AUTOMATION OF BUS CREW ROSTERS

by

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SUMMARY

This thesis is concerned with the problem of rostering operating crews for a city transportation system and in particular with the mathematical formulation of constructing an automated week day operator's roster.

The problem of constructing late p.m. straight shifts is discussed and formulated as a heuristic programme. Methods for forming straight shifts consisting of two or three pieces of work are derived.

The other problem considered is the formulation of a basic framework which defines the structure of the a.m. straight shifts and broken shifts. A technique for classifying the type of shifts necessary over the off-peak period is also discussed.
This thesis contains no material which has been accepted for the award of any other degree or diploma in any University. To the best of my knowledge and belief, the thesis contains no material previously published or written by any other person, except where due reference is made in the text of the thesis.

(R.M. POTTER)
ACKNOWLEDGEMENTS

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CHAPTER I
INTRODUCTION

The movement of vehicles and the allocation of duties of operating staff of many transport industries must be planned in advance. A time-table defines the vehicle movement and a roster specifies the duties.

The logical sequence in which bus operations are usually planned is as follows. First, the time-table is developed from consideration of desirable route structure, public demand, capital available for investment in equipment and operations and the policy to be adopted in meeting the demand and carrying out the operations.

When the time-table is developed and rules for operation have been formulated it is possible to produce a schedule of shifts to be worked by the drivers and conductors.

An operator shift schedule is determined by run-cutting. It is necessary to cut the runs defined by the time-table into pieces of work which can be combined to form man-shifts. The two usual types of shift allowed are broken and straight shifts. A broken shift consists of two pieces of work, one covering the a.m. peak period and the other covering the p.m. peak period. A straight shift however consists of one piece of work broken only to allow time for a crib or meal.
2.

For a large transportation enterprise, the scheduling problem is very formidable and although data processing techniques are commonly used as an aid in the preparation of time-tables and rosters, only little progress has been reported in the completely automated preparation of the operator's week-day roster. A significant challenge is to construct a computer programme which will produce in a short time schedules which are better or at least as good as those constructed by the schedule officers. These men base their methods on years of experience and their knowledge is hard to define explicitly and hard to surpass.


This thesis describes methods used to prepare an automated operator's week-day roster for the bus system in the city of Adelaide.*

* A description of this bus system and the agreements which must be satisfied is given in Appendix I.
The following steps are used in the run-cutting procedure:

1. Construction of the late finishing straight work schedule;
2. Calculation of the minimum number and type of work schedules necessary to man runs left from (1);
3. Construction of morning straight work schedules;
4. Allocation of the remaining duty to broken work schedules.

The construction of the late finishing straight-work schedules, i.e. p.m. straight shifts, can be divided into the following stages:

(i) Grouping the runs so that the minimum number of extra men are needed to cover the crib periods;
(ii) Forming late operators' shifts consisting of two pieces of work;
(iii) Forming late operators' shifts consisting of three pieces of work.

The contents of the chapters of this thesis are as follows. In Chapter 2 the problem of choosing the least number of men to cover the late operator cribs is formulated and a sequential stepping algorithm is discussed. The computer preparation of completed p.m. straight shifts consisting of two or three pieces of work is described in Chapters 3 and 4.
The assessment of the number and type of staff needed to cover the work left over from the preparations of p.m. straight shifts is discussed in Chapter 5.

Chapter 6 contains the method used to produce the most efficient off-peak roster.

The thesis is concluded with a short discussion chapter.
CHAPTER II

THE SEQUENTIAL STEPPING PROCEDURE

The agreement (8) of Appendix 1 states that operators must have a meal break or crib within five hours of starting or finishing their duty. Therefore, if buses are to be kept in continuous operation, provision must be made for replacing men taking cribs or meal breaks. This can be done economically by arranging for a man finishing a crib to replace a man starting a crib. This second man at the end of his crib would then replace an operator starting his crib, and so on. This procedure is termed sequential stepping and is illustrated in figure 2.1.

2.1 Use of Broken Shift Operators

An important consideration in sequential stepping is the use of broken shift men (illustrated in figure 2.2). Provided that the crib of a straight shift operator does not finish after the time limit of agreement (29) for broken shifts, viz. 8 p.m., the crib of a straight shift operator can be covered by a broken shift operator before he goes off duty. However, if the straight shift crib ended later than the limit of broken shifts, it would have to be covered by a straight shift driver. Use of broken shift men to cover cribs facilitates preparation of rosters, and the sequential stepping procedure is started for runs in which the crib ends after the broken shift limit.
FIGURE 21: THE SEQUENTIAL STEPPING PROCEDURE
Figure 2.2 The use of a broken shift operator in sequential stepping.
2.2 **Classification of the Sequential Groups**

The considerations given above lead to the requirement that the initial problems in schedule mechanization shall be the formation of groups of sequentially stepped runs for the evening straight shifts.

Bus runs are divided into three groups depending on the proximity of their relief points to the three depots as follows:

- **Group A:**
  - Runs with Hackney relief points (relief points 1, 2)
- **Group B:**
  - Runs with Port relief points (relief points 14, 15, ..., 20)
- **Group C:**
  - Runs with City relief points (relief points 5, 6, ..., 13).

The number of steps in a sequential group is limited on account of the length of crib and that there is a minimum amount of duty which must be paid for in a straight shift. In practice there are never more than nine steps. The smallest number of groups possible so far has been found to be 14. (The fewer the groups the less extra men needed to be rostered to cover the p.m. straight shift cribs).

2.3 **Selection of the Initial Relief Time**

The first task in sequential stepping is to find the relief time for the run on which the first group of steps will be based. The following conditions must be met:
7.

(i) the point at which it is intended to relieve the operator must be one of the specified relief points for the group being considered,
\[ \text{in.} \quad \frac{\text{rpm}_{m,n} \leq \text{rpm}_{i,j} \leq \text{rpm}_{\text{max},m}}{\text{rpm}_{\text{min},m}} \] (2.3.1)
where \( \text{rpm}_{\text{min},m} \): smallest relief point specified for group \( m \),
\( \text{rpm}_{\text{max},m} \): largest relief point specified for group \( m \),
\[ m = 1, 2, \ldots, 14; \]

(ii) the relief time, \( \text{rt}_{i,j} \), associated with the relief point \( \text{rpm}_{i,j} \) defined in (2.3.1) must be such that the remainder of the shift including the sign off allowance does not exceed the five hour maximum of agreement (8),
\[ \text{in.} \quad \frac{\text{rt}_{i,j} \geq \text{at}_{i} + \text{SOF} - 300 + \text{tt}_{cd}, \text{rpm}_{i,j}}{\text{rpm}_{i,j}} \] (2.3.2)
where \( \text{SOF} = 25 \) mins., from the agreement (24) and \( \text{cd} \) is the depot closest to \( \text{rpm}_{i,j} \);

(iii) the relief time, \( \text{rt}_{i,j} \), must be later than the latest sign off time of a broken shift operator,
\[ \text{in.} \quad \frac{\text{rt}_{i,j} \geq \text{SOF}_{1} + \text{tt}_{cd}, \text{rpm}_{i,j}}{8 \text{ p.m.} - \text{SOF}_{1}} \] (2.3.3)
where \( \text{SOF}_{1} = 15 + \text{tt}_{cd}, \text{rpm}_{i,j} \), from the agreement (26).

Let \( A(i) \) be a list of first relief times of runs which satisfy the equations (2.3.1), (2.3.2) and (2.3.3), where \( i \) is a count of such runs.
Thus when the list $A$ has been completed, the earliest time is selected as the initial relief time of the first group, i.e. define relief time, $r_t^z$, of the initial run, $r_{n_z}$, by

$$r_t^z = \min_i A(i)$$  \hspace{1cm} (2.3.4)

where $z$ is the position of this run in list $L_i$.

If there is a run which satisfies equations (2.3.1) and (2.3.2) but not equation (2.3.3) its crib can be covered by a broken shift operator.

The values of $r_t^1$, $r_p^1$, and $r_{n_1}$ for the runs satisfying equations (2.3.1) and (2.3.2) are stored in $J_T(t)$, $J_P(t)$, $J_G(t)$ respectively where $t$ is a count of all such runs.

A flow diagram setting out the logic used is shown in figure 2.3.

2.4 Construction of the First Sequential Group

Having selected the initial relief time, the next stage is to set up the sequential group associated with this relief time. Runs which meet the following conditions are possibilities for the second run of the first sequential group:

(i) the point at which it is intended to relieve the operator must be one of the specified relief points for the group being considered, i.e.

$$R_{p_{\min}} \leq r_p^1 \leq R_{p_{\max}}$$  \hspace{1cm} (2.4.1)
Figure 2.3 The logic for determining the first relief times.
(ii) the length of the crib must be greater than the minimum allowable crib time plus the time needed to travel from the relief point of the first run to the nearest depot plus the time needed to travel from this depot to the relief point on the second run,

\[ r_{1,j} - r_t > c_t + t_t + t_t \text{cd,rp}_2 + \text{cd,rp}_1 \]  \hspace{1cm} (2.4.2)

where \( c_t = 20 \) mins., from the agreement (10);

(iii) the run must not be one of those already chosen for relief by broken shift runs, or already used. Once a run has been used, a switching device defined by the function \( \text{irn}_1 \) is used, where

\[ \text{irn}_1 < 10^{12}: \text{ bus } i \text{ has not been used} \]
\[ \text{irn}_1 > 10^{12}: \text{ bus } i \text{ has been used.} \]  \hspace{1cm} (2.4.3)

Initially \( \text{irn}_1 = \text{rn}_1 \) and \( \text{irn}_1 \) is set equal to \( \text{rn}_1 + 10^{12} \) when bus \( i \) has been used;

(iv) in order that the maximum number of runs may be used to form a sequential group it was found that those runs which end relatively early, should not be used at the beginning of the sequential group,

\[ \text{i.e. } \text{if } b = 2 \text{ then } a_t > \ell t_m \]  \hspace{1cm} (2.4.4)

where \( \ell t_m \) is the latest sign off time for runs in group \( m \).

It should be noted that this time \( \ell t_m \) is completely arbitrary and may be varied from one group of sequential runs to another,
10.

Let \( B(i) \) be a list of the first relief times, \( r_{t_1} \), of runs which satisfy the equations (2.4.1), (2.4.2), (2.4.4), where \( i \) is a count of all such runs.

When the list \( B \) has been completed, the earliest time is selected as the second relief time of the first group.

i.e. define relief time, \( r_{t_y} \), of the next run, \( r_{n_y} \), by

\[ r_{n_y} = \min_{i} B(i) \]

where \( y \) gives the position of the run in list \( L_{1;k} \).

The next step is to identify the third relief point. This is done by replacing the relief time and relief point of the first run by the corresponding quantities for the second run and repeating the procedure above except that equation (2.4.4) need not be satisfied. With the third relief point identified there is no difficulty in obtaining the fourth and later relief points by further iterations of the same process. Indeed, the problem is not how to go on, but when to stop and this problem is dealt with in the next section.

2.4.1 Identification of the Final Run

One unwritten agreement that the M.T.T. has with the Union is that the latest starting time for a Port operator's crib is 9:15 p.m. and for a Hackney operator's crib is 10:00 p.m.,

i.e.

\[
\begin{align*}
t_{ct} &= \begin{cases} 
1275 \text{ mins. if Port operator} \\
1320 \text{ mins. if Hackney operator.} 
\end{cases} 
\end{align*}
\]

(2.4.5)
Thus as soon as $rt_z$ is greater than one of these two values, depending at which depot the operator signs off, the process must stop. However it is sometimes possible to combine runs as illustrated in figure 2.4. Operator on run 1 transfers to run 2 where the time between the two relief points may be less than the crib allowance. Nevertheless, it is possible to break run 1 at such a time before the time limit for broken shift operators and the length of the work is still less than the maximum of five hours.

2.5 Procedure for Ending the First Sequential Group

Let the last relief time before lct, found by the method described previously, be $rt_x$ and its corresponding run number and relief point be $rn_x, rp_x$ respectively.

For any run $i$, not already used by the sequential stepping it may be possible to find a time $rt_{1j}$ and relief point $rp_{1j}$ satisfying the following conditions:

$$rp_{\text{min},m} \leq rp_{1j} \leq rp_{\text{max},m}$$

(2.5.1)

and

$$rt_{1j} - rt_x \leq ttr_{rp_x, rp_{1j}}$$

(2.5.2)

where $ttr_{rp_x, rp_{1j}}$ is the time needed to travel between the relief points $rp_x$ and $rp_{1j}$.

However the following conditions must also be satisfied by some relief time, $rt_x$, and relief point $rp_x$ on run $rn_x$:
Figure 2.4 Late Reliefs in Sequential Stepping
\[ r_{x_i} \geq at_{x_i} + \text{SOF} - (300 - \tau) \]  
\[ \text{where SOF = 25 mins., by the agreement (24)} \]

and \( \tau = \min(t_1, t_2, t_3); \)

\[ r_{x_i} \leq 8 \text{ p.m.} - \text{SOF}_1 \]  
\[ \text{where SOF}_1 = 15 + \tau, \text{ by the agreement (26)}; \]

\[ r_{\text{min,m}} \leq r_{x_i} \leq r_{\text{max,m}}. \]  

Let \( B(i) \) be a list of the first relief times of runs satisfying equations (2.5.3), (2.5.4) and (2.5.5).

When list \( B \) has been completed, the earliest time is selected. This time then is the value of \( r_{x_i} \) and this procedure is repeated until no run can be found satisfying the equations (2.5.3), (2.5.4) and (2.5.5); the last such run found is the final run in the sequential group. The next stage is to form the remaining sequential groups.

2.6 Construction of Further Sequential Groups

The list of first relief times i.e. \( A(i) \), in which the values corresponding to the runs already used have been removed, is examined and the earliest time at a relief point within a specified range is selected. Then using the same procedures as before, another sequential group is constructed.

This logic is used to form all the groups.

Two groups of Hackney runs were initially formed, then five Port groups and finally seven City groups.
2.7 **Summary of Results**

Denote the grouping of the runs formed by sequential stepping thus,

\[ G(i,j) : j^{th} \text{ run number in the } i^{th} \text{ group} \quad (2.7.1) \]
\[ T(i,j) : \text{relief time at which the } j^{th} \text{ run in } i^{th} \text{ group was broken} \quad (2.7.2) \]
\[ P(i,j) : \text{relief point corresponding to } T(i,j). \quad (2.7.3) \]

A flow diagram setting out the logic used in the group construction is shown in figure 2.5. A set of results obtained from the computer are shown in Tables 1(a) and 1(b).
VALUES
OF rpy, rty, rtyz m, m t, let DEFINED
BY FIGURE 2.5

Figure 2.5: The Logic for the Construction of the Sequential Groups.
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RUNS 144, 145, 179, 180 ARE COVERED BY BROKEN SHIFT MEN.

TABLE I(b).
CHAPTER III
COMPLETION OF STRAIGHT SHIFTS - TWO PIECES OF WORK

3.1 Introduction

The aim of Chapter III is to indicate how p.m. straight shifts are formed by adding pieces of work to those chosen by the Sequential Stepping procedure. This addition is most desirable as one extra piece of work but two or even three pieces may be needed to form a shift of acceptable length.

Figure 2.1 illustrates the various types of runs in a sequential group, viz., first, last and intermediate runs. Different algorithms are used to complete the p.m. straight shifts depending on the type of run ending the shift.

3.2 Notation

The essence of forming straight shifts is the splitting up of each bus run into several pieces of work and recombining the pieces into shifts satisfying the agreements. The following data must be known for each piece of work:

(i) the run number,
(ii) the start and finish times,
(iii) the start and finish places.

Let \( R_{n_1}, R_{n_2}, R_{n_3}, \ldots \) define the run numbers of the first, second, third, \ldots pieces of work needed to complete a straight shift.
Let $T_2, T_1$ be the start and finish times for run $Rn_1$ and $P_2, P_1$ be the start and finish places for run $Rn_1$.

Similarly
let $T_4, T_3$ be the start and finish times for run $Rn_2$ and $P_4, P_3$ be the start and finish places for run $Rn_2$, etc.

Thus a p.m. straight shift consisting of two pieces of work is completely defined when $T_4, T_3, T_2, T_1, P_4, P_3, P_2, P_1, Rn_1, Rn_2$ and allowances for signing off, signing on and travelling are known. Such a shift is illustrated in figure 3.1. Similarly a p.m. straight shift consisting of three pieces of work is completely defined when $T_5, T_5, T_4, T_3, T_2, T_1, P_5, P_4, P_3, P_2, P_1, Rn_3, Rn_2, Rn_1$ and corresponding allowances are known.

3.3 Classification of Shifts

Various types of shifts are formed depending on the position of the runs in the sequential groups.

(i) Shifts with one run number defined:

The first and last run in each sequential group have only one piece of work defined as indicated in figures 3.2(a) and 3.2(b). Let the shifts which are formed from runs depicted in figure 3.2(a) be called $F$ type shifts. The following variables for $F$ type shifts are already known:

$$Rn_1 = G(K_1)$$

$$T_1 = \text{time bus with run number } G(K_1) \text{ returns to depot}$$
FIGURE 3.1 A P.M. STRAIGHT SHIFT CONSISTING OF TWO PIECES OF WORK.
16.

\[ P_1 = \text{arrival depot} \]
\[ T_2 = T(K,1) \]
\[ P_2 = P(K,1). \]

Let the shifts which are formed from runs depicted in figure 3.2(b) be called \( L \) type shifts.
The following variables for \( L \) type shifts are already defined:

\[ R_{n_1} = G(K,L) \]
\[ T_1 = T(K,L) \]
\[ P_1 = P(K,L) \]

\[ K = 1, 2, \ldots, 14 \]

\( L \) is number of runs in Group \( K \).

A method for finding values of \( T_2 \) and \( P_2 \) for \( L \) type shifts is described in a later section.

To complete \( F \) or \( L \) type shifts one or two new pieces of work must be chosen. An algorithm for choosing suitable new pieces is given in section 3.8.

(ii) Shifts with two run numbers defined.

Intermediate runs in the sequential groups have two pieces of work defined as illustrated in figures 3.3(a) and 3.3(b). Let the shifts which are formed from runs depicted in figure 3.3(a) be called \( I_1 \) type shifts.
The following variables for \( I_1 \) type shifts are already defined:
Figure 3.2(a) The first run in a sequential group with one piece of work completely defined.

Figure 3.2(b) The last run in a sequential group with one piece of work partially defined.
\[ R_n = G(K, r+1) \quad , \quad K = 1, 2, \ldots, 14; \quad 2 \leq r + 1 \leq L \]

\[ P_1 = \text{arrival depot} \]

\[ T_1 = \text{depot arrival time for bus with run number } R_n \]

\[ T_2 = T(K, r+1) \quad , \quad K = 1, 2, \ldots, 14; \quad 2 \leq r + 1 \leq L \]

\[ P_2 = P(K, r+1) \quad , \quad K = 1, 2, \ldots, 14; \quad 2 \leq r + 1 \leq L(3.3.1) \]

\[ T_3 = T(K, r) \quad , \quad K = 1, 2, \ldots, 14; \quad 1 \leq r \leq L \]

\[ P_3 = P(K, r) \quad , \quad K = 1, 2, \ldots, 14; \quad 1 \leq r \leq L \]

\[ R_n = G(K, r) \quad , \quad K = 1, 2, \ldots, 14; \quad 1 \leq r \leq L. \]

Let the shifts which are formed from runs depicted in figure 3.3(b) be called \( I_2 \) type shifts.

The variables for the \( I_2 \) type shifts are defined by the equation (3.3.1).

Values for \( T_4 \) and \( P_4 \) must be found for both \( I_1 \) and \( I_2 \) type shifts. The method for doing this is described later in this chapter.

\( I_2 \) type shifts must have at least one new piece of work chosen whereas \( I_1 \) types may or may not need an extra piece of work to be complete shifts.

3.4 \( T_2, P_2 \) Values - \( L \) Type Shifts

It was mentioned in Chapter II that the last runs in a sequential group have a crib prior to the time \( T_2 \).

Since these runs end at approximately 10.30 p.m. and the agreement (28) specifies a maximum of five hours work after a crib, the earliest time that a crib could be taken would be approximately 5.30 p.m., i.e. the crib would fall during the p.m. peak period. This is undesirable, therefore \( T_2 \)
FIGURE 3.5(a) INTERMEDIATE RUNS OF SEQUENTIAL GROUP WITH A CRIB ALLOWED

FIGURE 3.5(b) INTERMEDIATE RUNS OF A SEQUENTIAL GROUP WITHOUT A CRIB ALLOWED
is chosen as the most appropriate time after the peak, i.e. after 6.30 p.m.

Let $T_2'$ and $P_2'$ be the first relief time and its corresponding relief point on the run with run number $R_{n_1}$ such that

$3 \text{ p.m.} - \text{SOF} > T_2' > 6.30 \text{ p.m.}$ \hspace{1cm} (3.4.1)

$P_2' = 'a'$ \hspace{1cm} (3.4.2)

where \text{SOF} = 15 + tt \hspace{1cm} \text{by the agreement (26).}

The values of $T_2'$ and $P_2'$ satisfying equations (3.4.1) and (3.4.2) are defined to be the $T_2$ and $P_2$ values for the $L$ type shifts.

3.4.1 Residues

When the run with run number $R_{n_1}$ is cut at $T_2$, the residual work on this run is stored in the array $D$ where:

$D_{21} = 1 \hspace{1cm} \text{i.e. the position of the run in the list}$

$L_{1jk}$ (see Appendix 2)

* $P_x = 'a'$ means

(i) $P_x$ must be a Hackney relief point if there are any on the run, if not
(ii) $P_x$ must be a Port relief point if there are any on the run, if not
(iii) $P_x$ must be a City relief point if there are any on the run.

[This order of choice simplifies the formation of the broken shifts.]
\[ D_{l2} = T_2 \]
\[ D_{l3} = P_2 \]
\[ l = 1, 2, \ldots \]

where \( l \) is a count of the number of residual pieces of work.

### 3.5 \( T_4, P_4 \) Values - \( I_4 \) Type Shifts

When values for \( T_4 \) and \( P_4 \) are defined for \( I_4 \) type shifts, a completed p.m. straight shift may have been formed, i.e. the length of the shift is greater than \( 8 \frac{1}{4} \) hours.

Let \( t_1 = T_4 + SOF - 540 \) \hspace{1cm} (3.5.1)

i.e. \( t_1 \) is the earliest starting time of the shift, ensuring that its length is less than the 9 hours mentioned in the agreement (7).

Let \( t_2 = T_4 + tt - 300 \) \hspace{1cm} (3.5.2)

where \( tt \) is the time needed to travel from \( P_3 \) to its nearest depot. Thus \( t_2 \) is the earliest starting time for the shift, ensuring that the length of the piece of work before the crib is less than the 5 hours mentioned in the agreement (8).

Let \( t = \max(t_1, t_2) \) \hspace{1cm} (3.5.3)

For a valid shift, \( T_4 \) must satisfy

\[ T_4 \geq t + SON \] \hspace{1cm} (3.5.4)

where the signing on allowance, SON, depends on
(i) whether the bus is 2 man or 1 man operated, or
(ii) whether $P_4$ is a relief point or a depot, or
(iii) whether the operator has to obtain a ticket outfit before the commencement of the shift or between $T_2$ and $T_3$.

The appropriate value for SON is calculated from the agreement (14).

Since $P_1$ is a depot for all $I_1$ type shifts, SOF mentioned in equation (3.5.1) has the value 25 mins., defined by the agreement (24).

For a valid shift, $P_4$ must satisfy

$$P_4 \sim P_4.$$  \hspace{1cm} (3.5.5)

The actual values of SON, mentioned in equation (3.5.4),

\[ P_x \sim P_y \text{ means either} \]

(i) $P_x$ is the same depot as $P_y$ \hspace{1cm} (3.5.6)

or (ii) $P_x$ belongs to the group of relief points closest to $P_y$

or (iii) $P_y$ belongs to the group of relief points closest to $P_x$

[If $P_y$ is a Port relief point or Port depot, $P_x \sim P_y$ is extended to mean:

$P_x$ is initially tested to see if it satisfies any of the conditions given in (3.5.6); if it does not then $P_x$ may be Hackney depot or a Hackney relief point and all allowances associated with $P_x$ are increased by 50 mins., i.e. the travel time from Port Adelaide to Hackney.]
are

(i) 20 mins if \( P_4 \) is a depot; from the agreement (24)  \( (3.5.7) \)

(ii) \( 10 + t_{c,P_4} \) if \( P_4 \) is a relief point;

from the agreement (26)  \( (3.5.8) \)

where \( C \) is the depot ~ \( P_4 \).

If the departure time, \( T_D \), and departure depot, \( P_D \), for the run with run number \( Rn_2 \) satisfy the equations (3.5.4) and (3.5.5) the \( T_4 \) and \( P_4 \) values for \( I_1 \) type shifts are defined to be \( T_D \) and \( P_D \) respectively.

If \( T_4 \) satisfies

\[
T_4 - SON > t + 45
\]  \( (3.5.9) \)

it is possible that a third piece of work may be added to produce a shift approximately nine hours in length. The procedure for finding such third pieces of work, if they exist, is discussed in Chapter 4.

If the equation (3.5.9) is not satisfied then the \( I_1 \) type shift is completely defined.

However if \( T_D \) and \( P_D \) do not satisfy the equations (3.5.4) and (3.5.5), let \( T_4' \) be the first time and \( P_4' \) the corresponding relief point satisfying these equations. For maximum service to the public, men are rostered so that they work as much of the p.m. peak as possible. This means that shifts starting between 5.10 p.m. and 6.30 p.m. are not permitted, i.e. if
the $T_4'$ and $P_4'$ values for $I_1$ type shifts are defined to be $T_4'$ and $P_4'$ respectively. If $T_4$ does not satisfy equation (3.5.9), the shift is considered complete, otherwise a search for a third piece of work must be made. If the equation (3.5.10) is not satisfied by $T_4'$, a value for $T_4$, not falling in the peak period, must be found.

Let $T_4''$ be the first time and $P_4''$ its corresponding relief point satisfying the following conditions:

$$T_4'' > 6.30 \, \text{p.m.} \quad (3.5.11)$$
$$T_4'' < 8 \, \text{p.m.} - \text{SOF} \quad (3.5.12)$$
$$P_4'' = 'a' \quad (3.5.13)$$

where SOF is calculated from the agreement (26).

When values of $T_4''$ and $P_4''$ are found satisfying the equations (3.5.11) to (3.5.13), these values are taken as the $T_4$ and $P_4$ values for the $I_1$ type shifts.

### 3.6 $T_4, P_4$ Values - $I_2$ Type Shifts

Since no crib was allowed between the times $T_3$ and $T_2$, it must be given prior to the time $T_4$. Therefore all $I_2$ type shifts must consist of at least three pieces of work.

Let $T_4''$ be the first time and $P_4''$ its corresponding relief point satisfying the equations (3.5.11), (3.5.12), (3.5.13), then the $T_4$ and $P_4$ values for $I_2$ type shifts are defined to be $T_4''$ and $P_4''$ respectively.
3.7 Residues

When \( P_i \) is a relief point, there is residual work left on the run with run number \( Rn_2 \). This work is stored in the array \( D \) defined thus:

\[
\begin{align*}
D_i &= i, \quad \text{where } i \text{ is the position of the run in list } L_{1,k} \\
D_{i2} &= T_i \\
D_{i3} &= P_i \\
\end{align*}
\]

\( l = 1, 2, \ldots \)

where \( l \) is a count of the number of residual pieces.

The flow chart in figure 3.4 indicates the method used for finding the \( T_i \) and \( P_i \) values for \( I_1 \) and \( I_2 \) type shifts.

3.8 Fixed Runs

Some runs on the original headway sheet have no associated list of relief times. These are called fixed work runs because the work defined by:

(i) the depot arrival time and the corresponding depot,
(ii) the depot departure time and the corresponding depot,

cannot be cut at any relief point. Such runs could have been included in \( L_{1,k} \) with \( nt_0 = 0 \), but it is more convenient to consider them separately and define them using the array \( B \) discussed in Appendix 2.

3.9 Second Piece of Work – L and F Type Shifts

Values of \( T_i, P_i, T_3, P_3 \) and \( Rn_2 \) which define the second piece of work for these shifts must be determined. Possible pieces of work may be contained in the arrays \( B \) and \( D \).
Figure 3.4 $T_t, P_t$ values for $I_1$ and $I_2$ type shifts.
3.9.1 "Suitable" Fixed Runs

Each run in B is tested for "suitability".

Suppose the \( j^{th} \) run is being tested. The variables \( T_4, P_4, T_3, P_3, T_{CL}, P_{CL} \) and \( Rn_2 \) for this run are defined as follows:

\[
T_4 = B_{j2}, \quad P_4 = B_{j3}, \quad Rn_2 = B_{j1}, \quad T_3 = B_{j4}, \quad P_3 = B_{j5}, \quad T_{CL} = B_{j6}, \quad P_{CL} = B_{j9}
\]  

(3.9.1)

where \( T_{CL} \) is the time that the conductor leaves the bus and \( P_{CL} \) is the corresponding relief point.

Allowance must be made for the operator to obtain a ticket outfit between the times \( T_3 \) and \( T_2 \) if the following conditions hold:

\[
T_3 < T_{CL}\quad (3.9.2)
\]

\[
P_3 = P_1.\quad (3.9.3)
\]

Most fixed work buses automatically satisfy these equations because the conductor returns to depot with the bus.

Let \( A_\ell \) be the minimum permissible period between \( T_3 \) and \( T_2 \) viz:

\[
A_\ell = ct + SOF_1 + t_{CD} + P_2\quad (3.9.4)
\]

where \( ct \) is the crib allowance defined by the agreement (10), \( CD \) is the depot nearest the relief point \( P_2 \), \( SOF_1 \) is the sign off allowance defined by the agreement (17).

If an outfit is to be collected between the times \( T_3 \) and \( T_2 \), \( A_\ell \) must be increased by 5 minutes and the sign on
allowance must be reduced by 10 minutes thereby satisfying the agreement (20).

The values of \( t_1 \) and \( t_2 \) for these shifts are calculated by:

\[
\begin{align*}
  t_1 &= T_1 + \text{SOF} - 540 \\
  t_2 &= T_3 + \text{SOF}_1 - 300
\end{align*}
\]  

(3.9.5)  

(3.9.6)

where \( \text{SOF} \) is the sign off allowance defined by the agreement (24).

The earliest possible starting time, \( t \), satisfies the condition:

\[
t = \max(t_1, t_2).
\]  

(3.9.7)

The \( j^{th} \) run in list B is considered "suitable" if the values \( T_4, P_4, T_3, P_3 \) defined by the equation (3.9.1) satisfy the following conditions:

\[
\begin{align*}
  T_4 - \text{SON} &> t \\
  T_4 &< 5.10 \text{ p.m.} \\
  6 \text{ p.m.} &< T_3 < T_2 - A_t \\
  P_3 &\sim P_2 \\
  P_4 &\sim P_1
\end{align*}
\]  

(3.9.8)  

(3.9.9)  

(3.9.10)  

(3.9.11)  

(3.9.12)

where \( \text{SON} \) is calculated from either equation (3.5.7) or (3.5.8).

3.9.2 "Suitable" Residual Runs

Each run in D is tested for "suitability".

Suppose the \( j^{th} \) run in D is being tested. The variables \( n, T_X, P_x, T_3, P_3, T_{CL}, P_{CL} \) and \( R_{n_s} \) for this run
are defined as follows:

\[ n = D_{j1}, \quad P_x = L_n,-7,0, \quad T_x = L_n,-8,0, \quad R_{n2} = L_n,-9,0, \]

\[ T_3 = D_{j2}, \quad P_3 = D_{j3}, \quad T_{CL} = L_n,-2,0, \quad P_{CL} = L_n,-1,0, \]

where \( n \) is the position of this run in the list \( L_{1jk} \).

The allowance \( A_t \), permitted between \( T_3 \) and \( T_2 \), is calculated from the equation (3.9.4) and is increased by 5 minutes if the equations (3.9.2) and (3.9.3) are satisfied. The earliest starting time, \( t \), is calculated from the equations (3.9.5), (3.9.6) and (3.9.7) where \( SOF_1 \) is defined by the agreement (22).

The \( j^{th} \) run in list \( D \) is considered "suitable" if its values of \( T_x \) and \( P_x \) satisfy the equations (3.9.8) to (3.9.11). The values of \( T_4 \) and \( P_4 \) are then taken as \( T_x \) and \( P_x \) respectively.

If \( T_x \) and \( P_x \) satisfy equations (3.9.9) to (3.9.11) only, it may still be possible to cut the run at a suitable relief point.

Let \( T_x' \) be the first time and \( P_x' \) its corresponding relief point which satisfy the following conditions:

\[ T_x' - \text{SON} \geq t \]
\[ P_x' \leq P_1 \]
\[ T_x' < T_3 \]

where \( \text{SON} \) is calculated from the agreement (20). If values of \( T_x' \) and \( P_x' \) are found satisfying these equations the \( j^{th} \) run in list \( D \) is still considered
"suitable" and the $T_x', P_x'$ values for the shift are defined to be $T_x'$ and $P_x'$ respectively.

3.9.3 "Most Suitable" Run.

The "most suitable" run chosen from all "suitable" runs in lists B and D is considered to be that run for which the length of the second piece of work is a maximum, i.e. $T_8 - T_4$ is a maximum.

If the shift length is now greater than $\theta_4^1$ hours the shift is said to be complete. However, if it is less than $\theta_4^1$ hours a third piece of work may be added to produce a still feasible shift. The method for finding such third pieces of work, if they exist, is discussed in the next chapter.

Once a run from list B is added to a shift, it may not be tested for "suitability" again. Although a portion of a run from list D may be added to a shift, the residual work may still be reconsidered for "suitability". Therefore if part of the jth run in list D had been used, its corresponding values for $D_{j2}$ and $D_{j3}$ then satisfy:

$$D_{j2} = T_4$$

(3.9.17)

$$D_{j3} = P_4.$$ 

Agreement (31) implies that only limited runs from lists B and D may be tested for "suitability" if $P_1$ is a Port relief point or Port depot.

The flow chart used by the computer program to choose the second piece of work for F and L type shifts is illustrated in Figure 3.5.
Figure 3.5 The logic for adding a second piece of work to L and F type shifts
CHAPTER IV
COMPLETION OF STRAIGHT SHIFTS - THREE PIECES OF WORK

4.1 Introduction

Third pieces of work may be chosen to complete the following types shifts, L,F,I_1 and I_2. Each type of shift will be treated separately and the method for choosing the third piece of work will be discussed.

4.2 Third Piece of Work - I_1 Type Shifts

The form of an I_1 type shift consisting of three pieces of work is illustrated in the figure 4.1. The values of T_6,P_6,T_5,P_5 and R_6 which define the third piece of work for this shift type must be determined for the "most suitable" run.

4.2.1 "Suitable" Fixed Runs

Each run in list B is tested for "suitability". Suppose the jth run is being tested. The variables T_6,P_6,R_6,T_5,P_5,T_{CL} and P_{CL} for this run are defined as follows:

\[ T_6 = B_{j2}, \quad P_6 = B_{j3}, \quad R_6 = B_{j4}, \quad T_5 = B_{j5}, \quad P_5 = B_{j6}, \quad T_{CL} = B_{j8}, \quad P_{CL} = B_{j9}. \]  

Allowance must be made for the operator to obtain his ticket outfit between the times T_5 and T_4 if these times satisfy the relations:

\[ T_5 \leq T_{CL} \]  

\[ P_5 \sim P_4. \]
Figure 4.1: An I-type shift with three pieces of work defined.
Let \( A_\ell \) be the minimum period allowed between the times \( T_5 \) and \( T_\ell \). The value of \( A_\ell \) is calculated using the condition:

\[
A_\ell = SOF_2 + t_{\text{CD}, P_\ell}
\]  

(4.2.4)

where \( SOF_2 \) is the sign off allowance, determined by the agreement (17), for the run with run number \( Rn_3 \), and CD is the depot nearest the relief point \( P_4 \). If the equations (4.2.2) and (4.2.3) are satisfied, \( A_\ell \) is increased by 5 minutes and the sign on allowance is reduced by 10 minutes, thereby satisfying the agreement (20).

The earliest starting time of the shift allowed by the agreement (7) is \( t_1 \), and the earliest starting time of the shift allowed by the agreement (8) is \( t_2 \). The values of \( t_1 \) and \( t_2 \) for this shift are calculated by:

\[
t_1 = T_1 + SOF - 540
\]  

(4.2.5)

\[
t_2 = T_3 + t_{\text{CD}, P_3} - 300
\]  

(4.2.6)

where \( C \) is the depot nearest \( P_3 \) and \( SOF \) is the sign off allowance for the run with the run number \( Rn_1 \).

The earliest possible starting time, \( t \), satisfies the condition:

\[
t = \max (t_1, t_2).
\]  

(4.2.7)

The \( j \)th run in list \( B \) is considered "suitable" if the values \( T_6, P_6, T_5, P_5 \) defined by the equation (4.2.1) satisfy
\[ T_6 - S O N \geq t \] (4.2.8)
\[ T_5 \leq T_4 - A_\ell \] (4.2.9)
\[ T_6 < 5.10 \text{ p.m.} \] (4.2.10)
\[ P_6 \sim P_1 \] (4.2.11)
\[ P_5 \sim P_4 \] (4.2.12)

where \( S O N \) is calculated from either equation (3.4.7) or (3.4.8).

4.2.2 "Suitable" Residual Runs

Each run in \( D \) is tested for "suitability". Suppose the \( j^{th} \) run in \( D \) is being tested. The variables \( n, P_x, T_x, T_5, P_5, R_n, T_6, P_6, T_{CL} \) and \( P_{CL} \) are defined as follows:

\[ n = D_{j1}, P_x = L_n, -7, 0, \ T_x = L_n, -8, 0, \ R_n = L_n, -9, 0 \] (4.2.13)
\[ T_5 = D_{j2}, P_5 = D_{j3}, \ T_{CL} = L_n, -2, 0, \ P_{CL} = L_n, -1, 0, \]

where \( n \) is the position of this run in the list \( L_{ij} \).

The allowance \( A_\ell \), permitted between \( T_5 \) and \( T_4 \), is calculated from the equation (4.2.4). Since a crib is not given between the times \( T_5 \) and \( T_4 \), no allowance need be made for the operator to collect a ticket outfit in this period. The earliest possible starting time, \( t \), for the shift is calculated from the equations (4.2.5), (4.2.6) and (4.2.7).

The \( j^{th} \) run in \( D \) is considered "suitable" if the values of \( T_x, P_x, T_5 \) and \( P_5 \) satisfy the equations
(4.2.8) to (4.2.12) where SOF₂, needed for calculating $A_{L}$, is determined by the agreement (18). The values of $T_e$ and $P_e$ are then defined to be $T_x$ and $P_x$ respectively.

If the equations (4.2.9) and (4.2.12) are satisfied but equation (4.2.8) is not, it may still be possible to cut the $j^{th}$ run at a suitable relief point and produce a "suitable" run. Let $T_{x}'$ be the first time with a corresponding relief point $P_{x}'$ which satisfy the following conditions:

$$T_{x}' > S O N > t \hspace{1cm} (4.2.14)$$
$$P_{x}' > P_{1} \hspace{1cm} (4.2.15)$$
$$T_{x}' < T_{5} \hspace{1cm} (4.2.16)$$
$$T_{x}' < 5.10 \text{ p.m.} \hspace{1cm} (4.2.17)$$

If there exist values of $T_{x}'$ and $P_{x}'$ which satisfy these conditions, the $j^{th}$ run in D is considered "suitable". These values are then taken as the values of $T_e$ and $P_e$ for the shift.

4.2.3 "Most Suitable" Run

The "most suitable" run is selected from all "suitable" runs contained in lists B and D, by the method described in section 3.9.3, providing equation (3.9.17) is replaced by

$$D_{j2} = T_{e} \hspace{1cm} (4.2.18)$$
$$D_{j3} = P_{e}.$$
4.3 Third Piece of Work - I<sub>2</sub> Type Shifts

The form of an I<sub>2</sub> type shift consisting of three pieces of work is illustrated in the figure 4.2. The values for T<sub>6</sub>, P<sub>6</sub>, T<sub>5</sub>, P<sub>5</sub> and Rn<sub>3</sub> which define the third piece of work for this shift type must be determined for the "most suitable" run.

4.3.1 "Suitable" Fixed Runs

Each run in list B is tested for "suitability". The variables T<sub>6</sub>, P<sub>6</sub>, Rn<sub>3</sub>, T<sub>5</sub>, P<sub>5</sub>, T<sub>CL</sub> and P<sub>CL</sub> for the j<sup>th</sup> run are calculated from the equation (4.2.1). If the equations (4.2.2) and (4.2.3) are satisfied, allowance must be made for the operator to collect a ticket outfit between the times T<sub>5</sub> and T<sub>6</sub>.

The minimum period, A<sub>6</sub>, permitted between the times T<sub>5</sub> and T<sub>6</sub> is defined by the following equation:

\[ A_6 = ct + SOF_2 + t_{CD,P_6} \]  \hspace{1cm} (4.3.1)

where ct is the crib allowance specified by agreement (10) and the SOF<sub>2</sub>, CD values are determined from section 4.2.1.

The values of \( t_1 \) and \( t_2 \) for this shift are calculated by:

\[ t_1 = T_1 + SOF - 340 \]  \hspace{1cm} (4.3.2)

\[ t_2 = T_5 + SOF_2 - 300 \]  \hspace{1cm} (4.3.3)

where SOF<sub>2</sub> is the sign off allowance for the run with run number Rn<sub>3</sub>. The value of SOF<sub>2</sub> is determined by agreement (17) or (18). The earliest possible starting
Figure 4.2 An $I_2$ type shift with 3 pieces of work defined.
time, \( t \), satisfies the condition:

\[ t = \max(t_1, t_2) \]  \hspace{1cm} (4.3.4).

The \( j \)th run in list \( B \) is considered "suitable" if the values \( T_e, P_e, T_s, P_s \) satisfy the equations (4.2.8) to (4.2.12).

4.3.2 "Suitable" Residual Runs

The method discussed in section 4.2.2, with equations (4.2.4) to (4.2.7) replaced by the equations (4.3.1) to (4.3.4), is repeated to find the "suitable" runs contained in list \( D \).

If the equations (4.2.2) and (4.2.3) are satisfied, \( A_i \) is increased by 5 minutes, whereas SON is reduced by 5 minutes if agreement (21) applies or it is reduced by 10 minutes if agreement (22) applies.

4.3.3 "Most Suitable" Run

The "most suitable" run is selected from all "suitable" runs, contained in lists \( B \) and \( D \), by the method described in section 4.2.3.

4.4 Third Piece of Work - L or F Type Shifts

The method needed for choosing the "most suitable" third piece of work for L or F type shifts is the same as that discussed in section 4.2. If the equations (4.2.2) and (4.2.3) are satisfied by runs from both lists, allowance must be made for the operator to collect a ticket outfit.
4.5 Completed P.M. Straight Shifts

Methods for finding straight shifts comprising two or three pieces of work have now been discussed. If it had been possible for the addition of yet a fourth piece of work, and the shift still a feasible length, the method described in this chapter could be extended to find a "most suitable" run with a run number \( R_n \).

The flow chart used for the computer programme to choose the third piece of work for the p.m. straight shifts is illustrated in figure 4.3.
FIGURE 4.3 THE LOGIC USED FOR CHOOSING THE THIRD PIECE OF WORM FOR A P.M. STRAIGHT SHIFT.
CHAPTER V
THE BASIC FRAMEWORK

5.1 Introduction

Having determined the staff needed to man the evening services and to provide their late cribs, an assessment of the men required for the remaining day's work must be made. Thus, the number of broken operator and conductor shifts, a.m. straight operator and conductor shifts, and early p.m. straight operator and conductor shifts for each depot must be determined. The construction of the basic framework, which defines the number and type of shifts needed to cover both peak periods, is discussed in this chapter.

5.2 Conditions

The conditions which govern the staff requirement are the following:

(i) broken shifts can not commence before 5.50 a.m.,

\[ (5.2.1) \]

(ii) broken shifts can not finish after 8 p.m., \[ (5.2.2) \]

(iii) the spread of hours on a broken shift must not exceed 12 hours.

\[ (5.2.3) \]

5.3 Sign-off Lists

A list of all sign off times for both operators and conductors may be constructed from all runs which were defined by the headway sheet and which were not used in a
p.m. straight shift. If the run has been cut at a relief point, the sign off allowance needed for calculating the sign off time, is obtained from agreement (25) or (26). The appropriate conductor's sign off allowance is also calculated from these agreements. However, if either the conductor or operator return to a depot travelling on a bus, the sign off allowance has the value specified by the agreement (23) or (24).

Any sign off time occurring before 5 p.m. is not included in this list, because the piece of work ending at this time does not cover the p.m. peak period.

Since agreement (28) requires all employees to sign off and on at the same depot, six sub-lists are constructed and defined in the following way:

- group 1, denoted by C1j; increasing sign off list for City operators,
- group 2, denoted by C2j; increasing sign off list for City conductors,
- group 3, denoted by C3j; increasing sign off list for Port operators,
- group 4, denoted by C4j; increasing sign off list for Port conductors,
- group 5, denoted by C5j; increasing sign off list for Hackney operators,
- group 6, denoted by C6j; increasing sign off list for Hackney conductors,
where \( j = 1,2,\ldots,n_i \) and \( n_i \) is the number of elements in group \( i \).

5.4 **Sign-on Lists**

A corresponding list of sign on times for operators and conductors may also be formed for the runs not used by the p.m. straight shifts. The sign on allowance necessary for calculating the sign on times of the conductors and operators is obtained from agreement (15) or (19).

Any sign on time occurring after 9.30 a.m. is not included in this list, because the piece of work starting at this time does not cover the a.m. peak period.

The sub-lists corresponding to this list of sign on times are defined as follows:

- **group 7**, denoted by \( C7j \); increasing sign on list for City operators,
- **group 8**, denoted by \( C8j \); increasing sign on list for City conductors,
- **group 9**, denoted by \( C9j \); increasing sign on list for Port operators,
- **group 10**, denoted by \( C10j \); increasing sign on list for Port conductors,
- **group 11**, denoted by \( C11j \); increasing sign on list for Hackney operators,
- **group 12**, denoted by \( C12j \); increasing sign on list for Hackney conductors,
where \( j = 1,2,\ldots, n_i \) and \( n_i \) is the number of elements in group \( i \).

5.5 Pairing Sign-on and Sign-off Times

An ideal pairing situation would exist if there were the same number of sign off times as sign on times in corresponding sub-lists. Since this is not the case in practice, allowance is made for adjusting the sign on and sign off times, thus permitting a man to start his duty at one depot, finish it at another depot or a relief point not associated with this depot and then travel back to the sign on depot. Allowance is also made for "shandy" shifts. These are formed if a man is initially rostered as an operator but finishes his duty as a conductor, or vice versa. When this occurs the shift worked is classified as an operator's shift; that is, paid at the higher rate. These allowances enable each of the p.m. sub-lists a choice of two or four a.m. sub-lists from which possible pairing may occur.

Table II illustrates the order of preference, allowed by the computer programme, for each p.m. sub-list.

5.5.1 Notation

Let the a.m. sub-lists to be tested against the p.m. list, \( C_{ij} \), be denoted by \( P_{ij}^k \), where \( k \) is the preference \( C_{ij} \) has for the a.m. list; i.e., \( k = 1,2,\ldots,4 \); and \( \ell = 1,2,\ldots, m_k \) where \( m_k \) is the number of elements in list \( P_{ij}^k \).
<table>
<thead>
<tr>
<th>P.M. GROUPS</th>
<th>POSSIBLE A.M. GROUPS [IN ORDER OF PREFERENCE]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1j</td>
<td>C7j</td>
</tr>
<tr>
<td>C2j</td>
<td>C8j</td>
</tr>
<tr>
<td>C3j</td>
<td>C9j</td>
</tr>
<tr>
<td>C4j</td>
<td>C10j</td>
</tr>
<tr>
<td>C5j</td>
<td>C11j</td>
</tr>
<tr>
<td>C6j</td>
<td>C12j</td>
</tr>
</tbody>
</table>

**TABLE II**
5.5.2 Early p.m. Straight Shift and Broken Shift Structure

Let RT be the last sign off time in the p.m. sublist Cij, thus

\[ RT = C_{i1} \]  \hspace{1cm} (5.5.1)

The earliest possible starting time for a broken shift with RT as its sign off time is 12 hours before RT.

If RT is greater than 8 p.m. then it must be the sign off time of an early ending p.m. straight shift, since condition (5.2.2) states that no broken shift may end after 8 p.m. Let the last sign off time in list Pil be RT_k, thus

\[ RT_k = P_{i1}^k \]  \hspace{1cm} (5.5.2)

\[ k = 1,2,\ldots,4 \]

Since the spread of a broken shift must be less than 12 hours, RT and RT_k will define the basic framework for a broken shift, if

\[ RT - 720 \geq RT_k \]  \hspace{1cm} (5.5.3)

\[ k = 1,2,\ldots,4 \]

Every sign on time in Pil is less than RT_k, therefore RT will not define a broken shift with any time in Pil if it does not with RT_k. However, if RT_k has already been used, the new value of RT_k is taken as the latest unused sign on time in Pil.

If RT does not satisfy the equation (5.5.3) for any value of RT_k, \( k = 1,2,\ldots,4 \), it must be the sign off
time of an early ending p.m. straight shift. The basic structure of a broken shift or an early p.m. straight shift is produced from each element in the list $C_{ij}$, where $i = 1, 2, \ldots, 6; \ j = n_1, n_1 - 1, \ldots, 1$. Sometimes in practice, there are more sign off times in a p.m. sub-list than there are sign on times in the corresponding first preference group. Suppose list $C_{ij}$ has 100 sign off times and $P^2_l$ has only 95 sign on times. If the previous procedure is performed, the 4th sign off time in $C_{ij}$ may be paired with the latest sign on time in $P^2_l$. Therefore a broken shift with a spread of perhaps only 9 hours may be defined. This means that when the pairing of late times in the p.m. sub-list which has $P^2_l$ as its first preference is attempted with sign on times in $P^2_l$, all the late sign on times may have been unnecessarily paired with sign off in $C_{ij}$. To prevent this from happening, the following procedure is used.

Let $R_T$ be the latest unused sign off time in the group $C_{ij}$. If $R_T$ satisfies the following conditions:

\[ R_T - 720 \leq P^l_\ell \]  
\[ R_T < 6 \text{ p.m.} \]  

(5.5.4)  
(5.5.5)

let $R_{T_k}$ be the smallest unused sign on time in list $P^k_l, k = 2, 3, 4$. It is extremely likely that the value of $R_{T_k}$ defined in this way does not satisfy the equation (5.5.3), but each element larger than $R_{T_k}$ in $P^k_l, k = 2, 3, 4$, is tested in turn, until one sign on time is
found which satisfies equation (5.5.3). This time then defines the structure of a broken shift with a sign off time of RT.

One of the procedures, depending on the number of elements in the groups, is used to produce the broken shift or early p.m. straight shift structure. Figure 5.1 shows the flow chart of the computer programme for constructing these shifts.

5.5.3 A.M. Straight Shift Structure

Each time a member of the lists $P^k_i$ was used to construct a broken shift, its value was set equal to $10^{12}$. This dummy value prevented the sign on time from being used again. Let $rt$ be a time in the a.m. list $C_{ij}$, $i = 7, 8, ..., 12$. Since a broken shift can not start before 5.50 a.m., $rt$ must be the sign on time of an a.m. straight shift if

$$rt < 5.50 \text{ a.m.} \quad (5.5.6).$$

Since each element in the lists $P^k_i$, $i = 1, 2, ..., 6$; $k = 1, 2, ..., 4$, corresponds to an element in the lists $C_{ij}$, $i = 7, 8, ..., 12$, any sign on time not used from these lists will start an a.m. straight shift. Thus, $rt$ defines the basic structure of an a.m. straight shift if it is not equal to $10^{12}$ or if it satisfies the equation (5.5.6).

The number of a.m. straight operator and conductor shifts is the number of elements not used for constructing
Figure 5.1: The logic for pairing the sign-on and sign-off times.
broken shifts from the a.m. groups. The flow chart used for constructing the basic framework of the a.m. straight shifts is given in the figure 5.2.
Figure 5.2 The logic for the formation of A.M. straight shift structure.
6.1 Introduction

There are three periods during a week-day which are classified as off-peak periods, namely:

1. Early morning: before 7.00 a.m.,
2. Mid-day: between 9.30 a.m. and 4.45 p.m.,
3. Late evening: after 6.30 p.m.

The mid-day off peak period is called the intermediate period. The number and type of shifts for both operators and conductors at each depot during or before the a.m. peak and during or after the p.m. peak was determined by the basic framework, defined in the preceding chapter. However, the number and type of shifts which are to be worked during the intermediate period are not known. This chapter will discuss the classification of operator shifts over this period of the day and how this affects the formation of a.m. straight and broken operator shifts.

The time table determines the number of buses on the road at any time of the day. Figure 6.1 is a graph of the number of buses on the road during a typical week day, where
FIGURE 6.1 THE NUMBER OF BUSES ON THE ROAD DURING A TYPICAL WEEK-DAY.
$N_1$ is the number of late p.m. buses,
$N_2$ is the maximum number of buses operating at one time during the p.m. peak,
$N_3$ is the maximum number of buses operating at one time during the intermediate period,
$N_4$ is the maximum number of buses operating at one time during the a.m. peak,
$N_5$ is the number of early a.m. buses.

6.2 The P.M. Peak Roster

The late p.m. straight shifts were constructed in such a way that all the operators were rostered over the p.m. peak. Let $\delta$ be the number of extra men needed to cover the cribs given after 8 p.m. The number, $n_1$ of late p.m. straight shifts is the sum of $N_1$ and $\delta$, i.e.

$$n_1 = N_1 + \delta \quad (6.2.1).$$

The number, $n_2$, of men needed to operate the remaining p.m. peak buses is the difference between $N_2$ and $n_1$, i.e.

$$n_2 = N_2 - n_1 \quad (6.2.2).$$

The most economic use of staff is obtained if these $n_2$ operators are also rostered over the a.m. peak, i.e. if there are $n_3$ broken shifts.

6.3 The A.M. Peak Roster

If it were possible to form $n_2$ broken shifts from the basic framework, the number, $n_3$, of extra men needed to operate the remaining a.m. peak buses is the difference between $N_4$ and $n_2$, i.e.

$$n_3 = N_4 - n_2 \quad (6.3.1).$$
Thus the number of straight shifts which cover the a.m. peak is $n_3$.

The most efficient operator's roster is therefore produced if the number of p.m. straight shifts, broken shifts and a.m. straight shifts are $n_1, n_2$ and $n_3$ respectively. If in practice, $N_3$ is greater than $n_3$, and there are some early p.m. straight shifts, the number of possible broken shifts is less than $n_2$.

6.4 The Intermediate Period Roster

To prevent extra men being rostered during the intermediate period, the buses running during this period must be operated by all the a.m. straight shift and some of the broken shift men.

The agreements (7) and (9) limit the latest sign off time of the a.m. straight shifts to between 1.30 p.m. and 2.30 p.m. Therefore only $n_3$ of the intermediate runs can be operated until approximately 2 p.m. by a.m. straight shift men. The latest limiting sign off time for the morning piece of any broken shift is between 12.15 p.m. and 1 p.m. due to the agreements (10) and (32). The remaining $(N_3 - n_3)$ intermediate buses must be operated by broken shift men. In order that the maximum amount of work is covered, $(N_3 - n_3)$ broken shifts having morning portions which finish near mid-day are chosen. The residual work after approximately 2 p.m. for $n_3$ runs and after approximately mid-day for the other runs must be covered by large afternoon portions of broken shifts.
A summary of the most efficient rostering of the week-day buses of figure 6.1, is given in figure 6.2.

6.5 Conditions

The following two conditions arise from the most economic way of rostering the intermediate period:

(i) a.m. straight shifts must end as late as possible, i.e. between 1 p.m. and 2 p.m.,

(ii) \((N_3-n_3)\) broken shifts should be constructed in such a way that their morning portions end between 12.15 p.m. and 1 p.m.

6.6 Meals

The method used to provide the meal, necessary by the agreement (9), for the a.m. straight shift operators is similar to the sequential stepping algorithm discussed in chapter 2.

6.7 Optimum Broken Shifts

When all the a.m. straight shifts are produced, the remaining pieces of work are added to existing pieces of broken shift work. No emphasis is placed on how economical the resultant broken shift is, but providing the conditions

(1) no break without pay in a day's duty shall be less than 2 hours,

(2) the maximum length of any portion of a broken shift is 6 hours,

are satisfied, the broken shift is acceptable. An optimum pairing of the a.m. and p.m. pieces of work for all these broken shifts is accomplished by using Bennett's pairing algorithm [1].
FIGURE 6.2 THE MOST EFFICIENT COVERING OF A WEEKDAY'S WORK SCHEDULE (DRAWN TO THE SAME SCALE AS FIGURE 6.1).
CHAPTER VII
CONCLUSIONS

Various attempts to automate scheduling procedures for transit companies have met with only limited success. The general methods developed by Elias [4], for example, have been widely tested in the U.S.A. but have not proved of direct practical benefit. The approach to the M.T.T. scheduling problem has been fundamentally different in that it has been recognised that it is preferable to subdivide the total complex problem into many sub-problems of manageable size and to base the computer algorithms on the ingenious manual methods which have been developed by the schedule officers. Figure 7.1 indicates this breakdown of the complete rostering problem into a sequence of interrelated sub-problems.

In solving the rostering problem step by step, it has proved advantageous to start with steps 5 and 11, which were automated by Bennett [1], and then to proceed with steps 1 and 2, which have been automated as described in this thesis. Completion of steps 3 and 4, at present under investigation, will then enable the whole M.T.T. roster to be produced automatically with intervention of the schedule officers at prescribed check points.

The methods used in this thesis are typical of those classified as heuristic programming. The aim has
Figure 3.1: The Division of the Complete Roster Preparation into Sub-Problems
not been optimization, partly because of the complexity of the problem and partly because of the impossibility of specifying the criteria explicitly. Instead, the purpose of the research has been to develop computer algorithms which give good acceptable rosters. By basing the approach on the manual methods, many latent constraints have been automatically satisfied without the necessity for their precise definition.

The automation of bus crew rosters has proved worthwhile from the management point of view and the following benefits have been derived.

(1) Frequent reviews of both timetables and rosters are able to be made because the time needed to prepare new duty rosters has decreased from eight weeks to three weeks.

(2) Experimentation to test new ideas and a flexibility in approach have resulted from the reduction in roster preparation time.

(3) Top management has now a measure of control over traffic costs because

   (i) rosters are being prepared on a minimum cost basis,

   (ii) quick evaluation can be made of the effect or changes or proposed changes in industrial working conditions,
(iii) more accurate estimates of the effect of service adjustments, extensions of services or the introduction of new services, can be obtained.

Because of the general acceptance of the present work to the M.T.T., it is now intended to proceed with the automation and optimization of the time-table, a problem which will be challenging in concept and complexity but which should yield to the powerful heuristic programming techniques developed in this thesis.
APPENDIX I

Adelaide, the capital city of the State of South Australia has a population of 800,000.

The Municipal Tramways Trust (M.T.T.) is the authority for street public transport services in metropolitan Adelaide. The Trust operates most of the bus services itself, particularly over the heavily patronised routes, but also licenses private bus companies to serve some areas. The number of diesel buses run by the Trust is 350, and these buses are operated on a one-man/two-man basis. Two man crews, comprising driver and conductor, are used during times of heavy passenger loading - between 7.30 a.m. and 6.30 p.m. on week days and on Saturday mornings, whilst at all other times drivers are required to collect fares.

An Industrial Arbitration Court Award and agreements between the Employees Association and the M.T.T. determine the rules governing the formation of rosters. The award and agreements are changed occasionally. The methods developed in this thesis are based on the following agreements:

1) An employee may be required to work broken shifts, provided that all duty performed on any day outside the spread of ten consecutive hours shall be paid for at the following rates -
(2) if the spread is between 10 and 11 hours - time and a half should be paid;

(3) if the spread is between 11 and 12 hours - double time should be paid.

(4) No break without pay in a day's duty shall be less than 2 hours.

(5) Employees shall not be signed off more than twice in any one day.

(6) The ordinary hours of duty shall not be less than 7 hours on any shift.

(7) The hours of duty on straight shifts shall not exceed 9 hours.

(8) Employees shall not be rostered to work for more than 5 hours without a meal relief or crib.

(9) Where an unpaid meal relief is allowed, a minimum of 40 mins. and a maximum of 55 mins. shall be allowed.

(10) Where a meal relief of at least 40 mins. is not provided, a crib shall be taken in the Trust's time. The minimum crib allowance is 20 mins.

(11) All time worked in excess of 8 hours 15 mins. shall be paid for at the rate of time and a half.

(12) When an employee finishes work on one bus to take over duties on another bus, the rate of pay described for the working of the former bus ceases immediately he terminates working it. The time taken to travel
to the other bus shall be payable for the working of such second bus.

(13) When an employee signs on duty as a one-man bus operator he shall be allowed sign on time irrespective of the period he is engaged as a one-man operator for such shift. Sign on time shall be paid at the rate applicable to the first one-man bus driven during the shift.

(14) The following sign on and sign off allowances must be allowed for a bus operator of a 2-man bus:

(15) When a bus is taken from a depot, the sign on allowance is 10 mins.

(16) When the operator takes charge of the bus in traffic, having signed on at a depot, the sign on allowance is 5 mins. + the time needed to travel from the depot to the relief point.

(17) When the bus is returned to a depot, the sign off allowance is 10 mins.

(18) When the operator is relieved in traffic and signs off at a depot, the sign off allowance is 5 mins. + the time needed to travel from the relief point to the depot.

The following sign on and sign off allowances must be allowed for a bus operator of a 1 man bus:
(19) When the bus is taken from a depot and a ticket outfit is obtained, the sign on allowance is 20 mins.

(20) When the operator takes charge of the bus in traffic, having signed on and obtained an outfit at a depot, the sign on allowance is 10 mins. + the time needed to travel from the depot to the relief point.

(21) When the bus is taken from the depot, the sign on allowance is 10 mins.

(22) When the operator takes charge of the bus in traffic, having signed on at a depot, the sign on allowance is 5 mins. + the time needed to travel from the depot to the relief point.

(23) When the bus is returned to a depot, the sign off allowance is 10 mins.

(24) When the bus is returned to a depot and the money collected paid in, the sign off allowance is 25 mins.

(25) When the operator is relieved in traffic and signs off at a depot the sign off allowance is 5 mins. + the time needed to travel from the relief point to the depot.

(26) When the operator is relieved in traffic and signs off and pays in the money collected, the sign off allowance is 15 mins. + the time needed to travel from the relief point to the depot.
(27) An employee signing on at any depot and subsequently directed to commence duty at some other point shall be allowed travelling time from the point at which he signed on to the point at which he commenced duty.

(28) All employees shall, each day, finally sign off at the depot at which they sign on.

(29) For broken shifts the sign on time shall not be before 5:50 a.m. and the sign off time shall not be after 8 p.m.

(30) Port operators may only drive buses with the following run numbers, 30 inclusive to 78, 242 inclusive to 267, 385 inclusive to 435, 471 inclusive to 479.

(31) A ticket outfit must be collected from the sign off depot.

(32) The maximum duty for a portion of a broken shift is 6 hours.
The amount of data associated with the problem is fairly voluminous, and all this data must be held in core store if the cost of the computer processing is to stay within reasonable bounds. The following are the items of data involved for each bus run $i$:

- $rn_i$: run number assigned to bus $i$,
- $dt_i$: depot departure time of bus $i$,
- $ddt_i$: depot of departure of bus $i$,
- $at_i$: depot arrival time of bus $i$,
- $dat_i$: depot of arrival of bus $i$,
- $cj_i$: time when conductor joins bus $i$,
- $d_j$: place where conductor joins bus $i$,
- $cl_i$: time when conductor leaves bus $i$,
- $dl_i$: place where conductor leaves bus $i$,
- $nt_i$: number of relief points on run $rn_i$,
- $rp_{i,j}$: bus $i$'s $j^{th}$ relief point,
- $rt_{i,j}$: time corresponding to $rp_{i,j}$.

The time needed to travel from depot $k$, to relief point $l$ is represented by $tt_{k,l}$, where $k = 1, 2, 3$; $l = 1, 2, ..., 20$.

Since there are some 400 runs each of which have between twenty and ninety pieces of information, some
45,000 store locations are involved. Lists L and B, were defined for handling the large amount of data. The variables given as data by the headway sheet define L in the following way:

\[
\begin{align*}
L_{1,1} & = r_{n1} \\
L_{1,2} & = d_{t1} \\
L_{1,3} & = d_{dt1} \\
L_{1,4} & = a_{t1} \\
L_{1,5} & = d_{at1} \\
L_{1,6} & = c_{j1} \\
L_{1,7} & = d_{cj1} \\
L_{1,8} & = c_{j1} \\
L_{1,9} & = d_{j1} \\
L_{1,10} & = c_{j1} \\
L_{1,11} & = d_{j1} \\
L_{1,12} & = r_{j1} \quad \text{where } j > 0 \\
L_{1,13} & = r_{j1} \quad \text{where } j > 0.
\end{align*}
\]

For example, \(L_{10,4,1}\) is the fourth relief time given by the headway sheet for the tenth run.

The variables given as data for fixed pieces of work, i.e. those pieces of work for which \(nt_1\) is zero, define the list B in the following way:

\[
\begin{align*}
B_{11} & = r_{n1} \\
B_{12} & = d_{t1} \\
B_{13} & = d_{dt1} \\
B_{14} & = a_{t1} \\
B_{15} & = d_{at1} \\
\end{align*}
\]
For example, $B_{42}$ is the depot arrival time of the fourth fixed work run.
REFERENCES


