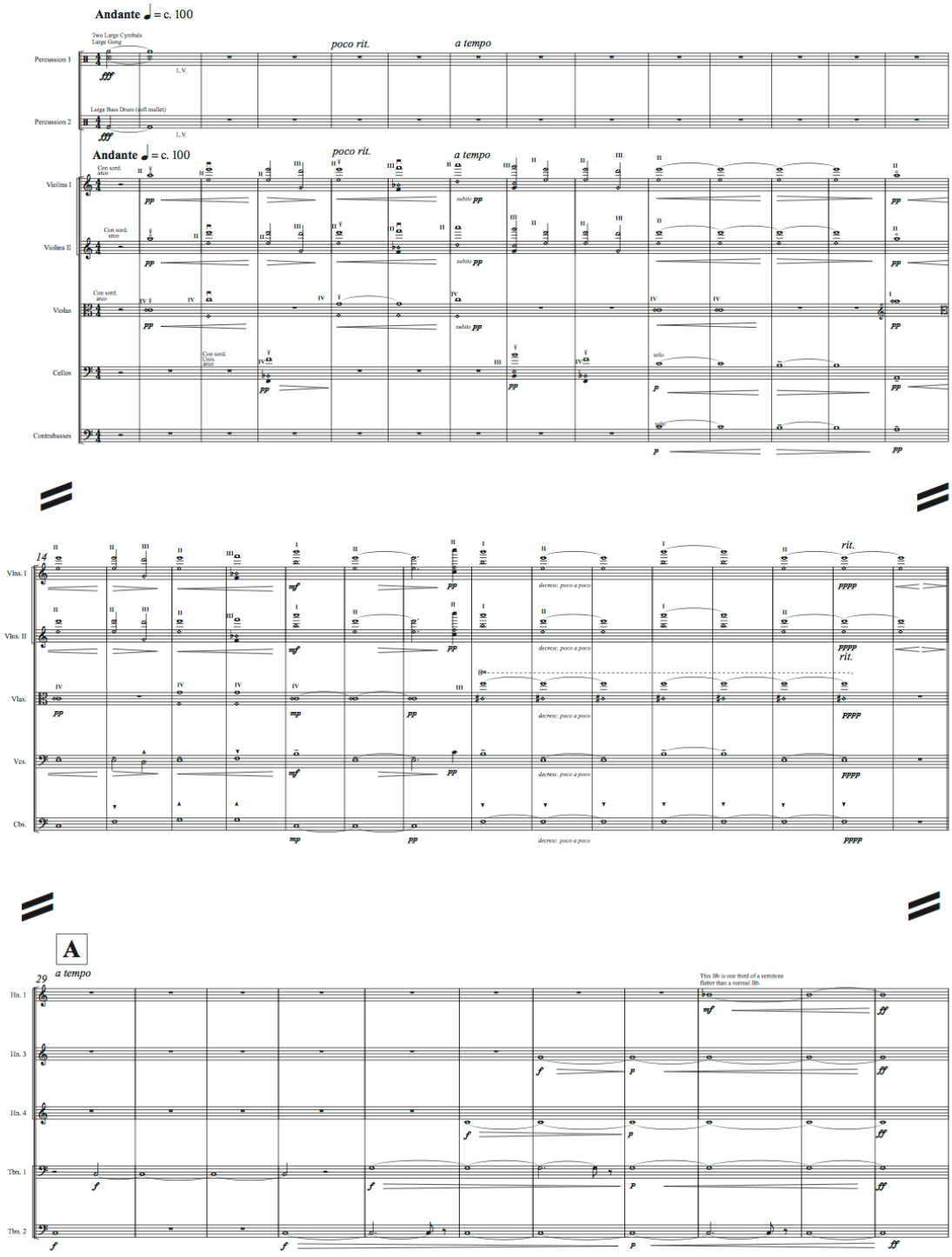
4 Horns in F
2 Trumpets in C
2 Tenor Trombones
1 Tuba

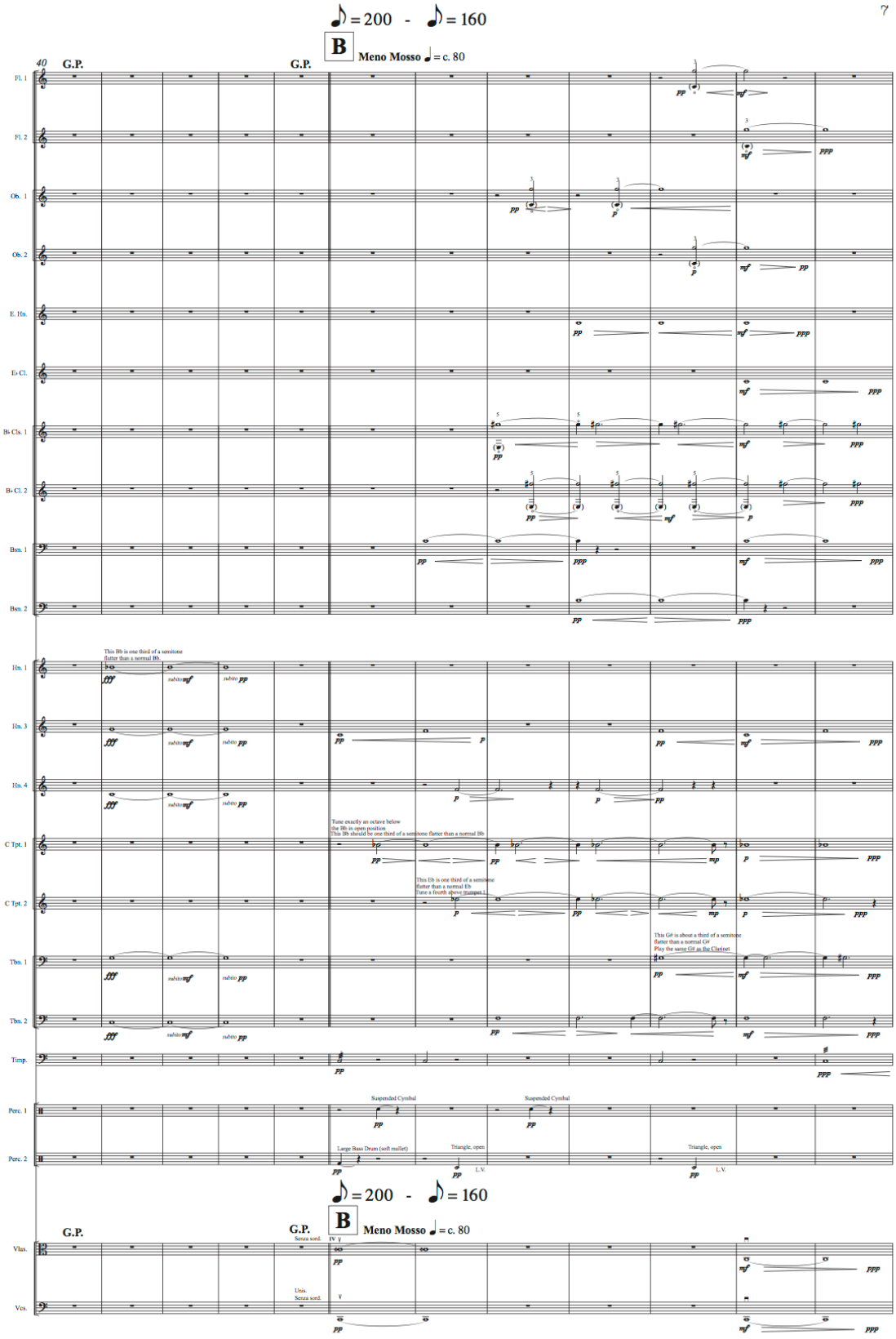
Timpani
2 Percussionists
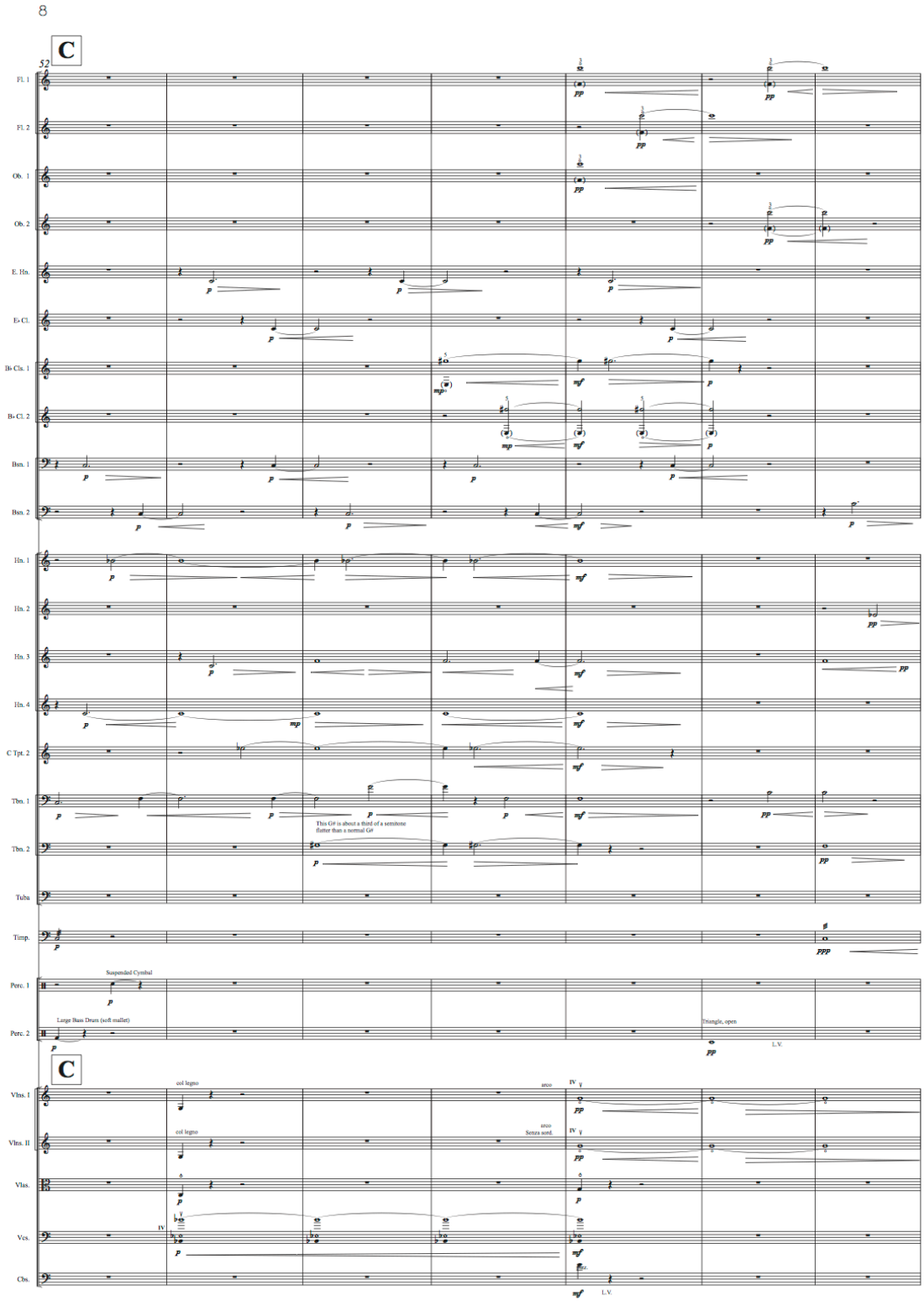
16 First Violins
12 Second Violins
10 Violas
6 Cellos
4 Contrabasses
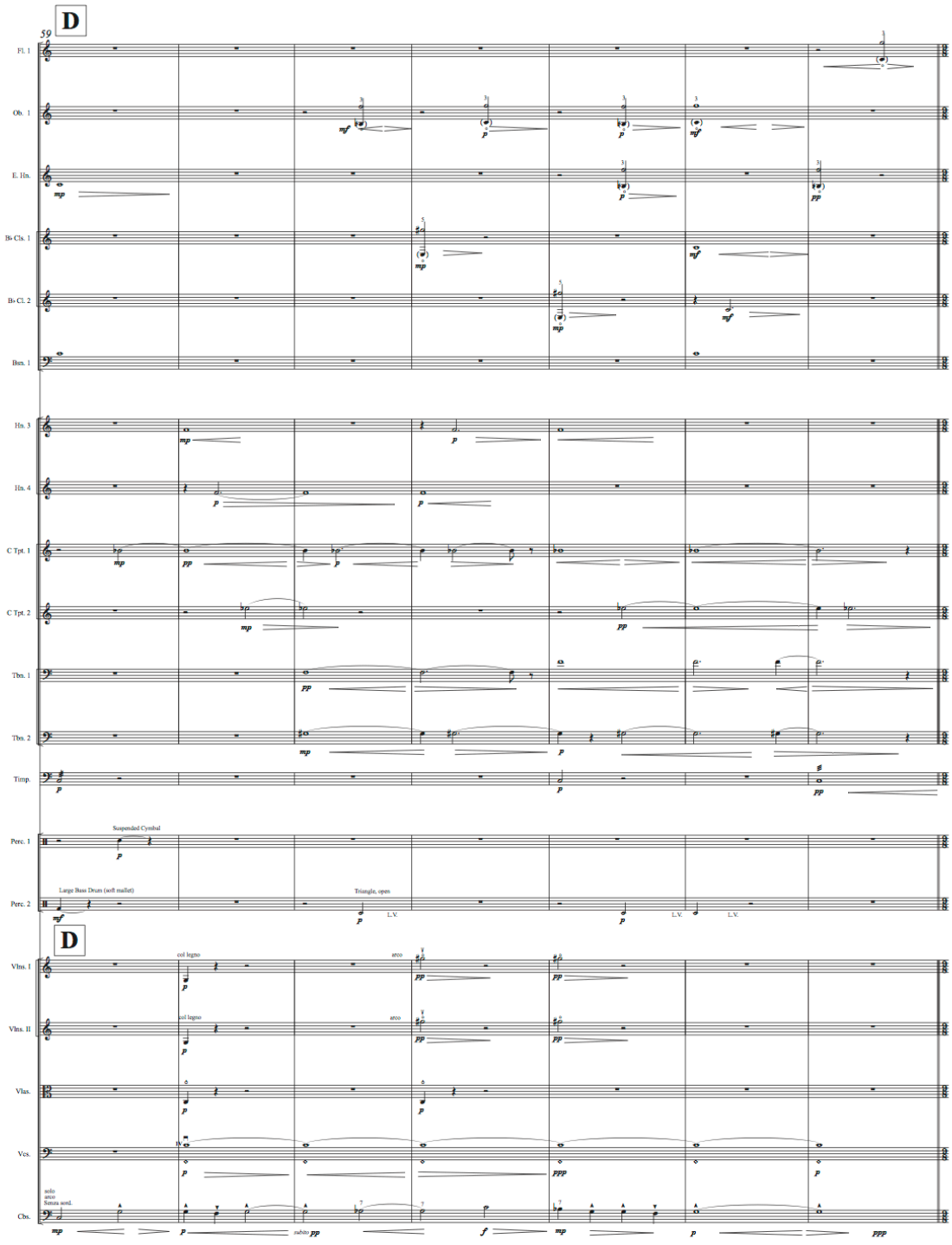
Remembering is a form of forgetting
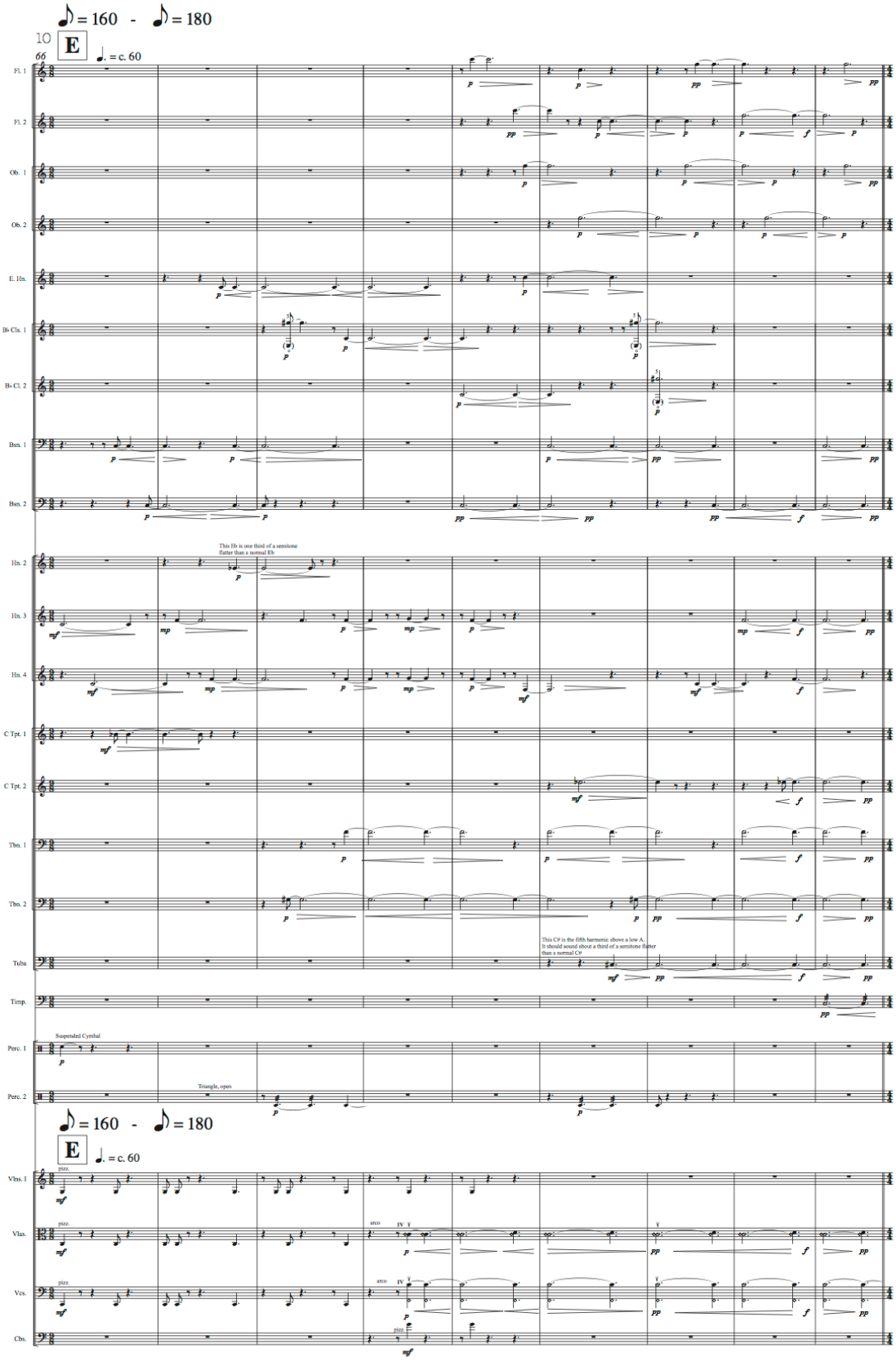
Dylan Crismani
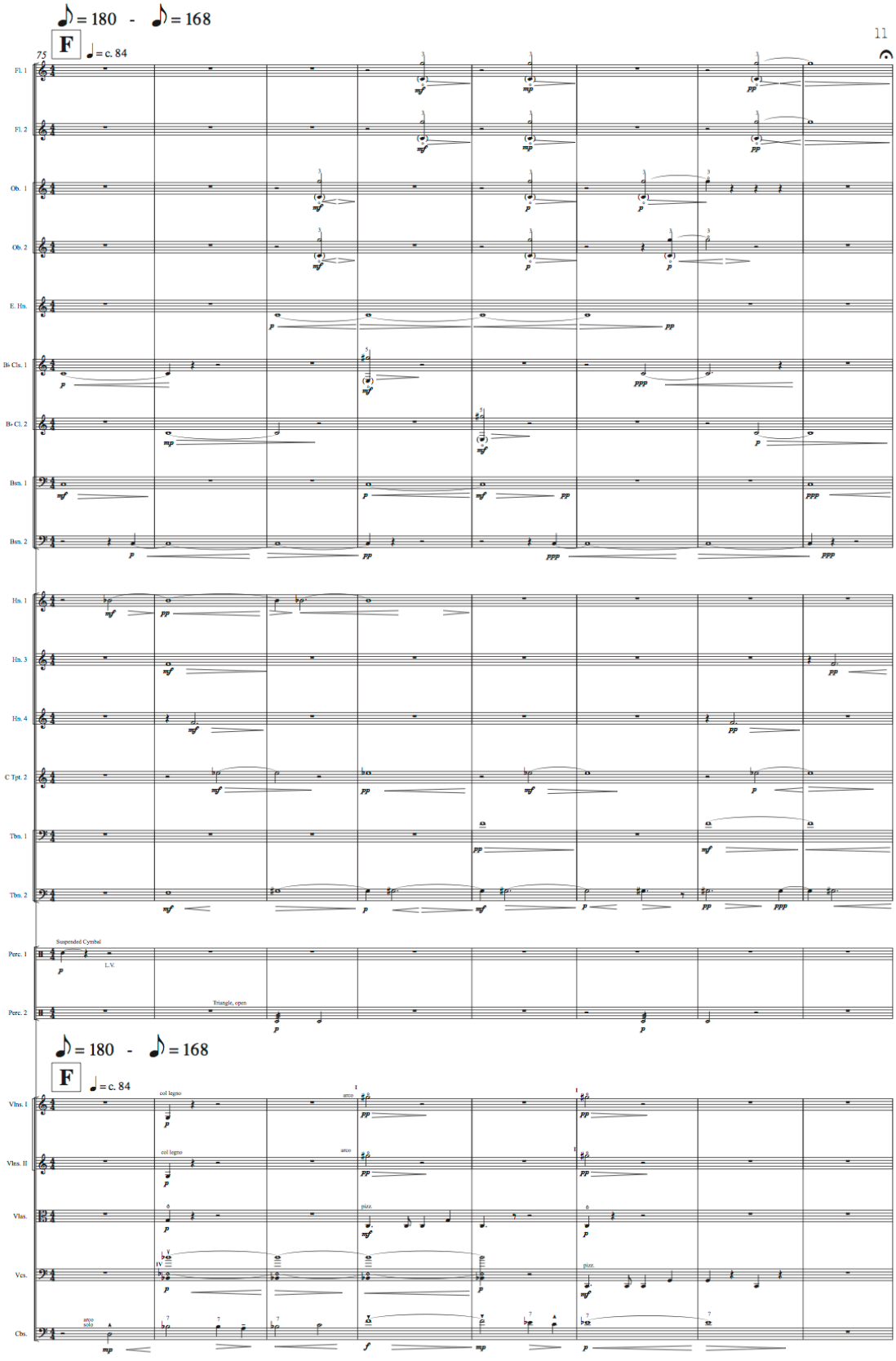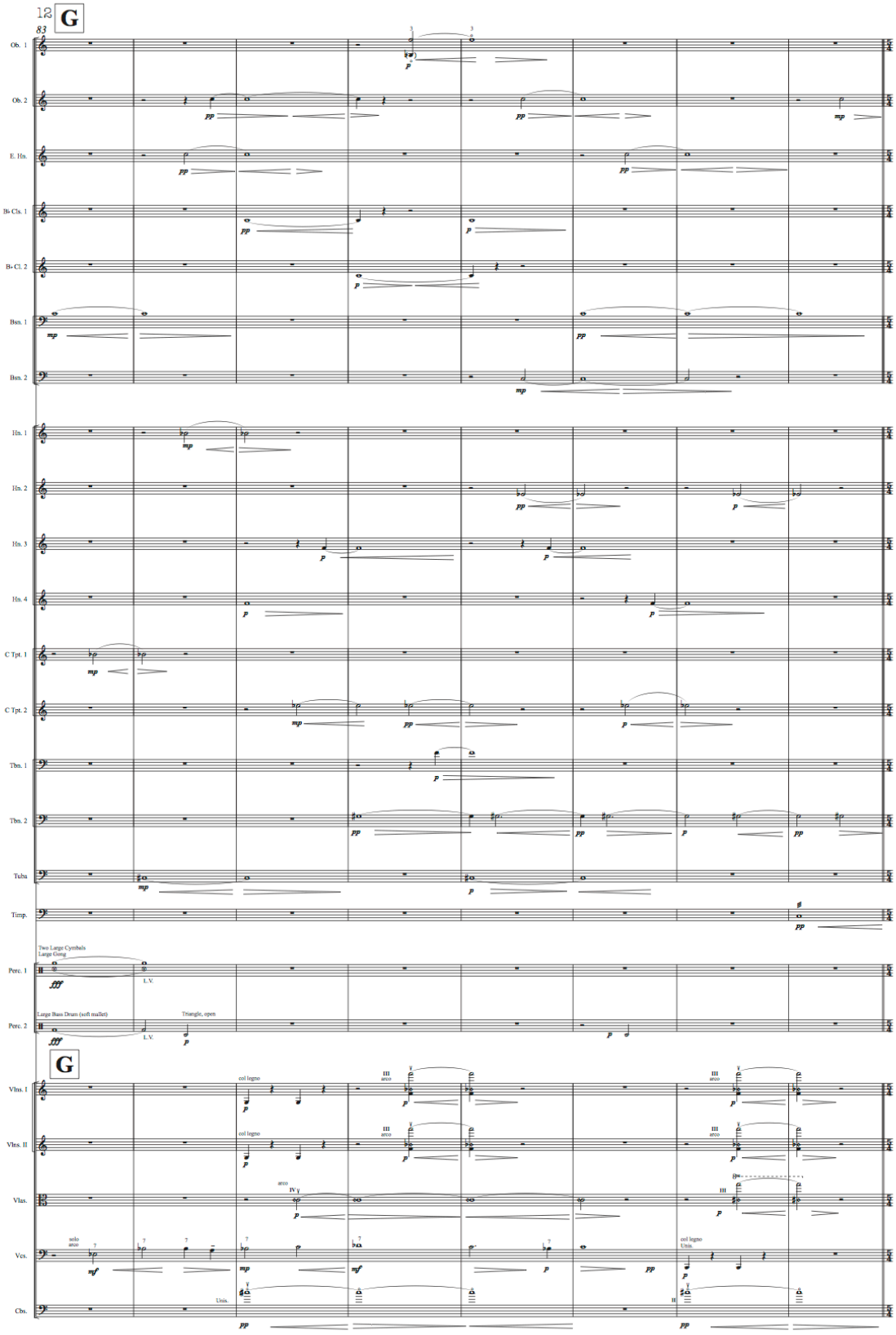



♪ = 200 - ♪ = 160
B
Meno Mosso ♩ = c. 80
40
G.P.
G.P.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Timp.
Perc. 1
Perc. 2
This Bb is one third of a semitone flatter than a normal Bb.
Tune exactly an octave below the Bb in open position
This Bb should be one third of a semitone flatter than a normal Bb
This Eb is one third of a semitone flatter than a normal Eb
Tune a fourth above trumpet 1
This G# is about a third of a semitone flatter than a normal G#
Play the same G# as the Clarinet
Suspended Cymbal
Suspended Cymbal
Large Bass Drum (soft mallet)
Triangle, open
Triangle, open
L.V.
L.V.
♪ = 200 - ♪ = 160
B
Meno Mosso ♩ = c. 80
G.P.
G.P.
Senza sord.
Vlns.
Vcs.
Unis.
Senza sord.
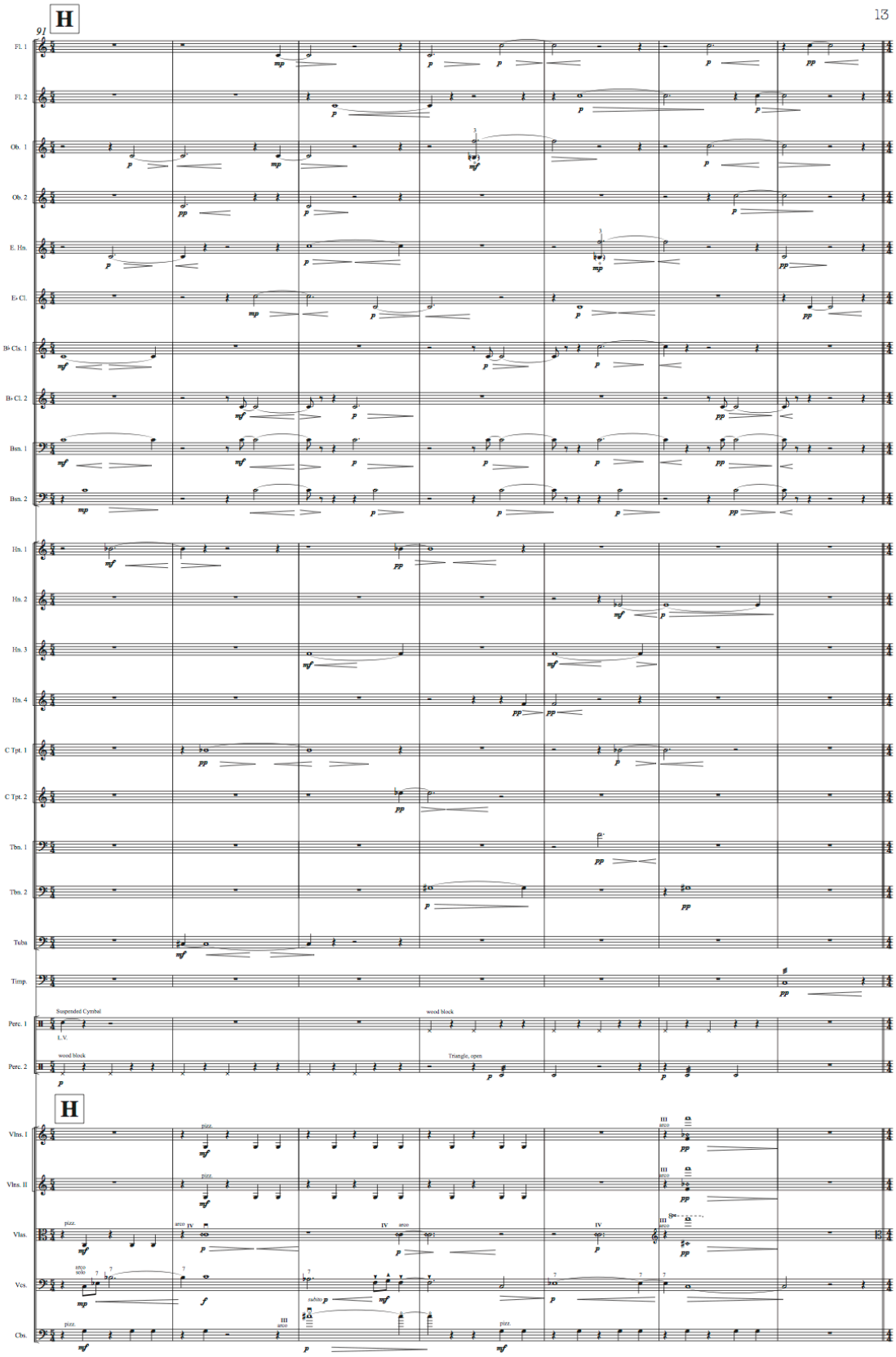

8
C
52
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 2
Tbn. 1
This G♯ is about a third of a semitone flatter than a normal G♯
Tbn. 2
Tuba
Timp.
Perc. 1
Suspended Cymbal
Large Bass Drum (soft mallet)
Perc. 2
Triangle, open
L.V.
C
Vlns. I
col legno
arco
Vlns. II
col legno
arco
Senza sord.
Vlas.
Vcs.
Cbs.
L.V.
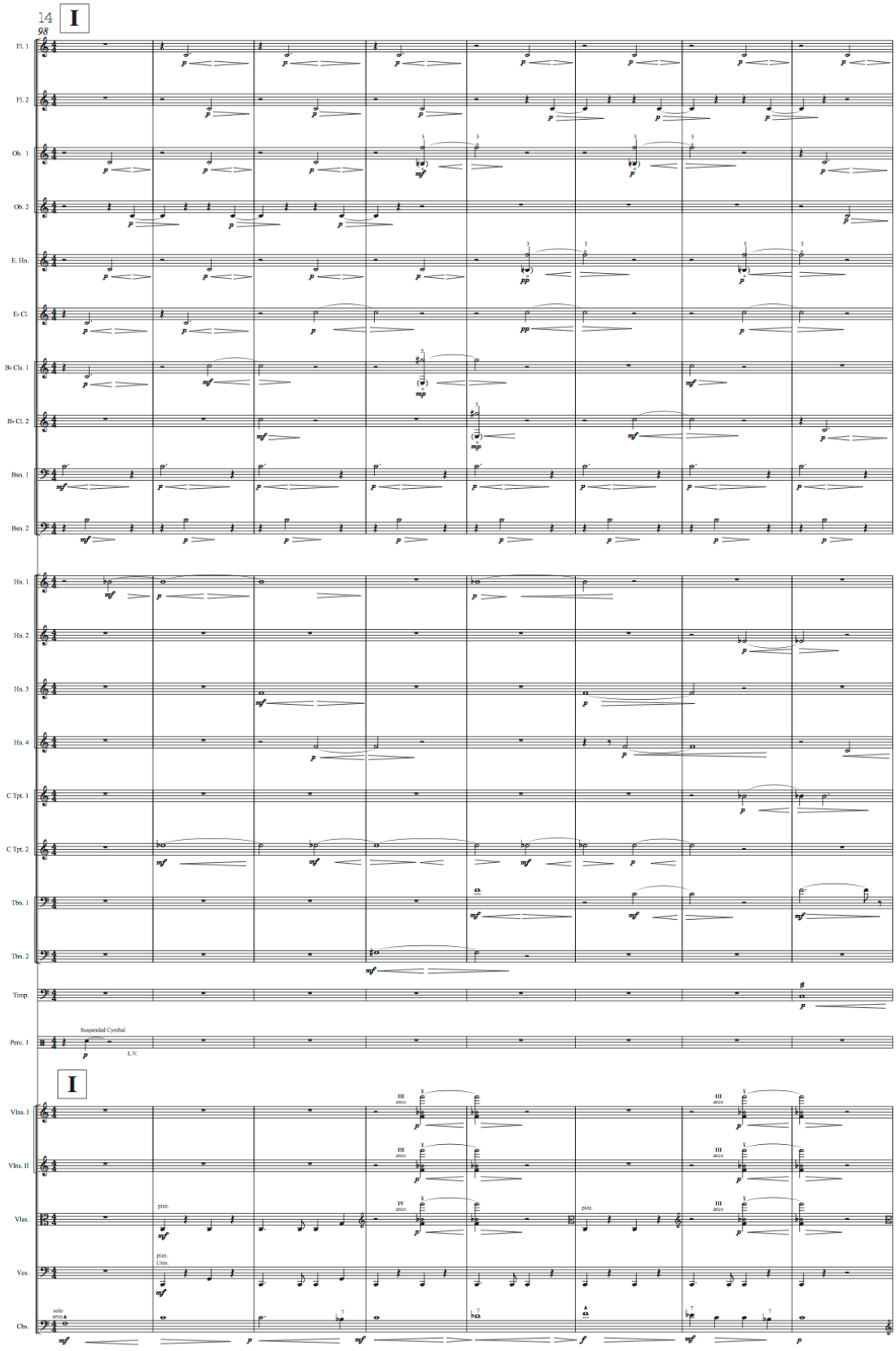

D
59
Fl. 1
Ob. 1
E. Hn.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Timp.
Perc. 1
Suspended Cymbal
Perc. 2
Large Bass Drum (soft mallet)
Triangle, open
L.V.
D
Vlns. I
col legno
arco
Vlns. II
Vlas.
Vcs.
Cbs.
solo
arco
Senza sord.
subito pp
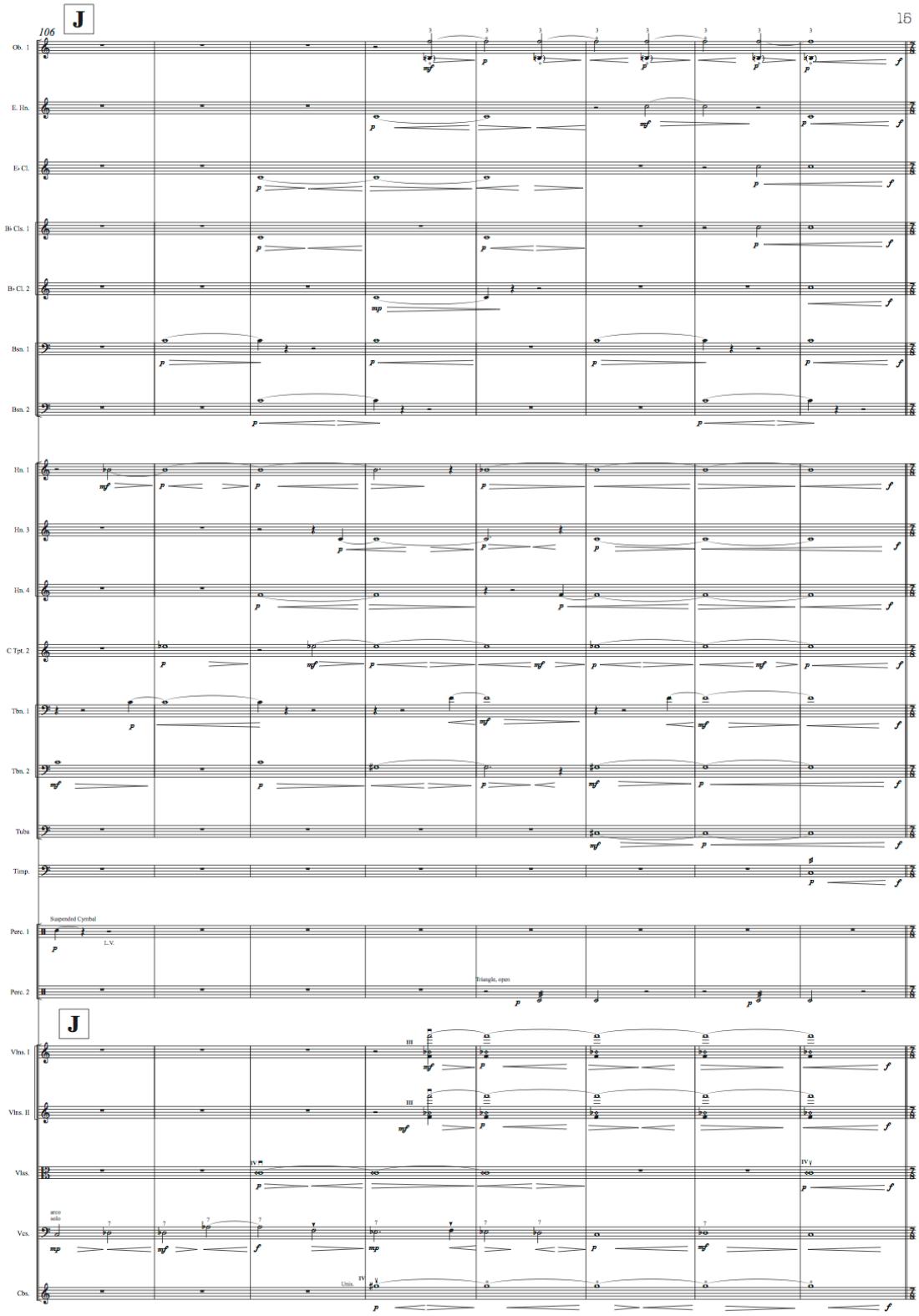

♪ = 160 - ♪ = 180
10
E
66
♩. = c. 60
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 2
This E♭ is one third of a semitone flatter than a normal E♭
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
This C♯ is the fifth harmonic above a low A. It should sound about a third of a semitone flatter than a normal C♯
Timp.
Suspended Cymbal
Perc. 1
Triangle, open
Perc. 2
♪ = 160 - ♪ = 180
E
♩. = c. 60
Vlns. I
Vlas.
Vcs.
Cbs.
pizz.
arco
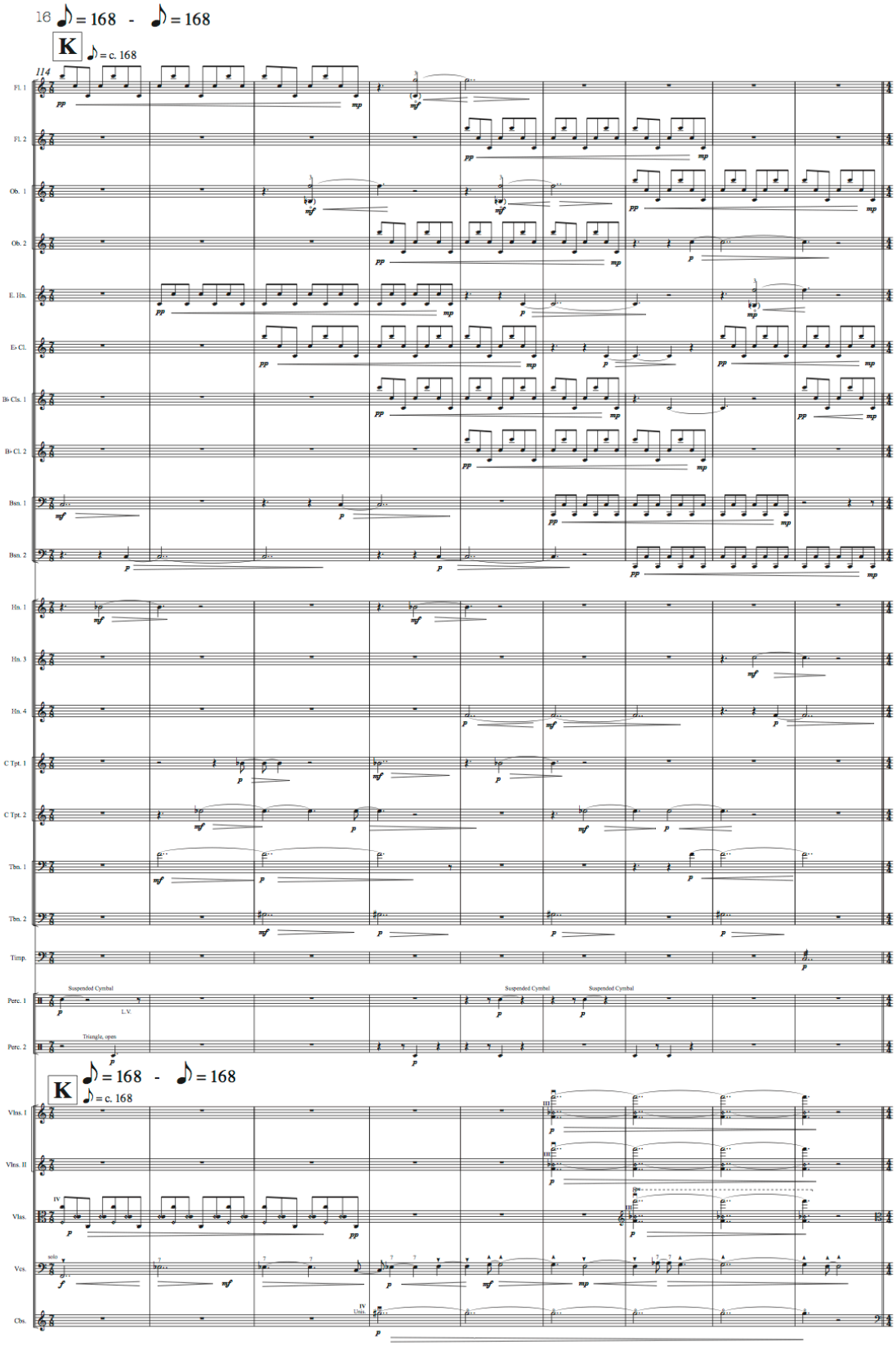

♪ = 180 - ♪ = 168
F
♩ = c. 84
75
11
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 2
Tbn. 1
Tbn. 2
Perc. 1
Suspended Cymbal
L.V.
Perc. 2
Triangle, open
♪ = 180 - ♪ = 168
F
♩ = c. 84
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
col legno
arco
pizz.
arco solo
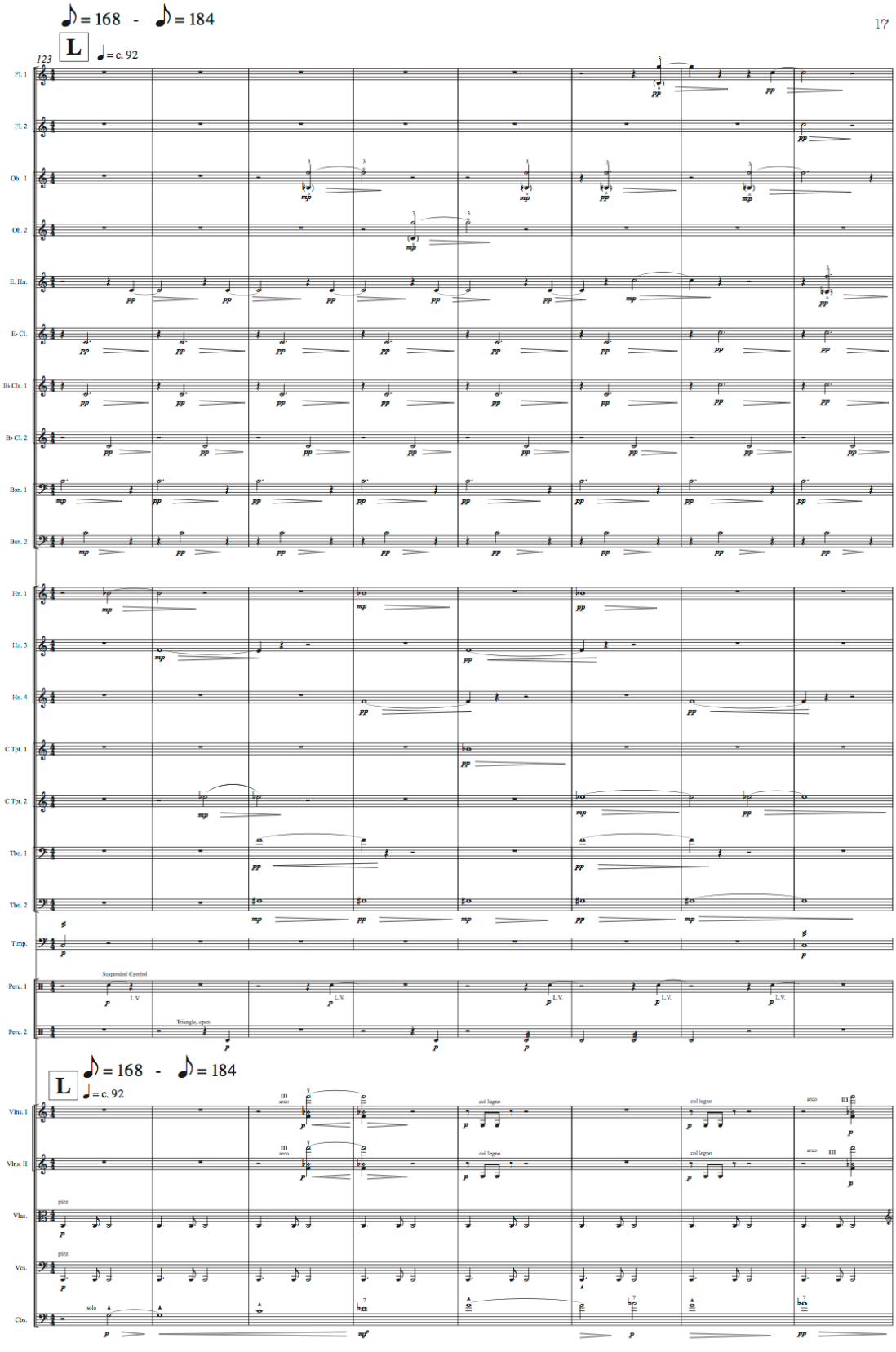

12
83
G
Ob. 1
Ob. 2
E. Hn.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Two Large Cymbals
Large Gong
Perc. 1
L.V.
Large Bass Drum (soft mallet)
Triangle, open
Perc. 2
G
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
col legno
arco
solo
Unis.

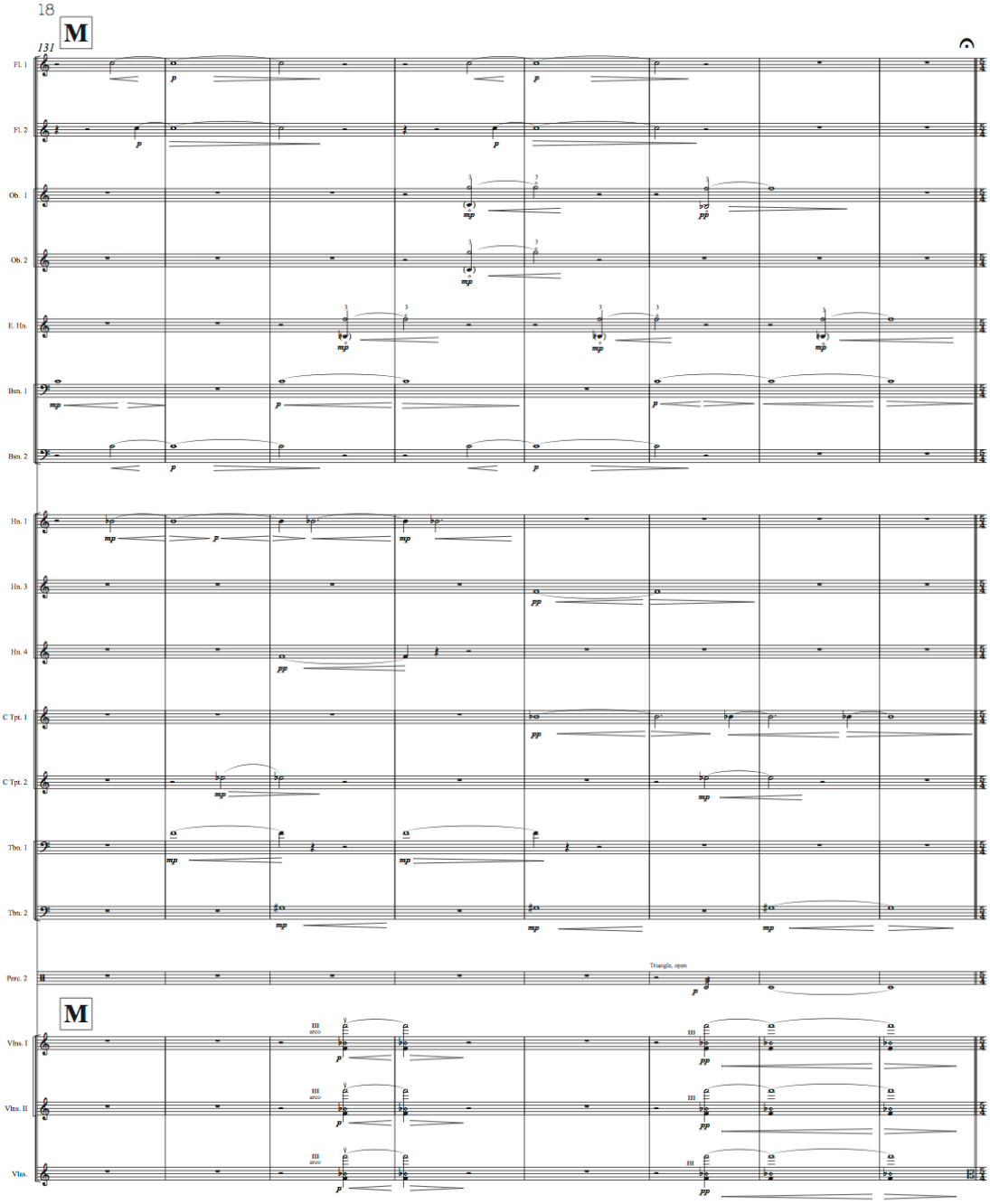

H
13
91
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Suspended Cymbal
L.V.
wood block
Perc. 1
wood block
Triangle, open
Perc. 2
H
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
pizz.
arco
solo
cubito
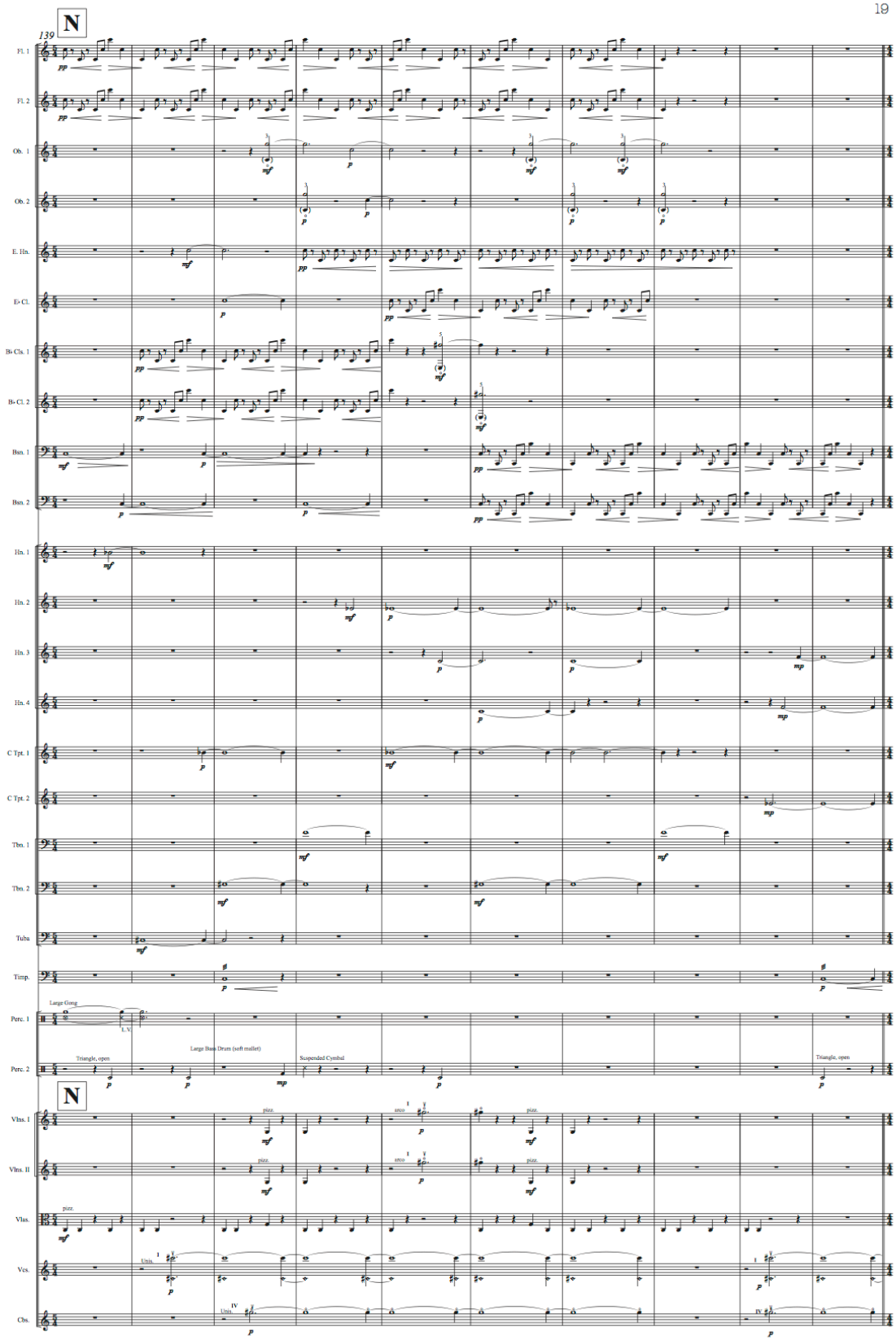

14
I
98
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Timp.
Suspended Cymbal
L.V.
Perc. 1
I
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
pizz.
arco
pizz.
Unis.
solo
arco
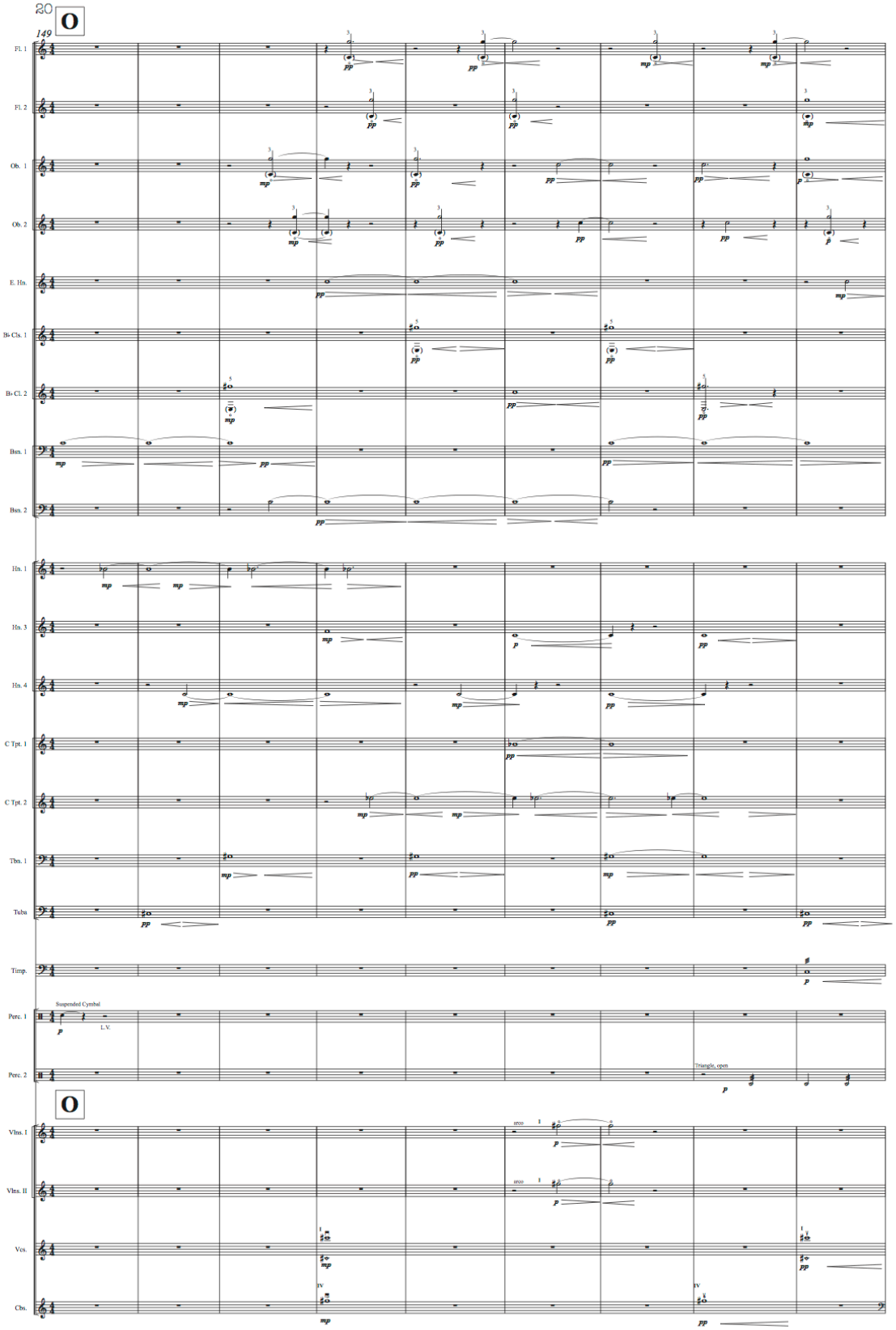

J
106
15
Ob. 1
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Perc. 1
Suspended Cymbal
L.V.
Perc. 2
Triangle, open
J
Vlns. I
Vlns. II
Vlas.
Vcs.
arco
solo
Cbs.
Unis
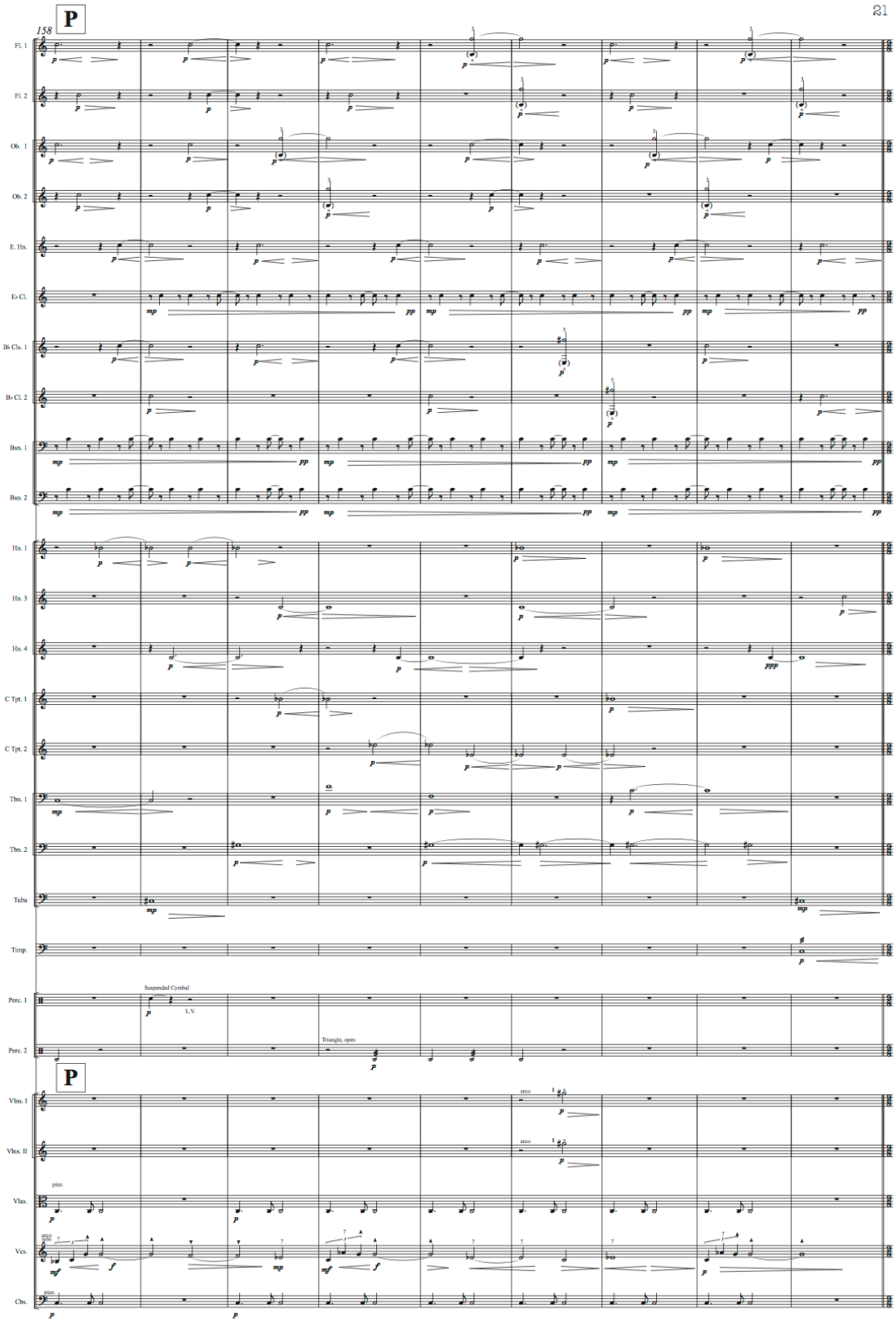

16 ♪ = 168 - ♪ = 168
K
♪ = c. 168
114
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Timp.
Suspended Cymbal
L.V.
Triangle, open
Perc. 1
Perc. 2
K
♪ = 168 - ♪ = 168
♪ = c. 168
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
solo
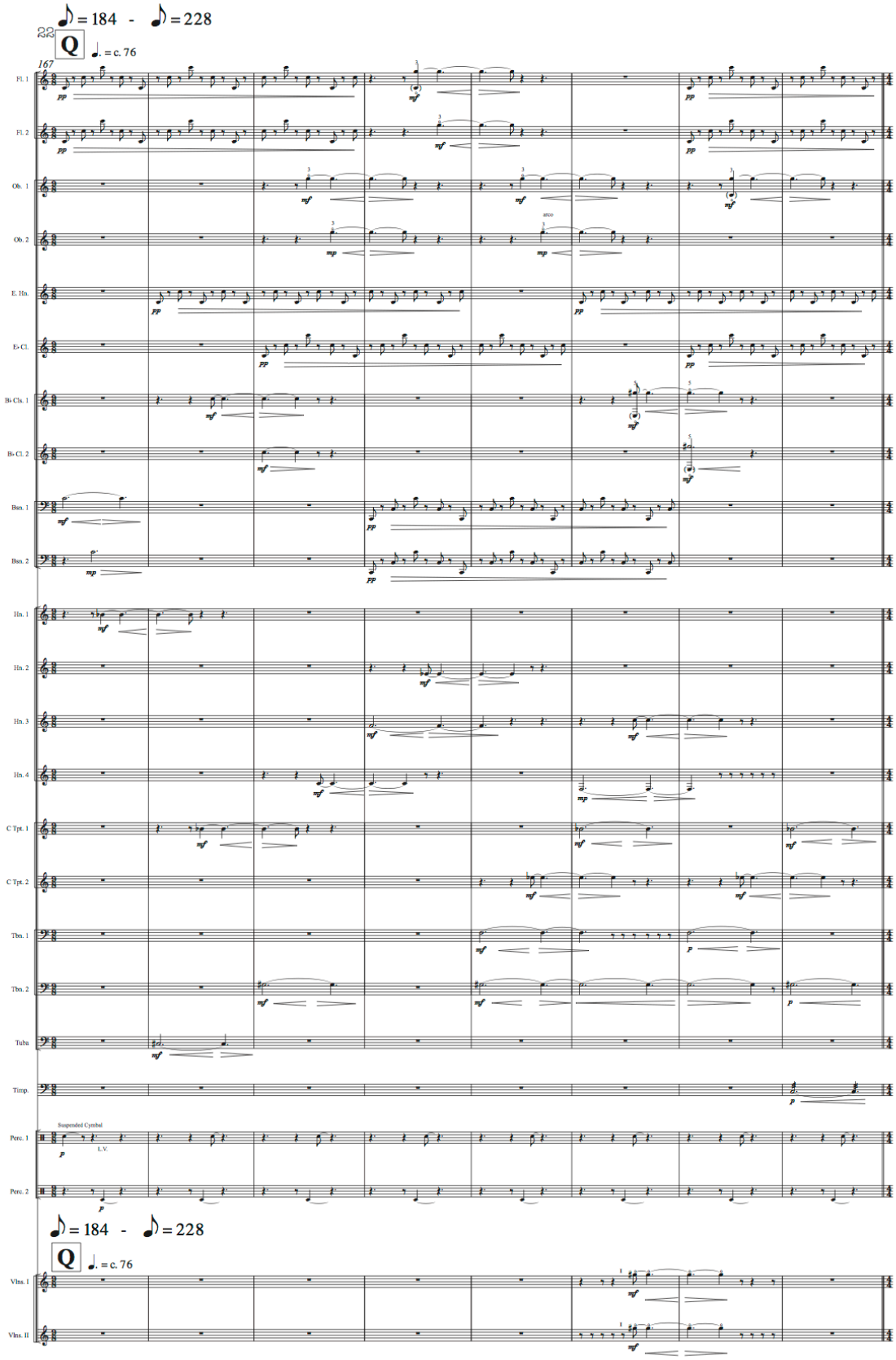

♪ = 168 - ♪ = 184
L
123
♩ = c. 92
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Timp.
Perc. 1
Suspended Cymbal
L.V.
Perc. 2
Triangle, open
♪ = 168 - ♪ = 184
L
♩ = c. 92
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
pizz.
solo
arco
col legno

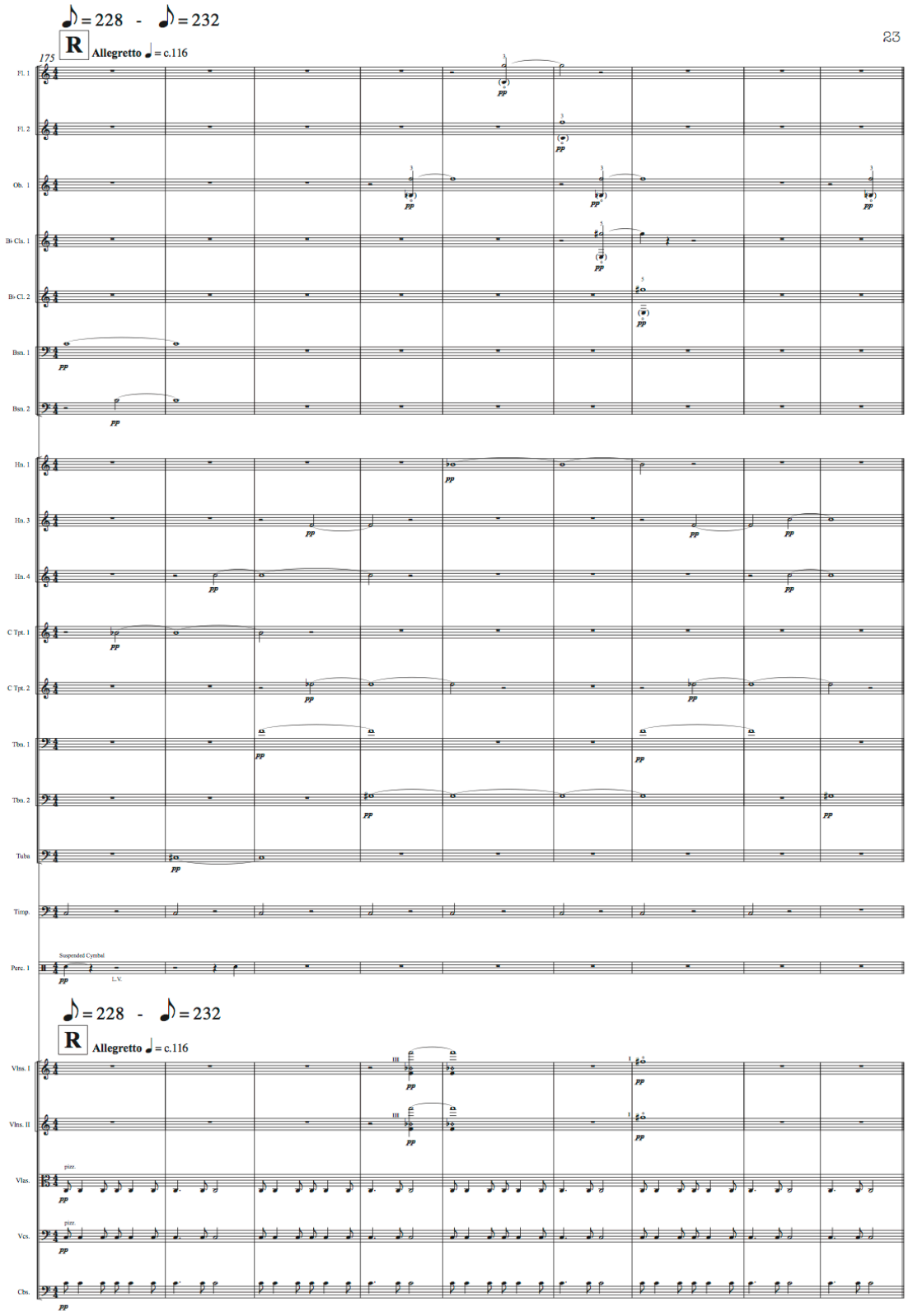

18
M
131
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Perc. 2
Triangle, open
M
Vlns. I
Vlns. II
Vlns.
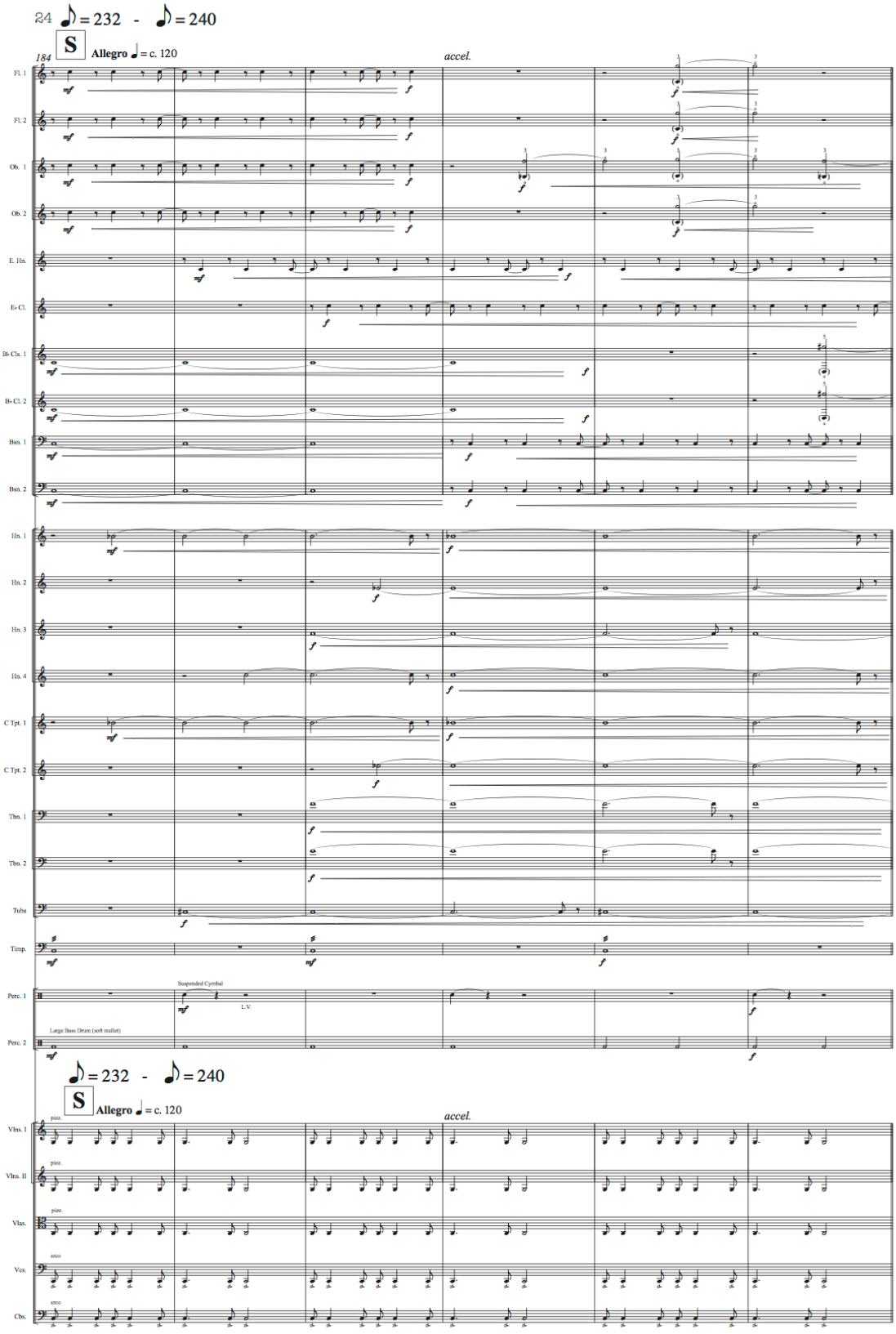

N
139
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Large Gong
L. V.
Perc. 1
Triangle, open
Large Bass Drum (soft mallet)
Suspended Cymbal
Triangle, open
Perc. 2
N
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
pizz.
arco
Unis.
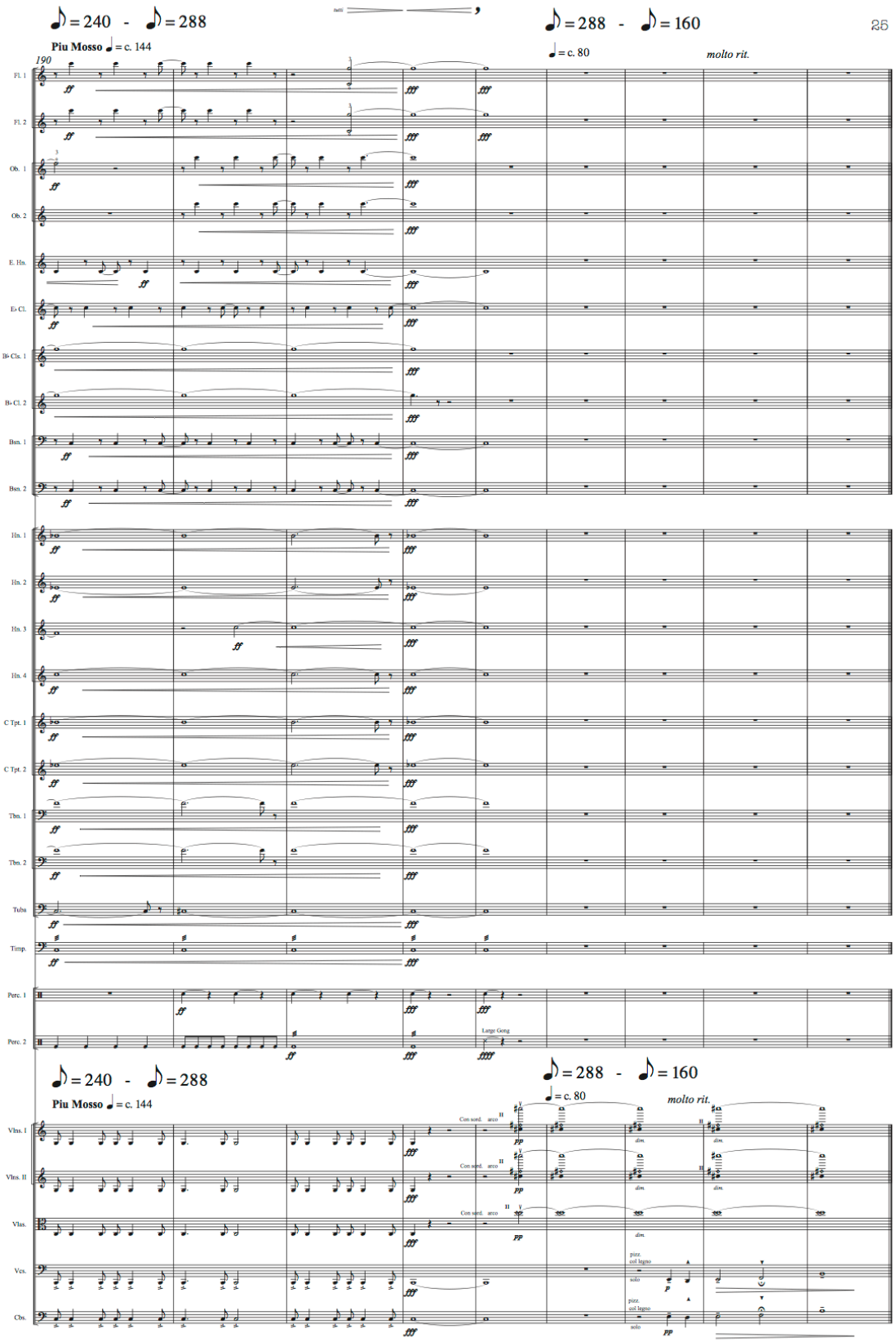

20
149
O
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tuba
Timp.
Perc. 1
Suspended Cymbal
L.V.
Perc. 2
Triangle, open
O
Vlns. I
Vlns. II
Vcs.
Cbs.


P
158
21
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Perc. 1
Perc. 2
Suspended Cymbal
L.V.
Triangle, open
P
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
arco
pizz.


♪ = 184 - ♪ = 228
22
Q
♩. = c. 76
167
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Suspended Cymbal
L.V.
Perc. 1
Perc. 2
♪ = 184 - ♪ = 228
Q
♩. = c. 76
Vlns. I
Vlns. II


♪ = 228 - ♪ = 232
R
Allegretto ♩ = c.116
175
23
Fl. 1
Fl. 2
Ob. 1
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Perc. 1
Suspended Cymbal
L.V.
♪ = 228 - ♪ = 232
R
Allegretto ♩ = c.116
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
pizz.
pizz.


24
♪ = 232 - ♪ = 240
S
Allegro ♩ = c. 120
184
accel.
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Perc. 1
Suspended Cymbal
L.V.
Perc. 2
Large Bass Drum (soft mallet)
♪ = 232 - ♪ = 240
S
Allegro ♩ = c. 120
accel.
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
pizz.
arco


♪ = 240 - ♪ = 288
Piu Mosso ♩ = c. 144
190
♪ = 288 - ♪ = 160
♩ = c. 80
molto rit.
25
Fl. 1
Fl. 2
Ob. 1
Ob. 2
E. Hn.
E♭ Cl.
B♭ Cls. 1
B♭ Cl. 2
Bsn. 1
Bsn. 2
Hn. 1
Hn. 2
Hn. 3
Hn. 4
C Tpt. 1
C Tpt. 2
Tbn. 1
Tbn. 2
Tuba
Timp.
Perc. 1
Perc. 2
Large Gong
Vlns. I
Vlns. II
Vlas.
Vcs.
Cbs.
Con sord.
arco
dim.
pizz.
col legno
solo

3.2 *The tale of Kiều* for scordatura string orchestra

The tale of Kiều

for scordatura string orchestra

Based on the poem Truyện Kiều by Nguyễn Du

Dylan Crismani

2

Tuning I

In ratios:

Violins 1 & 2
I 81:64
II 27:16
III 9:8
IV 3:2

Violins 3 & 4
I 729:512
II 243:128
III 81:64
IV 27:16

Violins 5 & 6
I 19683:16384
II 6561:4096
III 2187:2048
IV 729:512

Violins 7 & 8
I 177147:131072
II 59049:32768
III 19683:16384
IV 6561:4096

Violins 9, 10, 11 & 12
I 4782969:4194304
II 1594323:1048576
III 531441:524288
IV 177147:131072

Violas 1 & 2
I 27:16
II 9:8
III 3:2
IV 1:1

Violas 3 & 4
I 59049:32768
II 19683:16384
III 6561:4096
IV 2187:2048

Cellos 1 & 2
I 27:16
II 9:8
III 3:2
IV 1:1

Cellos 3 & 4
I 6561:4096
II 2187:2048
III 729:512
IV 243:128

Contrabass I
I 3:2
II 9:8
III 27:16
IV 81:64

Contrabass II
I 6561:4096
II 19683:16384
III 59049:32768
IV 177147:131072

In hertz:

Violins 1 & 2
I 662.28
II 441.52
III 294.35
IV 196.23

Violins 3 & 4
I 745.06
II 496.71
III 331.14
IV 220.75

Violins 5 & 6
I 628.65
II 419.1
III 279.4
IV 186.27

Violins 7 & 8
I 707.23
II 471.48
III 314.32
IV 209.55

Violins 9, 10, 11 & 12
I 596.72
II 397.81
III 265.21
IV 176.81

Violas 1 & 2
I 441.52
II 294.35
III 196.23
IV 130.82

Violas 3 & 4
I 471.48
II 314.32
III 209.55
IV 139.7

Cellos 1 & 2
I 220.76
II 147.18
III 98.12
IV 65.41

Cellos 3 & 4
I 209.55
II 139.7
III 93.12
IV 62.09

Contrabass 1
I 98.12
II 73.59
III 55.19
IV 41.39

Contrabass II
I 104.77
II 78.58
III 58.94
IV 44.20

Letter names:

Violins 1 & 2
I E
II A
III D
IV G

Violins 3 & 4
I F#
II B
III E
IV A

Violins 5 & 6
I D#
II G#
III C#
IV F#

Violins 7 & 8
I E#
II A#
III D#
IV G#

Violins 9, 10, 11 & 12
I D
II G
III C
IV E#

Violas 1 & 2
I A
II D
III G
IV C

Violas 3 & 4
I A#
II D#
III G#
IV C#

Cellos 1 & 2
I A
II D
III G
IV C

Cellos 3 & 4
I G#
II C#
III F#
IV B

Contrabass 1
I G
II D
III A
IV E

Contrabass II
I G#
II D#
III A#
IV E#

Tuning II

The relationship of a perfect fifth between strings IV to III, III to II, and II to I is retained for each instrumental table in this scordatura, as is the relationship of a perfect fourth for the two contrabasses. With this in mind, each table can either choose to re-tune each string using an electronic tuner, or re-tune string IV with an electronic tuner and then re-tune the remaining strings by ear.

Premise

The premise underlying this piece is that the 46-pitch just intonation tonnetz shown on the following page can be achieved by an orchestra composed of conventional instrumentalists, with no specialisation in the theory or practice of just intonation. Provided that the orchestra are willing to re-tune according to the specified scordatura, and that each instrumentalist can play up to the eighth harmonic on each string of their instrument, then the tuning model illustrated on the next page is theoretically possible.
There are a number of variables which will cause the piece to never be 100% intonationally accurate, such as string width, length, elasticity, bow position, and pressure. Despite this, 95% accuracy may be attainable, and in any case 100% accuracy is not the true goal of this piece of music. It's true goal is to open the door to new sounds, the theory of just intonation is simply the vehicle which can help this piece of music arrive at its destination.

Notation

The notation for this piece of music has been written in two ways. The first is the approximate sounding pitches. The second is as node points, or where the players should touch the strings in order for the sounding pitches to sound correctly. The node points have been scored as a kind of tablature, as the observant reader will quickly realise. The intention is that despite the scordatura tuning, the players will read the score as if no re-tuning has taken place. This treatment essentially turns a very large portion of the orchestra into transposing instruments, where the pitches written are not the same as those that will sound at 'concert pitch', so the notation of harmonics is simply a guide of where to place one's fingers.
In the interest of producing a clean, neat and readable score, no microtonal alterations such as exotic accidentals, or cent deviations have been included in the notation. Therefore the notated sounding pitches are approximations only, they will not sound exactly as they appear, in many cases they will sound rather sharper or flatter than notated.

Performance notes

Diamond shaped note heads indicate where one should place one's finger on the string, as if the string is in its normal tuning.

O.P. indicates that particular note is to be played with over pressure, creating more noise and texture than pitch.

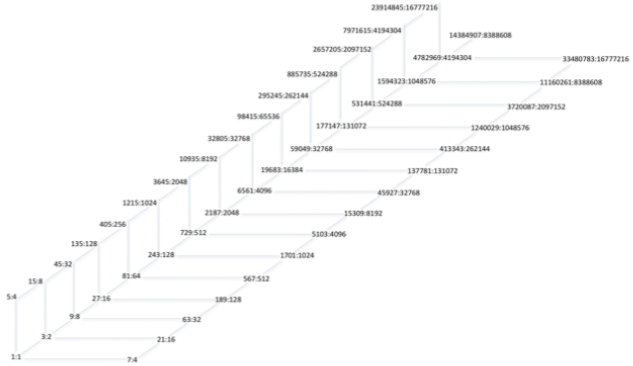
Norm indicates that the player should return to a normal level of bow pressure.

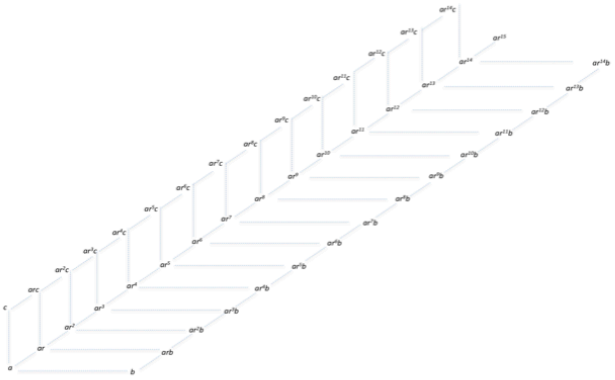
Glissandi that move up or down to a comma indicates that the glissando should end not necessarily on a pitched sound, but simply until there is no more string left to move on.

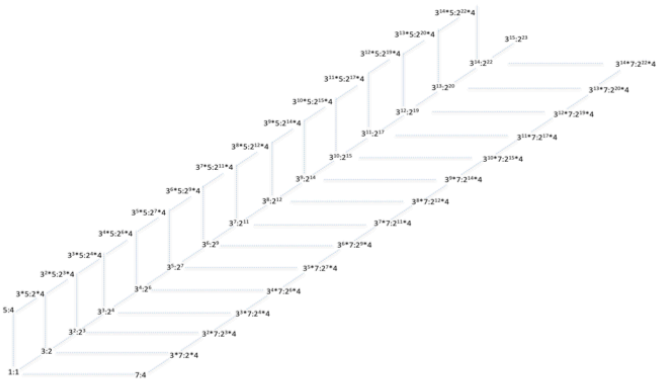
Ord. indicates a return to normal bow position afer being either sul pont. or flautando.

4

Lattice


23914845:16777216
7971615:4194304
14384907:8388608
2657205:2097152
4782969:4194304
33480783:16777216
885735:524288
1594323:1048576
11160261:8388608
295245:262144
531441:524288
3720087:2097152
98415:65536
177147:131072
1240029:1048576
32805:32768
59049:32768
413343:262144
10935:8192
19683:16384
137781:131072
3645:2048
6561:4096
45927:32768
1215:1024
2187:2048
15309:8192
405:256
729:512
5103:4096
135:128
243:128
1701:1024
45:32
81:64
567:512
15:8
27:16
189:128
5:4
9:8
63:32
3:2
21:16
1:1
7:4


c
arc
ar
a
b
arb


5:4
1:1
3:2
7:4
9:8
63:32
21:16

Trăm năm trong cõi người ta,
Chữ tài chữ mệnh khéo là ghét nhau.
Trải qua một cuộc bể dâu,
Những điều trông thấy mà đau đớn lòng.
Lạ gì bỉ sắc tư phong,
Trời xanh quen thói má hồng đánh ghen.

寮薢齷墝鴃些
笋才笋命窖羅恄饒
硬戈沒局滂桃
仍調**鼂簣**缶**疔疸悉**
遜之彼嗇斯豐
丟撐悁貝腾紅打慳

What tragedies take place
within each circling space of years!
'Rich in good looks' appears
to mean poor luck and tears of woe;
which may sound strange, I know,
but is not really so, I swear,
since Heaven everywhere
seems jealous of the fair of face.

(English translation by Michael Counsell)

6

Orchestra

6 first violins
6 second violins
4 violas
4 cellos
2 contrabasses

The tale of Kiều

I

The grave of Đạm Tiên

Dylan Crismani

Moderato, unsettling ♩ = 30

Sounding pitches
Violins 1 & 2
Tablature

Sounding pitches
Violins 3 & 4
Tablature

Sounding pitches
Violins 5 & 6
Tablature

Sounding pitches
Violins 7 & 8
Tablature

Sounding pitches
Violins 9 & 10
Tablature

Sounding pitches
Violins 11 & 12
Tablature

Sounding pitches
Violas 1 & 2
Tablature
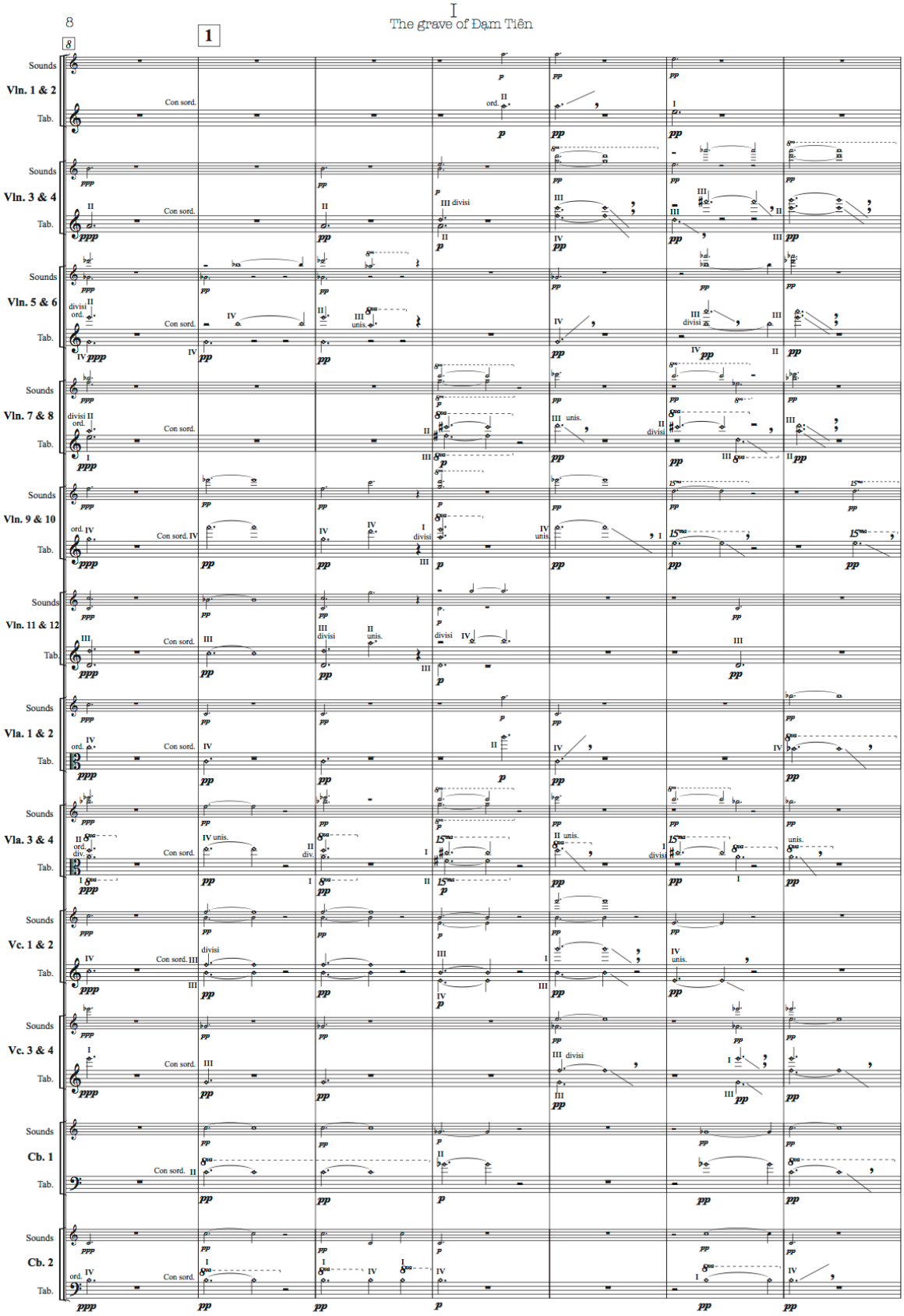
Sounding pitches
Violas 3 & 4
Tablature
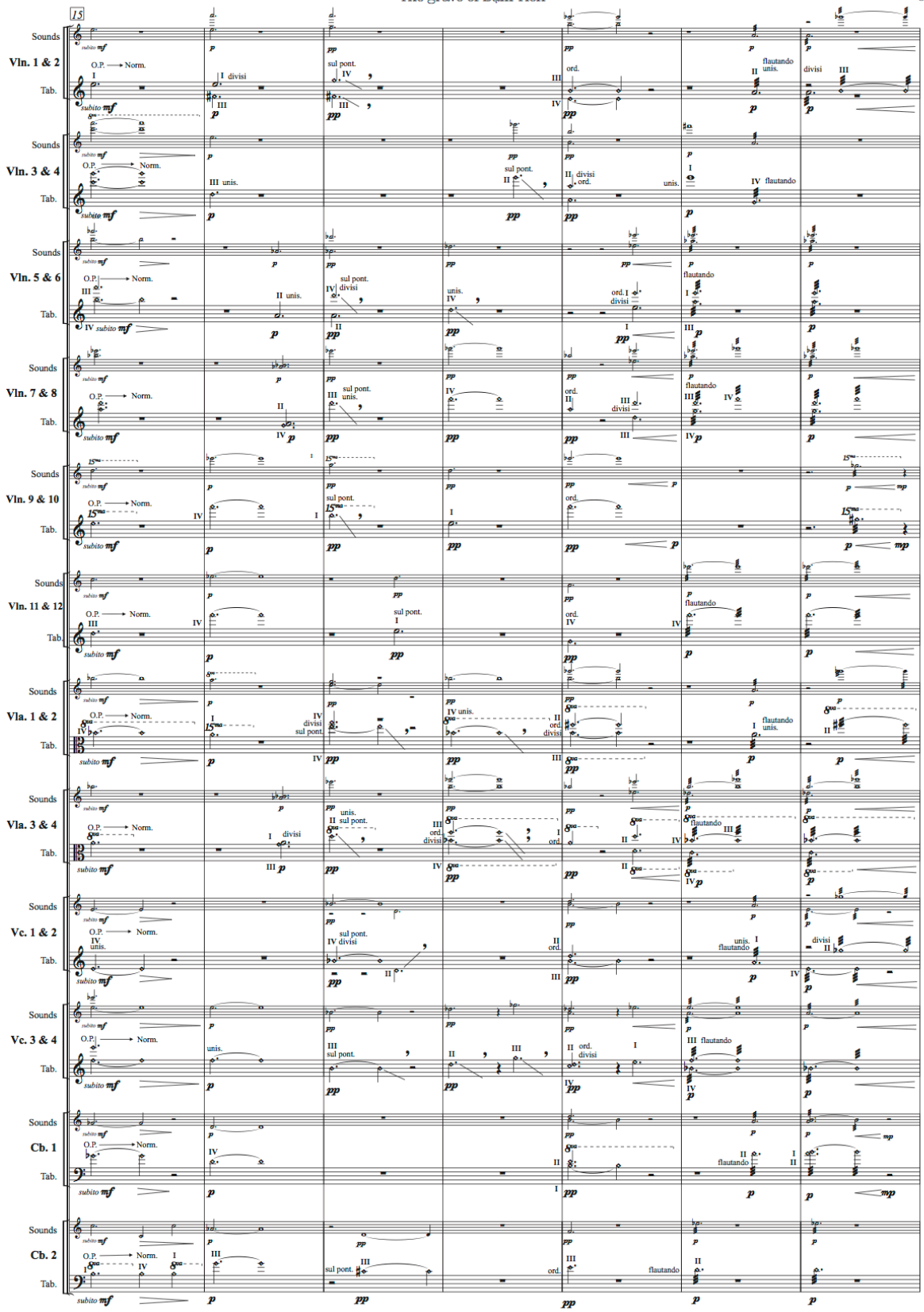
Sounding pitches
Cellos 1 & 2
Tablature

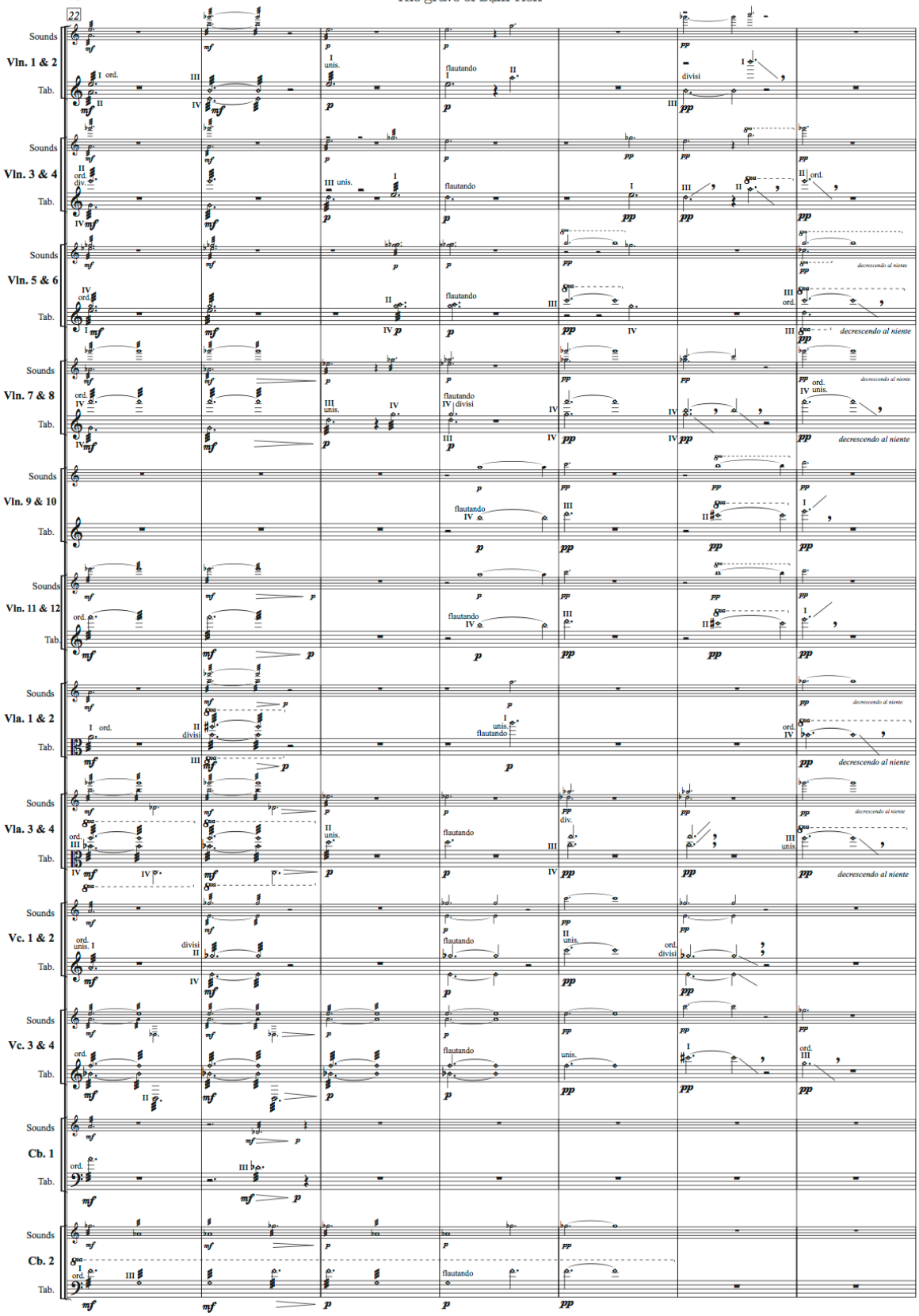
Sounding pitches
Cellos 3 & 4
Tablature
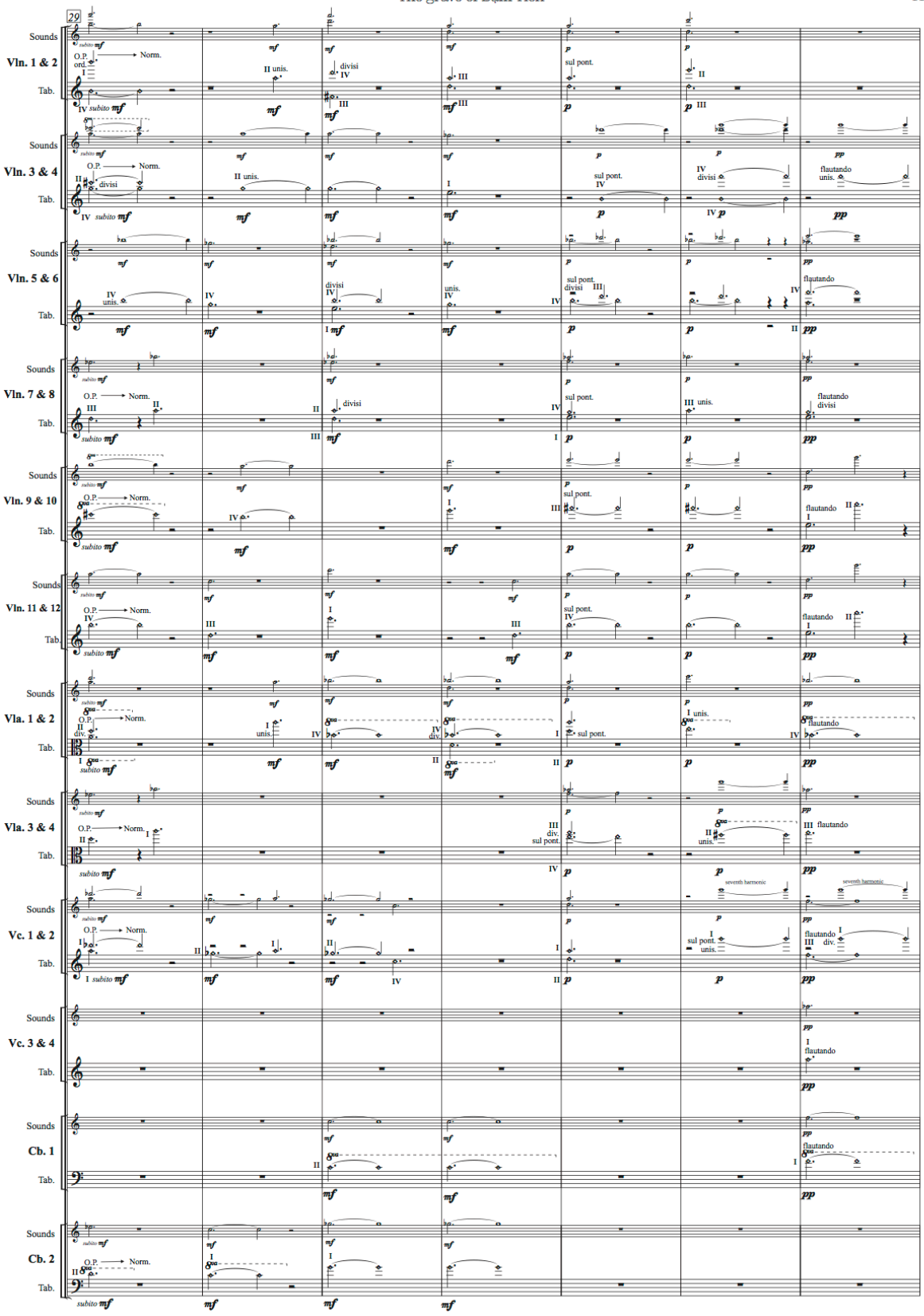
Sounding pitches
Contrabass 1
Tablature

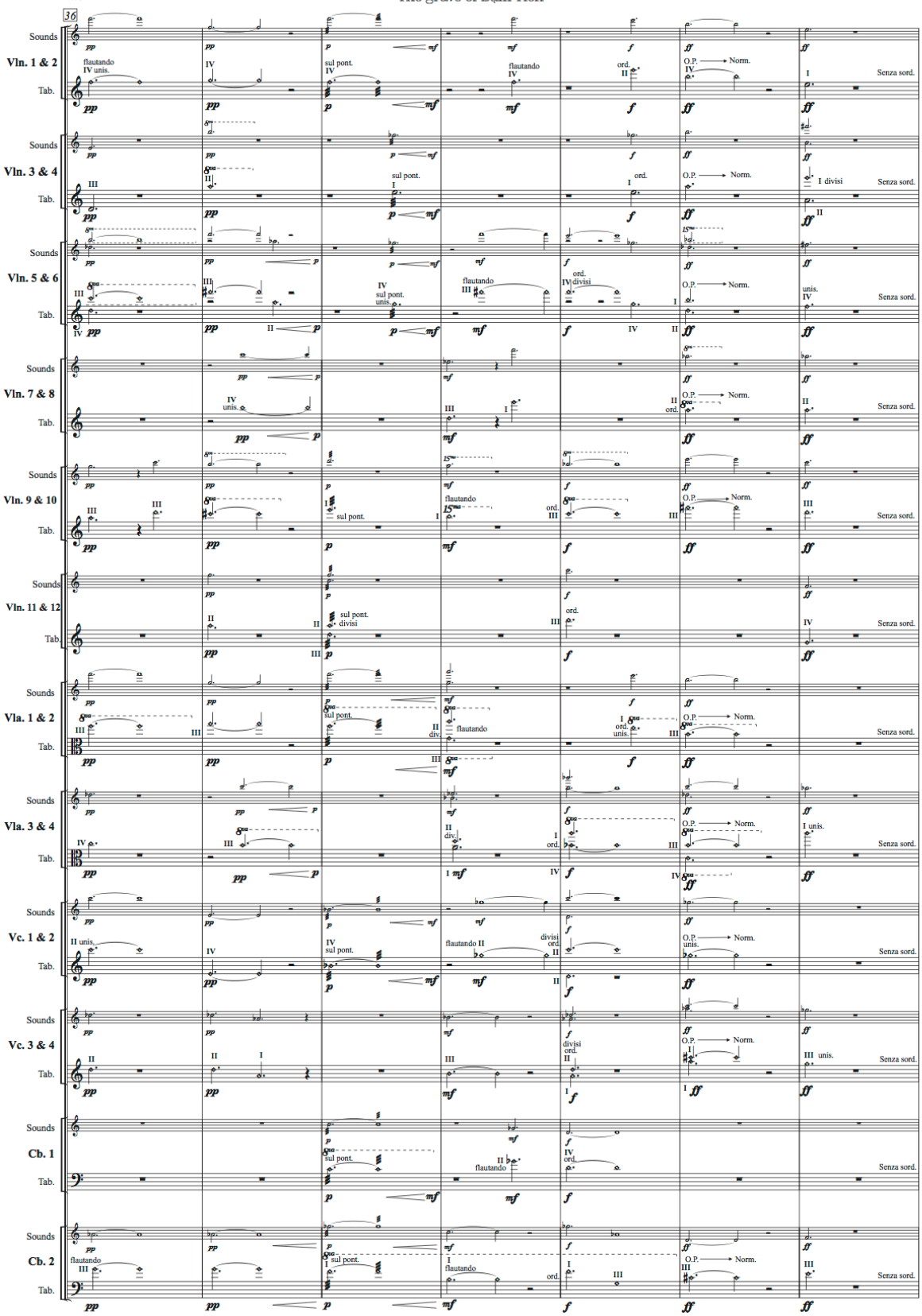
Sounding pitches
Contrabass 2
Tablature
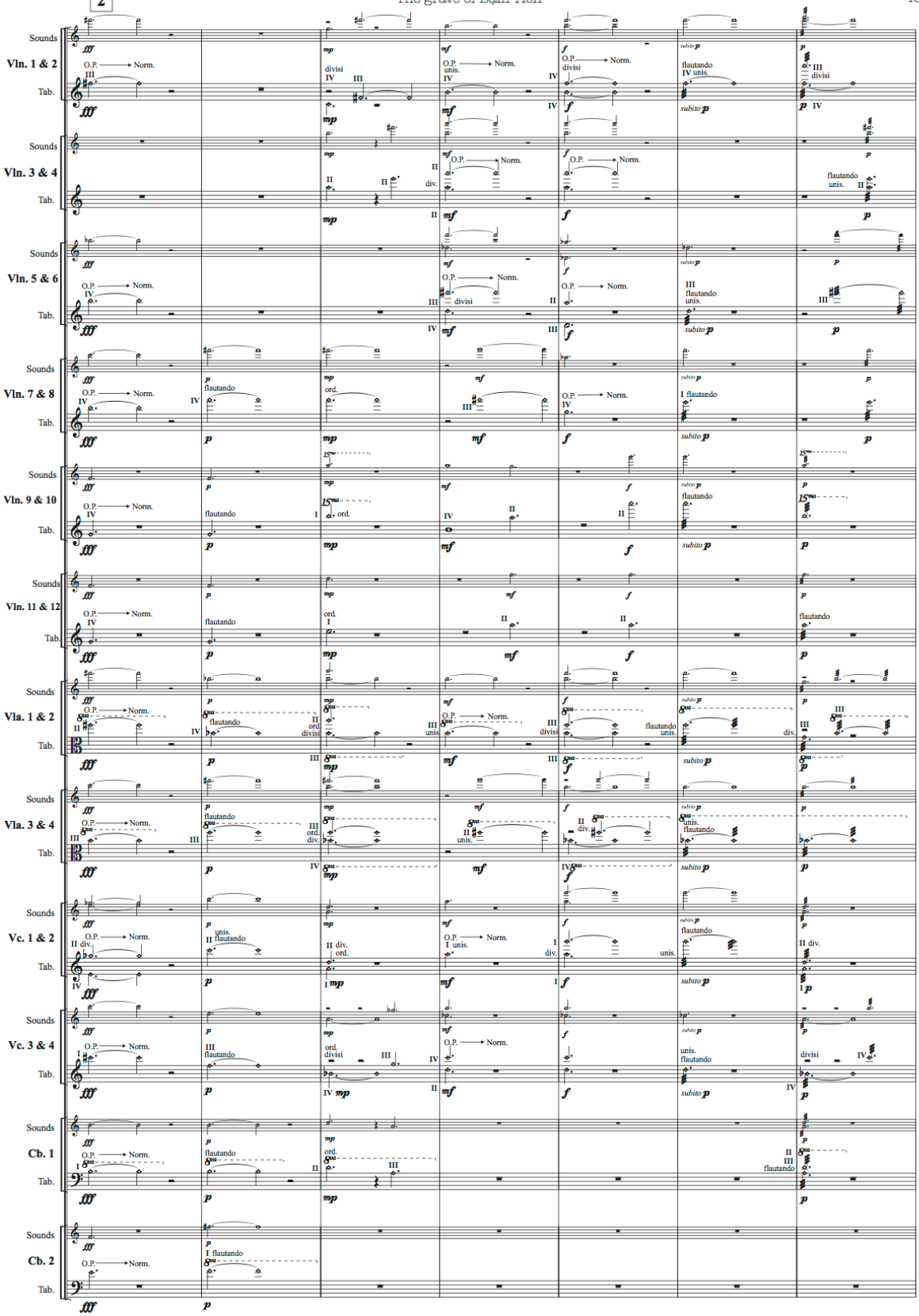

8
1
Sounds
Vln. 1 & 2
Tab.
Con sord.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2


15
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
O.P. → Norm.
subito mf
p
pp
mp
sul pont.
ord.
divisi
unis.
flautando
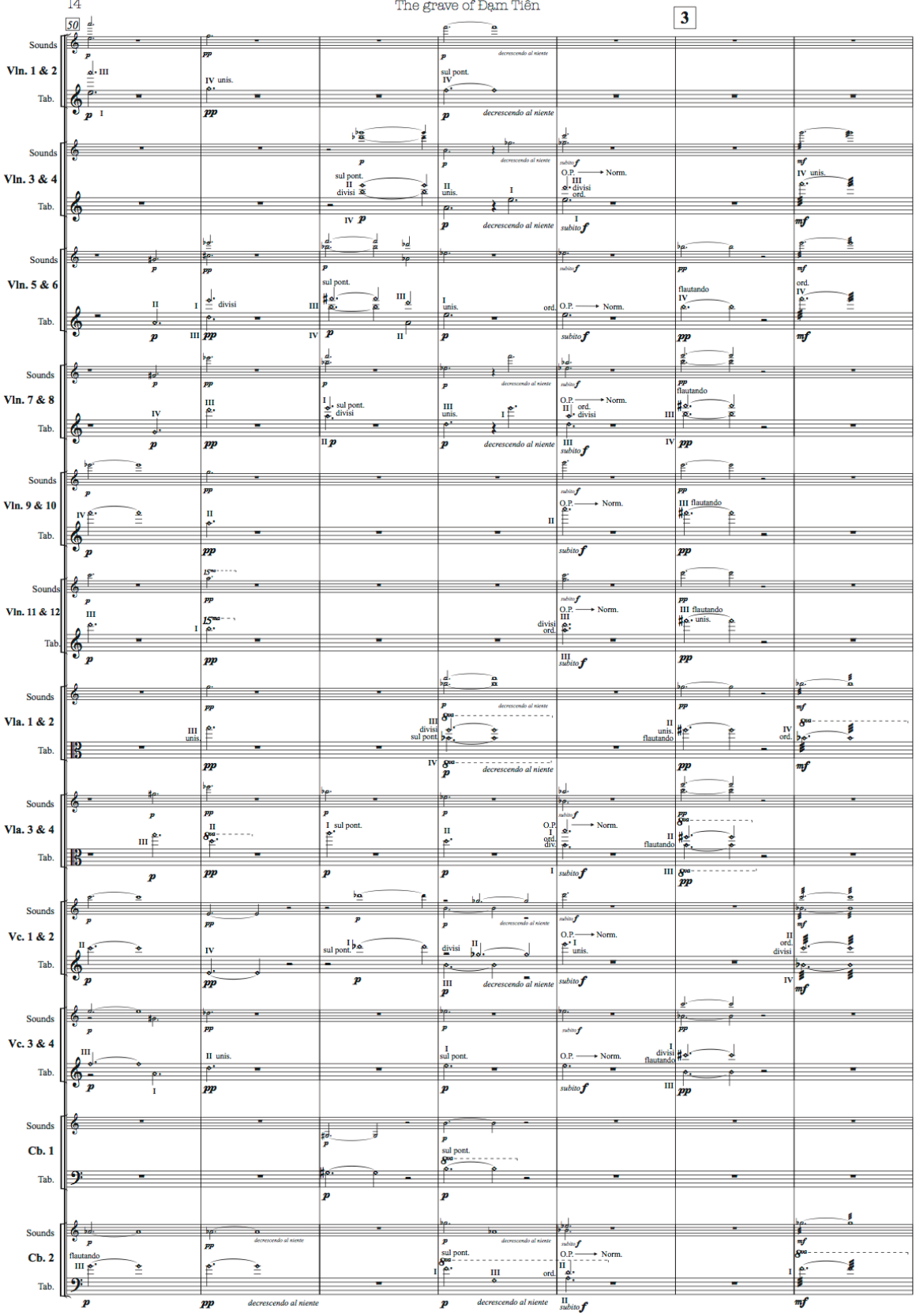

10
22
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
ord.
divisi
unis.
flautando
decrescendo al niente
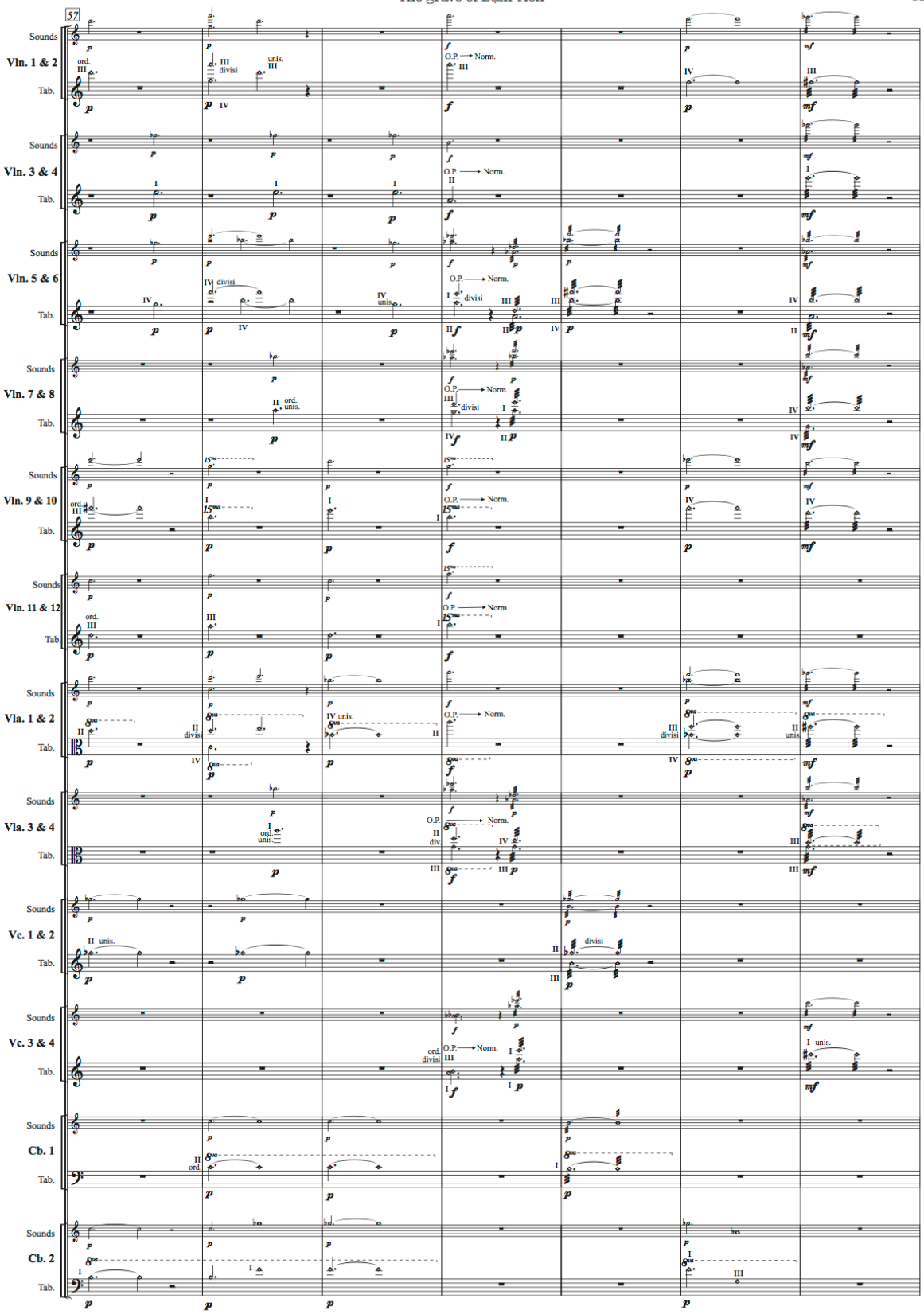

29
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
O.P. → Norm.
ord.
divisi
II unis.
sul pont.
flautando
unis.
seventh harmonic
subito mf
mf
p
pp


36
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
flautando
sul pont.
ord.
O.P. Norm.
Senza sord.
divisi
unis.


2
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
O.P. → Norm.
flautando
divisi
unis.
ord.
subito p
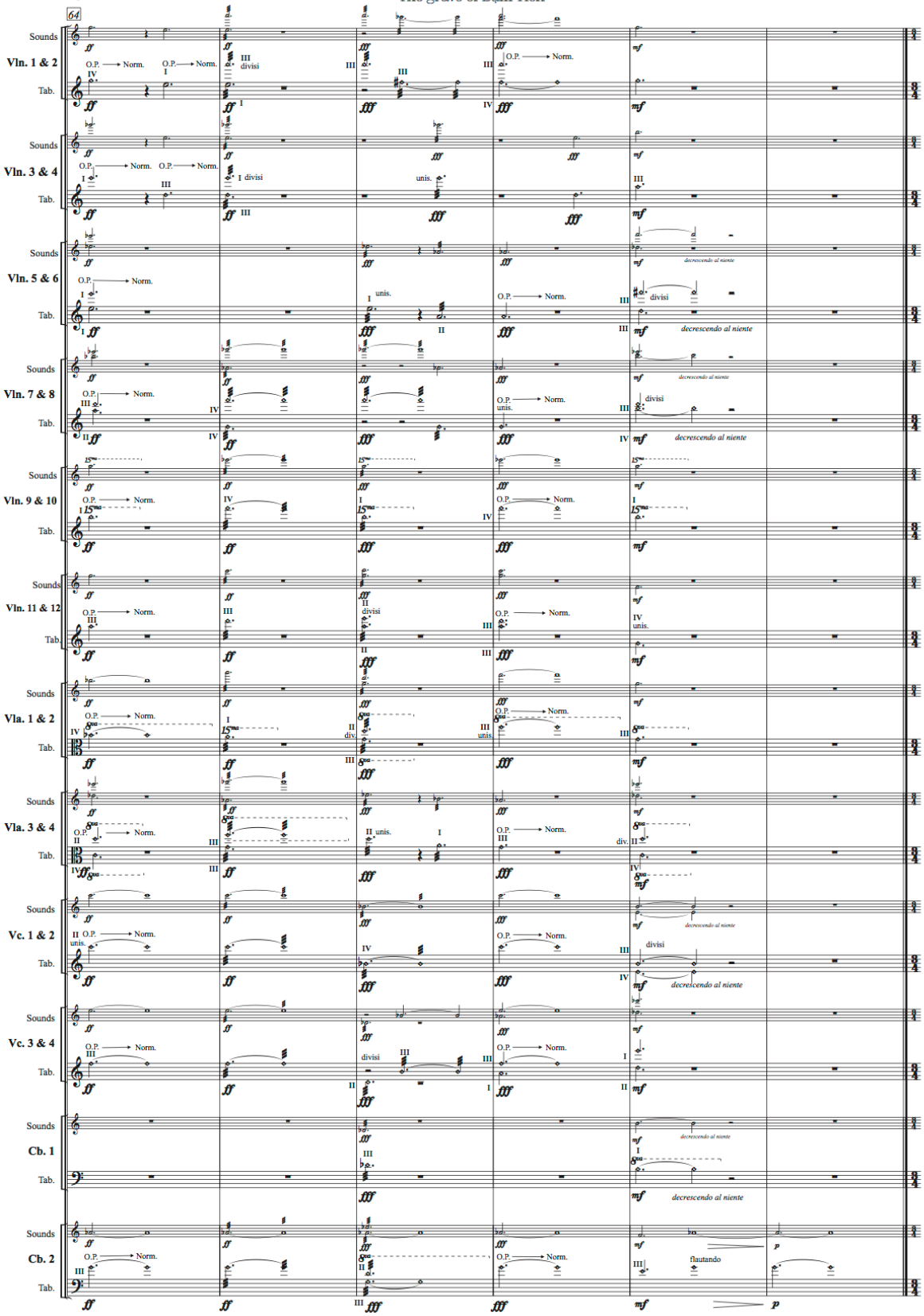
I
The grave of Đạm Tiên
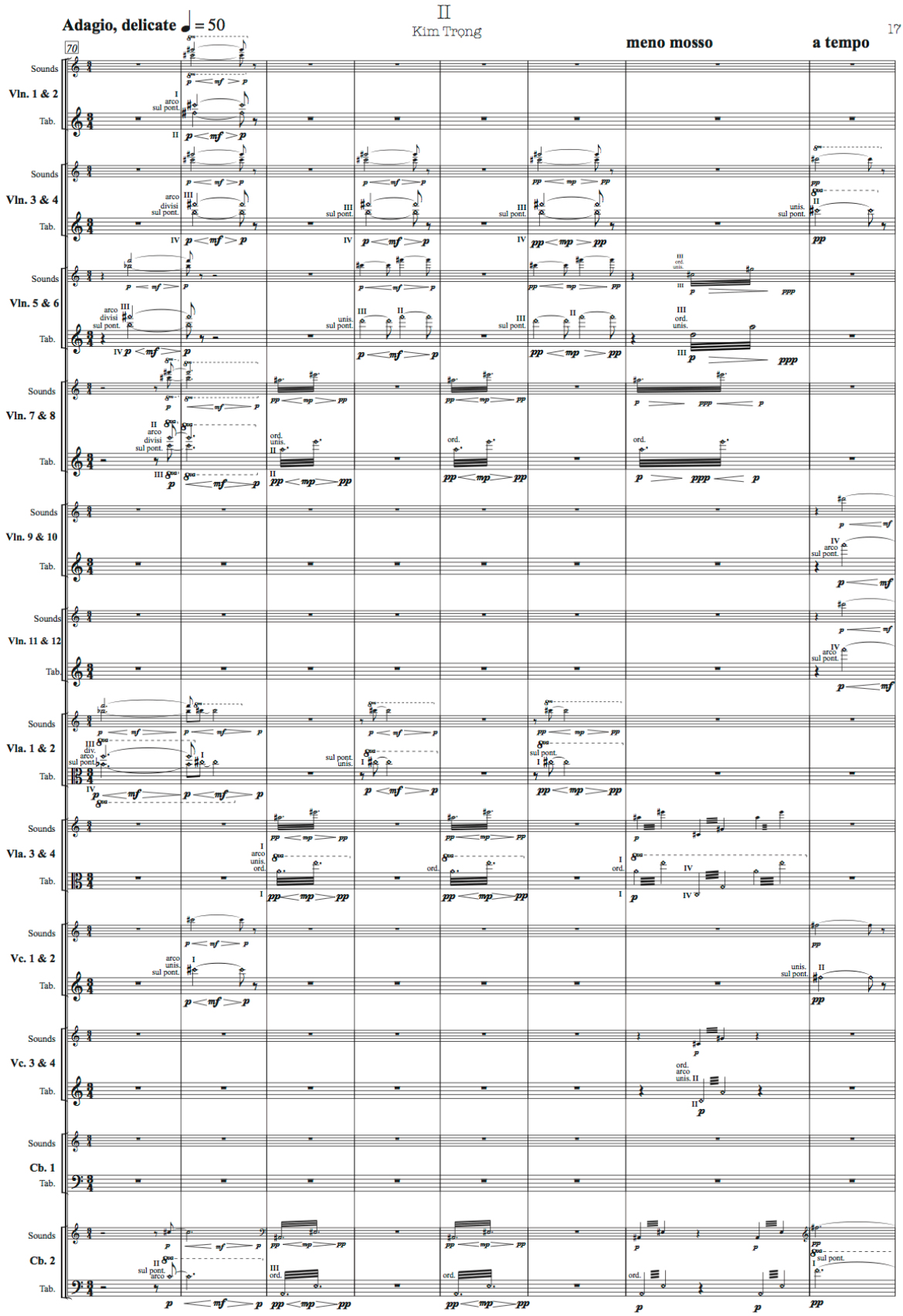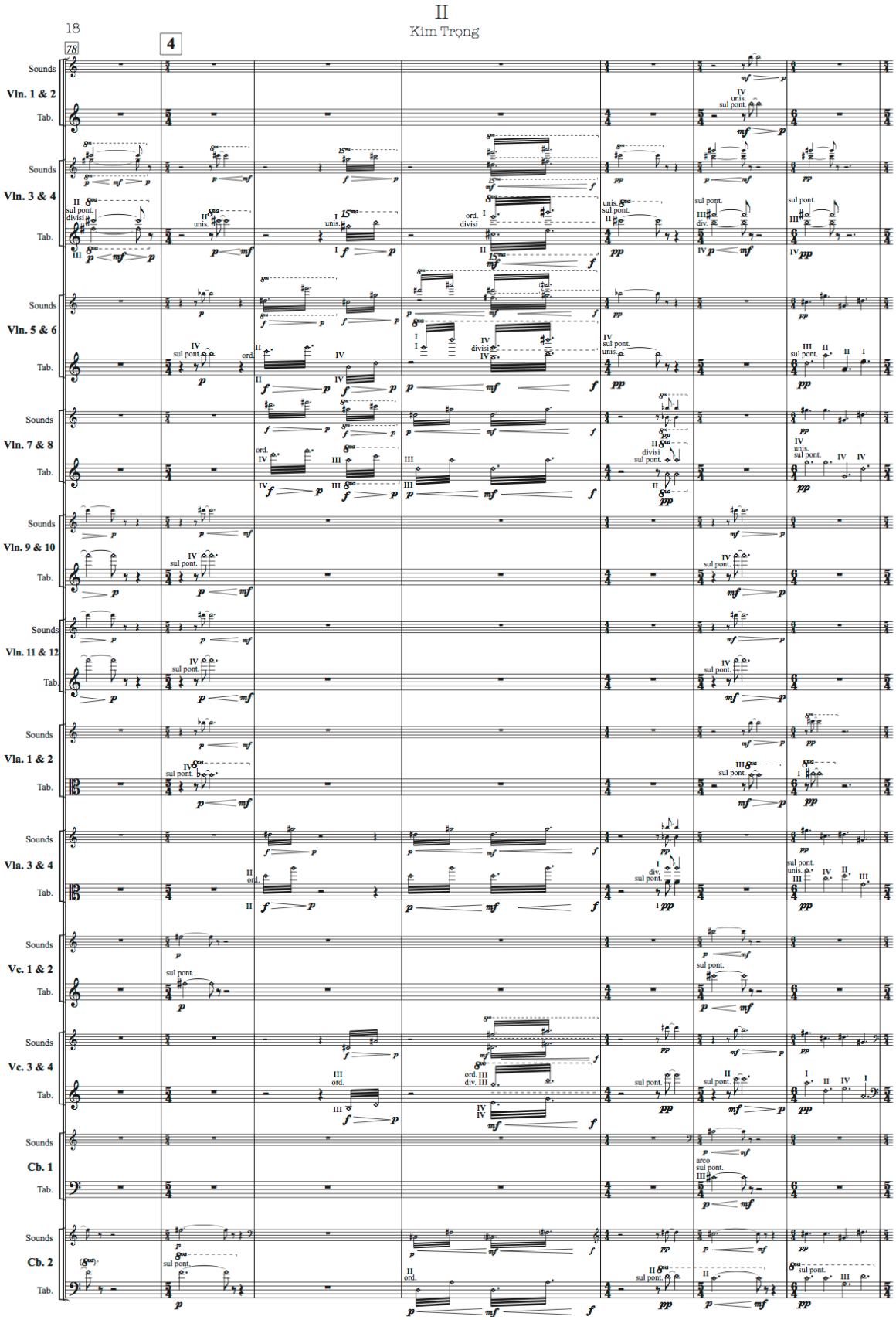



57
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
O.P. → Norm.
ord.
unis.
divisi
div.
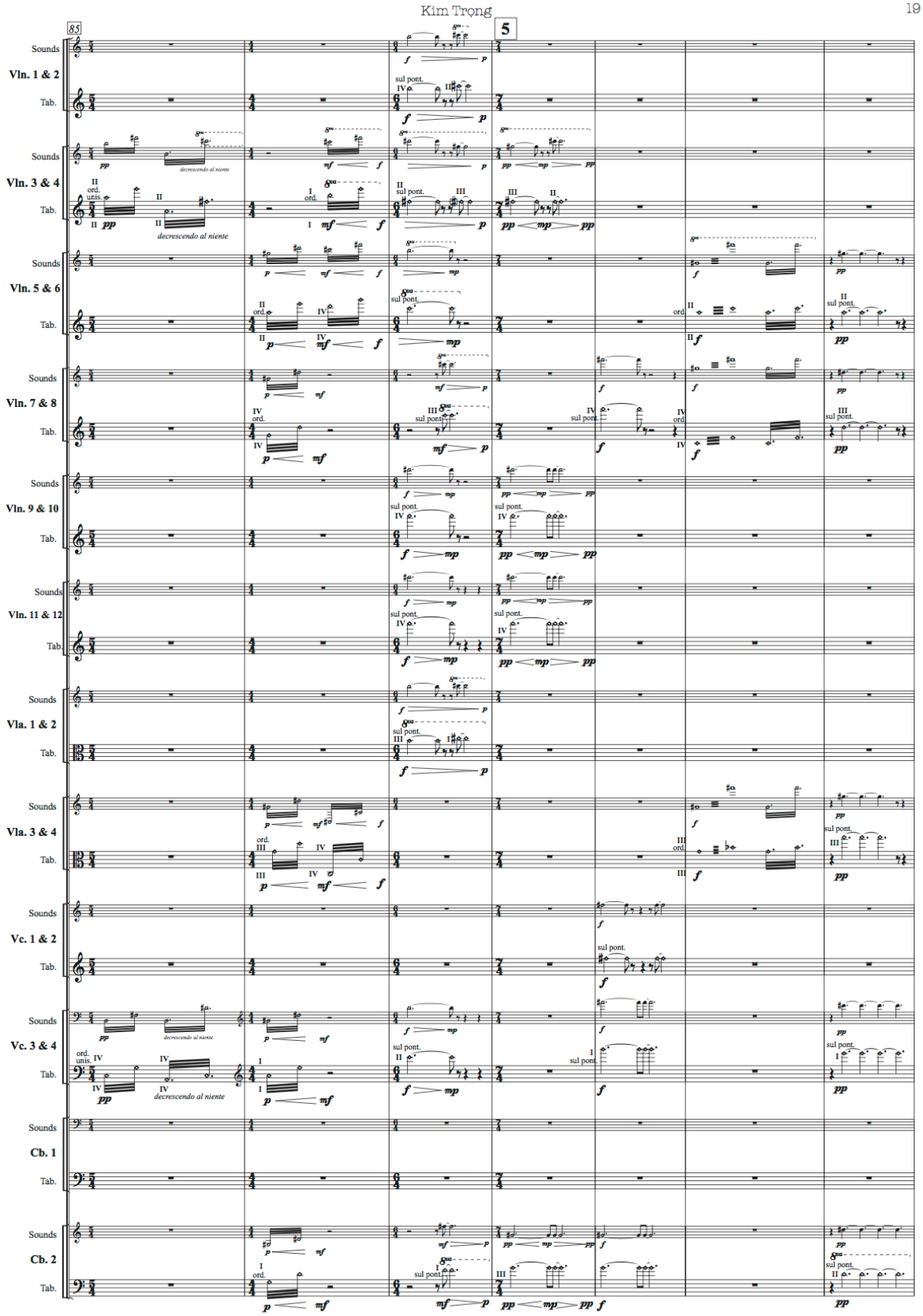
I
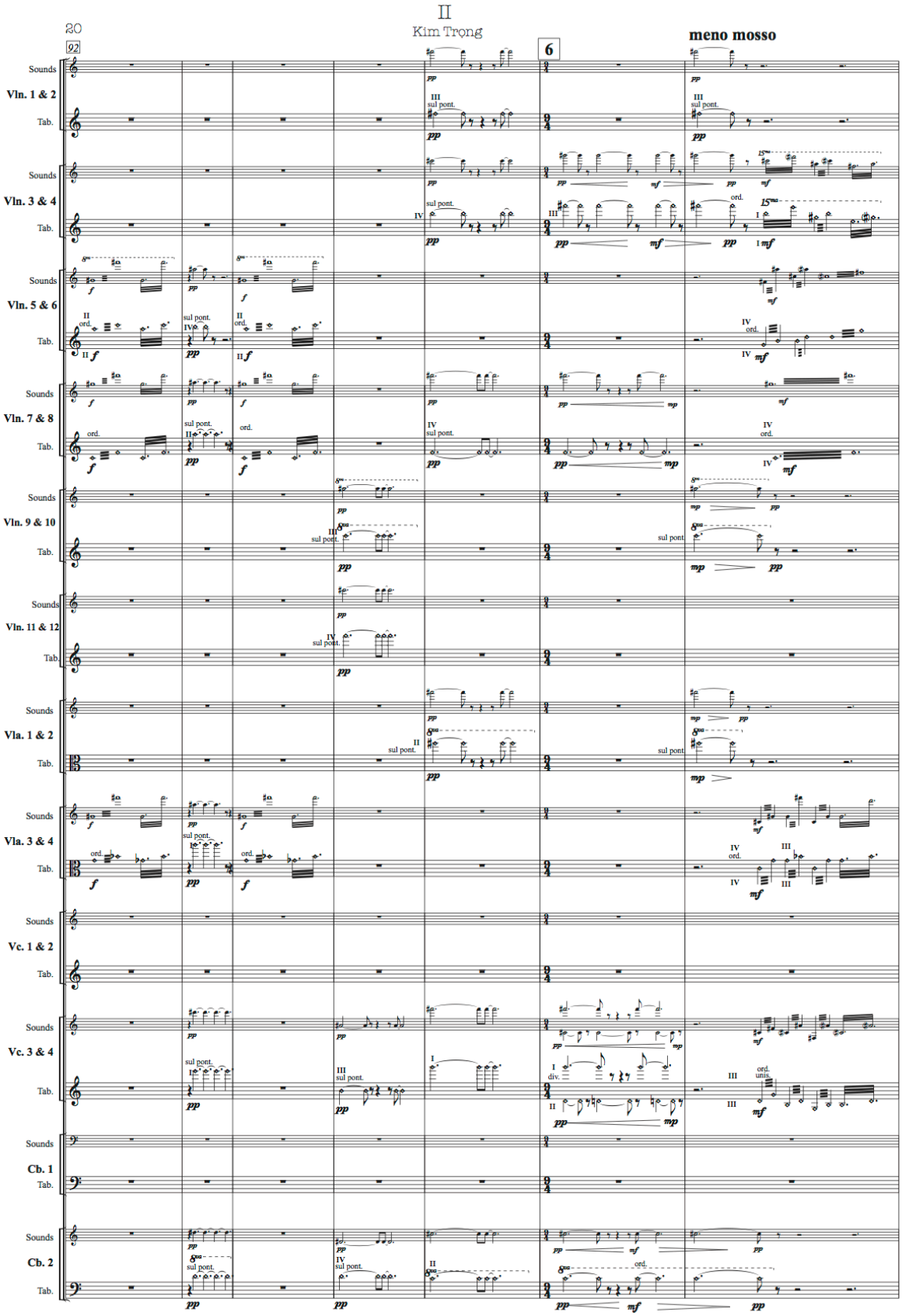
The grave of Đạm Tiên
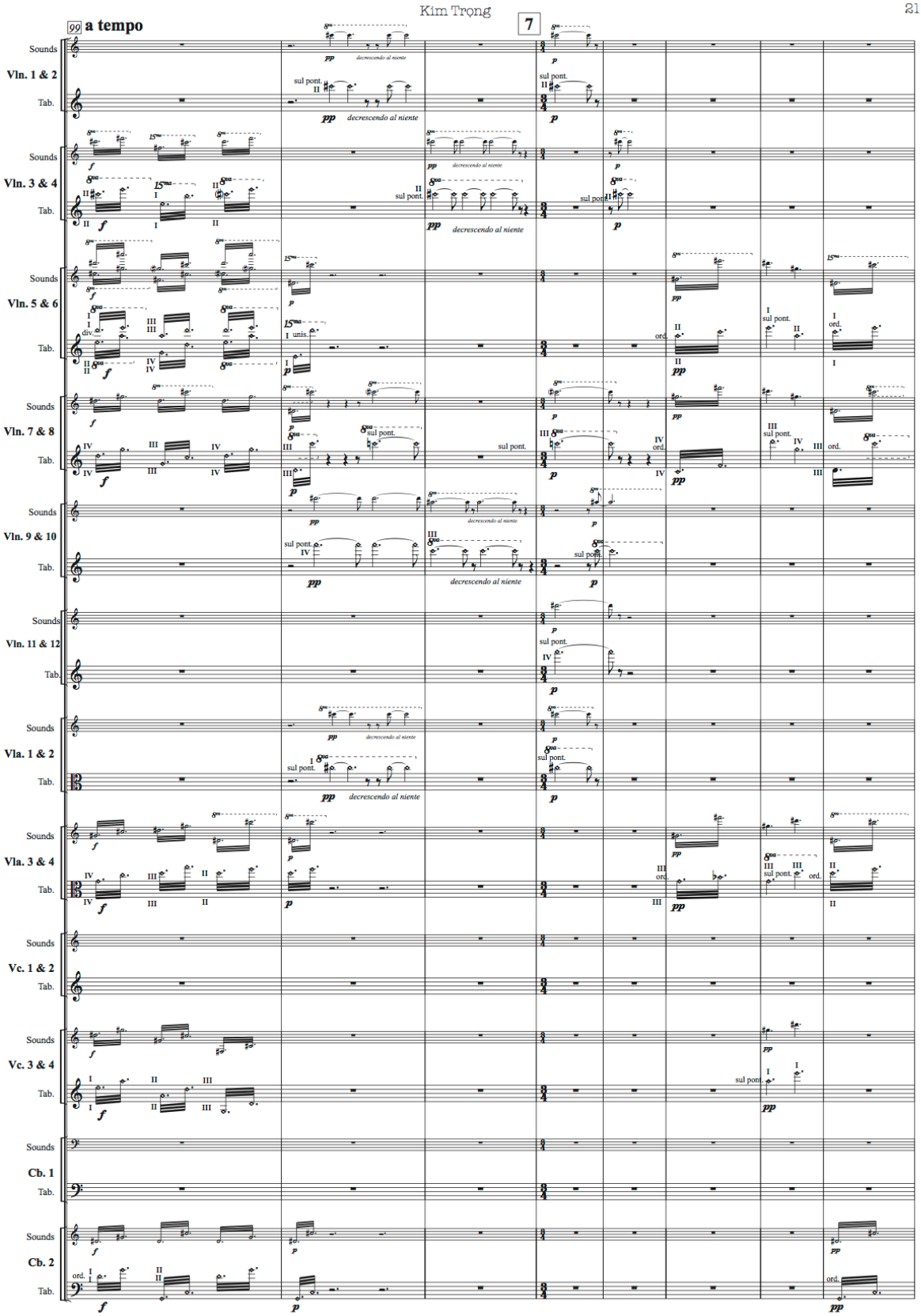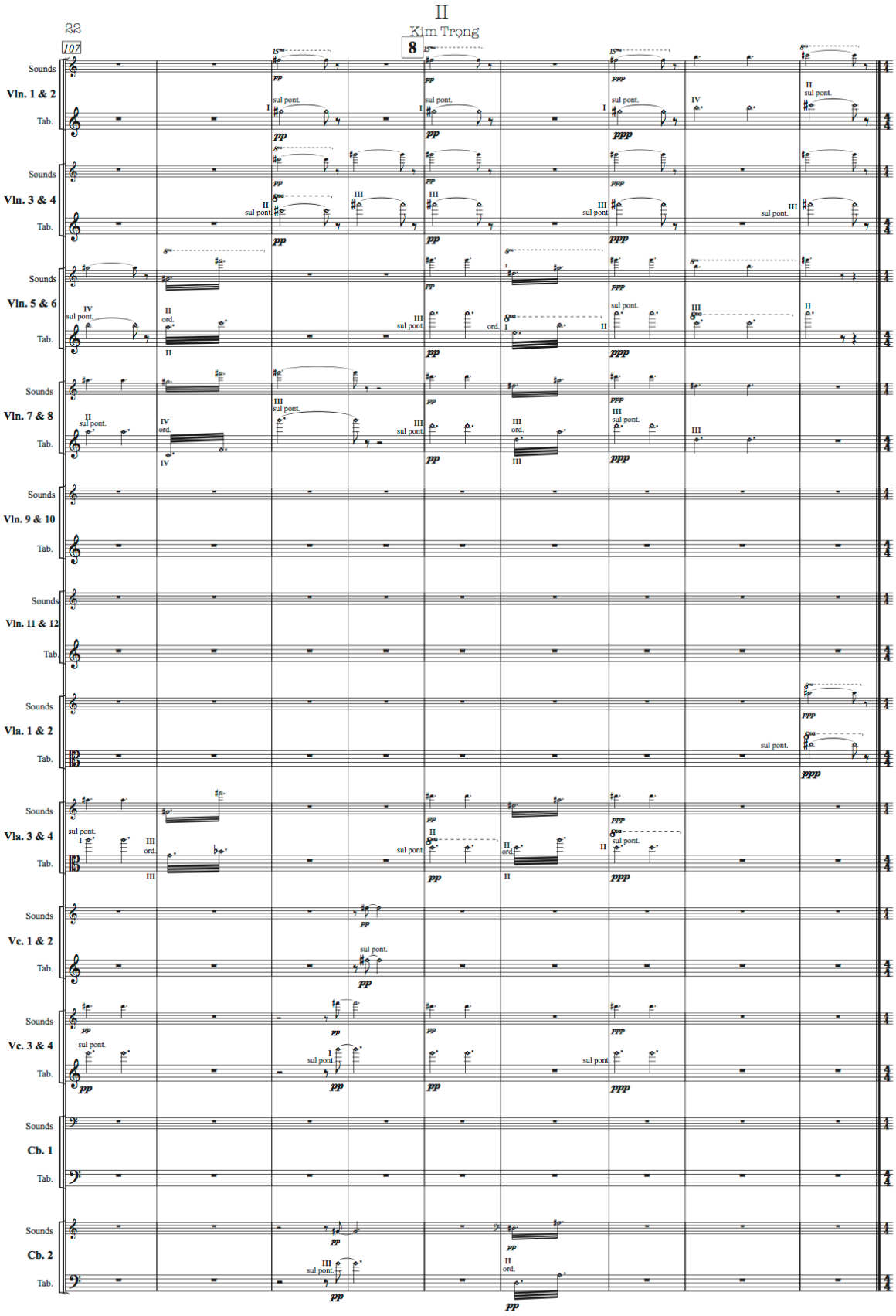



II
Kim Trọng
17
Adagio, delicate ♩ = 50
70
meno mosso
a tempo
Sounds
Vln. 1 & 2
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
Tab.
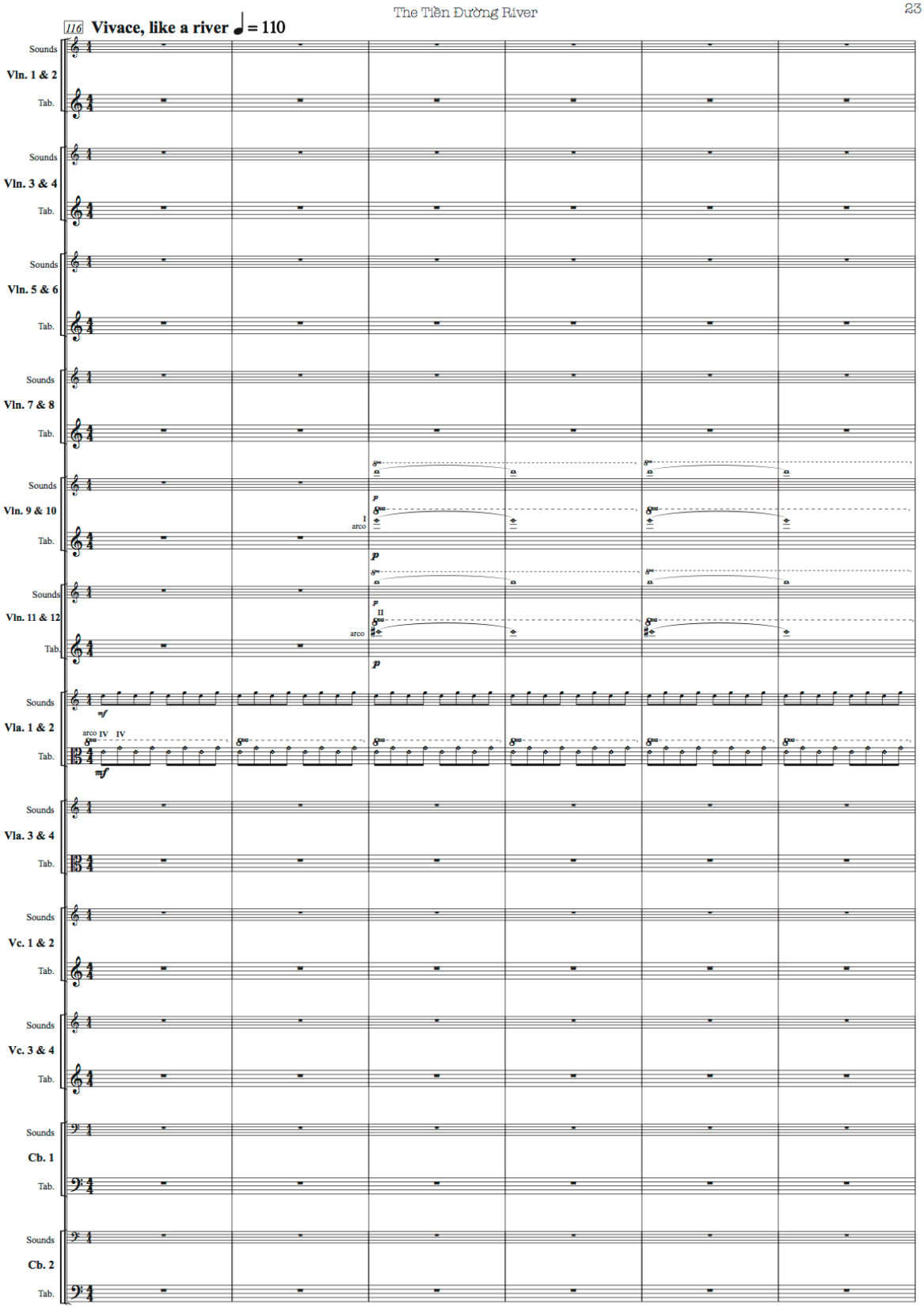

II
Kim Trọng
18
78
4
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
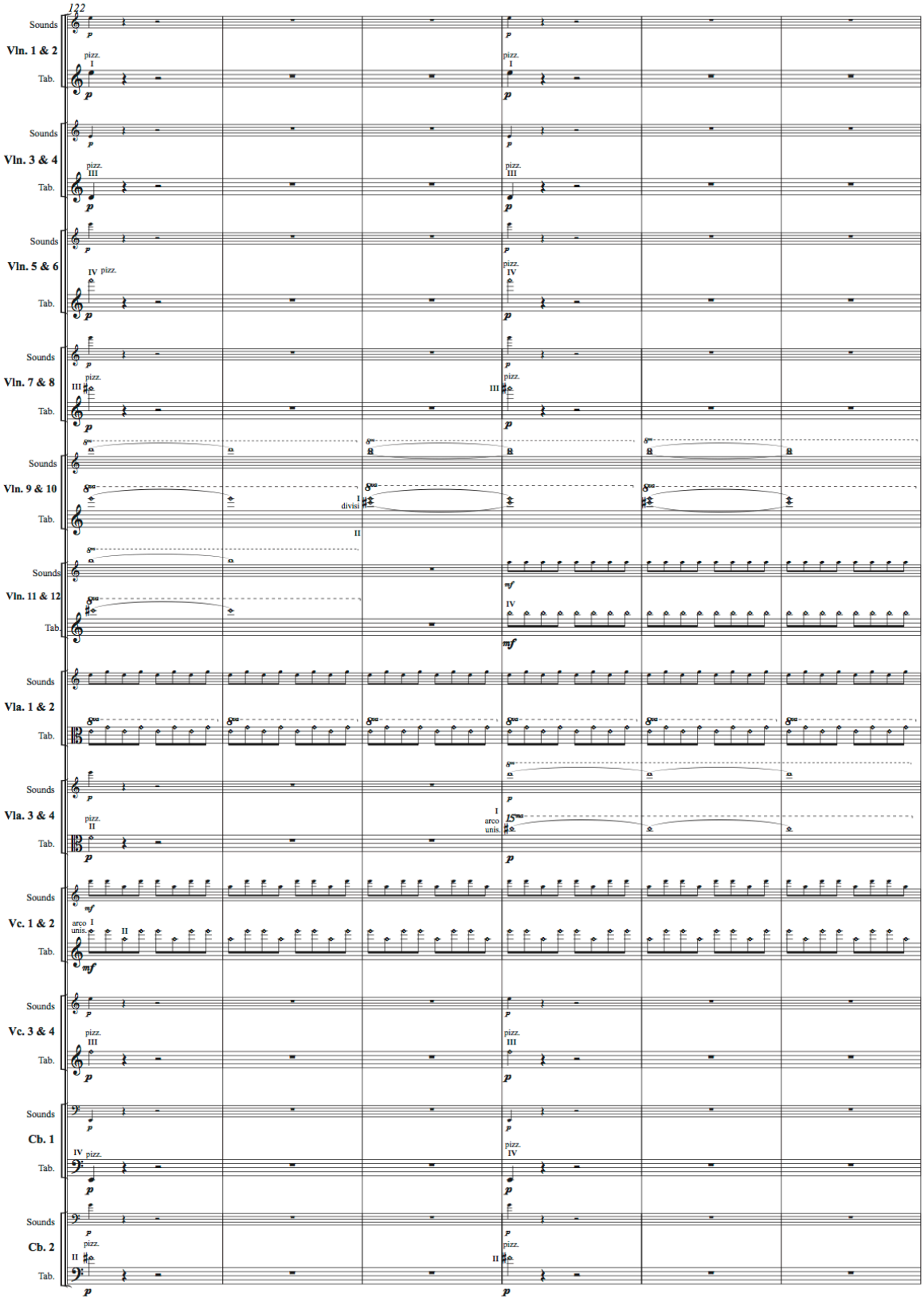

85
5
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
sul pont.
ord.
unis.
decrescendo al niente
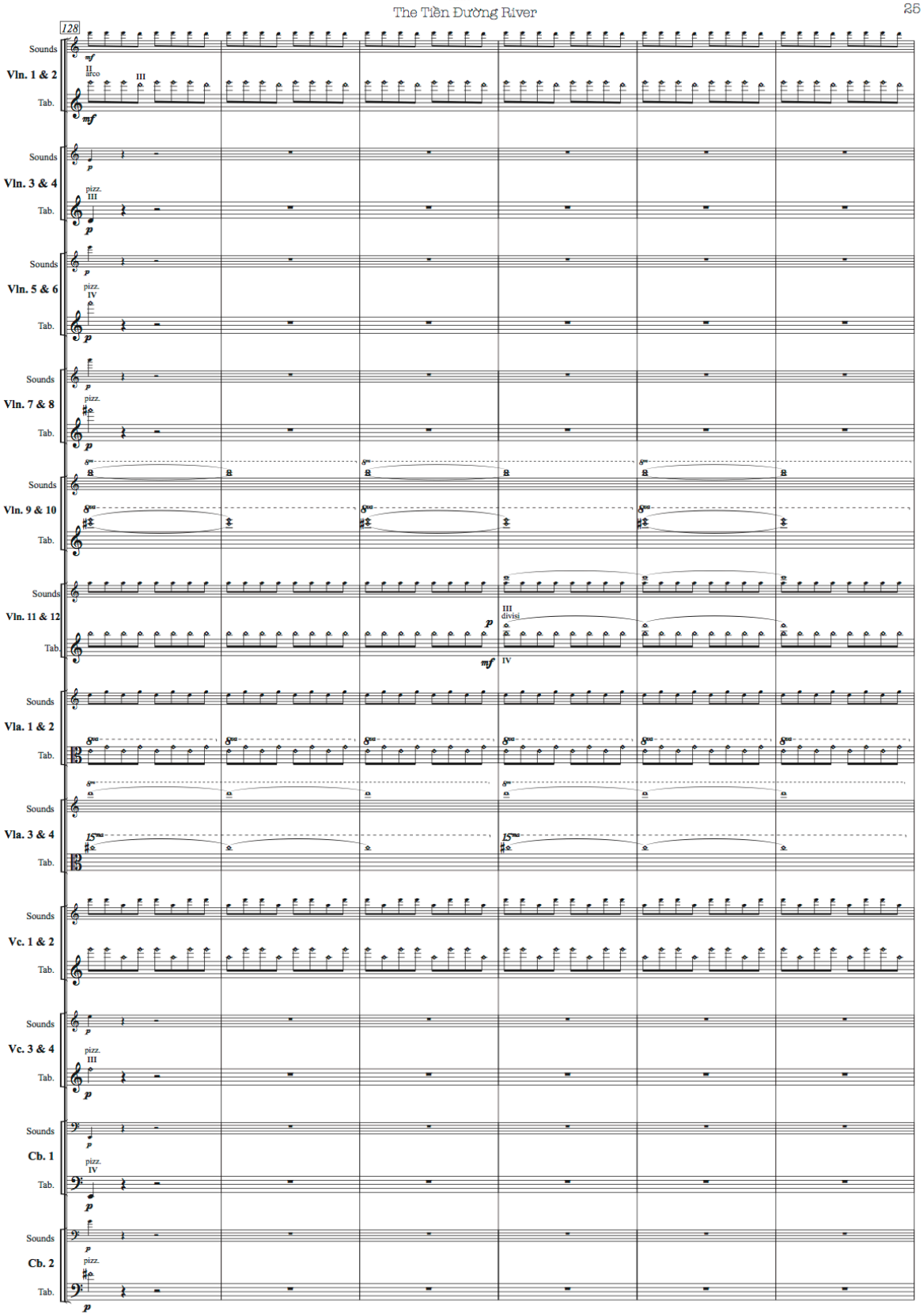

II
Kim Trọng
20
92
6
meno mosso
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
sul pont.
ord.
div.
unis.
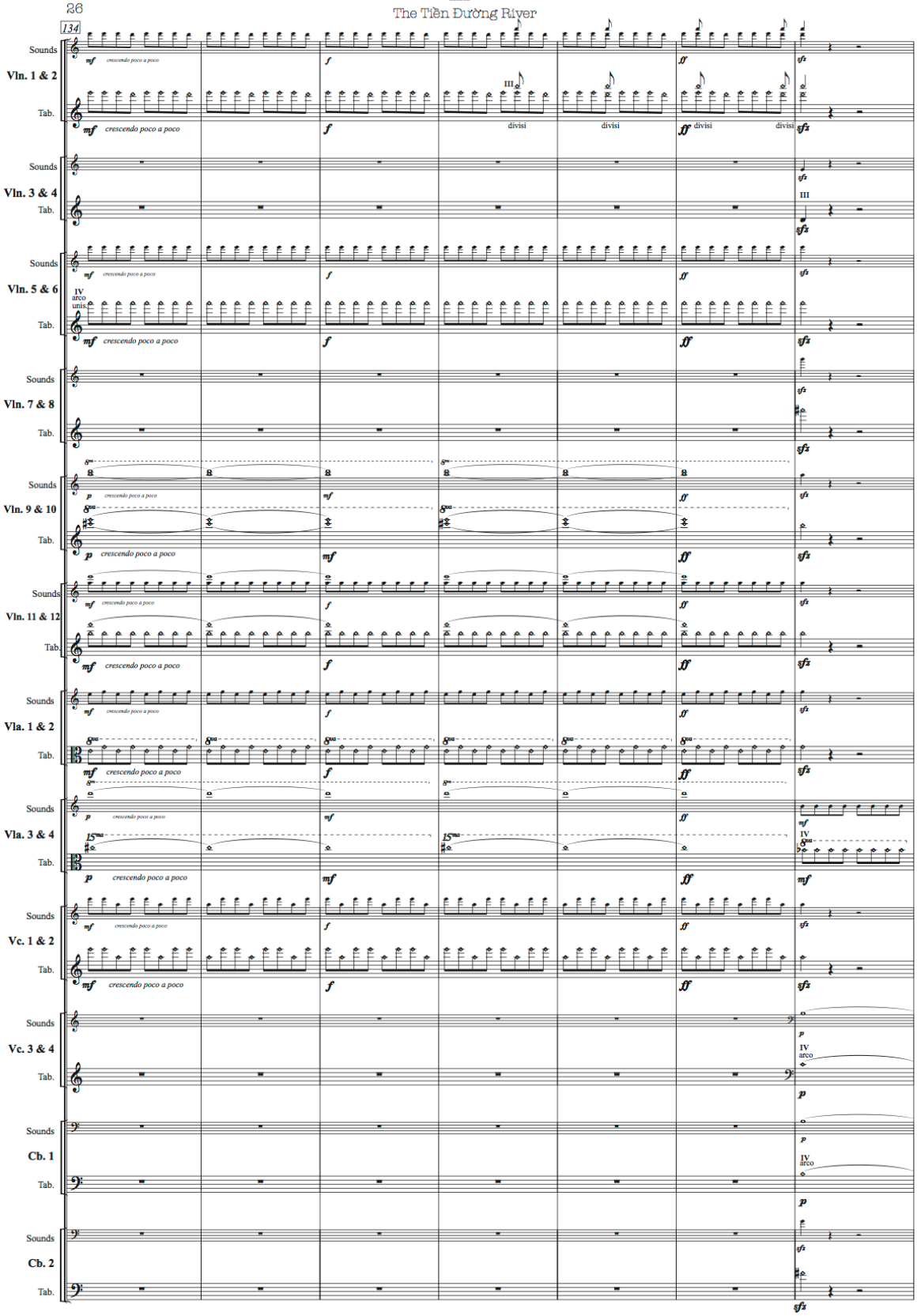

99 a tempo
7
Sounds
Vln. 1 & 2
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
Tab.
decrescendo al niente
sul pont.
ord.
div.
unis.
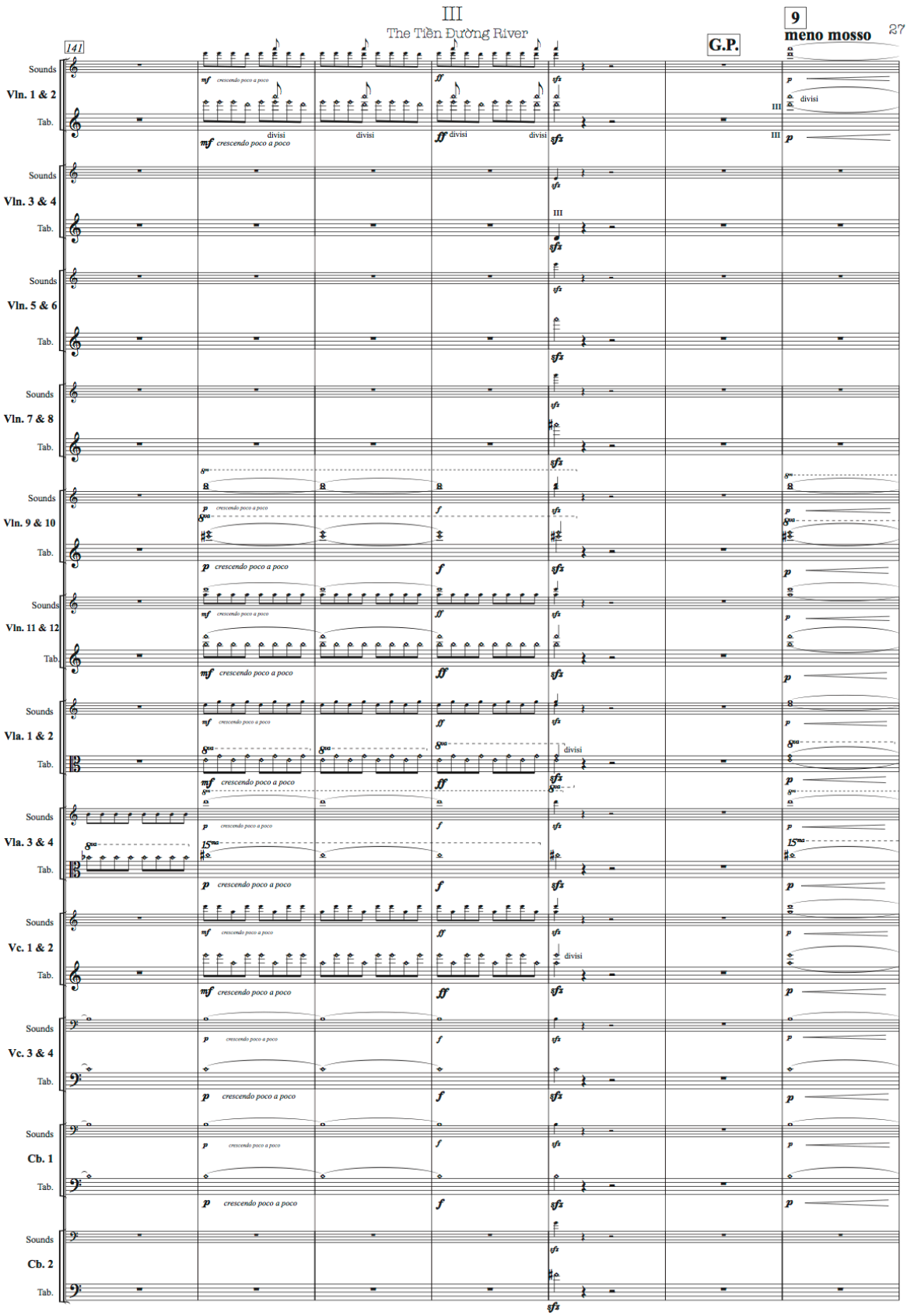

II
Kim Trọng
22
107
8
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
sul pont.
ord.
pp
ppp

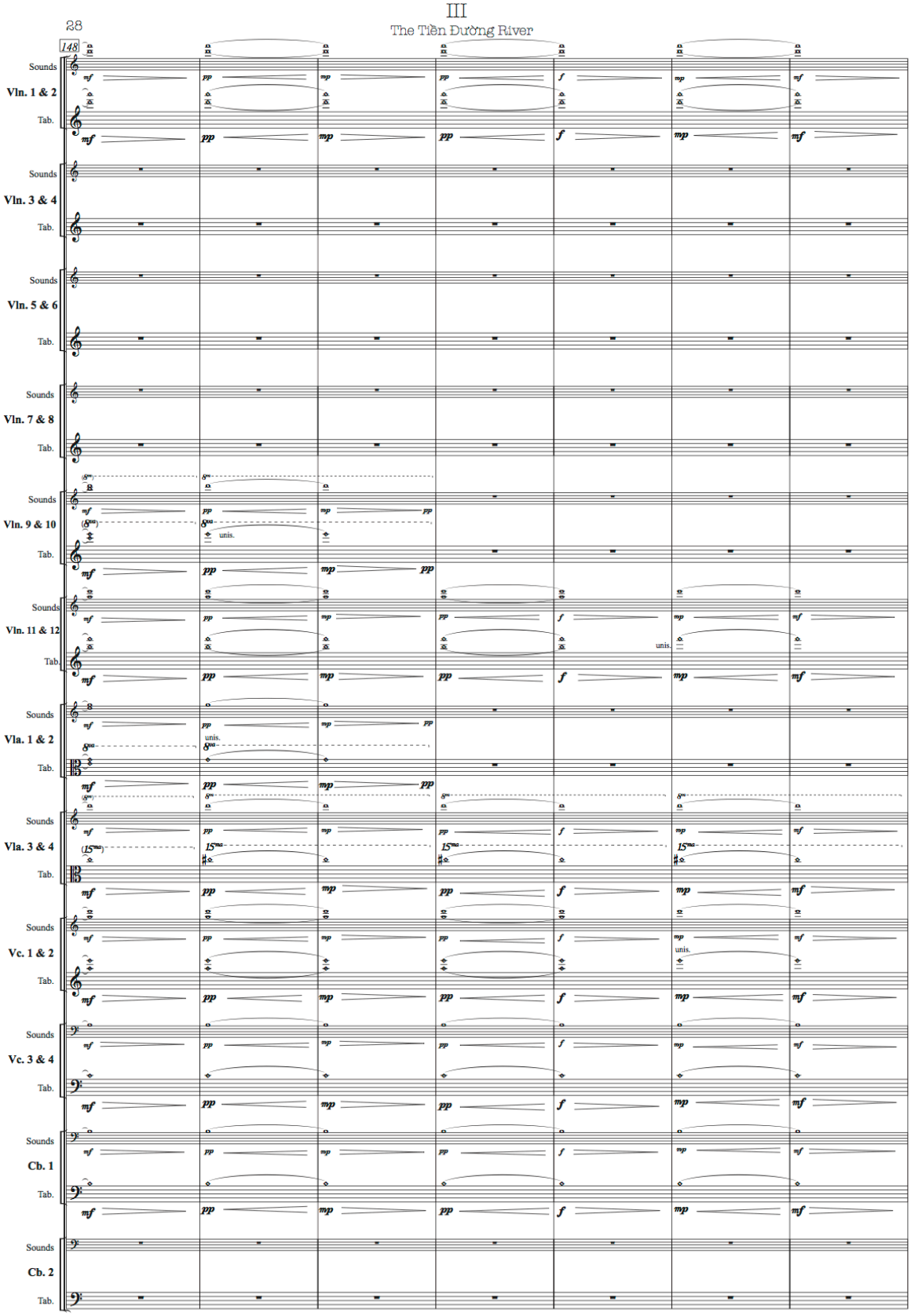

116
Vivace, like a river ♩= 110
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
I
arco
Vln. 11 & 12
II
Vla. 1 & 2
arco IV IV
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
Tab.
p
mf
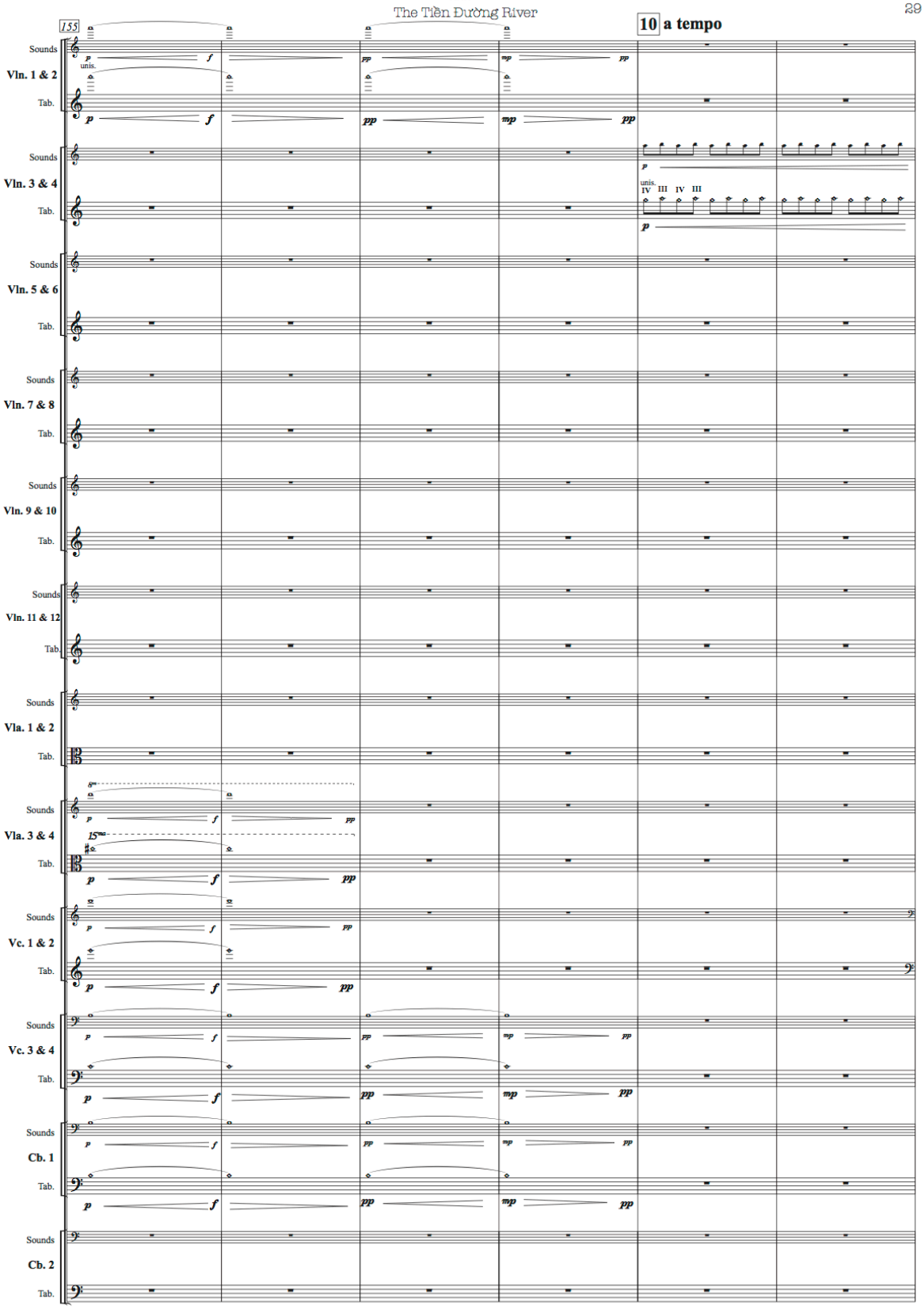

24
122
Sounds
Vln. 1 & 2
Tab.
pizz.
I
Vln. 3 & 4
III
Vln. 5 & 6
IV
Vln. 7 & 8
Vln. 9 & 10
I
divisi
II
Vln. 11 & 12
mf
Vla. 1 & 2
Vla. 3 & 4
arco
unis.
15ma
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
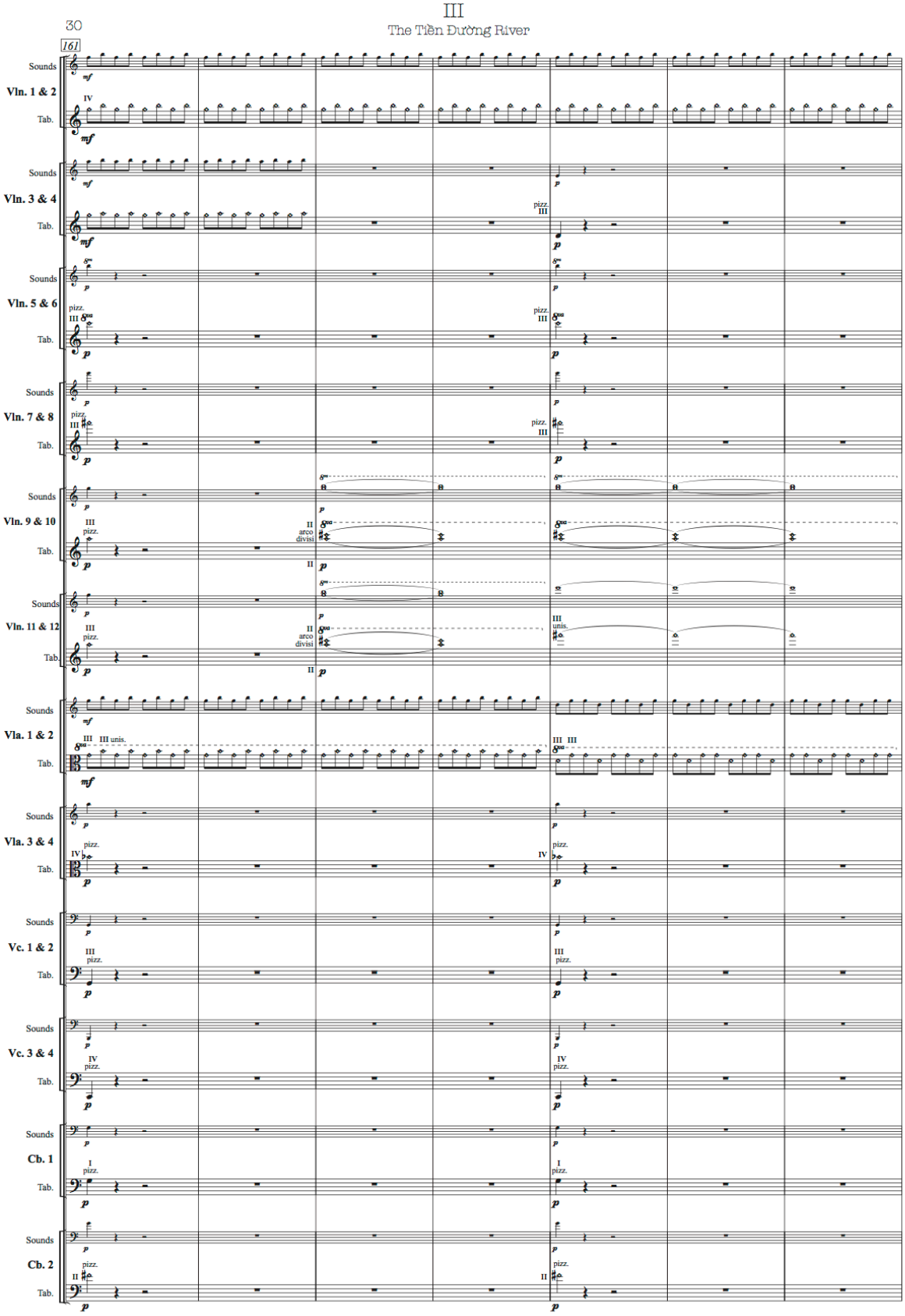

128
Sounds
Vln. 1 & 2
Tab.
mf
II
arco
III
Sounds
Vln. 3 & 4
Tab.
p
pizz.
III
Sounds
Vln. 5 & 6
Tab.
pizz.
IV
Sounds
Vln. 7 & 8
Tab.
pizz.
Sounds
Vln. 9 & 10
Tab.
Sounds
Vln. 11 & 12
Tab.
III
divisi
IV
Sounds
Vla. 1 & 2
Tab.
Sounds
Vla. 3 & 4
Tab.
Sounds
Vc. 1 & 2
Tab.
Sounds
Vc. 3 & 4
Tab.
pizz.
III
Sounds
Cb. 1
Tab.
pizz.
IV
Sounds
Cb. 2
Tab.
pizz.
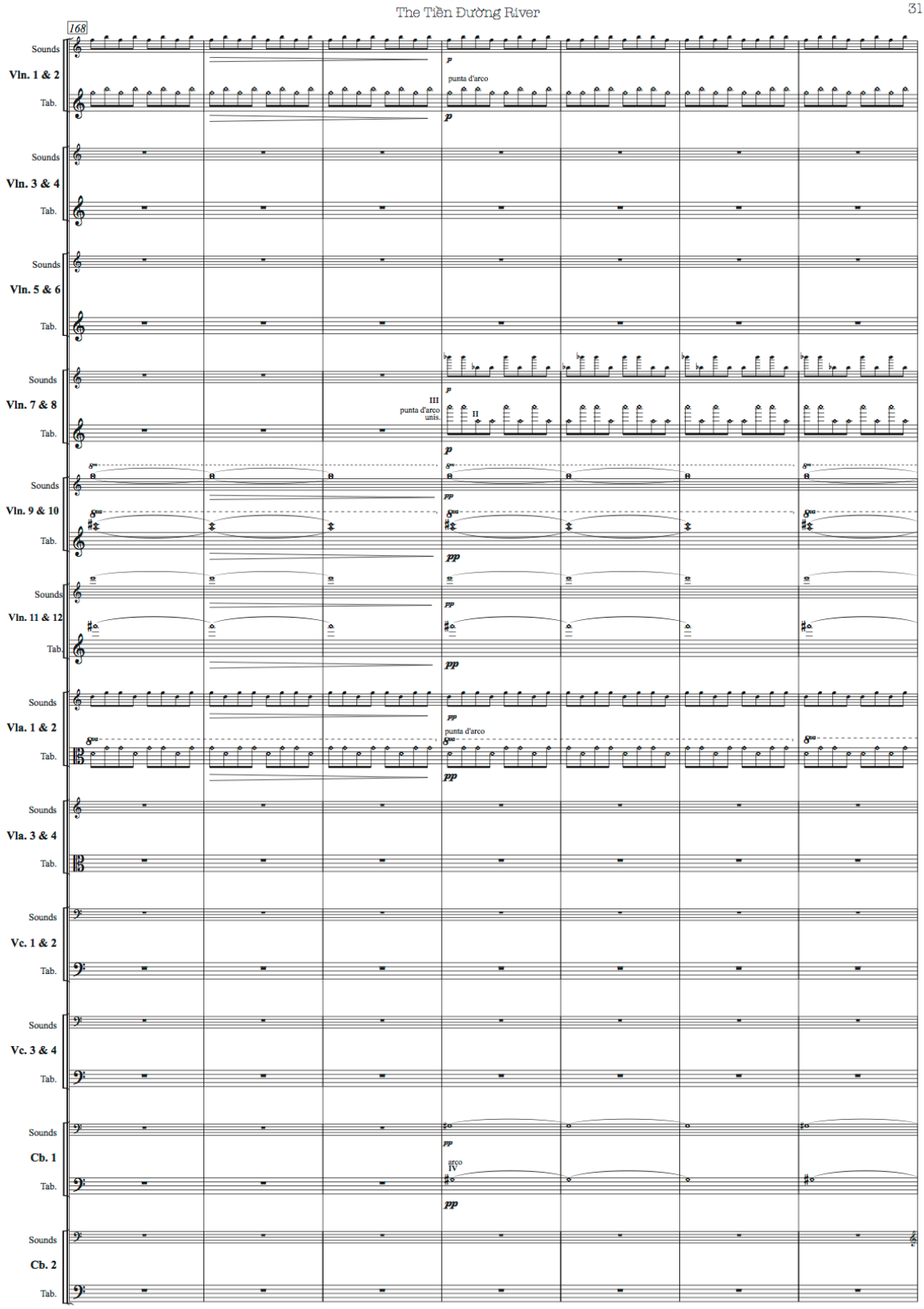

26
134
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
crescendo poco a poco
divisi
arco
unis.
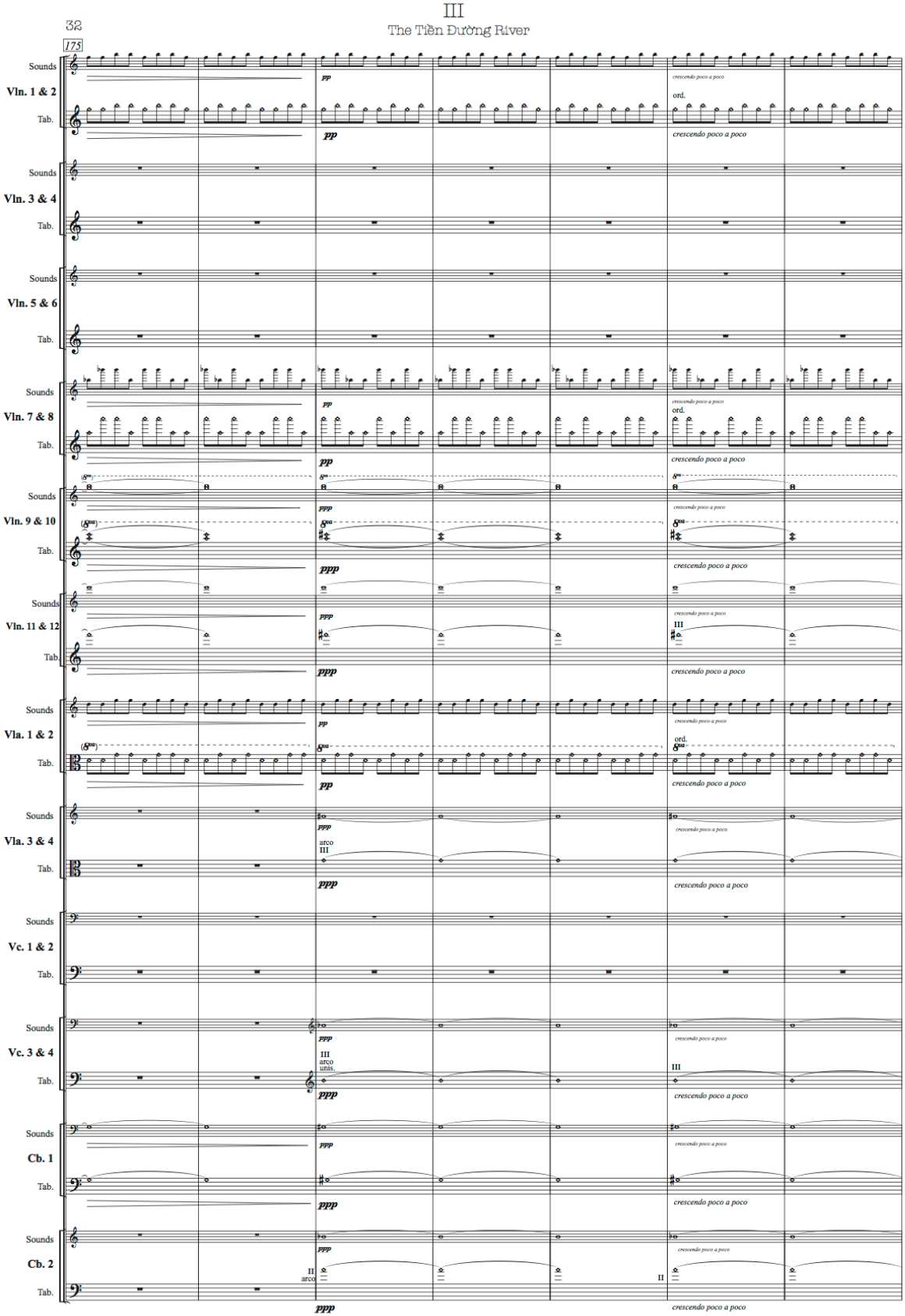

III
The Tiền Đường River
141
G.P.
9
meno mosso
27
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
mf crescendo poco a poco
ff
sfz
p
f
divisi
III
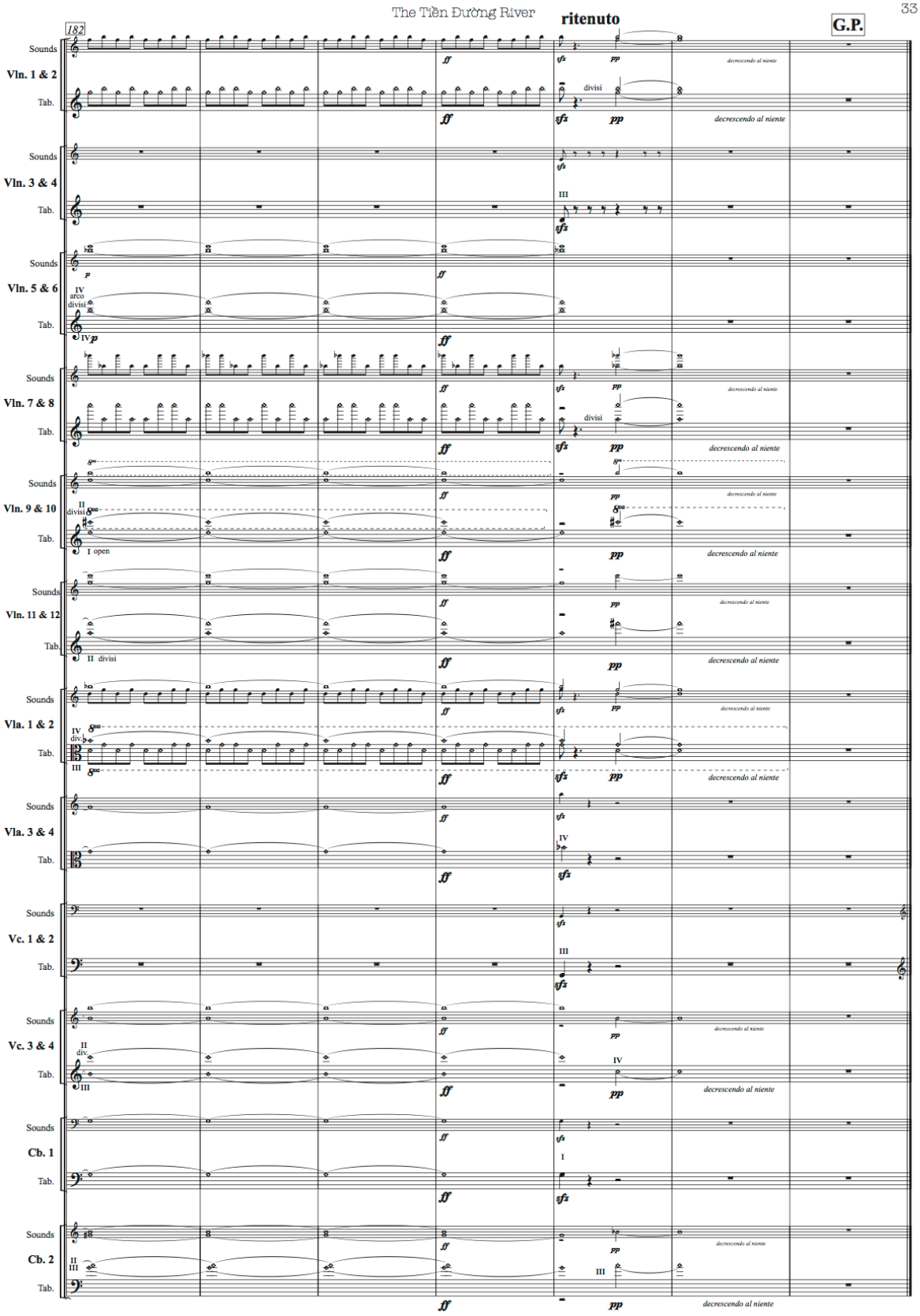

28
148
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
unis.
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2

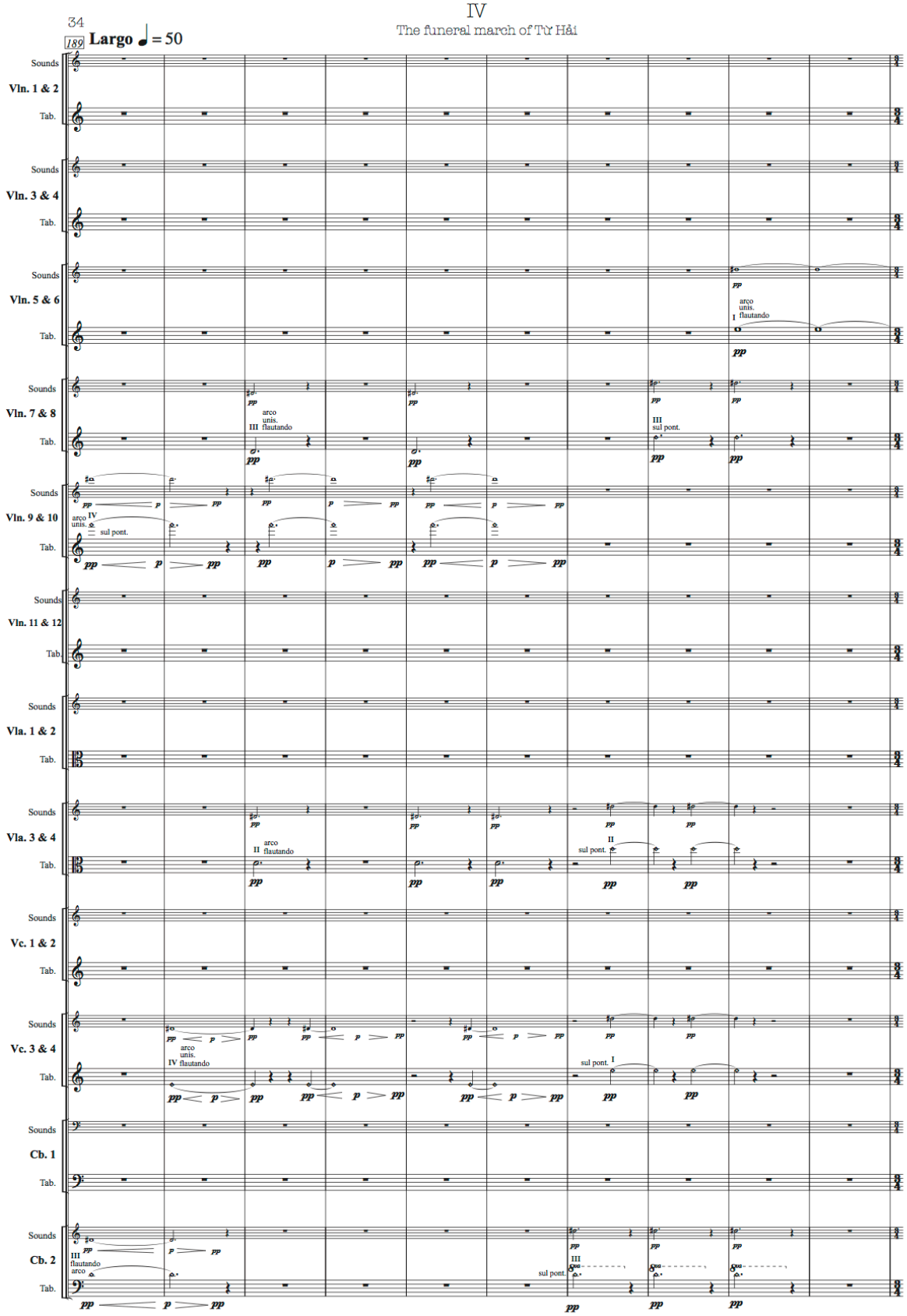

155
10 a tempo
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
unis.
IV III IV III

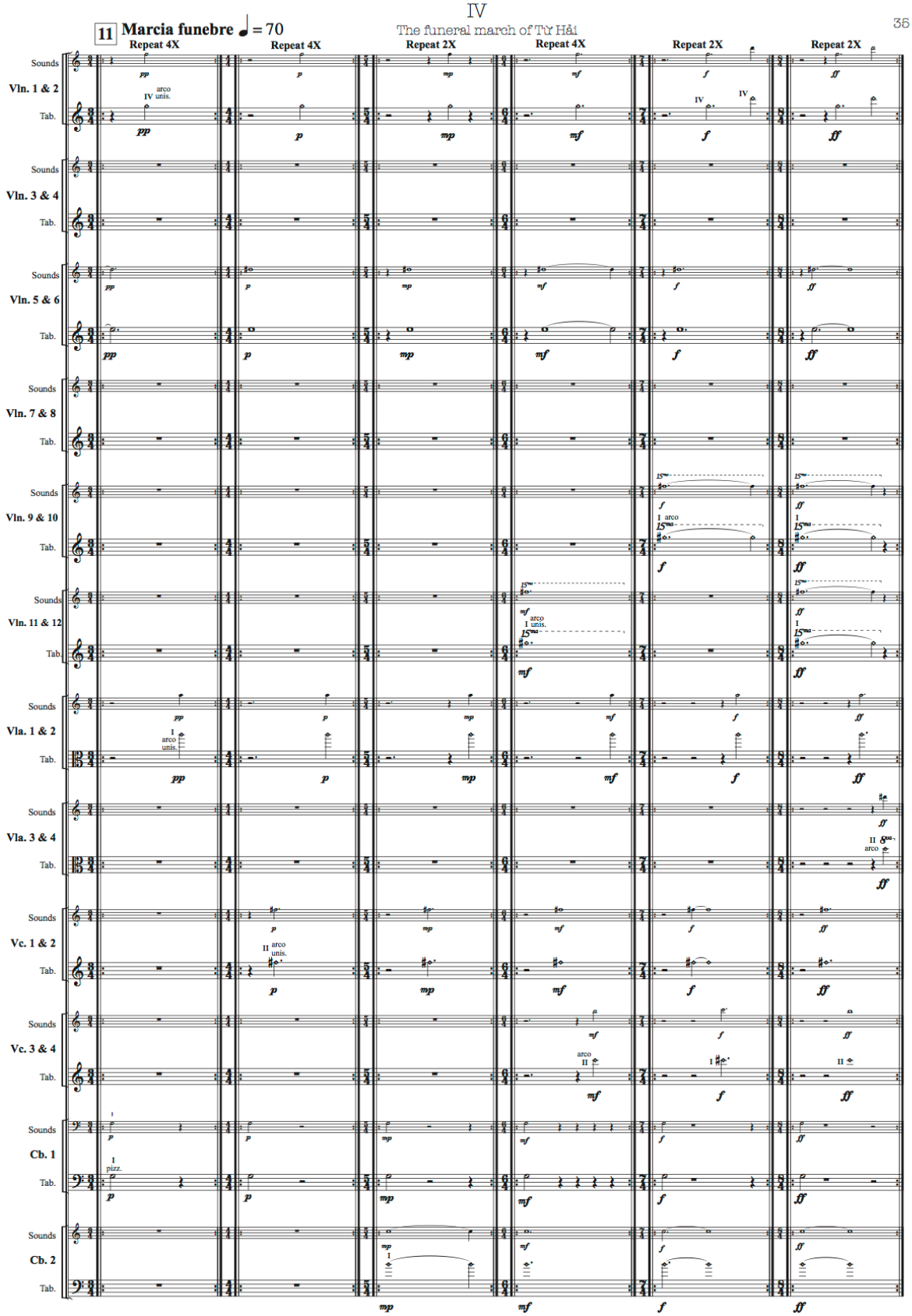

30
161
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
mf
p
pizz.
arco
divisi
unis.


168
Sounds
Vln. 1 & 2
Tab.
punta d'arco
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
III
punta d'arco
unis.
II
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
arco
IV
Cb. 2


32
175
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
pp
ppp
ord.
arco
unis.
crescendo poco a poco

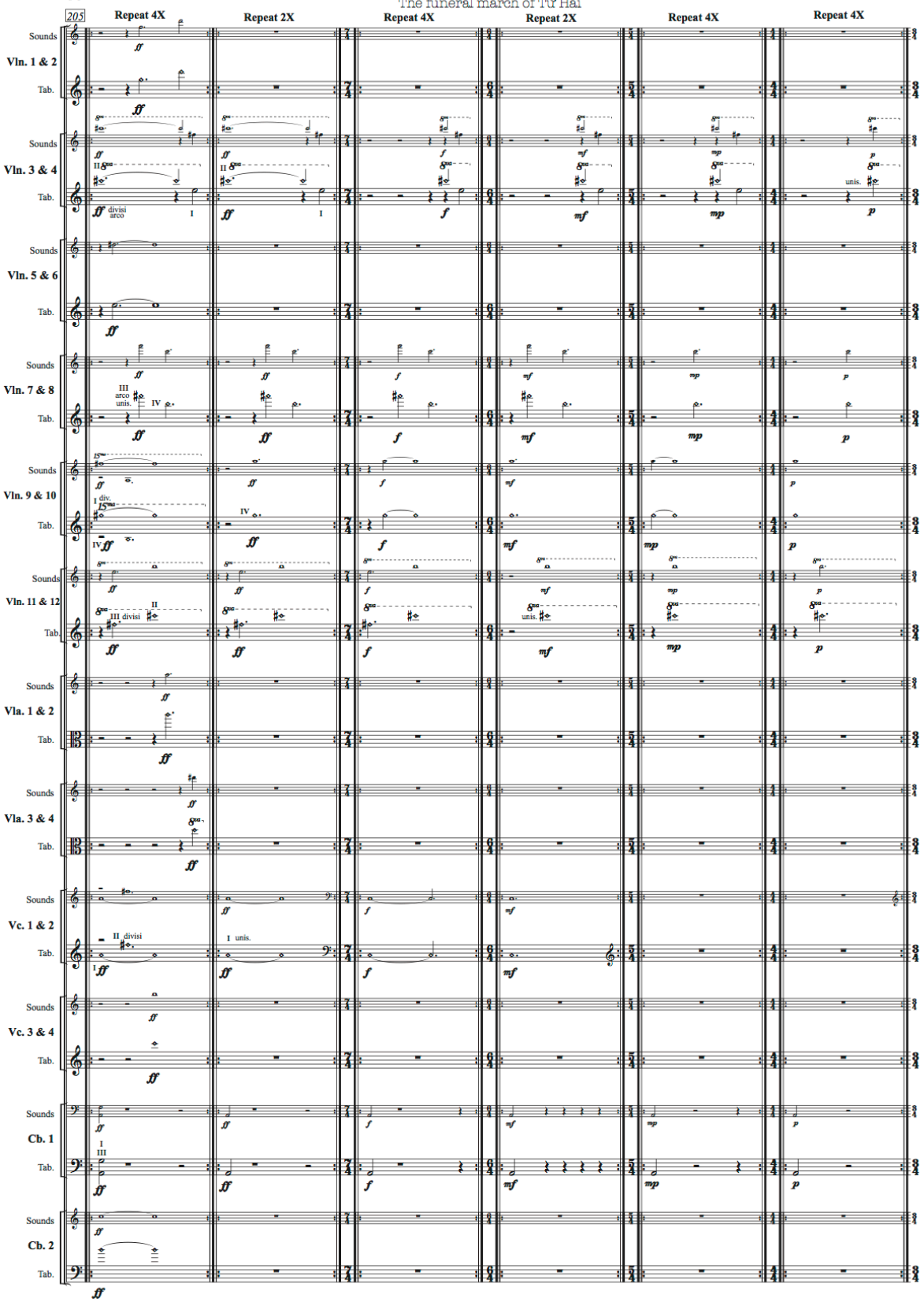

182
ritenuto
G.P.
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
divisi
arco
I open
II divisi
decrescendo al niente

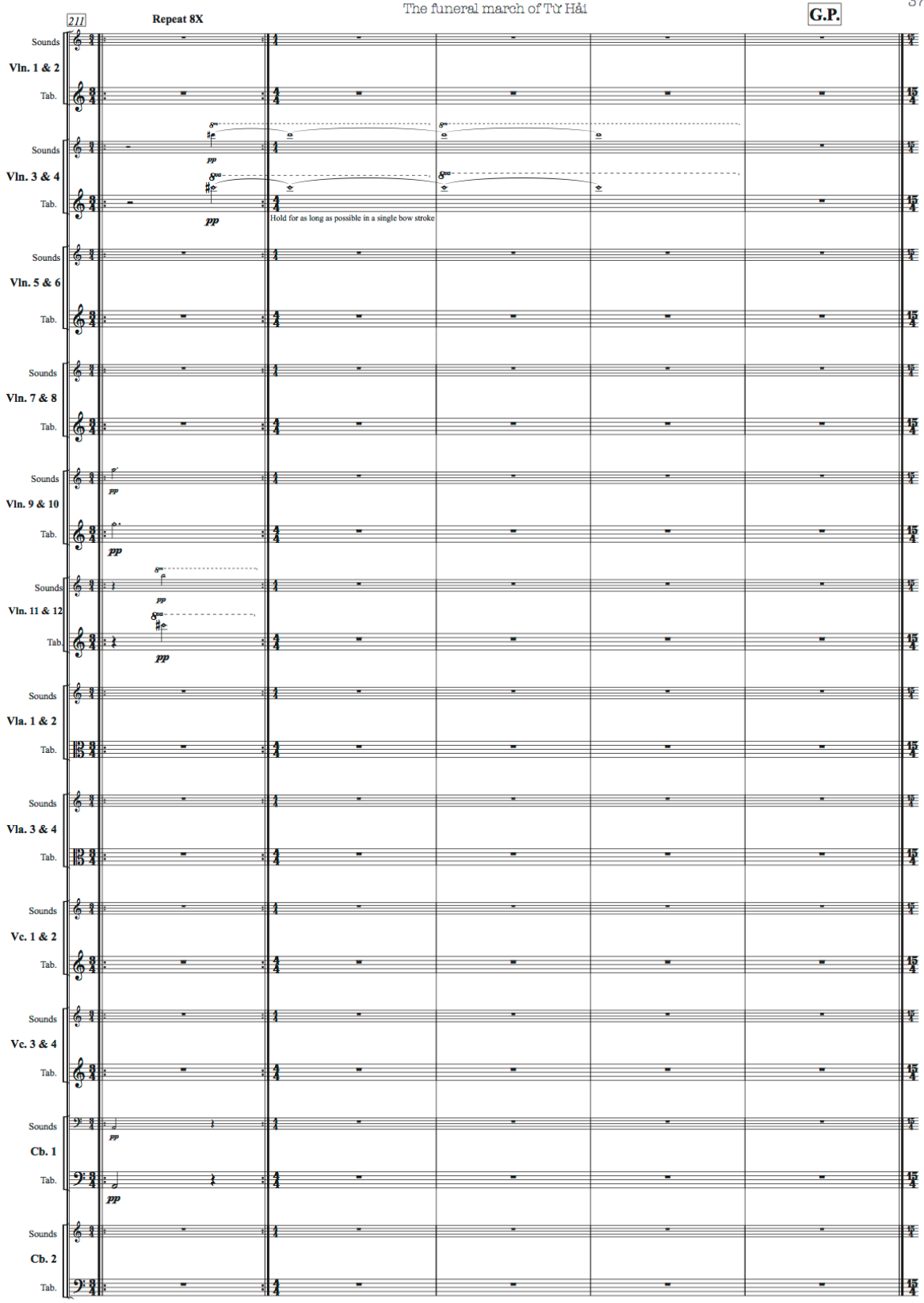

34
189
Largo ♩ = 50
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
arco unis. flautando
Vln. 7 & 8
arco unis. flautando
sul pont.
Vln. 9 & 10
arco unis. sul pont.
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
arco flautando
sul pont.
Vc. 1 & 2
Vc. 3 & 4
arco unis. flautando
sul pont.
Cb. 1
Cb. 2
flautando arco
sul pont.
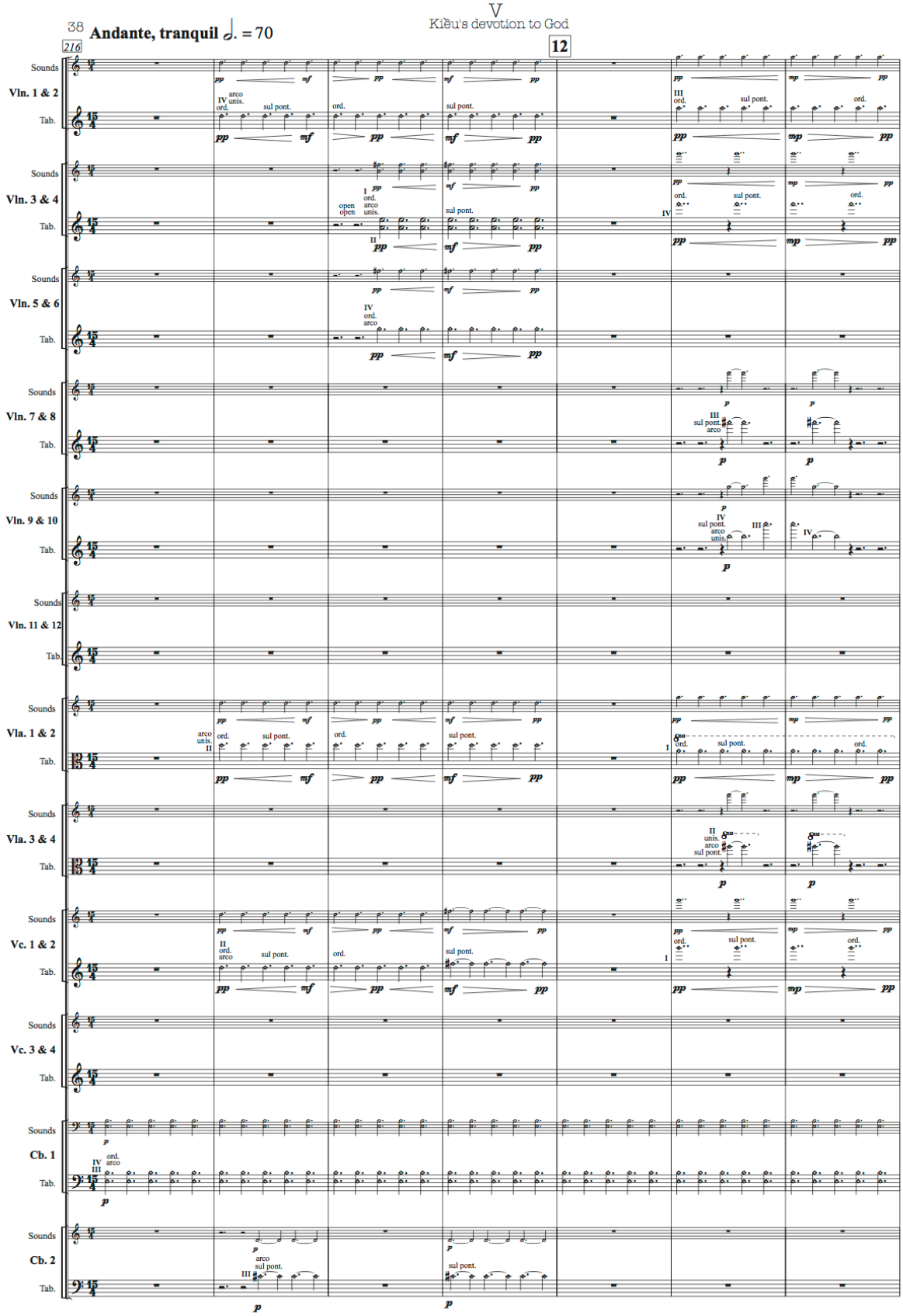

11 Marcia funebre ♩ = 70
Repeat 4X
Repeat 4X
Repeat 2X
Repeat 4X
Repeat 2X
Repeat 2X
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
arco
IV unis.
pizz.
arco unis.
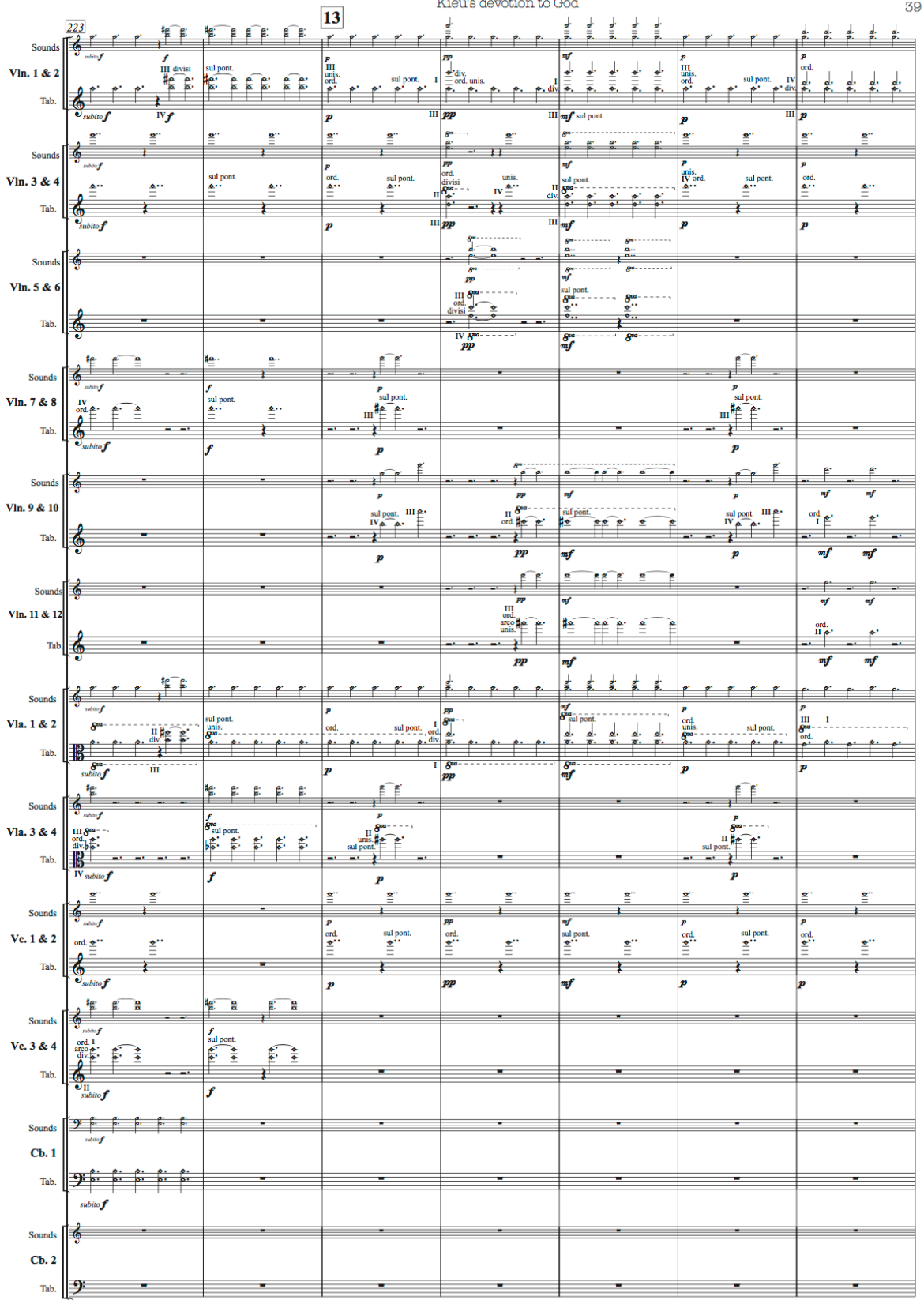
IV
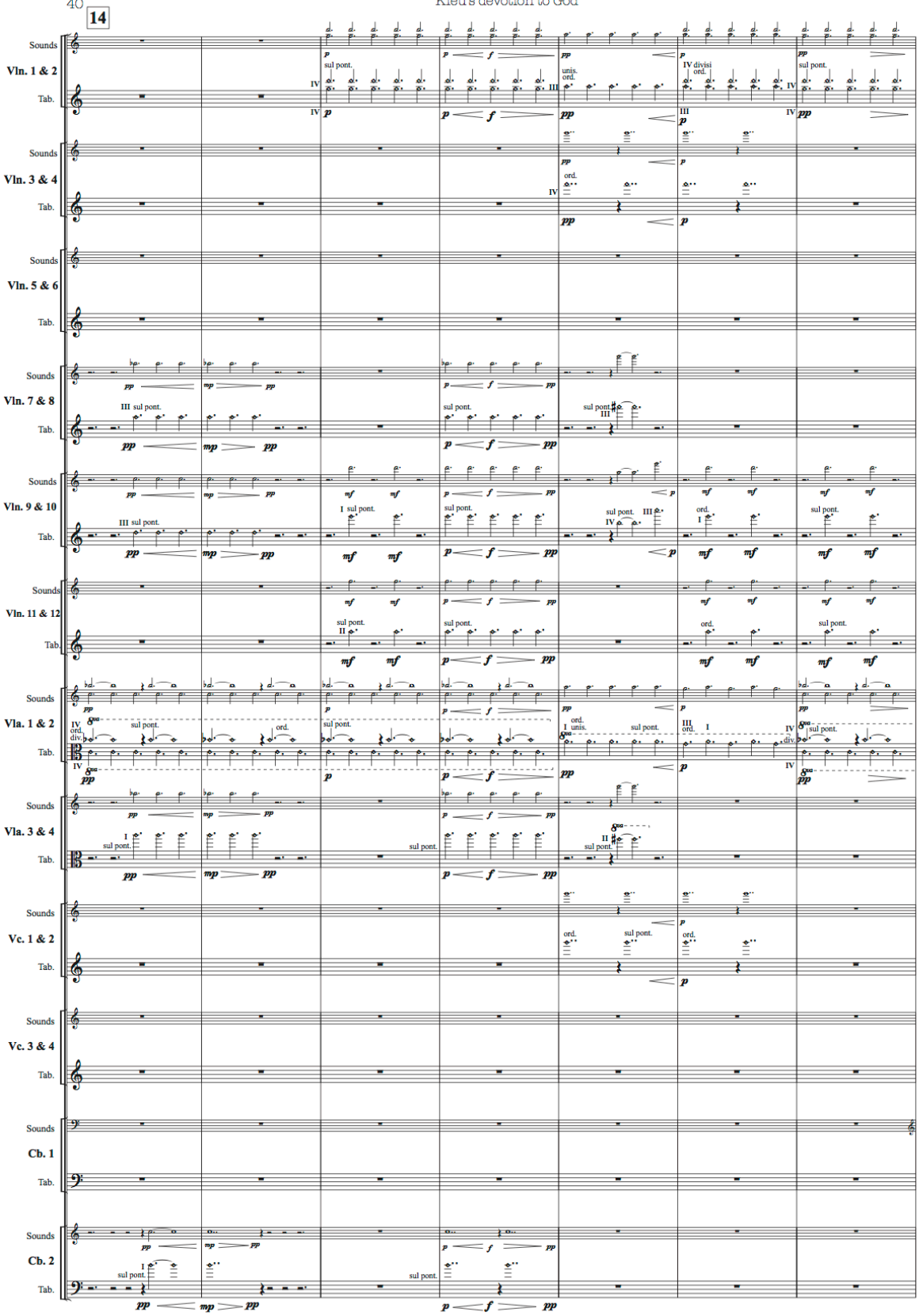
The funeral march of Từ Hải
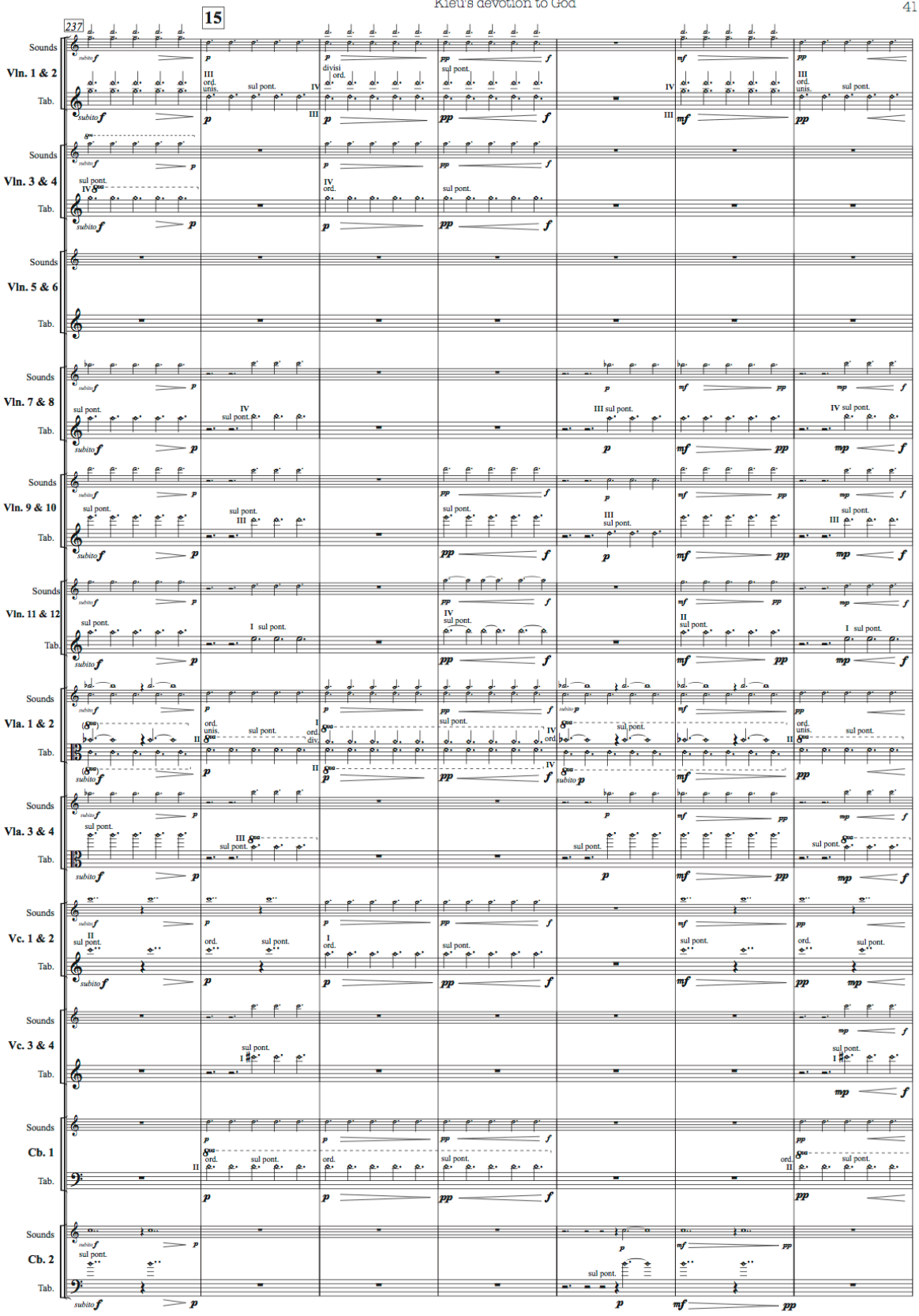



211
Repeat 8X
G.P.
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
pp
Hold for as long as possible in a single bow stroke
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2


38
Andante, tranquil ♩. = 70
216
12
Sounds
Vln. 1 & 2
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
Tab.
arco unis.
ord.
sul pont.
open open
sul pont. arco
arco
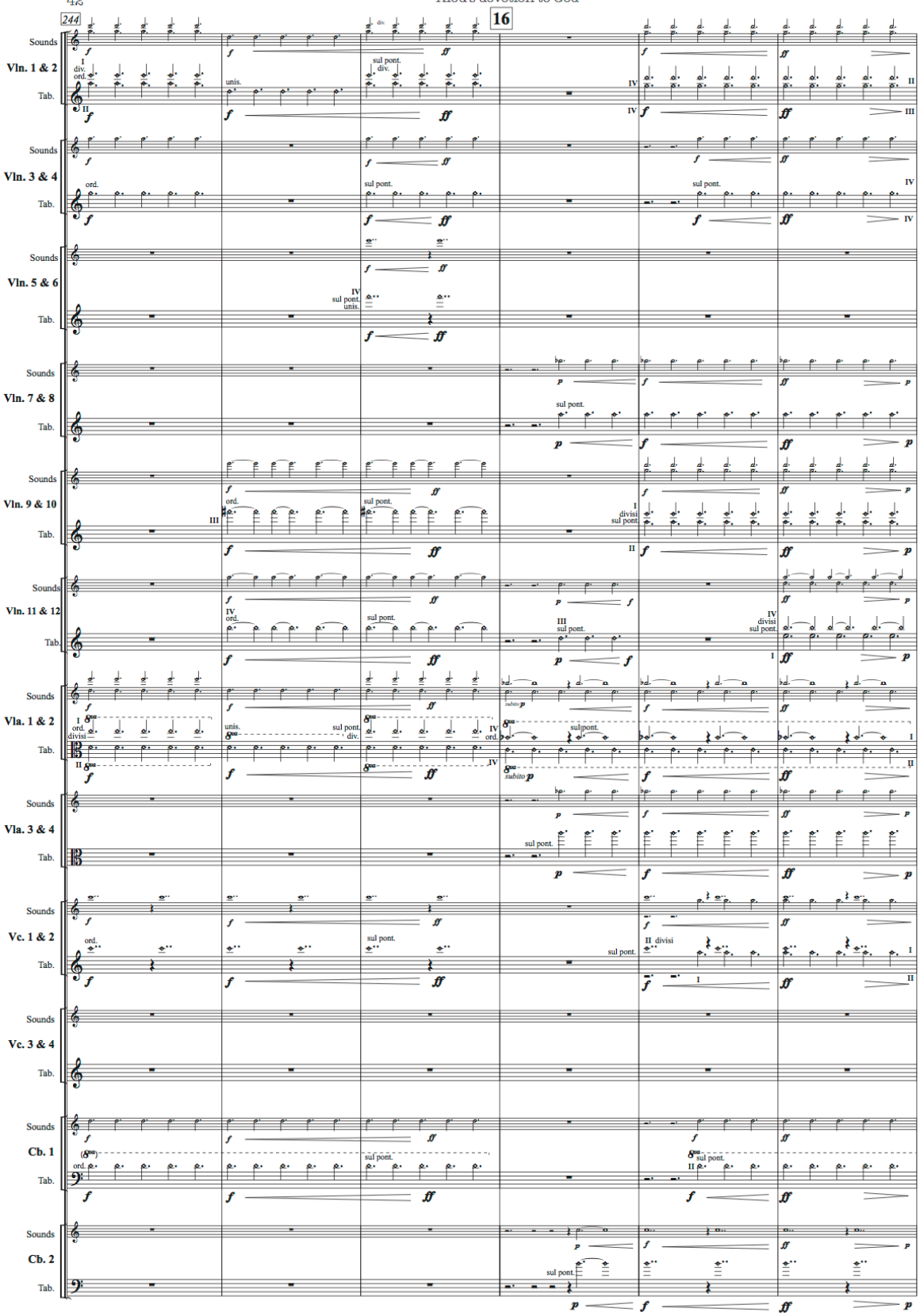

223
13
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
subito f
sul pont.
ord.
unis.
div.
divisi
arco
sul pont.
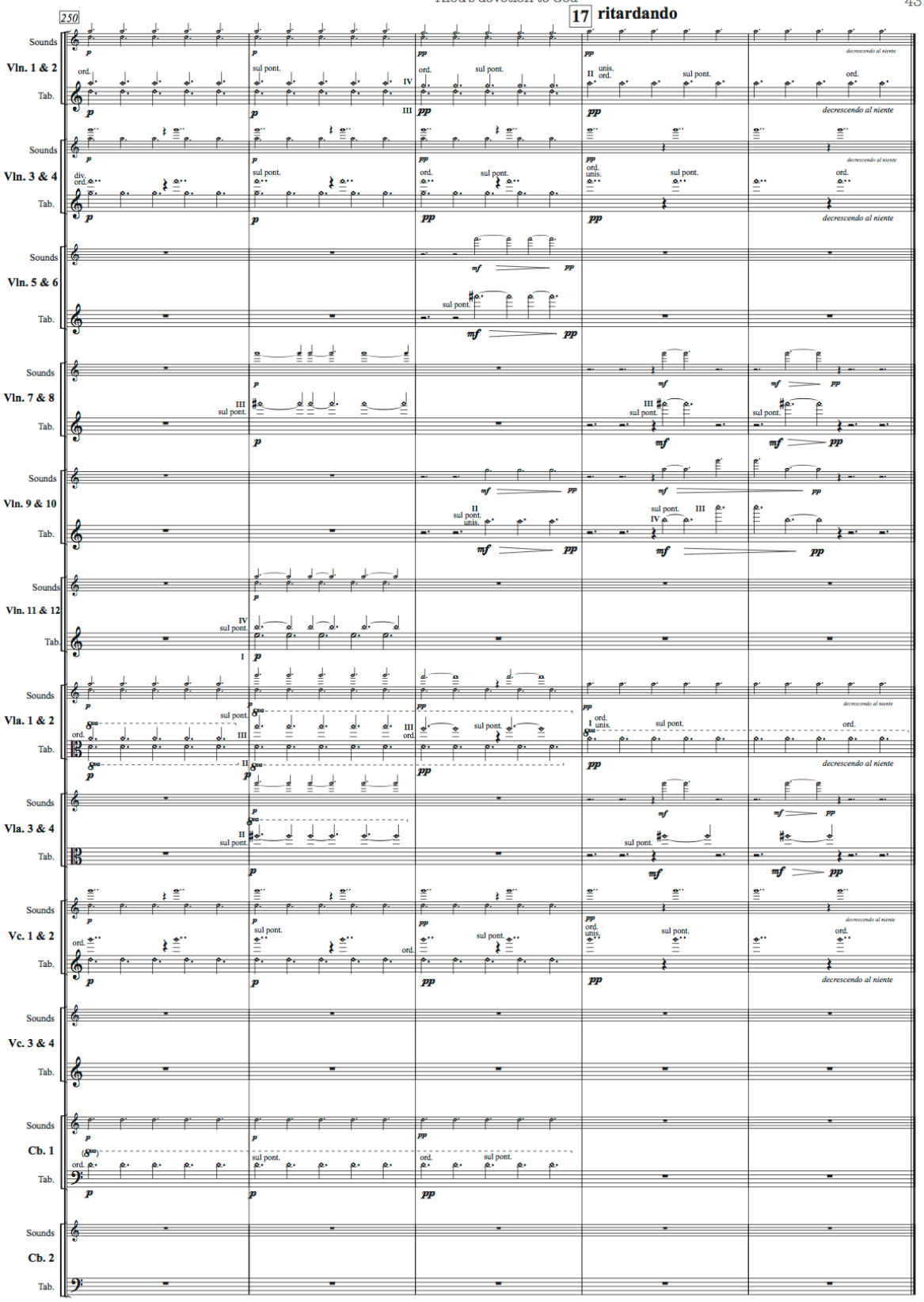

40
14
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
sul pont.
ord.
unis.
IV divisi


237
15
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
subito f
p
pp
f
mf
mp
sul pont.
ord.
unis.
divisi
div.

V

Kiều's devotion to God




250
17 ritardando
Sounds
Vln. 1 & 2
Tab.
Vln. 3 & 4
Vln. 5 & 6
Vln. 7 & 8
Vln. 9 & 10
Vln. 11 & 12
Vla. 1 & 2
Vla. 3 & 4
Vc. 1 & 2
Vc. 3 & 4
Cb. 1
Cb. 2
ord.
sul pont.
unis.
div.
decrescendo al niente

3.3. *13* for scordatura string quartet

13

for scordatura string quartet

full score

transposing score

Tuning

In Ratios:

Violin I
I 351:256
II 117:64
III 39:32
IV 13:8

Violin II
I 4,563:4,096
II 1,521:1,024
III 507:256
IV 169:128

Viola
I 27:16
II 9:8
III 3:2
IV 1:1

Cello
I 27:16
II 9:8
III 3:2
IV 1:1

In Hertz:

Violin I
I 624 Hz
II 468 Hz
III 312 Hz
IV 208 Hz

Violin II
I 570.37 Hz
II 380.25 Hz
III 253.5 Hz
IV 169 Hz

Viola
I 432 Hz
II 288 Hz
III 192 Hz
IV 128 Hz

Cello
I 216 Hz
II 144 Hz
III 96 Hz
IV 64 Hz

As approximate notes:

Violin I
I F
II Bb
III Eb
IV Ab

Violin II
I D
II G
III C
IV F

Viola
I A
II D
III G
IV C

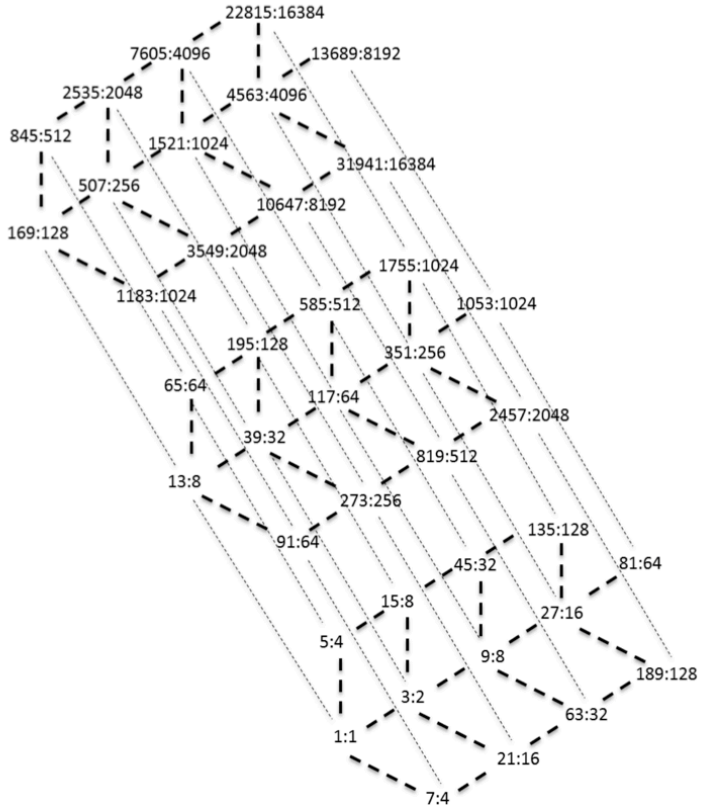
Cello
I A
II D
III G
IV C

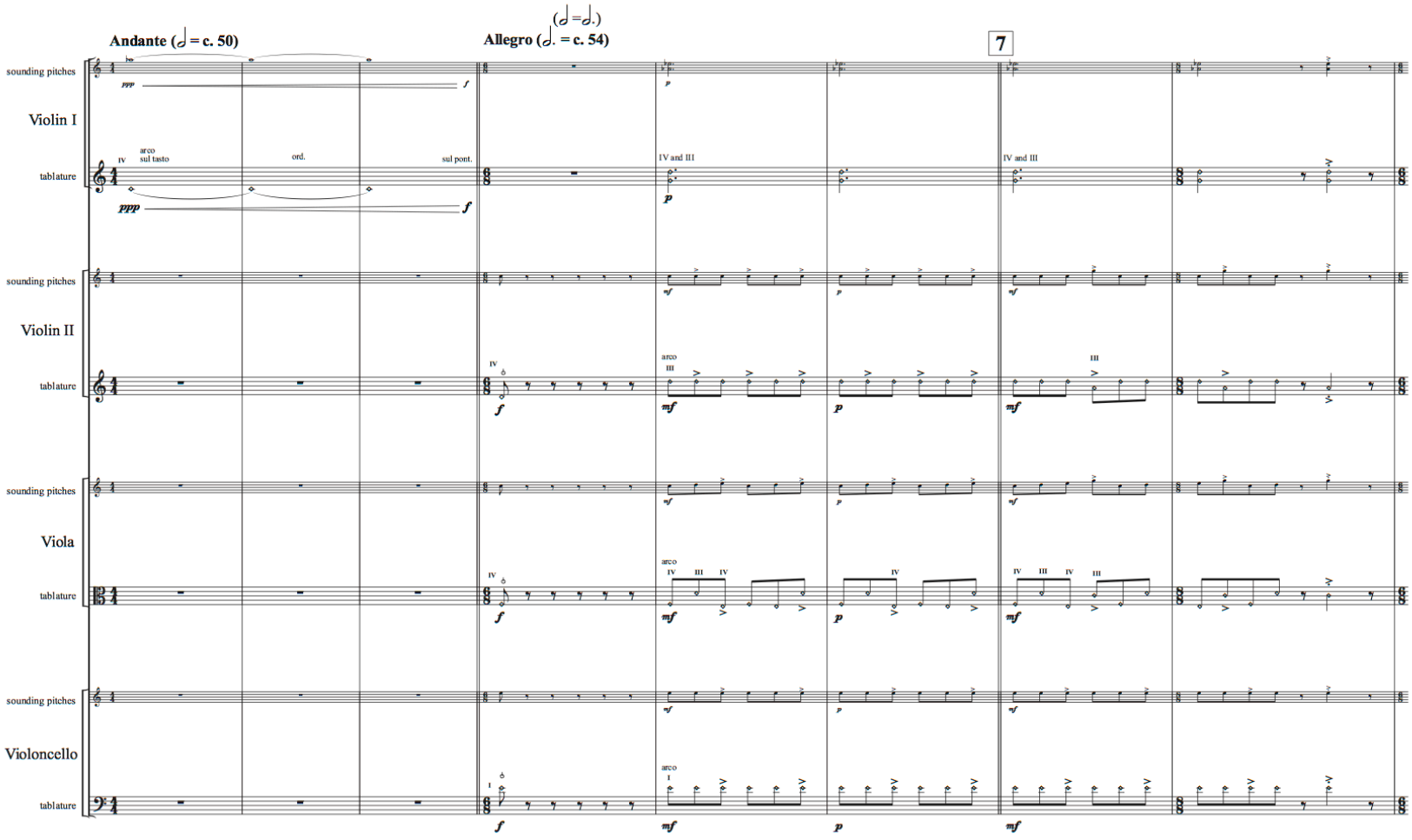
It is strongly recommended that the quartet tune string IV with the aid of electronic tuning devices. The remaining three strings may be tuned by ear, as perfect fifths above string IV.

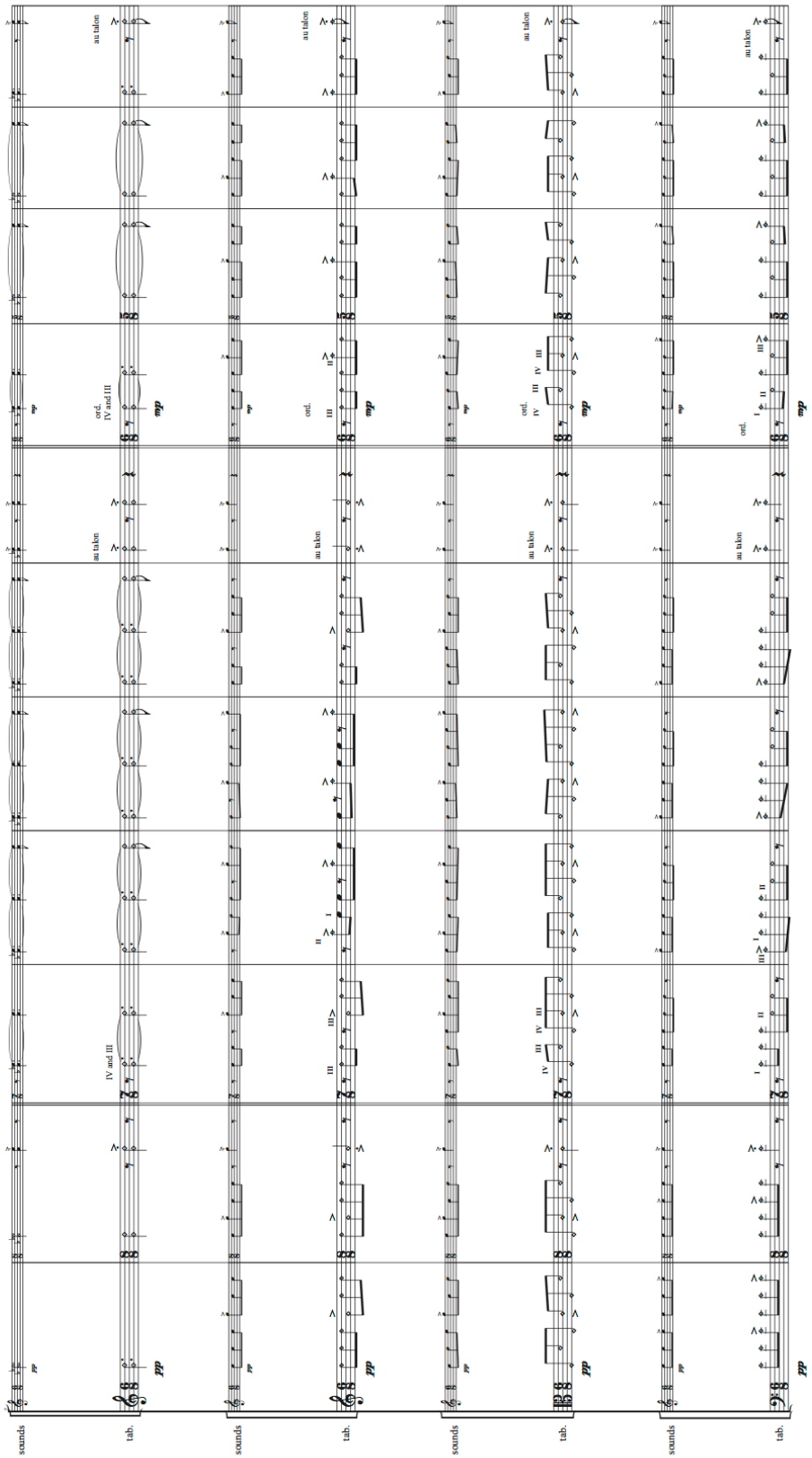
Notation

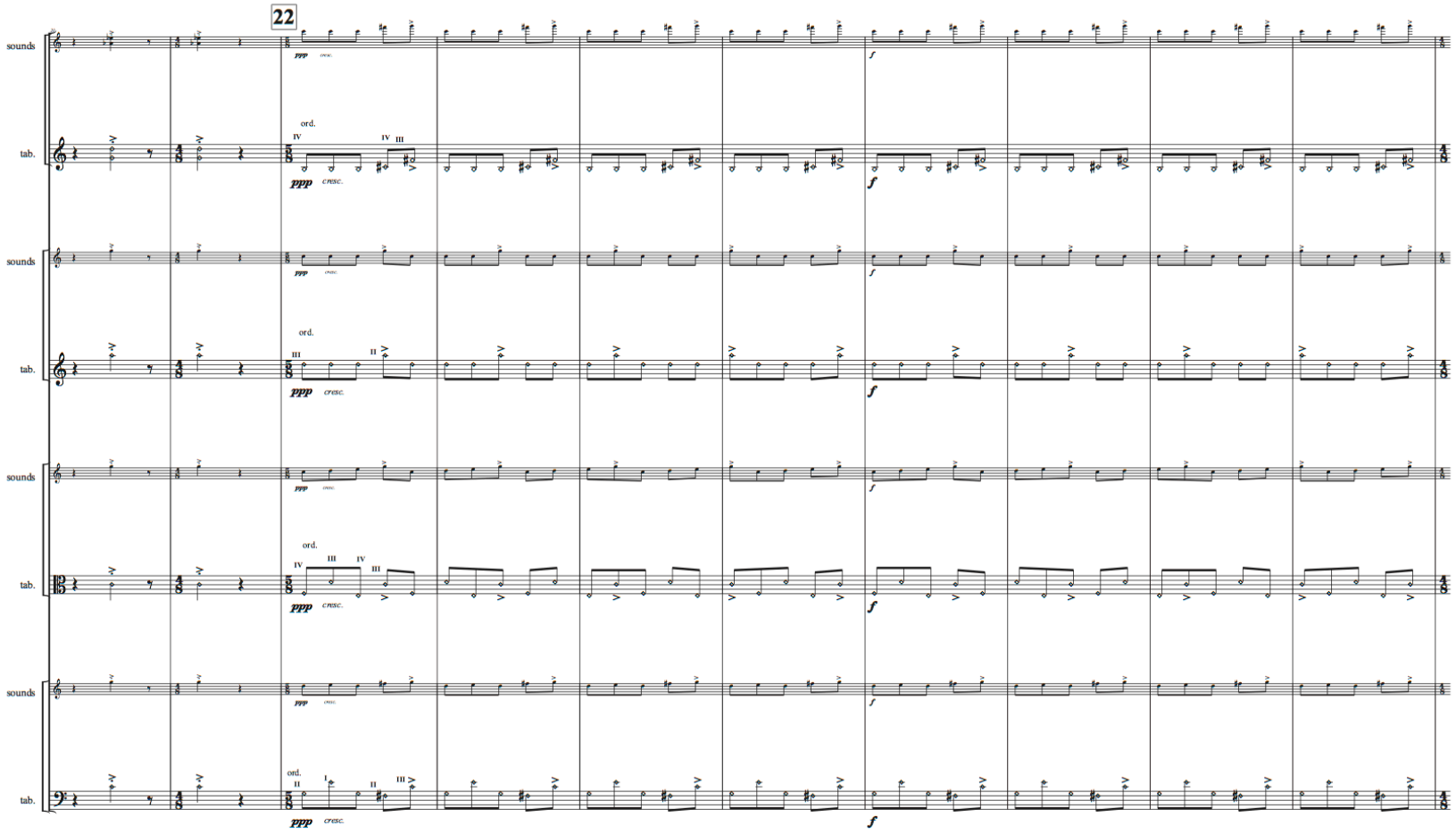
The notation for this piece of music has been written in two ways. The first is the approximate sounding pitches. The second is as node points, or where the players should touch the strings in order for the sounding pitches to sound correctly. The node points have been scored as a kind of tablature, as the observant reader will quickly realise. The intention is that despite the scordatura tuning, the players will read the score as if no re-tuning has taken place. This treatment essentially turns the first and second violins into transposing instruments, where the pitches written are not the same as those that will sound at 'concert pitch', so the notation of harmonics is simply a guide of where to place one's fingers.

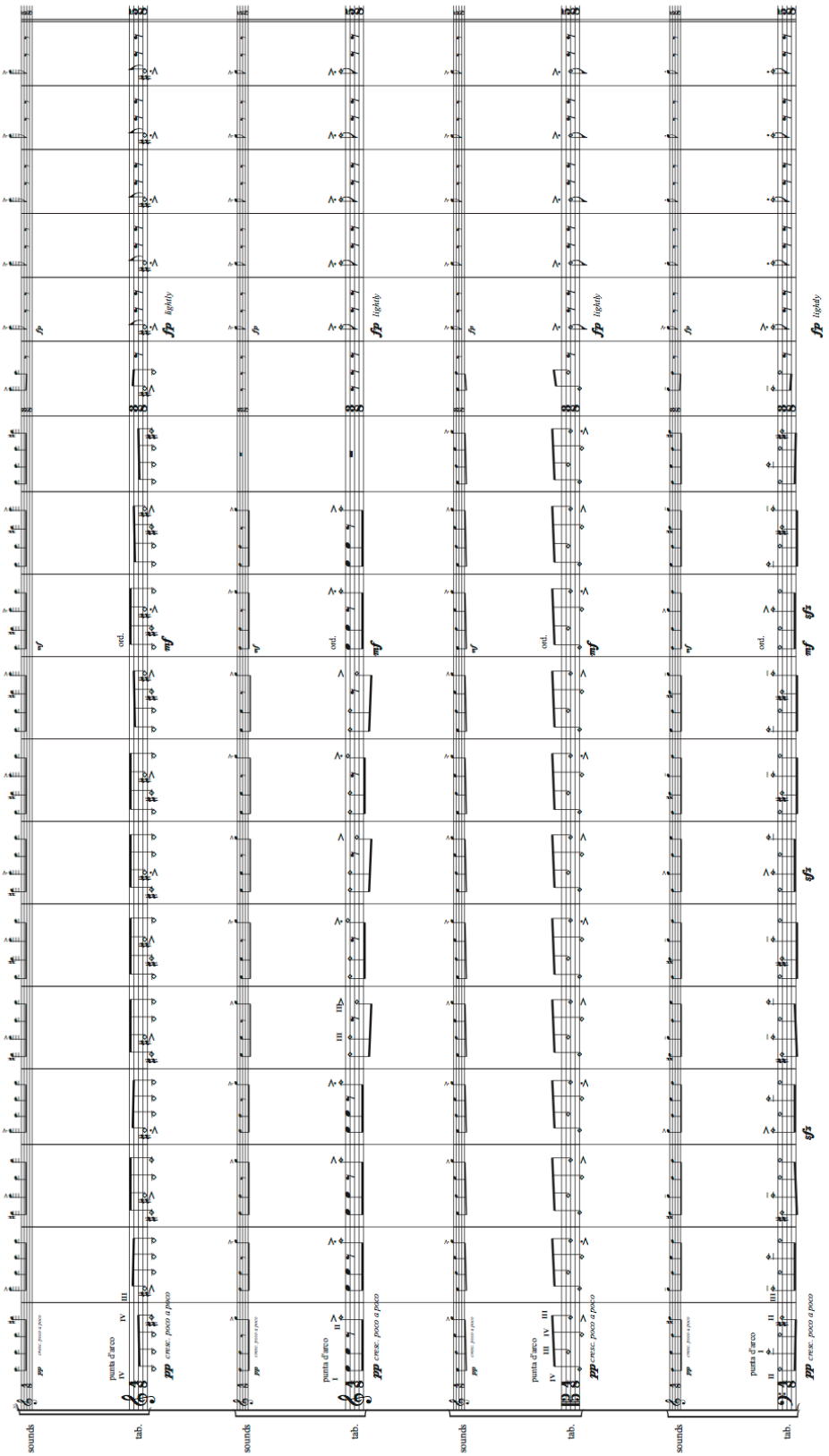
Lattice

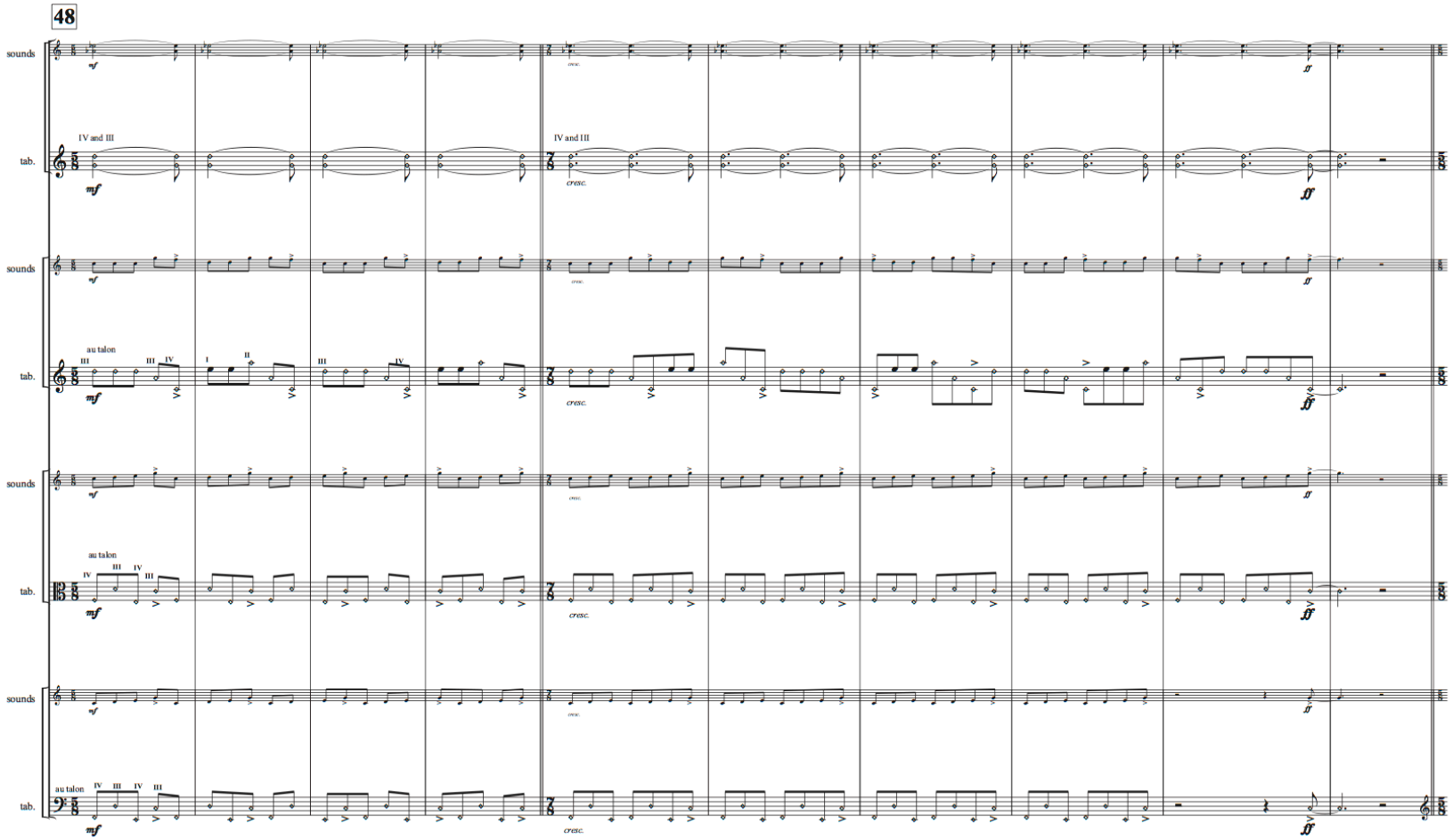

22815:16384
7605:4096
13689:8192
2535:2048
4563:4096
845:512
1521:1024
31941:16384
507:256
10647:8192
169:128
3549:2048
1755:1024
1183:1024
585:512
1053:1024
195:128
351:256
65:64
117:64
2457:2048
39:32
819:512
13:8
273:256
135:128
91:64
45:32
81:64
15:8
27:16
5:4
9:8
189:128
3:2
63:32
1:1
21:16
7:4

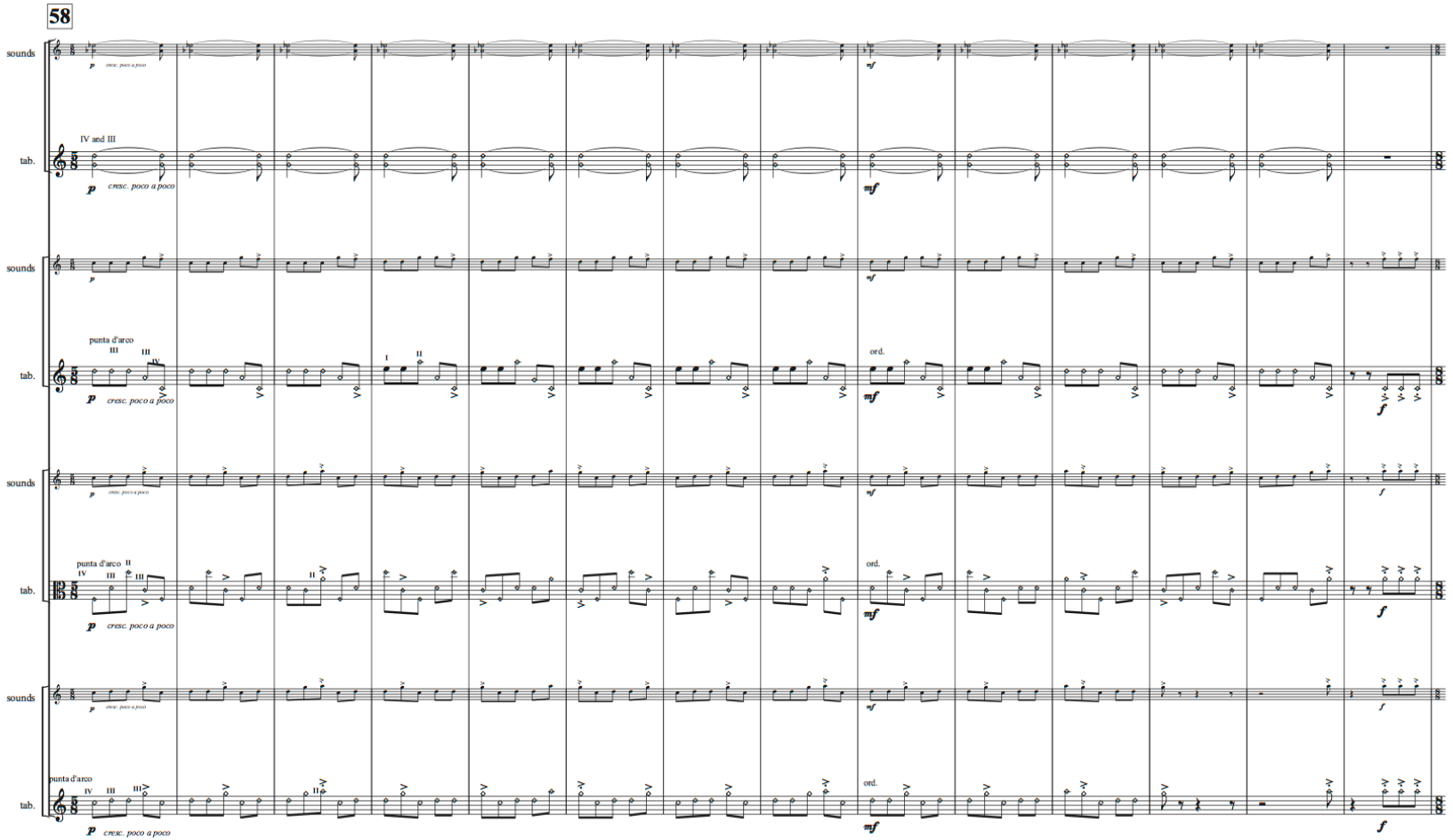

Andante (𝅗𝅥 = c. 50)
(𝅗𝅥=𝅗𝅥.)
Allegro (𝅗𝅥. = c. 54)
7
sounding pitches
Violin I
tablature
arco
sul tasto
ord.
sul pont.
IV and III
Violin II
Viola
Violoncello


sounds
tab.
au talon
ord.
IV and III
III
II
I
IV
sounds
tab.
au talon
ord.
III
II
sounds
tab.
au talon
ord.
IV
III
II
sounds
tab.
au talon
ord.
I
II
III

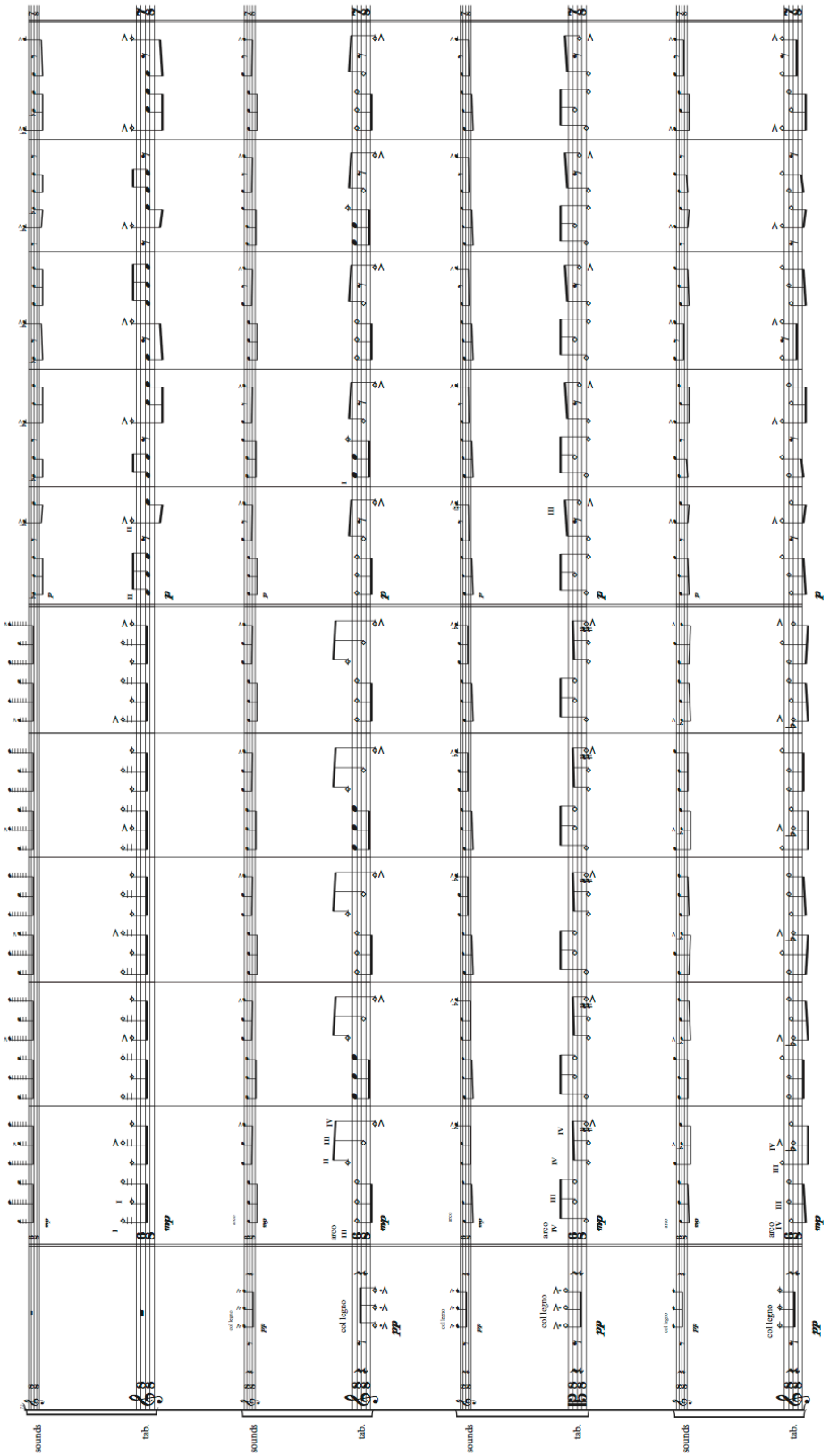

22
sounds
tab.
ord.
ppp
cresc.
f

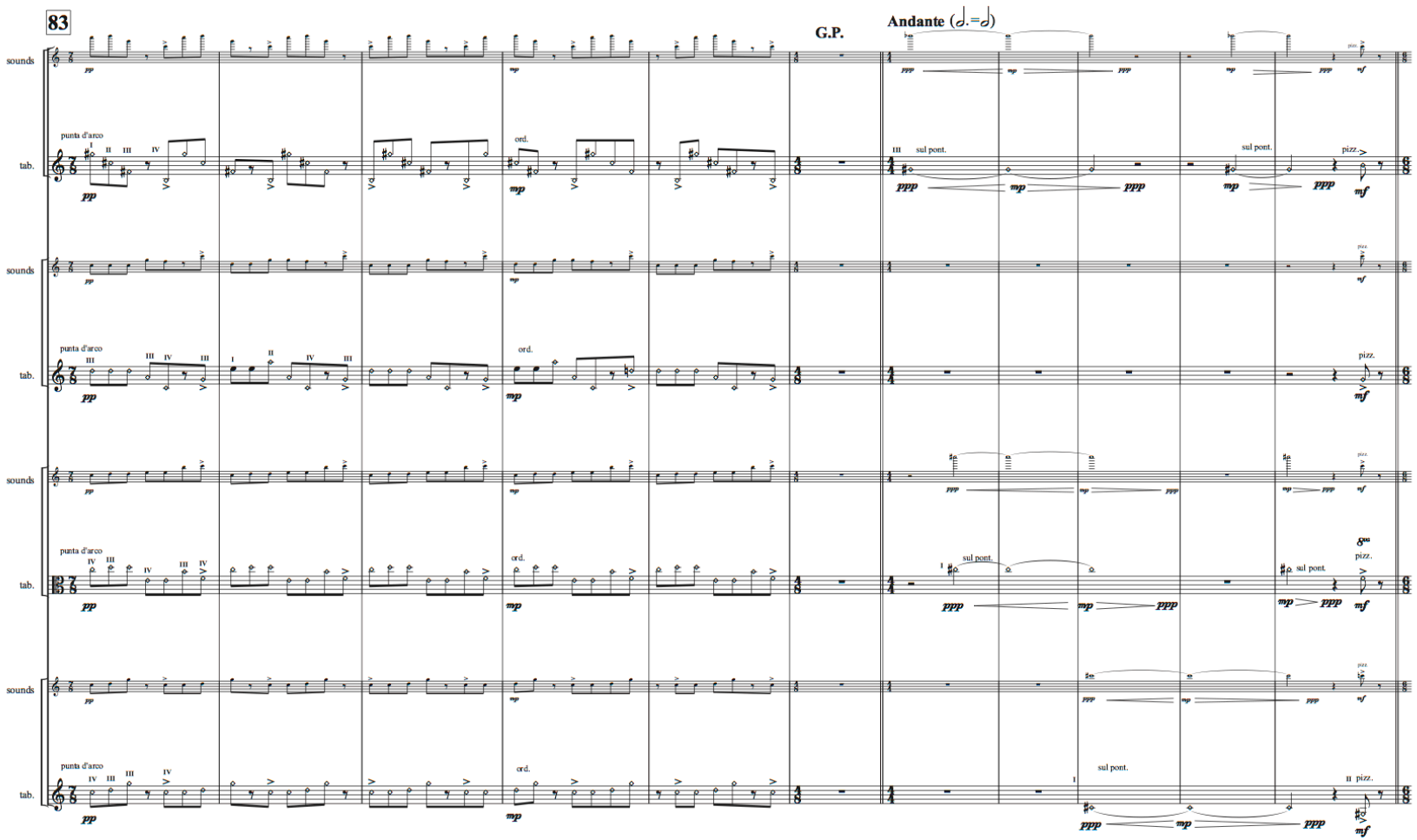

8
sounds
tab.
ord.
punta d'arco
cresc. poco a poco
lightly

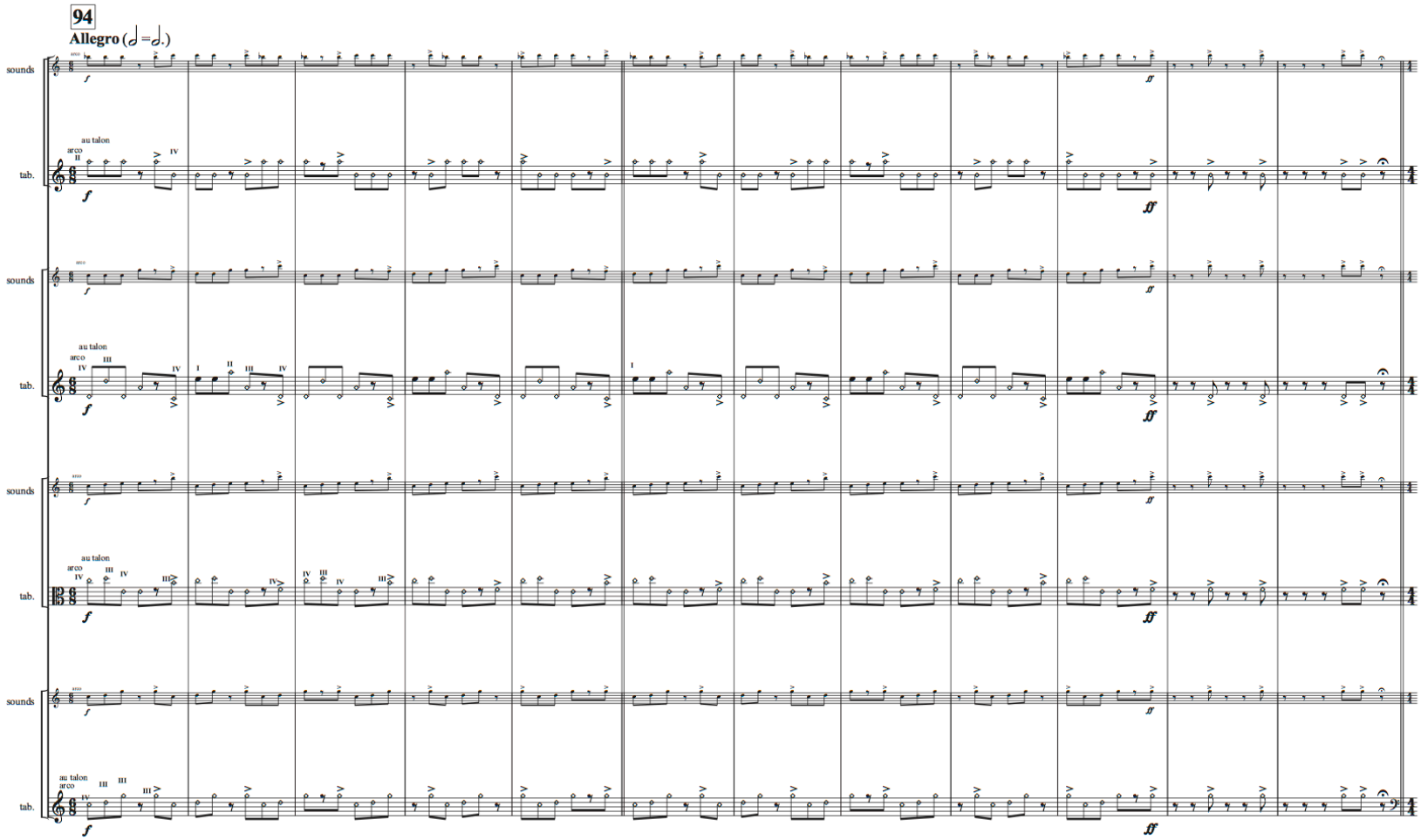

48
sounds
tab.
IV and III
au talon
mf
cresc.
ff

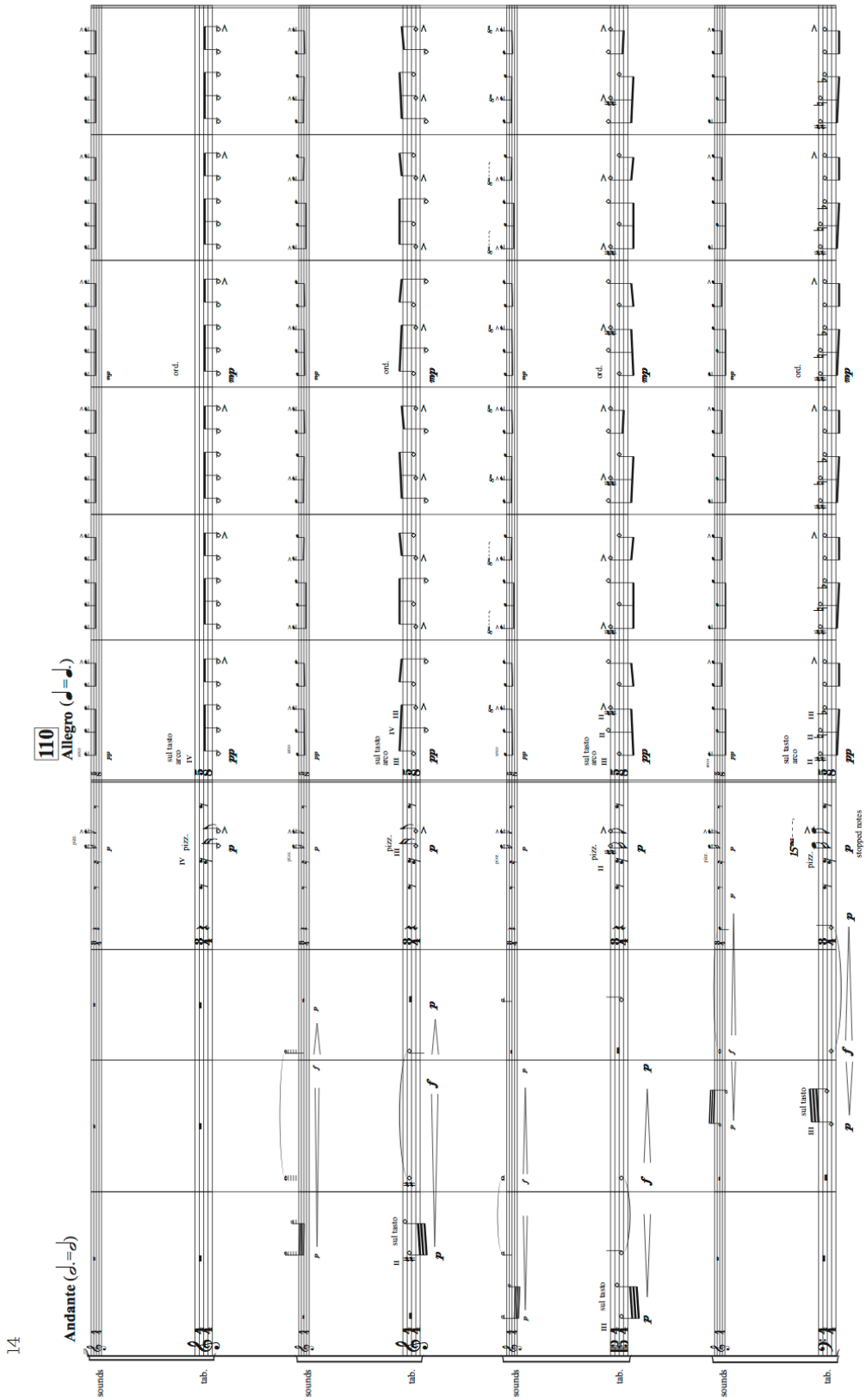

58
sounds
tab.
IV and III
punta d'arco
ord.
cresc. poco a poco

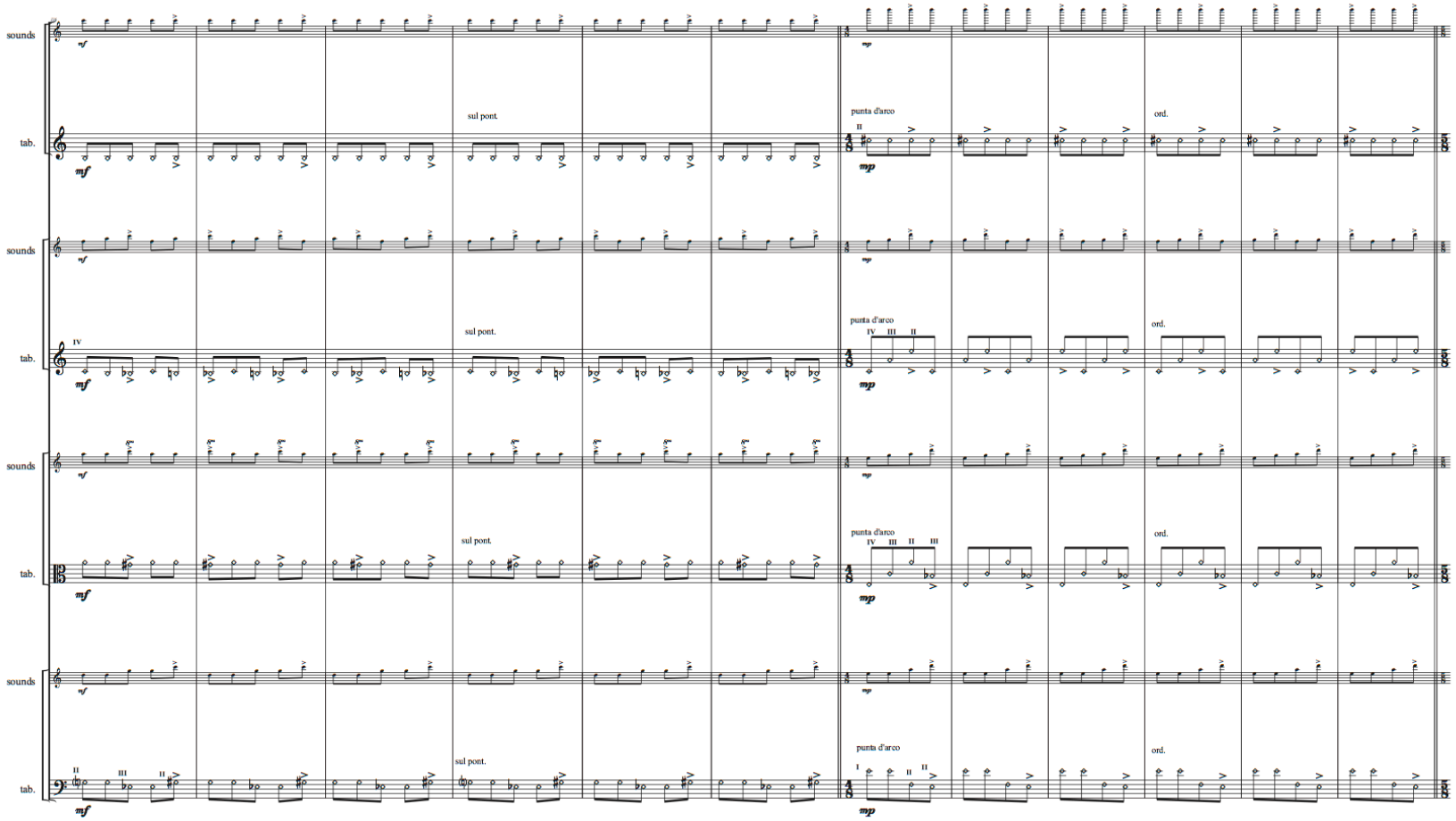
11

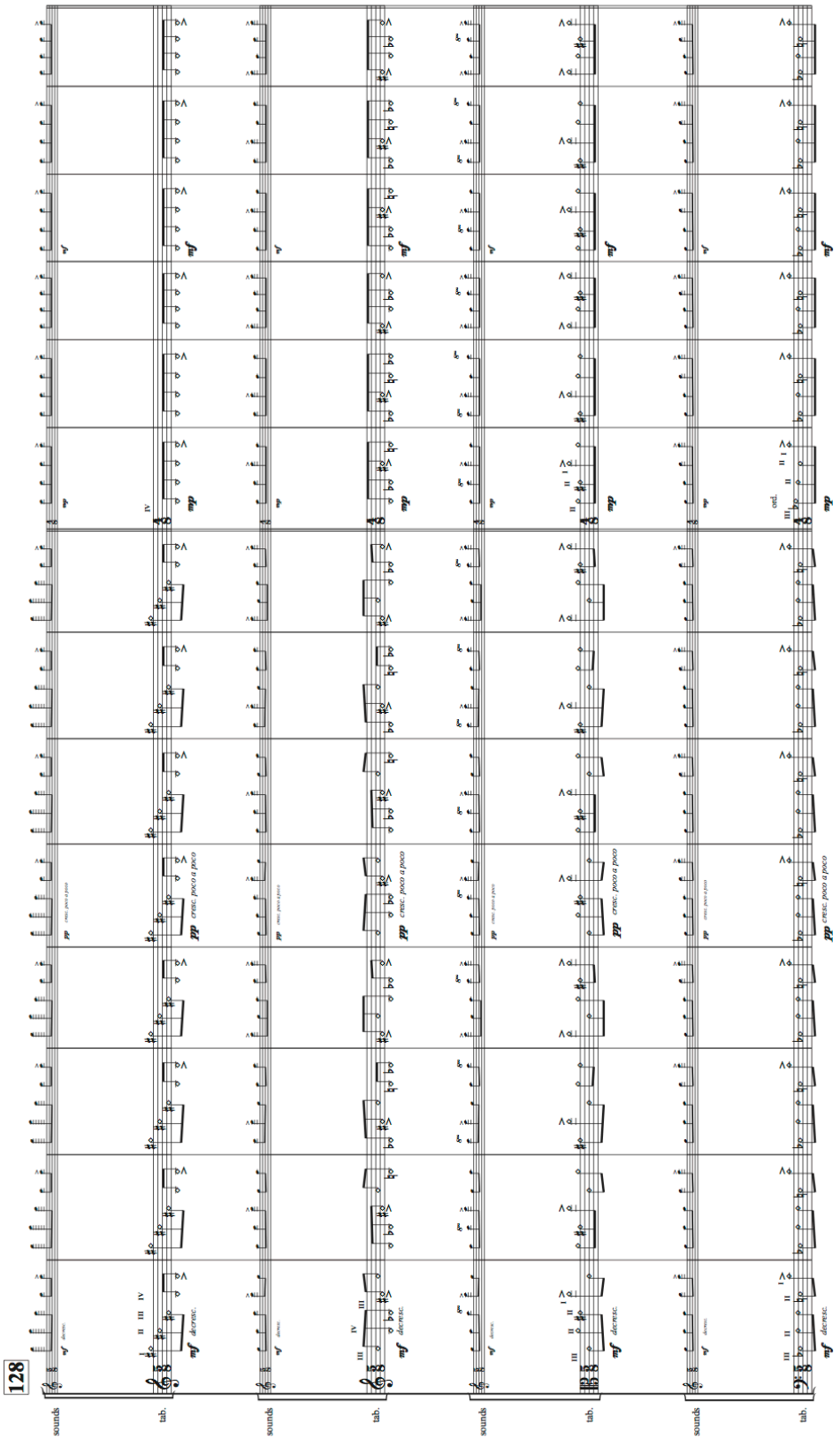



83
G.P.
Andante (𝅗𝅥.=𝅗𝅥)
sounds
tab.
punta d'arco
ord.
sul pont.
pizz.

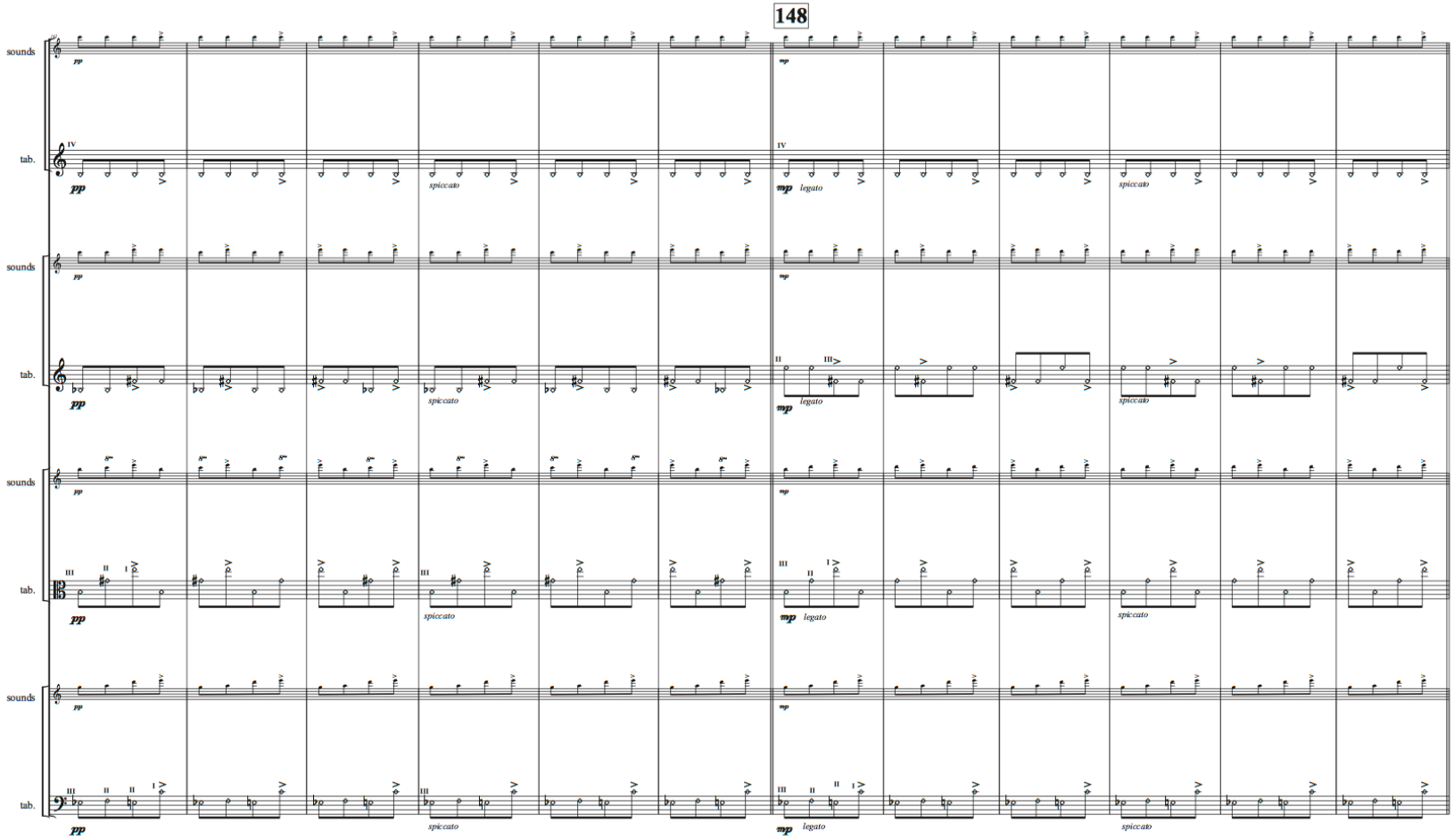

94
Allegro (𝅗𝅥=𝅗𝅥.)
sounds
tab.
au talon
arco
f
ff

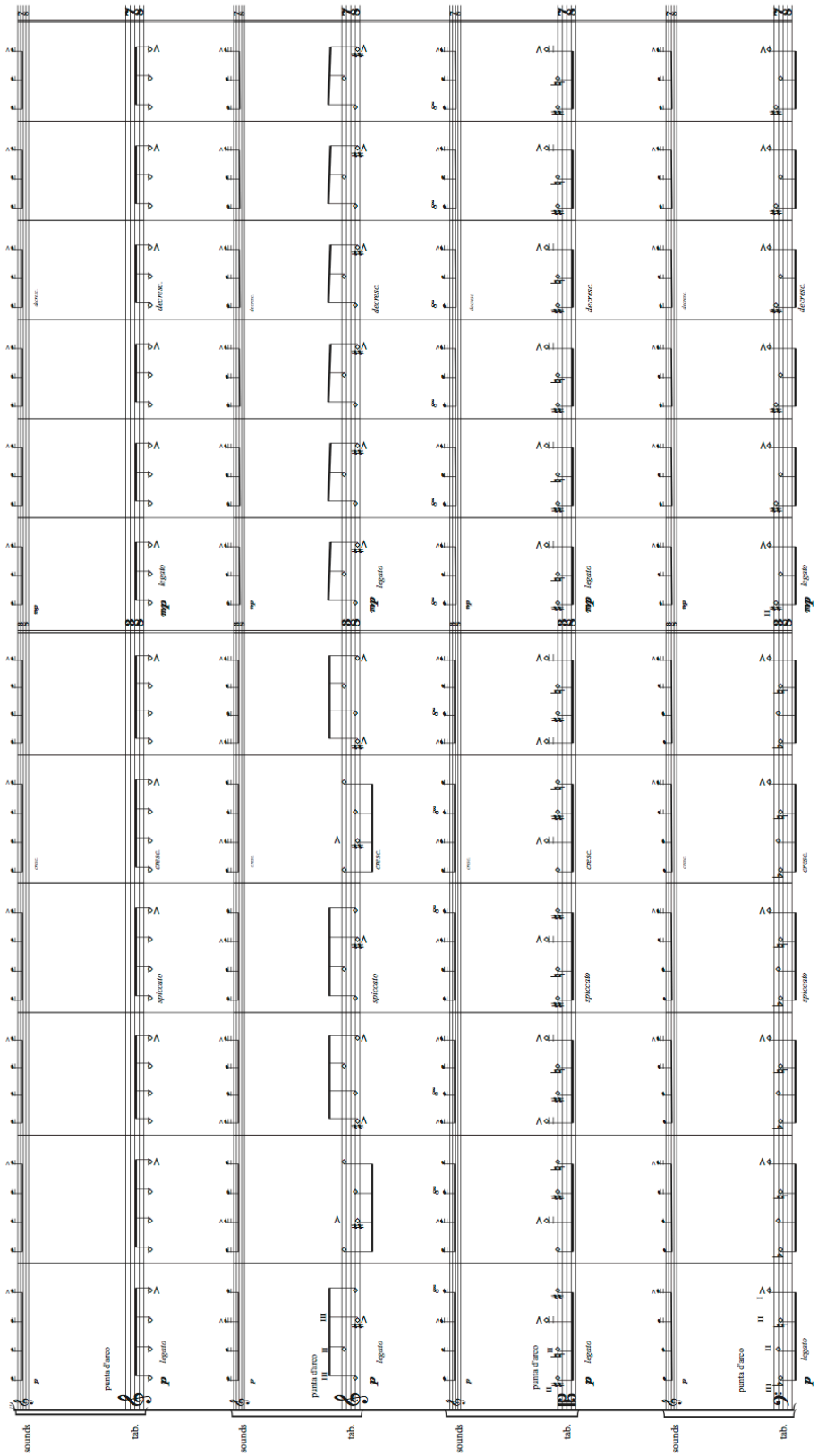

14
Andante (♩.=♩)
110
Allegro (♩=♩.)
sounds
tab.
pizz.
sul tasto
arco
ord.
stopped notes

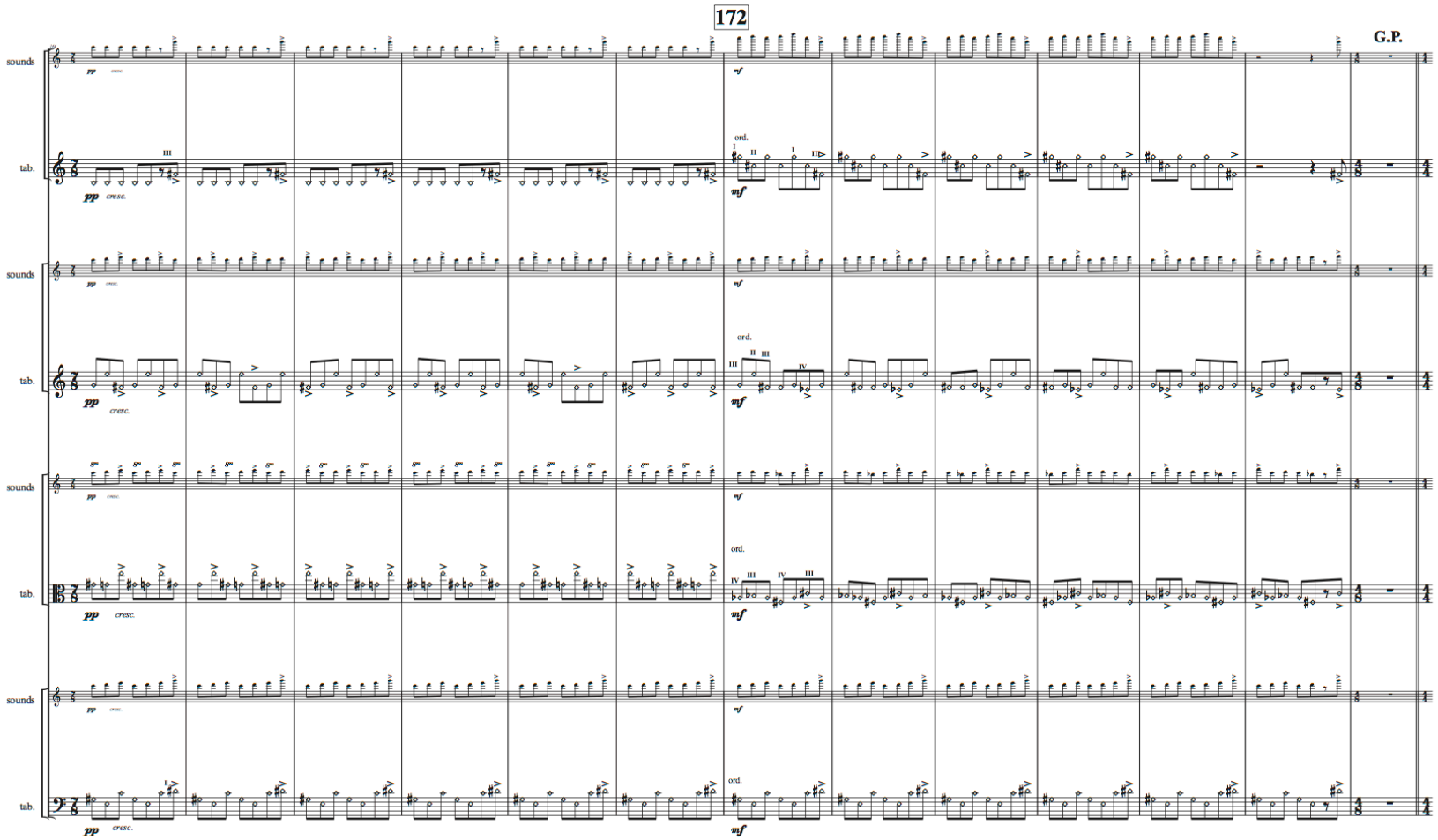

sounds
tab.
sounds
tab.
sounds
tab.
sounds
tab.
sounds
tab.
sul pont.
punta d'arco
ord.
mf
mp

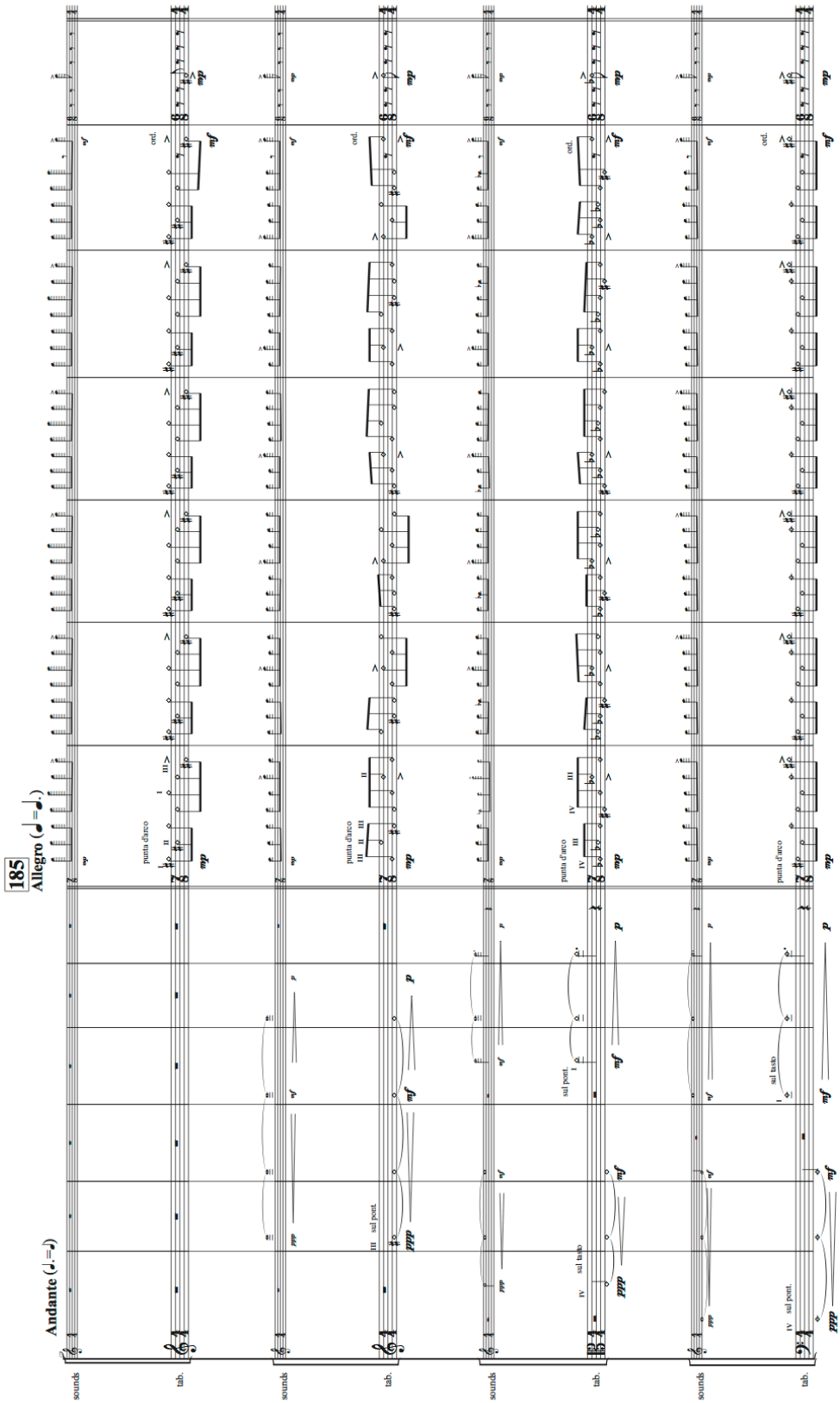

128
sounds
tab.
mf
mp
pp
ppp
cresc. poco a poco
CRESC. poco a poco
decresc.
ord.
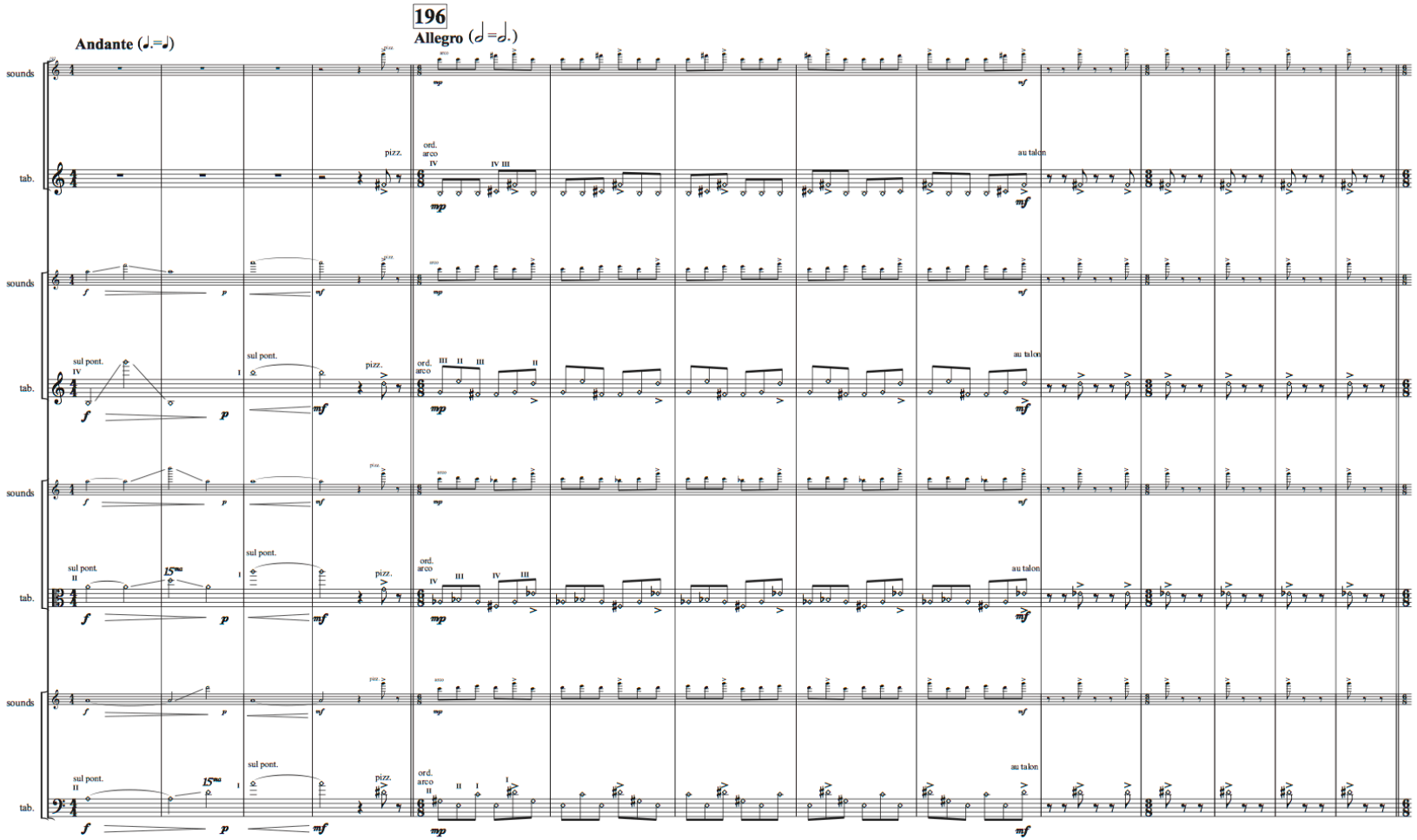
16

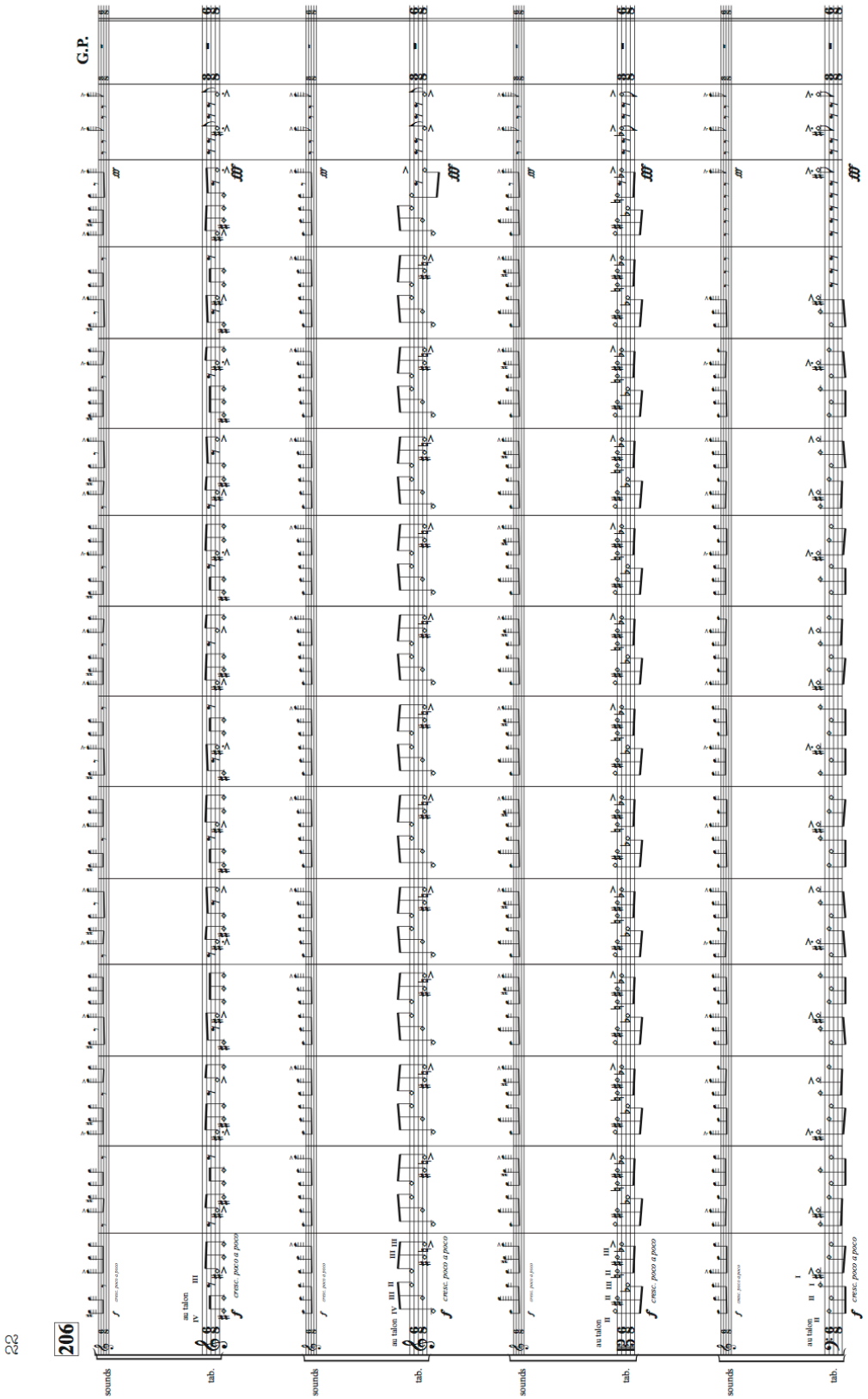

148
sounds
tab.
pp
mp
legato
spiccato

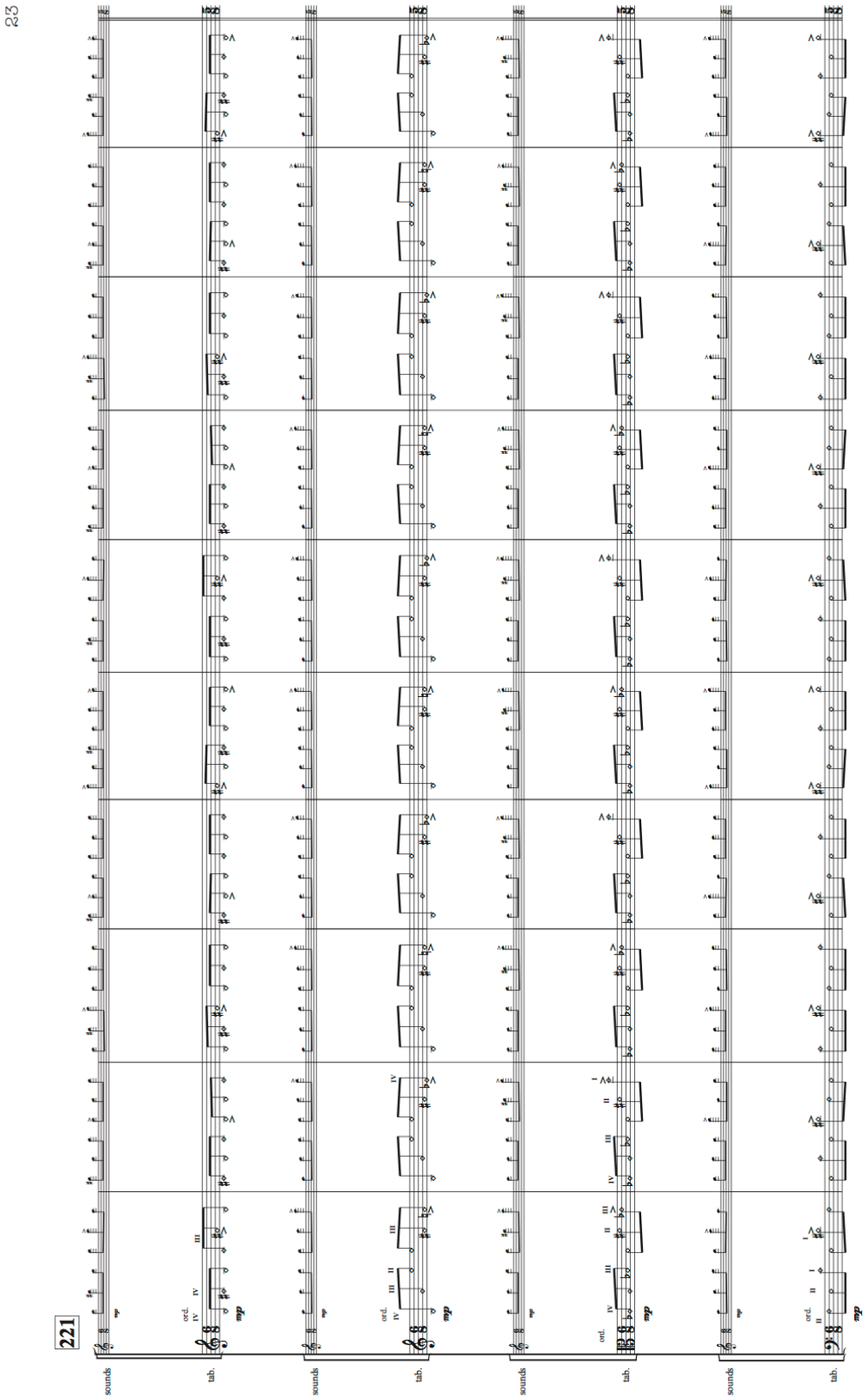

18
sounds
tab.
punta d'arco
legato
spiccato
cresc.
decresc.
mp
p

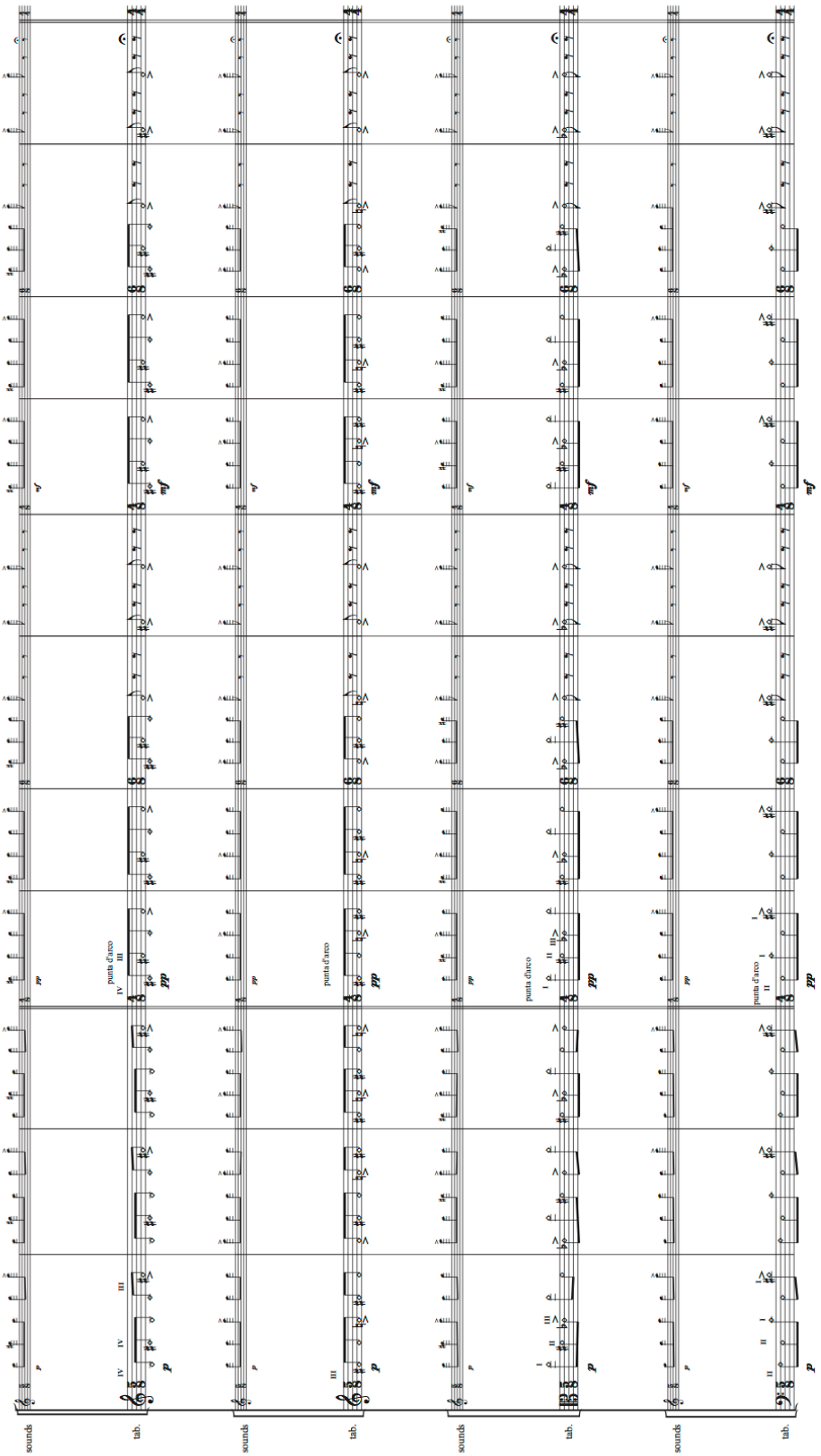

172
G.P.
sounds
tab.
pp
cresc.
mf
ord.
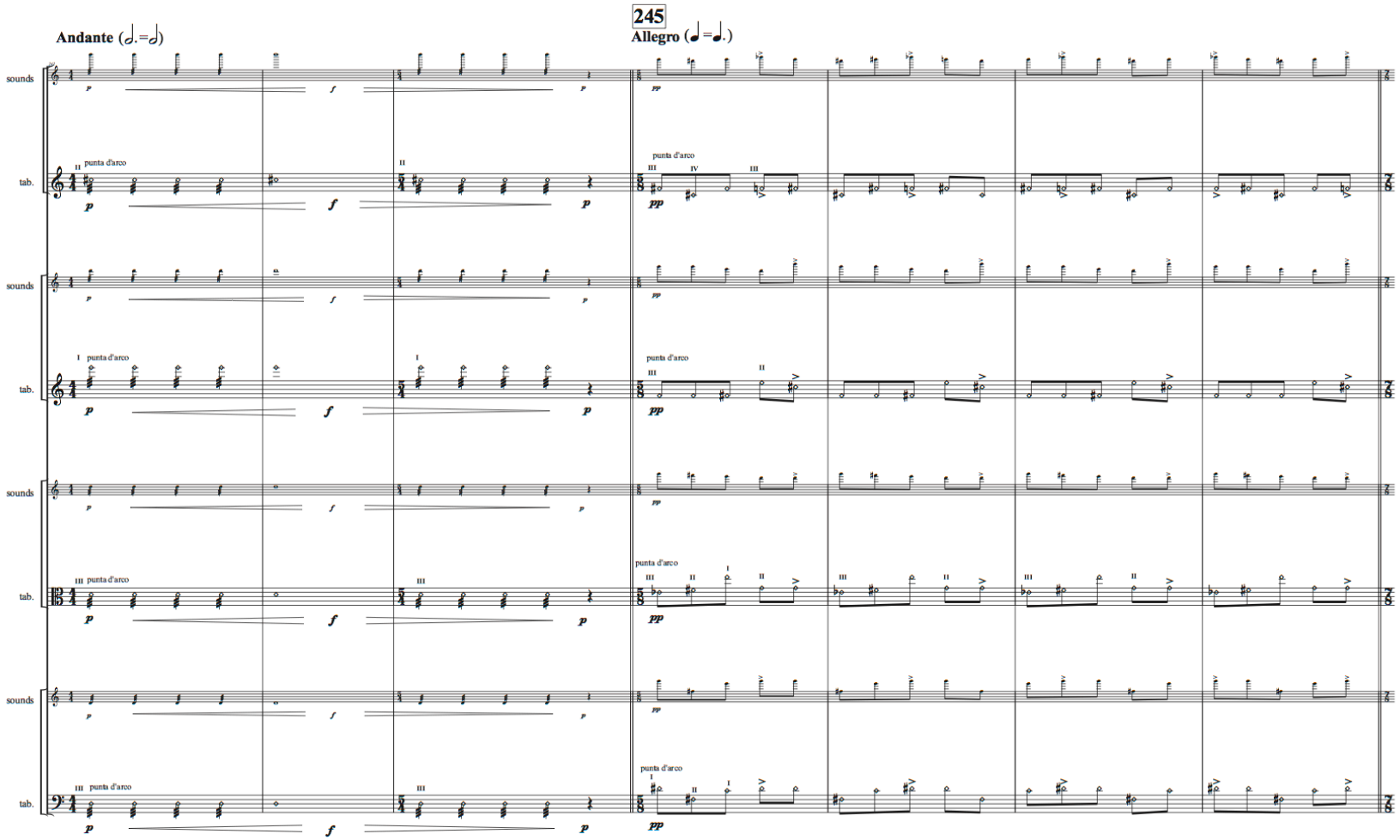

185
Allegro (♩ = ♩.)
Andante (♩. = ♩)
sounds
tab.
punta d'arco
ord.
sul ponti.
sul tasto
sul pont.

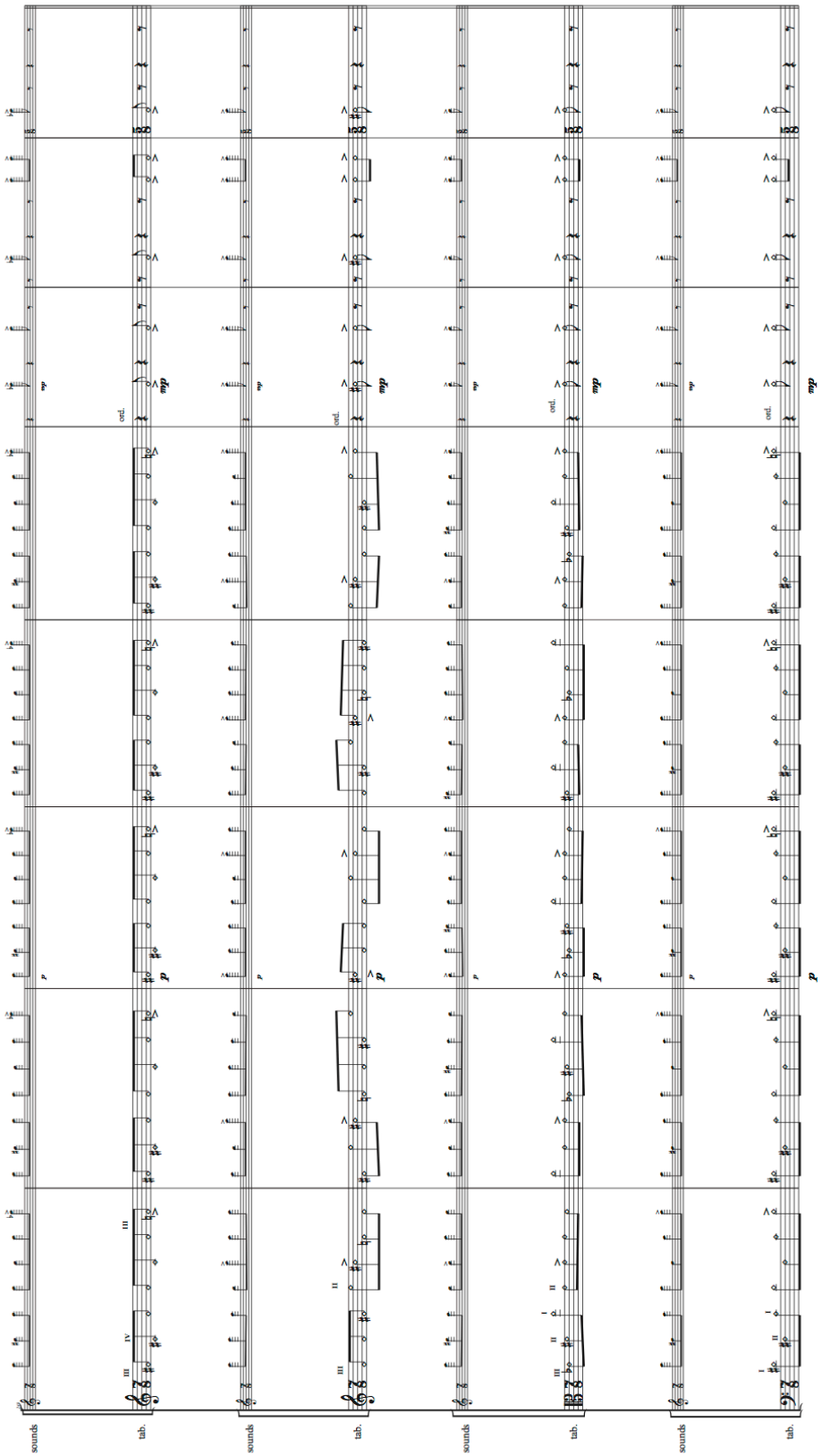

196
Andante (♩.=♩)
Allegro (𝅗𝅥=𝅗𝅥.)
sounds
tab.
pizz.
ord.
arco
au talon
sul pont.

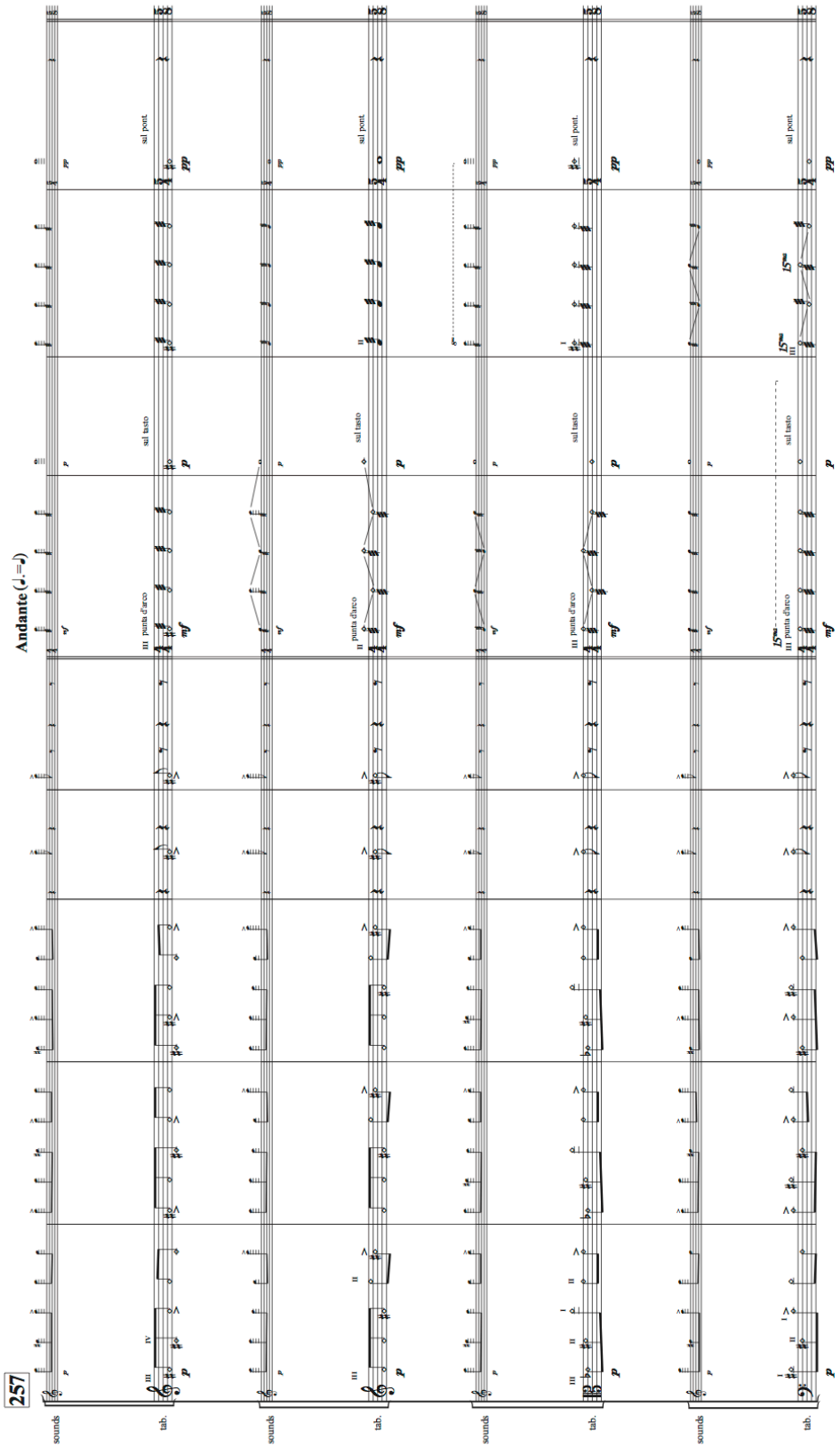

G.P.
206
sounds
tab.
au talon
cresc. poco a poco

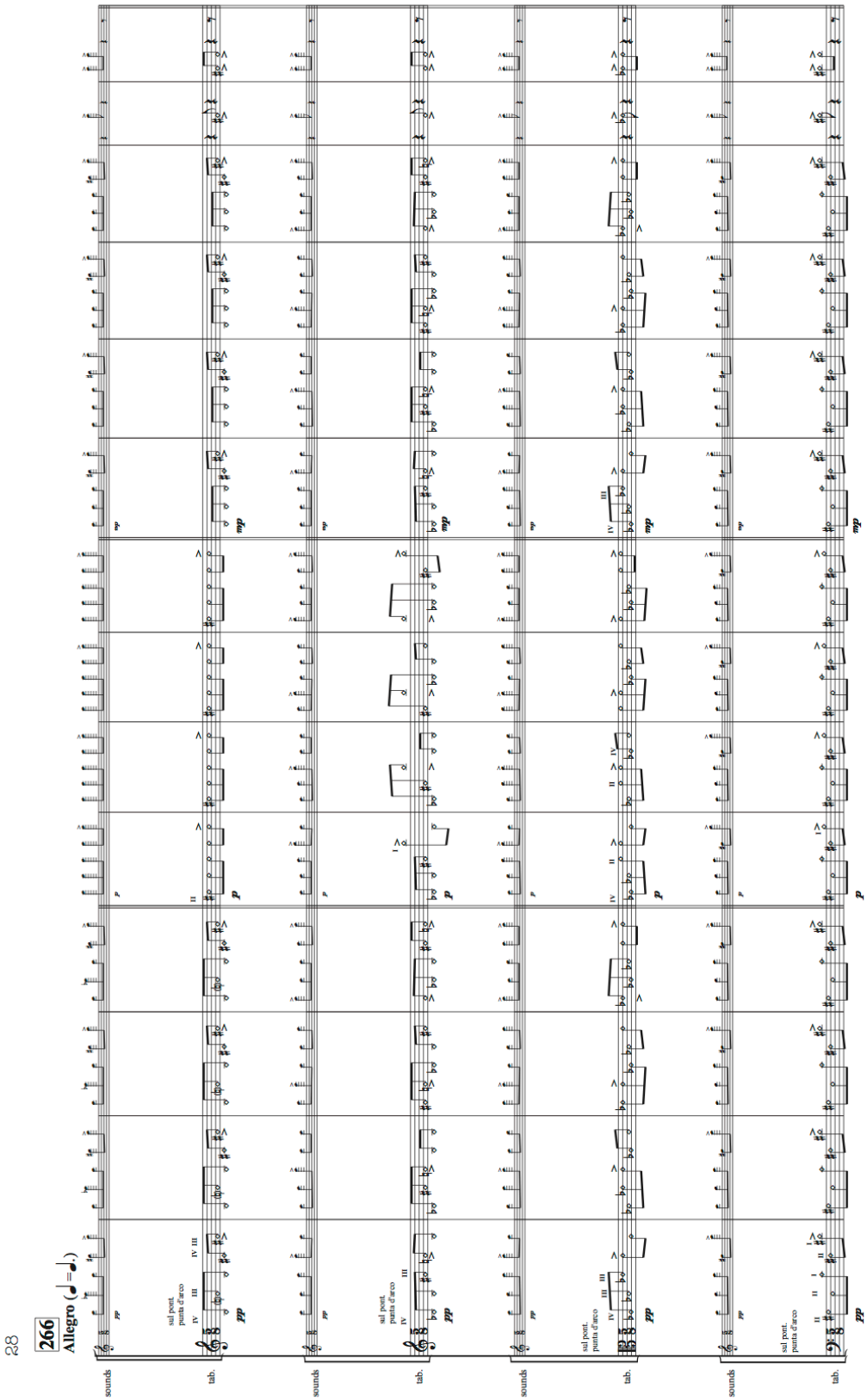

23
221
sounds
tab.
ord.
mp

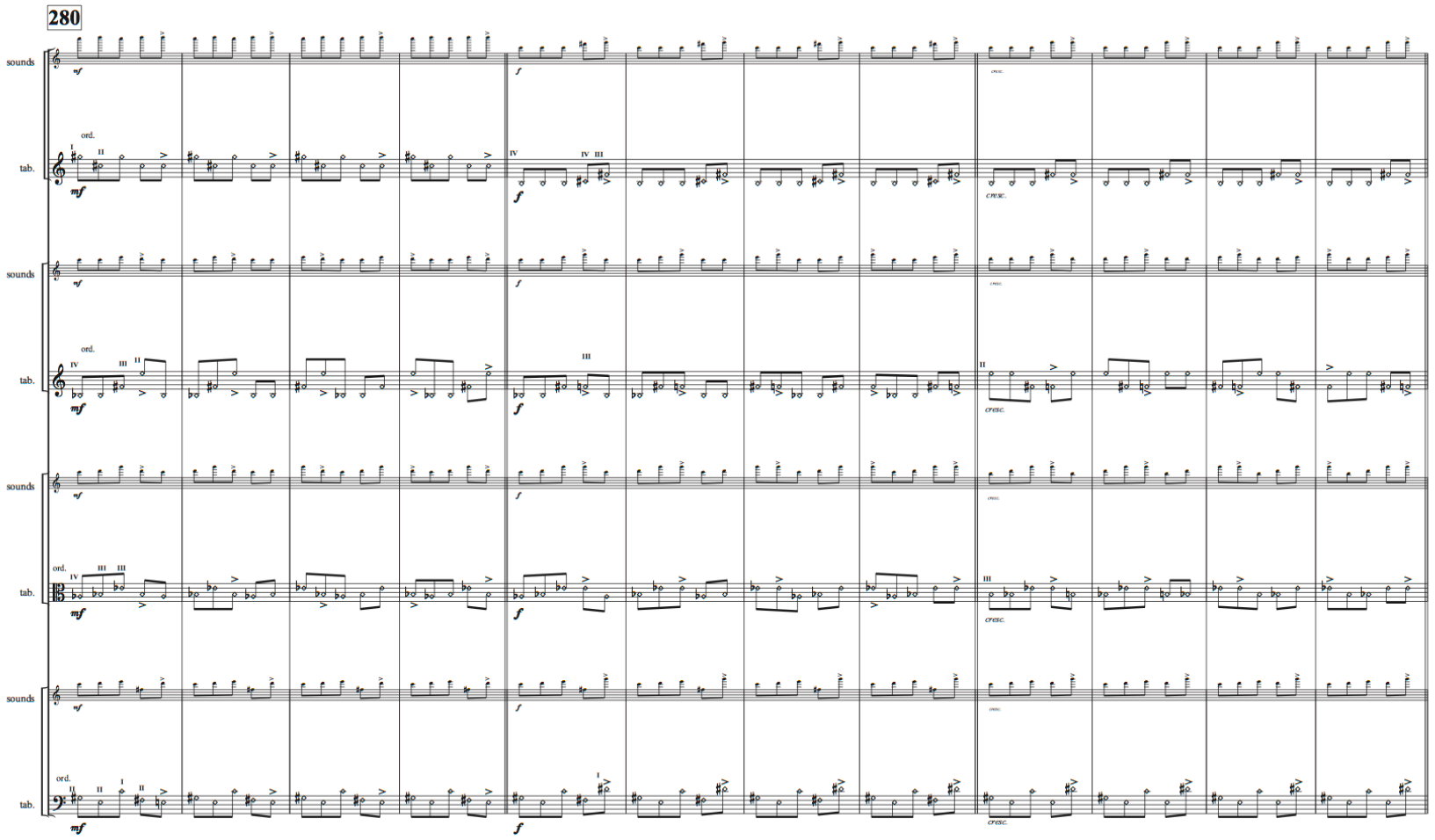

231
sounds
tab.
punta d'arco
punta d'arco
punta d'arco
punta d'arco

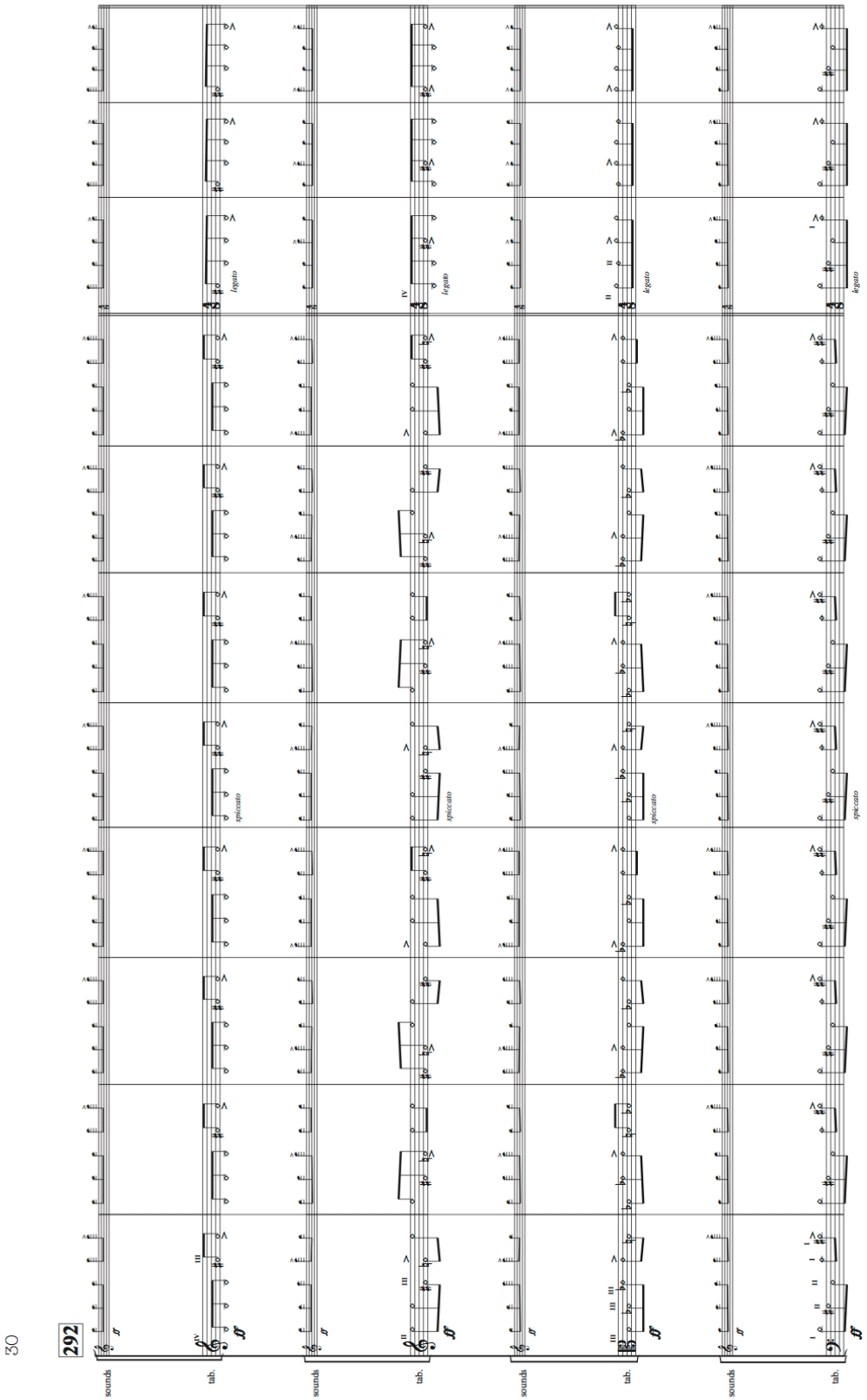

245
Andante (𝅗𝅥.=𝅗𝅥)
Allegro (♩=♩.)
sounds
tab.
punta d'arco

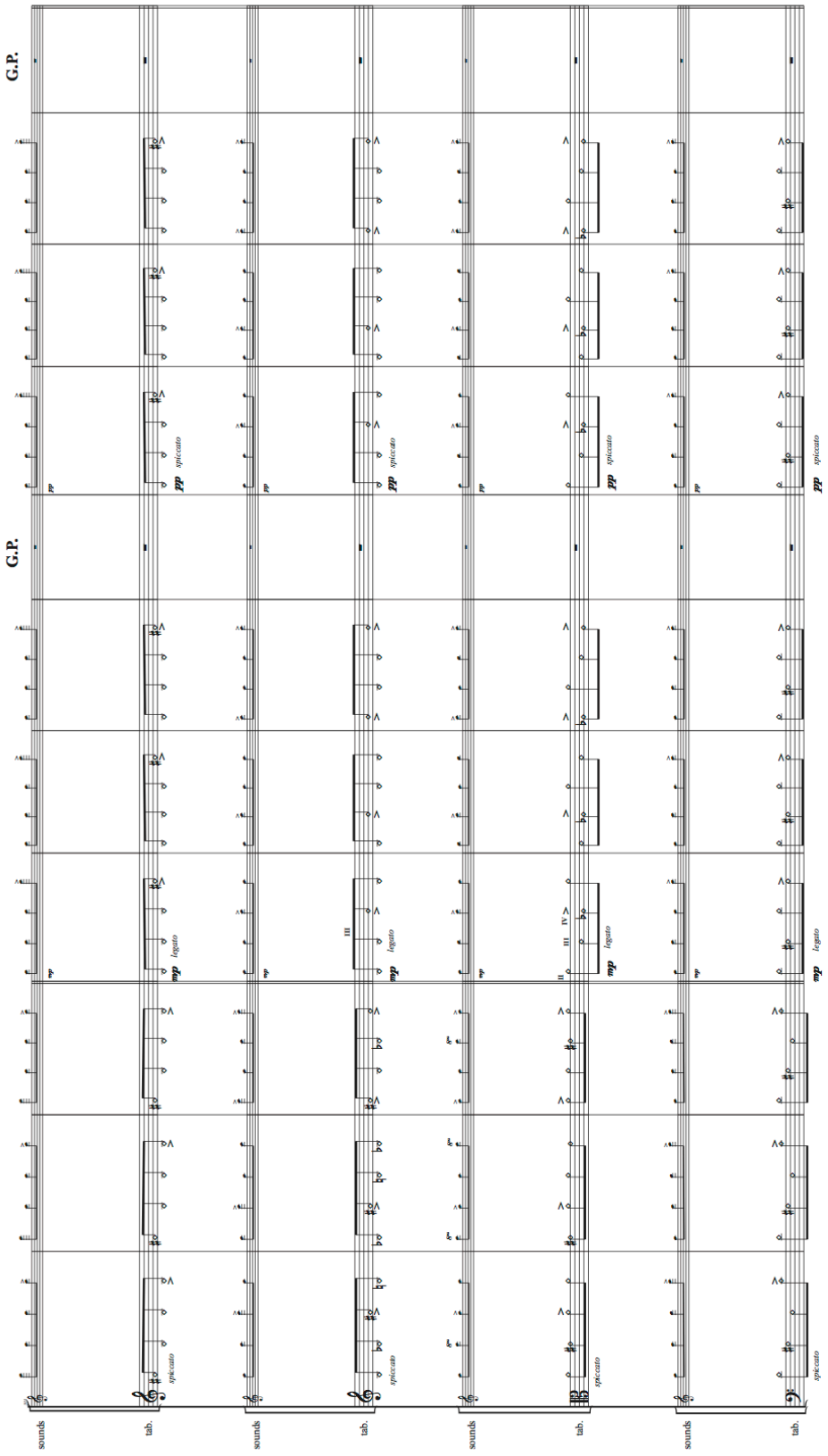

26
sounds
tab.
ord.
mp
p
f

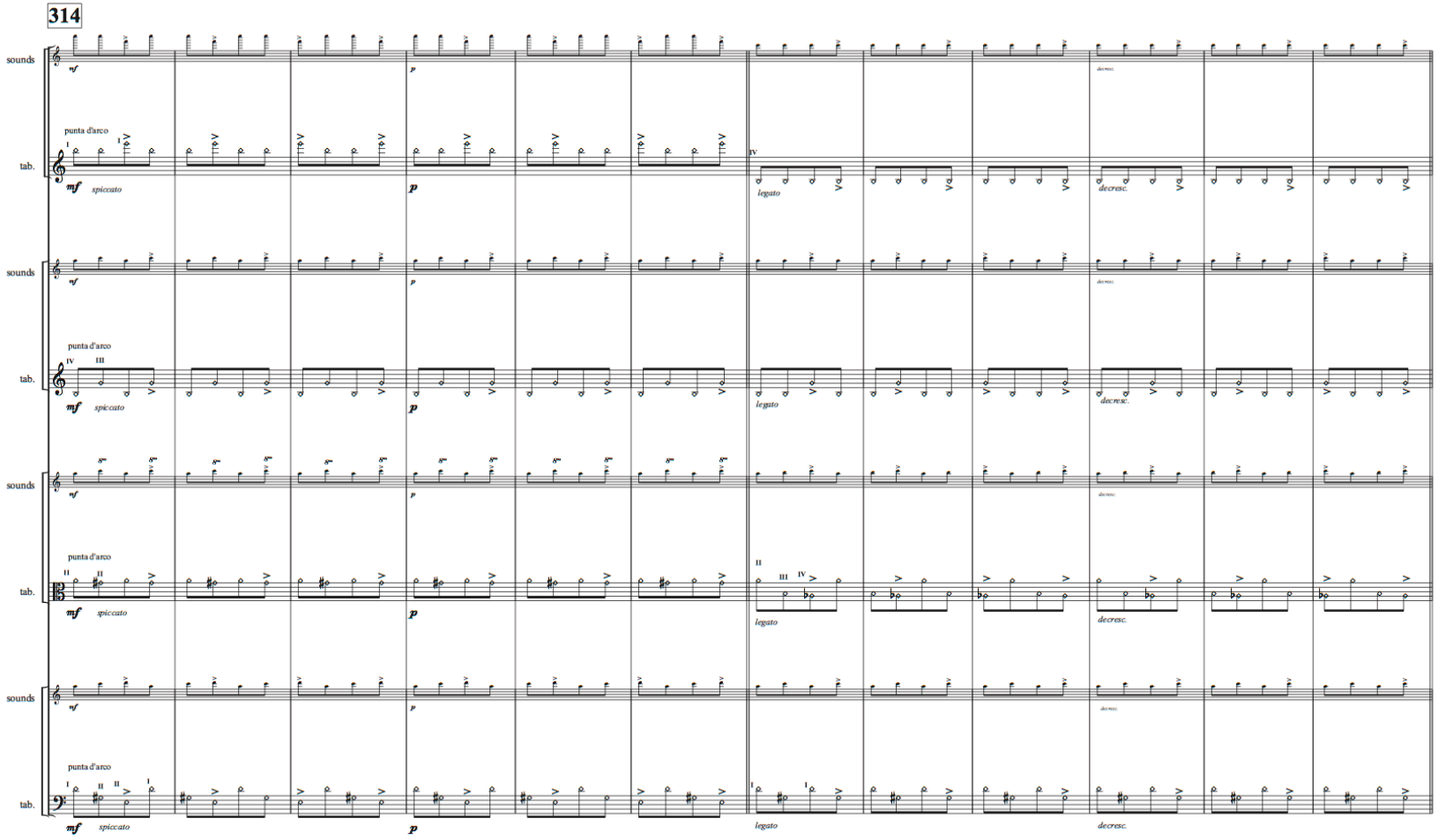

257
Andante (♩.=♩)
sounds
tab.
sul tasto
sul pont.
punta d'arco

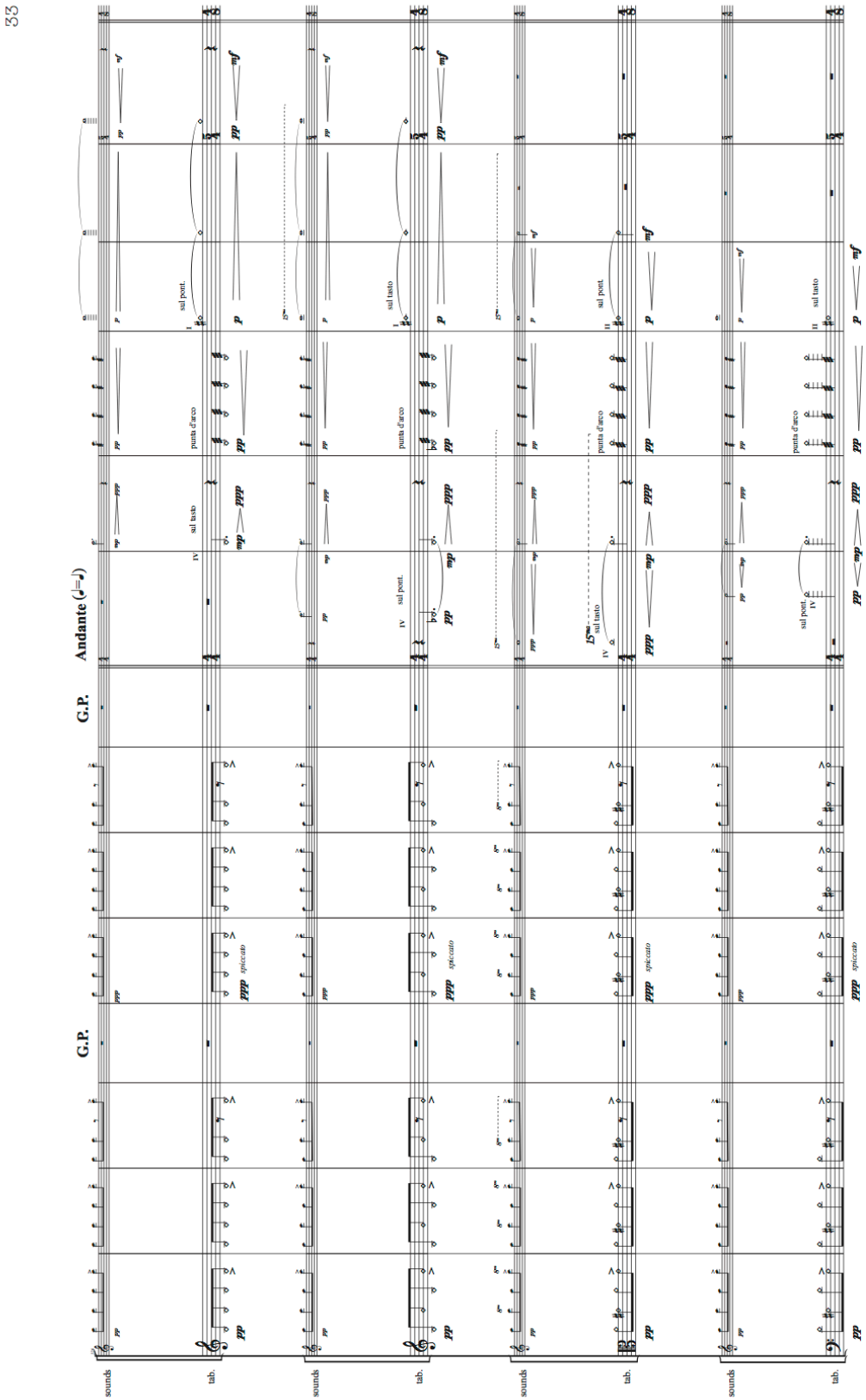

28
266
Allegro (♩ = ♩.)
sounds
tab.
sul pont.
punta d'arco

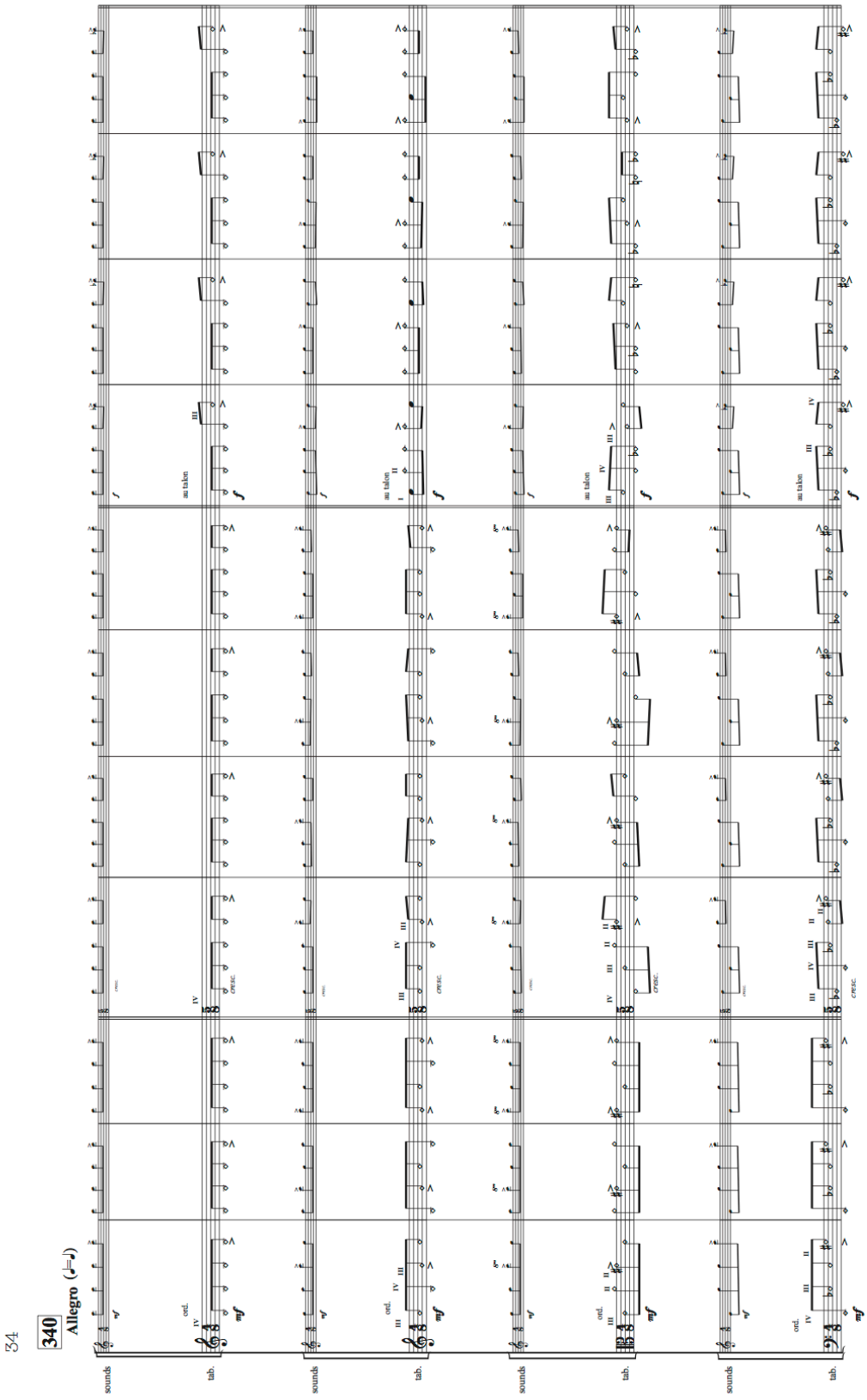

280
sounds
tab.
ord.
mf
f
cresc.

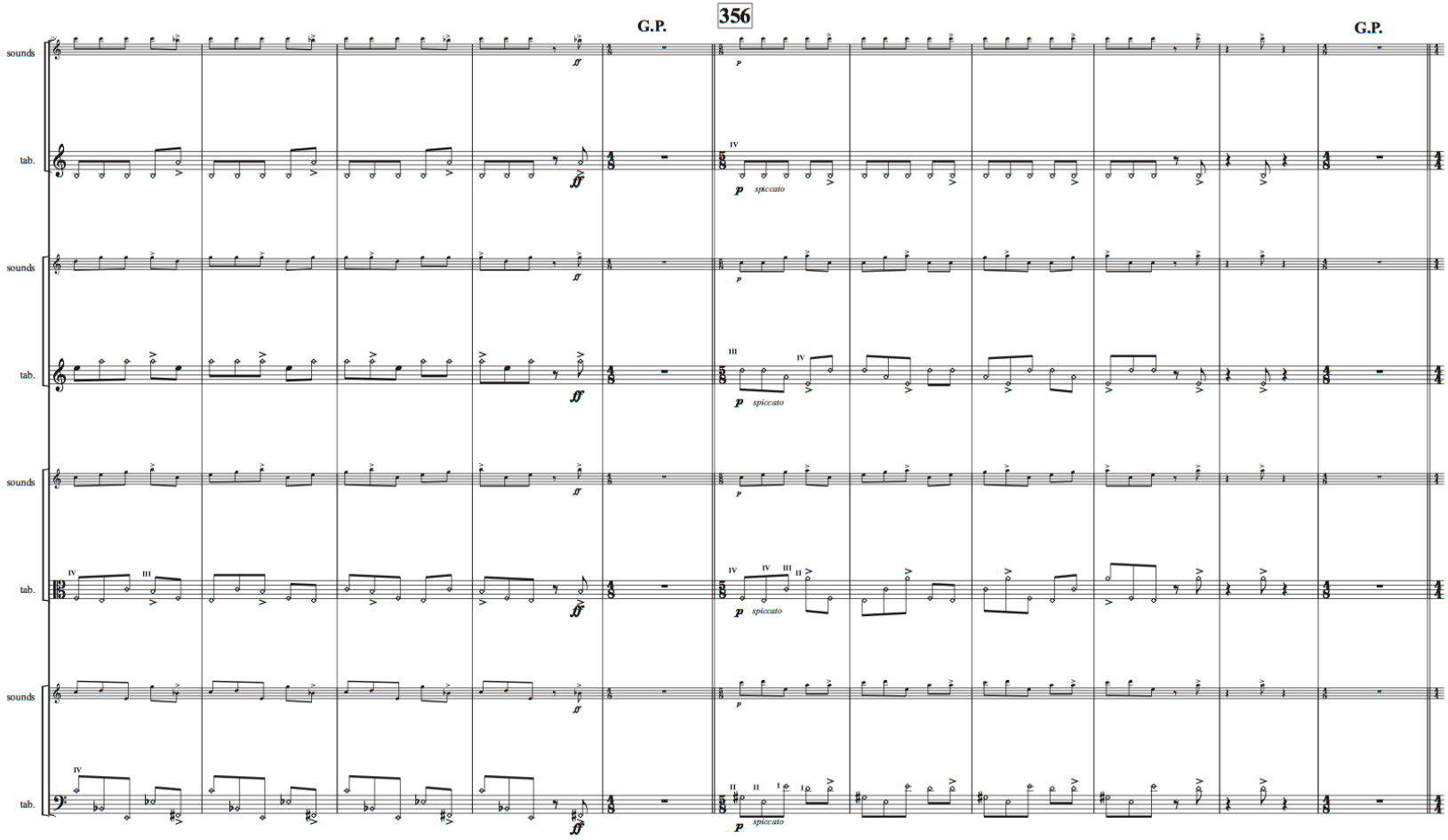

292
sounds
tab.
legato
spiccato

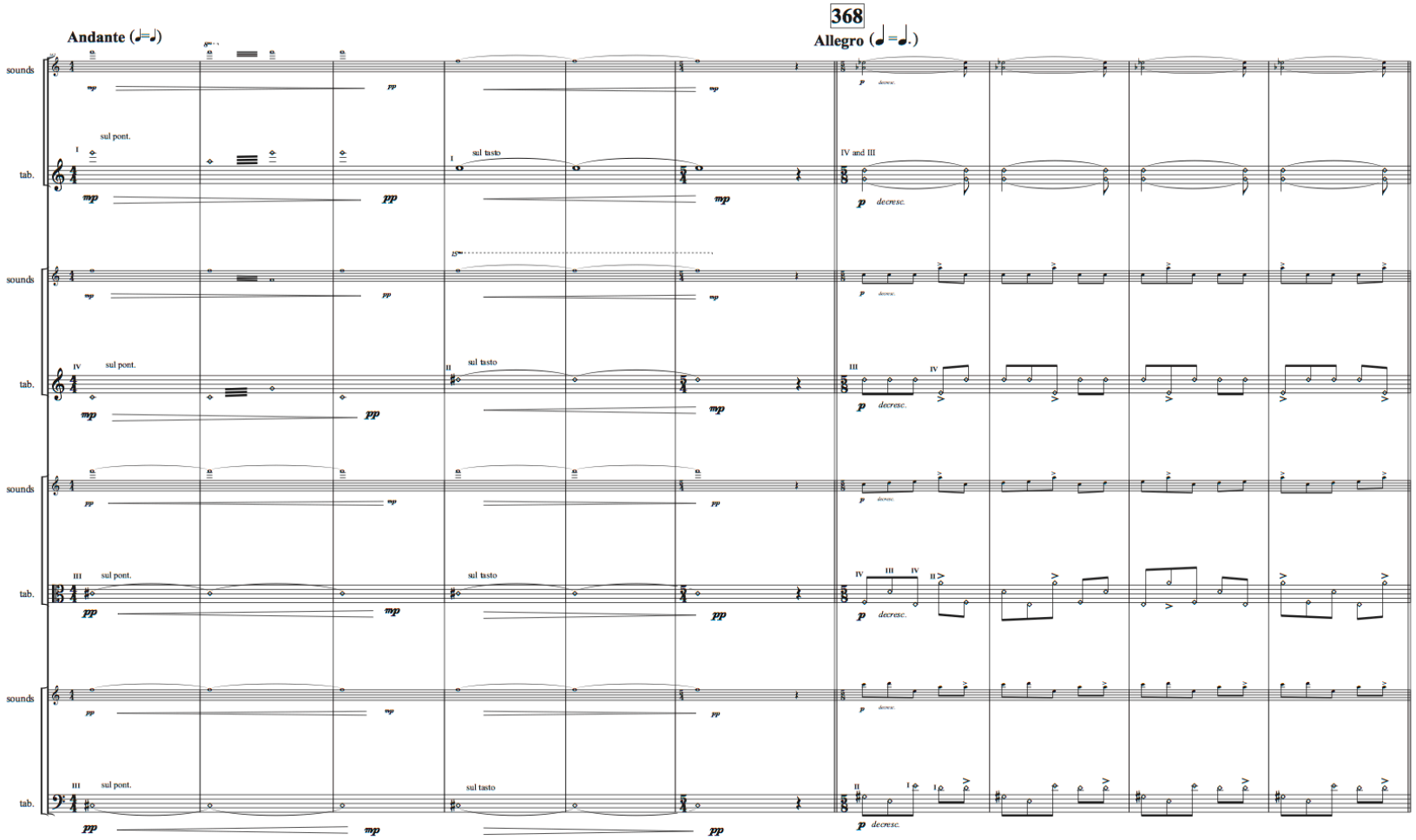

31
G.P.
G.P.
sounds
tab.
sounds
tab.
sounds
tab.
sounds
tab.
pp
ppp
spiccato
mp
legato

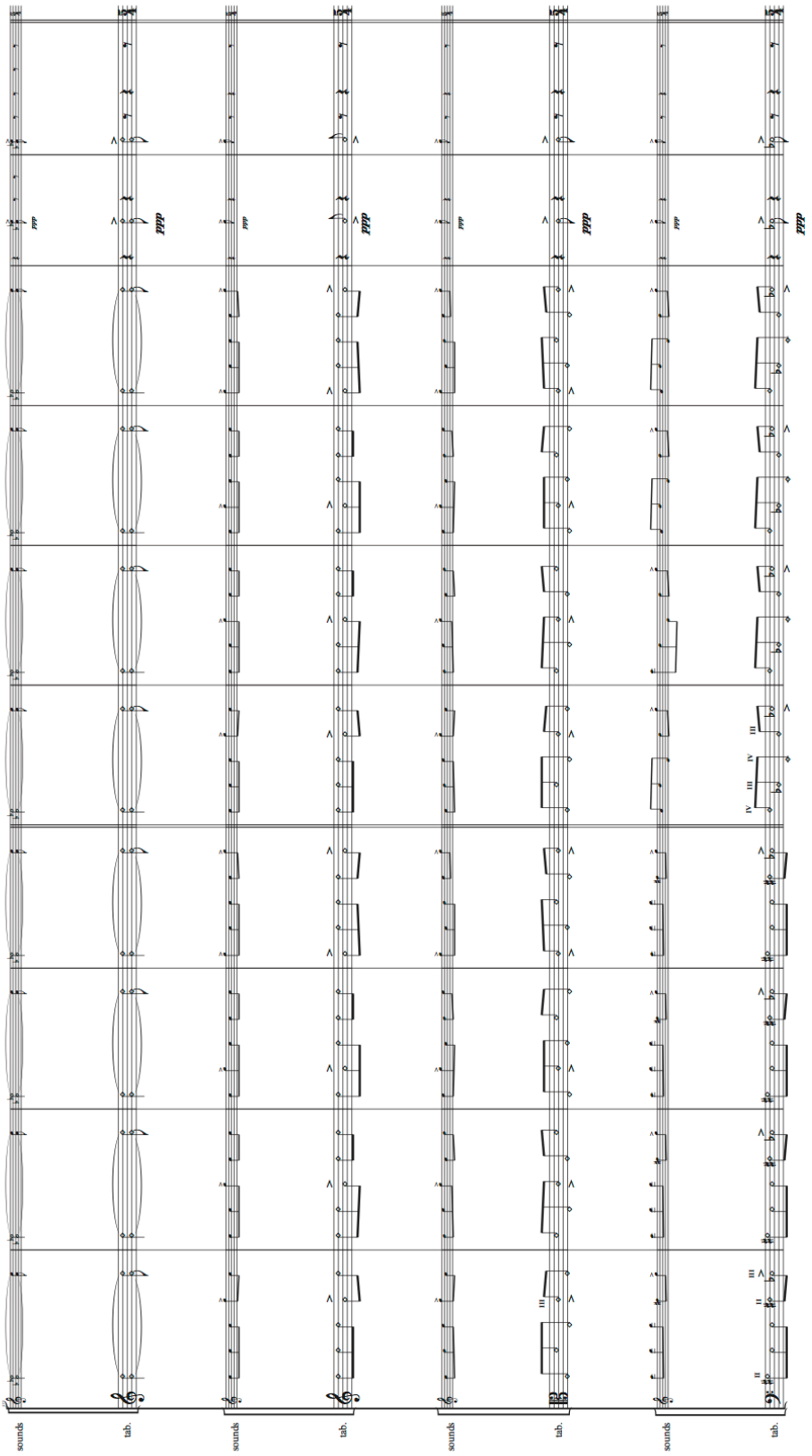

314
sounds
tab.
punta d'arco
mf
spiccato
p
legato
decresc.

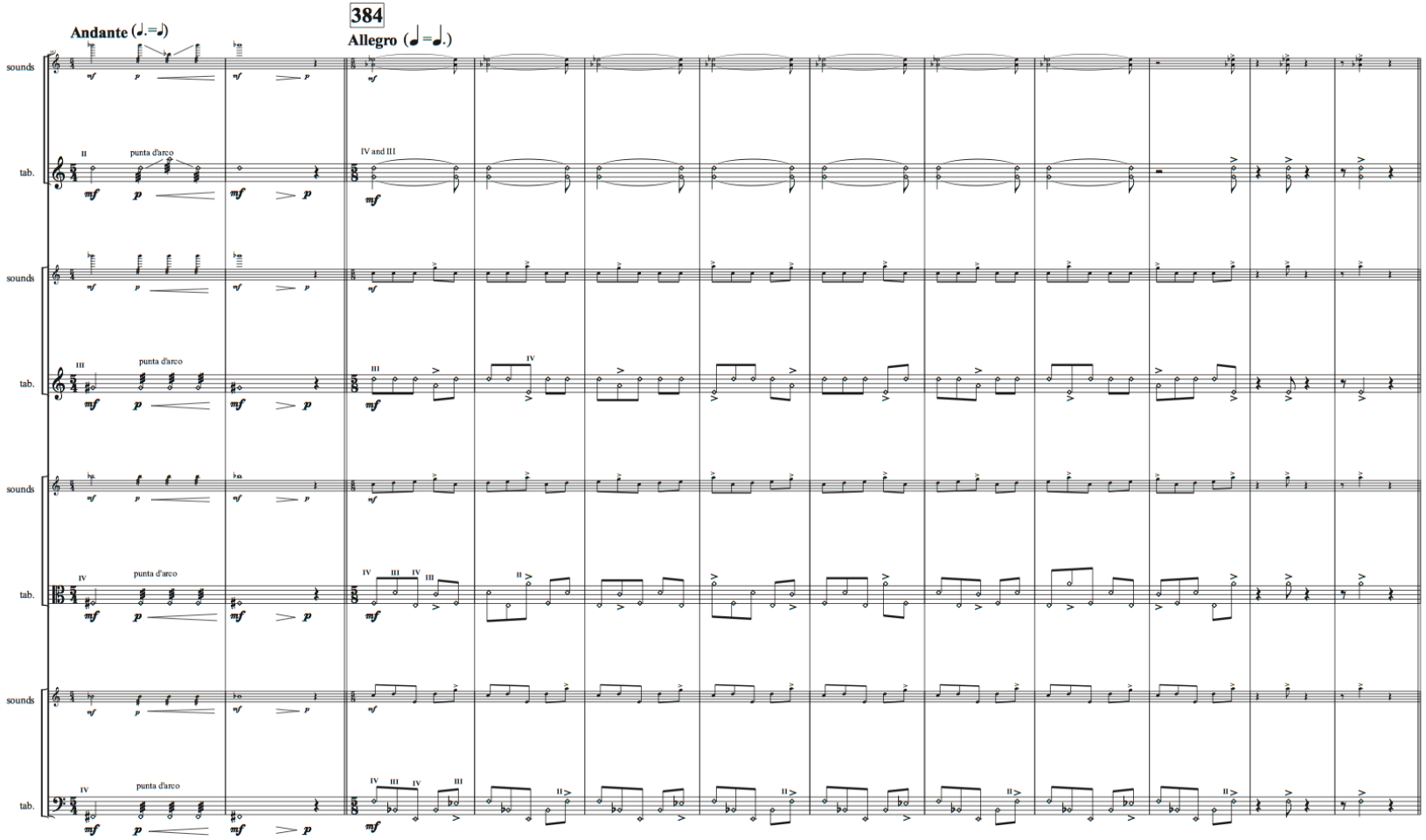

33
G.P.
Andante (♩=♩)
sul pont.
punta d'arco
sul tasto
spiccato
sounds
tab.

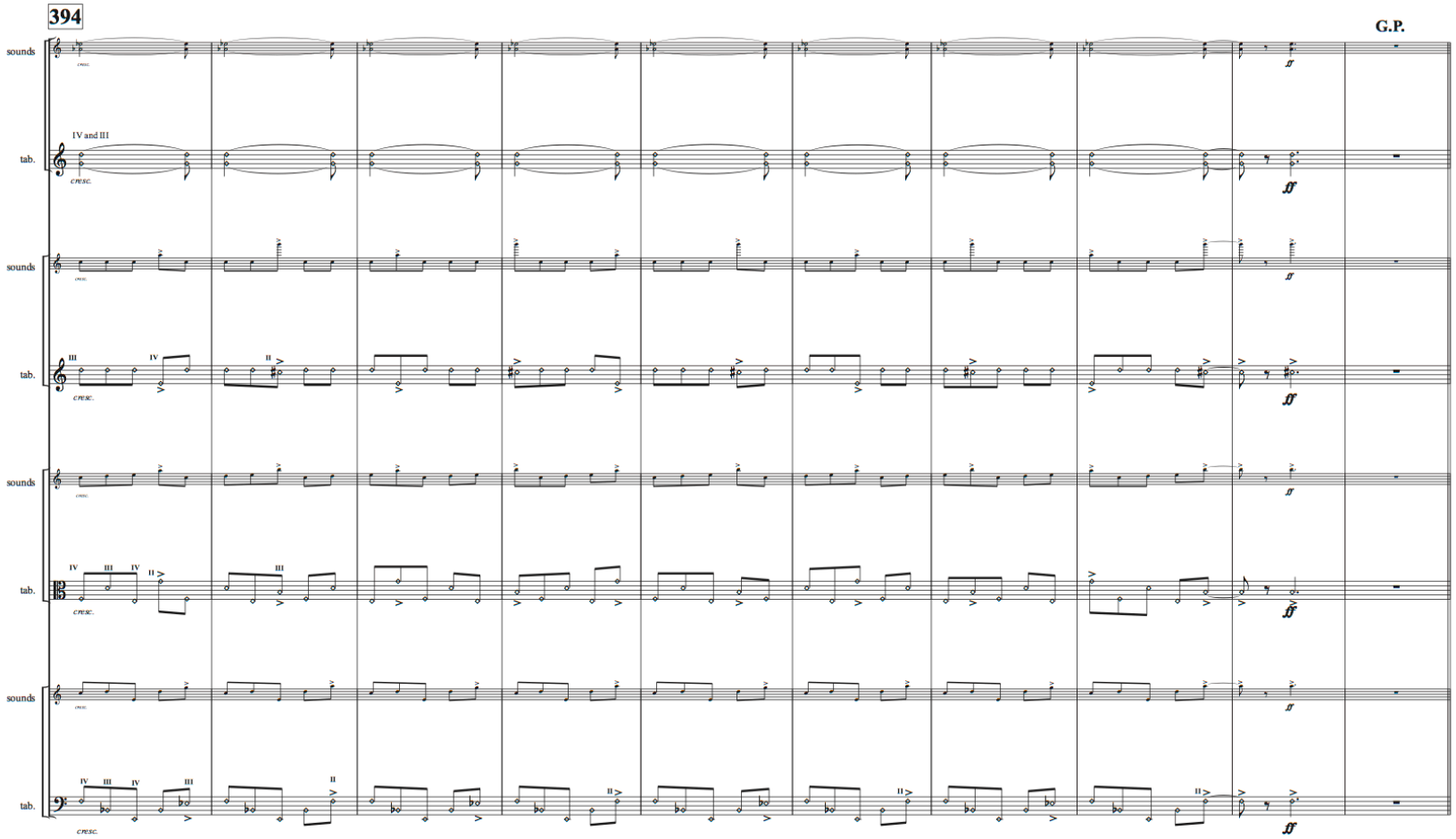

34
340
Allegro (♩=♩)
sounds
tab.
ord.
au talon
cresc.

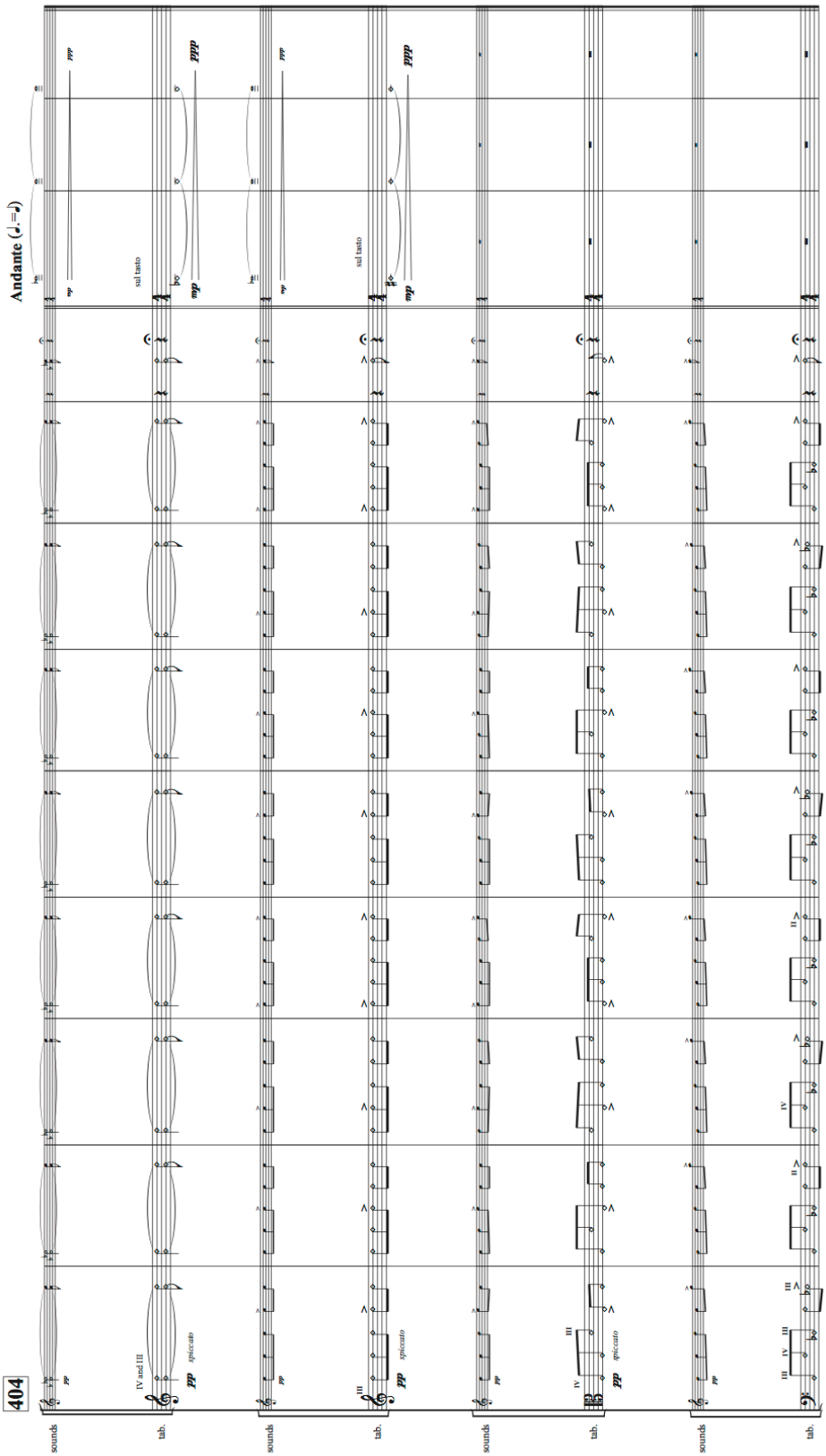

G.P.
356
G.P.
sounds
tab.
sounds
tab.
sounds
tab.
sounds
tab.
spiccato


Andante (♩=♩)
368
Allegro (♩=♩.)
sounds
tab.
sul pont.
sul tasto
IV and III
decresc.


37
sounds
tab.
sounds
tab.
sounds
tab.
sounds
tab.


Andante (♩.=♩)
384
Allegro (♩=♩.)
sounds
tab.
punta d'arco
IV and III


394
G.P.
sounds
tab.
IV and III
CRESC.
sounds
tab.
CRESC.
sounds
tab.
CRESC.
sounds
tab.
CRESC.
sounds
tab.
CRESC.


Andante (♩.=♩)
404
sul tasto
sul tasto
IV and III
sounds
tab.
sounds
tab.
sounds
tab.
sounds
tab.

3.4. *The Un-tempered Pianos* for two pianos and one pianist

The Un-tempered Pianos

for two pianos and one pianist

Dylan Crismani

2

Lattice

36015:32768
(583443:524288)
7203:4096
(583443:327680)
19:16
(1539:1280)
5145:4096
(83349:65536)
1029:1024
(83349:81920)
13:8
(1053:640)
735:512
(11907:8192)
147:128
(11907:10240)
12005:8192
(194481:131072)
2401:2048
(194481:163840)
105:64
(1701:1024)
21:16
(1701:1280)
33:32
(2673:2560)
1715:1024
(27783:16384)
343:256
(27783:20480)
15:8
(243:128)
3:2
(243:160)
245:128
(3969:2048)
49:32
(3969:2560)
35:32
(567:512)
7:4
(567:320)
11:8
(891:640)
5:4
(81:64)
1:1
(81:80)

TEMPI

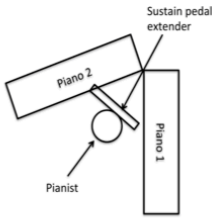
The Untempered Pianos for two pianos and pianist is a minimalist work and may cause fatigue even in experienced players. For this reason it should be noted that all tempi are flexible and the performer may exercise discretion to decide on which tempi are suitable for them. Rubato in all sections is also acceptable, given that the rubato is musically coherent. The length of each section is to be determined by the performer, and should take into consideration the speed with which they are attempting for each section. So, for example one pianist may decide to make minimal use of repeats and take quite a fast tempo as the piece will then be less fatiguing. Alternatively, another pianist may decide upon slower overall tempi, and greater use of the repeats. Both choices, among others would be acceptable interpretations.

ON THE TONE COLOUR OF THE PIANOS

Piano 1 should have a bright, crisp and clear tone colour, for example a Yamaha or a Steinway.
Piano 2 should have a mellow and slightly dull tone colour, for example a Kawai.

PHYSICAL ARRANGEMENT OF THE PIANOS

The two pianos should be arranged as per the diagram below in an L or V shape. Although it should be clear from the notation, it is intended that piano 2, the mellower of the two pianos, is to be played by the left hand, and the bright piano, or piano 1, is to be played by the right hand. During the time spent writing and performing this piece I found difficulty and pain in my knees trying to depress the sustain pedal on both pianos simultaneously for extended periods as the entire piece is to be played with both sustain pedals depressed all the time. My solution to this was a short plank of timber available from the local hardware shop placed over the pedal of piano number 2. This allowed my knees greater flexibility and resulted in no pain at all. By doing this, the pianist still has control over exactly how depressed the sustain pedal is as the plank of timber acts as an extention of the pedal, it doesn't merely keep it depressed all the way all the time.



HEALTH WARNING: Pitches will not sound as they appear on the score. The score is written as a performance aid for the pianist only and is not an accurate representation of the actual sounding pitches.

4

Tuning instructions

Piano 1 tuning

1:1	33:32	35:32	19:16	5:4	21:16	11:8	3:2	13:8	105:64	7:4	15:8

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32	64	128	256	512	1024
C#	33	66	132	264	528	1056
D	35	70	140	280	560	1120
D#	38	76	152	304	608	1216
E	40	80	160	320	640	1280
F	42	84	168	336	672	1344
F#	44	88	176	352	704	1408
G	48	96	192	384	768	1536
G#	52	104	208	416	832	1664
A	52.5	105	210	420	840	1680
A#	56	112	224	448	896	1792
B	60	120	240	480	960	1920

Piano 1 tuning, plus 81:80

81:80	2673:2560	567:512	1539:1280	81:64	1701:1280	891:640	243:160	1053:640	1701:1024	567:320	243:128

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.4	64.8	129.6	259.2	518.4	1036.8
C#	33.41	66.82	133.64	267.28	534.56	1069.12
D	35.44	70.88	141.76	283.52	567.04	1134.08
D#	38.48	76.96	153.92	307.84	615.68	1231.36
E	40.5	81	162	324	648	1296
F	42.53	85.06	170.12	340.24	680.48	1360.96
F#	44.55	89.1	178.2	356.4	712.8	1425.6
G	48.6	97.2	194.4	388.8	777.6	1555.2
G#	52.65	105.3	210.6	421.2	842.4	1684.8
A	53.16	106.32	212.64	425.28	850.56	1701.12
A#	56.7	113.4	226.8	453.6	907.2	1814.4
B	60.75	121.5	243	486	972	1944

Piano 2 tuning

1029:1024	36015:32768	147:128	2401:2048	5145:4096	343:256	735:512	12005:8192	49:32	1715:1024	7203:4096	245:128

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.16	64.32	128.64	257.25	514.5	1029
C#	35.17	70.34	140.68	281.36	562.72	1125.44
D	36.75	73.5	147	294	588	1176
D#	37.52	75.04	150.08	300.16	600.32	1200.64
E	40.2	80.4	160.8	321.6	643.2	1286.4
F	42.88	85.76	171.52	343.04	686.08	1372.16
F#	45.94	91.88	183.76	367.52	735.04	1470.08
G	46.89	93.78	187.56	375.12	750.24	1500.48
G#	49	98	196	392	784	1568
A	53.95	107.18	214.36	428.72	857.44	1714
A#	56.29	112.58	225.16	450.32	900.64	1801.28
B	61.25	122.5	245	490	980	1960

Piano 2 tuning, plus 81:80

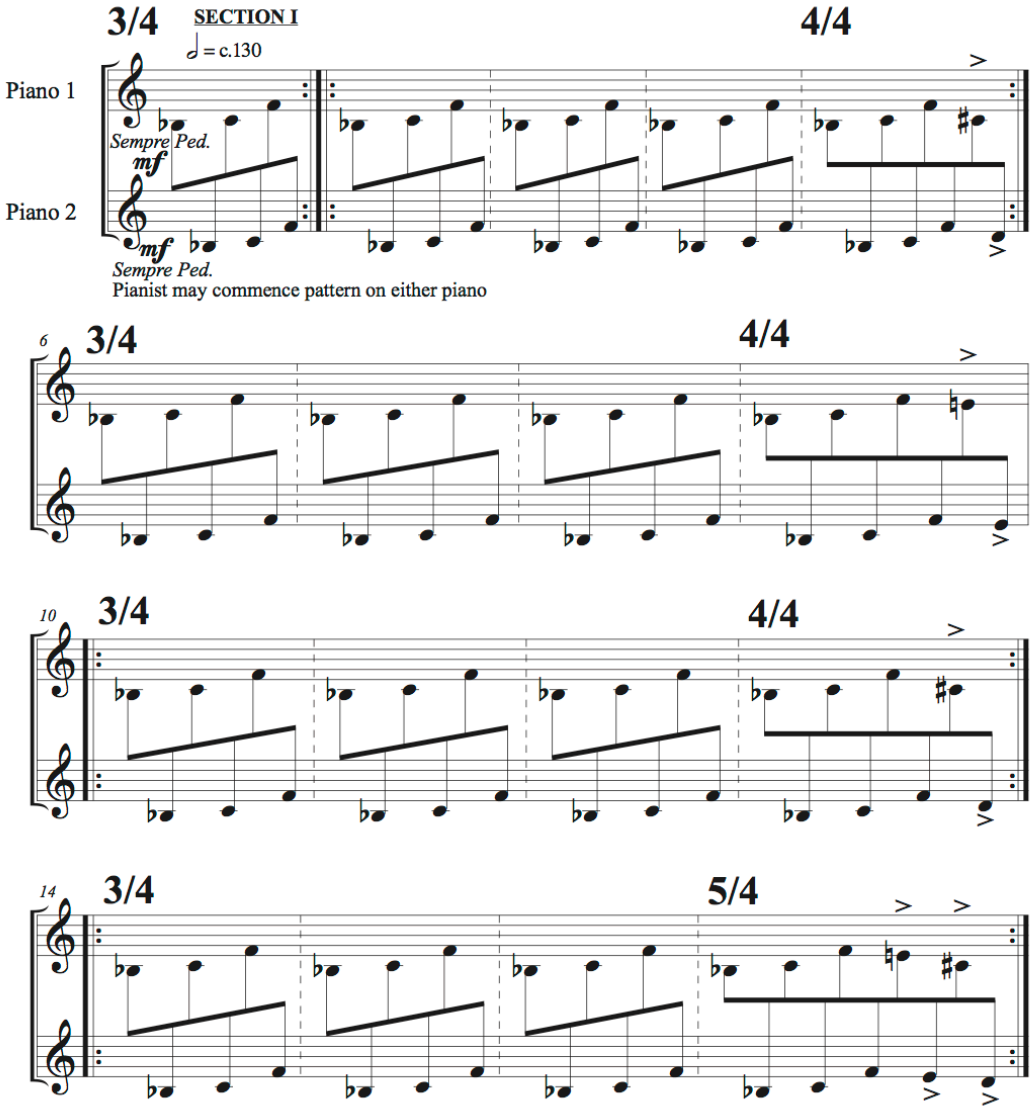
83349 81920	583443 524288	11907 10240	194481 163840	83349 65536	27783 20480	11907 8192	194481 131072	3969 2560	27783 16384	583443 327680	3969 2048

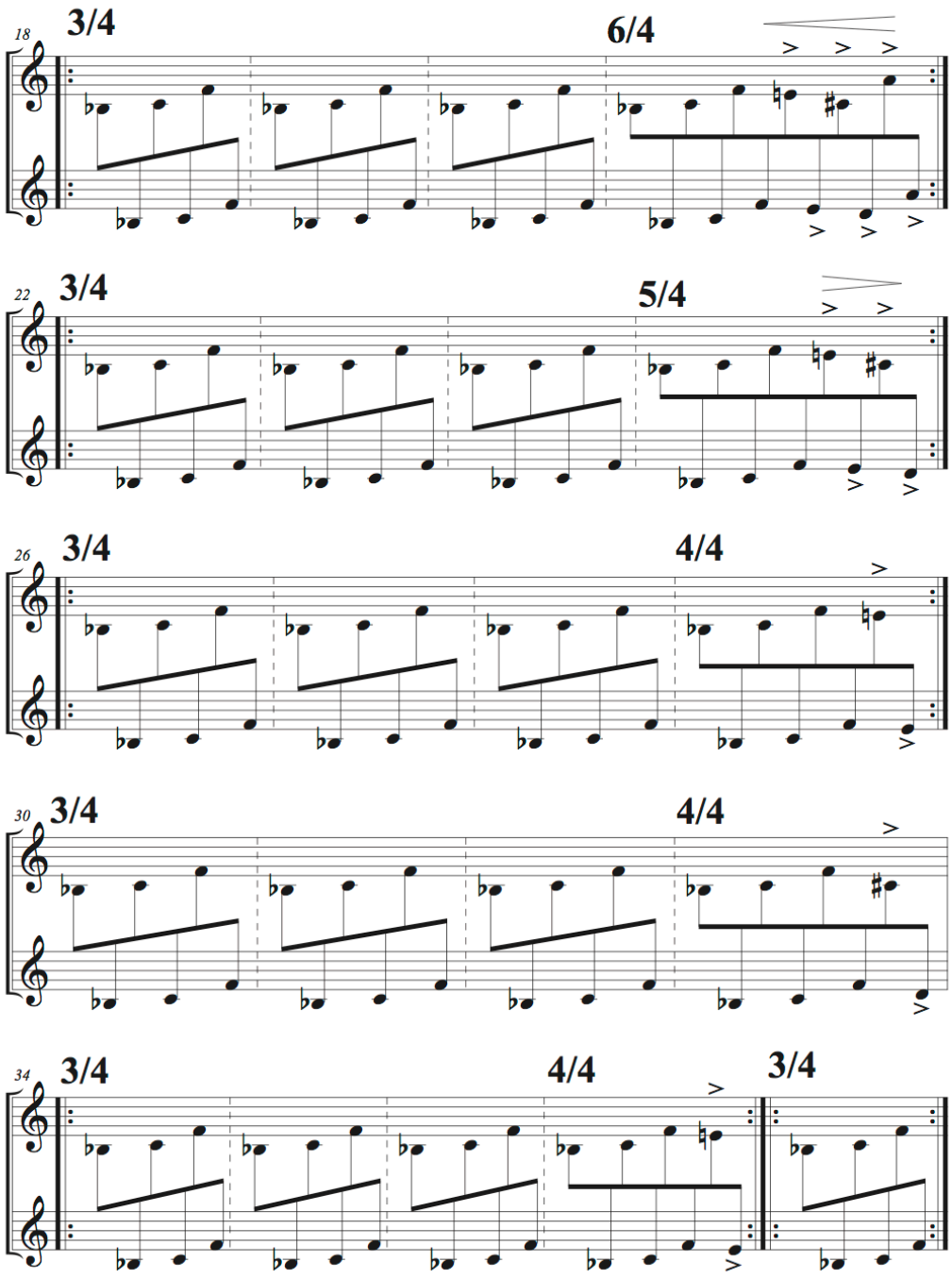
Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.72	65.44	130.88	261.76	523.52	1047.04
C#	35.79	71.58	143.16	286.32	572.64	1145.28
D	37.4	74.8	149.6	299.2	598.4	1196.8
D#	38.17	76.34	152.68	305.36	610.72	1221.44
E	40.9	81.8	163.6	327.2	654.4	1308.8
F	43.63	87.26	174.52	349.04	698.08	1396.16
F#	46.74	93.48	186.96	373.92	747.84	1495.68
G	47.72	95.44	190.88	381.76	763.52	1527.04
G#	49.86	99.72	199.44	398.88	797.76	1595.52
A	54.53	109.06	218.12	436.24	872.48	1744.96
A#	57.26	114.52	229.04	458.08	916.16	1832.32
B	62.33	124.66	249.32	498.64	997.28	1994.56

The Untempered Pianos

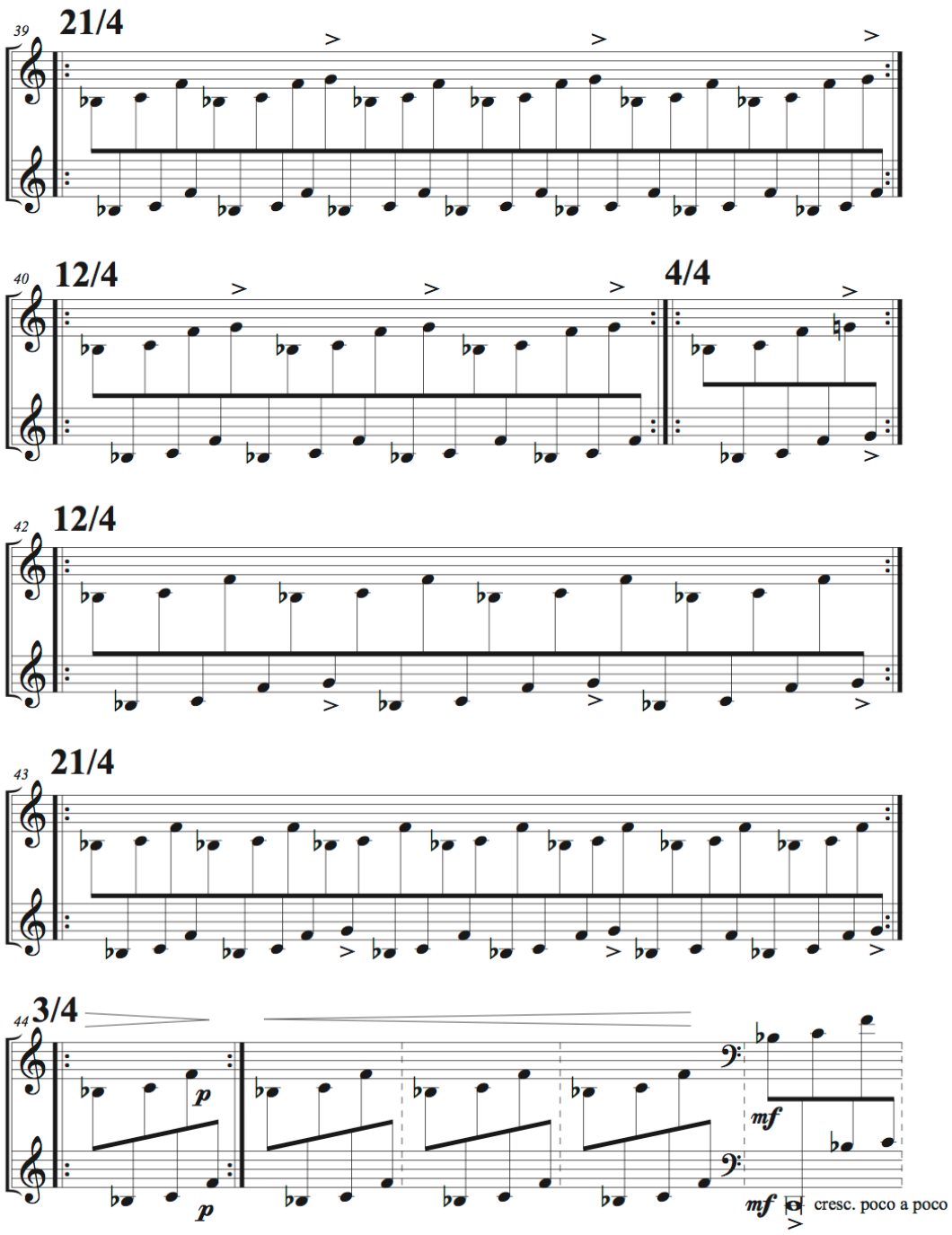
Transposing score

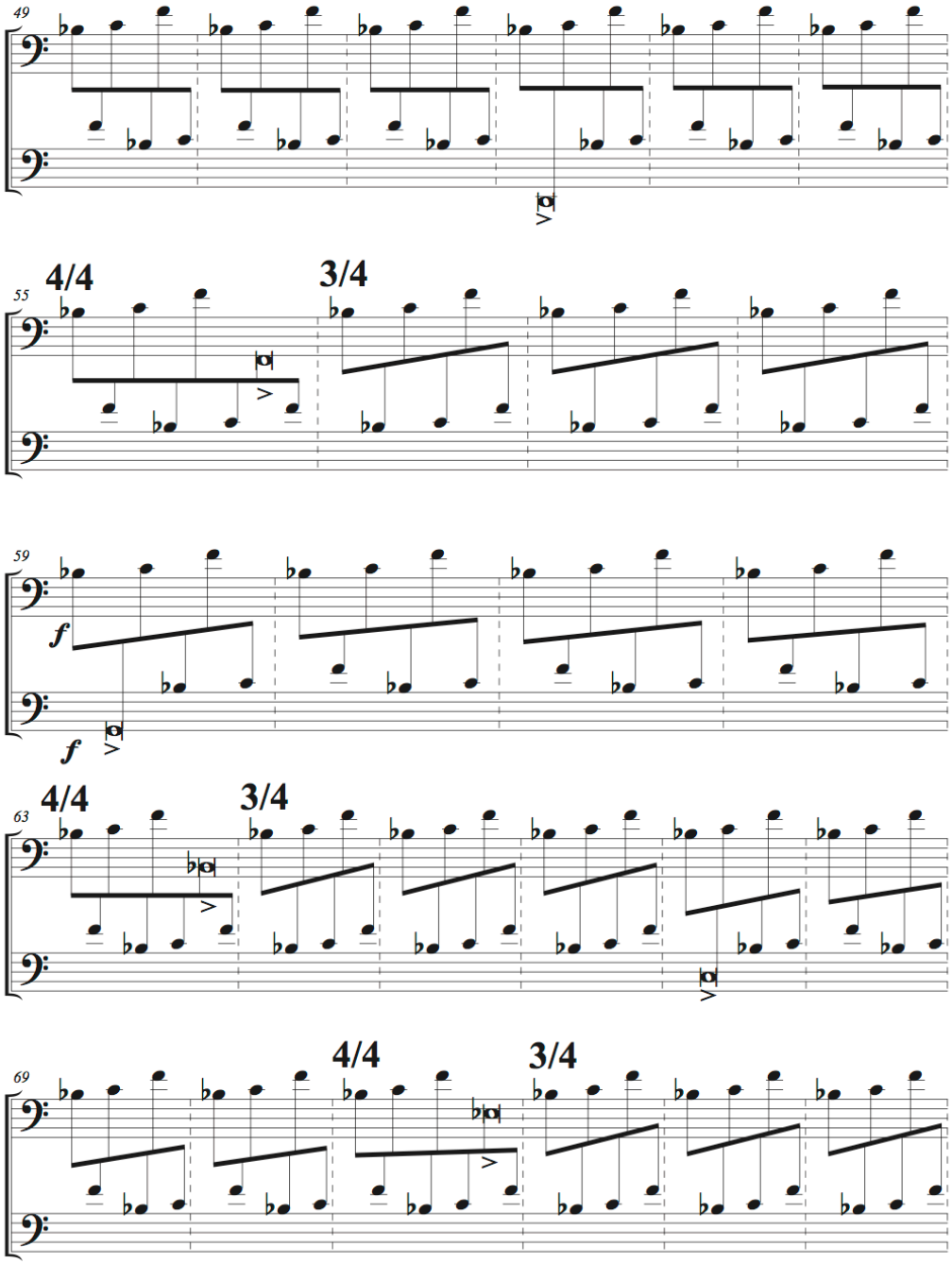
Dylan Crismani



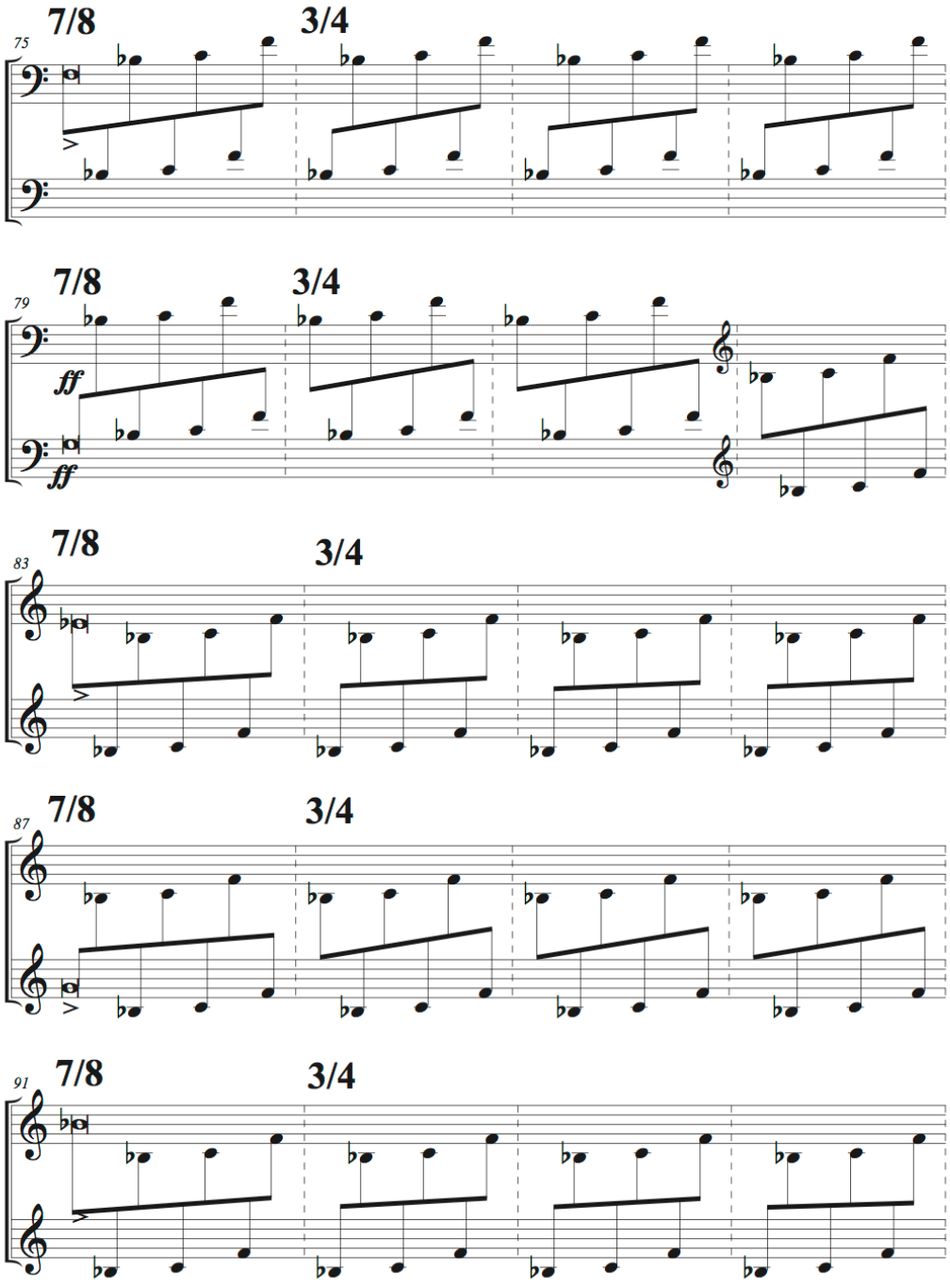

18
3/4
6/4
22
3/4
5/4
26
3/4
4/4
30
3/4
4/4
34
3/4
4/4
3/4

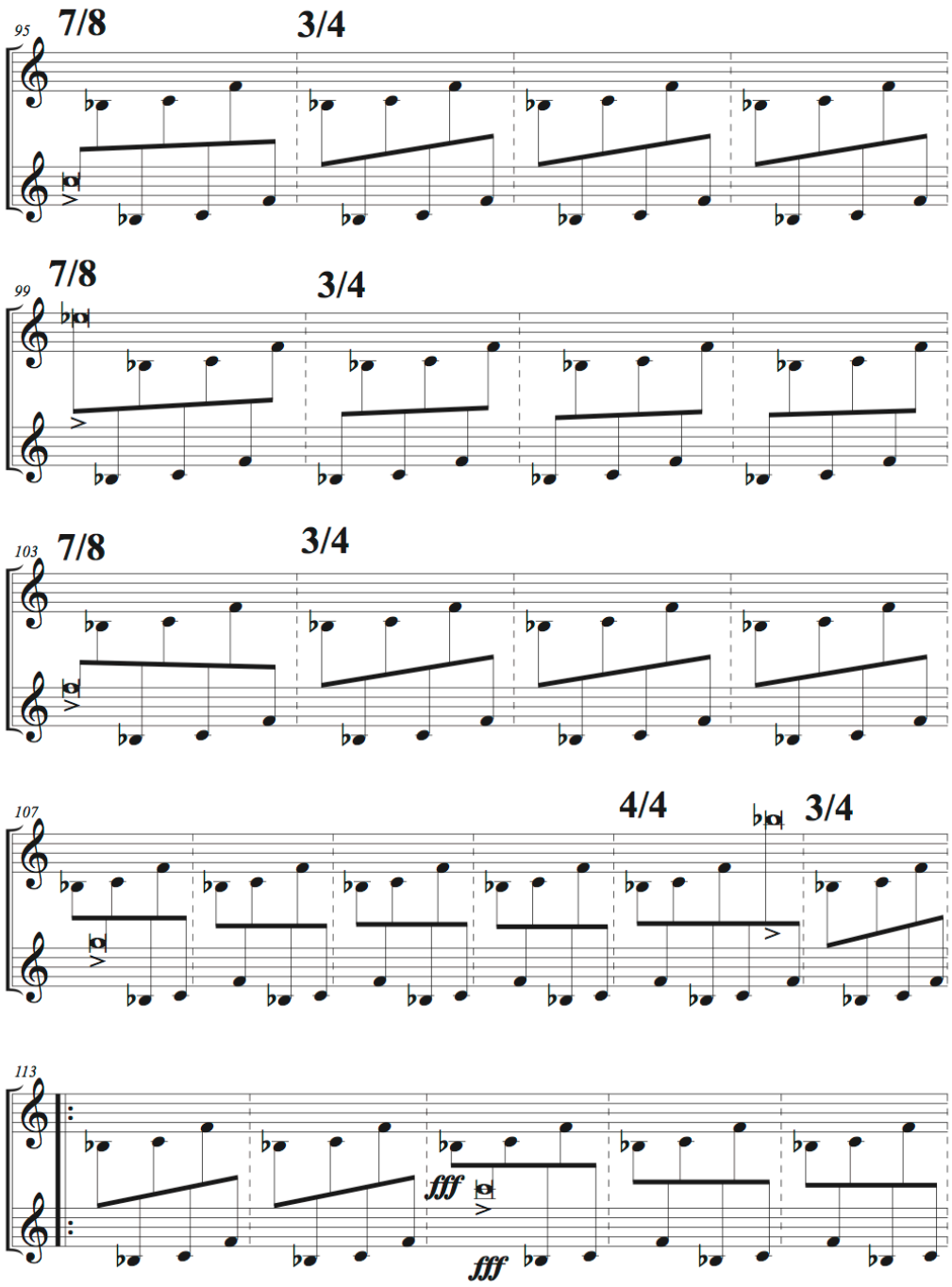
8



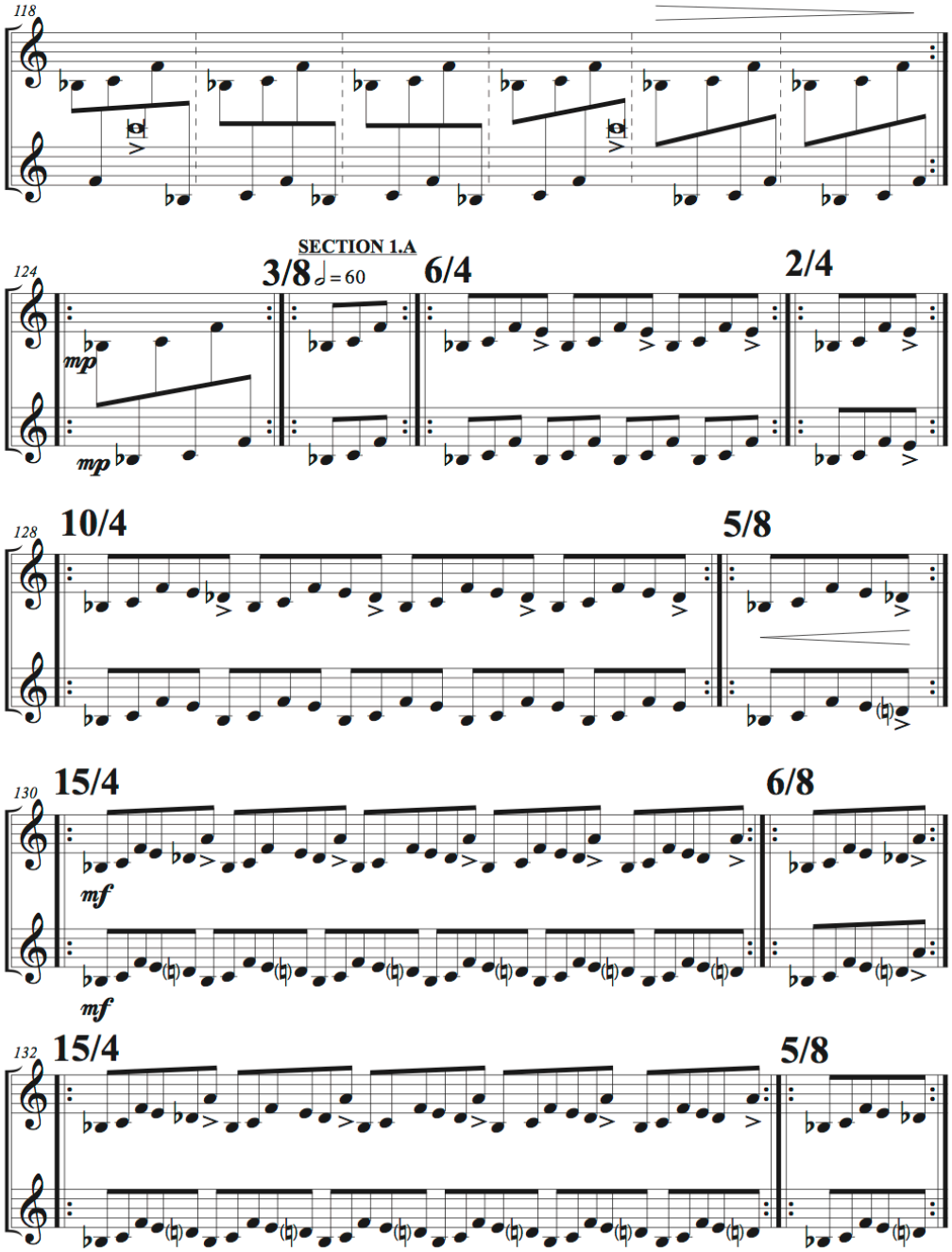

49
55
4/4
3/4
59
f
f
63
4/4
3/4
69
4/4
3/4
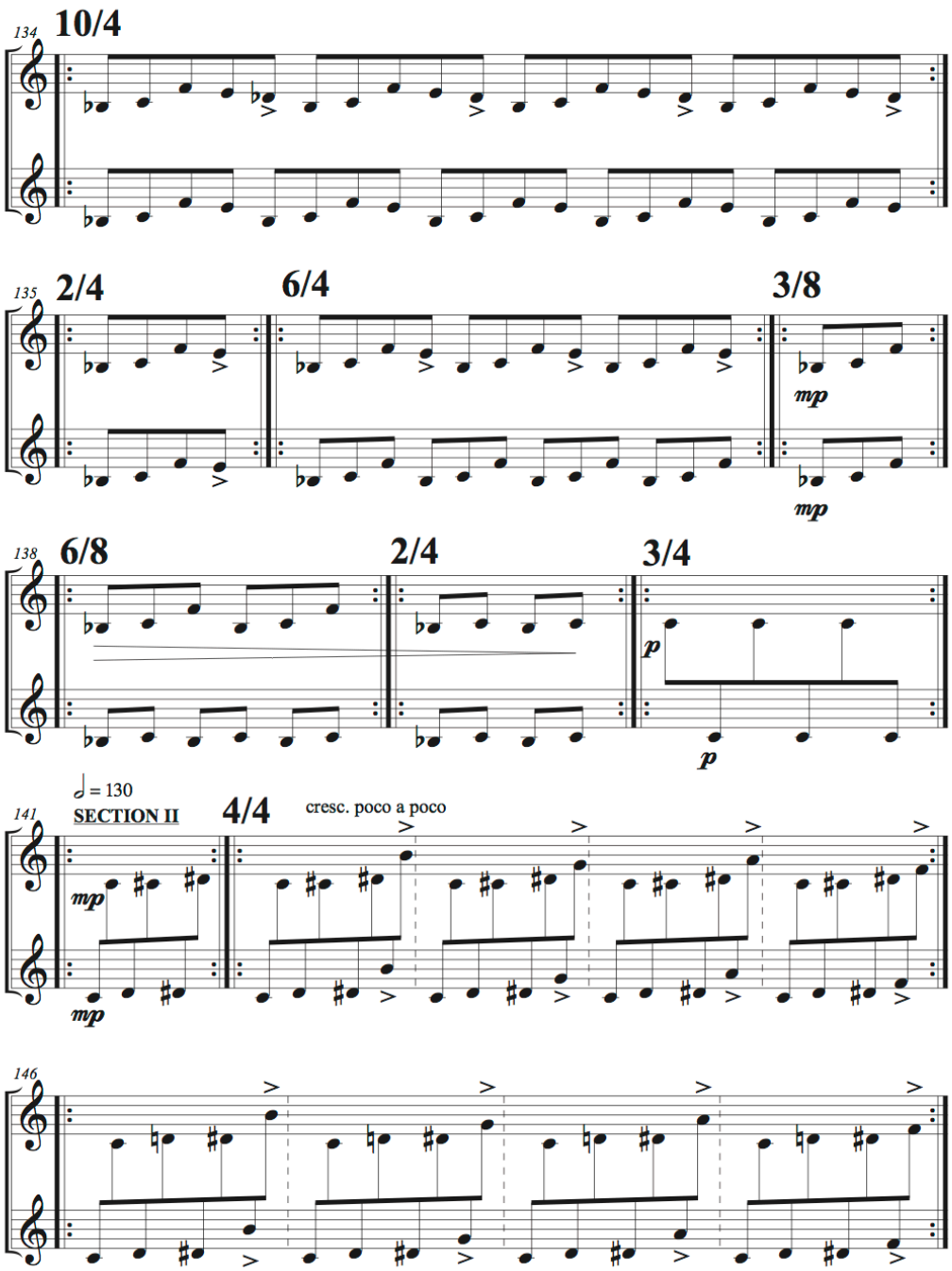
10



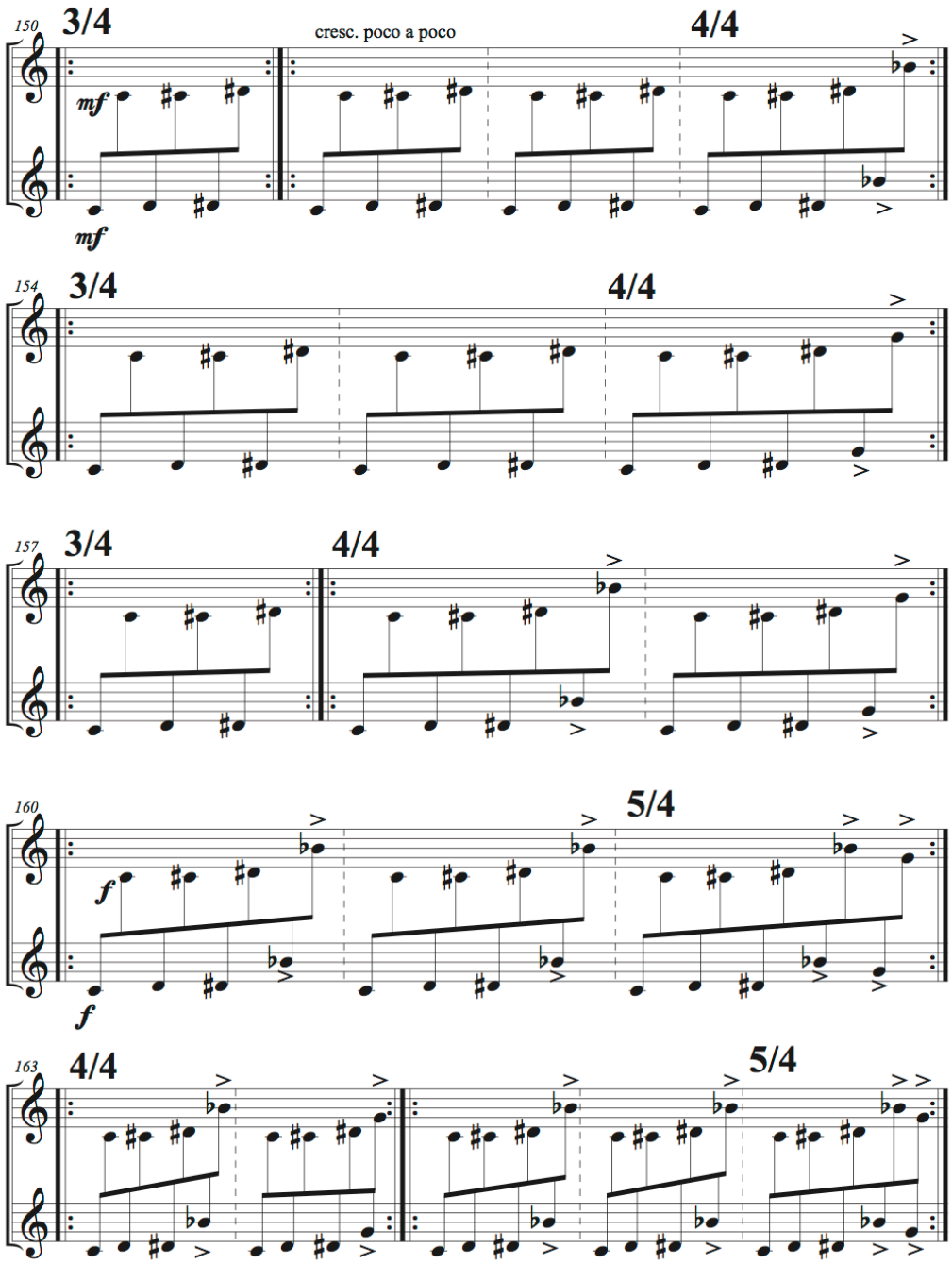

95
7/8
3/4
99
7/8
3/4
103
7/8
3/4
107
4/4
3/4
113
fff
fff
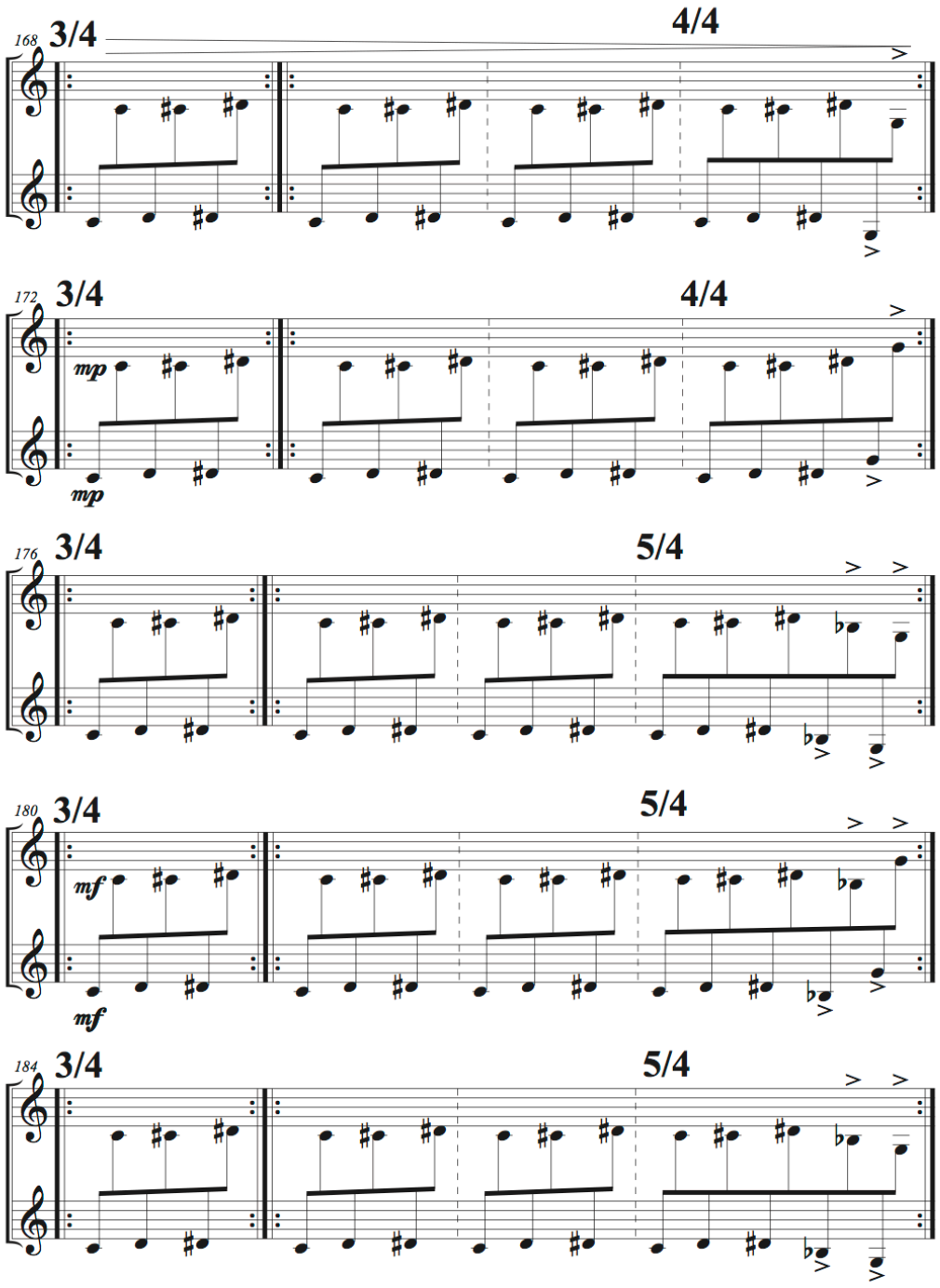
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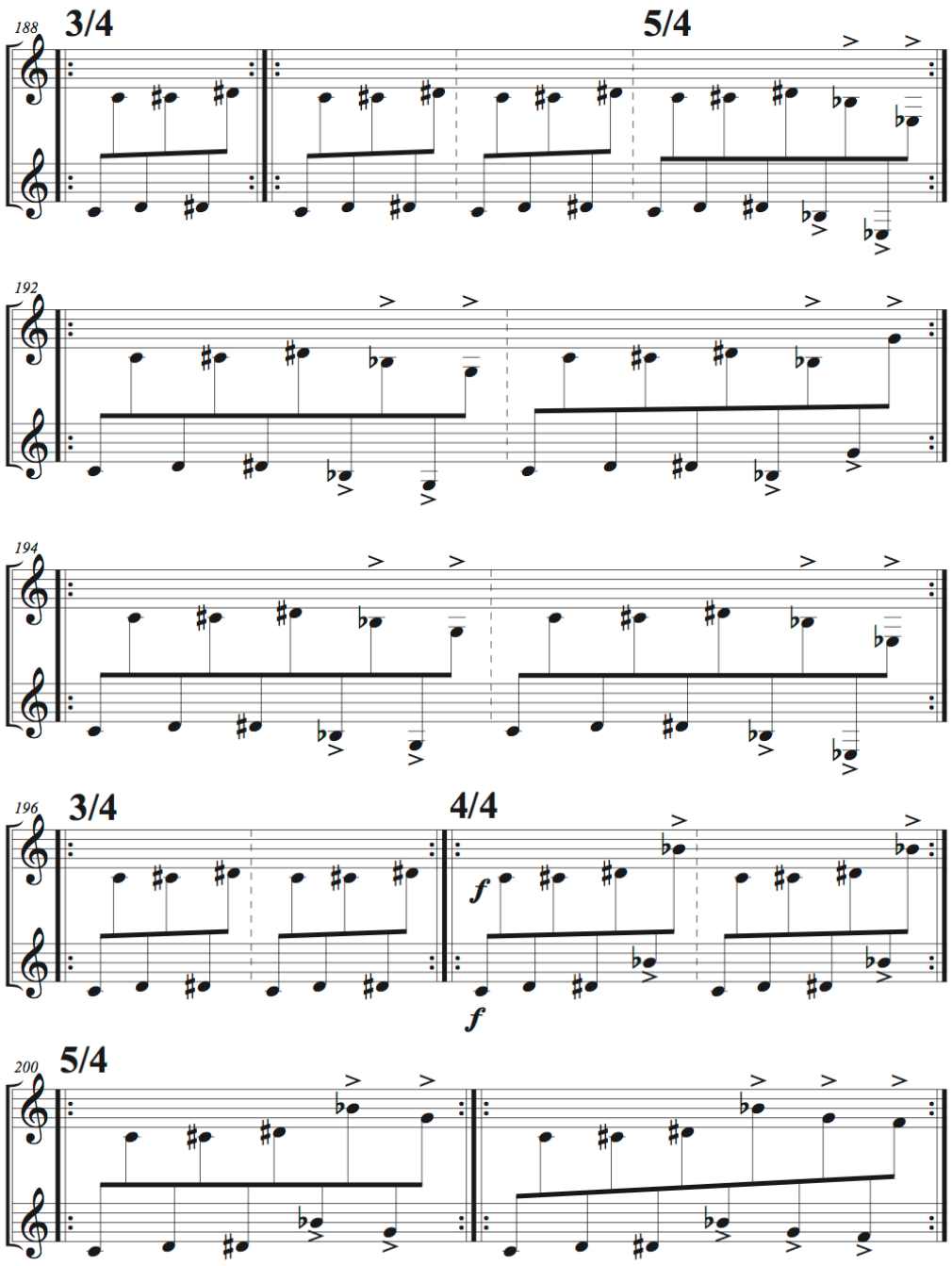

118
SECTION 1.A
124
3/8 ♩ = 60
6/4
2/4
mp
mp
128
10/4
5/8
130
15/4
6/8
mf
mf
132
15/4
5/8

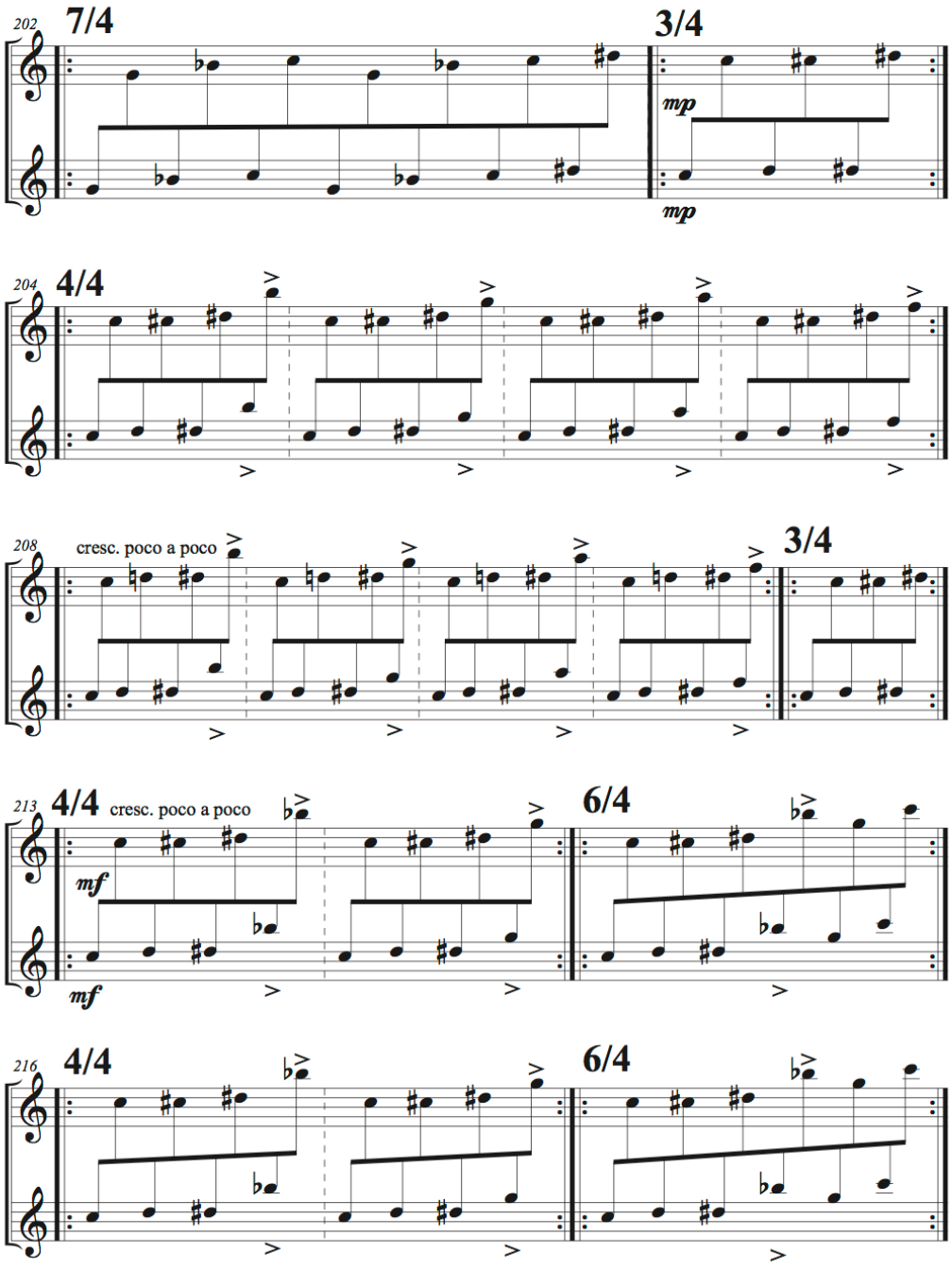

134
10/4
135
2/4
6/4
3/8
mp
mp
138
6/8
2/4
3/4
p
p
♩ = 130
141
SECTION II
4/4
cresc. poco a poco
mp
mp
146
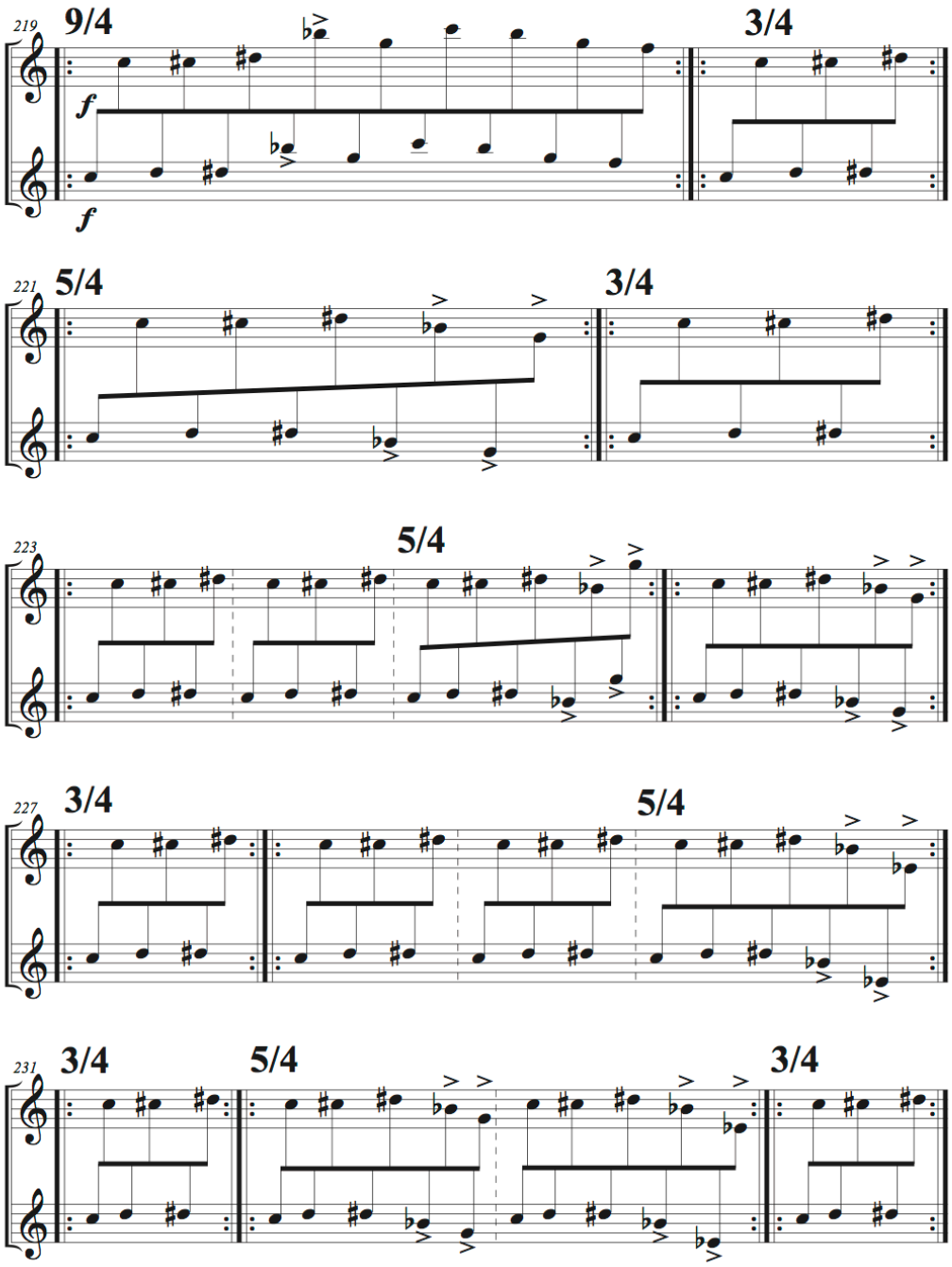
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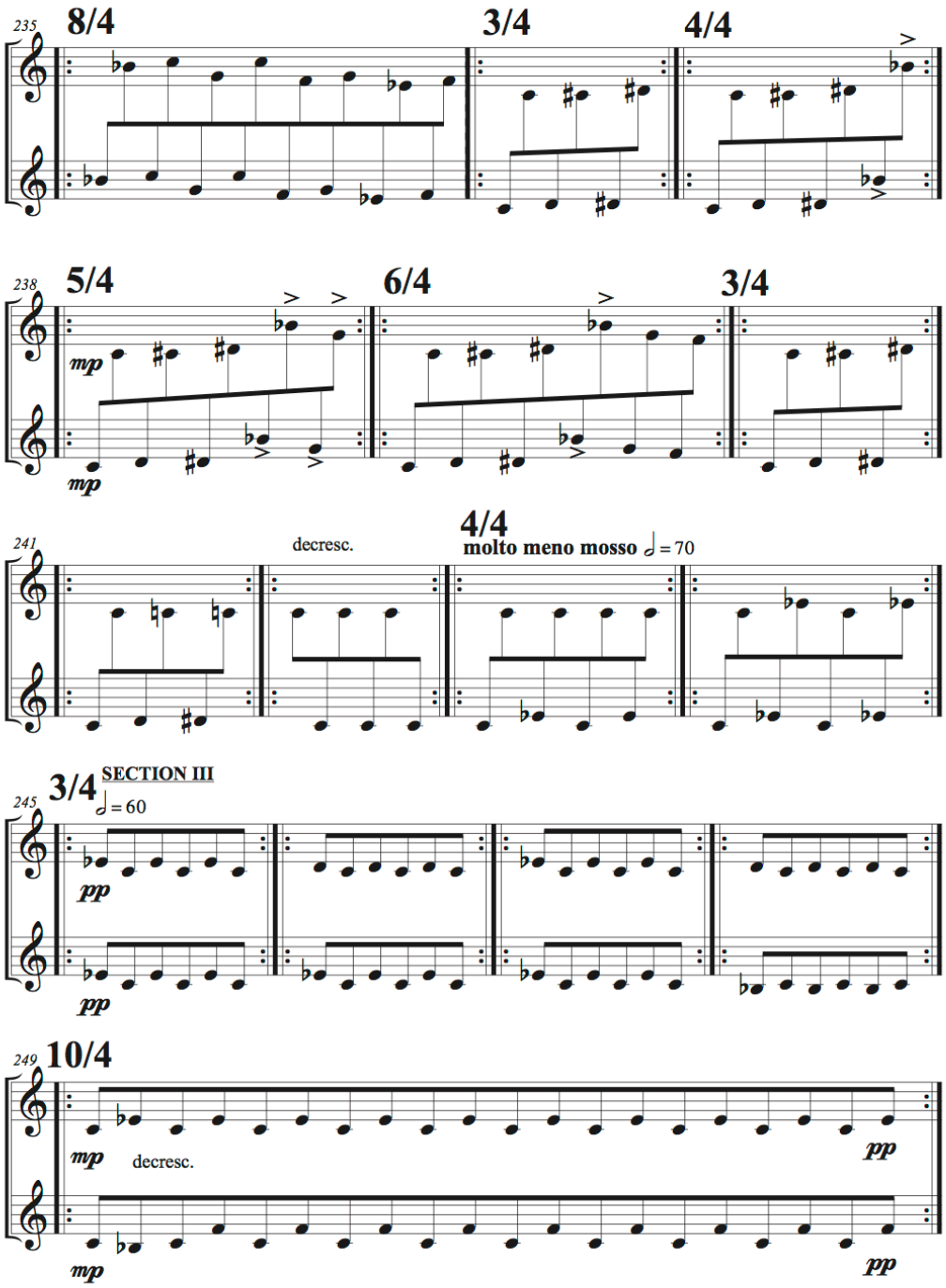

150
3/4
cresc. poco a poco
4/4
mf
mf
154
3/4
4/4
157
3/4
4/4
160
5/4
f
f
163
4/4
5/4

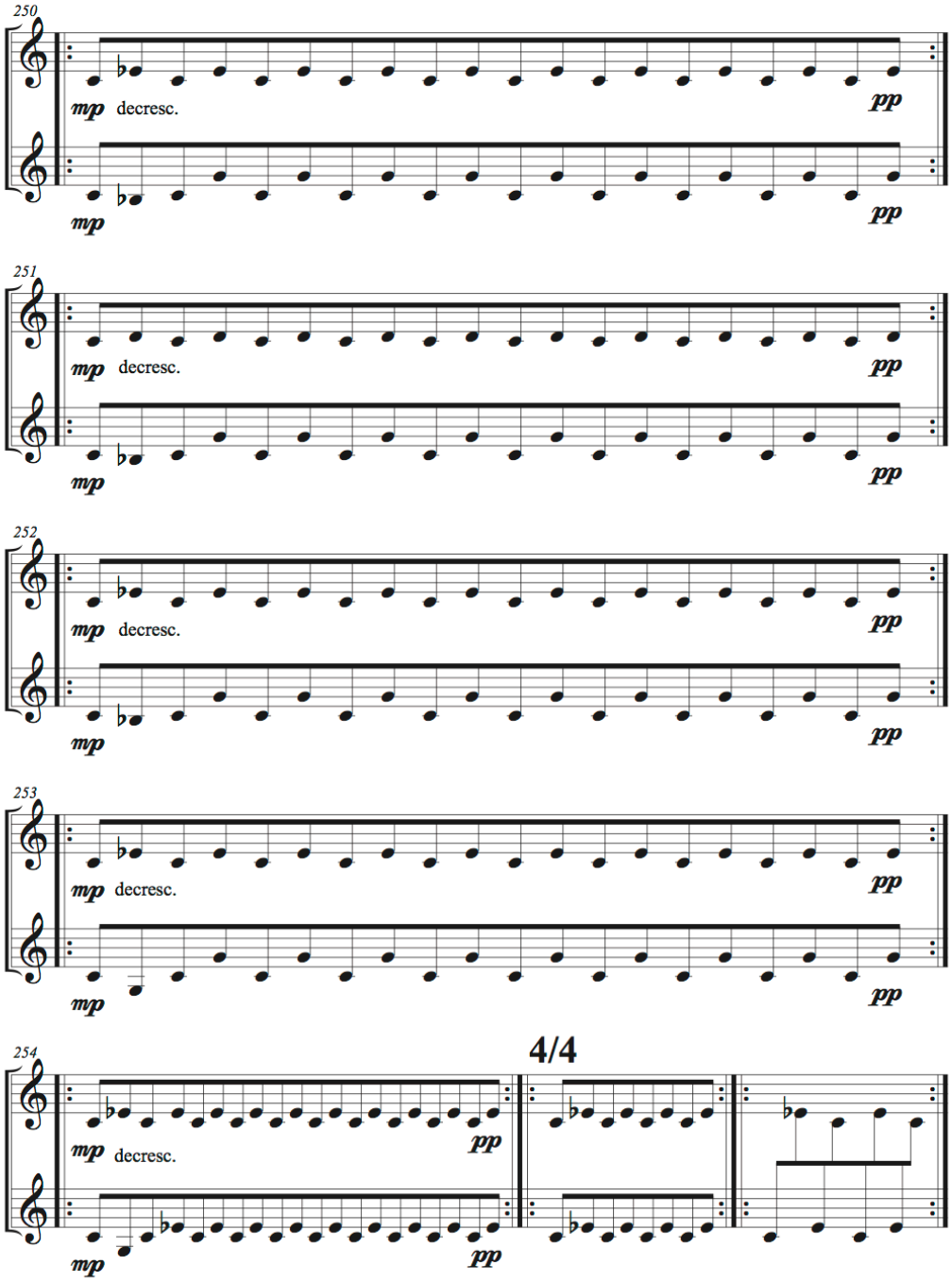

168
3/4
4/4
172
3/4
4/4
mp
mp
176
3/4
5/4
180
3/4
5/4
mf
mf
184
3/4
5/4
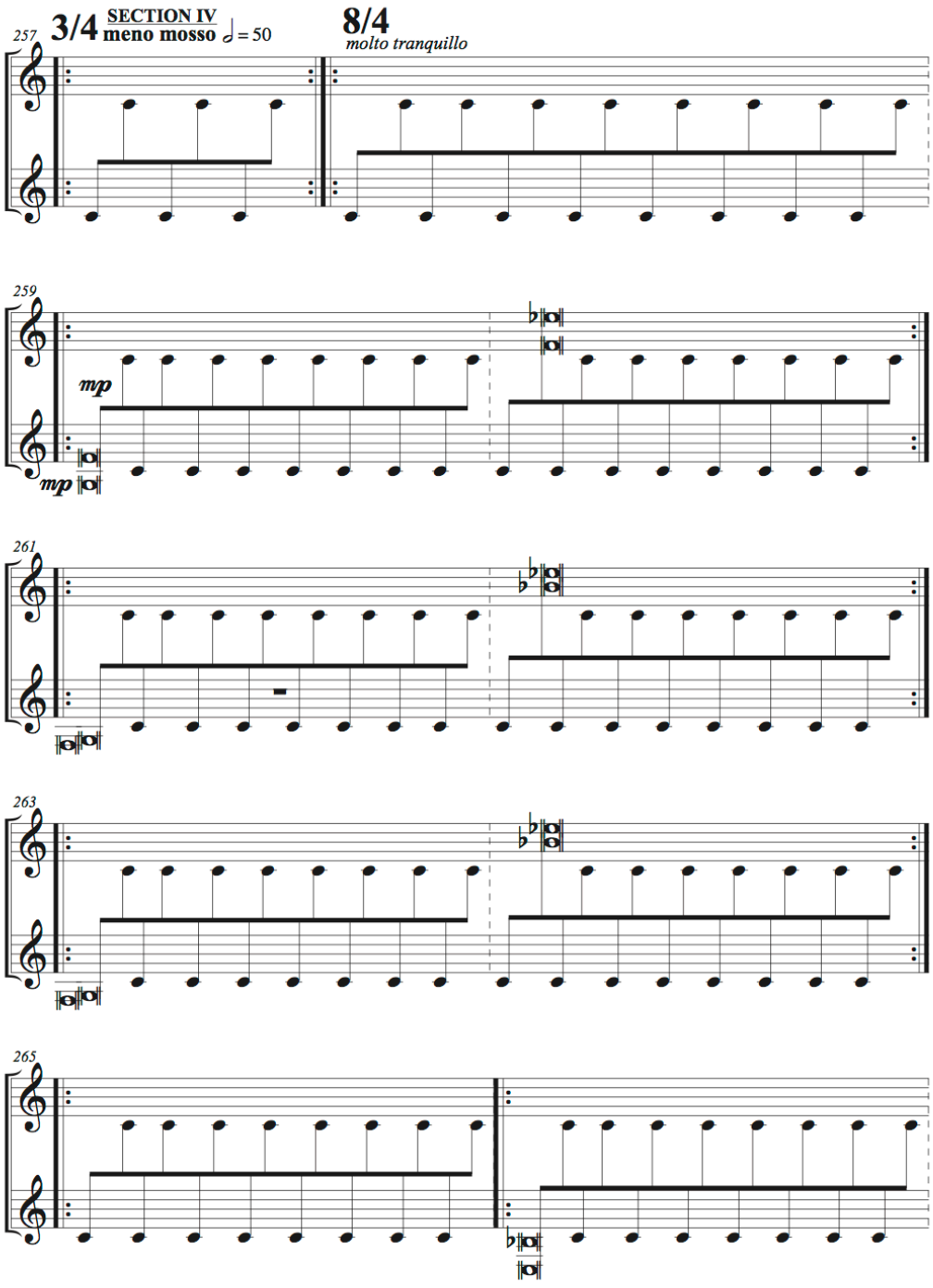

188
3/4
5/4
192
194
196
3/4
4/4
f
f
200
5/4

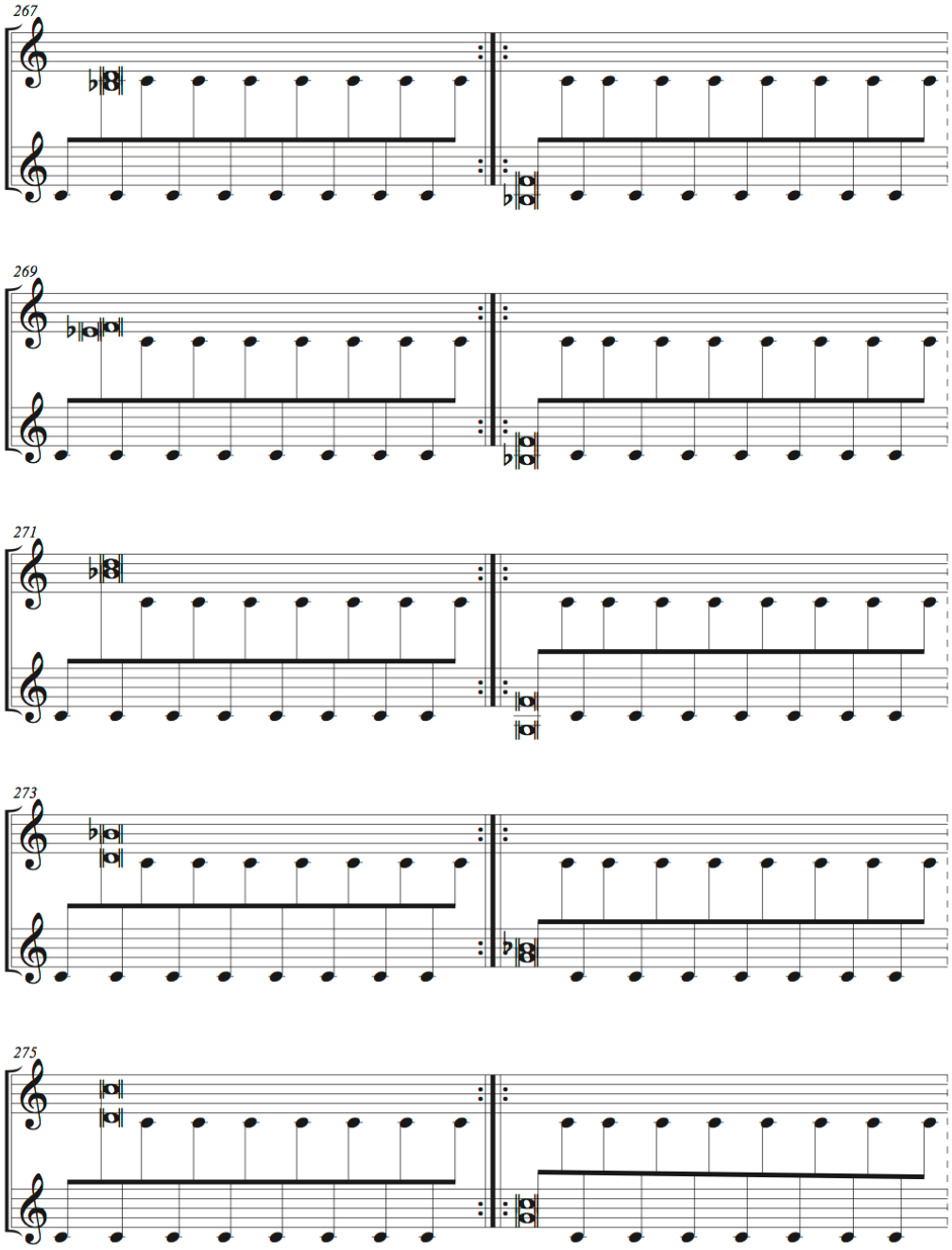

202
7/4
3/4
mp
mp
204
4/4
208
cresc. poco a poco
3/4
213
4/4
cresc. poco a poco
mf
mf
6/4
216
4/4
6/4

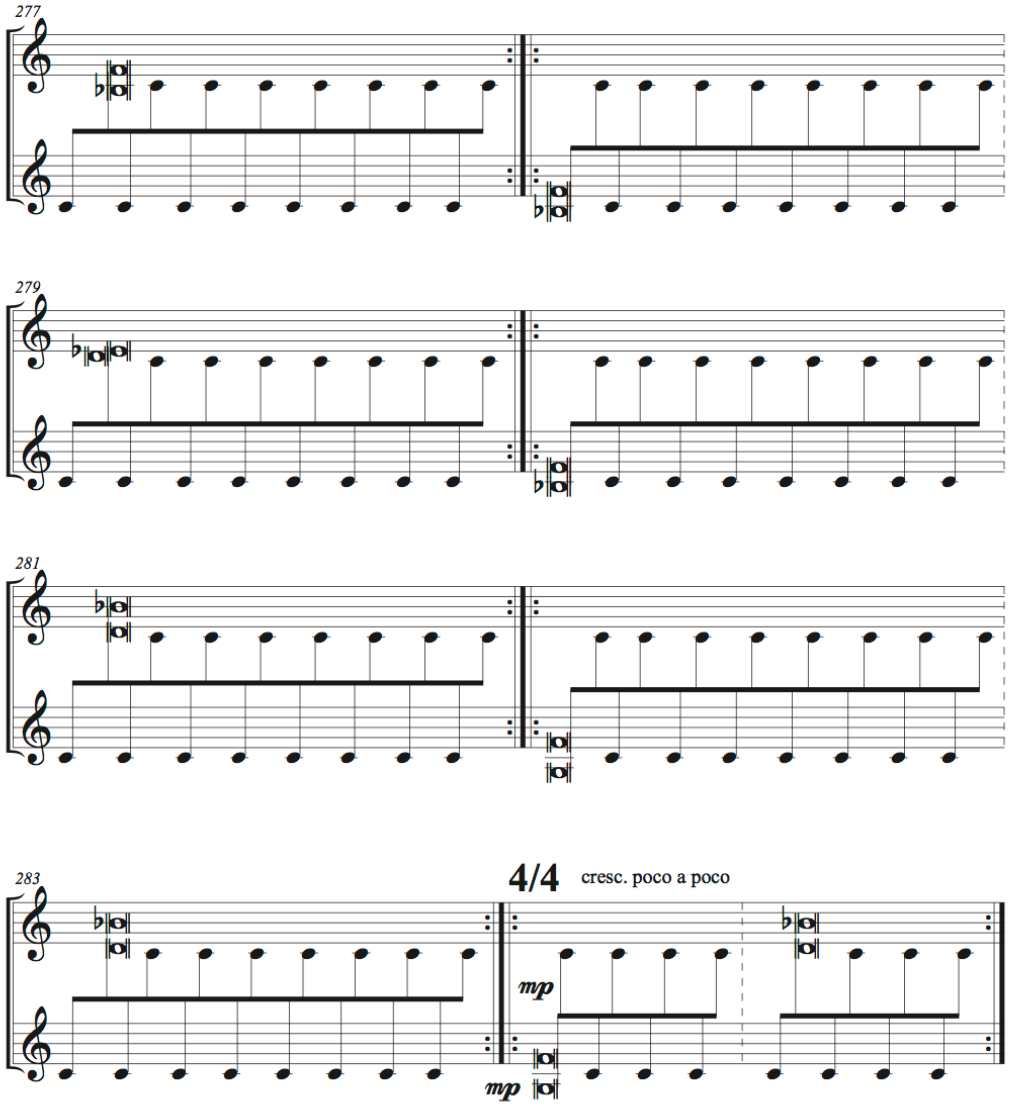

219
9/4
3/4
f
f
221
5/4
3/4
223
5/4
227
3/4
5/4
231
3/4
5/4
3/4

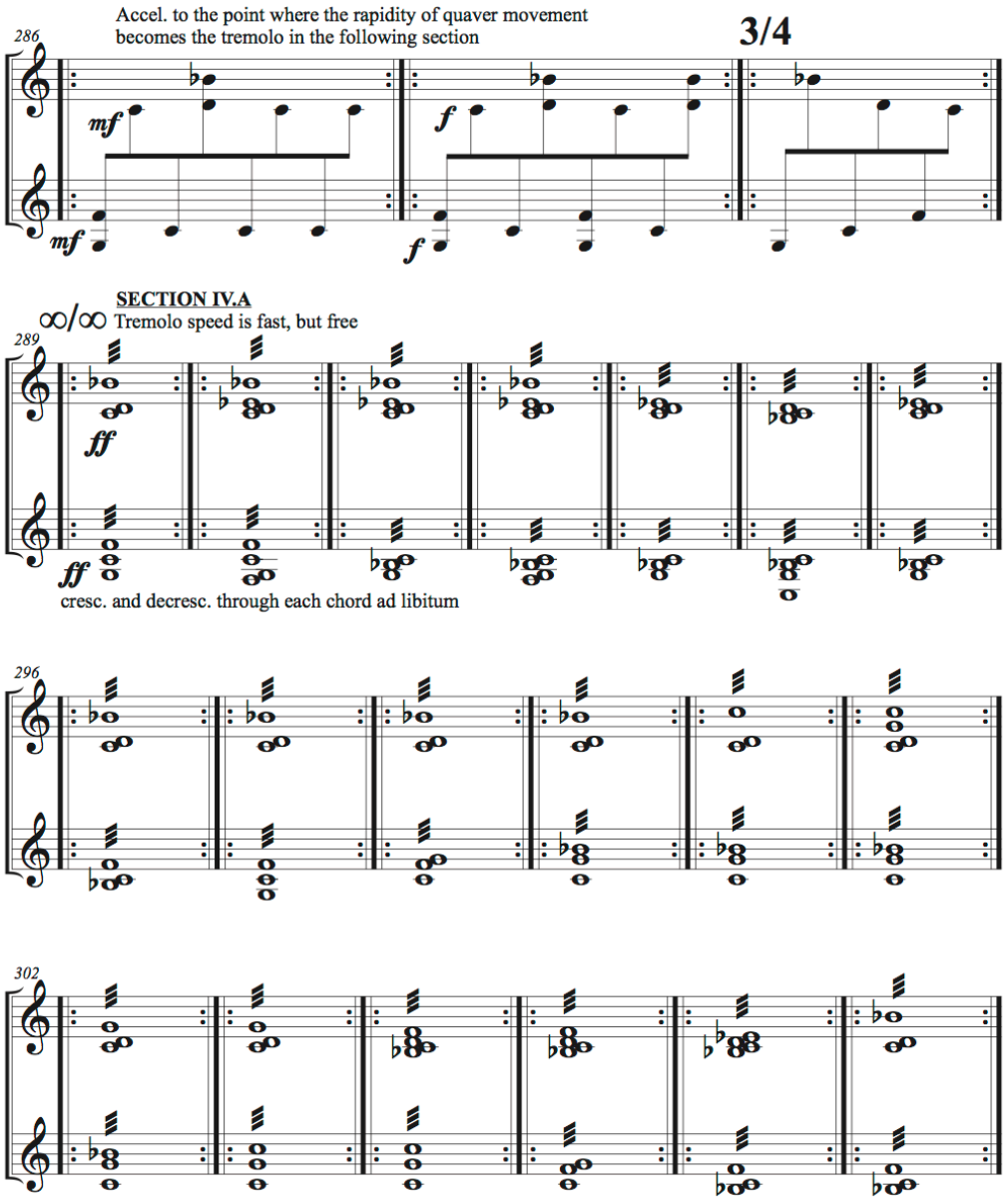

235
8/4
3/4
4/4
238
5/4
6/4
3/4
mp
mp
241
decresc.
4/4
molto meno mosso ♩ = 70
245
3/4
SECTION III
♩ = 60
pp
pp
249
10/4
mp
decresc.
pp
mp
pp

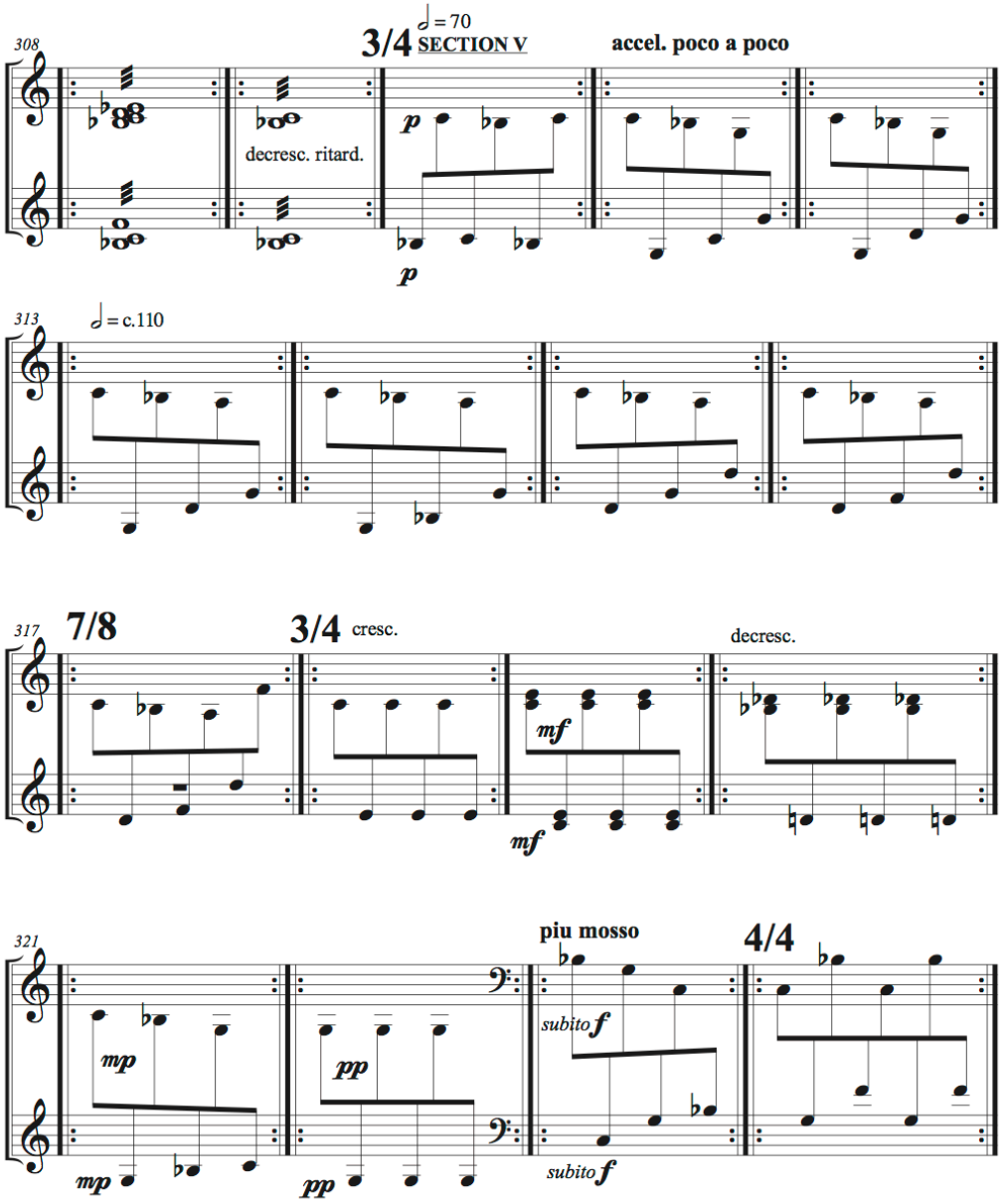

250
mp decresc.
pp
mp
pp
251
mp decresc.
pp
mp
pp
252
mp decresc.
pp
mp
pp
253
mp decresc.
pp
mp
pp
254
4/4
mp decresc.
pp
mp
pp

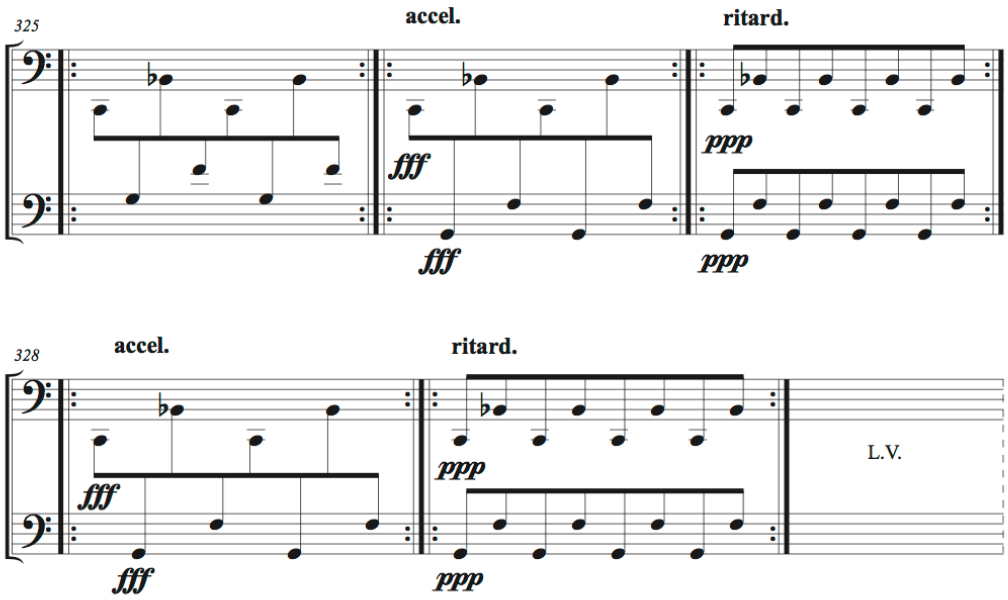

257
3/4
SECTION IV
meno mosso
= 50
8/4
molto tranquillo
259
mp
mp
261
263
265


267
269
271
273
275


277
279
281
283
4/4
cresc. poco a poco
mp
mp


Accel. to the point where the rapidity of quaver movement becomes the tremolo in the following section
3/4
286
mf
mf
f
f
SECTION IV.A
∞/∞ Tremolo speed is fast, but free
289
ff
ff
cresc. and decresc. through each chord ad libitum
296
302


308
3/4
SECTION V
accel. poco a poco
decresc. ritard.
313
317
7/8
3/4
cresc.
decresc.
321
piu mosso
4/4
subito


325
accel.
ritard.
fff
ppp
fff
ppp
328
accel.
ritard.
fff
ppp
fff
ppp
L.V.

3.5 *K* for two pianos, two pianists, and two hands

K

for two pianos, two pianists, and two hands

Dylan Crismani

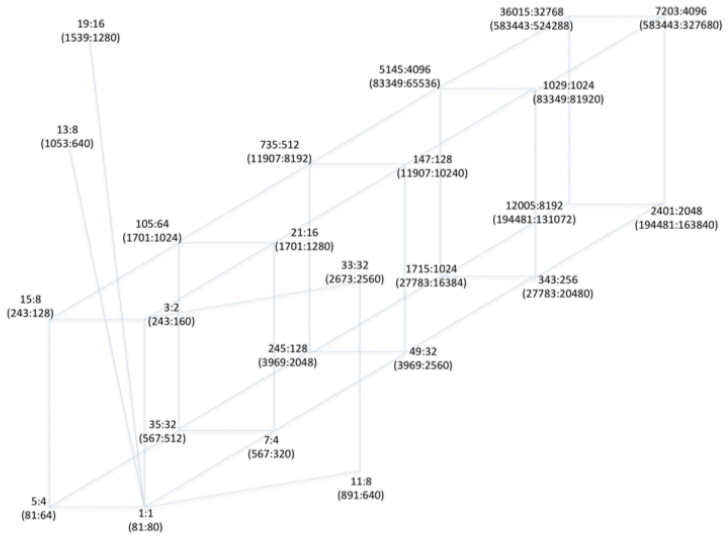
Performance notes

On repeats:
Performers may choose from two options. The first option is to play each bar marked with repeats twice. The second option is for the performers to repeat bars at their own discretion. Most changes between one bar to the next only involve a change in the material of one pianist, therefore they can change at any time they feel is 'right' and signal to the other pianist that they have moved to the next bar. Option number two is encouraged if the performers find they have adequate time in rehearsals to achieve this. Option two will create more unique performances. Option one may be best in cases where rehearsal time is limited.

On tempi:
All tempi are flexible, especially if large numbers of repeats are being undertaken, but tempi shouldn't be slackened once set.

On hands:
The score is written in two single clefs, and in theory should be playable with one hand only by each pianist. Certain passages, however, may lend themselves to being split between the two hands. This is acceptable.

Lattice



Tuning instructions

Piano 1 tuning

1:1	33:32	35:32	19:16	5:4	21:16	11:8	3:2	13:8	105:64	7:4	15:8

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32	64	128	256	512	1024
C#	33	66	132	264	528	1056
D	35	70	140	280	560	1120
D#	38	76	152	304	608	1216
E	40	80	160	320	640	1280
F	42	84	168	336	672	1344
F#	44	88	176	352	704	1408
G	48	96	192	384	768	1536
G#	52	104	208	416	832	1664
A	52.5	105	210	420	840	1680
A#	56	112	224	448	896	1792
B	60	120	240	480	960	1920

Piano 1 tuning, plus 81:80

81:80	2673:2560	567:512	1539:1280	81:64	1701:1280	891:640	243:160	1053:640	1701:1024	567:320	243:128

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.4	64.8	129.6	259.2	518.4	1036.8
C#	33.41	66.82	133.64	267.28	534.56	1069.12
D	35.44	70.88	141.76	283.52	567.04	1134.08
D#	38.48	76.96	153.92	307.84	615.68	1231.36
E	40.5	81	162	324	648	1296
F	42.53	85.06	170.12	340.24	680.48	1360.96
F#	44.55	89.1	178.2	356.4	712.8	1425.6
G	48.6	97.2	194.4	388.8	777.6	1555.2
G#	52.65	105.3	210.6	421.2	842.4	1684.8
A	53.16	106.32	212.64	425.28	850.56	1701.12
A#	56.7	113.4	226.8	453.6	907.2	1814.4
B	60.75	121.5	243	486	972	1944

Tuning instructions

Piano 2 tuning

1029:1024	36015:32768	147:128	2401:2048	5145:4096	343:256	735:512	12005:8192	49:32	1715:1024	7203:4096	245:128

Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.16	64.32	128.64	257.25	514.5	1029
C#	35.17	70.34	140.68	281.36	562.72	1125.44
D	36.75	73.5	147	294	588	1176
D#	37.52	75.04	150.08	300.16	600.32	1200.64
E	40.2	80.4	160.8	321.6	643.2	1286.4
F	42.88	85.76	171.52	343.04	686.08	1372.16
F#	45.94	91.88	183.76	367.52	735.04	1470.08
G	46.89	93.78	187.56	375.12	750.24	1500.48
G#	49	98	196	392	784	1568
A	53.95	107.18	214.36	428.72	857.44	1714
A#	56.29	112.58	225.16	450.32	900.64	1801.28
B	61.25	122.5	245	490	980	1960
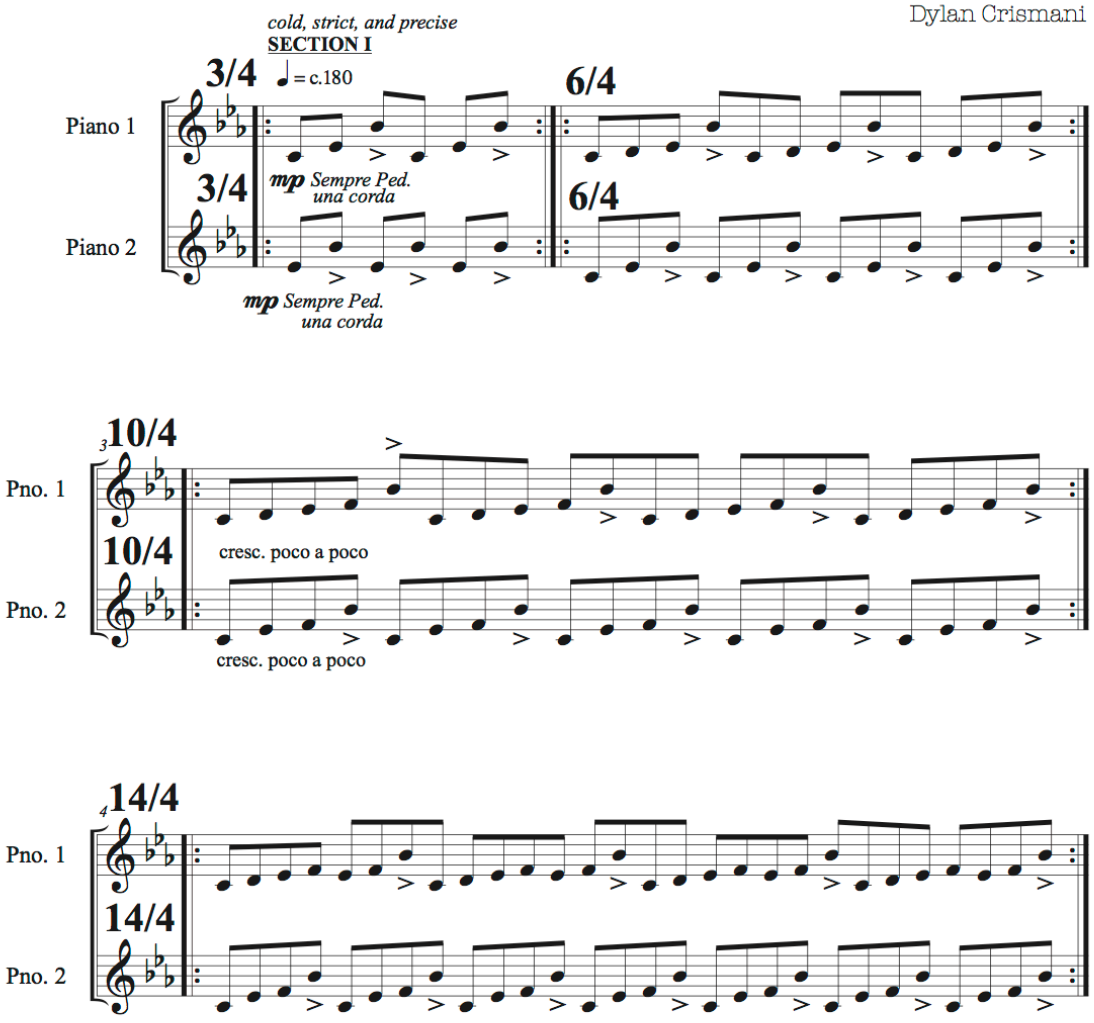
Piano 2 tuning, plus 81:80

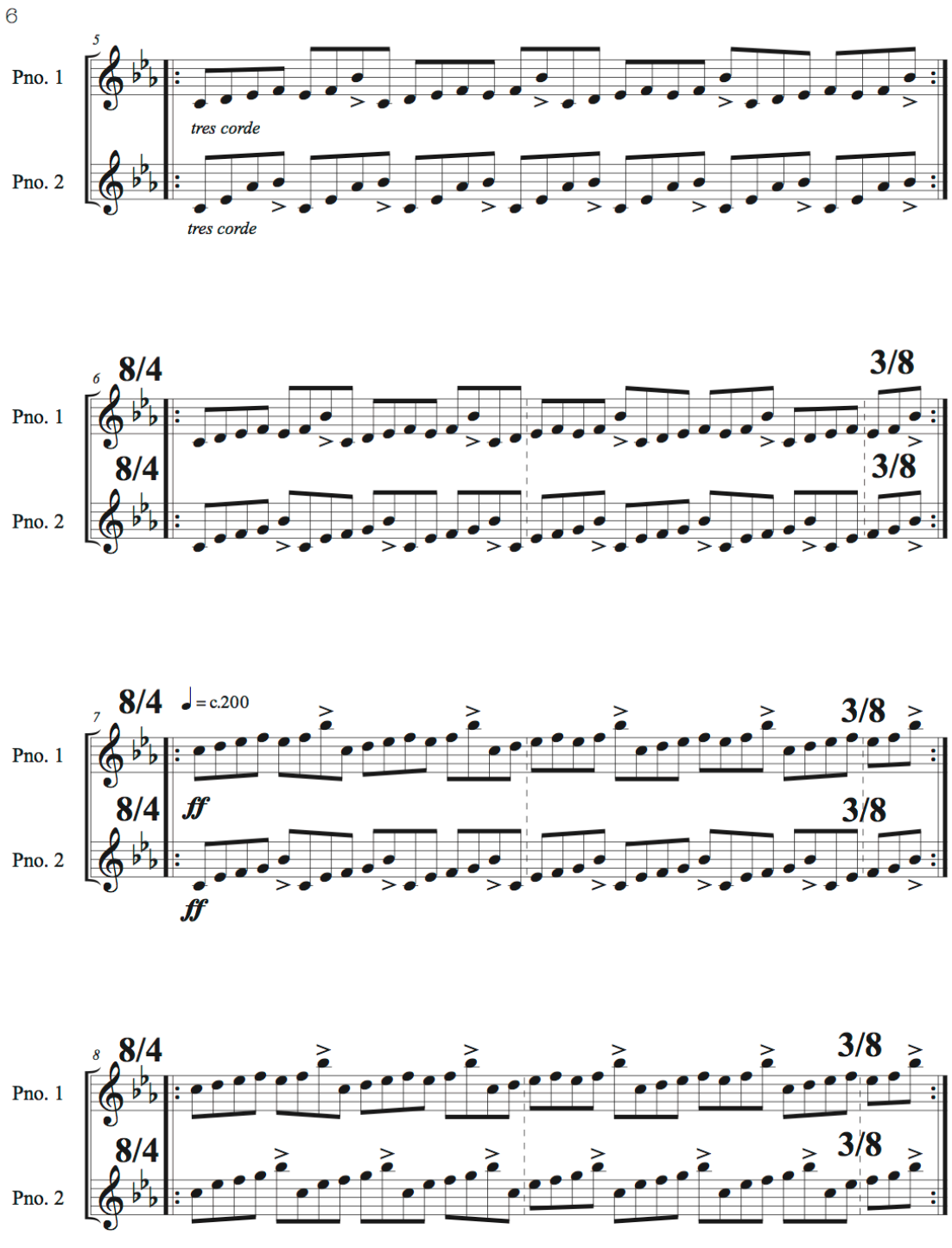
83349 81920	583443 524288	11907 10240	194481 163840	83349 65536	27783 20480	11907 8192	194481 131072	3969 2560	27783 16384	583443 327680	3969 2048

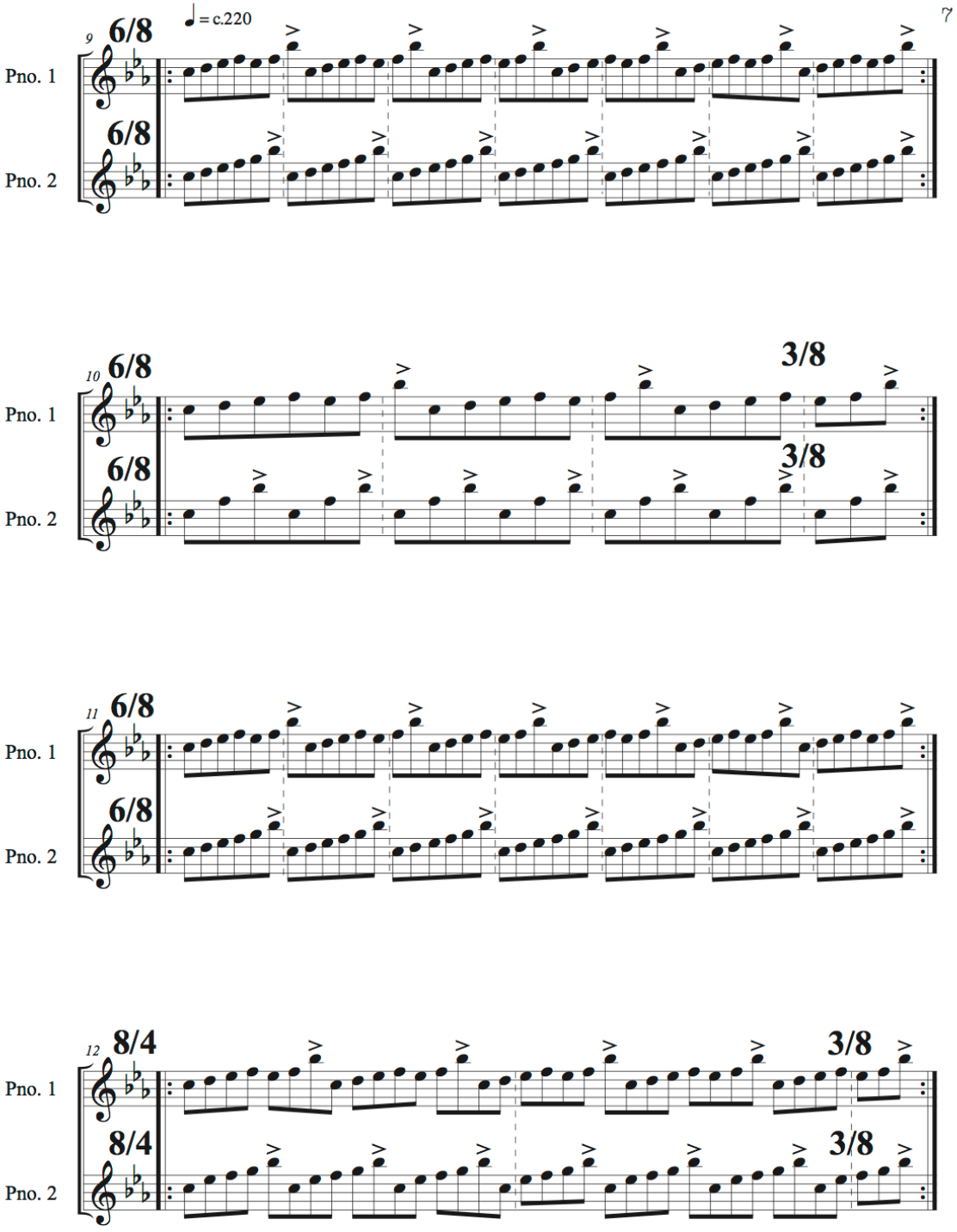
Corresponding key	Octave 1 (Hertz)	Octave 2 (Hertz)	Octave 3 (Hertz)	Octave 4 (Hertz)	Octave 5 (Hertz)	Octave 6 (Hertz)
C	32.72	65.44	130.88	261.76	523.52	1047.04
C#	35.79	71.58	143.16	286.32	572.64	1145.28
D	37.4	74.8	149.6	299.2	598.4	1196.8
D#	38.17	76.34	152.68	305.36	610.72	1221.44
E	40.9	81.8	163.6	327.2	654.4	1308.8
F	43.63	87.26	174.52	349.04	698.08	1396.16
F#	46.74	93.48	186.96	373.92	747.84	1495.68
G	47.72	95.44	190.88	381.76	763.52	1527.04
G#	49.86	99.72	199.44	398.88	797.76	1595.52
A	54.53	109.06	218.12	436.24	872.48	1744.96
A#	57.26	114.52	229.04	458.08	916.16	1832.32
B	62.33	124.66	249.32	498.64	997.28	1994.56

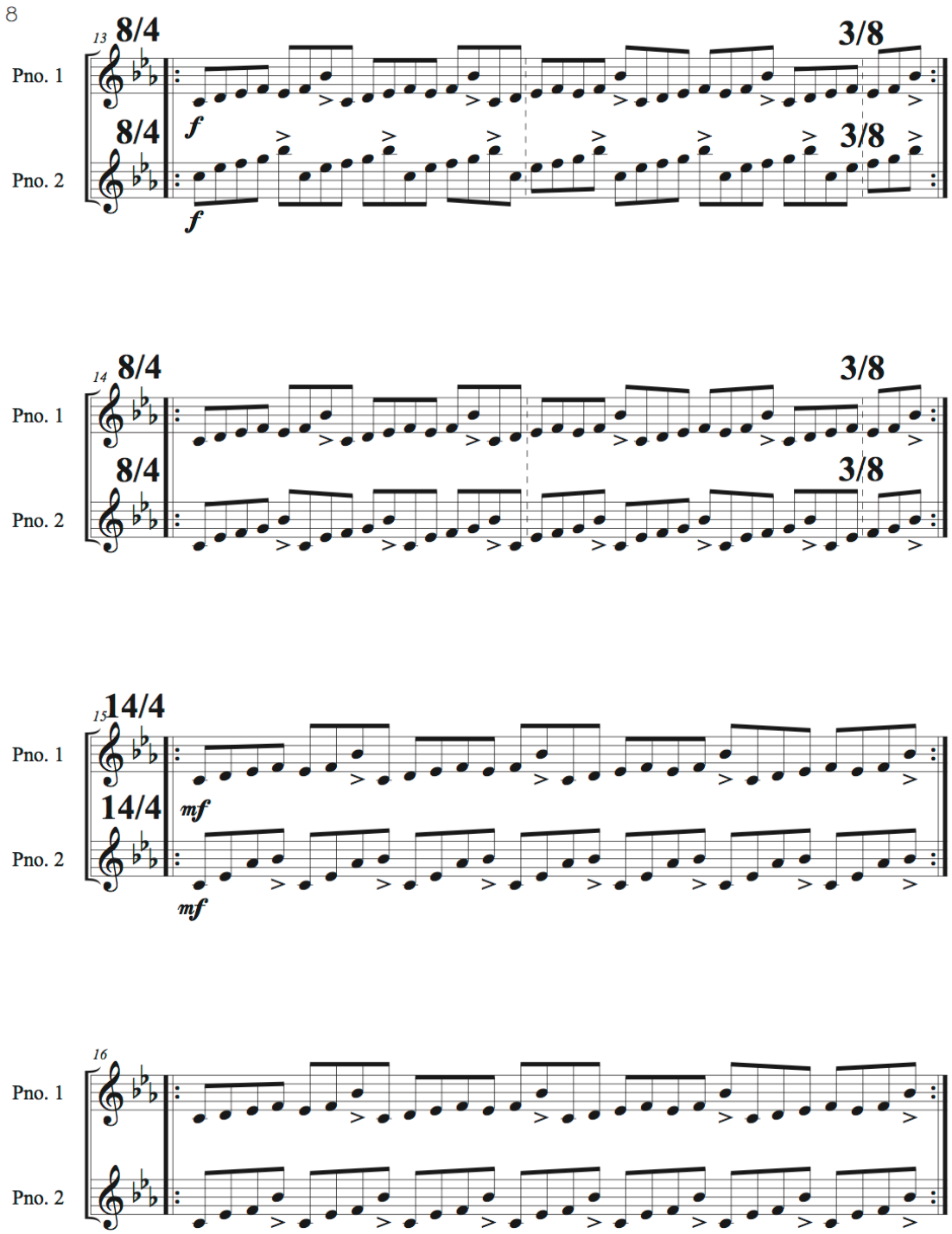
K
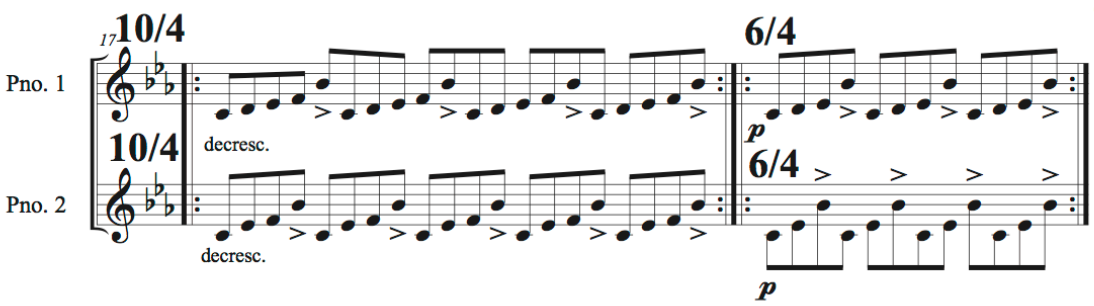
Dylan Crismani

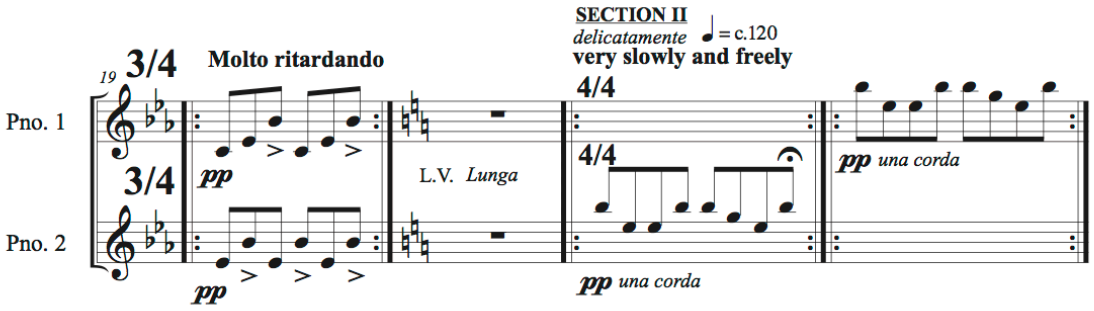


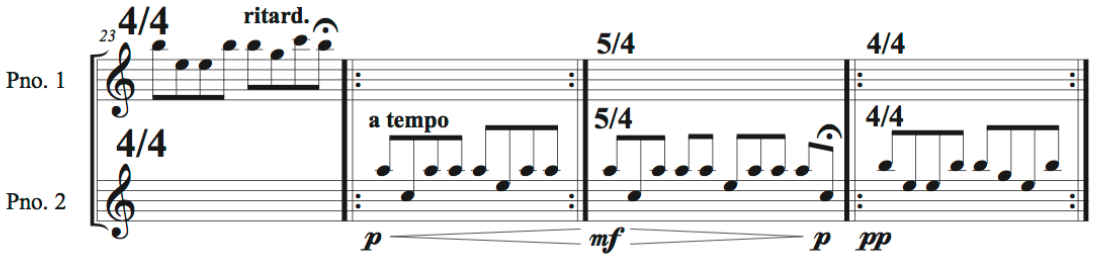

6
5
Pno. 1
tres corde
Pno. 2
tres corde
6
8/4
3/8
Pno. 1
8/4
3/8
Pno. 2
7
8/4
♩= c.200
3/8
Pno. 1
8/4
ff
3/8
Pno. 2
ff
8
8/4
3/8
Pno. 1
8/4
3/8
Pno. 2

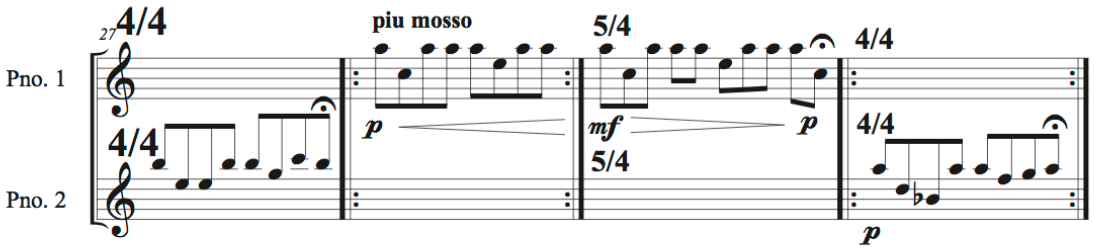

9
6/8
♩= c.220
7
Pno. 1
6/8
Pno. 2
10
6/8
3/8
Pno. 1
6/8
3/8
Pno. 2
11
6/8
Pno. 1
6/8
Pno. 2
12
8/4
3/8
Pno. 1
8/4
3/8
Pno. 2

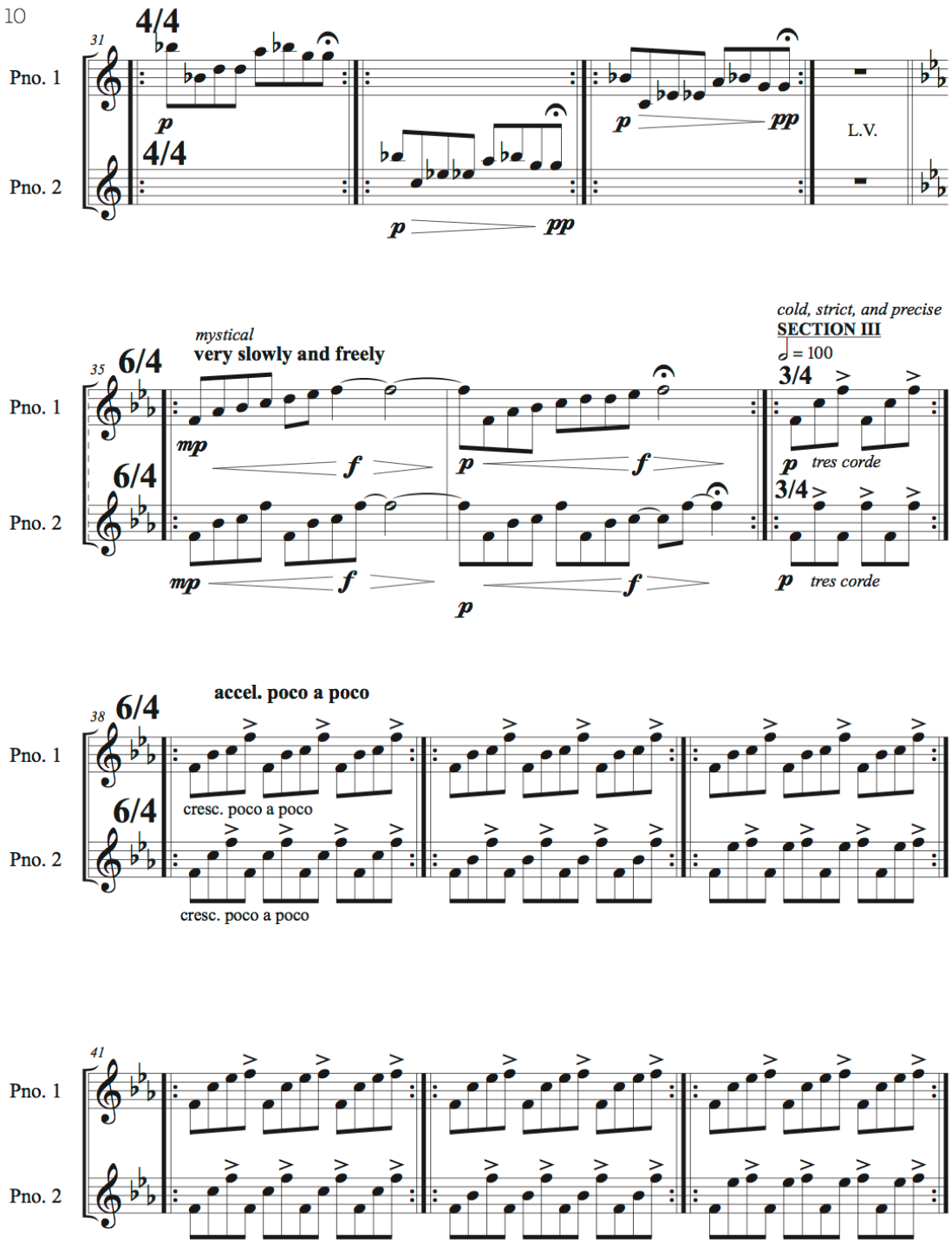

8
13
8/4
3/8
Pno. 1
8/4
f
3/8
Pno. 2
f
14
8/4
3/8
Pno. 1
8/4
3/8
Pno. 2
15
14/4
Pno. 1
14/4
mf
Pno. 2
mf
16
Pno. 1
Pno. 2

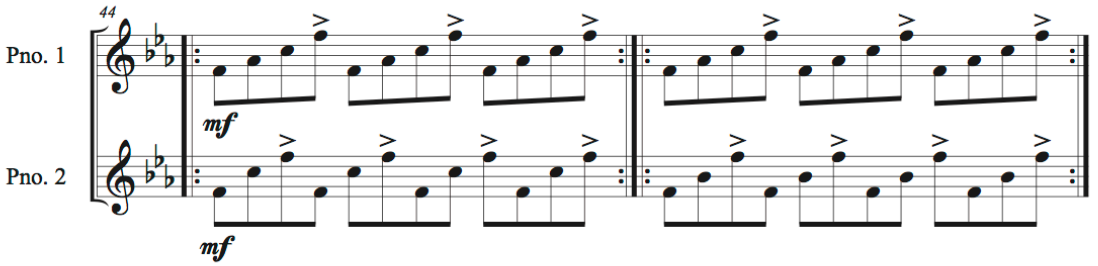

17
10/4
6/4
Pno. 1
10/4
6/4
Pno. 2
decresc.
decresc.
p
p

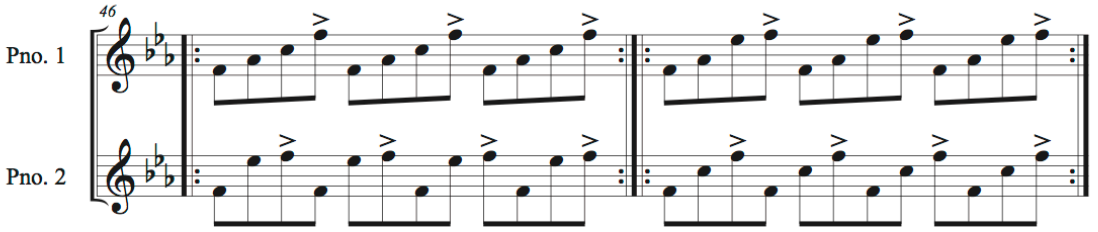

SECTION II
delicatamente
= c.120
very slowly and freely
19
3/4
Molto ritardando
4/4
Pno. 1
3/4
4/4
Pno. 2
pp
L.V. Lunga
pp una corda
pp
pp una corda

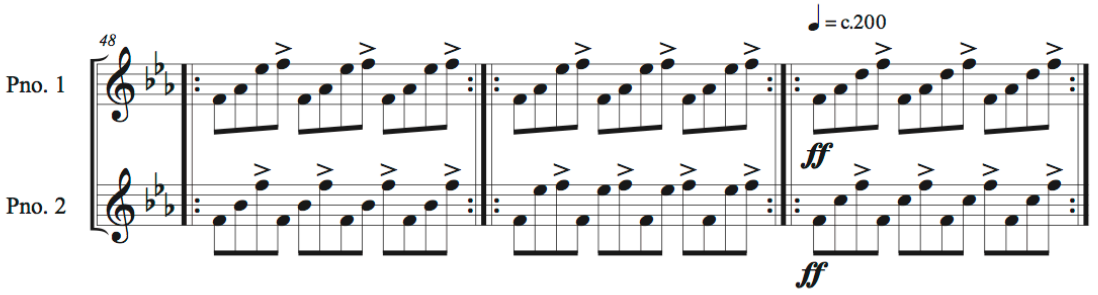

23
4/4
ritard.
5/4
4/4
Pno. 1
a tempo
4/4
5/4
4/4
Pno. 2
p
mf
p
pp

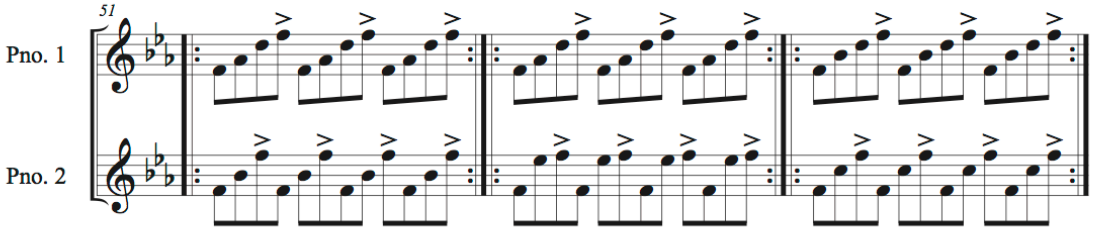

27
4/4
piu mosso
5/4
4/4
Pno. 1
4/4
5/4
4/4
Pno. 2
p
mf
p
p


10
31
4/4
Pno. 1
Pno. 2
p
4/4
p
pp
L.V.
p
pp
35
6/4
mystical
very slowly and freely
mp
f
p
f
6/4
mp
f
p
f
p
cold, strict, and precise
SECTION III
= 100
3/4
p
tres corde
3/4
p
tres corde
38
6/4
accel. poco a poco
cresc. poco a poco
6/4
cresc. poco a poco
41
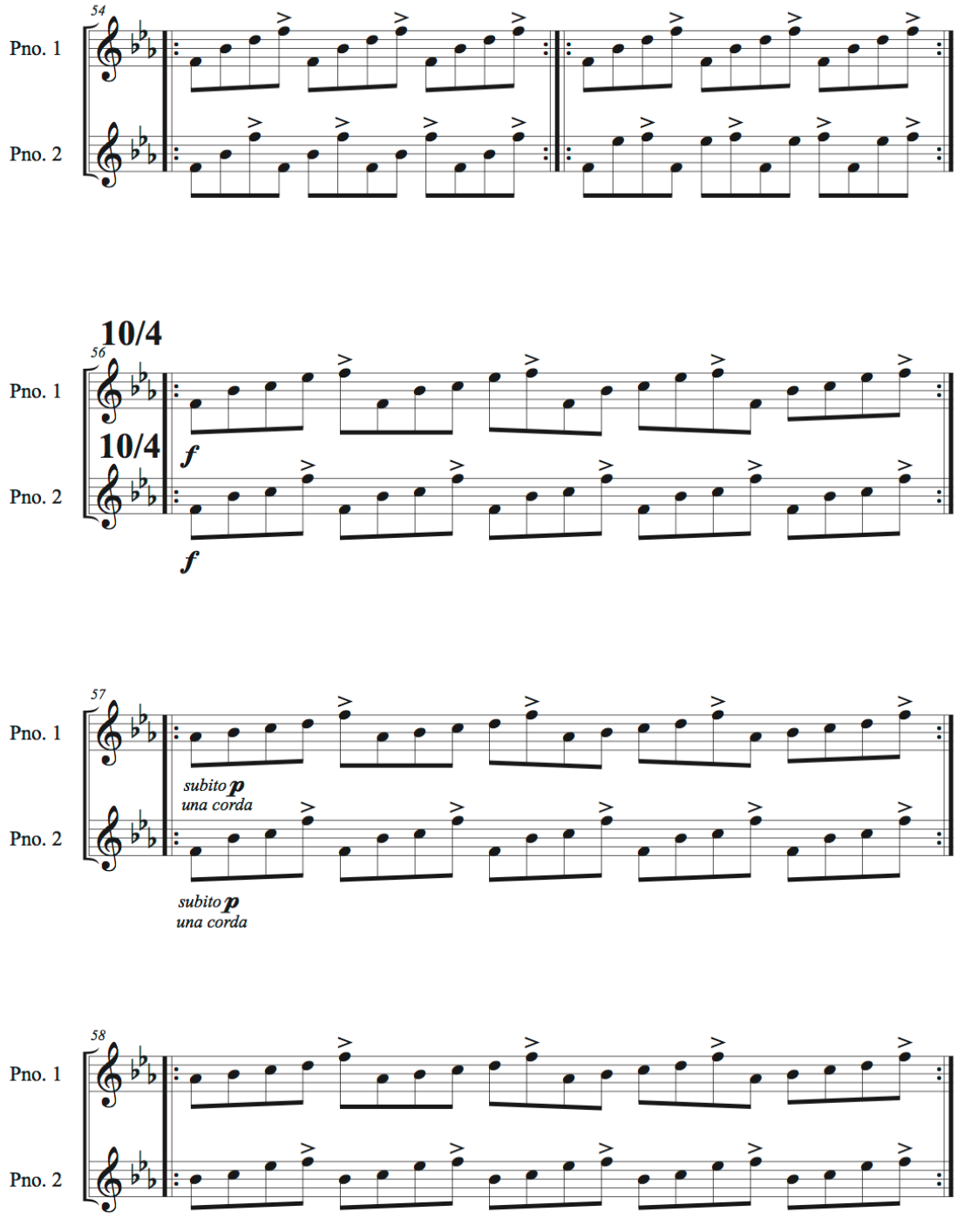

44
Pno. 1
Pno. 2
mf
mf

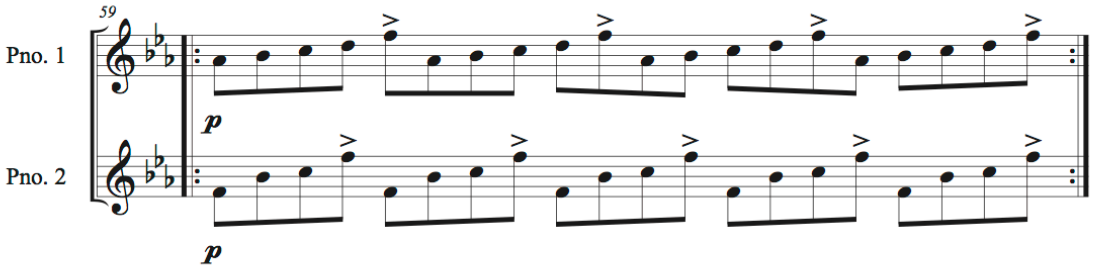

46
Pno. 1
Pno. 2

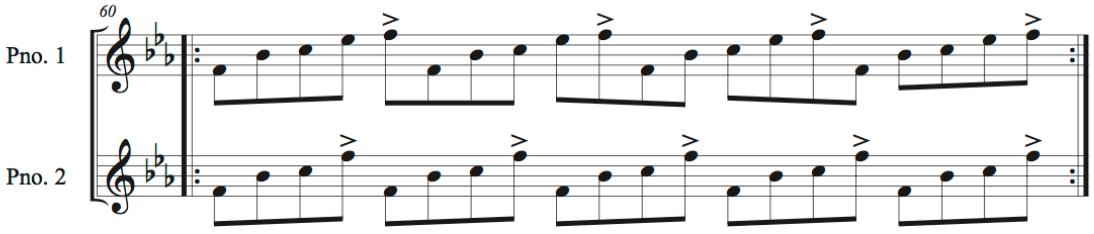

48
♩ = c.200
Pno. 1
Pno. 2
ff
ff

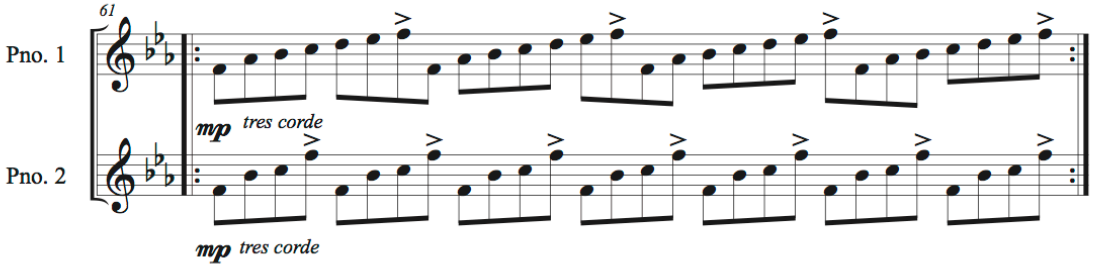

51
Pno. 1
Pno. 2

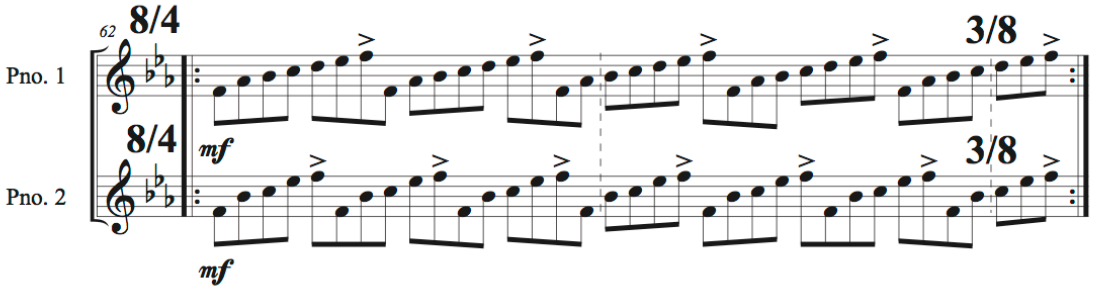
12


54
Pno. 1
Pno. 2
10/4
56
Pno. 1
10/4
f
Pno. 2
f
57
Pno. 1
subito p
una corda
Pno. 2
subito p
una corda
58
Pno. 1
Pno. 2

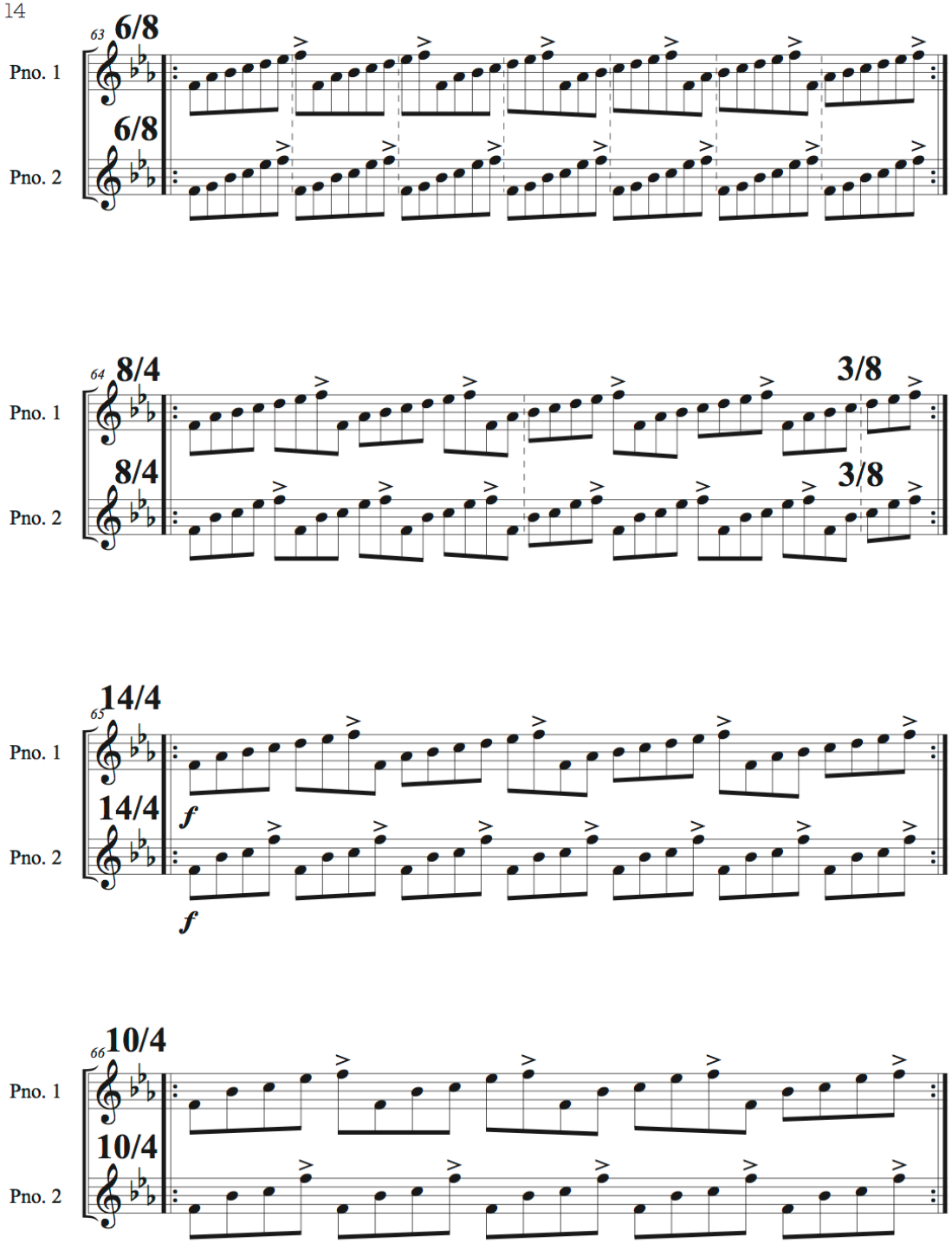

59
Pno. 1
Pno. 2
p
p
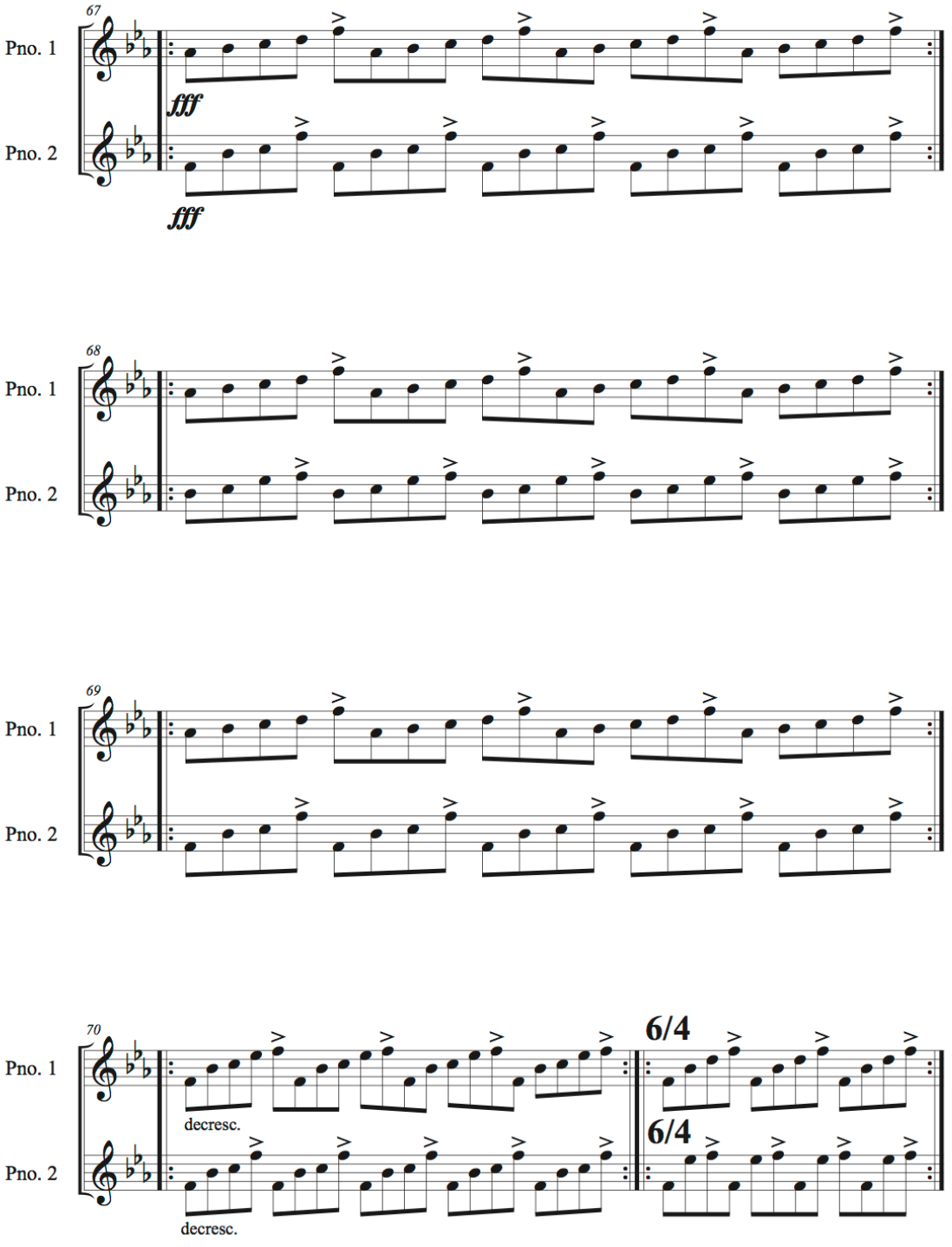

60
Pno. 1
Pno. 2

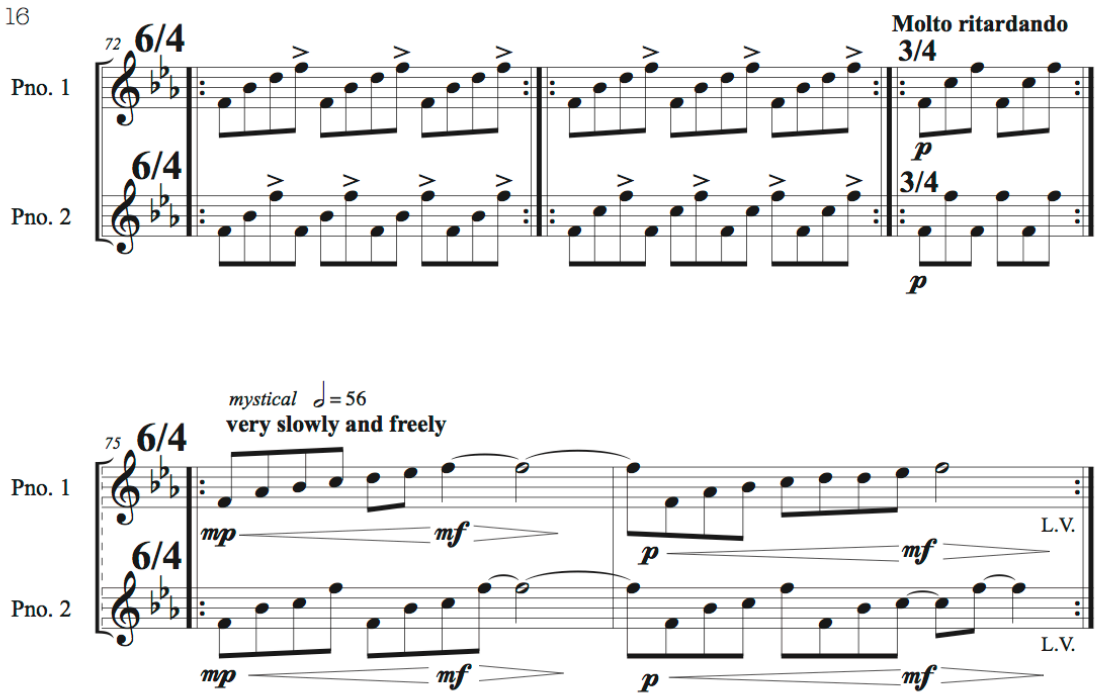

61
Pno. 1
Pno. 2
mp tres corde
mp tres corde


62
8/4
8/4
Pno. 1
Pno. 2
mf
mf
3/8
3/8

14


63
6/8
Pno. 1
6/8
Pno. 2
64
8/4
3/8
Pno. 1
8/4
3/8
Pno. 2
65
14/4
Pno. 1
14/4
f
Pno. 2
f
66
10/4
Pno. 1
10/4
Pno. 2


67
Pno. 1
Pno. 2
fff
fff
68
Pno. 1
Pno. 2
69
Pno. 1
Pno. 2
70
Pno. 1
Pno. 2
decresc.
decresc.
6/4
6/4


16
72
6/4
Pno. 1
6/4
Pno. 2
Molto ritardando
3/4
p
3/4
p
mystical
= 56
very slowly and freely
75
6/4
Pno. 1
6/4
Pno. 2
mp
mf
p
mf
L.V.
mp
mf
p
mf
L.V.

Chapter four: Discourse on the results

Introduction to the discussion

The discussion chapter outlines how the portfolio of compositions answers the research questions, and addressed the research aims. This chapter does not contain any discourse regarding research aim, and question number three, as this is addressed through chapter two. The remaining research aims, and questions have been addressed here, but each piece in the creative portfolio does not necessarily address every aim, and question.

4.1 Discussion of the results: *Remembering is a form of forgetting* for symphony orchestra

Remembering is a form of forgetting was the first piece of music written for the creative portfolio. The purpose of this piece was to address the first five research aims. The University of Adelaide provided five, thirty-minute rehearsals with the Elder Conservatorium Symphony Orchestra. Following the five rehearsals was a premiere of the work in the Elder Conservatorium of Music 2016 Lunch Hour Concert Series; the recording of the premiere is contained in appendix A. The practical approaches investigated by *Remembering is a form of forgetting*, including limited assignment of pitches to instruments, use of natural harmonics where possible, tuning to other instruments in the orchestra, and use of rehearsal tapes were met with varying degrees of success.

Assigning each instrumentalist only a single pitch, or a very limited number of pitches in order to remove intonational difficulties was well received and applied by the orchestra. This approach appeared to positively affect playability in the context of pitch. Unfortunately ease of playability in this area was counteracted by an unforeseen difficulty regarding playability in the realm of rhythm and musical texture. The concept of Klangfarbenmelodie, first discussed by Schoenberg in Harmonielehre,[1] informed the timbral structuring of *Remembering is a form of forgetting*. With the exception of the solo melodies, this piece is a-melodic, and essentially a continuously changing fabric of texture and simple harmony. The way in which this was achieved was through intentionally tying notes across bar lines, commencing notes in rhythmically weaker places

[1] Schoenberg, Arnold. *Harmonielehre*. Liepzig. Universal Edition, 1911.

within bars, and avoiding any type of distinct rhythmic patterning. This is especially true in measures 45 through 113, 131 through 138, and 149 through 157. Despite visual cues from the conductor, the orchestra seemed to find the total lack of composed-in rhythmic signposting disorienting. This is a problem which was more carefully considered in later pieces in the portfolio of compositions.

It was determined through *Remembering is a form of forgetting* that natural string harmonics, to a large degree, remove control of real time tuning from string players. A number of artificial harmonics appear in the score, which is due to the conductor, Luke Dollman who himself is a string player, believing that these particular harmonics would work better as artificial harmonics, this was a necessary compromise. Nevertheless, the success of string harmonics, both natural and artificial led to later pieces in the folio, including the major work, focusing on string ensembles being composed for entirely in natural harmonics. The tuning of harmonics on wind and brass instruments proved to have greater variability than string harmonics, due to the fact that instrumentalists are still, essentially, tuning these harmonics in real time, through the fundamental. This is problematic in terms of achieving consistency. For example, the correct tuning of a 405:256 G# is dependent upon the clarinetist taking an 81:64 E as the fundamental, but in reality the fundamental taken is likely to be anywhere between 5:4 to 80:63, or 81:64, and the fundamental is not necessarily one of these integer ratios, but possibly an irrational number.

Adapting Johnston's system of notation that is used is in string quartets numbers two[2] and three,[3] whereby instrumentalists tune certain intervals to one another, failed within the context of this piece, and was eventually abandoned. A number of factors influenced this failure. Firstly, *Remembering is a form of forgetting* spent a limited amount of time in rehearsal, and an adequate amount of this limited time was not allocated, by the conductor, to exploring this concept, despite his having been briefed on it prior to the rehearsal process. Furthermore, I was not assertive enough in demanding that this be a focus in rehearsals. Secondly, Johnston applies this concept in a much smaller group of musicians,

[2] Johnston, Ben. String Quartet No. 2. Baltimore. Smith publications, 1964.
[3] Johnston, Ben. String Quartet No. 3. Baltimore. Smith publications, 1966.

who possess specialised intonation skills, this undoubtedly increases the chances of this concept achieving any kind of success.

Finally, the density of the musical landscape, affected the instrumentalists' ability to focus on a single sound being produced by another player who was not necessarily within close, physical, proximity. It is likely that the application of Johnston's concept would be met with greater success, in an orchestral context, provided that adequate time was spent focusing on intonation in rehearsal, the musical landscape kept sparse, and the orchestra adequately briefed and trained in this method of tuning intervals. This was a shortfall of *Remembering is a form of forgetting.* Had Johnston's concept been more carefully composed into the piece, perhaps the conductor would have found it impossible to avoid using rehearsal time on this aspect of the composition, or the instrumentalists could have navigated this concept with greater ease.

The use of tuning tapes, as Doty recommends, is a way that variability of intonation can potentially be minimized. Players were offered two options, the first was access to a sound file containing the correctly tuned pitches, the second option was simply a hertz value for each note combined with a digital tuner. Instrumentalists demonstrated equal preference for each of these two options. It is difficult to accurately quantify the success of this approach in the context of a student ensemble who possess greatly varying skill levels.

The rehearsal and performance of *Remembering is a form of forgetting* demonstrates that, by implementing the techniques discussed in this chapter and the methodology, it is possible for a conventional orchestral ensemble to tune a number of intervals purely, with no specialist training. Despite this proof of concept, the possibilities of using just intonation in such a context remain largely un-explored, as *Remembering is a form of forgetting* had a limited scope in terms of what it was investigating. The main aim of this piece was to simply achieve differently tuned intervals, and this was realized; investigating ways in which unique intervals can inform musical ideas, and processes is addressed in the remainder of the creative compositional portfolio. *Remembering is a form of forgetting* revealed that the methodology presents a feasible approach to

composing playable music in just intonation, but, as with anything, it is not without its limitations.

Notation in *Remembering is a form of forgetting* was kept relatively standard, as per research aim number four. String harmonics were scored using small diamond note heads showing where to touch the string, with the sounding pitch being shown above on the same stave. As depicted in the score itself, and chapter two, table 2.1 soloists had a small number of additional symbols in their notation, which were designed to show varying degrees of departure from normal tuning. This was an adaption of the notation Ligeti uses in The Hamburg Concerto.[4] In my opinion this particular approach was not met with much success. Firstly, this approach allows the performers too much control over intonation, and secondly the table does not provide the performers with adequate information about how they might achieve this different tuning of intervals. For this system to work with greater efficacy perhaps small chalk markings need to be made on the instrument's fingerboard.

4.2 Discussion of the results: *The tale of Kiều* for scordatura string orchestra

Achieving just intonation in *The tale of Kiều*

In February 2019 *The tale of Kiều* for scordatura string orchestra was performed by the Basel Chamber Orchestra, conducted by Franck Ollu. *The tale of Kiều* addresses the aim of how an ensemble of classically trained, non-just intonation specialists, can produce just intoned music through the combination of natural harmonics and scordatura. During the early stages of conceiving the composition, natural harmonics were chosen as the simplest route to achieving pure intonation due to the fact that open harmonics cannot be tuned in real-time by a string player in the same way that a stopped note can be. The importance of playability dictated that only the first eight harmonics of each string would be

[4] Ligeti, Gyorgy. *Hamburg Concerto,* for solo horn and chamber orchestra*.* Mainz. Schott Music. 1998-99.

used as these are considered to be among the more accessible open harmonics.[5] It quickly became apparent, however, that the scale resulting from strings tuned 1:1, 3:2, 9:8, 27:16, and 81:64, and the first eight open harmonics of each of these strings would sound not too dissimilar from 12-TET. This presented the problem of either accepting the familiar, or considering some other way to access more pitches. Scordatura appeared to be the most logical option to pursue, considering the context of this project and its specified aims.

The second way in which *The tale of Kiều* for scordatura string orchestra addresses how an ensemble of classically trained musicians can play in just intonation with no specialist training is based on a suggestion from David Doty. Doty recommends that instead of sending your score out into the world littered with cent deviations and exotic accidentals, which are unlikely to be understood, composers should instead aim to develop practical solutions which will assist performers in achieving correct intonation.[6] One such way is to assign individual performers a small number of pitches only, in the same was as *Remembering is a form of forgetting* tested this idea. This, theoretically, allows performers to focus on achieving better intonation, as they have fewer notes to worry about tuning correctly. The system of scordatura combined with natural harmonics which is investigated in *The tale of Kiều* for scordatura string orchestra assigns 13 pitches to each instrumental table. This is, technically, one pitch more than 12-TET. Despite this, the way in which the pitches are assigned, as eight harmonics per string, is far more restrictive than what normal violin playing technique would allow.[7] Using conventional playing technique, a violinist may conservatively be capable of playing slightly more than two octaves per string, where each octave contains twelve pitches. The tuning system applied in *The tale of Kiều* for scordatura string orchestra requires instrumentalists to produce eight pitches, per string, across of a span of three octaves. Once octave repetition of the fundamental, and the third harmonic are taken into account, the actual number

[5] Strange, Patricia and Allen. "Harmonics.". *The Contemporary Violin.* Berkeley. University of California Press, 2001. P. 117.
[6] Doty, David B. *The Just Intonation Primer: An introduction to the theory and practice of Just Intonation.* Second ed. San Francisco. The Just Intonation Network, 1994. P. 71.
[7] The violin, in this instance, has simply been taken as one example, but the same principle applies to all the instruments in the string orchestra.

of discrete pitches per string is reduced to only four, a quarter the number of pitches that would be expected of a violinist in a conventional orchestral setting.

The possibilities of the tuning system developed for *The tale of Kiều*

The tuning system developed for *The tale of Kiều* presented three interesting possibilities which have been investigated. These possibilities are: variable tuning of chords in real time, exploration of pitches which lie within a single critical band, and the exploration of closely related versus distantly related harmonic relationships in very similar intervals.[8]

Variable tuning of intervals and chords

Working with such a large set of pitches, combined with a large number of instrumentalists made the variable tuning of intervals and chords, in real time, a very realistic possibility; this concept was investigated in the first movement of *The tale of Kiều.* The way in which this concept was applied is discussed in depth, on pages 249-254, with reference to how symmetrical modes have been applied in this piece of music.

Exploration of a number of pitches which lie within a single critical band

Movements one, two and five explore the relationships of pitches lying within single critical bands. There are seven commas in the lattice for *The tale of Kiều*, which allow the investigation of critical bands. Table 4.1 illustrates the seven commas that are available in the lattice, and how many times each comma occurs.

[8] Closely and distantly refer to physical proximity within the lattice. For example 1:1 to 3:2 could be considered closely related, where as 1:1 – 177147:131072 may be considered more distantly related, as the two ratios are further apart in the geometric series.

Table 4.1, the seven commas within the lattice for *The tale of Kiều*, and the frequency that they occur.

Integer ratio	**Name of comma**	**No. of times the comma occurs within the lattice**	**Series which the comma is derived from**	**Difference between**
32805:32768	Schisma	7	3:2 and 5:4	1:1 and 32805:32768
5120:5103	Garibaldi comma	9	5:4 and 7:4	5:4 and 5103:4096
81:80	Syntonic comma	12	3:2 and 5:4	5:4 and 81:64
531441:524288	Pythagorean comma	4	3:2	1:1 and 531441:524288
3645:3584	Un-named comma	9	5:4 and 7:4	7:4 and 3645:2048
59049:57344	Harrison's comma	5	3:2 and 7:4	7:4 and 59049:32768
63:32	Septimal comma	13	3:2 and 7:4	1:1 and 63:32

It is worth highlighting that there is a reasonably even distribution of commas across three geometric series. There are two commas occurring between each combination of the 3:2, 5:4, and 7:4, series with the Pythagorean comma subsisting independently within each chain of fifths. Therefore the factors affecting which commas occur most frequently in a composition include the composer's conceptual approach, combined with whether the composer has some intention to utilize certain combinations of geometric series. In movement one, the main compositional goal was to change the tuning of a number of chords, creating different iterations of around thirty chords. The harmonic nature of the chords is quartal, and the way in which they are arranged is according to voice leading principles; see appendix E, part one for piano sketches of the first movement of *The tale of Kiều*. This operation was achieved whilst using modes that are symmetrical, un-transposable, or both. This approach caused a large number of septimal and syntonic commas to occur, a less significant number of Harrison's and Pythagorean commas occurred, with the Schisma, Garibaldi comma, and un-named comma occurring in smaller amounts again. Appendix D, part one provides a comprehensive bar by bar list of the
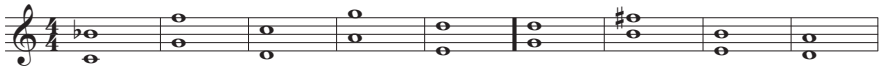
commas as they occur in movement one. In movement two the main compositional goal was to alternate between static chords, and tremolo chords that have a fluttering effect. As a general rule the static chords, if arranged as a mode were originally intended to be symmetrical in design, while the tremolo chords were not intended to be symmetrical in design. The symmetrical design of the static chords was at times compromised in order to include one of the commas shown in table 4.1. This movement investigates the relationship between the 5:4 series and the 7:4 series more so than the other movements. It is therefore unsurprising that the Garibaldi comma is the most frequently occurring, with less frequent instances of the syntonic comma, the septimal comma, and an un-named comma (3645:3584). Appendix D, part two provides a list that shows which commas are used throughout the second movement of *The tale of Kiều*, and the frequency with which they occur. In movement five the main compositional goal was to alternate between different tunings of dyads based on the size of the ratios, or between closely related ratios in the lattice, and more distantly related ratios. This compositional approach appears to have resulted in a reasonably even distribution of commas between the 3:2 – 5:4, and the 3:2 – 7:4 series, and within each series individually. The least utilised commas are those subsisting between the 5:4 and 7:4 series. The complete list of commas, as they unfold in movement five is contained in appendix D, part three.

Exploring closely related versus distantly related harmonic relationships in very similar intervals

The lattice for *The tale of Kiều* for string orchestra extends the central 1:1(a) and 3:2 (r) geometric series to ar^{15}, and the two corresponding 5:4, and 7:4 series to ar^{14}. The length of these three chains of fifths allows a number of complex, or distantly related harmonic relationships, as well as simple, or closely related harmonic relationships; the fifth movement of *The tale of Kiều* for scordatura string orchestra aims to exploit this. Initially the fifth movement of *The tale of Kiều* was conceived simply as a series of dyads. The original piano sketch of the fifth movement is contained in appendix E, part two. These sketches reference

the dyadic opening of *The Well Tuned Piano*[9] which is also mainly composed of fifths and sevenths. The sketches contained in appendix E, part two can be reduced to a small number of perfect fifths and minor sevenths; these are shown in example 4.1.

Example 4.1, the basic dyadic harmonies which form the fifth movement of *The tale of Kiều*.



True dyadic harmony is not retained in the final orchestration. The way in which the orchestration of this movement changes the original pure dyadic harmony concept is by alternating between simple and complex harmonic versions of a single interval. Bars 217-252 analyzed in Table 4.2 exemplify how this oscillation between simple and complex dyads unfolds in the fifth movement of *The tale of Kiều* for scordatura string orchestra.

Table 4.2, analysis of dyads in selected bars from the fifth movement of *The tale of Kiều.*

	C-Bb	G-F	D-C	A-G	E-D	G-D	B-F#	E-B	D-A	A-E
Bar 217	-	-	-	-	S/C	-	-	-	-	-
Bar 218	-	-	-	-	Simple	-	Simple	-	-	-
Bar 219	-	-	-	-	Complex	-	Complex	-	-	-
Bar 221	-	-	-	S/C/C	-	-	-	-	-	-
Bar 223	-	-	-	Simple	-	-	S/S	-	-	-
Bar 224	-	-	-	Simple	-	-	Complex	-	-	-
Bar 226	-	-	-	Simple	-	-	-	S/S/S	-	Simple
Bar 227	-	-	-	Complex	-	-	-	Complex	-	Complex
Bar 229	-	-	-	Simple	-	S/C	-	-	Simple	-
Bar 230	S/C	-	-	-	-	-	-	-	-	-
Bar 232	Simple	-	-	-	-	S/C	-	-	-	-
Bar 233	Complex	-	-	-	-	Complex	-	-	-	-
Bar 238	-	-	S/C	-	-	-	-	-	-	-
Bar 239	-	-	-	-	-	-	-	-	Simple	-
Bar 240	-	-	-	-	-	-	-	-	Complex	-
Bar 246	-	-	Simple	-	-	-	-	-	-	Complex
Bar 249	Complex	-	-	-	-	Complex	-	-	Complex	-

[9] Young, La Monte. *The Well Tuned Piano: A Basic Descriptive Score of the Main Themes and Chordal Areas*. Transcribed by Michael Harrison under the direction of La Monte Young. Unpublished. 1988.

Bar 252	-	Complex	-	-	-	Complex	-	-	S/C	-

S/C indicates that a given bar contains a simple relationship followed by a complex relationship, and S/S/S or S/C/C indicate three different simple relationships in succession or one simple and two complex relationships in succession, respectively. More detailed reference tables, showing the precise integer ratios played by each instrument in the fifth movement of *The tale of Kiều*, are provided in Appendix D, part four.

The limitations of the tuning system developed for *The tale of Kiều*

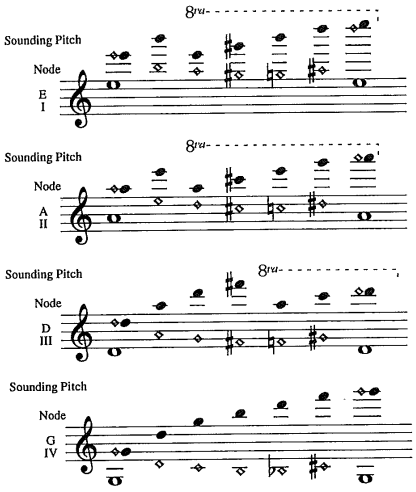
The methodology developed for *The tale of Kiều* for string orchestra is rich in possibility; it is perhaps even richer in difficulties. The limitations that arise can be separated into four main categories: the limitation of human ability, the limitation of human perception, limitation imposed by physical phenomena, and the limitation of uneven distribution of pitches across octaves.

The limitation of human ability

The limitation of human ability, in this context, refers to the skill level or proficiency of the instrumentalists. The main problem to consider in this situation is the fact that the entire piece is written in natural harmonics. Example 4.2 illustrates the more accessible open harmonics on the four strings of a violin, according to Patricia and Allen Strange.[10]
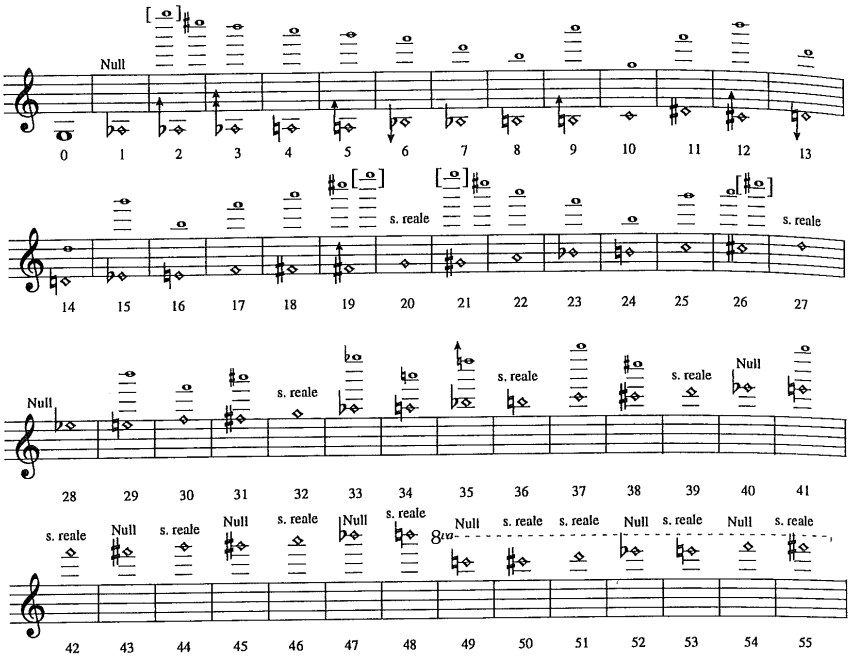
[10] Strange, Patricia and Allen. "Harmonics.". *The Contemporary Violin.* Berkeley. University of California Press, 2001. P. 117.

Example 4.2, the more accessible harmonics on all four strings, illustration taken from *The Contemporary Violin*.



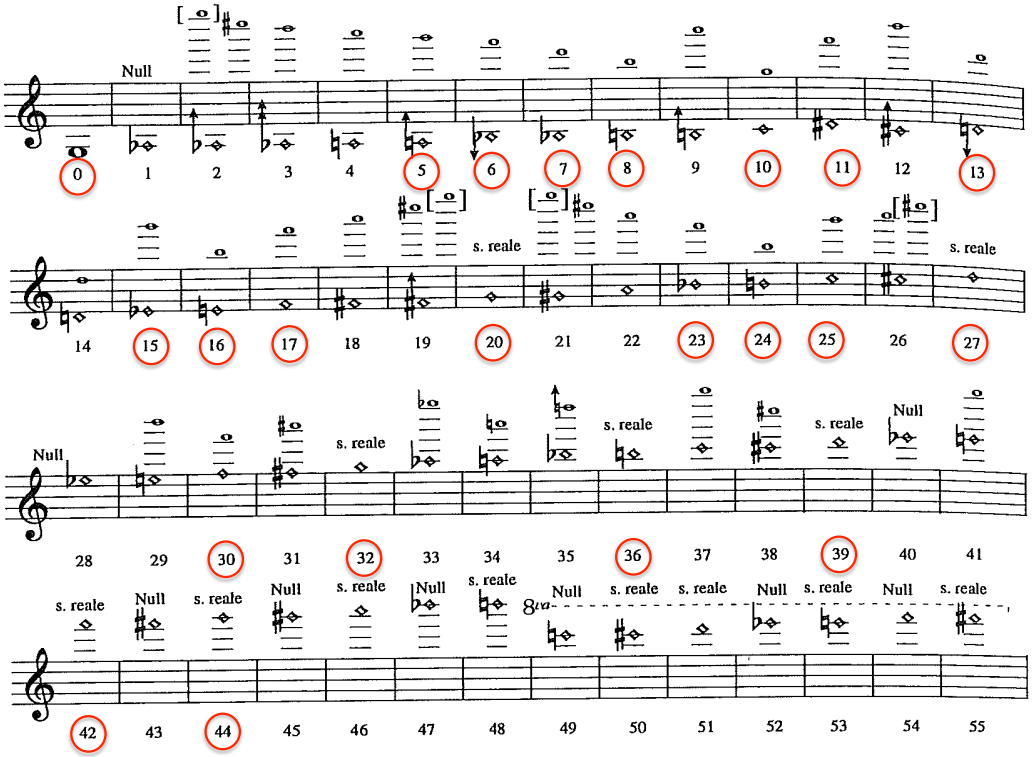
Having taken the information in example 4.2 into account, *The tale of Kiều* was written using these harmonics as the starting point. Using this information, combined with the precise location of up to 55 harmonics per string, shown in example 4.3, numerous positions for each harmonic could be determined.

Example 4.3, microtonal natural harmonics, illustration taken from *The Contemporary Violin*.



Example 4.3 shows that a number of these 55 microtonal harmonics are simply alternative finger positions for the same eight sounding pitches. The multiple positions have been marked in example 4.4.

Example 4.4, microtonal natural harmonics, with markings showing which pitches are repeats of the first eight harmonics. Illustration taken from *The Contemporary Violin*.


Null
s. reale
Null
s. reale
Null
s. reale

1. With the exception of contrabasses, double stops are never used.
2. Rhythmic movement is quite limited in all movements except movement three.
3. Rhythmic patterns in movement three are in single left hand positions, and are repetitive.
4. In general, all tempi are rather slow. This has also been done in order to allow listeners time to experience certain predicted results such as the auditory effects of critical bands, variable chord tuning structures, and so on.

The tactics used in *The tale of Kiều* to maintain simplicity, and playability appear to have been successful following the performance of the piece by the Basel Chamber Orchestra. The orchestra mastered the piece within three rehearsals (three hours) with few complications.

The limitation of human perception

The human ear is capable of discriminating two pitches within a few cents of one another, in the case of musicians. This ability, however, begins to dwindle above 5000Hz[11]. The system of scordatura, and harmonics which was devised for *The tale of Kiều* contains a small number of notes near the 5000Hz range. In addition to this, as has already been demonstrated, certain pairs or groups of notes lie within a single critical band. The way these notes, which lie within a single critical band, are perceived varies between individual ratios, and octaves. The extent to which some pitches lying within a single critical band can be perceived at all also varies. Overall, the varying effects of critical bands in *The tale of Kiều* are perceivable. The piece is characterized by a dissonance, or distortion in the sonic texture.

The limitation imposed by physical phenomena

A wide number of variables affect the timbre, and spectrum of a bowed string, and therefore the precision of intonation, both perceived and real. Such factors,

[11] Strange, Patricia and Allen. "Tuning Systems." *The Contemporary Violin.* Berkeley. University of California Press, 2001. P. 141.

discussed at length by Rossing, and Fletcher and Rossing in *The Science of String Instruments*[12] and *The Physics of Musical Instruments,*[13] will affect music belonging to any tuning system. The problem, or limitation, caused by this particular methodology lies in the use of natural harmonics. Under normal circumstances a string player has a high degree of control over many of these variables, and thus the intonation; much of the methodology of this project focuses on removing intonational control from the instrumentalists. The phenomenon of inharmonicity, therefore, could potentially be more problematic for this piece of music compared to a piece of less experimental music, as certain physical variables affecting intonation are beyond the control of the performer.

The limitation of uneven pitch distribution across octaves

The system of scordatura and harmonics for *The tale of Kiều* was not designed to achieve an equal number of notes per octave. The system was designed to solve the problem of how conventional instrumentalists playing conventional instruments could play music in just intonation. A result of attempting to solve this problem in the specific way that this project has adopted has caused the limitation of uneven distribution of pitches across octaves to occur. As with any limitation, the limitation of uneven distribution of pitches across octaves could be considered problematic, or as a new parameter that simply needs to be observed. Individual perceptions aside, the factors influencing this uneven distribution are physical in nature and include, the structure of the harmonic series; and the fundamental frequencies that are assigned to individual strings. Pitch class repetition is embedded in the harmonic series; the harmonic series of each string, up to the triple octave, contains four pitch class repetitions of the fundamental, two pitch class repetitions of a perfect fifth, and only one iteration of both the major third, and minor seventh. The central geometric series in the lattice for *The tale of Kiều* represents the open tuning of the orchestra, and thus the fundamental frequency of each string. As this particular geometric series represents not only the fundamental frequency of each string, but also five out of

[12] Guettler, Knut. "Bows, Strings, and Bowing." In: Rossing T. (eds) *The Science of String Instruments*. New York. Springer, 2010.
[13] Fletcher, Neville H. and Thomas D. Rossing. "Bowed String Instruments." In: *The Physics of Musical Instruments.* New York. Springer-Verlag, 1991.

eight harmonics being utilized in this system, it quickly becomes evident that this set of pitches will be the most numerous once distributed across a number of octaves. Pitch class repeats of the fifth and seventh harmonics don't occur until the tenth and fourteen harmonics are reached, respectively. As the methodology only calls for harmonics to the triple octave (eighth harmonic), therefore pitches from the 5:4 chain, and the 7:4 chain are less numerous, only occurring once per string. This point is clearly illustrated in examples 2.26, and 2.27, which are contained in chapter two, section 2.2.

Notation in *The tale of Kiều*

This aim of how music can be composed in just intonation with limited alteration to standard notation is addressed in *The tale of Kiều* for scordatura string orchestra by treating a large number of instruments as transposing instruments. By re-tuning these instruments to a specific scordatura and using open harmonics, the instrumentalists need only know where to place their fingers on the string. Due to the methodology applied, the intonational accuracy of pitches is determined primarily by the physical behaviour of the string, and secondarily by the instrumentalist. The second aspect to take into account regarding notation is that the sounding pitches have all been scored as 12-TET approximations. These sounding pitches could have been scored using cent deviations and exotic accidentals. Given the context of the music, however, those alterations would be of little use to anyone other than perhaps a music analyst whose role is redundant in the context of the performance of this music anyhow. Arguments concerning the relevance of music analysts in this project aside, anyone wishing to analyse the music through specific integer ratios can nevertheless do so, by utilizing the relevant tuning information contained in the introductory notes in the score. It is arguable whether the method of notation that has been developed for *The tale of Kiều* is more time-consuming for the composer, and useful for performers and analysts than Johnston's method of notation. Johnston's method is certainly admirable in the sense that it elegantly conveys a vast spectrum of tuning data to the informed reader. My method of notation actively seeks to remove this extraneous information from the score itself, providing only what is absolutely necessary for the performers, with additional information contained

in performance notes, and then further separate sets of data in form of ratios alone are produced for the purpose of analysis. In reality, however, how often would a composer realistically need to produce these analytical sets of data? Perhaps for training pupils, or at the request of another scholar, but these circumstances would likely be the exception, not the rule. If this secondary data need not be produced every time a score is composed, perhaps then, my method of notation provides a quicker alternative to Johnston's. Despite this, there are obvious benefits to Johnston's system, such as not needing to play entire scores in natural harmonics, which my system does not allow. My method of notation was well received by the Basel Chamber Orchestra and the conductor, Franck Ollu.

How prime numbers have been applied in *The tale of Kiều*

Prime numbers have been used to generate the tuning model for *The tale of Kiều;* to determine the duration of the second movement; as a data management system in movements one, two, and five; and to determine tempo relationships between movements.

Tuning

As has already been discussed in chapter two, the tuning model is derived from three connected geometric series based on the co-prime ratios of 3:2, 5:4 and 7:4.

Duration of movement two

Movement two is broken into five sections and a coda. Without the coda, the five sections are structured as shown in table 4.3.

Table 4.3, durations of the five sections in movement two.

Section	Description	Total length
I	9 bars of 3 quavers each	27 quavers
II	Effectively 9 bars of 5 quavers	45 quavers
III	9 bars of 7 quavers	63 quavers

IV	5 bars of 9 quavers	45 quavers
V	9 bars of 3 quavers	27 quavers

Table 4.3 shows that the rhythmic duration of each section is determined by combinations of the generating primes three, five, and seven; furthermore sections one, two, four, and fives are then arranged symmetrically around section three.

Rhythmic durations as a system of pitch identification, and therefore data management, in movements one, two, and five.

Movements one and five are comprised of only three rhythmic durations; a dotted minim, a dotted minim tied to a minim, and a semibreve tied to a dotted minim. These values correspond quite directly to the prime numbers three, five, and seven, respectively. This same idea is applied in movement two, but the base value of a single unit changes from crotchets to quavers. Assigning fixed rhythmic values to specific ratios, based on the numerator of the co-prime ratio allows the precise value of a musical pitch on a five-line staff to be identified quickly.[14] This system of pitch identification is significant because the style of notation used in this research project is intentionally standard, and there are a large number of pitches in the tuning model for *The tale of Kiều* for scordatura string orchestra. As discussed in chapter two, section two, one of the steps in the composition of *The tale of Kiều* was to transpose sounding pitches into tablature showing instrumentalists where to place their fingers. The process of transposition was not a particularly quick task. The duration of this task would have been considerably lengthened without such a system of quick pitch identification. In addition to being of use to a composer, it also potentially holds value for the music analyst if they are aware that the system exists, and that it is in use in a given composition. Indeed during my analysis of *The tale of Kiều* it was frustrating when for some reason I had decided to deviate from the pitch-data management system!

[14] That is a musical pitch, with no exotic accidentals, or cent deviations attached to it.

Tempo relationships between movements.

The intended tempi, and actual tempi used throughout *The tale of Kiều* for scordatura string orchestra are shown in table 4.4.

Table 4.4, the intended, and actual tempo for each movement in *The tale of Kiều.*

Movement no.	**Intended tempo**	**Actual tempo**
1	Crotchet = 30	Crotchet = 100
2	Crotchet = 50	Crotchet = 60
3	Crotchet = 110	Crotchet = 140
4	Crotchet = 50/70	Crotchet = 50/80
5	Crotchet = 70	Crotchet = 180

Table 4.4 Shows that the tempo for each movement was originally based on one of the four prime numbers used to generate the tuning matrix, with the exception of eleven. Different tempi were trialled for movement three, based on the numbers two, three, five, and seven, but eventually eleven was chosen, despite not being integrated into other aspects of the piece.

The reason the tempi were changed to those numbers appearing in the far right column of table 4.4 is because of the Basel Composition Competition. In order to enter the composition into this competition the duration requirement was twenty minutes or under. As the piece was shortlisted for the finals, the tempi needed to be adjusted in order to meet the duration requirements. The most drastic of the tempo adjustments took place in movements one and five, and perhaps some subtlety was lost as a result of these changes. The trade-off, however, was that the piece would actually be played, and a loss of subtlety in exchange for a performance from a professional orchestra is arguably a reasonable trade.

Macro-periodicity

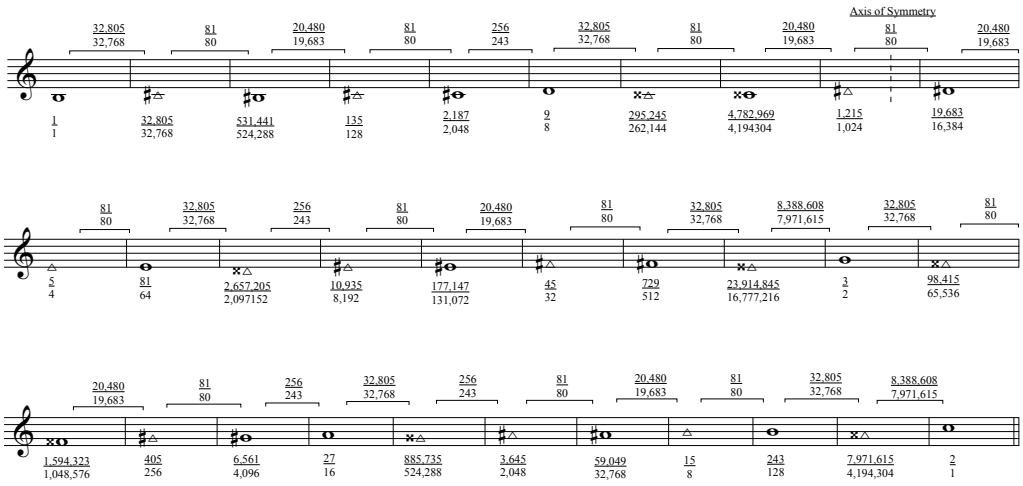
Initially the third movement of *The tale of Kiều* was composed using the same technique as *13*; the difference in size of ensembles, however, is obviously a factor that requires consideration. For string quartet, long repetitive polyrhythmic quaver passages have more chance of succeeding; this can be

attributed to the smaller number, and skill level, of the musicians in the ensemble. While such writing may be achievable in an orchestral setting, the desired effect of such writing would likely be less successful than in a smaller ensemble. With this consideration in mind, strict macro-periodic writing was abandoned, but the relatively fast quaver movement was retained throughout this movement.

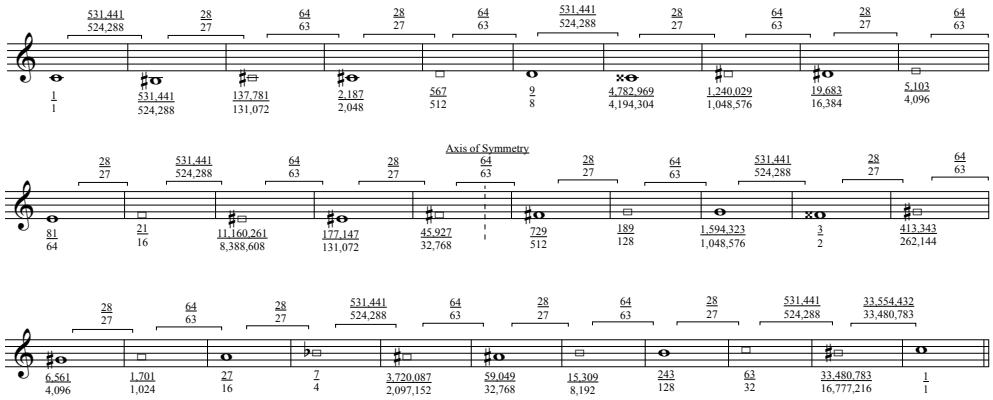
The application of symmetrical modes in *The tale of Kiều*

The tuning model developed for *The tale of Kiều* theoretically contains five Maximal Un-transposable Harmonic Modes, although only three of these were given consideration during the compositional process, these three MUHMs are illustrated in examples 4.5 through 4.7.

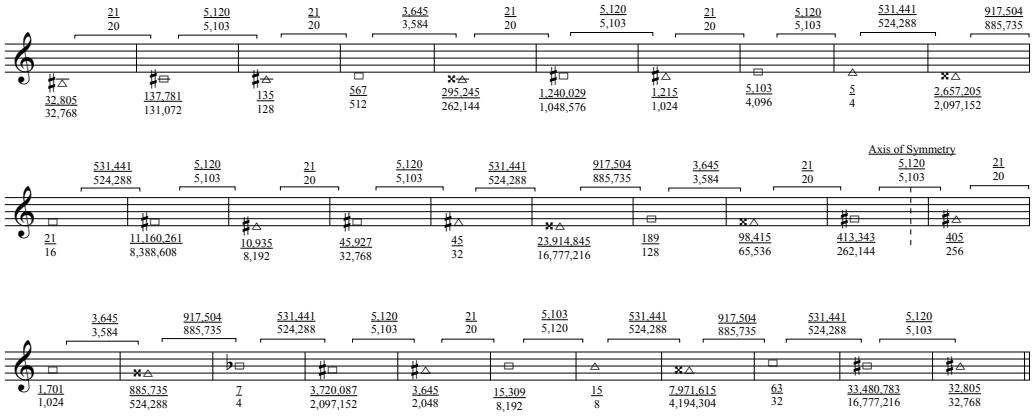
Example 4.5, the Maximal Un-transposable Harmonic Mode derived from the 3:2 and 5:4 series.



Example 4.6, the Maximal Un-transposable Harmonic Mode derived from the 3:2 and 7:4 series.



Example 4.7, the Maximal Un-transposable Harmonic Mode derived from the 5:4 and 7:4 series.



The two MUHMs which were not considered during the compositional process exist between the 3:2 – 5:4 and 3:2 – 7:4 series, and are found by replacing 1:1 with 14384907:8388608.

The application of symmetrical modes in movement one

There are three types of mode that have been used in the first movement of *The tale of Kiều*:

1. Modes which have no transpositions, and are not symmetrical.
2. Modes which have transpositions, and are symmetrical.
3. Modes which have no transpositions, and are symmetrical (UHMs).

The characteristics of each mode used in movement one is illustrated in table 4.5.

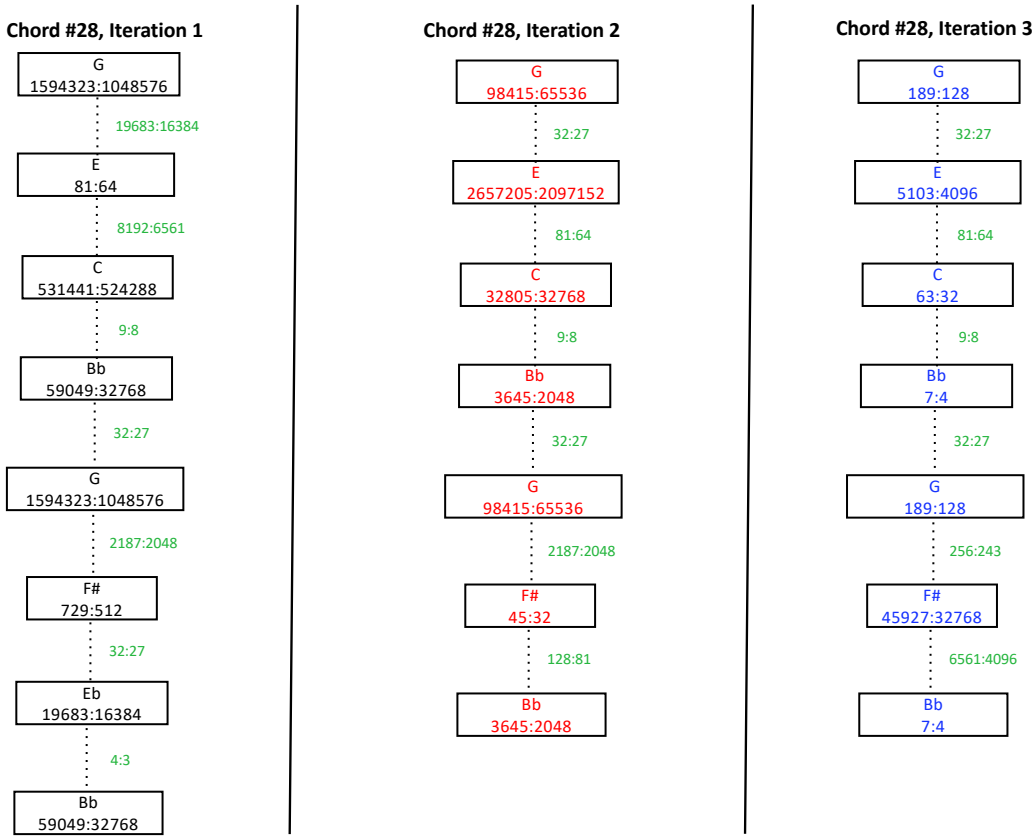
Table 4.5, chart showing the characteristics of each mode used in movement one of *The tale of Kiều*.

Chord no.	Iteration no.	Transpositions	Symmetrical	UHM	Series	Bar no.
1	1	0	Yes	Yes	3:2, 5:4	1
1	2	0	No	No	3:2, 5:4, 7:4	2
2	1	0	Yes	Yes	3:2, 5:4	3
2	2	5	No	No	3:2, 7:4	4
3	1	0	Yes	Yes	3:2, 5:4	5
3	2	3	No	No	3:2, 7:4	6
3	3	0	Yes	Yes	3:2, 7:4	7
4	1	3	No	No	3:2	8
4	2	2	No	No	3:2, 5:4, 7:4	9
5	1	3	No	No	3:2, 5:4, 7:4	10
6	1	0	Yes	Yes	3:2, 5:4	11
6	2	3	No	No	3:2, 5:4, 7:4	12
7	1	0	Yes	Yes	3:2, 5:4	13
7	2	0	Yes	Yes	3:2, 7:4	14
7	3	0	No	No	3:2, 5:4, 7:4	15
8	1	6	Yes	No	3:2, 7:4	16
8	2	0	No	No	3:2, 5:4, 7:4	17
9	1	0	Yes	Yes	3:2, 7:4	18
9	2	4	No	No	3:2, 5:4, 7:4	19
10	1	6	Yes	No	3:2, 7:4	20
10	2	3	No	No	3:2, 5:4, 7:4	21
11	1	6	Yes	No	3:2, 7:4	22
11	2	3	No	No	3:2, 5:4, 7:4	23
12	1	8	Yes	No	3:2, 7:4	24
12	2	6	No	No	3:2, 5:4, 7:4	25
13	1	4	Yes	No	3:2, 7:4	26
13	2	1	No	No	3:2, 5:4	27
14	1	0	Yes	Yes	3:2, 7:4	28
14	2	2	No	No	3:2, 5:4	29
15	1	6	No	No	3:2, 5:4, 7:4	30
15	2	0	No	No	3:2, 5:4, 7:4	31
15	3	0	No	No	3:2, 7:4	32
16	1	4	Yes	No	3:2, 5:4	33
16	2	4	No	No	3:2, 5:4, 7:4	34
16	3	0	Yes	Yes	3:2, 7:4	35
17	1	2	Yes	No	3:2, 7:4	36
17	2	2	No	No	3:2, 5:4, 7:4	37
19	1	0	Yes	Yes	3:2, 7:4	38
19	2	1	No	No	3:2, 5:4, 7:4	39
20	1	4	Yes	No	3:2, 7:4	40
20	2	3	No	No	3:2, 5:4	41
21	1	19	No	No	3:2	42
21	2	3	No	No	3:2, 5:4	43
21	3	3	No	No	3:2, 5:4, 7:4	44
23	1	1	No	No	3:2, 5:4, 7:4	45
24	1	2	Yes	No	3:2, 5:4	46
24	2	2	No	No	3:2, 5:4, 7:4	47
24	3	2	Yes	No	3:2, 7:4	48
25	1	1	No	No	3:2, 5:4, 7:4	49
26	1	5	Yes	No	3:2, 7:4	50
26	2	0	No	No	3:2, 5:4	51
27	1	6	Yes	No	3:2, 5:4	52
27	2	3	No	No	3:2, 5:4, 7:4	53
28	1	16	No	No	3:2	54
28	2	13	No	No	5:4	55
28	3	19	No	No	7:4	56
29	1	4	Yes	No	3:2, 5:4	57
29	2	1	No	No	3:2, 5:4, 7:4	58
29	3	0	Yes	Yes	3:2. 7:4	59
30	1	3	No	No	3:2	60
30	2	8	No	No	5:4	61
30	3	10	No	No	7:4	62
31	1	4	Yes	No	3:2, 5:4	63
31	2	0	No	No	3:2, 7:4	64
31	3	6	Yes	No	3:2, 7:4	65
32	1	0	No	No	3:2, 5:4, 7:4	66

33	1	2	Yes	No	3:2, 7:4	67
33	2	0	No	No	3:2, 5:4	68

Various combinations of these symmetrical modes, asymmetrical modes, modes with no transpositions, and UHMs have been used throughout the first movement as a way to change the tuning of basic chord structures in real time. This concept of multiple iterations of the same chord structure with variable tuning is what conceptually underpins the entire first movement, and it is applied in three ways. Firstly the concept of beat-less purity, which some prize just intonation for, is briefly investigated through chord nos. 4, iteration 1; 21, iterations 1, 2, and 3; 28, iterations 1, 2, and 3; and 31, iterations 2, and 3. Example 4.8 shows how this concept is applied in chord number twenty eight.

Example 4.8, analysis of the three iterations of chord #28 from *The tale of Kiều*.



Example 4.8 shows that each iteration of chord number twenty-eight is made up of integer ratios from a single geometric series, not combinations of the three series. The second way in which the variable tuning of chords is applied in movement one is quite the opposite to the first way in which this is done. The

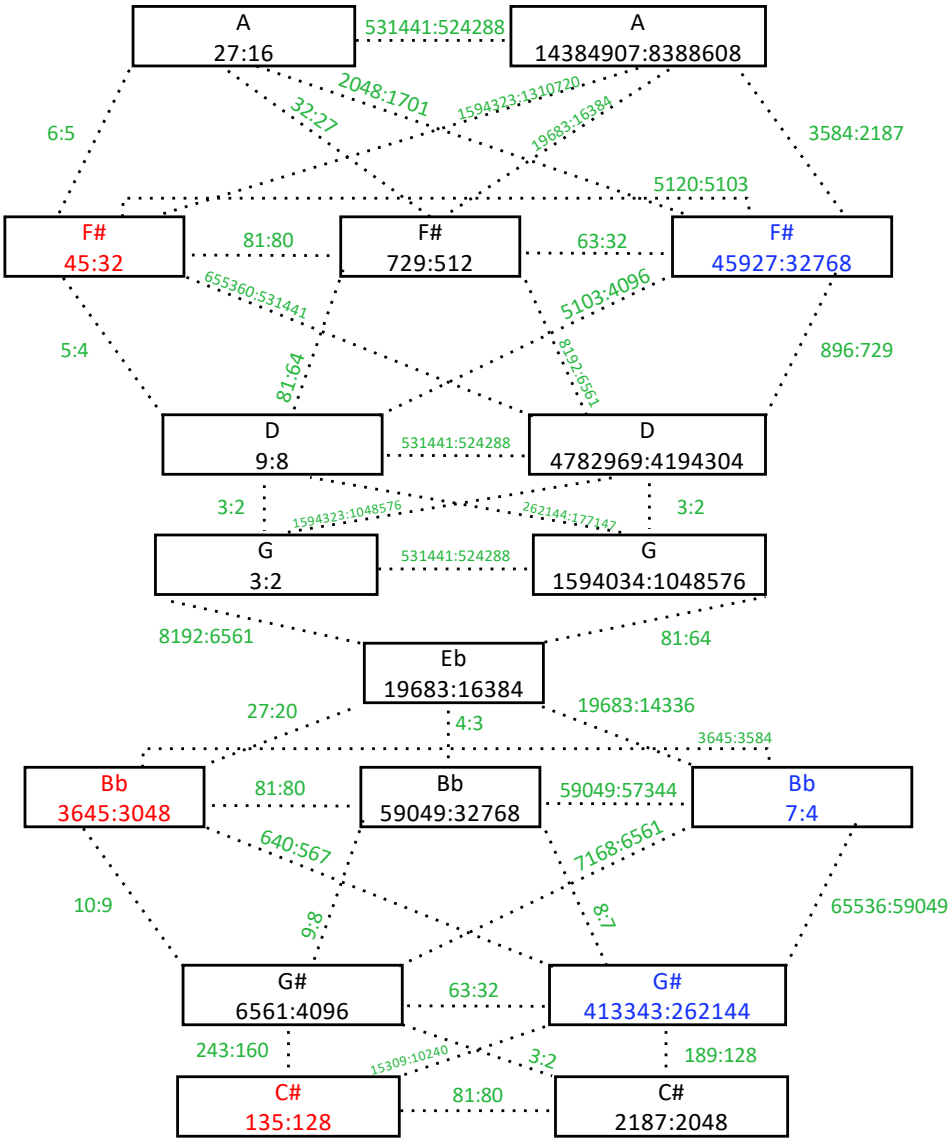
goal in these chords, which are listed in table 4.6, is to create harmonic, or textural complexity, with many interacting patterns of beating, and roughness.

Table 4.6, list of chords whose main harmonic goal is complex texture, or minimal sonic texture.

Chord number	Iteration number	Texture	Bar number
4	1	Minimal	8
4	2	Complex	9
5	1	Complex	10
7	3	Complex	15
8	2	Complex	17
9	2	Complex	19
10	2	Complex	21
11	2	Complex	23
12	2	Complex	25
15	1	Complex	30
15	2	Complex	31
16	2	Complex	34
17	2	Complex	37
19	2	Complex	39
21	1	Minimal	42
21	2	Minimal	43
21	3	Minimal	44
23	1	Complex	45
24	2	Complex	47
25	2	Complex	49
27	2	Complex	53
28	1	Minimal	54
28	2	Minimal	55
28	3	Minimal	56
29	2	Complex	58
30	2	Minimal	61
30	3	Minimal	62
32	1	Complex	66

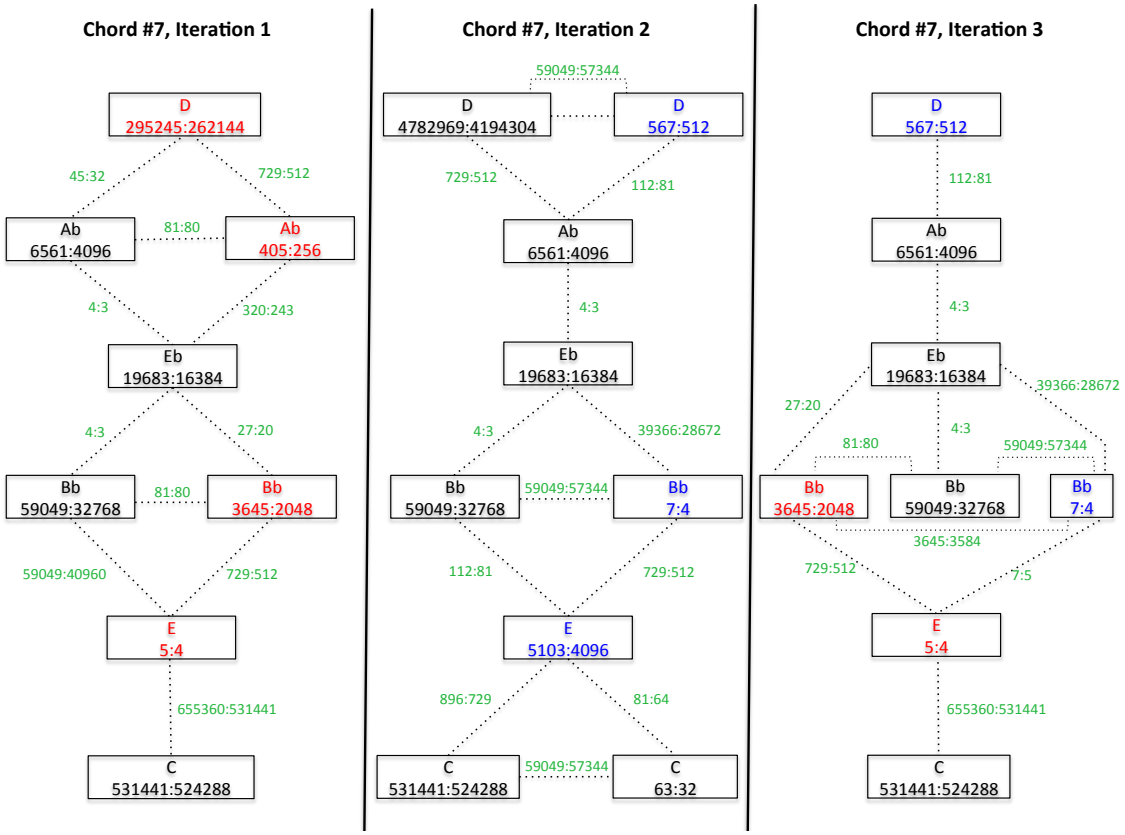
The creation of these complex sonic textures is dependent on the seven commas in the tuning model. Therefore, the best way to generate such textures, in the context of this piece, is to combine pitches from all three geometric series'. Example 4.9 shows how this concept is achieved in chord number thirty-two.

Example 4.9, analysis of chord #32 from *The tale of Kiều*.



Example 4.9 shows that chord number thirty-two contains three Pythagorean commas, two syntonic commas, two septimal commas, one Harrison's comma, one Garibaldi comma, and one un-named comma. The third way in which variable tuning of chords has been applied in movement one of *The tale of Kiều* is through the use of symmetrical modes. Example 4.10 shows how this concept has been applied in chord number seven, bars 13-15.

Example 4.10, analysis of the three iterations of chord #7 from *The tale of Kiều*.



Example 4.10 shows that Iterations one, and two of chord number seven are both UHMs, this is indicated by the combination of only two geometric series, the even number of pitches from both series, in addition to containing ratios that lie on the outer edges of the tuning matrix. Iteration three is a combination the previous two chords. Example 4.10 illustrates that in reality these symmetrical modes often contain elements of complex texture also, but complex texture is not the main goal, in this case it is a by-product. Appendix D, part five contains a complete set of descriptive charts of the chords, and their variable tunings which are used throughout movement one.

The application of symmetrical modes in movement two

The second movement of *The tale of Kiều* is mostly composed using the pitches B, G#, F#, F, Eb, C#, and Bb. The complete set of integer ratios is shown in table 4.7.

Table 4.7, integer ratios utilized in the UHMs from the second movement of *The tale of Kiều*.

Pitch name	Ratio from 3:2 series	Ratio from 5:4 series	Ratio from 7:4 series
Bb	59049:32768	3645:2048	7:4 3720087:2097152
C#	2187:2048	135:128	137781:131072
Eb	19683:16384	1215:1024	1240029:1048576
F	177147:131072	10935:8192	21:16
F#	729:512	23914845:16777216 45:32	45927:32768
G#	6561:4096	405:256	413343:262144
B	243:128	7971615:4194304 15:8	15309:8192

The symmetrical modes used in the second movement of *The tale of Kiều* are focused mainly on the combination of the 5:4 series, and the 7:4 series. Initially there was a planned symmetrical structuring of the deployment of modes within the five sections of this movement. This can be seen in the notes written above the bars in appendix E, part three which is a piano sketch of this movement. The symmetry lies in which combinations of geometric series are being employed, not in applying identical modes, or halves of modes either side of some point of symmetry as is done in movement four. The overall modal structure of the movement, as it unfolds in the score, is illustrated in table 4.8.

Table 4.8, the modal structuring of the five sections of movement two.
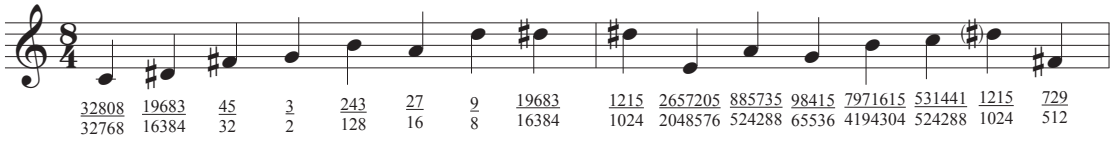
Section	**Mode**	**Mode**	**Mode**	**Mode**	**Mode**	**Mode**	**Mode**	**Mode**	**Mode**
	Bar 70	Bar 71	Bar 72	Bar 73	Bar 74	Bar 75	Bar 76	Bar 77	Bar 78
I	7:5	7:5	3:2	7:5	3:2	7:5	3:2	7:5	7:5
	Bar 79	Bar 80	Bar 81	Bar 82	Bar 83	Bar 84	Bar 85	Bar 86	Bar 87
II	7:5	Blend	5:4	5:4	7:5	Blend	5:4	3:2	7:5
	Bar 88	Bar 89	Bar 90	Bar 91	Bar 92	Bar 93	Bar 94	Bar 95	Bar 96
III	7:5	7:5	7:4	3:2	7:4	5:4	7:4	7:5	7:5
			Bar 97	Bar 98	Bar 99	Bar 100	Bar 101		
IV			7:5	Blend	Blend	Blend	7:5		
	Bar 102	Bar 103	Bar 104	Bar 105	Bar 106	Bar 107	Bar 108	Bar 109	Bar 110
V	7:5	7:5	7:4	3:2	7:4	5:4	7:4	Blend	Blend
			Bar 111	Bar 112	Bar 113	Bar 114	Bar 115		
Coda			5:4	3:2	5:4	Blend	7:4		

Table 4.8 shows that the symmetrical structure of the deployment of modes has been retained in sections one, and four only, with the remaining sections being asymmetric in design, this is partially due to the inclusion of commas to generate more complex texture amongst what is otherwise a set of pitches that are related by relatively small ratios. Appendix D, part six contains a more comprehensive representation of the modes which are used throughout movement two, along with basic descriptive information about the characteristics of each mode.

The application of symmetrical modes in movement four

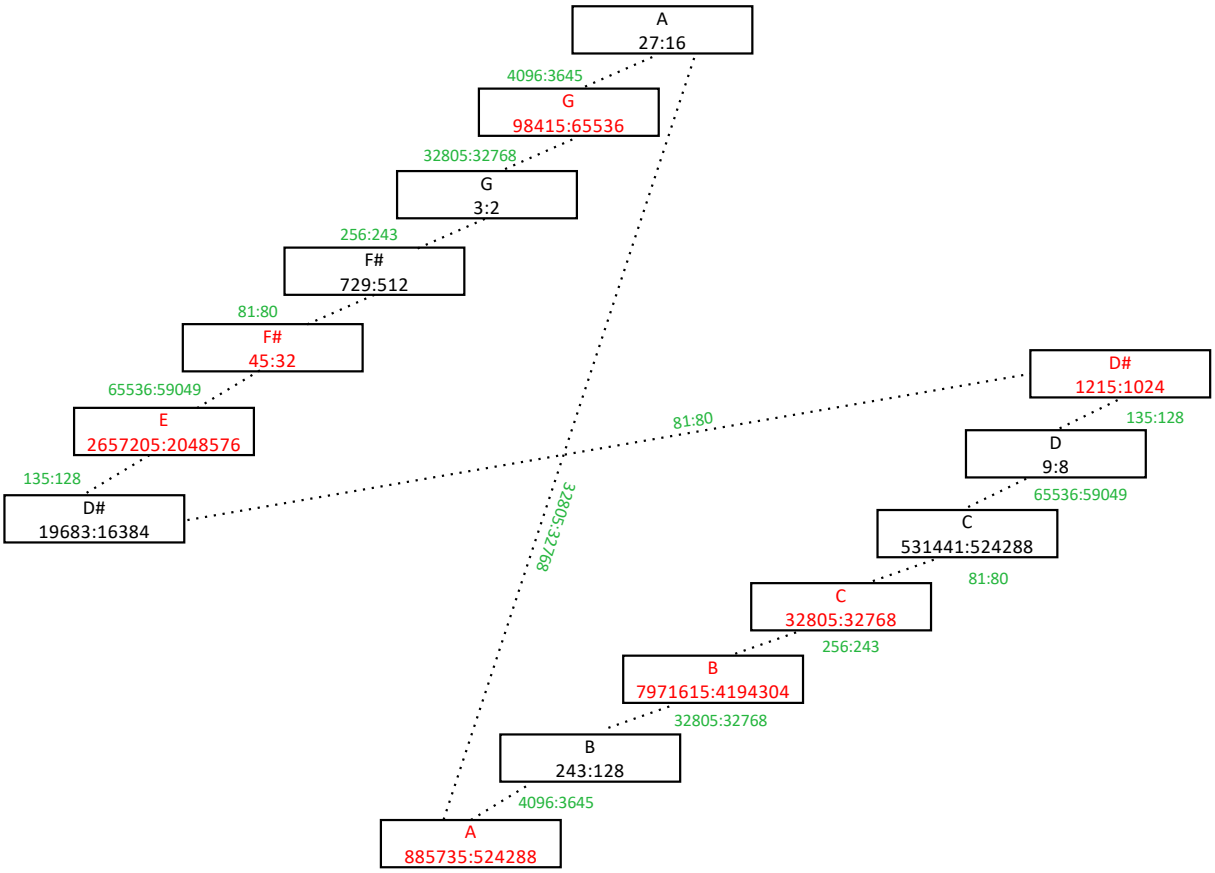
The fourth movement of *The tale of Kiều* is based on the phrase which is shown in example 4.11, which is a symmetrical mode, but not a UHM, as three possible transpositions of this mode exist within the lattice.

Example 4.11, the central phrase of movement four of *The tale of Kiều*.
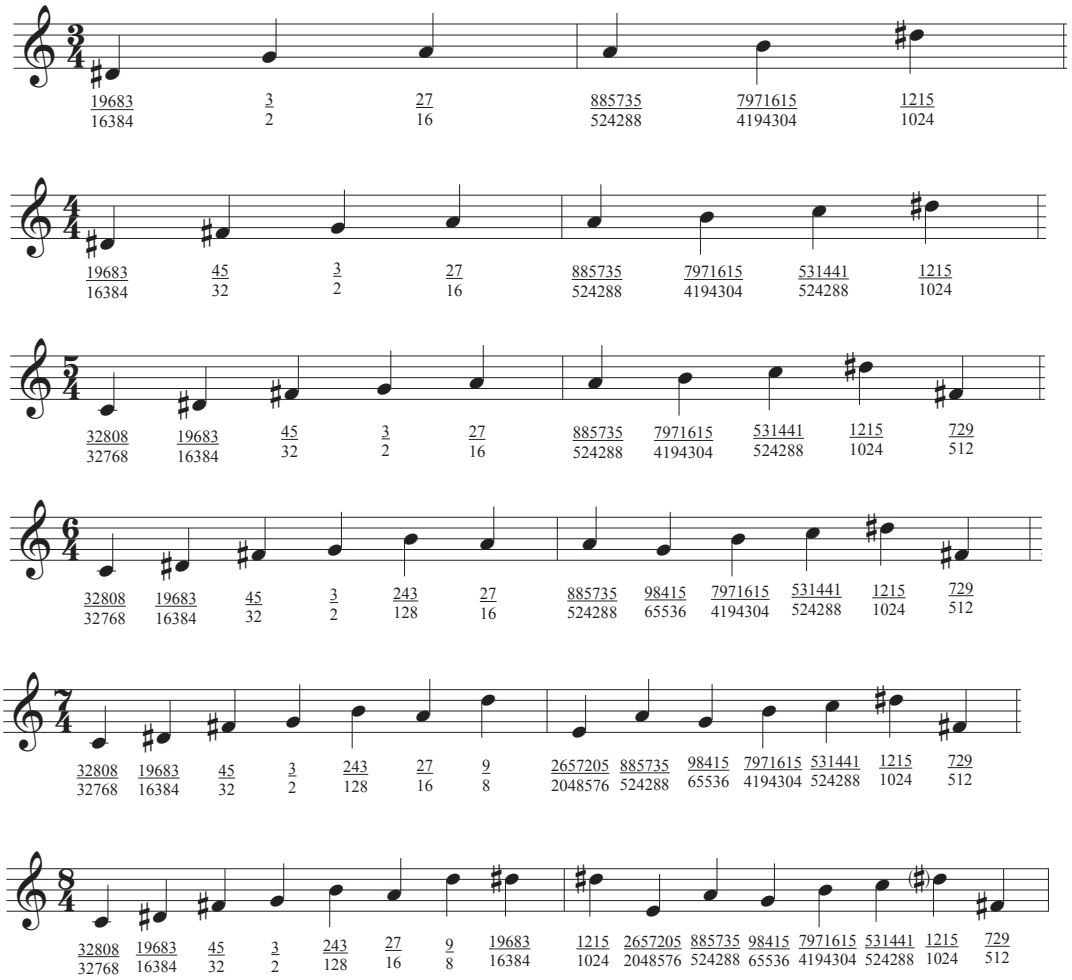


If this central phrase is represented in numbers alone, in ascending order, the symmetry is more easily observed. Such a representation is provided in example 4.12.

Example 4.12, a helical representation of the central phrase from movement four of *The tale of Kiều*.



Example 4.12 illustrates the intervallic structure of the central phrase, which shows that there are three syntonic commas, and three schismas in this phrase. The central phrase is truncated a number of times, into phrases of 7/4, 6/4, 5/4 and 4/4. These shorter phrases are also symmetrical. The shorter phrases are split down the centre point and placed either side of the 8/4 phrase, causing the entire movement to be inversionally symmetrical. The various phrases are shown in example 4.13.

Example 4.13, all phrases which combined make up the structure of movement four of *The tale of Kiều*.



The concepts underpinning this movement are taken from Tenney's *Chromatic Canon*[15] for two just intoned pianos and Andriessen's Symphony for open strings[16] (string orchestra). The inversionally symmetrical structure of this movement is borrowed from the way Tenney constructs *Chromatic Canon*. The element borrowed from Andriessen is the whole orchestra working together to produce a single melody, as opposed to a melody being something that is assigned to a single instrument only.[17]

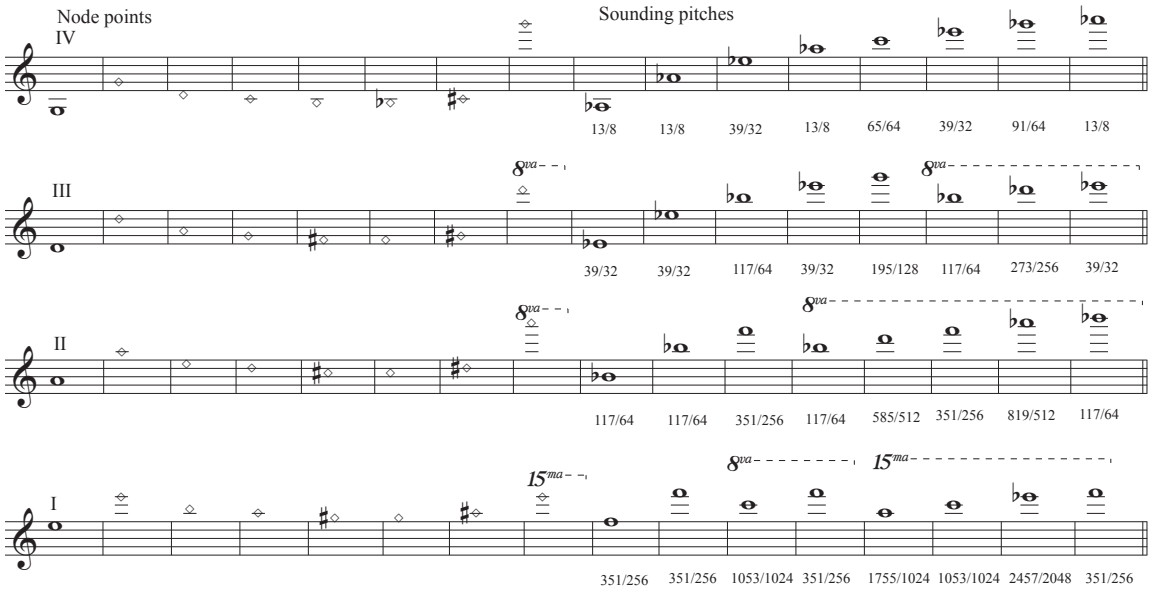
[15] Tenney, James. *Chromatic Canon* for two pianos. 1983. Sonic Art Editions, Baltimore, MD.
[16] Andriessen, Louis. Symphony for open strings. 1978. Donemus, Amsterdam.
[17] It is acknowledged that this phenomenon exists in other musics, such as Javanese Gamelan for example, but in this case Andriessen has been taken as the example as he incorporates this concept into Western Art Music, and in the case of Symphony for open strings he uses exactly the same ensemble as *The tale of Kiều.*

While on the surface *The tale of Kiều* may appear naively simplistic, hopefully the reader can now understand how the concepts discussed have informed the composition of this deceptively minimalist piece.

4.3 Discussion of the results: *13* for scordatura string quartet

In July 2018 the Australian String Quartet (ASQ) work-shopped *13* for scordatura string quartet; the recording of this workshop is contained in appendix B. *13* for scordatura string quartet investigates the approach of achieving just intonation in a string quartet through the use of natural harmonics, open strings, and scordatura. The transposition charts developed in the methodology are designed to guide the instrumentalists with where to place their fingers in order to produce sounds related by relatively small-integer ratios. Initially only one node position was used for each harmonic, an example of an early transposition chart is shown in example 4.14.

Example 4.14, an early transposition chart for violin 1 from *13*.



There are instances, however, where multiple different positions are available for a single harmonic; this is due to the symmetrical patterning of harmonics along the string. Incorporating all possible natural harmonic positions into the transposition charts allows most patterns in the piece to be composed in single left hand positions.

Scordatura in 13

The scordatura, while certainly credible and realistic did have a somewhat unexpected result. Cello, viola, and second violin all have the tension of their strings slackened, with the first violin being the only instrument with the overall tension of strings increased. The result was that the sound quality of the first violin appeared much brighter and more prominent than the remainder of the quartet[18]. In hindsight, this should have been an easy prediction to make, this was an oversight. Such an observation in future compositions for scordatura groups is certainly worth careful consideration.

Possibilities and limitations in 13

The main limitation of the natural-harmonics-only approach centres around stopped notes being entirely removed from the scope of the composition. The result of this tuning model is a reflection of the layout of the harmonic series. The way in which the limited range of usable pitches is distributed across octaves is uneven; there are large intervals between the first harmonics on each string, and as the series continues the size of intervals becomes gradually smaller with each new harmonic. The other consequence of this natural phenomenon is that a large amount of the lattice is quite high pitched, while a smaller portion of the lattice exists in lower registers.

The main benefit of this approach is that natural harmonics largely remove intonational control from the instrumentalists, which is necessary if the performers are not just intonation specialists. This lack of intonational control was particularly noticeable in the andante sections where pitches lying within a single critical band are sounded simultaneously; table 4.10 contains a table outlining which commas are being investigated throughout the andante sections. The ASQ could clearly hear that certain notes were almost in unison, but were unable to alter the intonation in order to achieve a relationship of 1:1. While this was initially disturbing for the quartet, once assured that this was the intention

[18] It is possible, but highly unlikely, that this brightness is a feature of the first violinists instrument, and not the scordatura. The ASQ perform on a matched set of Guadagnini instruments, and considering the quartet commented on the brightness of the first violin, it is likely that the scordatura caused this timbral variance.

of the composition they were more at ease with this musical phenomenon which must have been unusual for them.

Practical playability of 13

The consensus of the ASQ was that *13* is playable, but would require many months of practice and rehearsal in order to achieve high quality results; this is worth contemplation. The ASQ are a professional quartet, but their usual repertoire is not particularly experimental.[19] It is possible that a quartet that devotes more of its time to modern experimental compositions would require less rehearsal time for such a piece of music. At this point in time, however, such an idea is nothing more than conjecture. A composer must take rehearsal time into consideration. Realistically, living composers do not ordinarily get months dedicated to a piece of music with a ten to fifteen-minute duration. Furthermore, if such time commitment is necessary, then it seems that while trying to simplify and streamline the process of composing music in just intonation for non-specialised performers, I have not done so. Perhaps I have simply created an equally specialised alternative to the kind of string quartets written by Ben Johnston, albeit marginally more accessible for the performer.

Tuning model for 13

The tuning model developed for *13* was born of pragmatism, while attempting to simultaneously investigate the number thirteen. Doty has the following to say of intervals generated by the number thirteen:

> . . . intervals based on eleven and thirteen lie entirely outside Western musical experience. These intervals are controversial. One is likely to love or hate them on first hearing.[20]

Doty continues to say that he finds thirteen unbearably harsh and has been unable to integrate it into his music. Johnston on the other hand perhaps has a better tolerance for thirteen, or greater exposure to it. Johnston describes

[19] This is not a criticism of the ASQ, simply my subjective observation.
[20] Doty, David B. *The Just Intonation Primer: An introduction to the theory and practice of Just Intonation.* Second ed. San Francisco. The Just Intonation Network, 1994. P. 55.

thirteen as having a melancholy, dark quality and an association with death.[21] Upon the first re-tuning of my pianos to the lattice developed for *13*, Doty's and Johnston's discourse on this number suddenly became tangible, and Doty being 'unable to integrate it' into his music began to make sense. Because of the foreignness of this interval to the western trained ear, I also found it quite difficult to utilize. The problem I faced was that this interval was one of the key building blocks for this particular lattice. Not only was I facing one instance of the relationship 13:8, but twenty-six replications of this relationship in the lattice. This could be perceived as a limitation or a possibility, depending on one's approach to the situation. Interestingly the interval 169:128, or the thirteen harmonic of the thirteenth harmonic, which forms one third of the lattice, does not seem unbearably harsher than 13:8. Instead it appears to the ear simply as almost imperceptibly flatter than 4:3, which is quite familiar to the Western musical experience. The main possibility or limitation, depending on one's viewpoint, of this particular tuning model is, in my opinion, the generation of musical intervals through the use of the number thirteen.

13 addresses how a suitable style of notation can be applied in the context of this project. The score makes use of two staves. The first stave is a normal size, and it contains diamond shaped note heads, which show the instrumentalists where to disturb the string in order to produce natural harmonics. The second stave is an ossia, and shows sounding pitches. The combination of the two versions of notation helps to resolve any ambiguities there may be in either tablature, or sounding pitches. It also means that the information necessary for analysis is more quickly and easily found than if the sounding pitches were omitted. The one downside here is that, as I have decided not to use Johnston's system of notation, the sounding pitches appear on the staff as approximations. A certain level of work would be required on the part of an analyst in order to determine exactly which pitches are sounding, but this is not dissimilar to the amount of work necessary to decipher what is happening in a Ben Johnston string quartet. The result of omitting exotic accidentals and cent deviations is a clear, neat, and

[21] Keislar, Douglas. "Six American Composers on Nonstandard Tunings" *Perspectives of New Music,* 29:1 (Winter 1991)

un-convoluted score, which is well worth the sacrifice in my opinion. The feedback from the ASQ was simply that tablature, plus sounding pitches as a smaller ossia stave, formatted in portrait, is their preference.

13 is an investigation into the numbers three, five, seven, and thirteen. As can be seen in the tuning model, these are the prime numbers used to form the lattice. This has already been discussed in chapter two, section 2.3. The same prime numbers that form the lattice also influence other temporal aspects of the work including macroperiodicity, and tempi.

Macroperiodicity

Initially *13* was conceived as a set of alternating macroperiods, comparable to *K* for two pianos, two pianists and two hands. While *13* is characterized by extended macroperiodic episodes, it was necessary to intersperse the piece with resting points for the performers. This was not a matter of allowing the players to rest due to physical fatigue, instead it was a matter of allowing their minds periods of respite from counting quavers in a sea of perpetual motion. These resting points, which are sometimes static motion, and at other times punctuated rhythms also serve as contrasting devices to the continuous quaver movement of the macroperiods. The macroperiods investigated in *13* are shown in in table 4.9.

Table 4.9, macroperiods investigated in *13*.

Measure no.	Length of period 1	Length of period 2	Length of macroperiod
5 – 6	2 quavers	3 quavers	6 quavers
7 - 10	3 quavers	4 quavers	12 quavers
11 – 15	4 quavers	7 quavers	28 quavers
16 – 19	4 quavers	5 quavers	20 quavers
22 – 29	4 quavers	5 quavers	20 quavers
30 – 42	4 quavers	13 quavers	52 quavers
48 – 51	4 quavers	5 quavers	20 quavers
52 – 56	5 quavers	7 quavers	35 quavers
58 – 70	5 quavers	13 quavers	65 quavers
73 – 82	5 quavers	6 quavers	30 quavers
83 – 87	5 quavers	7 quavers	35 quavers
94 – 103	5 quavers	6 quavers	30 quavers
110 – 121	3 quavers	5 quavers	15 quavers
122 – 127	3 quavers	4 quavers	12 quavers
128 – 135	4 quavers	5 quavers	20 quavers
136 – 159	3 quavers	4 quavers	12 quavers
166 – 171	3 quavers	7 quavers	21 quavers
172 – 177	6 quavers	7 quavers	42 quavers
185 – 192	6 quavers	7 quavers	42 quavers
196 – 200	5 quavers	6 quavers	42 quavers
206 – 218	6 quavers	13 quavers	78 quavers
221 – 230	5 quavers	6 quavers	30 quavers
231 – 233	3 quavers	5 quavers	15 quavers
234 – 236	3 quavers	4 quavers	12 quavers
238 – 240	3 quavers	4 quavers	12 quavers
245 – 248	4 quavers	5 quavers	20 quavers
249 – 253	5 quavers	7 quavers	35 quavers
257 – 259	3 quavers	5 quavers	15 quavers
266 – 277	4 quavers	5 quavers	20 quavers
280 – 299	4 quavers	5 quavers	20 quavers
300 – 308	3 quavers	4 quavers	12 quavers
310 – 312	3 quavers	4 quavers	12 quavers
314 – 328	3 quavers	4 quavers	12 quavers
330 – 332	3 quavers	4 quavers	12 quavers
340 – 342	3 quavers	4 quavers	12 quavers
343 – 354	4 quavers	5 quavers	20 quavers
356 – 359	4 quavers	5 quavers	20 quavers
368 – 379	4 quavers	5 quavers	20 quavers
384 – 391	4 quavers	5 quavers	20 quavers
394 – 401	4 quavers	5 quavers	20 quavers
404 – 411	4 quavers	5 quavers	20 quavers

Table 4.9 shows that the macroperiods investigated in *13* explore the various combinations of the generative primes, with the exception of a combination of thirteen and seven. Appendix H, part one contains an early draft of *13* where

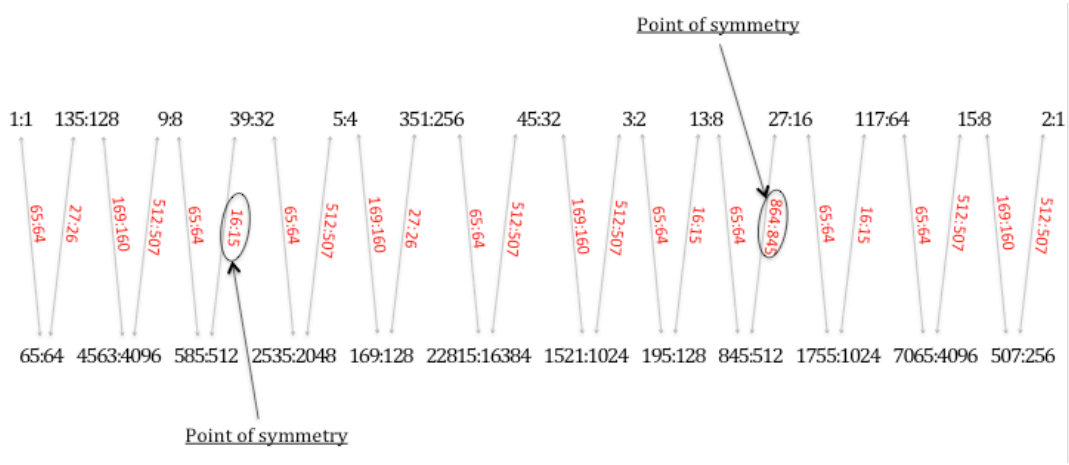
macroperiods are easily identified by the unmanageably large time signatures, and there is a lack of punctuation, or pauses between the continuous quaver motion. The alteration of these two aspects of the score was aimed at making *13* more playable.

Tempo relationships

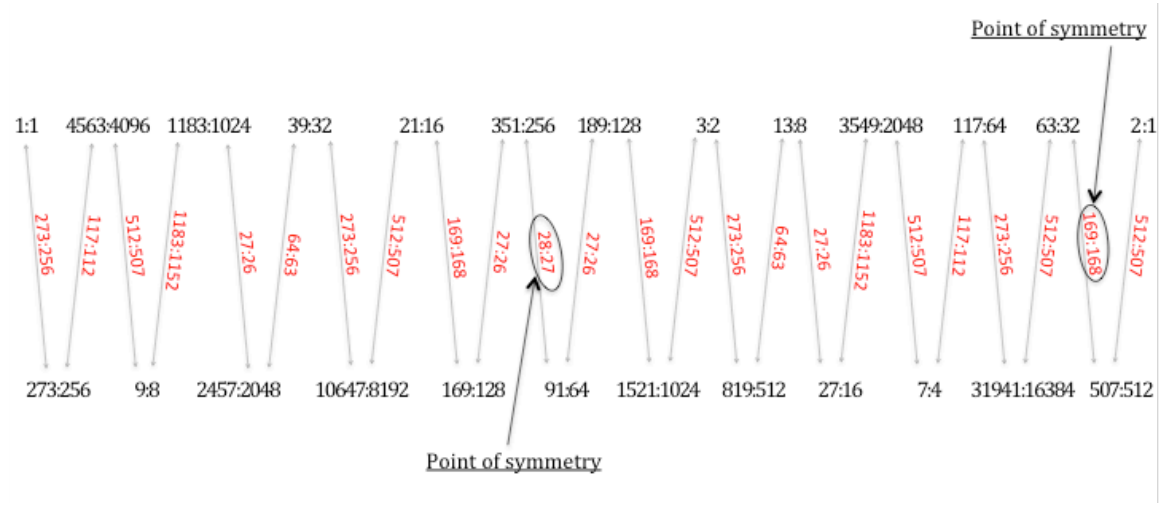
The third aspect that relates the musical material in *13* to its title is the relationship between tempi. There are only two tempi which alternate throughout the piece, andante, where minim equals 50bpm and allegro, where dotted minim equals 54bpm. This is, realistically as close as one can get to a relationship of 13 to 8 in tempo relationships without specifying the tempo in quavers which is a little awkward. The equivalent tempi expressed as quavers are andante, quaver equals 200bpm, and allegro where quaver equals 325. Although my times signatures in the final score may at first appear to the reader to be imprecise compared to the care that has been taken to ensure precision in the tuning aspect of the project, it was again a decision born of pragmatism. It is doubtful that any quartet would count compound bars by individual quaver, these bars would inevitably be counted in groups of three and four. To save a quartet the trouble of dividing awkward quaver based time signatures into workable numbers, the final time signatures appearing in *13* are based on larger values in order to troubleshoot this problem before it occurs. The compromise is, in my opinion, a trifling imprecision.

The tuning model devised for *13* contains four MUHMs, which are shown in examples 4.15 through 4.18.

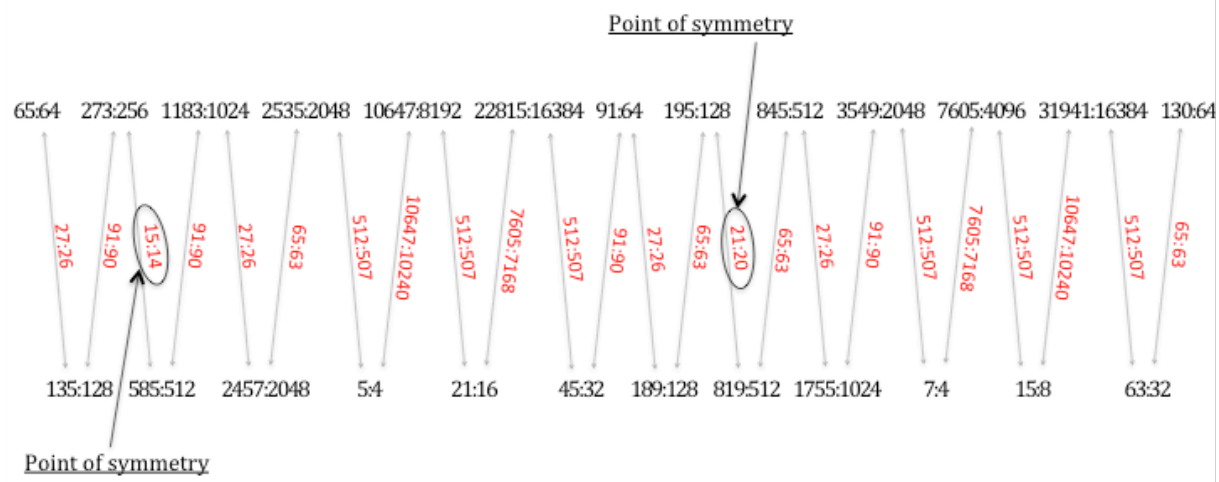
Example 4.15, MUHM-13.1



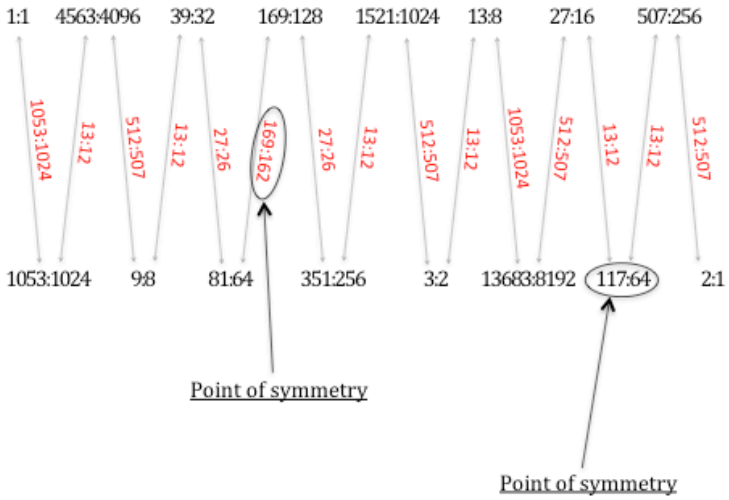
Example 4.16, MUHM-13.2



Example 4.17, MUHM-13.3



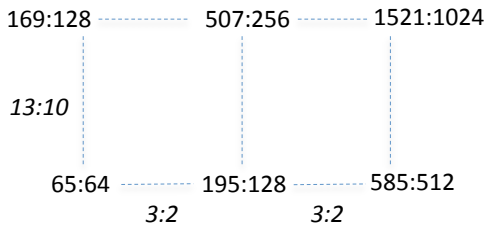
Example 4.18, MUHM-13.4



Examples 4.15 through 4.18 have been provided here as a courtesy to the reader, not as a tool for which to commence analysis of *13* as these MUHMs did not form the basis of *13.* Conceptually *13* represents a transitional period in the project, it heralds a shift from composing without any specific kind of modes to composing using modes with limited or no transpositions, and inversionally symmetrical modes. *Remembering is a Form of Forgetting,* which was the first composition in the project, did not utilize any of these distinct modes. *13* is the next step in the conceptual development of the project. It does not utilize UHMs according to the strict definition set forth in chapter one. Instead, *13* begins to explore symmetrical sections of the lattice, which consequently form inversionally symmetrical modes, and also modes which have limited, or no transpositions within the lattice. Additionally, *13* investigates the relationship between very small intervals, those lying within, or just outside a single critical band. The discussion that follows herein highlights some of the more symmetrical moments (and moments utilizing large numbers of pitches which are, from my perspective, 'different tunings of the same note'). To assist the reader in understanding the following discussion, a number of appendices have been included to illustrate the symmetrical sets of pitches which occur in the allegro sections of *13.* Appendix G, part one shows pitches as they occur in the lattice from which the symmetrical sets can be deduced, with the symmetrical sets being illustrated in Appendix G, part two. Appendix G, part three contains a table outlining information regarding

the dimensions of the sets, where they appear throughout the piece, the generating primes, and what prime number space they appear within.
It is only the macroperiodic, allegro sections of *13* which intentionally utilize modes containing symmetrical structures. These sections often contain more than one symmetrical set, and pitches which are auxiliary to the symmetry of the groups. These auxiliary pitches have often been included to alter the sonic texture, through critical bands.
The symmetry of the macroperiodic sections is initially two dimensional, in that it utilizes two perpendicular chains of intervals. Most frequently occurring is the plane generated by the combination of 13:8 and 3:2; this is due to the fact that the two violins are set 13:8 and 169:128 $(13:8)^2$ apart from the viola and cello. This 13:8 – 3:2 plane is largely in use from measures 5-127. As the allegro sections develop, a third dimension begins to be implied from measure 78. To explain what is meant by *implied* Example 4.19 illustrates set 17, which first appears in measure 83.

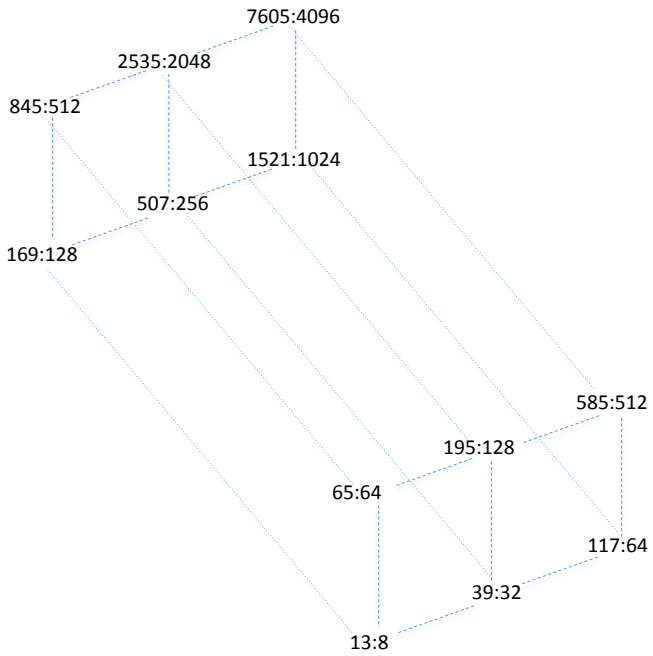
Example 4.19, set 17, from *13* for scordatura string quartet.



Set 17

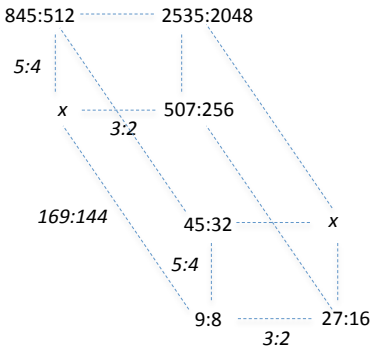
While example 4.19 appears to be a two dimensional plane, it is clear that 65:64 – 585:512 exist in the 5:4 - 13:8 plane, while 169:128 -1521:1024 exist in the 13:8 – 3:2 plane. Example 4.20 provides a more complete picture of the third dimension which is being implied by set 17.

Example 4.20, the three-dimensional prism that set 17 implies.



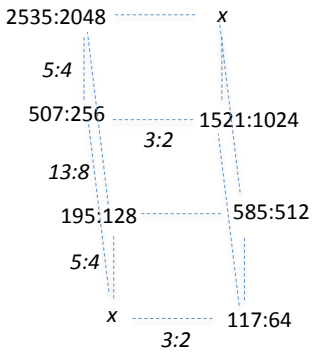
The two series 13:8 – 117:64, and 845:512 – 7605:4096 make explicit the third dimension that one cannot see when looking at example 4.19 alone. Symmetry in a third dimension is not made explicit until measure 206, where set 53 is introduced. Sets 53, 67, and 68 which are illustrated in examples 4.21 through 4.23 are the only sets which appear three-dimensionally symmetrical.

Example 4.21, set 53 from *13* for scordatura string quartet.



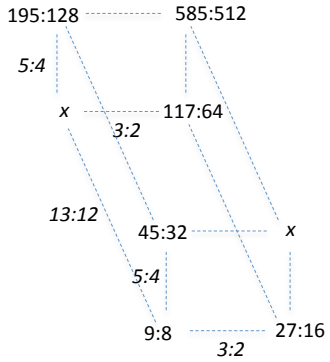
Set 53

Example 4.22, set 67 from *13* for scordatura string quartet.



Set 67

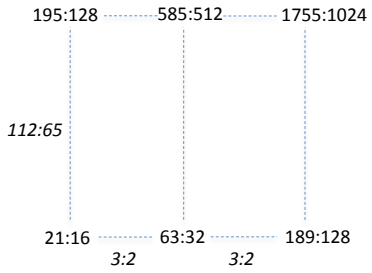
Example 4.23, set 68 from *13* for scordatura string quartet.



Set 68

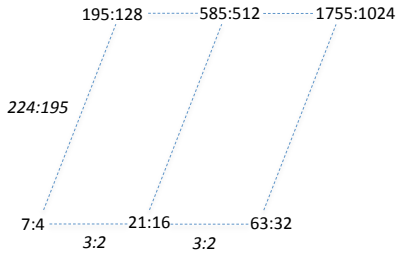
A fourth dimension is also implied through measures 172-219. This occurs in sets 50 through 52 which are illustrated in examples 4.24 through 4.26; the implication of a fourth dimension is due to the fact that in order to exist, these sets require the four prime numbers 3, 5, 7, and 13 to generate the ratios they are composed of.

Example 4.24, set 50 from *13* for scordatura string quartet.



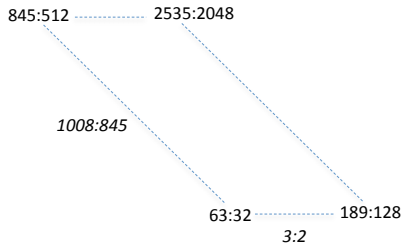
Set 50

Example 4.25, set 51 from *13* for scordatura string quartet.



Set 51

Example 4.26, set 52 from *13* for scordatura string quartet.



Set 52

If the reader now directs their attention to appendix G, part one they will notice that as the allegro sections unfold, there is no sense of establishing a key, and modulations, but rather a kind of continual development is taking place instead. Interspersed with the allegro sections are the andante sections, whose harmonic purpose is to explore pitches lying roughly within a single critical band of one another. The continual development of the allegro is counter balanced by the descending fifth cycle which occurs in the andante throughout the course of the piece. The musical events which take place in the andante are expressed in table 4.10.

Table 4.10, table showing the commas explored in the andante sections of *13.*

Measure nos.	Pitch name	Violin 1 ratio	Violin 2 ratio	Viola/Cello ratio	Comma name	Comma ratio
106-109	B	-	7605:4096	15:8	Tridecimal neutral third comma	512:507
106-109	D	-	4563:4096	9:8	Tridecimal neutral third comma	512:507
179-184	E	-	2535:2048	5:4	Tridecimal neutral third comma	512:507
179-184	E		2535:2048	81:64	-	864:845
192-195	A	-	845:512	27:16	-	864:845
192-195	A	-	13683:8192	27:16	Tridecimal neutral third comma	512:507
243-245	D	585:512	4563:4096	-	Tridecimal minor diesis	40:39
243-245	D	585:512	-	9:8	13th partial chroma	65:64
243-245	D	-	4563:4096	9:8	Tridecimal neutral third comma	512:507
262-265	G	195:128	1521:1024	-	Tridecimal minor diesis	40:39
262-265	G	195:128	-	3:2	13th partial chroma	65:64
262-265	G	-	1521:1024	3:2	Tridecimal neutral third comma	512:507
334-339	C	65:64	507:256	-	Tridecimal minor diesis	40:39
334-339	C	65:64		1:1	13th partial chroma	65:64
334-339	C	-	507:256	1:1	Tridecimal neutral third comma	512:507
334-339	C	1053:1024	31941:16384	-	-	96:91
334-339	C	1053:1024		63:32	-	117:112
334-339	C	-	31941:16384	63:32	Tridecimal neutral third	512:507

					comma	
362-367	F	351:256	169:128	-	Tridecimal comma	27:26
362-367	F	351:256	-	21:16	-	117:112
362-367	F	-	169:128	21:16	Schulter's comma	169:168
362-367	F	351:256	10647:8192	-	-	96:91
362-367	F	-	10647:8192	21:16	Tridecimal neutral third comma	512:507
382-383	Bb	117:64	3549:2048	-	-	96:91
382-383	Bb	117:64	-	7:4	-	117:112
382-383	Bb	-	3549:2048	7:4	Tridecimal neutral third comma	512:507
413-415	Eb	39:32	1183:1024	-	-	96:91

Table 4.10 elucidates the descending fifth cycle, and illustrates the named, and unnamed commas which are investigated in the andante sections. There is scope within the lattice to investigate the various stages of human perception within critical bands. Narrower commas such as the tridecimal neutral third, and Schulter's comma produce subtle beating, wider commas such as the 13th partial chroma, and the tridecimal minor diesis produce roughness in the sonic texture, and the ear can perceive two separate notes in the widest intervals of 117:11, and 96:91. The allegro sections were also composed with the intention of investigating pitches lying within a single critical band. Despite this being the intention, the nature of the allegro sections, which is fast moving lines of quavers, causes the subtle sonic texture of beating in the critical bands to be lost. This is also in part due to the acoustic nature of the instruments; *13* was composed on two pianos, and the difference in sound between a string quartet and two pianos is obviously quite marked due to the various factors influencing the harmonic spectrum of different instruments. Appendix G, part four contains a list outlining the commas which were implemented throughout the allegro sections of *13.*

4.4 Discussion of the results: *The Un-tempered Pianos* for two pianos, and one pianist, and *K* for two pianos, two pianists and two hands.

Although Partch, Riley, Young, Johnston, and presumably numerous less well known others, have already shown that it is possible to adapt a piano[22] from being equally tempered to just intoned, *The Un-tempered Pianos* and *K* nevertheless tackled this concept. The greatest advantage of this approach to achieving just intoned music is that no real-time tuning of pitches takes place at all, pitches are all fixed beforehand, and performers have no control over intonation at all. The second very desirable advantage of this approach is that the performers require no knowledge of just intonation, provided that the pianos are re-tuned by someone with a rudimentary understanding of the topic. The main limitation of this approach to achieving just intonation is unrelated to the music itself, instead it lies in the ease with which the music can be presented to an audience. Initially I recorded *The Un-tempered Pianos* and *K* on my own pianos, in my home. Later, an opportunity arose for concert pianist and composer Gabriella Smart to perform *The Un-tempered Pianos* as part of the 2018 Blue Touch concert series. This was an exciting prospect, but the danger of a performance in 12-TET was a very real threat. Universities and individuals alike seem to be afraid of their expensive pianos being ruined by some foreign tuning, this is understandable. There is also the problem of the time consuming nature of re-tuning and presenting works in 12-TET alongside works in just intonation. In such a context, two sets of instruments are required. Gabriella was happy to perform *The Un-tempered Pianos* in 12-TET, which presented me with an obvious ethical dilemma. Because in this case it was an option, I decided to transport my own two pianos to the venue, to ensure that the piece would be presented in the way in which it was conceived. The premiere of *K*, however, was performed in 12-TET. This was a compromise that I was willing to make, in order to have the piece premiered by the highly esteemed concert pianists Lisa Moore, and Sonya Lifschitz. Composer Martin Bresnick, Moore's husband, criticized me for this decision. On this particular occasion bringing my own two pianos to the venue was not an option, and although presenting the piece in 12-TET makes it an

[22] In the case of Partch, keyboard instrument instead of a piano is perhaps more accurate.

entirely different piece of music I would prefer to present my music to the public in 12-TET than not at all. So, composing music for just intoned keyboard instruments has some coveted advantages, but in order to truly be a success the composer must be a purist and uncompromisingly committed to their artistic vision.

A further limitation of this particular approach to achieving just intonation in a chamber setting was regarding the register of the piano. Fine tuning the upper and lower octaves of the piano is quite a challenge to the ear, the digital tuner, and the tuning wrench; inharmonicity it seems is indeed quite real. A fine tuning device on each string, such as is found on a violin's tailpiece, would perhaps be useful in such a scenario, but sadly it would probably also be immensely expensive to implement such devices on a piano! A less expensive solution to this problem was to re-tune only the middle register of the two piano keyboards.

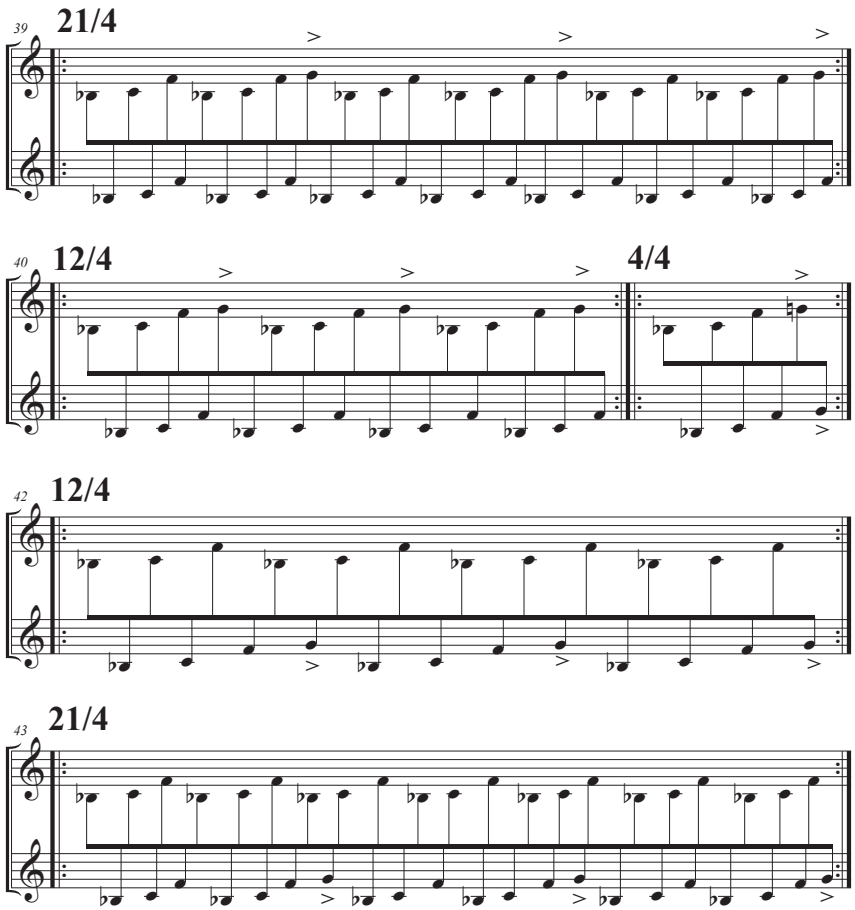
While Partch, Riley, Young and Johnston may have already tackled the concept of composing for just intoned keyboard instruments, Partch is the only one of these four who decided to include more than twelve pitches to the octave. Technically the tuning model conceived for *The Un-tempered Pianos* and *K* contains forty-eight pitches to the octave, although one set of twenty-four is inseparable from the other set of twenty-four. In the context of existing repertoire, *The Un-tempered Pianos* and *K* have increased scope through sheer number of pitches. That is not to say that quantity trumps quality, but it helps to highlight the departure from existing repertoire. In my opinion the most interesting possibility of such a tuning model is the relationship of the syntonic comma, which imposes itself upon every key of the two pianos. As explained in the methodology chapter, the syntonic comma was but one of a number of possible commas that could have been chosen, not to mention the possibility of using a combination of commas throughout the tuning system. Other commas that exist within the lattice, which are of interest, and are investigated in *The Un-tempered Pianos* and *K* include the gamelan residue and the slendro diesis. The main limitation of this tuning system is rooted in the fact that it imposes a tuning that is not 12-TET on what is arguably *the* instrument of 12-TET, the piano. One possible argument is that such a system doesn't allow the things that 12-TET

allows such as equal steps and modulations. These limitations are really only limitations if the composer chooses to view them as such. Many styles of music from Javanese gamelan, to Hindustani classical or the traditional vocal music of countless indigenous Australian musicians do not contain equal steps and modulations. A number of modulations are actually possible within this tuning system, so such an argument is redundant anyhow.
Composing music for just intoned pianos, in terms of musical notation, surely represents the easiest way to write music with limited adaption to the notation. This is because there really is no real need to change the notation; pianists playing pianos will never be able to make microtonal inflections, unless some direction were given to disturb the node points, or some preparation which does such a thing were indicated. The only scenario where more accurate representations of the actual sounding pitch may be useful is for a music analyst. Implementing such a style of notation is not only beyond the scope of this project, it actually contradicts aim number four of the project.

The tuning model for *The Un-tempered Pianos* and *K* explores prime numbers up to nineteen, skipping seventeen. This particular tuning model is really an exploration of the possibilities of seven and three. The relationships of 7:4 and 81:80 are the most prominent relationship in the model. Two further parameters in *The Un-tempered Pianos* and *K* are determined by the prime numbers which form the lattice, these parameters are macroperiods, and tempo relationships.

Macroperiods in The Un-tempered Pianos
Bars 39 – 43, and bars 126 – 136 of *The Un-tempered Pianos* are macroperiodic. Bars 39 – 43 are shown in example 4.27.

Example 4.27, bars 39 -43 excerpt from *The Un-tempered Pianos.*



Example 4.27 shows that there are two macro periods being explored in this five bar excerpt, arranged symmetrically around bar 41. Table 4.11 illustrates the length of periods, and macroperiods from bars 39 – 43.

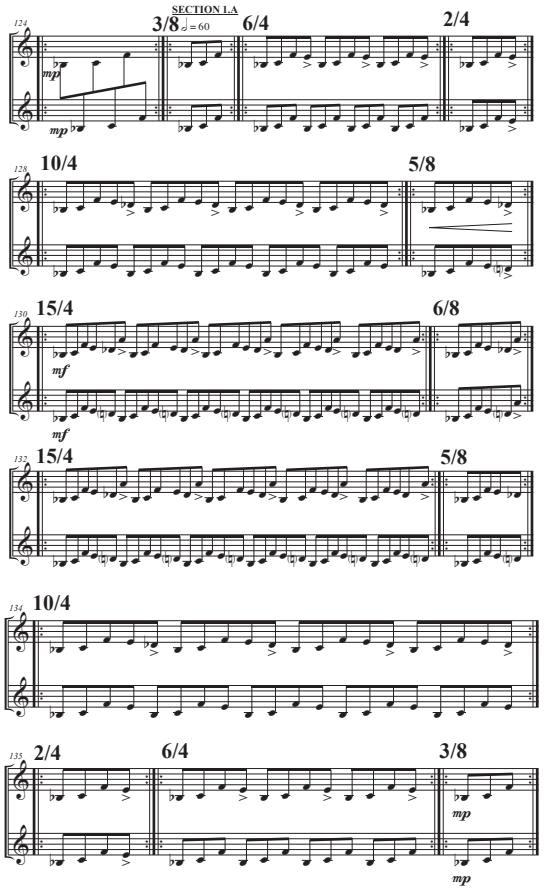
Table 4.11, length of periods and macroperiods in bars 39 – 43 of *The Un-tempered Pianos.*

Measure no.	Length of period 1	Length of period 2	Length of macroperiod
39	7 quavers	3 quavers	42 quavers
40	4 quavers	3 quavers	24 quavers
42	4 quavers	3 quavers	24 quavers
43	7 quavers	3 quavers	42 quavers

Table 4.11, and example 4.27 show that bars 39 – 43 are really a brief macroperiodic exploration of the relationship between two, three and seven.

Bars 126 – 136 of *The Un-tempered Pianos* also explore a number of macroperiods, these measures explore the relationship between two, three and five. Example 4.28 shows measures 124 – 137.

Example 4.28, bars 124 -137 excerpt from *The Un-tempered Pianos.*



Example 4.28 shows that there are three macro periods in this short excerpt, again the macroperiods appear in a symmetrical arrangement around bar 131. Table 4.12 illustrates the length of periods, and macroperiods from bars 126 to 136.

Table 4.12, length of periods and macroperiods in bars 126 – 136 of *The Un-tempered Pianos.*

Measure no.	Length of period 1	Length of period 2	Length of macroperiod
126	4 quavers	3 quavers	12 quavers
128	5 quavers	4 quavers	20 quavers
130	6 quavers	5 quavers	30 quavers
132	6 quavers	5 quavers	30 quavers
134	5 quavers	4 quavers	20 quavers
136	4 quavers	3 quavers	12 quavers

Tempo relationships in The Un-tempered Pianos

The relationship between tempi in *The Un-tempered Pianos* are also dictated by the prime numbers which form the basis of the lattice, with the exception of nineteen. Table 4.13 illustrates the tempi, prime factors and relative ratios.

Table 4.13, the tempi, prime factors, and related ratios in *The Un-tempered Pianos.*

Tempo marked	Measure nos.	Prime factors	Relative ratio
130	1 – 124	2, 5, 13	13:12
60	125 – 140	2, 3, 5	13:12
130	141 – 242	2, 5, 13	13:12, 14:13
70	243 – 244	2, 5, 7	14:13, 7:6
60	245 – 256	2, 3, 5	7:6, 6:5
50	257 – 288	2, 5	6:5
Free	289 – 309	n/a	n/a
70	310 – 312	2, 5, 7	11:7
110	313 – 324	2, 5, 11	11:7

In most cases the influencing prime is quite clear, and can be deduced by simply dividing the marked tempo by ten. Minim equals sixty is the exception here, and is representative of three, this is because in the case of measures 125 – 140 and 245 – 256 minim equals thirty was far too slow, it was therefore doubled.

Macroperiods in K

K is an investigation into macroperiodicity. Due to the proliferation of macroperiods, score excerpts shall not be included here. Instead the reader is kindly asked to refer to the score for *K* in chapter three. Table 4.14 outlines the length of periods, and macroperiods as they appear in *K.*

Table 4.14, length of periods and macroperiods as they appear throughout *K.*

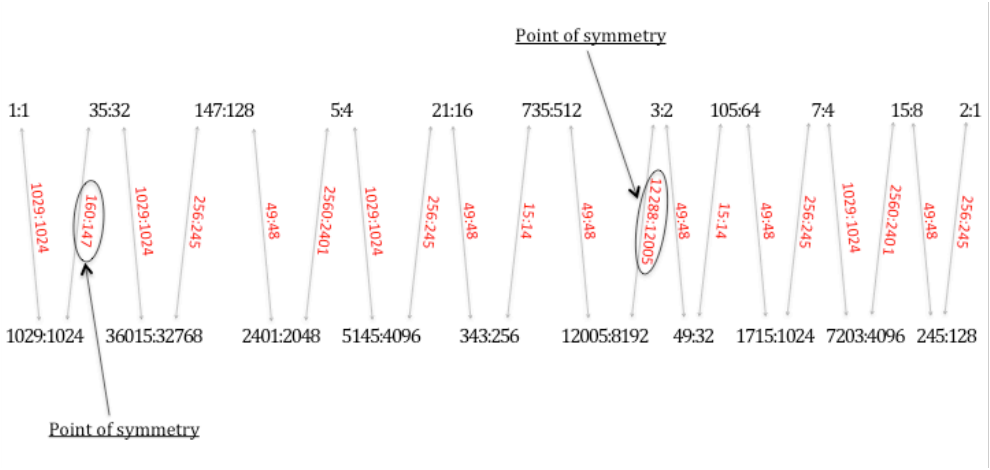
Measure no.	Length of period 1	Length of period 2	Length of macroperiod
1	3 quavers	2 quavers	6 quavers
2	4 quavers	3 quavers	12 quavers
3	5 quavers	4 quavers	20 quavers
4 – 5	7 quavers	4 quavers	28 quavers
6 – 8	7 quavers	5 quavers	35 quavers
9	7 quavers	6 quavers	42 quavers
10	7 quavers	3 quavers	21 quavers
11	7 quavers	6 quavers	42 quavers
12 – 14	7 quavers	5 quavers	35 quavers
15 – 16	7 quavers	4 quavers	28 quavers
17	5 quavers	4 quavers	20 quavers
18	4 quavers	3 quavers	12 quavers
19	3 quavers	2 quavers	6 quavers
20 – 36	n/a	n/a	n/a
37	3 quavers	2 quavers	6 quavers
38 – 55	4 quavers	3 quavers	12 quavers
56 – 60	5 quavers	4 quavers	20 quavers
61	7 quavers	4 quavers	28 quavers
62	7 quavers	5 quavers	35 quavers
63	7 quavers	6 quavers	42 quavers
64	7 quavers	5 quavers	35 quavers
65	7 quavers	4 quavers	28 quavers
66 – 70	5 quavers	4 quavers	20 quavers
71 – 73	4 quavers	3 quavers	12 quavers
74	3 quavers	2 quavers	6 quavers

As can be seen clearly in table 4.14 the macroperiods have again been arranged in symmetrical patterns in sections one (measures 1 – 19) and three (measures 37 – 76).

Visually representing the Maximal Un-transposable Harmonic Mode

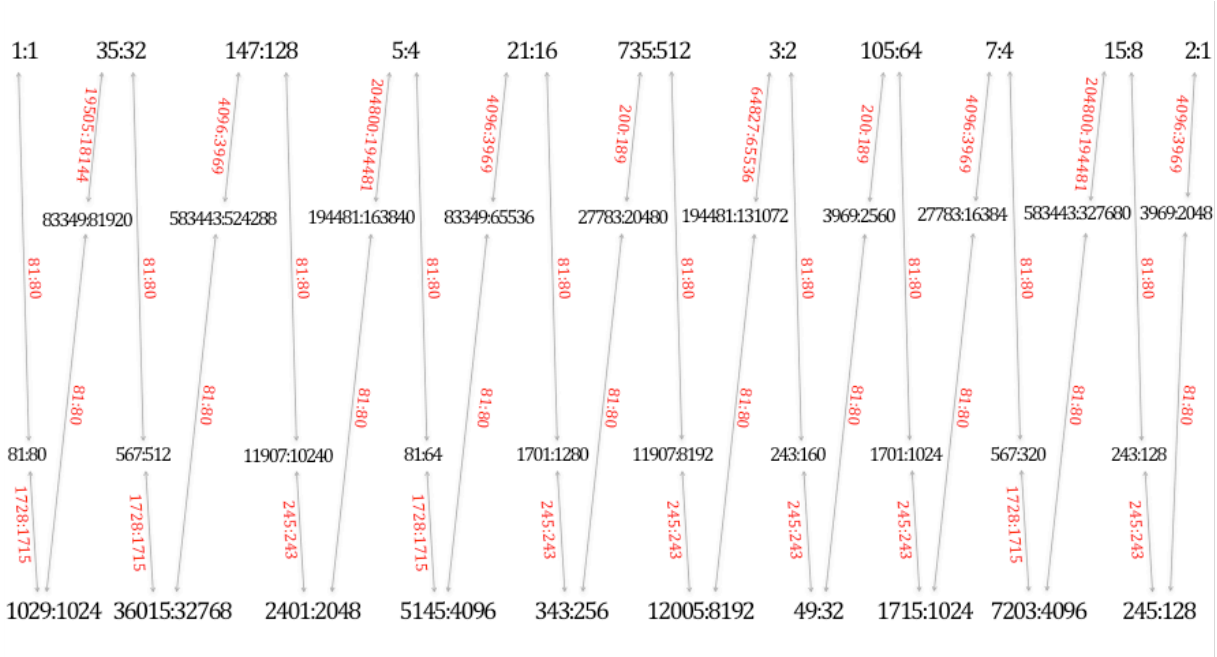
The tuning model for *The Un-tempered Pianos* and *K* contains one maximal un-transposable harmonic mode, all other UHMs in the system are truncations of the maximal UHM. A simplified expression, which doesn't contain the second set of twenty-four pitches that are a syntonic comma sharper than the original twenty four-pitch set, of the MUHM is shown in example 4.29.

Example 4.29, a simplified expression of the MUHM of *The Un-tempered Pianos* and *K.*



Example 4.29 is a simple way of observing this MUHM, and of understanding the mode's characteristics, but it is inaccurate. It shows only set one of a mode which contains two sets of pitches with identical intervallic structures. A more accurate modal diagram could be represented in two ways. The first way one could look at it is that the addition of set two creates a new, different UHM to occur. This idea is illustrated by example 4.30.

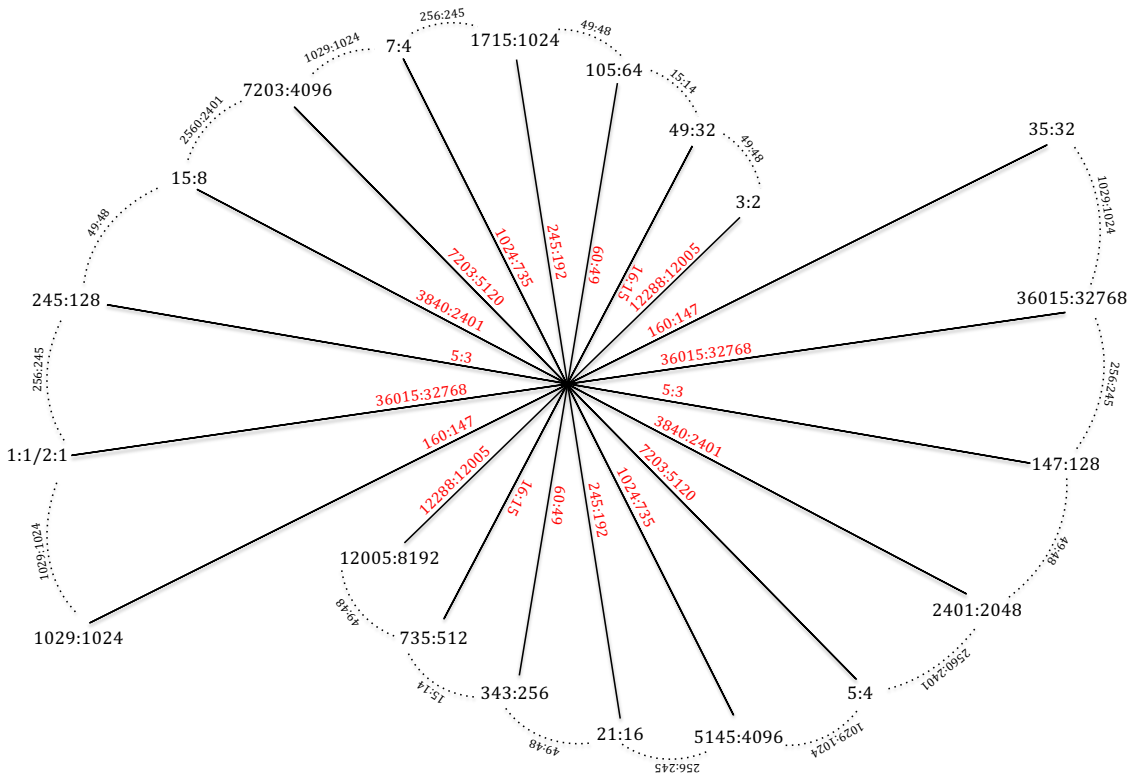
Example 4.30, one possible representation of the MUHM of *The Un-tempered Pianos* and *K.*


1:1
35:32
147:128
5:4
21:16
735:512
3:2
105:64
7:4
15:8
2:1
1950:18144
4096:3969
204800:194481
4096:3969
200:189
64827:65536
200:189
4096:3969
204800:194481
4096:3969
83349:81920
583443:524288
194481:163840
83349:65536
27783:20480
194481:131072
3969:2560
27783:16384
583443:327680
3969:2048
81:80
81:80
81:80

The second way is perhaps a more appropriate representation, as the way the pianos are tuned for these two pieces of music does not allow set two to have its own separate group of keys that can be operated independently of set one. As outlined in chapter two, the methodology attaches pitch set one to pitch set two. As the two sets are not assigned their own separate keys, to actually play the mode which appears in example 4.30, as a set of individual pitches, is impossible,[23] it is purely theoretical. Additionally, as previously discussed, the ear really only perceives the second set of pitches, in perpetual combination with the first set of pitches, as a sonic texture.

Another way of conceptualizing the maximal UHM of *The Un-tempered Pianos* is as a twenty, or forty-pointed star. The twenty pointed star is shown in example 4.32.

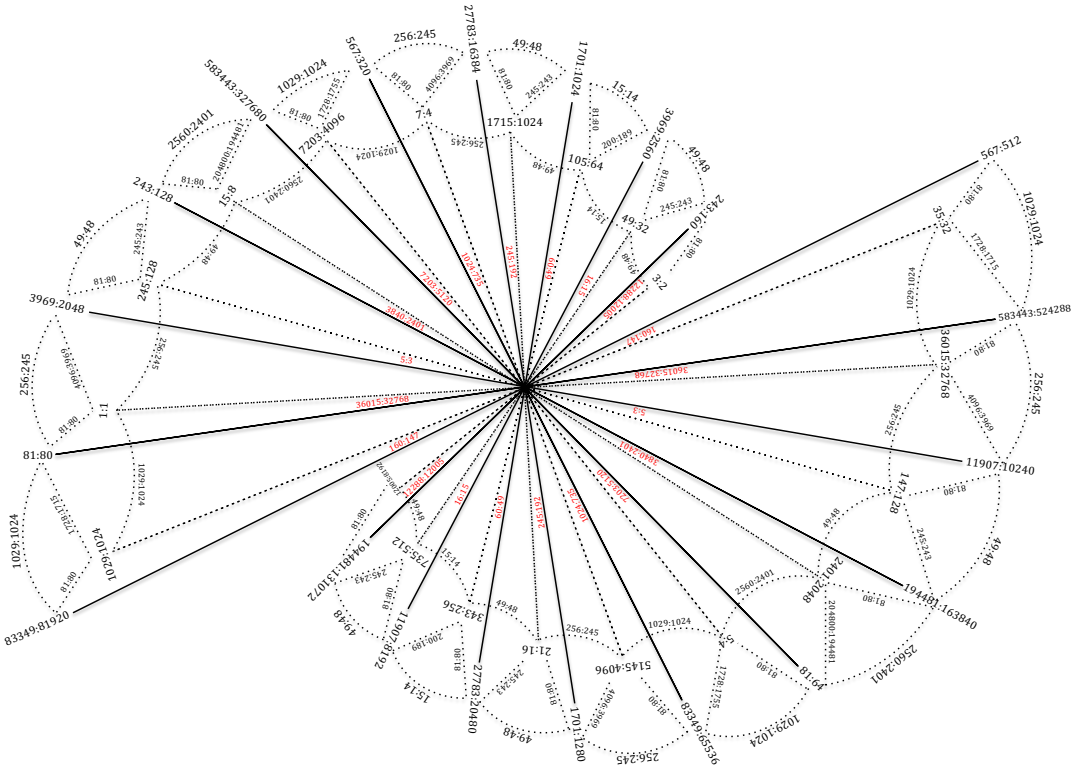
Example 4.32, the twenty-pointed star.



[23] The word 'impossible' is being used within the context of this project, and this piece specifically. It is of course possible to line four pianos up side by side and re-tune them to create 40 discrete pitches within an octave, but the system I have created only calls for two pianos.

The observant reader will quickly realise that example 4.32 is another simplified expression of the MUHM. The complete expression of the MUHM is shown in example 4.33.

Example 4.33, the forty-pointed star.



The benefit of using the star-style representation of the MUHM is that it allows the theorist to illustrate the arrangement of equidistant pitch pairs, while simultaneously showing the ascending order of the mode, and visually representing the specific type of symmetry of the mode.[24] To show the equidistance of pairs from the point of symmetry, along with the different distances of each pair, 10 lines of slightly differing lengths are arranged around the symmetry point, at either end of the line the pairs are arranged in ascending order. Once the arrangement is complete the two equal halves of the mode rotating around a point becomes visually quite obvious. The specific kind of symmetry also becomes visually obvious, a 180-degree rotation of one half of the mode, re-creates the other half. In musical terms, the second half of the mode is the inversion of the first half of the mode. The symmetry of the mode is two-dimensional, unlike the symmetry of the lattice, which is three-dimensional.

[24] The specific kind of symmetry is of course the symmetry of a rectangle.

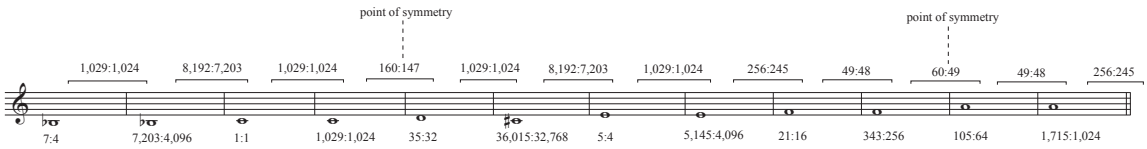
The application of symmetrical modes in The Un-tempered Pianos

The following modal analysis of *The Un-tempered Pianos* is simplified. The relationship of the syntonic comma is not expressed for reasons already outlined and illustrated in examples 4.29 and 4.31.

Section one

The first UHM begins to be introduced from bar number one, but is not heard in its complete state, as shown in example 4.34, until bars 18 – 21.

Example 4.34, the opening UHM from *The Un-tempered Pianos.*



The mode shown in example 4.34 is characterized by the gamelisma, or gamelan residue (1029:1024), and the slendro diesis, or the septimal sixth tone (49:48), and obviously the syntonic comma which is ever-present in this piece. There are four gamelismas, and two septimal sixth-tones. The mode also contains the smaller approximation of the neutral third (60:49), the septimal minor semitone (256:243) and two un-classified intervals. Following the initial opening, a new mode, which is not a UHM, is introduced at bars 39-43. This symmetrical five-bar interlude has already been discussed with relation to macroperiodicity.
Bars 48-123 do not investigate UHMs. A long ascending melody of breves, shared between the two pianos builds to a climax, signalling the end of section 1.

Section 1.A

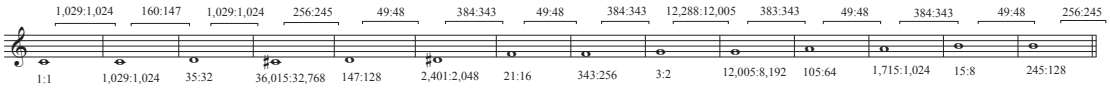
Section 1.A is composed purely by process. The process involves gradually re-introducing all the notes of the opening UHM, and then removing them; this takes place from bars 125-139. The process is derived from the process used in Tenney's *Chromatic Canon*, although my application of this process is slightly different. Tenney spends the first half of *Chromatic Canon* gradually building a group of notes, which are then matched with their corresponding inversion, where in *The Un-tempered Pianos* matching pitches with their inversion is not

composed in such strict order. There are two main conceptual differences between Tenney's process and my own. Tenney's musical material appears to be derived from the idea of major and minor being the inverse of each other, and it also appears to be composed according to serial procedures. The musical material in section 1.A of *The Un-tempered Pianos* is derived from the concept of UHMs, and macroperiodicity.

Section two

Section two modulates to a 14-pitch UHM, which is shown in example 4.35, using a transitional device, which is simply an ostinato of two alternating Cs related by the gamelisma. This UHM is introduced at bars 141 – 149. The new UHM is again characterized by the gamelisma, and the slendro diesis. There are four septimal sixth tones, and two gamelismas, which is the opposite of the opening UHM. Two new unclassified intervals also appear in this mode: 384:343, and 12288:12005.

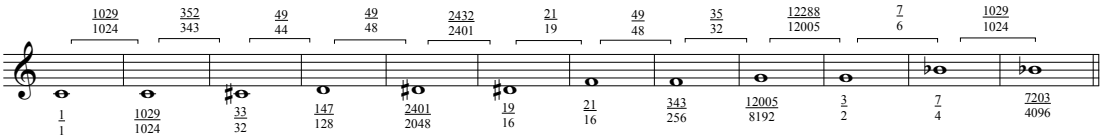
Example 4.35, the second UHM to appear in *The Un-tempered Pianos.*



The UHM appearing in example 4.35 is used in combination with the pitch 19:16, which has been included as an auxiliary colour tone. When 19:16 is taken into account, another comma appears, with the ratio 2,432:2,401; this is quite close to the Nuwell comma (2,430:2,401) but remains to be unclassified.

Modulation to a new tonal centre takes place at bar 151. This new mode, shown in example 4.36, is not transposable nor is it symmetrical; it is used to contrast the UHM which precedes it. Section two modulates between these modes twice, before transitioning into section three through the same C-gamelisma transitional device used previously in bar 140.

Example 4.36, the contrasting mode from section two, full statement of the mode appears in bar 201, truncations appear from bar 151.



In addition to containing a number of unclassified intervals, the mode shown in example 4.36 contains a septimal neutral second (35:32), a septimal minor third (7:6), and an undecimal whole tone (49:44).

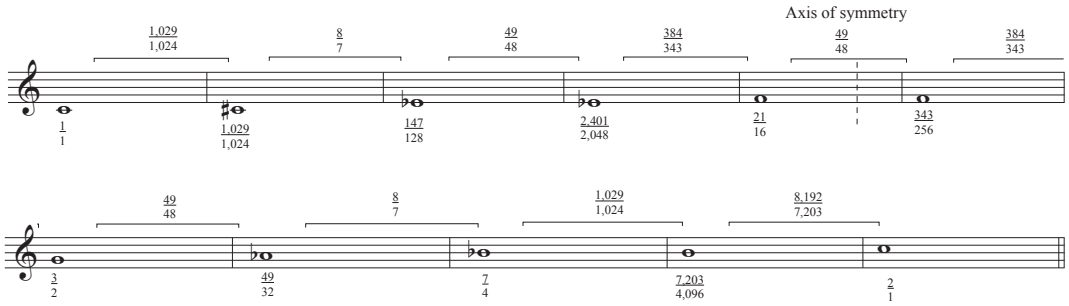
Section three

Section three does not investigate macroperiodicity, or UHMs. Perhaps it does not quite warrant its own section title, as it is, in reality, simply a more developed transitional device than the one used in previous sections. This section focuses on the gamelisma, and the slendro diesis.

Section four

Sections four, and IV.A are an homage to La Monte Young and *The Well-Tuned Piano*. Despite having access to fifths, thirds and sevenths, thirds have been omitted (those derived from the number five, that is) as is Young's preference. This means eliminating half of the lattice. If, hypothetically, all those pitches derived from the number five were actually removed from the lattice, then a new maximal UHM would subsist, this is shown in example 4.37.

Example 4.37, the maximal UHM that would exist if the number five were removed from the tuning model.



As all pitches derived from the number five have not actually been removed from the lattice, the mode shown in example 4.37 is technically a mode of limited transposition with symmetrical properties. Within this tuning model, there is one possible transposition of this mode.

Section four centres around the gamelisma between the two Cs on pianos one and two. On this occasion the gamelisma does not act as a transitional device, but as a background texture over which the dyadic harmonies unfold. The dyads are mostly related by the simple ratios 3:2 and 7:4, although there are some more complex intervals used also. The mode shown in example 4.37 is the mode which is used throughout sections four, and 4.A with the exception of 49:32, which does not appear.

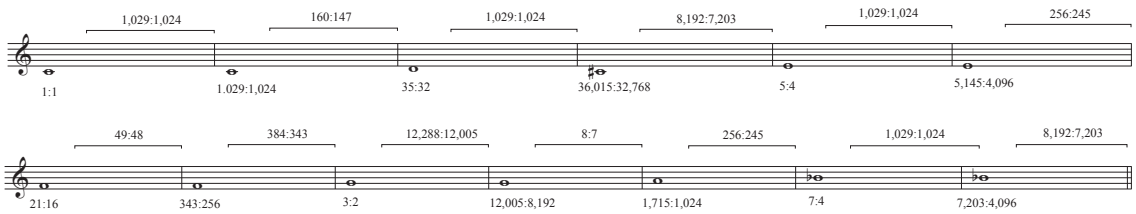
Section 4.A

Section 4.A contains the same pitch material as section four, but instead of the slow tranquil way in which section IV unfolds, section 4.A is characterized by rapid tremolos, or in La Monte Young's terms 'harmonic clouds'.

Section five

Section five continues to focus on the relationship of the harmonic seventh, while also re-introducing thirds that are derived from the number five. The mode that is explored in this section, shown in example 4.38, is an un-even truncation of the maximal UHM. The harmonic ratio 105:64, which pairs with 343:256, has not been used.

Example 4.38, the mode used throughout section five.



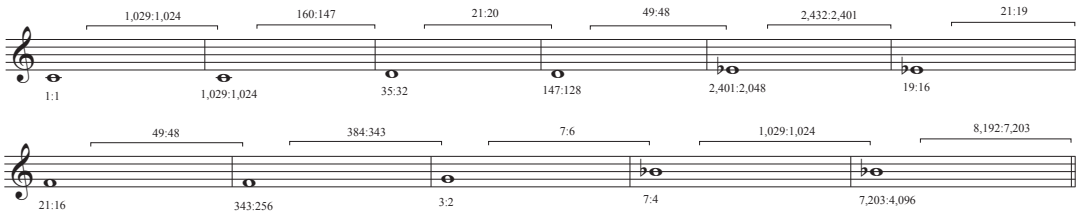
Modal analysis of *K*

K does not investigate UHMs. The purpose of this piece was to investigate macroperiodicity, which has already been discussed. Despite not utilizing any UHMs, the modes that have been used in K shall still be reviewed.

Section one

The mode used in section one of *K* is similar to the mode used in section four of *The Un-Tempered Pianos*. This mode is un-transposable within the tuning model, but not symmetrical, it is shown in example 4.39.

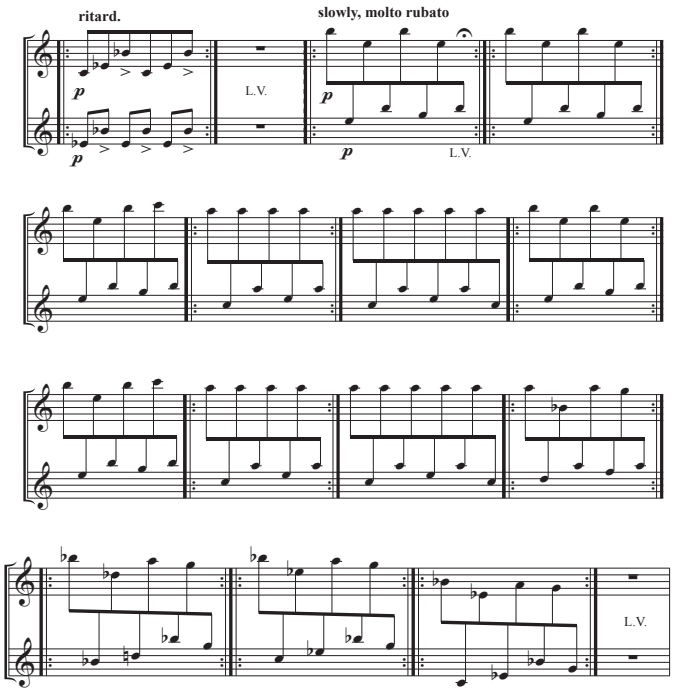
Example 4.39, the mode used in section one of *K*.



Section two

Section two, as it appears in the final rendition of *K* is not how it was originally conceived. The first conception is shown in example 4.40.

Example 4.40, the original conception of section two of *K.*



The original concept was to highlight the gamelisma and slendro diesis while a descending chain of fifths simultaneously unfolds. The revised version can have little emphasis on this relationship as phrases are confined to individual pianos. This call and response which occurs between the two pianos in the final version of the score was re-written in such a way for the purposes of playability. Perhaps two pianists could execute the patterns shown in example 4.40, at the time however, it seemed to risky to attempt, so it was re-written to accommodate two pianists more easily. The result of this revision is perhaps trivial. If this section is considered chordal, as opposed to modal, then tables 4.15 and 4.16 show the difference, in ratios, between the two iterations of the same material.

Table 4.15, section two of *K:* original chord structure.

Bar no.	Integer Ratios
21 – 23	5:4 – 5145:4096 – 3:2 – 15:8 – 245:128 – 1:1
24 – 25	105:64 – 1715:1024 – 1:1 – 5:4
26 – 27	5:4 – 5145:4096 – 3:2 – 15:8 – 245:128 – 1:1
28 – 29	105:64 – 1:1 – 5:4
30	12005:8192 – 105:64 – 1715:1024 – 7203:4096 – 35:32 – 21:16
31	12005:8192 – 3:2 – 1715:1024 – 7:4 – 7203:4096 – 36015:32768 – 35:32
32 – 33	1:1 – 2401:2048 – 19:16 – 12005:8192 – 3:2 – 1715:1024 – 7:4 – 7203:4096

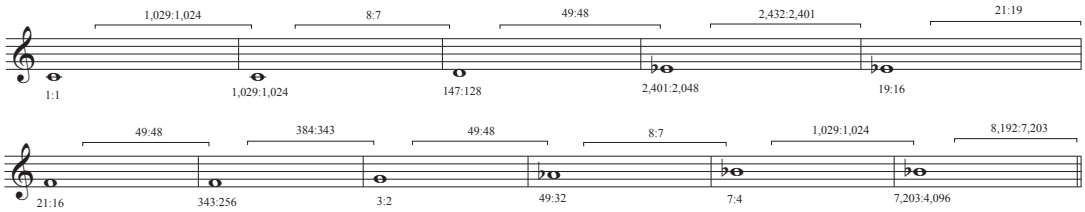
Table 4.16, section two of *K:* revised chord structure.

Bar no.	Integer Ratios
21	5:4 – 3:2 – 15:8
22	5145:4096 – 12005:8192 – 245:128
23	5145:4096 – 12005:8192 – 245:128 – 1029:1024
24 – 25	105:64 – 1:1 – 5:4
26 – 27	5:4 – 3:2 – 15:8
28 – 29	1715:1024 – 1029:1024 – 5145:4096
30	3:2 – 105:64 – 7:4 – 35:32 – 21:16
31	12005:8192 – 1715:1024 – 7203:4096 – 36015:32768
32	1:1 – 19:16 – 3:2 – 105:64 – 7:4
33	1029:1024 – 2401:2048 – 12005:8192 – 7203:4096

Section three

The mode used in section three, shown in example 4.41, contains all the pitches necessary to make the hypothetical UHM discussed in relation to section four of *The Un-tempered Pianos*, where one half of the lattice is removed, it also includes the 19th harmonic, and is therefore not symmetrical.

Example 4.41, the mode used in section three of *K.*



The mode shown in example 4.41 is quite similar to the mode used in section one of *K*, one of the main differences being that it takes a tonic of F, instead of C; a Schenkerian approach could suggest that in this way the piece, overall reaches some resolution through this dominant, tonic shift.

Conclusion

The section that follows herein is concerned with how successful the project was overall, and how concepts exposed in this project can be further developed either by myself in future compositions, or by other readers who also feel that such concepts warrant creative investigation.

Throughout the course of this project, the theory seemed to constantly run ahead of the compositions. I felt as though the compositions were continually trying to catch up to the theoretical developments. This personal compositional battle is represented here in the final thoughts section. While trying to complete the project within its pre-determined scope and timeframe, my thoughts about new approaches to lattices and mode construction continue to outpace the compositions themselves. As a result, I will briefly present some concepts for future modes and lattices here, without supporting compositions, for the interested reader.

Lattice construction, and mode building strategies

Throughout the course of this project, I have developed concepts for lattice and mode construction. The main concepts explored include: building lattices based on the ease with which performers can achieve the notes in the lattice, building lattices based on the desire to explore the qualities of a specific number or set of numbers, building lattices based on the desire to explore the quality of a given interval. These concepts represent a handful of strategies for answering a problem that has no solution.[1] There are of course many potential solutions to this problem, some of which I have discovered but have not had the scope to include in the main body of this research.

Hyper-prisms (four or more-dimensional shapes)

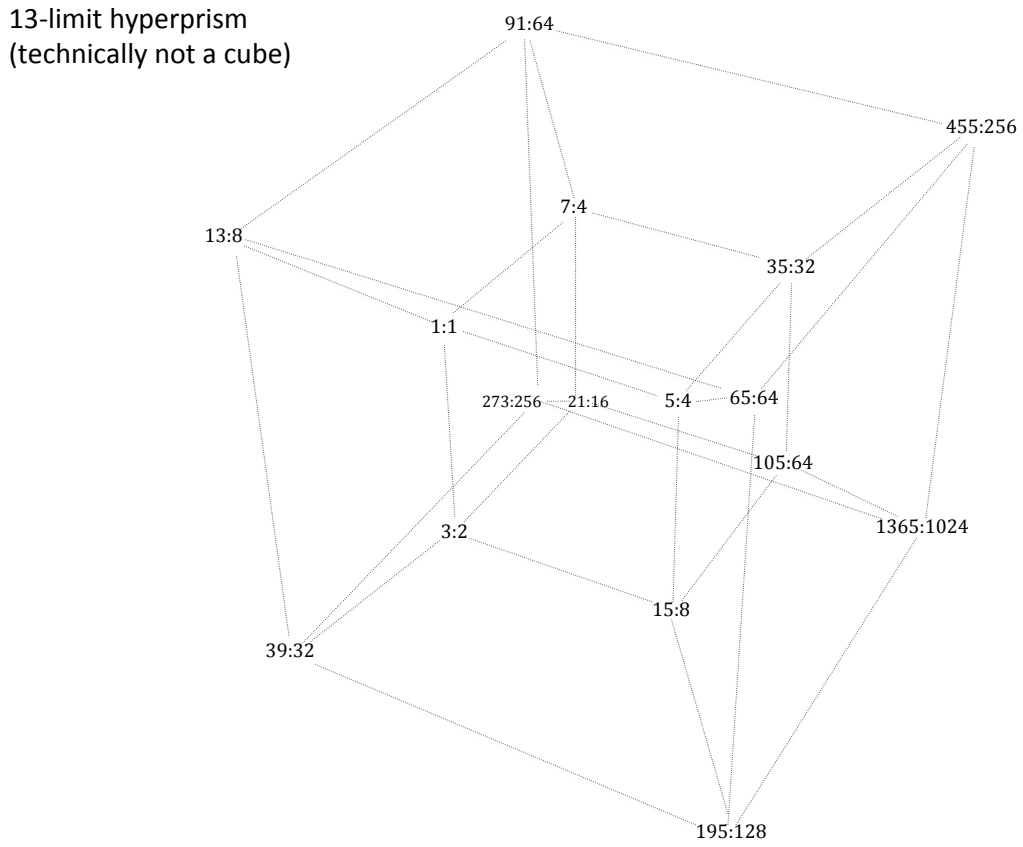
The tuning model developed for *The Un-tempered Pianos,* and *K,* is similar to a hyper-prism, in that it can be represented by two prisms either lying side by side, or one inside of the other. The prism generated by the syntonic comma in this

[1] The problem being addressed here is how does a composer develop, or arrive at a lattice for any given piece of music? This is a problem I unofficially (i.e. it is not an explicit research question) tackled throughout the course of this project.

lattice does not represent a new dimension as it contains no new prime number with regard to the rest of the lattice, therefore perhaps a mathematician would not consider it to be a true hyper-prism.

A basic just intonation hyper-prism, illustrated in example 5.1, would generate a lattice of 16 pitches. More sophisticated prisms, containing geometric series, which expand in various directions, and possibly further dimensions, would allow more pitches to be incorporated into such a lattice.

Example 5.1, A basic just intonation 13-limit hyper-prism.



Although the hyper-prism appearing in example 5.1 appears to be a hypercube, technically it is not. A matrix which immediately bears a semblance to this example is the lattice belonging to *13* for string quartet. The lattice for *13* was of course born of pragmatism, and not pure theory. If pure theory had trumped playability, then perhaps the tuning scheme for *13* would have appeared as something similar to what is seen in example 5.1.

Modes of Synthesized Inversional Symmetry (MOSIS)

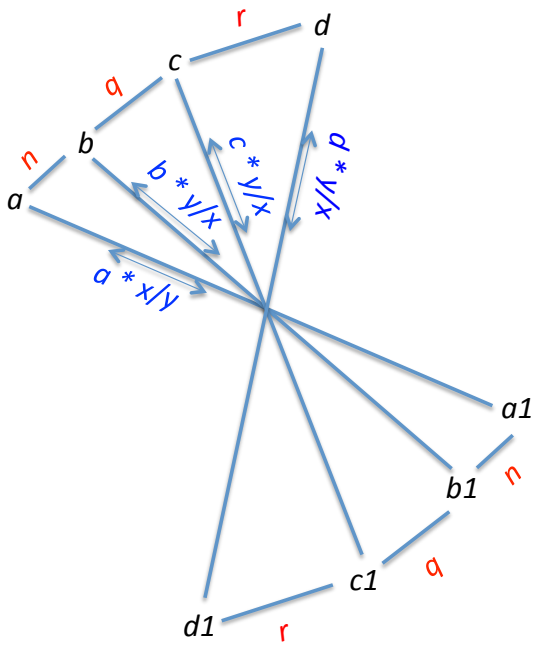
Any set of intervals can be infinitely replicated in just intonation; this can be achieved by translating a set of ratios into a new dimension. Alternatively a translation can be performed within the same dimensions which the first set of ratios exists within. Simply translating a set of ratios to a different position on an existing lattice, or onto a new lattice does not, however, guarantee that combining the two sets will generate a symmetrical mode. In order to guarantee symmetry, the replication process must simultaneously incorporate a 180-degree rotation about a given point. To move to a new dimension, a new co-prime ratio is necessary, and the value of the new co-prime ratio must be asserted. One ratio in set one can then be multiplied by the asserted co-prime ratio. The remaining ratios in set one can then multiplied by the reciprocal of the new co-prime ratio in order to achieve the 180 rotation. This is one way of finding the value of the ratios in set two. Suppose that set one is composed of the values *a, b, c, d* which are related by the intervals *n, q, r* then set one as a mode can be represented as the mode shown in example 5.2.

Example 5.2, set one.



If *a1* is found through the operation of multiplying *a* by *x/y*, then the remaining values of set two can be found by multiplying the remaining values of set one by *y/x.* This is illustrated in example 5.3.

Example 5.3, diagram showing the relationship between set one and set two.



An alternative method for finding the values of set two is to assert the value of *x/y* and then calculate the value of *b1, c1, d1* using the values of the intervals *n, q, r,* whose values are known. This is a theoretical way to synthesize symmetry in a mode; its usefulness is yet to be tested in compositions.

As a concept MOSIS are important because it provides the composer with a quick way to find a corresponding symmetrical set, when such a set may not be immediately apparent. Consider the scenario where a composer has an asymmetrical mode, and the ratios which form the mode are multi-dimensional and distantly related. In such a scenario it may be difficult to determine a corresponding set of ratios that would unify the asymmetrical mode, making it symmetrical. MOSIS provides a way of doing this at a time where it seems there is currently no published method of doing so.

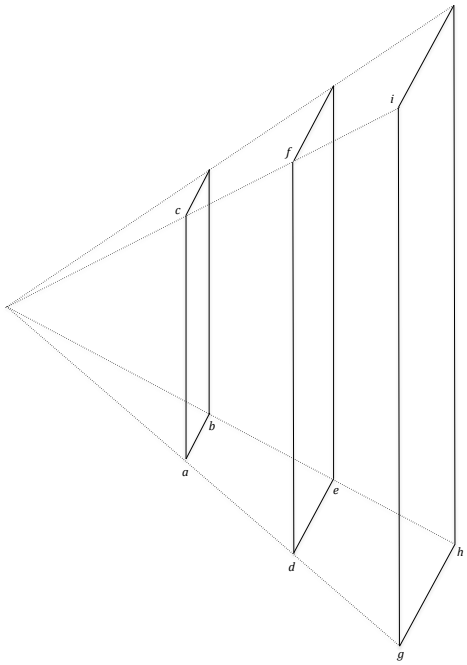
There appears to be an inverse link between UHMs and MOSIS, however, using one model to predict the other is problematic. If one were to use MOSIS in order to produce a particular shape on a lattice, it would prove rather difficult, perhaps impossible to produce a specific polygon or polyhedron without already knowing the dimensions of that shape within the lattice. Conversely, it is also particularly difficult to determine the way in which a shape in a lattice will be broken into

two the symmetrical halves which form a MOSIS, without having already constructed the MOSIS. The relationship between UHMs and MOSIS seems to be fertile ground for future research.

Expanding polygons

Another possible, and purely theoretical, way in which a lattice could be generated is by beginning with some rudimentary shape, and then expanding or contracting it a number of times. The way in which a composer or theorist would achieve this is illustrated in example 5.4.

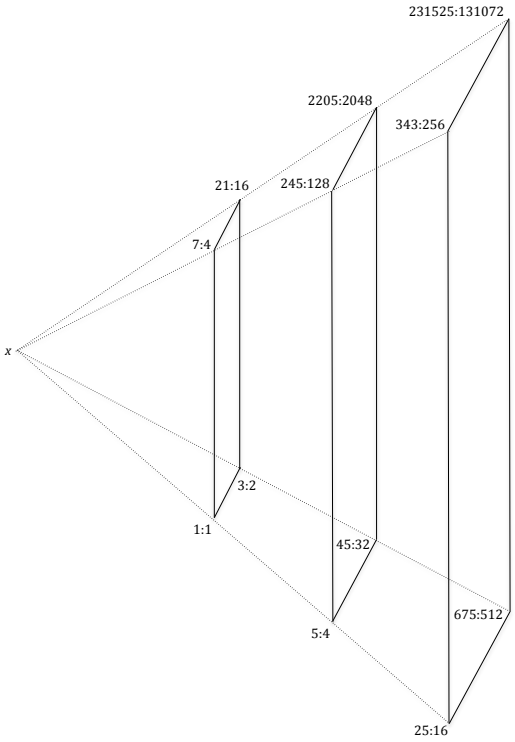
Example 5.4, expanding a rectangle.



$a \rightarrow d = ad$
$a \rightarrow e = ad + ab^2$
$a \rightarrow f = ad + ac^2$
$a \rightarrow g = ad^2$
$a \rightarrow h = ad^2 + ab^3$

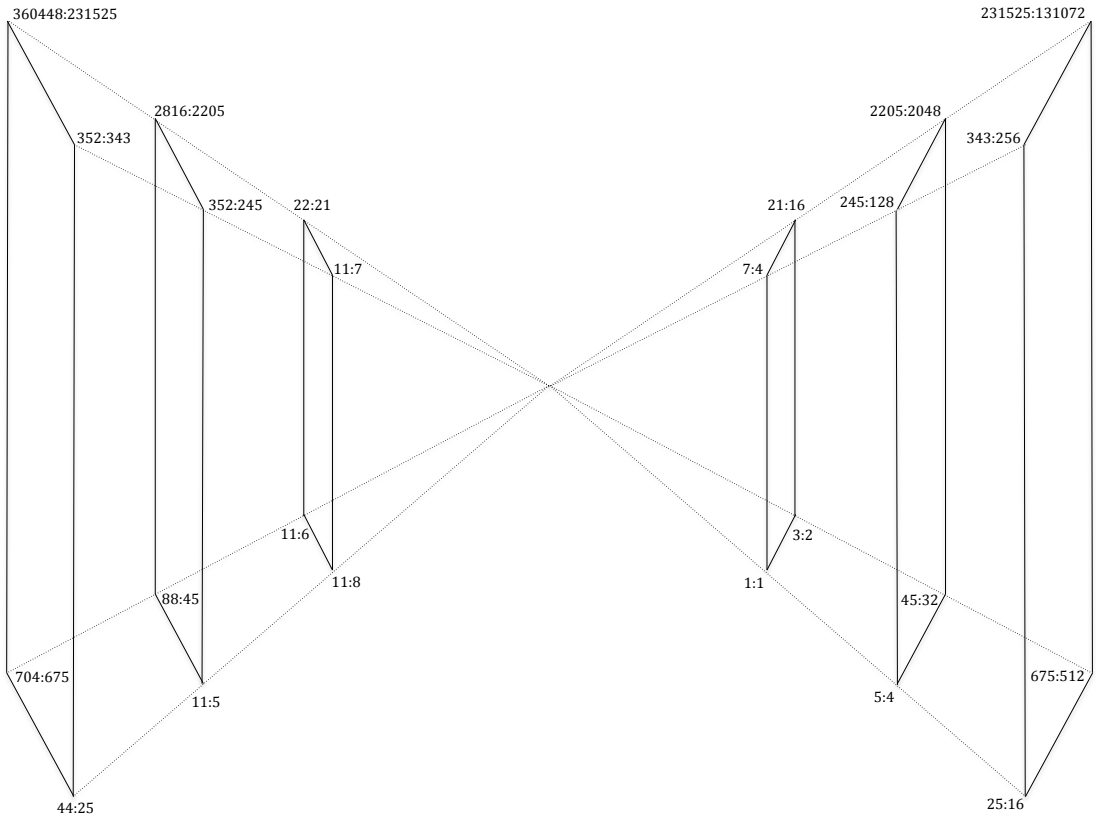
If example 5.4 is translated into just intonation terms, using 3:2, and 7:4 as the generating factors, then example 5.5 can be arrived at.

Example 5.5, example 5.4 translated into just intonation terms.



If the three concepts of the hypercube, MOSIS, and expanded polygons were to be combined simultaneously, then a lattice such as the one shown in example 5.6, would become possible.

Example 5.6, the conceptual combination of the hypercube, MOSIS, and expanded polygons.



Such methods for generating new tuning models, certainly do not guarantee even-ness of steps, nor do they guarantee that the entirety of the twelve-key-per-octave piano keyboard will be completely occupied. This is not the purpose of these concepts; the purpose of these concepts is simply to find intelligible ways in which new tuning models may be arrived at.

Final thoughts about the project

The project has addressed the research aims, and answered the research questions often through different approaches for each aim and question. The results each time have caused outcomes which vary in their effectiveness, or have had some unexpected side-effect. Despite addressing the aims and research questions, there are a number of problems that were encountered during the project which should briefly be considered.

Problems encountered

The problems with the project can be divided into the two categories: Problems arising from the theory, and shortcomings of the composer and theorist.[2]

Problems arising from the theory

In the context of this project, the root of many problems lies in the fact that there is a vast difference between theory, and reality. This was obvious in many aspects of the portfolio, for example *The Un-tempered Pianos,* and *K* could both only utilize the middle register of the piano keyboards, due to difficulties re-tuning the extreme upper and lower registers. Although evident in other pieces in the creative portfolio, the next problem was particularly apparent in *13* for scordatura string quartet. There was a noticeable difference between what I had written, and how the string quartet actually sounded during the workshopping of *13.* The timbre of many of the harmonics in *13,* combined with the dissonance of the interval of 13:8 makes for some rather challenging listening in this piece. The timbre of a pitch, after all, has a great influence over the perceived consonance or dissonance of the sound.[3] A further consideration here is perhaps the speed with which many new pitches are introduced to the listener. The allegro sections of *13* bombard the listener with a large number pitches whose relationships are far from simple, and not familiar parts of the Western listening experience.

Another problem arising from theory, and also concerns *13* for string quartet, is the number thirteen, in relation to macroperiods. In my opinion, macroperiodic writing can be an effective compositional tool, but it appears that beyond a certain combination of prime numbers macroperiodic writing become unintelligible in the context of music being performed by conventional instrumentalists on conventional instruments. It seems that around 13:4, or 13:5 is where things begin to become less intelligible for the listener. I must stress that this is my personal observation and experience only; it is beyond the scope of this project to attempt to quantify such a statement. It is entirely possible that with greater exposure to larger prime combinations in macroperiods, then the

[2] In this case the composer and theorist is the author, Dylan Crismani.
[3] Sethares, William A. *Tuning, Timbre, Spectrum, Scale.* Second ed. London. Springer-Verlag, 2005. P. VI.

sound would seem more familiar. It is also possible that larger prime number combinations such as 29:23 would sound intelligible if played by a computer, at super human speeds. Alternatively our ears might perform some complex calculations for us, and we would perceive this relationship as 5:4, in the same way that the ear can perform complex calculations that cause us to perceive a 12-TET fifth as barely different to 3:2.

The problem of human perception also affects the truncation of modes with many pitches. Truncating modes in a twelve-pitch system causes perceivable differences between modes; however when the mode being truncated contains thirty pitches, removing two pitches at a time causes an almost imperceptible difference. Initially I considered this to be a problem, as there was a certain same-ness to the sound quality of many of the modes I was working with. Having taken some time to reflect carefully on this, I realise that having access to a large number of modes which are almost imperceptibly different from one another is similar to the painter who has access to limitless shades of a single colour; there is great potential here that is yet to be explored.

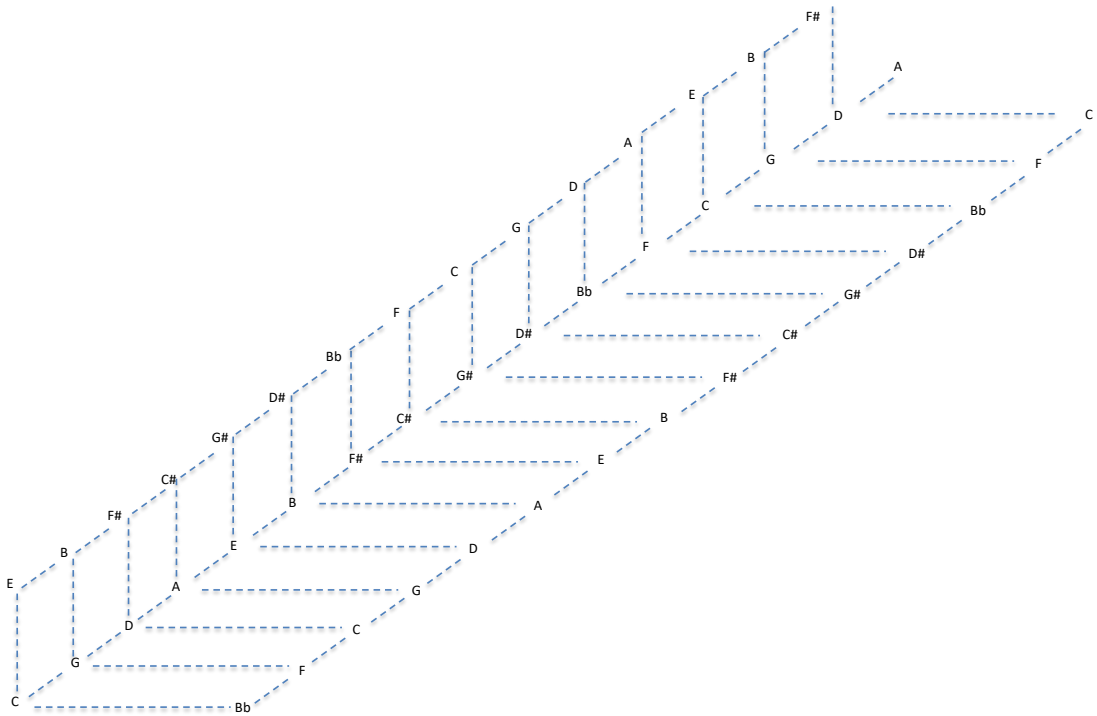
The problem of combinatorial explosions is a pertinent one for any tuning model with a large number of pitches. As the number of pitches increases, the number of possible combinations grows exponentially, to the point of intractability. The theorist and composer need to apply limitations to the groups of pitches they are working with in order that a group of notes can be managed with relative ease. The specific set of limitations will obviously vary from one problem or piece of music to another. Take *The tale of Kiều* as an example. The tuning model for this piece contains forty-six pitches. The limiting factor which aids mode management is the use of UHMs; modes must be un-transposable whilst also being inversionally symmetrical. This decreases the maximum number of possible pitches being combined from forty-six to thirty.

The total number of possible combinations[4] of fifteen pitch-pairs is 32,768. There are three possible combinations of geometric series. 1:1 and 5:4, 5:4 and 7:4, and 7:4 and 5:4. Therefore there are 98,214 possible symmetrical modes within this system. Additionally the two combinations of geometric series which include the 1:1 series has two possible centre points depending on whether or

[4] This includes transposable, and un-transposable.

not 14384907:8388608 is included instead of 1:1. This would generate further symmetrical modes. Applying the next limiting factor of un-transposability would still not decrease the number of modes to a manageable figure. There is, however, a limiting factor that can be applied which decreases the number of modes in this system significantly, and would cause the theorist or composer to be studying a unique subset of UHMs. Suppose the composer decided to come full circle from just intonation, and back into 12-TET where there are only twelve pitches then the tuning model for *The tale of Kiều* might appear as illustrated in example 5.7.

Example 5.7, A pitch-name representation of the lattice for *The tale of Kiều.*



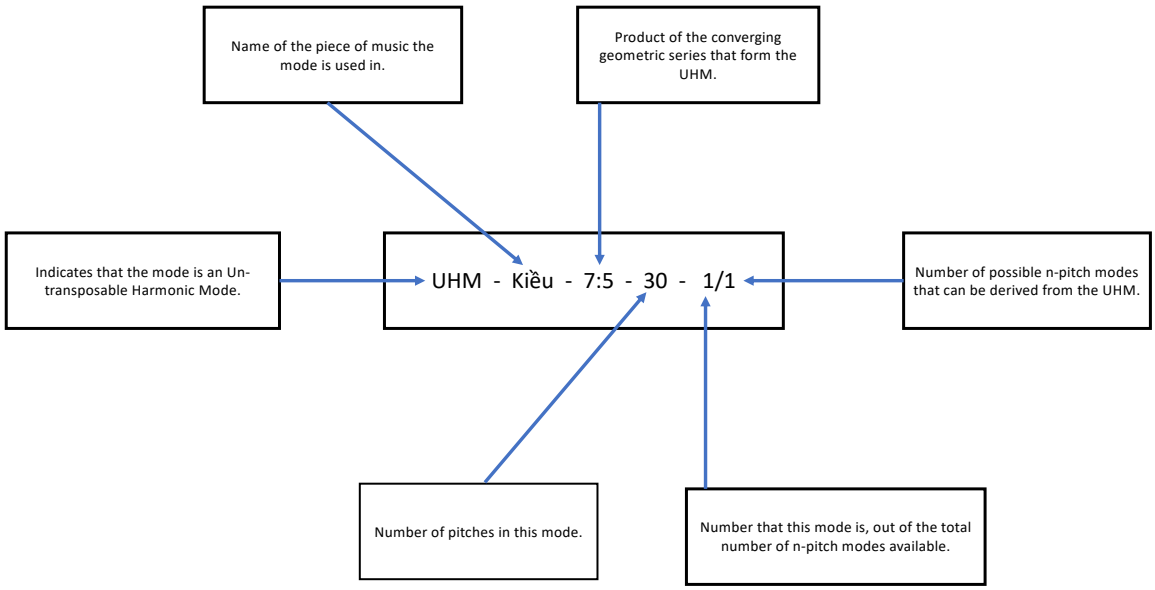
Despite there being forty-six distinct pitches in the lattice, this representation shows only twelve. If this idea were translated into some limitation, the limitation would be that if one version of a C were being used, then all versions of C (within the thirty-pitch set) must also be used. Such a limitation would decrease the number of pitch-pairings from fifteen to seven. The total number of modes that each combination of two geometric series would yield is ninety-six per combination. Furthermore, this limitation would cause every single one

these particular modes to contain the special sonic texture which occurs within critical bands.

Shortcomings of the composer and theorist

A problem that the reader has likely already identified is the lack of any system of classification for UHMs. I have identified a possible system which could be used. Such a system of UHM classification could in the future appear as something similar to what is shown in example 5.8.

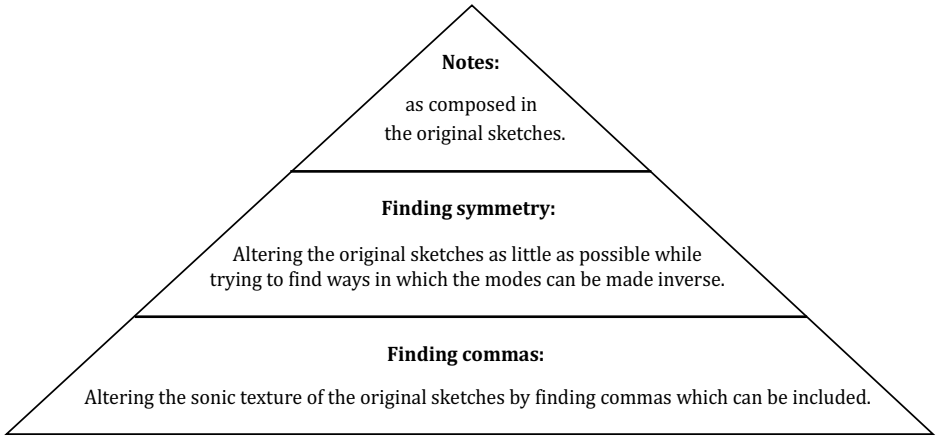
Example 5.8, a potential method of classifying UHMs.



The method of mode classification shown in example 5.7 provides the reader with a large amount of information about the mode in question, but it is rather more clunky and awkward than simply being able to label something as a pentatonic or octatonic mode with five different tunings of each pitch for example.

During the composition of *The tale of Kiều*, and *13* there was an unofficial hierarchy of musical ideas dictating how problems were handled, and how compositional processes were dealt with. This is illustrated in example 5.9.

Example 5.9, The hierarchy of musical ideas pyramid



The model shown in example 5.9 is a model where the system served my needs as a composer, instead of the composer serving the system. The pyramid dictates that the notes, as composed in the original sketches, are of the greatest importance, and must be adhered to. Of secondary importance is attempting to create symmetry from the original set of pitches, if they are not already symmetrical. The third most important thing is then finding, and including commas.

The tiers of the pyramid could be re-arranged, or added to. It is impossible to posit whether one arrangement of the pyramid is better than another, and obviously different circumstances will dictate which arrangement is the most suitable, or if other tiers need to be included. Taking an artists perspective, and desiring a certain degree of control over my finished product, I believe that the arrangement I chose for this project was suitable, but not without problems. One problem that arose while writing *13* and *The tale of Kiều* was compromising between symmetry and including commas. Often the inclusion of a musical comma meant sacrificing the pure theoretical symmetry in the mode. There are ways around this problem, such as the tuning model that was adapted for *The Un-tempered Pianos* and *K*. In this tuning model the syntonic is so ingrained in the system, composers need not concern themselves with finding commas at all. The other way around this problem is to re-arrange the pyramid so that the composer's originally imagined sounds become less important than the considerations of symmetry, and the inclusion of commas. This approach would obviously have drastic effects upon the final sound of the composition. Another

way that this problem could be solved is by building unique instruments, either digital or physical that highlight symmetries and commas for any given tuning system; unfortunately such a solution was beyond the scope of this research. Alternatively, before composing anything, the composer could compile a comprehensive list of all the modes within a given tuning system which contain the desired amount of commas, and symmetry, and develop a composition from that starting point.

A final thought

Dear reader, the time has come for me to say farewell to you, but before I depart I would like to share my final thought on the project. Although at times the project was criticized with thoughtlessly delivered words of discouragement, professional musicians have shown me, by playing my music, that they have faith in this new and experimental work; furthermore, audiences have been surprisingly receptive to the compositions. So I have confidence that the compositions and theories presented to you here are in some way on the right track. Looking at the future, there are new ideas out there for me to explore and it is time for me to use this project as a scaffold and investigate those new ideas.

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Appendix A

(Please see attached compact disc labelled Appendix A)

Appendix B

(Please see attached compact disc labelled Appendix B)
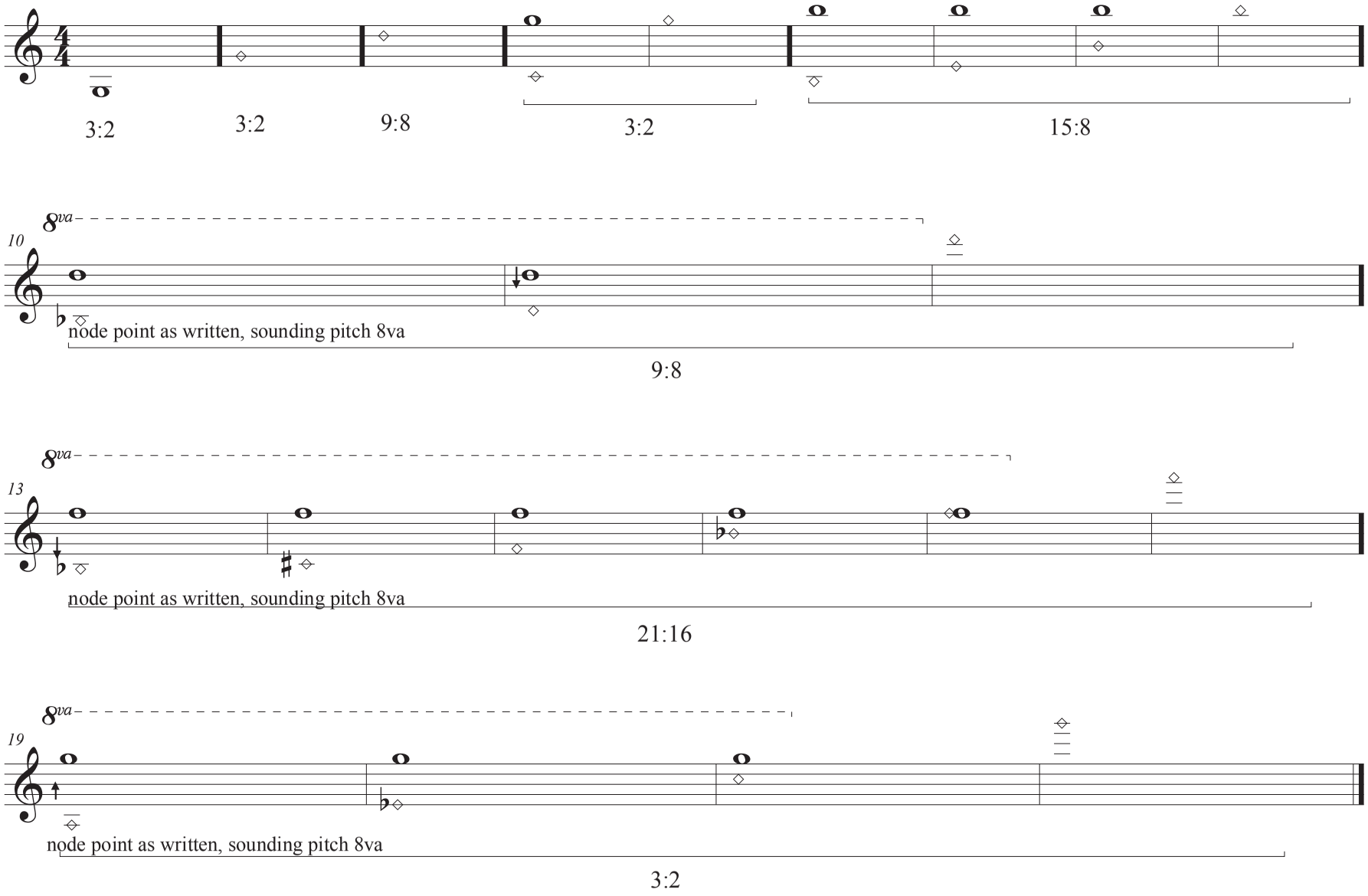
Appendix C

Appendix C, part one

(Transposing chart for violins 1 & 2)

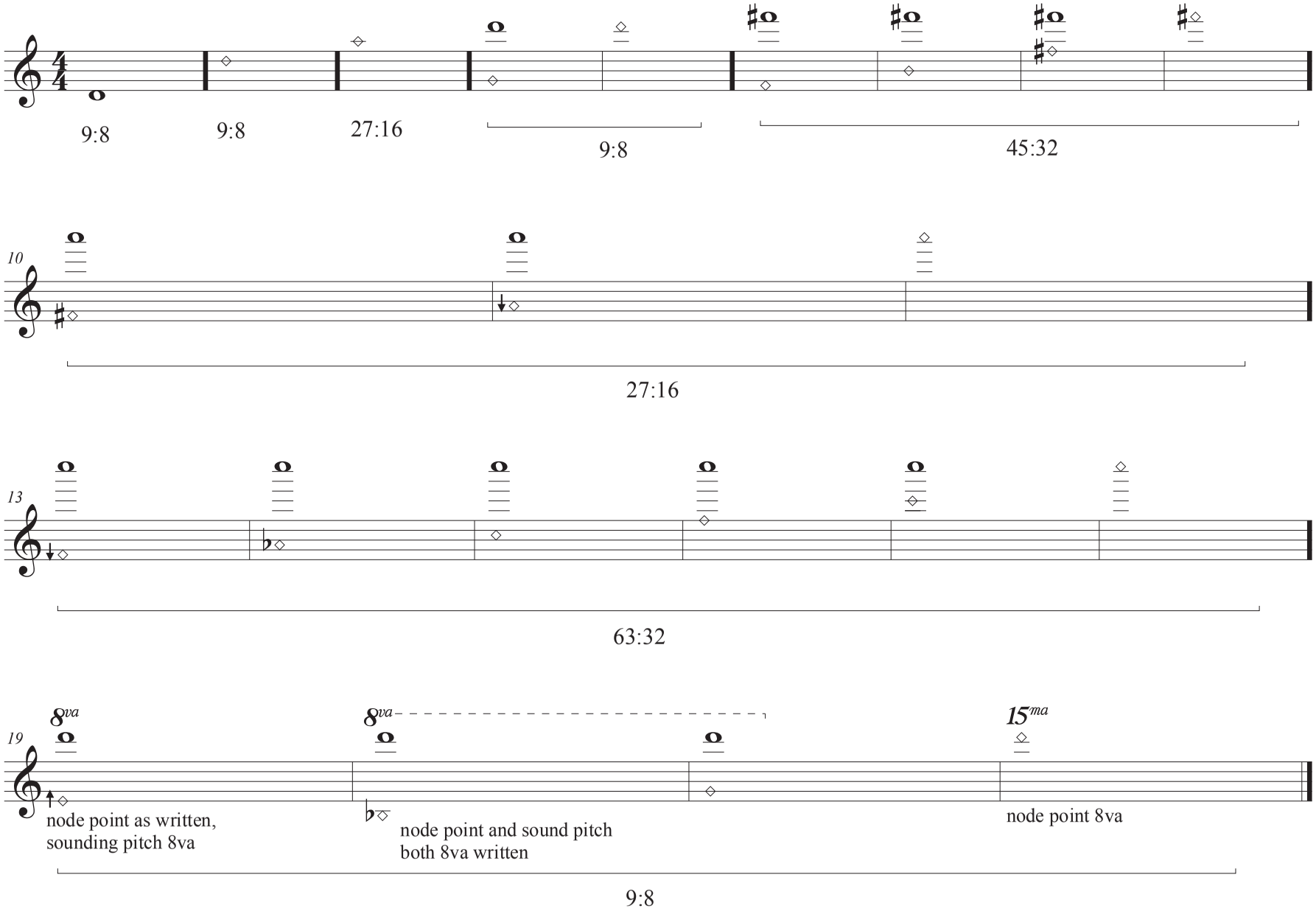
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Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



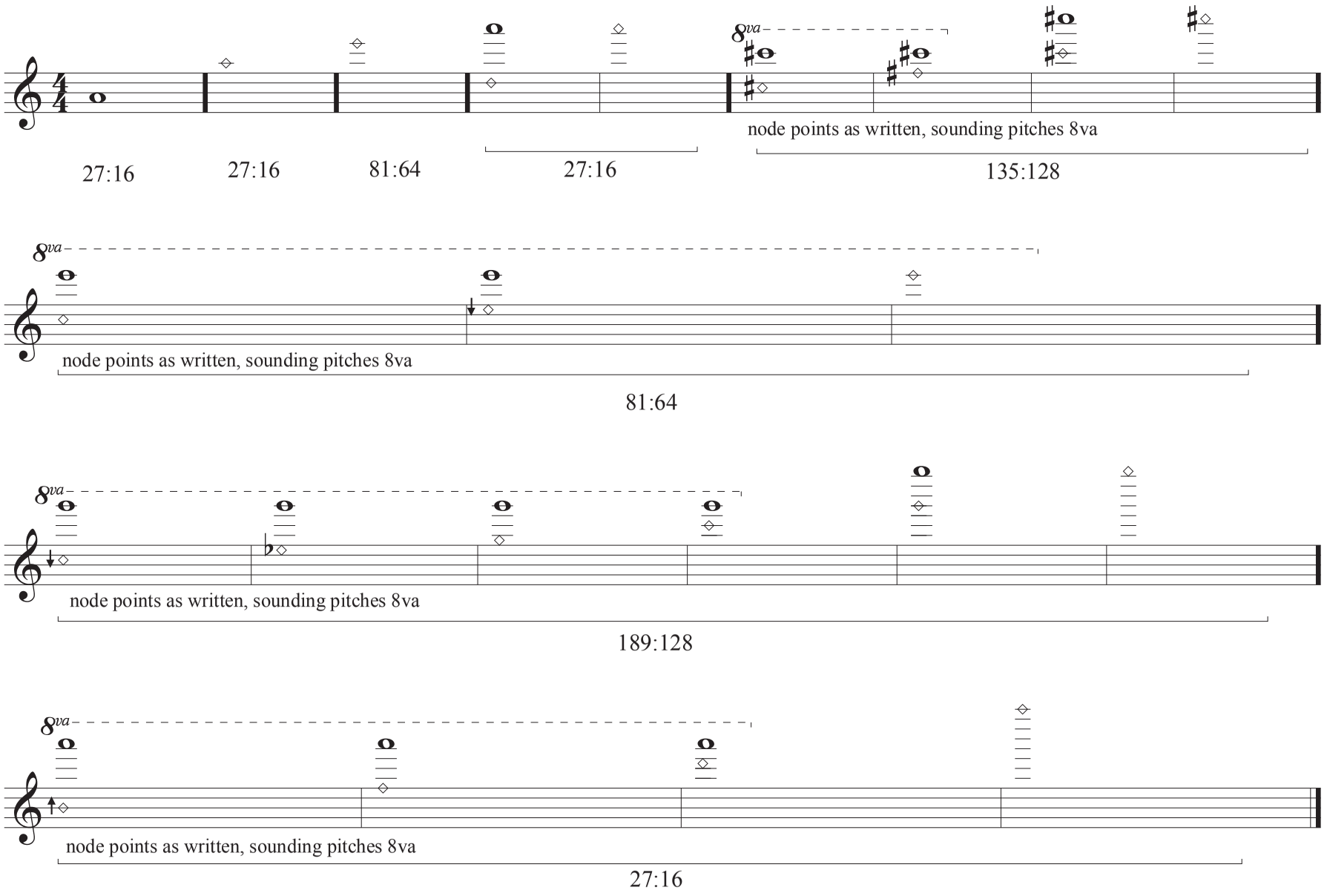
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Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



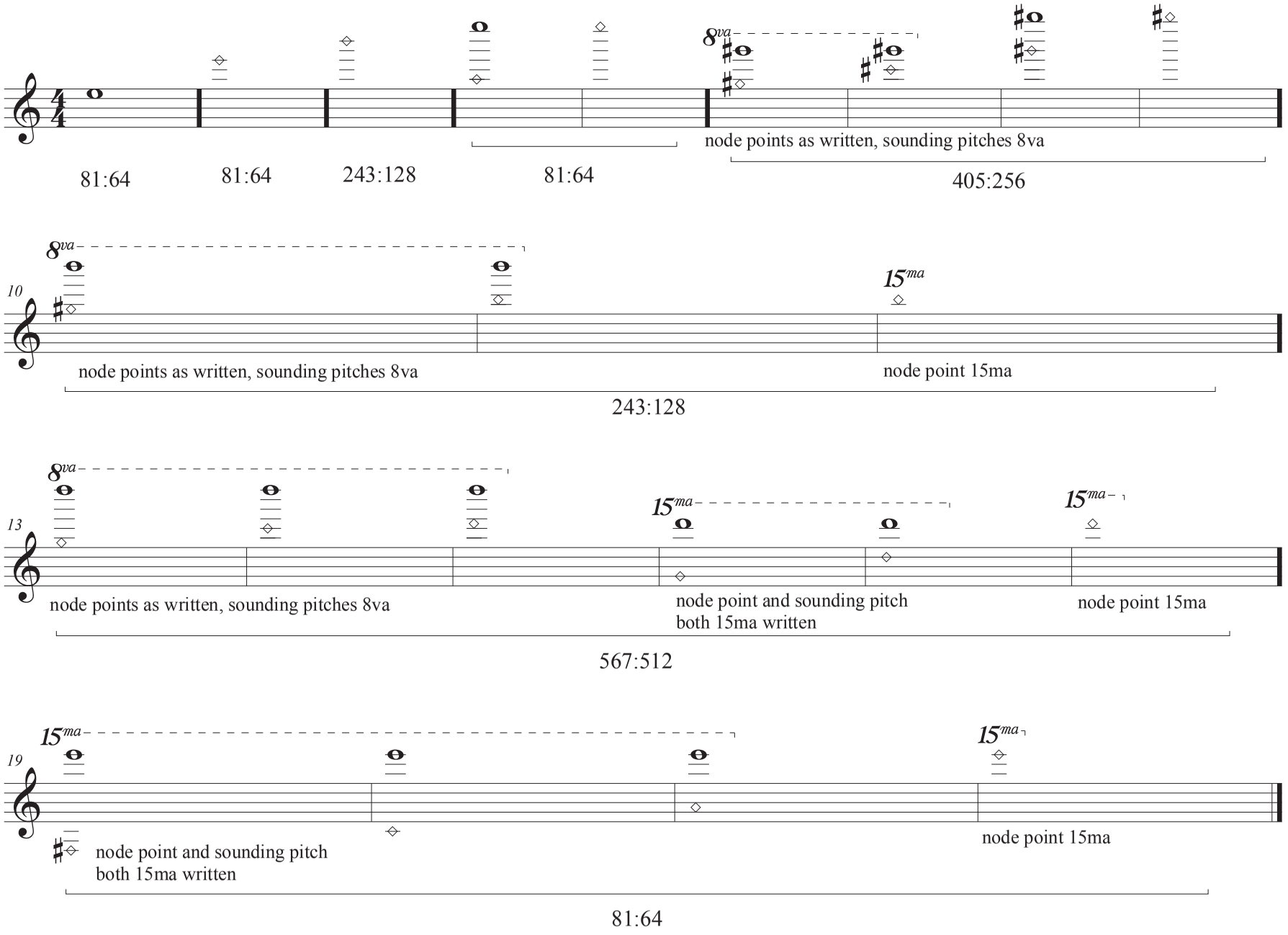
A

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



E

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)
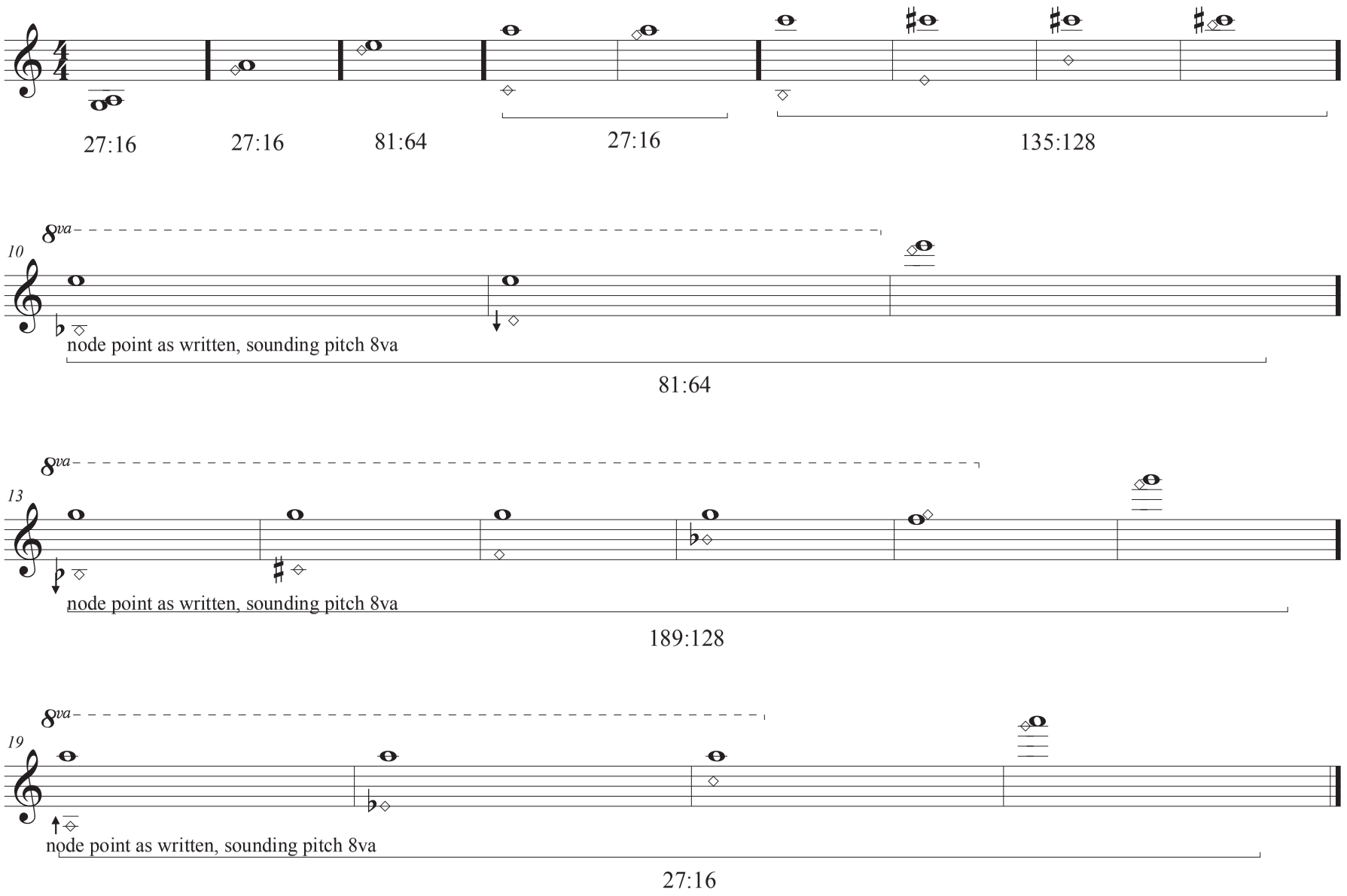


Appendix C, part two

(Transposing chart for violins 3 & 4)

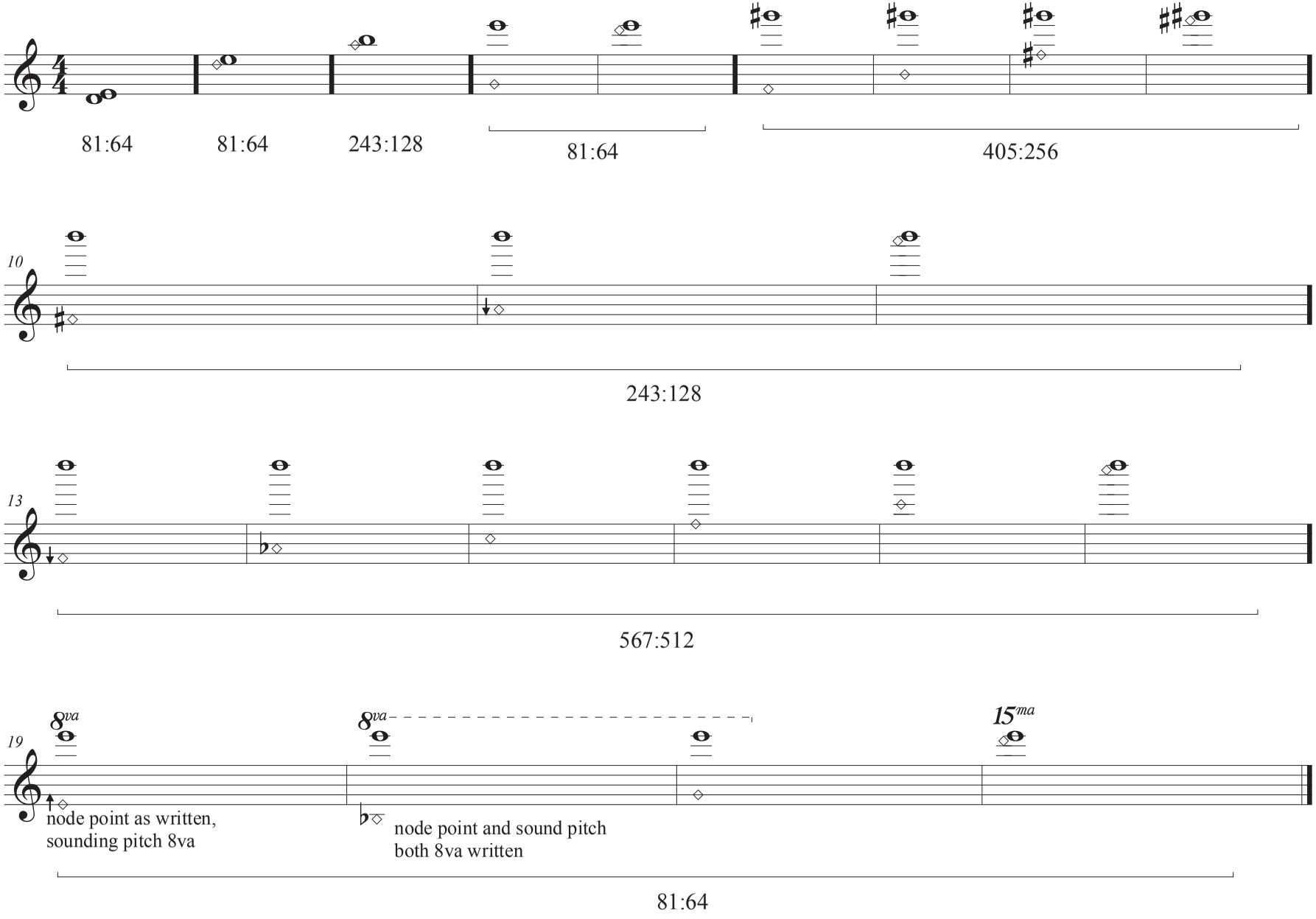
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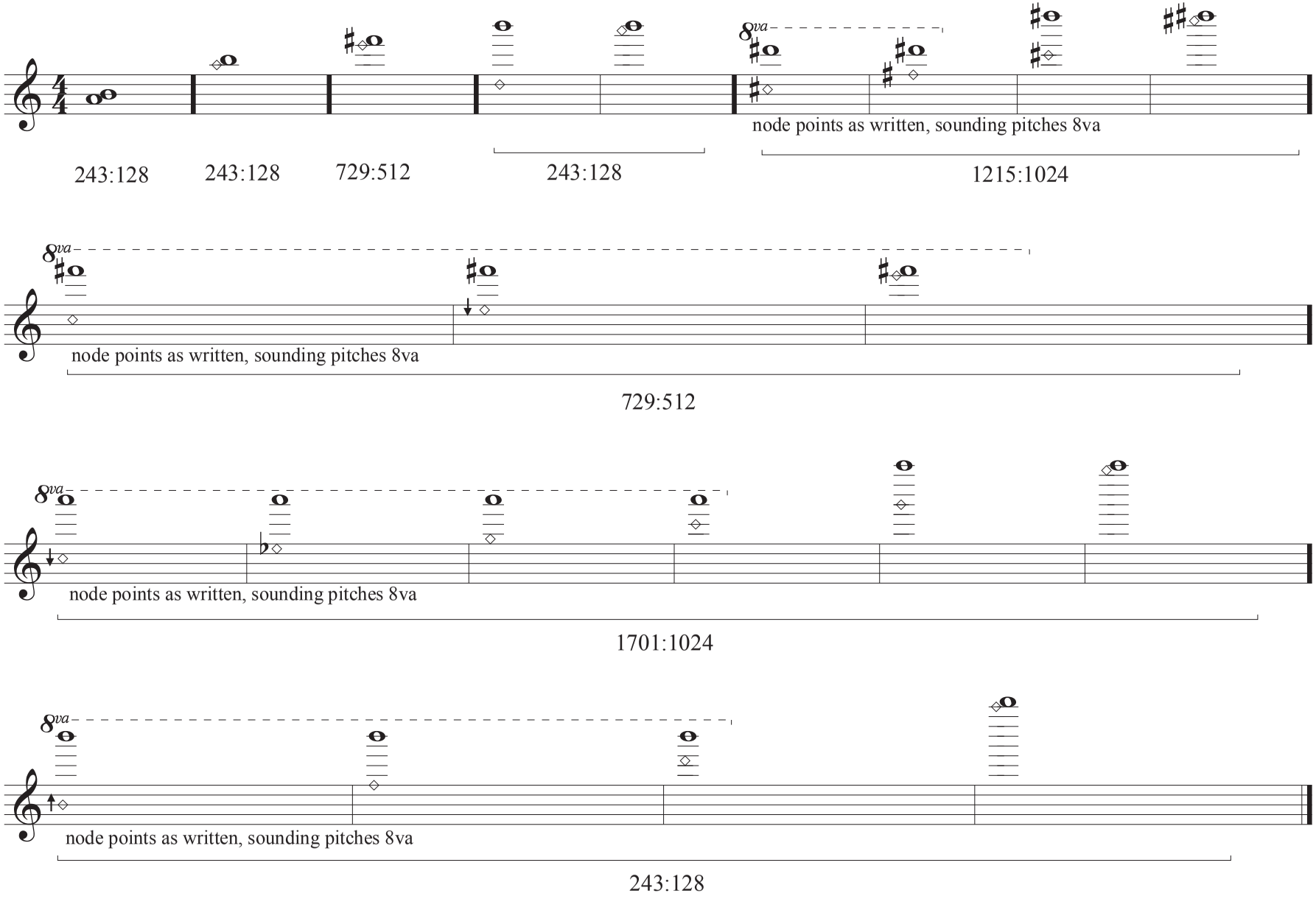
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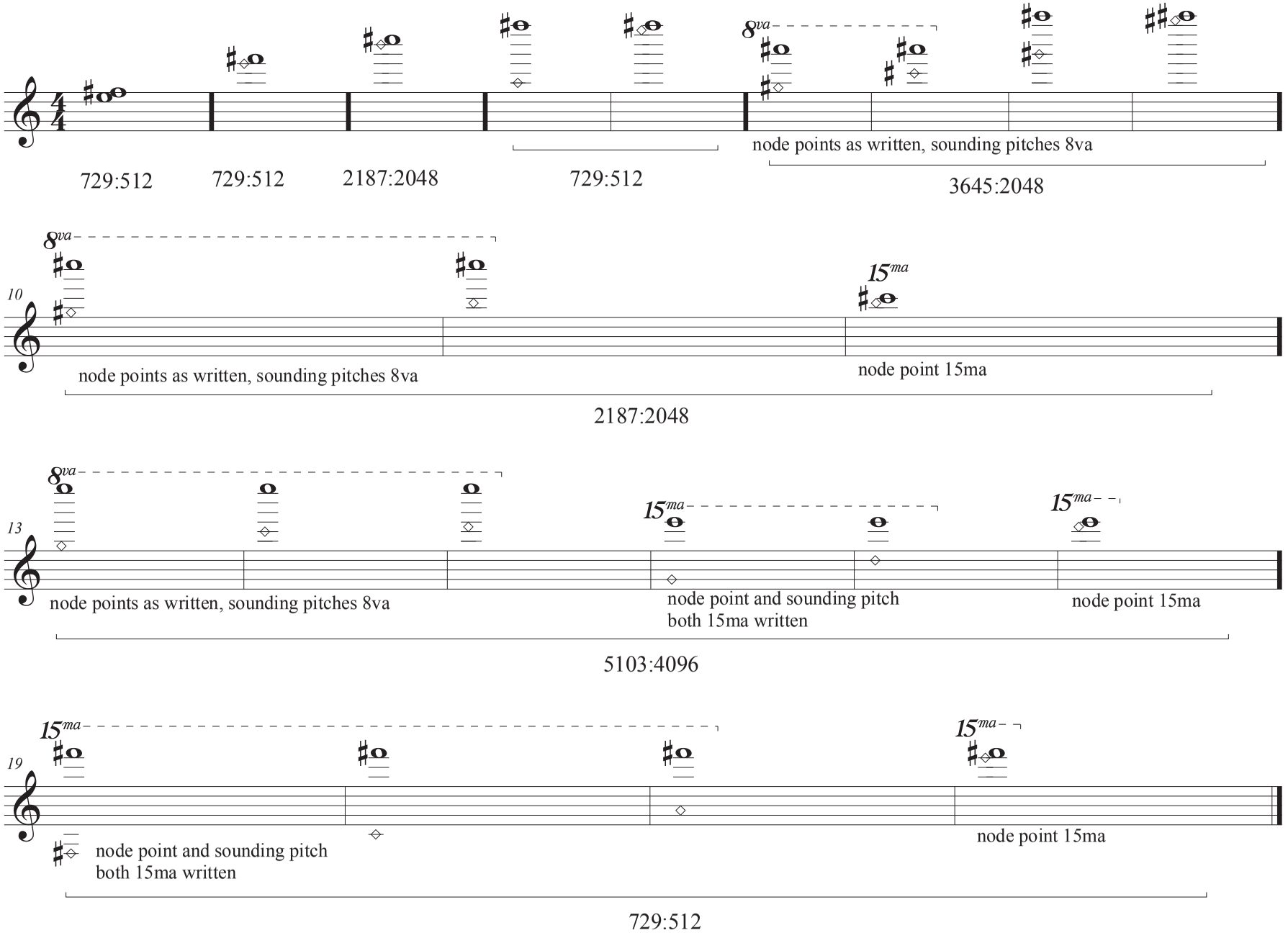
A

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



E

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)
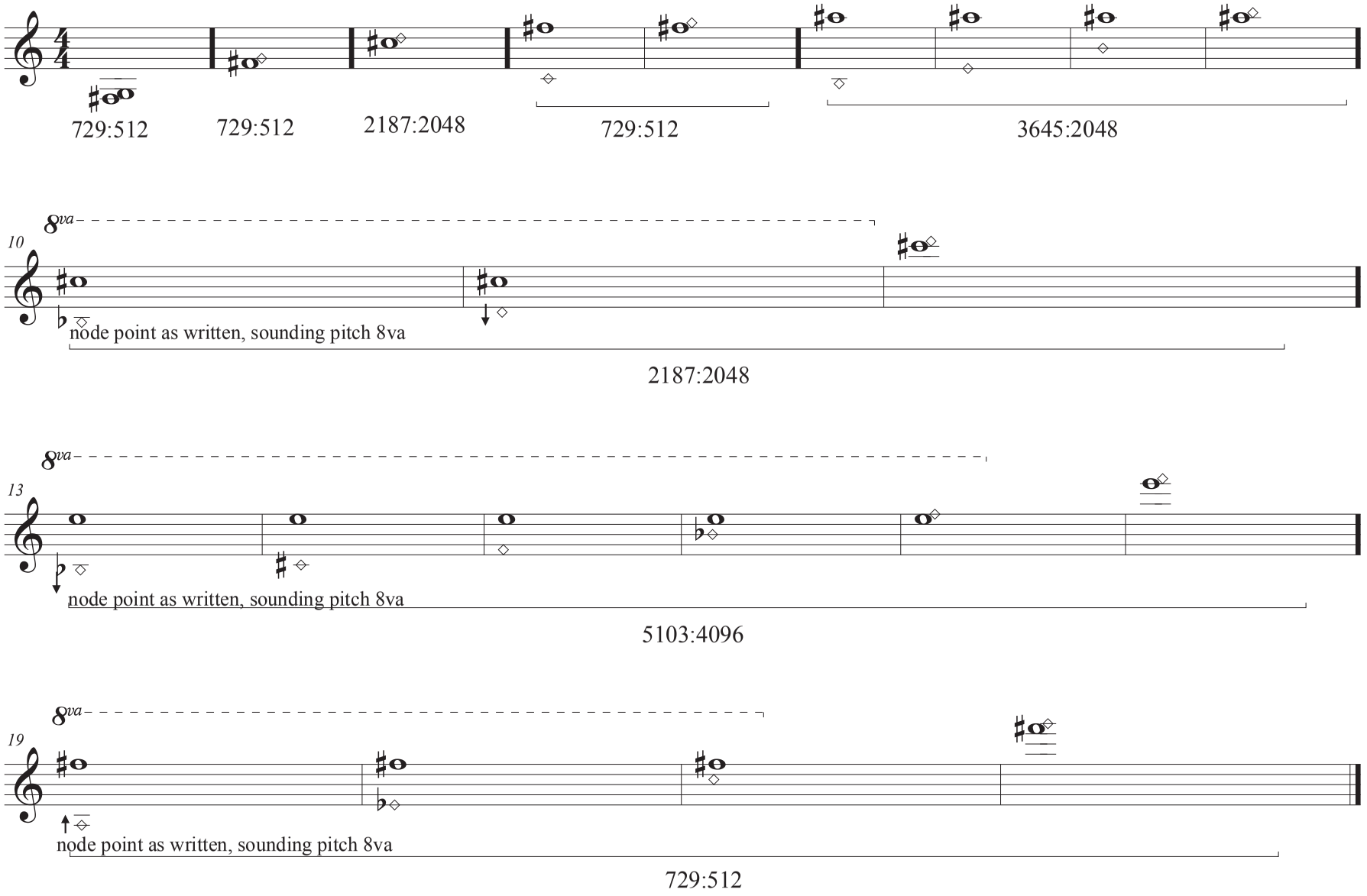


Appendix C, part three

(Transposing chart for violins 5 & 6)

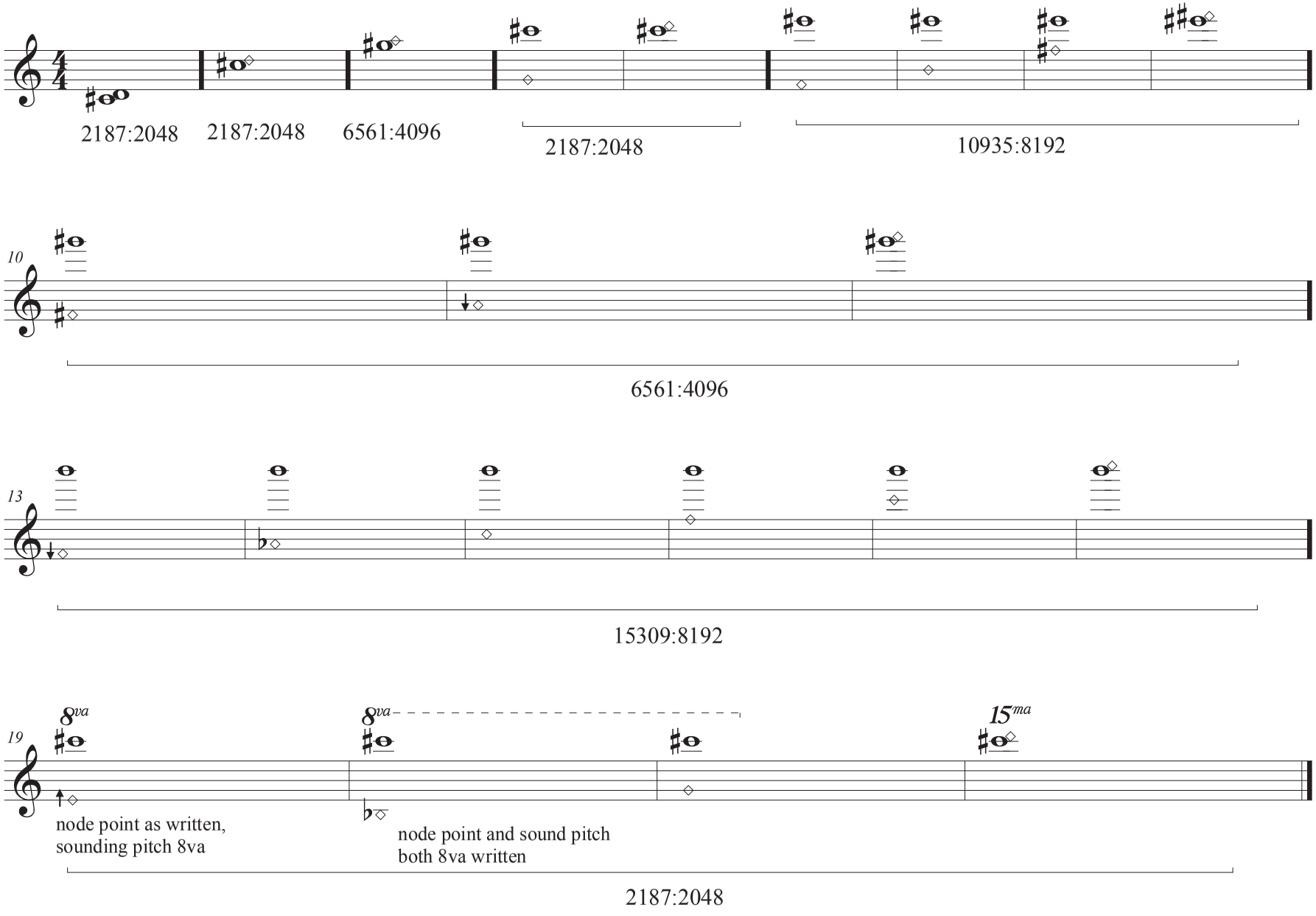
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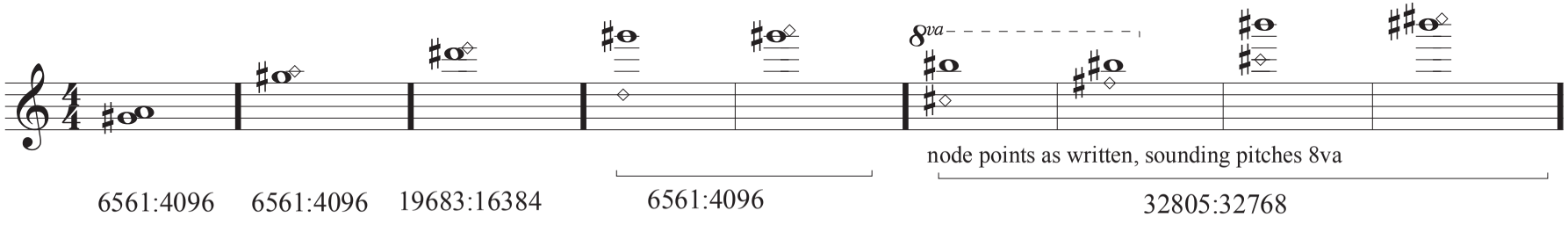
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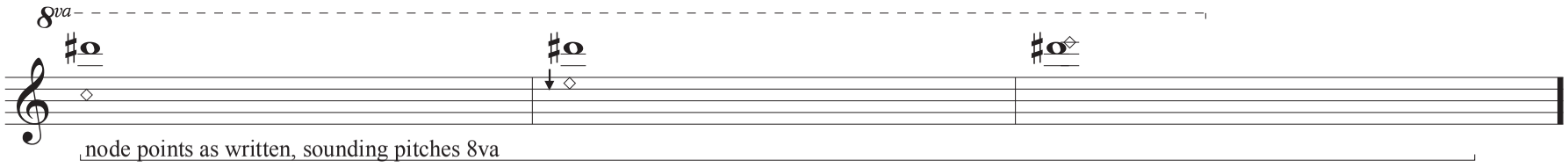
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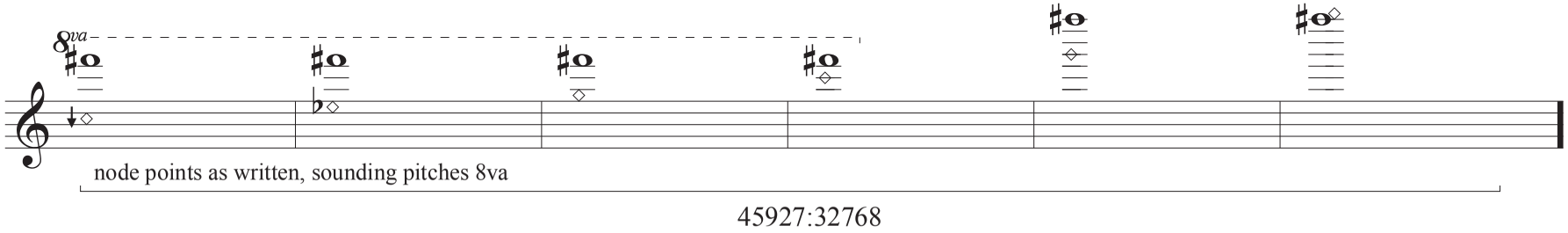


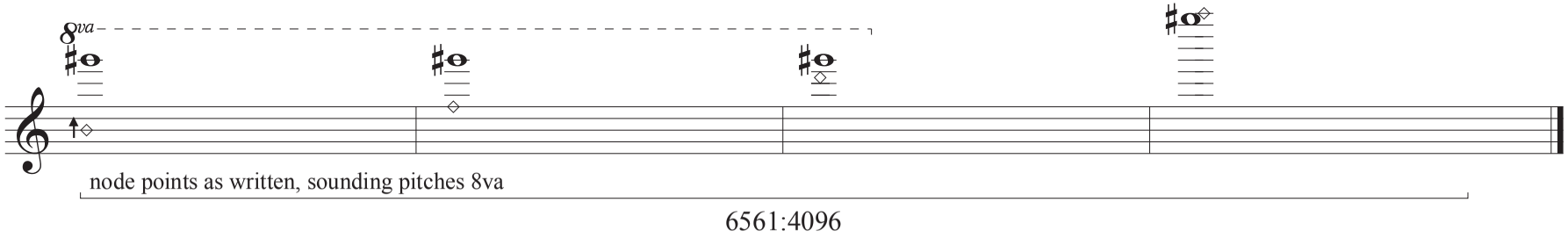
A

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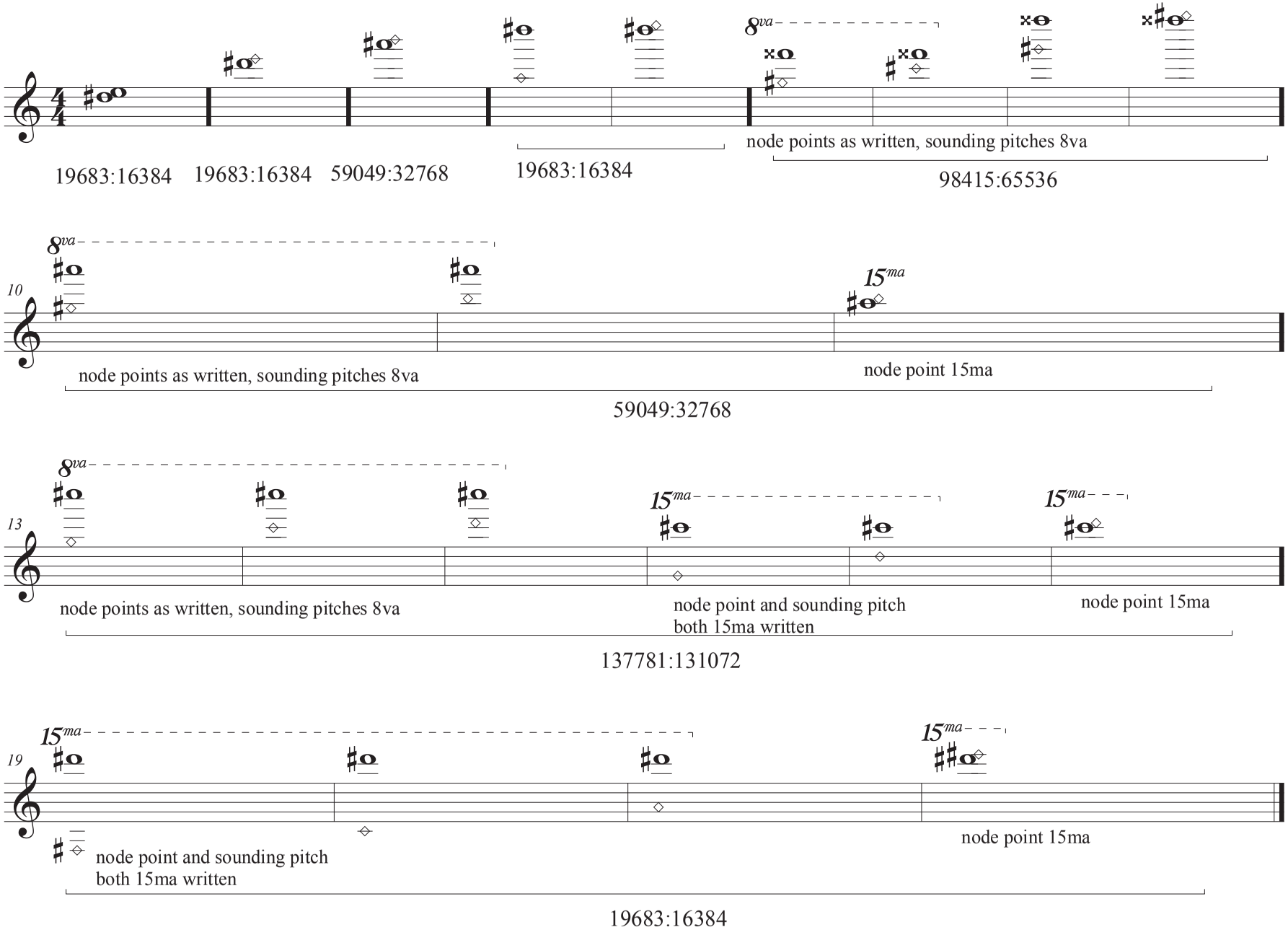






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The Violin: Harmonics Classification and New Techniques)

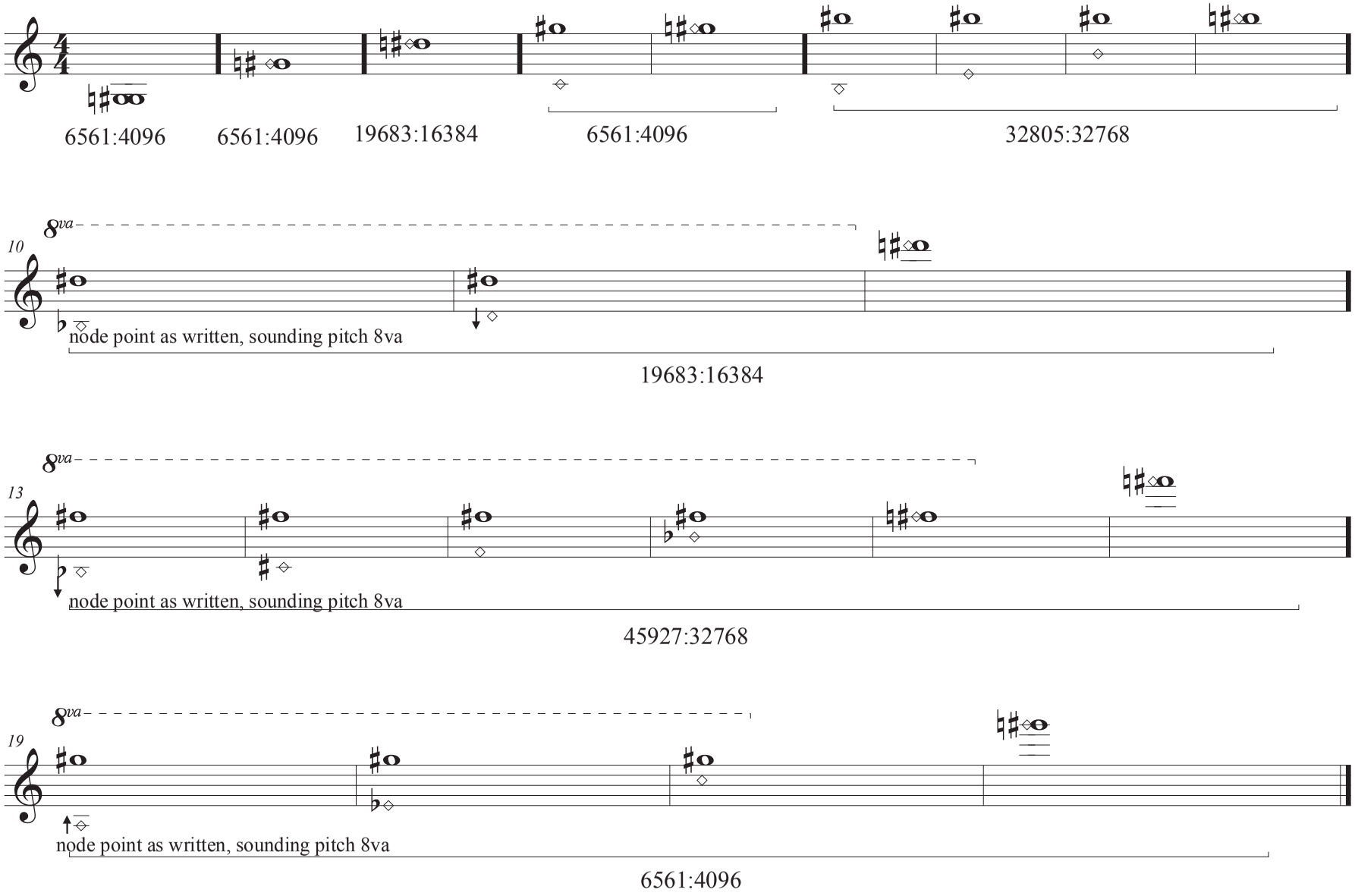


Appendix C, part four

(Transposing chart for violins 7 & 8)

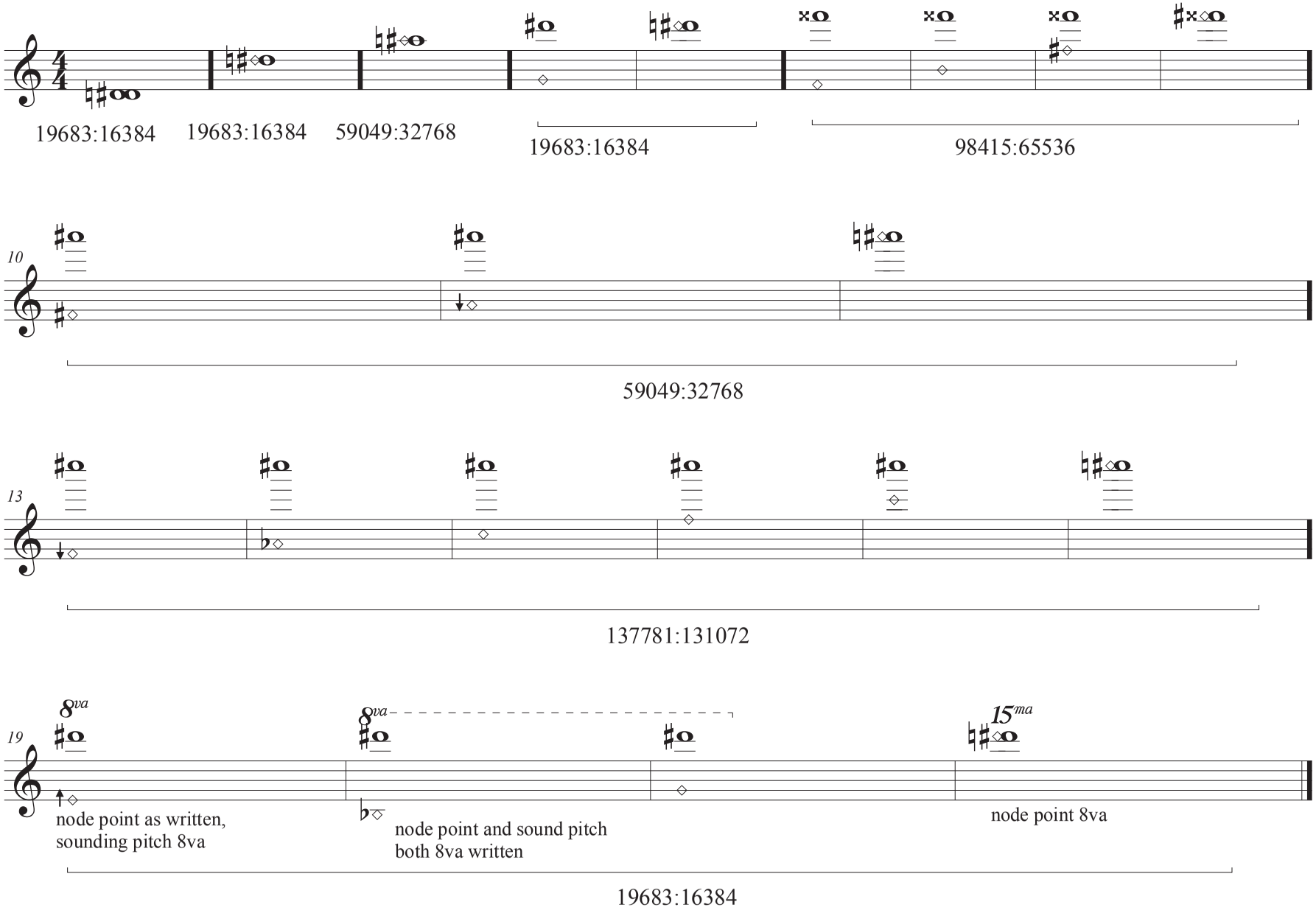
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The Violin: Harmonics Classification and New Techniques)



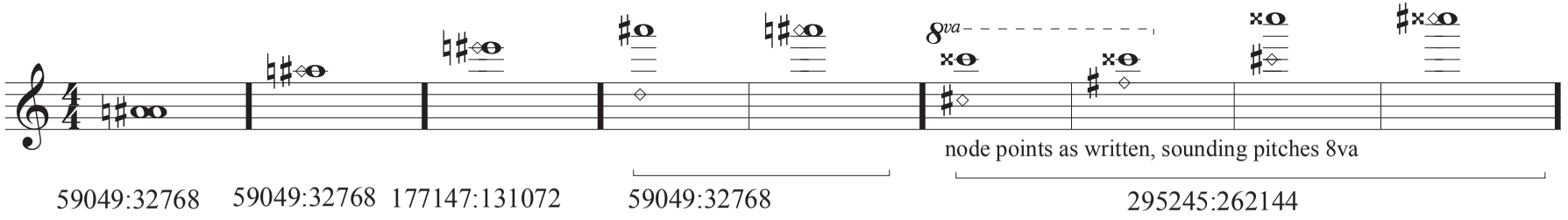
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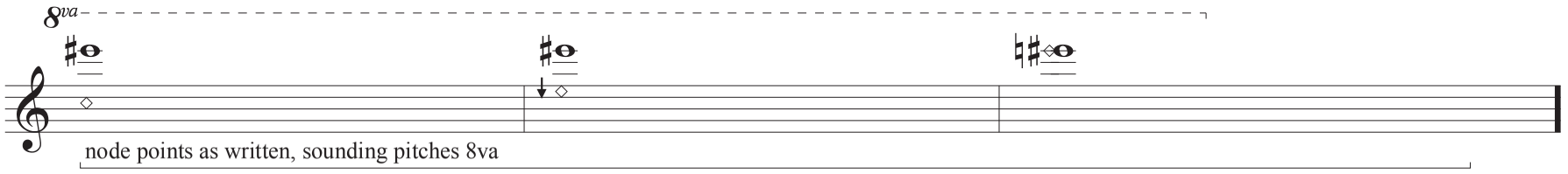
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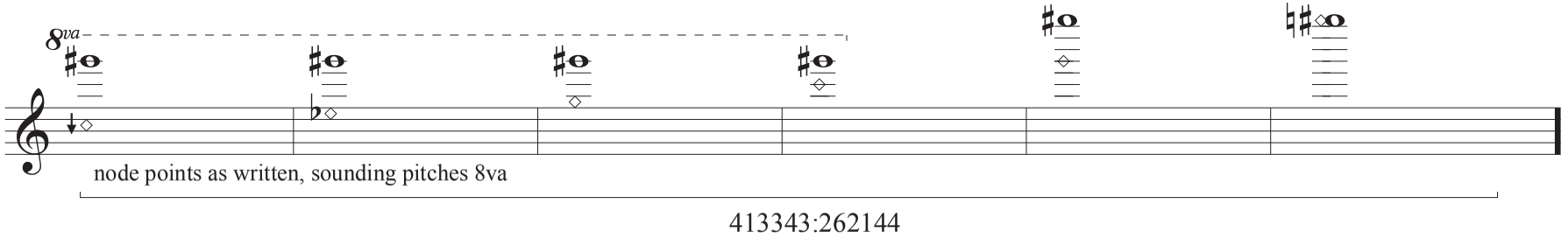


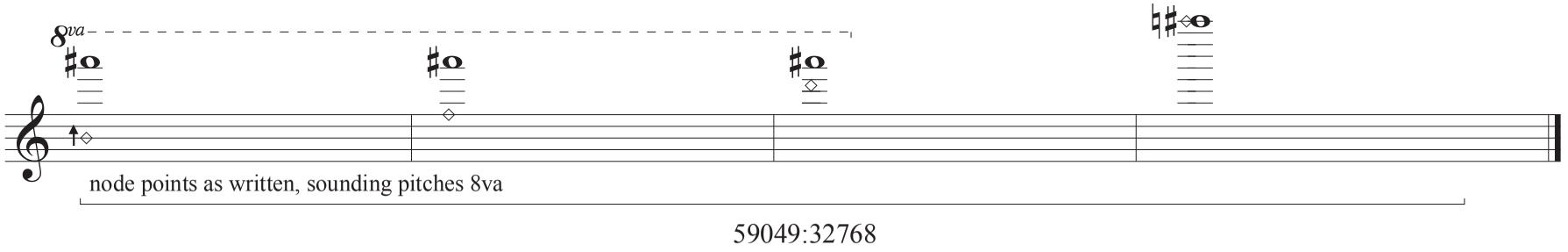
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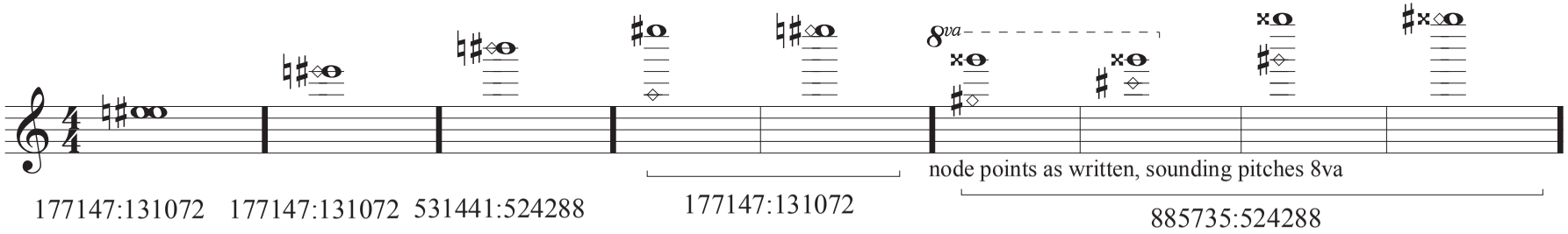


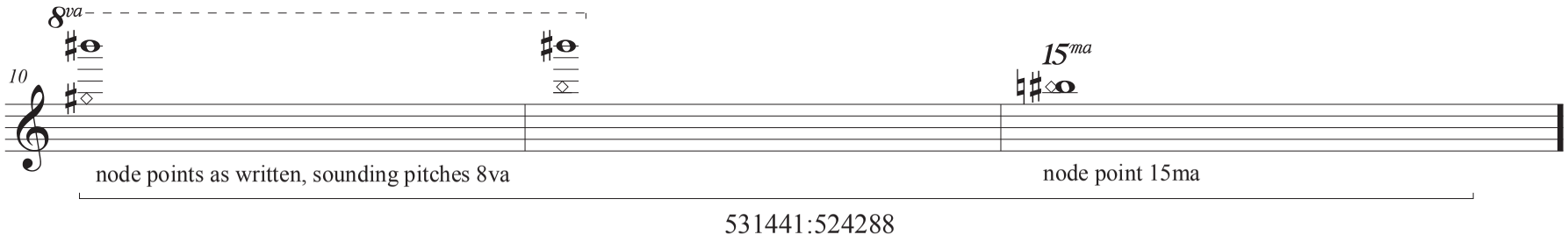


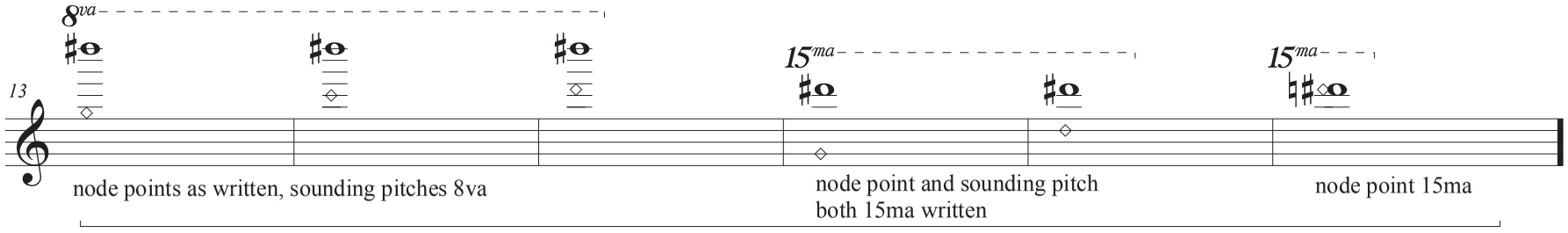


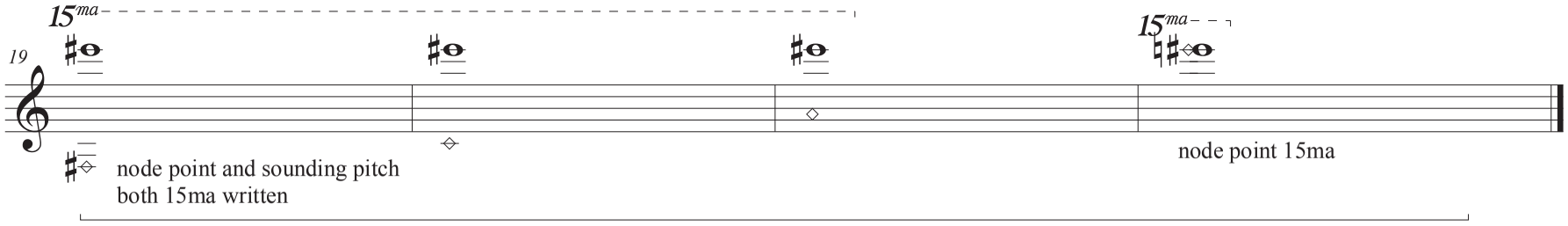
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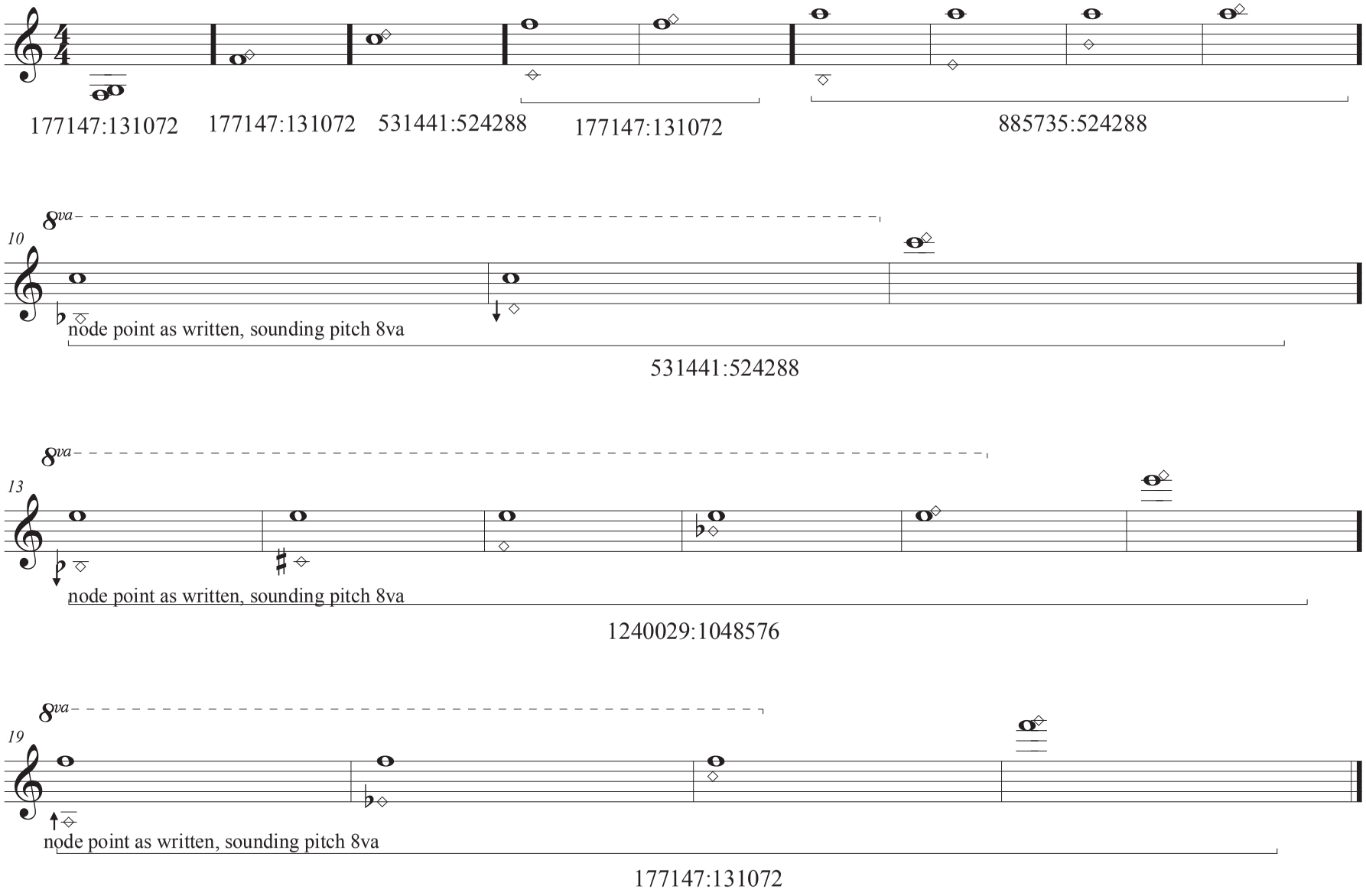


Appendix C, part five

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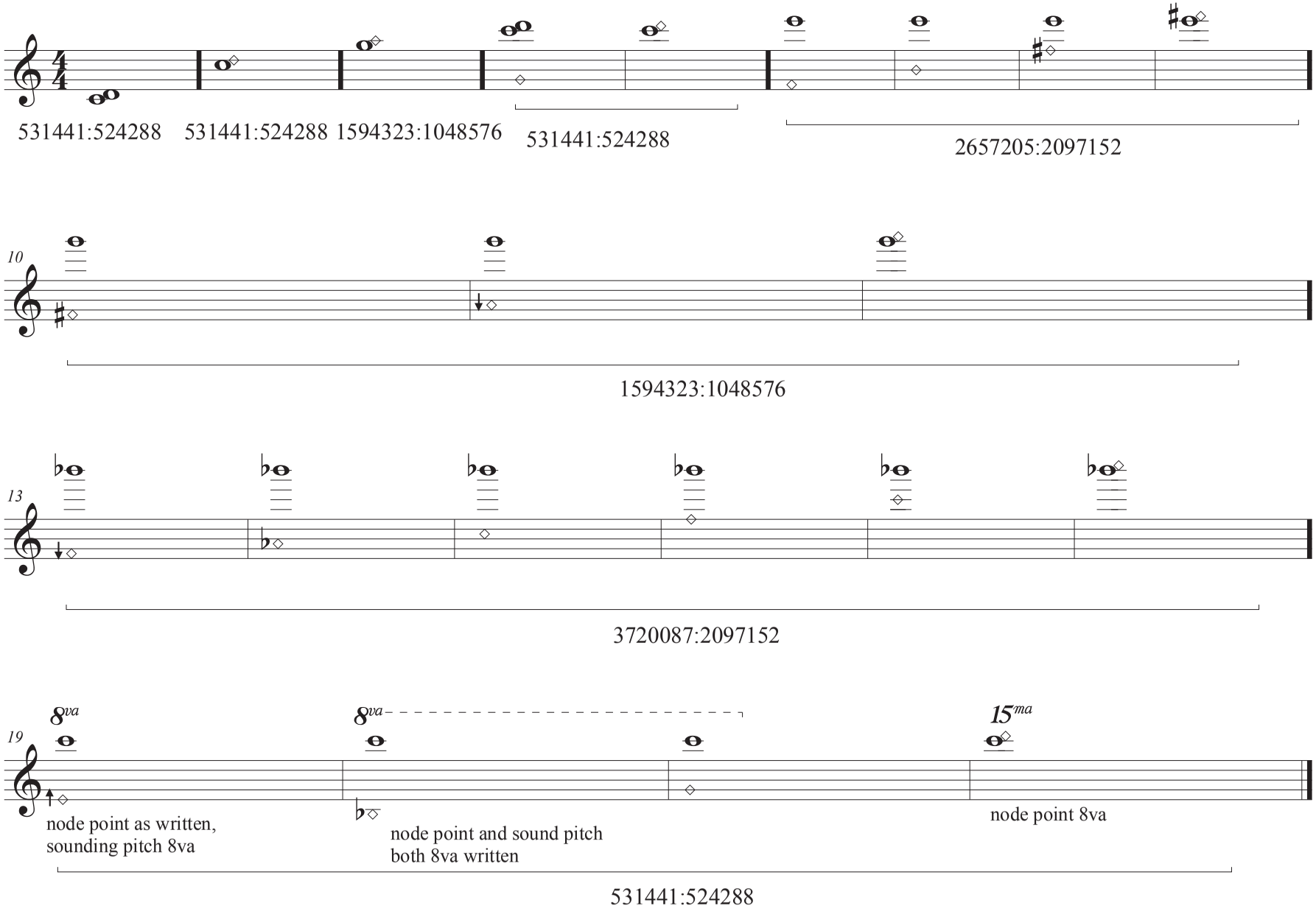
G

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



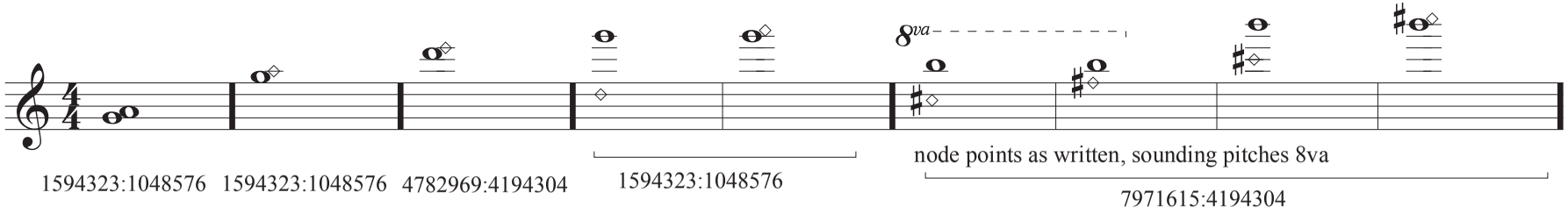
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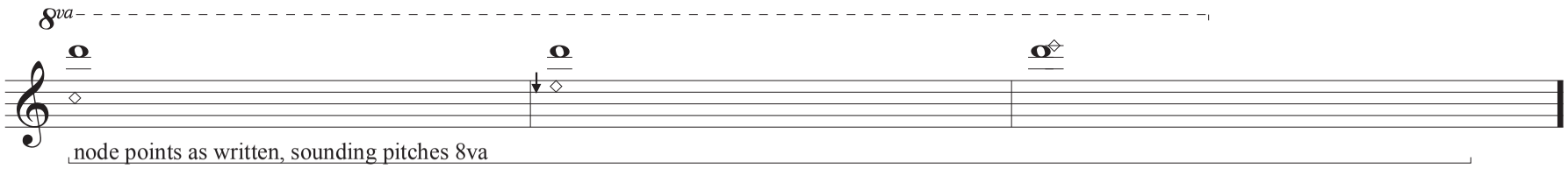
Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



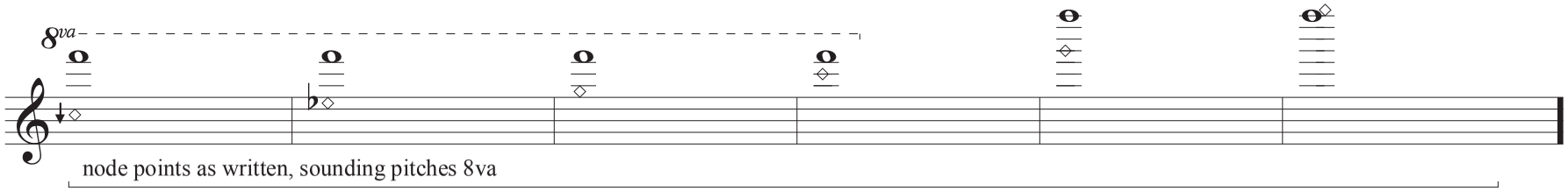
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Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)

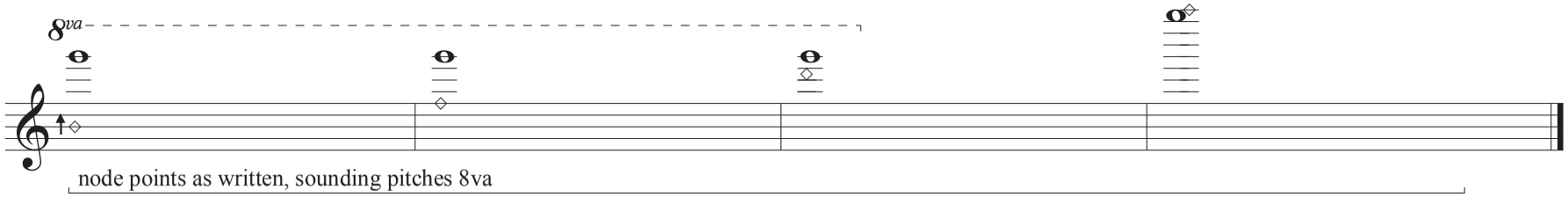




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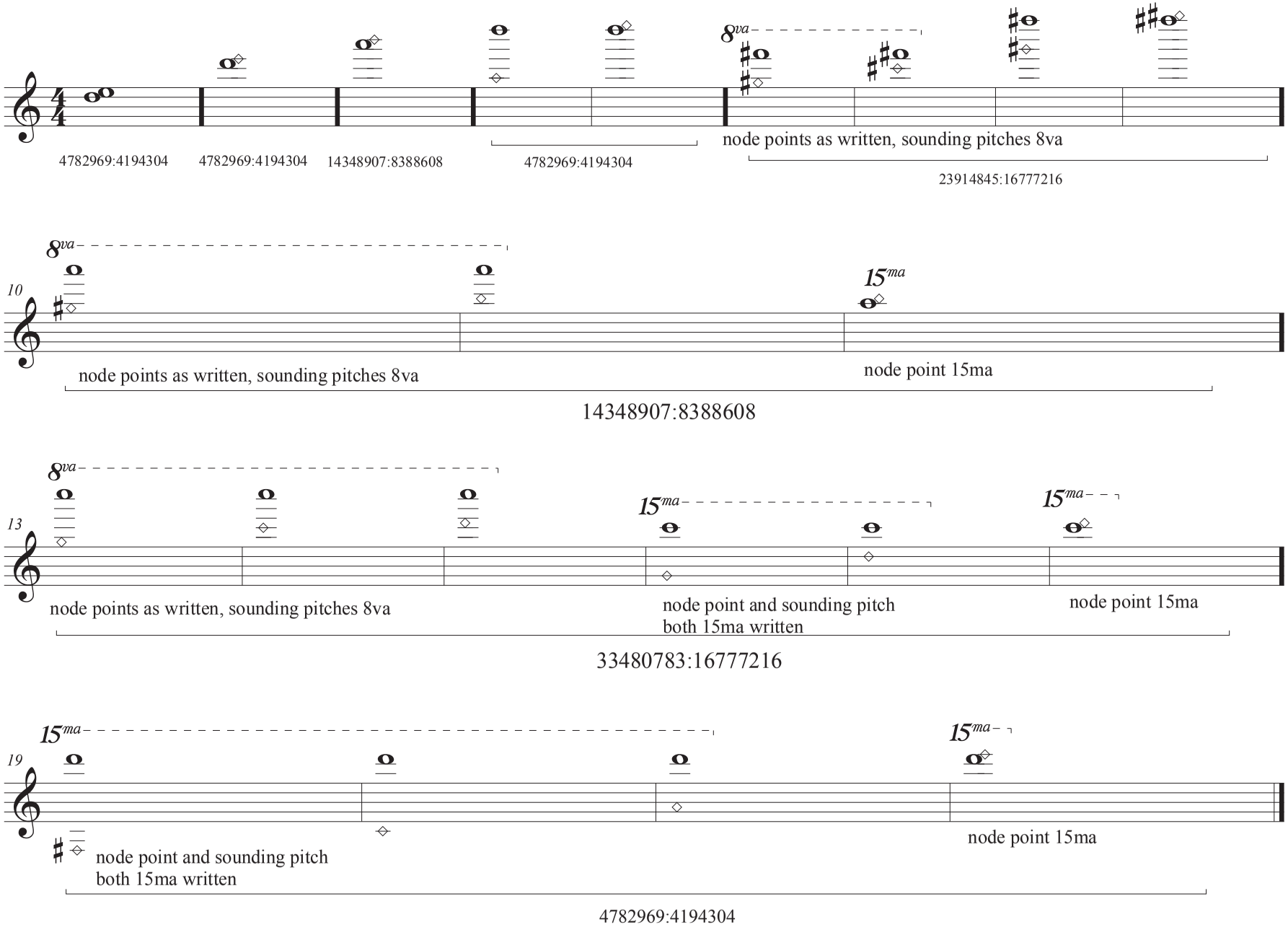
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Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

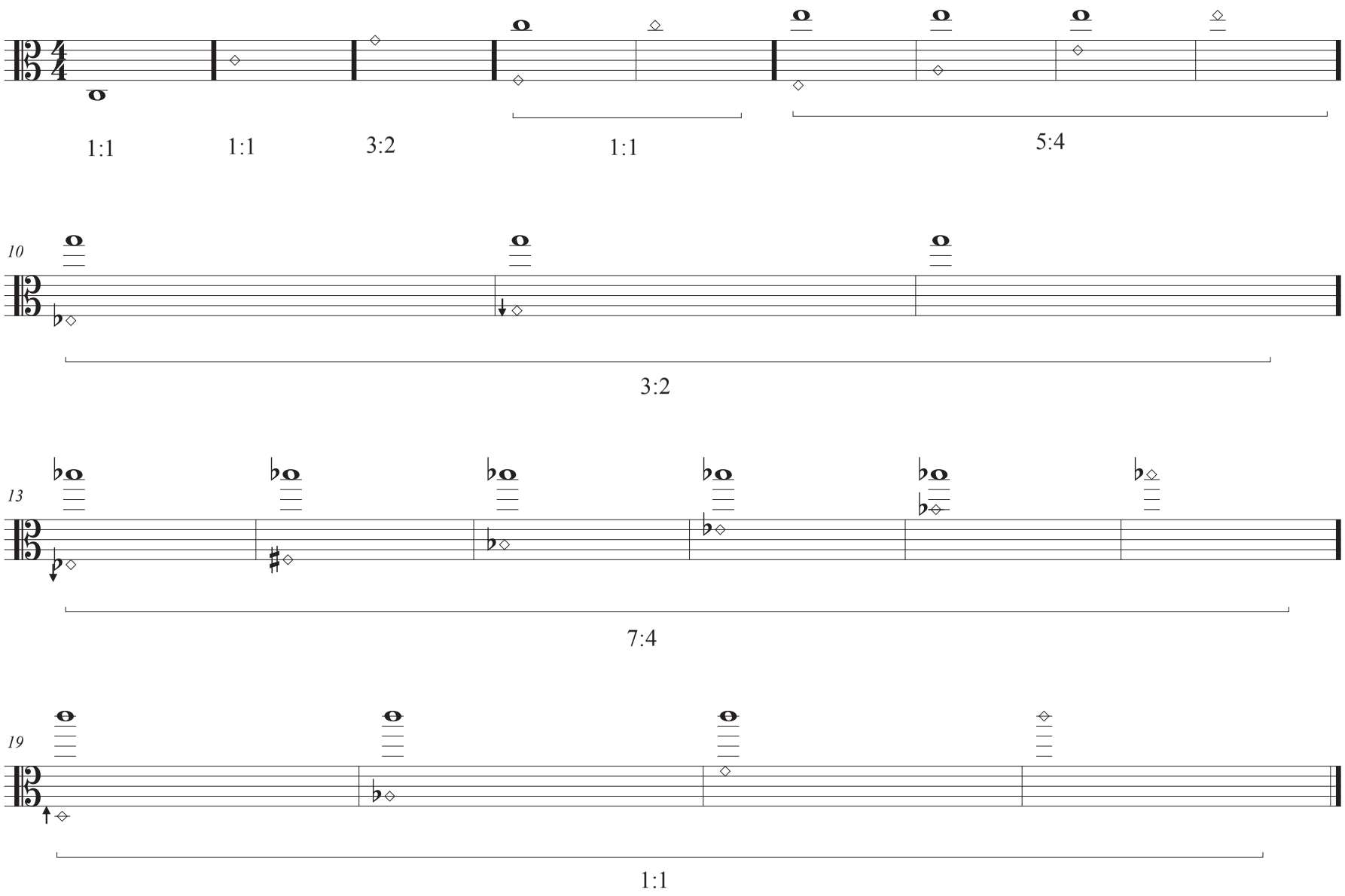


Appendix C, part six

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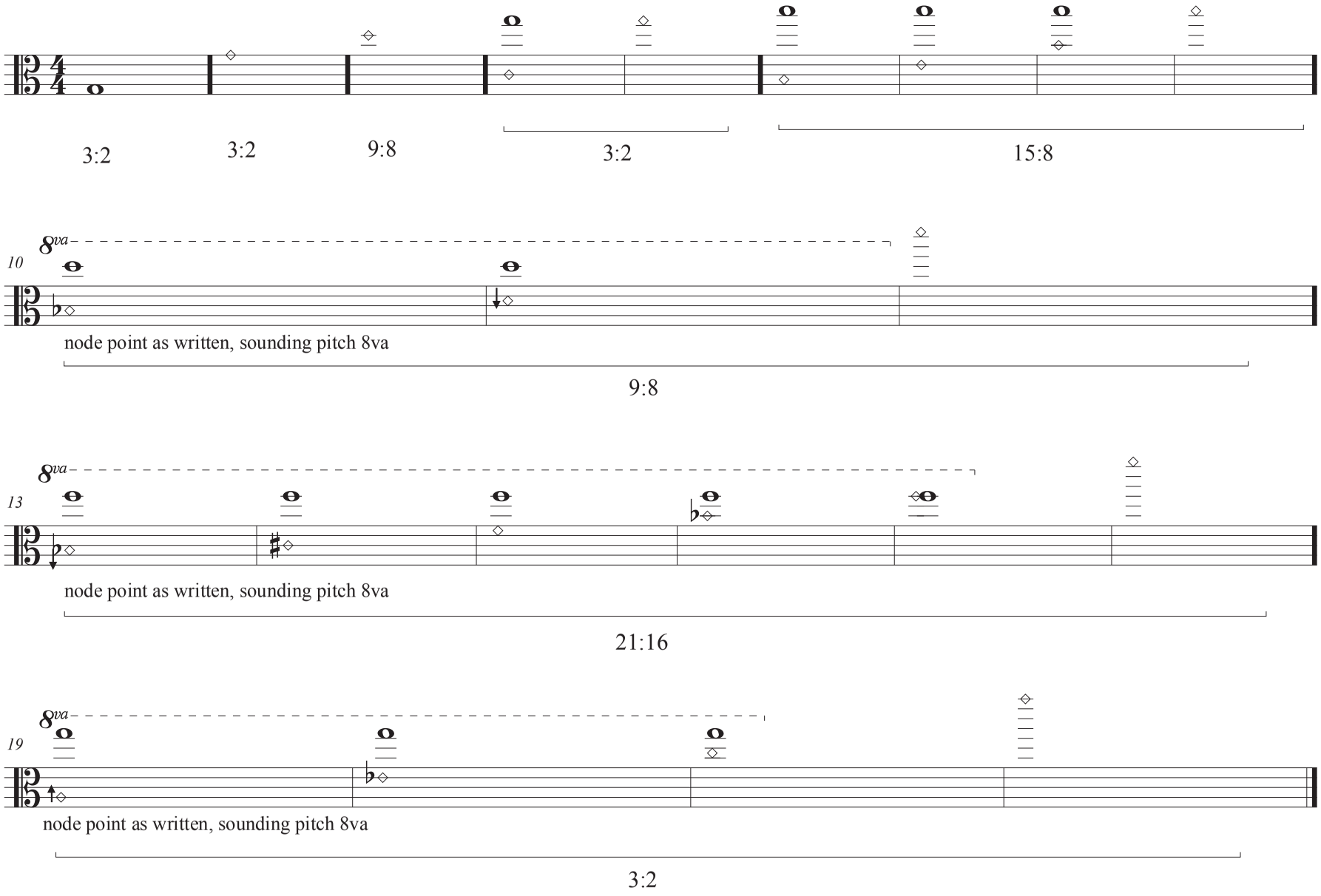
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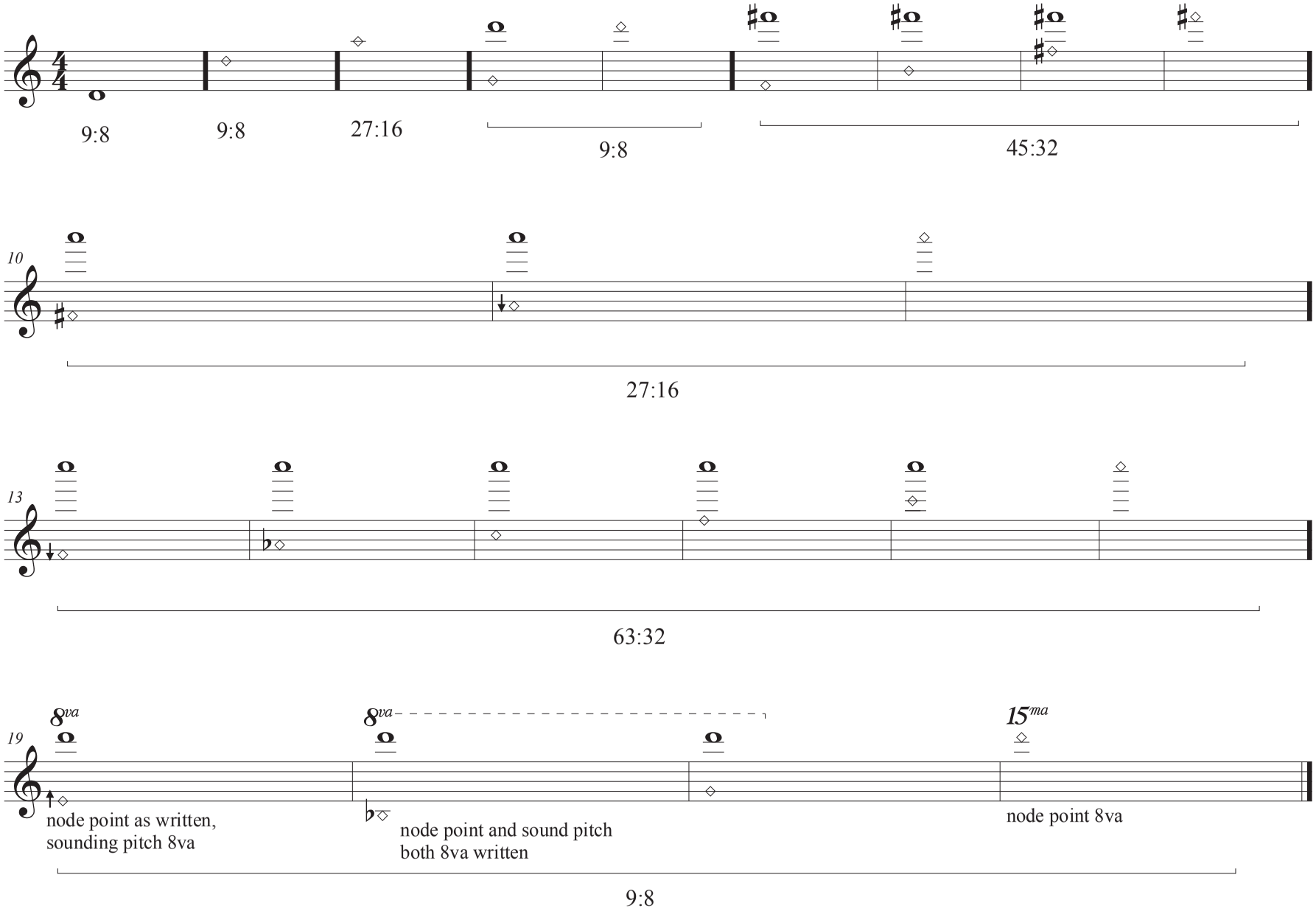
G

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



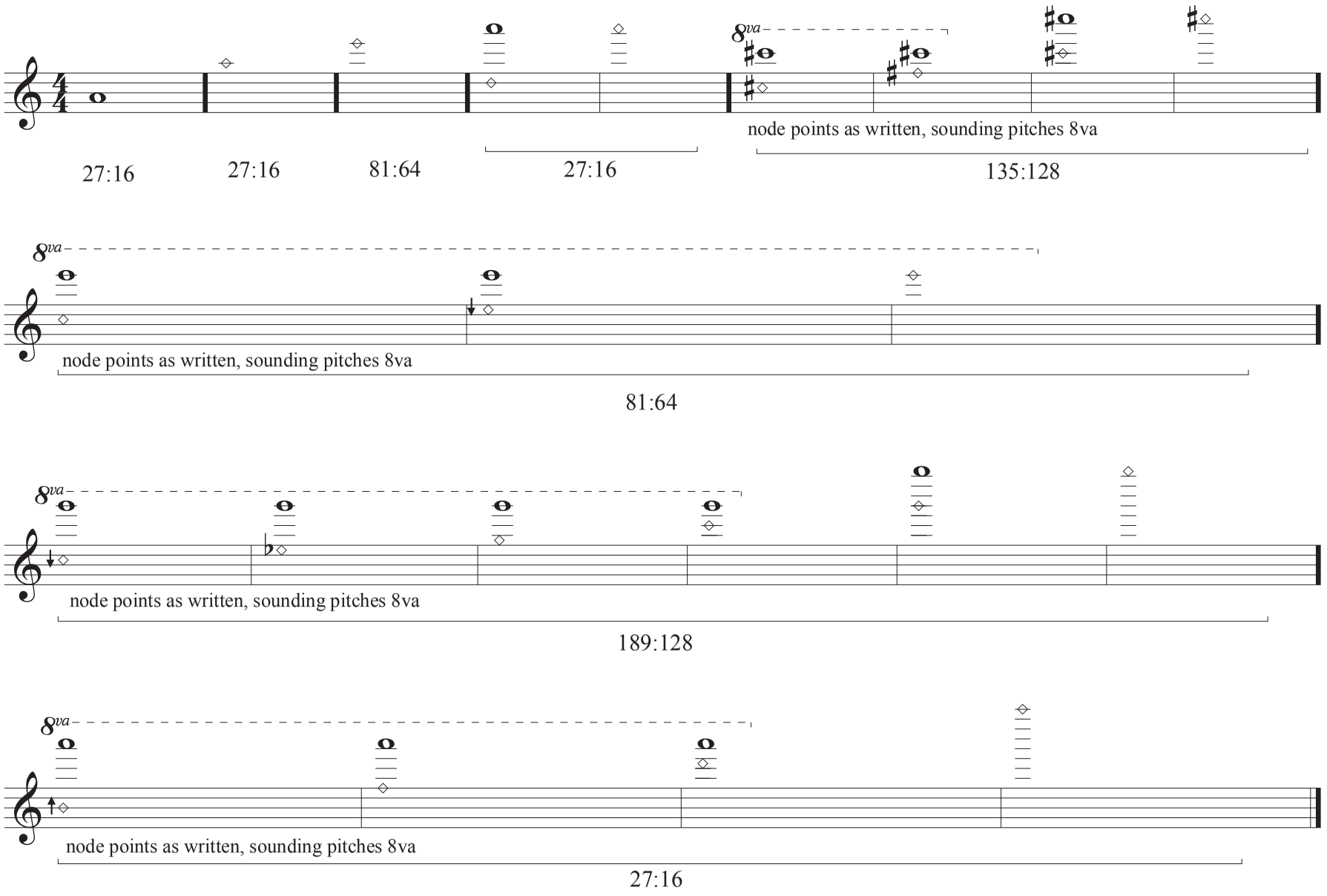
D

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



A

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)

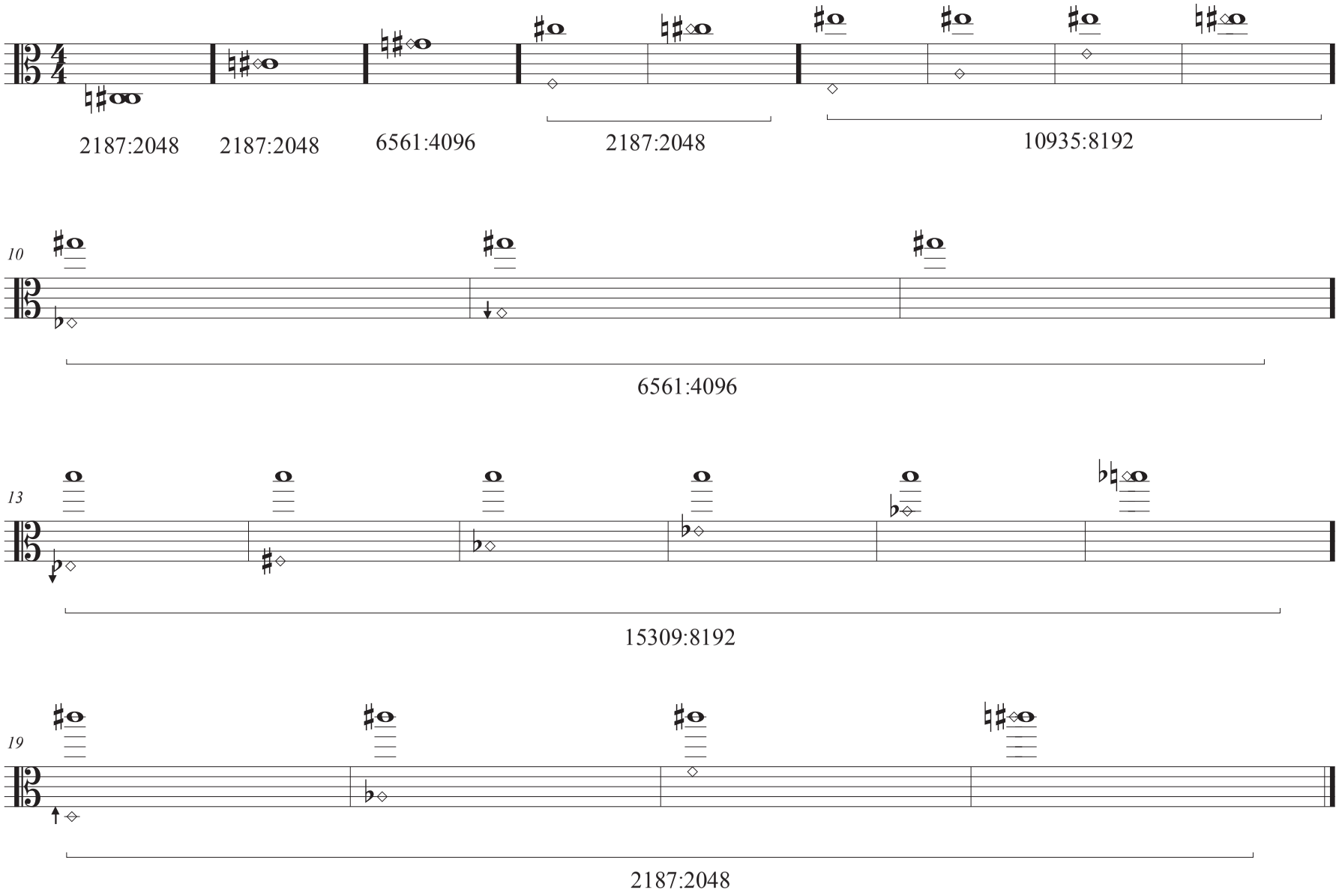


Appendix C, part seven

(Transposing chart for violas 3 & 4)

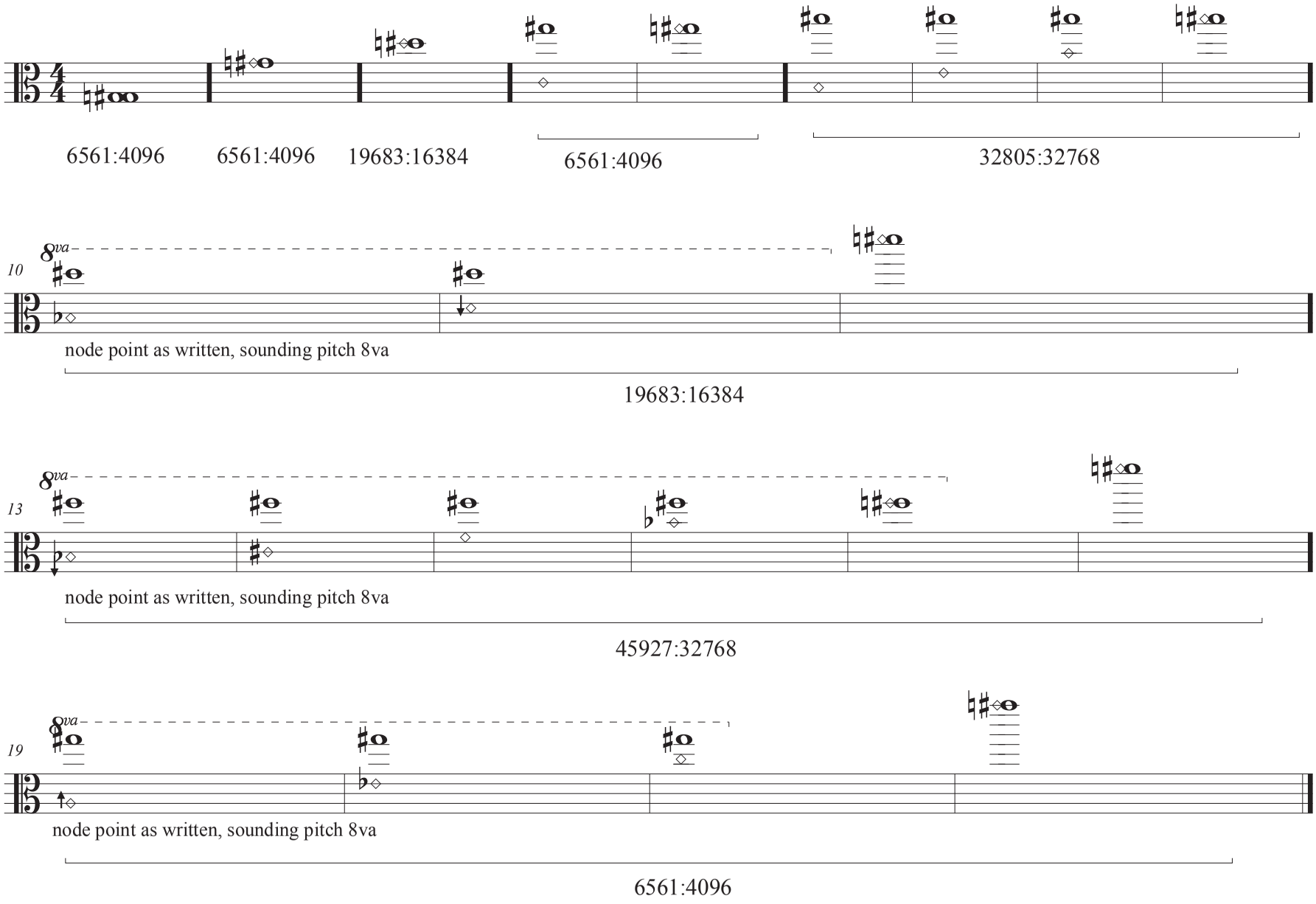
C

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



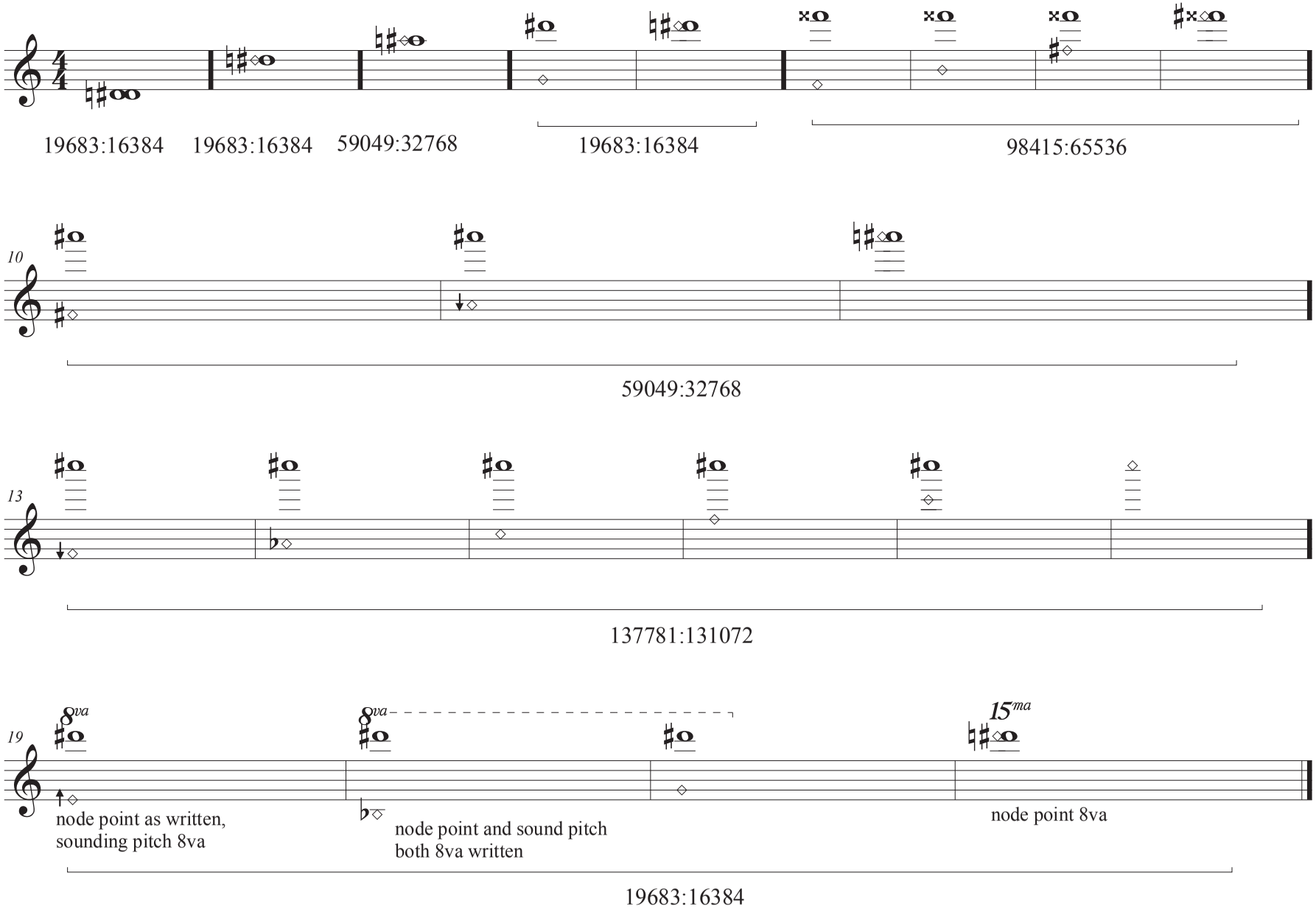
G

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



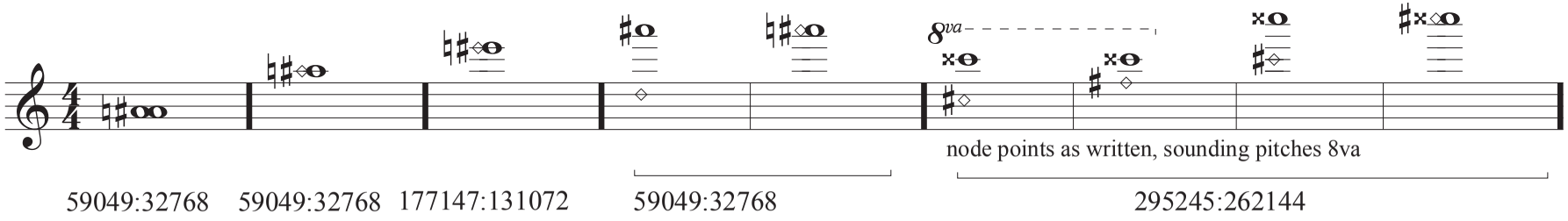
D

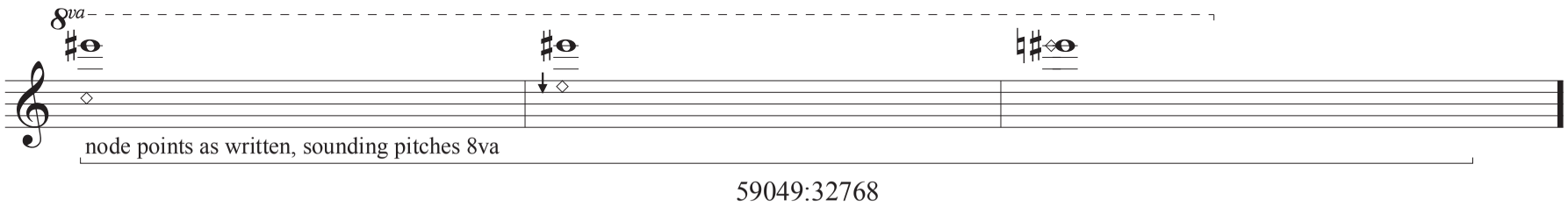
Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)

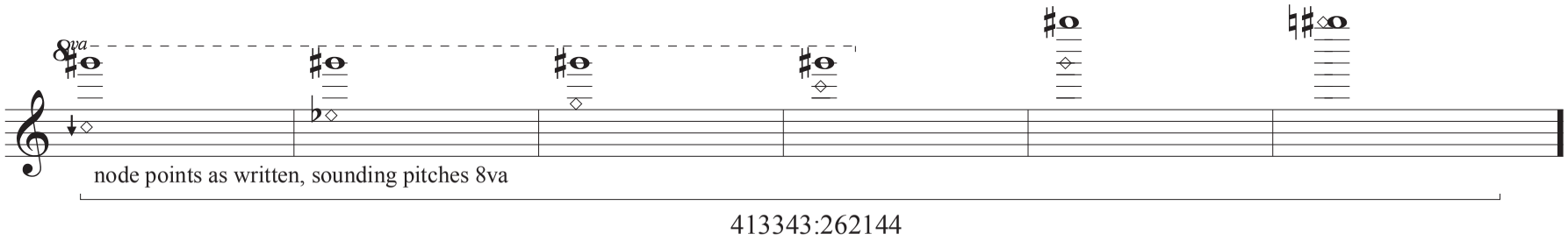


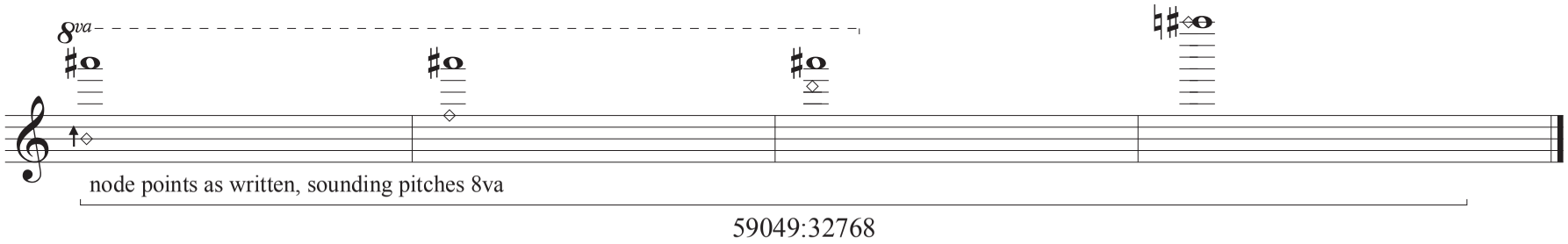
A

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)







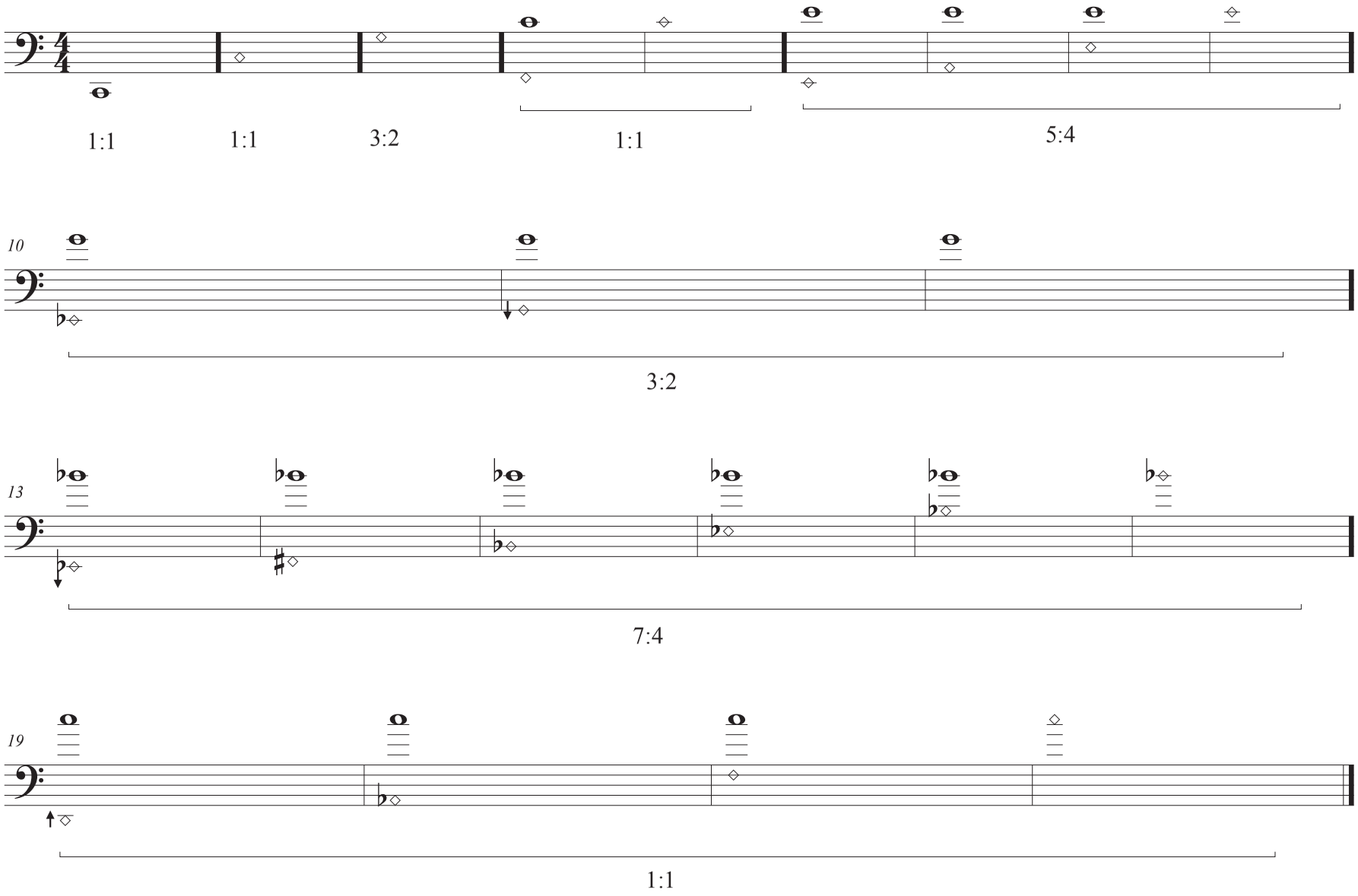


Appendix C, part eight

(Transposing chart for cellos 1 & 2)
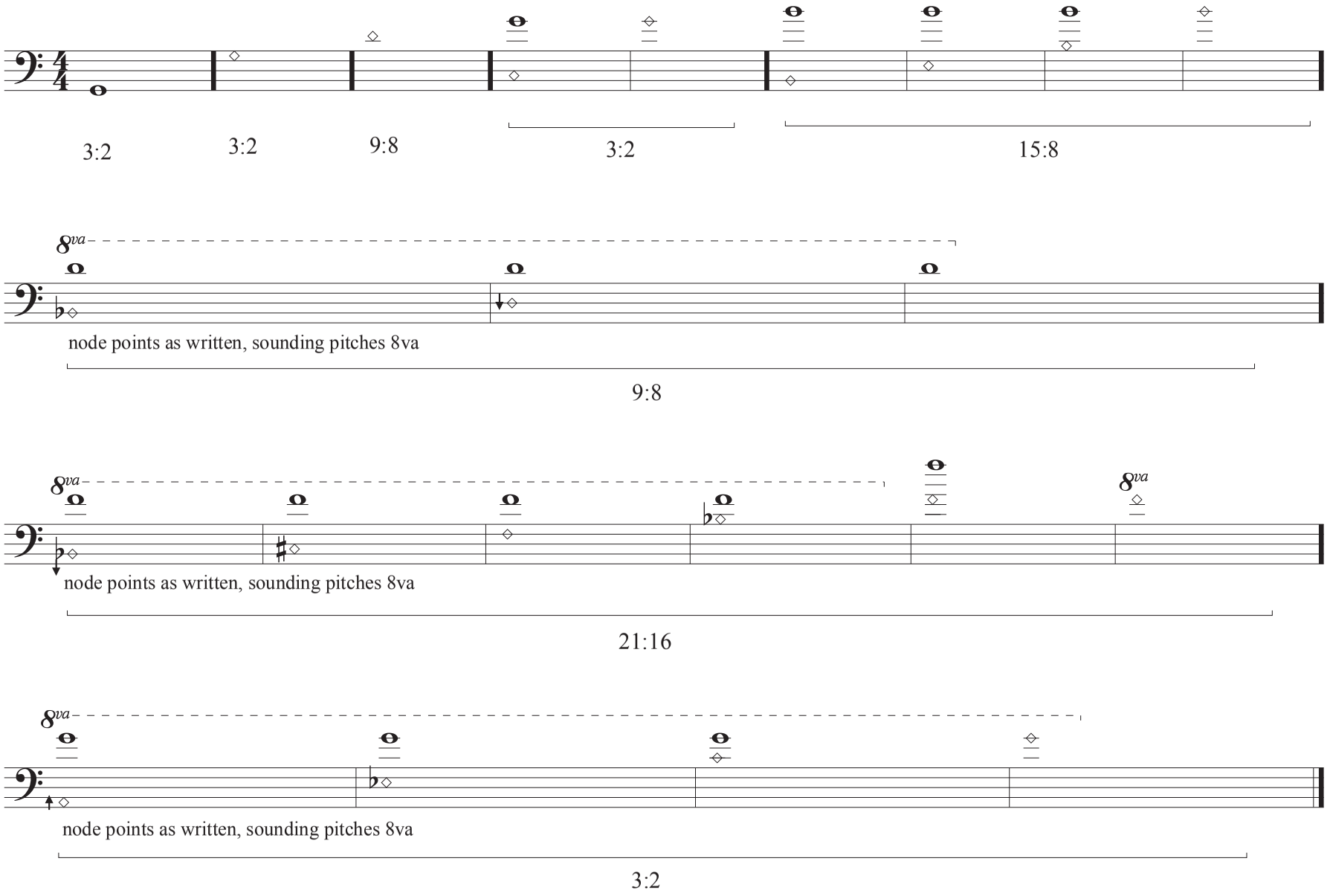
C

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



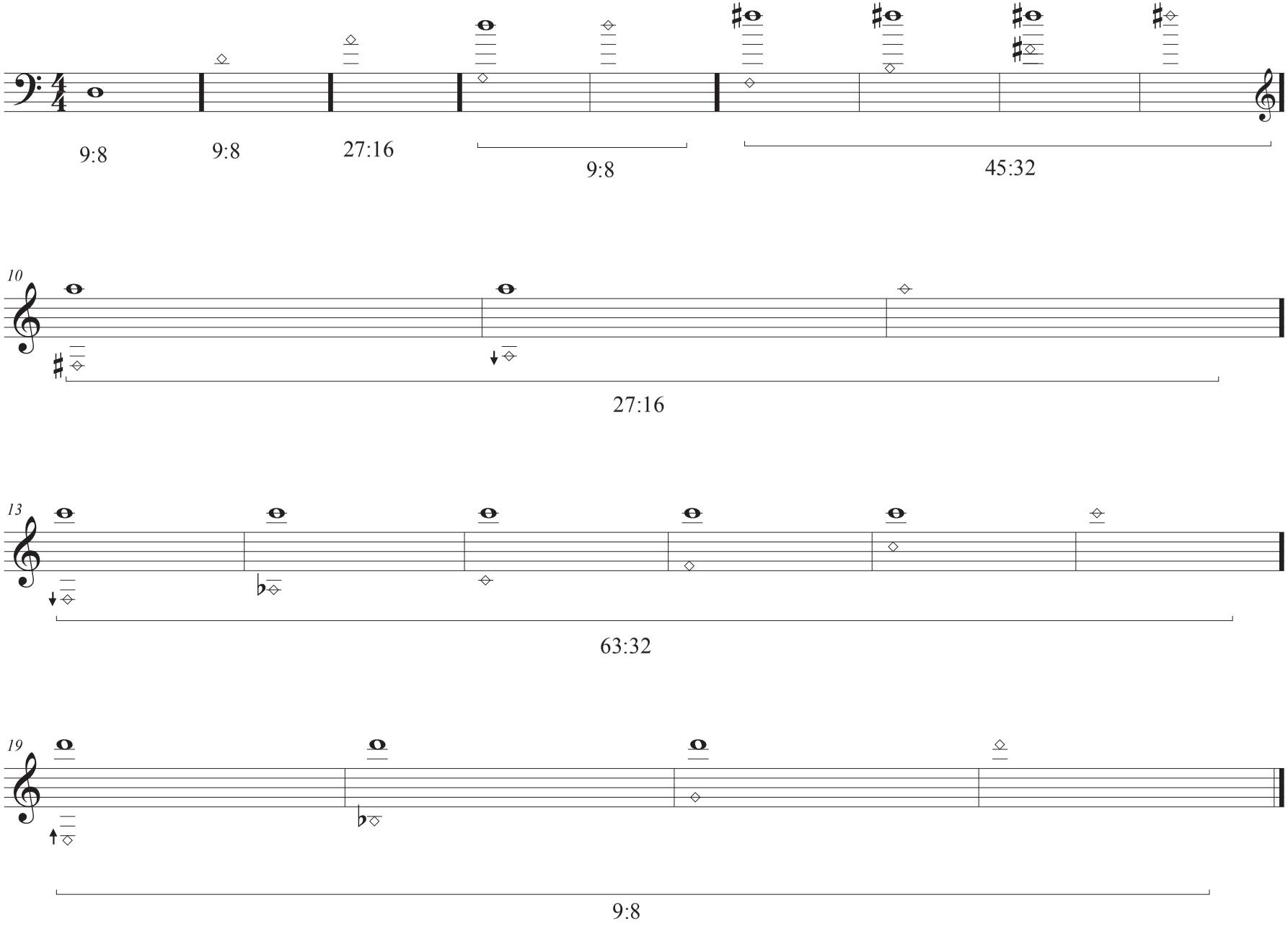
G

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



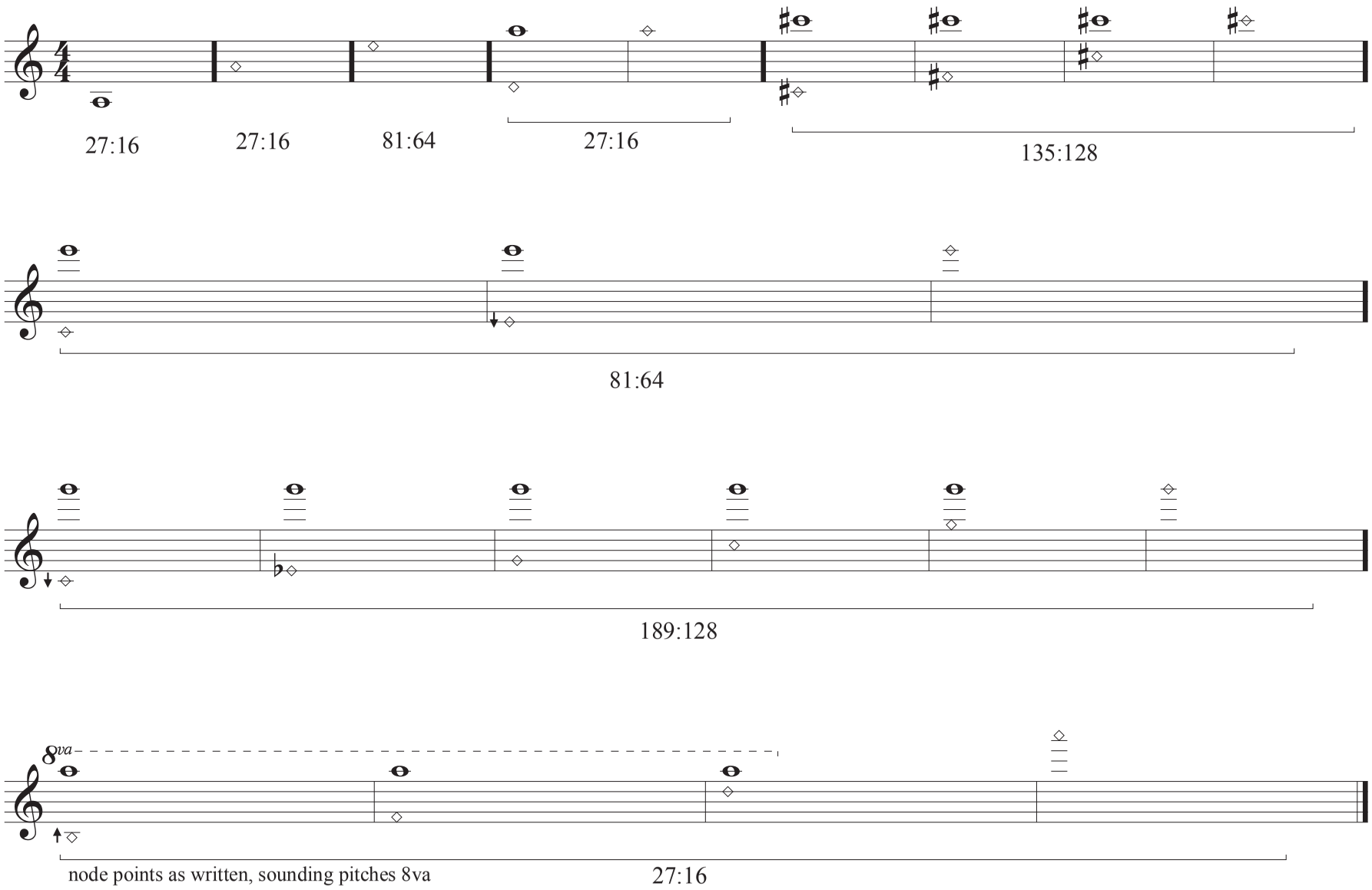
D

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



A

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)
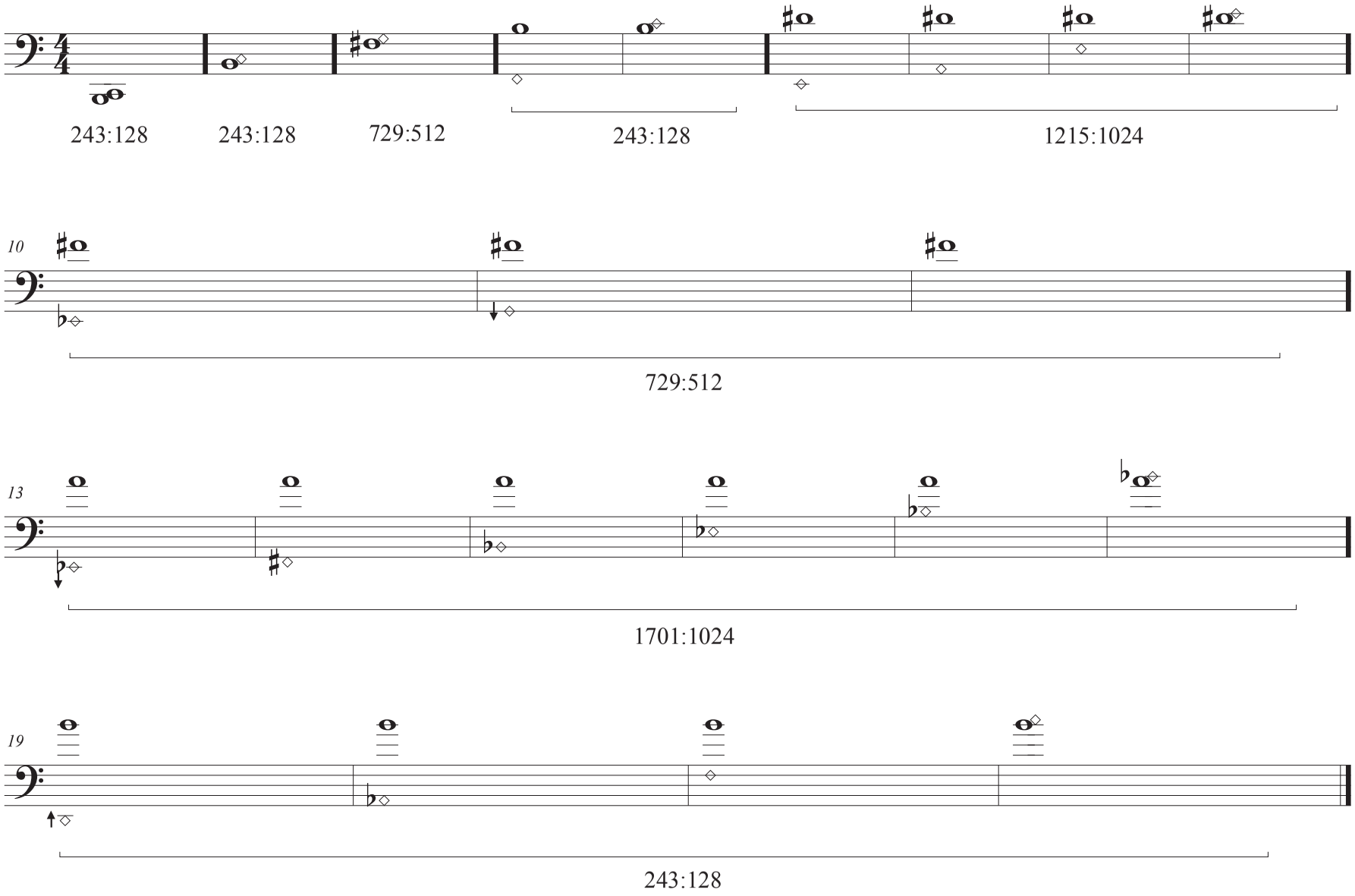


Appendix C, part nine

(Transposing chart for cellos 3 & 4)

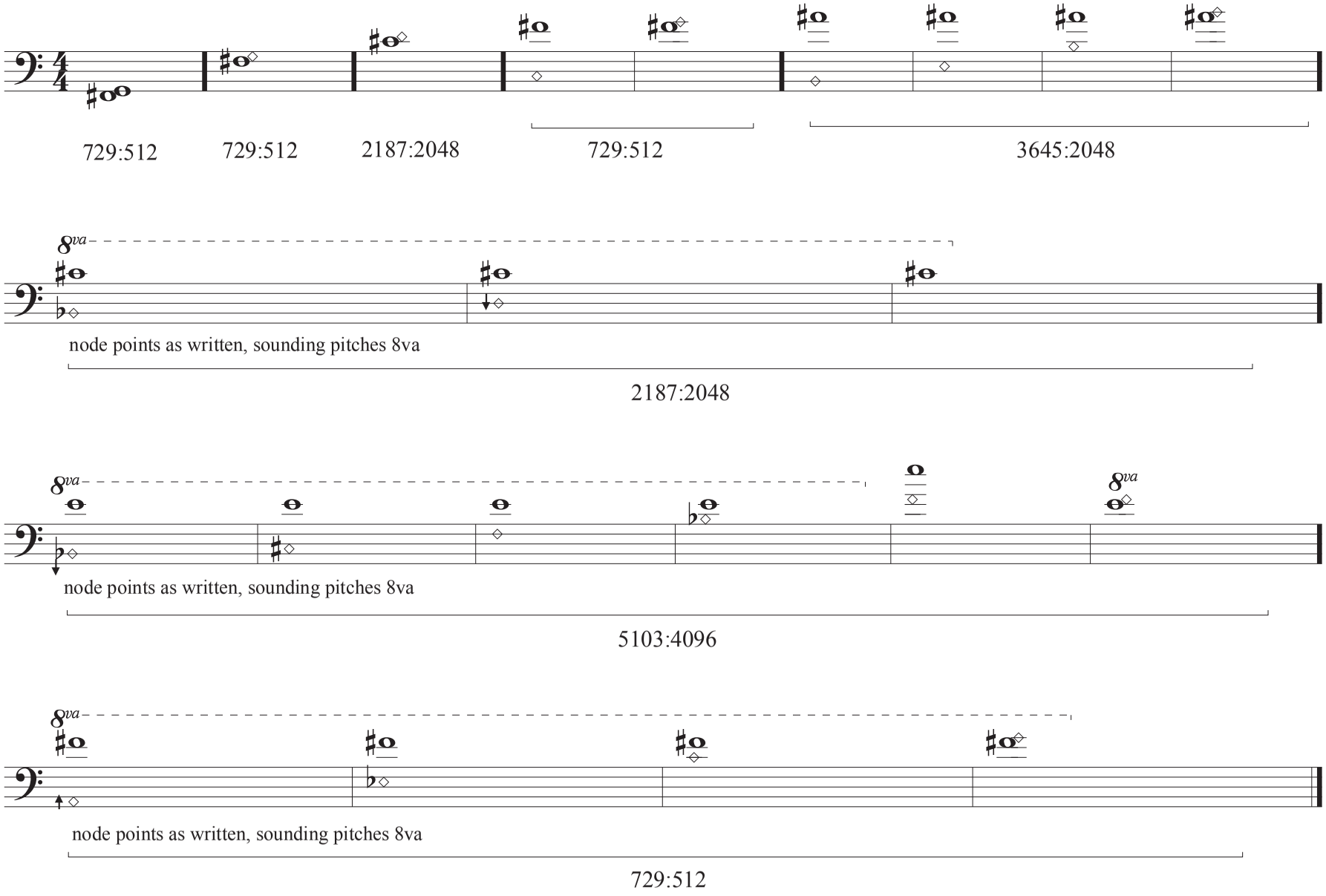
C

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)
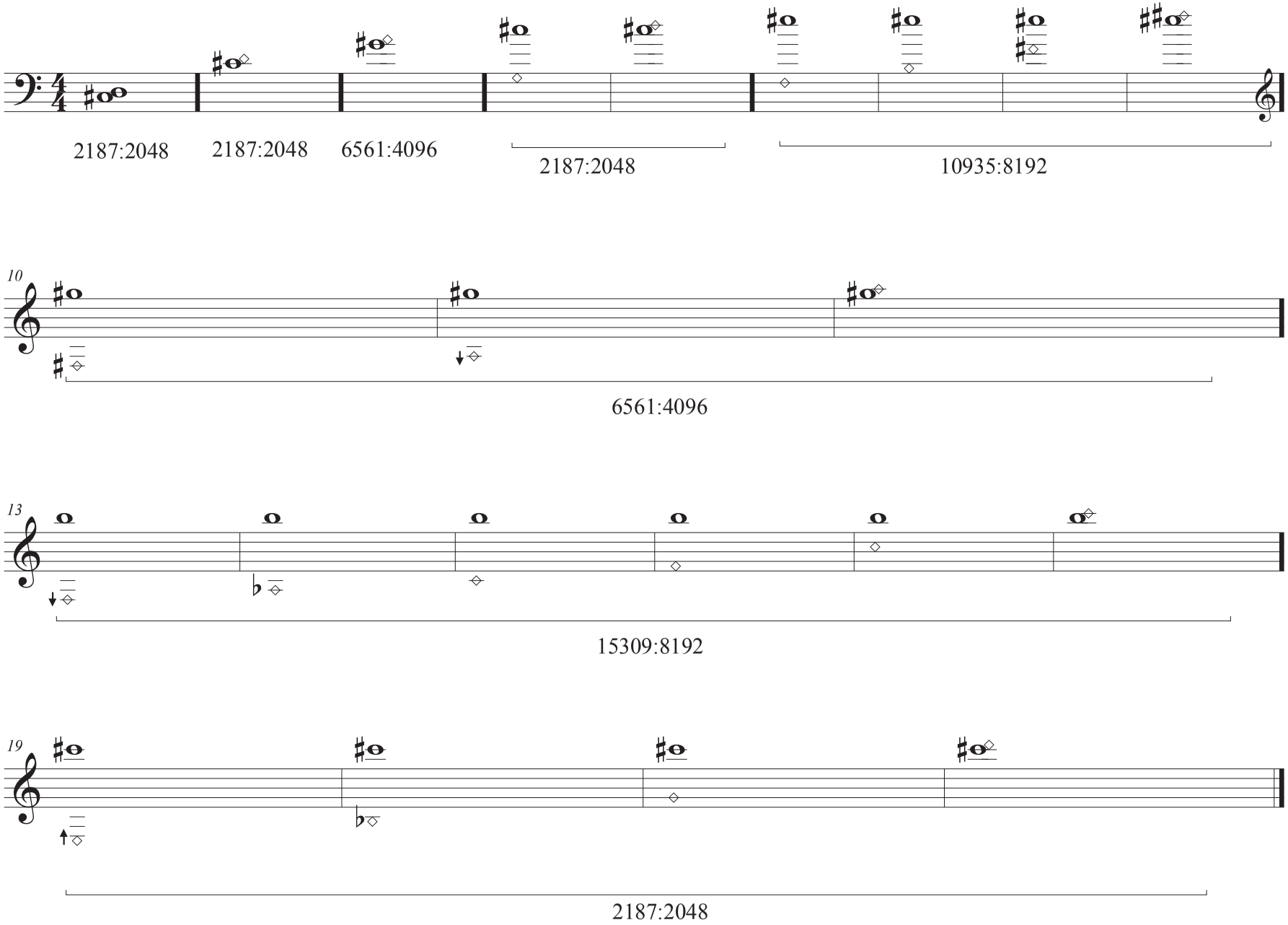


G

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



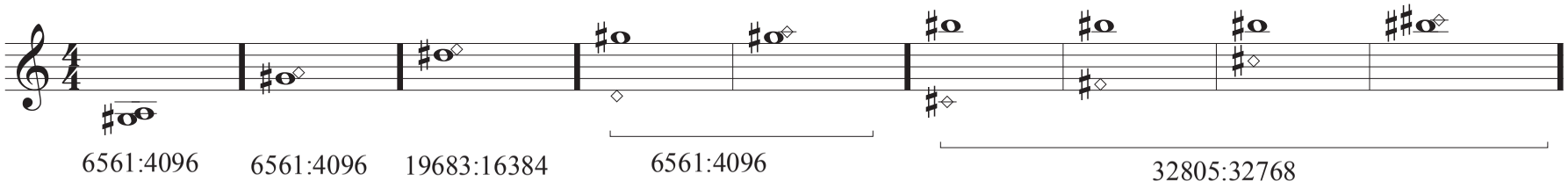
D

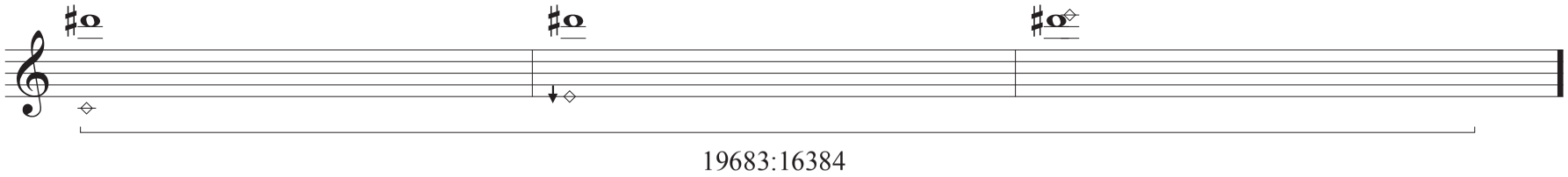
Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)

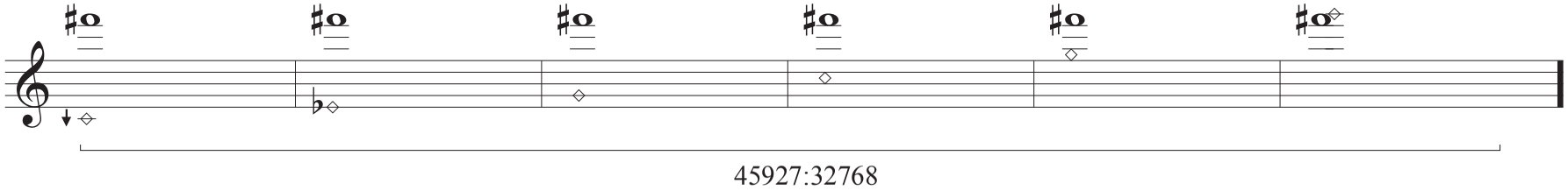


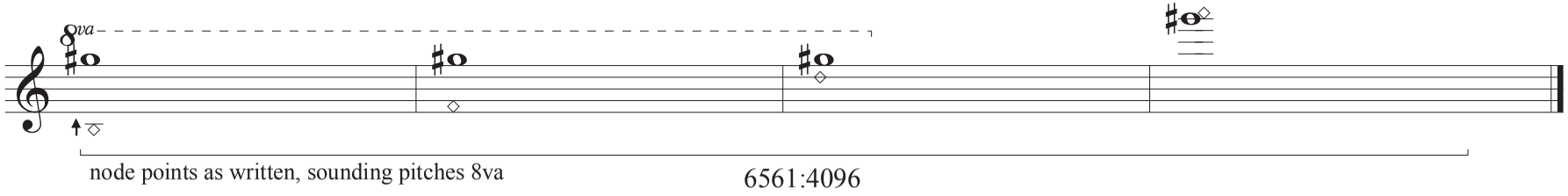
A

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)







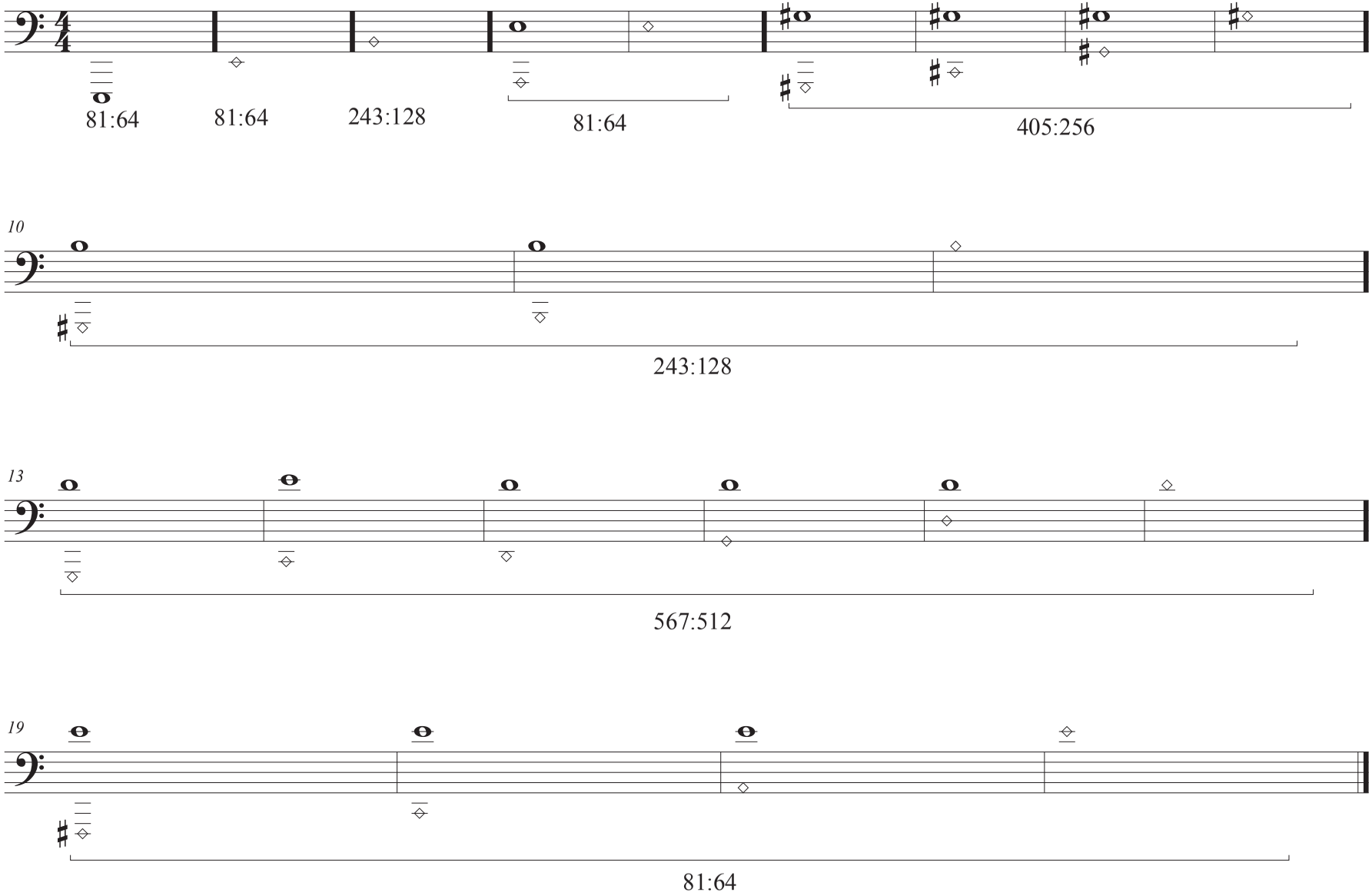


Appendix C, part ten

(Transposing chart for contrabass 1)

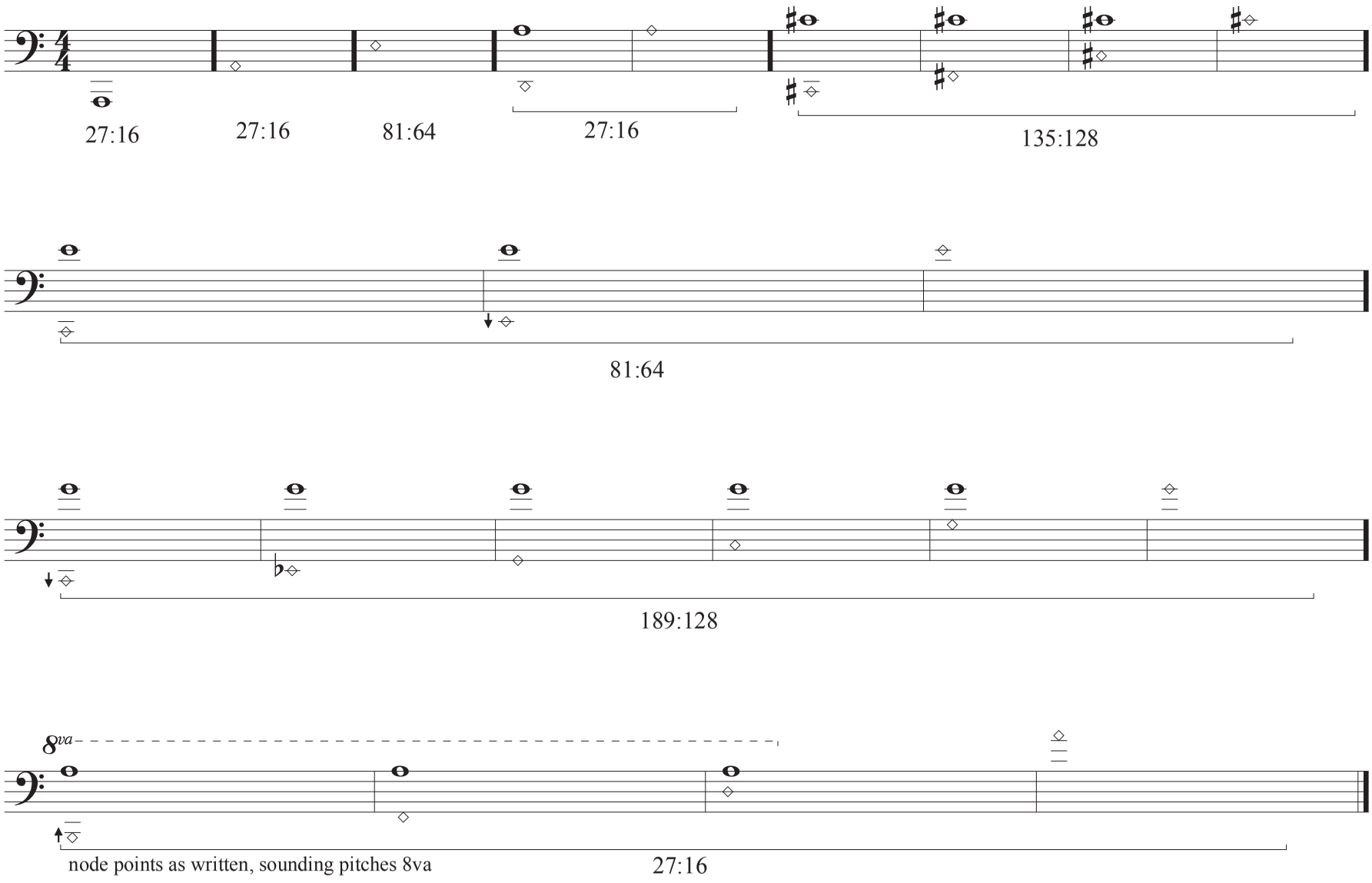
E

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



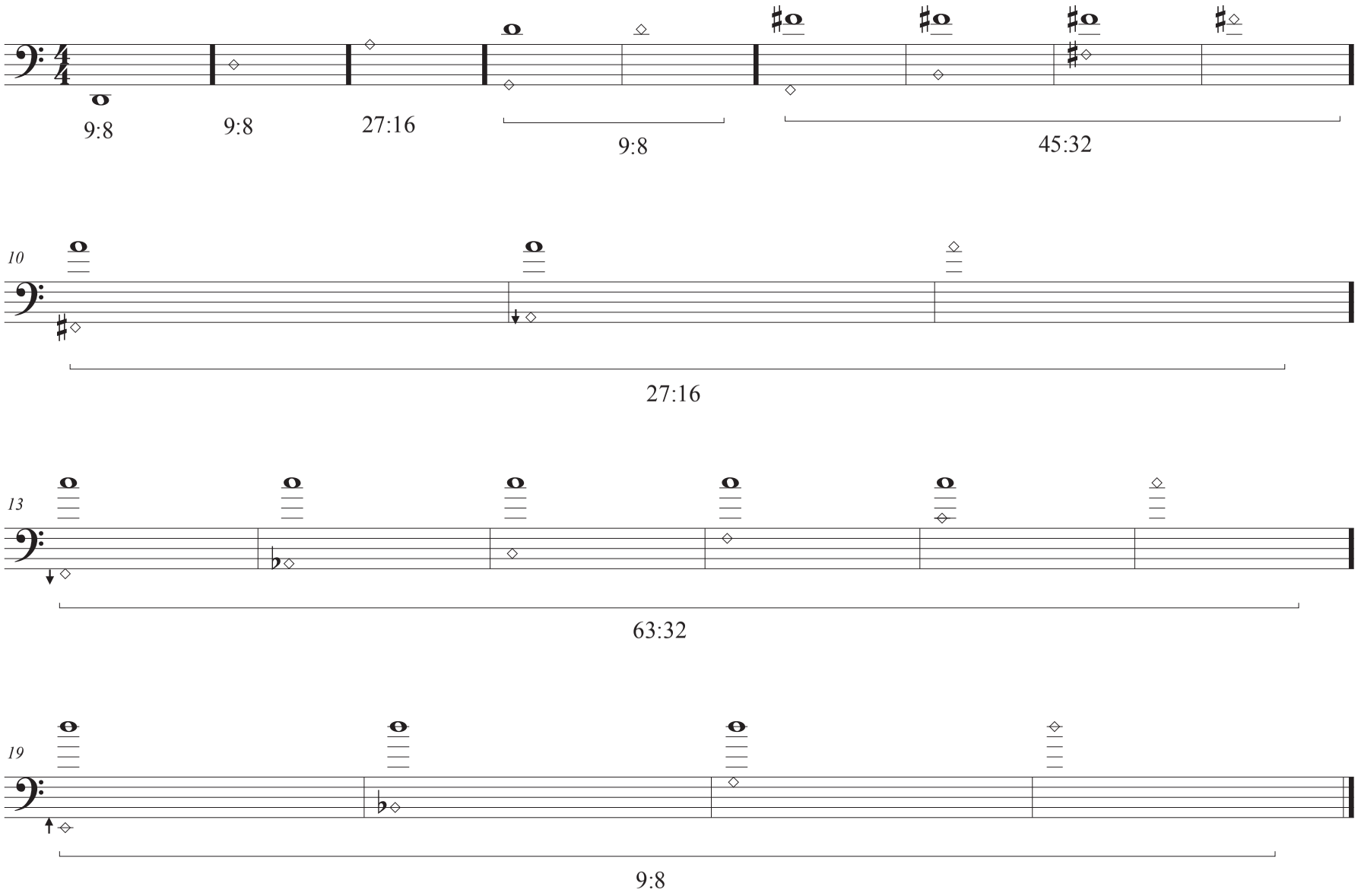
A

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)
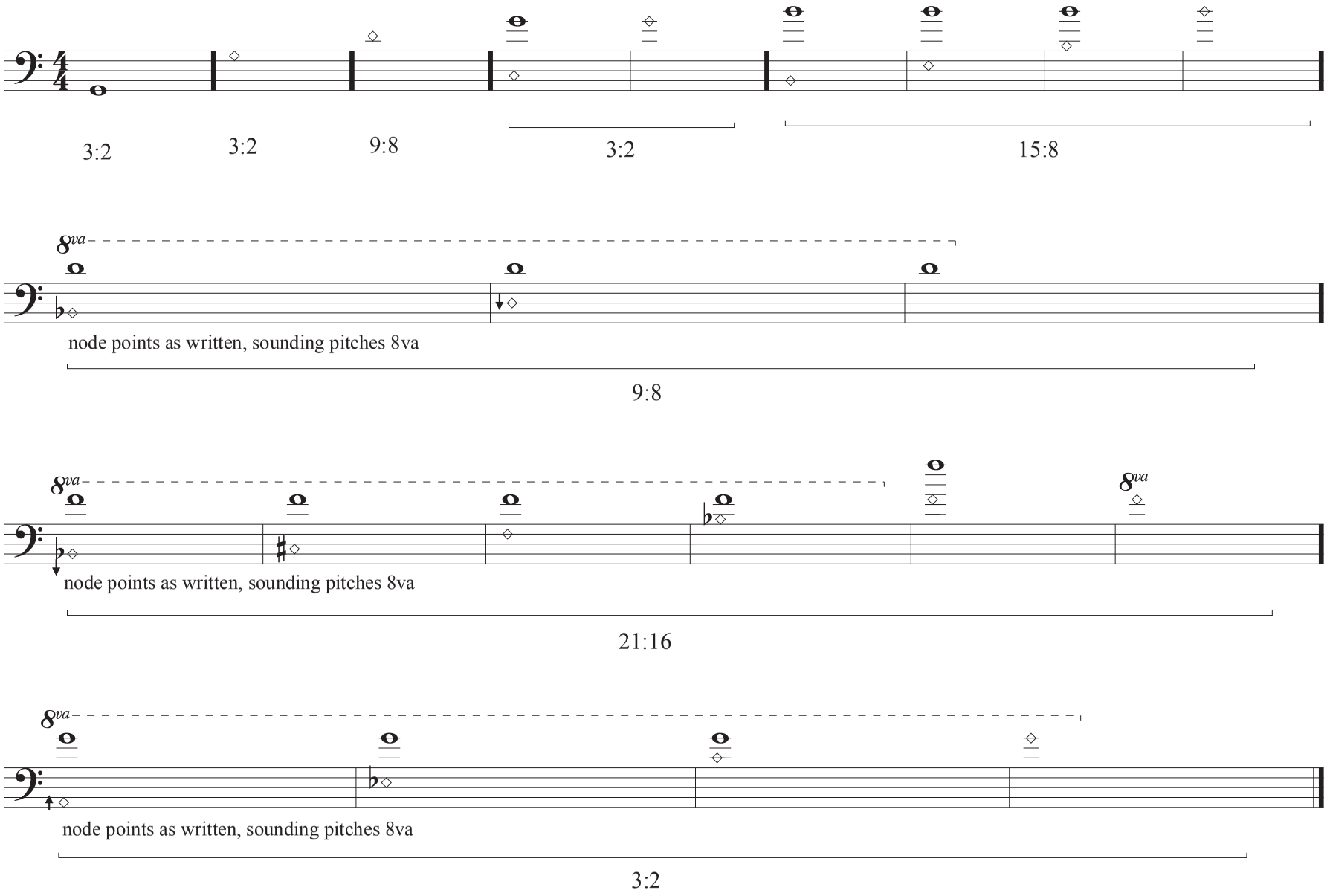


D

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)



G

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)

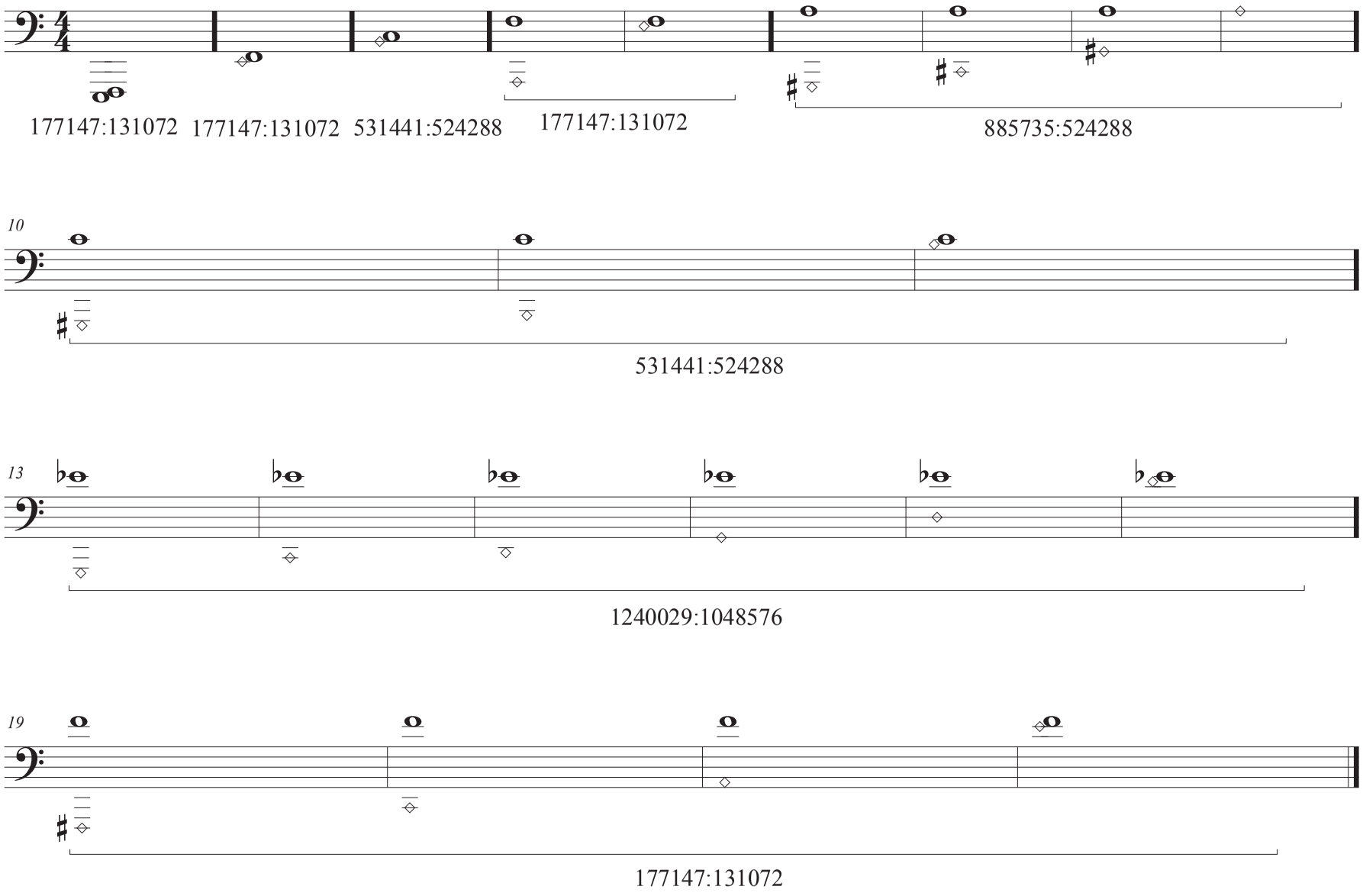


Appendix C, part eleven

(Transposing chart for contrabass 2)

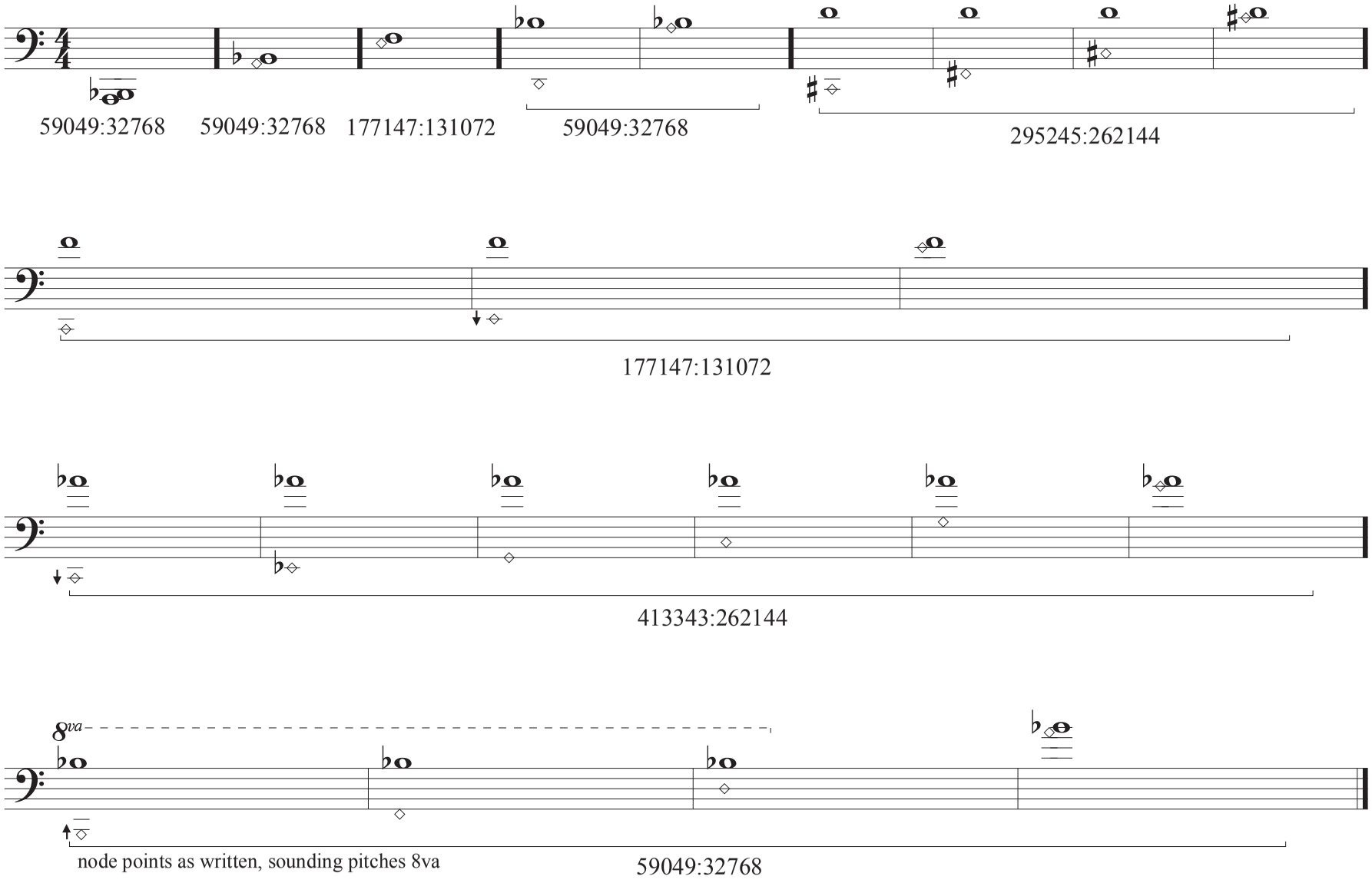
E

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)
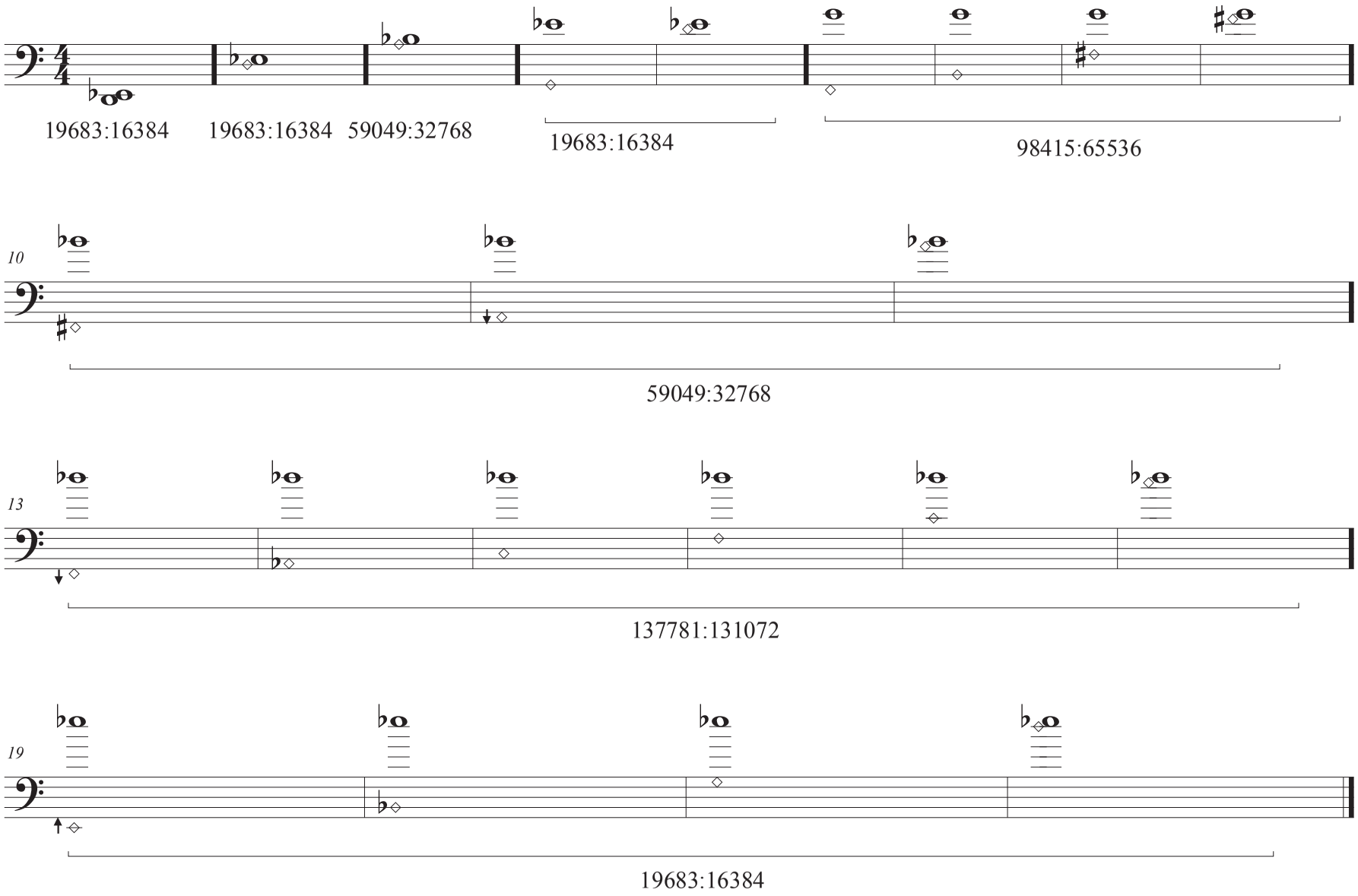


A

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



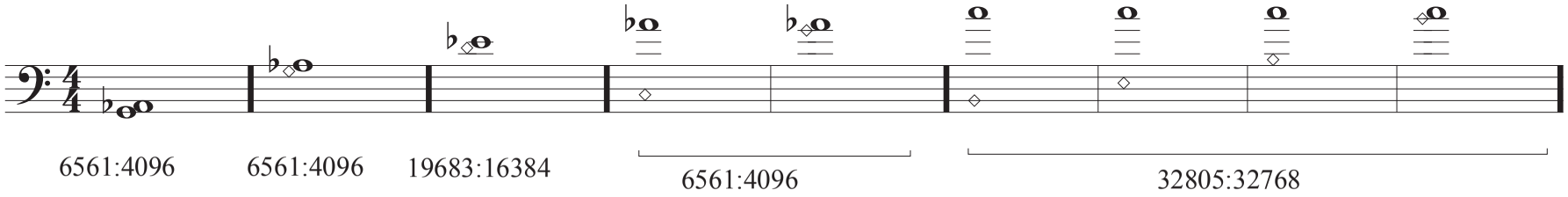
D

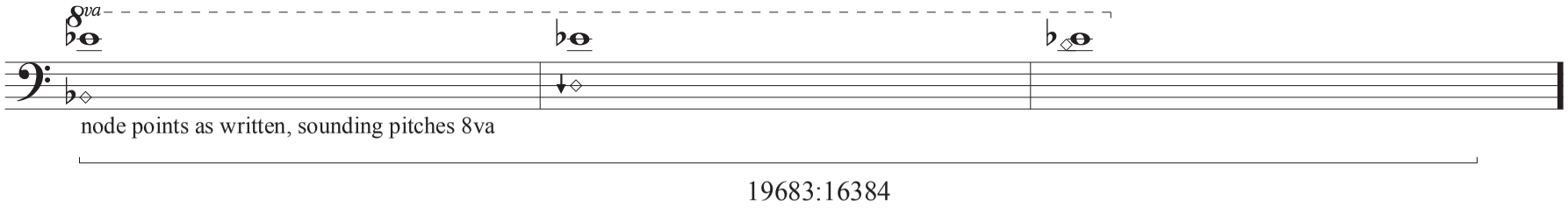
Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)

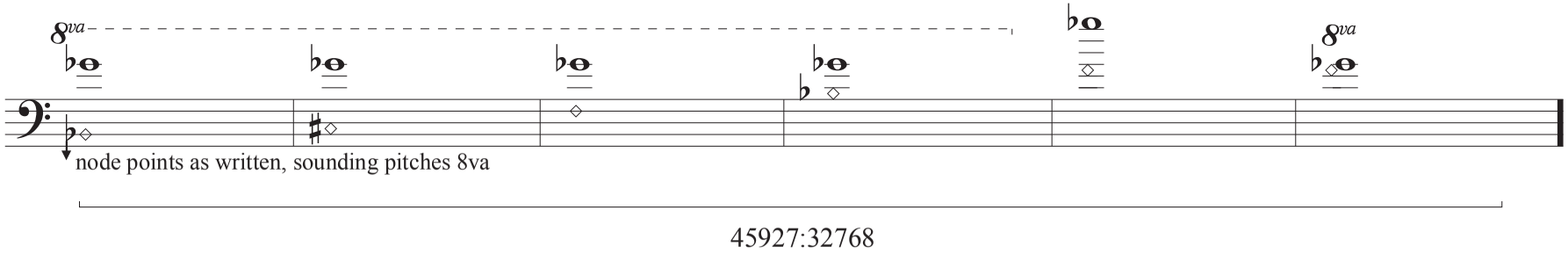


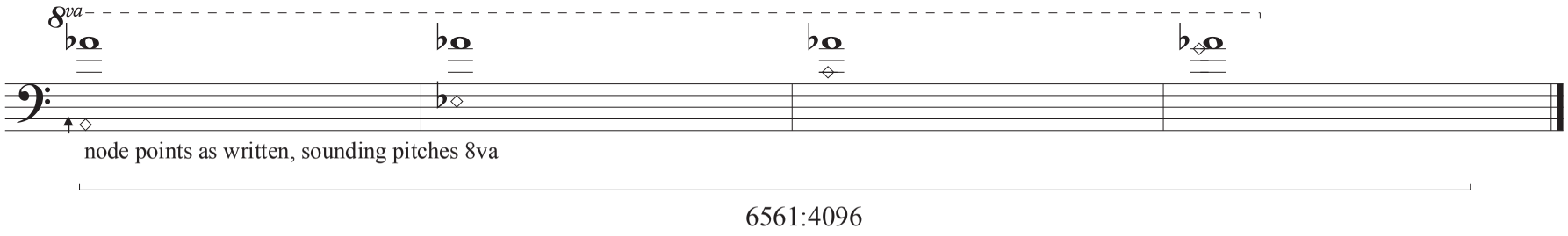
G

Harmonics up to the triple octave (adapted from Enzo Porta's *The Violin: Harmonics Classification and New Techniques*)









Appendix D

Appendix D, part one

(List of commas as they occur in movement one of *The tale of Kiều)*

Appendix D, part one:
List of commas as they occur in movement one of *The tale of Kiều*

	Iteration no.	Bar no.	Comma	No. of times comma occurs in the chord
1	1	1	Schisma	1
			Pythagorean comma	2
1	2	2	Syntonic comma	1
			Pythagorean comma	1
			Harrison's comma	1
2	1	3	Syntonic comma	1
2	2	4	Harrison's comma	1
3	2	6	Harrison's comma	1
			Septimal comma	1
			Pythagorean comma	1
3	3	7	Harrison's comma	1
4	1	8	Pythagorean comma	1
4	2	9	Garibaldi comma	1
			Septimal comma	1
			Schisma	1
			Un-named comma	1
5	1	10	Septimal comma	1
			Pythagorean comma	1
			Schisma	1
			Un-named comma	1
			Harrison's comma	1
			Syntonic comma	1
6	1	11	Syntonic comma	2
			Schisma	1
6	2	12	Septimal comma	1
7	1	13	Syntonic comma	6
7	2	14	Harrison's comma	3
7	3	15	Syntonic comma	3
			Garibaldi comma	1
			Septimal comma	1
			Harrison's comma	2
			Un-named comma	1
8	1	16	Septimal comma	2
8	2	17	Pythagorean comma	2
			Schisma	1
			Syntonic comma	2
			Un-named comma	1
9	1	18	Septimal comma	1
9	2	19	Syntonic comma	1
			Septimal comma	1
10	1	20	Septimal comma	2
10	2	21	Septimal comma	2
			Garibaldi comma	2
11	1	22	Septimal comma	5
11	2	23	Garibaldi comma	3
			Septimal comma	3
			Syntonic comma	1
12	1	24	Septimal comma	3
12	2	25	Schisma	1
			Septimal comma	3
			Syntonic comma	1
			Un-named comma	1
			Garibaldi comma	1
13	1	26	Septimal comma	1

Appendix D, part one:
List of commas as they occur in movement one of *The tale of Kiều*

			Harrison's comma	1
13	2	27	Schisma	1
14	1	28	Septimal comma	1
14	2	29	Syntonic comma	3
			Pythagorean comma	1
			Schisma	2
15	1	30	Septimal comma	1
			Syntonic comma	2
			Schisma	1
			Un-named comma	1
15	2	31	Pythagorean comma	1
			Septimal comma	1
			Garibaldi comma	1
			Harrison's comma	1
			Syntonic comma	1
			Un-named comma	1
15	3	32	Pythagorean comma	1
			Harrison's comma	1
			Septimal comma	1
16	1	33	Syntonic comma	2
			Schisma	1
16	2	34	Syntonic comma	1
			Schisma	1
			Un-named comma	1
16	3	35	Harrison's comma	2
17	1	36	Harrison's comma	1
17	2	37	Septimal comma	1
			Pythagorean comma	1
			Schisma	1
19	1	38	Septimal comma	1
			Harrison's comma	1
19	2	39	Un-named comma	1
			Harrison's comma	1
			Syntonic comma	2
			Pythagorean comma	1
20	1	40	Septimal comma	5
20	2	41	Syntonic comma	1
23	1	45	Syntonic comma	2
			Septimal comma	2
			Garibaldi comma	2
			Pythagorean comma	2
24	1	46	Syntonic comma	2
			Schisma	1
24	2	47	Harrison's comma	1
			Syntonic comma	1
24	3	48	Harrison's comma	2
25	1	49	Un-named comma	1
			Syntonic comma	2
			Harrion's comma	2
			Septimal comma	1
26	1	50	Harrison's comma	1
			Septimal comma	2
26	2	51	Pythagorean comma	1
27	1	52	Syntonic comma	3
27	2	53	Harrison's comma	1
			Syntonic comma	2

Appendix D, part one:
List of commas as they occur in movement one of *The tale of Kiều*

			Garibaldi comma	1
			Septimal comma	1
29	1	57	Syntonic comma	2
29	2	58	Pythagorean comma	1
			Septimal comma	3
			Syntonic comma	2
			Garibaldi comma	1
			Schisma	1
			Un-named comma	1
29	3	59	Septimal comma	1
			Harrison's comma	1
30	1	53	Pythagorean comma	1
			Syntonic comma	1
31	1	63	Syntonic comma	2
31	2	64	Pythagorean comma	1
			Harrison's comma	1
			Septimal comma	1
31	3	65	Septimal comma	3
32	1	66	Pythagorean comma	3
			Un-named comma	1
			Syntonic comma	3
			Harrison's comma	1
			Garibaldi comma	1
			Septimal comma	2
33	1	67	Septimal comma	2
			Harrison's comma	1
33	2	68	Syntonic comma	2

Appendix D, part two

(List of commas as they occur in movement two of *The tale of Kiều)*

Appendix D, part two:
List of commas as they occur in movement two of *The tale of Kiều*

Bar no.	Comma	No. of times comma occurs in the chord
70	Un-named comma	2
71	Garibaldi comma	2
73	Garibaldi comma	2
75	Garibaldi comma	2
77	Garibaldi comma	2
78	Garibaldi comma	1
79	Garibaldi comma	2
	Un-named comma	1
80	Syntonic comma	1
81	Syntonic comma	2
83	Garibaldi comma	2
84	Garibaldi comma	1
	Syntonic comma	3
	Septimal comma	2
87	Garibaldi comma	3
88	Garibaldi comma	2
89	Garibaldi comma	1
93	Syntonic comma	1
95	Garibaldi comma	1
96	Garibaldi comma	2
97	Septimal comma	1
98	Syntonic comma	1
	Garibaldi comma	1
	Septimal comma	2
99	Septimal comma	1
	Syntonic comma	1
100	Septimal comma	1
	Garibaldi comma	1
102	Garibaldi comma	1
107	Syntonic comma	1
109	Syntonic comma	1
110	Syntonic comma	1
	Septimal comma	1
	Garibaldi comma	2
111	Syntonic comma	1
113	Syntonic comma	1
114	Garibaldi comma	1
115	Syntonic comma	1

Appendix D, part three

(List of commas as they occur in movement five of *The tale of Kiều)*

Appendix D, part three:
List of commas as they occur in movement five of *The tale of Kiều*

Bar no.	**Comma**	**No. of times comma occurs in the chord**
217	Schisma	1
219	Schisma	1
	Syntonic comma	1
221	Un-named comma	1
	Schisma	1
	Harrison's comma	1
223	Septimal comma	1
224	Garibaldi comma	2
227	Septimal comma	2
	Schisma	2
	Un-named comma	2
229	Septimal comma	1
	Pythagorean comma	1
	Harrison's comma	1
230	Pythagorean comma	1
	Schisma	1
	Syntonic comma	1
	Harrison's comma	1
232	Pythagorean comma	2
233	Harrison's comma	1
	Pythagorean comma	3
238	Syntonic comma	1
	Pythagorean comma	1
	Un-named comma	1
	Harrison's comma	1
240	Pythagorean comma	1
	Schisma	1
246	Septimal comma	1
	Schisma	2
	Un-named comma	1
249	Pythagorean comma	2
	Schisma	2
	Septimal comma	1
	Harrison's comma	1
	Un-named comma	1
252	Septimal comma	1
	Un-named comma	1
	Pythagorean comma	1
	Harrison's comma	1

Appendix D, part four

(Tables showing the integer ratios from each bar in movement five of *The tale of Kiều)*

Appendix D, part four:

Tables showing the integer ratios from each bar in movement five of *The tale of Kiều*

Bar 217

Instrument	Beats 1-5 Ratios	Beats 6-10 Ratios	Beats 11-15 Ratios
Violins 1 & 2	9:8	9:8	9:8
Violas 1 & 2	9:8	9:8	9:8
Cellos 1 & 2	9:8	9:8	9:8
Contrabass 1	243:128 81:64	243:128 81:64	243:128 81:64
Contrabass 2	-	295245:262144	295245:262144

Bar 218

Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	9:8	9:8	9:8	9:8	9:8
Violins 3 & 4	-	-	729:512 243:128	729:512 243:128	729:512 243:128
Violins 5 & 6	-	-	729:512	729:512	729:512
Violas 1 & 2	9:8	9:8	9:8	9:8	9:8
Cellos 1 & 2	9:8	9:8	9:8	9:8	9:8
Contrabass 1	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64

Bar 219

Instrument	Beats 1-15
Violins 1 & 2	9:8
Violins 3 & 4	729:512 243:128
Violins 5 & 6	729:512
Violas 1 & 2	9:8
Cellos 1 & 2	45:32
Contrabass 1	243:128 81:64
Contrabass 2	295245:262144

Bar 221					
Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	27:16	27:16	27:16	27:16	27:16
Violins 3 & 4	189:128	189:128	189:128	189:128	189:128
Violins 7 & 8	-	-	98415:65536	98415:65536	-
Violins 9 & 10	-	-	885735:524288	885735:524288	1594323:1048576
Violas 1 & 2	27:16	27:16	27:16	27:16	27:16
Violas 3 & 4			98415:65536	98415:65536	
Cellos 1 & 2	189:128	189:128	189:128	189:128	189:128
Contrabass 1	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64

Bar 223					
Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	27:16	27:16	27:16	45:32 15:8	45:32 15:8
Violins 3 & 4	189:128	189:128	189:128	189:128	189:128
Violins 7 & 8	45927:32768	45927:32768	45927:32768		
Violas 1 & 2	27:16	27:16	27:16	45:32 15:8	45:32 15:8
Violas 3 & 4	45927:32768 15309:8192				
Cellos 1 & 2	189:128	189:128	189:128	189:128	189:128
Cellos 3 & 4	45927:32768 15309:8192	45927:32768 15309:8192	45927:32768 15309:8192		
Contrabass 1	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64

Bar 224			
Instrument	Beats 1-5 Ratios	Beats 6-10 Ratios	Beats 11-15 Ratios
Violins 1 & 2	45:32 15:8	45:32 15:8	45:32 15:8
Violins 3 & 4	189:128	189:128	189:128
Violins 7 & 8	45927:32768	45927:32768	45927:32768
Violas 1 & 2	27:16	27:16	27:16
Violas 3 & 4	45927:32768 15309:8192	45927:32768 15309:8192	45927:32768 15309:8192
Cellos 3 & 4	45927:32768 15309:8192	45927:32768 15309:8192	45927:32768 15309:8192

Bar 226					
Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	243:128 27:16	27:16	27:16	27:16	27:16
Violins 3 & 4	243:128 81:64	-	-	189:128	189:128
Violins 5 & 6	-	15309:8192 5103:4096	15309:8192 5103:4096	-	-
Violins 9 & 10	-	-	-	7971615:4194304	7971615:4194304
Violins 11 & 12	-	-	-	2657205:2097152	2657205:2097152
Violas 1 & 2	81:64 27:16	27:16	27:16	27:16	27:16
Cellos 1 & 2	189:128	189:128	189:128	189:128	189:128

Bar 227					
Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	243:128 27:16	243:128 27:16	243:128 27:16	243:128 27:16	243:128 27:16
Violins 3 & 4	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64	243:128 81:64
Violins 5&6	15309:8192 5103:4096	15309:8192 5103:4096	15309:8192 5103:4096	15309:8192 5103:4096	15309:8192 5103:4096
Violins 9 & 10	7971615:4194304	7971615:4194304	7971615:4194304	7971615:4194304	7971615:4194304
Violins 11 & 12	2657205:2097152	2657205:2097152	2657205:2097152	2657205:2097152	2657205:2097152
Violas 1 & 2	81:64 27:16	27:16	27:16	27:16	27:16
Cellos 1 & 2	189:128	189:128	189:128	189:128	189:128

Bar 229					
Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	9:8 27:16	9:8 27:16	9:8 27:16	9:8 27:16	9:8 27:16
Violins 3 & 4	189:128	189:128	189:128	189:128	189:128
Violins 9 & 10	-	4782969:4194304	-	4782969:4194304	-
Violins 11 & 12	-	1594323:1048576	-	1594323:1048576	-
Viola 1 & 2	3:2	27:16	3:2	27:16	3:2
Cello 1 & 2	189:128	189:128	189:128	189:128	189:128

Bar 230

Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 7 & 8	-	-	59049:32768	59049:32768	59049:32768
Violins 9 & 10	-	-	531441:524288	531441:524288	531441:524288
Violas 1 & 2	7:4 1:1	7:4 1:1	7:4 1:1	7:4 1:1	7:4 1:1
Violas 3 & 4	-	-	59049:32768	59049:32768	59049:32768
Contrabass 2	-	-	32805:32768	32805:32768	32805:32768

Bar 232

Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	9:8 3:2	9:8 3:2	9:8 3:2	9:8 3:2	9:8 3:2
Violins 9 & 10	-	4782969:4194304	-	4782969:4194304	-
Violin 11 & 12	-	1594323:1048576	-	1594323:1048576	-
Violas 1 & 2	7:4 1:1	7:4 1:1	7:4 1:1	7:4 1:1	7:4 1:1

Bar 233

Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	9:8 3:2	9:8 3:2	9:8 3:2	9:8 3:2	9:8 3:2
Violins 7 & 8	59049:32768	59049:32768	59049:32768	59049:32768	59049:32768
Violins 9 & 10	4782969:4194304	4782969:4194304	4782969:4194304	4782969:4194304	4782969:4194304
Violins 11 & 12	1594323:1048576	1594323:1048576	1594323:1048576	1594323:1048576	1594323:1048576
Violas 1 & 2	7:4 1:1	7:4 1:1	7:4 1:1	7:4 1:1	7:4 1:1
Violas 3 & 4	59049:32768	59049:32768	59049:32768	59049:32768	59049:32768
Contrabass 2	32805:32768	32805:32768	32805:32768	32805:32768	32805:32768

Bar 238					
Instrument	Beats 1-3 Ratios	Beats 4-6 Ratios	Beats 7-9 Ratios	Beats 10-12 Ratios	Beats 13-15 Ratios
Violins 1 & 2	9:8	9:8	9:8	9:8	9:8
Violins 7 & 8	-	-	32805:32768	32805:32768	32805:32768
Violins 9 & 10	-	-	531441:524288	531441:524288	531441:524288
Violins 11 & 12	-	-	4782969:4194304	4782969:4194304	4782969:4194304
Violas 1 & 2	9:8	9:8	9:8	9:8	9:8
Violas 3 & 4	-	-	32805:32768	32805:32768	32805:32768
Cellos 1 & 2	63:32	63:32	63:32	63:32	63:32
Cellos 3 & 4	-	-	32805:32768	32805:32768	32805:32768
Contrabass 2	9:8	9:8	9:8	9:8	9:8

Bar 239	
Instrument	Beats 1-15
Violins 1 & 2	9:8
Violins 3 & 4	27:16
Violas 1 & 2	27:16
	9:8
Cellos 1 & 2	27:16
Contrabass 1	9:8

Bar 240	
Instrument	Beats 1-15
Violins 1 & 2	9:8
Violins 3 & 4	27:16
Violins 9 & 10	4782969:4194304
Violins 11 & 12	885735:524288
Violas 1 & 2	27:16
	9:8
Cellos 1 & 2	27:16
Contrabass 1	9:8

Bar 246	
Instrument	Beats 1-15
Violins 1 & 2	81:64
	27:16
Violins 3 & 4	27:16
Violins 5 & 6	5103:4096
Violins 9 & 10	2657205:2097152
Violins 11 & 12	885735:524288
Violas 1 & 2	81:64
	9:8
Cellos 1 & 2	63:32
Contrabass 1 & 2	9:8

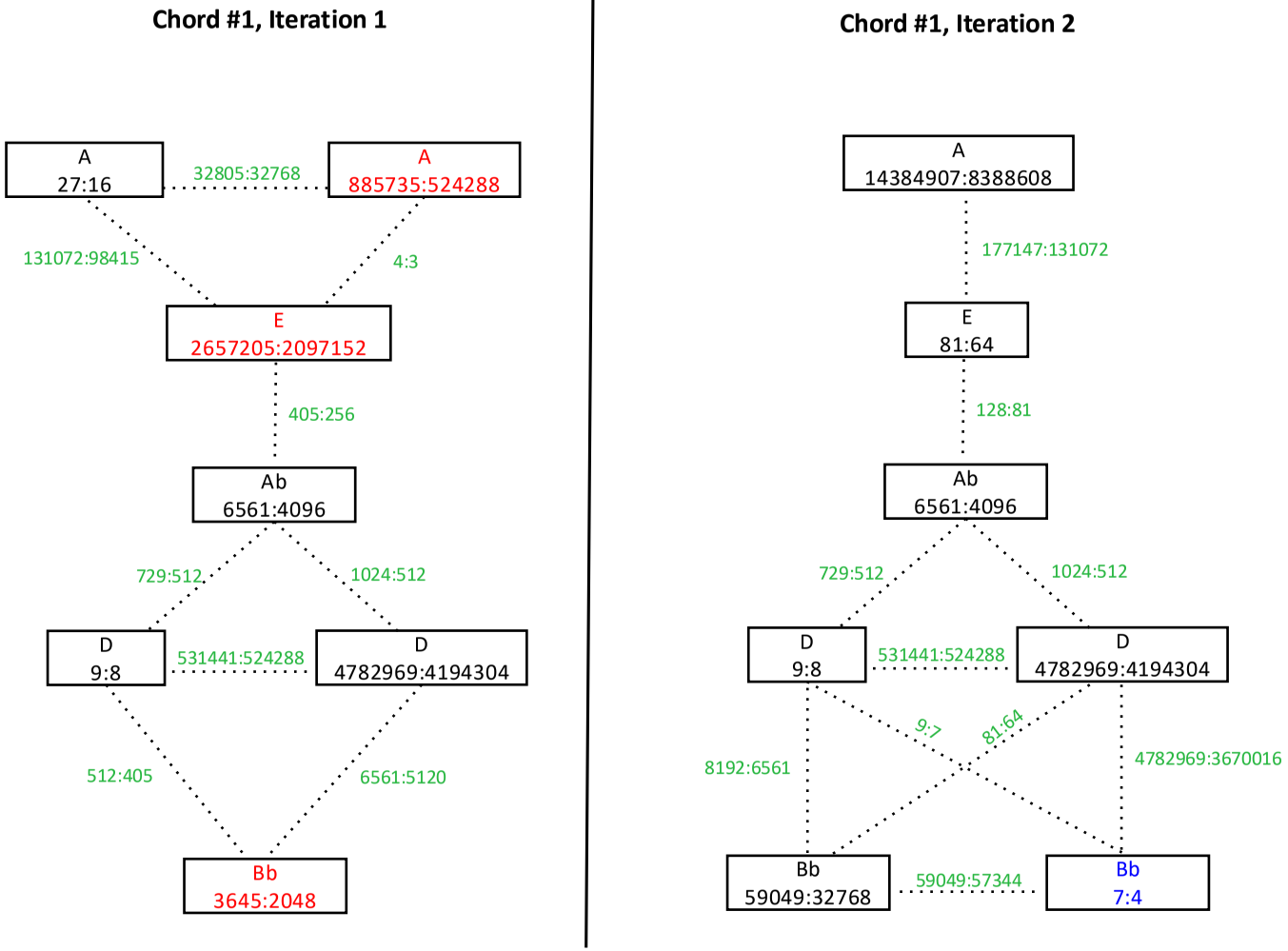
Bar 249	
Instrument	Beats 1-15
Violins 1 & 2	9:8
	3:2
Violins 3 & 4	27:16
Violins 7 & 8	59049:32768
Violins 9 & 10	4782969:4194304
	1594323:1048576
Violins 11 & 12	885735:524288
	4782969:4194304
Violas 1 & 2	7:4
	1:1
Violas 3 & 4	59049:32768
Cellos 1 & 2	63:32
	27:16
Contrabass 1	9:8
Contrabass 2	32805:32768

Bar 252					
Instrument	Beats 1-3	Beats 4-6	Beats 7-9	Beats 10-12	Beats 13-15
	Ratios	Ratios	Ratios	Ratios	Ratios
Violins 1 & 2	3:2	3:2	3:2	3:2	3:2
	9:8	9:8	9:8	9:8	9:8
Violins 3 & 4	189:128	189:128	189:128	189:128	189:128
	27:16	27:16	27:16	27:16	27:16
Violins 5 & 6	-	-	10935:8192	10935:8192	10935:8192
Violins 9 & 10	-	-	1594323:1048576	1594323:1048576	1594323:1048576
Violas 1 & 2	21:16	21:16	21:16	21:16	21:16
	9:8	9:8	9:8	9:8	9:8
Cellos 1 & 2	189:128	189:128	189:128	189:128	189:128
	9:8	9:8	9:8	9:8	9:8
Contrabass 1	9:8	9:8	9:8	9:8	9:8

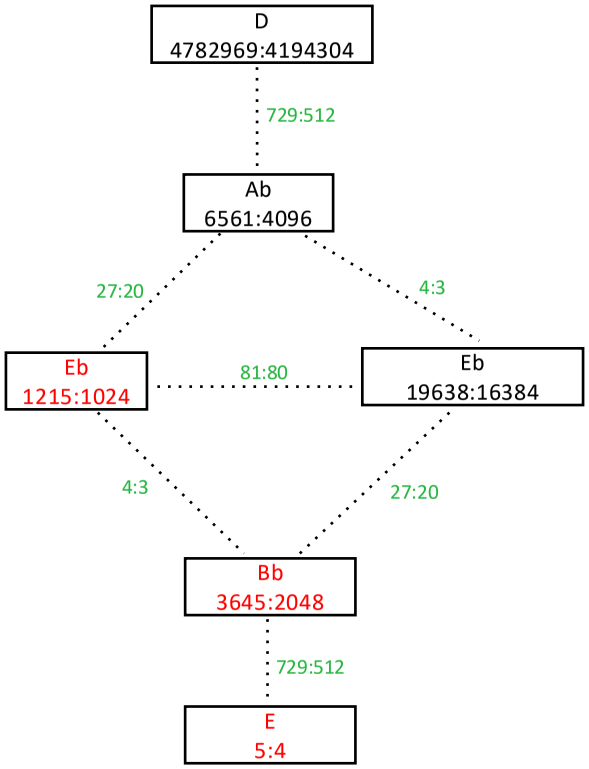
Appendix D, part five

(Set of descriptive charts of the chords, with variable tunings, used throughout movement one of *The tale of Kiều)*

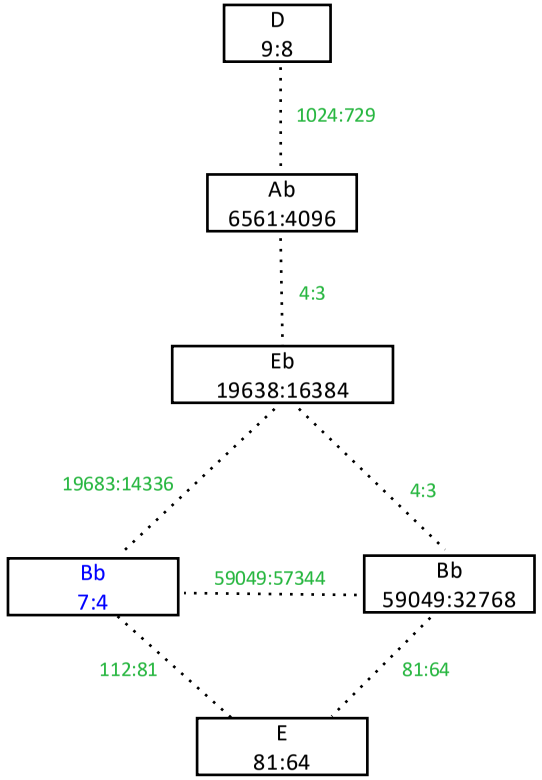
Appendix D, part five

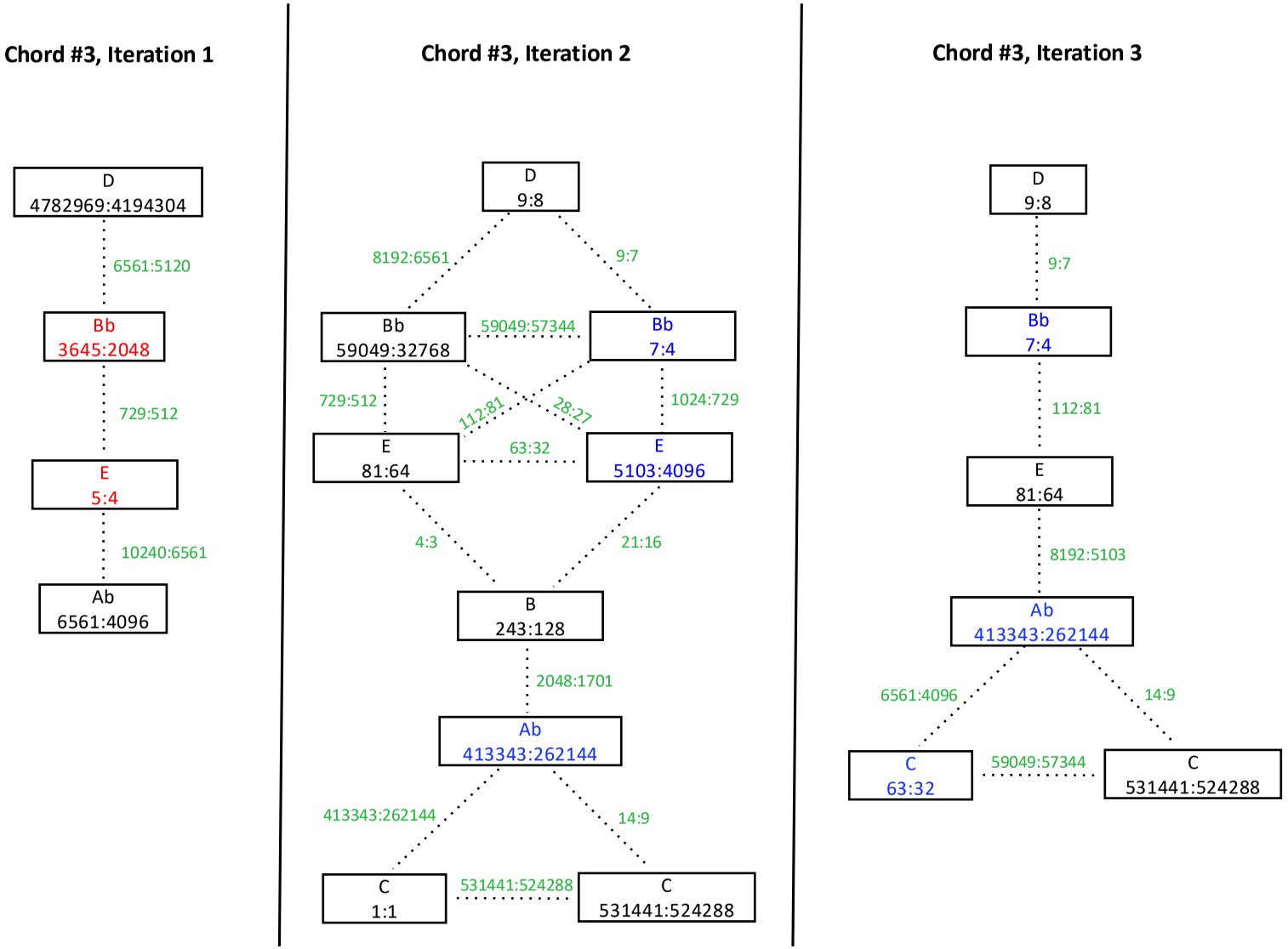


Chord #2, Iteration 1

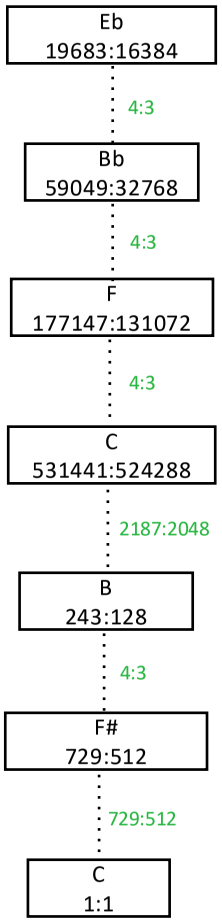


Chord #2, Iteration 2

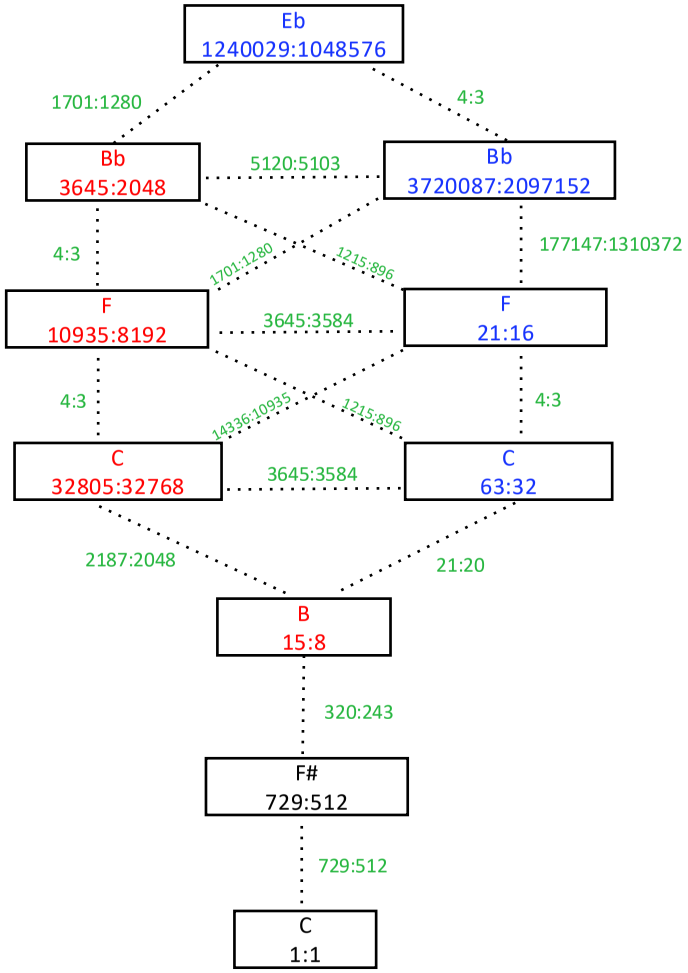



Chord #3, Iteration 1
D
4782969:4194304
6561:5120
Bb
3645:2048
729:512
E
5:4
10240:6561
Ab
6561:4096
Chord #3, Iteration 2
D
9:8
8192:6561
9:7
Bb
59049:32768
59049:57344
Bb
7:4
729:512
112:81
28:27
1024:729
E
81:64
63:32
E
5103:4096
4:3
21:16
B
243:128
2048:1701
Ab
413343:262144
413343:262144
14:9
C
1:1
531441:524288
C
531441:524288
Chord #3, Iteration 3
D
9:8
9:7
Bb
7:4
112:81
E
81:64
8192:5103
Ab
413343:262144
6561:4096
14:9
C
63:32
59049:57344
C
531441:524288

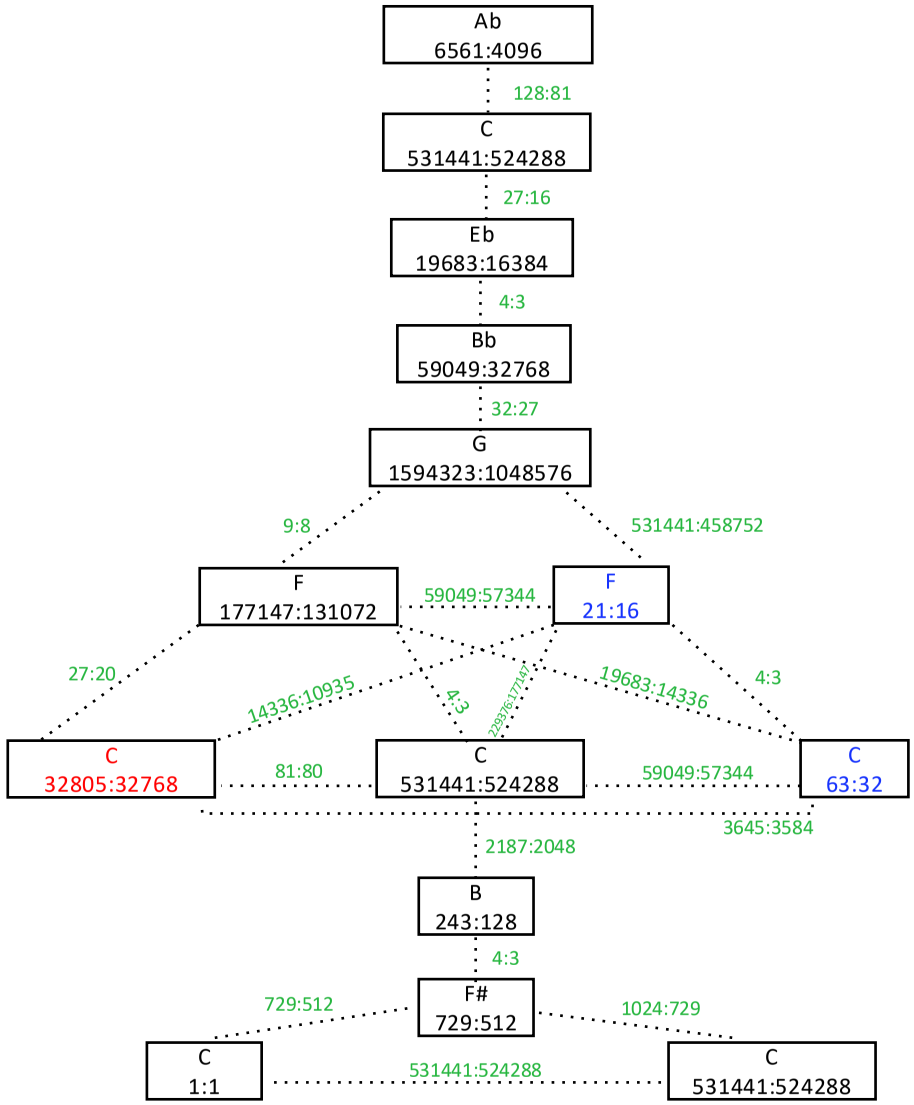
Chord #4, Iteration 1



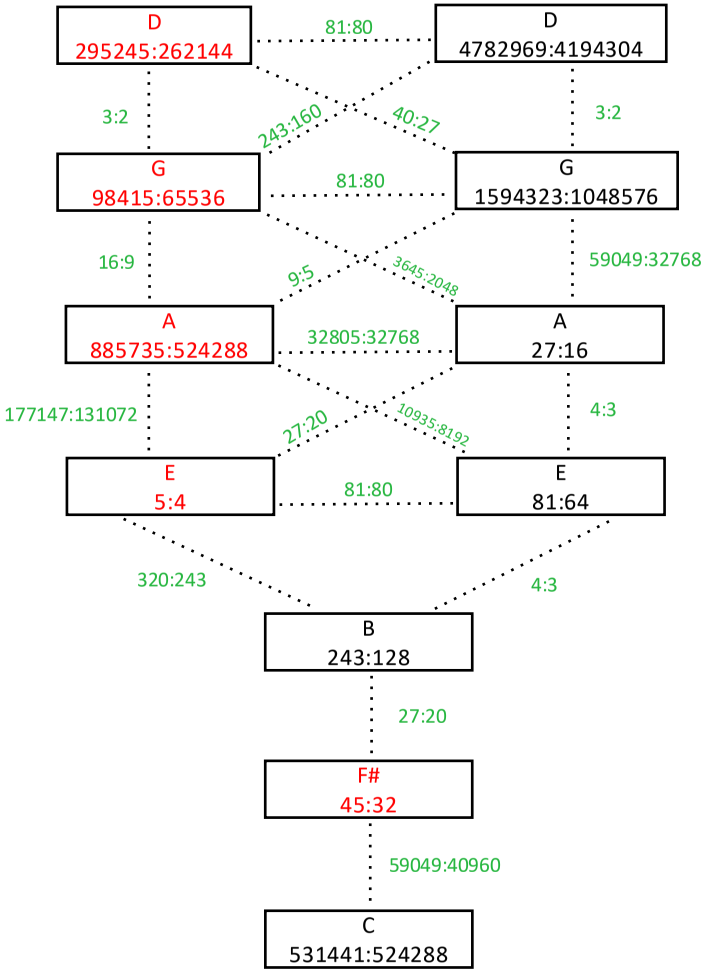
Chord #4, Iteration 2



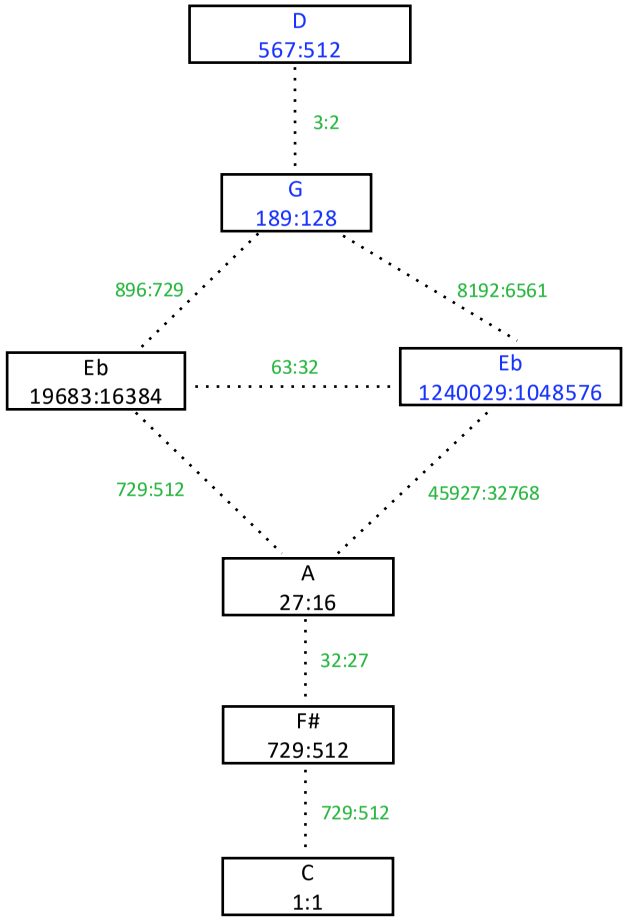
Chord #5

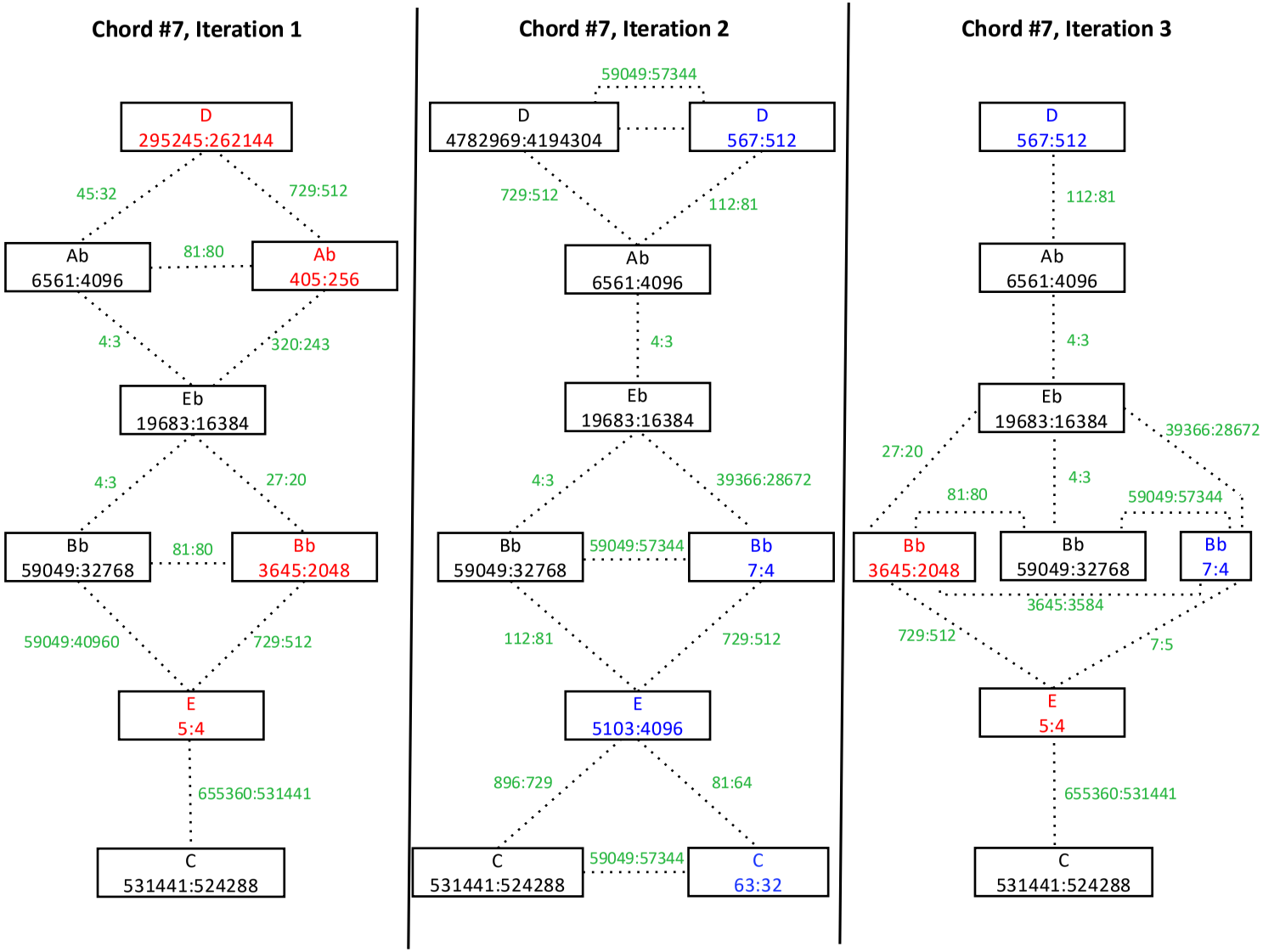

Ab
6561:4096
128:81
C
531441:524288
27:16
Eb
19683:16384
4:3
Bb
59049:32768
32:27
G
1594323:1048576
9:8
531441:458752
F
177147:131072
59049:57344
F
21:16
27:20
14336:10935
4:3
2239761771441
19683:14336
4:3
C
32805:32768
81:80
C
531441:524288
59049:57344
C
63:32
3645:3584
2187:2048
B
243:128
4:3
F#
729:512
729:512
1024:729
C
1:1
531441:524288
C
531441:524288

Chord #6, Iteration 1

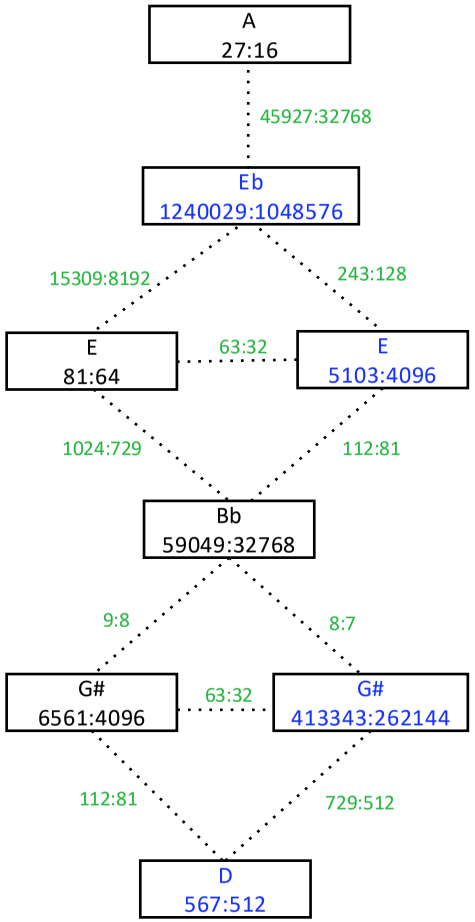


Chord #6, Iteration 2

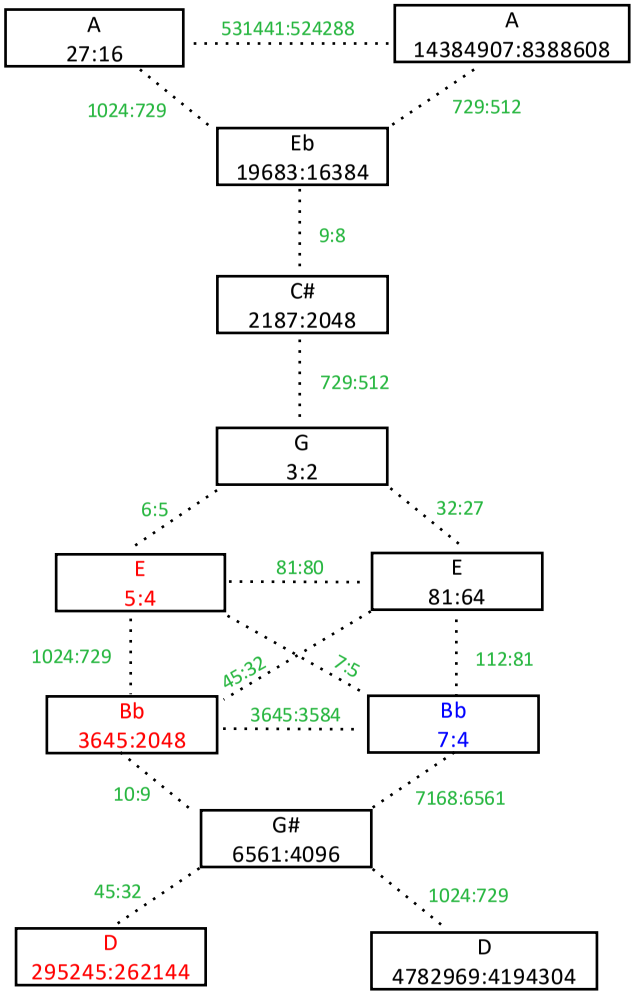



Chord #7, Iteration 1
D
295245:262144
45:32
729:512
Ab
6561:4096
81:80
Ab
405:256
4:3
320:243
Eb
19683:16384
4:3
27:20
Bb
59049:32768
81:80
Bb
3645:2048
59049:40960
729:512
E
5:4
655360:531441
C
531441:524288
Chord #7, Iteration 2
59049:57344
D
4782969:4194304
D
567:512
729:512
112:81
Ab
6561:4096
4:3
Eb
19683:16384
4:3
39366:28672
Bb
59049:32768
59049:57344
Bb
7:4
112:81
729:512
E
5103:4096
896:729
81:64
C
531441:524288
59049:57344
C
63:32
Chord #7, Iteration 3
D
567:512
112:81
Ab
6561:4096
4:3
Eb
19683:16384
27:20
39366:28672
81:80
4:3
59049:57344
Bb
3645:2048
Bb
59049:32768
Bb
7:4
3645:3584
729:512
7:5
E
5:4
655360:531441
C
531441:524288

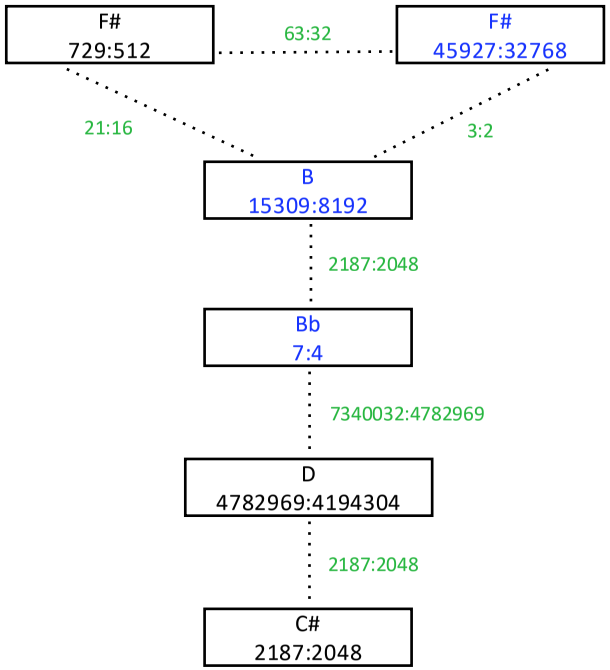
Chord #8, Iteration 1



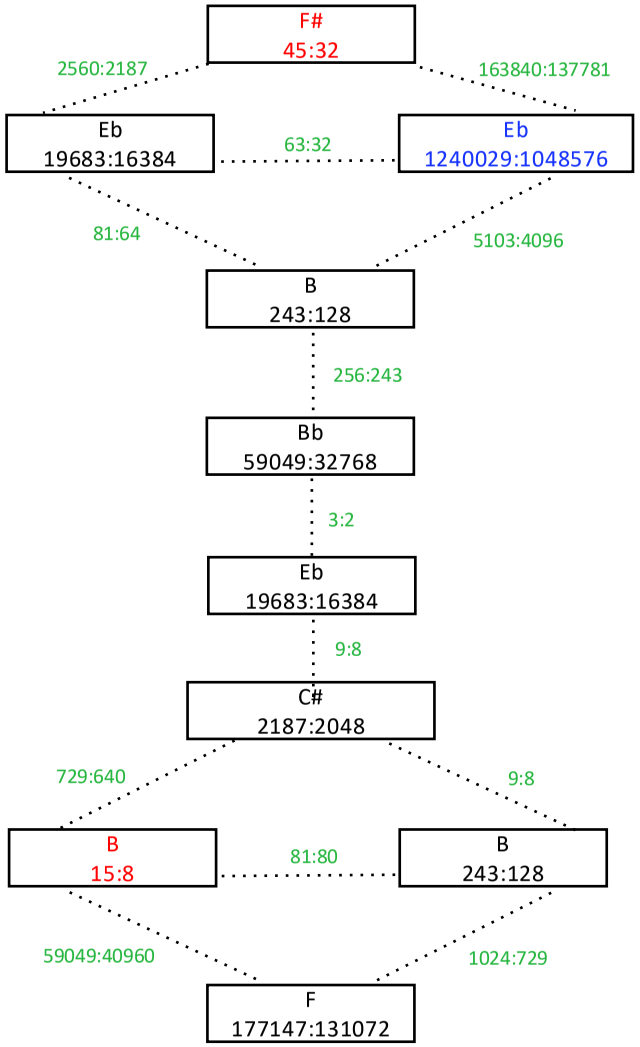
Chord #8, Iteration 2



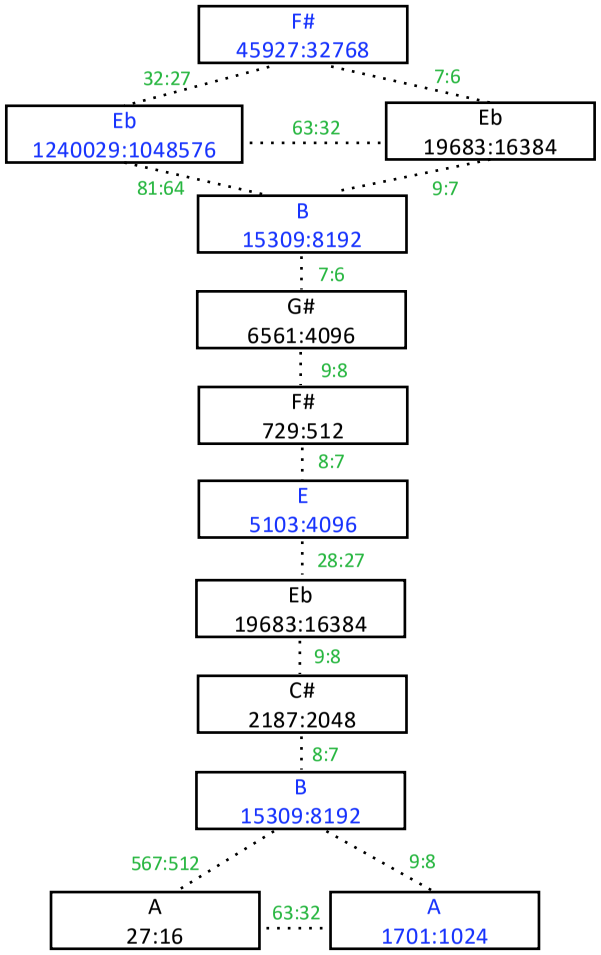
Chord #9, Iteration 1



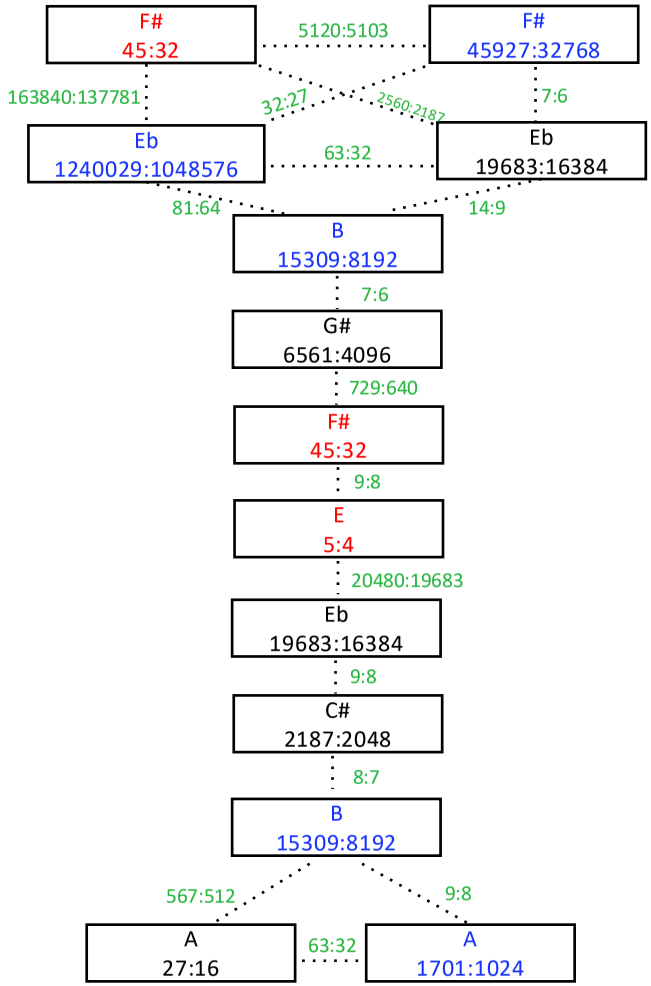
Chord #9, Iteration 2



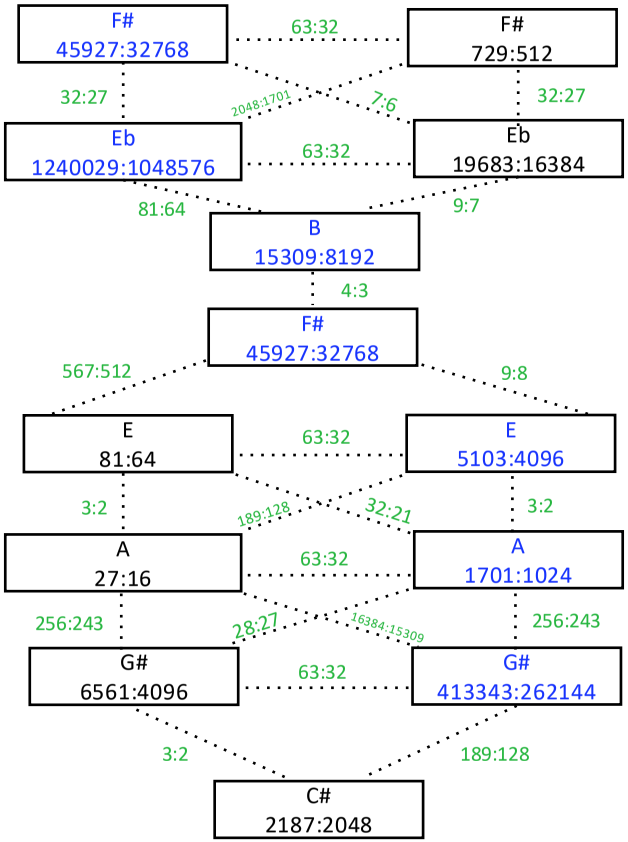
Chord #10, Iteration 1



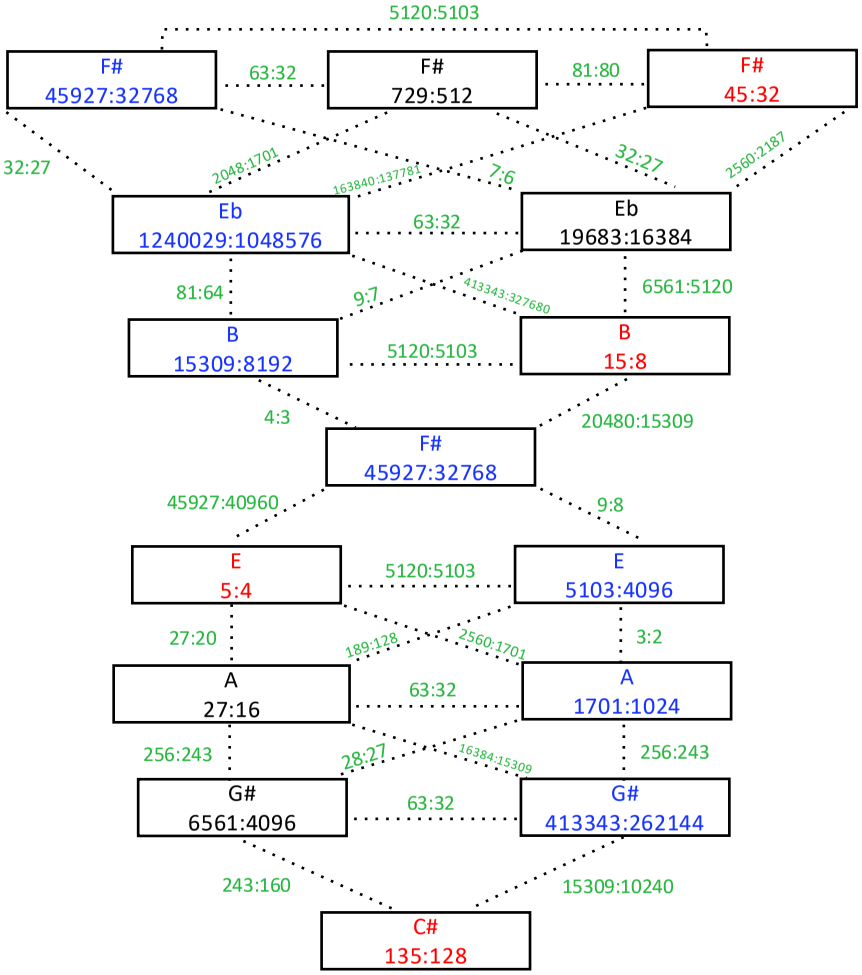
Chord #10, Iteration 2



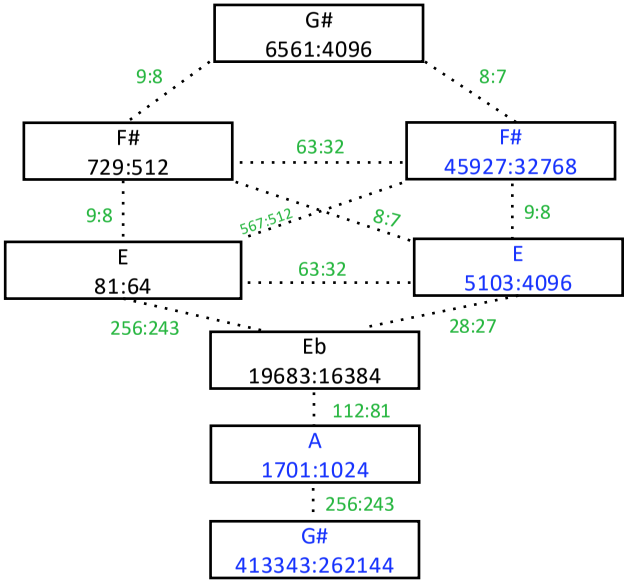
Chord #11, Iteration 1



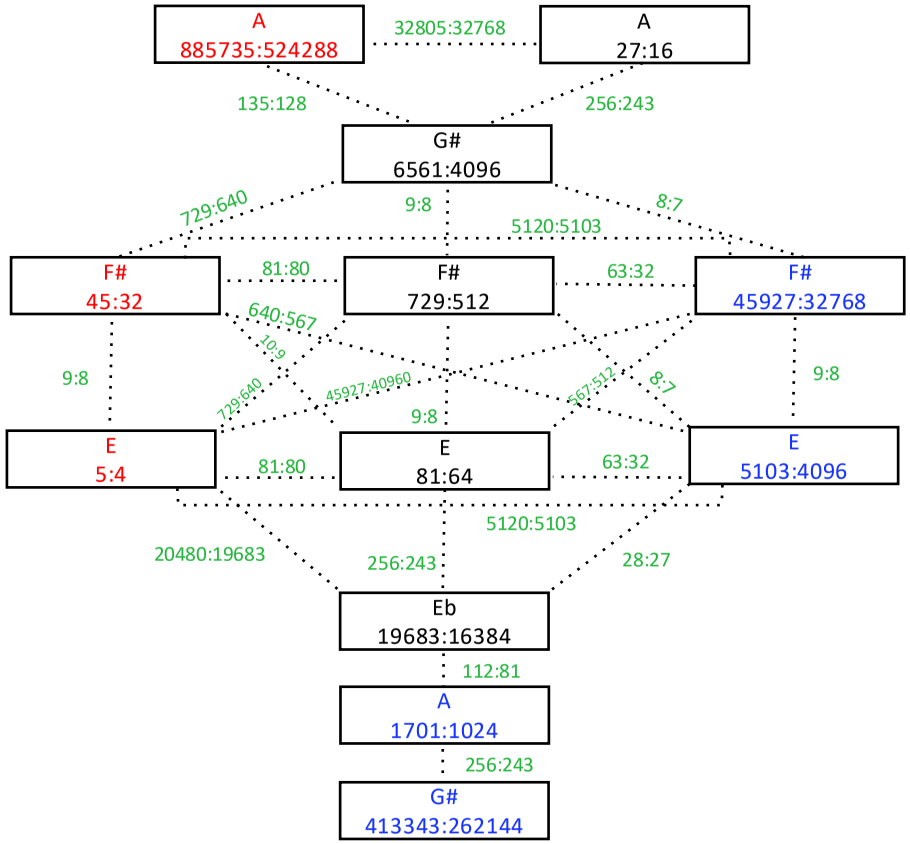
Chord #11, Iteration 2



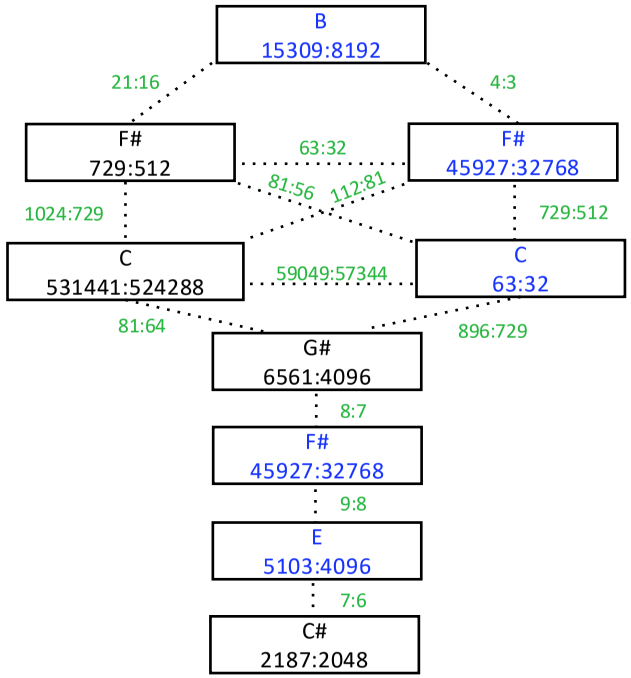
Chord #12, Iteration 1



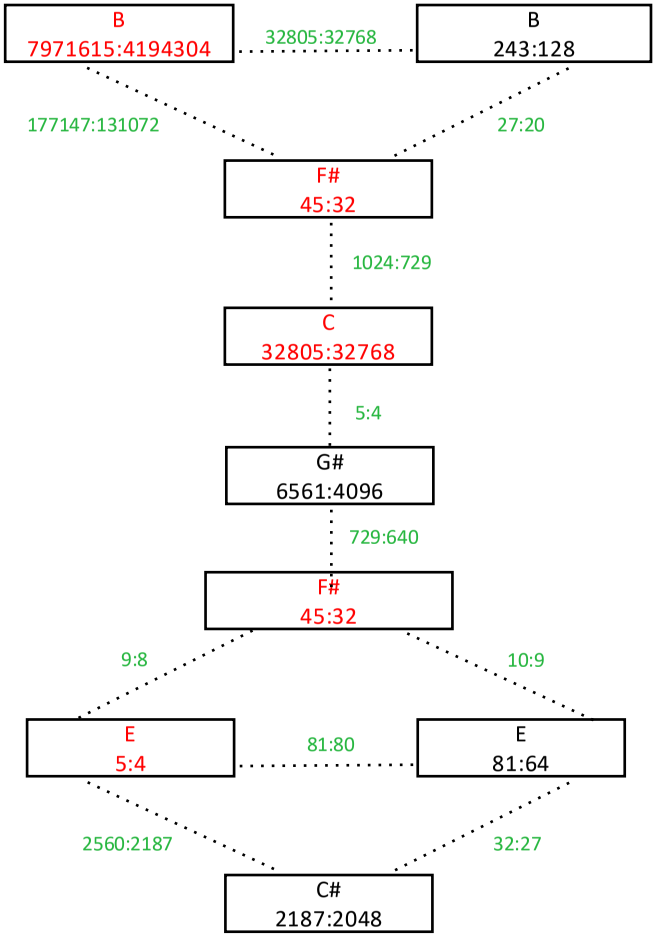
Chord #12, Iteration 2



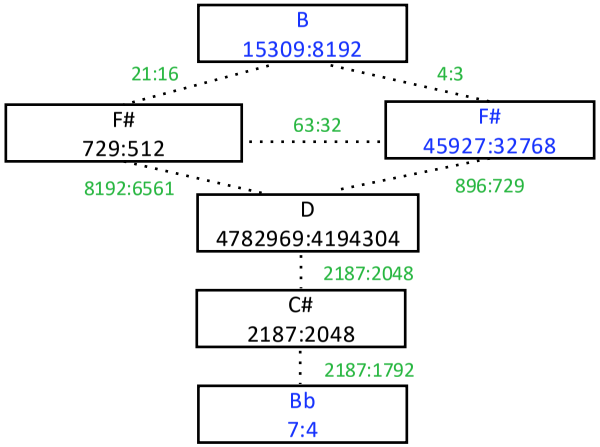
Chord #13, Iteration 1



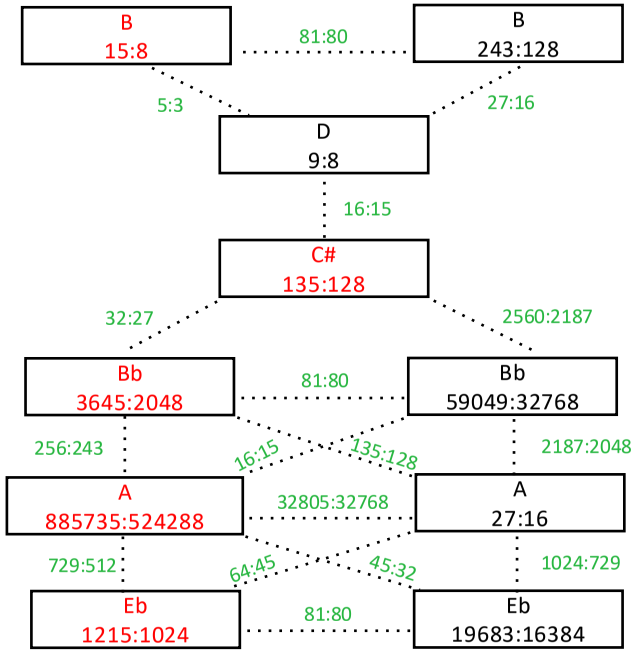
Chord #13, Iteration 2



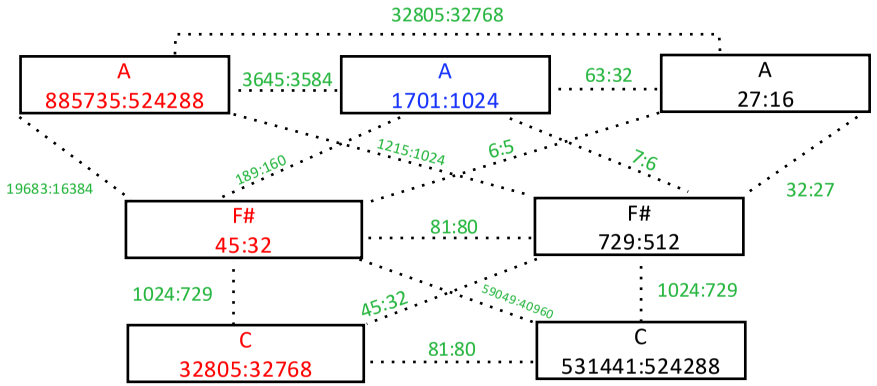
Chord #14, Iteration 1



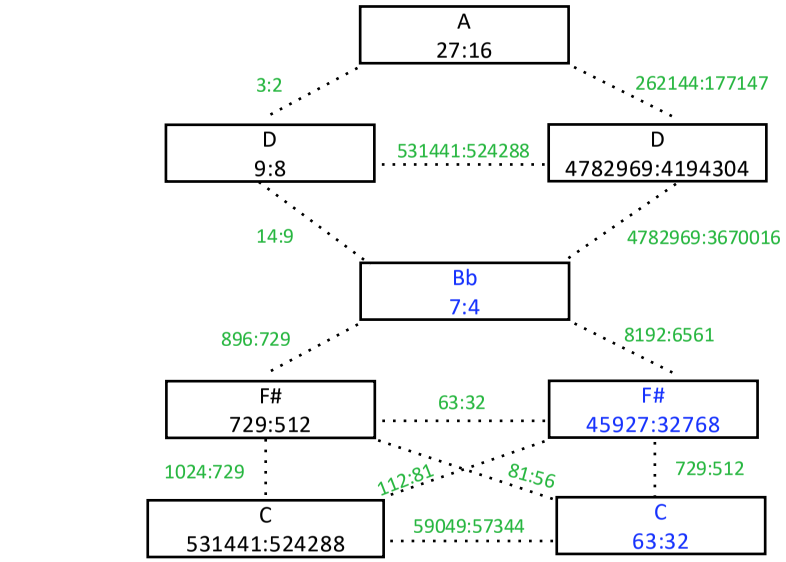
Chord #14, Iteration 2



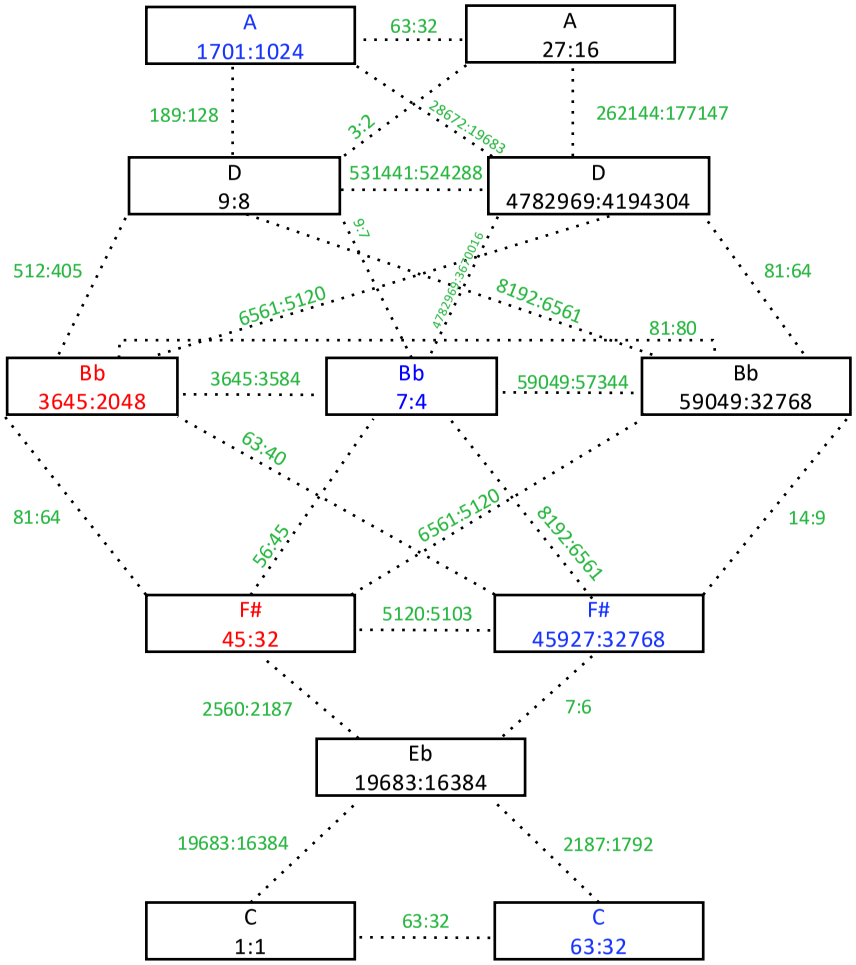
Chord #15, Iteration 1

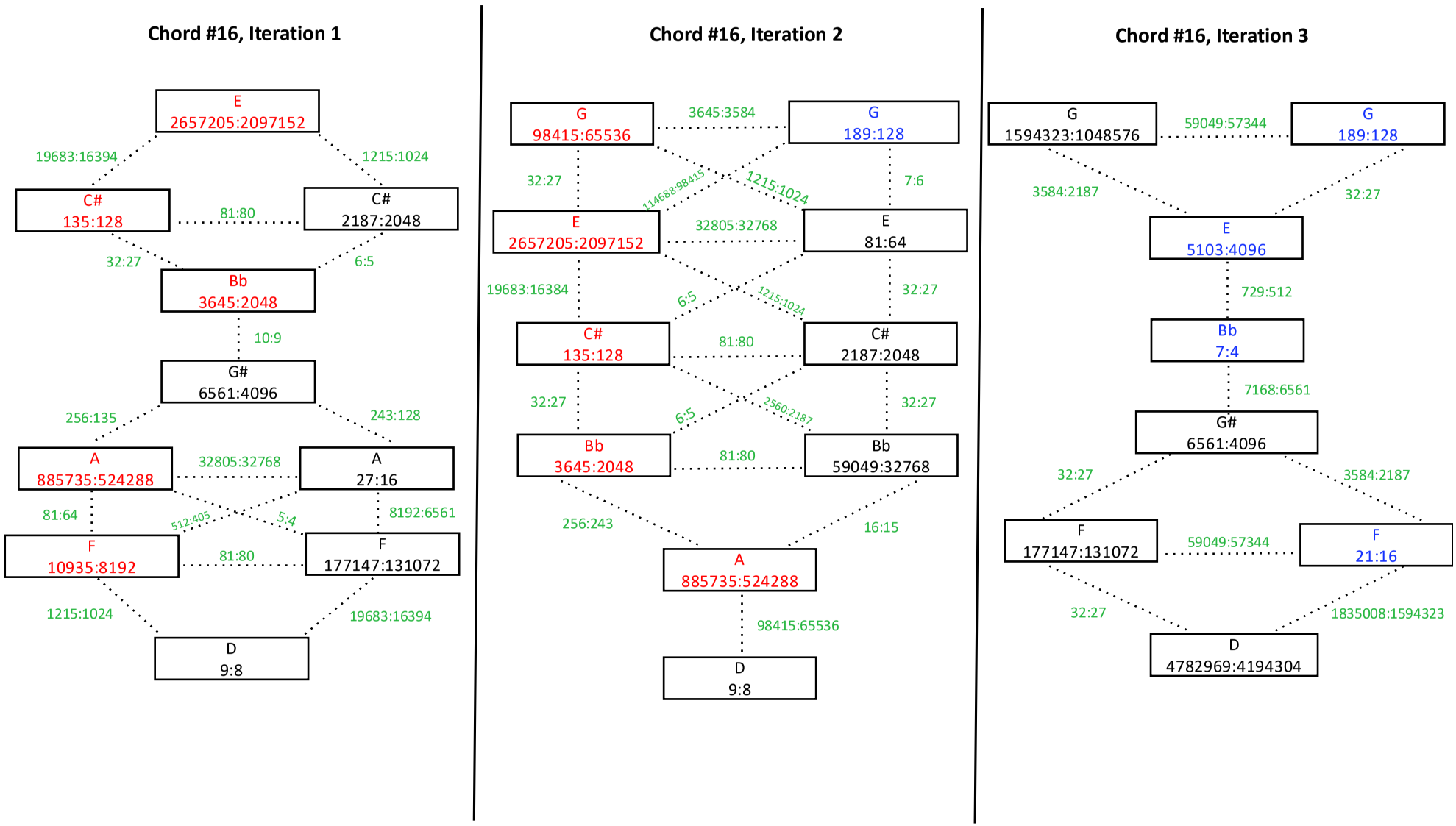


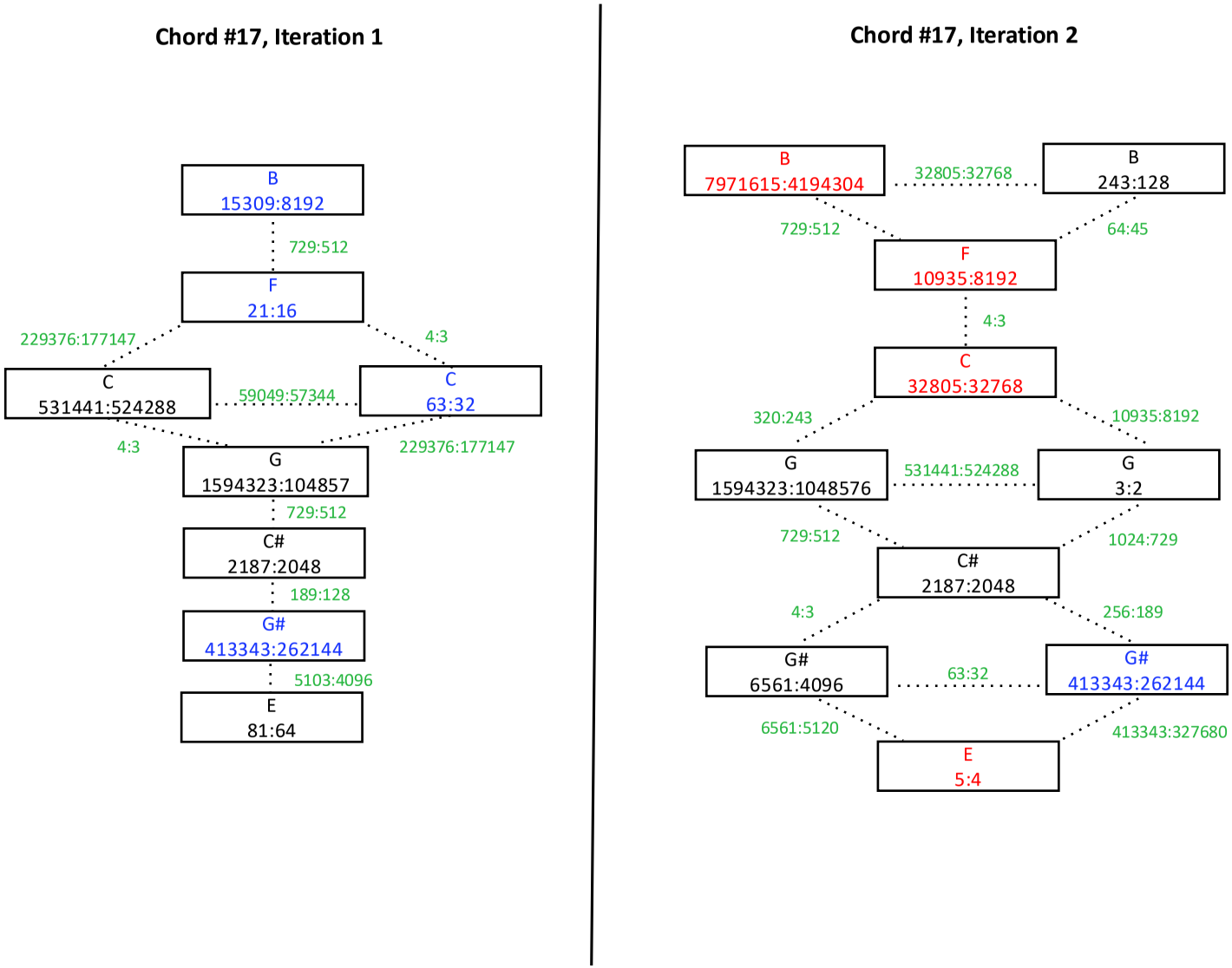
Chord #15, Iteration 2



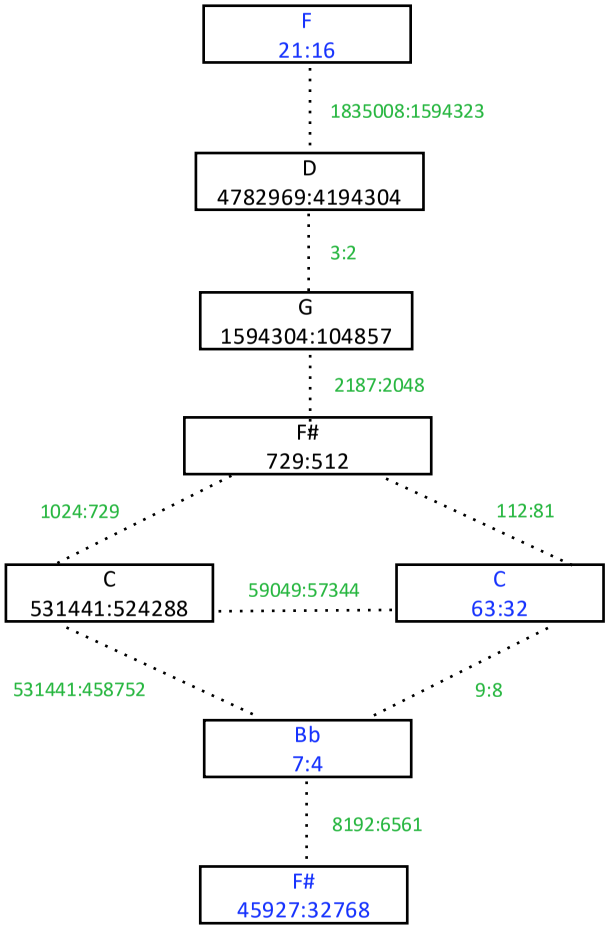
Chord #15, Iteration 2



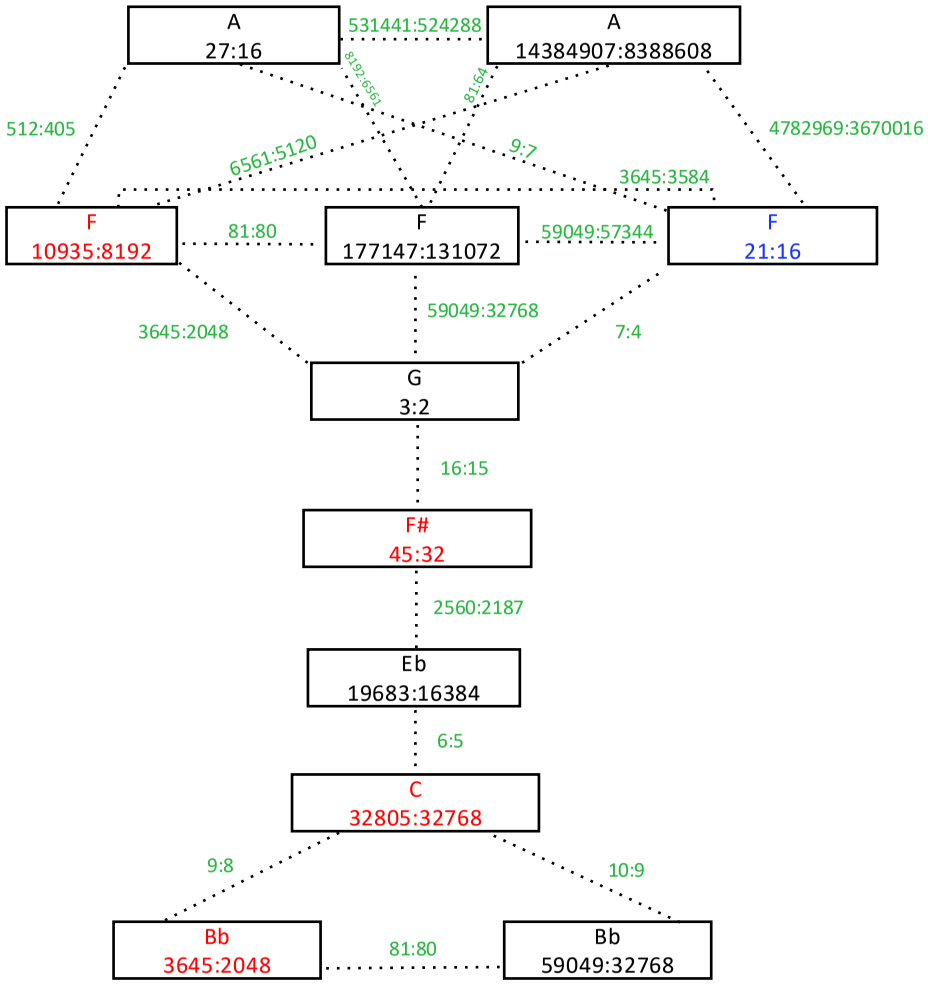

Chord #16, Iteration 1
E
2657205:2097152
19683:16394
1215:1024
C#
135:128
81:80
C#
2187:2048
32:27
6:5
Bb
3645:2048
10:9
G#
6561:4096
256:135
243:128
A
885735:524288
32805:32768
A
27:16
81:64
512:405
5:4
8192:6561
F
10935:8192
81:80
F
177147:131072
1215:1024
19683:16394
D
9:8
Chord #16, Iteration 2
G
98415:65536
3645:3584
G
189:128
32:27
114688:98415
1215:1024
7:6
E
2657205:2097152
32805:32768
E
81:64
19683:16384
6:5
1215:1024
32:27
C#
135:128
81:80
C#
2187:2048
32:27
6:5
2560:2187
32:27
Bb
3645:2048
81:80
Bb
59049:32768
256:243
16:15
A
885735:524288
98415:65536
D
9:8
Chord #16, Iteration 3
G
1594323:1048576
59049:57344
G
189:128
3584:2187
32:27
E
5103:4096
729:512
Bb
7:4
7168:6561
G#
6561:4096
32:27
3584:2187
F
177147:131072
59049:57344
F
21:16
32:27
1835008:1594323
D
4782969:4194304


Chord #17, Iteration 1
B
15309:8192
729:512
F
21:16
229376:177147
4:3
C
531441:524288
59049:57344
C
63:32
4:3
229376:177147
G
1594323:104857
729:512
C#
2187:2048
189:128
G#
413343:262144
5103:4096
E
81:64
Chord #17, Iteration 2
B
7971615:4194304
32805:32768
B
243:128
729:512
64:45
F
10935:8192
4:3
C
32805:32768
320:243
10935:8192
G
1594323:1048576
531441:524288
G
3:2
729:512
1024:729
C#
2187:2048
4:3
256:189
G#
6561:4096
63:32
G#
413343:262144
6561:5120
413343:327680
E
5:4

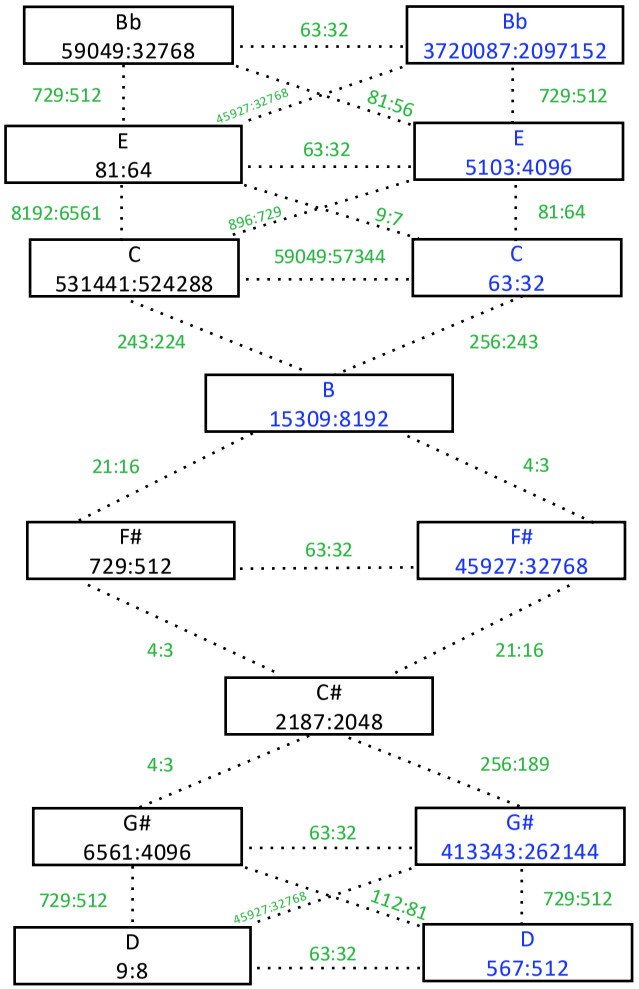
Chord #19, Iteration 1



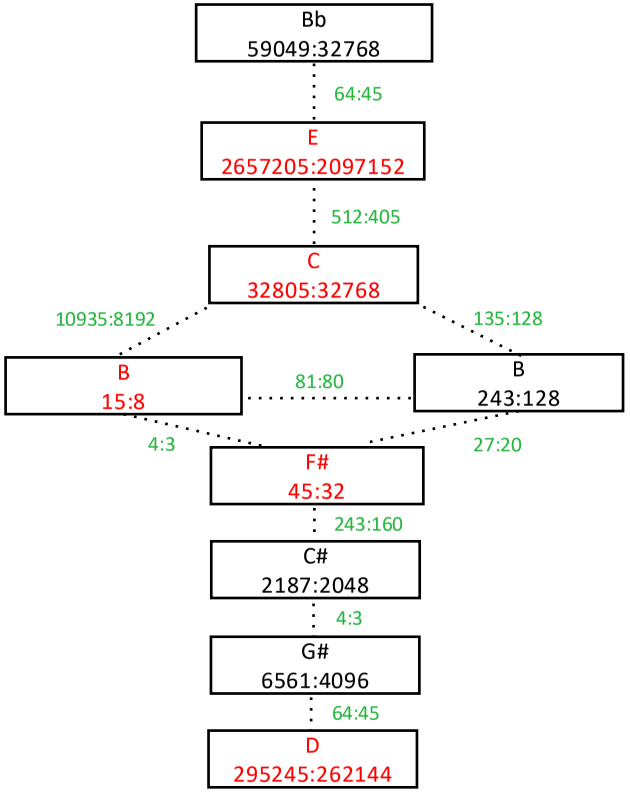
Chord #19, Iteration 2

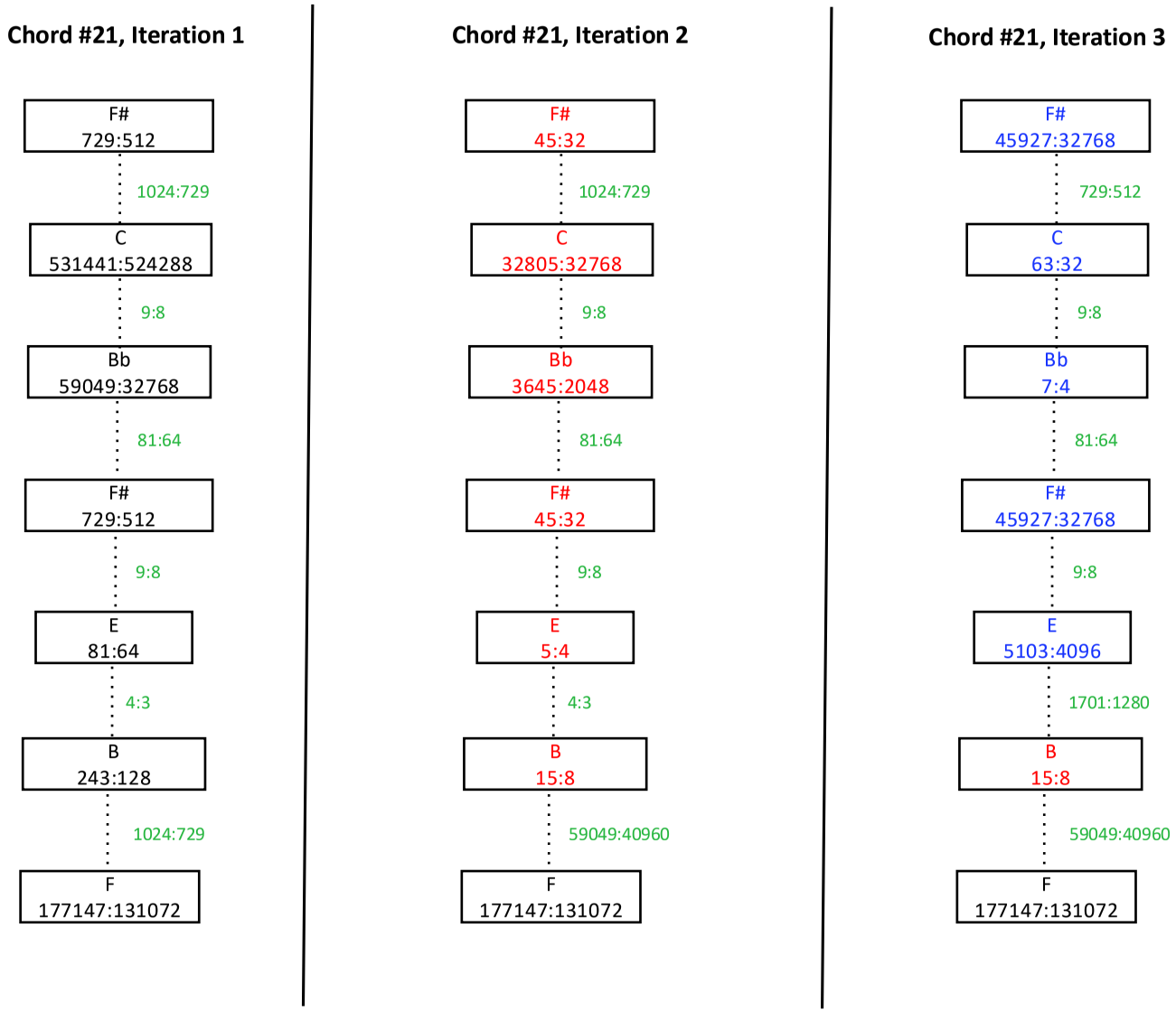


Chord #20, Iteration 1

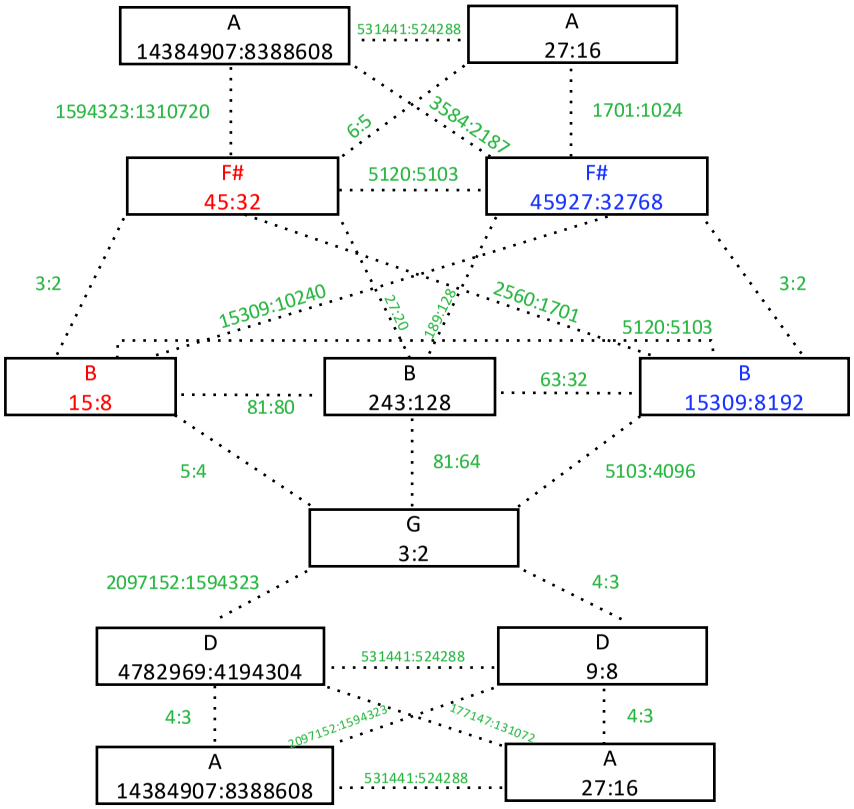


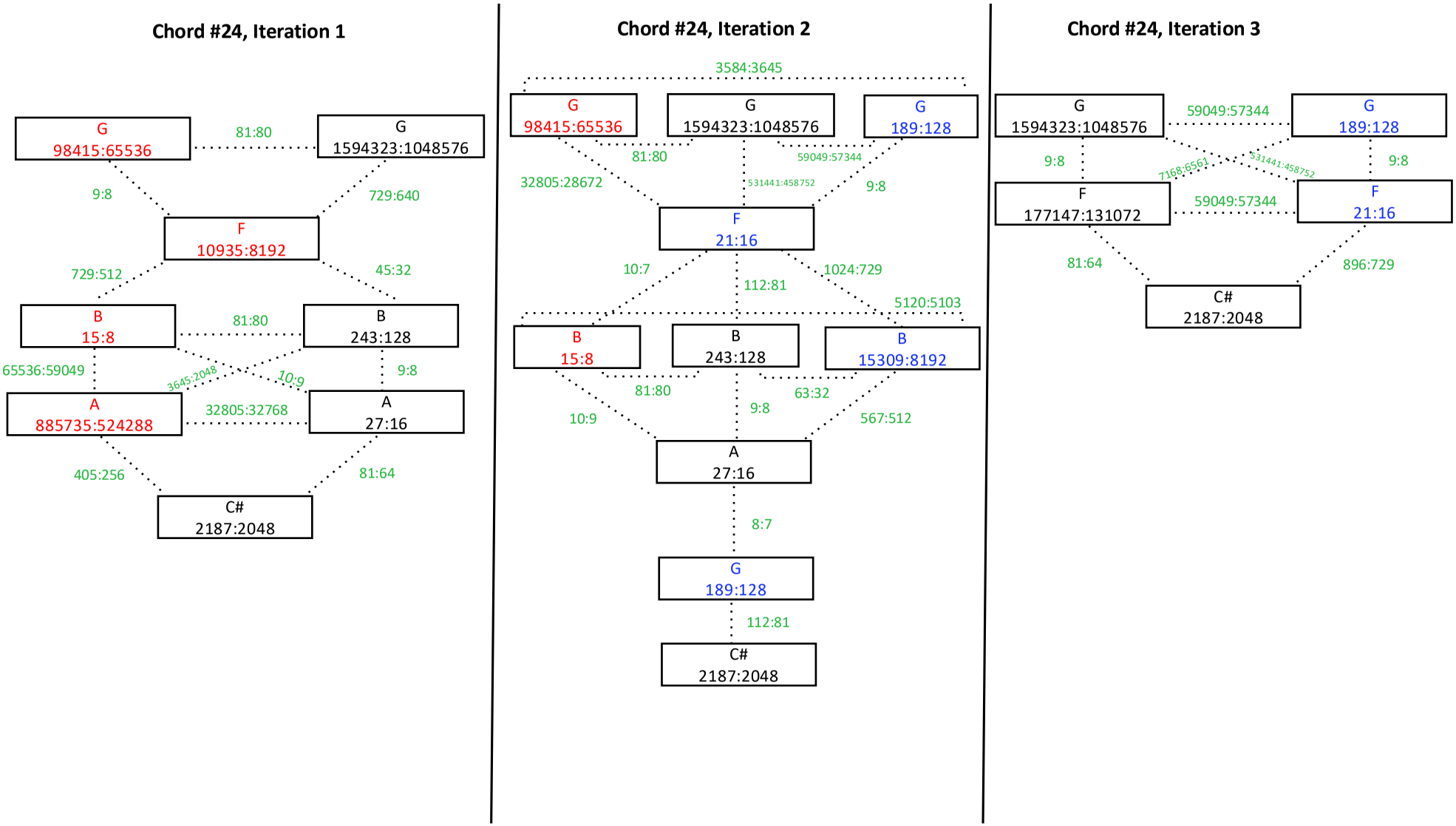
Chord #20, Iteration 2



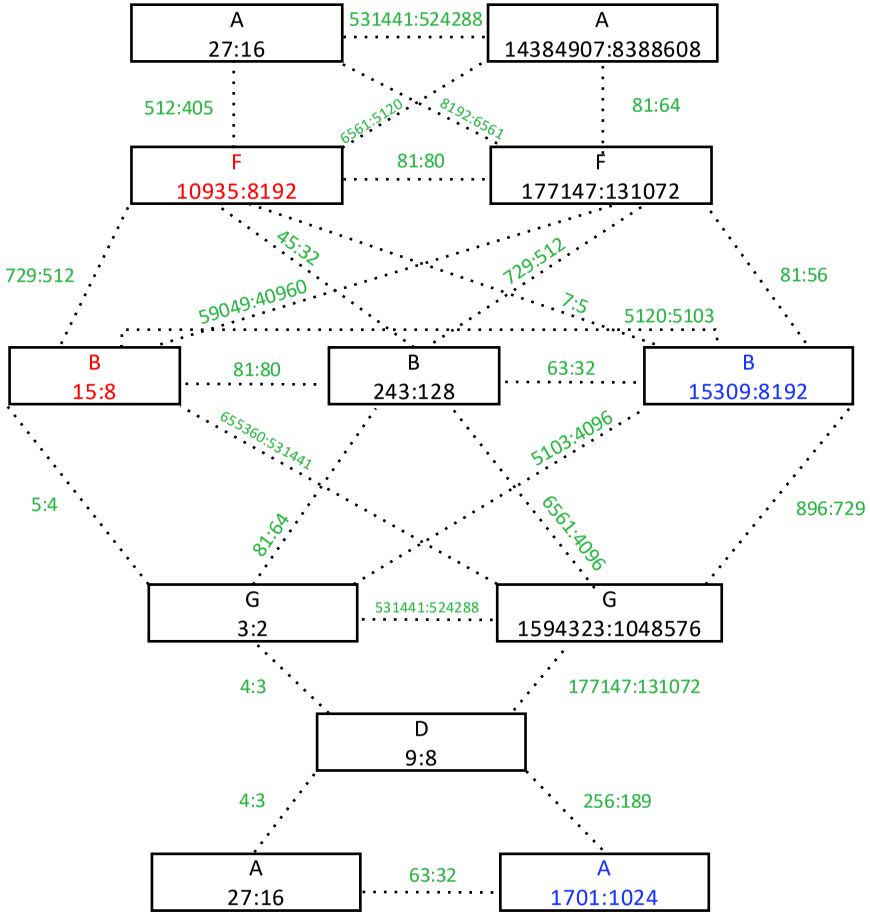

Chord #21, Iteration 1
F#
729:512
1024:729
C
531441:524288
9:8
Bb
59049:32768
81:64
F#
729:512
9:8
E
81:64
4:3
B
243:128
1024:729
F
177147:131072
Chord #21, Iteration 2
F#
45:32
1024:729
C
32805:32768
9:8
Bb
3645:2048
81:64
F#
45:32
9:8
E
5:4
4:3
B
15:8
59049:40960
F
177147:131072
Chord #21, Iteration 3
F#
45927:32768
729:512
C
63:32
9:8
Bb
7:4
81:64
F#
45927:32768
9:8
E
5103:4096
1701:1280
B
15:8
59049:40960
F
177147:131072

Chord #23

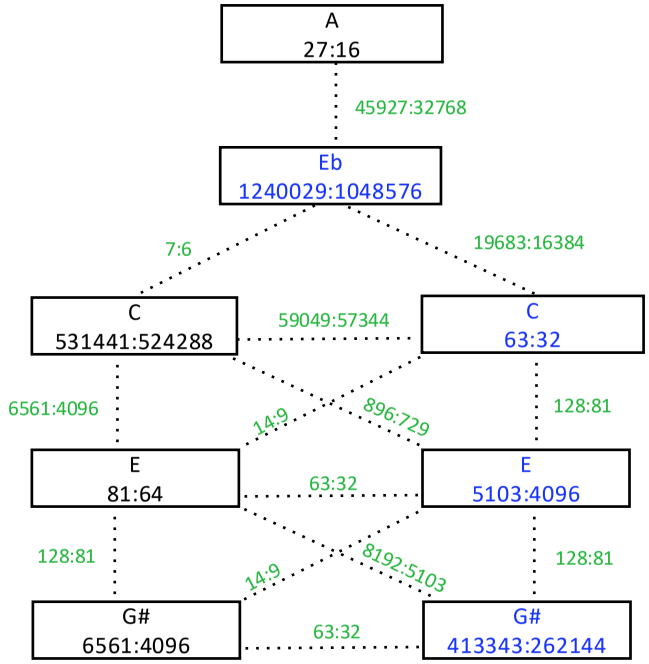

A
14384907:8388608
531441:524288
A
27:16
1594323:1310720
6:5
3584:2187
1701:1024
F#
45:32
5120:5103
F#
45927:32768
3:2
15309:10240
27:20
189:128
2560:1701
3:2
5120:5103
B
15:8
81:80
B
243:128
63:32
B
15309:8192
5:4
81:64
5103:4096
G
3:2
2097152:1594323
4:3
D
4782969:4194304
531441:524288
D
9:8
4:3
2097152:1594323
177147:131072
4:3
A
14384907:8388608
531441:524288
A
27:16


Chord #24, Iteration 1
G
98415:65536
81:80
G
1594323:1048576
9:8
729:640
F
10935:8192
729:512
45:32
B
15:8
81:80
B
243:128
65536:59049
3645:2048
10:9
9:8
A
885735:524288
32805:32768
A
27:16
405:256
81:64
C#
2187:2048
Chord #24, Iteration 2
3584:3645
G
98415:65536
G
1594323:1048576
G
189:128
81:80
59049:57344
32805:28672
531441:458752
9:8
F
21:16
10:7
112:81
1024:729
5120:5103
B
15:8
B
243:128
B
15309:8192
81:80
63:32
10:9
9:8
567:512
A
27:16
8:7
G
189:128
112:81
C#
2187:2048
Chord #24, Iteration 3
G
1594323:1048576
59049:57344
G
189:128
9:8
7168:6561
531441:458752
9:8
F
177147:131072
59049:57344
F
21:16
81:64
896:729
C#
2187:2048

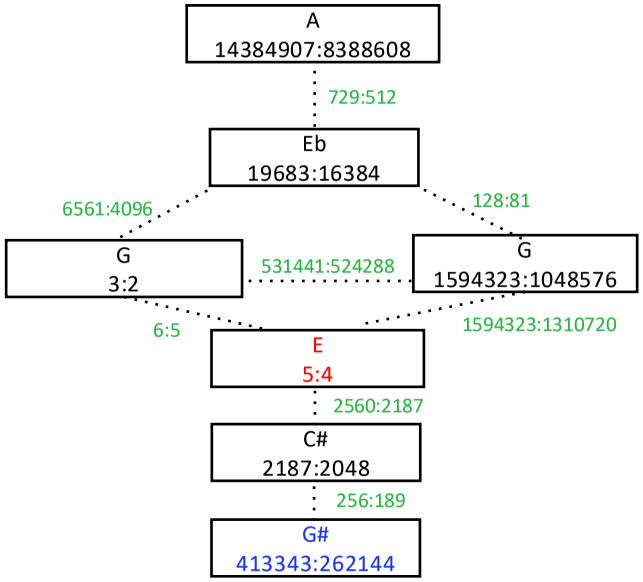
Chord #25


A
27:16
531441:524288
A
14384907:8388608
512:405
6561:5120
8192:6561
81:64
F
10935:8192
81:80
F
177147:131072
729:512
45:32
729:512
81:56
59049:40960
7:5
5120:5103
B
15:8
81:80
B
243:128
63:32
B
15309:8192
655360:531441
5103:4096
5:4
81:64
6561:4096
896:729
G
3:2
531441:524288
G
1594323:1048576
4:3
177147:131072
D
9:8
4:3
256:189
A
27:16
63:32
A
1701:1024

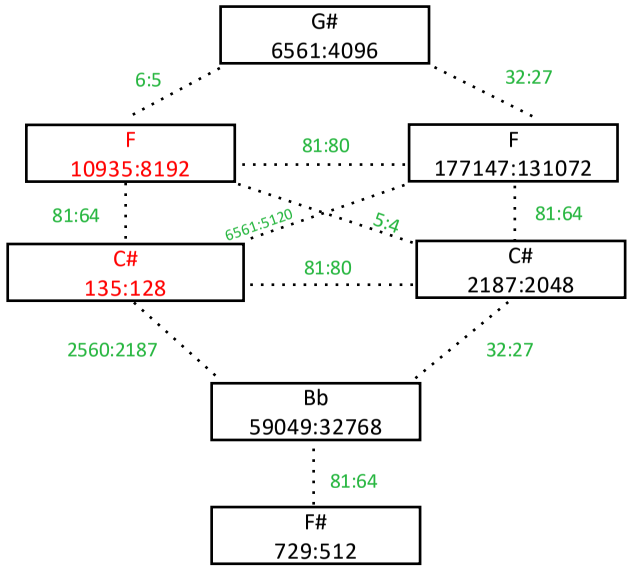
Chord #26, Iteration 1



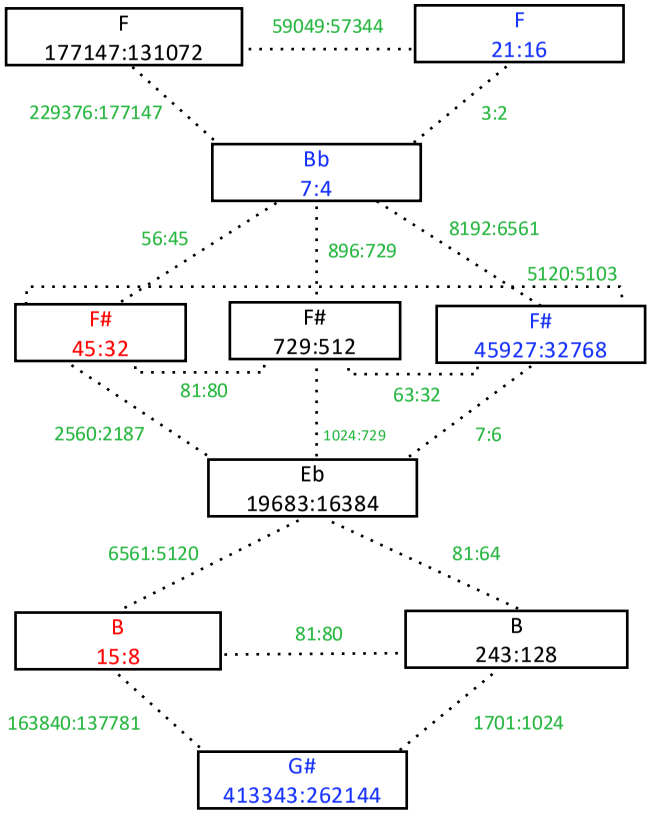
Chord #26, Iteration 2

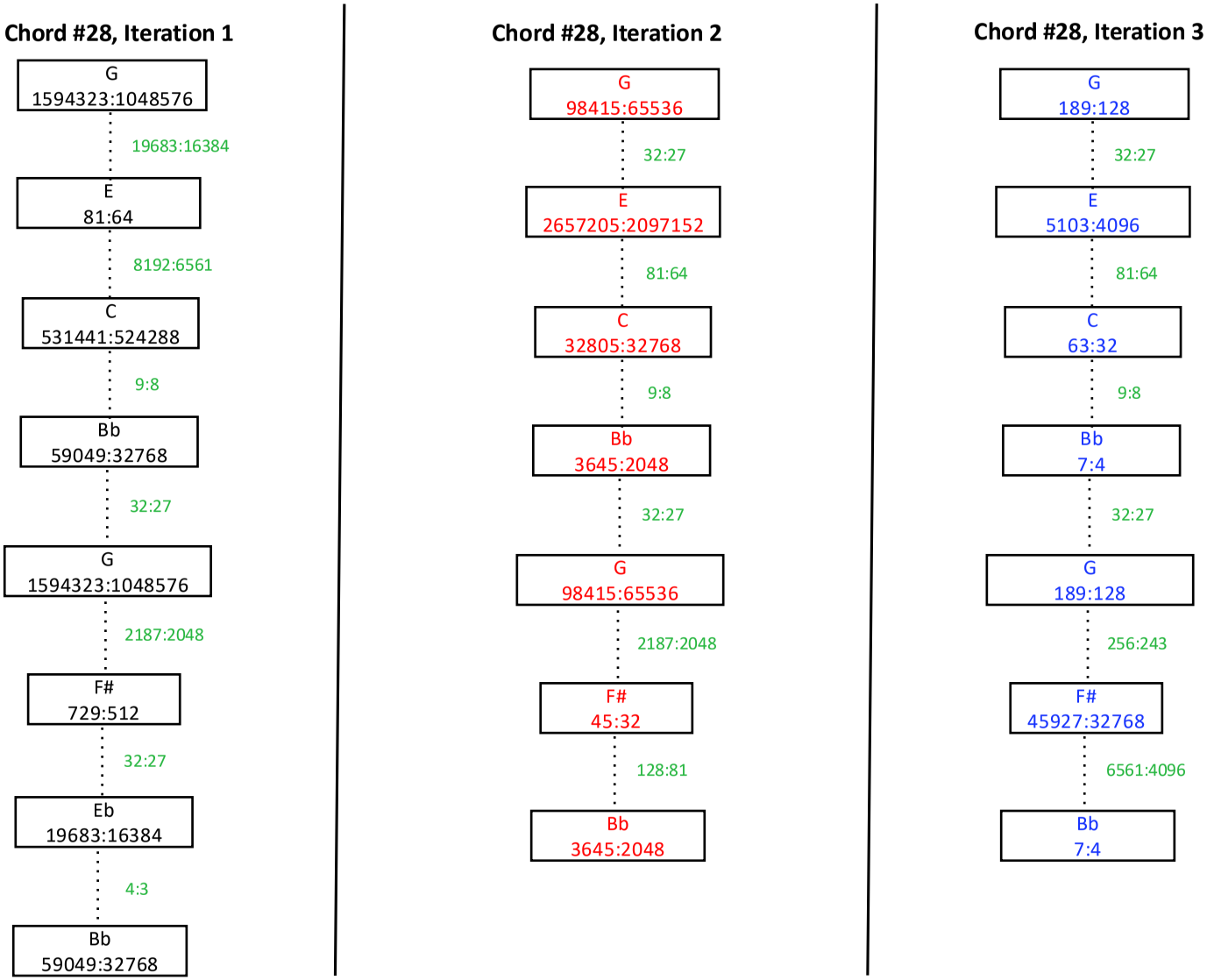


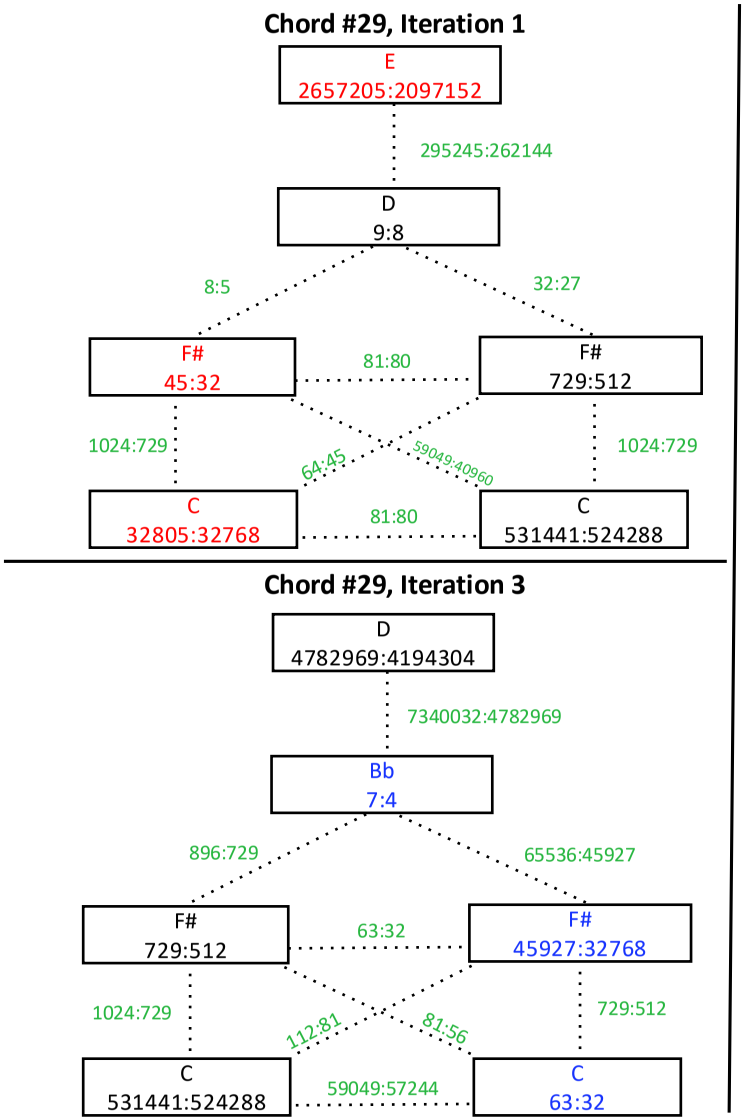
Chord #27, Iteration 1

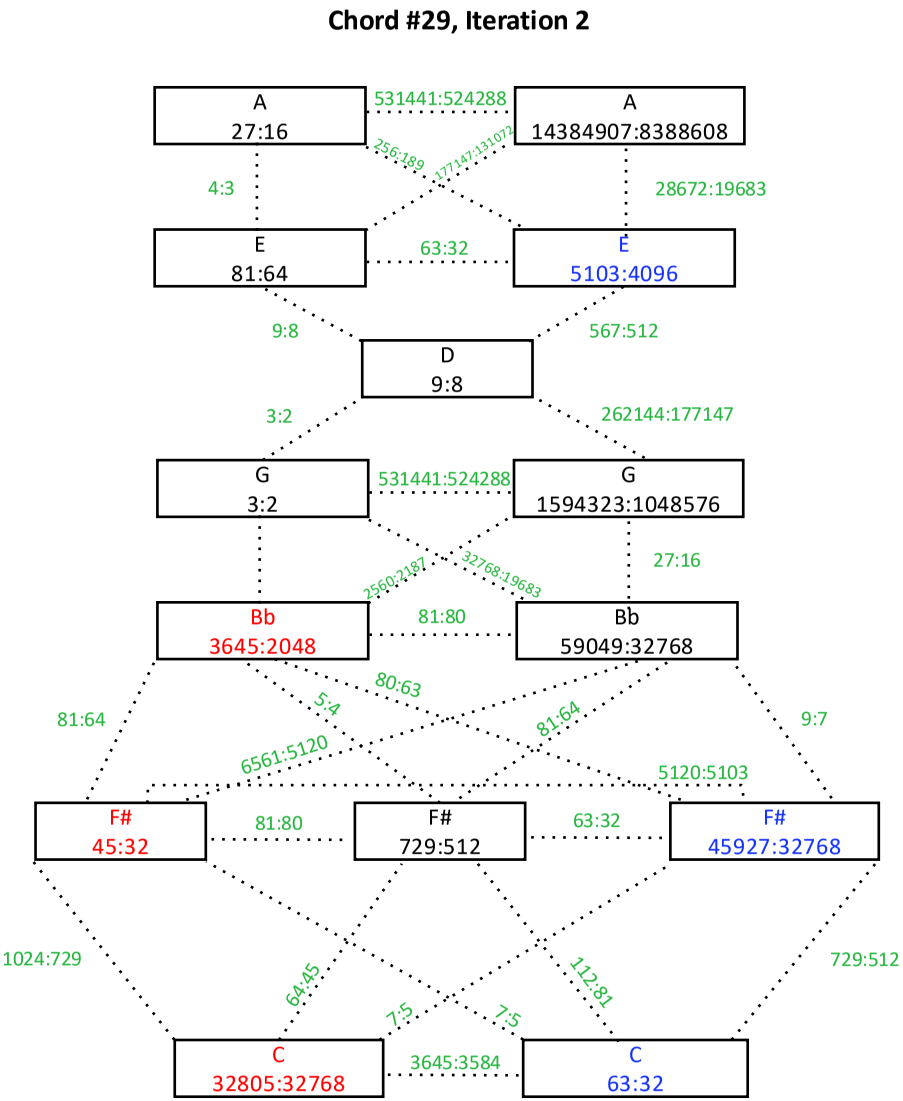


Chord #27, Iteration 2

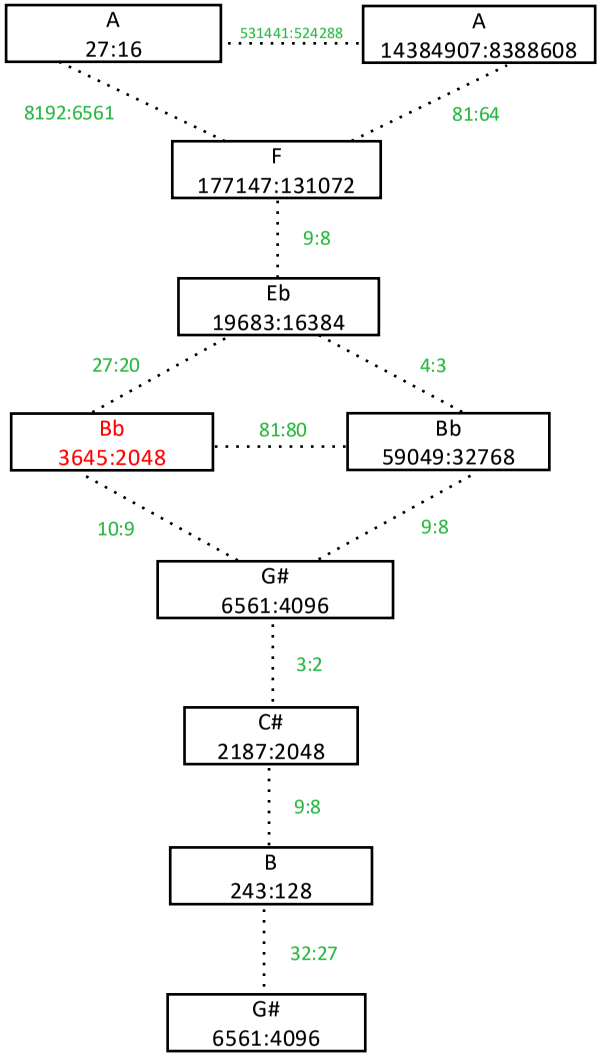



Chord #28, Iteration 1
G
1594323:1048576
19683:16384
E
81:64
8192:6561
C
531441:524288
9:8
Bb
59049:32768
32:27
G
1594323:1048576
2187:2048
F#
729:512
32:27
Eb
19683:16384
4:3
Bb
59049:32768
Chord #28, Iteration 2
G
98415:65536
32:27
E
2657205:2097152
81:64
C
32805:32768
9:8
Bb
3645:2048
32:27
G
98415:65536
2187:2048
F#
45:32
128:81
Bb
3645:2048
Chord #28, Iteration 3
G
189:128
32:27
E
5103:4096
81:64
C
63:32
9:8
Bb
7:4
32:27
G
189:128
256:243
F#
45927:32768
6561:4096
Bb
7:4

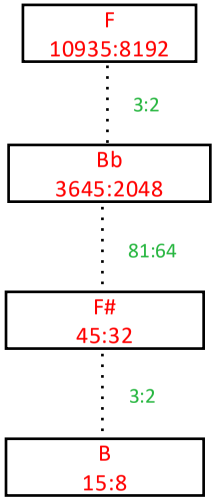

Chord #29, Iteration 1
E
2657205:2097152
295245:262144
D
9:8
8:5
32:27
F#
45:32
81:80
F#
729:512
1024:729
64:45
59049:40960
1024:729
C
32805:32768
81:80
C
531441:524288
Chord #29, Iteration 3
D
4782969:4194304
7340032:4782969
Bb
7:4
896:729
65536:45927
F#
729:512
63:32
F#
45927:32768
1024:729
112:81
81:56
729:512
C
531441:524288
59049:57244
C
63:32


Chord #29, Iteration 2
A
27:16
531441:524288
A
14384907:8388608
256:189
177147:131072
4:3
28672:19683
E
81:64
63:32
E
5103:4096
9:8
567:512
D
9:8
3:2
262144:177147
G
3:2
531441:524288
G
1594323:1048576
2560:2187
32768:19683
27:16
Bb
3645:2048
81:80
Bb
59049:32768
80:63
5:4
81:64
81:64
9:7
6561:5120
5120:5103
F#
45:32
81:80
F#
729:512
63:32
F#
45927:32768
1024:729
64:45
7:5
7:5
112:81
729:512
C
32805:32768
3645:3584
C
63:32

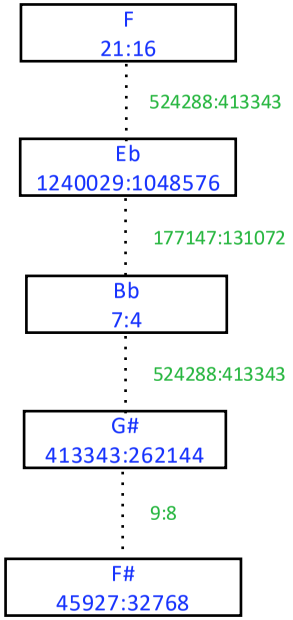
Chord #30, Iteration 1

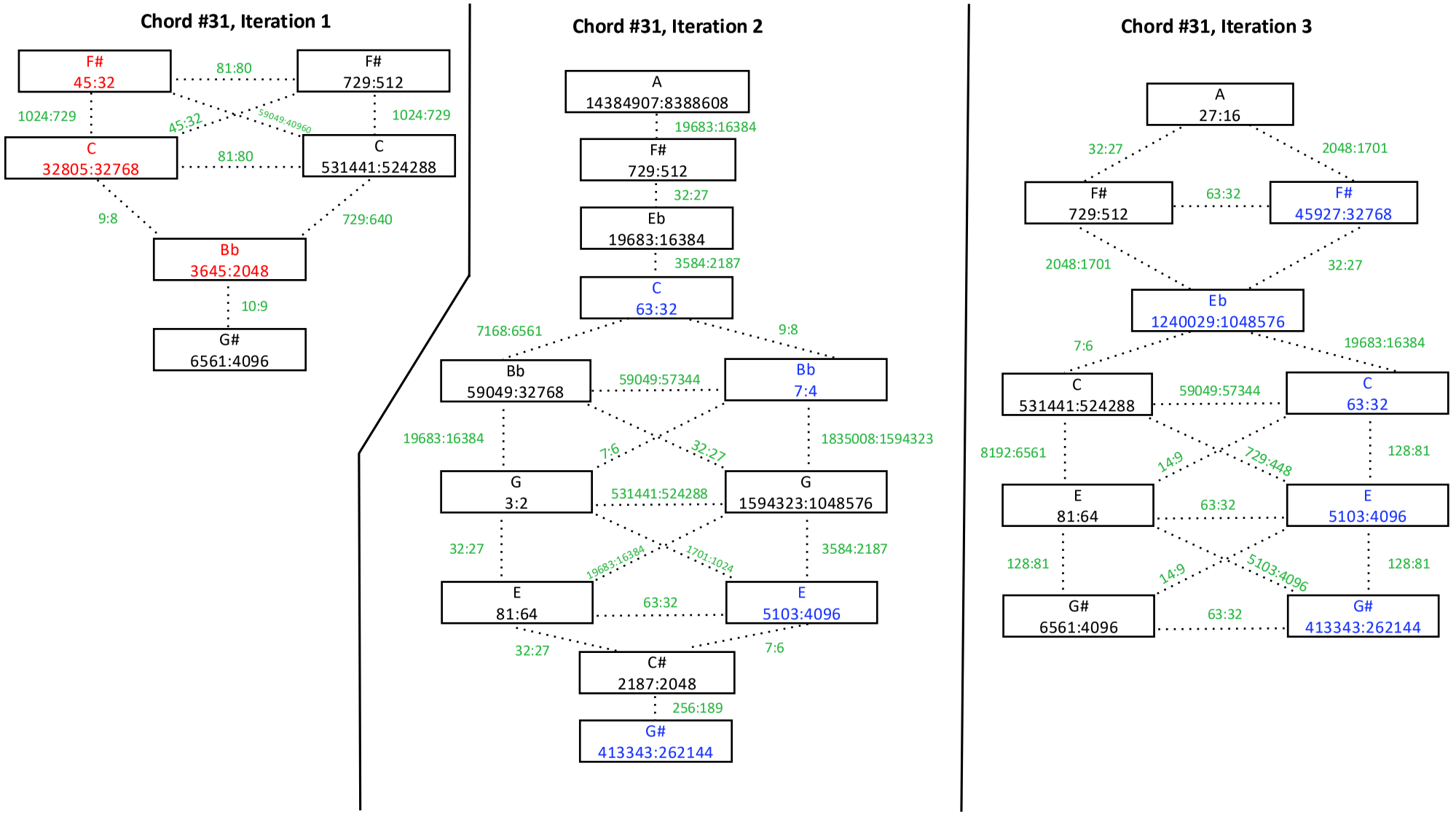


Chord #30, Iteration 2

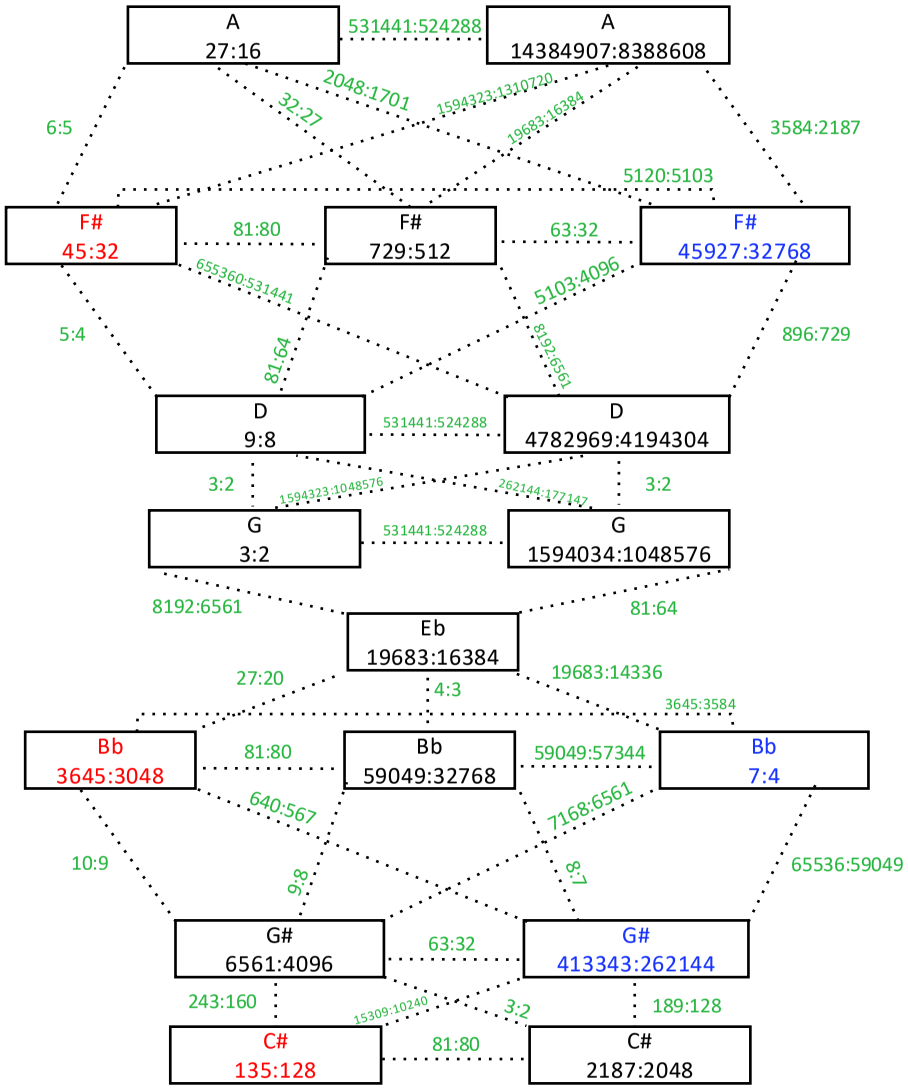


Chord #30, Iteration 3

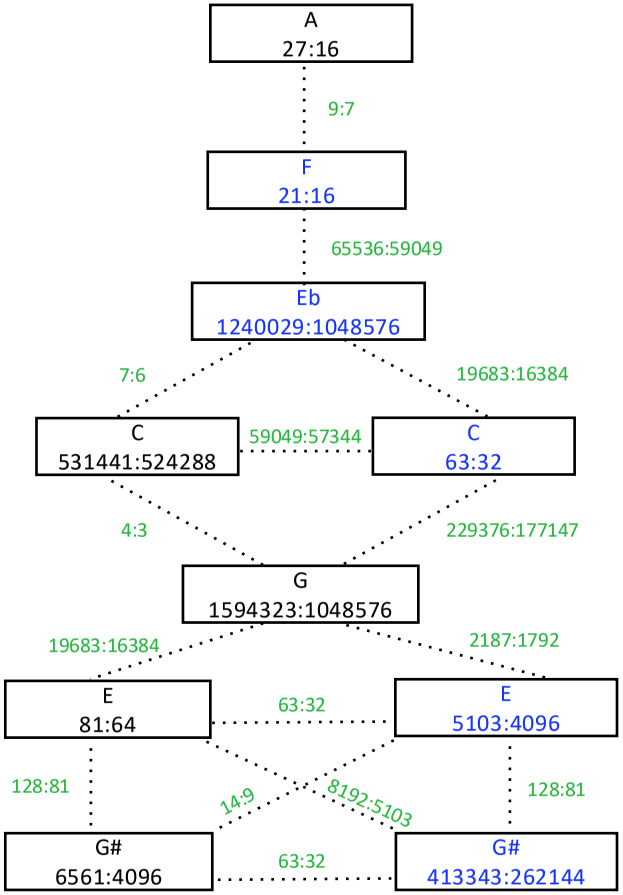



Chord #31, Iteration 1
F#
45:32
81:80
F#
729:512
1024:729
45:32
59049:40960
1024:729
C
32805:32768
81:80
C
531441:524288
9:8
729:640
Bb
3645:2048
10:9
G#
6561:4096
Chord #31, Iteration 2
A
14384907:8388608
19683:16384
F#
729:512
32:27
Eb
19683:16384
3584:2187
C
63:32
7168:6561
9:8
Bb
59049:32768
59049:57344
Bb
7:4
19683:16384
7:6
32:27
1835008:1594323
G
3:2
531441:524288
G
1594323:1048576
32:27
19683:16384
1701:1024
3584:2187
E
81:64
63:32
E
5103:4096
32:27
7:6
C#
2187:2048
256:189
G#
413343:262144
Chord #31, Iteration 3
A
27:16
32:27
2048:1701
F#
729:512
63:32
F#
45927:32768
2048:1701
32:27
Eb
1240029:1048576
7:6
19683:16384
C
531441:524288
59049:57344
C
63:32
8192:6561
14:9
729:448
128:81
E
81:64
63:32
E
5103:4096
128:81
14:9
5103:4096
128:81
G#
6561:4096
63:32
G#
413343:262144

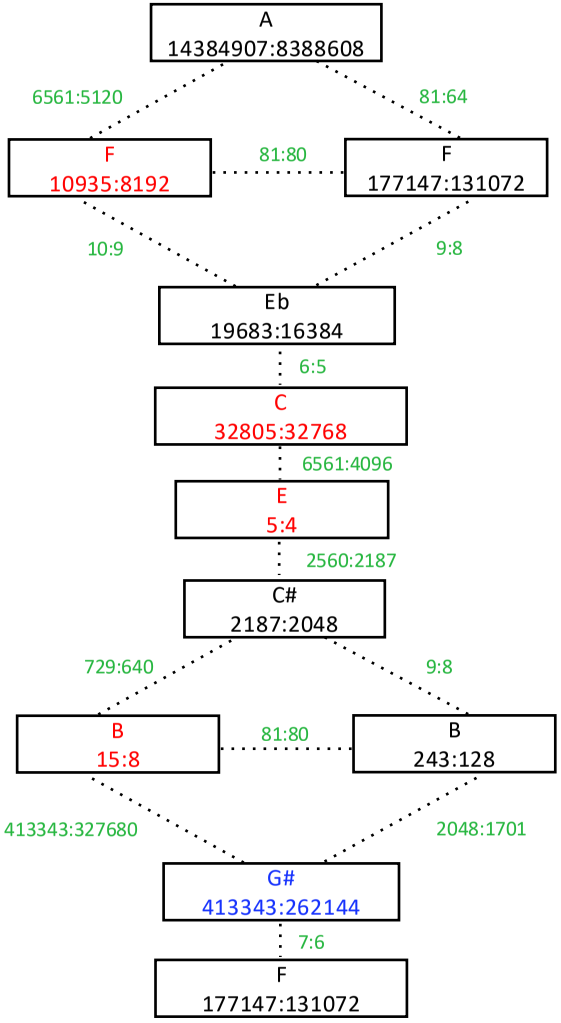
Chord #32


A
27:16
531441:524288
A
14384907:8388608
2048:1701
1594323:1310720
32:27
19683:16384
6:5
3584:2187
5120:5103
F#
45:32
81:80
F#
729:512
63:32
F#
45927:32768
655360:531441
5103:4096
5:4
81:64
8192:6561
896:729
D
9:8
531441:524288
D
4782969:4194304
3:2
1594323:1048576
262144:177147
3:2
G
3:2
531441:524288
G
1594034:1048576
8192:6561
81:64
Eb
19683:16384
27:20
4:3
19683:14336
3645:3584
Bb
3645:3048
81:80
Bb
59049:32768
59049:57344
Bb
7:4
640:567
7168:6561
10:9
9:8
8:7
65536:59049
G#
6561:4096
63:32
G#
413343:262144
243:160
15309:10240
3:2
189:128
C#
135:128
81:80
C#
2187:2048

Chord #33, Iteration 1



Chord #33, Iteration 2



Appendix D, part six

(Table showing basic descriptive information about the modes used in movement two of *The tale of Kiều)*

Appendix D, part six:
Table showing basic descriptive information about the modes used in movement two of *The tale of Kiều.*

Bar no.	Mode	Symmetrical	Transposable	Symmetry compromised for a comma?
70	21:16 – 10935:8192 – 7:4 – 3645:8192	Yes	Yes	No
71	137781:131072 – 135:128 – 413343:262144 - 405:256	Yes	Yes	No
72	177147:131072 – 59049:32768	Yes	Yes	No
73	137781:131072 – 135:128 – 413343:262144 - 405:256	Yes	Yes	No
74	177147:131072 – 59049:32768	Yes	Yes	No
75	137781:131072 – 135:128 – 413343:262144 - 405:256	Yes	Yes	No
76	2187:2048 – 177147:131072 – 6561:4096 – 59049:32768	Yes	Yes	No
77	1240029:1048576 – 1215:1024 – 45927:32768 – 45:32	Yes	Yes	No
78	1240029:1048576 – 1215:1024 – 405:256	No	Yes	Yes
79	1240029:1048576 – 1215:1024 – 45927:32768 – 45:32 – 7:4 – 3645:8192	No	Yes	Yes
80	2187:2048 – 19683:16384 – 45927:32768 – 3645:2048 – 59049:32768	No	Yes	No
81	1215:1024 – 19683:16384 – 729:512 – 3645:2048 – 59049:32768	No	Yes	No
82	1215:1024 – 19683:16384 – 3645:2048 – 59049:32768	Yes	Yes	No
83	137781:131072 – 135:128 – 1240029:1048576 – 15309:8192 – 15:8	No	Yes	Yes
84	137781:131072 – 135:128 – 2187:2048 – 19683:16384 – 413343:262144 – 405:256 – 6561:4096	No	Yes	Yes
85	729:512 – 243:128	Yes	Yes	No

Appendix D, part six:
Table showing basic descriptive information about the modes used in movement two of *The tale of Kiều.*

86	2187:2048 – 19683:16384 – 729:512 – 6561:4096	Yes	Yes	No
87	137781:131072 – 135:128 –1240029:1048576 – 1215:1024 – 45927:32768 – 405:256 – 15309:8192 – 15:8	Yes	Yes	No
88	1240029:1048576 – 1215:1024 – 413343:262144 – 405:256	Yes	Yes	No
89	45927:32768 – 45:32 – 413343:262144	No	Yes	Yes
90	45927:32768 – 6561:4096	Yes	Yes	No
91	19683:16384 – 6561:4096	Yes	Yes	No
92	45927:32768 – 6561:4096	Yes	Yes	No
93	19683:16384 – 3645:2048 – 59049:32768	No	Yes	Yes
94	45927:32768 – 6561:4096	Yes	Yes	No
95	1240029:1048576 – 3720087:2097152 – 3645:2048	No	Yes	Yes
96	137781:131072 – 135:128 – 45927:32768 – 45:32	Yes	Yes	No
97	137781:131072 – 2187:2048 – 45927:32768 – 405:256	No	Yes	Yes
98	137781:131072 – 2187:2048 45927:32768 – 45:32 – 729:512 – 405:256 – 6561:4096 – 3720087:2097152	No	Yes	No
99	137781:131072 – 2187:2048 – 1215:1024 – 19683:16384 – 729:512 – 6561:4096	No	Yes	No
100	137781:131072 – 135:128 – 1240029:1048576 – 19683:16384	No	Yes	Yes
101	1215:1024 – 3720087:2097152	Yes	Yes	No
102	137781:131072 – 135:128 – 1240029:1048576	No	Yes	Yes
103	137781:131072 – 1215:1024 – 3720087:2097152	No	Yes	Yes
104	45927:32768 – 6561:4096	Yes	Yes	No

Appendix D, part six:
Table showing basic descriptive information about the modes used in movement two of *The tale of Kiều.*
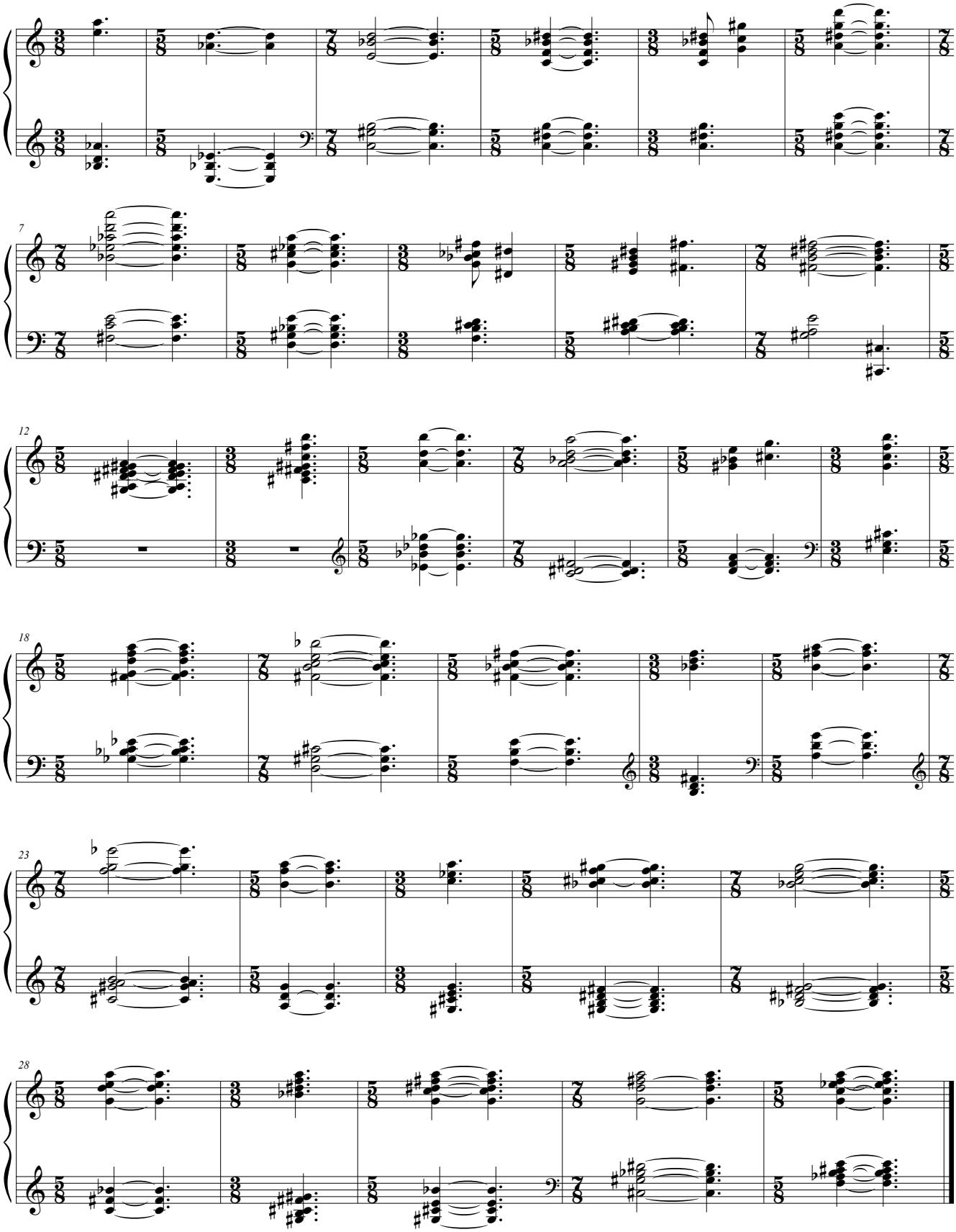
105	19683:16384 – 45927:32768	Yes	Yes	No
106	137781:131072 – 19683:16384	Yes	Yes	No
107	19683:16384 – 3645:2048 – 59049:32768	No	Yes	Yes
108	45927:32768 – 6561:4096	Yes	Yes	No
109	1215:1024 – 45927:32768 – 405:256 – 6561:4096 – 413343:262144	No	Yes	Yes
110	45927:32768 – 45:32 – 405:256 – 6561:4096 – 413343:262144	No	Yes	Yes
111	19683:16384 – 405:256 – 6561:4096	No	Yes	Yes
112	19683:16384 – 59049:32768	Yes	Yes	No
113	19683:16384 – 405:256 – 6561:4096	No	Yes	Yes
114	19683:16384 – 15309:8192 – 15:8	No	Yes	No
115	135:128 – 405:256 – 6561:4096	No	Yes	Yes

Appendix E

Appendix E, part one

(Piano sketch of movement one of *The tale of Kiều)*

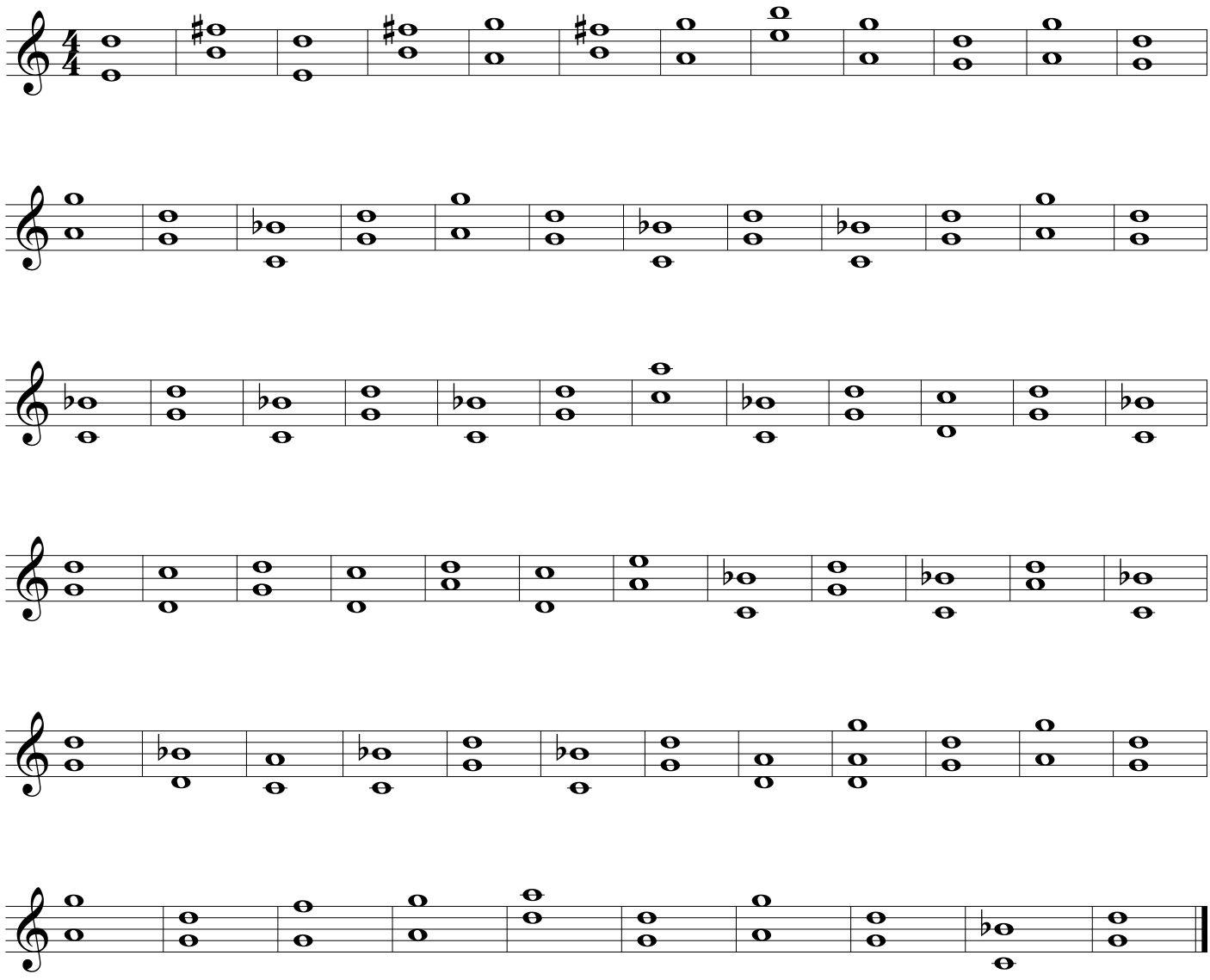
Appendix E, part one:
Piano sketch of movement one of *The tale of Kiều*



Appendix E, part two

(Piano sketch of the outline of movement five of *The tale of Kiều)*

Appendix E, part two:
Piano sketch of the outline of movement five of The tale of Kiều
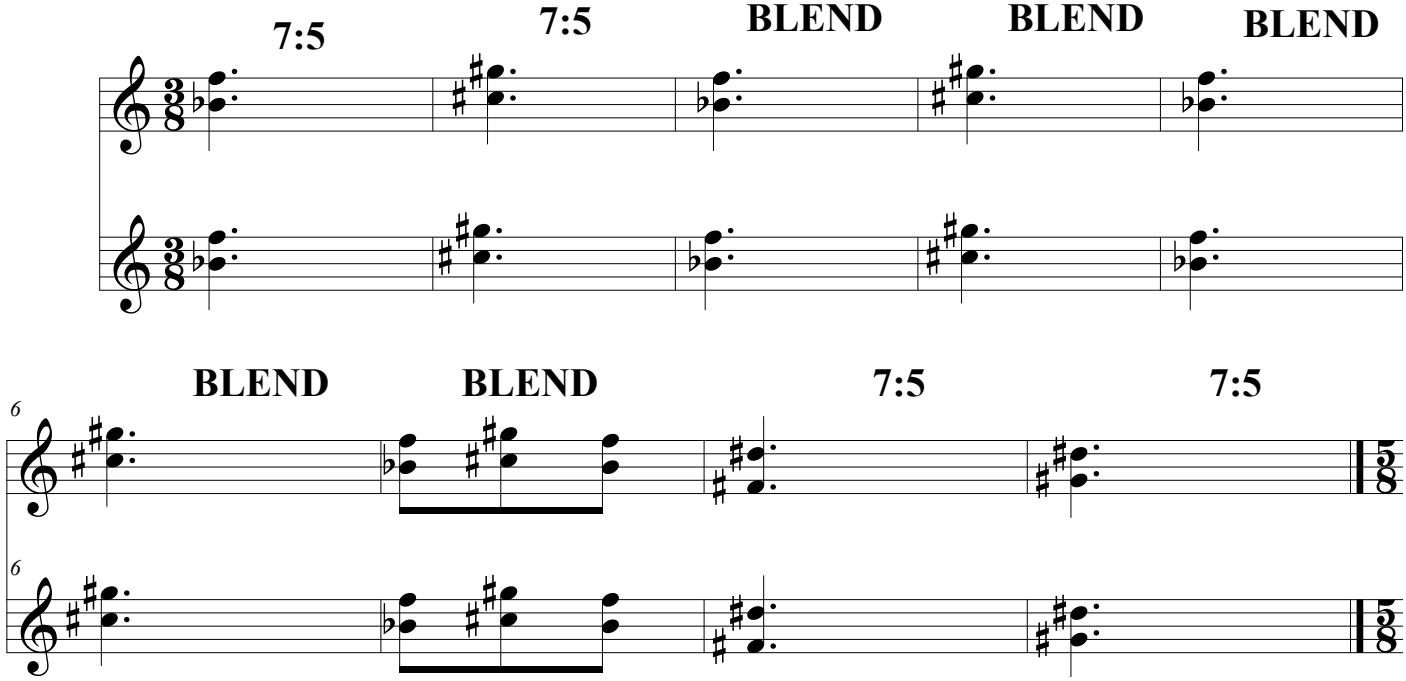


Appendix E, part three

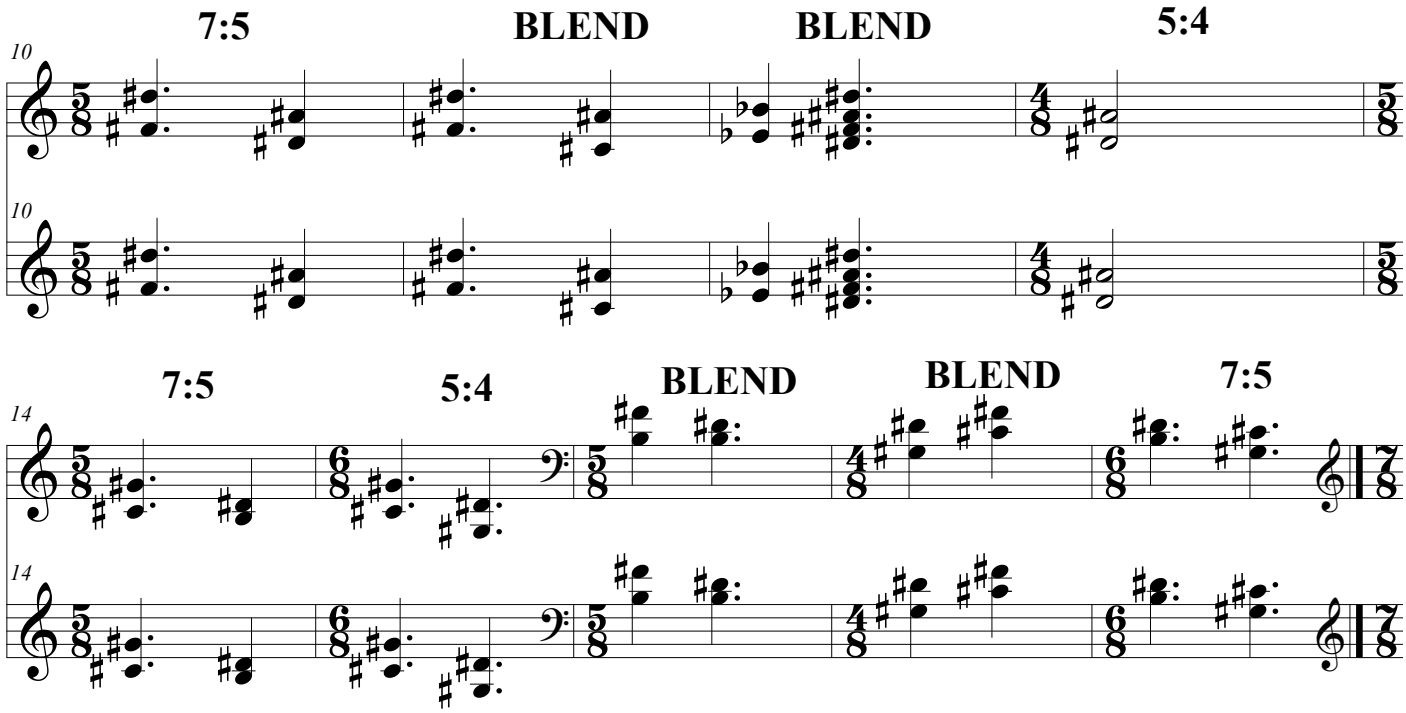
(Piano sketch of movement two of *The tale of Kiều)*

Appendix E, part three:
Piano sketch of the second movement of The tale of Kiều
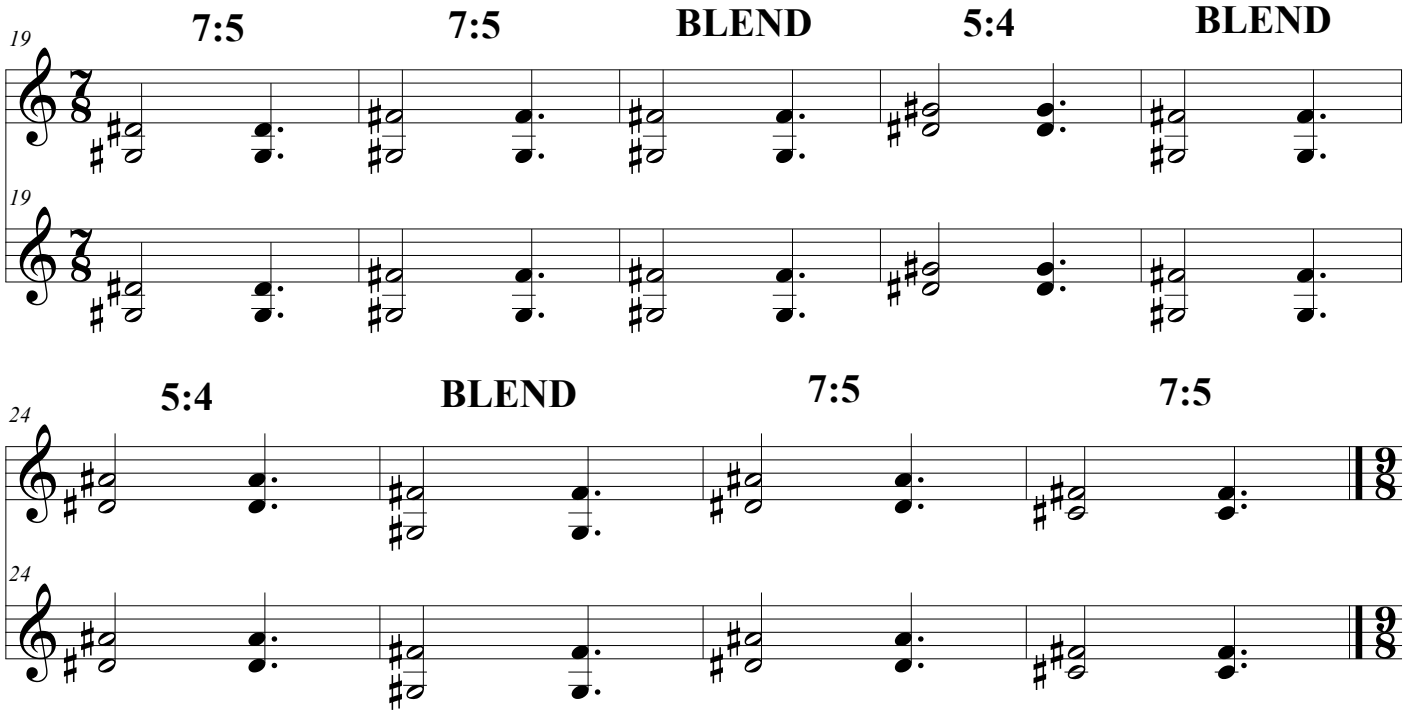
Section I, 27 quavers
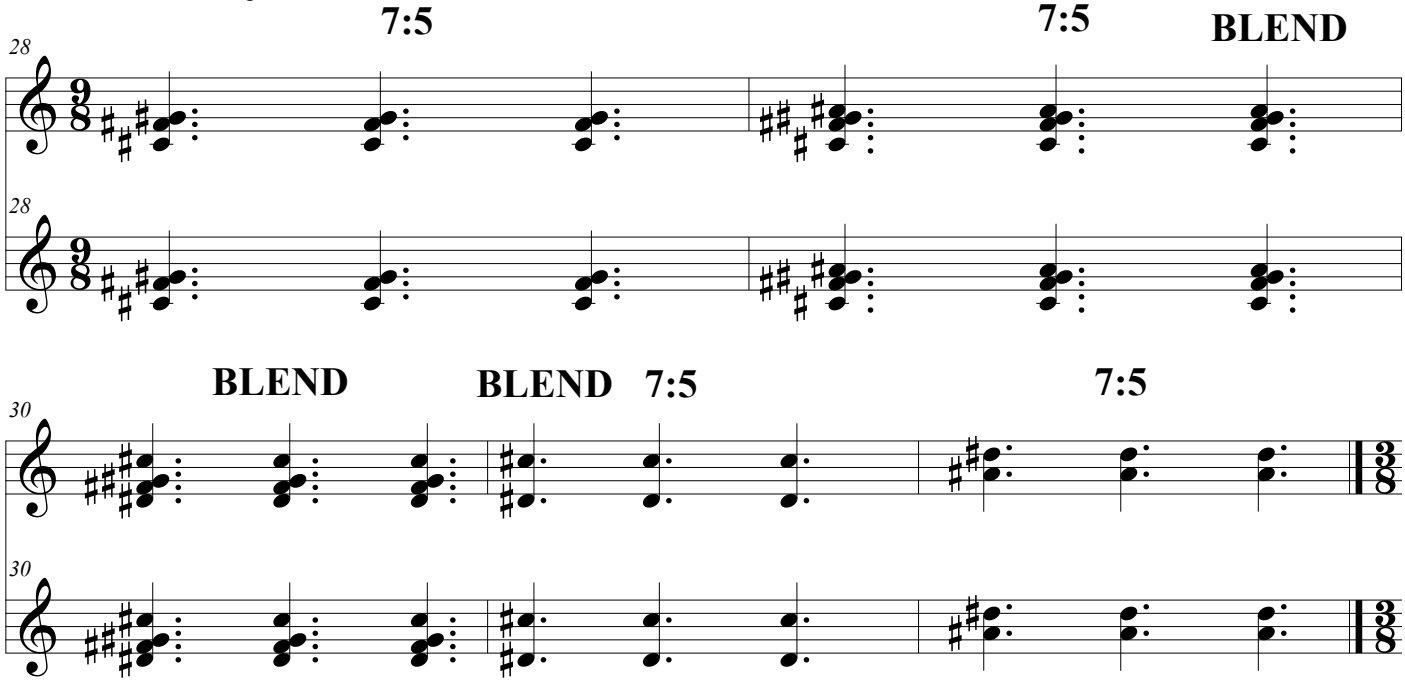


Section II, 45 quavers
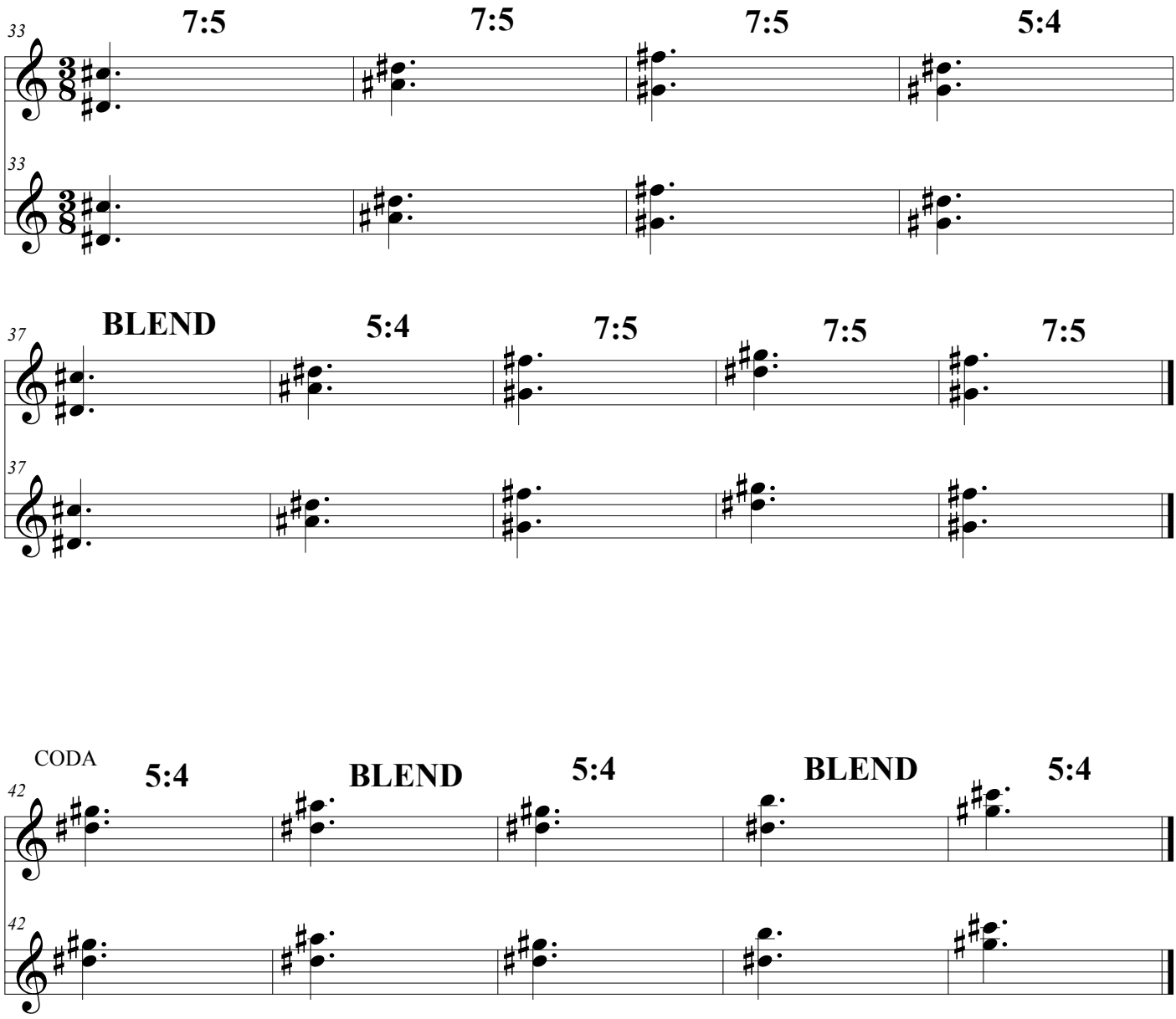


Section III, 63 quavers



Section IV, 45 quavers



Section V, 27 quavers



CODA

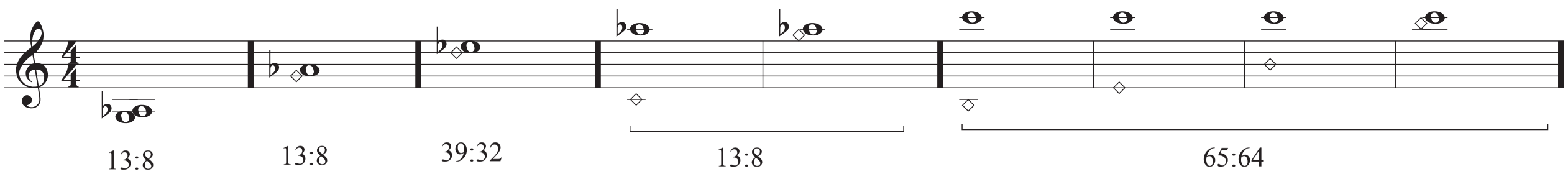
Appendix F

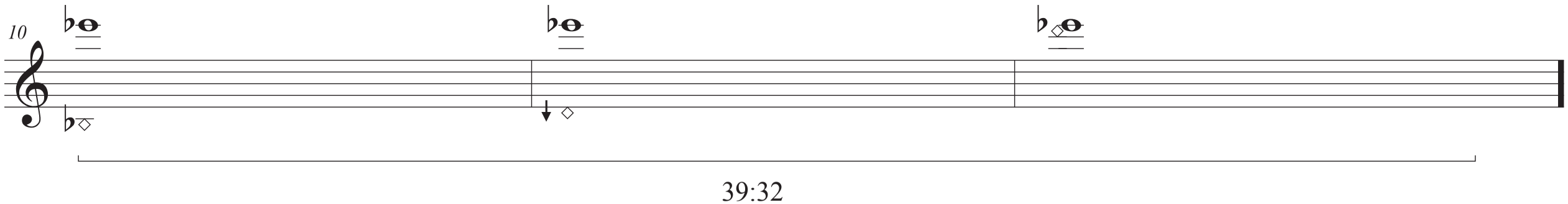
Appendix F, part one

(transposing chart for violin one)

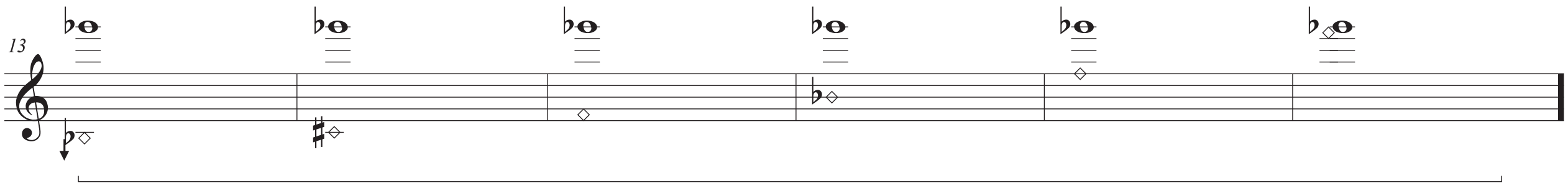
G

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

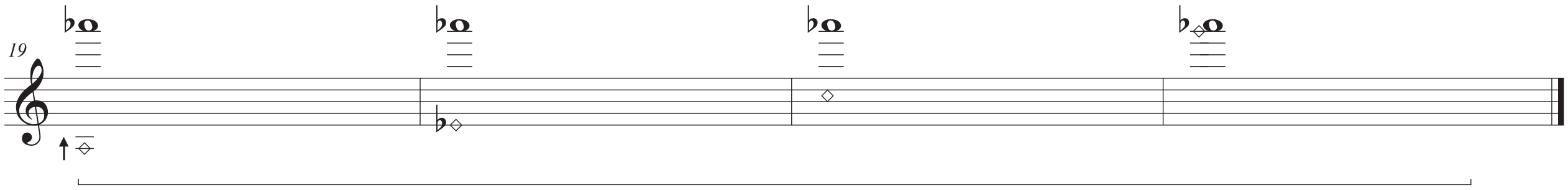




39:32



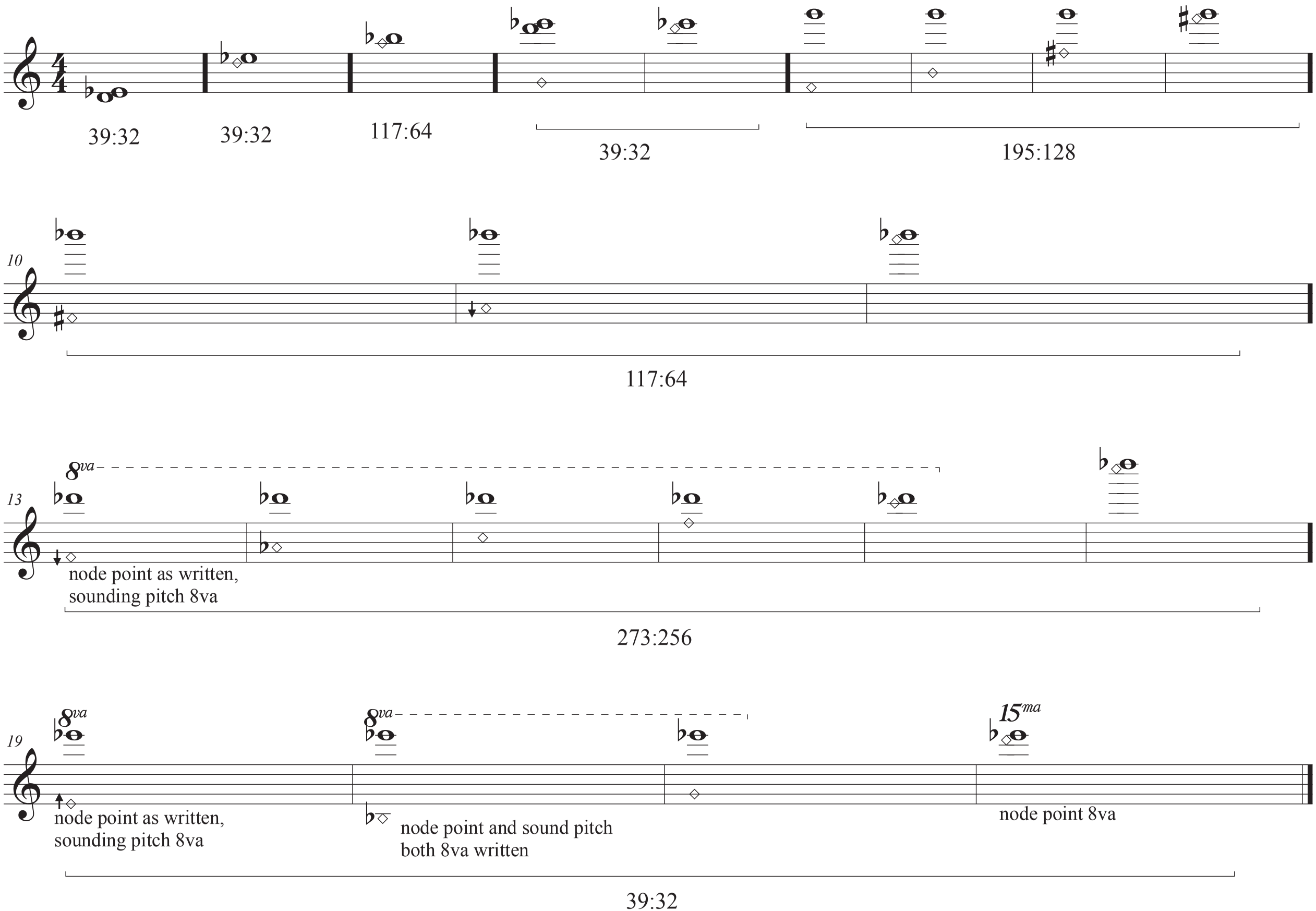
91:64



13:8

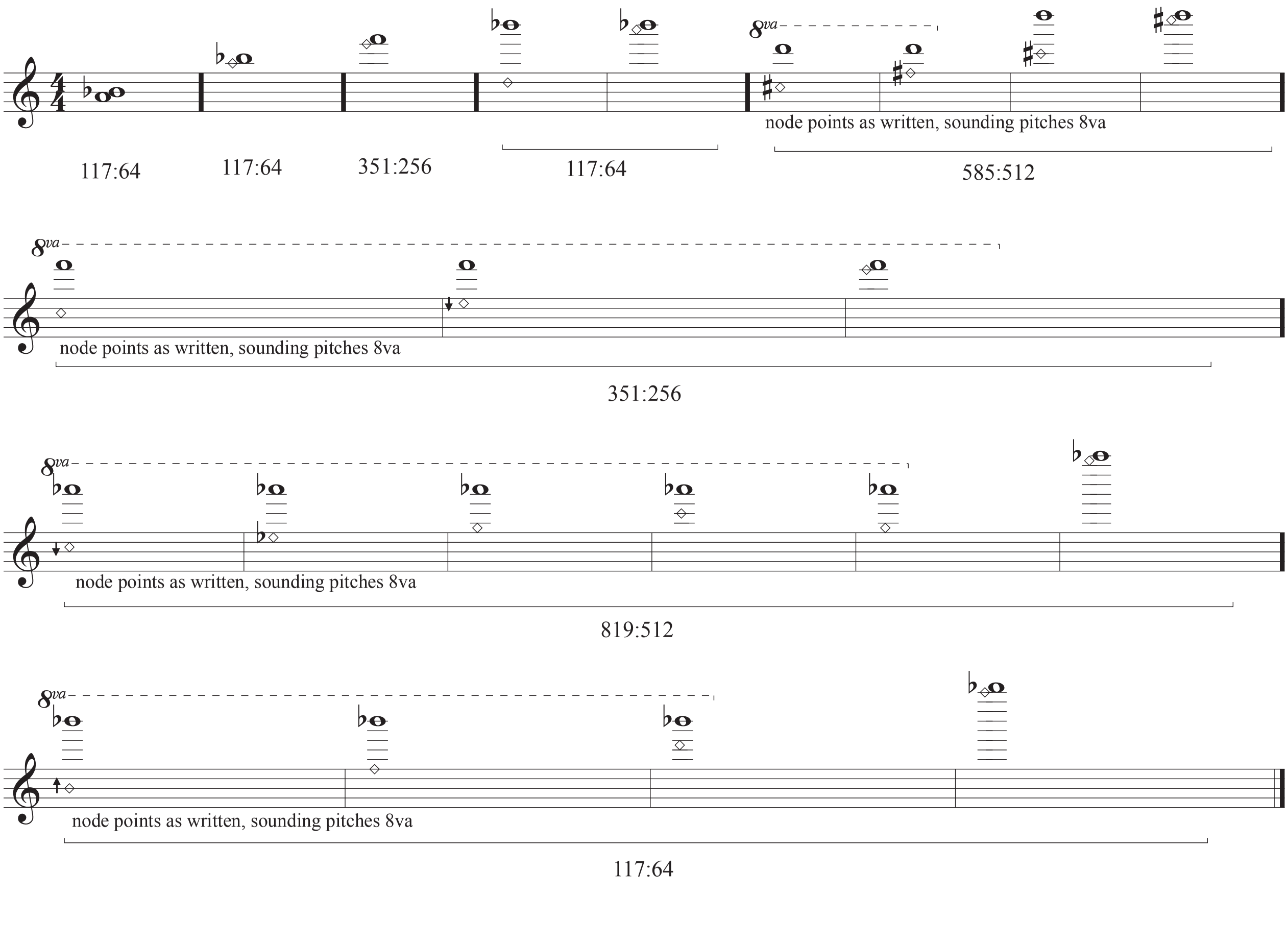
D

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



A

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



E

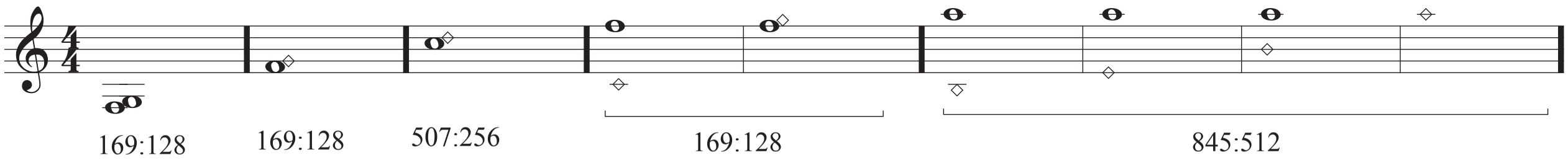
Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

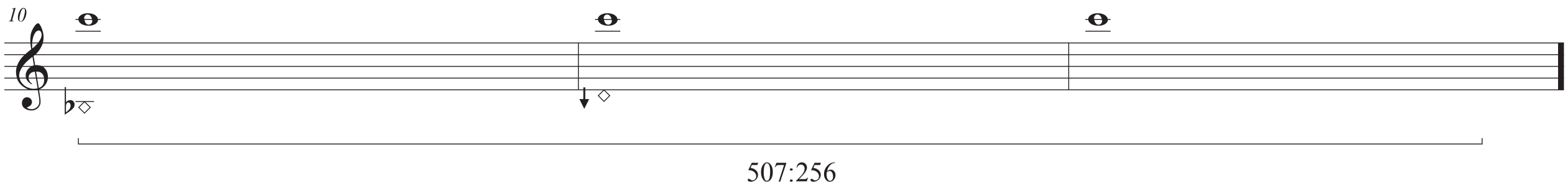
Appendix F, part two

(transposing chart for violin two)

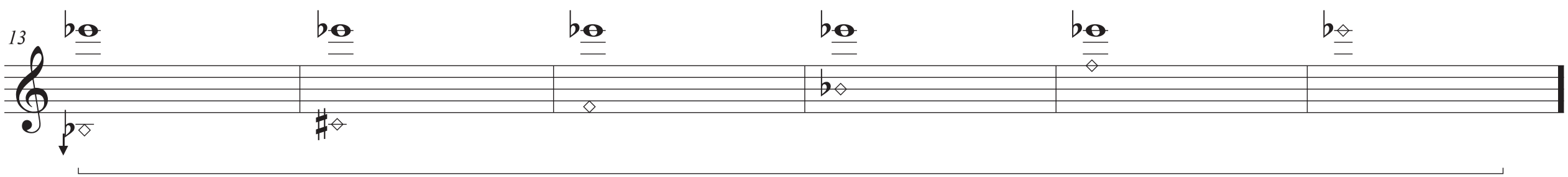
G

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

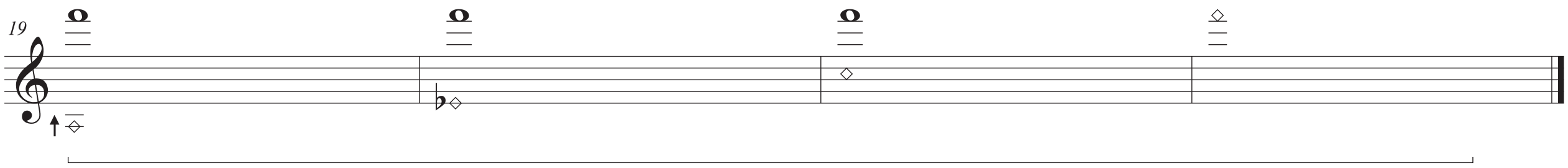




507:256



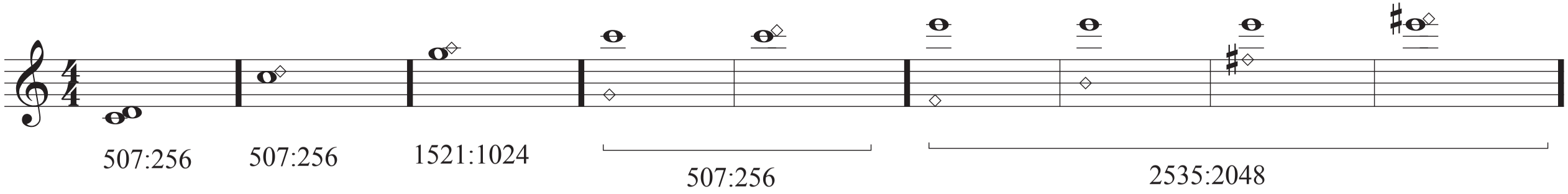
1183:1024

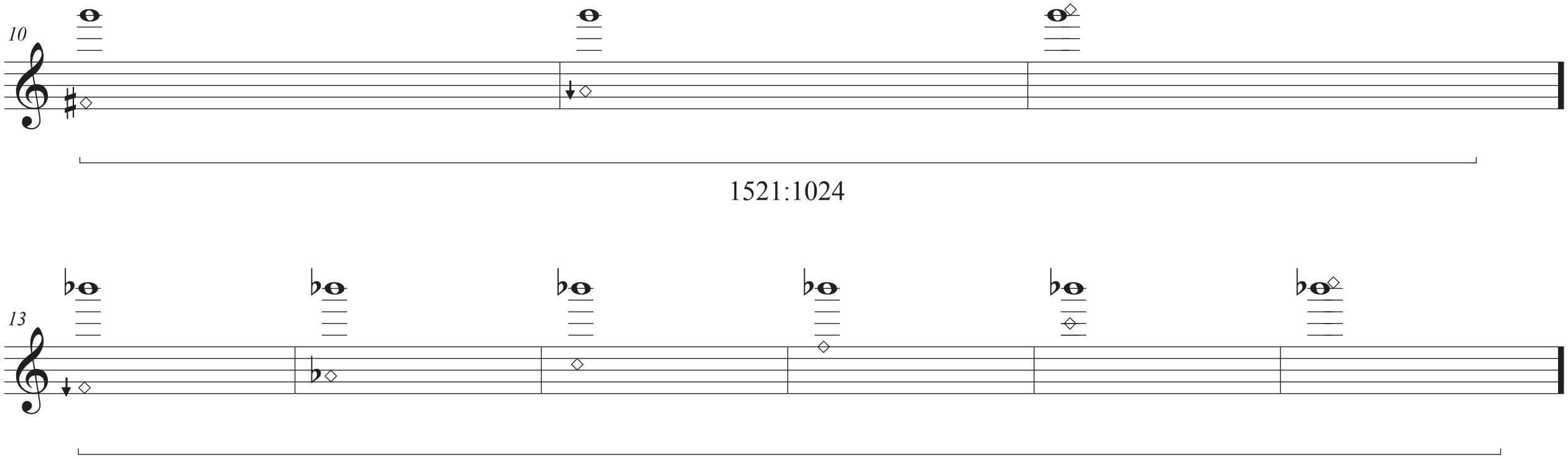


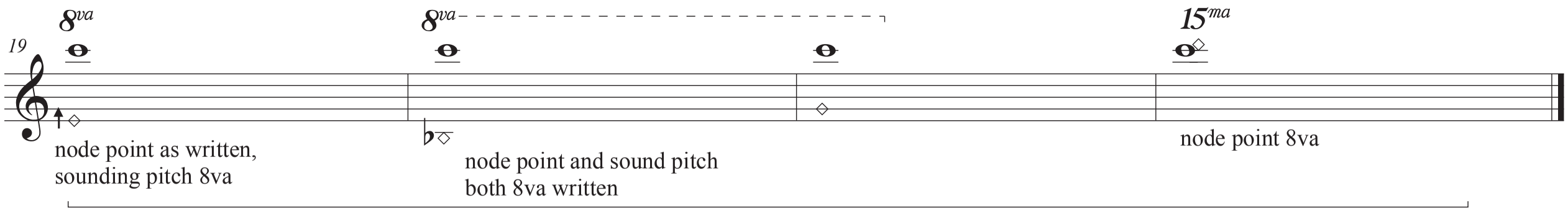
169:128

D

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



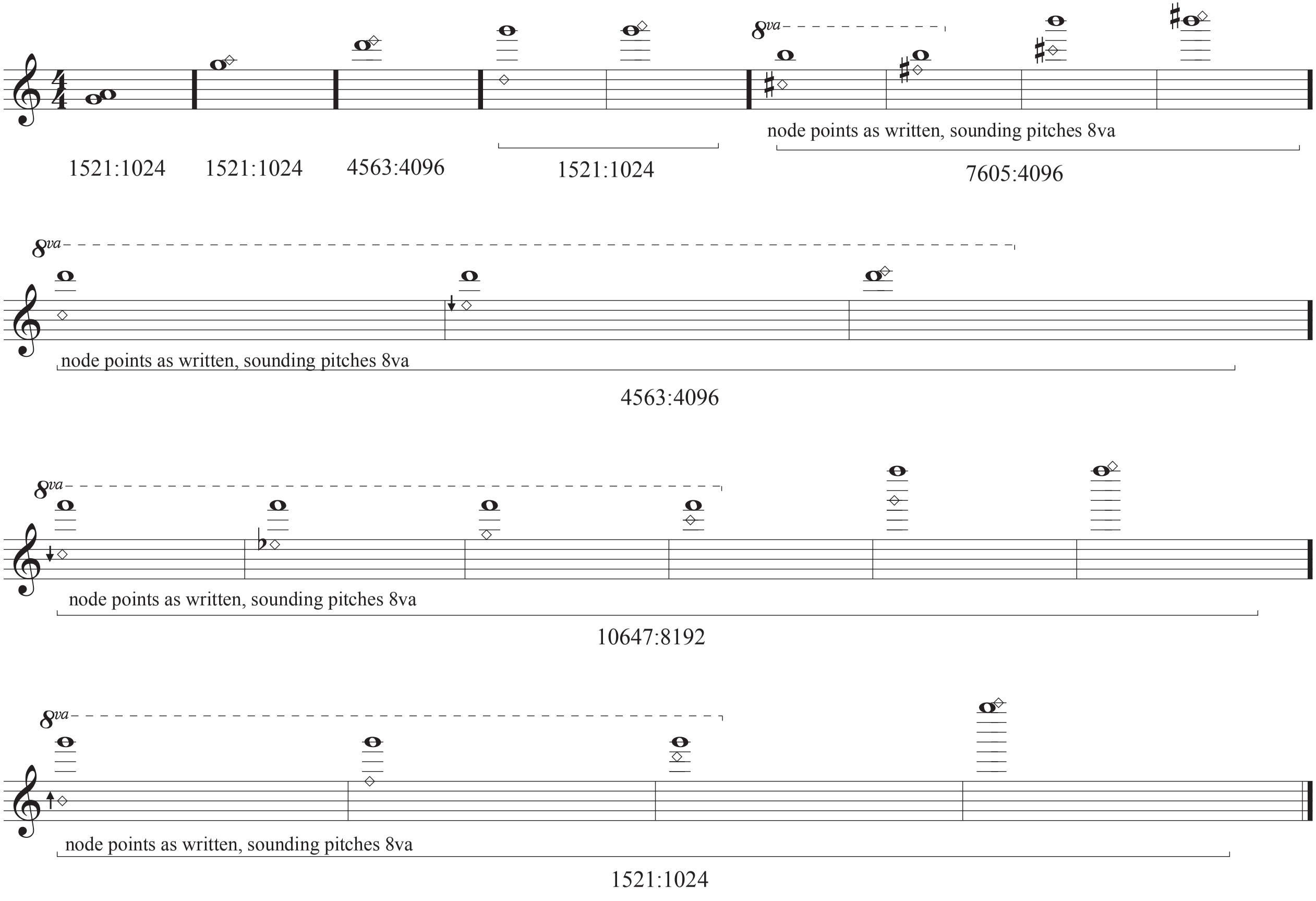




507:256

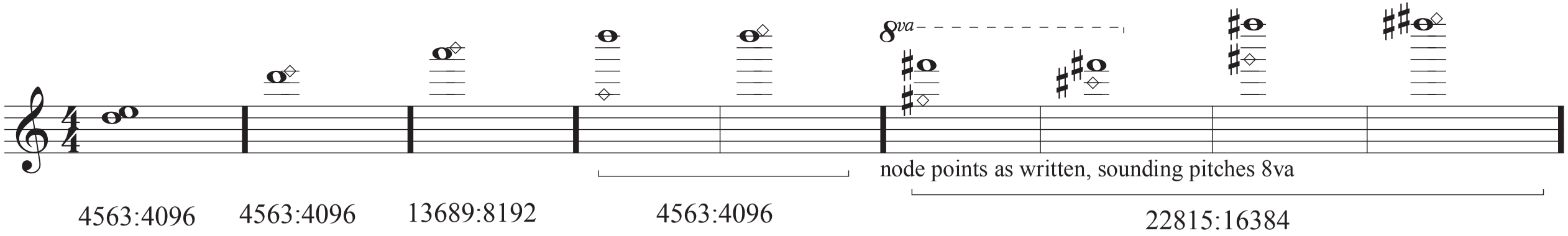
A

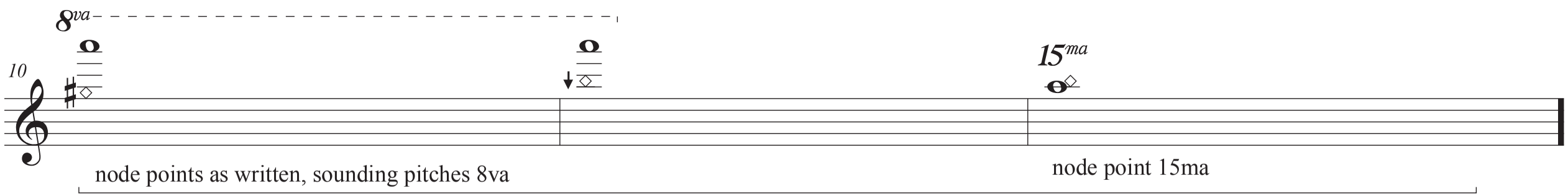
Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



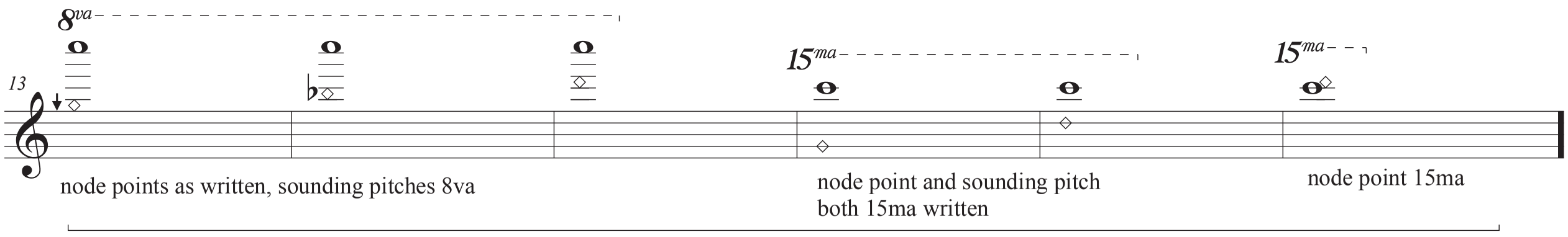
E

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

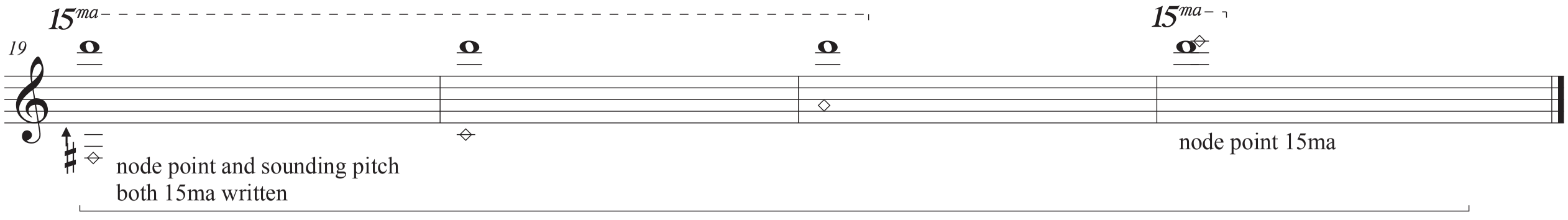




13689:8192



31941:16384



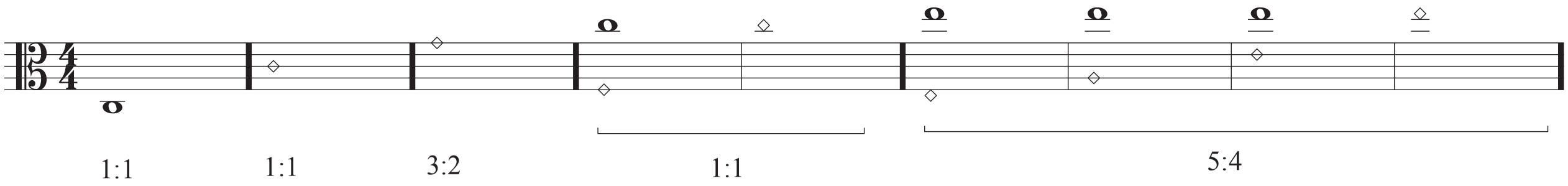
4563:4096

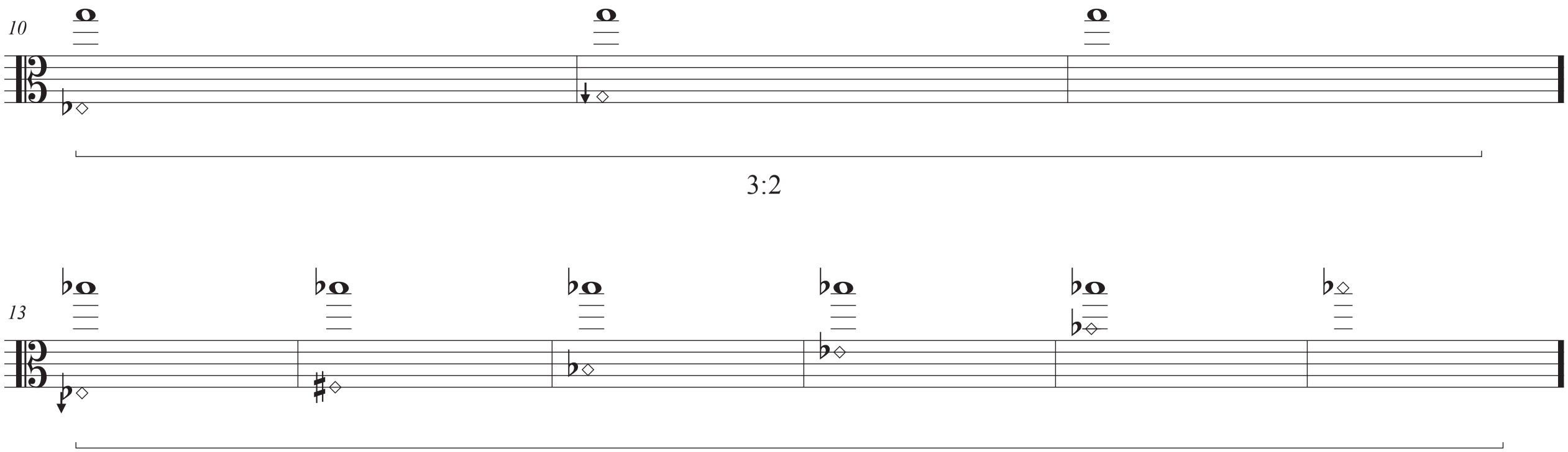
Appendix F, part three

(transposing chart for viola)

C

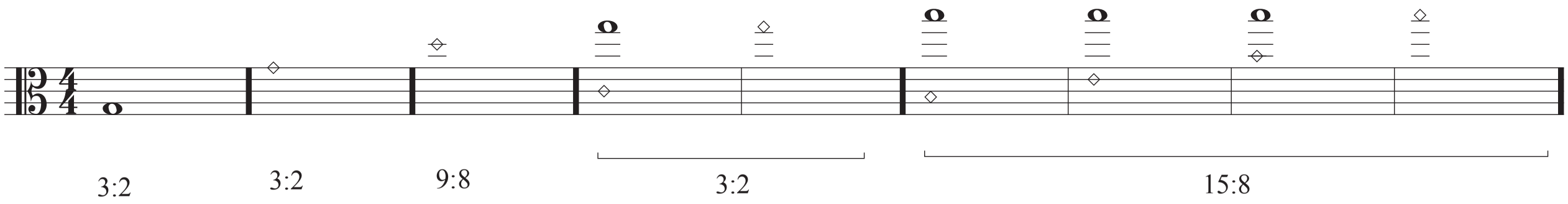
Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

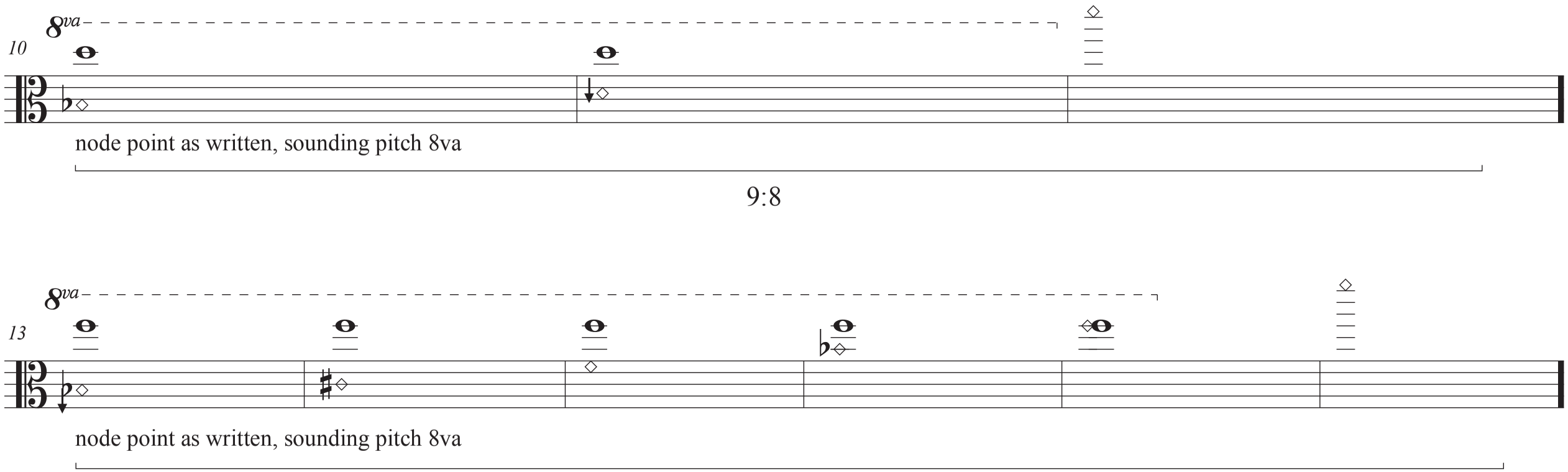




G

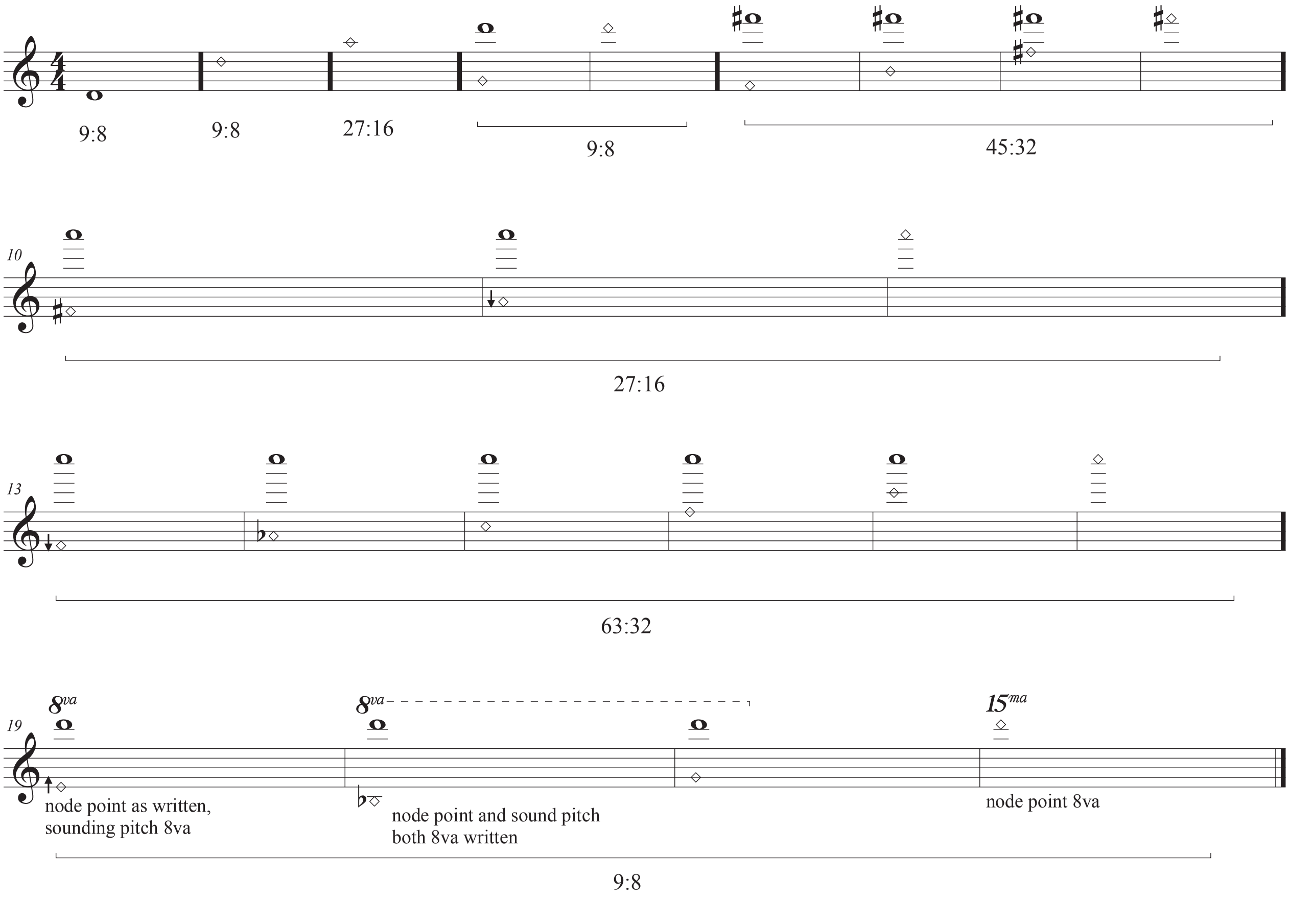
Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)





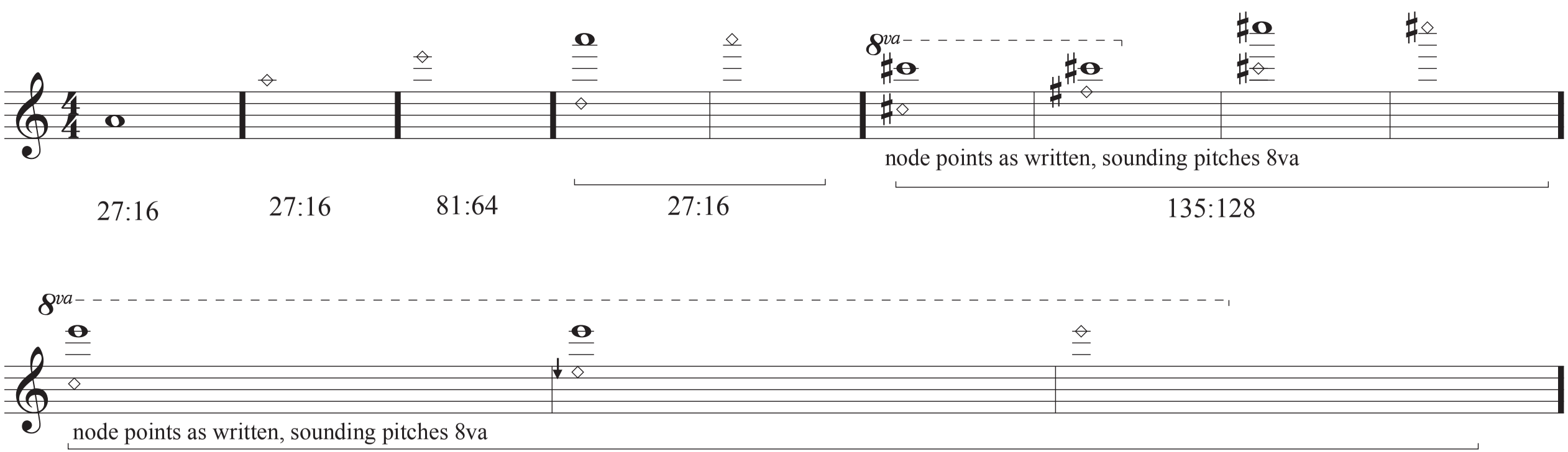
D

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

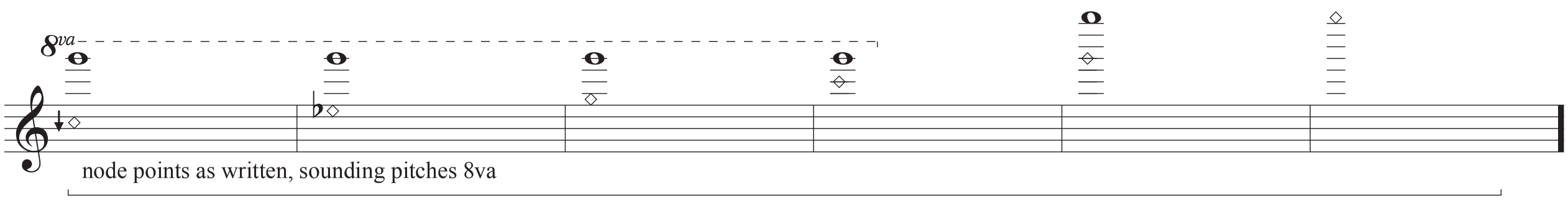


A

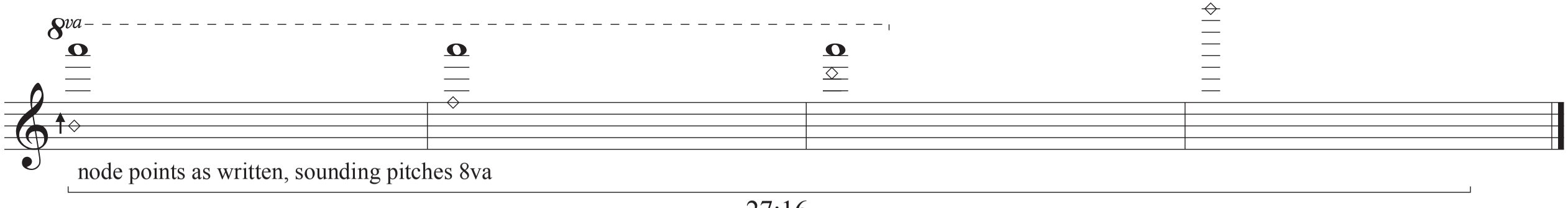
Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)



81:64



189:128



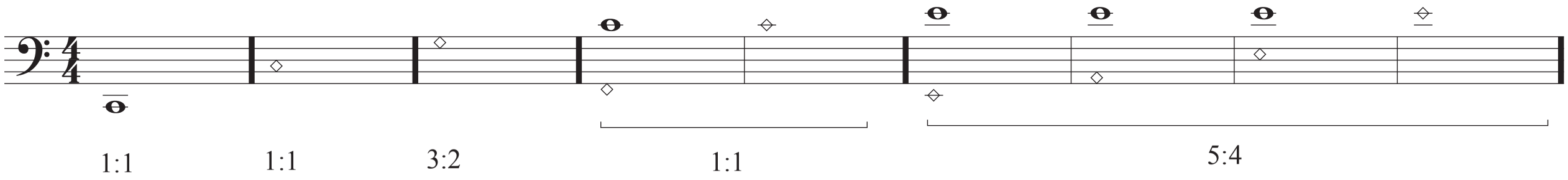
27:16

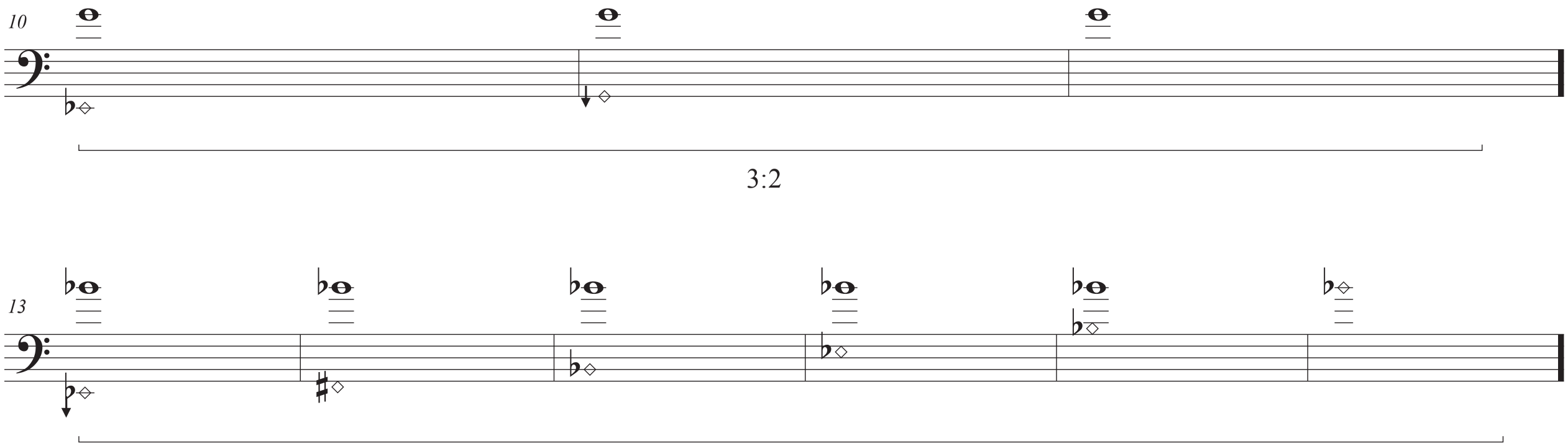
Appendix F, part four

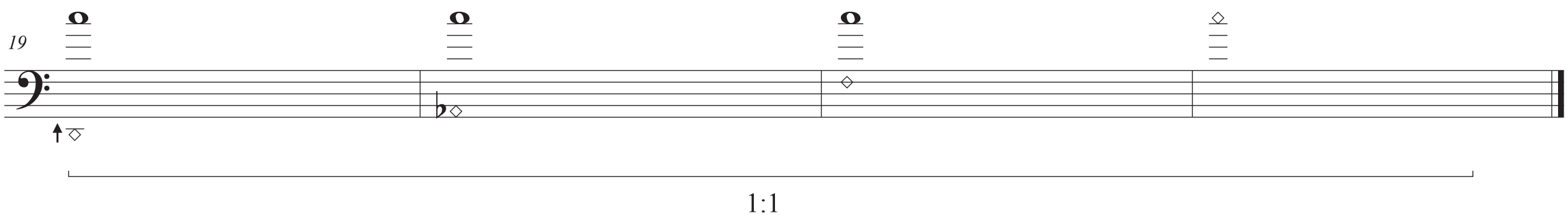
(transposing chart for cello)

C

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

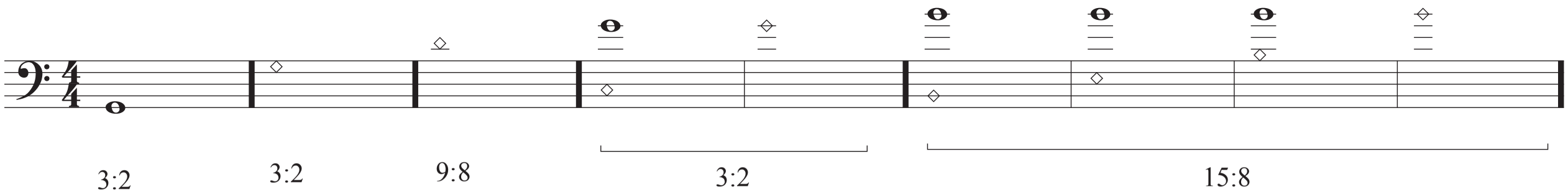


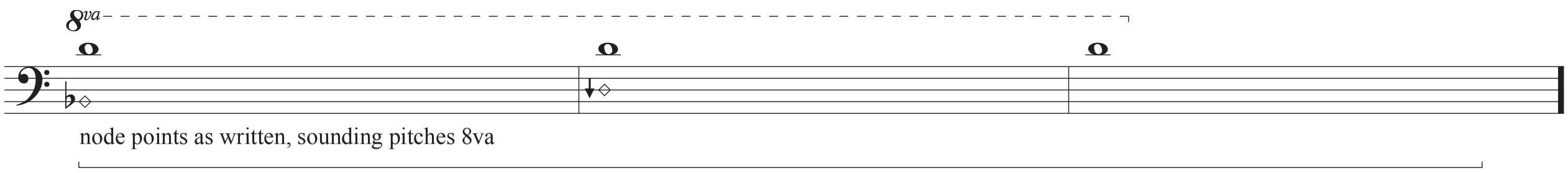




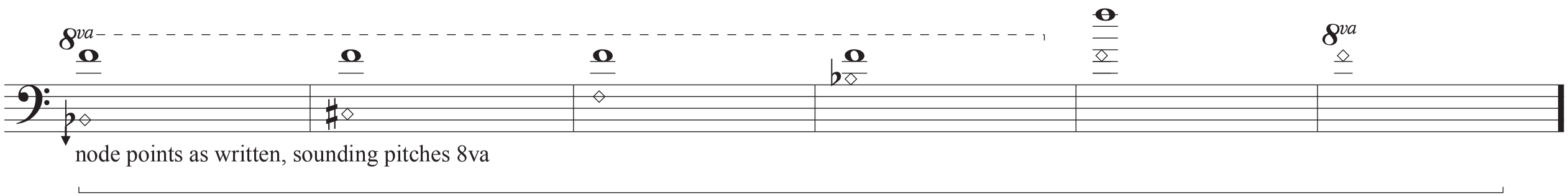
G

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

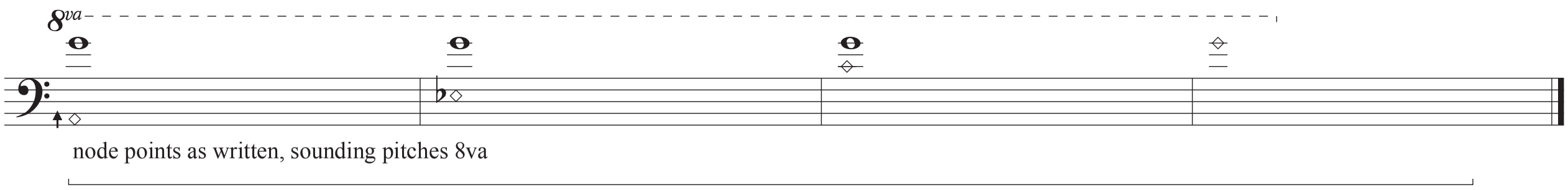




9:8



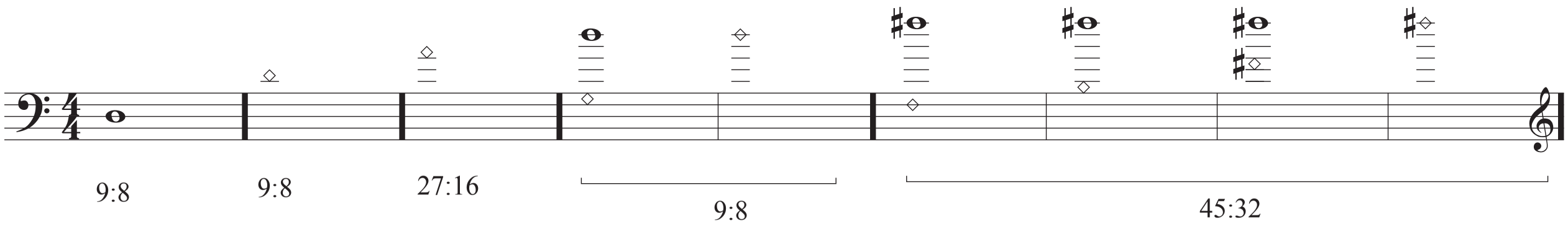
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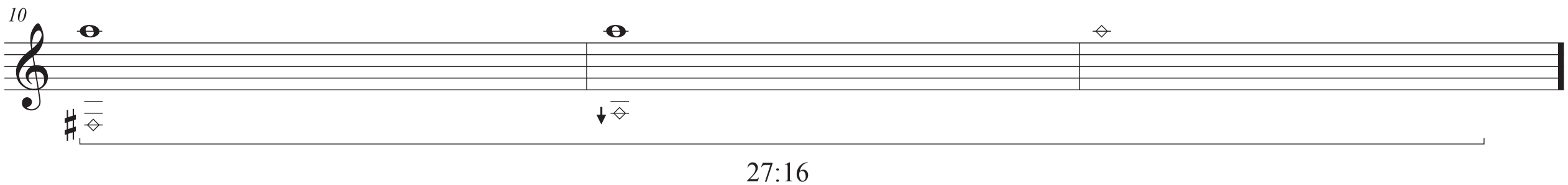


3:2

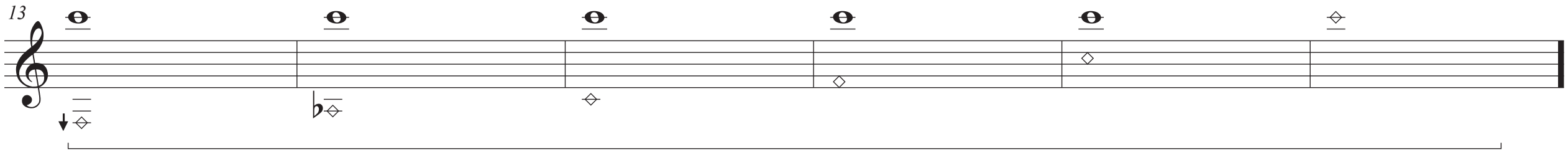
D

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

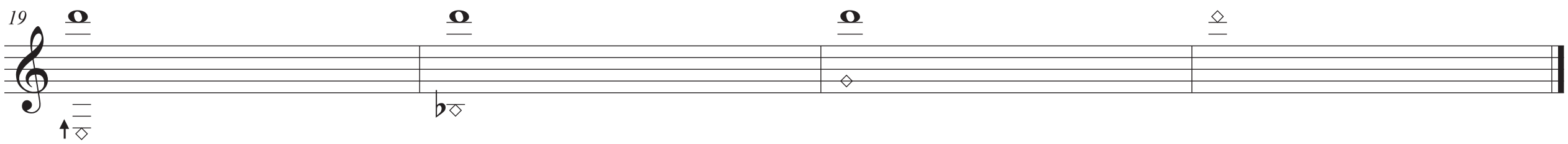




27:16



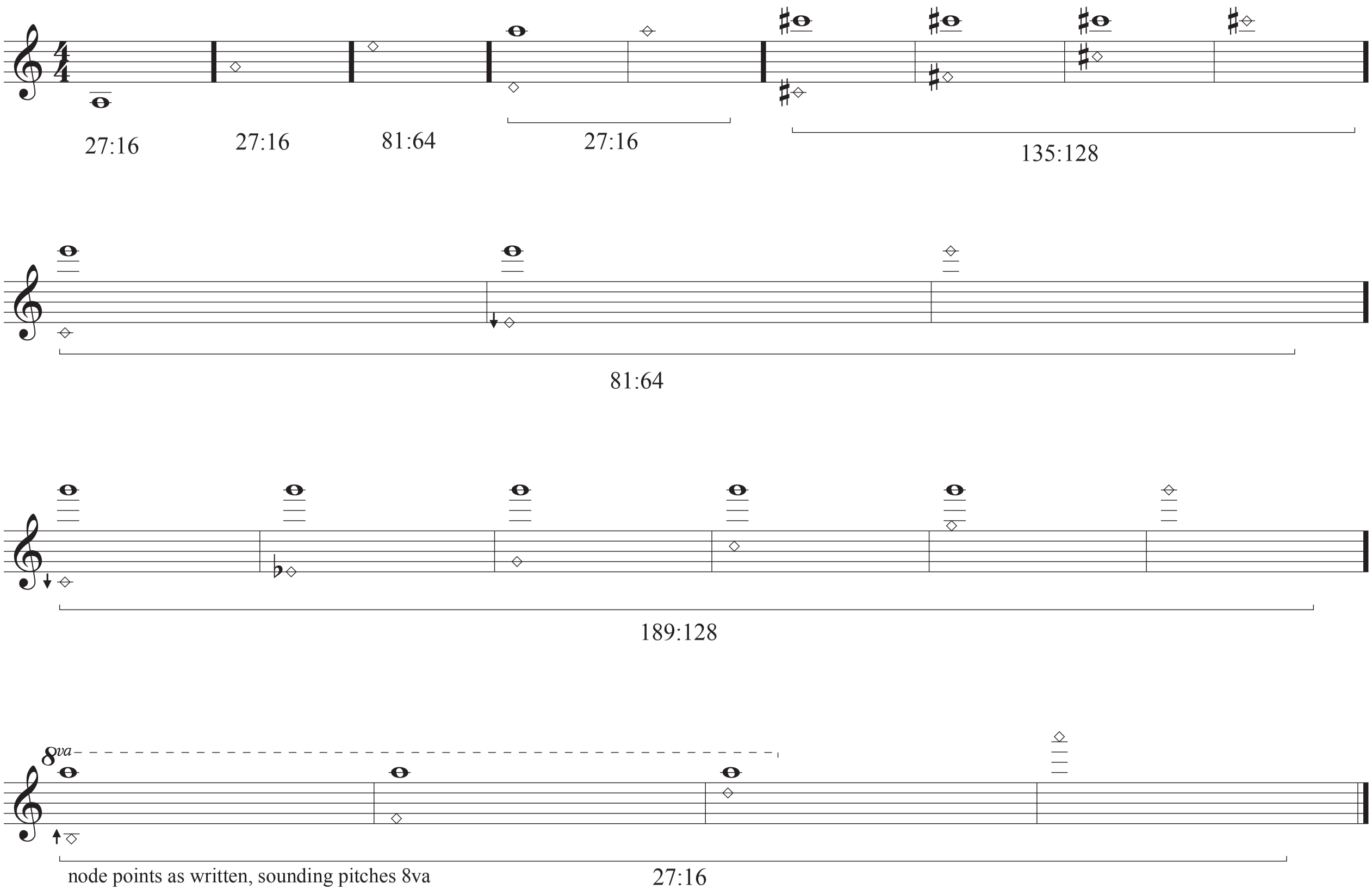
63:32



9:8

A

Harmonics up to the triple octave (adapted from Enzo Porta's
The Violin: Harmonics Classification and New Techniques)

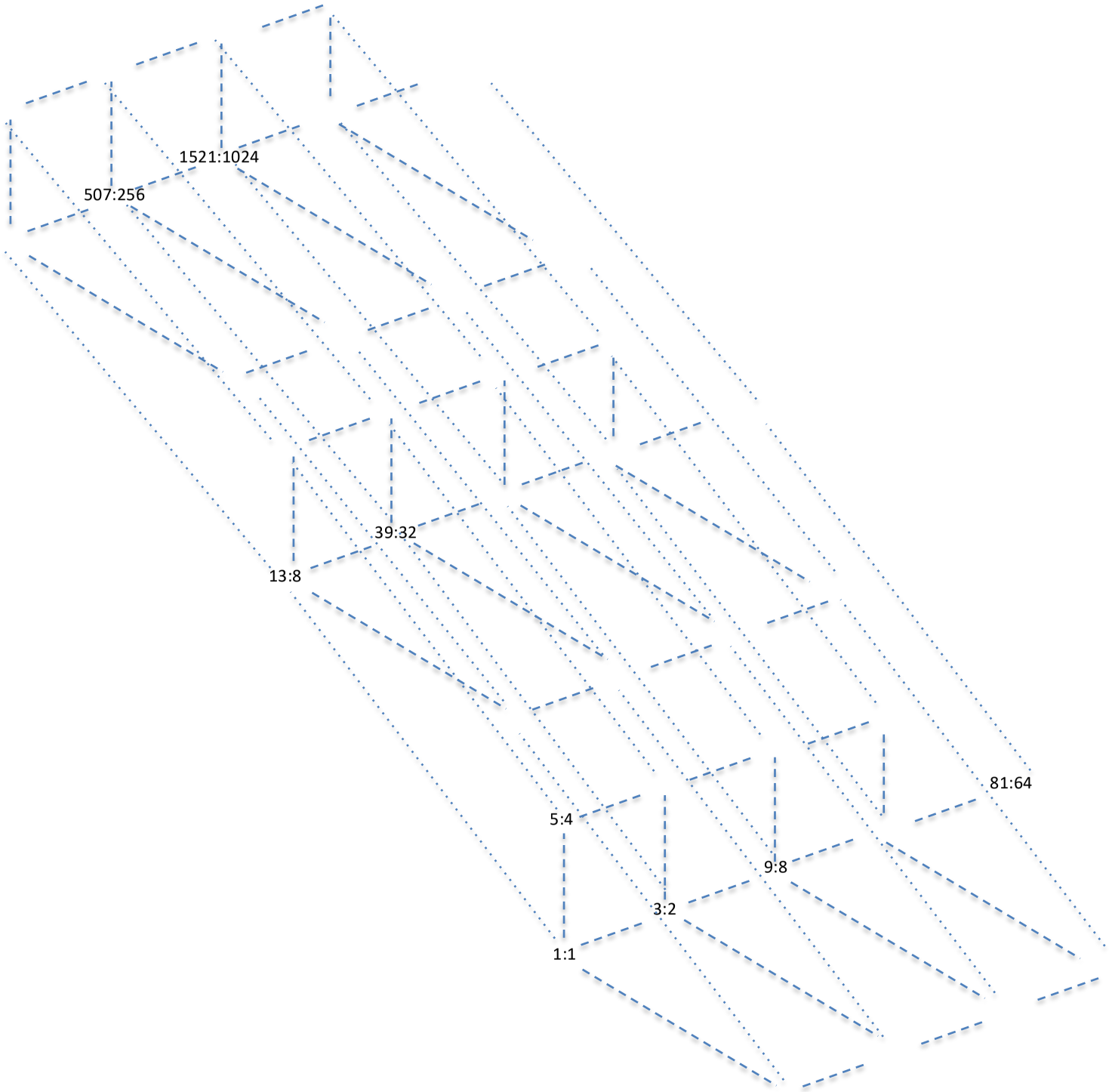


Appendix G

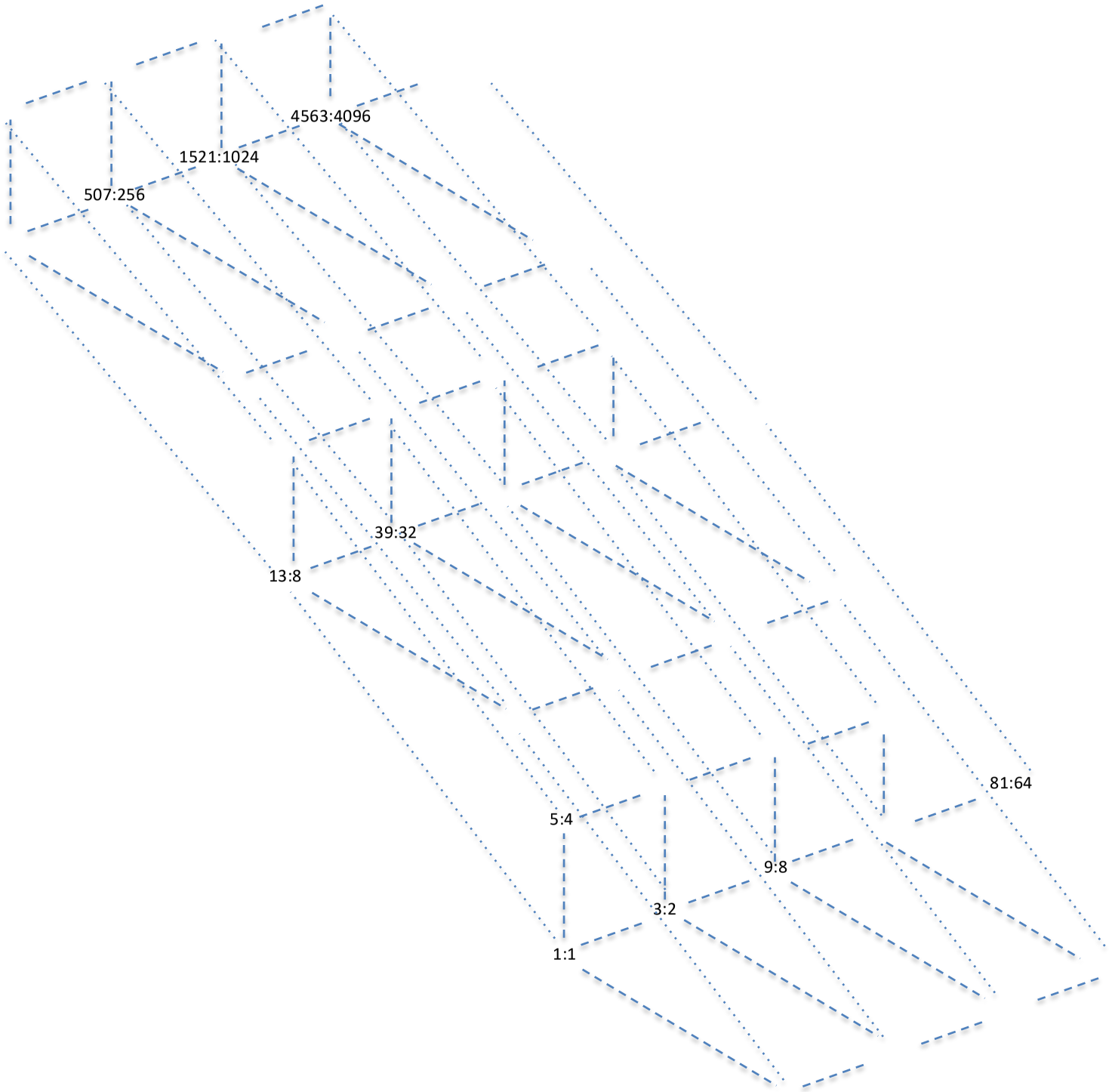
Appendix G, part one

(Integer ratios occurring in the allegro sections of *13,* from which the symmetrical sets can be deduced)

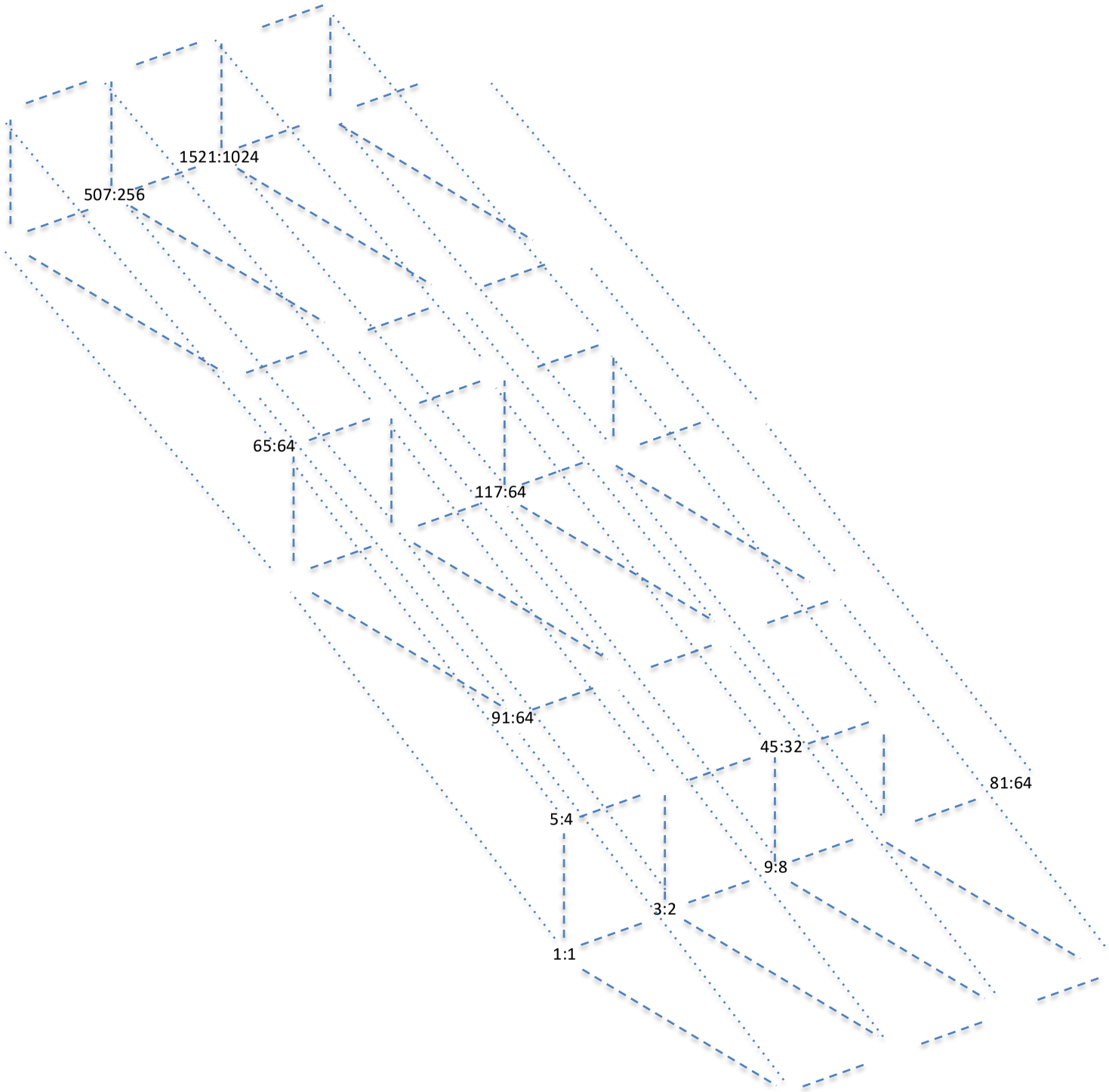
Measures 5-10



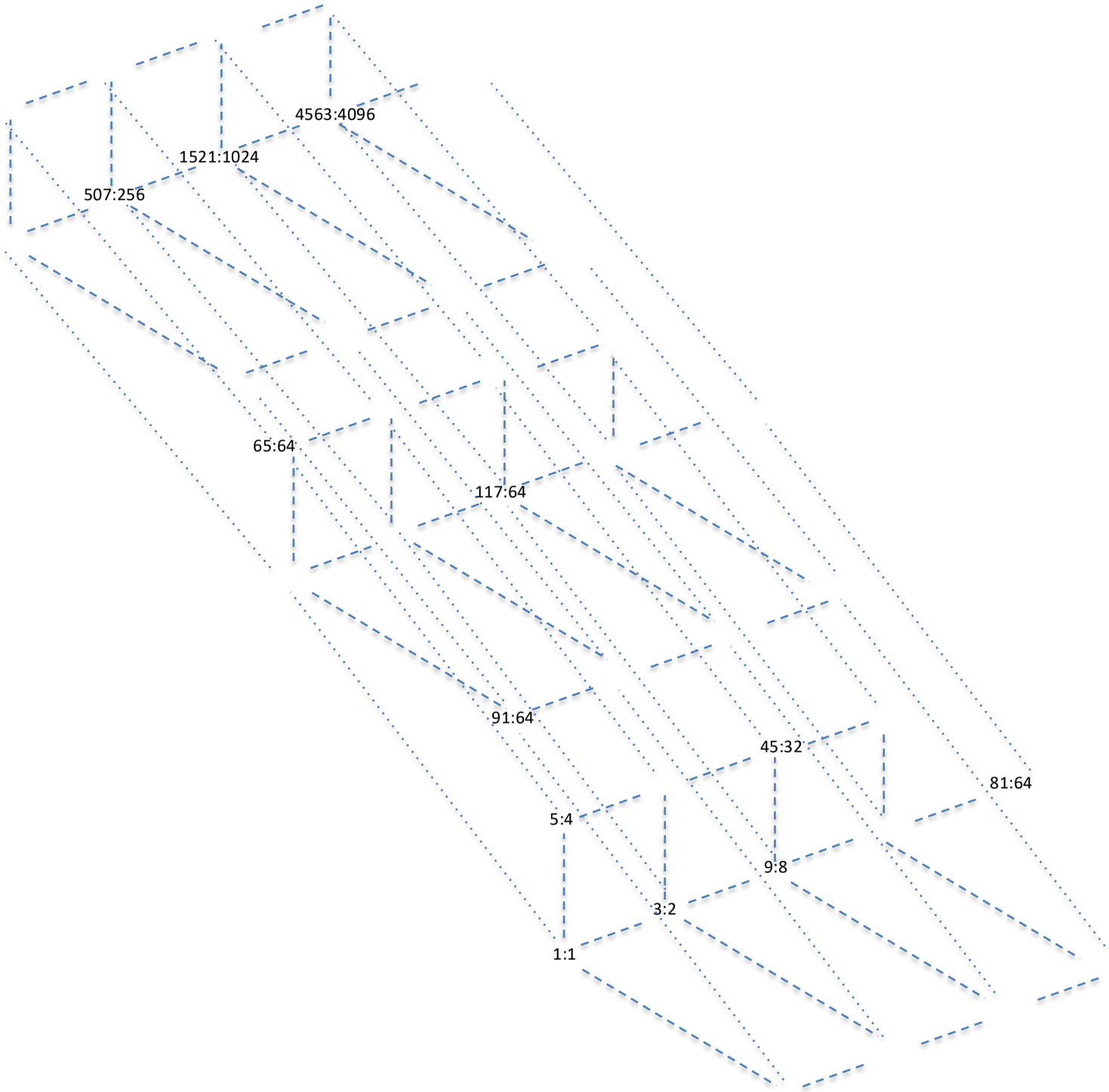
Measures 11-19



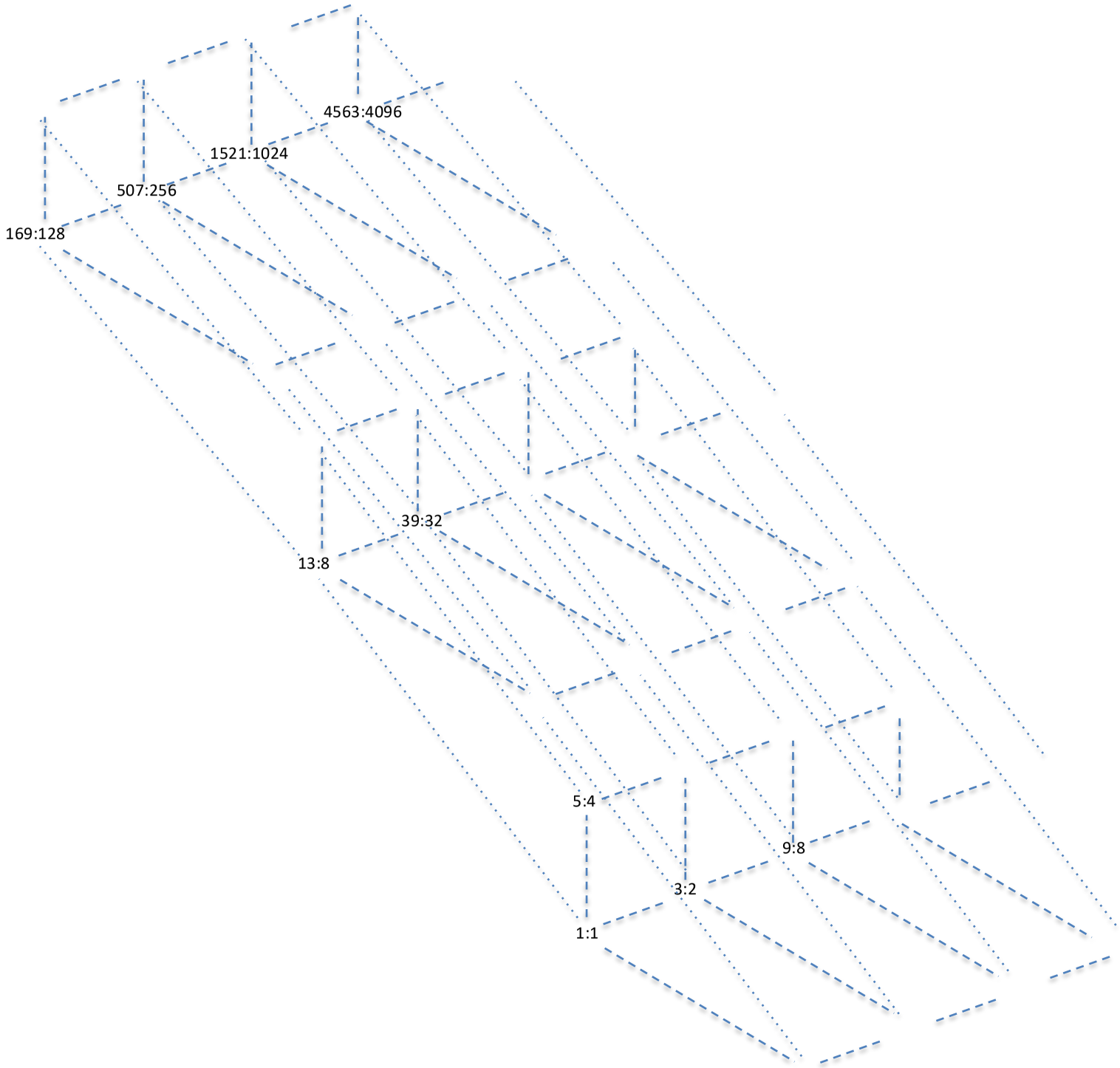
Measures 22-29



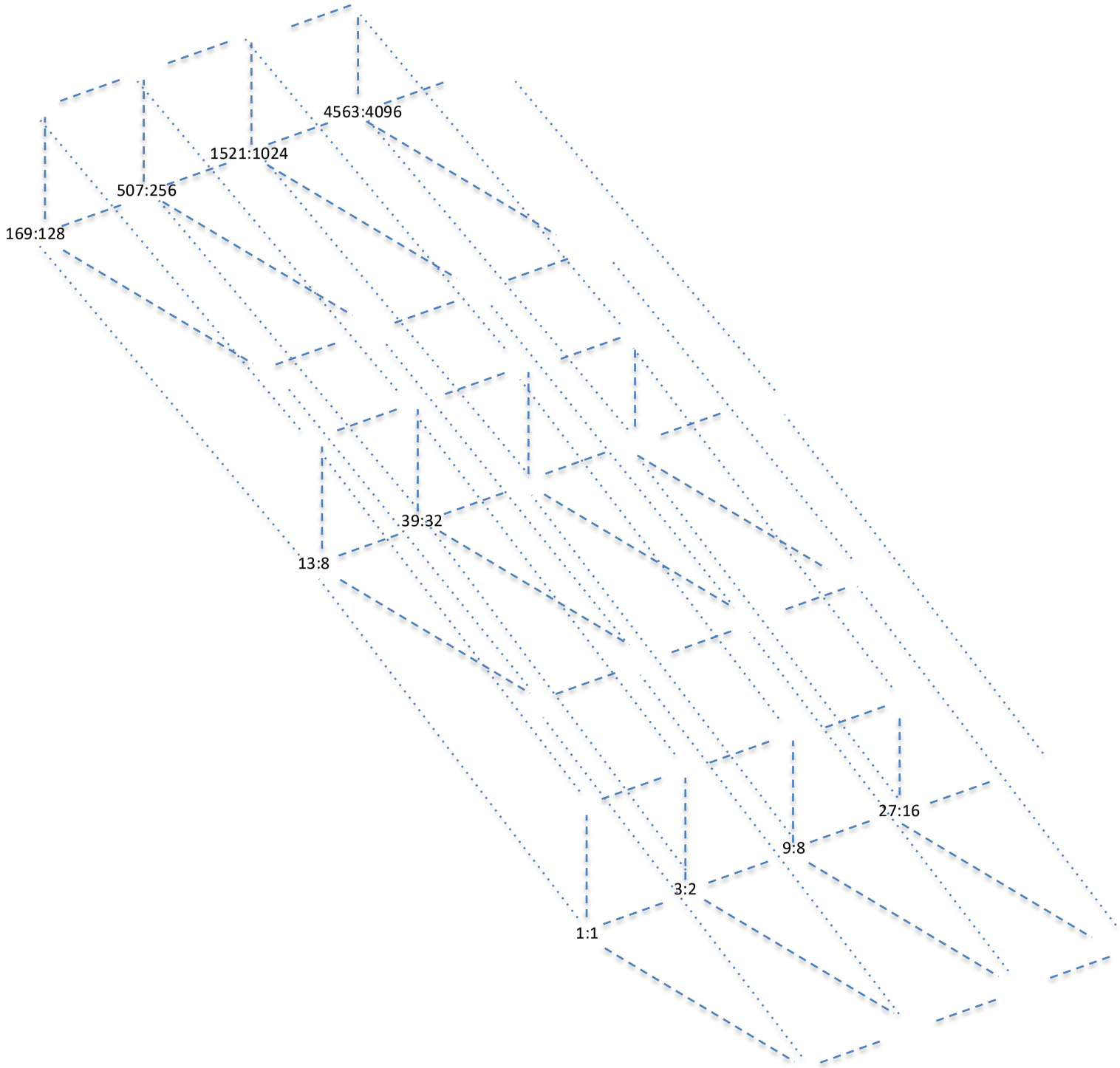
Measures 30-41



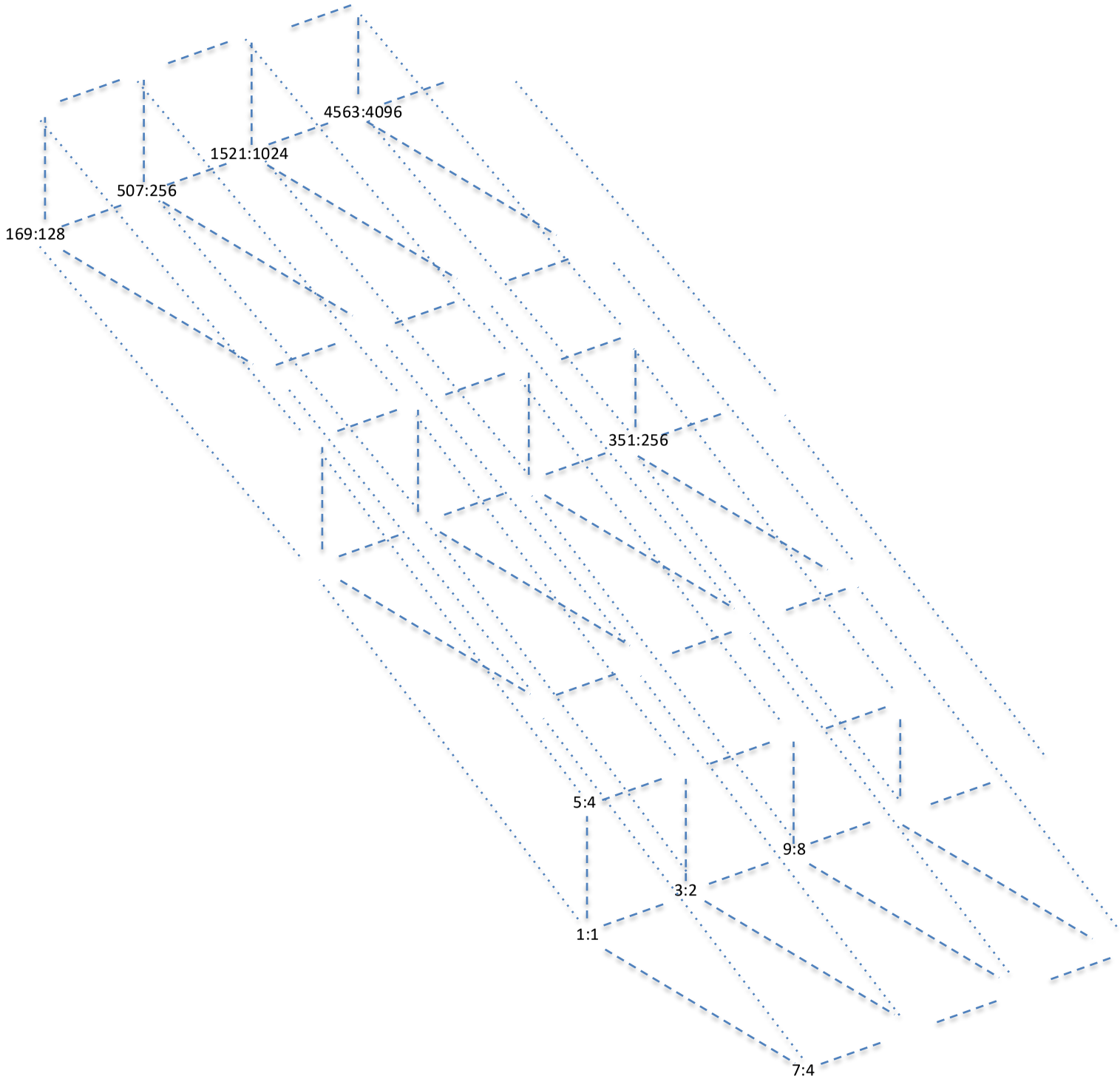
Measures 48-56



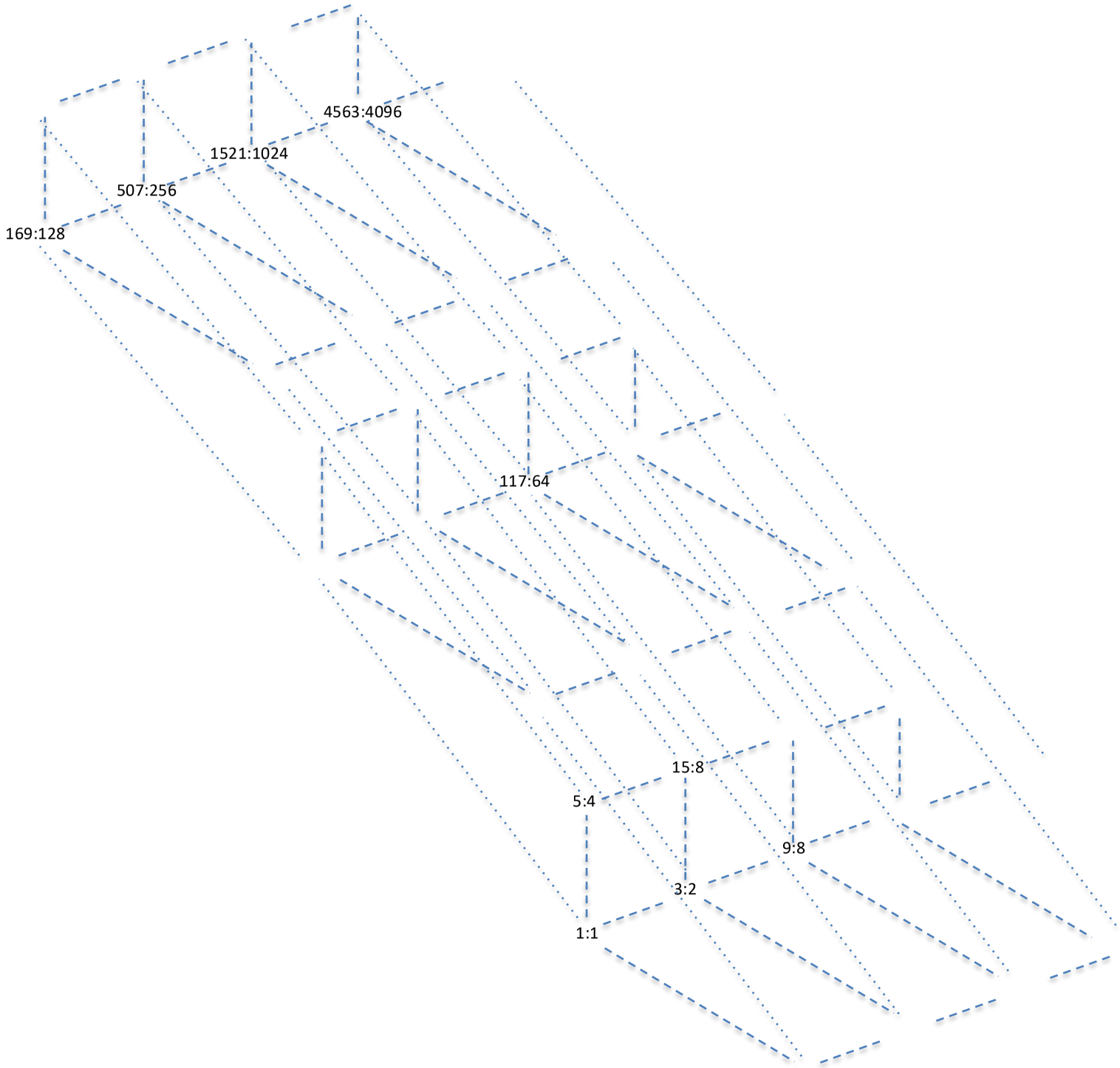
Measures 58-70



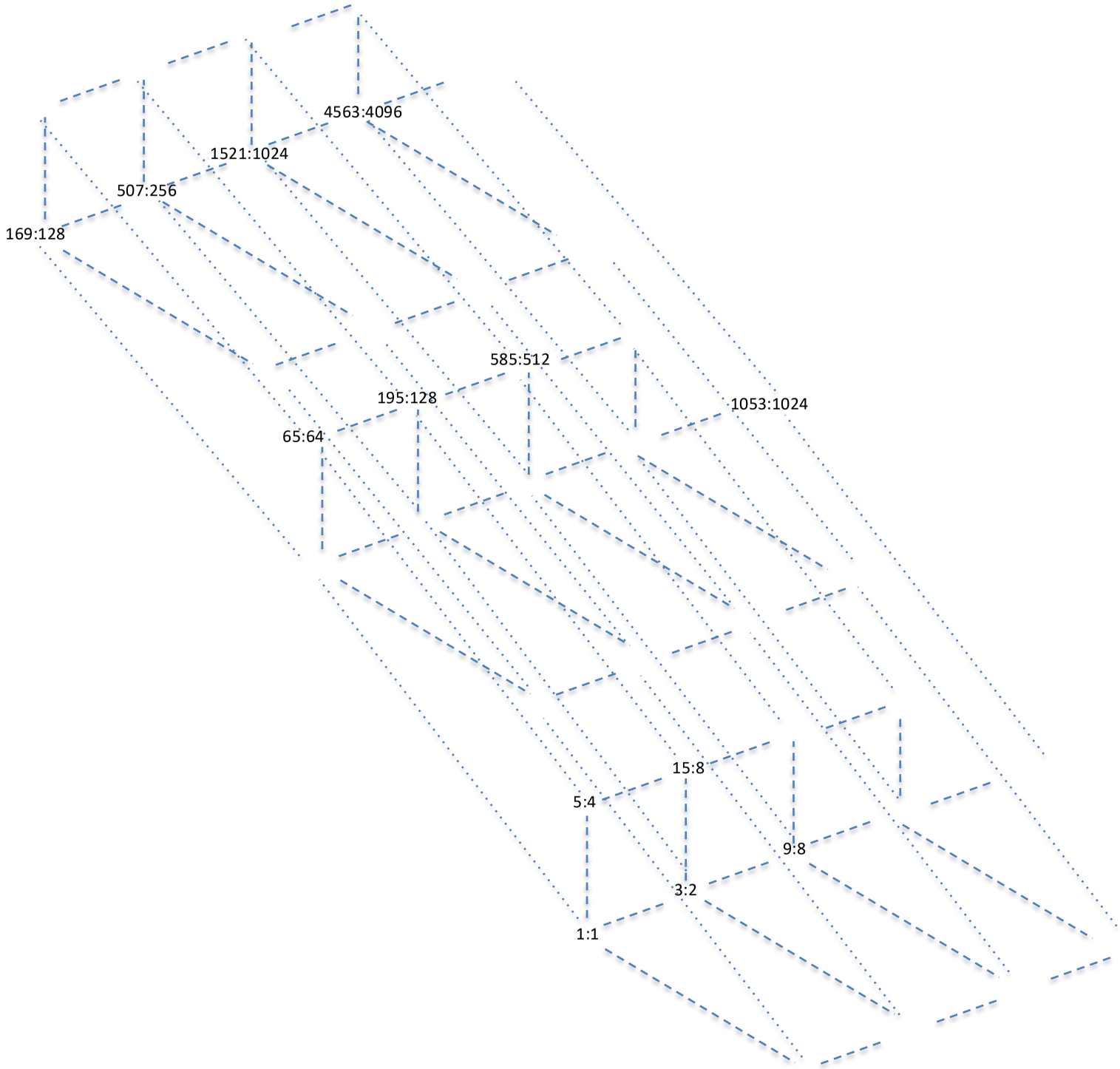
Measures 73-77



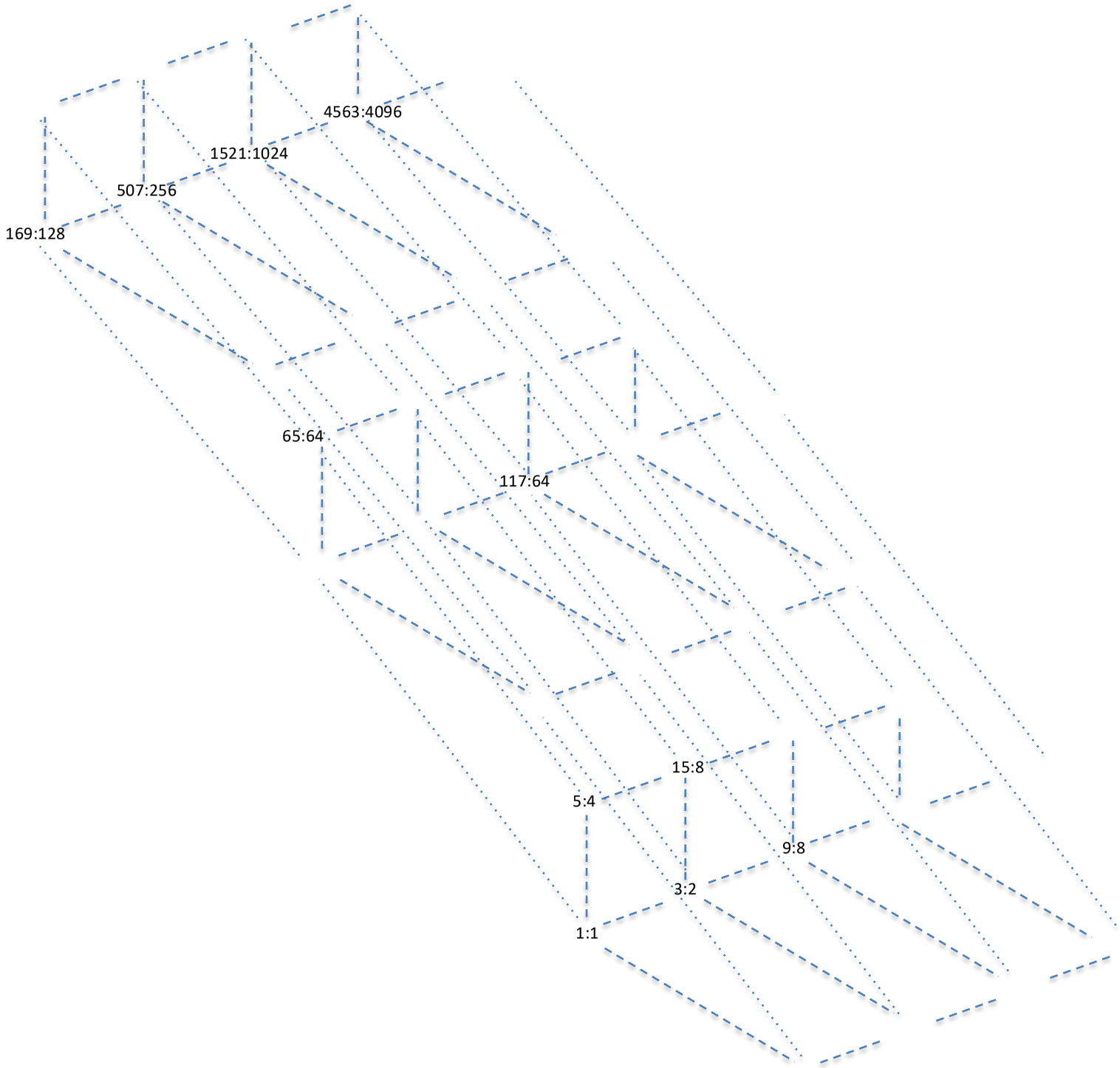
Measures 78-82



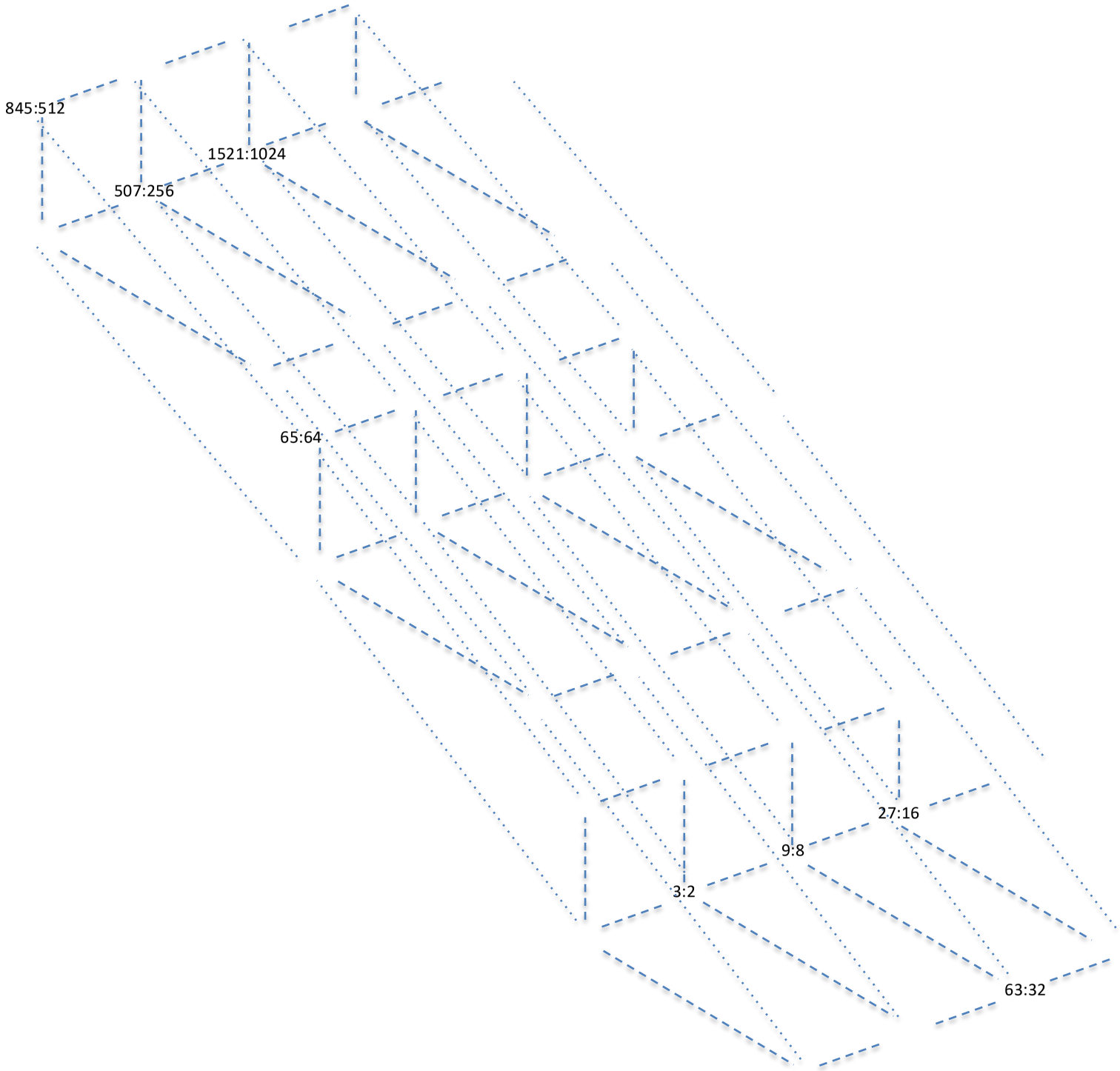
Measures 83-87



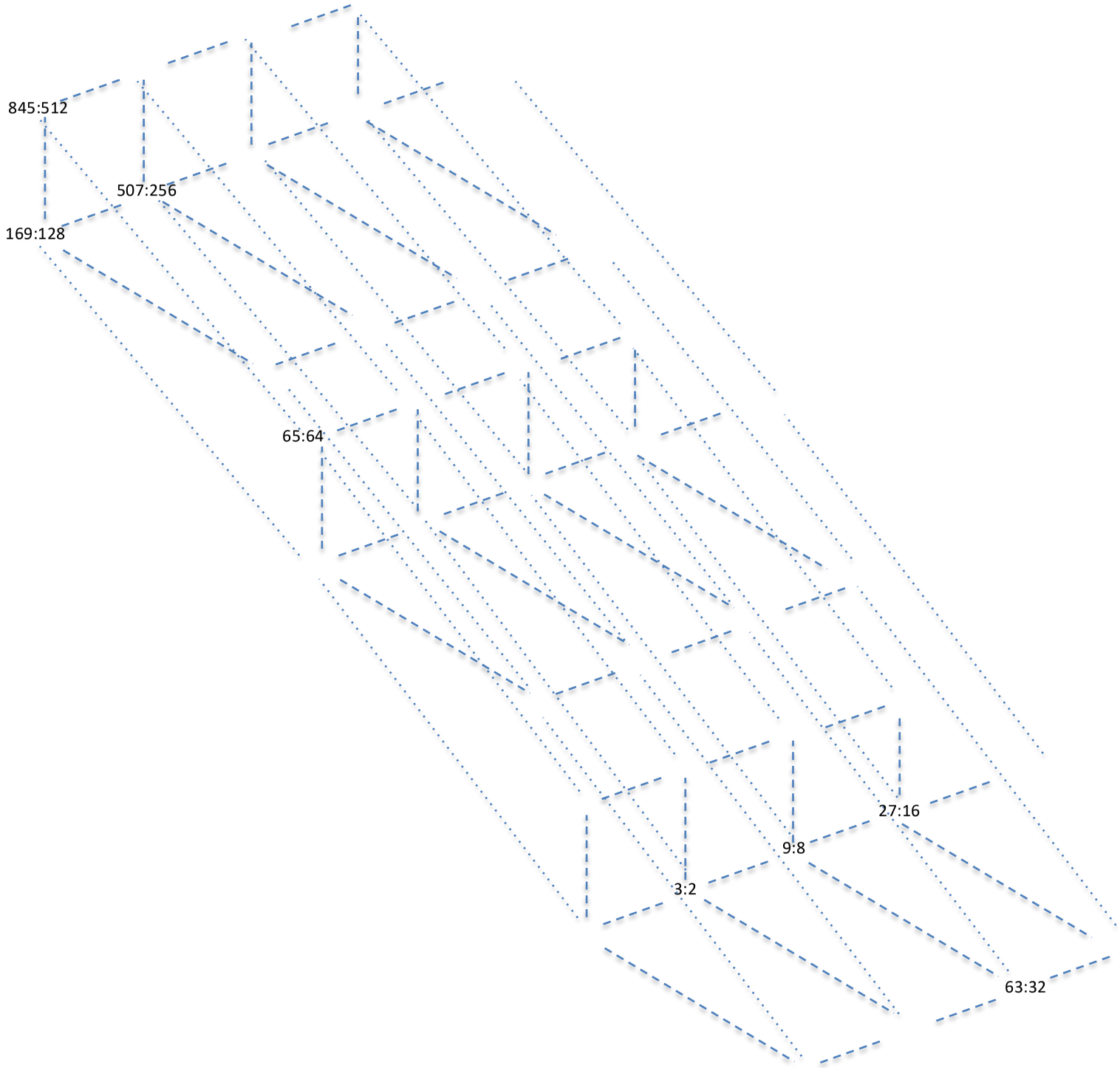
Measures 94-103



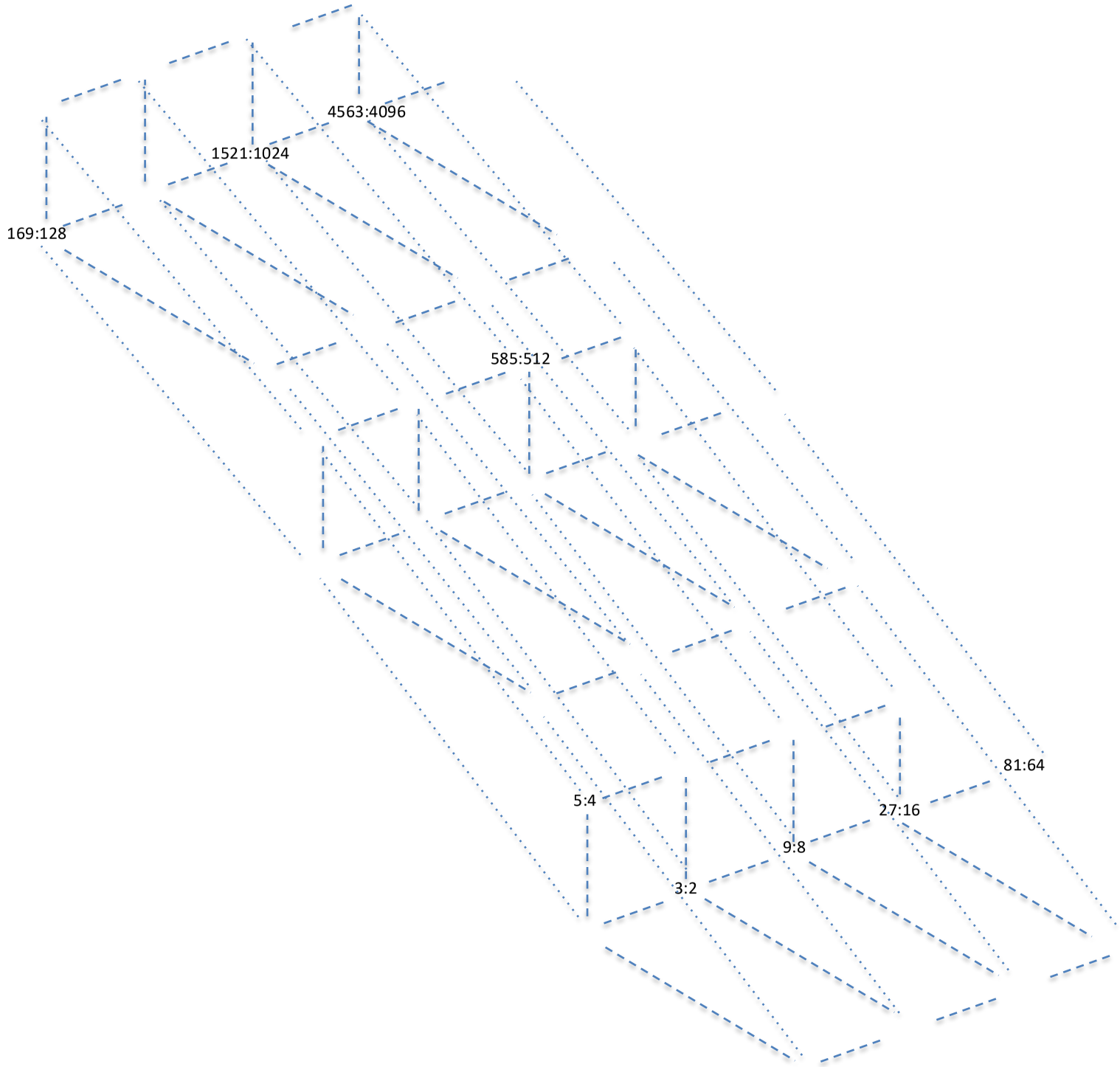
Measures 110-115



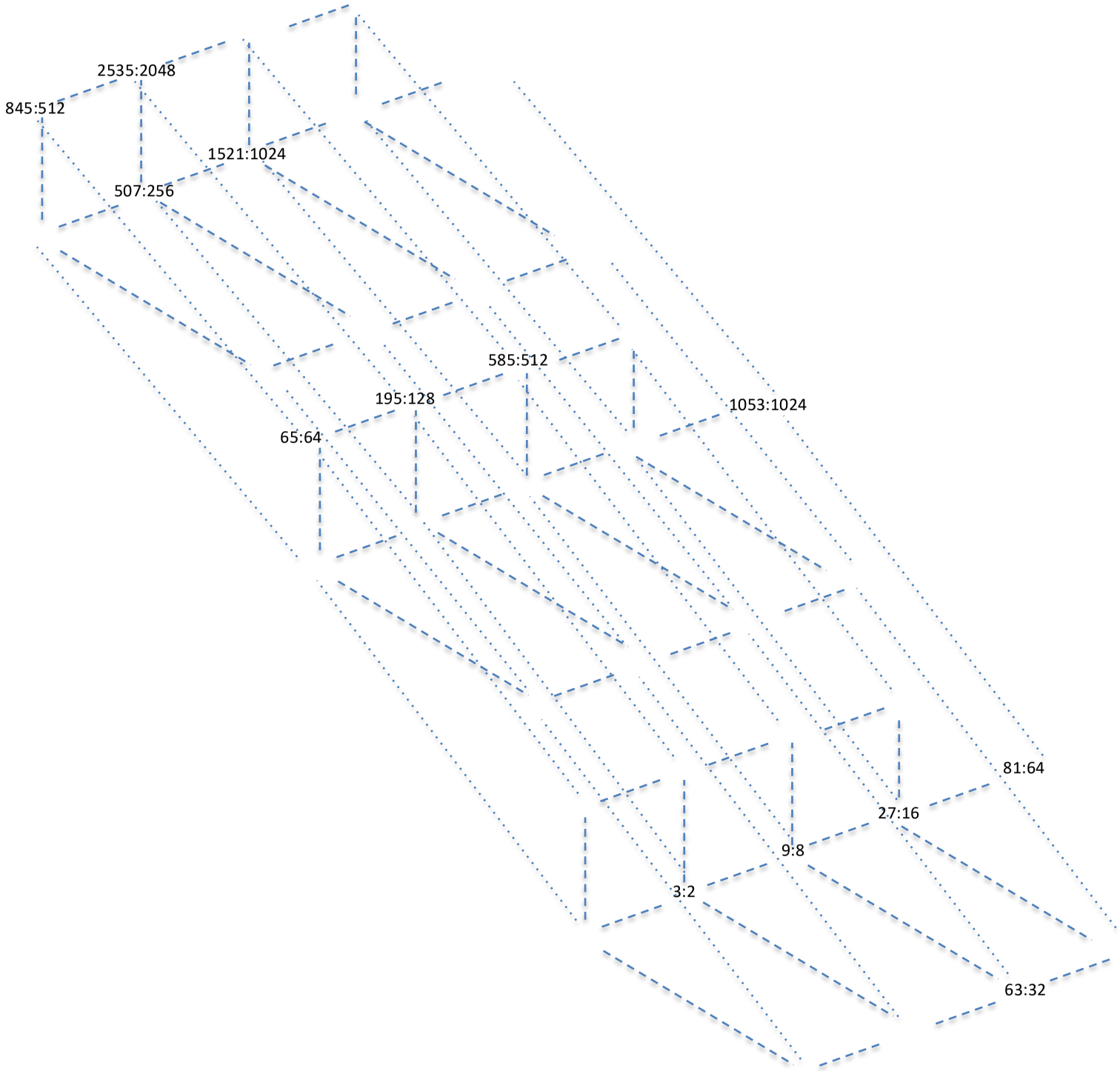
Measures 116-121



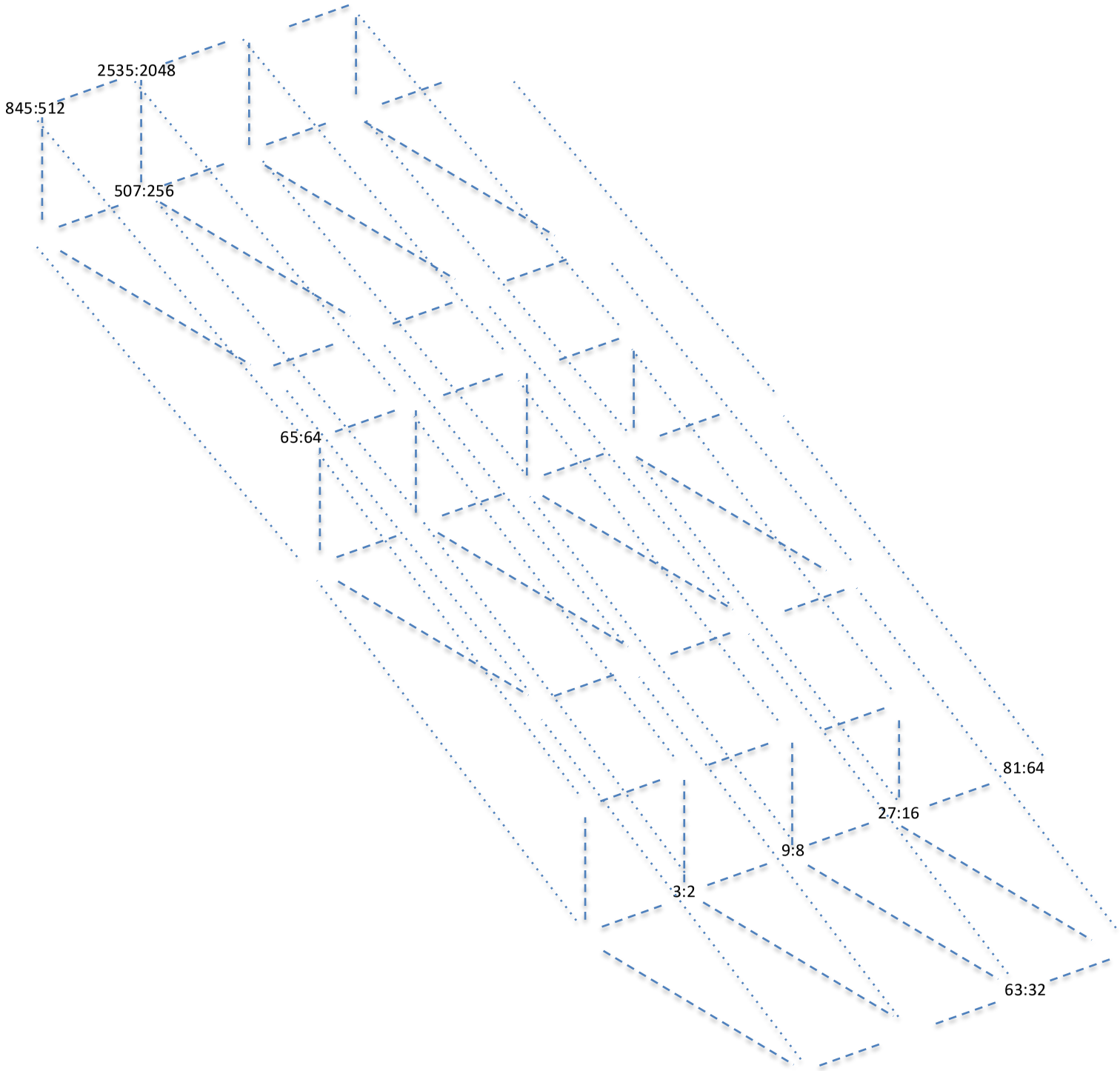
Measures 122-127



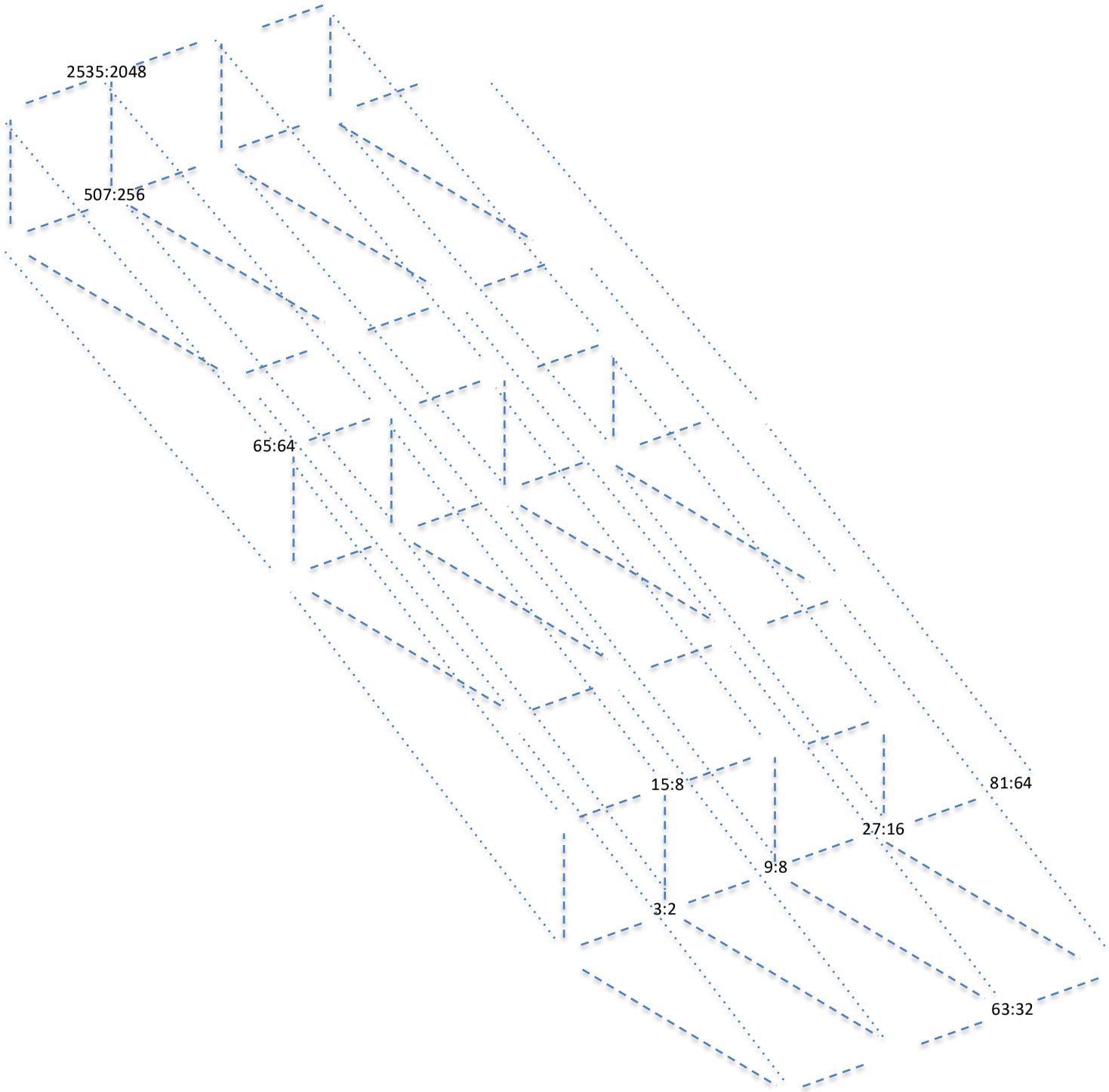
Measures 128-135



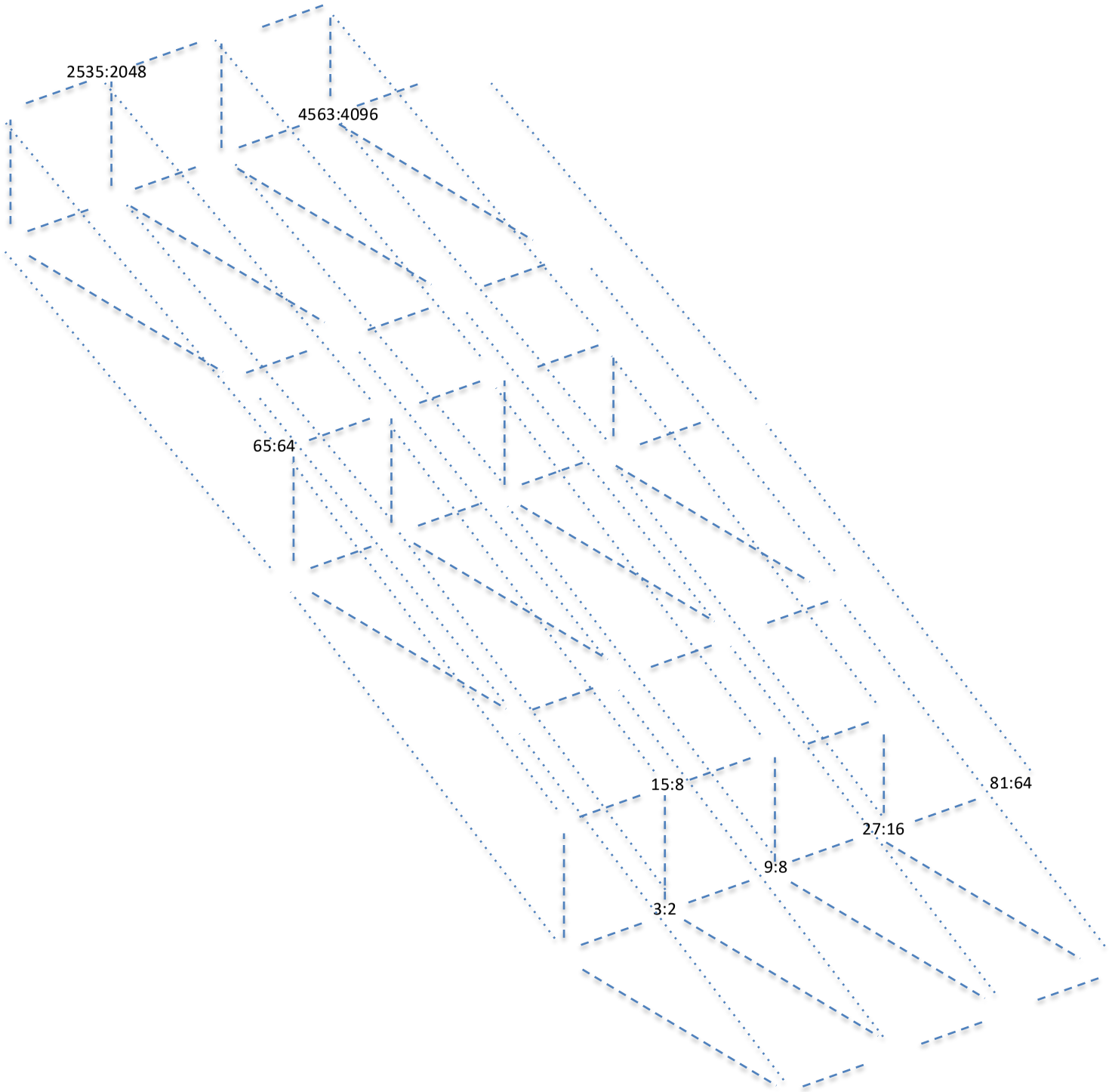
Measures 136-141



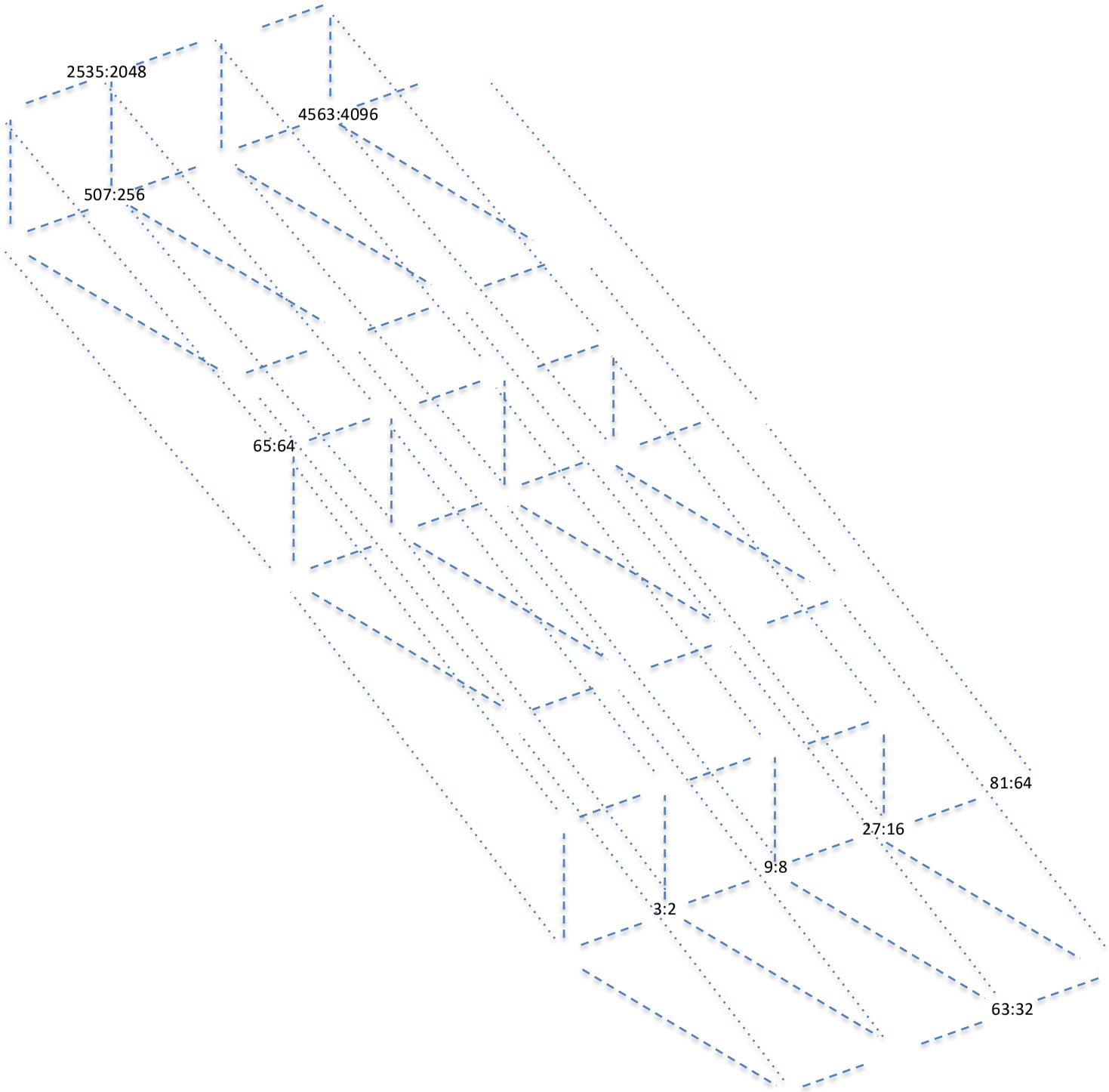
Measures 142-147



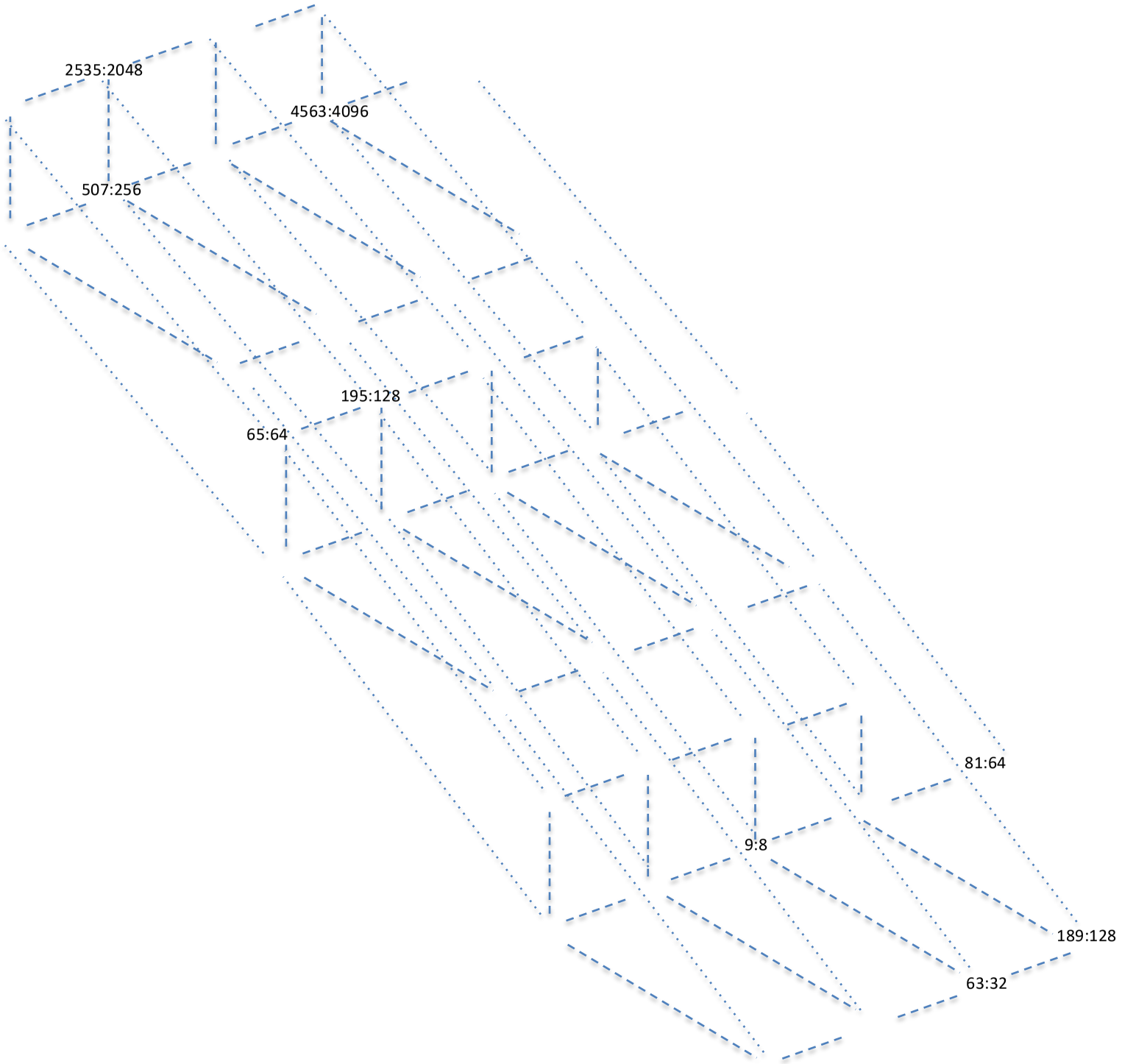
Measures 148-153



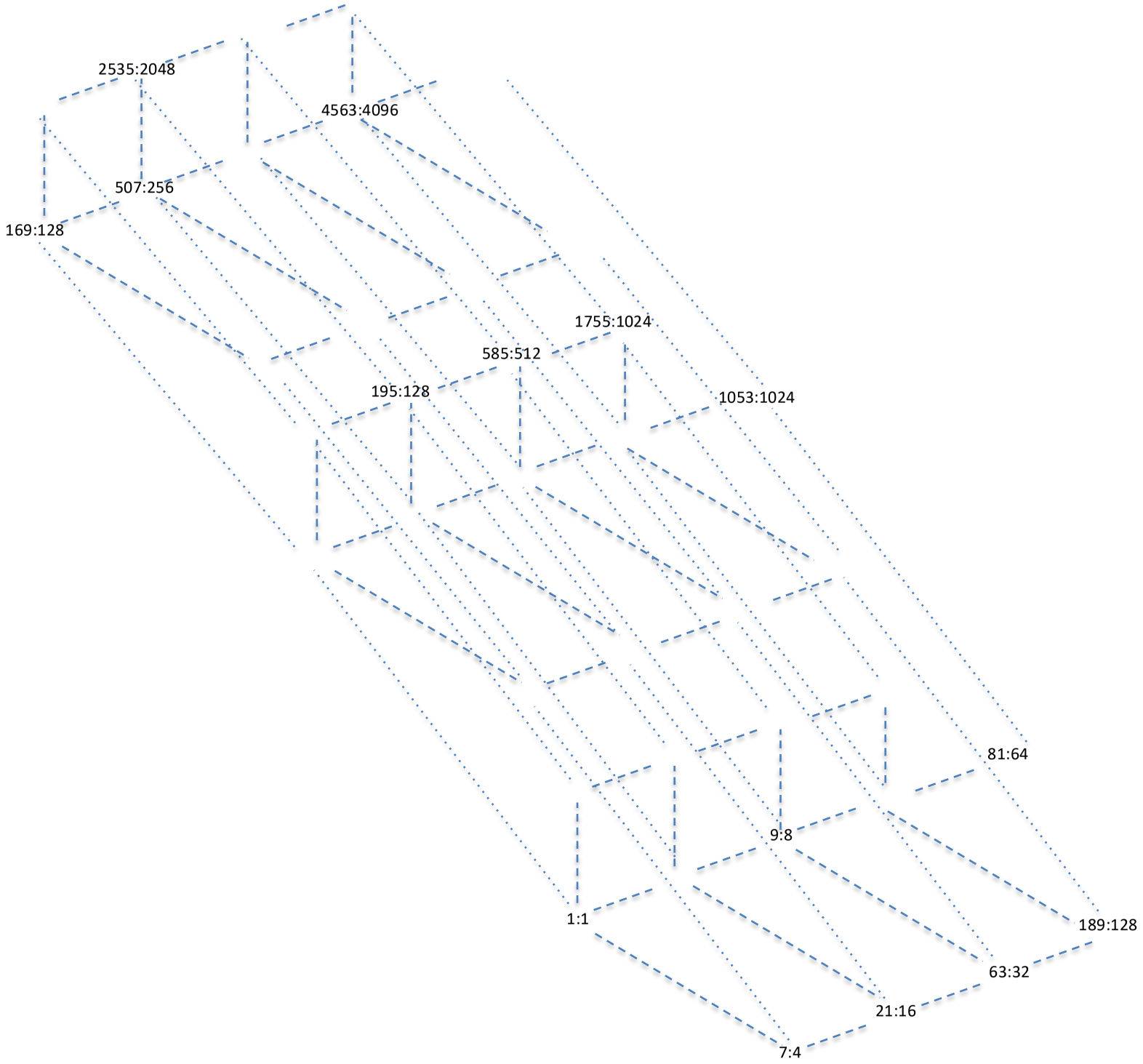
Measures 154-159



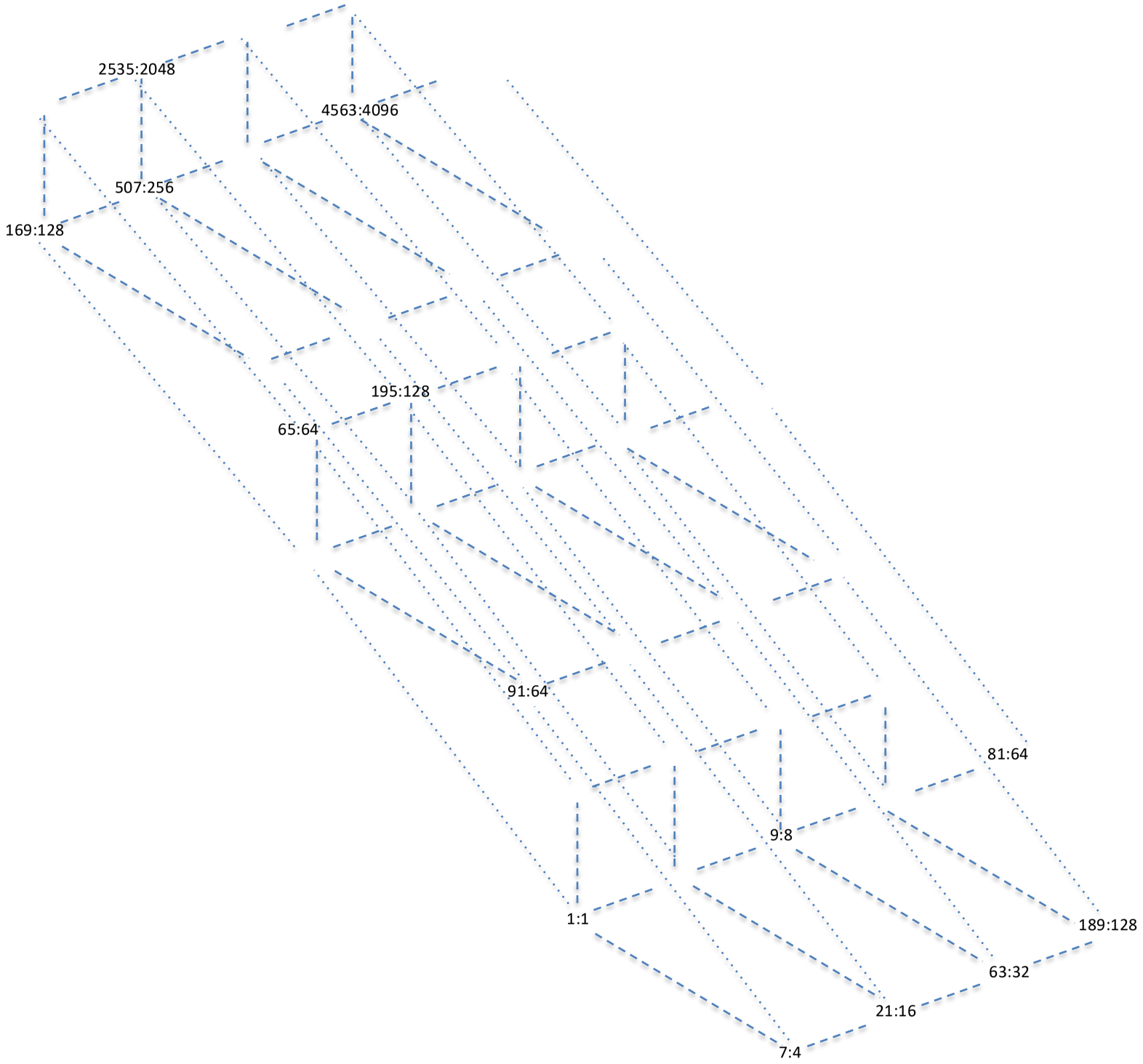
Measures 166-171



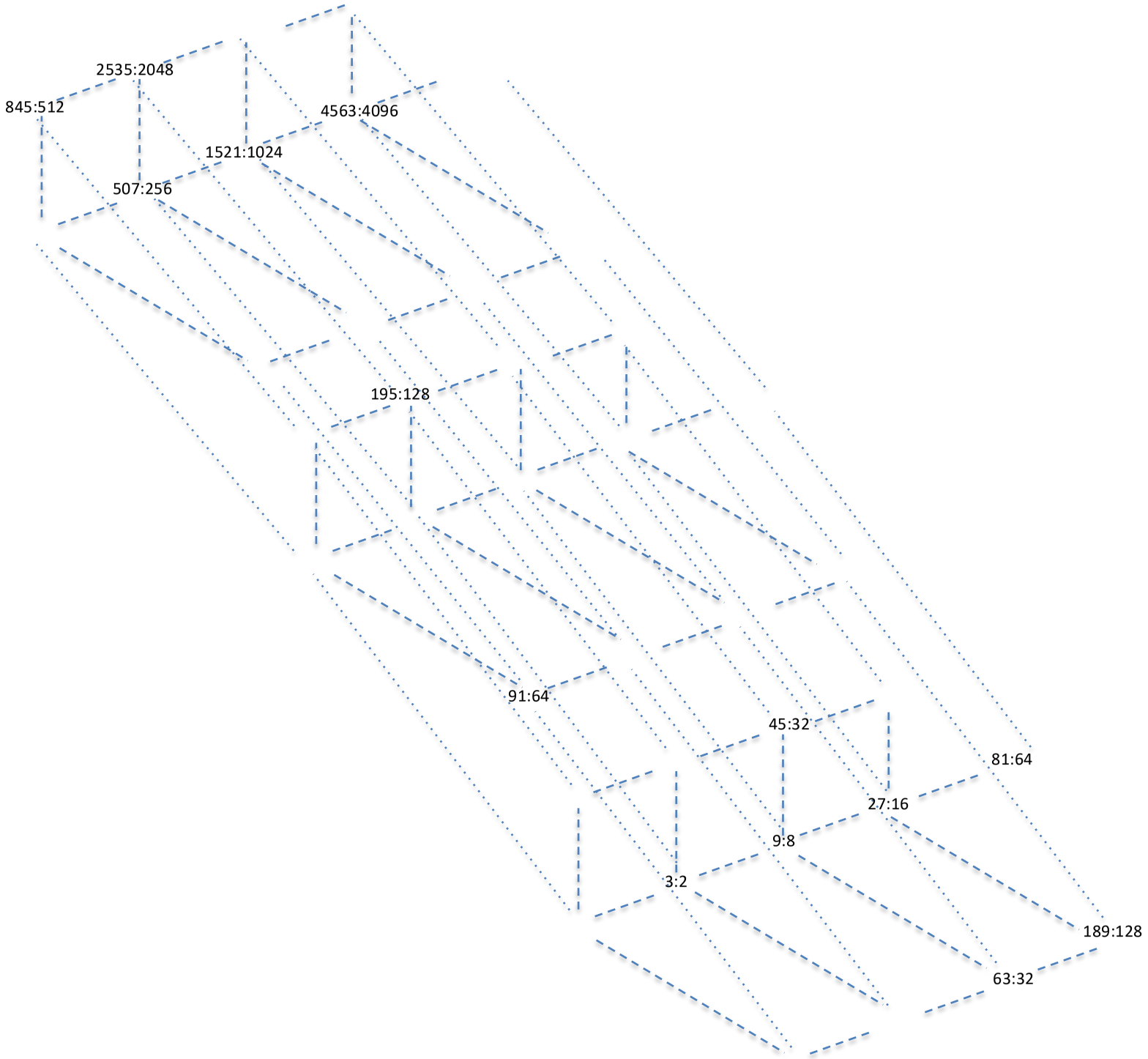
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Measures 185-191



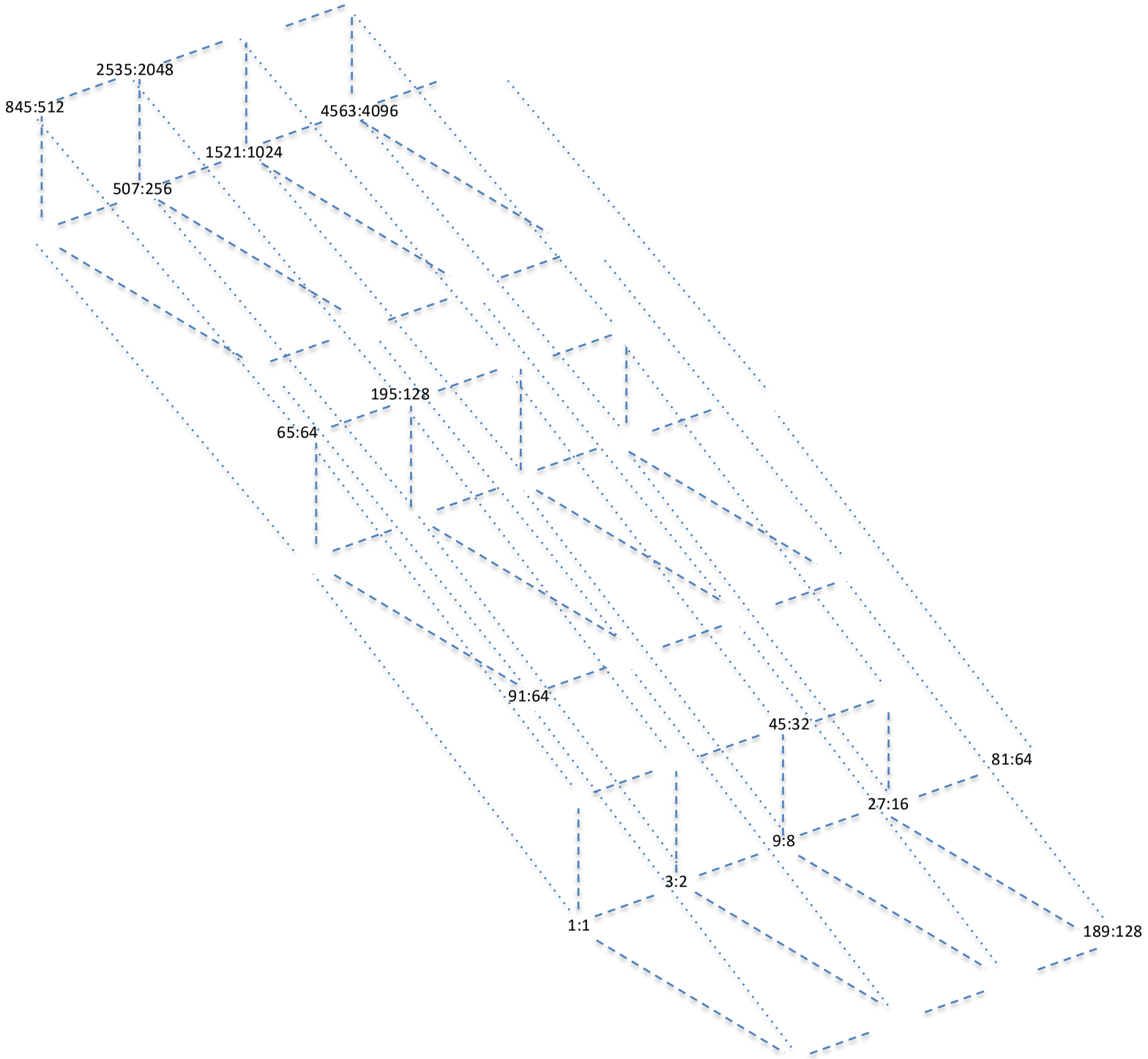
Measures 196-200



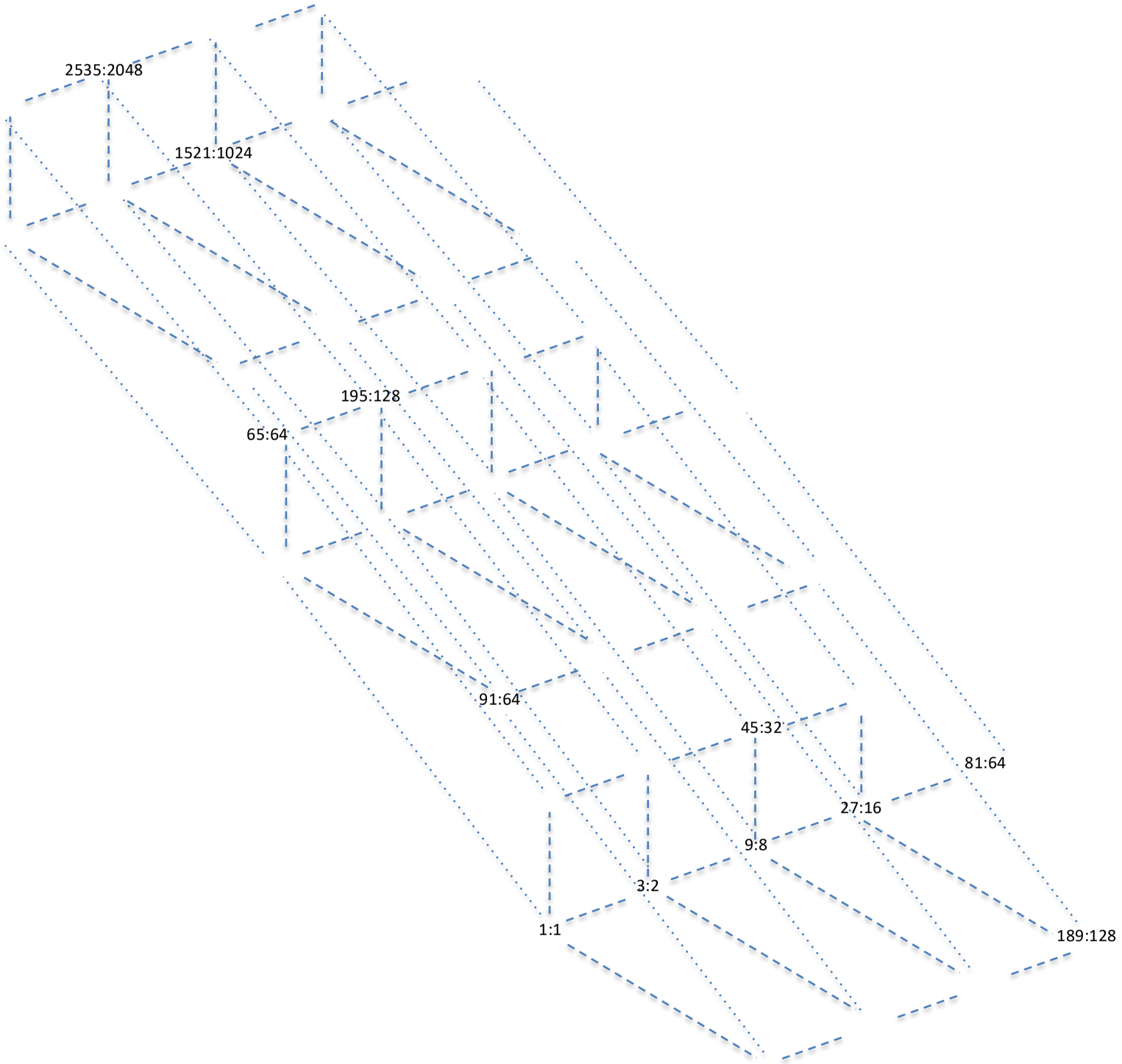
Measures 206-219



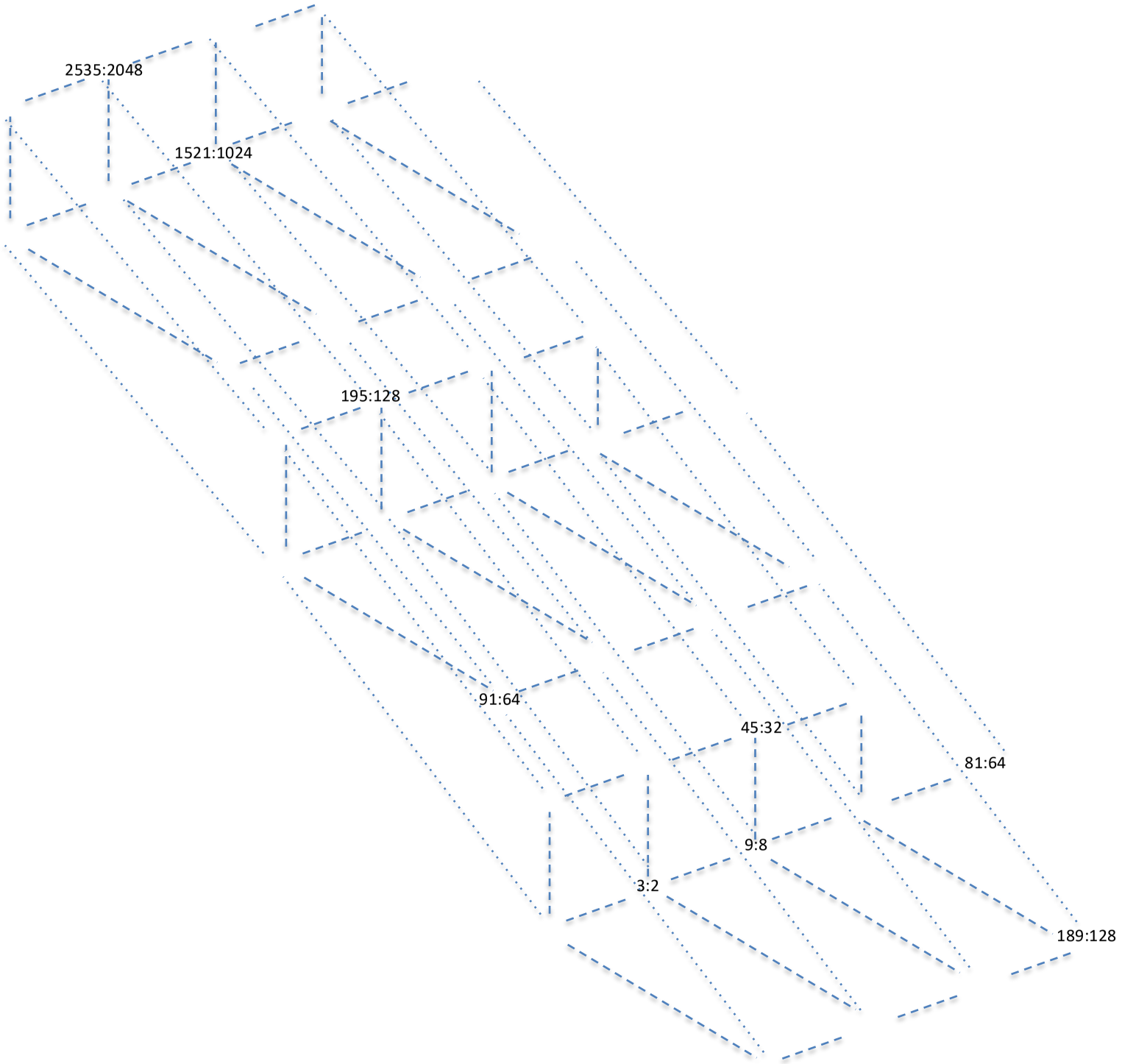
Measures 221-230



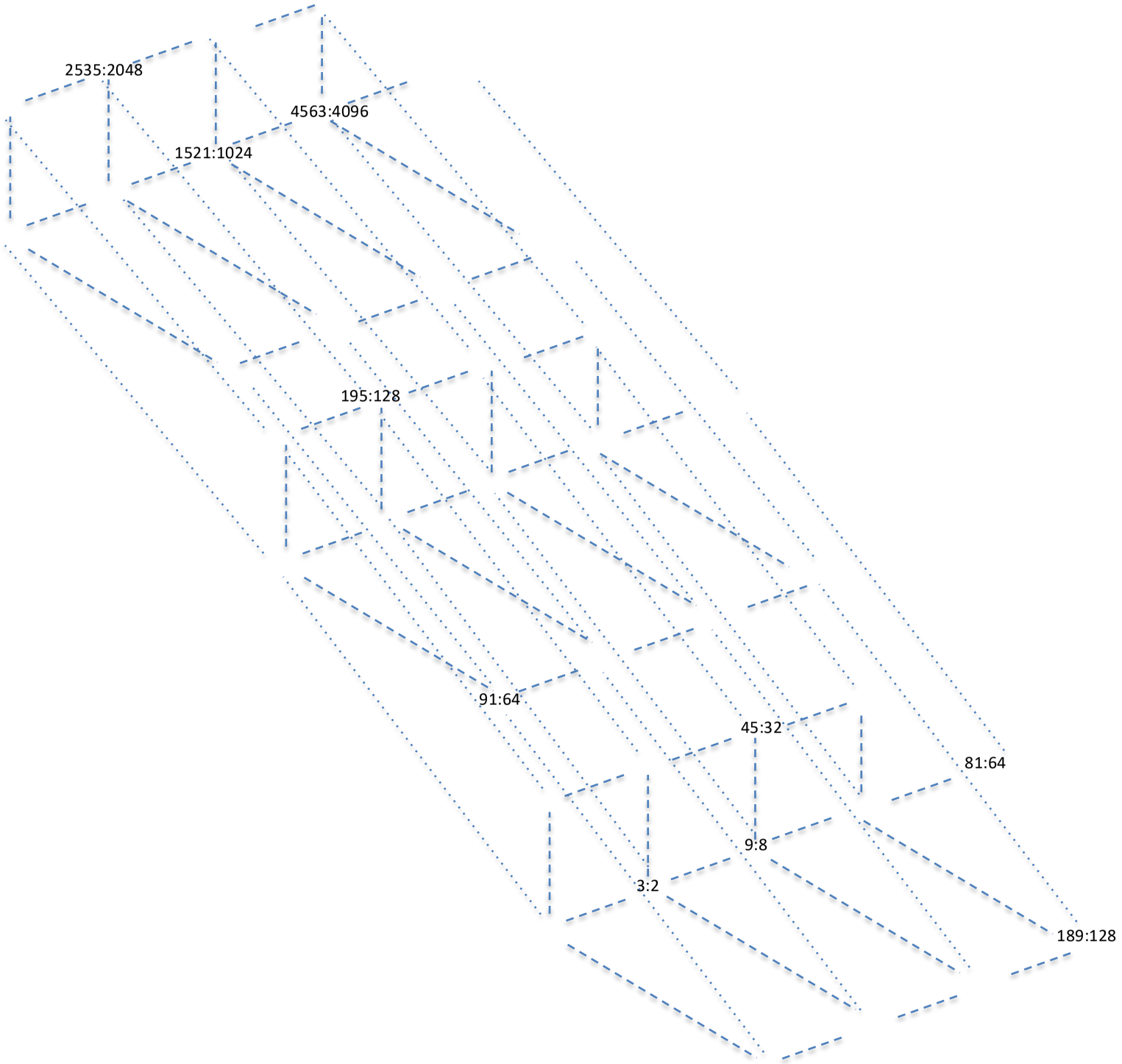
Measures 231-233



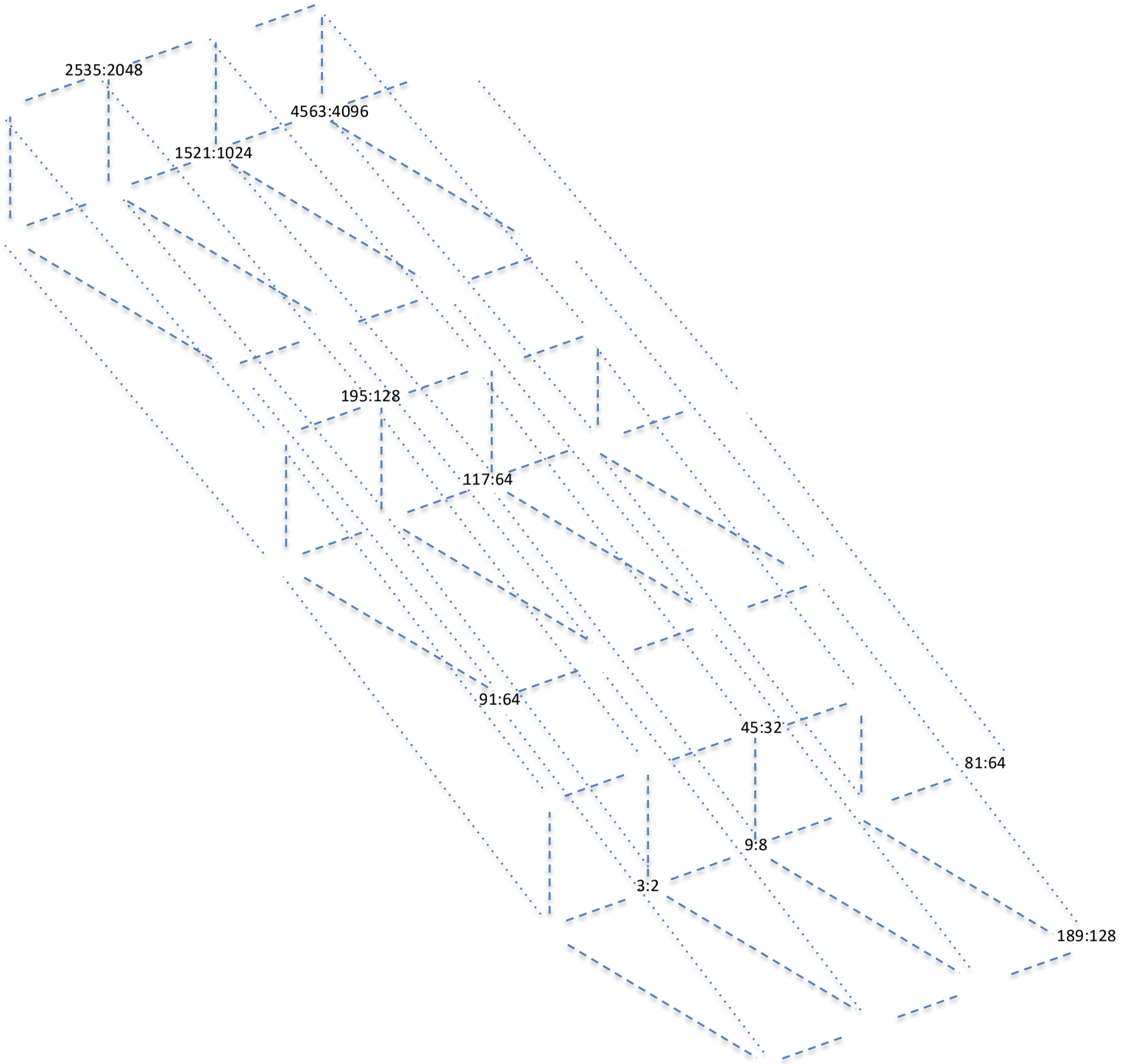
Measures 234-240



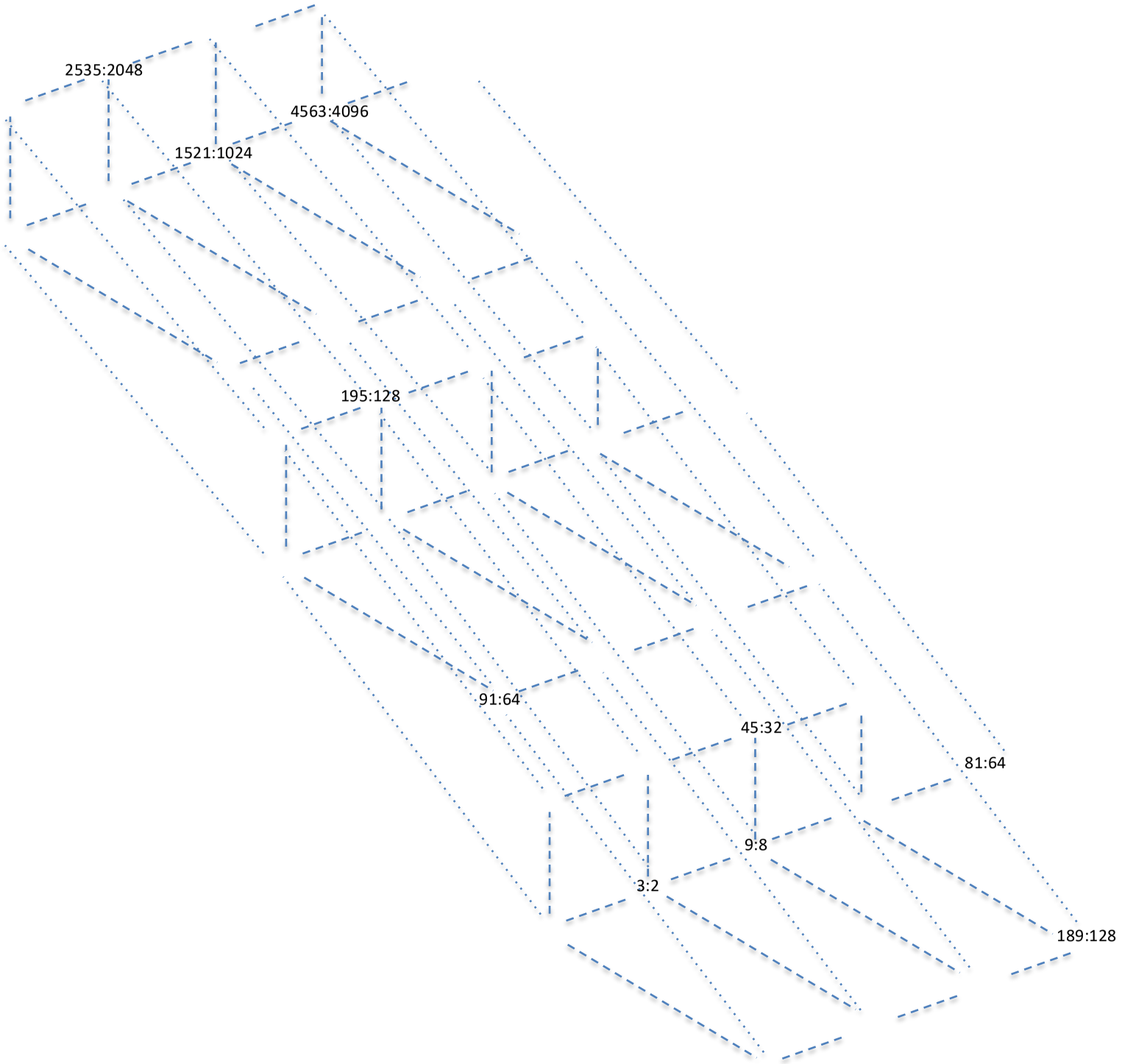
Measures 245-248



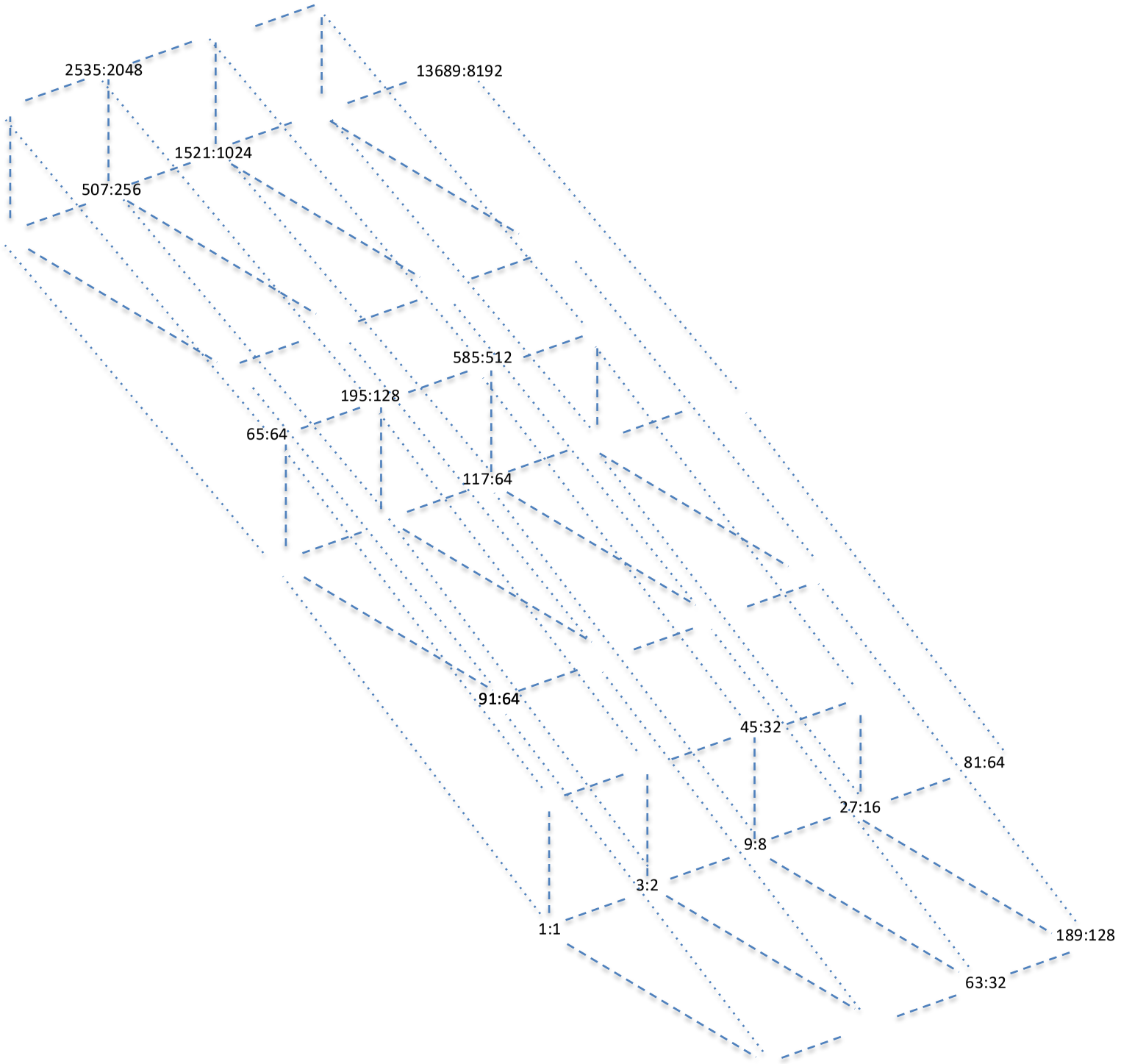
Measures 249-253



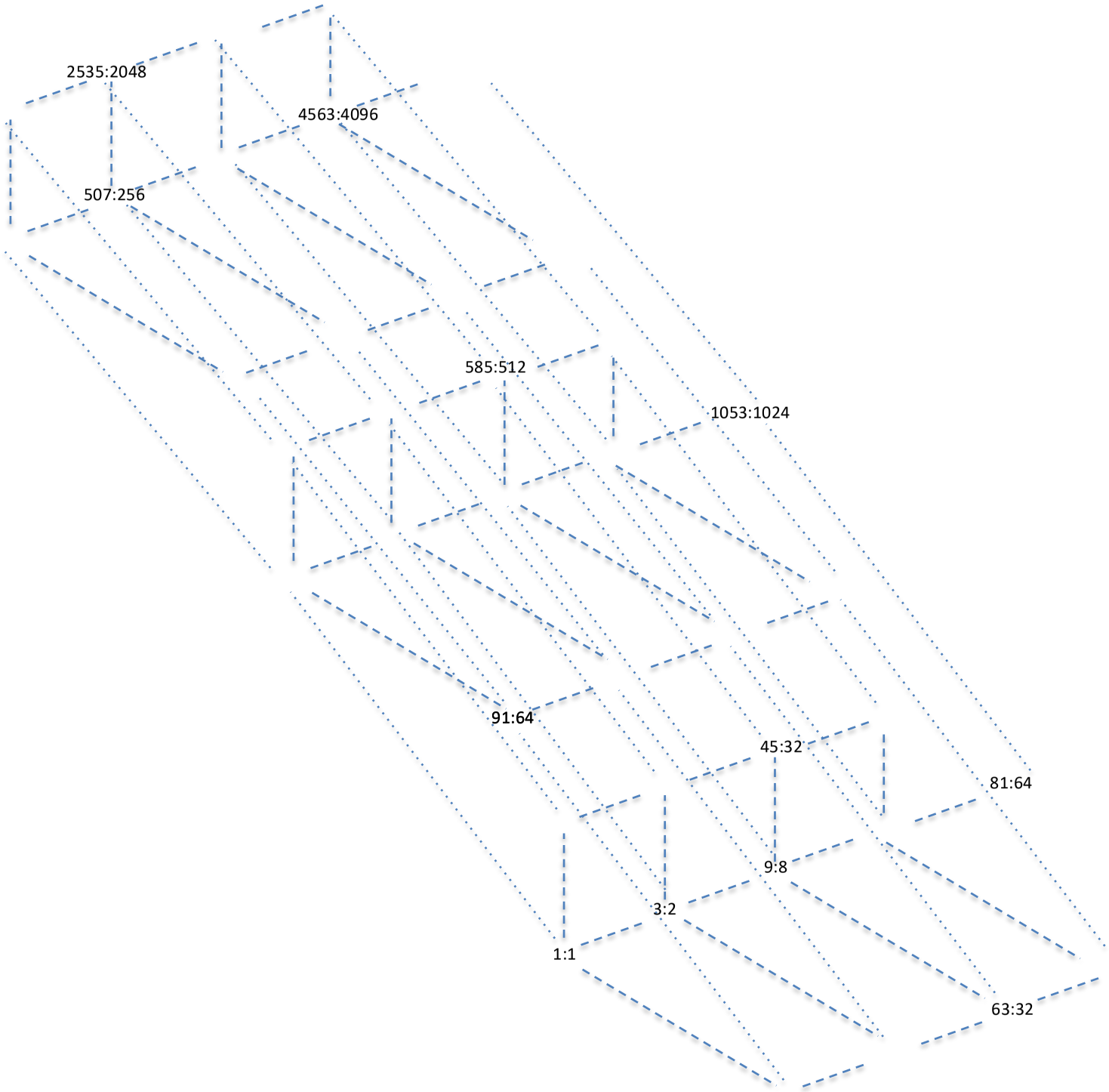
Measures 257-259



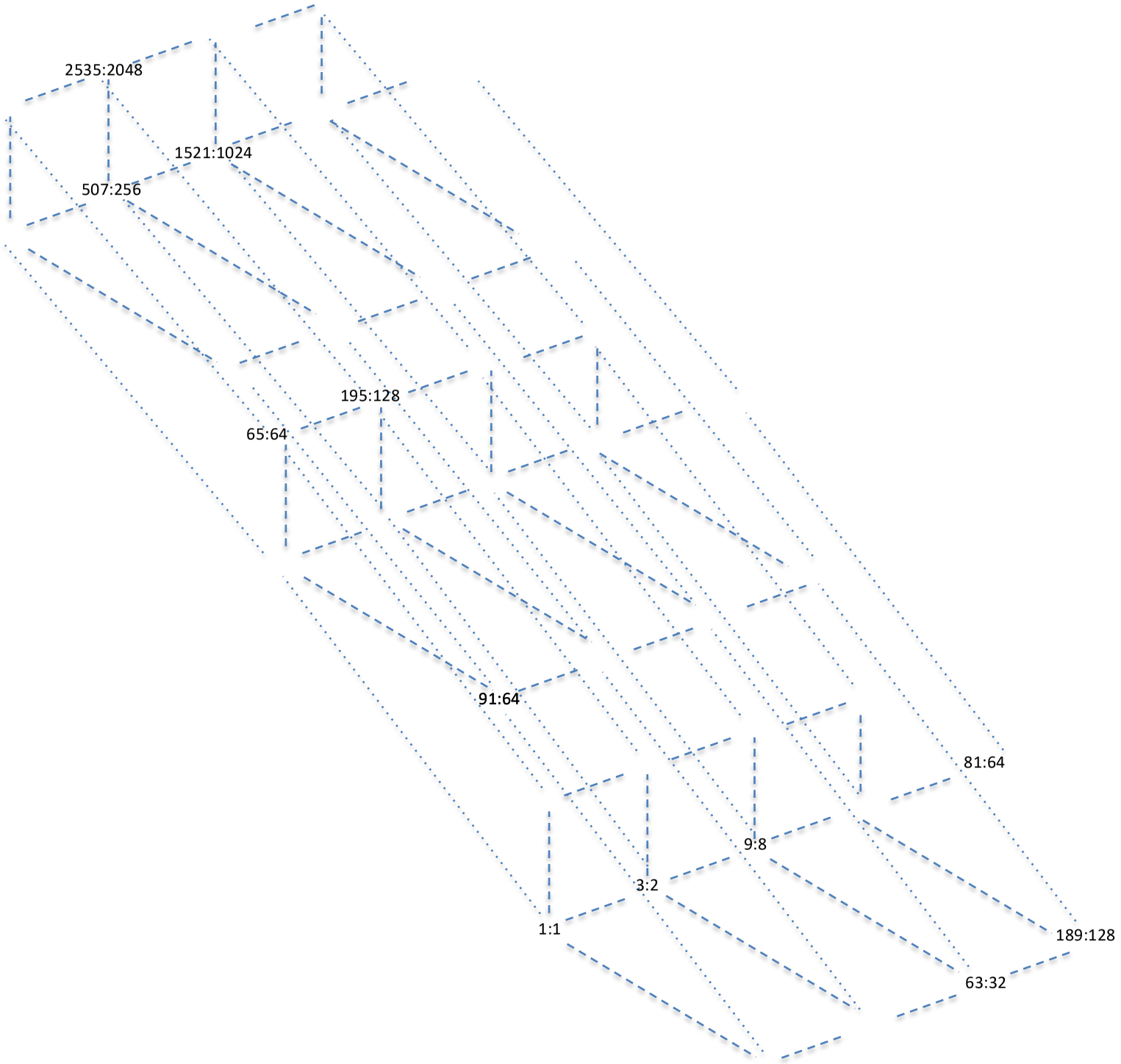
Measures 266-277



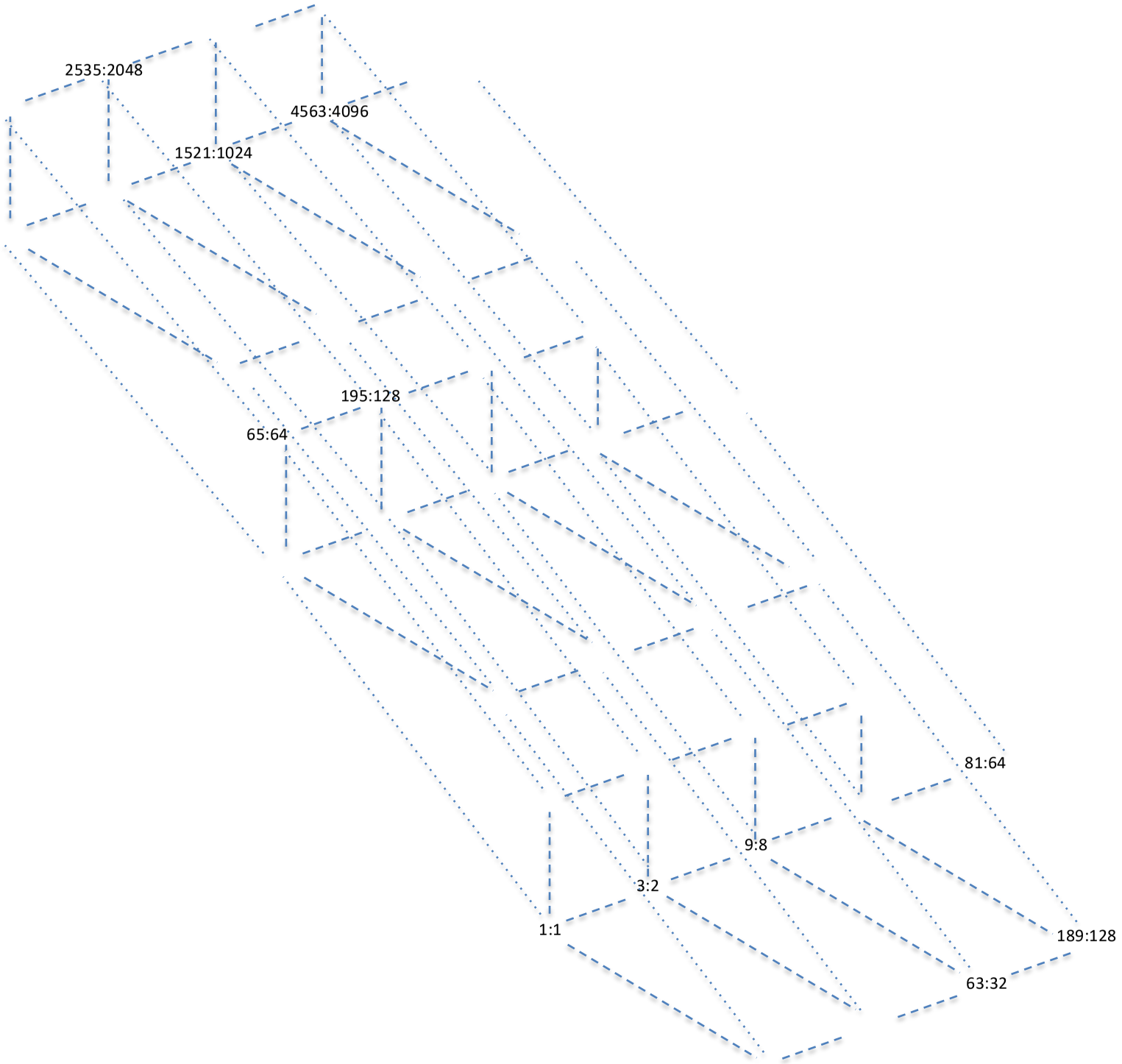
Measures 280-283



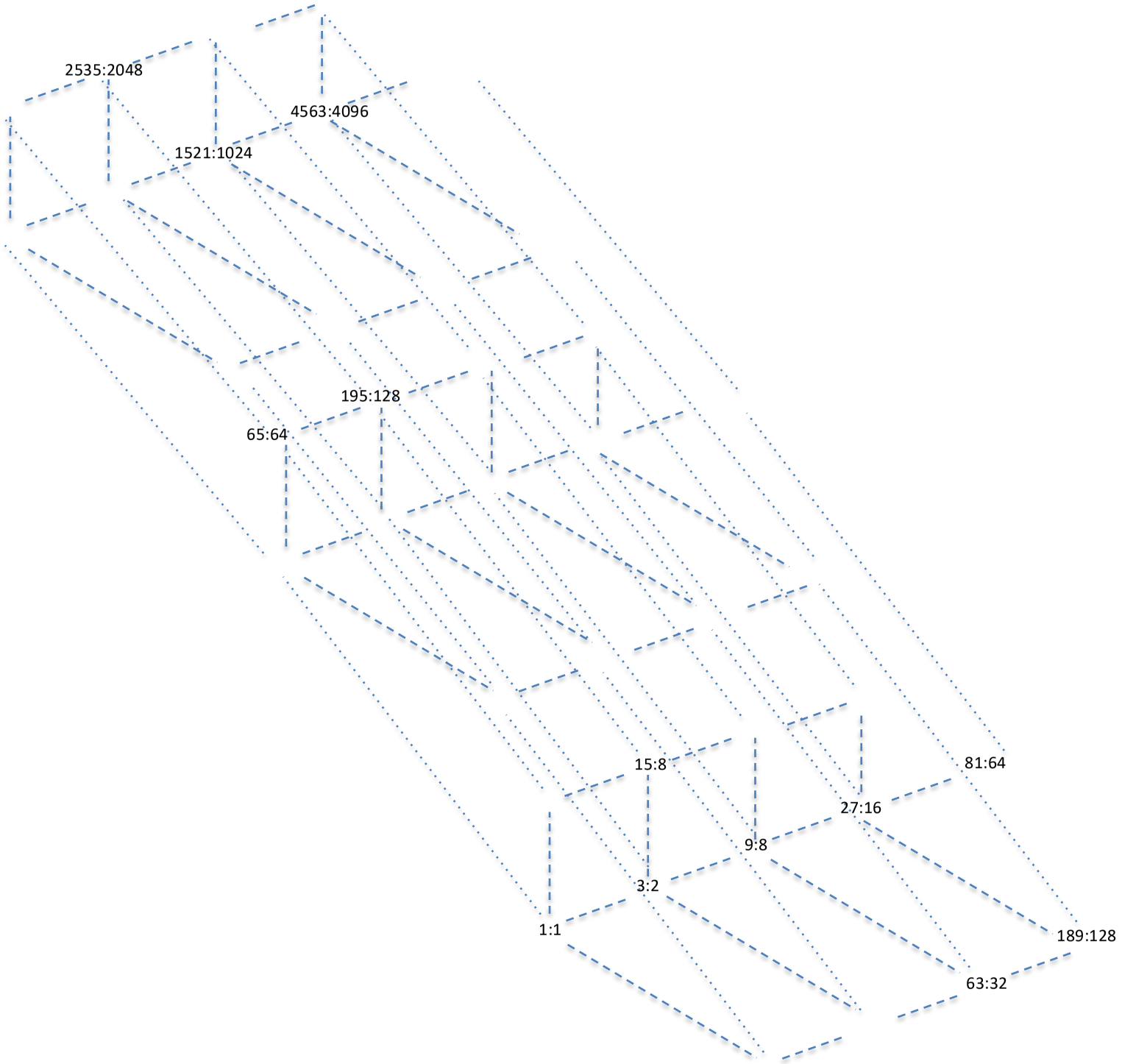
Measures 284-287



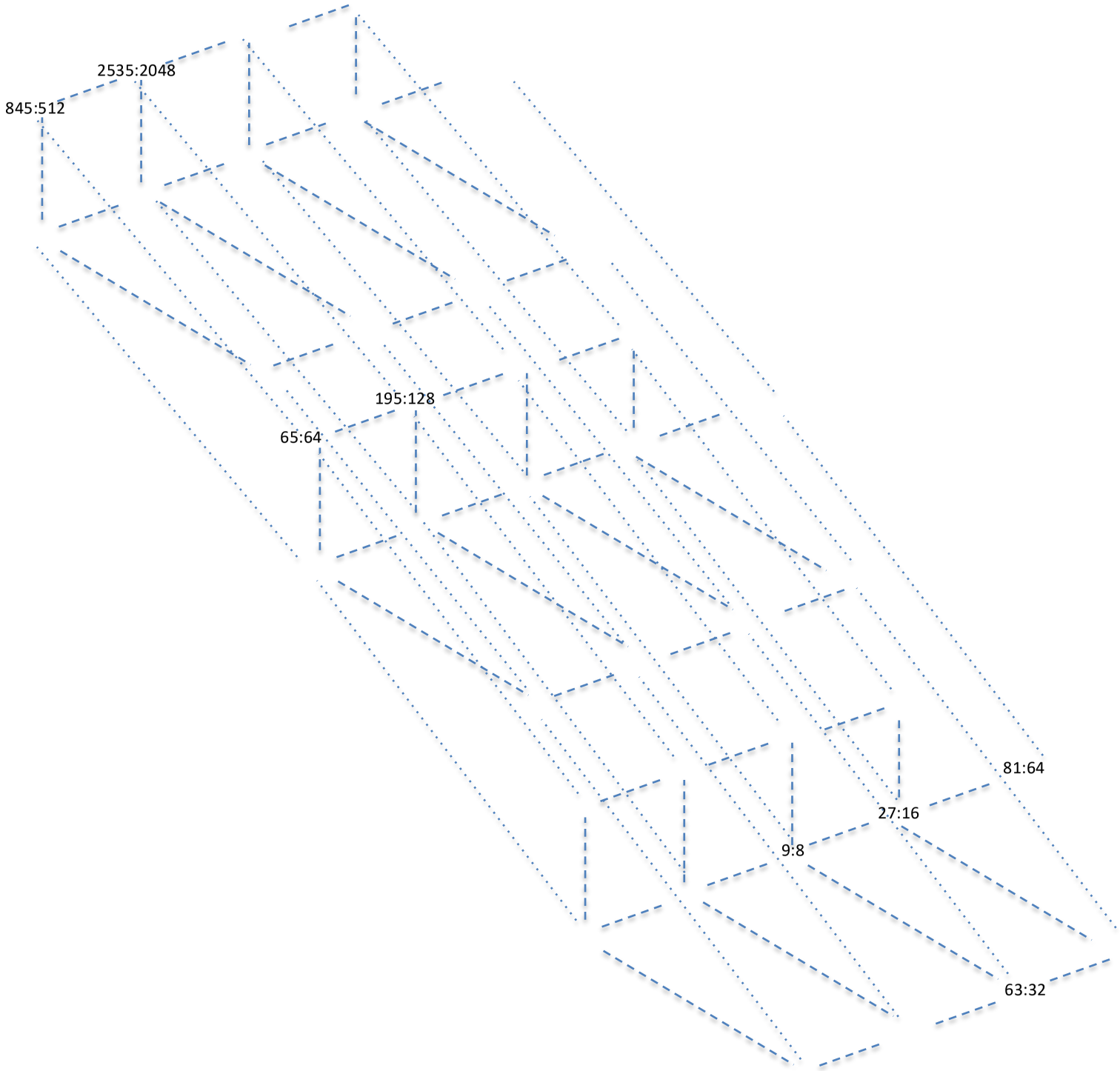
Measures 288-291



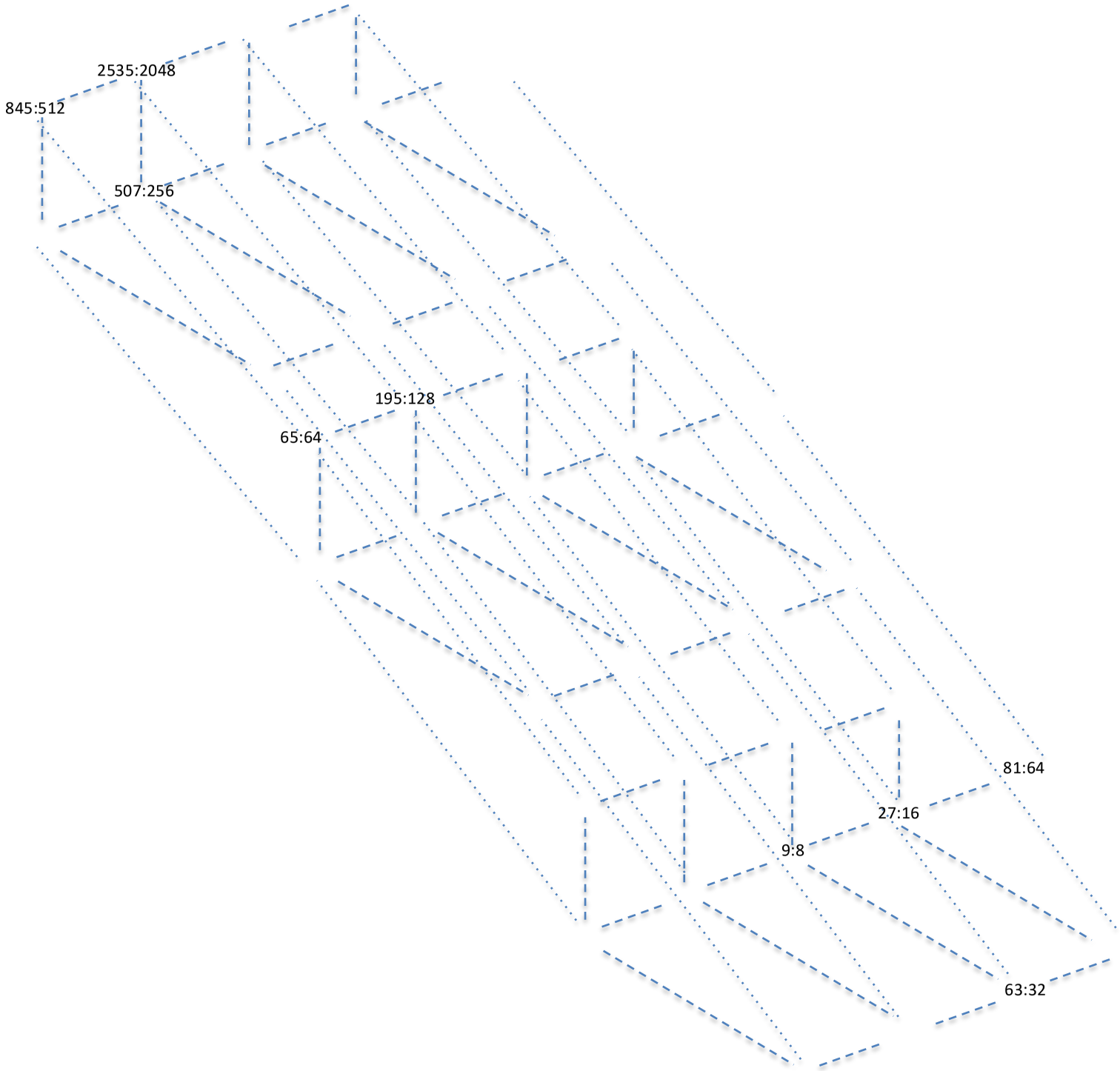
Measures 292-299



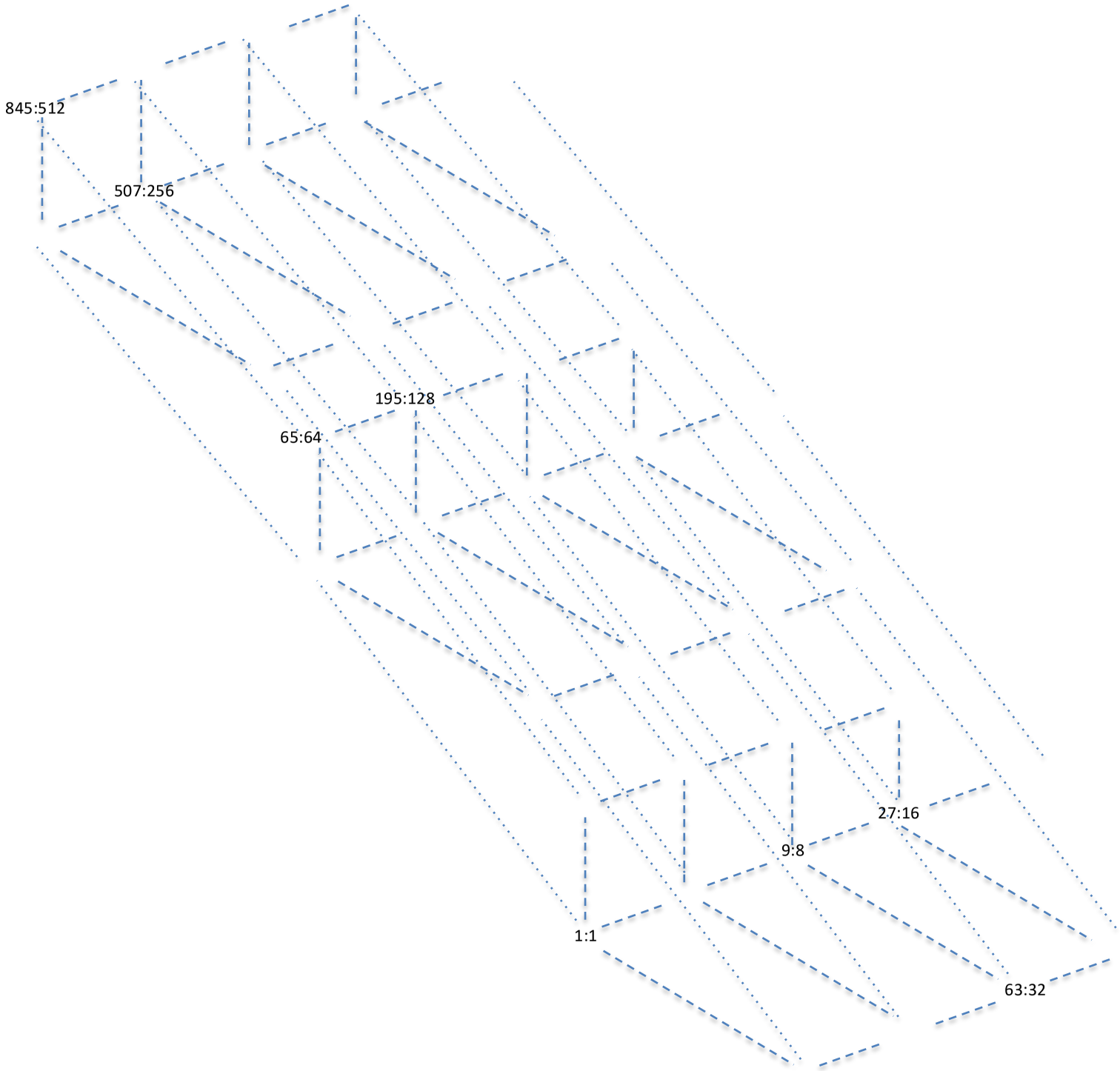
Measures 300-302



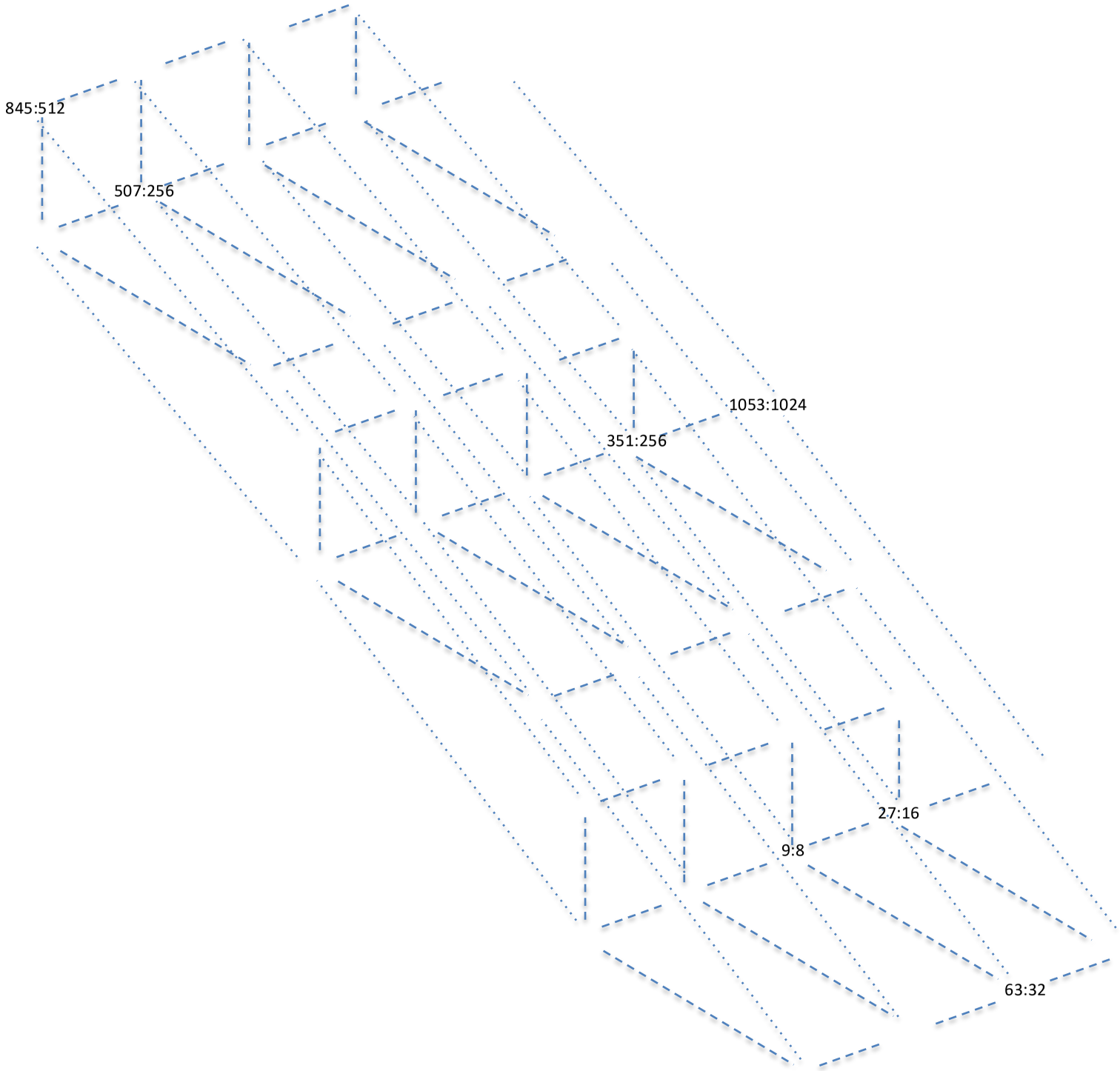
Measures 303-305



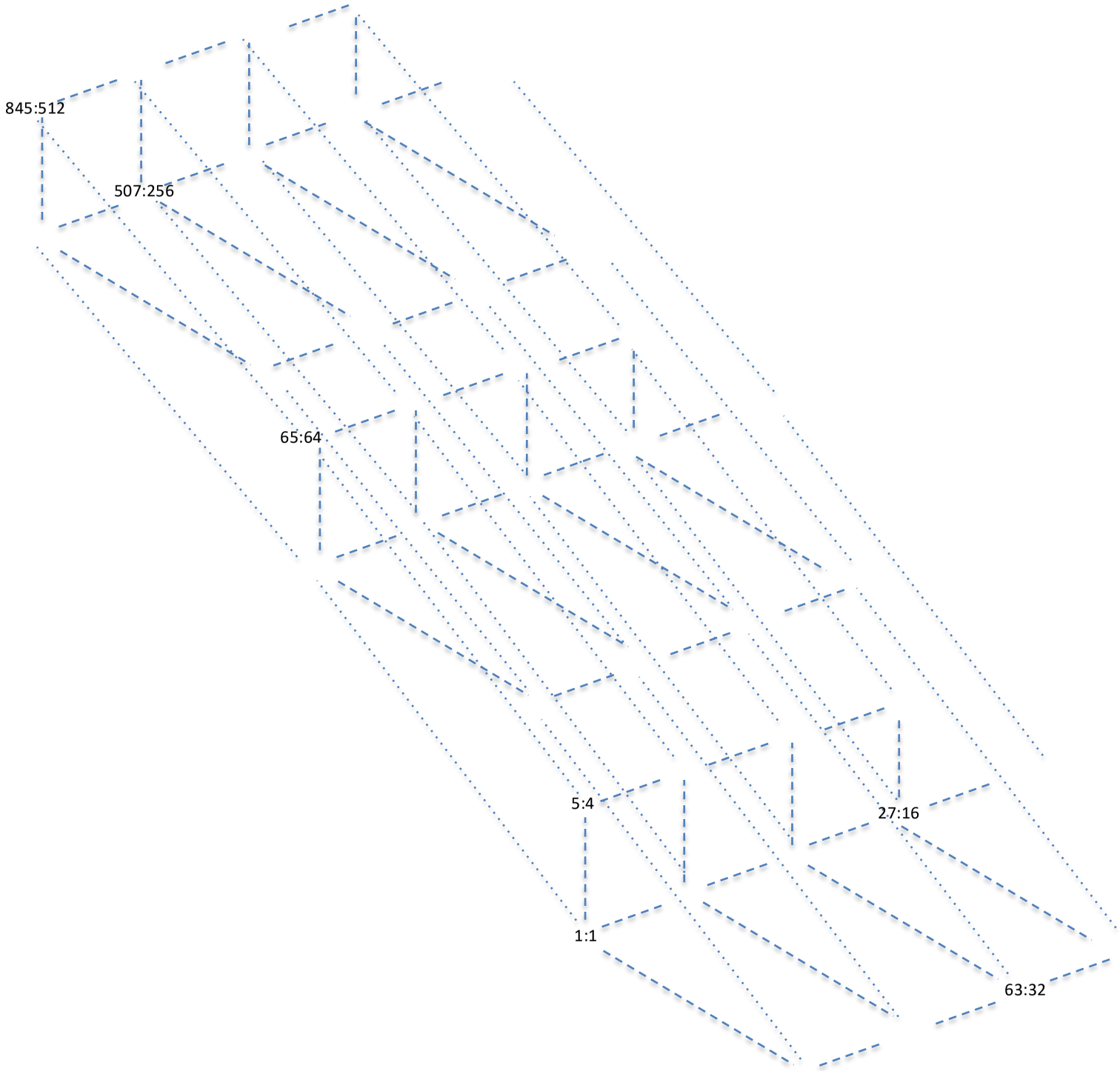
Measures 306-312



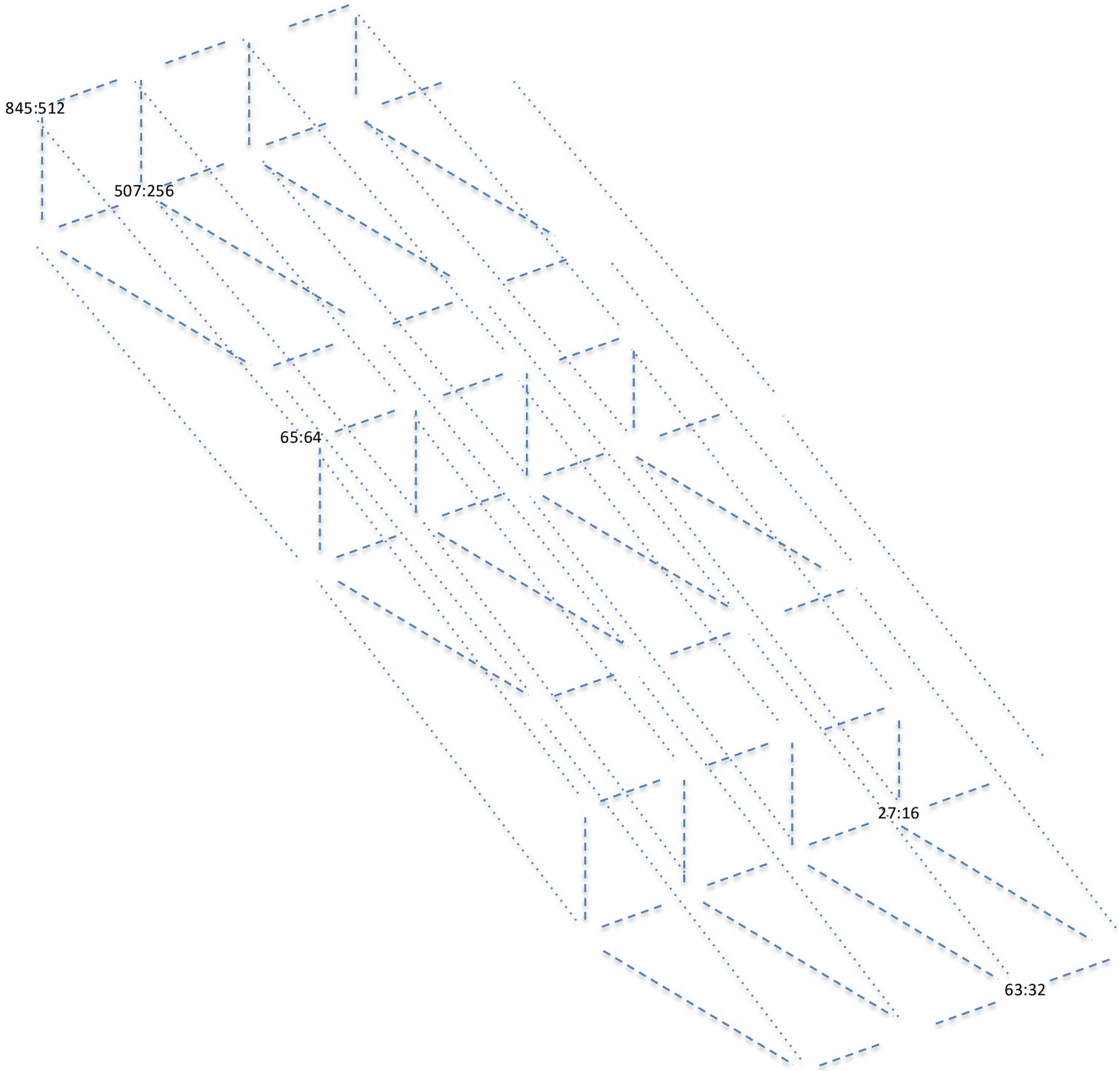
Measures 314-319



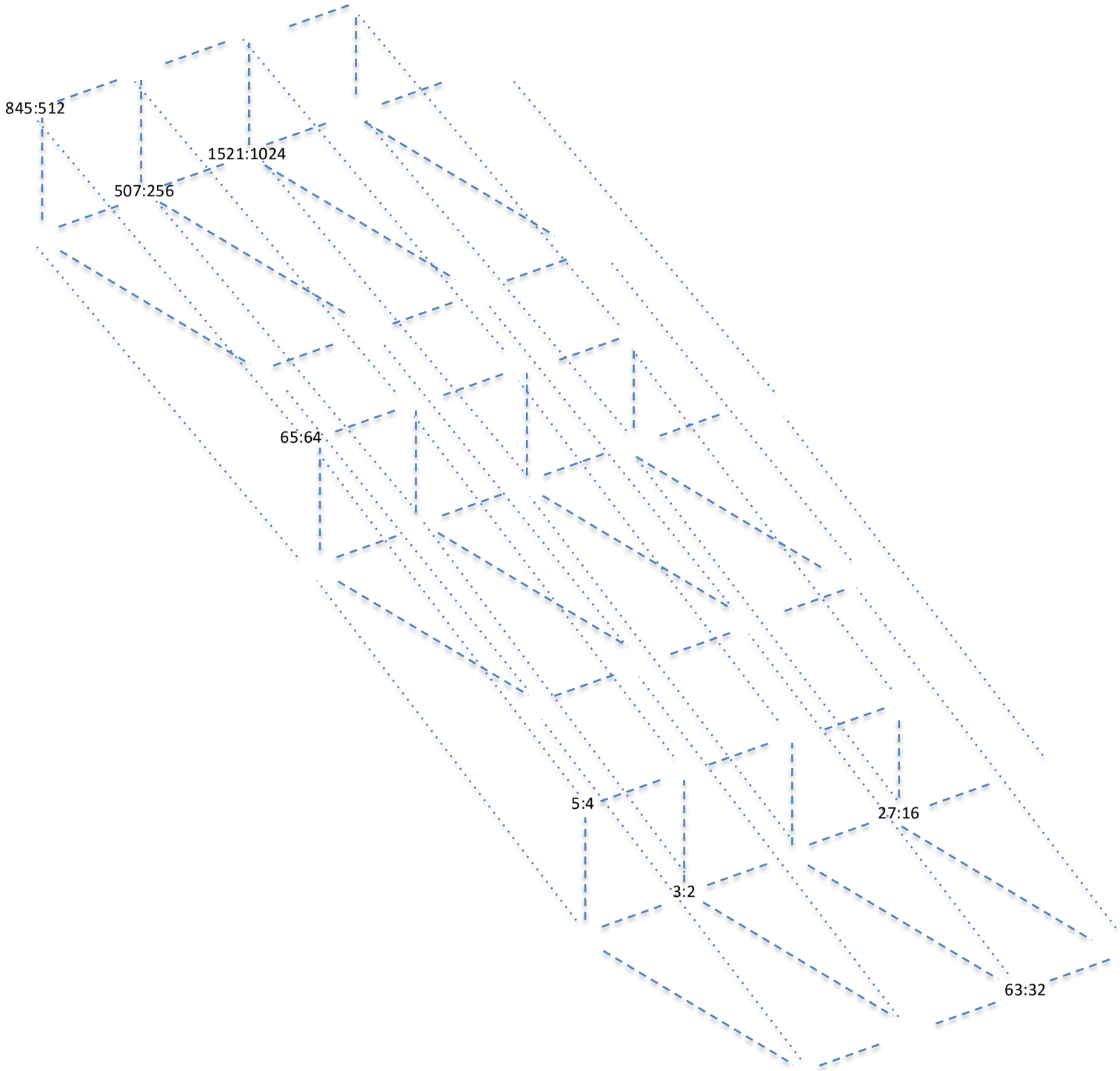
Measures 320-325



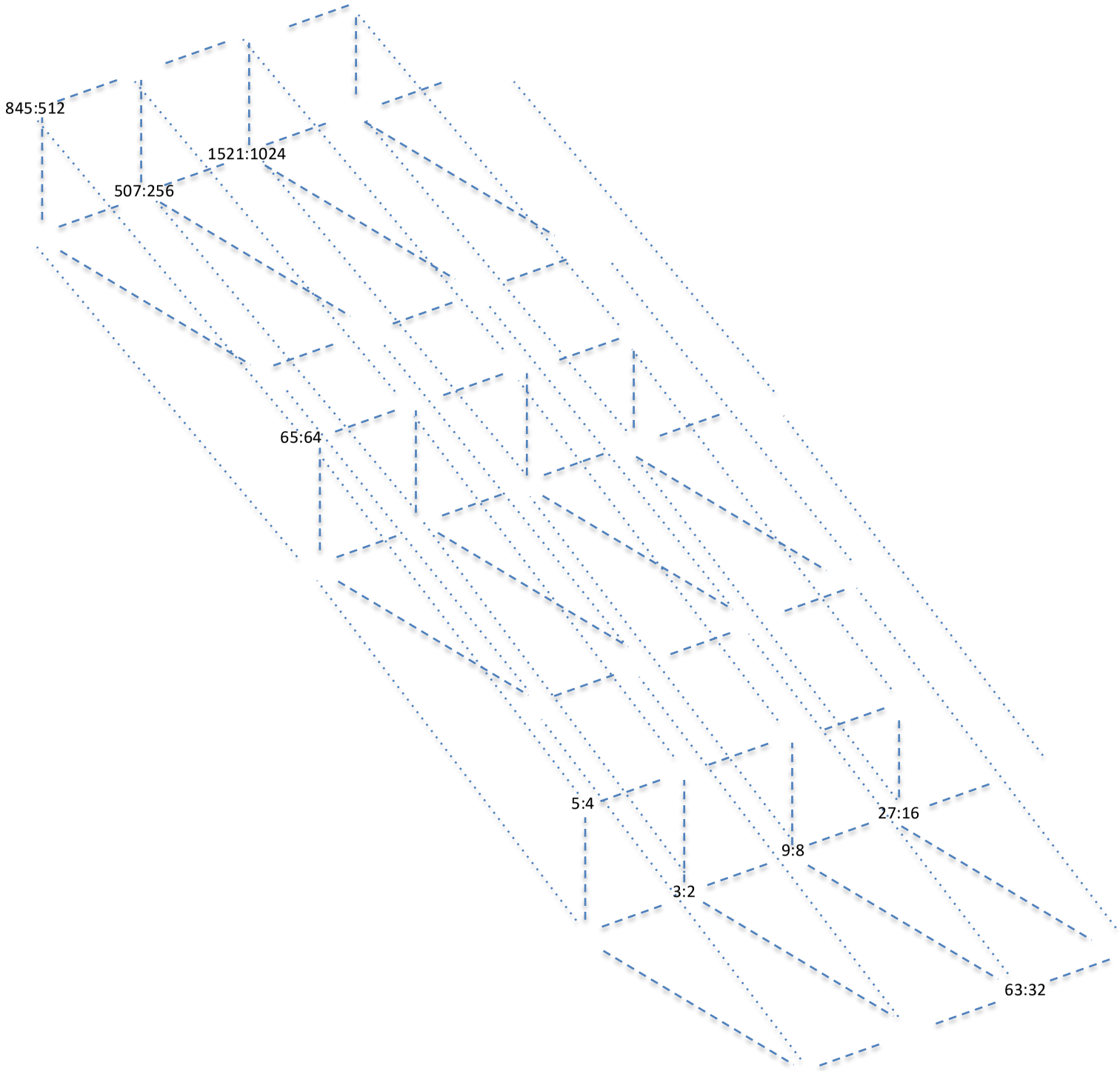
Measures 326-332



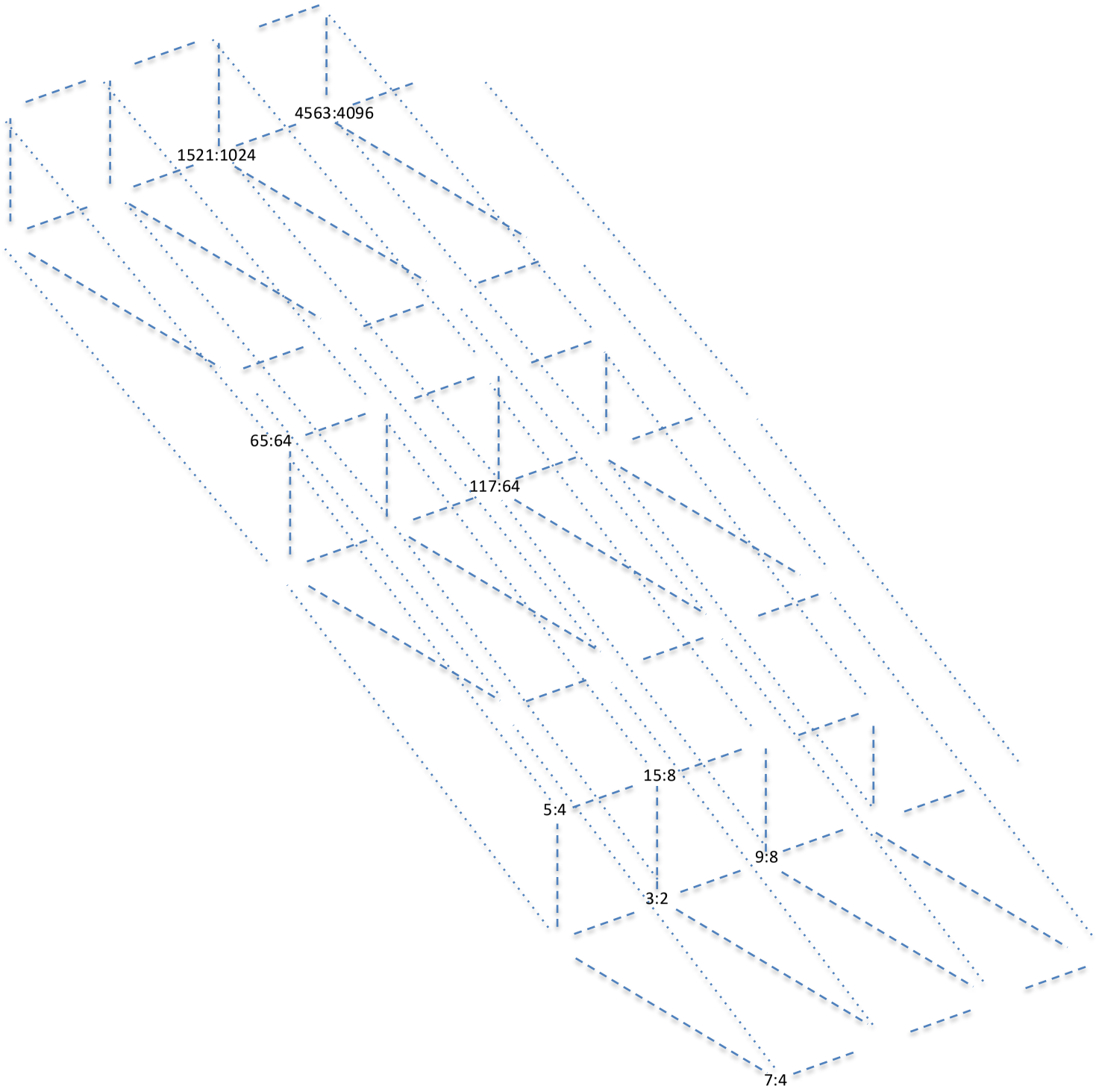
Measures 340-342



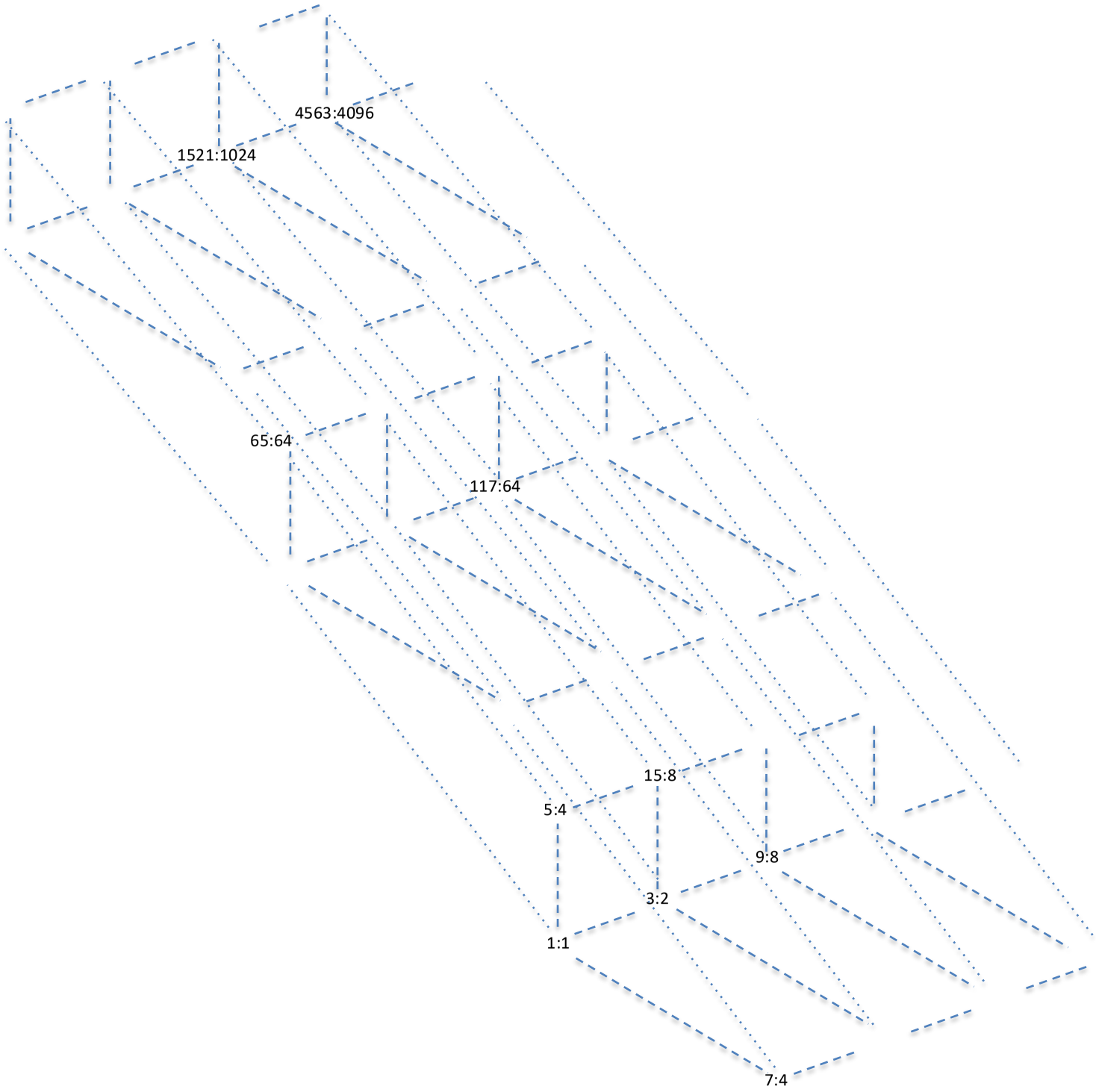
Measures 343-346



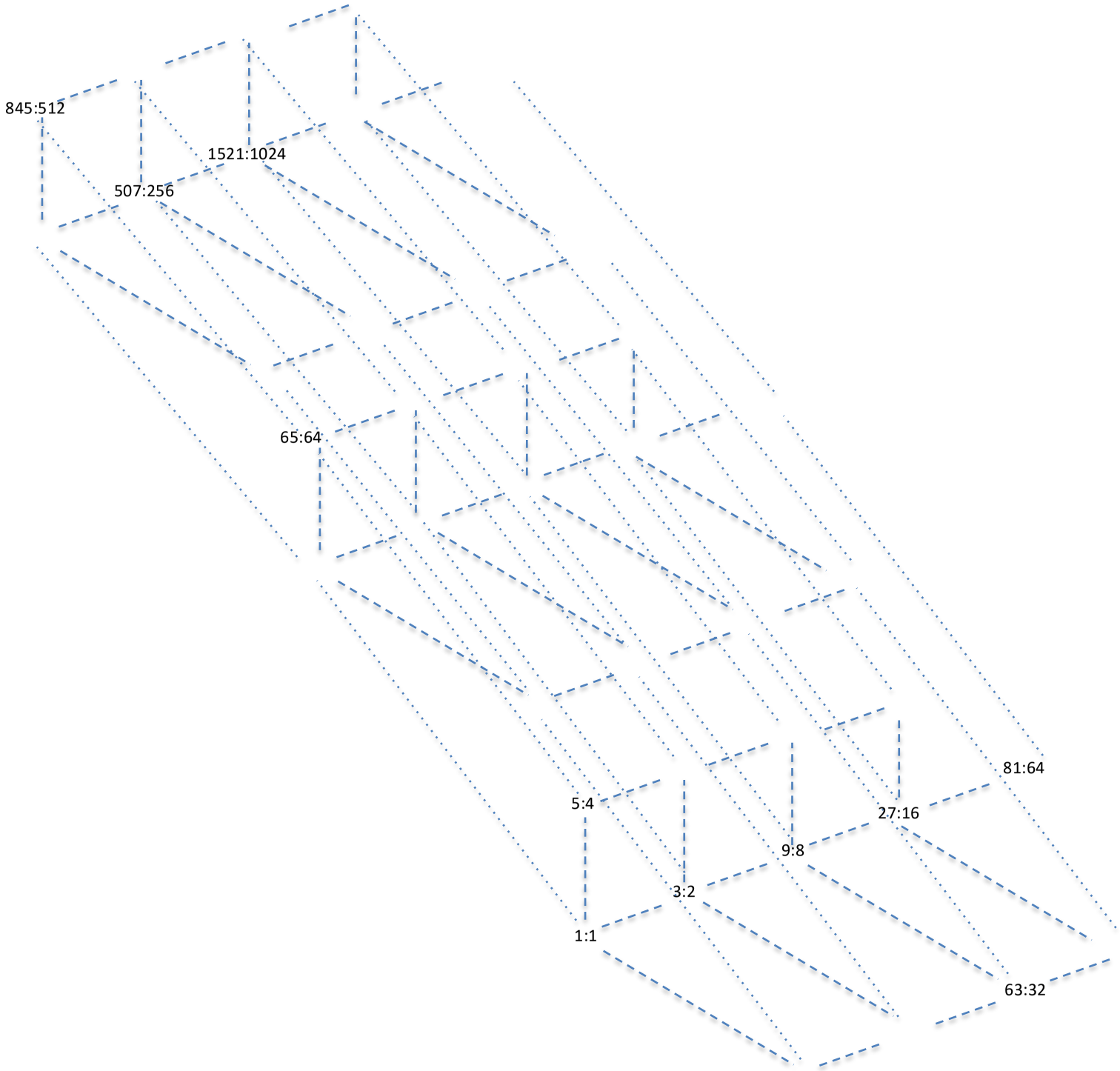
Measures 347-350



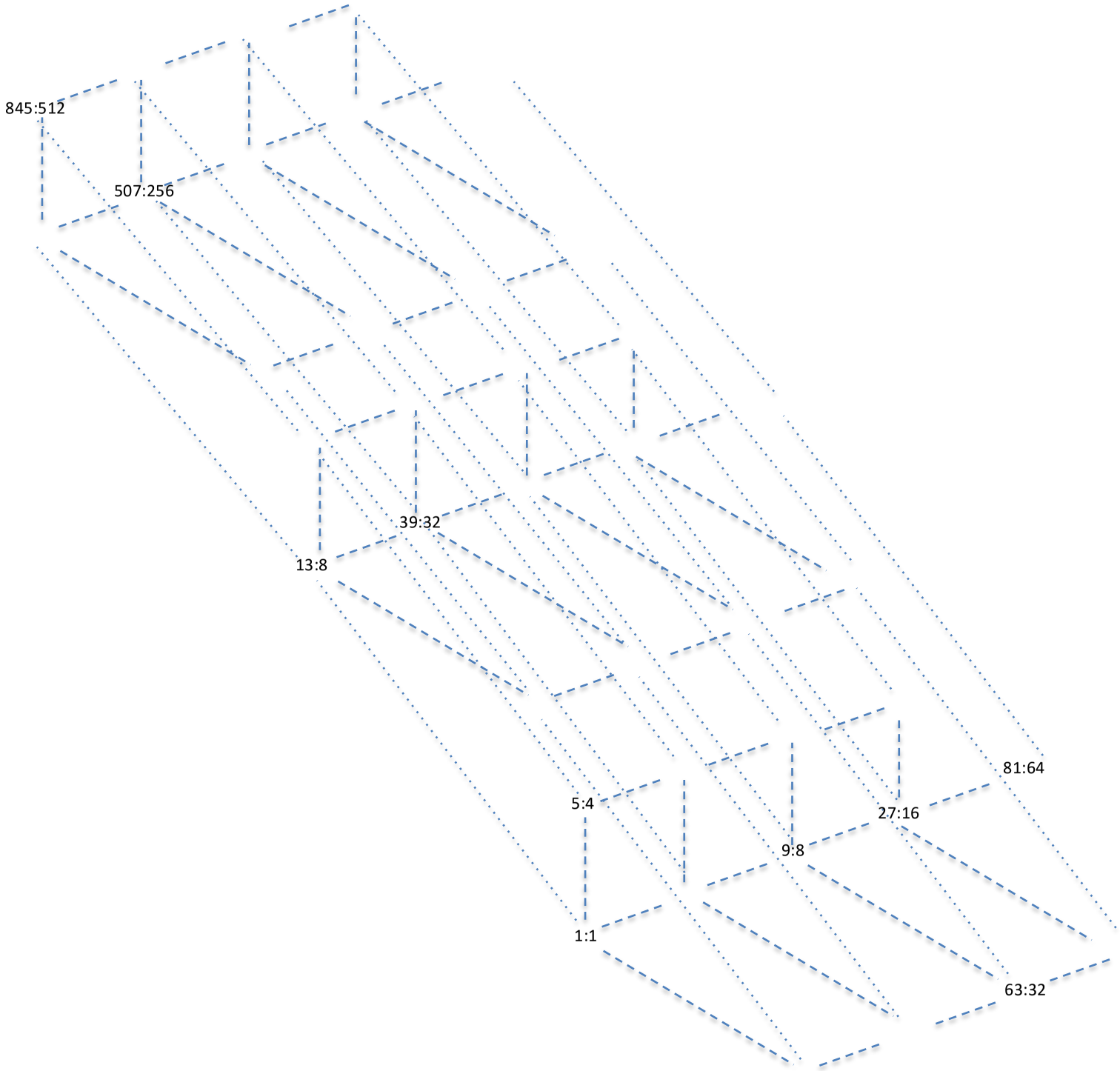
Measures 351-354



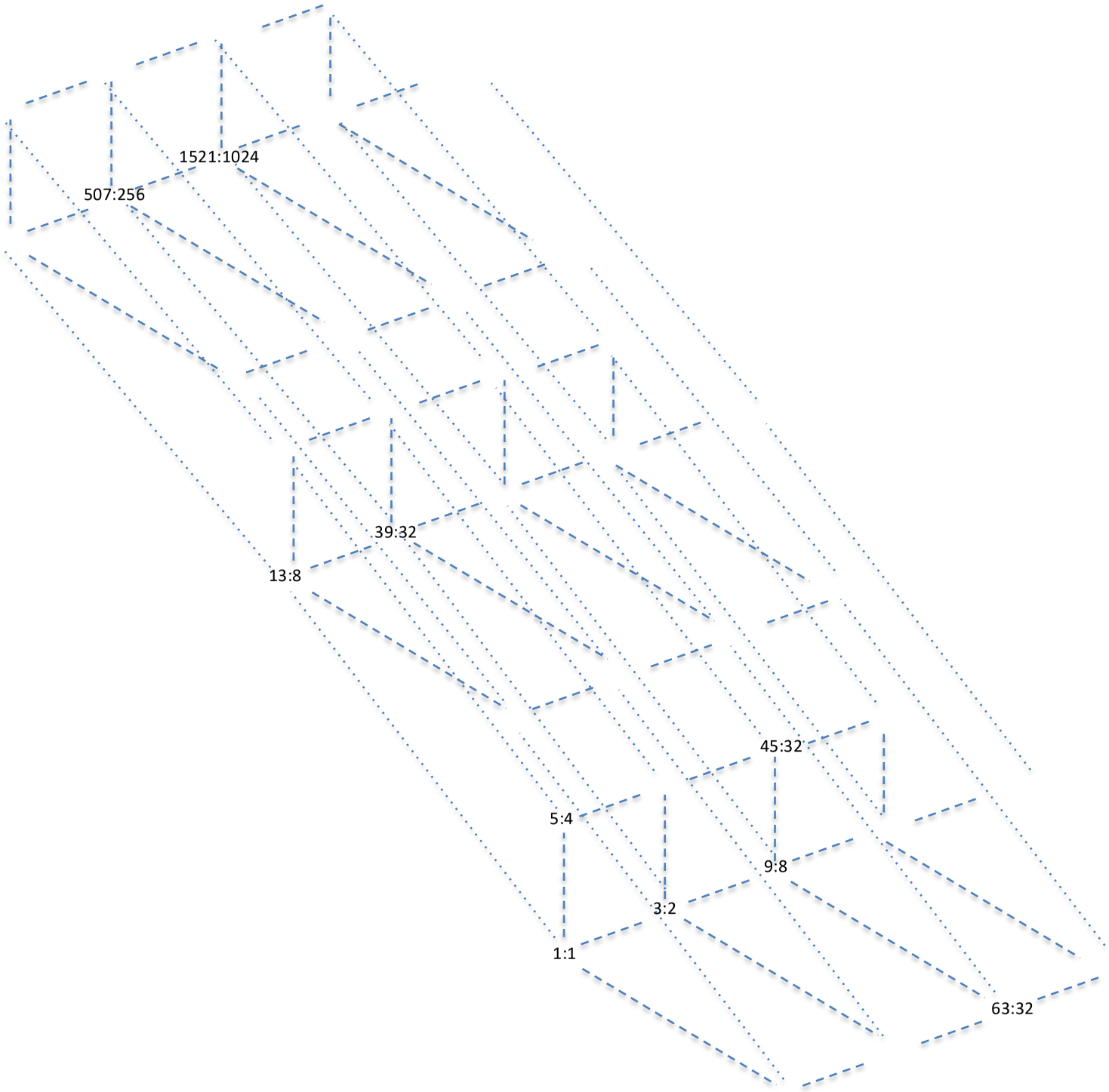
Measures 356-359



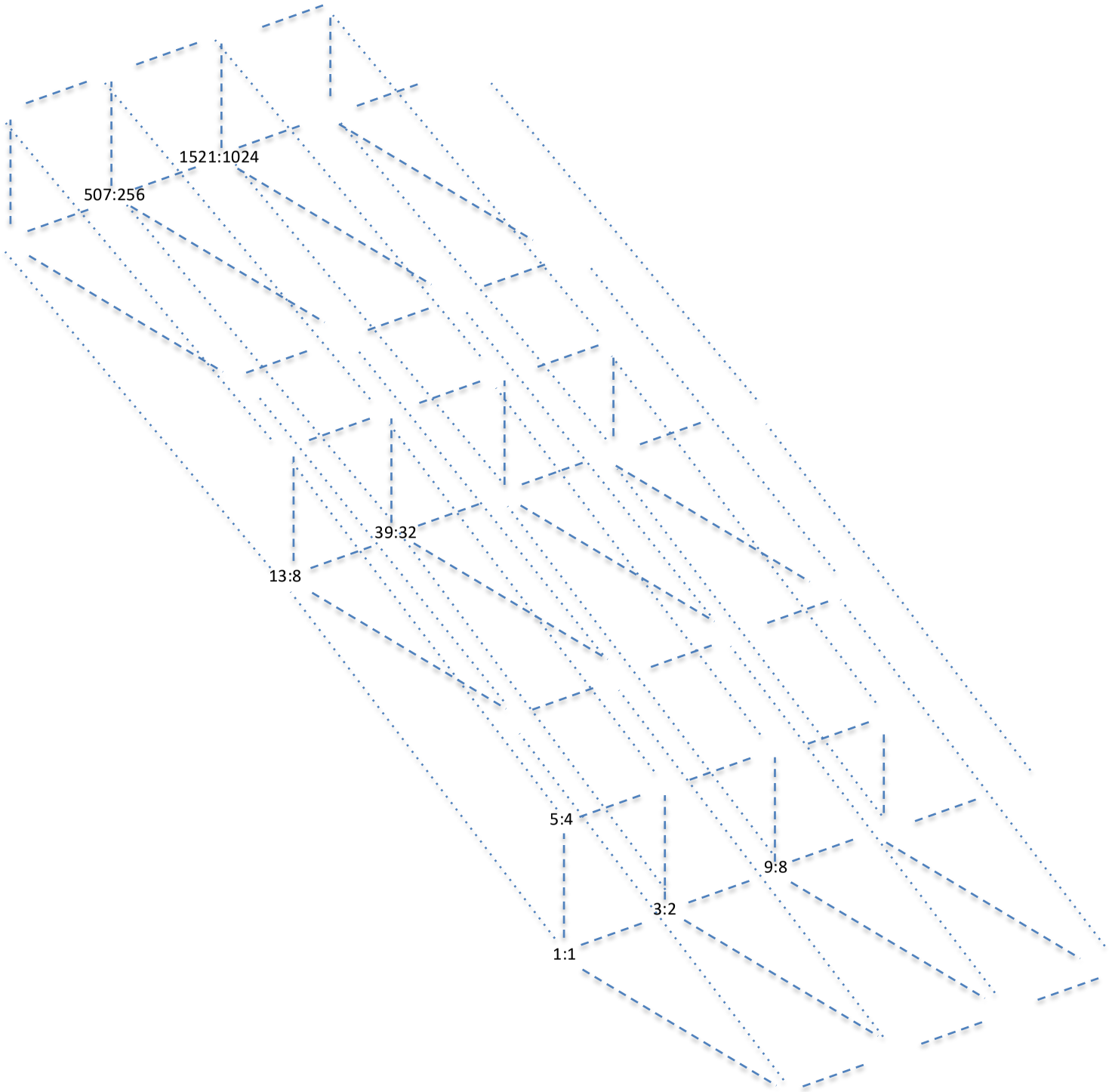
Measures 368-371



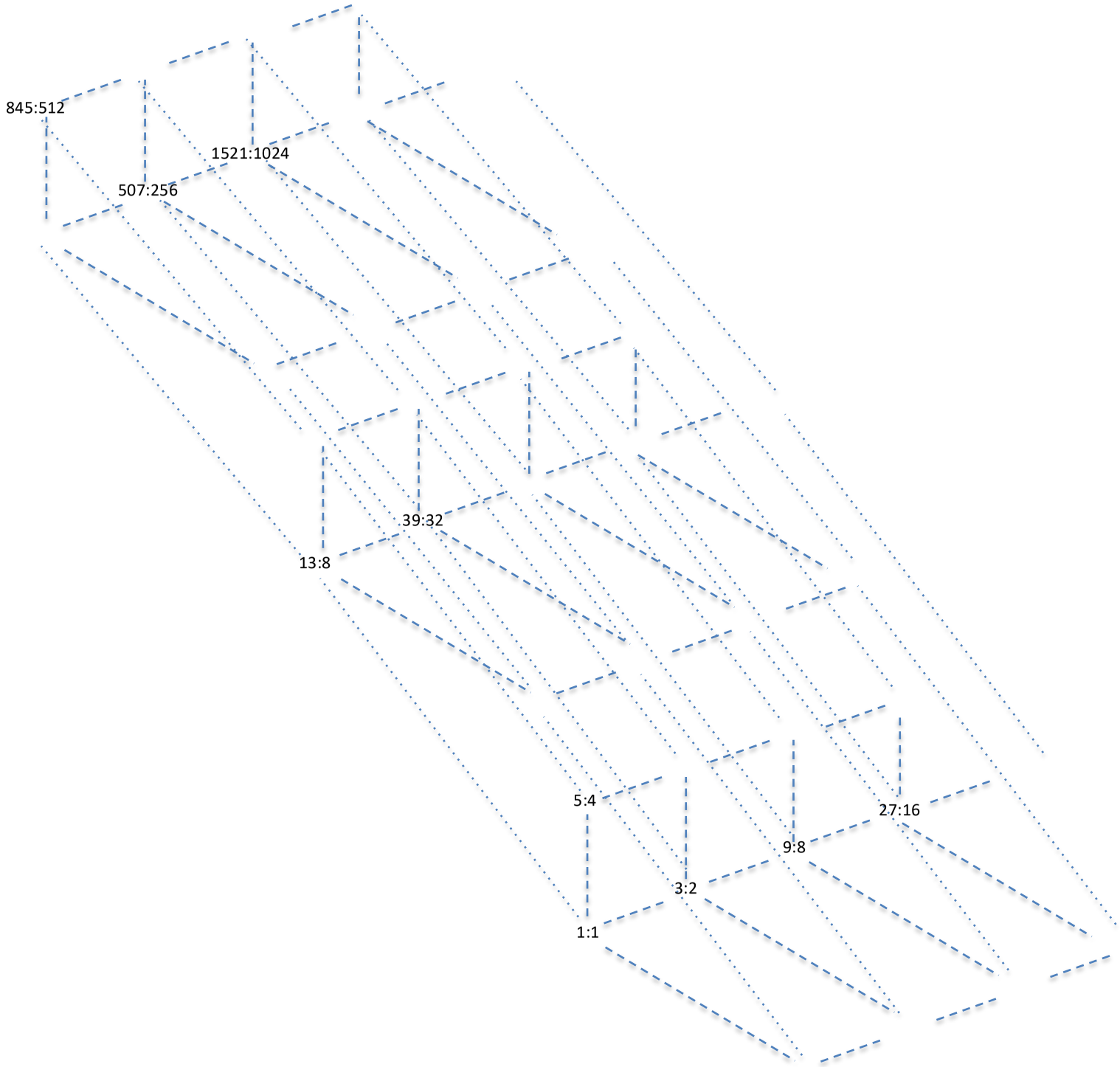
Measures 372-375



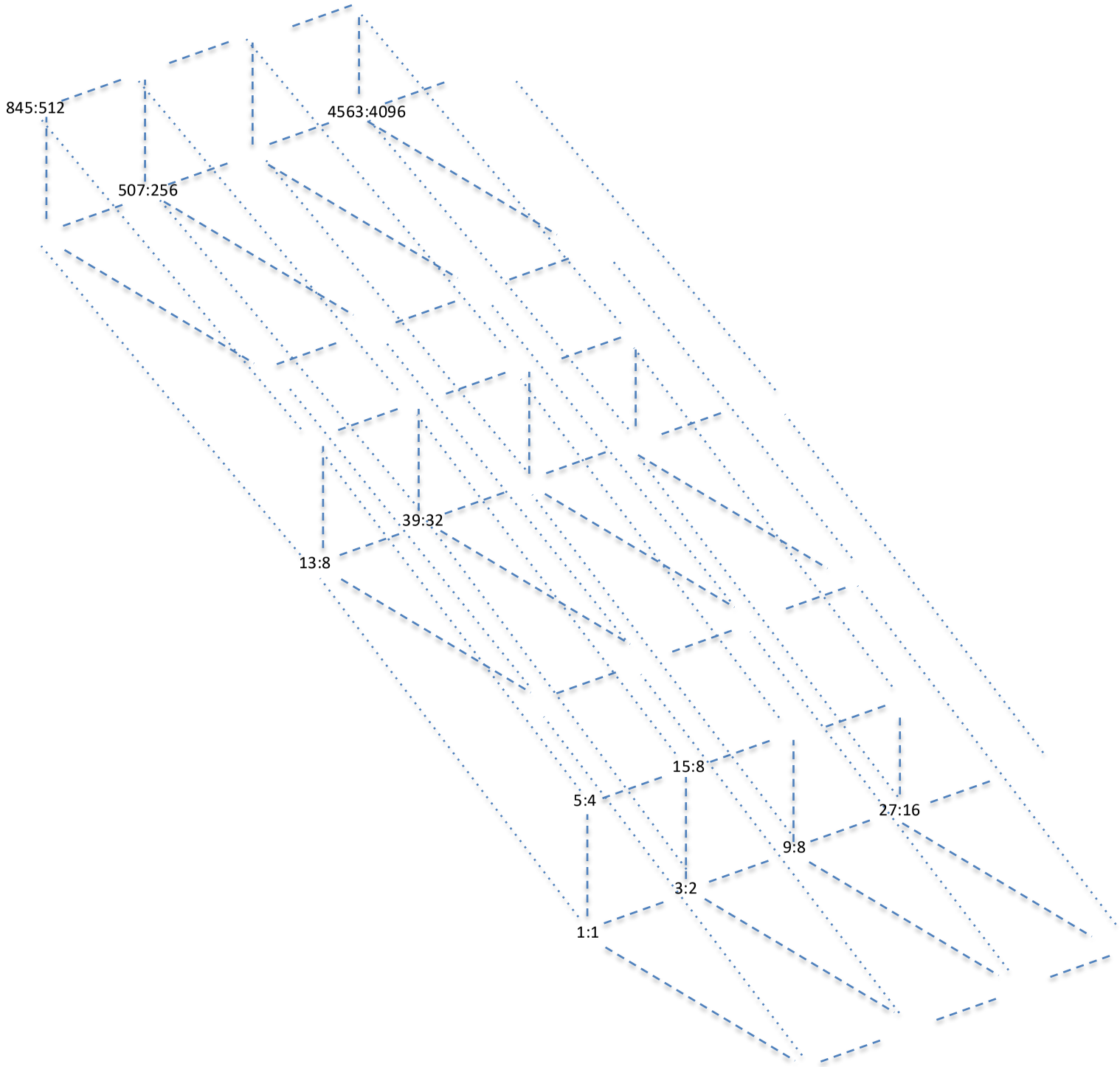
Measures 376-379



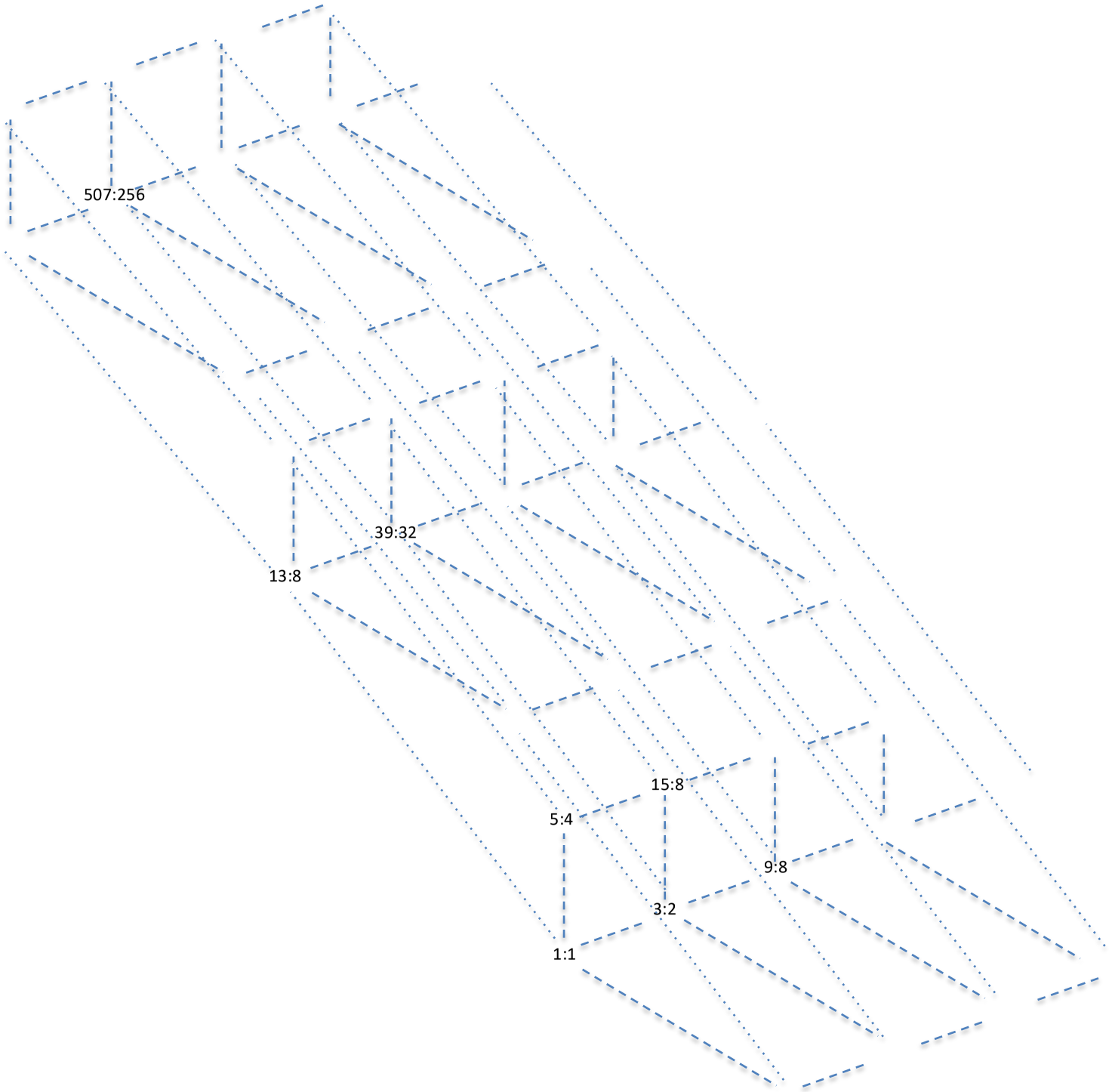
Measures 384-391



Measures 394-401



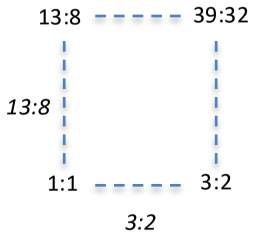
Measures 404-411



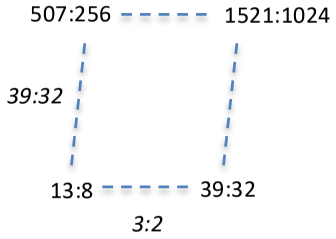
Appendix G, part two

(Symmetrical sets occurring in the allegro sections of *13*)

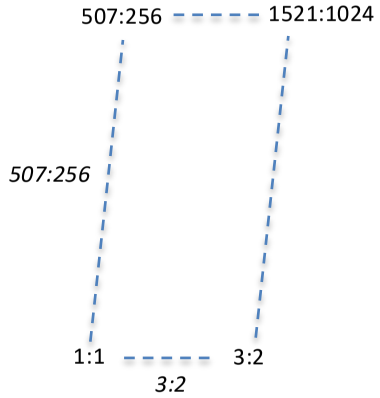
Measures 5-10



Set 1

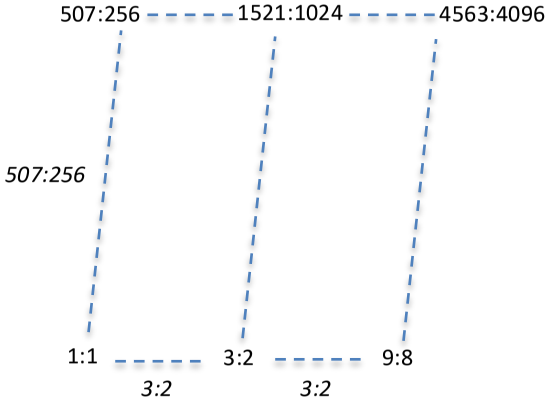


Set 2



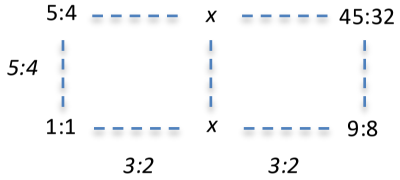
Set 3

Measures 11-19

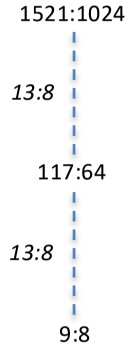


Set 4

Measures 22-29

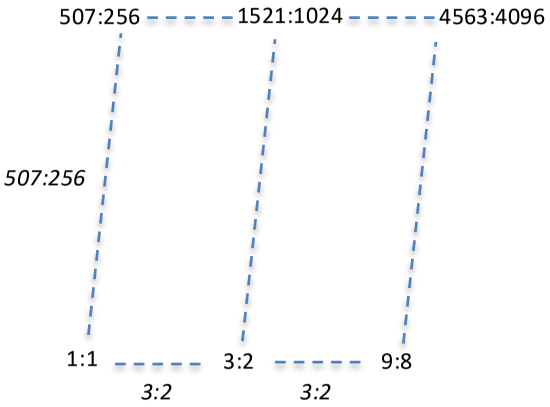


Set 5



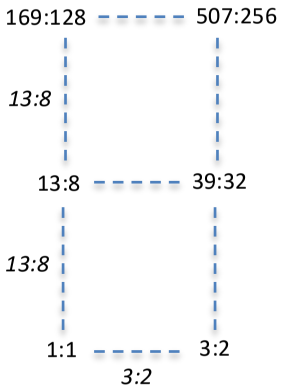
Set 6

Measures 30-41

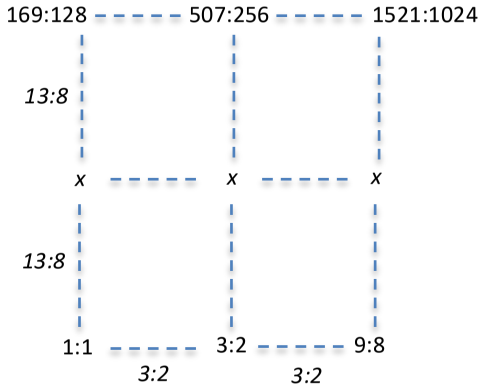


Set 4

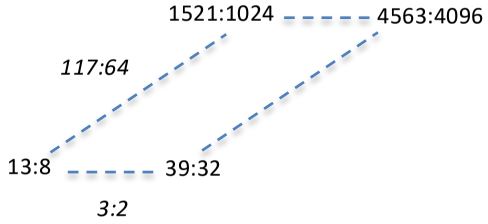
Measures 48-56



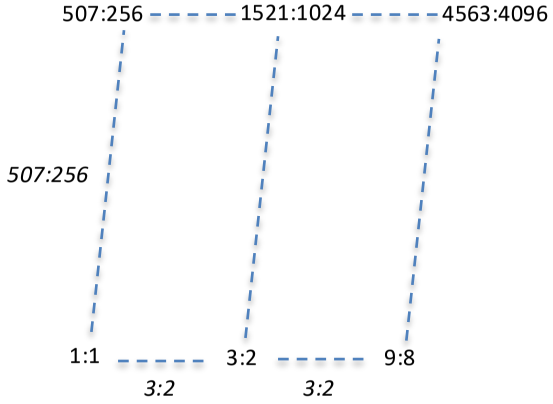
Set 7



Set 8

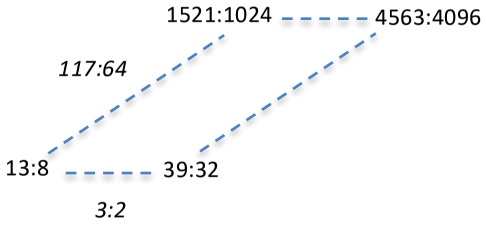


Set 9

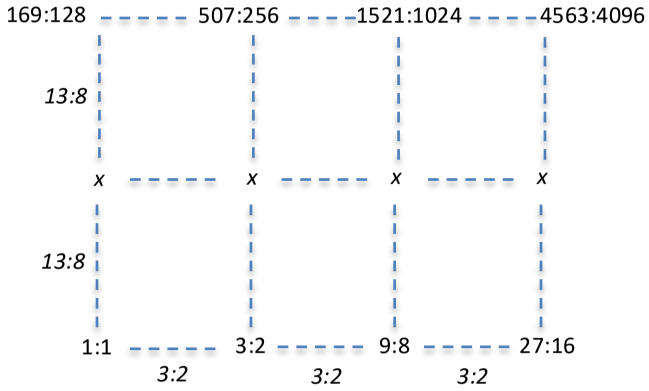


Set 4

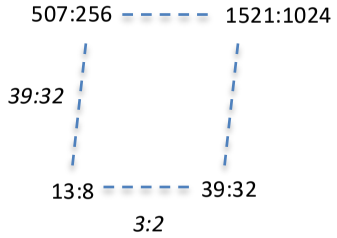
Measures 58-70



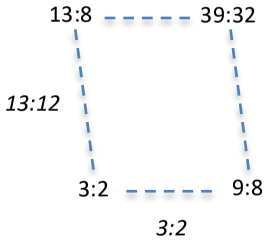
Set 9



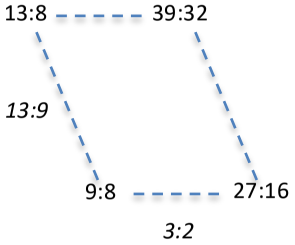
Set 10



Set 2

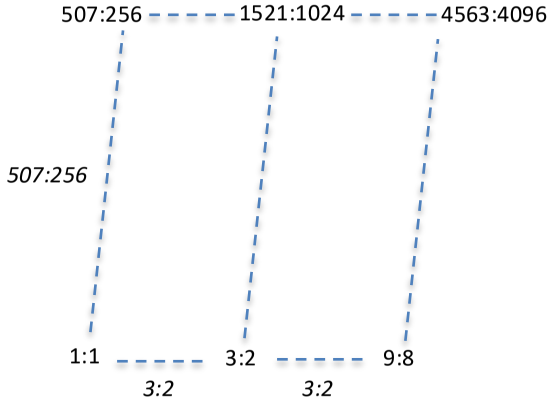


Set 11



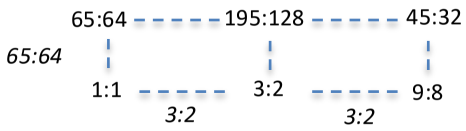
Set 12

Measures 73-77

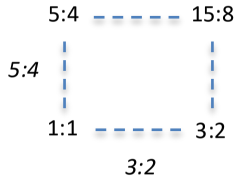


Set 4

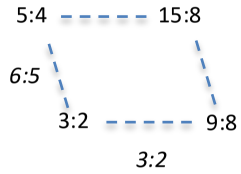
Measures 78-82



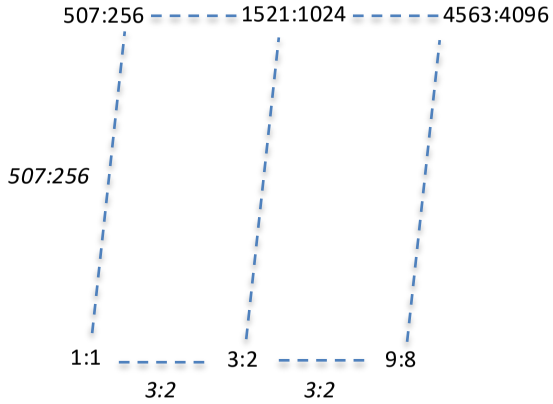
Set 13



Set 14

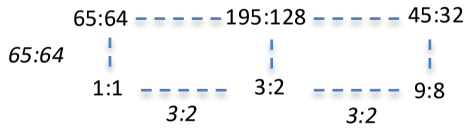


Set 15

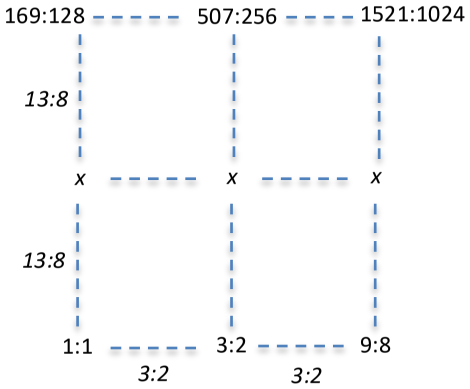


Set 4

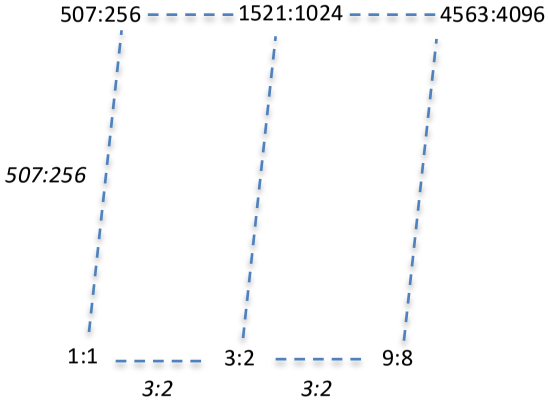
Measures 83-87



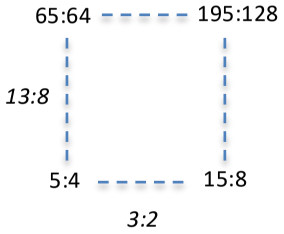
Set 13



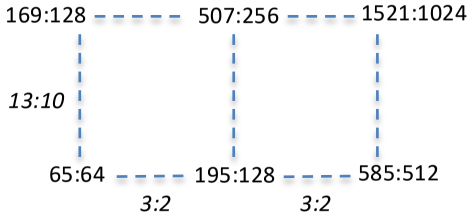
Set 8



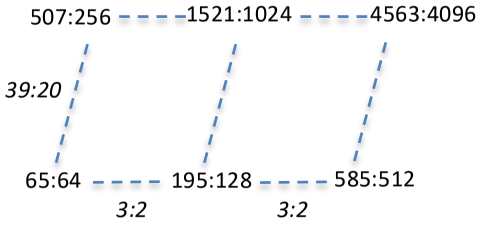
Set 4



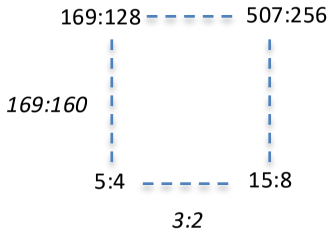
Set 16



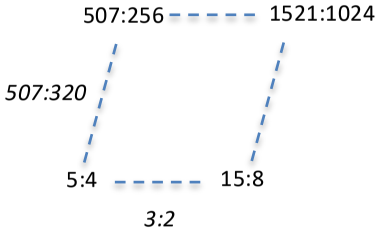
Set 17



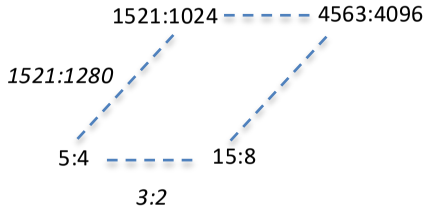
Set 18



Set 19

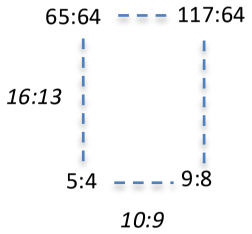


Set 20

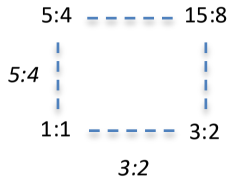


Set 21

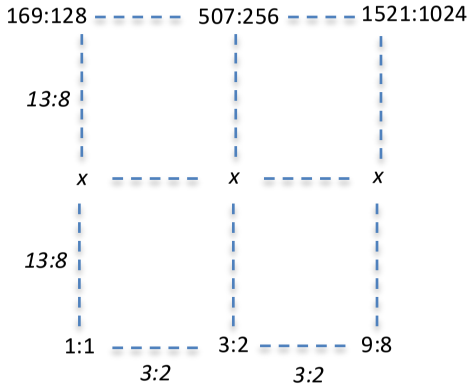
Measures 94-103



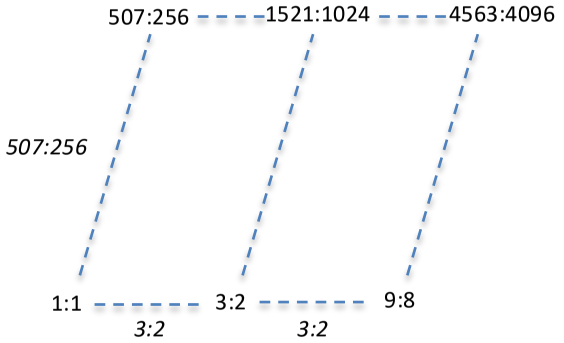
Set 16



Set 14

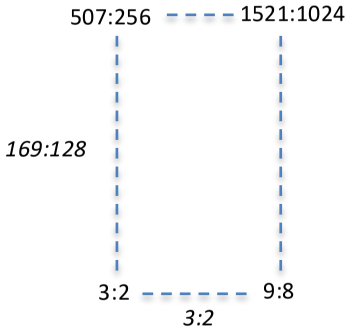


Set 8

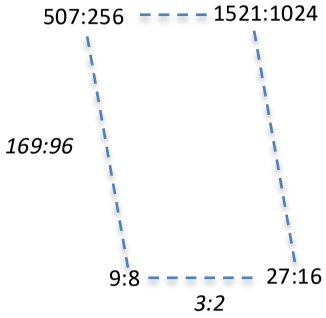


Set 4

Measures 110-115

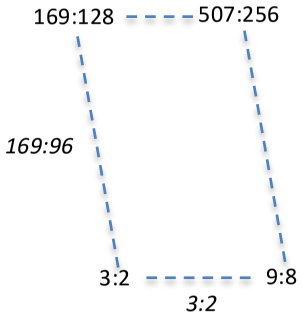


Set 22

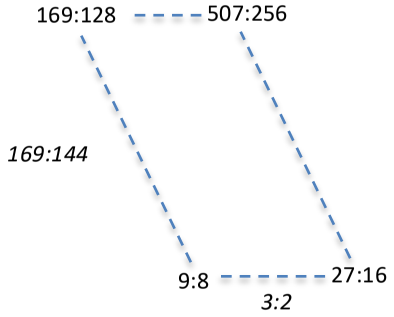


Set 23

Measures 116-121

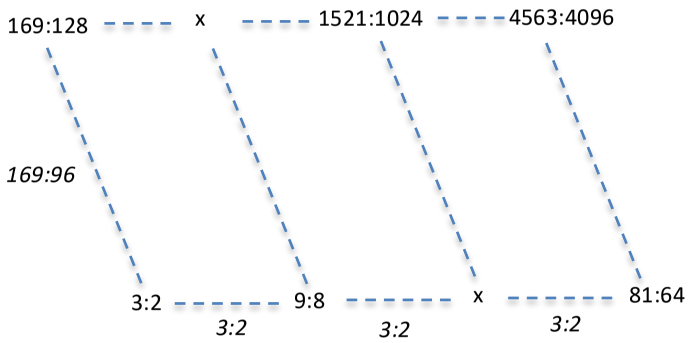


Set 24



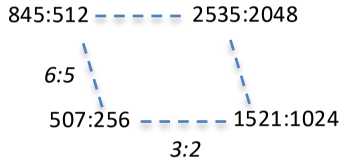
Set 25

Measures 122-127

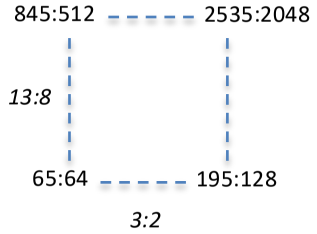


Set 26

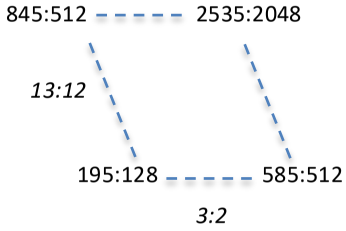
Measures 128-135



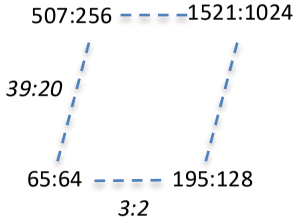
Set 27



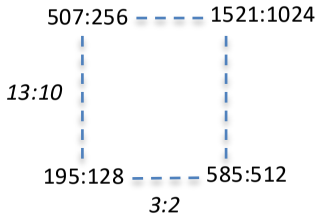
Set 28



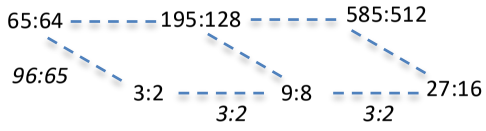
Set 29



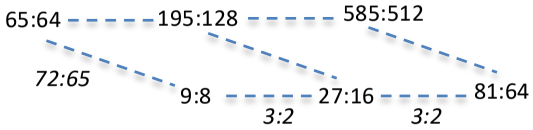
Set 30



Set 31

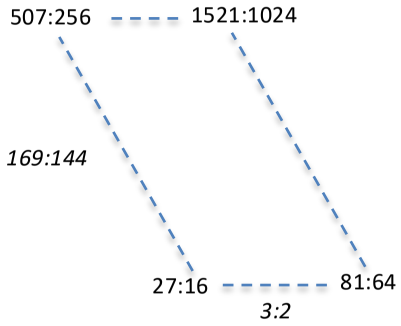


Set 32

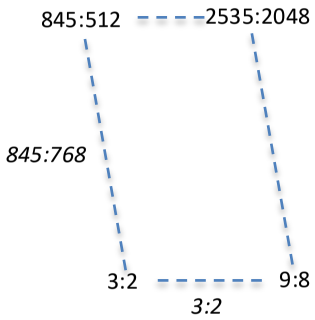


Set 33

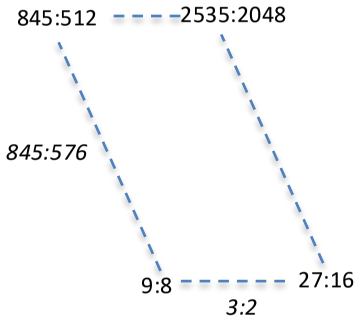
Measures 128-135 continued



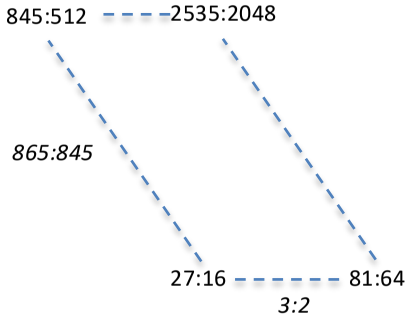
Set 34



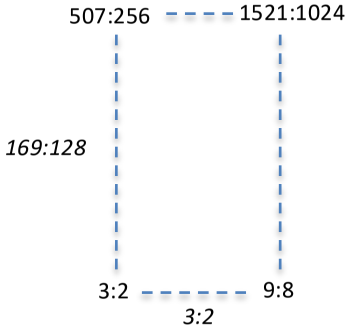
Set 35



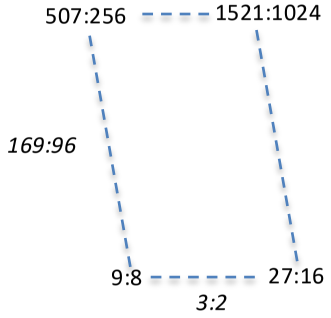
Set 36



Set 37

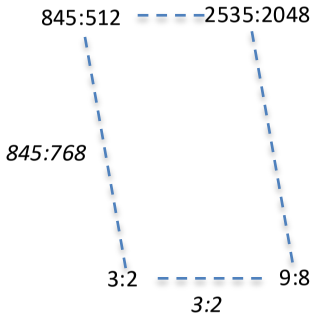


Set 22

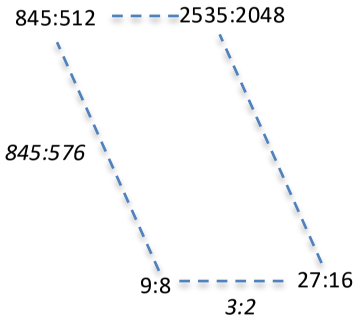


Set 23

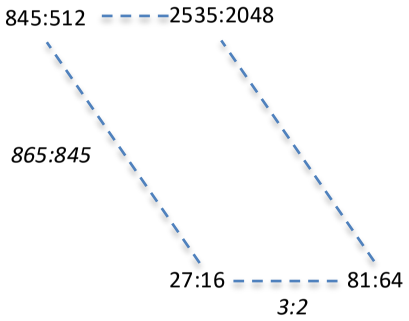
Measures 136-141



Set 35

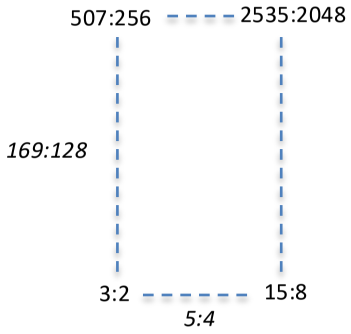


Set 36



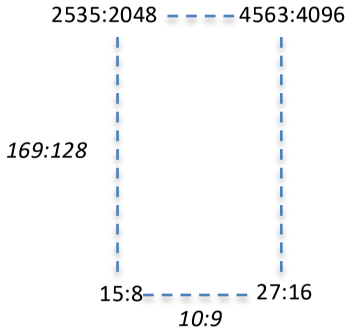
Set 37

Measures 142-147



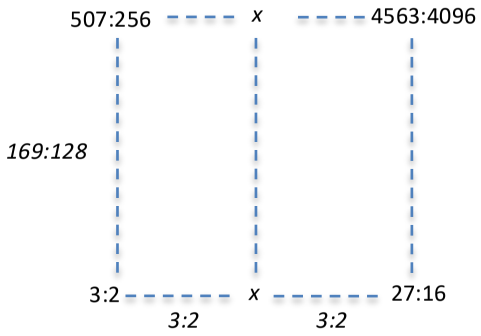
Set 38

Measures 148-153

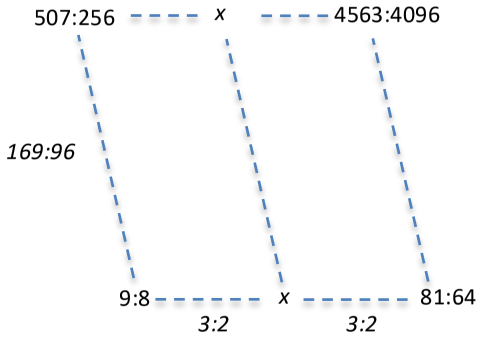


Set 39

Measures 154-159

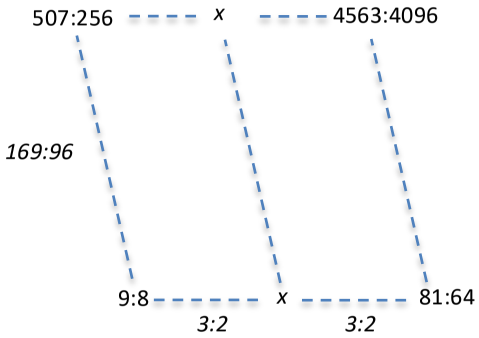


Set 40

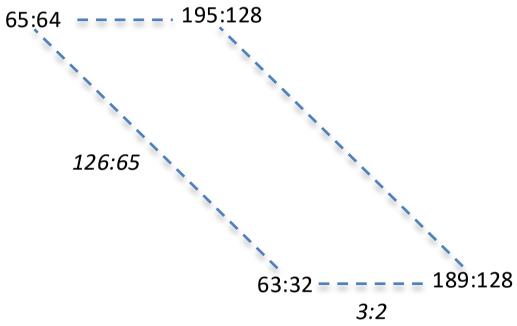


Set 41

Measures 166-171

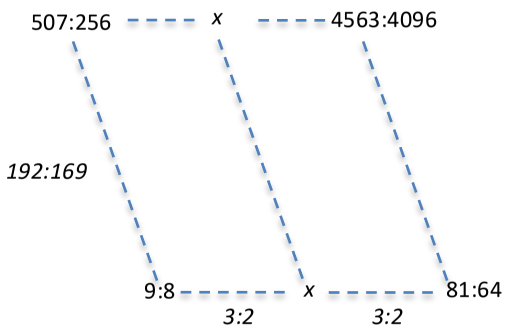


Set 41

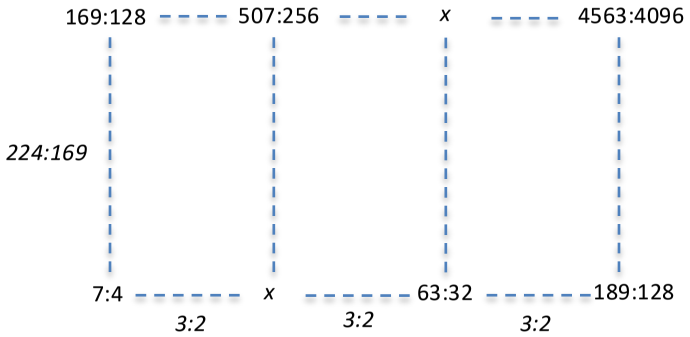


Set 42

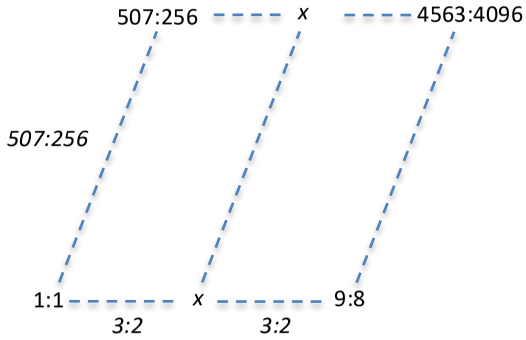
Measures 172-176
Measures 185-191



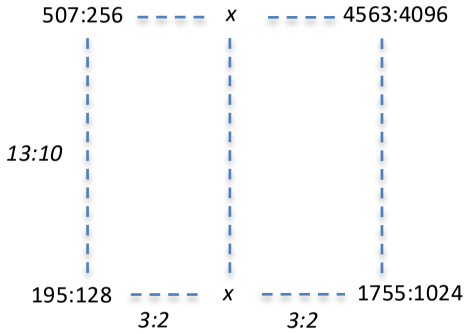
Set 41



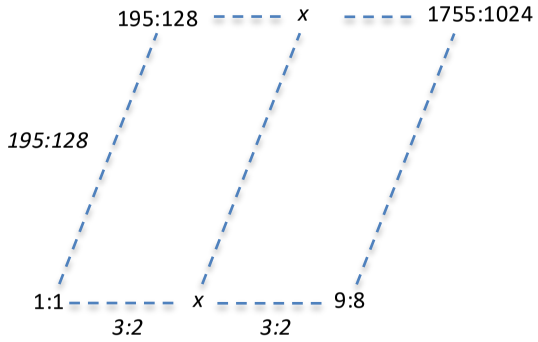
Set 43



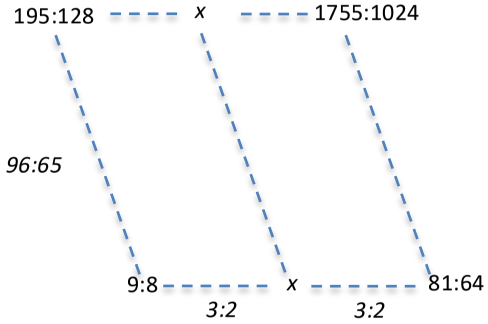
Set 44



Set 45

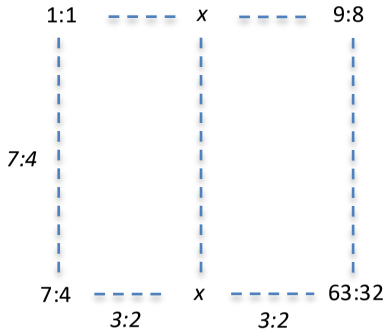


Set 46

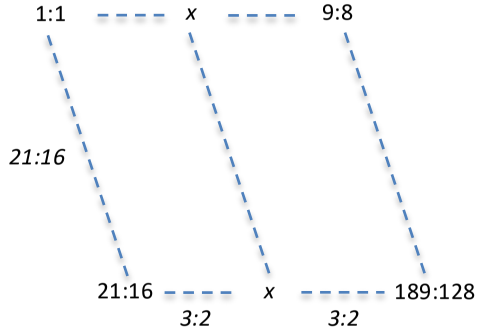


Set 47

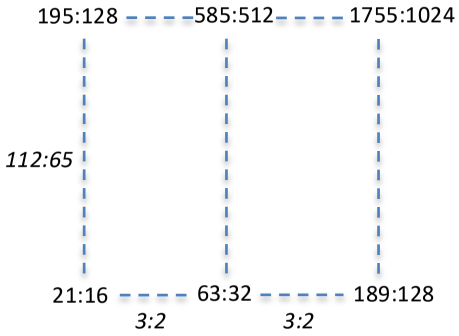
Measures 172-176
Measures 185-191 continued



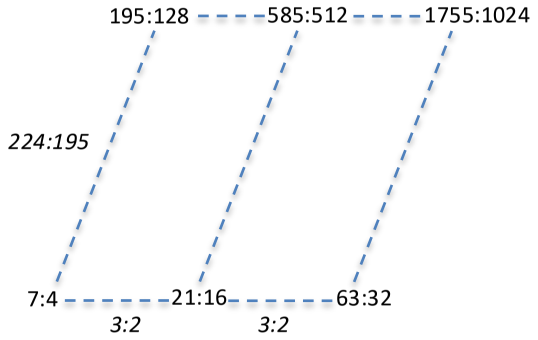
Set 48



Set 49

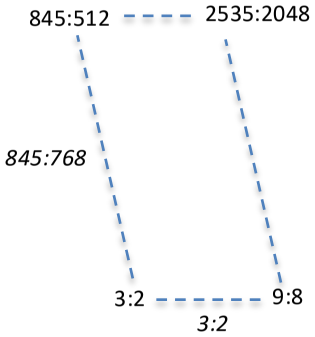


Set 50

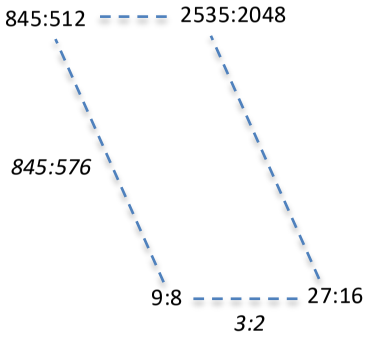


Set 51

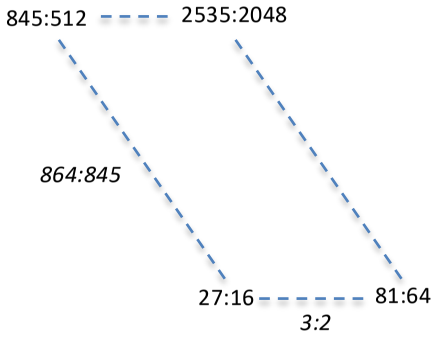
Measures 206-219



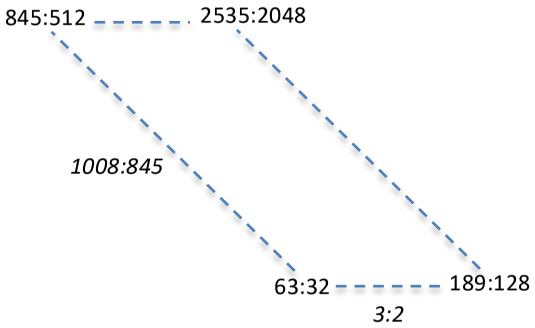
Set 35



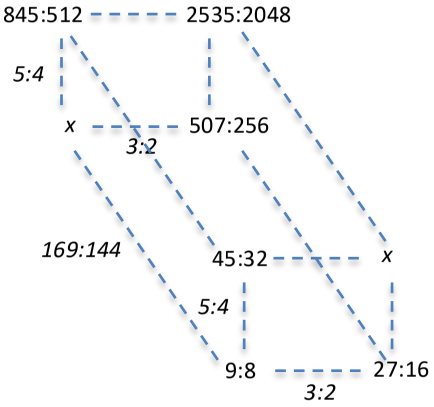
Set 36



Set 37

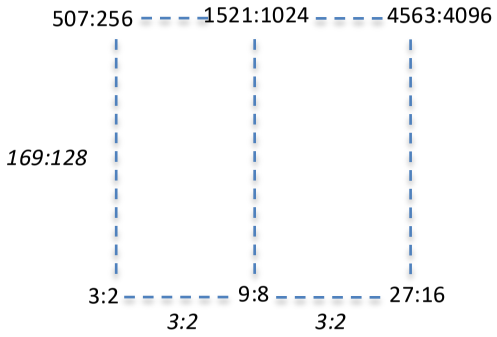


Set 52

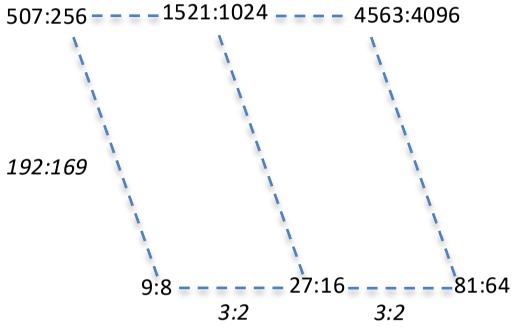


Set 53

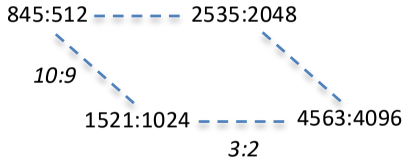
Measures 206-219 continued



Set 54

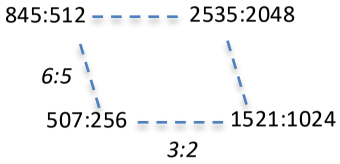


Set 55

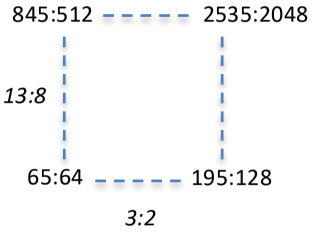


Set 56

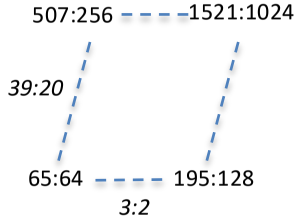
Measures 221-230



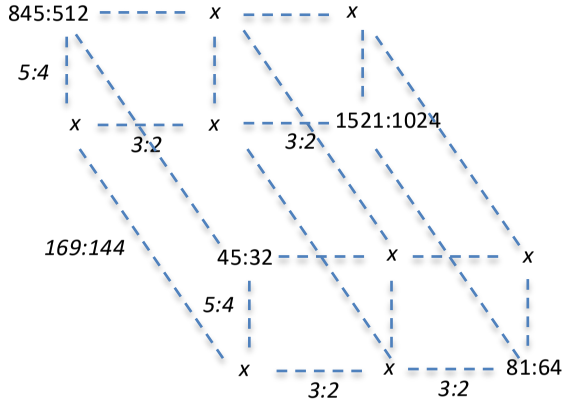
Set 27



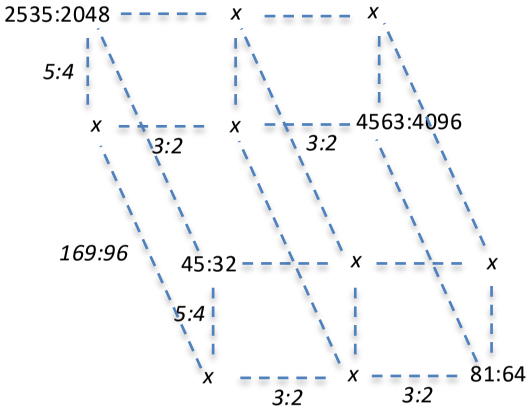
Set 28



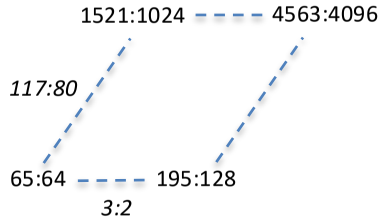
Set 30



Set 57

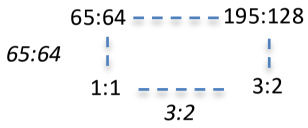


Set 58

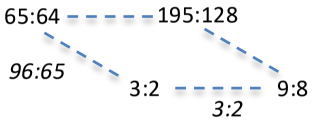


Set 59

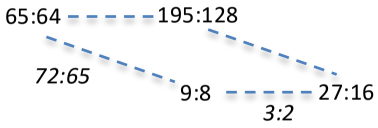
Measures 221-230 CONTINUED



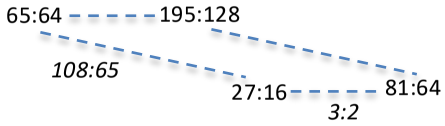
Set 60



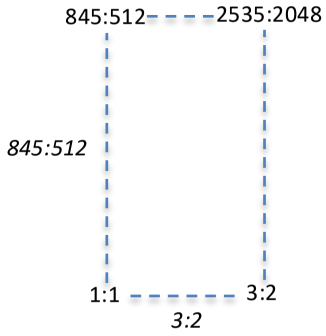
Set 61



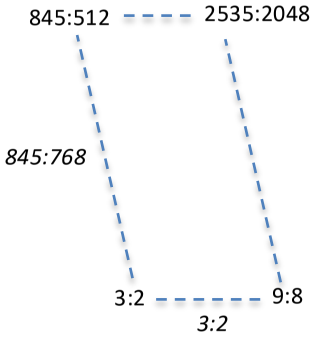
Set 62



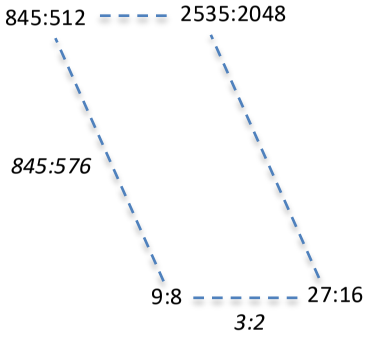
Set 63



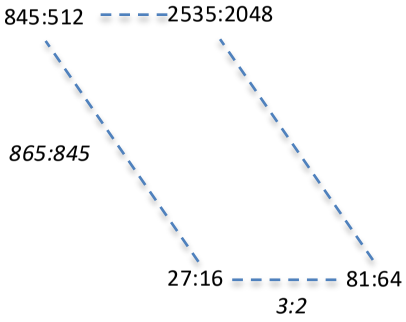
Set 64



Set 35

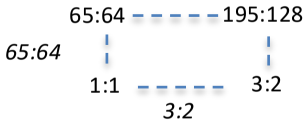


Set 36

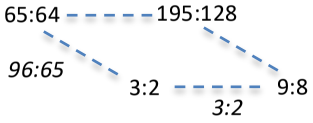


Set 37

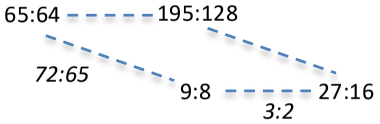
Measures 231-233



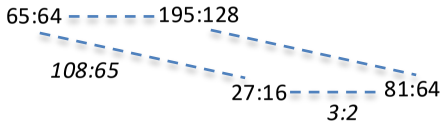
Set 60



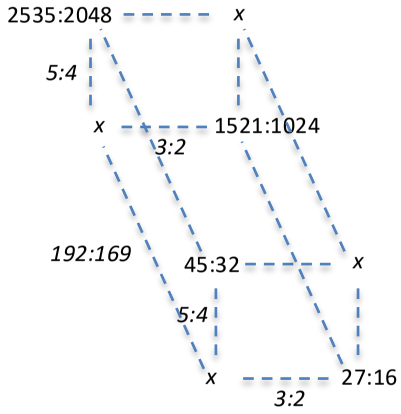
Set 61



Set 62

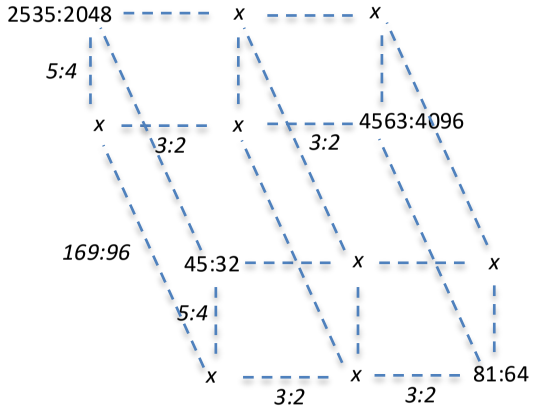


Set 63



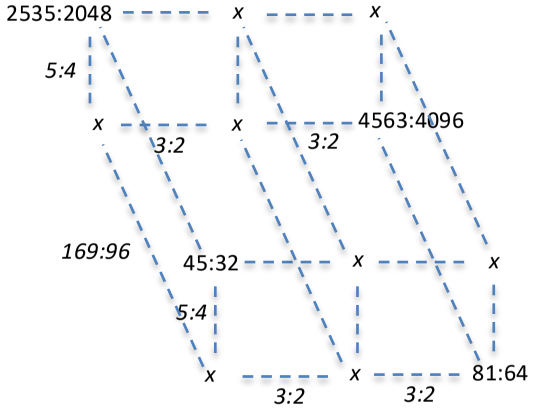
Set 65

Measures 249-253

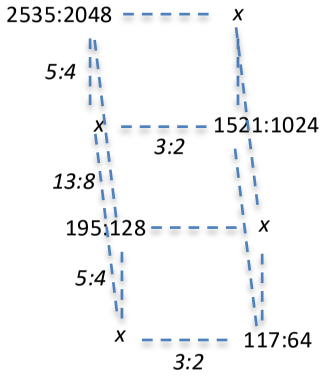


Set 58

Measures 245-248

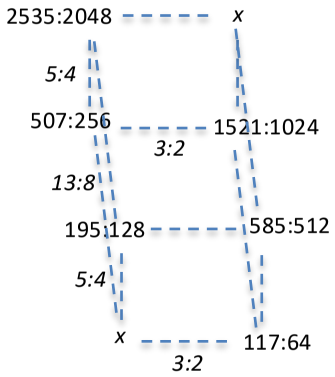


Set 58

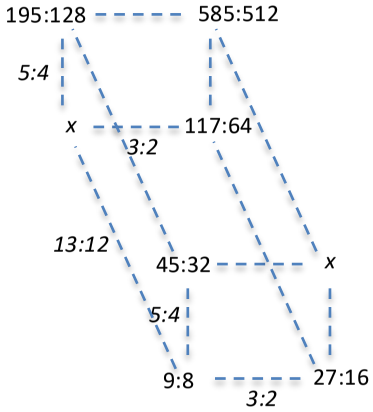


Set 66

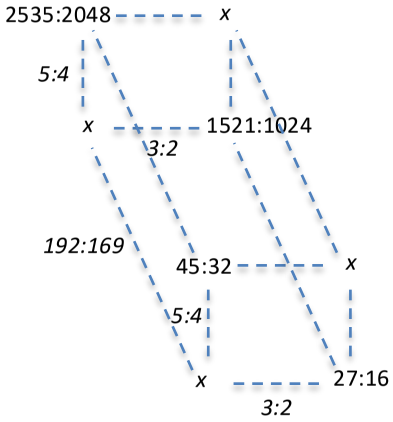
Measures 266-277



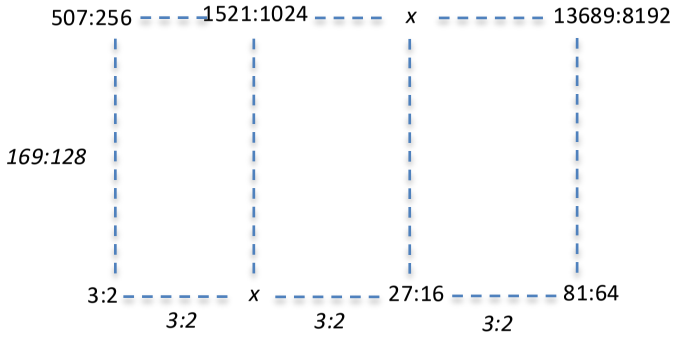
Set 67



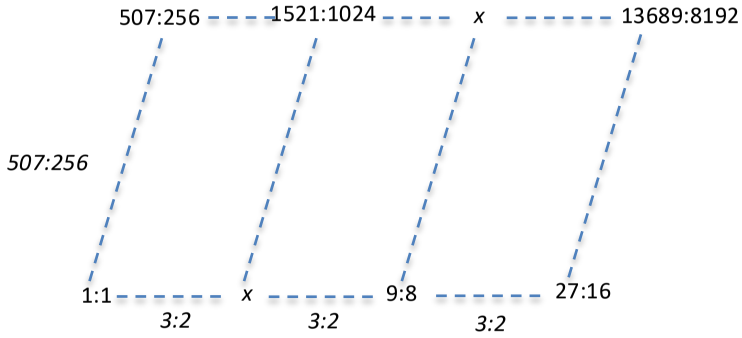
Set 68



Set 69

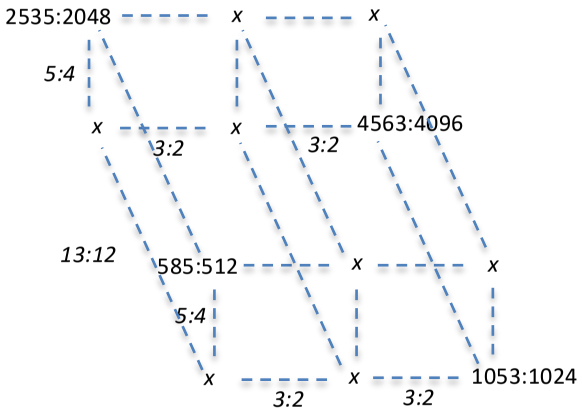


Set 70

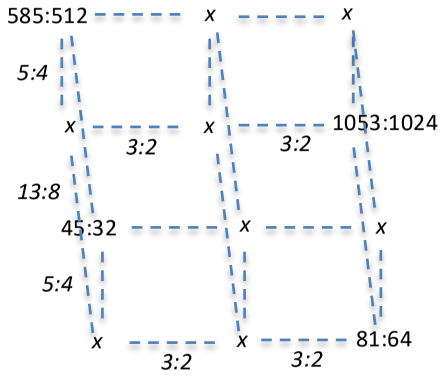


Set 71

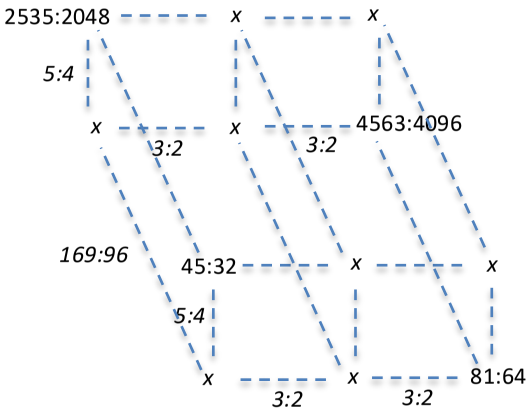
Measures 280-283



Set 72

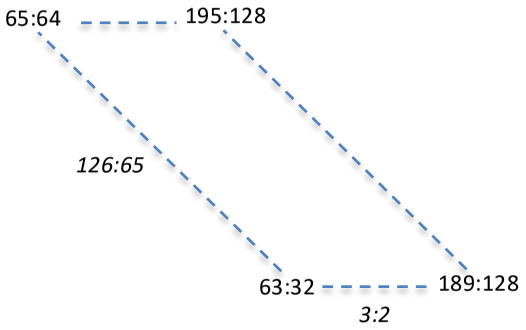


Set 73

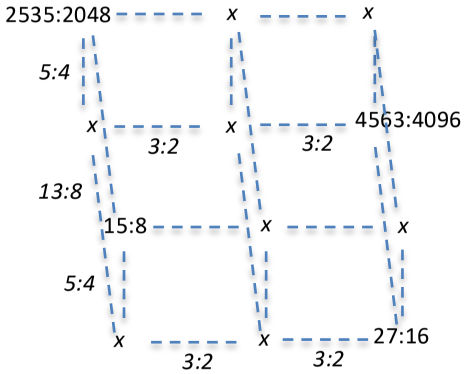


Set 58

Measures 292-299

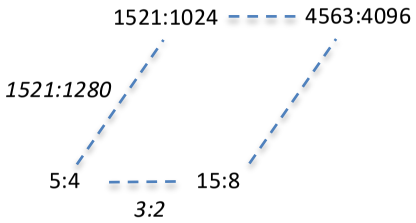


Set 42



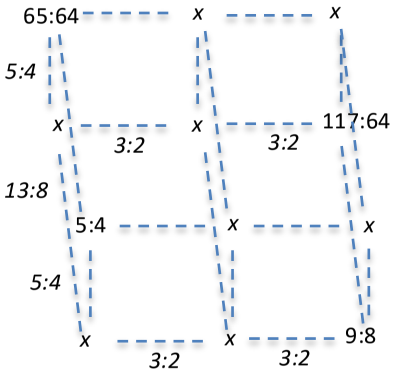
Set 74

Measures 347-350



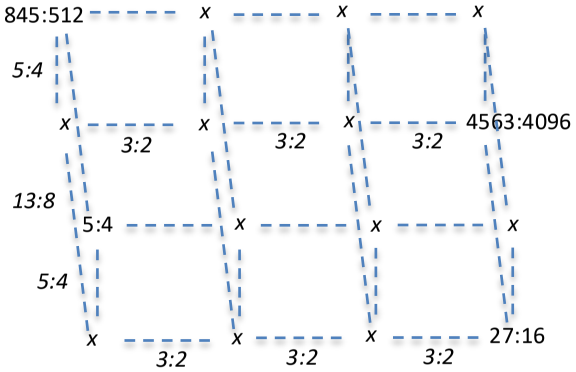
Set 75

Measures 351-354

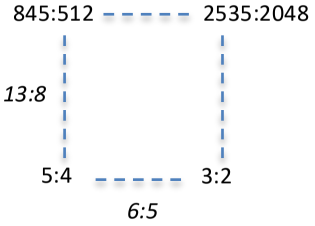


Set 16

Measures 394-401



Set 76



Set 77

Appendix G, part three

(Table containing basic descriptive information about the modes from the allegro sections of *13*)

Appendix G, part three:

Table containing basic descriptive information about the modes from the allegro sections of *13.*

Set no.	Appears in measure nos.	No. of possible transpositions	Generating prime nos.	True Dimensions	Appears as (dimensions)	Occurring in x space
1	5-10	17	3, 13	2	2	3:2, 13:8
2	5-10, 58-70	13	3, 13	2	2	3:2, 13:8
3	5-10	6	3, 13	2	2	3:2, 13:8
4	11-19, 30-41 48-56, 73-77, 78-82, 83-87, 94-103	3	3, 13	2	2	3:2, 13:8
5	22-29	8	3, 5	2	2	3:2, 5:4
6	22-29	12	13	1	1	3:2, 13:8
7	48-56	9	3, 13	2	2	3:2, 13:8
8	48-56, 83-87, 94-103	6	3, 13	2	2	3:2, 13:8
9	48-56, 58-70	11	3, 13	2	2	3:2, 13:8
10	58-70	3	3, 13	2	2	3:2, 13:8
11	58-70	13	3, 13	2	2	3:2, 13:8
12	58-70	11	3, 13	2	2	3:2, 13:8
13	78-82, 83-87	3	3, 5, 13	3	2	5:4, 13:8, 3:2
14	78-82, 94-103	8	3, 5	2	2	3:2, 5:4
15	78-82	8	3, 5	2	2	3:2, 5:4
16	83-87, 94-103 351-354	17	3, 5, 13	3	2	3:2, 5:4, 13:8
17	83-87	3	3, 5, 13	3	2	3:2, 5:4, 13:8
Set no.	Appears in measure nos.	No. of possible transpositions	Generating prime nos.	True Dimensions	Appears as (dimensions)	Occurring in x space
18	83-87	3	3, 5, 13	3	2	3:2, 5:4, 13:8
19	83-87	2	3, 5, 13	3	2	3:2, 5:4, 13:8
20	83-87	2	3, 5, 13	3	2	3:2, 5:4, 13:8
21	83-87	1	3, 5, 13	3	2	3:2, 5:4, 13:8
22	110-115, 128-135	6	3, 13	2	2	3:2, 13:8
23	110-115, 128-135	4	3, 13	2	2	3:2, 13:8
24	116-121	4	3, 13	2	2	3:2, 13:8

25	116-121	3	3, 13	2	2	3:2, 13:8
26	122-127	0	3, 13	2	2	3:2, 13:8
27	128-135, 206-219	8	3, 5, 13	2	2	3:2, 5:4, 13:8
28	128-135, 221-230	17	3, 13	3	2	3:2, 13:8
29	128-135	11	3, 5, 13	2	2	3:2, 5:4, 13:8
30	128-135, 221-230	6	3, 5, 13	3	2	3:2, 5:4, 13:8
31	128-135	5	3, 5, 13	3	2	3:2, 5:4, 13:8
32	128-135	3	3, 5, 13	3	2	3:2, 5:4, 13:8
33	128-135	1	3, 5, 13	3	2	3:2, 5:4, 13:8
34	128-135	3	3, 13	2	2	3:2, 13:8
35	128-135, 136-141 206-219, 221-230	2	3, 5, 13	3	2	3:2, 5:4, 13:8
36	128-135, 136-141 206-219, 221-230	1	3, 5, 13	3	2	3:2, 5:4, 13:8
37	128-135, 136-141 206-219, 221-230	0	3, 5, 13	3	2	3:2, 5:4, 13:8
Set no.	Appears in measure nos.	No. of possible transpositions	Generating prime nos.	True Dimensions	Appears as (dimensions)	Occurring in x space
38	142-147	3	5, 13	2	2	3:2, 5:4, 13:8
39	148-153	2	3, 5, 13	3	2	3:2, 5:4, 13:8
40	154-159	4	3, 13	2	2	3:2, 13:8
41	154-159, 166-171	2	3, 13	2	2	3:2, 13:8
42	166-171, 172-176 185-191, 292-299	1	3, 5, 13	3	2	3:2, 5:4, 13:8
43	172-176, 185-191	6	3, 13	2	2	3:2, 7:4, 13:8
44	172-176, 185-191	4	3, 13	2	2	3:2, 13:8
45	172-176, 185-191	3	3, 5, 13	3	2	3:2, 5:4, 13:8

46	172-176, 185-191	1	3, 5, 13	3	2	3:2, 5:4, 13:8
47	172-176, 185-191	3	3, 5, 13	3	2	3:2, 5:4, 13:8
48	172-176, 185-191	5	3, 7	2	2	3:2, 7:4
49	172-176, 185-191	2	3, 7	2	2	3:2, 7:4
50	172-176, 185-191	3	3, 5, 7, 13	4	2	3:2, 5:4, 7:4, 13:8
51	172-176, 185-191	1	3, 5, 7, 13	4	2	3:2, 5:4, 7:4, 13:8
52	206-219	0	3, 5, 7, 13	4	2	3:2, 5:4, 7:4, 13:8
53	206-219	1	3, 5, 13	3	3	3:2, 5:4, 13:8
54	206-219	5	3, 13	2	2	3:2, 13:8
55	206-219	3	3, 13	2	2	3:2, 13:8
56	206-219	8	3, 5, 13	2	2	3:2, 5:4, 13:8
57	221-230	1	3, 5, 13	3	3	3:2, 5:4, 13:8
58	221-230, 245-248 249-253, 280-283	1	3, 5, 13	3	2	3:2, 5:4, 13:8
Set no.	Appears in measure nos.	No. of possible transpositions	Generating prime nos.	True Dimensions	Appears as (dimensions)	Occurring in x space
59	221-230	3	3, 5, 13	3	2	3:2, 5:4, 13:8
60	221-230, 231-233	5	3, 5, 13	3	2	3:2, 5:4, 13:8
61	221-230, 231-233	5	3, 5, 13	3	2	3:2, 5:4, 13:8
62	221-230, 231-233	3	3, 5, 13	3	2	3:2, 5:4, 13:8
63	221-230, 231-233	1	3, 5, 13	3	2	3:2, 5:4, 13:8
64	221-230	2	3, 5, 13	3	2	3:2, 5:4, 13:8
65	231-233	2	3, 5, 13	3	2	3:2, 5:4, 13:8
66	249-253	5	3, 5, 13	3	2	3:2, 5:4, 13:8
67	266-277	5	3, 5, 13	3	3	3:2, 5:4, 13:8
68	266-277	5	3, 5, 13	3	3	3:2, 5:4, 13:8
69	266-277	2	3, 5, 13	3	2	3:2, 5:4, 13:8
70	266-277	1	3, 13	2	2	3:2, 13:8
71	266-277	0	3, 13	2	2	3:2, 13:8

72	280-283	3	3, 5, 13	3	2	3:2, 5:4, 13:8
73	280-283	5	3, 5, 13	3	2	3:2, 5:4, 13:8
74	292-299	2	3, 5, 13	3	2	3:2, 5:4, 13:8
75	347-350	1	3, 5, 13	2	2	3:2, 5:4, 13:8
76	394-401	1	3, 5, 13	3	2	3:2, 5:4, 13:8
77	394-401	2	3, 5, 13	3	2	3:2, 5:4, 13:8

Appendix G, part four

(List of the commas as they occur throughout the allegro sections of *13*)

Appendix G, part four:
List of the commas as they occur throughout the allegro sections of *13.*

Bar nos.	**Comma**	**No. of times comma occurs in the chord**
5-10	Tridecimal neutral third comma	2
	Syntonic comma	1
11-19	Tridecimal neutral third comma	3
	Syntonic comma	1
22-29	Tridecimal neutral third comma	2
	Medium tridecimal comma	1
	Syntonic comma	1
30-41	Tridecimal neutral third comma	3
	Syntonic comma	1
	Medium tridecimal comma	1
48-56	Tridecimal neutral third comma	3
58-70	Tridecimal neutral third comma	3
73-77	Tridecimal neutral third comma	3
78-82	Tridecimal neutral third comma	3
83-87	Tridecimal neutral third comma	3
	13^{th} Partial chroma	3
	Tridecimal major diesis	1
94-103	Tridecimal neutral third comma	3
	13^{th} partial chroma	1
116-121	Schulter's comma	1
122-127	Tridecimal neutral third comma	2
	Syntonic comma	1
128-135	Tridecimal neutral third comma	1
136-141	Schulter's comma	1
142-147	Schulter's comma	1
148-159	Tridecimal neutral third comma	1

Appendix G, part four:
List of the commas as they occur throughout the allegro sections of *13.*

154-159	Tridecimal neutral third comma	1
	Schulter's comma	1
166-171	Tridecimal neutral third comma	1
	Schulter's comma	1
172-176 and 185-191	Schulter's comma	2
	Tridecimal neutral third comma	1
	13th partial chroma	1
	Tridecimal major diesis	1
	Septimal comma	1
196-200	Tridecimal neutral third comma	2
	Schulter's comma	2
	13th partial chroma	1
	Septimal comma	1
206-219	Medium tridecimal comma	1
	Tridecimal neutral third comma	2
	Septimal comma	1
221-230	Septimal comma	1
	13th partial chroma	2
	Tridecimal neutral third comma	3
231-233	Septimal comma	1
	Medium tridecimal comma	1
	Tridecimal neutral third comma	1
	13th partial chroma	2
234-240	Tridecimal neutral third comma	1
	Medium tridecimal comma	1
	13th partial chroma	1
	Septimal comma	1
245-248	Medium tridecimal comma	1
	Tridecimal neutral third comma	2
	13th partial chroma	1
	Septimal comma	1
249-253	Tridecimal neutral third comma	1

Appendix G, part four:
List of the commas as they occur throughout the allegro sections of *13.*

	Medium tridecimal comma	1
	13th partial chroma	1
	Septimal comma	1
257-259	13th partial chroma	1
	Septimal comma	1
	Tridecimal neutral third comma	2
	Medium tridecimal comma	1
266-277	Septimal comma	2
	13th partial chroma	3
	Medium tridecimal comma	1
	Tridecimal neutral third comma	2
280-283	Tridecimal major diesis	1
	13th partial chroma	1
	Septimal comma	1
	Medium tridecimal comma	1
	Tridecimal neutral third comma	2
284-287	Tridecimal neutral third comma	2
	13th partial chroma	2
	Septimal comma	2
288-291	13th partial chroma	2
	Septimal comma	2
	Tridecimal neutral third comma	2
292-299	13th partial chroma	2
	Septimal comma	2
	Tridecimal neutral third comma	2
303-305	Schulter's comma	1
306-312	13th partial chroma	1
	Septimal comma	1
	Tridecimal neutral third comma	1
	Schulter's comma	1
314-319	Schulter's comma	1
320-325	13th partial chroma	1
	Septimal comma	1
	Tridecimal neutral third comma	1

Appendix G, part four:
List of the commas as they occur throughout the allegro sections of *13.*

	Schulter's comma	1
326-332	Schulter's comma	1
340-342	Schulter's comma	1
	Tridecimal neutral third comma	1
343-346	Schulter's comma	1
	Tridecimal neutral third comma	1
347-350	Tridecimal neutral third comma	2
351-354	Tridecimal neutral third comma	2
	13th partial chroma	1
356-359	Tridecimal neutral third comma	2
	13th partial chroma	1
	Septimal comma	1
	Syntonic comma	1
368-371	Syntonic comma	1
	Septimal comma	1
	Tridecimal neutral third comma	1
372-375	Septimal comma	1
	Tridecimal neutral third comma	2
376-379	Tridecimal neutral third comma	2
384-391	Tridecimal neutral third comma	2
394-401	Tridecimal neutral third comma	2
404-411	Tridecimal neutral third comma	1

Appendix H
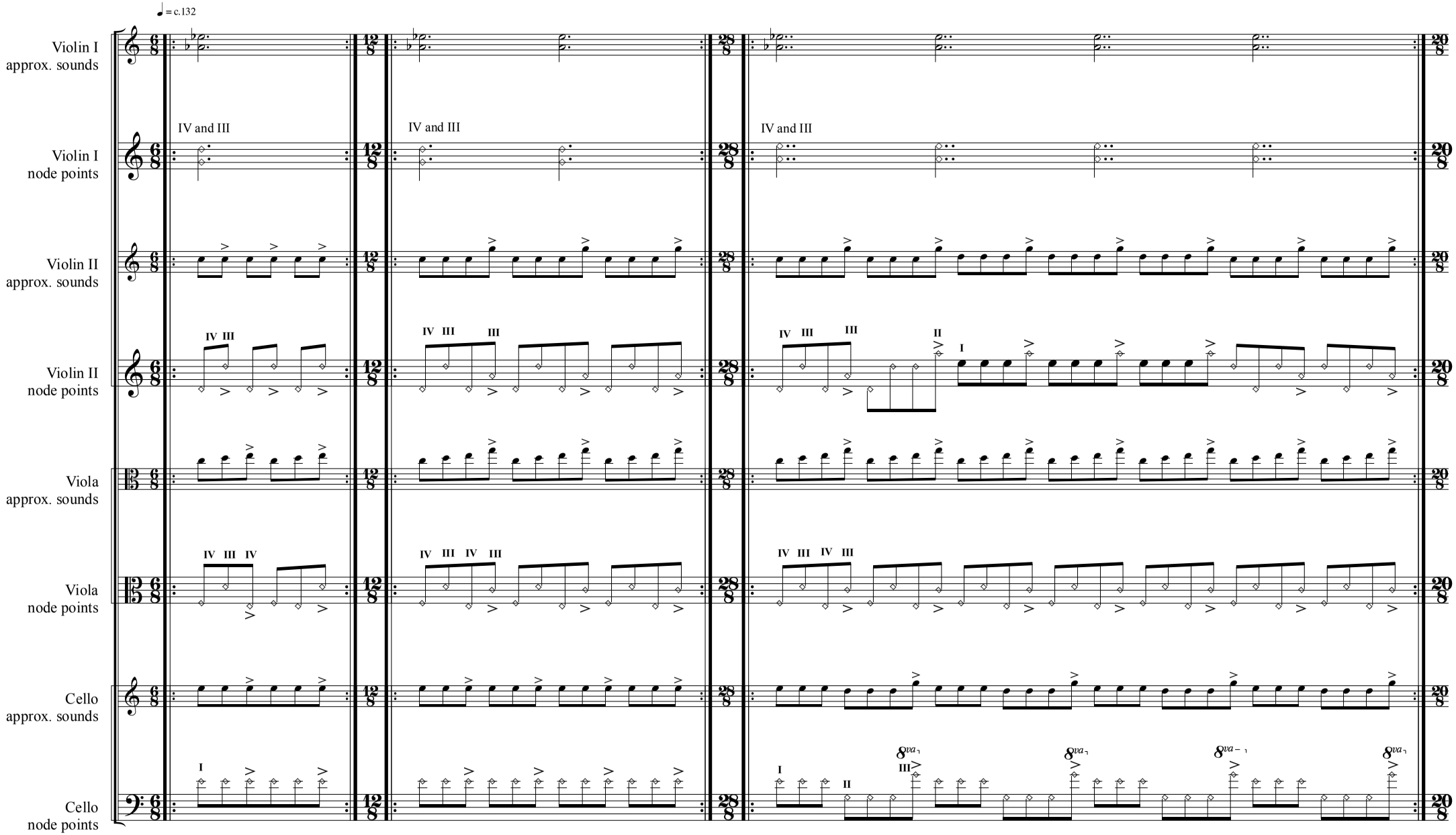
Appendix H, part one
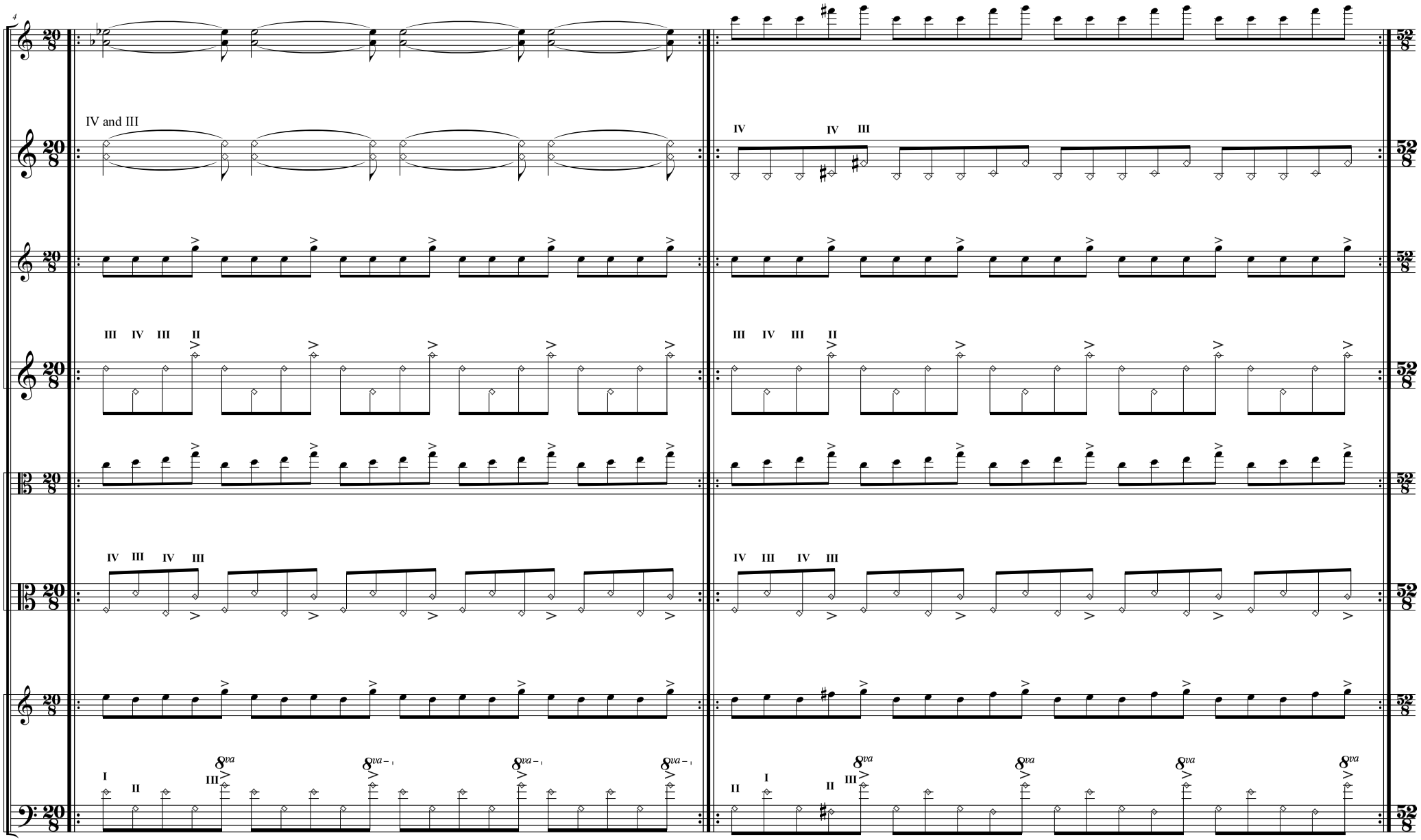
(draft of *13* with unmanageably large time signatures)

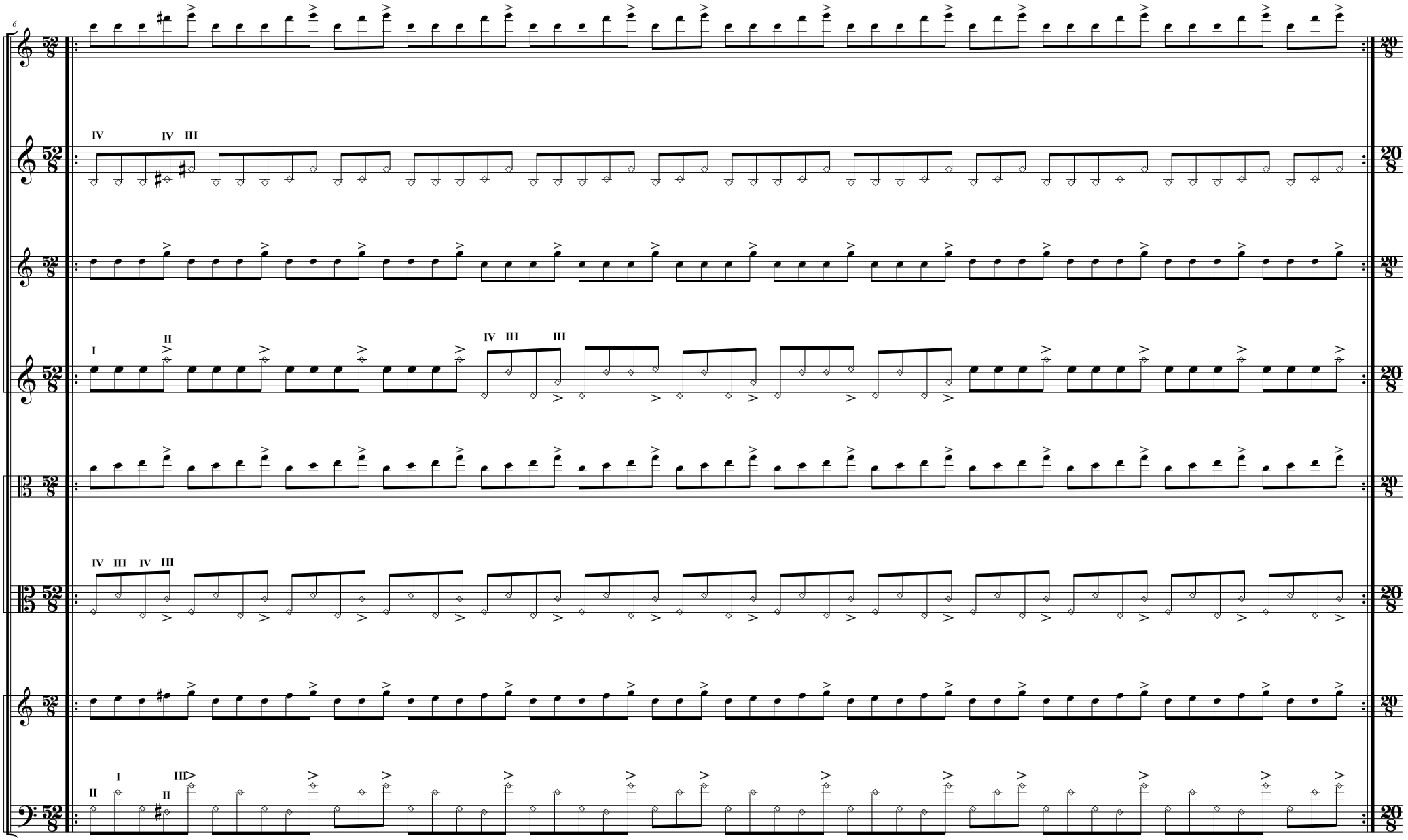
#13

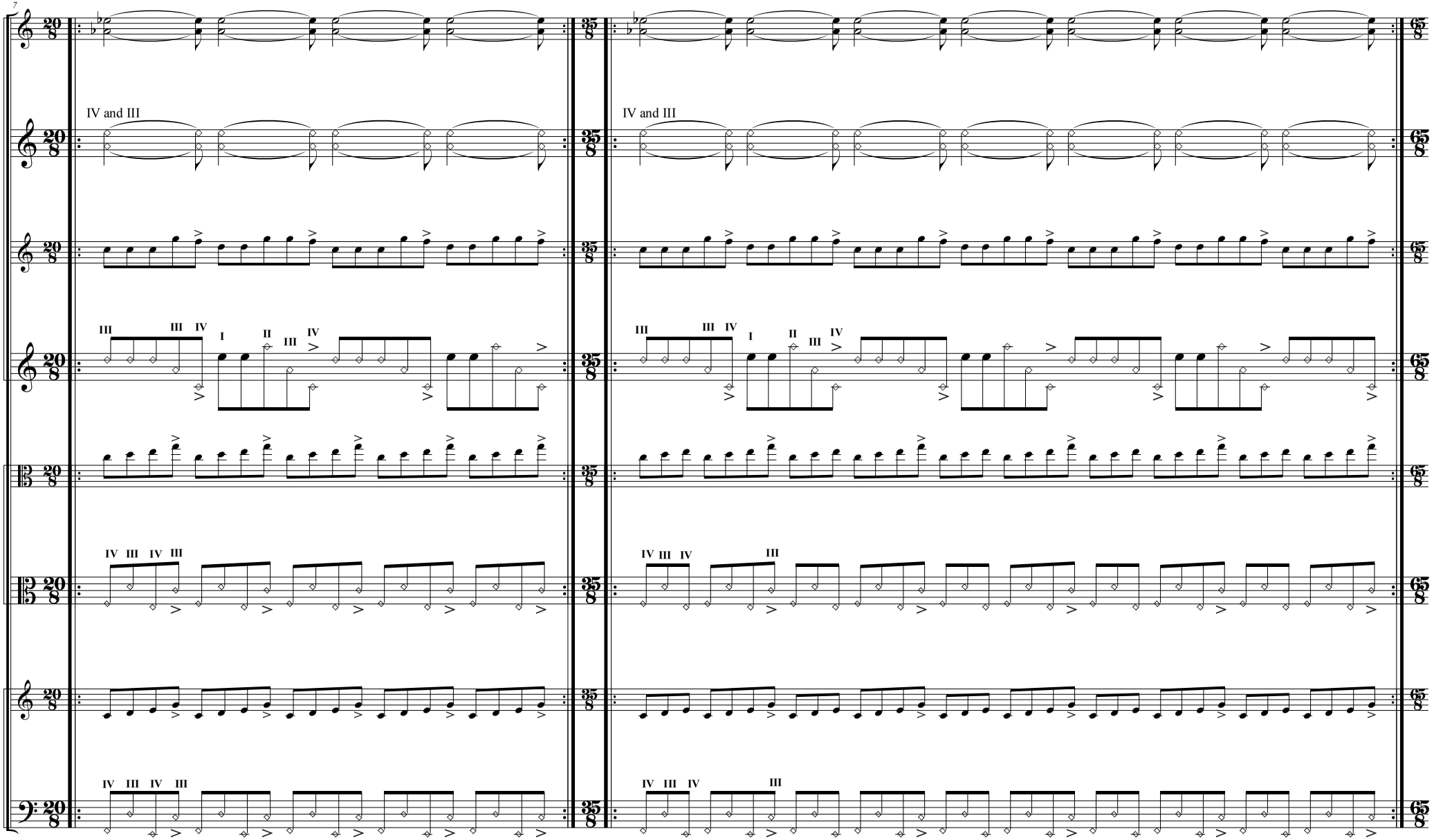
transposing score

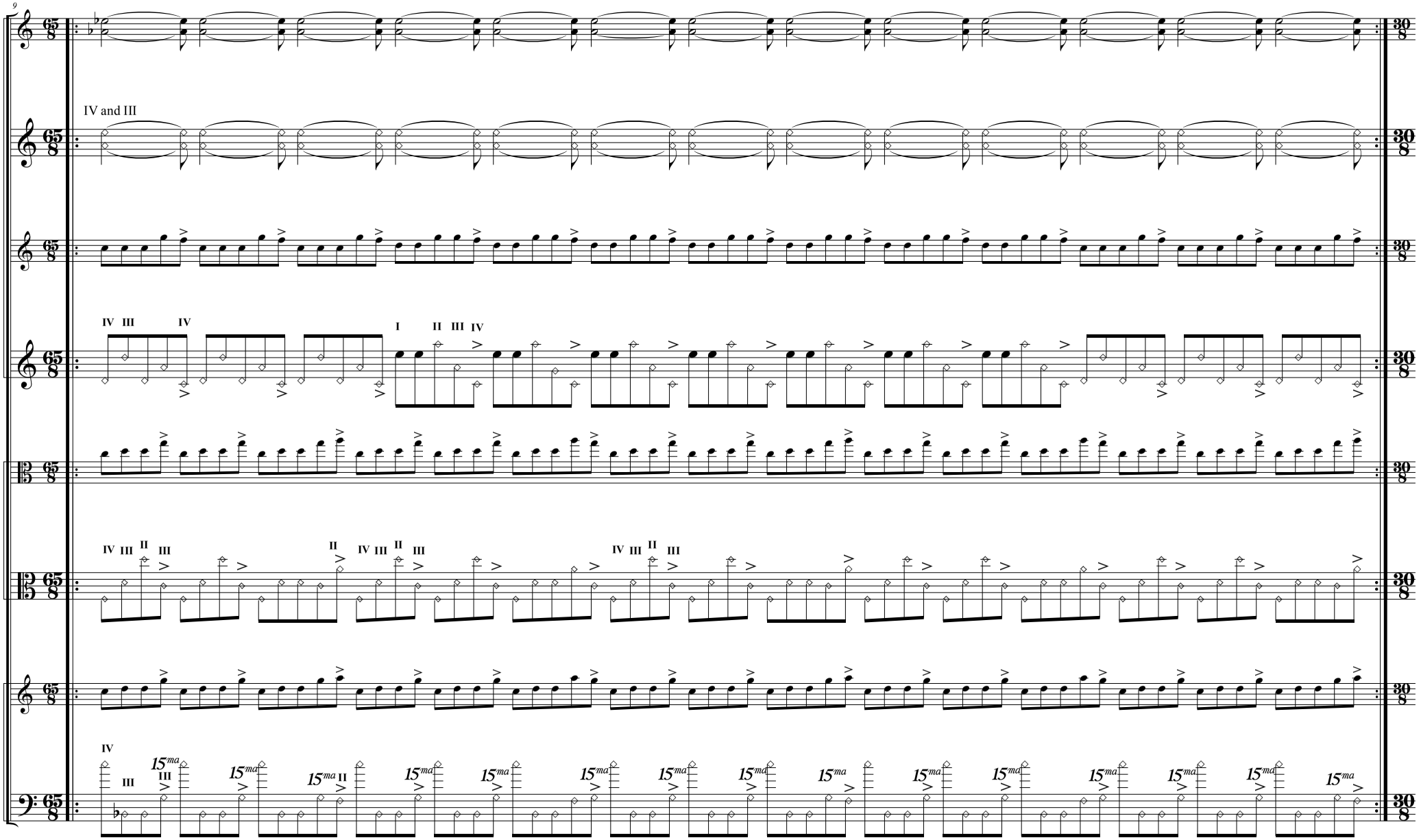
Dylan Crismani

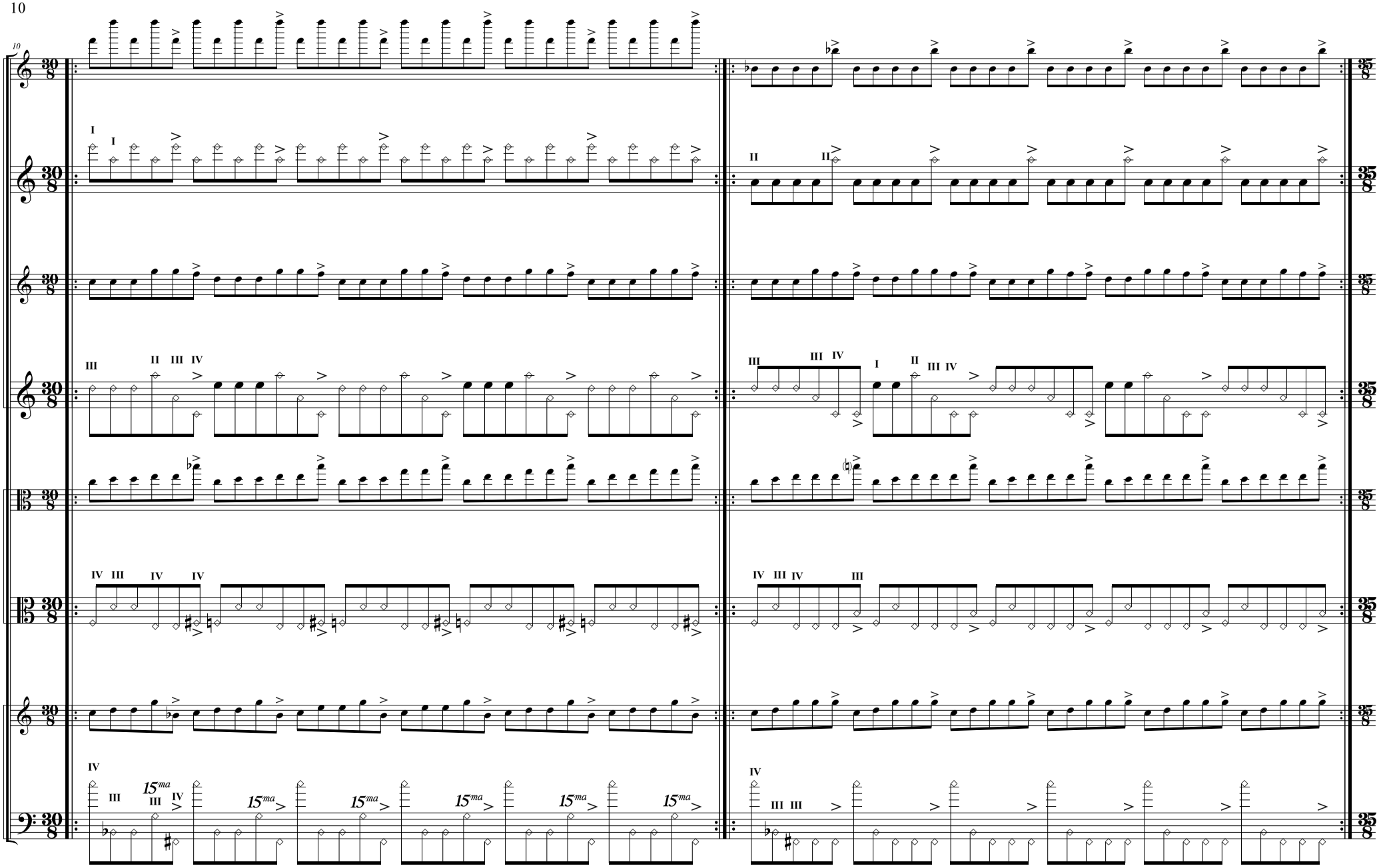


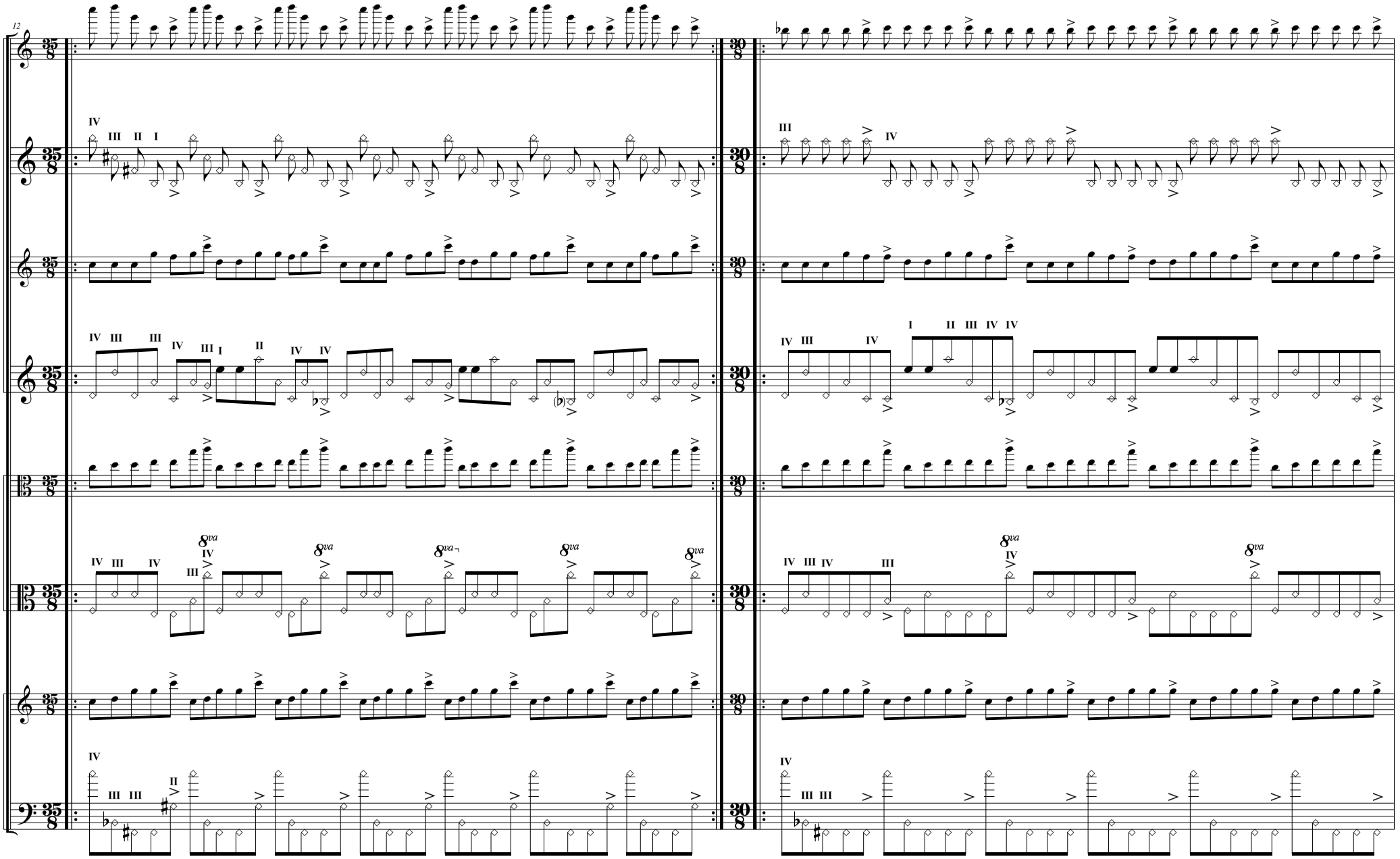

Vln. I
Vln. I
IV and III
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

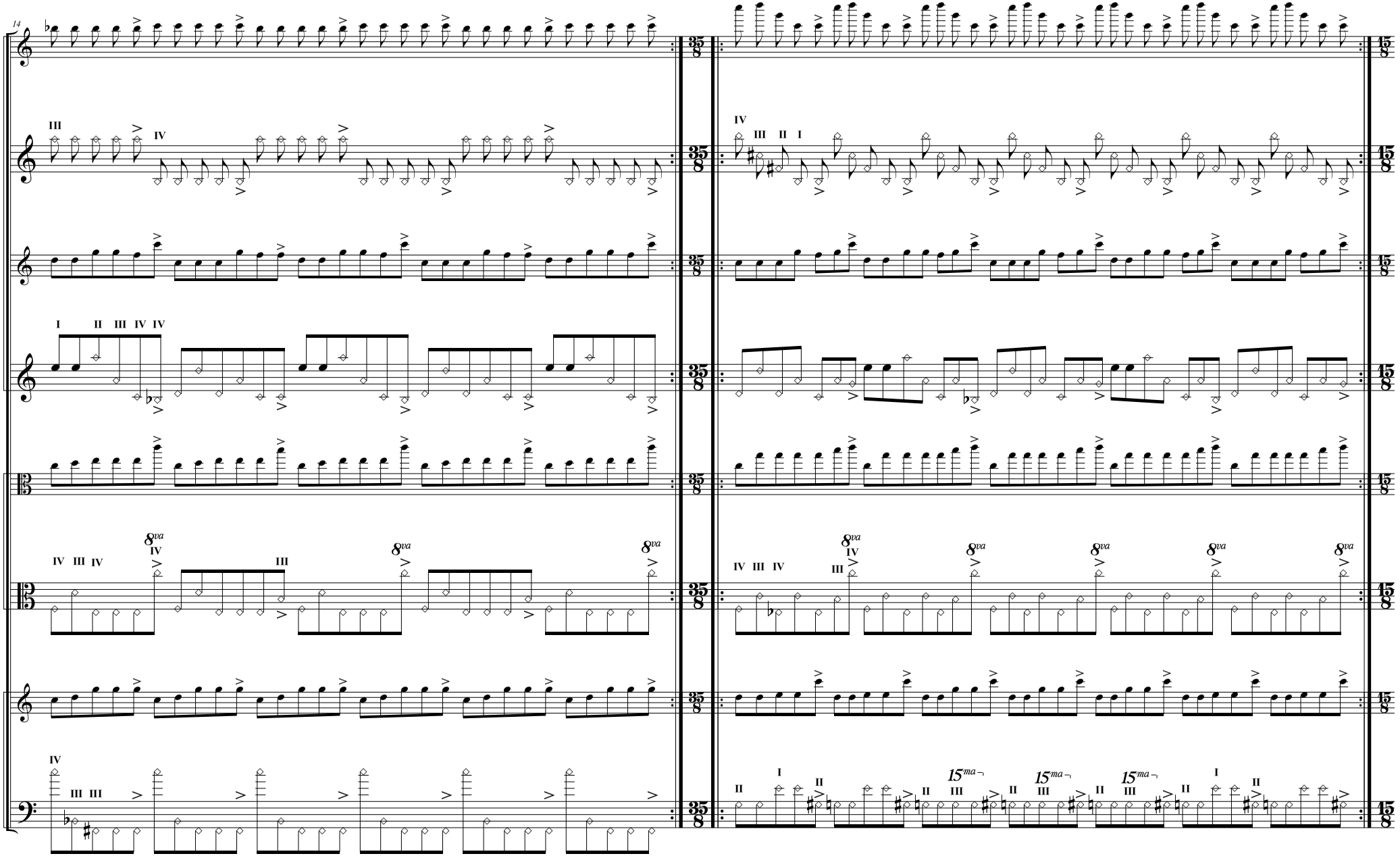

6
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

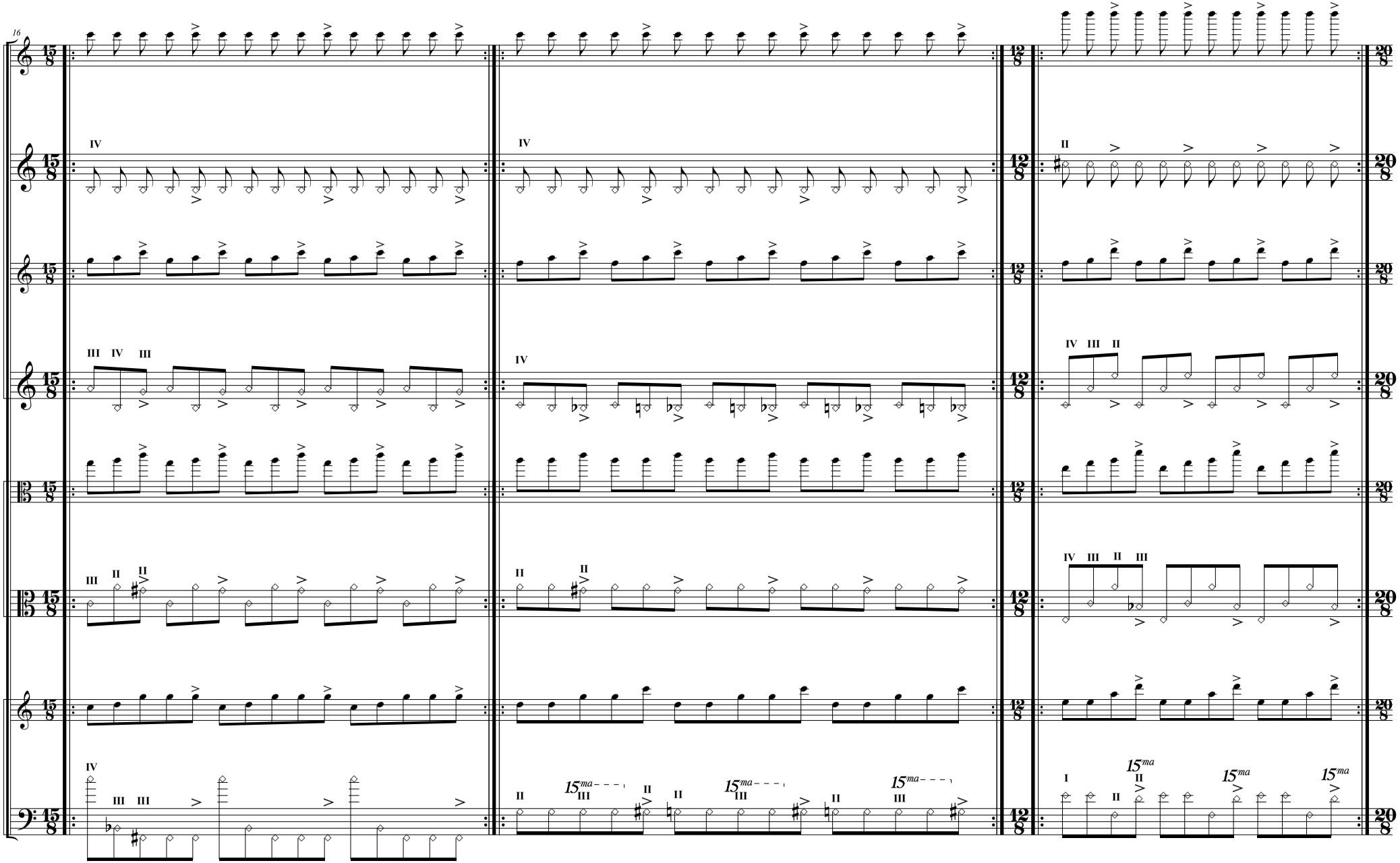

7
Vln. I
IV and III
Vln. I
IV and III
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

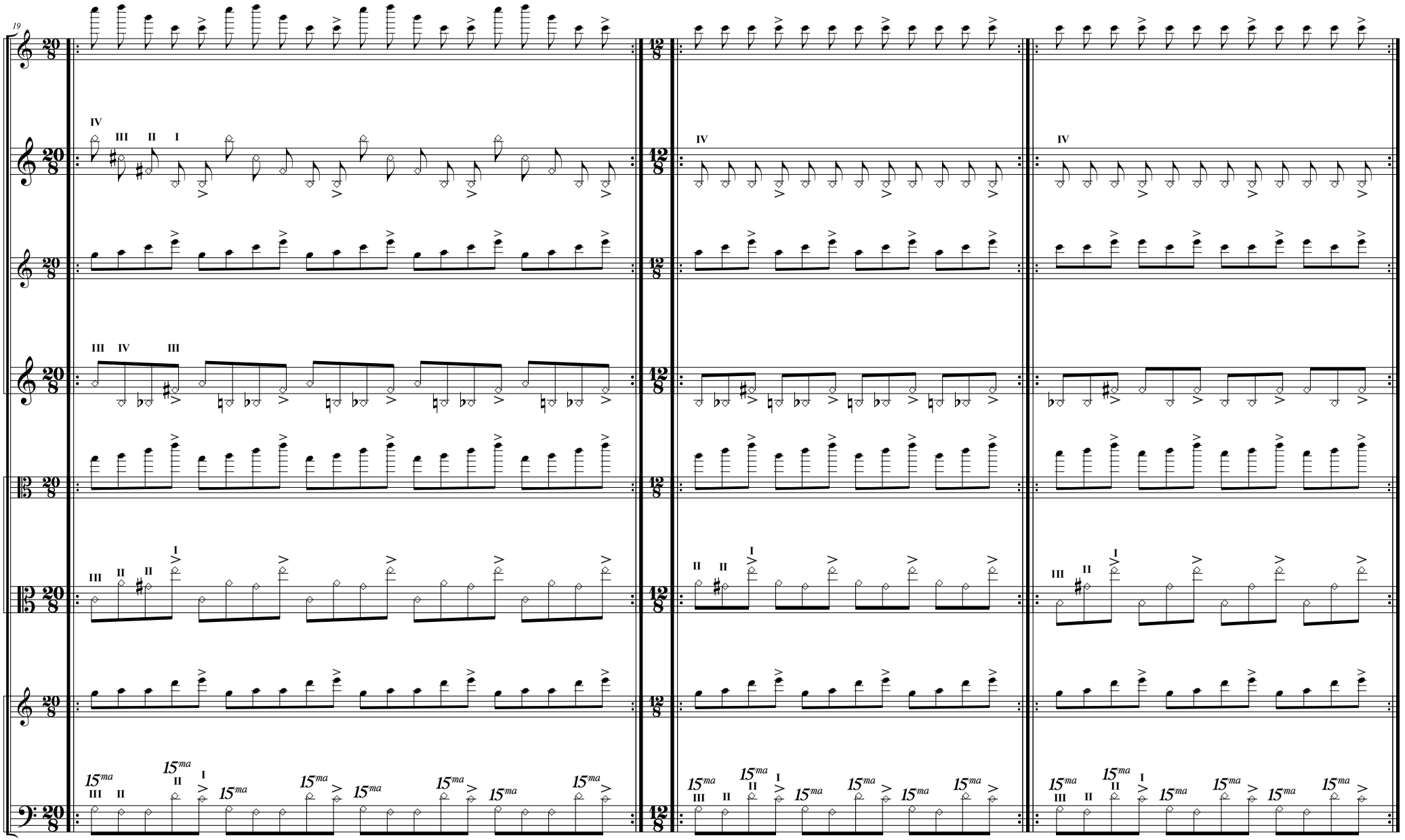

9
Vln. I
IV and III
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

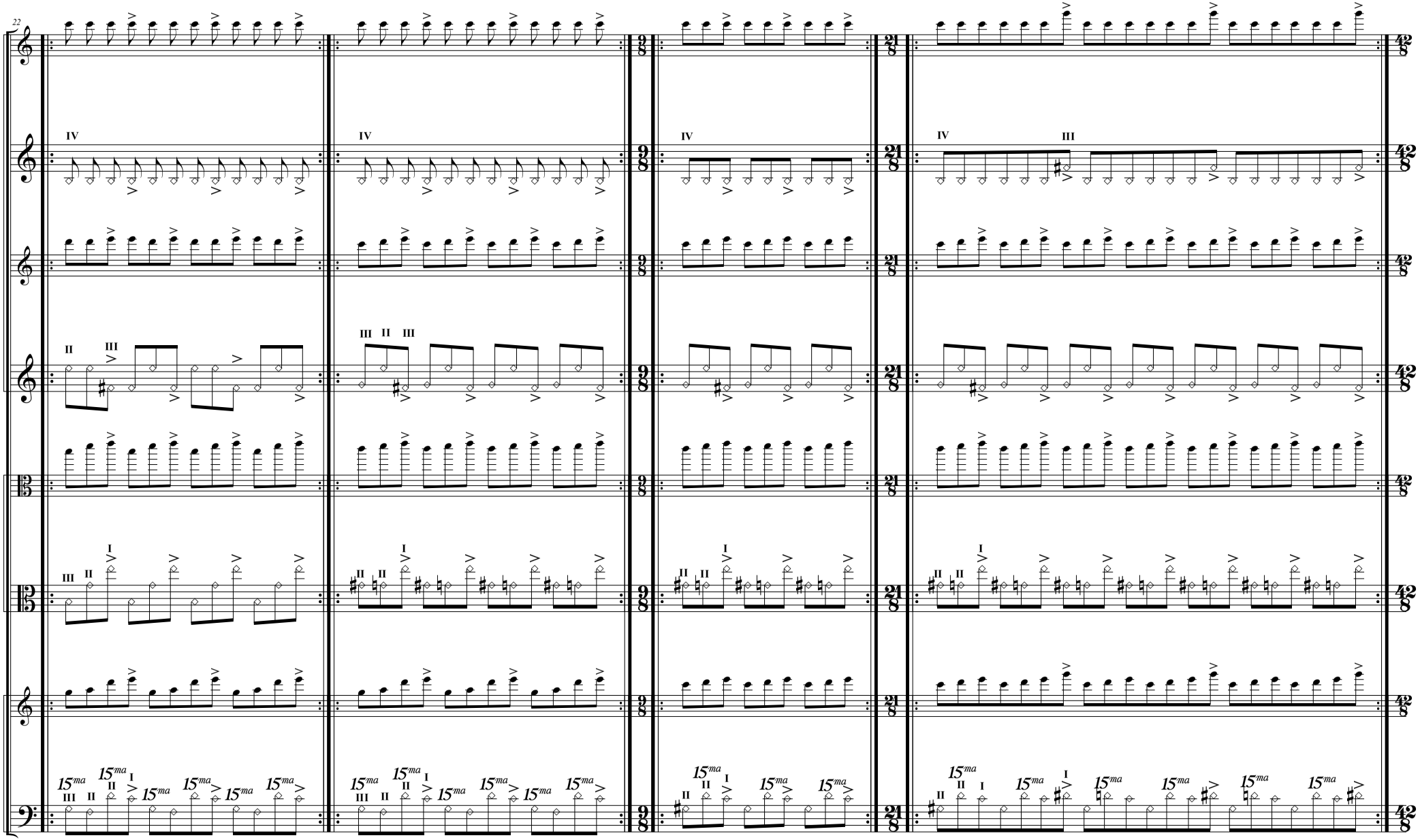

10
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
15ma

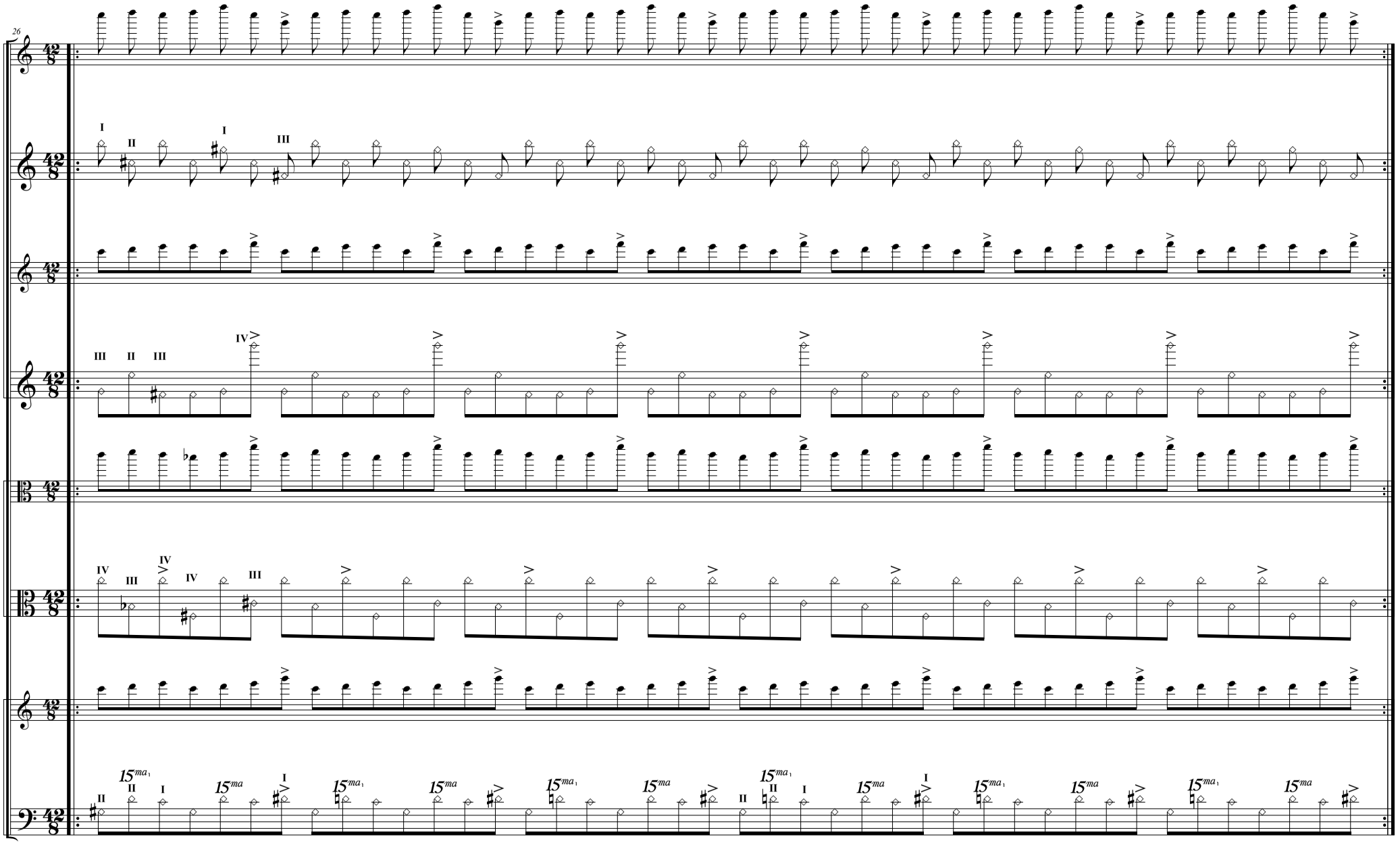

12
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
8va

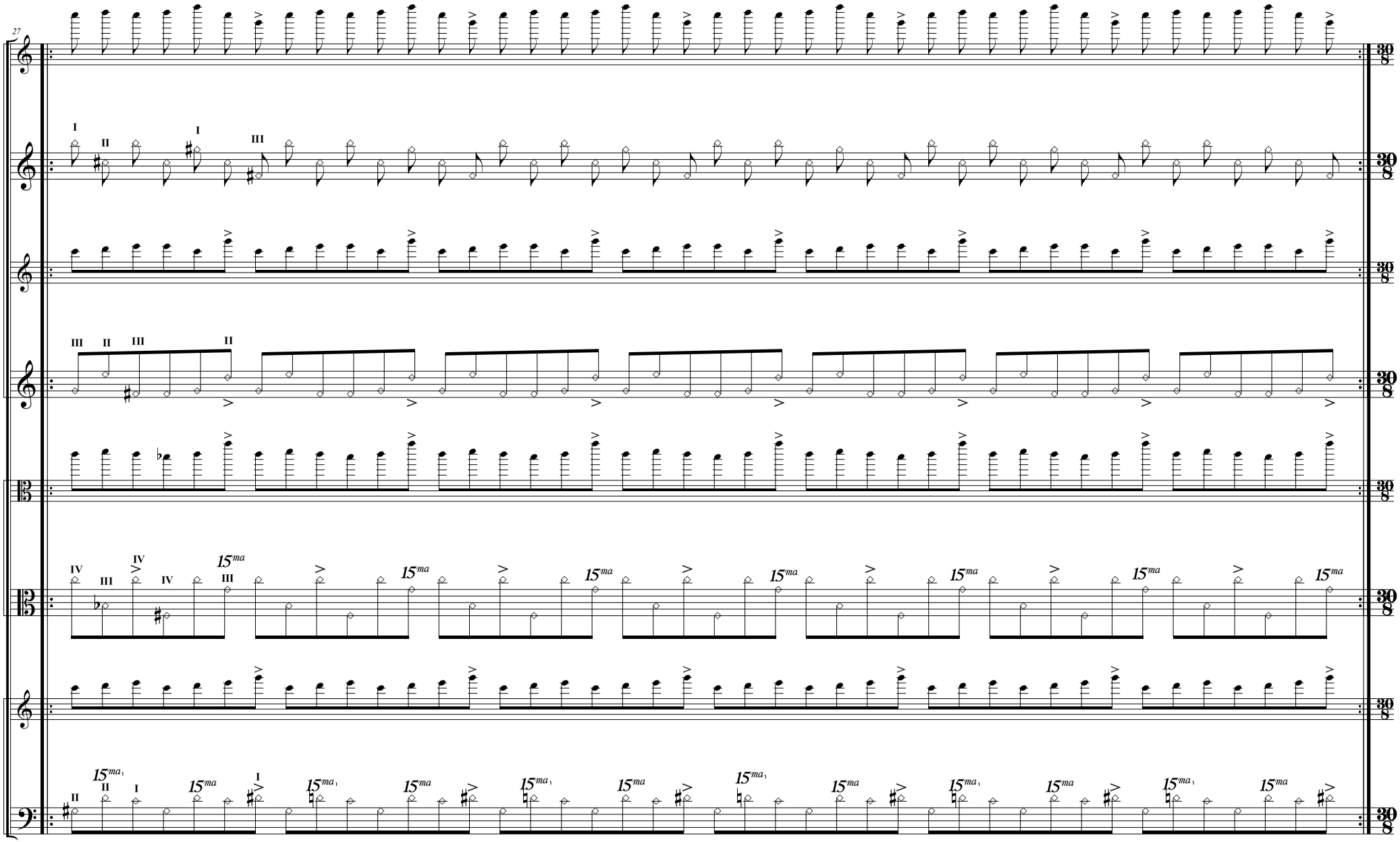

14
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
35/8
15/8
8va
15ma

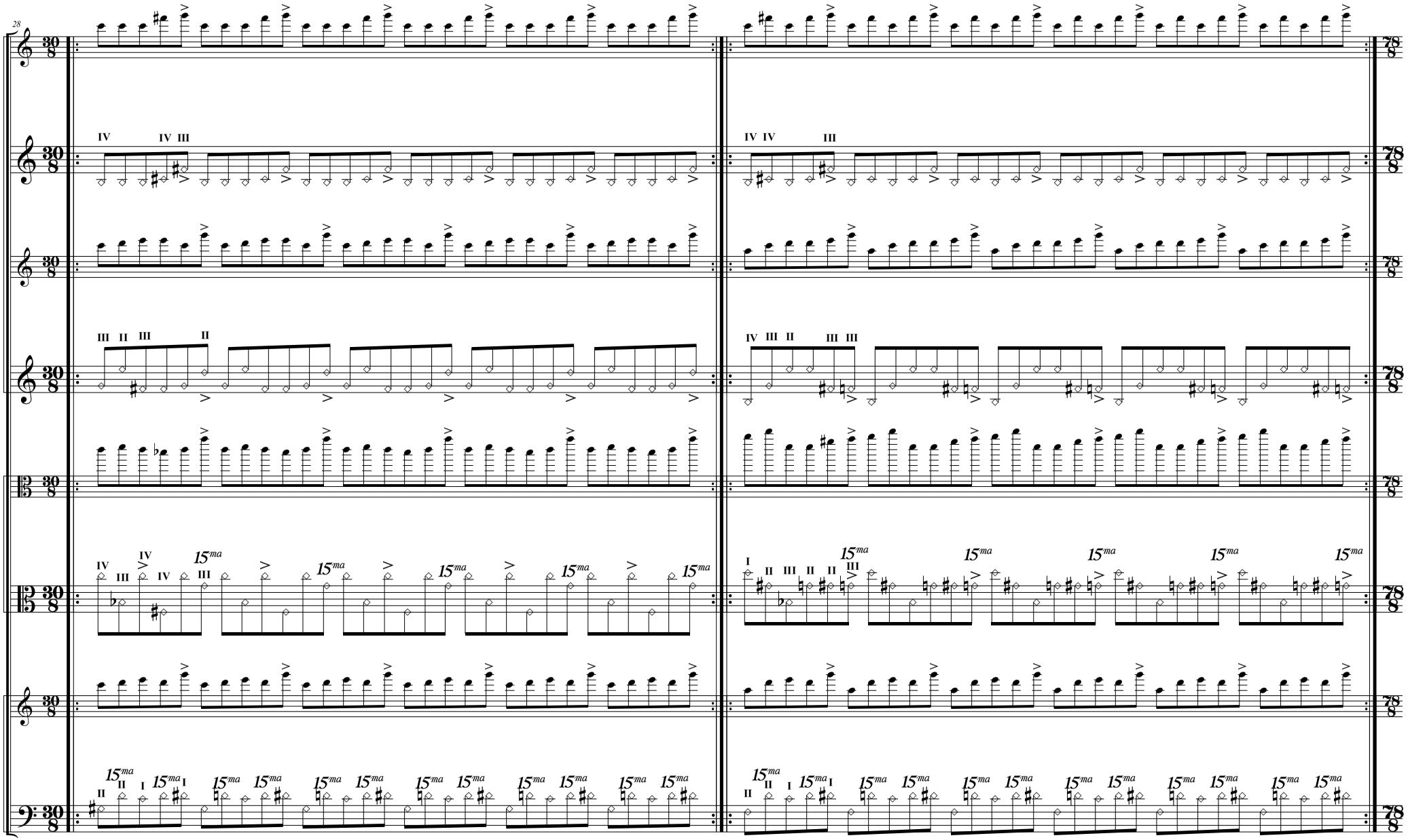

16
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
15/8
12/8
20/8
15ma

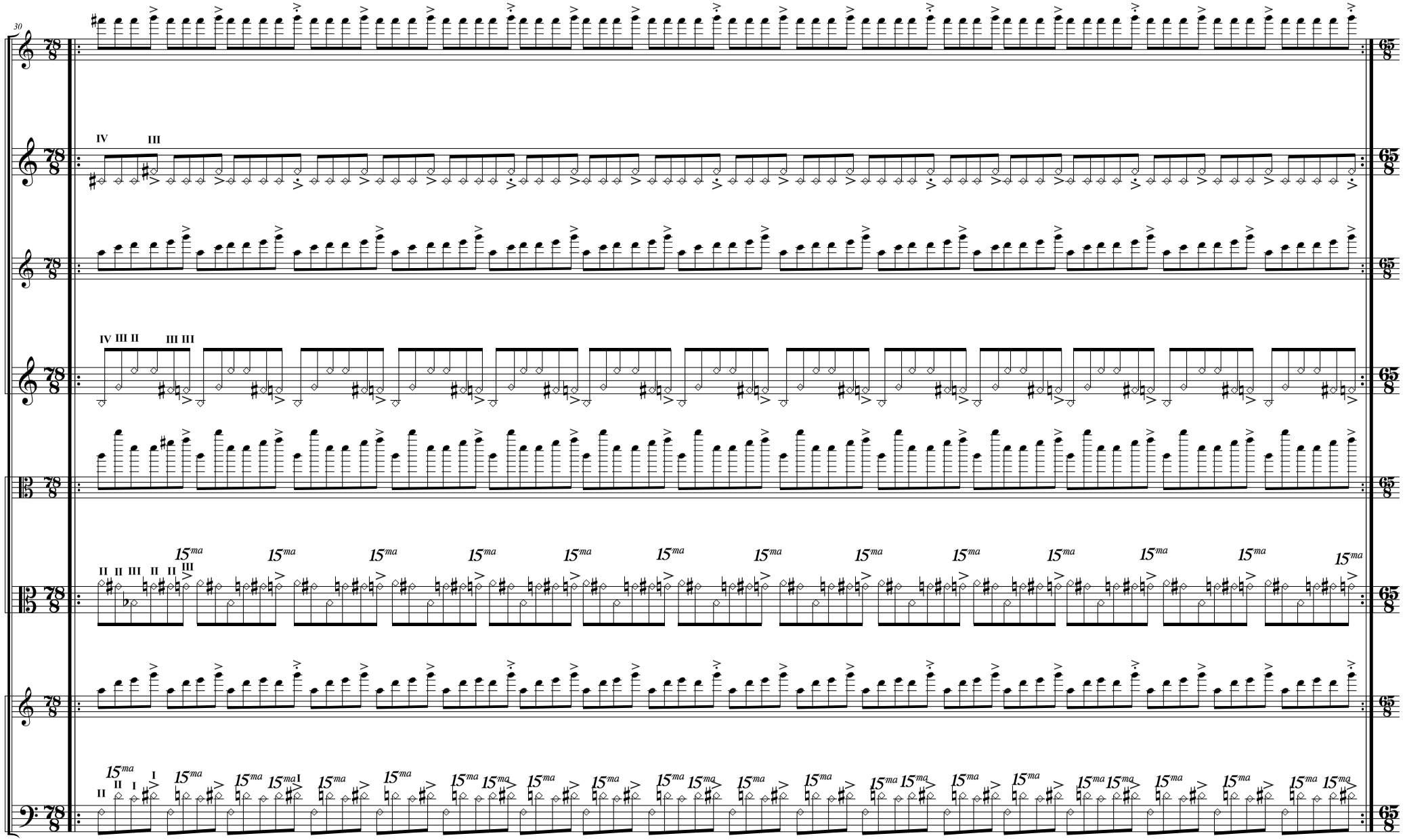

19
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
15ma

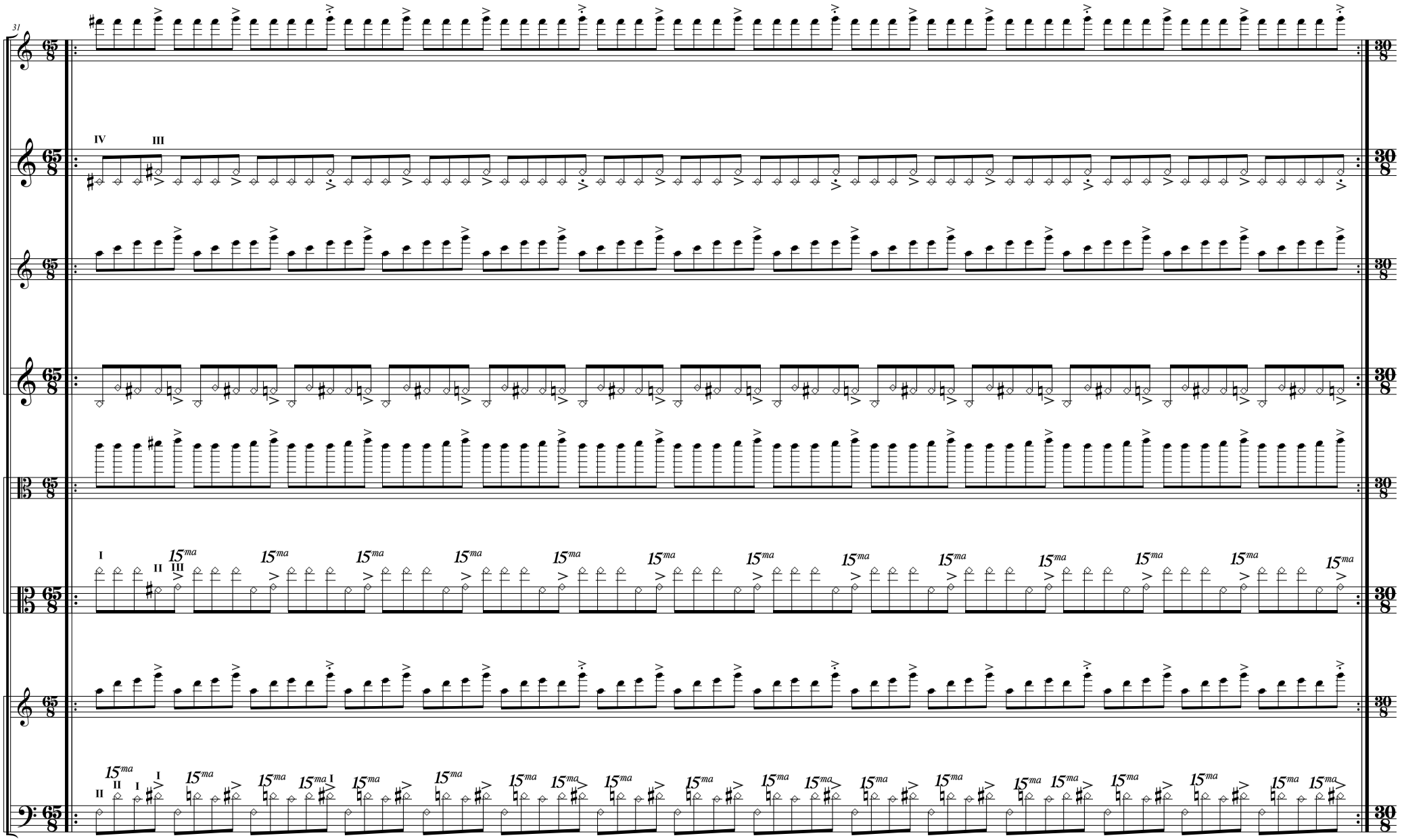

22
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
9/8
21/8
42/8

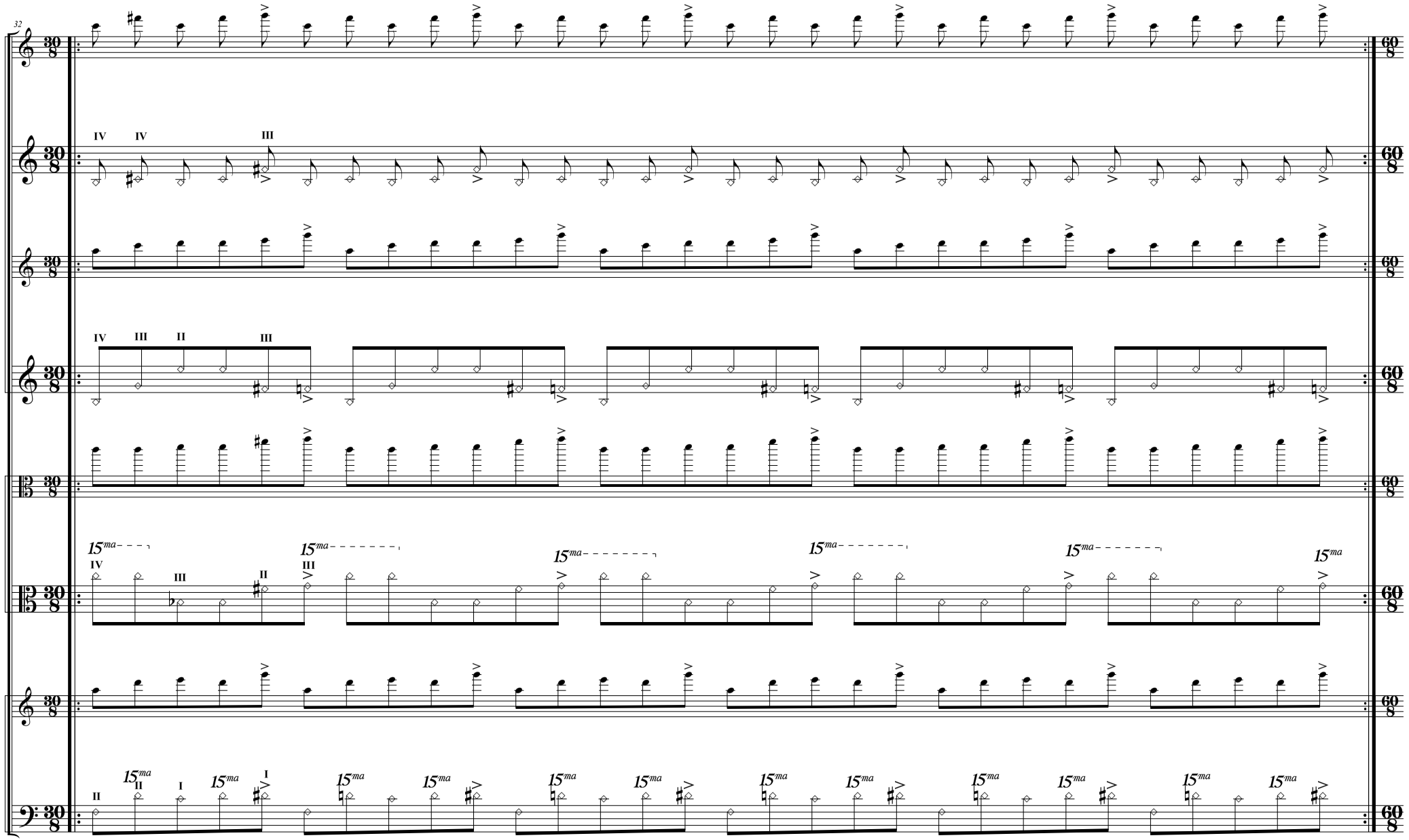

26
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

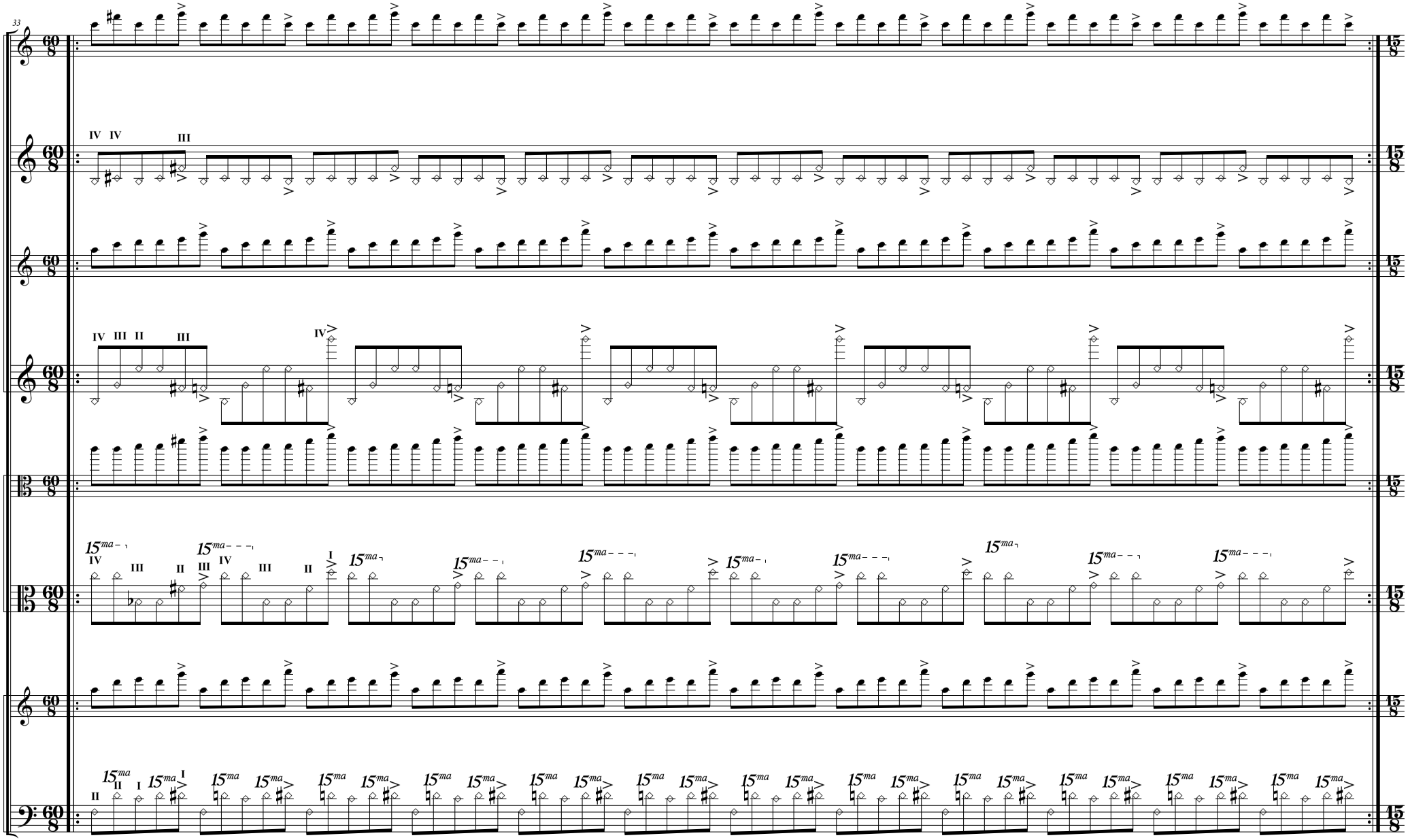

27
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

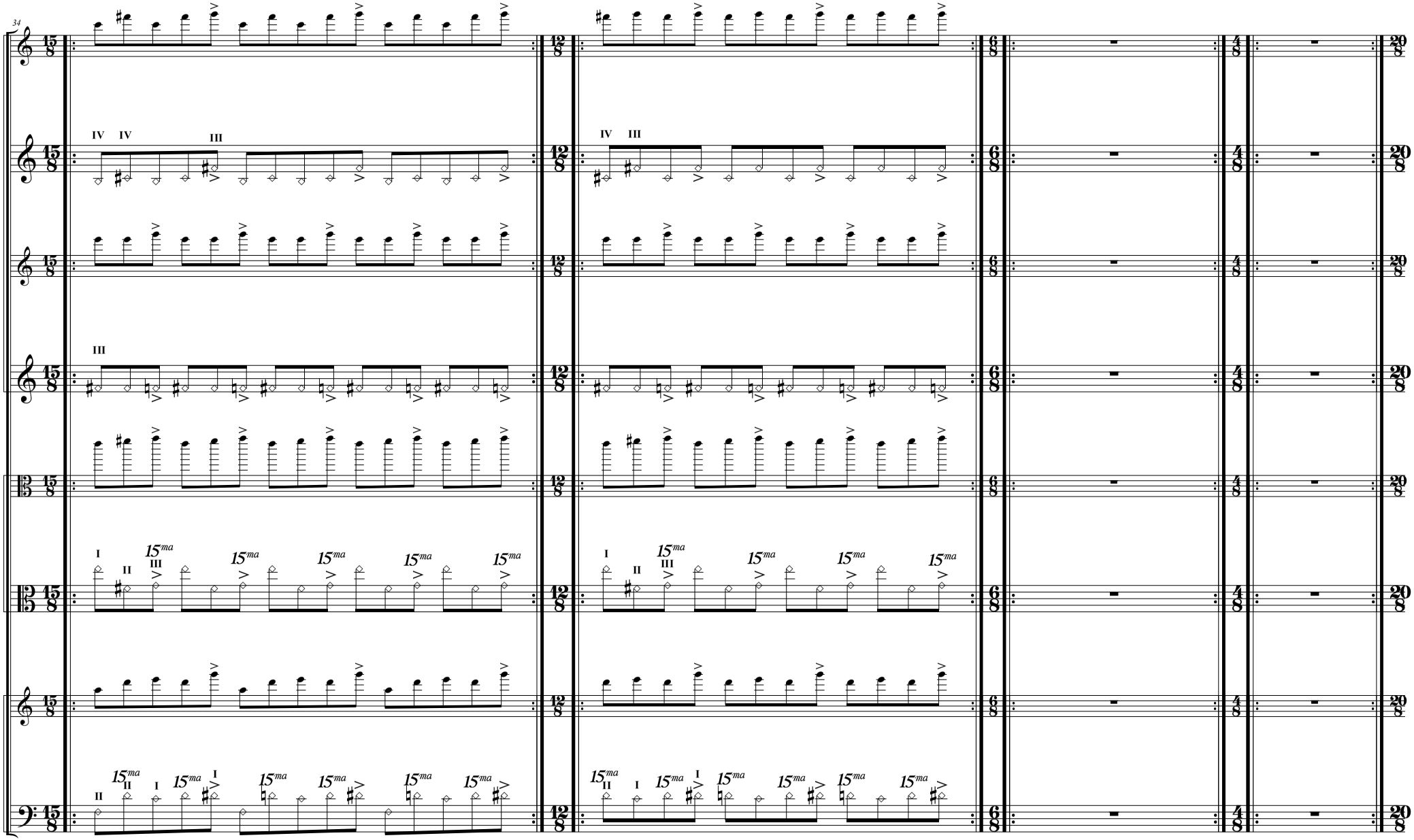

28
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
15ma

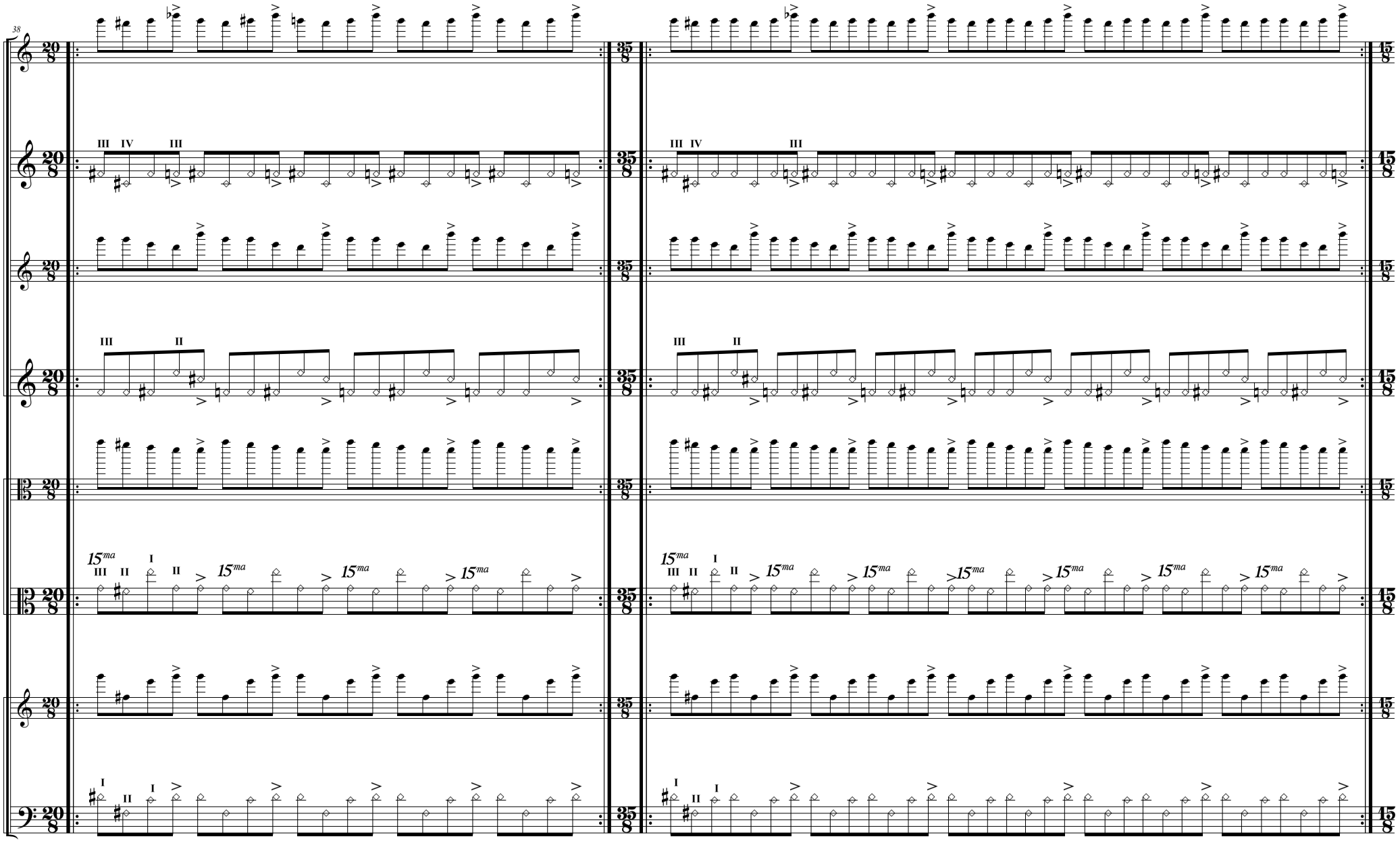

30
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
IV
III
IV III II
III III
II II III II II III
15ma
II
II I
I

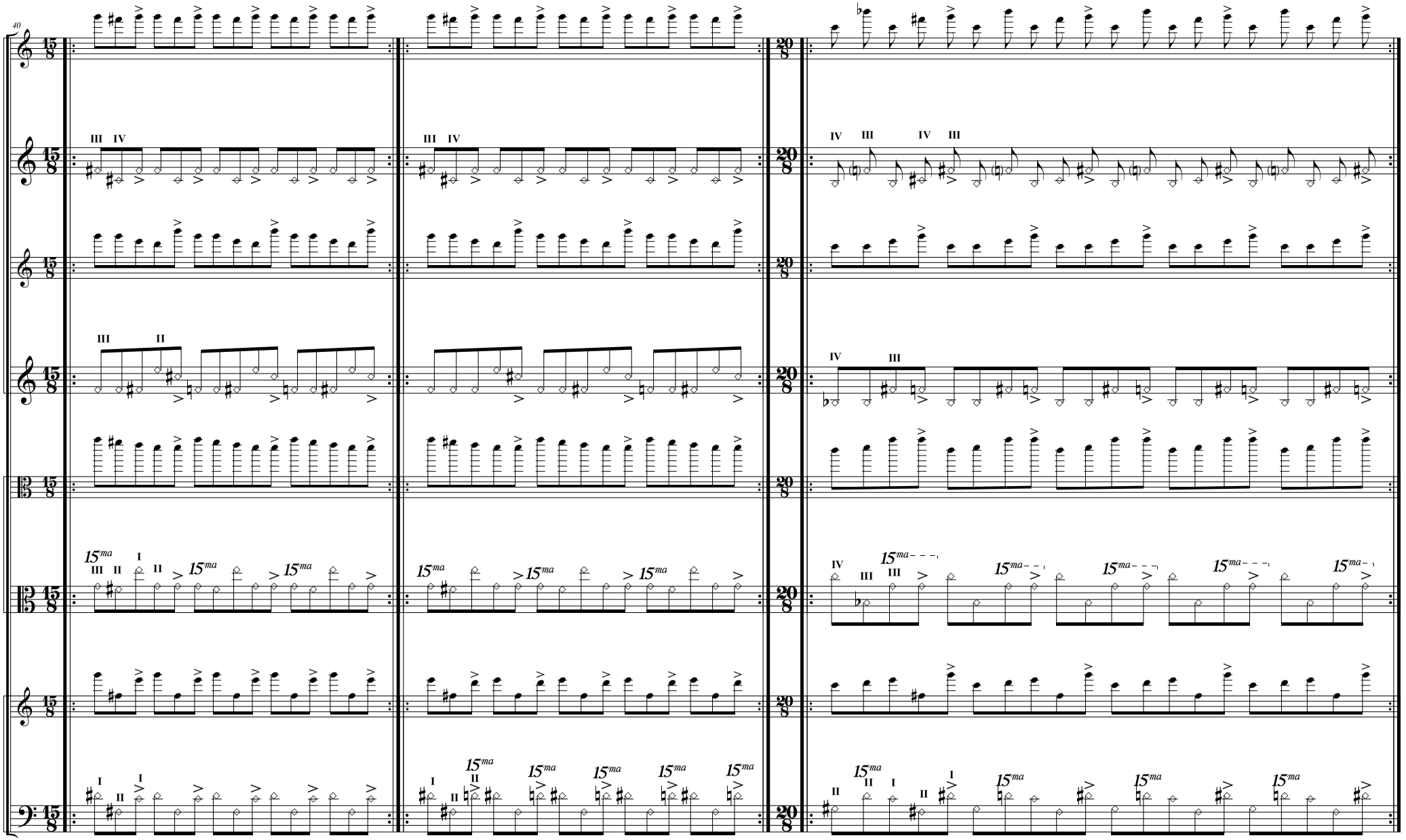

31
Vln. I
Vln. I
IV
III
Vln. II
Vln. II
Vla.
Vla.
I
II
III
15ma
Vc.
Vc.
II
I

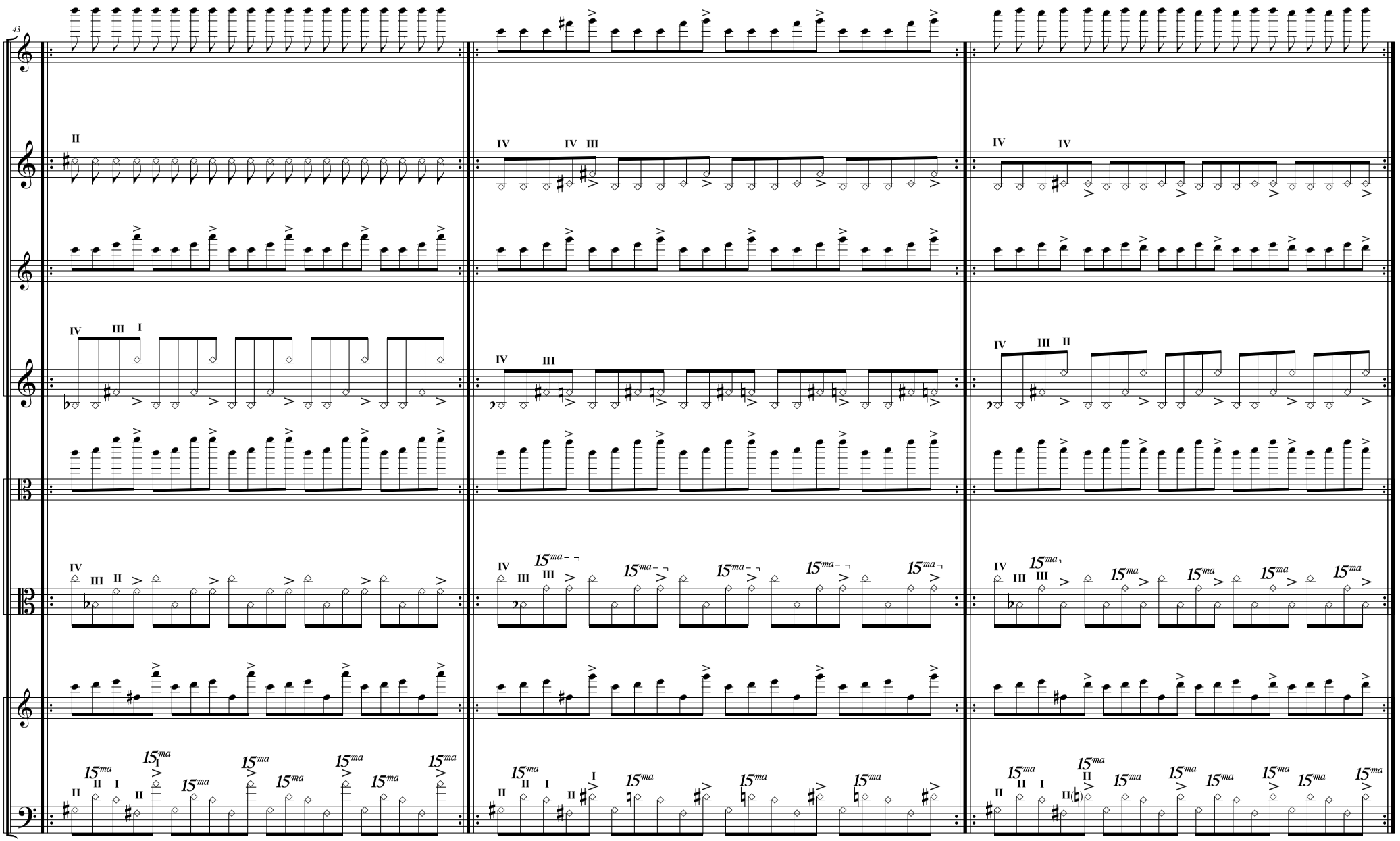

32
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
IV IV III
IV III II III
15ma
IV III II III
II I I

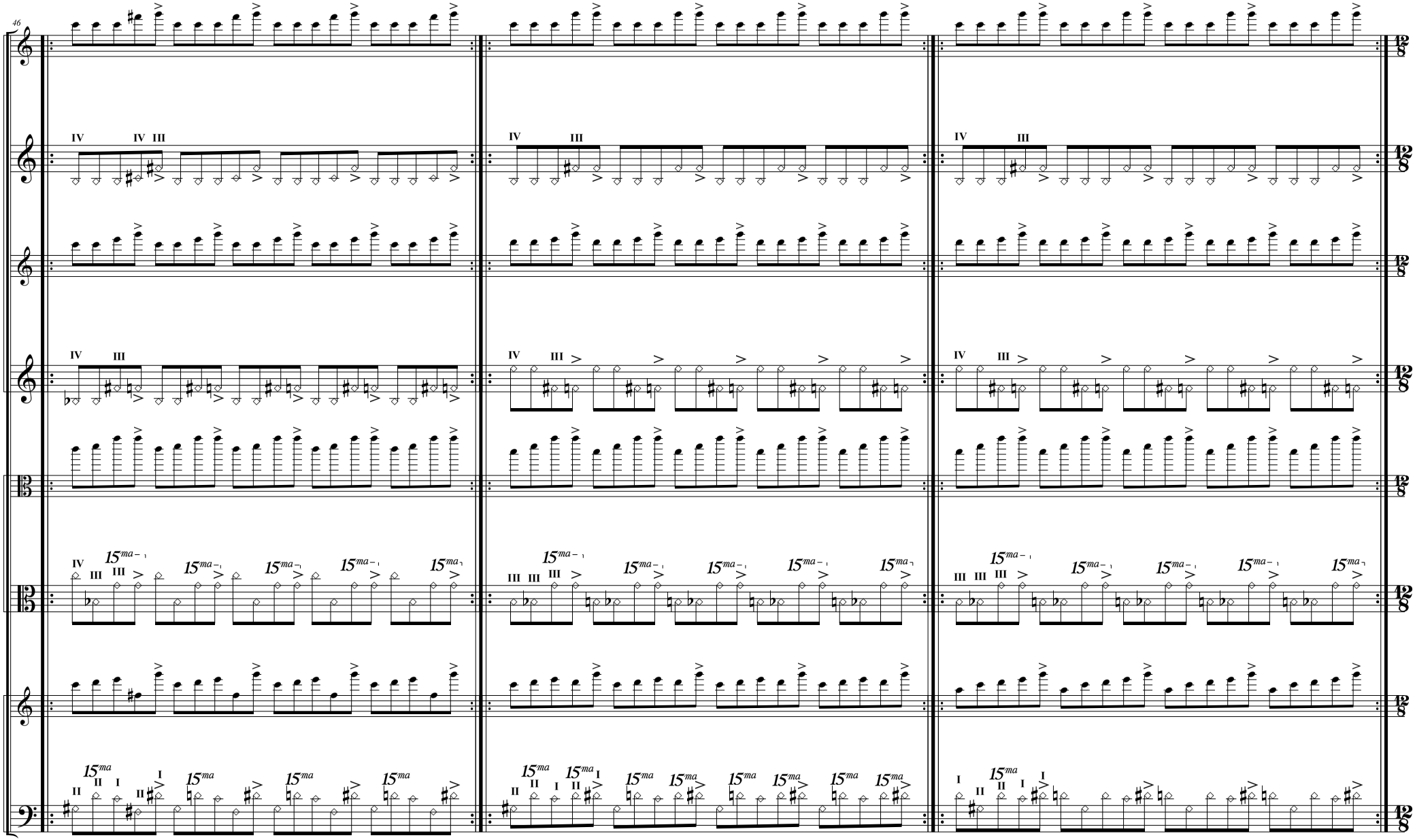

33
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

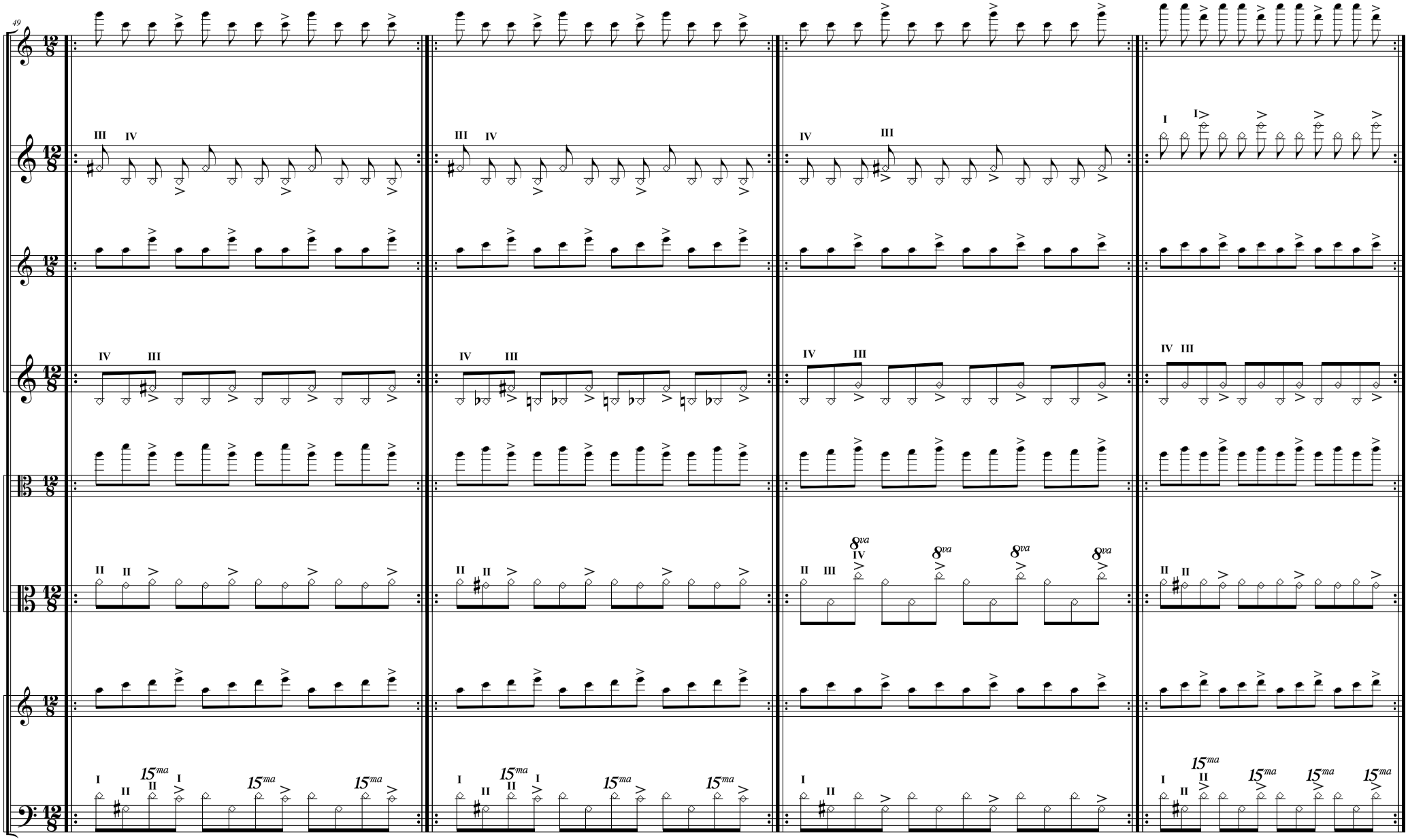

34
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

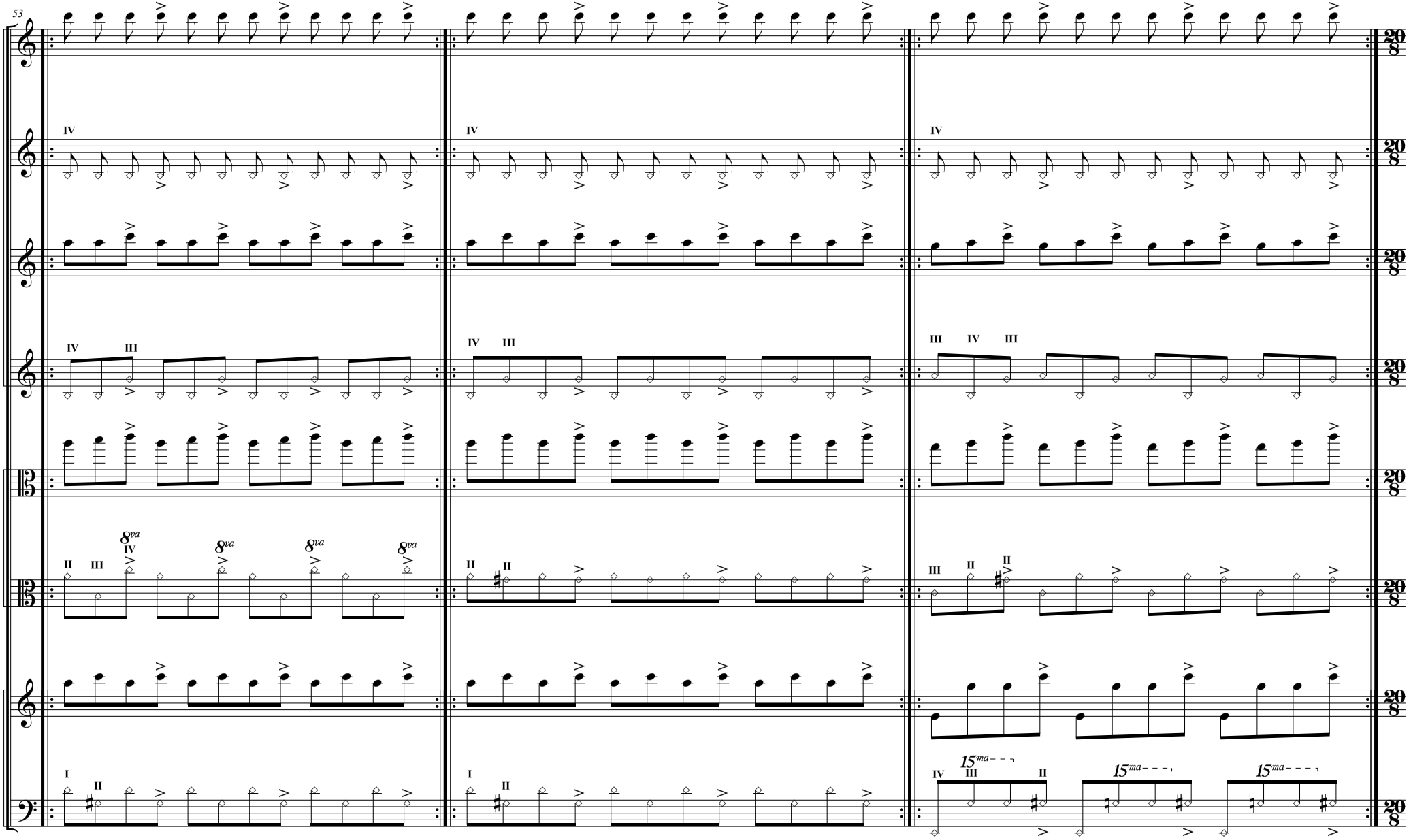

38
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
20/8
35/8
15/8
III IV III
III II
15ma
I II

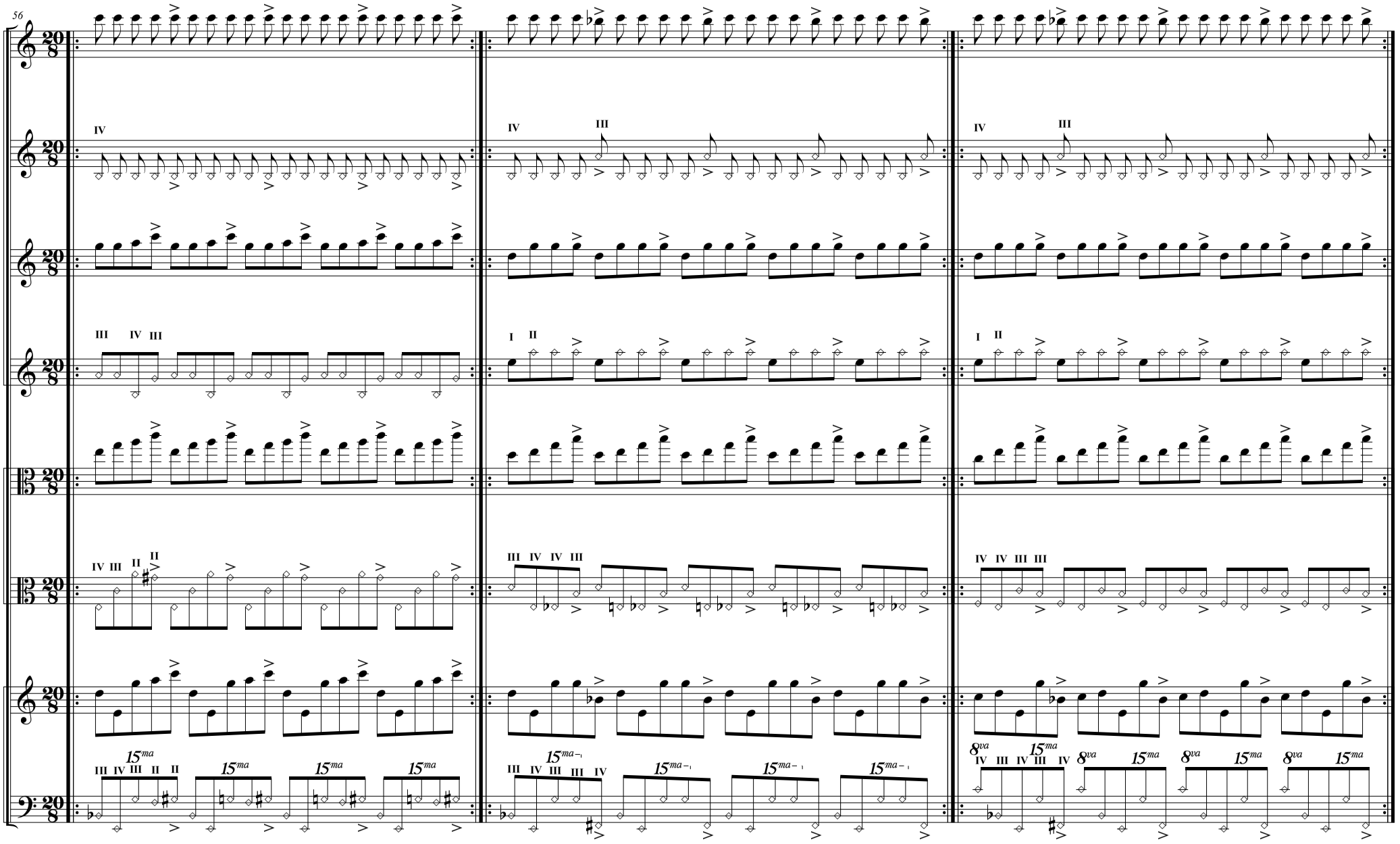

40
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
15/8
20/8
15ma

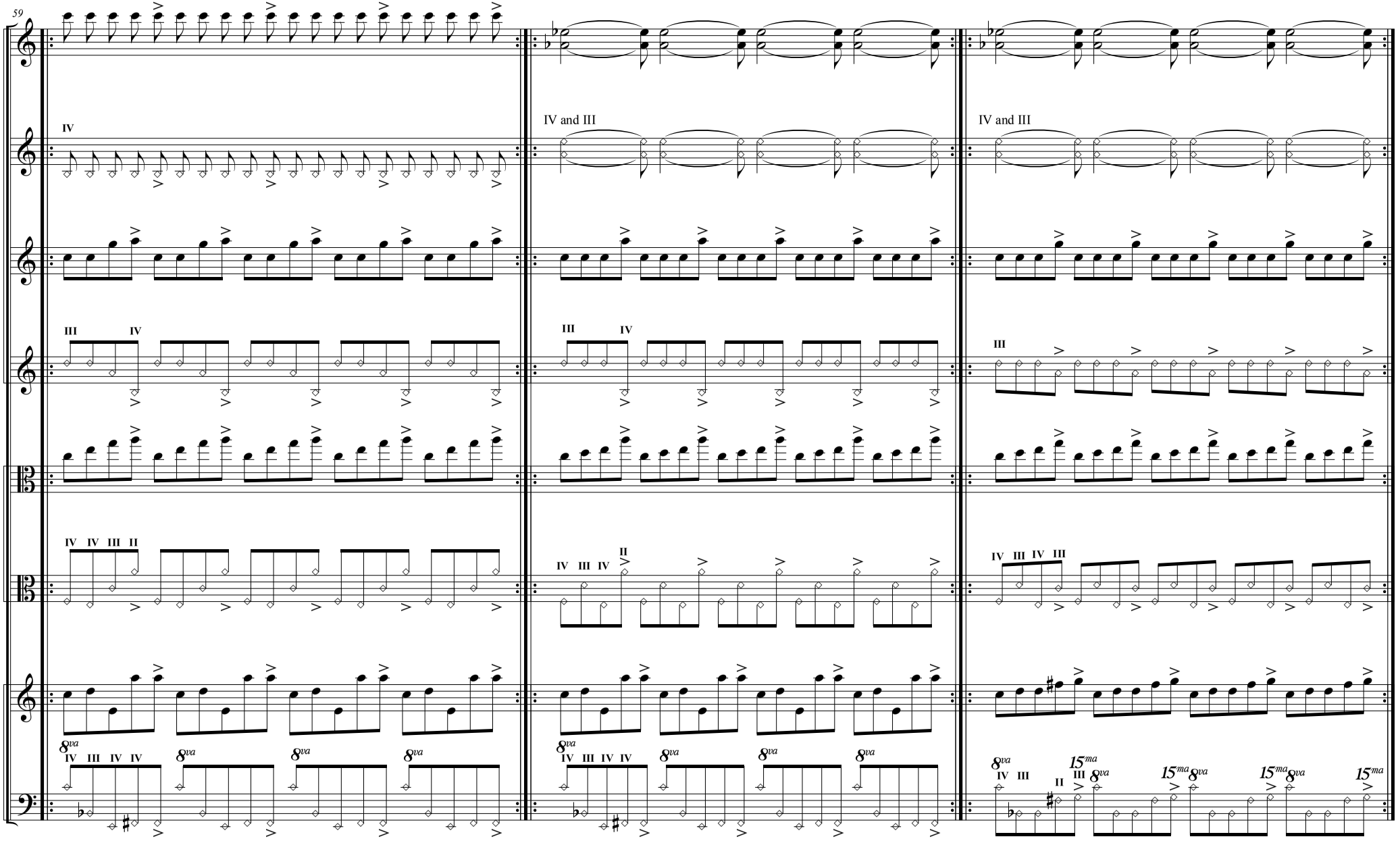

43
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

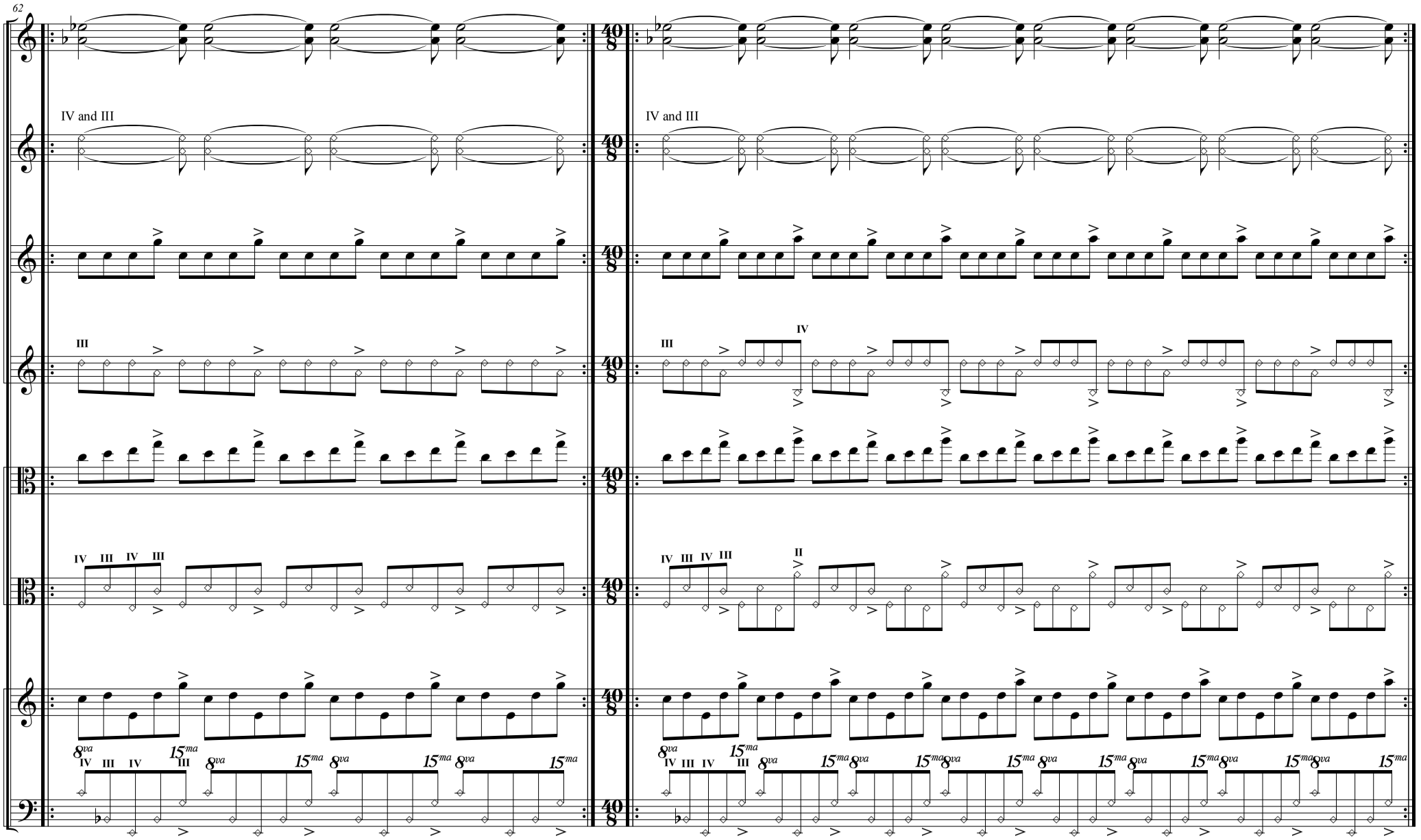

46
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.

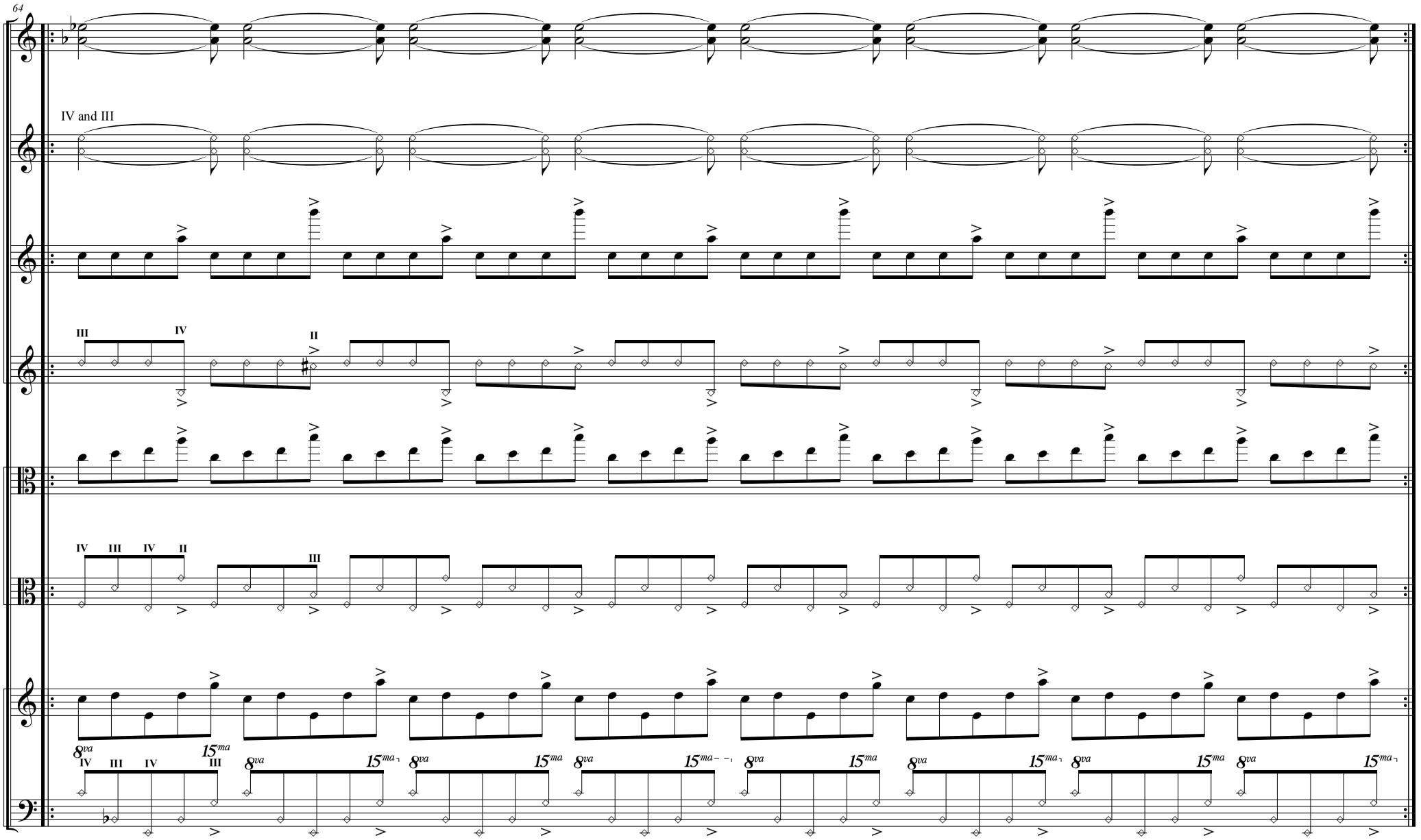

49
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
8va
15ma

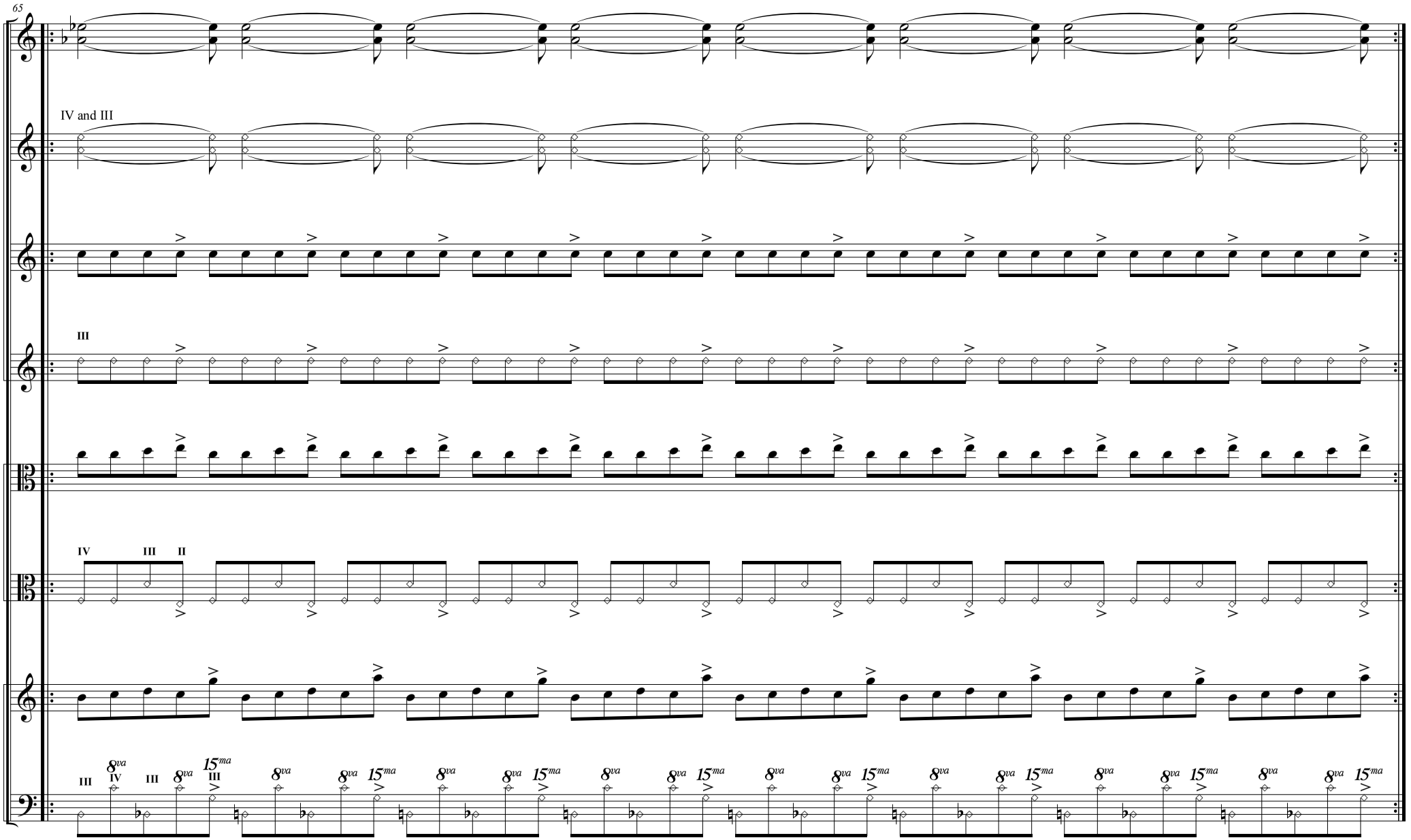

53
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
8va
15ma


56
Vln. I
Vln. I
Vln. II
Vln. II
Vla.
Vla.
Vc.
Vc.
20/8
IV
III
IV III
I II
IV III II II
III IV IV III
IV IV III III
III IV III II II
III IV III III IV
IV III IV III IV
15ma
8va


59
Vln. I
Vln. I
IV
IV and III
IV and III
Vln. II
Vln. II
III IV
III IV
III
Vla.
Vla.
IV IV III II
IV III IV II
IV III IV III
Vc.
Vc.
8va
IV III IV IV
15ma
III
II


62
Vln. I
IV and III
Vln. I
IV and III
Vln. II
Vln. II
III
III
IV
Vla.
Vla.
IV III IV III
IV III IV III
II
Vc.
Vc.
8va
15ma


64
Vln. I
IV and III
Vln. I
Vln. II
III
IV
II
Vln. II
Vla.
IV
III
IV
II
III
Vla.
Vc.
8va
15ma
IV
III
IV
III
Vc.


65
Vln. I
IV and III
Vln. I
Vln. II
III
Vln. II
Vla.
IV III II
Vla.
Vc.
III IV III III
8va 8va 15ma
Vc.