An Deng and Yadong Zhou

Modeling electroosmosis and surcharge preloading consolidation. II: Validation and simulation results

Journal of Geotechnical and Geoenvironmental Engineering, 2016; 142(4):04015094-1-04015094-11

GT4201

Modeling Electroosmosis and Surcharge Preloading Consolidation.II: Validation and Simulation Results

An Deng[1] and Yadong Zhou[2]

1. Lecturer, School of Civil, Environmental and Mining Engineering, Univ. of Adelaide, Adelaide, SA 5005, Australia (corresponding author). E-mail: an.deng@adelaide.edu.au
2. Lecturer, School of Civil Engineering, Tianjin Chengjian Univ., Tianjin, 300384, China; Formerly, Graduate Student, Geotechnical Research Institute, MOE Key Laboratory for Geomechanics and Embankment Engineering, Hohai Univ., Nanjing, Jiangsu 210098, China. E-mail: zyd476300@126.com

Abstract: The results of the numerical simulations for electroosmosis–surcharge preloading consolidation, obtained using the EC2 model, are presented. EC2 accounts for hydraulic and electroosmotic flows under the conditions of changing physical and geoelectrical properties of saturated compressible porous media. Verification checks for EC2 show excellent agreement of the pore pressure and degree of consolidation with analytical solutions for one- and two-dimensional small-strain consolidation. Similarly, excellent agreement is attained for two-dimensional large-strain consolidation occurring to a kaolinite layer subjected to an experimental test. The EC2 model is then used to investigate consolidation optimization for soil layers in three example problems. The electroosmosis–surcharge preloading combined method outperformed single process methods, particularly where hydraulic conductivity is on the order of magnitude 10^{-8} m/s or lower. Applying voltage in steps optimizes electric power usage, as does ceasing power supply when the degree of consolidation reaches approximately 80%.

Keywords: electroosmosis; surcharge preloading; large strain; nonlinear; consolidation; validation; simulation.

INTRODUCTION

This is the second of a companion pair of papers that describe a new approach for the simulation of large-strain consolidation of a soil layer subject to the electroosmosis–surcharge preloading method. The approach is coded in the numerical model EC2 (Electroosmotic Consolidation 2). EC2 involves an algorithm of two-dimensional (2D) consolidation and solves the vertical settlement with the associated material inhomogeneity and geometric nonlinearities. EC2 accounts for the superposition of the hydraulic and electroosmotic flows between the elements and eliminates calculation inaccuracy due to assumptions made for the entire layer. EC2 is flexible with respect to the geometry of the layer, the layout of the electrode grid, the time-dependent loading and power transmission, the choices of boundary conditions, and the constitutive relationships of the compressibility and permeability. The first paper presents the model's development. This paper presents model verification checks and the results of the simulations, which illustrate the optimization of electroosmosis–surcharge preloading consolidation for some interesting design scenarios.

MODEL VALIDATION

The capacity of the model EC2 is validated against analytical and experimental results. The analytical validation examines the model's accuracy for small-strain consolidation, and the experimental validation examines the model's accuracy for large-strain consolidation. The verification checks conducted under both small- and large-strain environments help confirm the model's suitability for both thin and thick soil layers.

Analytical Validation (Small Strain)

Two analytical solutions, Wan and Mitchell (1976) and Shang (1998), were revisited, as both apply to the load-current orthogonal pattern of EC2 (Deng and Zhou 2015). The two solutions assumed small-strain consolidation and provide mathematical forms for the pore pressure and degree of consolidation. The two solutions differ in boundary conditions. Wan and Mitchell (1976) sealed the top and bottom surfaces of a soil layer of interest, confined the fluid flows to the horizontal direction, and resolved the one-dimensional (1D) pore pressures. Shang (1998) considered both the vertical and horizontal flows and derived solutions for the 2D pore pressures. Verification checks against the two pore pressures help confirm EC2's capacity in variable boundary environments.

For the soil layer $B_1 \times D$ shown in the companion paper, Wan and Mitchell (1976) provided mathematical forms for the pore pressure u_x^t at position x and time t and the degree of consolidation U_{avg}^t at time t as

$$u_x^t = -\frac{k_\text{e}\gamma_\text{w} x}{k_\text{h} B_1} V_\text{m} + \sum_{l=1}^{\infty} \frac{2}{\left(l-\frac{1}{2}\right)\pi} \left[\Delta q + \frac{(-1)^{l-1}}{\left(l-\frac{1}{2}\right)\pi} \frac{k_\text{e}\gamma_\text{w}}{k_\text{h}} V_\text{m} \right] \sin \frac{\left(l-\frac{1}{2}\right)\pi x}{B_1} \text{e}^{-\left(l-\frac{1}{2}\right)^2 \pi^2 T_\text{v}} \tag{1}$$

$$U_{\text{avg}}^t = 1 - \frac{\sum_{l=1}^{\infty} \frac{2}{\left(l-\frac{1}{2}\right)^2 \pi^2} \left[\Delta q + \frac{(-1)^{l-1}}{\left(l-\frac{1}{2}\right)\pi} \frac{k_\text{e}\gamma_\text{w}}{k_\text{h}} V_\text{m} \right] \text{e}^{-\left(l-\frac{1}{2}\right)^2 \pi^2 T_\text{v}}}{\Delta q + \frac{k_\text{e}\gamma_\text{w}}{2k_\text{h}} V_\text{m}} \tag{2}$$

where x is the distance to the cathode; k_e and k_h are the electroosmotic and hydraulic conductivities, respectively; γ_w is a unit weight of water; B_1 is the interval between the opposite polarity; V_m is the effective voltage; Δq is the vertical load increment; and T_v is the time factor and is equal to $c_\text{v} t / B_1^2$, in which c_v is the coefficient of consolidation. Equations (1) and (2) were derived based on the assumption that the values of c_v, k_h and k_e are constant—a small-strain consolidation. For the same layer, Shang (1998) improved the

mathematical expressions as

$$u(x,z,t)=\xi(x,z,t)-\frac{k_{\mathrm{e}}V_{\mathrm{m}}\gamma_{\mathrm{w}}}{k_{\mathrm{h}}B_{1}}x \tag{3}$$

$$U_{\mathrm{avg}}^{t}=1-\frac{\int_{0}^{D}\int_{0}^{B_1}\xi dxdz}{\int_{0}^{D}\int_{0}^{B_1}\xi_{0}dxdz} \tag{4}$$

where $\xi(x,z,t)$ is a dummy variable and equal to

$$\xi(x,z,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}E_{mn}\left\{\sin\frac{(2m+1)\pi x}{2B_{1}}\exp\left[-T_{\mathrm{x}}\frac{(2m+1)^{2}\pi^{2}}{4}\right]\right.$$
$$\left.\times\sin\frac{(2n+1)\pi z}{2D}\exp\left[-T_{\mathrm{z}}\frac{(2n+1)^{2}\pi^{2}}{4}\right]\right\} \tag{5}$$

where $T_{\mathrm{x}}=c_{\mathrm{x}}t/B_{1}^{2}$ and $T_{\mathrm{z}}=c_{\mathrm{z}}t/D^{2}$ are time factors in the horizontal and vertical dimensions, respectively, and E_{mn} is equal to

$$E_{mn}=\frac{16\Delta q}{\pi^{2}}\frac{1}{2m+1}\frac{1}{2n+1}+\frac{32k_{\mathrm{e}}i_{\mathrm{e}}\gamma_{\mathrm{w}}B_{1}}{\pi^{3}k_{\mathrm{h}}}\frac{1}{(2m+1)^{2}}\frac{1}{2n+1}(-1)^{n} \tag{6}$$

where i_{e} is the voltage gradient.

An example problem was designed to examine the capacity of model EC2 against the two analytical solutions. As shown in Table 1, the problem involves a soil layer (B_1=1 m, W=1 m, and D=1 m) subjected to a vertical load increment $\Delta q=1\times10^{-4}$ kPa and an electric voltage $V_{\mathrm{m}}=1\times10^{-5}$ volt. The values were set sufficiently low so that an infinitesimal strain of 0.000025% occurs. The infinitesimal strain was small enough such that the relevant physical and geoelectrical properties did not change to a notable extent. To be in line with the assumptions for the solutions, the input values are 1 for r_{k} (isotropic permeability) and 1 for G_{s} (no self-weight). All other properties in Table 1 fall into the range of values for clays.

The soil layer was modeled using a 51×51 mesh. Zhou et al. (2013) found that an element number of 50 or above gives accurate enough results for the same layer.

Figure 1(a) shows the pore pressure isochrones across the soil layer. Similar to the coordinates used in Wan and Mitchell (1976) and Shang (1998), the abscissa adopts a normalized dimensionless distance of x (to the cathodes) over the electrodes interval B_1, and the ordinate represents the pore pressure normalized against the maximum pore pressure u_{max}. The value of u_{max} is equal to the load Δq at t=0 or the suction pressure $-\left(k_e V_m \gamma_w\right)/k_h$ at the end of consolidation. Table 1 shows that both pressures have an equal absolute value of 1×10^{-4} kPa so that the normalized pore pressures lie in the range of 1.0 to −1.0. Three groups of isochrones are shown corresponding to three times: 10, 31 and 85 d. Each group has four curves of the analytical and model results. The isochrones for Shang (1998) and the corresponding EC2 model represent pore pressures at the mid-height of the soil layer.

In Fig. 1(a), model EC2's results show excellent agreement with the curves from Wan and Mitchell (1976). Similarly, there is close agreement between the results of EC2 and Shang (1998), albeit small gaps occur. Shang's results lead to relatively accelerated consolidation compared with those of EC2, particularly at the mid-stage of consolidation (t=31 d). The acceleration of consolidation arises from the difference of the pore pressure input values at the upper boundary, $u(x,0,t)$ [Eq. (3)]. Shang (1998) presumed a value of zero for the dummy variable $\xi(x,0,t)$ and a suction force of $-(k_e V_m \gamma_w x)/(k_h B_l)$ for $u(x,0,t)$, whereas the EC2 model used $u(x,0,t)$=0 at the upper boundary. The suction force is an extra requirement to accelerate soil consolidation. In Fig. 1(b), excellent agreement is shown with respect to the results for the degree of consolidation. The agreement confirms EC2's capacity in simulating consolidation with respect to 1D and 2D consolidation where small-strain settlements occur.

Experimental Validation (Large Strain)

Experimental validation is a further step to verify EC2's capacity and simulation accuracy under large-strain consolidation. Past experimental tests were reviewed to develop a validation program and to help shed light on the choices of the process deployed to enhance consolidation.

Previous Experimental Setups

Based on the summary by Lo et al. (1991), Table 2 shows an updated review of the experimental setups used for the prototype testing of electroosmotic consolidation. The setups include the electroosmotic cells developed, the electrodes used, directions of the electric field, ranges of the electric gradient, soils tested, elapsed time of tests, and additional processes used to improve consolidation.

The electroosmotic cells include cylindrical cells and rectangular boxes. The cylindrical cells are modified from triaxial or oedometer cells or cut from fabricated tubes. A loading cap or a plunger is usually used to 'clamp' together electrode plates and a cylindrical soil sample and offers close electrode-soil contacts. The load-current parallel pattern, however, as discussed in the companion paper, results in only 1D consolidation and is not as feasible as an orthogonal pattern (mimicked by a box) in practice.

The electric gradient falls in the range of 10 to 100 volts/m and as high as 1,200 volt/m in Casagrande (1949), who attempted a direct current of up to 225 volts to a 0.18 m long soil specimen. Additional processes used to improve consolidation include surcharge preloading, injection of saline solutions, polarity reversal, and current intermittence. The surcharge loads range from 20 to 383 kPa and sometimes are applied in steps, such as in Morris et al. (1985). Similar stepped loading is applied to power transmission, such as in Bo et al. (2001). These designs are taken in account when sketching the experimental program of this study and creating example problems.

Experimental Setup

The experimental setup used a rectangular box (Fig. 2). It consists of an electroosmotic cell (a), a loading cantilever (b), and peripheral devices (c–p). The electroosmotic cell was fabricated from 10 mm thick transparent polycarbonate sheets. The sheets are electrically nonconductive to avoid short-circuiting boundaries. The good-sized cell box has an internal space of 480 mm (L)×150 mm (W)×250 mm (D). Acting as anodes (c), a row of six stainless steel round rods [ϕ12 mm×250 mm (L)] of equal intervals was installed vertically 15 mm to the left-hand end of the box. At the other end, cathodes (d) composed of a row of six stainless steel tubes [ϕ15 mm outside diameter×1 mm wall×250 mm (L)] were installed. The wall was perforated through with ϕ3 mm openings that are arranged in triangular patterns at full length. The tubes were wrapped with two layers of permeable cotton fabric (e) to prevent the passage of fine or colloid solid particles. The anode rods and the cathode tubes stood on the bottom of the box. The bottom next to the cathode was perforated with a row of ϕ5 mm openings (f) whereby the influx into the cathode tubes could escape. Both the anodes and the cathodes use steel materials because of the benefits of low cost, convenience to replace, and stable performance (Mohamedelhassan and Shang 2001).

The loading cantilever is similar to those in oedometers or direct shear test apparatuses. It involves a static weight (g), a static weight hanger (h), an amplification leverage, and a steel loading pad (i) [350 mm (L)×120 mm (W)×40 mm (H)]. To transfer loads uniformly to the soil surface, three wooden strips (10) [450 mm (L)×30 mm (W)×30 mm (H)] were sandwiched between the loading pad and the soil surface. A polycarbonate sheet (11) [450 mm (L)×150 mm (W)×10 mm (H)] was placed below the wooden strips and above the soil surface to even the loads. The polycarbonate sheet was perforated with ϕ5 mm openings at 15 mm intervals and, in conjunction with two layers of cotton fabric (e), provided

the drained upper boundary. In lieu of a sand cushion, the combination of the perforated sheet, the wooden strips, and the loading pad created room above the soil layer to mount potential needles (l). Electric potentials, however, were not achievable because of the abnormality of the multimeters (n). On the front and back panels of the box, two pairs of U-shaped steel channels were mounted to clamp the box and prevent possible bulging failures.

The peripheral devices include a direct current (DC) power supply (m) with an output capacity of 60 volts and 5 amps, electrical wirings, two sets of digital AC/DC multimeters (n), five potential needles (l), two settlement dial gauges (o), and a graduated cylinder (p). The needles were ϕ2 mm wires that were insulated, except for the outermost 5 mm of the wire, positioned at 75 mm or 100 mm intervals and penetrated 110 mm into the soil layer. The dial gauges (o) had a measurement range of 50 mm and were capable of covering settlements of a 250 mm high soil layer in a few days. The two dial gauges sat next to the respective box ends to gauge settlements around the anodes and cathodes.

Material

The material used in the experimental test was remolded kaolinite (q). Zhou et al. (2013) tested its physical and geoelectrical properties. Some properties (e.g., the moisture content and the void ratio) were retested because of possible reconstitutions of the kaolinite. Its properties are summarized in Table 3. The retested properties (w, e_0, C_c, a_v, k_{h0} and C_k) have results in agreement with corresponding results in Zhou et al. (2013) and thus imply consistent testing.

The kaolinite was placed and tamped in lifts into the consolidation box. Every effort was made to minimize voids and bubbles, including vibrating and mixing the kaolinite into a viscous paste. The kaolinite layer in the box was 220 mm high. A DC voltage of 40 volts (i.e., a voltage gradient of 89 volts/m) was applied between the electrodes. Loads were

placed in multiple steps, 12.5, 25, and 50 kPa, at 24-hour intervals to examine EC2's capacity in complicated scenarios.

Settlement

Two sets of settlement results are presented in Fig. 3: experimental observations and EC2's numerical results for the kaolinite layer after 72 hours. The experimental observations are the averages of the results of the two dial gauges. The averages reflect the mean settlements of the layer when subjected to a load through a rigid platform (the polycarbonate sheet and the wooden strips), i.e., an iso-strain condition. Observations were made every 1–2 hours and sometimes at long intervals if restricted, such as at nighttime. The kaolinite layer undergoes a total settlement of 40.9 mm, i.e., a consolidation strain of 18.6%. The strain is significantly higher than the strain of 0.000025% calculated in the small-strain validation and confirms that the prototype testing is a valid case of large-strain consolidation.

EC2's results were obtained from calculations conducted on a 51×51 mesh of the kaolinite layer. Each element measures 0.0088 (*b*)×0.0043 (*d*) m, which is smaller than the elements used in the small-strain validation. Therefore, the calculations are at least equally accurate. The other input values for the EC2 model are the physical and geoelectrical properties shown in Table 3. An iso-strain consolidation was conducted to reflect the testing condition. In Fig. 3, EC2's results shows excellent agreement with the experimental observations throughout the consolidation process of the soil layer. EC2's results are even 'smart' in reflecting the jumps of settlement where load increments occur. The agreement confirms the capacity of EC2 to simulate large-strain consolidation.

NUMERICAL SIMULATIONS

Numerical simulations were conducted on three example problems 1) to determine the efficiency of the combined method compared with single process methods, 2) to determine

the consolidation efficiency and level of electric power savings when adding enhancing processes, and 3) to choose a process in terms of the soil types. Example problem 1 involves comparisons between the combined method and the two single process methods, the electroosmosis and the surcharge preloading methods. Example problem 2 helps optimize soil consolidation by improving its design through additional processes such as stepped loading or current intermittence. Example problem 3 helps further improve consolidation design through factoring in the soil hydraulic conductivity. The simulations conducted for the three example problems help consolidate soils using a relatively shorter elapsed time and less energy.

Simulation Cases

For example problem 1, four simulation cases were designed and are shown in Table 4. The cases differ in values for the voltage gradient i_e and the load increment Δq. Case 1 uses a loading of Δq=100 kPa, whereas case 2 uses electroosmosis of i_e=30 volts/m. Cases 3 and 4 use preloading–electroosmosis combined processes with different input values, i_e=30 volts/m and Δq=100 kPa for case 3, and i_e=20 volts/m and Δq=60 kPa for case 4. The reduction in values for case 4 helps examine its impact on consolidation. Case 3 acts as a benchmark case throughout the three example problems.

Example problem 2 has five new cases, shown in Table 5. The cases were created by referring to processes added to enhance consolidation (Table 2). The processes include stepped loading, stepped current density, load lagging, current lagging, and current intermittence. These processes were designed to take place at an early stage of consolidation, such as 10 or 20 d, when the excess pore pressure remains high and the processes help its dissipation. The input values for the five cases were capped at 30 volts/m for i_e and 100 kPa for Δq so that all cases ended up with equal final settlements.

Three consolidation metrics were assessed: the average degree of consolidation U_{avg}, the energy consumption E, and the energy consumption index E_i. The last two metrics are decisive for assessing the trade-off between the time elapsed, the water discharged and the power consumed. The energy consumption E (in kW·h) is a numeric accumulation of the electric power consumed throughout the process of consolidation. Similar to household power consumption, E is the product of the electric power P (in kW) and time t, where P is the product of the voltage V_m and the electric current I. The value of I is a function of the time and position of interest and obtained in terms of the electrical resistance R. The energy consumption index E_i [in kW·h/(L·m^3)] was defined in Zhou et al. (2013) to quantify the rate of doing 'work,' i.e., the energy consumption per unit volume of soil layer and per unit volume of water discharge.

Example problem 3 creates two soils with different hydraulic conductivities (Table 6). The values for the electroosmotic conductivity remain unchanged to reflect its relative insensitivity to the soil types. The insensitivity was evidenced in Casagrande (1949) and discussed in Acar et al. (1994). The soils are subjected to the same processes as in cases 1–3.

The soil's physical and geoelectrical properties for the eleven cases are presented in Table 7. The values were determined in terms of the range of results for fine-grained soils. All simulation cases involve a soil layer $B_l \times W \times D$=1×1×5 m. The values for the layer agree with the choices of design in practice, such as in Bjerrum et al. (1967). To reflect field situations, the void ratio of the layer is variable as a function of the depth and has a compression index of 1.0. As a result, the values for the initial void ratio are 2.0 at the ground surface and 1.8 at the bottom of the layer. Corresponding to the initial void ratio, the values are 4.0×10^{-9} m/s for the initial vertical hydraulic conductivity and 2.0×10^{-9} m^2/s·volt for the initial electroosmotic conductivity. The values change with the depth at a hydraulic conductivity index of 1.0 and the exponent for an electroosmotic conductivity of 3.5. The

soil hydraulic conductivity ratio is 2 to reflect the anisotropic property of the soil layer. An iso-stress condition applies to all cases.

Example Problem 1: Combined vs. Single Process Methods

Pore Pressures

Pore pressure isochrones for cases 1–4 are presented in Fig. 4. The isochrones show pore pressures at the mid-height of the soil layer and include three times: 10 d, 30 d, and the time upon equilibrium. In response to Δq=100 kPa in case 1, positive pore pressures are initially generated. The pressures dissipate over time and eventually vanish, showing a movement in agreement with the chart solutions for Terzaghi's 1D consolidation theory. If subjected to a plain voltage gradient of i_e=30 volts/m (case 2), the layer starts with small negative pore pressures. The pressures grow into a diagonal isochrone because of the drained condition at the cathode and the undrained condition at the anode, which is consistent with the pressure development described by Esrig (1968).

The combined processes help dissipate the pore pressures. At an early stage of consolidation, such as 10 d in Fig. 4(a), the pore pressures of the combined processes dissipate more rapidly than those of the preloading process, which agrees with the results obtained by Shang (1998). This suggests that the electric field within the layer facilitates its pressure dissipation and water discharge. The electric field creates negative pore pressures, as shown in case 4. The negative pressures offset the existing positive pressures generated by the preloading and accelerate pressures dissipation. It is noteworthy that the offsetting does not mean an algebraic operation. The pore pressures in the combined process are not equal to the summation of the pore pressures developed in the respective single processes. That is, the preloading and electroosmosis are coupled and influence each other when combined to consolidate soils. This is confirmed by analyzing the pore pressures at equilibrium.

At equilibrium [Fig. 4(c)], the combined process creates final negative pore pressures higher than electroosmosis does. The difference is associated with the change of the void ratio e. The value for e in the combined process decreases further in relation to that in the electroosmosis process because of the compaction effort added by the preloading. A lower e value leads to a higher ratio of k_e to k_h, as k_e is not as sensitive as k_h to the change of e (Acar et al. 1994). According to Esrig (1968), the ratio k_e/k_h determines the gradient of the negative pore pressure distribution at equilibrium, and the higher the ratio is, the higher the gradient. As a result, higher negative pore pressures occur in the combined process.

The isochrones are not straight but concave downward. The curved isochrones are different from the linear developments reported by Esrig (1968). The difference arises from the presumption made about the void ratio, which is constant in Esrig's solutions and changes in this study. The changing void ratio leads to the heterogeneity of the permeability and, thus, the nonlinearity of all pressure isochrones within the layer.

Settlement

The results for soil settlement are presented in Fig. 5. In response to the load Δq=100 kPa (case 1), the preloading process raises settlement 0.17 m at the anode and 0.71 m at the cathode at 10 d [Fig. 5(a)]. The settlements grow to 0.51 m at the anode and 0.71 m at the cathode at 30 d [Fig. 5(b)] and translate into a less differential settlement. The differential settlement continues to decrease and eventually vanishes at equilibrium [Fig. 5(c)]. The electroosmosis process (case 2) shows different progress of soil settlements. The soils adjacent to the anode settle quickly and more frequently, whereas the soils near the cathode barely settle. This is due to the directed fluid flow (from the anode to the cathode) in the soil layer. As a result, the soil surface deforms linearly at equilibrium [Fig. 5(c)].

The combined process helps achieve additional and steady settlements. Case 3's settlements are clearly in excess of those of cases 1 and 2 and, importantly, progress evenly.

Drawing a parallel between the graphs, case 3's soil surface undergoes balanced settlements and improves the situations of either single process. Similar balanced settlements occur in case 4 and confirm the improved consolidation performance gained by using the combined method. Moreover, case 4 shows additional and fast settlements relative to case 2, albeit subjected to a lower current density. This implies that a well-designed combined process outperforms an electroosmosis process.

At equilibrium [Fig. 5(c)], the soil surfaces show significant differential settlements. The differential settlements reflect theoretical results for soil deformation in an ideal condition—no shear stress transfer between laterally adjacent elements. In reality, however, the soil particles do move or rearrange sideway because of exertions of gravity, surcharge loads, and perhaps seepage forces. As a result, on the top surface of the soil layer, some places undergo lower settlements than calculated, whereas the other places do the opposite. Alongside this, the settlement profiles of the soil layer might not be in strict compliance with those shown in Fig. 5. To account for this, the average settlement, S_{avg}, is used as an additional metric to assess the performance of cases 1–4.

The average settlements for cases 1–4 are presented in Fig. 6. As expected, the average settlements induced by case 3 are in clear excess of those by either case 1 or 2. Case 4 also demonstrates settlements greater than case 1 or 2 does and confirms the benefit of combining preloading and electroosmosis. The preloading process leads to settlements greater than the electroosmosis process, although it is complex to compare the respective input values of Δq=100 kPa and i_e=30 volt/m. Case 3's average settlements are not exactly the summations of those of cases 1 and 2 but are slightly less. This confirms the foregoing statement that the preloading and electroosmosis influence each other in the combined process.

Average Degree of Consolidation

The results for the average degree of consolidation U_{avg} are presented in Fig. 7. Four graphs are presented with respect to (a) the soil layer, (b) soils near the anode, (c) soils at the center, and (d) soils near the cathode. Figure 7(a) shows that cases 1–4 reach the equilibrium of consolidation at approximately 200 d. Well prior to that time, the four cases accomplish major consolidation, such as U_{avg}=90% between 40 d to 60 d. Overall, the electroosmosis process lags behind the other three processes.

The degree of consolidation is associated with the places of interest and the choices of the consolidating process. For the soils near the anode [Fig. 7(b)], the electroosmosis process leads consolidation, and the preloading process significantly lags behind the other three processes. For instance, to attain U_{avg}=40%, it takes 7.9 d for the electroosmosis process, 10.2 d for the combined processes, and 14.5 d for the surcharge preloading process. The lagging occurs mainly in the early-stage (i.e., <30 d) of consolidation. The soils near the cathode [Fig. 7(d)] do the opposite, where the electroosmosis process consolidates the soils much slower than the other three processes. To attain the same U_{avg}=40%, the times are 20.5 d for electroosmosis and 2.1 d for preloading. In contrast to the performance of the single processes, the combined processes offer relatively steady and rapid rates of consolidation.

Example Problem 2: Effectiveness of a Changing Consolidation Process

Average Degree of Consolidation and Energy Consumption

The results for the average degree of consolidation U_{avg} and the energy consumption E are shown in Fig. 8. The results apply to cases 3 and 5–9 and help highlight the consolidation efficiency between typical choices of processes used to enhance consolidation. Figure 8(a) shows that the stepped loading and stepped current processes decrease the consolidation rates in relation to the continuous process in case 3 (the benchmark). The difference, however, is minor and gradually vanishes when U_{avg} approaches 100%. For instance, it takes 46 d for the continuous process to attain U_{avg}=90% and 53 d for the stepped loading process and 50 d for

the stepped current process to attain the same U_{avg}. In contrast, the energy consumptions vary between the processes, and the stepped current process dramatically saves energy. To attain the same U_{avg}=90%, energy consumptions are 484 kW·h for the stepped current process and 650 kW·h for the continuous process, which means energy savings of 25.5%. Similar trendlines occur to the current lagging and current intermittence processes in Fig. 8(b). To attain U_{avg}=90%, the energy saving rates are 15.3% for the current lagging process and 29.5% for the current intermittence process. The consolidation 'delay' for the current lagging, however, seems 'excessive' at 62 d to attain U_{avg}=90%, which is 34.7% longer than the time for the continuous process.

The energy consumptions of all cases are not demanding until U_{avg} exceeds 80% or so. This result indicates that major consolidations have been accomplished using a relatively low portion (approximately 20%) of the total energy. As a further explanation of the curves in Fig. 8, the energy takes active effects for up to 40 d of consolidation and thereafter is used mainly to 'heat' soils. It is plausible to cease energy supply when U_{avg} reaches 80% or so.

Energy Consumption Index

The results for the energy consumption index E_i are shown in Fig. 9. The lower the index is, the more efficient the power is to drain water per unit volume of soil, and vice versa. Within 10 d of consolidation, the stepped current process (case 6) outperforms the other five. On average, this process maintains E_i as low as 0.01 kW·h/(L·m^3). The next lowest E_i occurs for the continuous process (case 3) and the current intermittence process (case 9), with an equal value of 0.85 kW·h/(L·m^3). These E_i values suggest that a lower voltage gradient (10 volts/m for case 6) drain water more efficiently than a higher one does (30 volts/m for cases 3 and 9), where the cases are subjected to equal surcharge loads. If the loads decrease, the E_i values further increase, as occurred in the stepped loading and loading lagging processes. These results suggest that it is energy efficient to apply a portion of voltage and a full surcharge

load in the early stage of consolidation and then increase the voltage and maintain the load for the next stages.

Not until a consolidation time of 30 d or so do the E_i values appear to rise drastically. Although the amount of energy needs to start to escalate at this stage, it is favorable to see from Fig. 8 that approximately 80% degree of consolidation is attained and a significantly lower volume of water is left to drain out. Hence, the E_i curves confirm the choice of ceasing power supplies at an early stage (e.g., 30 d) of consolidation.

Based on the above discussion for example problem 2, the stepped current process (case 6) appears to outperform the other processes by offering consolidation optimization with respect to soil settlement, consolidation time, and energy need. The current intermittence process (case 9) is a 'must-attempt' choice, particularly when energy saving is weighed rather than the elapsed time. Certainly, loading is another equally important factor when assessing optimization. Applying loads in steps (cases 5 and 7), however, reduces the rate of consolidation. Moreover, the stepped loading, in the view of construction management, will not save the materials cost but influence its schedule (i.e., backfilling time log), as the total weight of the surcharge remains fixed. To these ends, consolidation optimization is mainly 'powered' by smart supplies of electricity.

Example Problem 3: Effects of the Soil Hydraulic Conductivity

The effects of the soil hydraulic conductivity k_v on consolidation are demonstrated in Fig. 10. The figure shows the results for the average degree of consolidation U_{avg} and the average settlement S_{avg} for three soils subjected to three processes, the combined process, preloading, and electroosmosis. It is shown that the consolidation is sensitive to k_v. As expected, the more permeable the soils are, the more accelerated the consolidation will be when the consolidation process remains the same. For instance, in the combined process, the

values of U_{avg} at 10 d are 76% for k_v=1×10^{-8} m/s, 45% for k_v=4×10^{-9} m/s, and 25% for k_v=2×10^{-9} m/s. Similar trends occur in the other two single processes.

The soil settlements are dependent on the combined effects of the processes and the hydraulic conductivity. In the combined and electroosmosis processes [Figs. 10(a and c)], the less permeable the soils are, the more settlements the soils attain. In the preloading process [Fig. 10(b)], the settlements remain equal. Given these, it is plausible to deploy the combined process to gain more settlements than attained by the preloading, where the soil hydraulic conductivity is on the order of 10^{-8} m/s or lower, such as reclaimed sewage sludge and seabed sediments.

CONCLUSIONS

This paper verifies the EC2 model and investigates the effects of the consolidating processes and the soil hydraulic conductivity on consolidation and power consumption in electroosmosis–surcharge preloading practice. The model's consolidation algorithm is two-dimensional and can accommodate large strains with associated material nonlinearities. The algorithm also accounts for temporal and spatial changes of the porosity, seepage velocity, and electrical resistivity of a consolidating layer. A finite difference program has successfully executed the algorithm and provided consolidation results.

Verification checks of EC2 were performed for the consolidation of a saturated kaolinite layer subjected to the combined method. Both the one- and two-dimensional consolidation problems were verified against analytical solutions (small-strain consolidation) and experimental test results (large-strain consolidation), respectively. The pore pressure isochrones and transient average settlements obtained from model EC2 are in excellent agreement with the respective results. In addition to validating the model, the experimental test helps setup a prototype-testing apparatus. The apparatus has successfully reproduced the

superposition of surcharge loading and current transmission, as well as two-dimensional consolidation under different boundary conditions.

Simulations were performed for two-dimensional consolidation in an anisotropic soil layer with linear compressibility and hydraulic conductivity, as well as the nonlinear coefficient of consolidation, the electroosmotic conductivity, and the electrical resistivity. The simulations cover single and combined consolidating processes, changing soils, variable surcharge loads and current density, and different sequences applying the loads and the current, including stepped loading or current density, lagging, and current intermittence. The combined process accelerates pore pressure dissipation and consolidates the layer steadily, rapidly and further. Ceasing power supply where the degree of consolidation helps achieve approximately 80% energy efficiency. A similar energy-efficient practice is to apply a portion of the voltage and a full surcharge load at an early stage of consolidation. The combined process is preferable where the hydraulic conductivity of the soil layer is on the order of 10^{-8} m/s or lower.

ACKNOWLEDGMENTS

The writers commenced the study in Hohai University, and continued and completed it after the first writer moved to the University of Adelaide. The writers appreciate the technical supports provided in Hohai University. The methodology and computer code in this study were developed under the enlightenment gained from Fox and Berles (1997) and Fox et al. (2003) which are much appreciated. The writers are also thankful to the anonymous reviewers for their comments to improve the paper.

NOTATION

The following symbols are used in this paper:

a_v = coefficient of compressibility;

B_l = interval between opposite polarities;

b = element length in x-axis;

C_c = compression index;

C_k = hydraulic conductivity index;

D = initial height of soil layer;

d = initial height of element;

E = energy consumption;

E_i = energy consumption index;

e = void ratio;

e_0 = initial void ratio;

G_s = specific gravity of solids;

I = electric current;

i_e = voltage gradient;

k_e = electroosmotic conductivity;

k_{e0} = initial electroosmotic conductivity;

k_h = hydraulic conductivity;

k_{h0} = initial hydraulic conductivity;

k_v = vertical hydraulic conductivity;

k_{v0} = initial vertical hydraulic conductivity;

P = electric power;

q = volume of water discharge;

q_0 = initial overburden effective stress at upper boundary;

R = electrical resistance;

R_i = number of elements in a row;

R_j = number of elements in a column;

r_k = hydraulic conductivity ratio;

S_{avg} = average settlement of soil layer;

$\bar{S}$ = final settlement of soil layer;

T_v = dimensionless time factor;

t = time;

U_{avg} = average degree of consolidation;

u = pore pressure;

u_{max} = maximum pore pressure;

V_m = effective voltage;

V_v = volume of soil layer;

x = horizontal coordinate;

z = vertical coordinate;

γ_w = unit weight of water;

ρ = electrical resistivity of soil;

ρ_s = electrical resistivity of solids;

ρ_w = electrical resistivity of pore fluid;

ξ = dummy variable; and

Δq = load increment at upper boundary.

Superscripts

a = exponent for electroosmotic conductivity; and

t = time.

REFERENCES

Acar, Y. B., Hamed, J. T., Alshawabkeh, A. N., and Gale, R. J. (1994). "Removal of cadmium (II) from saturated kaolinite by the application of electrical current." *Geotechnique*, 44(2), 239–254.

Bergado, D. T., Balasubramaniam, A. S., Patawaran, M. A. B., and Kwunpreuk, W. (2000). "Electro-osmotic consolidation of soft Bangkok clay with prefabricated vertical drains." *Ground Improv.*, 4, 153–163.

Bjerrum, L., Moum, J., and Eide, O. (1967). "Application of electro-osmosis to a foundation Problem in a Norwegian quick clay." *Geotechnique*, 17(3), 214–235.

Bo, M. W., Choa, V., and Zeng, X. Q. (2001). "Laboratory investigation on electro-osmosis properties of Singapore marine clay." *Soils Found.*, 41(5), 15–23.

Casagrande, L. (1949). "Electro-osmosis in soils." *Geotechnique*, 1(3), 159–177.

Deng, A., and Zhou, Y. D. (2015). "Modeling electroosmosis and surcharge preloading consolidation.I: Model formulation." *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)GT.1943-5606.0001417.

Esrig, M. I. (1968). “Pore pressure, consolidation and electrokinetics.” *J. Soil Mech. Found. Div.*, 94(SM4), 899–922.

Esrig, M. I., and Gemeinhardt, J. P. (1967). “Electrokinetic stabilization of an illitic clay.” *J. Soil Mech. Found. Div.*, 93(SM3), 109–128.

Fox, P. J., and Berles, J. D. (1997). “CS2: A piecewise-linear model for large strain consolidation.” *Int. J. Numer. Analty. Meth. Geomech.*, 21(7), 453–475.

Fox, P. J., Di Nicola, M., and Quigley, D. W. (2003). “Piecewise-linear model for large strain radial consolidation.” *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)1090-0241(2003)129:10(940), 940–950.

Gray, D. H. (1970). “Electrochemical hardening of clay soils.” *Geotechnique*, 20(1), 81–93.

Gray, D. H., and Mitchell, J. K. (1967). “Fundamental aspects of electro-osmosis in soils.” *J. Soil Mech. Found. Div.*, 93(SM6), 209–236.

Jeyakanthan, V., Gnanendran, C. T., and Lo, S. C. R. (2011). “Laboratory assessment of electro-osmotic stabilization of soft clay.” *Can. Geotech. J.*, 48(12), 1788–1802.

Johnston, I. W., and Butterfield, R. (1977). “A laboratory investigation of soil consolidation by electro-osmosis.” *Aus. Geotech. J.*, G7, 21–32.

Kondner, R. L., and Boyer, W. C. (1957). “Research on the use of electro-osmosis in the stabilization of fine grained soil.” *Proc. Highway Research Board*, 36, 783–793.

Laursen, S. (1997). “Laboratory investigation of electroosmosis in bentonites and natural clays.” *Can. Geotech. J.*, 34(5), 664–671.

Lefebvre, G., and Burnotte, F. (2002). “Improvements of electroosmotic consolidation of soft clays by minimizing power loss at electrodes.” *Can. Geotech. J.*, 39(2), 399–408.

Lo, K. Y., Inculet, I. I., and Ho, K. S. (1991). “Electroosmotic Strengthening of Soft Sensitive Clays.” *Can. Geotech. J.*, 28(1), 62–73.

Mohamedelhassan, E., and Shang, J. Q. (2001). “Effects of electrode materials and current intermittence in electro-osmosis.” *Ground Improv.*, 5(1), 3–11.

Morris, D. V., Hillis, S. F., and Caldwell, J. A. (1985). “Improvement of sensitive silty clay by electroosmosis.” *Can. Geotech. J.*, 22(1), 17–24.

Nicholls, R. L., and Herbst, R. L. (1967). “Consolidation under electrical-pressure gradients.” *J. Soil Mech. Found. Div.*, 93(SM5), 139–151.

Ou, C. Y., Chien, S. C., and Lee, T. Y. (2013). “Development of a Suitable Operation Procedure for Electroosmotic Chemical Soil Improvement.” *J. Geotech. Geoenviron. Eng.*, 10.1061/(ASCE)GT.1943-5606.0000819, 993–1000.

Shang, J. Q. (1998). “Electroosmosis-enhanced preloading consolidation via vertical drains.” *Can. Geotech. J.*, 35(3), 491–499.

Shang, J. Q., and Dunlap, W. A. (1996). “Improvement of soft clays by high-voltage electrokinetics.” *J. Geotech. Engrg.*, 10.1061/(ASCE)0733-9410(1996)122:4(274), 274–280.

Wan, T. Y., and Mitchell, J. K. (1975). “New apparatus for consolidation by electro-osmosis.” *J. Geotech. Eng. Div.*, 101(GT5), 503–507.

Wan, T. Y., and Mitchell, J. K. (1976). “Electro-osmotic consolidation of soils.” *J. Geotech. Eng. Div.*, 102(5), 473–491.

Zhou, Y. D., Deng, A., and Wang, C. (2013). “Finite-difference model for one-dimensional electro-osmotic consolidation.” *Comput. Geotech.*, 54, 152–165.

Figure Captions

Fig. 1. Validation: (a) pore pressure; (b) degree of consolidation

Fig. 2. Profile of the electroosmotic consolidation cell and the peripheral devices

Fig. 3. Validation of the consolidation settlements

Fig. 4. Pore pressures at times (a) 10 d; (b) 30 d; and (c) at equilibrium

Fig. 5. Settlements at times (a) 10 d; (b) 30 d; and (c) at equilibrium

Fig. 6. Average settlements over time

Fig. 7. Average degree of consolidation: (a) soil layer; (b) soils near the anode; (c) soils at the center; and (d) soils near the cathode

Fig. 8. Average degree of consolidation and energy consumption: (a) stepped actions and (b) lagging and intermittence

Fig. 9. Energy consumption index over time

Fig. 10. Average degree of consolidation and average settlement: (a) combined process; (b) preloading; and (c) electroosmosis

Table 1. Model validation: input values for the model geometry and the soil properties

Property	Value
Model geometry $B_l \times W \times D$ (m^3)	1×1×1
Vertical load increment Δq (kPa)	1×10^{-4}
Voltage V_m (volts)	1×10^{-5}
Coefficient of compressibility a_v (kPa^{-1})	0.005
Hydraulic conductivity k_h (m/s)	2×10^{-9}
Electroosmotic conductivity k_e (m^2/s·volt)	2×10^{-9}
Hydraulic conductivity ratio r_k	1
Initial void ratio e_0	2
Specific gravity of solids G_s	1
Mesh $R_i \times R_j$	51×51
Boundary conditions	Cathode: drained; anode and lower boundary: undrained; and upper boundary: undrained for Wan and Mitchell (1976) or drained for Shang (1998).

Table 2. Review of experimental setups for electroosmotic consolidation

Author(s)	Apparatus	Sample size (mm)	Electrodes	Electric field pattern	Electric gradient (volt/m)	Soils	Elapsed time of test	Additional process
Casagrande (1949)	Lucite box	254 (*L*)	Iron sheet anode, and platinum/copper gauze cathode	Parallel and horizontal	10–1,200	Remolded clay, silt, sand, bentonite, kaolin, and gel	Up to 99 days	None
Kondner and Boyer (1957)	Box	1,219 (*L*)×457 (*W*)×559 (*D*)	Brass rods	Parallel and horizontal	42	Remolded swamp muck	14 days	Surcharge, 22.1 kPa
Esrig and Gemeinhardt (1967)	Lucite tube	ϕ44.5×127 (or 63.5) (*L*)	Carbon rod anode and steel/brass mesh cathode	Parallel and horizontal	100	Remolded illitic soil	8 hours to 11 days	Uses of $CaCl_2$
Nicholls and Herbst (1967)	Oedometer	ϕ 63.5	Steel ring anode and steel nail cathode	Radial	0–133	Remolded silty clay	15 hours	Surcharge, 48–383 kPa
Gray (1970)	Cubic box	100 (*L*)×100 (*W*) ×100 (*D*)	Copper plate cathode, and aluminum plate or graphite rod anode	Parallel and horizontal	30–80	Bentonite-flour mixtures and remolded fat clay	Up to 20 hours	Uses of NaCl
Wan and Mitchell (1975; 1976)	Lucite box	200 (*L*)×200 (*W*) ×60 (*D*)	AgCl plate	Parallel and horizontal	75 and 125	Remolded kaolinite	Up to 16.7 hours, or 100 hours if involving reversal	Surcharge, 49–98 kPa and polarity reversal
Johnston and Butterfield (1977)	Triaxial cell	ϕ102×102 (*H*)	Stainless steel mesh	Parallel and vertical	6–28	Undisturbed London clay	Up to 30 days	None
Morris et al. (1985)	Plexiglas cylinder	ϕ77×250 (*H*)	Stainless steel wool	Parallel and vertical	45	Undisturbed silty clay	Up to 24 days	Stepped loading up to 30 kPa
Lo et al. (1991)	Plexiglas tube	ϕ102×229 (*H*)	Copper	Parallel and vertical	15–30	Undisturbed clay	Up to 30 hours, or 57 hours if involving reversal	Polarity reversal
Shang and Dunlap (1996)	Plexiglas cylinder	ϕ300×320 (*H*)	Copper rods with plexiglas insulator	Not described	Insulated 15 kV AC/DC	Remolded marine sediment	27 and 90 days	Intermittent current
Laursen (1997)	Glass tube	ϕ10×60 (*L*)	Copper and Ag-AgCl wire/plate	Parallel and horizontal	5 mA for current density	Remolded bentonite and clay	Not described	Uses of NaCl and $CaCl_2$
Bergado et al. (2000)	Cylinder cells	ϕ450×950 (*H*)	Tubes (material not described)	Point–point and horizontal	0–120	Bangkok clay	Up to 8 days	Preconsolidated at 5–75 kPa and electrode reversal
Bo et al. (2001)	Triaxial cell	ϕ70×140 (*H*)	Copper discs	Parallel and vertical	14–57	Undisturbed Singapore marine clay	Up to 21 days	Stepped loading of voltage
Mohamedelhassan and Shang (2001)	Plexiglas box	250 (*L*)×100 (*W*) ×250 (*D*)	Carbon, steel, and copper	Parallel and horizontal	16–60	Remolded marine sediment	Up to 8 hours	Preconsolidated at 9.3 kPa, current intermittence, and uses of saline solution
Lefebvre and Burnotte (2002)	PVC cylinder	ϕ254×130 (*H*)	Perforated steel tubes	Parallel and horizontal	30–37	Undisturbed over- and normally consolidated clays	140 hours	Surcharge loads of 20–175 kPa and use of saline solution
Jeyakanthan et al. (2011)	Plexiglas triaxial cell	ϕ60×120 (*H*)	Perforated copper discs	Parallel and vertical	37 and 40	Remolded silty clay	Up to 13 days	None
Zhou et al. (2013)	Plexiglas box	330 (*L*)×250 (*W*) ×170 (*D*)	Steel mesh cathode and graphite rods anode	Parallel and horizontal	42–121	Remolded kaolinite	Up to 72 hours	None
Ou et al. (2013)	Plexiglas box	440 (*L*)×230 (*W*) ×320 (*D*)	Platinum coated titanium meshed plates	Parallel and horizontal	50	Remolded kaolinite	Up to 48 hours	Preconsolidated at 30 kPa and uses of NaCl and $CaCl_2$
This study	Plexiglas box	450 (*L*)×150 (*W*) ×220 (*D*)	Stainless steel solid rods anode, and stainless steel tubes cathode	Parallel and horizontal	100	Remolded kaolinite	Up to 72 hours	Stepped loading up to 50 kPa

Note: *H*=height; *L*=length; *W*=width; *D*=depth.

Table 3. Model validation: physical and geoelectrical properties of kaolinite

Property	Value
Plastic limit w_P (%)	22.2
Liquid limit w_L (%)	43.7
Specific gravity of the solid G_s	2.62
Moisture content w (%)	60.3%
Initial void ratio e_0	1.57
Compression index c_c	0.22
Coefficient of compressibility a_v (MPa^{-1})	0.55
Hydraulic conductivity k_{h0} (m/s)	2.1×10^{-9}
Hydraulic conductivity index C_k	0.99
Hydraulic conductivity ratio r_k	1
Electroosmotic conductivity k_{e0} (m^2/s·volt)	5.3×10^{-9}
Exponent for electroosmotic conductivity a	3.5
Electrical resistivity of the pore fluid ρ_w (Ω·m)	4.5
Electrical resistivity of the solid ρ_s (Ω·m)	608

Table 4. Example 1 simulation cases

Case	Process	Voltage gradient i_e (volt/m)	Load Δq (kPa)
1	Preloading	Nil	100
2	Electroosmosis	30	Nil
3	Combined	30	100
4	Combined	20	60

Table 5. Example problem 2 simulation cases

Case	Process	Voltage gradient i_e (volt/m)	Load Δq (kPa)
5	Stepped loading	30 from t=0 d	25, 50 and 100 from t=0, 10 and 20 d, respectively
6	Stepped current density	10, 20 and 30 from t=0, 10 and 20 d, respectively	100 from t=0 d
7	Load lagging	30 from t=0 d	100 from t=10 d
8	Current lagging	30 from t=10 d	100 from t=0 d
9	Current intermittence	30 on/off at 10 d interval till 100 d or the end of consolidation	100 from t=0 d

Table 6. Example problem 3 simulation cases

Case	Vertical hydraulic conductivity k_{v0} (m/s)	Electroosmotic conductivity k_{e0} (m^2/s·volt)	Consolidating processes
10	1×10^{-8}	2×10^{-9}	Combined and single processes:
11	2×10^{-9}	2×10^{-9}	i_e=30 volt/m and/or Δq=100 kPa

Table 7. Numerical simulation: physical and geoelectrical properties of the soil layer

Property	Value
Model geometry $B_1 \times W \times D$ (m^3)	1×1×5
Initial load q_0 (kPa)	50
Specific gravity of the solid G_s	2.67
Initial void ratio at the ground surface e_0	2.0
Compression index C_c	1.0
Initial vertical hydraulic conductivity at the ground surface k_{v0} (m/s)	4.0×10^{-9}
Initial electroosmotic conductivity at the ground surface k_{e0} (m^2/s·volt)	2.0×10^{-9}
Hydraulic conductivity index C_k	1.0
Exponent for electroosmotic conductivity a	3.5
Hydraulic conductivity ratio of r_k	2
Electrical resistivity of the pore fluid ρ_w (Ω·m)	4.0
Electrical resistivity of the solid ρ_s (Ω·m)	1,000
Mesh $R_i \times R_j$	51×51
Boundary conditions	Anode and lower boundary: undrained; and cathode and upper boundary: drained.