

ACCEPTED VERSION

An Deng and Yadong Zhou

Modeling electroosmosis and surcharge preloading consolidation I: Model formulation

Journal of Geotechnical and Geoenvironmental Engineering, 2016; 142(4):04015093-1-04015093-8

© 2015 American Society of Civil Engineers

This material may be downloaded for personal use only. Any other use requires prior permission of the American Society of Civil Engineers. This material may be found at [http://dx.doi.org/10.1061/\(ASCE\)GT.1943-5606.0001417](http://dx.doi.org/10.1061/(ASCE)GT.1943-5606.0001417) .

PERMISSIONS

<http://ascelibrary.org/page/informationforasceauthorsreusingyourownmaterial>

Draft Manuscript

Authors may post the final draft of their work on open, unrestricted Internet sites or deposit it in an institutional repository when the draft contains a link to the bibliographic record of the published version in the [ASCE Library](#) or [Civil Engineering Database](#). "Final draft" means the version submitted to ASCE after peer review and prior to copyediting or other ASCE production activities; it does not include the copyedited version, the page proof, or a PDF of the published version.

18 February 2020

<http://hdl.handle.net/2440/123398>

GT4200

Modeling Electroosmosis and Surcharge Preloading Consolidation. I: Model

Formulation

An Deng¹ and Yadong Zhou²

1. Lecturer, School of Civil, Environmental and Mining Engineering, Univ. of Adelaide, Adelaide, SA 5005, Australia (corresponding author). E-mail: an.deng@adelaide.edu.au
2. Lecturer, School of Civil Engineering, Tianjin Chengjian Univ., Tianjin, 300384, China; Formerly, Graduate Student, Geotechnical Research Institute, MOE Key Laboratory for Geomechanics and Embankment Engineering, Hohai Univ., Nanjing, Jiangsu 210098, China. E-mail: zyd476300@126.com

Abstract: A numerical model, EC2, is developed to simulate the consolidation of a soil layer subjected to combined electroosmosis and surcharge preloading. EC2 includes the capabilities of a previous model, EC1, in accounting for large-strain-induced nonlinear changes of the physical and geoelectrical properties occurring to the layer, with the additions of two-dimensional consolidation, coupled loading and electric fields, time-dependent loading and current density, and an external hydraulic gradient. A highlight of the EC2 model is the superposition of multiple streams between small elements, which improves the prediction accuracy of the rate of flow and the degree of consolidation. The consolidation algorithm of EC2 is one-dimensional and conducted using finite difference and piecewise-linear methods. This paper presents the theoretical and numerical development of the EC2 model. A companion paper presents the validation of the model and the results of simulations that illustrate the optimization of the consolidating processes for some interesting design scenarios.

Keywords: electroosmosis; surcharge preloading; large strain; finite difference; nonlinear; consolidation; numerical models.

INTRODUCTION

Electroosmosis is a phenomenon that describes the movement of pore fluid in a soil regime in response to the transmission of direct current through the soil. The direct current is transmitted between a negatively charged electrode (cathode) and a positively charged electrode (anode), which are installed in pairs in the soil. The current transmission is associated with the migration of ions in the pore fluid. Positive ions (cations) move toward the cathode, and negative ions (anions) move toward the anode. As the ions migrate, they carry a portion of the pore fluid with them by viscous action (Esrig 1968). Most pore fluid contains a surplus of cations and thus ends up travelling toward the cathode. Given the boundaries designed for a soil layer of interest, such as an undrained anode and a drained cathode, the pore fluid that gathers around the cathode is drained up, which achieves consolidation of the soil layer.

Electroosmosis-induced consolidation is particularly favorable for practices where problematic fine-grained soils outcrop at the ground surface. Such practices include massive reclamations of seabed sediment; thick deposits of swamp muck; and large-scale disposals of industrial sludge, municipal sewage, or mine tailings. These fine-grained soils are of high moisture content, high compressibility, and low permeability, and therefore, they pose challenges for conventional ground improvement methods, such as surcharge preloading, wick drains, and vacuum preloading (Jeyakanthan et al. 2011). Alternatively, the use of electroosmosis is promising in that it is largely insensitive to soil permeability, which presents a striking contrast with fluid flow driven by a hydraulic gradient. It was reported

that the rates of flow driven by electroosmosis in fine-grained soils are orders of magnitude larger than those by hydraulic consolidating (Acar et al. 1994).

To enhance the efficiency of consolidation, electroosmosis has been conducted in conjunction with surcharge preloading with vertical drains (Vey 1949; Kondner and Boyer 1957; Nicholls and Herbst 1967; Wan and Mitchell 1976; Shang 1998; Bergado et al. 2000; Micic et al. 2001; Iwata and Jami 2010). In practice, the conjunction is usually configured in accordance with the schematic shown in Fig. 1. It involves two scenarios: an electrode grid installed into the soil layer of interest and a large-scale surcharge placed on the layer's surface. Specifically, rows of anodes and cathodes are installed vertically to a depth of D . Laterally, the rows align in an alternate fashion at an interval of B_1 . The electrodes are positioned relatively closer to the y -axis than the x -axis so that the voltage field acts ideally on the x -axis (Shang 1998). The cathodes are often prefabricated into the formation of a vertical drain, i.e., a perforated metal pipe enclosed with layers of geofabrics. The anodes are installed with direct exposure to adjacent soil masses. On the ground surface, the large-scale surcharge provides additional effort to enhance consolidation. Often, a cushion of sand (a drain blanket) is layered immediately above the ground surface and below the surcharge to establish a drained boundary.

The electrodes grid behaves differently from vertical drains deployed in surcharge preloading, albeit geometrically identical. There are major dissimilarities that lie in a) the gradients involved to move the pore fluid and b) the pore pressures generated in consolidating. In addition to the hydraulic gradient, a voltage gradient is present in the soil layer, which arises from the electric potential difference between the anodes and the cathodes. The voltage gradient raises additional stream—electroosmosis, which the vertical drains do not involve. Additionally, negative pore pressures likely build up (Esrig 1968) and are supposed to complicate the dissipation of pore pressures. To these ends, further research is needed to

explore and resolve consolidation problems related to electroosmosis–surcharge preloading practice.

PREVIOUS STUDIES

Previous studies were conducted mainly on single or combined uses of two consolidating processes: surcharge preloading and electroosmosis. For this study, the combined uses were reviewed, particularly in analytical and numerical studies. Experimental studies are reviewed in the companion paper (validation and simulation results).

Where surcharge loads and current are combined in soil consolidation practices, two load-current arrangements are involved: a parallel pattern and an orthogonal pattern. The parallel pattern involves the loads and the current acting in parallel, so there is 1D flow. The orthogonal pattern has the loading and the current transmitting normal to each other, as shown in Fig. 1(a), and apparently improves its feasibility in practice. Plus, in contrast to the parallel pattern, the orthogonal pattern results in 2D flow and helps accelerate the consolidation process.

The parallel pattern was examined in mathematical expressions for pore pressures (Vey 1949; Banerjee and Vitayasupakorn 1980), settlements (Kondner and Boyer 1957), and degree of consolidation (Iwata and Jami 2010). Regarding the orthogonal pattern, Nicholls and Herbst (1967) predicted pore pressures in the circumstance of radial flow (a hexagonal grid pattern). As a next step to radial flow, Wan and Mitchell (1976) examined parallel flow (a square grid pattern) and developed mathematical expressions for excess pore pressures. However, they sealed the upper and bottom surfaces of the soil layer of interest and confined fluid flows to one dimension. To improve this model, Shang (1998) extended Esrig's (1968) solutions for 2D consolidation and derived analytical solutions for excess pore pressures.

The above analytical studies were developed along the line of small strain. Large-strain settlements, however, have been acknowledged in consolidating fine-grained soils, particularly wet thick layers. Up to 20% settlements have occurred in experimental investigations (Casagrande 1949; Kondner and Boyer 1957; Nicholls and Herbst 1967; Mesri and Olson 1971; Shuang and Ho 1998; Win et al. 2001). Numerical studies (Feldkamp and Belhomme 1990; Fox and Berles 1997; Yuan and Hicks 2013; Zhou et al. 2013) have also predicted the development of large-strain consolidation. A large-strain compression of a soil layer is significant in changing the soil's physical (e.g., hydraulic conductivity and void ratio) and geoelectrical (e.g., electrical resistivity and electroosmotic conductivity) properties. The changes were confirmed in laboratory studies, such as Kondner and Boyer (1957) and Wan and Mitchell (1976). Additionally, both the stress conditions and the physical properties of the soil layer become nonhomogeneous because of directed fluid flows. Both the property changes and the material heterogeneity across the soil layer may be non-negligible, so a large-strain-based consolidation concept is expected. The large-strain concept is plausible in that it treats the soil properties and stress conditions as independent variants temporally and spatially, which offers, within engineering accuracy, predictions of the property heterogeneity. The heterogeneity is then integrated into a computer program to simulate consolidation.

The large-strain concept above has been successfully implemented in establishing consolidation models, such as the series of models CS2 (Fox and Berles 1997; Fox and Pu 2012), RCS1 (Fox et al. 2003), and CS3 (Fox et al. 2014), developed to simulate clay layer consolidation under variable scenarios. Adapted from the model CS2, EC1 (Zhou et al. 2013) was recently developed to simulate the 1D consolidation of clayey soils when subjected to electroosmosis. A similar adaption is conducted in this study to establish a model for the 2D consolidation of a soil layer subjected to the electroosmosis–surcharge

preloading method, which is called EC2 (Electroosmotic Consolidation 2), a next step to EC1. EC2 models large-strain consolidation by using a piecewise-linear approach. Fox and Berles (1997) and Fox et al. (2003) have discussed the advantage of the piecewise-linear approach and verified its high accuracy for simulations.

In lieu of evaluating the electrochemical effects, such as the types of anode metal, cation depletion, heating, chemical changes, and evolution of gas at electrodes, this study focuses on establishing and validating a numerical model with which soil consolidation can be simulated with sufficient accuracy. Specifically, the established EC2 model is able to account for large strain, the soil self-weight, the relative velocity of the fluid phase, and the nonlinear changes of the soil properties (compressibility, hydraulic and electroosmotic conductivity) associated with the combined effects of electroosmosis and surcharge preloading. In the companion paper of this study, the performance of the model EC2 is examined by comparing its numerical solutions with analytical solutions and experimental results. Three example problems involving large-strain settlement and nonlinear constitutive relationships are illustrated to show how soil consolidation progresses under various scenarios and to optimize the consolidation. The studies presented in the two papers aim to enhance the prediction accuracy of soil consolidation and improve practical designs, particularly the designs for consolidating deep soil deposits.

MODEL DESCRIPTION

Geometry

The geometry of the model was configured based on RCS1's geometry (Fox et al. 2003). The hatched area of Fig. 1(b) is used as a unit cell for modeling purposes. The cell is represented in Fig. 2. As mentioned above, the cell is presented continuously in an alternate fashion throughout the electrode grid, and therefore, it is justifiable to single out the cell as a

representative unit. The initial geometry (the profile view) of the cell, prior to the application of a voltage and a vertical stress increment (time $t < 0$), is shown in Fig. 2(a). A saturated homogeneous soil layer of initial height D , fixed interval B_1 between a cathode and an anode, and a unit width is treated as an idealized two-phase material in which the solid particles and the pore fluid are incompressible. Only vertical compression takes place. The mass conservation and continuity equations are valid throughout the consolidation process of the soil layer. Electric current is transmitted between the cathode and the anode. An initial overburden effective stress q_0 ($t < 0$) and a load increment Δq ($t \geq 0$) are placed on the ground surface. The loads are applied at a large scale so that the additional stresses are uniform across the soil layer. As discussed in RCS1 (Fox et al. 2003), the layer can consolidate under the condition of iso-stress (uniform applied stress) or iso-strain (uniform settlement).

In Fig. 2(a), a vertical Eulerian coordinate, z , is defined as positive upward (against gravity) from a fixed datum plane coincident with the bottom of the soil layer. The soil layer is sliced laterally into R_i and vertically into R_j elements, forming an $R_i \times R_j$ mesh. In the y -axis, no flow occurs (Shang 1998), and slicing is not required. It is common in finite difference method to configure elements in the same dimension to favor inputting the initial values; therefore, the mesh has a uniform element length $b = B_1/R_i$ and height $d = D/R_j$. The b values remain unchanged over time, as lateral displacements are negligible. The d values are variable and are determined using the geometric relations shown in the mesh.

An element ij is positioned in terms of two elevations, its central node z_{ij}^t and upper-right corner $z_{c,ij}^t$ at time t . Based on the geometric relations shown in Fig. 2, z_{ij}^t and $z_{c,ij}^t$ are calculated as

$$z_{ij}^t = \frac{z_{c,(i-1)j}^t + z_{c,(i-1)(j-1)}^t + z_{c,ij}^t + z_{c,i(j-1)}^t}{4} \quad (1)$$

$$z_{c,ij}^t = z_{c,(i-1)j}^t + z_{c,i(j-1)}^t - z_{c,(i-1)(j-1)}^t + \frac{A_{ij}^t - (z_{c,(i-1)j}^t - z_{c,(i-1)(j-1)}^t)b}{b/2} \quad (2)$$

where subscripts $i=1, 2, \dots, R_i$; $j=1, 2, \dots, R_j$; and A_{ij}^t is the cross-sectional area of element ij at time t . The values of the subscripts remain the same throughout unless new values are provided. The element height at time t , d_{ij}^t , is

$$d_{ij}^t = \frac{z_{c,(i-1)j}^t - z_{c,(i-1)(j-1)}^t + z_{c,ij}^t - z_{c,i(j-1)}^t}{2} \quad (3)$$

To solve the problem, Fox et al. (2003) assumed cross sections of the rightmost column elements R_j to remain rectangular throughout the process of consolidation, i.e. $z_{R_i j}^t = z_{(R_i-1)j}^t$. The assumption is tolerable, as the settlements of the soils neighboring the cathode are relatively low.

Constitutive Relationships

Model EC2 adopts constitutive relationships in models CS2 and RCS1. As shown in Fig. 3, the constitutive relationships include compressibility and permeability curves specified using discrete data points extracted from mathematical approximations or derivative functions of the soil properties. The compressibility curve [Fig. 3(a)] is defined by $R_m (\geq 2)$ pairs of the corresponding void ratio \hat{e} and the vertical effective stress $\hat{\sigma}'$. The permeability relationship [Fig. 3(b)] is defined by $R_n (\geq 2)$ pairs of the corresponding void ratio \bar{e} and the hydraulic conductivity \bar{k}_h (or electroosmotic conductivity \bar{k}_e). The values of \bar{k}_h and \bar{k}_e increase, in different mathematical forms, as the value of \bar{e} increases. Both the compressibility and permeability relationships develop monotonically, which agrees with findings in previous studies (Taylor 1948; Casagrande 1949; Carrier et al. 1983; Feldkamp and Belhomme 1990; Zhou et al. 2013). The monotonicity indicates that the model does not take into account unloading/reloading effects.

The compressibility curve [Fig. 3(a)] is commonly represented by a linear semi-logarithmic relationship between e and σ' . The constant of the linear relationship is known as the compression index C_c . The linear relationship is applicable to normally consolidated soils in a strict sense and overconsolidated soils as a first approximation of the compressibility relationship over the appropriate effective stress ranges (Fox 1999).

The hydraulic conductivity relationship [Fig. 3(b)] is represented in a linear logarithmic graph, as done by Jeyakanthan et al. (2011), or a linear semi-logarithmic form, as shown in Eq. (4). The linear semi-logarithmic relationship was initially suggested by Taylor (1948) as an empirical form applicable for clays and later validated by Vey (1949) and Mesri and Olson (1971) for the range of void ratio changes encountered in engineering practice. It has become one of the most common forms used to represent the constitutive relationship of hydraulic conductivity. The parameter C_k is the hydraulic conductivity index and determined experimentally.

$$\Delta e / \Delta \log k_h = C_k \quad (4)$$

Casagrande (1949) defined the electroosmotic conductivity k_e as the factor of proportionality between the voltage gradient i_e and the gradient induced rate of flow, where i_e is the rate of change of the electric potential with respect to distance in the direction of greatest change across a medium. Based on the classical Helmholtz and Smoluchowski theory summarized by Mitchell and Soga (2005), Zhou et al. (2013) developed and verified a simplified form for k_e as

$$k_e = k_{e0} \left(\frac{(1 + e_0)e}{e_0(1 + e)} \right)^a \quad (5)$$

where the initial void ratio e_0 , the initial electroosmotic conductivity k_{e0} , and the exponent a are determined experimentally.

Stress and Pore Pressure

Model EC2 follows the concepts in CS2 (Fox and Berles 1997) and RCS1 (Fox et al. 2003) to calculate the total stress, the effective stress and the pore pressure. The total vertical stress at node ij , σ_{ij}^t , is the sum of the overburden stress and the self-weight of the overlying elements.

$$\sigma_{ij}^t = \left(h_{w,iR_j}^t - \frac{z_{c,(i-1)R_j}^t + z_{c,iR_j}^t}{2} \right) \gamma_w + q_0 + \Delta q_i^t + \frac{A_{ij}^t \gamma_{ij}^t}{2b} + \sum_{l=j+1}^{R_j} \frac{A_{il}^t \gamma_{il}^t}{b} \quad (6)$$

where h_{w,iR_j}^t is the total head at the upper boundary; γ_w is a unit weight of water; and γ_{ij}^t is the saturated unit weight of element ij . The vertical effective stress, σ_{ij}^t , is computed from the void ratio e_{ij}^t using the piecewise-linear approximation of the compressibility curve [Fig. 3(a)] as

$$\sigma_{ij}^t = \hat{\sigma}_{m-1}^t + \frac{\hat{e}_{m-1} - e_{ij}^t}{a_{v,m-1}}, m=2,3, \dots, R_m \quad (7)$$

where the coefficient of compressibility $a_{v,m-1}$ is the slope (absolute value) of the linear segment of the compressibility curve between the points $(\hat{\sigma}_{m-1}^t, \hat{e}_{m-1})$ and $(\hat{\sigma}_m^t, \hat{e}_m)$. Given the total and effective stresses, the excess pore pressure at node ij , Δu_{ij}^t , is

$$\Delta u_{ij}^t = \sigma_{ij}^t - \sigma_{ij}^t - \left(h_{w,iR_j}^t - z_{ij}^t \right) \gamma_w \quad (8)$$

Electrical Resistivity and Electric Potential

Soil electrical resistivity is considered as a proxy of variability of the intrinsic compositions of the soil (Banton et al. 1997) and is determined as a function of the compositions. Samouelian et al. (2005) summarized the compositions as the nature of the solid particles (mineralogy and gradation), arrangement of the voids (porosity, pore size distribution and connectivity), degree of saturation, moisture content, electrical resistivity of the pore fluid (solute concentration), and temperature. These properties affect the value of the electrical

resistivity in different ways and to various extents. Agreeing with soil properties defined for EC2, the electrical resistivity is mainly dependent on the void ratio, electrical resistivity of solids ρ_s , and electrical resistivity of pore fluid ρ_w . Given this, a model developed by Komine (1997) is used to compute the electrical resistivity for element ij , ρ_{ij}^t , as

$$\rho_{ij}^t = \frac{1}{\frac{1}{\rho_s} \frac{1}{1+e_{ij}^t} + \frac{1}{\rho_w} \frac{e_{ij}^t}{1+e_{ij}^t}} \quad (9)$$

In terms of the Pouillet's law, the electrical resistance for element ij , R_{ij}^t , is

$$R_{ij}^t = \frac{\rho_{ij}^t b}{d_{ij}^t} \quad (10)$$

The value of R_{ij}^t is used to calculate electric potential within the soil layer. Zhou et al. (2013) suggested that the electric potential at node ij , V_{ij} , takes a value in relation to the effective voltage V_m^t —the end-to-end potential drop, and is expressed as

$$V_{ij}^t = V_m^t \left(1 - \frac{\sum_{l=1}^{i-1} R_{lj}^t + \frac{R_{ij}^t}{2}}{\sum_{l=1}^{R_1} R_{lj}^t} \right) \quad (11)$$

Fluid Flow and Settlement

The fluid flow is calculated based on the procedures in Fox et al. (2003). Element ij and its neighboring elements are extracted and re-plotted in Fig. 4. The elements become distorted because of directed fluid flow and uneven settlement. The distortion increases the inclination (by an angle of θ_{ij}) of the vertical hydraulic conductivity vector $k_{v,ij}$. The angle θ_{ij} is calculated as

$$\theta_{ij}^t = \tan^{-1} \left(\frac{z_{ij}^t - z_{i-1,j}^t}{b} \right) \quad (12)$$

The amended hydraulic conductivity vector $k_{z,ij}$, acting in the z -axis, is defined to compute vertical hydraulic flow rate. The values of $k_{z,ij}$ are calculated using the hydraulic conductivity ellipse (Harr 1962), as follows:

$$k_{z,ij}^t = \frac{k_{v,ij}^t}{\cos^2 \theta_{ij}^t + \frac{\sin^2 \theta_{ij}^t}{r_k}} \quad (13)$$

where the hydraulic conductivity ratio $r_k = k_x / k_v$. The values of r_k are assumed constant during consolidation (Fox et al. 2003). The equivalent hydraulic conductivity is defined to account for properties changing within the soil layer and to calculate the inter-element fluid flow. The equivalent vertical hydraulic conductivity $k_{zs,ij}^t$ between elements ij and $i(j+1)$ is expressed as

$$k_{zs,ij}^t = \frac{k_{z,ij}^t k_{z,i(j+1)}^t (d_{ij}^t + d_{i(j+1)}^t)}{k_{z,ij}^t d_{i(j+1)}^t + k_{z,i(j+1)}^t d_{ij}^t}, \quad i=1, 2, \dots, R_i, \text{ and } j=1, 2, \dots, R_j-1 \quad (14)$$

At the upper and lower boundaries, $k_{zs,iR_j}^t = k_{z,iR_j}^t$ and $k_{zs,i0}^t = k_{z,i1}^t$. The vertical hydraulic flow rate $q_{z,ij}^t$ from element ij to element $i(j+1)$ is

$$q_{z,ij}^t = k_{zs,ij}^t i_{z,ij}^t b, \quad i=1, 2, \dots, R_i, \text{ and } j=0, 1, \dots, R_j \quad (15)$$

where the vertical hydraulic gradient $i_{z,ij}^t$ is

$$i_{z,ij}^t = \frac{h_{i(j+1)}^t - h_{ij}^t}{z_{i(j+1)}^t - z_{ij}^t}, \quad i=1, 2, \dots, R_i, \text{ and } j=1, 2, \dots, R_j-1 \quad (16)$$

where h_{ij}^t is the total head for node ij . If the top boundary is drained,

$i_{z,iR_j}^t = 2(h_{iR_j}^t - h_{w,iR_j}^t) / d_{iR_j}^t \cos \theta_{iR_j}^t$; otherwise $i_{z,iR_j}^t = 0$. If the lower boundary is drained,

$i_{z,i0}^t = (h_{w,i1}^t - h_{i1}^t) / z_{i1}^t$, where $h_{w,i1}^t$ is the total head at the lower boundary; otherwise $i_{z,i0}^t = 0$.

Likewise, define the equivalent lateral hydraulic conductivity $k_{xs,ij}^t$ and equivalent the electroosmotic conductivity $k_{es,ij}^t$ between elements ij and $(i+1)j$ as

$$k_{xs,ij}^t = \frac{2k_{x,ij}^t k_{x,(i+1)j}^t}{k_{x,ij}^t + k_{x,(i+1)j}^t}, \quad i=1, 2, \dots, R_i-1, \text{ and } j=1, 2, \dots, R_j \quad (17)$$

$$k_{es,ij}^t = \frac{2k_{e,ij}^t k_{e,(i+1)j}^t}{k_{e,ij}^t + k_{e,(i+1)j}^t}, \quad i=1, 2, \dots, R_i-1, \text{ and } j=1, 2, \dots, R_j \quad (18)$$

At the respective boundaries, $k_{xs,0j}^t = k_{x,1j}^t$, $k_{xs,R_i j}^t = k_{x,R_i j}^t$, $k_{es,0j}^t = k_{e,1j}^t$, and $k_{es,R_i j}^t = k_{e,R_i j}^t$. The

horizontal hydraulic flow rate $q_{x,ij}^t$ and the horizontal electroosmotic flow rate $q_{e,ij}^t$ are

calculated as

$$q_{x,ij}^t = k_{xs,ij}^t i_{x,ij}^t (z_{c,ij}^t - z_{c,i(j-1)}^t) \cos \theta_{ij}^t, \quad i=0, 1, \dots, R_i, \text{ and } j=1, 2, \dots, R_j \quad (19)$$

$$q_{e,ij}^t = k_{es,ij}^t i_{e,ij}^t (z_{c,ij}^t - z_{c,i(j-1)}^t) \cos \theta_{ij}^t, \quad i=0, 1, \dots, R_i, \text{ and } j=1, 2, \dots, R_j \quad (20)$$

where the horizontal hydraulic gradient $i_{x,ij}^t$ and the voltage gradient $i_{e,ij}^t$ are equal to

$$i_{x,ij}^t = \frac{h_{ij}^t - h_{(i+1)j}^t}{\sqrt{b^2 + (z_{i+1,j}^t - z_{ij}^t)^2}}, \quad i=1, 2, \dots, R_i-1, \text{ and } j=1, 2, \dots, R_j \quad (21)$$

$$i_{e,ij}^t = \frac{V_{ij}^t - V_{i+1,j}^t}{\sqrt{b^2 + (z_{i+1,j}^t - z_{ij}^t)^2}}, \quad i=1, 2, \dots, R_i-1, \text{ and } j=1, 2, \dots, R_j \quad (22)$$

The value of $i_{x,ij}^t$ is negative (Esrig 1968), so $q_{x,ij}^t$ flows opposite to $q_{e,ij}^t$. If the cathode is

drained, $i_{e,R_i j}^t = 2V_{R_i j}^t / b$ and $i_{x,R_i j}^t = 2(h_{R_i j}^t - h_{w,R_i j}^t) / b$, where $h_{w,R_i j}^t$ is the total head at the

cathode; otherwise, $i_{e,R_i j}^t = 0$ and $i_{x,R_i j}^t = 0$. If the anode is drained, $i_{e,0j}^t = 2(V_m^t - V_{1j}^t) / b$ and

$i_{x,0j}^t = 2(h_{w,1j}^t - h_{R_i j}^t) / b$, where $h_{w,1j}^t$ is the total head at the anode; otherwise, $i_{e,0j}^t = 0$ and

$i_{x,0j}^t = 0$. The conditions hold when some phenomena are neglected, such as the well

resistance, smear zone, and electrochemical effects. Neglecting these effects is acceptable where disturbances to a vertical drain and its neighboring soils are marginal and electrochemical reactions such as redox at the electrodes are minor.

The superposition of $q_{z,ij}^t$, $q_{x,ij}^t$ and $q_{e,ij}^t$ leads to pore fluid escape and changes of the elements' cross-sectional area. The new area $A_{ij}^{t+\Delta t}$ at time $(t+\Delta t)$ for element ij , relative to its area A_{ij}^t at time t , is

$$A_{ij}^{t+\Delta t} = A_{ij}^t - \left(q_{z,ij}^t - q_{z,i(j-1)}^t + q_{x,ij}^t - q_{x,(i-1)j}^t + q_{e,ij}^t - q_{e,(i-1)j}^t \right) \Delta t \quad (23)$$

The new void ratio $e_{ij}^{t+\Delta t}$, relative to the initial void ratio $e_{0,ij}$ and the initial cross-sectional area A_{ij} , becomes

$$e_{ij}^{t+\Delta t} = \frac{A_{ij}^{t+\Delta t} (1 + e_{0,ij})}{A_{ij}} - 1 \quad (24)$$

which, in conjunction with Eq. (23), provides an explicit solution for the void ratio.

Three settlements are calculated, the accumulative settlement of column i , S_i ; the average settlement of the soil layer, S_{avg} ; and the average degree of consolidation of the soil layer, U_{avg} . At time t , these settlements are calculated as

$$S_i^t = D - z_{c,iRj}^t \quad (25)$$

$$S_{\text{avg}}^t = \frac{\sum_{i=1}^{R_i} S_i^t}{R_i} \quad (26)$$

$$U_{\text{avg}}^t = \frac{S_{\text{avg}}^t}{\bar{S}} \quad (27)$$

where \bar{S} is the final settlement of the soil layer when all streams of the fluid flow reach equilibrium or consolidation is completed. In the EC2 model, the equilibrium arrives when

two consecutive average settlements have a sufficiently small difference, i.e., on the order of magnitude $\times 10^{-3}$ m. The value of \bar{S} is also achievable through 1D settlement theory.

$$\bar{S} = \frac{a_v}{1 + e_0} \Delta \sigma' D \quad (28)$$

where the change of the effective stress $\Delta \sigma'$ is equal to the sum of the load increments Δq at $t=0$ and the absolute value of the average pore pressure \bar{u}^f at equilibrium. Esrig (1968) provided a solution for \bar{u}^f as

$$\bar{u}^f = -\frac{k_e \gamma_w}{2k_h} V_m \quad (29)$$

which helps rewrite Eq. (28) as

$$\bar{S} = \frac{a_v}{1 + e_0} \left(\Delta q + \frac{k_e \gamma_w}{2k_h} V_m \right) D \quad (30)$$

The numeric procedure for \bar{S} is suitable for both large- and small-strain consolidation problems, as no assumption is made on soil properties. The analytical procedure is based on small-strain assumptions made for the 1D settlement theory and is limited to problems with small strain. In the companion paper, the analytical procedure is used to find \bar{S} in verification checks conducted for example problems with small strain, whereas the numeric procedure is used for simulations with large strain.

Time Increment

Model EC2 adapts the criteria used in models CS2 (Fox and Berles 1997) and EC1 (Zhou et al. 2013) to determine the time increment Δt . The adaption is valid in that EC2 is developed in a form analogous to CS2 and EC1, including the definition of the fixed coordinate, element meshing, flow mass continuity, constitutive relations, and iterative algorithm. The value of Δt is calculated as the minimum of three criteria:

$$\Delta t = \min \left\{ \frac{\alpha \gamma_w a_{v,ij}^t (d_{ij}^t)^2}{k_{z,ij}^t (1 + e_{ij}^t)}, \frac{\alpha \gamma_w a_{v,ij}^t b^2}{k_{x,ij}^t (1 + e_{ij}^t)}, \left| \frac{0.001 A_{ij} (e_{0,ij} - e_{ij}^f)}{(1 + e_{0,ij}) (q_{z,ij}^t - q_{z,i(j-1)}^t + q_{x,ij}^t - q_{x,(i-1)j}^t + q_{e,ij}^t - q_{e,(i-1)j}^t)} \right| \right\} \quad (31)$$

where α is a constant equal to or less than 0.5, and Δt is most accurate for $\alpha \cong 0.4$; e_{ij}^f is the final void ratio. The value of e_{ij}^f is predetermined from the compressibility curve (Fig. 3a) in terms of $\Delta \sigma'$ and then updated based on the subsequent iterative algorithm until it converges. The first and second criteria in Eq. (31) are defined to govern the frequency of the calculation in response to vertical and lateral flows, respectively, which is a step to align with the solutions for Terzaghi's 1D consolidation theory (Al-Khafaji and Tooley 1986). The third criterion is defined to limit element settlement to 0.1% of the final settlement. The percent is set tenfold lower than 1% for EC1, which is to cope with the additional settlement due to surcharge loading and to maintain computation accuracy. For a soil layer, $D=5$ m, $B_1=1$ m, $V_m^t=30$ volts, and $R_i=R_j=51$. The third criterion gives the lowest Δt of several seconds in an early stage (i.e., <20 d), and afterwards the first two give a lower Δt of several hundred seconds.

EC2 COMPUTER PROGRAM

Figure 5 shows a flow chart illustrating the basic algorithm for the computer program of EC2. EC2 extends the algorithm used in previous models CS2 and EC1 by upgrading iterations from one to two dimensions. The input properties include the initial dimension of the soil layer ($B_1 \times D$), the element numbers ($R_i \times R_j$), the effective voltage (V_m), the specific gravity of the solids (G_s), the electrical resistivity of the pore fluid (ρ_w) and the solids (ρ_s), the constitutive relationships, the boundary conditions, and the termination criteria for the

program. After EC2 reads the initial input values for element ij (b , d_{ij} , z_{ij} , $z_{c,ij}$, V_{ij} , $e_{0,ij}$, e_{ij}^f), the calculation loop begins at a time step. Following each time step increment, the pore pressure (u), effective stress (σ'), void ratio (e), electrical resistivity (ρ), hydraulic conductivity (k_x , k_z), and electroosmotic conductivity (k_e) are calculated for each element in terms of the specified constitutive relationships. The calculations involve the flow rates, the new heights of each element, the average settlements of the soil layer, and the average degree of consolidation. The calculations terminate when $t \geq t_f$ or $\Delta S_{avg} \leq m$, where t_f and m are a user-specified elapsed time and a sufficiently small settlement value, respectively. A further confirmation check is executed between the output value of void ratio (e_{ij}^{fl}) and its input value (e_{ij}^f) for each element. If disagreement exists, e_{ij}^f reads e_{ij}^{fl} , and the loop is executed again. The program ends when the two void ratios converge. Then, the average settlement (\bar{S}) and the average degree of consolidation (U_{avg}^t) are calculated.

CONCLUSIONS

EC2 is a numerical model for the consolidation of a saturated soil layer subjected to the electroosmosis–surcharge preloading method. EC2 involves an algorithm of 2D consolidation and accounts for electroosmosis, hydraulic permeation, the parallel electric field, the soil self-weight, and general constitutive relationships. The algorithm also accounts for nonlinear changes of the properties (including electrical resistivity, electric potential, electroosmotic conductivity, hydraulic conductivity, and compressibility), the time-dependent loading and electric density, and an external hydraulic gradient acting at the boundaries of the soil layer.

EC2 provides the following quantities as a function of time: a) rate of flow at the boundaries and b) the degree of consolidation of the soil layer. EC2 provides the following

quantities as a function of time and position within the soil layer: a) settlement, b) the void ratio, c) the pore pressure, d) the vertical effective stress, e) the moisture content, and f) the electric potential and current density. The companion paper presents the validation of EC2 and the results of simulations that illustrate the optimization of electroosmosis–surcharge preloading consolidation for some interesting design scenarios.

ACKNOWLEDGEMENTS

The writers commenced the study in Hohai University, and continued and completed it after the first writer moved to the University of Adelaide. The methodology and computer code in this study were developed under the enlightenment gained from Fox and Berles (1997) and Fox et al. (2003) which are much appreciated. The writers are also thankful to the anonymous reviewers for their comments to improve the paper.

NOTATION

The following symbols are used in this paper:

A_{ij} = vertical cross-sectional area of elements ij ;

a_v = coefficient of compressibility;

B_1 = interval between opposite polarities;

B_2 = interval between identical polarities;

b = element length in x -axis;

C_c = compression index;

C_k = hydraulic conductivity index;

D = initial height of soil layer;

d = initial height of element;

e = void ratio;

e_0 = initial void ratio;

e^f = final void ratio (input);

e^{fl} = final void ratio (output);

G_s = specific gravity of solids;

h_{ij} = total head for element ij ;

$h_{w,i0}$ = total head at lower boundary;

h_{w,iR_j} = total head at upper boundary;

$h_{w,0j}$ = total head at anode;

$h_{w,R_i,j}^t$ = total head at cathode;

i = horizontal element coordinate;

i_e = voltage gradient;

$i_{e,ij}$ = voltage gradient between elements ij and $(i+1)j$;

i_h = hydraulic gradient;

$i_{x,ij}$ = hydraulic gradient between elements ij and $(i+1)j$;

$i_{z,ij}$ = hydraulic gradient between elements ij and $i(j+1)$;

j = vertical element coordinate;

k = hydraulic or electroosmotic conductivity;

k_e = electroosmotic conductivity;

k_{e0} = initial electroosmotic conductivity;

$k_{es,ij}$ = equivalent electroosmotic conductivity between elements ij and $(i+1)j$;

k_h = hydraulic conductivity;

k_s = equivalent hydraulic or electroosmotic conductivity;

k_v = vertical hydraulic conductivity;

k_x = horizontal hydraulic conductivity;

$k_{xs,ij}$ = equivalent hydraulic conductivity between elements ij and $(i+1)j$;

k_z = amended vertical hydraulic conductivity;

$k_{zs,ij}$ = equivalent hydraulic conductivity between elements ij and $i(j+1)$;

m = small number of settlement difference;

q = rate of flow;

q_0 = initial overburden effective stress at upper boundary;

$q_{z,ij}$ = rate of hydraulic flow between elements ij and $i(j+1)$;

$q_{x,ij}$ = rate of hydraulic flow between elements ij and $(i+1)j$;

$q_{e,ij}$ = rate of electroosmotic flow between elements ij and $(i+1)j$;

R = electrical resistance;

R_i = number of elements in a row;

R_j = number of elements in a column;

R_m = number of data points for compressibility curve;

R_n = number of data points for permeability curves;

r_k = hydraulic conductivity ratio;

S_{avg} = average settlement of soil layer;

S_i = settlement of column i ;

\bar{S} = final settlement of soil layer;

t = time;

U_{avg} = average degree of consolidation;

u = pore pressure;

u^f = final pore pressure;

V_{ij} = electric potential at element ij ;

V_m = effective voltage;

x = horizontal coordinate;

y = coordinate orthogonal to x and z ;

z = vertical coordinate;

$z_{c,ij}$ = corner elevation of element ij ;

z_{ij} = node elevation of element ij ;

α = constant for time increment;

γ = saturated unit weight of soil;

γ_w = unit weight of water;

θ = angle of inclination of element;

ρ = electrical resistivity of soil;

ρ_s = electrical resistivity of solids;

ρ_w = electrical resistivity of pore fluid;

σ = vertical total stress;

σ' = vertical effective stress;

Δe = change of void ratio;

Δq = load increment at upper boundary;

Δt = time step increment;

Δu = excess pore pressure; and

$\Delta \sigma'$ = change of vertical effective stress.

Superscripts

a = exponent for electroosmotic conductivity;

t = time;

\wedge = data points for compressibility curve; and

$\bar{\quad}$ = data points for permeability curves.

Subscripts

i = i th element in x -axis;

j = j th element in z -axis.

m = m th data point for compressibility curve; and

n = n th data point for permeability curve.

REFERENCES

- Acar, Y. B., Hamed, J. T., Alshawabkeh, A. N., and Gale, R. J. (1994). "Removal of cadmium (II) from saturated kaolinite by the application of electrical current." *Geotechnique*, 44(2), 239–254.
- Al-Khafaji, A. W., and Tooley, J. R. (1986). *Numerical methods in engineering practice*, Holt, Rinehart and Winston, Inc., New York.
- Banerjee, S., and Vitayasupakorn, V. (1980). "Appraisal of electro-osmotic oedometer tests." *J. Geotech. Engrg.*, 10.1061/(ASCE)0733-9410(1984)110:8(1007), 1007–1023.
- Banton, O., Seguin, M. K., and Cimon, M. A. (1997). "Mapping field-scale physical properties of soil with electrical resistivity." *Soil Sci. Soc. Am. J.*, 61(4), 1010–1017.
- Bergado, D. T., Balasubramaniam, A. S., Patawaran, M. A. B., and Kwunpreuk, W. (2000). "Electro-osmotic consolidation of soft Bangkok clay with prefabricated vertical drains." *Ground Improv.*, 4, 153–163.
- Carrier, W. D., Bromwell, L. G., and Somogyi, F. (1983). "Design capacity of slurried mineral waste ponds." *J. Geotech. Engrg.*, 10.1061/(ASCE)0733-9410(1983)109:5(699), 699–716.
- Casagrande, L. (1949). "Electro-osmosis in soils." *Geotechnique*, 1(3), 159–177.
- Esrig, M. I. (1968). "Pore pressure, consolidation and electrokinetics." *J. Soil Mech. Found. Div.*, 94(SM4), 899–922.
- Feldkamp, J. R., and Belhomme, G. M. (1990). "Large-strain electrokinetic consolidation: Theory and experiment in one dimension." *Geotechnique*, 40(4), 557–568.
- Fox, P., and Pu, H. (2012). "Enhanced CS2 model for large strain consolidation." *Int. J. Geomech.*, 10.1061/(ASCE)GM.1943-5622.0000171, 574–583.
- Fox, P., Pu, H., and Berles, J. (2014). "CS3: Large strain consolidation model for layered soils." *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)GT.1943-5606.0001128, 04014041.

- Fox, P. J. (1999). "Solution charts for finite strain consolidation of normally consolidated clays." *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)1090-0241(1999)125:10(847), 847–867.
- Fox, P. J., and Berles, J. D. (1997). "CS2: A piecewise-linear model for large strain consolidation." *Int. J. Numer. Analyt. Meth. Geomech.*, 21(7), 453–475.
- Fox, P. J., Di Nicola, M., and Quigley, D. W. (2003). "Piecewise-linear model for large strain radial consolidation." *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)1090-0241(2003)129:10(940), 940–950.
- Harr, M. E. (1962). *Groundwater and seepage*, McGraw–Hill, New York.
- Iwata, M., and Jami, M. S. (2010). "Analysis of combined electroosmotic dewatering and mechanical expression operation for enhancement of dewatering." *Dry. Technol.*, 28(7), 881–889.
- Jeyakanthan, V., Gnanendran, C. T., and Lo, S. C. R. (2011). "Laboratory assessment of electro-osmotic stabilization of soft clay." *Can. Geotech. J.*, 48(12), 1788–1802.
- Komine, H. (1997). "Estimation of chemical grouted soil by electrical resistivity." *Ground Improv.*, 1(2), 101–113.
- Kondner, R. L., and Boyer, W. C. (1957). "Research on the use of electro-osmosis in the stabilization of fine grained soil." *Proc. Highway Research Board*, 36, 783–793.
- Mesri, G., and Olson, R. E. (1971). "Mechanisms controlling permeability of clays." *Clay Clay Miner.*, 19(3), 151–158.
- Micic, S., Shang, J. Q., Lo, K. Y., Lee, Y. N., and Lee, S. W. (2001). "Electrokinetic strengthening of a marine sediment using intermittent current." *Can. Geotech. J.*, 38(2), 287–302.
- Mitchell, J. K., and Soga, K. (2005). *Fundamentals of soil behavior*, John Wiley & Sons, Inc.

- Nicholls, R. L., and Herbst, R. L. (1967). "Consolidation under electrical-pressure gradients." *J. Soil Mech. Found. Div.*, 93(SM5), 139–151.
- Samouelian, A., Cousin, I., Tabbagh, A., Bruand, A., and Richard, G. (2005). "Electrical resistivity survey in soil science: A review." *Soil. Till. Res.*, 83(2), 173–193.
- Shang, J. Q. (1998). "Electroosmosis-enhanced preloading consolidation via vertical drains." *Can. Geotech. J.*, 35(3), 491–499.
- Shuang, J. Q., and Ho, K. S. (1998). "Electro-osmotic consolidation behaviour of two Ontario clays." *Can. Geotech. J.*, 29(2), 181–194.
- Taylor, D. W. (1948). *Fundamentals of soil mechanics*, John Wiley & Sons, Inc, New York.
- Vey, E. (1949). "The mechanics of soil consolidation by electro-osmosis." *Proc. Highway Research Board*, 29, 578–589.
- Wan, T. Y., and Mitchell, J. K. (1976). "Electro-osmotic consolidation of soils." *J. Geotech. Eng. Div.*, 102(5), 473–491.
- Win, B. M., Choa, V., and Zeng, X. Q. (2001). "Laboratory investigation on electro-osmosis properties of Singapore marine clay." *Soils Found.*, 41(5), 15–23.
- Yuan, J., and Hicks, M. A. (2013). "Large deformation elastic electro-osmosis consolidation of clays." *Comput. Geotech.*, 54, 60–68.
- Zhou, Y. D., Deng, A., and Wang, C. (2013). "Finite-difference model for one-dimensional electro-osmotic consolidation." *Comput. Geotech.*, 54, 152–165.

Figure Captions

Fig. 1. Schematic for the electroosmosis–surcharge preloading method: (a) profile view; (b) plan view

Fig. 2. Geometry for EC2: (a) initial configuration ($t < 0$); (b) after application of the voltage gradient and/or surcharge preloading ($t \geq 0$)

Fig. 3. Soil constitutive relationships: (a) compressibility; (b) permeability [adapted from Fox and Berles (1997)]

Fig. 4. Fluid flows between elements (adapted from Fox et al. 2003, © ASCE)

Fig. 5. Flow chart for the EC2 model (adapted from Fox and Berles 1997 and Zhou et al. 2013)