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Three-Tether Wave Energy Converter: Hydrodynamic Modelling, Performance Assessment and Control

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*Thesis submitted in partial fulfillment of the
requirements for the degree of Doctor of Philosophy
in February, 2018.*

Ph. D. Thesis

24th April 2018

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Typeset by the author using $\text{\LaTeX}$.
Printed in Australia.

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Abstract

Hydro, wind and solar power have become major contributors to the global renewable energy market. However, ocean wave power is emerging as a strong contender in the renewable energy mix due to its high power density and minimal environmental impact. Wave energy has the potential to provide an off-grid electricity solution to remote island communities, and fulfil offshore power needs of small industrial projects. One of the best wave energy resources in the world is concentrated along the southern margin of Australia, and if harnessed, wave power could contribute up to 27 per cent of the country's electricity demand by 2050.

Over the past few decades, a large number of concepts and designs have been suggested to convert wave energy into electricity. Despite a huge effort made by industry and the scientific community, the technology for extracting power from ocean waves still remains at a pre-commercial stage of development. The main challenge is to design an economically viable wave energy converter (WEC) where its life-cycle costs (investments, operation and maintenance) can be justified by the amount of generated electricity.

This thesis focuses on the performance improvement of a particular class of wave energy converters, namely, a bottom-referenced fully submerged point absorber, by means of the three-tether mooring configuration. The main contribution is made towards the design, optimisation and control of the converter in order to answer three research questions: (i) what distinctive features of the fully submerged WECs can be utilised to increase their power absorption efficiency; (ii) how geometric parameters of the converter, such as the tether arrangement, shape, and aspect ratio affect the system performance; and (iii) what factors influence the practical implementation of the optimal control strategies on the three-tether WEC. To explore these questions, numerical frequency- and time-domain models have been developed using state-of-the-art techniques based on linear hydrodynamic theory.

In order to gain background knowledge and build a core understanding of the submerged systems, the difference between floating and fully submerged

point absorbers is investigated. Attention is given to the distinctive features observed in the hydrodynamic properties, power production limits, and control performance. Recommendations are provided on the choice of the buoy size and shape, depending on the wave climate of the deployment site. The advantages of employing multiple degrees of freedom in energy harvesting, especially for submerged converters, are demonstrated.

The design considerations of the three-tether WEC are investigated from a number of perspectives including the tether arrangement, mass, shape, and aspect ratio of the buoy. A clear correlation between an optimal tether inclination angle and the buoy aspect ratio is identified. The comparison of three-tether WECs with different buoy geometries is performed not only based on their power output, but also taking into account a range of cost-related performance metrics. Moreover, the benefits of the three-tether converter over its single-tether counterpart are demonstrated through the detailed techno-economic analysis of both prototypes.

The final aspect of this dissertation is devoted to the development of the advanced control system for the three-tether WEC. The causal velocity-tracking controller is taken as a basis and extended to the multivariable control problem. It is demonstrated that the designed controller is able to improve the power absorption of the three-tether WEC as compared to a quasi-standard control approach while imposing a series of technical requirements on the power take-off machinery.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

Nataliia Y. Sergiienko

Acknowledgements

First of all I would like to thank my principle supervisor Professor Benjamin Cazzolato, who accepted me into this project and who has patiently supported and supervised me during the course of this research. I have learnt a lot from his invaluable experience in the field of vibration and control of dynamic systems as well as experimental studies.

Next, I owe huge thanks to my co-supervisor Associate Professor Maziar Arjomandi, for his ability to ask right questions and encouraging to publish high quality research papers. I would also like to thank Dr Boyin Ding, the third person of my supervisory panel, who has provided great support and stimulating conversations during my candidature.

A special thanks goes to Peter Hardy, my colleague and friend, for his helpful comments, ideas, suggestions and insightful discussions. I am also grateful to Dr Markus Wagner and other members of his team for the productive collaboration around the topic of modelling and optimisation of wave energy farms.

Finally, I wish to thank colleagues and friends who have contributed to the enjoyable and motivating atmosphere at work during these years.

Authors' Comments

All publications within this thesis are identical to the original article as published or submitted, with the following exceptions:

1. The typesetting and referencing have been altered so that there is a consistent format throughout the entire thesis.
2. The positions of the figures and tables may differ from the final published version of the papers to improve readability.
3. The numbering of tables, figures and equations of each publication has been changed to include the number of the chapter.

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Acronyms

ACE	Average climate capture width per characteristic capital expenditure
AEP	Annual energy production
BEM	Boundary element method
CapEx	Capital expenditure
DOF	Degree of freedom
EKF	Extended Kalman filter
IMC	Internal model control
IRF	Impulse response function
LCoE	Levelised cost of electricity
LQG	Linear quadratic Gaussian
MIMO	Multiple-input multiple-output
MPC	Model predictive control
OpEx	Operational expenditure
OWC	Oscillating water column
OWSC	Oscillating wave surge converter
PA	Point absorber
PTO	Power take-off
RAO	Response amplitude operator
RCW	Relative capture width
SDC	Spring-damper control
SISO	Single-input single-output
TRL	Technology readiness level
VTC	Velocity tracking control
WEC	Wave energy converter

Chapter 1

Introduction

1.1 Background and prospects

Energy is the driving force of modern society and its availability determines the quality of life and economic development of nations. Over the last 25 years the global energy consumption has almost doubled and every year it continues to grow at a rate of approximately 1.8% p.a. [3]. This is caused not only by the ever-increasing world population, but also by the booming economies of some developing countries, e.g. India and China. Currently, there are three main groups of energy sources that are used for electricity generation, transportation and industry:

fossil fuels represented by oil, coal, and natural gas, account for 85.5% [3] of the total energy demand. Such an active use of fossil fuels has led to a negative impact on the environment including air pollution, acid rains and exhaustion of natural resources [2];

nuclear energy currently contributes 4.5% [3] and its use is a controversial topic due to the several reasons. From one side, it is a clean source of energy that does not involve combustion of fossil fuels and does not cause significant air pollution; and from other side, there are increasing concerns about the safety, health, cost, disposal of radioactive waste, and weapon proliferation;

renewable power accounts for almost 10% [3] of the global energy consumption and includes hydro, solar, wind, marine power, biomass, etc. Sustainable energy sources are a vast and virtually inexhaustible with minimal global warming emissions. Power generation from renewable sources rose by 14.1% over the past year (2016) demonstrating

a social acceptance and economic viability of renewable technology deployment.

Continuous use of fossil fuels over the last two centuries has been accompanied by the emission of carbon dioxide, methane and other greenhouse gases into the atmosphere which has led to global climate change and a rise in the Earth's mean surface temperature. It has been estimated that in order to limit the global temperature increase to 2 °C above pre-industrial levels, almost complete decarbonisation of global electricity systems is required by 2050 [2]. To address this problem, at least 67 countries have set renewable energy targets that will stimulate greenhouse gas emission reductions and encourage the additional generation of electricity from sustainable sources [16]. The energy sector is forced to look for new solutions and cutting-edge technologies towards pollution-free power generation. In this rapid deployment of renewable energy, ocean wave power is emerging as a potentially viable source of electricity, attracting the attention from researchers, government and industry sectors.

1.2 Ocean wave energy

Ocean surface waves carry a significant amount of renewable energy that could make a major contribution to the global energy needs moving towards a sustainable future. One of the reasons why ocean waves are attractive for harvesting is that they have the highest power density among the sources of renewable energy [5] and can be considered as a concentrated form of solar power. According to the most recent estimates, the global wave energy potential is approximately 37,000 TWh/yr (or 3.7 TW) [18] which is twice the worldwide present-day electricity consumption [3]. However, this estimate does not take into account geographical, economic and technical constraints that could prevent the effective conversion of wave power into electricity. Therefore, according to a more realistic assessment, the amount of wave energy suitable for harvesting is an order of magnitude less [24]. A map showing the worldwide distribution of average annual wave energy transport is demonstrated in Figure 1.1.

Australia has one of the best wave energy resources in the world concentrated along its southern margin due to the long coastline, large uninterrupted fetch and strong Southern Ocean winds [14]. The wave energy resource that intersects the 25 m depth contour around Australia has been estimated to exceed 1800 TWh/yr [13]. However, if to take into account proximity to the existing grid infrastructure, marine protected areas and capacity of available

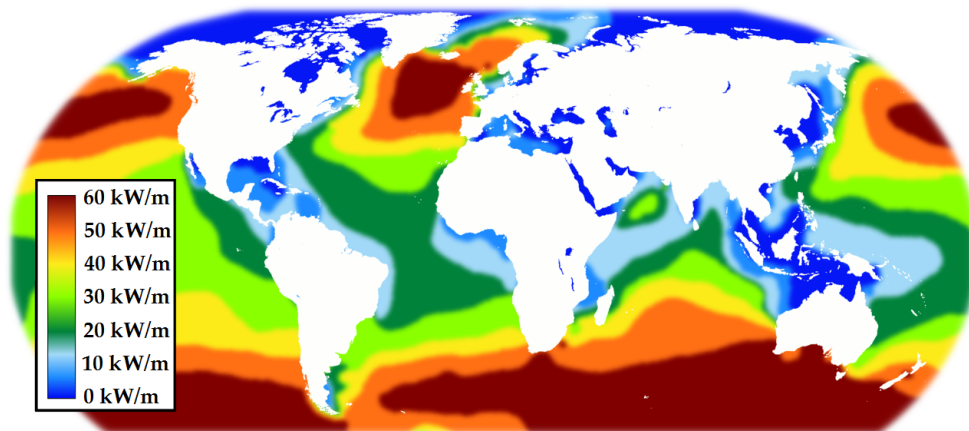


Figure 1.1: The global annual average wave energy transport, adapted from [1].

wave energy prototypes, approximately 67 TWh/yr [13] can be harvested from ocean waves contributing up to 27 per cent of the present Australian electricity demand.

1.3 Wave energy technology

Over the past 40 years, more than 1,000 wave energy conversion concepts have been proposed to harvest and convert wave power into a usable form of energy, e.g. electricity. However, only a few prototypes have been tested at a full scale, and none of these technologies have reached economic viability. Such a relatively slow development of the wave energy technology is mostly associated with the harsh operating environment, where the converter is exposed to extreme wave loads affecting its survivability and longevity. This, in turn, has a negative impact on the economic performance of the system.

Existing wave energy prototypes differ in the operating principle (oscillating water column, oscillating body, overtopping device or submerged pressure differential), number of modes of motion involved in the power absorption (heave, pitch, surge or their combination), reaction source (seafloor, fixed structure or converter itself), relative dimensions and orientation to the incident wave (point absorber, terminator or attenuator), etc. Even though the wave energy sector is yet to reach convergence on the most suitable technology, oscillating bodies constitute a major class of wave energy converters [15] being the focus of this thesis. The following guidelines for the design of the oscillating WECs (some of which are still debated) have been suggested in the literature [6, 11, 21] to accelerate their further development:

- (i) the wave absorption system should operate at or close to the water surface where the wave power is concentrated [10, 23];
- (ii) the device should be designed to withstand high loads and large motion amplitudes [12];
- (iii) the absorbing body should have good wave generating properties leading to high power absorption levels [8];
- (iv) the WEC should exploit more than one mode of motion in order to maximise its power production [4, 7, 8, 19];
- (v) the converter should be equipped with an advanced control system to utilise the full capacity of the installed structure (buoy) [9, 17, 22];
- (vi) when operating in an array, WECs should share basic facilities including mooring systems, electricity cables, etc. [21];
- (vii) the development of WECs should go through the successive stages from the modelling of the conceptual prototype and its experimental testing in small scale to the demonstration of the array performance [20].

As a result, any new design or concept of the oscillating wave energy converter (WEC) should follow these recommendations in order to show the long-term economic potential of the proposed technology.

1.4 Thesis

1.4.1 Thesis aims and scope

The CETO system developed by Carnegie Clean Energy, Australia is one of the few wave energy converters that has achieved a technology readiness level of 8 [20]. This is a bottom-referenced fully submerged quasi-point absorber that harvests wave power from heave motion. Being fully submerged this WEC has a number of advantages over other prototypes: (i) it is invisible from the shore, and (ii) has higher survival in storm conditions. However, this system does not realise the full capacity of the installed structure (buoy) being actuated in only one degree of freedom. Therefore, the main motivation behind the work undertaken in this thesis is to explore the possibility of improving the performance of wave energy converters similar to the CETO system by adding more controllable hydrodynamic modes through a three-tether mooring configuration.

This thesis is dedicated to the investigation of the behaviour of the submerged three-tether axisymmetric wave energy converter including hydrodynamic design, techno-economic assessment and development of control strategies. The main objectives of this research are summarised as follows:

- (i) to generalise and organise the existing knowledge about wave energy converters that operate under water, and to build a new understanding about the performance and dynamics of these systems;
- (ii) to develop a set of recommendations and general guidelines for the hydrodynamic design of fully submerged systems;
- (iii) to investigate the effect of the tether arrangements on the performance of the three-tether wave energy converter;
- (iv) to identify an optimal aspect ratio of the submerged converter that absorbs power from heave and surge motion modes;
- (v) to investigate possible financial benefits of employing three tethers instead of one;
- (vi) to build an understanding about the controllability of the three-tether system.

1.4.2 Thesis outline

With the exception of Chapters 2, 3 and 8, this thesis is formatted as a collection of manuscripts that have been published or currently under review in high-quality journals and conference proceedings.

Chapter 2 presents a literature review of the existing wave energy technologies with a special focus on the oscillating bodies. The fundamental wave energy absorption principles and factors that influence the converter design are also specified. The critical review of literature specific to the scientific contributions presented in this thesis is expanded in Chapters 4–7.

A basic linear theory of ocean waves is presented in Chapter 3, followed by the mathematical techniques for modelling wave/structure interaction. The state-of-the-art frequency- and time-domain models of the oscillating wave energy converters that have been used in published results presented in this thesis are also demonstrated.

Distinctive properties of fully submerged axisymmetric converters are investigated in detail in Chapter 4. The relationship between the power absorption ability, scalability, controllability and the submergence depth of oscillating converters is studied.

Considerations on the design of the three-tether system are addressed in Chapter 5. The sensitivity of the power production to the geometric and mass characteristics of the WEC is investigated under realistic sea conditions.

A thorough analysis of the performance, dynamic responses and load forces of the three-tether system is presented in Chapter 6 and compared with the behaviour of the single-tether counterpart. This technical content is supplemented by an economic component with the aim to identify the commercial potential of the three-tether technology.

In Chapter 7, a velocity tracking controller for the improvement of the power production of the developed WEC is proposed. An in-depth analysis of the system dynamics is performed leading to a specific design of the low-level controller. Considerations on the practical implementation of this control strategy are presented as well.

Finally, the main conclusions and a list of contributions to the research field made throughout this thesis are documented in Chapter 8 along with recommendations for future work.

1.4.3 List of publications included as part of the thesis

1. Sergiienko, N. Y., Cazzolato, B. S., Ding, B. and Arjomandi, M. (2016). "An optimal arrangement of mooring lines for the three-tether submerged point-absorbing wave energy converter". *Renewable Energy* 93, pp. 27–37.
2. Sergiienko, N. Y., Cazzolato, B. S., Ding, B. and Arjomandi, M. (2016). "Three-tether axisymmetric wave energy converter: Estimation of energy delivery". In: *Proceedings of the 3rd Asian Wave and Tidal Energy Conference*. Vol. 1. Singapore, pp. 163–171.
3. Sergiienko, N. Y., Cazzolato, B. S., Ding, B., Hardy, P. and Arjomandi, M. (2017). "Performance comparison of the floating and fully submerged quasi-point absorber wave energy converters". *Renewable Energy* 108, pp. 425–437.
4. Sergiienko, N. Y., Cazzolato, B. S., Hardy, P., Arjomandi, M. and Ding, B. (2017). "Internal-model-based velocity tracking control of a submerged three-tether wave energy converter". In: *Proceedings of the 12th European Wave and Tidal Energy Conference*. Ed. by A. Lewis. Cork, Ireland, pp. 1126-1–1126-8.

5. Sergiienko, N. Y., Rafiee, A., Cazzolato, B. S., Ding, B., and Arjomandi, M. (2018). "Feasibility study of the three-tether axisymmetric wave energy converter". *Ocean Engineering* 150, pp. 221–233.
6. Sergiienko, N. Y., Cazzolato, B. S., Arjomandi, M. and Ding, B. (2018). "Considerations on the control design for a three-tether wave energy converter". Submitted to *IEEE Transactions on Sustainable Energy*.

1.4.4 Additional publications relevant to the thesis but not forming part of it

1. Wu, J., Shekh, S., Sergiienko, N. Y., Cazzolato, B. S., Ding, B., Neumann, F., and Wagner, M. (2016). Fast and effective optimisation of arrays of submerged wave energy converters. In: *Proceedings of the 2016 on Genetic and Evolutionary Computation Conference*. Denver, CO, USA, pp. 1045–1052.
2. Arbonès D.R., Ding B., Sergiienko N.Y., and Wagner M. (2016) Fast and effective multi-objective optimisation of submerged wave energy converters. In: *Proceedings on 14th International Conference on Parallel Problem Solving from Nature*. Springer, Cham, pp. 675–685.
3. Ding, B., da Silva, L. S. P., Sergiienko, N., Meng, F., Piper, J. D., Bennetts, L., Wagner, M., Cazzolato, B. and Arjomandi, M. (2017). "Study of fully submerged point absorber wave energy converter-modelling, simulation and scaled experiment". In: *The 32nd International Workshop on Water Waves and Floating Bodies (IWWWFB)*. Dalian, China.
4. Ding, B., Sergiienko, N., Bleckly, B., da Silva, L. S. P., Cazzolato, B., and Arjomandi, M. (2017). "Power-take-off control in a scaled experiment of a point absorber wave energy converter". In: *Proceedings of the 12th European Wave and Tidal Energy Conference*. Ed. by A. Lewis. Cork, Ireland, pp. 1077-1–1077-7.

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Chapter 2

Wave energy conversion: A literature review

The present thesis focuses on the performance assessment of the three-tether bottom-referenced fully submerged quasi-point absorber. The purpose of this chapter is to justify choosing to research this particular wave energy technology by reviewing existing prototypes, factors that influence their power absorption efficiency and techno-economic performance, and to demonstrate the motivation behind the research carried out in the framework of this dissertation.

2.1 Methods to extract wave energy

The first mention of converting ocean waves into electricity dates back to the 18th century, where the first wave conversion technique was patented in 1799 [43]. The modern era of the wave energy research started after the global oil crisis during the 1970s, predominantly in the United Kingdom [24]. A broader research community was attracted to this topic when the canonical paper by Salter [75] was published in *Nature*. Since then, research in this area was carried out with varying financial support from government agencies [22] which has led to more than one thousand [51, 69] patented technologies for wave energy conversion. Despite such a huge effort, the WECs have not converged to one, or several, dominant prototypes with demonstrated commercial viability.

Existing wave energy converters can be categorised in different ways as shown in [44, 53, 58]. One of the possible classifications is demonstrated in Figure 2.1.

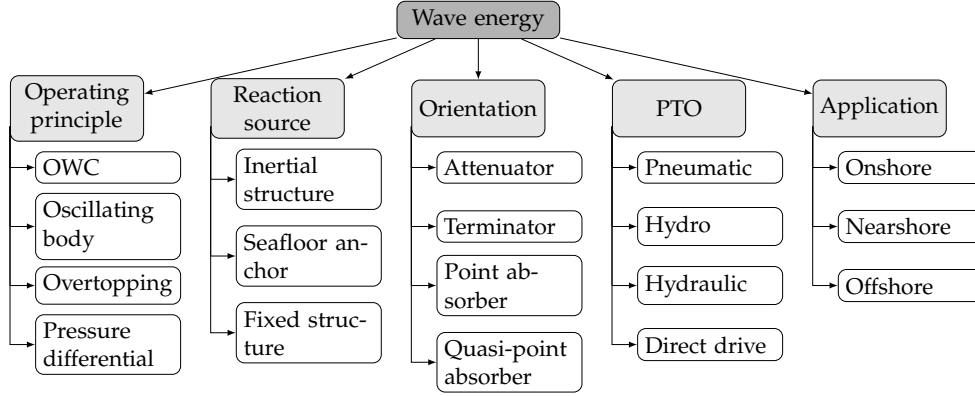


Figure 2.1: Classification of existing wave energy converters, adapted from [47].

2.1.1 Operating principle

Operating principle defines how a wave energy converter interacts with an incoming wave field, and how the energy contained in a wave is absorbed. The classification by working principle was proposed by Falcão [29], where all converters are divided into three categories: oscillating water columns, oscillating bodies and overtopping devices. In this work, one more category is distinguished, namely submerged pressure differentials, which can be considered as a subset of oscillating bodies but have rather specific characteristics.

An **oscillating-water-column** (OWC) is a fixed or floating hollow structure which uses the wave-induced change of the water level inside the chamber to compress and decompress trapped air [28]. The difference in pressure inside and outside the chamber forces the air to flow through a turbine which is coupled to the generator (Figure 2.2a). If built close to the shore, OWCs can serve as breakwater structures in order to protect the coastline [56].

An **oscillating body** is a collective term used to describe wave energy converters that harvest power from the wave-induced oscillations of the floating or submerged structures predominantly in heave or surge (refer to Figure 2.2b). More details on this WEC type will be provided in Section 2.2.

An **overtopping device** is a floating or bottom fixed structure that makes ocean waves to break and elevate to a storage reservoir where the water is returned to the ocean by flowing through a number of turbines (refer to Figure 2.2c). This operating principle is similar to the hydro power plant but applied to an offshore floating platform.

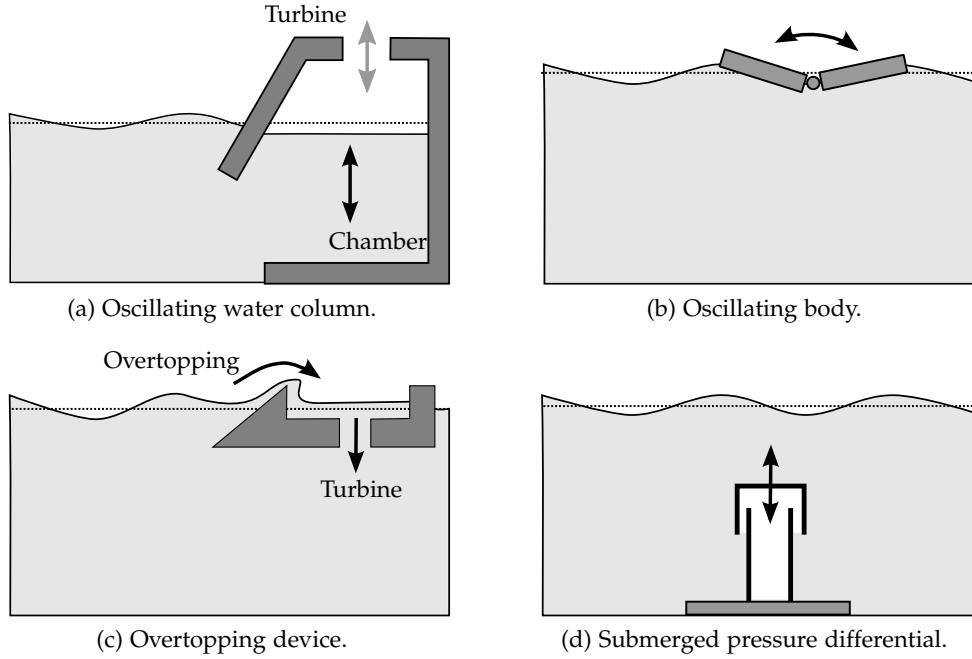


Figure 2.2: Types of WECs depending on the operating principle.

A **submerged pressure differential** consists of one or several air-filled chambers where the pressure varies depending on the phase of the incident wave (crest or trough), see Figure 2.2d. In the case of deformable chambers (membranes), the varying pressure leads to a constant airflow inside the system that is converted into electricity using the air-turbine. This principle is utilised in Bombora WEC [3] and M3 wave [8]. In the case of rigid structures, the air chamber consists of one fixed and one moving part where the varying pressure causes a change in the chamber volume. An up and down motion of the moving part may then be converted into electricity by a linear electric generator, as it is implemented in Archimedes Wave Swing [87].

Although the performance (power output) of these WECs differs significantly even within one category, the approximate hydrodynamic efficiency of these four groups is listed in Table 2.1 based on the collection of published results [5, 8]. Interestingly, overtopping devices are one of the least efficient wave power converters, while the performance of oscillating bodies varies considerably depending on the motion of the device (refer to Section 2.2.3).

As for the number of prototypes, the concept of the oscillating body is the most favoured (78%) by wave developers as shown in Figure 2.3.

Table 2.1: Hydrodynamic efficiency of different WECs [5, 8].

WEC type	Efficiency, %
OWC	29
Oscillating body	
- heaving	16
- surging	37
Overtopping	17
Pressure differential	38*

* Based on one measurement only.

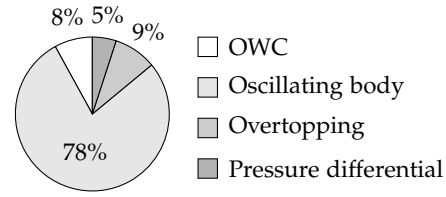


Figure 2.3: Breakdown of WECs depending on the operating principle (as of 2017), based on data from [47].

2.1.2 Reaction point

An extraction of wave energy is only possible when there is a relative motion between two or more working parts of the WEC, whether it be water, concrete/steel structure, or the seafloor. This motion is then utilised by the power take-off machinery for electricity generation. Therefore, WECs can be categorised by the type of the reaction source employed by the system:

inertial structure, such as a suspended plate, buoyant spine or a rotating mechanism located inside the buoy hull, provides a reaction point to the floating device (mainly buoy), where the relative motion between these parts is converted into useful electricity (Figure 2.4a). Such systems are called self-reacting (or self-referenced) WECs;

seafloor reference is mostly used by the tightly-moored converters, where the seabed anchorage is implemented as a deadweight or a pile (Figure 2.4b);

fixed structure, such as a concrete foundation, land, or a jack-up platform, is a preferred option for the onshore and nearshore installations (Figure 2.4c).

For some types of wave energy converters, such as OWCs or heaving devices, the type of the reaction source (fixed or self-referenced) does not have a significant effect on the system efficiency. However, as will be explained in Section 2.2.3, the performance of some oscillating converters strongly depends on the reference point.

2.1.3 Orientation

Depending on the relative dimensions and orientation to the propagating wave, wave energy converters can be grouped as follows:

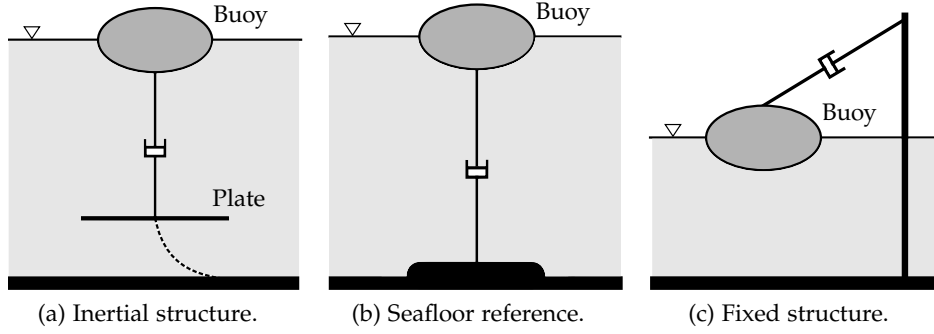


Figure 2.4: Types of WECs depending on the reaction point.

- a terminator** device operates perpendicular to the wave propagation direction (e.g. overtopping devices);
- an attenuator** is oriented parallel to the wave direction and its length is larger than the length of a dominant wave;
- a point absorber** has dimensions significantly smaller than a wavelength and can absorb power regardless the direction of wave propagation;
- a quasi-point absorber** introduced by Falnes and Hals [32] in order to describe axisymmetric WECs that are insensitive to the wave direction (similar to point absorbers), but have relatively large dimensions compared to the wavelength (similar to terminators).

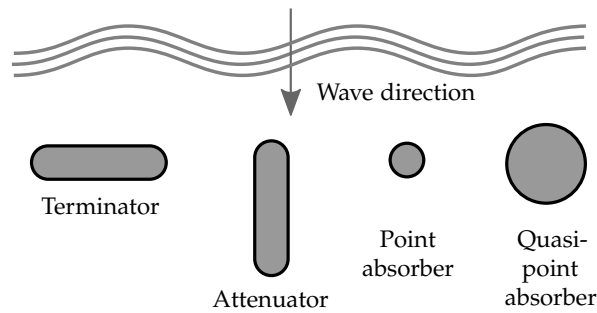


Figure 2.5: Types of WECs depending on their size and orientation relative to the incident wave.

According to a study performed by the International Renewable Energy Agency [47], point absorbers account for more than half of the existing prototypes (see Figure 2.6).

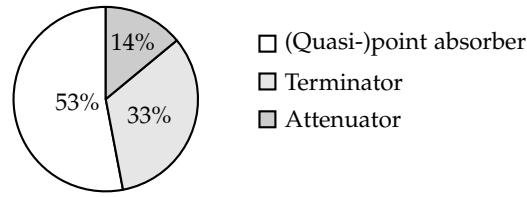


Figure 2.6: Breakdown of WECs depending on their size and orientation relative to the incident wave [47].

2.1.4 Power take-off machinery

An interaction of the wave energy converter with waves leads to a transformation of wave power to the mechanical (body motion), or fluid (air or water flow) power. The objective of the power take-off system is to transform this power to useful electricity. The main types of PTO systems and conversion of the wave energy into electricity are shown in Figure 2.7:

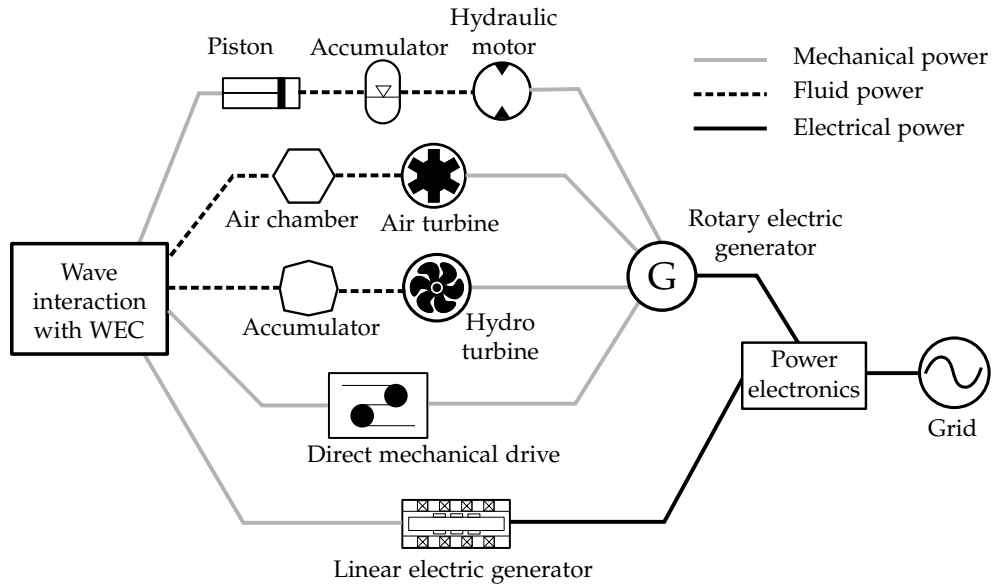


Figure 2.7: Power take-off principles utilised for the wave energy conversion, adapted from [80].

hydraulic PTO system utilises cyclic motion of the hydraulic cylinder driven by the buoy to force pressurised fluid through a controlled manifold to the hydraulic motor, which is connected to the rotary electric generator. As hydraulic circuits are designed to operate with large forces at low speeds, they are well-suited for wave energy conversion purposes and make up the largest part of all PTO technologies;

2.1 Methods to extract wave energy

Table 2.2: Indicative efficiency of different PTO systems [80].

PTO system	Efficiency, %
Hydraulic	65
Pneumatic	55
Hydro	85
Direct mechanical drive	90
Direct electrical drive	95

hydro turbines are used to transform the fluid flow into electricity utilising well-established technologies from hydropower plants. This PTO type is primarily employed in overtopping devices;

pneumatic PTO refers to the air-turbines that are driven by an oscillating air pressure and directly coupled to a generator. This technology is utilised in the OWCs and pressure-differential devices;

direct-drive refers to PTO systems where mechanical power from the buoy oscillations is directly transformed into electricity. While in a direct electrical drive system the buoy is directly coupled to the moving part of the linear generator, direct mechanical drive is equipped with an additional mechanism to couple the buoy motion to a rotary generator.

The main challenge of the PTO development is that the machinery is required to operate at low speeds and large forces, while conventional electrical generators are designed for high-speed motion and low torques. As shown in Figure 2.8, hydraulic systems are the most applicable for absorbing wave energy despite their relatively low efficiency (see Table 2.2).

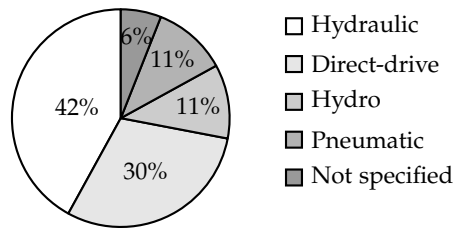


Figure 2.8: Breakdown of WECs depending on the utilised power take-off principle [47].

Power take-off systems also play an important role in the control of wave energy converters. The dynamic behaviour of a PTO machinery dictates what control strategies can be applied to the WEC. However, at the initial stage of the wave energy converter development (numerical analysis of the annual

power output, or experimental validation of the WEC hydrodynamics), it is usually assumed that the PTO can behave similar to a linear system with spring and damping forces.

2.1.5 Application

Depending on the installation location, converters can be divided into three groups:

offshore locations are more attractive for the wave power harvesting due to the higher energy content in deep water waves [24]. However, offshore devices are more difficult to install and maintain, and they should withstand greater loads due to the extreme wave conditions;

nearshore refers to the relatively shallow water where converters are often attached to the seabed (e.g. submerged pressure differentials);

onshore sites are beneficial as they are close to utility network allowing an easy access to the wave power plant, and onshore devices are less likely to be damaged in storms.

The majority of existing WEC technologies are designed specifically for offshore applications as shown in Figure 2.9.

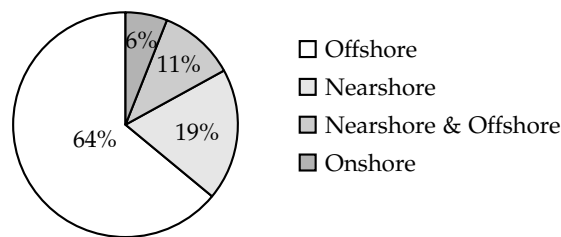


Figure 2.9: Breakdown of WECs depending on the classification of the installation site [47].

Despite the great diversity of existing wave energy converter prototypes, it can be concluded that an oscillating point absorber with a hydraulic PTO system installed offshore is the most common technology for the research and development based on Figures 2.6, 2.8 and 2.9.

2.2 Oscillating bodies

2.2.1 Wave energy absorption principles

Any body placed in water, whether it is a ship, offshore platform, or wave energy converter, interacts with incident waves: some part of the wave field is reflected from the structure, while another part continues propagating forward. If the body is not stationary and experiences an oscillatory motion, it generates (radiates) waves that spread out from the body along the water surface. It is interesting to note that the radiated wave does not depend on the size of the oscillating body, but it is a function of the body dimensions and amplitude of the oscillations that determine how large a generated wave is [30]. Destructive interference between incident and radiated waves leads to the wave energy absorption phenomenon: in order to absorb a wave (power), a 'counter-wave' should be generated by the wave energy device to interfere and cancel an incident wave [30, 82]. This principle is clearly demonstrated in Figure 2.10 showing 100% power absorption by a 2D-body that oscillates vertically and horizontally. As a result, almost all energy contained in a wave can be absorbed by an oscillating terminator-type device.

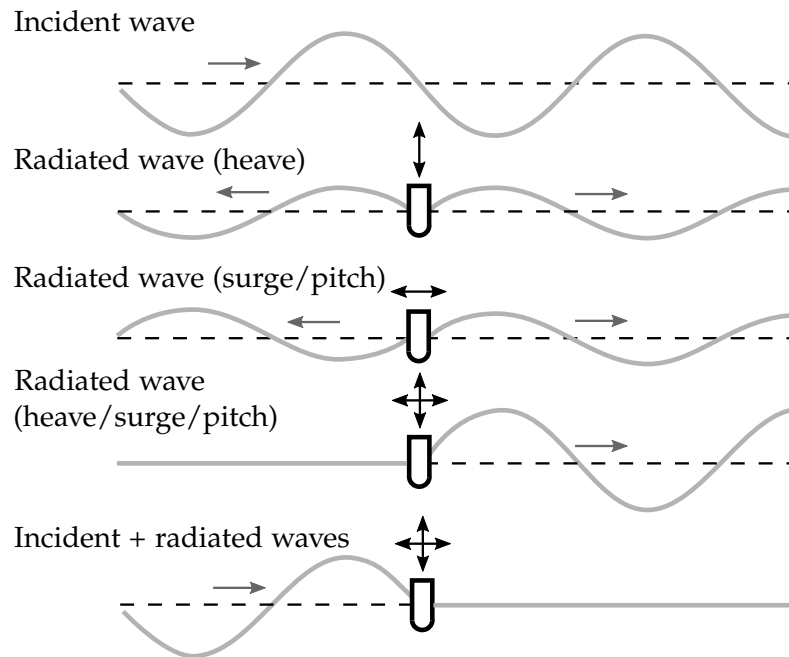


Figure 2.10: Principle of wave power absorption by the 2D-body, adapted from [30].

As shown in Figure 2.10, the pattern of the radiated wave depends on

the degrees of freedom of the oscillating system (for the definition of motion modes adopted in the marine industry refer to Figure 2.11). Thus, heave is considered as a symmetric, or source-type, mode due to the fact that an axisymmetric body oscillating in heave radiates circular waves (Figure 2.12a), while motion in surge or pitch generates dipole waves (Figure 2.12b) [32]. This difference in the radiated wave patterns is a key factor that defines power absorption limits of the oscillating wave energy converters [32, 82].

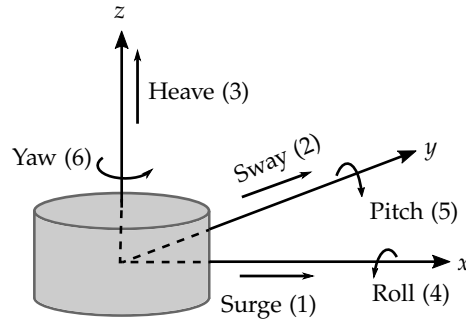
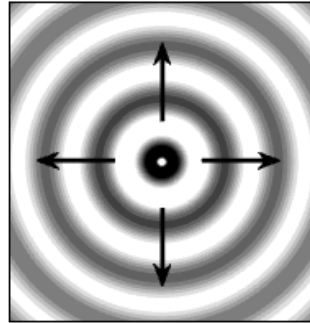
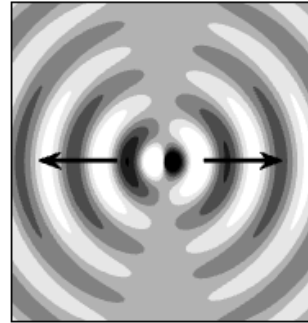


Figure 2.11: Six degrees of freedom of the rigid body modes (mode numbers).



(a) Monopole or circular waves (heave).



(b) Dipole waves (surge or pitch).

Figure 2.12: Types of waves radiated by an axisymmetric body in different modes, adapted from [82].

2.2.2 Power limits

An infinitely long oscillating terminator-type device (or an array of point absorbers) is able to harvest 100% of the incident wave power, when it oscillates in heave and surge simultaneously [30]. However, for an axisymmetric

body, the amount of energy that can be removed from the sinusoidal wave is limited by the radiation properties of the system [12, 26, 63]:

$$P_{\max} = \alpha \frac{J}{k} = \alpha \frac{\lambda}{2\pi} J, \quad (2.1)$$

where $\alpha = 1$ for oscillations in heave, and $\alpha = 2$ for surge or pitch, J is the wave energy transport per unit frontage of the incident wave, λ is the wavelength, and k is the wavenumber.

As a result, the efficiency of an oscillating WEC is maximised when surge, heave and pitch modes are involved in the power production, reaching the level of $P = (3\lambda/2\pi)J$. In this case, 1/3 of the total power will be absorbed from oscillations in heave, while remaining 2/3 will be captured from surge and pitch. Due to the fact that both surge and pitch are dipole-type modes that radiate waves of the same pattern, only one of the two may be considered for power absorption purposes, as it is not possible to improve the efficiency of the system including the second dipole mode [30].

The limit presented in Equation (2.1) shows how much energy can be removed from the ocean wave regardless the WEC size or motion amplitude. However, any converter is designed to operate within a constrained working envelope which imposes an additional upper bound on the amount of absorbed power [13, 32]:

$$P_{\max} = \frac{\pi}{4} \rho g \frac{V_s H}{T}, \quad (2.2)$$

where V_s is an available volume stroke, ρ is a fluid density, g is the acceleration due to gravity, H is the wave height and T is the wave period. Equation (2.2) is only valid for the floating converters that absorb power from heave and operate in deep water waves. More details on other modes of motion and fully submerged bodies are provided in Chapter 4.

The graphical representation of the two upper limits from Equations (2.1) and (2.2), which is usually referred to as Budal's diagram [13], is demonstrated in Figure 2.13. The curve of the actual absorbed power (black solid line) approaches these bounds at the low- and high-frequency limits.

Despite a clear difference between heave and surge modes of oscillation in terms of their radiation patterns and power absorption limits, there is no single answer on which mode is the most attractive for conversion purposes [32, 34, 82]. For relatively small systems with a horizontal extension of $\approx \lambda/15$, the source-type mode is more efficient in its interaction with incident waves than the dipole modes (surge and pitch) [83]. The opposite is true for the large converters ($\approx \lambda/5$) as their efficiency in surge is higher than in the

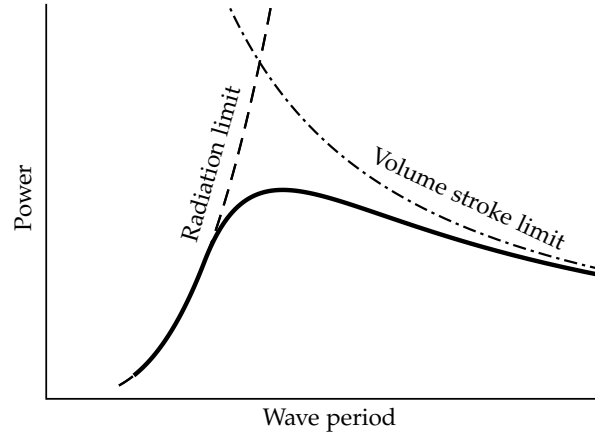


Figure 2.13: An example of the upper limits on the power that can be absorbed by an axisymmetric converter oscillating in heave. The black solid line corresponds to the absorbed power by an optimally controlled WEC.

heave mode. Regardless of all this, it can be concluded that the combination of several motion modes is potentially beneficial for any WEC type and provides a more efficient use of the oscillating structure (buoy).

2.2.3 Existing prototypes

Despite an obvious advantage of multi-mode converters, the majority of oscillating WECs generate power from only one degree-of-freedom. This can be explained by the relative simplicity of the power take-off design when only one actuator/generator is employed. More complex engineering solutions must be implemented for multi-DOF systems in order to couple and control several PTO units. Depending on the utilised mode of motion, existing oscillating WECs are divided into three major categories: (i) heaving buoys (Figure 2.14a), (ii) oscillating wave surge converters (Figure 2.14b), and (iii) devices extracting power from oscillations in multiple degrees of freedom (Figure 2.14c).

The comparison of various oscillating WECs is presented below, and any reference to the system efficiency refers specifically to the *hydrodynamic efficiency* of the converter. The latter is usually calculated as a relative capture width (or capture width ratio) to demonstrate the ratio between the input wave power to the system and the amount of absorbed power [22]:

$$\eta_{hydro} = RCW = \frac{P}{JD}, \quad (2.3)$$

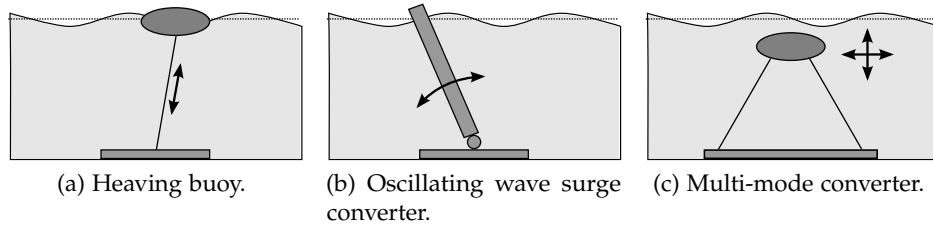


Figure 2.14: Categories of the wave energy converters depending on the utilised degree(s) of freedom.

where P is the captured power [kW], J is a wave resource [kW/m] and D is a characteristic width of the device (e.g. diameter) [m].

Heaving buoys

Heaving devices are usually designed as axisymmetric buoys (predominantly of cylindrical shape) that operate on or just below the water surface and extract power from the vertical motion in waves. The axisymmetric shape of the oscillator allows power to be absorbed regardless the direction of wave propagation which is beneficial for offshore installations with a relatively wide wave-angle distribution. A total of 74 wave energy companies listed by the European Marine Energy Centre [81] focus on the development of heaving point absorbers. These include Ocean Power Technology's Power-buoy [64] (self-referenced two-body floating system with a hydraulic PTO), the CorPower wave energy converter [18] (bottom-referenced floating WEC with a pneu-mechanical drive train), CETO system [15] (bottom-referenced submerged converter with a hydraulic PTO), Seabased WEC [77] (bottom-referenced floating system connected to a linear generator), etc.

Although all heaving devices share a similar operating principle, their efficiency varies from 3% (small Seabased WEC) to approximately 46% (Wave-bob) [7, 10] with an average value of 16% and a standard deviation of 10%. The latter has been calculated based on the analysis of 19 heaving prototypes in the wave resource of 25 kW/m [5]. Also, there is a strong correlation between the characteristic dimensions of the WEC and its efficiency which increases with the diameter of the heaving device [5].

Oscillating wave surge converters

Oscillating wave surge converters (OWSC) consist of a flapping structure (e.g. a plate) that is hinged at the seabed or at a submerged reference platform. Due to the fact that these converters pitch about an axis which is close to the

bottom, the hydrodynamic interaction is similar to the surging motion [82] which is responsible for the name given to this type of system. OWSCs are designed to operate in shallow water waves as the converter's performance is highly dependent on the wave direction and closer to the shore waves travel almost orthogonal to the coastline [33]. Moreover, in nearshore locations, the horizontal velocity of water particles is much higher than their vertical velocity which is beneficial for surging devices.

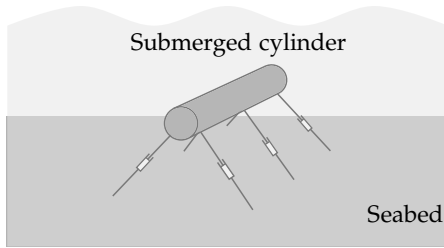
There are currently 14 different oscillating wave surge converters developed around the world [81], including Oyster (bottom-fixed vertical flap with a hydraulic PTO), bioWAVE (bottom-fixed vertical cylinder with a hydraulic PTO), Langlee (two flaps attached to the floating reference frame with a hydraulic PTO), etc. The floating OWSCs have a lower power output than their bottom-fixed counterparts as the reference platform which is designed to be stable also have a tendency to move in waves and does not always provide a sufficiently high impedance reaction point [7]. Considering only fixed OWSCs, their reported efficiency is in the range of 17-72% depending on the width of the device, with an average value of 37% and a standard deviation of 20% [5].

Despite the large deviation in reported performance, it can be concluded that OWSCs have better power absorption properties and approximately twice that of the hydrodynamic efficiency of heaving devices. This is expected given different radiated wave patterns for heave and surge/pitch motion as explained in Section 2.2.2.

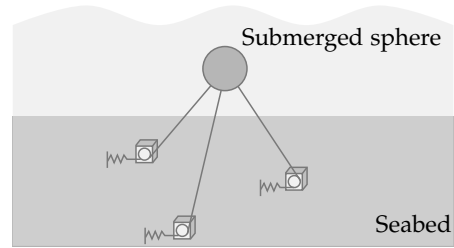
Multi-mode wave energy converters

The number of wave energy converters that capture power from several motion modes (e.g. surge, heave and pitch) is relatively low as compared to uni-modal WEC types. It is necessary to clearly distinguish between multi-mode converters and converters with multiple degrees of freedom, as sometimes in the literature these terms are used interchangeably. Some WECs have more than one degree of freedom, e.g. hinged devices similar to Pelamis [65] and DEXA [91], but they absorb power from only one hydrodynamic mode and are considered as variants of heaving buoys. Therefore, such systems will not be taken into account in this section.

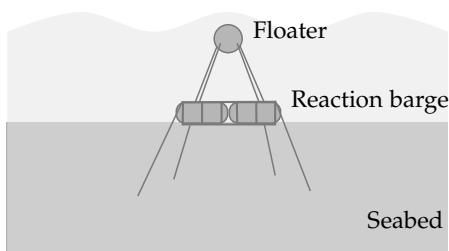
One of the first developments of multi-mode wave energy converters relates to the Bristol cylinder invented by Evans et al. [27] from the University of Bristol, UK. This is a fully submerged long circular cylinder with its main axis parallel to the incident wave, which uses the principle described in [66, 84] to achieve a 100% power absorption. When the cylinder axis moves in



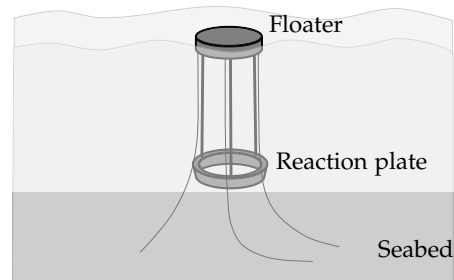
(a) Bristol cylinder [72].



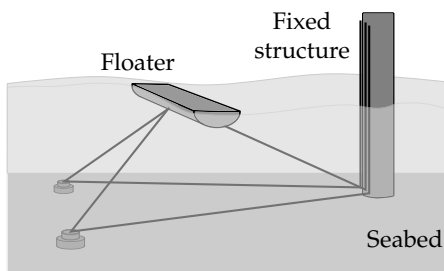
(b) Submerged sphere attached to three cables [78].



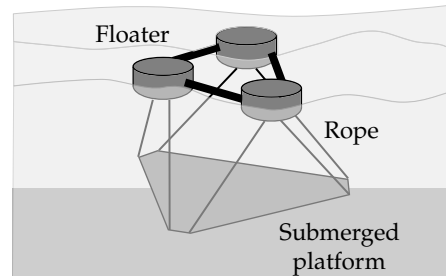
(c) WaveSub by Marine Power Systems [89].



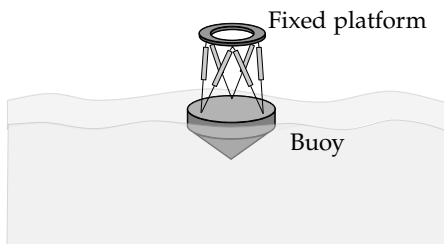
(d) Triton WEC by Oscilla Power [74].



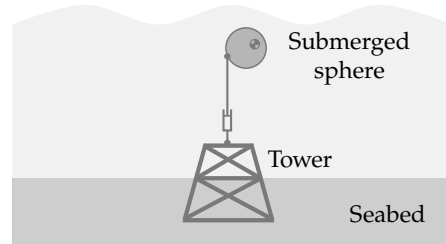
(e) NEMOS WEC [62].



(f) WEC based on Stewart-Gough platform [39, 88].



(g) WEC suggested by Shanghai Jiao Tong University, China [40].



(h) Spherical point absorber with asymmetric mass distribution [57].

Figure 2.15: Examples of wave energy converters that capture power in several modes of motion.

a circle undergoing heave and surge oscillations with a phase difference of $\pi/2$ radians, radiated waves generated above the cylinder on the water surface propagate away from the oscillating body only in one direction. This combination of surge and heave motion is used to cancel the transmitted wave behind the cylinder achieving a full absorption of the incoming wave. The designed size of the cylinder is 75-100 m in length and 12-15 m in diameter, and the power is captured by means of 6 hydraulic pistons located along the device, three on each side as shown in Figure 2.15a. Although the efficiency of this WEC can reach 65% in irregular waves, an initial design had a high estimated cost of produced power due to the high cost of installation and power take-off [72]. As a result, it has not been tested in full scale and has not been commercialised. In order to offer a more affordable solution of the power take-off, an alternative version of the Bristol cylinder was developed [21] where the power is captured from surge motion only (similar to OWSCs) and the PTO system is placed inside the cylinder itself. Despite the difficulties associated with a practical implementation of the Bristol cylinder, its initial design served as a vivid example of the very efficient terminator device (similar to the Edinburgh duck [75]).

In contrast to the Bristol cylinder, Srokosz [78] has suggested to apply an idea of the multi-mode motion to the point absorbing device. The wave energy converter consists of a spherical fully submerged buoy which is connected to three inclined tethers allowing power to be captured from all translational degrees of freedom (see Figure 2.15b). It is assumed that each tether is connected to the spring-loaded cable drum attached to the power generator that converts tether motion into electricity, similar to the direct mechanical drive mechanism. The power output of the system has been analysed for three different arrangements of the tethers: (i) one tether (tether angle from the vertical is 0°), (ii) three tethers are inclined by 45° , and (iii) by 60° . As a result, the three-tether configuration of the power take-off provides an increase in power absorption from two to three times in regular waves as compared to the single-tether mooring. Even though the analysis was dimensionless and purely theoretical, it demonstrated that this system could be an effective absorber of wave power as its capture width in regular waves can be greater than the diameter of the sphere leading to a hydrodynamic efficiency higher than 100%.

The concept of the spherical device with multi-mooring system has been also utilised in the WaveSub wave energy converter [16, 89] developed by Marine Power Systems in collaboration with the University of Bath, UK. The device consists of two main parts: a submerged inertia platform (reaction barge) and a subsurface spherical buoy (see Figure 2.15c). The buoy is connec-

ted to the hydraulic power take-off system located on the platform through the four flexible tethers. The inertia platform is kept in place by means of four taut mooring lines attached to the seabed. The main power absorption principle of the WaveSub is to follow the circular orbit of the water particle while capturing energy from combination of heave and surge motion modes. The system adapts the concept of varying geometry WECs as the submergence depth of the buoy and of the barge can be tuned to maximise power production, or detuned to avoid high wave loads in the storm conditions. The device is currently under development and just recently undertook 1/4 scale testing in the open sea [54]. Due to the fact that the WaveSub is a commercial product, any information on its efficiency is not publicly available.

A similar approach to the two-body system with multiple mooring lines has been adapted to design Triton WEC by Oscilla Power [60]. In contrast to the WaveSub, the Triton WEC employs an asymmetric buoy floating on the water surface and a toroidal submerged reaction plate (see Figure 2.15d). The power is captured from the heave, surge, sway, pitch and roll motion of the floating body that reacts against a relatively stable submerged platform by means of the three flexible tether lines [74]. The power take-off is placed inside the float and designed as a direct mechanical drive system for each tether separately. The Triton WEC took the 4th place (out of 92 initial submissions) in the Wave Energy Prize [85] competition arranged by the US Department of Energy in 2015/2016. Based on the results from the 1/20 scale testing [86], this WEC has an efficiency of 23.5% in irregular sea states, which is slightly higher than an average efficiency of just heaving devices (Table 2.1). However, it seems that such configuration does not take a full advantage of multi-mode energy capture system due to the several reasons: (i) almost parallel arrangement of tethers makes the PTO system insensitive to the surge and sway oscillations, and (ii) a non-rigidly fixed reaction platform significantly decreases power absorption from pitch and roll as compared to the bottom-fixed case.

Another commercial prototype of the multi-mode system is the NEMOS wave energy converter developed by NEMOS GmbH [62]. The elongated floating body is attached to the sea bottom by three tethers as shown in Figure 2.15e. Wave-induced motion of the buoy transmits mechanical energy by tethers to winches and generators placed on the fixed tower above the mean water level. One of the tethers can behave as an actuator to control the motion of buoy and achieve the maximum power output. The orientation of this terminator device can be adjusted according to the direction of the wave propagation. Moreover, this WEC has an ability to submerge the floater in order to protect the system in heavy storms. According to the 1/5 scale

testing [68], the hydrodynamic efficiency of the NEMOS can reach up to 60% in natural sea conditions.

Several research groups [39, 40, 52, 88] have adapted the Stewart-Gough platform for the wave energy conversion purposes. This parallel mechanism with six actuators allows full spatial control (6-DOF) of the system. A WEC depicted in Figure 2.15f [39, 88] consists of three rigidly interconnected floating buoys that are attached to the lower submerged platform by means of six cables (legs). Each leg is equipped with an electric generator to control the motion of the floats and generate electricity. The lower submerged platform works as a reaction plate and is connected to the seabed via a system of slack mooring lines. The details of this WEC design have not been finalised yet; however, it has been found that there is a slight increase (up to 3%) in the converter efficiency when the floats are controlled in heave and surge as compared to the heave mode only.

A similar wave energy converter with six controllable degrees of freedom (see Figure 2.15g) has been suggested by Gao and Yu [40] from Shanghai Jiao Tong University, China, where a conical buoy is attached to the fixed frame by means of six hydraulic cylinders. It is assumed that the power take-off can provide a resistive load only (damping force). The parameters of the system, such as draft, diameter, position of the centre of gravity are optimised to provide the maximum power output for a particular deployment site. The authors claim a 50%-efficiency of the device with the optimised geometry in irregular sea states based on data from the numerical analysis (linear wave theory).

A different concept has been introduced in [57], where a submerged spherical WEC absorbs power from several motion modes while employing only one power take-off unit (see Figure 2.15h). This is achieved by an asymmetric mass distribution (offset mass) of the buoy hull that enhances motion coupling between surge, heave and pitch modes, which in turn are coupled to the tether elongation. However, having only one controllable input imposes a series of limitations on the system performance. The resonance frequency of this WEC in heave can be adjusted changing the PTO control parameters, while the resonance in surge is heavily dependent on the initial length of the tether which should be tuned to a targeted wave climate and cannot be changed during the life-time of the converter. This results in a narrow bandwidth of the system, and the authors claim that in a limited range of wave frequencies, the power output of the spherical point absorber with asymmetric mass distribution can be 2.4 times higher than that of the generic spherical buoy. Their study is limited to monochromatic waves and further investigation is required to demonstrate the efficiency of this converter

under more realistic wave conditions.

It is worth noting that all designs of the multi-mode WECs presented in this section employ inclined degrees of freedom to provide the controllability of the converter in both heave and surge modes. The only difference is in the number of tethers utilised to control the buoy and extract wave power (see Figure 2.16):

- (i) a one-tether configuration provides a full control authority in heave mode only, while indirect control of other degrees of freedom is only possible through the motion coupling mechanism, or non-linearities, in particular geometric;
- (ii) two tethers inclined at 45° are usually used to demonstrate a possible mooring configuration of the 2D-system, such the Bristol cylinder, where the actual number of required power take-off units is strongly correlated with a length to diameter ratio of the terminator device;
- (iii) three is the minimal number of tethers that allow full controllability of the 3D-WEC in all translational degrees of freedom (Srokosz' sphere or NEMOS), or in planar case, control of heave, surge, and pitch;
- (iv) the four-tether solution used in the WaveSub WEC introduces an over-constrained control problem if 3-DOF control is the target, or under-constrained if 6DOF control is required;
- (v) the six-tether solution is the only mechanism which provides controllability for all 6 rigid-body DOFs. However, such control comes at significantly increased capital cost, and therefore, is unlikely to be commercialised.

The number of tethers, and consequently the number of power take-off machineries, is directly related to the capital and operational expenditures of the wave energy converter. Therefore, according to the law of diminishing returns, there should be an optimal configuration where an additional tether and costs associated with it are not justified by the increased power production of the multi-mode converter.

In terms of the efficiency of multi-mode converters, it is difficult to draw a conclusion given the small number of existing prototypes. These systems definitely absorb more energy than heaving only devices and have a higher power to mass ratio. In relation to the oscillating wave surge converters, it is not obvious that multi-mode devices demonstrate better performance. However, as stated above, the development of the converter is not only driven by its hydrodynamic efficiency but also by the cost of electricity that this

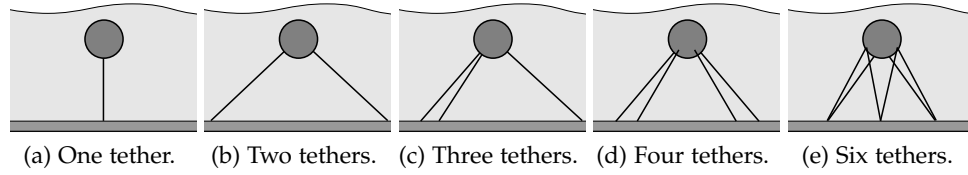


Figure 2.16: Various solutions of the tether arrangements utilised in the multi-mode WECs.

technology can generate, and more detailed techno-economic assessment is required to identify the most cost-effective solution.

2.2.4 Economics of wave energy converters

In the early development of the wave energy converters, most attention has been, and still is, paid to the hydrodynamic efficiency of the device and its annual power output. Maximisation of power production is the main objective function that drives the WEC design including its shape, size, mass-related parameters, etc. For example, McCabe [55] optimised the shape of the floating surging converter using several objective functions, including power, power per unit of characteristic length, and power per unit volume. This study is interesting from the scientific point of view but has a limited practical application as optimised shapes had very complex geometries (many protuberances and sharp corners) which would be difficult to manufacture.

Potential power production is only one of the key indicators that determine the investment attractiveness of a system. Other factors are related to the financial side of the system development including capital (CapEx) and operational expenditures (OpEx), and decommissioning costs. Thus, wave power plants with high hydrodynamic efficiency do not necessarily provide the lowest cost of electricity [67]. For instance, Costello et al. [20] investigated a barge-type wave energy converter, and identified an optimal scale of the device and optimal number of converters in the array based on the non-economic (power output per unit surface) and economic (net present value) objective functions. The results obtained from two optimisation routines differ significantly highlighting the importance of considering the economics of WEC development. Another study [65] has been performed to compare two different technologies, namely Pelamis and WaveStar, based on their economic metrics for the range of deployment sites in Europe. Interestingly, the cost of produced electricity is not only device dependent, but also is affected by the wave climate at a particular site.

The levelised cost of electricity (LCoE) is the most important measure for energy investment and includes all costs associated with the system life-cycle [11, 17]:

$$\text{LCoE} = \frac{\text{CapEx} + \sum_{y=0}^Y \text{PV}(\text{OpEx}_y)}{\sum_{y=0}^Y \text{PV}(\text{AEP}_y)}, \quad (2.4)$$

where PV indicates a present value, AEP is the annual energy production and Y is a total number of operation years.

However, many assumptions are made in order to calculate the LCoE for the wave energy prototypes with low to medium technology readiness levels (TRL). Therefore, some authors have suggested to use the techno-economic measures (see Table 2.3), such as the annual energy production per unit mass or per unit surface area [7, 10], which captures some information on the cost of the system while ignoring operating and maintenance expenses. Another alternative to the LCoE, called Average Climate Capture Width per Characteristic Capital Expenditure (ACE), has been developed by US Department of Energy [85] for the Wave Energy Prize competition to measure the commercial performance of the prototypes. De Andres et al. [23] have identified a strong correlation between LCoE, AEP per unit mass and ACE measures, while there is no clear dependence on AEP per unit surface area. Consequently, power per mass or ACE are indicative when cost estimates are not available and can serve as objective functions for the device optimisation purposes.

Table 2.3: Economic metrics for wave energy technologies, adapted from [19].

	Yield included	CapEx included	OpEx included	Revenue included
<i>Productivity</i>				
Annual energy production (AEP)	+	–	–	–
Capture width	+	–	–	–
Capture width ratio	+	+/-	–	–
<i>Cost of energy</i>				
AEP per unit mass	+	+/-	–	–
AEP per unit surface area	+	+/-	–	–
Average climate capture width per characteristic capital expenditure (ACE)	+	+/-	–	–
Levelised cost of electricity (LCoE)	+	+	+	+/-

+ included explicitly, – not included, +/- included implicitly.

In reality, capital expenditures do not just cover the cost of materials, but also represent the total cost associated with a system development,

construction, transportation, installation, commissioning, and potentially decommissioning. As wave energy converters are designed to operate in a harsh environment with a high variability of sea conditions, their design and capital costs are generally driven by (i) the safety factors of moorings, especially for bottom-fixed wave energy converters [70, 71]; (ii) strength of the buoy hull, especially for floating devices that are subject to slamming events [48]; (iii) maximum stroke provided by the PTO machinery to avoid hard stop events and withstand extreme responses of the buoy, (iv) peak-to-average ratio of absorbed power, etc. Despite the fact that wave devices operate at a partial load for most of the time, they should be designed to withstand larger forces and survive extreme conditions [59]. As a result, Equation (2.4) can be re-written in general terms as:

$$\text{LCoE} = \frac{\text{Cost (Survival mode)}}{\text{Power output (Power production mode)}}. \quad (2.5)$$

It is expected that with an increasing number of built and deployed prototypes, the cost of the device and subsequently, the cost of the produced electricity will decrease significantly as shown in Figure 2.17.

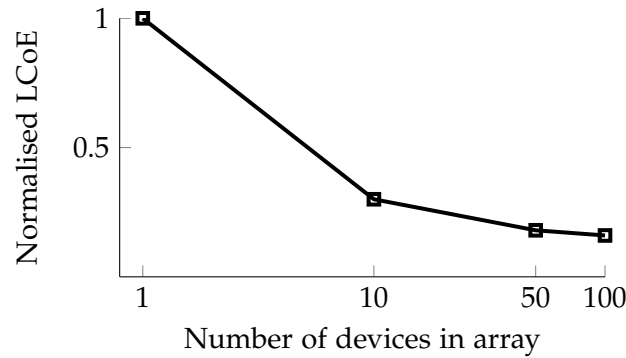


Figure 2.17: The dependence of the LCoE level on the deployment scale of the wave array of two-body floating point absorbers, adapted from [61].

To summarise, the development of the wave energy converter is dictated by energy production, cost and survivability. Therefore, a potential device should possess the following properties: (i) high hydrodynamic efficiency, preferably employing several motion modes, (ii) low loads on mooring system, (iii) improved survivability, e.g. keeping the converter fully submerged, and (iv) shared structures and facilities when operating in arrays.

2.2.5 Control of wave energy devices

Control systems of wave energy converters are usually regarded as a topic separated from the device design serving mainly as a tool that helps to increase the power production of the system. However, a number of studies [41, 42] have demonstrated that the choice of control strategy can considerably affect the geometry of the device as well as the layout of the entire wave farm. The primary goals of control and existing control algorithms are explained below.

As described in Section 2.2.1, the wave power is captured by an oscillating body when there is a destructive interference between the incoming and radiated waves. However, efficient power absorption takes place only when the system oscillates and radiates waves with the correct (optimum) phase and amplitude in relation to the incident wave front [30]:

- (i) the velocity of the oscillating body should be in phase with the dynamic pressure of the incoming wave;
- (ii) the motion amplitude of the oscillator should be adjusted so that the amplitude of the radiated wave is half the amplitude of the incident wave (for one mode of the 2D-body).

Consequently, motion of the wave energy device should be controlled to maximise converted power subject to constraints imposed by the equipment and machinery, such as the maximum amplitude, velocity, load force, etc. [73]. Control is implemented by the power take-off system that exerts a load force on the oscillating body thereby tuning its dynamic response. The development of control strategies for the wave energy applications depends on parameters such as (i) the power take-off capabilities, (ii) availability and complexity of the system model, (iii) availability of information about an incident wave field, (iv) number of controlled variables, etc. Based on these characteristics, control algorithms can be grouped as shown in Figure 2.18.

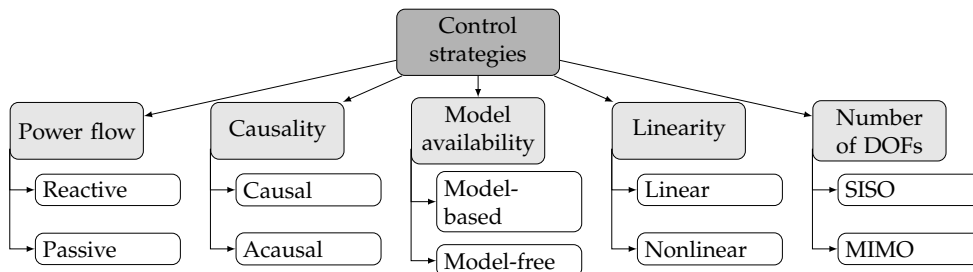


Figure 2.18: Classification of existing wave energy control strategies.

Power flow

The first classification relates to the ability of the power take-off to provide either unidirectional or bi-directional power flow (the fundamental control law is given in brackets):

reactive control ($F_{pto}(t) = m_{pto}\ddot{x}(t) + b_{pto}\dot{x}(t) + k_{pto}x(t)$) implies that the machinery not only receives energy from the converter but also has to put some energy back during a part of a sinusoidal cycle (works partly as a motor) [30]. It requires a bi-directional power flow through the PTO and can be achieved by introducing the energy storage in the system, e.g. a series of accumulators for a hydraulic system. Reactive control was first applied to the Edinburgh duck (also known as the Salter's duck) [75], where the load force was proportional to the duck's velocity and position, imposing a complex-conjugate control law;

passive control ($F_{pto}(t) = b_{pto}\dot{x}(t)$) means that the machinery is only required to extract power from the system, but not to return it [45], which is a reasonable assumption for some types of machineries. Two variations of passive control exist: a *phase control* presented by latching and declutching [6, 9] and a *resistive control* where only a load resistance is adjusted to the current sea state [14, 25].

In terms of the efficiency of these control strategies, Falnes and Hals [32] have demonstrated that the reactive loading outperforms other controllers as shown in Figure 2.19 and uses the full capacity of the oscillating body [32]. Both reactive and latching controls satisfy the phase optimality condition, while the passive loading does not have this capability that leads to a low efficiency of WECs when this strategy is applied.

Causality

It is relatively easy to set the optimal phase and amplitude of the WEC oscillations when the incoming wave is sinusoidal with uniform frequency and height. However, real sea states have a stochastic nature and in order to maximise the absorbed power by applying an optimal control force to the PTO, the wave excitation forces should be known in advance [73]. It means that the wave energy control problem resulting from the optimality conditions is, in general, **acausal** (or non-causal) [31] relying on the future values of the wave excitation force. This future information can be gained by installing additional equipment and sensors at some distance from a converter in the direction of an incoming wave [31, 79] and/or using wave prediction algorithms (e.g. auto-regressive models [35, 36] or augmented

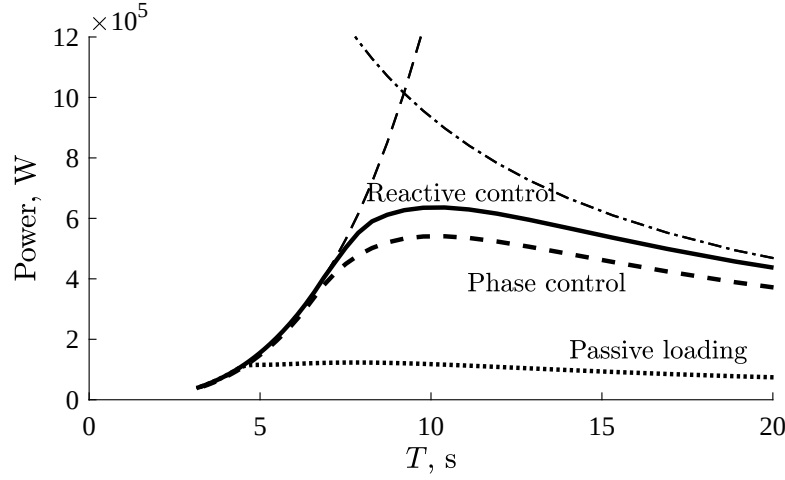


Figure 2.19: Power absorbed by the spherical heaving buoy in regular waves using passive and reactive control strategies, adapted from [32].

Kalman filters [46]). As a result, the performance of acausal controllers is heavily dependent on the real-time wave forecasting which deteriorates with increasing prediction horizon [37].

Other controllers that obviate uncertainties associated with wave predictions are called **causal**. For example, Lattanzio and Scruggs [49] and Scruggs et al. [76] suggested to include the knowledge about the spectrum of the operating sea state in the system model forming a basis of the linear-quadratic-Gaussian control problem. Another method to achieve the causality of the system has been offered by Fusco and Ringwood [38], where a non-causal transfer function between the optimal velocity and the excitation force is replaced by the frequency-dependent coefficient of proportionality. The controllers that tune their parameters to the peak (or energy) frequency of the incident wave [4, 50] also belong to the class of causal controllers.

Comparing performance of these two classes of controllers [45], acausal strategy represented by the model-predictive control (MPC) demonstrates the best power generation efficiency provided perfect wave force estimation is available. However, it comes at the cost of the power take-off complexity that requires high peak-to-average power ratios, large energy storage and high control forces [90]. As a result, similar to the techno-economic development of the wave energy devices, an implementation of advanced control strategies to increase the power production must be subjected to the economic assessment.

Multivariable control

A vast majority of wave energy converters absorb power from only one hydrodynamic mode (heave or surge), so the development of control systems has also focused on the devices that operate in one degree of freedom considering single-input-single-output (SISO) plants. Moreover, when developing new control principles and ideas, it is easier to work with systems that have relatively simple dynamics where no attention should be paid to the coupling between state variables. As a result, almost all effective controllers in wave energy sector have been designed for single-DOF heaving devices [45, 90].

However, the number of converters that absorb power from several motion modes is increasing, and the following features should be taken into account when developing controllers for the multiple-input-multiple-output (MIMO) WECs:

- (i) the modes have different natural frequencies;
- (ii) there is a strong hydrodynamic coupling between surge and pitch;
- (iii) depending on the arrangement of the power take-off, the system can be designed underactuated when the number of control inputs is less than the number of controlled variables (e.g. NEMOS WEC in Figure 2.15e).

One of the first WEC MIMO controllers was applied to the Salter's duck [75] in regular wave conditions, where the control force in heave and surge was equal to the spring load, while control force in pitch had a spring-damper behaviour. However, more advanced solutions should be offered to enhance the power production of multivariable WECs in irregular sea states. Recently, a research group at the Michigan Technological University (US) [1, 2, 92, 93] has developed several controllers (LQG, MPC and multi-resonant feedback control strategies) for the wave energy converter that harvests power from heave, surge and pitch modes assuming independent control inputs for each degree of freedom. It is worth noting that the authors have come to the conclusion that it is possible to achieve the same level of power production if actuation in pitch is removed, which supports the statement about two dipole-modes presented in Section 2.2.2 .

Whereas the previous control laws were for fully-actuated plants, Scruggs et al. [76] developed an LQG optimal controller for an underactuated MIMO device. Their plant consisted of the floating cylindrical buoy tied to three tethers (similar to Figure 2.15b) that are attached to individual electric generators. In this case, the operational space of the PTO system is different from the wave-induced motion of the WEC which is an interesting multivariable

control problem itself. It has been revealed that this control strategy allows a significant improvement of the power efficiency, however it increases the demands on the performance of each generator. Thus, in the range of frequencies close to the surge resonance, two out of three generators should operate as motors actuating the WEC. This means that they do not deliver power to the grid but absorb energy from electrical network. This once again demonstrates that the design of the power take-off machinery cannot be separated from the development of control system.

Overall, control systems have the potential to improve the performance of wave energy converters, in particular reactive control strategies, which can reduce the cost of generated electricity. The number of control approaches developed for multi-mode wave devices is limited and more work should be done to integrate existing SISO controllers into MIMO plants.

2.3 Concluding remarks and perspectives

This chapter reviewed the broad research areas related to this thesis, in particular, the overview of existing wave energy converters, their advantages and associated challenges, and main factors that affect their techno-economic performance.

The overwhelming majority of WECs (with the exception of submerged pressure differentials) are designed as floating devices that operate on the water surface and have a significant water plane area. This is due to the fact that the wave energy content is highest at the ocean surface and diminishes exponentially with depth. However, as shown in Section 2.2.3, there are several submerged prototypes, such as the Bristol cylinder or Srokosz's sphere, that demonstrate high hydrodynamic efficiency in addition to other benefits, e.g. the minimal visual impact. Except for the power absorption potential, several important aspects of the fully submerged oscillating WECs have received little or no attention from the research community. These include the effect of the buoy shape and dimensions on the performance of the individual converter or the wave farm, unique features that should be considered for the control system development, arguments for and against designing a fully submerged system with multi-mode power absorption. These properties are investigated in Chapter 4 of this thesis by comparing the performance of the bottom-referenced floating and submerged quasi-point absorbers that harvest power from one or several motion modes in regular and irregular waves.

All oscillating multi-mode wave energy converters employ the same fundamental hydrodynamic principles and have the same theoretical maximum

power output, as demonstrated in Section 2.2.2. However, the engineering solutions for designing these converters vary in shape and dimensions of the absorbing body, implementation of the multi-DOF power take-off machinery (the number of tethers and their arrangement), controllability of the system, etc. leading to different power efficiencies in practice. A three-tether configuration, utilised in several WECs, has the minimal number of tethers to provide the full controllability of the buoy in all translational degrees of freedom (or heave, surge and pitch for the planar case) while being the most cost-effective solution for the axisymmetric converter. However, this tether arrangement is investigated and documented only for the spherical buoy [78], while other prototypes employing a multi-tether PTO systems (e.g. the WaveSub or NEMOS WEC) are commercial, and therefore, access to their R&D analysis and performance results is limited. Therefore, Chapter 5 is designed to build an understanding of the power output of the submerged WECs connected to three tethers and investigate the sensitivity of the system performance to the design parameters of the absorbing body (buoy).

As outlined in Section 2.2.4, the economics of WECs has come to play an increasing role in the design and assessment of new concepts. Starting from the very initial design stage, WEC developers consider economic performance indicators to compare their technology with existing converters. In this thesis, the three-tether mooring configuration is proposed as an alternative solution to the single-tether arrangement in order to improve efficiency of the bottom-referenced heaving WEC. Therefore, in Chapter 6, the techno-economic analysis of the three-tether system is presented in comparison with the single-tether WEC in order to identify possible financial benefits of the proposed multi-mode concept.

An advanced controller applied to a wave energy converter aims to maximise its power output while handling constraints imposed by the power take-off machinery and other power conversion chain equipment. Most of the control solutions for wave energy conversion purposes have been designed for the oscillating systems with motion constrained to one degree of freedom. However, WECs that extract power from several motion modes require special architecture within the control system due to the coupling induced by the hydrodynamics and the power take-off unit. As demonstrated in Section 2.2.5, several control approaches have been offered for multivariable converters with independent control inputs over each mode of oscillation (surge, heave and pitch). In contrast, the three-tether configuration requires a solution to the underactuated control problem with a strong coupling between all degrees of freedom. As a result, the main objectives of Chapter 7 are to build an understanding about the controllability of the three-tether system, explore

ways to apply controllers previously developed for the heaving device to the WEC under consideration, and investigate possible complications associated with the practical implementation of this controller.

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Chapter 3

Background theory

The main purpose of this chapter is to present methods adopted in the wave energy sector to describe ocean waves and to model the interaction of oscillating bodies with an incident wave front. This is done to justify the choice of modelling approaches utilised throughout this thesis.

3.1 Characterisation of ocean waves

3.1.1 Wave types

An ocean wave is usually characterised by its wave period (T), wavelength (λ), wave height (H) which is twice the wave amplitude, and derived parameters such as the wave steepness ($s = H/\lambda$), wavenumber ($k = 2\pi/\lambda$) and wave frequency ($\omega = 2\pi/T$). The depth of water (h) also can be regarded as a parameter characterising the wave, as the relationship between the wavelength and water depth determines the trajectory of the water particles within a wave, and as a result, the shape of a wave. Therefore, depending on the water depth, ocean waves can be divided into three categories as shown in Figure 3.1:

- (i) deep-water waves travel through water deeper than half of the wavelength [19], where water particles experience a circular motion not being affected by the seabed. In the wave energy sector, the term ‘deep water’ usually refers to the water depth greater than 40 m [6];
- (ii) shallow-water waves travel through the water depth less than 1/20th of the wavelength, and the trajectory of water particles is flattened becoming elliptical due to the proximity to the seabed;
- (iii) transitional waves exist between these two extremes.

Waves travelling from the deep water to shallow sea slow down and dissipate energy as a result of frictional losses with the seabed [7, 10]. For instance, the waves with a power density of 50 kW/m in deep water can contain less than 20 kW/m when travel closer to the shore [3]. Therefore, it may appear that deep water locations are more attractive for wave energy production [6], but this is still a matter of debate in the wave energy community and strongly depends on the type of converter.

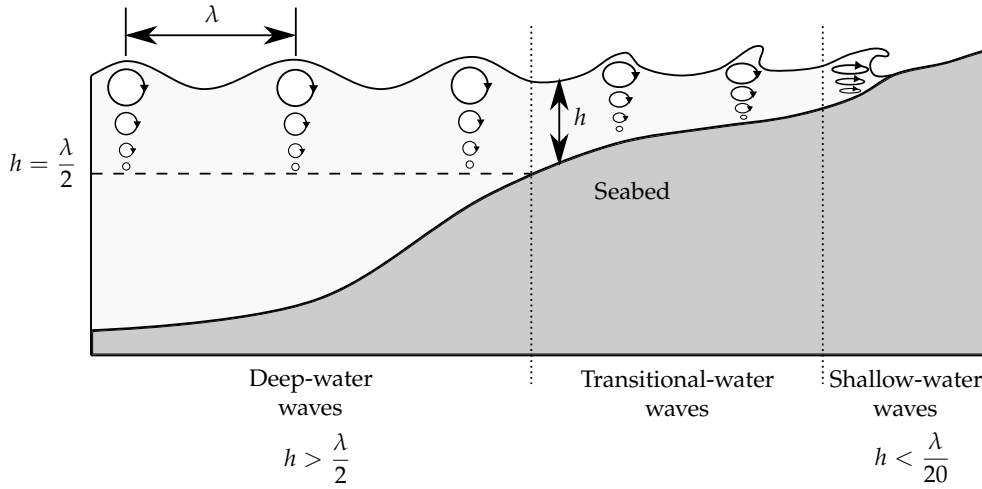


Figure 3.1: Classification of the gravity waves depending on the water depth, adapted from [12].

3.1.2 Wave modelling approaches

An accurate modelling of ocean waves, in particular their surface elevation and flow velocities, plays an important role in the design of coastal and offshore structures. Theoretical, numerical and experimental studies use wave models as the input for estimating wave loads and extreme responses needed for the design and operation.

Regular waves

As shown in Figure 3.1, when a wave travels from deep to shallow water, its shape alters due to the increase of its height and decrease of its speed and length. Thus, deep-water waves with a steepness less than 0.01 can be approximated by a sinusoidal wave applying linear wave theory (inviscid, irrotation and incompressible fluid and a uniform water depth) [10]. Intermediate (transitional) water waves have more nonlinear wave profiles due to the sharper crests and flatter troughs. Such waves are usually described by the

Stokes' wave theory, where the order of nonlinearity is directly dependent on the wave steepness. Shallow water waves of long wavelength as compared to the water depth, can be approximated by the cnoidal wave theory. The suitability of different wave models depending on the relative water depth and wave steepness has been proposed by Le Méhauté [15] and shown in Figure 3.2.

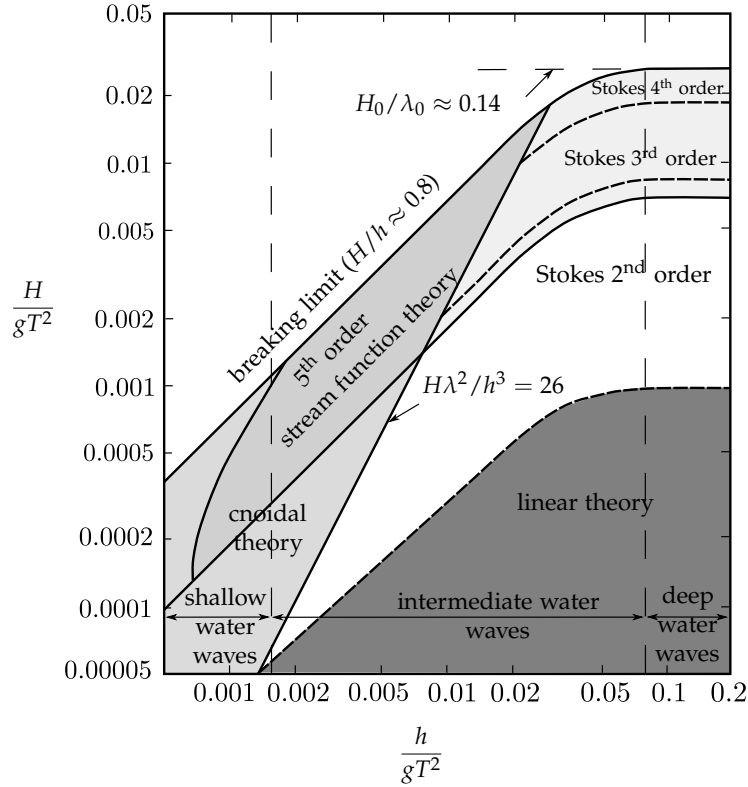


Figure 3.2: Wave model suitability, adapted from [15].

It is generally assumed that wave energy converters operate in the range of sea states where linear wave theory is considered valid. However, as shown in [20, 24], the operational conditions of WECs are within not only linear wave theory, but also cover Stokes waves of 2nd and 3rd order.

Irregular waves (spectral formulation)

The formation of the wind-generated waves occurs according to the following scenario. Firstly, due to the constant input of energy from the local wind, waves start to form and grow (wind waves). Then, waves reach a certain limit and assumed to be fully developed when the energy from wind and energy losses due to the wave breaking are balanced out. The distance travelled

by wind (fetch) and wind strength directly affect whether waves will reach a *fully developed* state or not. Finally, when generated waves are powerful enough to travel long distances even in the absence of wind, they are called *swell* waves [12]. The difference between wind and swell waves is important for the spectral modelling of the wave climate.

The ocean surface elevation is usually characterised by the wave spectrum, which is assumed to be stationary from 20 min to 3-6 h [3]. The latter is a simplified representation of the sea state which shows a distribution of wave energy at different frequencies. In general, the wave spectrum is regarded as a linear superposition of sinusoidal or higher-order waves with different wave frequencies, amplitudes, phases and directions [23]. The main characteristics of the wave spectrum include: (i) a significant wave height which is an average height of the third highest waves (H_s), and (ii) spectral peak wave period (T_p).

The most commonly used spectra include [23]:

Bretschneider, or generalised Pierson-Moskowitz spectrum, describes fully developed seas (Figure 3.3a);

JONSWAP (the Joint North Sea Wave Observation Project) spectrum has been offered to model developing sea states with a finite fetch. This spectrum has a more pronounced peak as compared to the Pierson-Moskowitz spectrum, where an additional parameter $1 < \gamma < 7$ that corresponds to the peak enhancement factor is introduced (Figure 3.3b);

Ochi-Hubble is a two-peak spectrum which is used to model sea states that are formed from swell (low-frequency peak) and local wind-generated waves (higher-frequency peak), (Figure 3.3c).

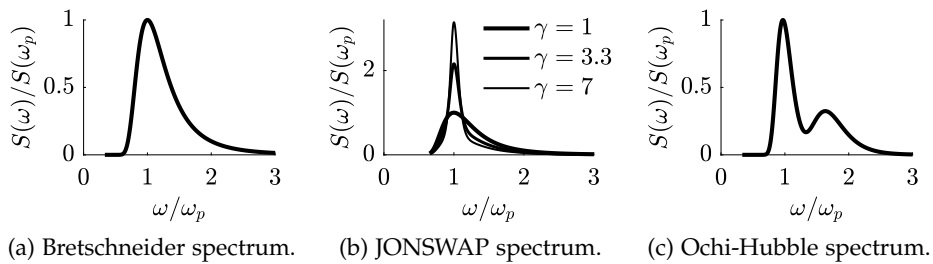


Figure 3.3: Examples of the most commonly used wave spectra.

The wave spectrum is a short-term description of the wave climate. For longer durations, e.g. a year, it is more common to use a scatter diagram

which shows the frequency of occurrence of significant wave heights and peak (or energy) wave periods for a particular sea location (Figure 3.4a). While the scatter diagram shows the average sea state probability across all directions (wave angle insensitive), a wave rose (Figure 3.4b) demonstrates the frequency of occurrence of significant wave heights depending of the wave angle. This statistical information is then used to assess the wave resource and an annual average power output of a particular wave energy converter for a potential deployment site.

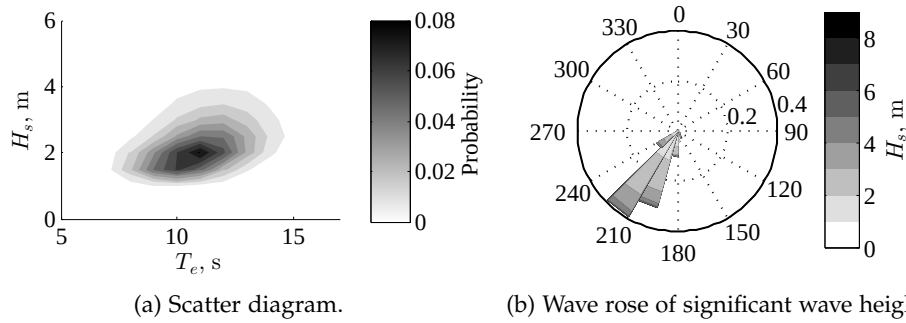


Figure 3.4: Example of the wave climate characterisation at a particular location.

The scatter diagram combined with the Bretschneider spectrum has been used in Chapters 4–6 when assessing the annual performance of the three-tether WEC at several sites around Australia and Europe.

3.2 Wave-body interaction approaches

The interaction of wave energy converters with ocean waves is a highly non-linear process, and there is a range of mathematical and computational tools that offer solutions to this hydrodynamic problem. Penalba et al. [20] have distinguished three main groups of modelling approaches utilised in the wave energy sector: (i) Navier-Stokes and smoothed-particle hydrodynamics; (ii) potential flow models; and (iii) models obtained from experimental data. These groups are summarised in Figure 3.5, demonstrating the compromise between the computational complexity and accuracy of each model.

The choice of the appropriate modelling method is highly dependent on the type of information required (e.g. to estimate average power output) and the application of the model (e.g. to build a control system). For example, a wave energy converter operating in the power production mode experiences relatively low velocities and wave forces that can be estimated by linear wave

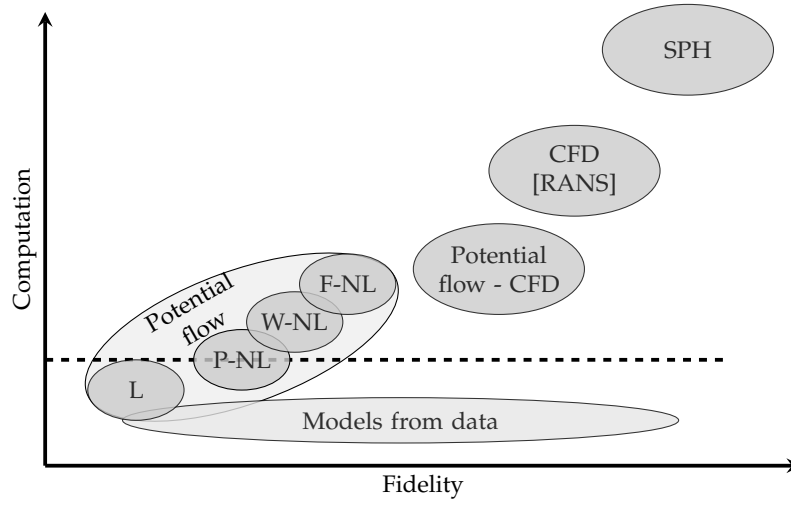


Figure 3.5: Modelling approaches used in the wave energy sector depending on their fidelity and computational cost, adapted from [20]. L refers to linear, P-NL is partially nonlinear, W-NL is weakly nonlinear, F-NL is fully nonlinear, CFD refers to computational fluid dynamics, RANS is Reynolds-Average Navier-Stokes equation, and SPH is smoothed-particle hydrodynamics. The black dashed line corresponds to the limit for models with low computation requirements.

theory with sufficient accuracy. However, the nonlinear behaviour becomes more important when the wave device operates in extreme sea states in the survival mode. The relevance of different modelling approaches depending on the operating conditions of the wave converters is illustrated in Figure 3.6 [20, 22].

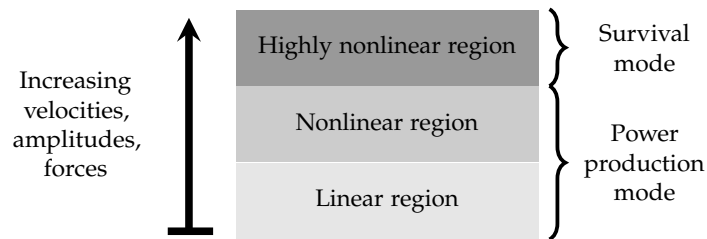


Figure 3.6: Operating modes of the wave energy converter, adapted from [22].

The most common linear and nonlinear external forces and physical effects in wave energy converters that are utilised to model wave-body interaction are listed below:

excitation force* is the incident wave load acting on a fixed body which

is calculated by superposition of the Froude-Krylov and diffraction forces. Froude-Krylov force is a result of the unsteady pressure field generated by undisturbed wave assuming that the body is absent, while the diffraction force is caused by the floating body disturbing the wave field. Linear approaches take into account only the mean position of the body, while nonlinear methods calculate the excitation force over the time-dependent instantaneous wetted surface [13];

radiation force* corresponds to the force acting on the moving body in the absence of incident waves. It can be broken into two main components: added mass forces proportional to the body acceleration, and hydrodynamic damping forces proportional to the body velocity;

hydrostatic force* is a result of the fluid pressure acting on the submerged part of the body surface. The force can be calculated over the mean (linear) or instantaneous (nonlinear) wetted surface;

wave drift is a second-order force caused by the non-symmetric wave loading on the body, or by the interaction between the wave and body motions;

viscous force* represented by the form drag and skin friction is a force acting opposite to the relative motion between body and fluid. The relevance of viscous effects for wave energy converters increases with decreasing device dimensions relative to the wavelength [4];

parametric excitation is caused by the nonlinear coupling between several degrees of freedom of the moving body. This phenomenon is mostly observed when the wave period is half of the resonance period of pitch or roll [11];

slamming is a highly nonlinear process of a very short duration when the bottom structure of the converter impacts the sea surface. This force is important for the structural analysis of the floating wave energy converters under extreme wave conditions;

mooring force is a positioning force that counteracts the horizontal wave drift force keeping the floating buoy in place. Mooring force is more important for the slack-moored devices;

machinery force* is a force provided by the power take-off system to control the buoy motion.

Forces marked with an asterisk (*) are the most important forces that affect the system dynamics in the power production mode, while other loads are more relevant for the survival mode of operation. Despite the fact that nonlinear approaches have become more common in the wave energy community, models based on linear assumptions remain essential where a large number of evaluations and iterations are required to optimise the design of a WEC under various sea conditions or assess its annual average energy production. As a result, it has been suggested in [20] to utilise linear and partially nonlinear modelling approaches supplemented by viscous drag forces for the power assessment, model-based control, and optimisation purposes.

3.3 Linear wave theory

Linear wave theory is a mathematical formulation of the propagation of gravity waves on the surface of an ideal fluid. The theory assumes that the water depth is uniform, and the fluid flow is incompressible, irrotational and inviscid. Furthermore, the wave amplitude is considered small compared to the wavelength.

3.3.1 Governing equations

There are two fundamental equations that describe the hydrodynamic interaction within the volume of fluid, defined by its density $\rho(x, y, z, t)$ and particle velocity $\vec{v}(x, y, z, t)$ [17]: (i) the conservation of mass, or continuity equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad (3.1)$$

and (ii) the conservation of momentum, known as the Navier-Stokes equation:

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} = -\frac{1}{\rho} \nabla p_{\text{tot}} + \nu \nabla^2 \vec{v} + \frac{1}{\rho} \vec{f}, \quad (3.2)$$

where p_{tot} is the pressure of the fluid, ν is the kinematic viscosity coefficient, and $\vec{f}$ is an external force per unit volume.

If only gravitational force is considered $\vec{f} = \rho \vec{g}$, assuming an ideal fluid which is inviscid, $\nu = 0$, and incompressible $\rho = \text{const}$, Equations (3.1) and (3.2) can be simplified to [7]:

$$\nabla \cdot \vec{v} = 0, \quad (3.3)$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho} \nabla p_{\text{tot}} + \vec{g}. \quad (3.4)$$

The additional assumption of irrotational flow described as $\nabla \times \vec{v} \equiv 0$ introduces a scalar property of the fluid, so-called velocity potential $\phi(x, y, z, t)$, which is defined as:

$$\vec{v} = \nabla \phi. \quad (3.5)$$

In order to apply the condition of irrotational motion and convert the two vector equations (3.3) and (3.4) into the scalar form, the following identities have been considered: $\vec{g} = -\nabla(gz)$, where z -axis is pointing upwards, and the vector identity $\vec{v} \times (\nabla \times \vec{v}) \equiv \frac{1}{2} \nabla v^2 - \vec{v} \cdot \nabla \vec{v}$ has been utilised to obtain $\vec{v} \cdot \nabla \vec{v} = \frac{1}{2} \nabla v^2$. As a result, the condition of irrotational field converts Equations (3.3) and (3.4) into [7]:

$$\nabla^2 \phi = 0, \quad (3.6)$$

$$\nabla \left(\frac{\partial \phi}{\partial t} + \frac{\nabla \phi \cdot \nabla \phi}{2} + \frac{p_{\text{tot}}}{\rho} + gz \right) = 0. \quad (3.7)$$

Equation (3.6), also known as the Laplace equation, must be satisfied in the fluid domain, and the solution for the velocity potential must satisfy certain boundary conditions.

3.3.2 Boundary conditions

Consider the fluid domain shown in Figure 3.7, which has a constant water depth h and the air-water interface presented by the wave elevation η . The domain is specified by the xyz Cartesian coordinate system, where the origin is located on the mean water level ($z = 0$), z -axis points upwards and a wave propagates in the xy -plane, $\eta = \eta(x, y, t)$.

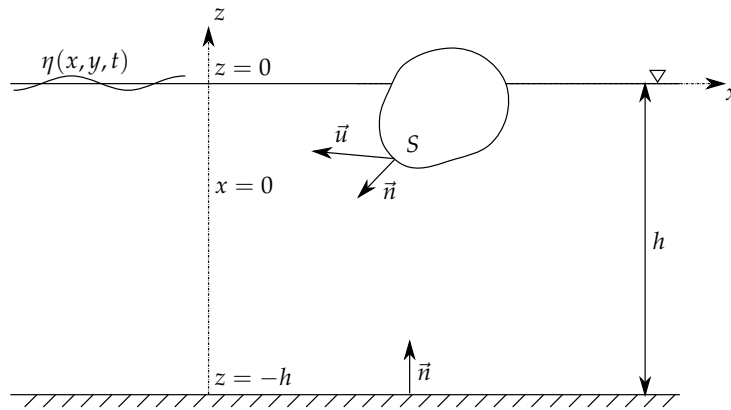


Figure 3.7: Fluid domain considered in linear wave theory, adapted from [7]. η is the wave elevation, $\vec{n}$ is the unit normal on wetted body surface S , $\vec{u}$ is the velocity of the particular point on S .

There are three main boundary conditions for the Laplace equation that should be satisfied in the fluid domain: on the body surface, at the seafloor and at the free surface.

If the body is moving with velocity $\vec{u}$, the component of the fluid velocity normal to the body surface, v_n , must be equal to the body velocity in the same direction $u_n \equiv \vec{n} \cdot \vec{u} = v_n = \vec{n} \cdot \nabla \phi \equiv \partial \phi / \partial n$. So the boundary on the moving body surface is:

$$\frac{\partial \phi}{\partial n} = u_n. \quad (3.8)$$

If the solid body is fixed, then:

$$\frac{\partial \phi}{\partial n} = 0. \quad (3.9)$$

Similarly, the vertical velocity of the water particles at the seafloor has to be zero which specifies the sea bottom boundary condition:

$$\left[\frac{\partial \phi}{\partial z} \right]_{z=-h} = 0. \quad (3.10)$$

If the surface tension on the air-fluid interface is neglected and only gravitational forces are considered, the general non-linear dynamic free-surface boundary condition can be obtained from Equation (3.7):

$$g\eta + \left[\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi \right]_{z=\eta} = 0. \quad (3.11)$$

There is also a linearised kinematic boundary condition on the interface between fluid and the air, which implies that the water particles on the free-surface follow the wave motion:

$$\left[\frac{\partial \phi}{\partial z} \right]_{z=0} = \frac{\partial \eta}{\partial t}, \quad (3.12)$$

which gives a general linearised form of the free surface boundary condition:

$$\left[\frac{\partial^2 \phi}{\partial t^2} + g \frac{\partial \phi}{\partial z} \right]_{z=0} = 0. \quad (3.13)$$

Having obtained the solution for the velocity potential $\phi(x, y, z, t)$, which is an auxiliary mathematical function, it is possible to derive important physical quantities, such as the fluid velocity

$$\vec{v} = \vec{v}(x, y, z, t) = \nabla \phi, \quad (3.14)$$

the hydrodynamic pressure

$$p = p(x, y, z, t) \approx -\rho \frac{\partial \phi}{\partial t}, \quad (3.15)$$

and the wave elevation

$$\eta = \eta(x, y, t) = -\frac{1}{g} \left[\frac{\partial \phi}{\partial t} \right]_{z=0}. \quad (3.16)$$

3.3.3 Harmonic waves

The solution for the velocity potential that satisfies all boundary conditions can be represented as a sinusoidal function for deep water waves [7]. This assumption allows one to separate the spatial and temporal variables and simplify the analysis of the wave/body interaction [10]:

$$\phi = \phi(x, y, z, t) = \Re\{\hat{\phi}(x, y, z)e^{i\omega t}\}, \quad (3.17)$$

where $\hat{\phi}(x, y, z)$ is the complex amplitude of the velocity potential, and ω represents the wave frequency.

As a result, the Laplace equation takes the form of

$$\nabla^2 \hat{\phi} = 0 \quad (3.18)$$

in the fluid domain. The boundary condition on the wetted surface becomes:

$$\frac{\partial \hat{\phi}}{\partial n} = \hat{u}_n, \quad (3.19)$$

on the seafloor:

$$\left[\frac{\partial \hat{\phi}}{\partial z} \right]_{z=-h} = 0, \quad (3.20)$$

and on the free surface:

$$g\hat{\eta} + i\omega|\hat{\phi}|_{z=0} = 0, \quad (3.21)$$

$$\left[-\omega^2 \hat{\phi} + g \frac{\partial \hat{\phi}}{\partial z} \right]_{z=0} = 0. \quad (3.22)$$

3.3.4 Wave-body interaction

A body immersed in water is subject to hydrodynamic forces due to the pressure from the surrounding fluid. These forces can be determined by integrating the fluid pressure over the wetted surface of the body [19]:

$$F_j = - \iint_S p n_j dS, \quad (3.23)$$

where n_j is a j -th component of the unit vector of the normal ($\vec{n}$) to surface (S) of the body. Using the relationship between the fluid pressure and the velocity potential in Equation (3.15), the force can be presented as:

$$F_j = \rho \iint_S \frac{\partial \phi}{\partial t} n_j dS. \quad (3.24)$$

The corresponding complex amplitude of the hydrodynamic force is:

$$\hat{F}_j = i\omega\rho \iint_S \hat{\phi} n_j dS. \quad (3.25)$$

According to linear wave theory it is possible to decompose the wave field around the body into the scattered (incident and diffracted) and radiated components:

$$\hat{\phi} = \underbrace{\hat{\phi}_s}_{\text{scattered}} + \underbrace{\hat{\phi}_r}_{\text{radiated}} = \underbrace{\hat{\phi}_0}_{\text{incident}} + \underbrace{\hat{\phi}_d}_{\text{diffracted}} + \hat{\phi}_r. \quad (3.26)$$

Excitation problem

The excitation force due to the incident and diffracted wave fields is equal to:

$$\hat{F}_{exc,j} = i\omega\rho \iint_S (\hat{\phi}_0 + \hat{\phi}_d) n_j dS. \quad (3.27)$$

As it is assumed that the body is fixed, the appropriate boundary condition is:

$$\frac{\partial \hat{\phi}_s}{\partial n} = 0 \quad \text{or} \quad -\frac{\partial \hat{\phi}_d}{\partial n} = \frac{\partial \hat{\phi}_0}{\partial n} \quad \text{on } S. \quad (3.28)$$

Radiation problem

The solution to the radiation problem, where the body is forced to oscillate in the absence of the incident wave, the radiated velocity potential should satisfy:

$$\frac{\partial \hat{\phi}_r}{\partial n} = \sum_{j=1}^6 u_j n_j \quad \text{on } S, \quad (3.29)$$

where u_j is a body velocity component in the j direction.

When the body oscillates in all six degrees of freedom, the radiated wave can be considered as a superposition of radiated waves generated by oscillations in each hydrodynamic mode:

$$\hat{\phi}_r = \sum_{j=1}^6 \varphi_j \hat{u}_j, \quad (3.30)$$

where $\varphi_j = \varphi_j(x, y, z)$ is a complex coefficient of proportionality. So the boundary condition specified in Equation (3.29), transforms to

$$\frac{\partial \varphi_j}{\partial n} = n_j \quad \text{on } S. \quad (3.31)$$

The term φ_j represents the complex amplitude of the radiated velocity potential due to the body oscillation in mode j with a unit velocity $\hat{u}_j = 1$.

It should be noted that φ_j is independent on the amplitude of oscillations and also should satisfy the Laplace equation in the fluid domain:

$$\nabla^2 \varphi_j = 0 \quad (3.32)$$

and the seafloor

$$\left[\frac{\partial \varphi_j}{\partial z} \right]_{z=-h} = 0 \quad (3.33)$$

and the free surface boundary conditions:

$$\left[-\omega^2 \varphi_j + g \frac{\partial \varphi_j}{\partial z} \right]_{z=0} = 0. \quad (3.34)$$

As a result, the radiation force that acts on the body in j' direction due the oscillations in mode j :

$$\hat{F}_{rad,j'} = i\omega\rho \iint_S \varphi_j \hat{u}_j n_{j'} dS, \quad (3.35)$$

or can be written as:

$$\hat{F}_{rad,j'} = -Z_{j'j} \hat{u}_j, \quad (3.36)$$

where $Z_{j'j}$ is the $j'j$ element of the radiation impedance matrix, given by:

$$Z_{j'j} = -\omega\rho \iint_S \varphi_j n_{j'} dS \quad (3.37)$$

Using the wet surface boundary condition from Equation (3.31):

$$Z_{j'j} = -\omega\rho \iint_S \varphi_j \frac{\partial \varphi_{j'}}{\partial n} dS. \quad (3.38)$$

As the radiation impedance coefficient is complex, it can be represented by its real and complex parts:

$$Z_{j'j} = B_{j'j} + i\omega A_{j'j}, \quad (3.39)$$

where B_{jj} is the hydrodynamic radiation damping coefficient and A_{jj} is the added mass.

As a result, the linear wave/structure interaction problem requires a solution of the Laplace equation in the fluid domain subject to the boundary conditions on the seafloor, free surface and body surface. For the relatively simple geometries, such as a sphere or a circular cylinder (horizontal and vertical), there are mathematical techniques [17] that allow the representation of the velocity potential as a series of separated eigenfunctions or as a sum of singularities placed within any structure present in the fluid domain. In this case, integral equations (3.27) and (3.38) are replaced by the infinite sum of algebraic equations with some unknown coefficients that need to be found. More complex geometries can be analysed using boundary element methods (BEM), such as ANSYS AQWA [1], NEMOH [2], or WAMIT [16]. These packages solve the radiation and scattering problems using Green's theorem where the velocity potential in the fluid domain is represented by the distribution of sources of unknown strength and the integration is performed over the discretised wetted surface of the structure.

3.3.5 Limitations of the linear theory

Linear wave theory is based on the assumption that the wave amplitude is much smaller than the wavelength, and that the oscillation amplitude of the moving body is small. The regions of validity of inviscid linear theory are demonstrated in Figure 3.8 which depend on the characteristic dimensions (D) of the body, wave height (H) and wavelength (λ) [4]. When the trajectory of the water particles is much larger than the diameter of the structure ($H/D \gg 1$), the wave flow separation occurs. Here, viscous effects dominate other hydrodynamic forces (Region VI), and the behaviour of the wave field cannot be described by the inviscid linear theory. Furthermore, small structures with dimensions much smaller than the wavelength ($\pi D/\lambda < 0.5$) have a little effect on the wave field and wave diffraction becomes insignificant (Regions I, III, V). However, when the wavelength and the body diameter are of similar order, the wave field is significantly modified by the structure and wave diffraction effects become more important (Region II). For moderate values of H/D and $\pi D/\lambda > 0.5$ (Region IV), both viscous and potential flow forces are significant and should be considered in the wave load calculations.

As a result, the primary use of linear wave theory is limited to regions II and IV, while the importance of viscous effects for the oscillating bodies is usually determined by the Keulegan-Carpenter number [14], which is equal

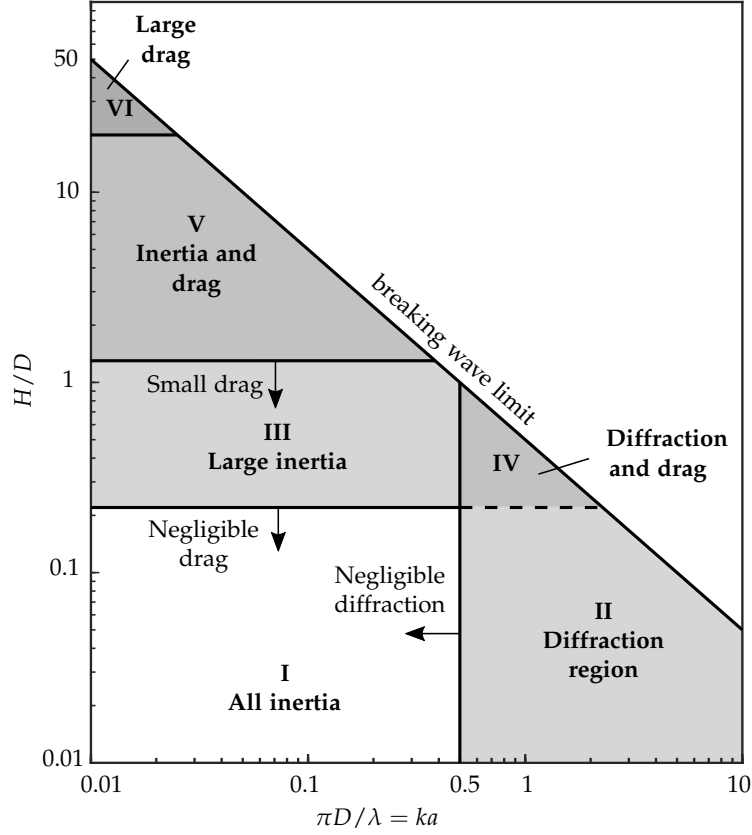


Figure 3.8: Regions of importance for diffraction and viscous forces, adapted from [4].

to:

$$KC = \frac{u_0 T}{D}, \quad (3.40)$$

where u_0 is the amplitude of the relative velocity between the fluid and the structure, and T is the period of oscillation. The linear behaviour of the wave energy converter can be observed as far as the Keulegan-Carpenter number does not exceed the value of π [9]. For $KC > \pi$, the system experiences significant viscous losses, and the drag forces must be taken into account in modelling.

3.4 State-of-the-art modelling of WECs in the time domain

Assuming that a wave energy converter is allowed to move in six degrees of freedom and its spatial position is defined by the vector $\mathbf{x} \in \mathbb{R}^{6 \times 1}$, the

equation describing the dynamic motion of this converter can be written as:

$$\mathbf{M}\ddot{\mathbf{x}}(t) = \mathbf{F}_{exc}(t) + \mathbf{F}_{rad}(t) + \mathbf{F}_{buoy}(t) + \mathbf{F}_{visc}(t) + \mathbf{F}_u(t), \quad (3.41)$$

where $\mathbf{M} \in R^{6 \times 6}$ is the mass (or inertia) matrix, $\mathbf{F}_{exc}$, $\mathbf{F}_{rad}$, $\mathbf{F}_{buoy}$, $\mathbf{F}_{visc}$, $\mathbf{F}_u \in R^{6 \times 1}$ are the excitation, radiation, buoyancy, viscous and control forces, respectively.

3.4.1 Excitation force

The undisturbed incident wave elevation measured at some reference position on the water surface (x_0, y_0) is usually taken as an input to the analysis of the dynamics of the wave energy converter. The reference point generally coincides with the mean position of the floating body or the projection of the centre of mass on the water plane for the submerged body. The relationship between the incident wave elevation $\eta(x_0, y_0)$ and the excitation force acting on the body $F_{exc,j}$ in mode j is described by the force response amplitude operator (the transfer function) $\hat{H}_{F_j}(\omega)$ in the frequency domain :

$$\hat{F}_{exc,j}(\omega) = \hat{H}_{F_j}(\omega) \hat{\eta}(x_0, y_0, \omega), \quad (3.42)$$

where the hat symbol, ' $\hat{\cdot}$ ', indicates the frequency domain equivalent, or alternatively, in the time domain it can be represented by the convolution operation:

$$F_{exc,j}(t) = h_{F_j}(t) * \eta(x_0, y_0, t) = \int_{-\infty}^{\infty} h_{F_j}(t) \eta(x_0, y_0, t - \tau) d\tau, \quad (3.43)$$

where $h_{F_j}(t)$ is the impulse response function (IRF) of the wave excitation force and the wave elevation. However, this IRF has non-causal properties [8] since it is not vanishing for $t < 0$ as shown in Figure 3.9. This means that in order to estimate the current excitation force, it is necessary to have future measurements of the wave elevation.

Causal impulse response

There are two ways to make the IRF of the excitation force causal: (i) to introduce a small time shift to the transfer function, or (ii) to shift the wave elevation measurement point to the upstream location.

According to the Fourier transform, in order to introduce a time shift of τ_c to the transfer function, it should be multiplied by $e^{i\omega\tau_c}$ in frequency domain:

$$\hat{H}_{F_j,c}(\omega) = \hat{H}_{F_j}(\omega) e^{-i\omega\tau_c}, \quad (3.44)$$

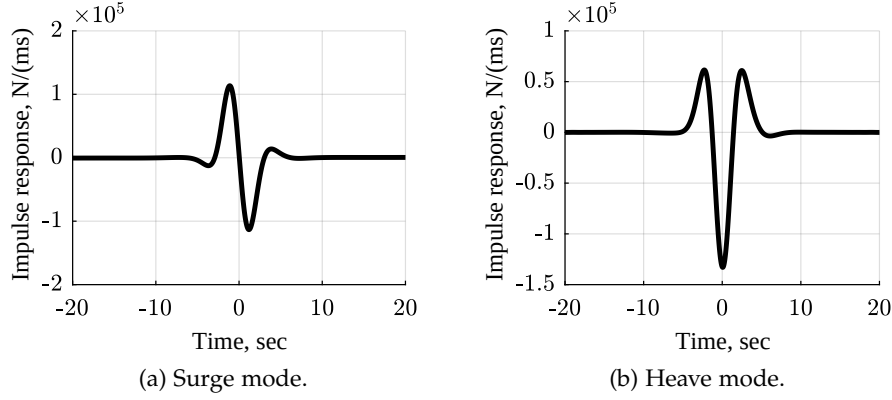


Figure 3.9: An example of the non-causal impulse response of the excitation force and the incident wave elevation for a fully submerged sphere.

and in time domain:

$$h_{F_j}(t - \tau_c) = h_{F_j}(t) * \delta(t - \tau_c), \quad (3.45)$$

where $\delta(t)$ is the Dirac delta function.

If there is a need to take the incident wave elevation measurements as an input to the system model, the convolution integral in Equation (3.43) can be replaced by the state-space representation with a time shift [25]:

$$\dot{\mathbf{z}}(t) = \mathbf{A}\mathbf{z}(t) + \mathbf{B}\eta(t) \quad (3.46)$$

$$F_{exc,j}(t - \tau_c) = \mathbf{C}\mathbf{z}(t) + \mathbf{D}\eta(t), \quad (3.47)$$

where the value of a time shift can be chosen based on plots similar to Figure 3.9.

The time shift also can be introduced by measuring the wave elevation at a distant location L upstream, which leads to [8]:

$$\hat{F}_{exc,j}(\omega) = \hat{H}_{F_j,L}(\omega)\hat{\eta}(x_0 - L, y_0, \omega), \quad (3.48)$$

where the modified frequency response function is:

$$\hat{H}_{F_j,L}(\omega) = \hat{H}_{F_j}(\omega)e^{-ikL}. \quad (3.49)$$

However, both these methods do not guarantee that the associated transfer function is causal, and it is difficult to choose one time shift or one distant location which is suitable for all elements of the excitation force vector (e.g. heave, surge, or pitch).

Spectral representation

The time-domain simulations also can be performed taking the excitation force time series as an input signal, instead of the wave elevation. If the sea state is represented by the energy spectral density $S_\eta(\omega)$, the complex wave amplitude for each frequency can be calculated as:

$$\hat{\eta}(\omega_k) = \sqrt{2S_\eta(\omega_k)\Delta\omega_k} e^{i\phi_\eta(\omega_k)}, \quad (3.50)$$

where ϕ_η is a phase angle known from the real data or can be randomly generated assuming the uniform distribution within $[0, 2\pi]$ radians. This complex wave amplitude can be directly related to the complex amplitude of the wave excitation force using:

$$\hat{F}_{exc,j}(\omega_k) = \hat{H}_{F_j}(\omega_k)\hat{\eta}(\omega_k), \quad (3.51)$$

and the resultant time-series that contain N frequency components is:

$$F_{exc,j}(t) = \sum_{k=1}^N \Re\{\hat{F}_{exc,j}(\omega_k)e^{i\omega_k t}\}. \quad (3.52)$$

In order to obtain $\mathbf{F}_{exc}(t) \in R^{6 \times 1}$, Equations (3.43), (3.47) and (3.52) should be applied to each hydrodynamic mode j . The spectral representation of the excitation force is used in Chapters 4–7 when modelling the response of the three-tether WEC in the time domain.

3.4.2 Radiation force

In the frequency domain the radiation force is usually written as [7]:

$$\hat{\mathbf{F}}_{rad}(\omega) = -\mathbf{A}(\omega)\hat{\mathbf{x}}(\omega) - \mathbf{B}(\omega)\hat{\dot{\mathbf{x}}}(\omega), \quad (3.53)$$

where $\mathbf{A}(\omega) \in R^{6 \times 6}$ is the added mass matrix, $\mathbf{B}(\omega) \in R^{6 \times 6}$ is the matrix of hydrodynamic damping coefficients, $\hat{\mathbf{x}}$ and $\hat{\dot{\mathbf{x}}}$ are complex amplitudes of the body acceleration and velocity.

Considering only the radiation force in mode j due to the oscillations in mode j' :

$$\hat{F}_{rad,jj'}(\omega) = -A_{jj'}(\omega)\hat{x}_{j'}(\omega) - B_{jj'}(\omega)\hat{\dot{x}}_{j'}(\omega), \quad (3.54)$$

where j and $j' = 1 \dots 6$.

In the time-domain, Cummins [5] showed that the linearised radiation force can be modelled as:

$$F_{rad,jj'}(t) = -A_{jj',\infty}\ddot{x}_{j'} - \int_0^t k_{jj'}(t-\tau)\dot{x}_{j'}(\tau)d\tau, \quad (3.55)$$

where $A_{jj',\infty} = \lim_{\omega \rightarrow \infty} A_{jj'}(\omega)$ is the infinite frequency added mass and $k_{jj'}(t)$ is the retardation function (or radiation impulse response) with a Fourier transform $k_{jj'}(j\omega) = \mathfrak{F}\{k_{jj'}(t)\} = B_{jj'}(\omega) + i\omega (A_{jj'}(\omega) - A_{jj',\infty})$.

It is possible to approximate the fluid-memory term from Equation (3.55) by a state-space model [21]:

$$F_{r,jj'}(t) = \int_0^t k_{jj'}(t-\tau) \dot{x}_{jj'}(\tau) d\tau \approx \begin{aligned} \dot{\mathbf{p}}(t) &= \mathbf{A}_r \mathbf{p}(t) + \mathbf{B}_r \dot{x}_{jj'}(t) \\ F_{r,jj'}(t) &= \mathbf{C}_r \mathbf{p}(t), \end{aligned} \quad (3.56)$$

where the velocity of the body $\dot{x}_{jj'}(t)$ is taken as an input, and $\mathbf{p}(t)$ is an auxiliary state vector that does not have any physical meaning. Perez and Fossen [21] have developed a MATLAB MSS FDI toolbox that uses pre-calculated values of the added mass $A_{jj'}(\omega)$ and damping coefficients $B_{jj'}(\omega)$ to identify the approximate time-domain model of the radiation force. The order of the model is detected automatically to guarantee the required accuracy and quality of the fit. An example of the identified state-space model of order 3 is demonstrated in Figure 3.10.

In order to obtain the radiation force vector $\mathbf{F}_{rad}(t) \in R^{6 \times 1}$, each pair of non-zero elements in $\mathbf{A}(\omega)$ ($A_{jj'} \neq 0$) and $\mathbf{B}(\omega)$ ($B_{jj'} \neq 0$) should be approximated by the state-space model. For example, the axisymmetric system has nine non-vanishing elements of the radiation impedance matrix and nine impulse response functions due to the hydrodynamic coupling between surge/pitch and sway/roll modes.

In this thesis, a MATLAB MSS FDI toolbox is used to model the radiation force acting on the three-tether WEC in time domain.

3.4.3 Viscous forces

Wave excitation and radiation forces discussed above are calculated based on linear wave theory assuming inviscid fluid flow. However, it is common to include the non-zero viscous forces in the time-domain formulation as they introduce significant power losses during operation of the wave energy converter [20]. The viscous force in mode j is usually approximated by the form drag using Morison equation [18]:

$$F_{visc,j}(t) = -\frac{1}{2} \rho C_{d,j} A_{\perp,j} \dot{x}_{rel,j}(t) |\dot{x}_{rel,j}(t)|, \quad (3.57)$$

where $\dot{x}_{rel,j}$ is the velocity of the body relative to the fluid, $A_{\perp,j}$ is the cross section area of the structure perpendicular to the fluid flow, and $C_{d,j}$ is the drag coefficient which is usually estimated from the experimental studies and depends on the geometry of the structure, roughness of the surface,

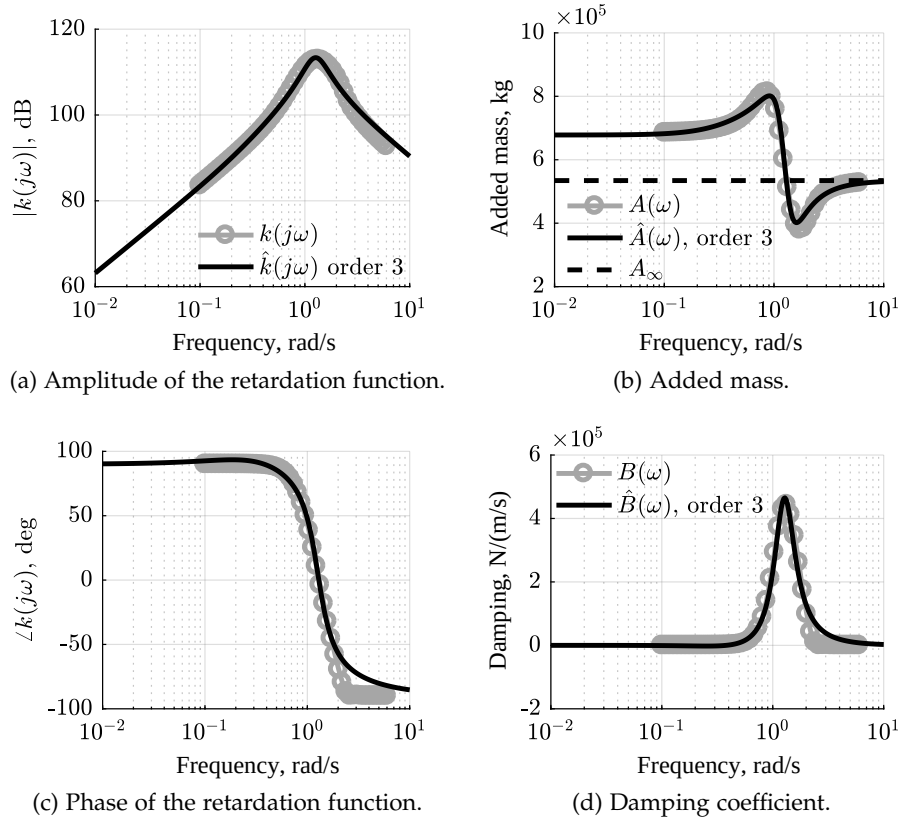


Figure 3.10: Identification results of the radiation force of a heave mode of a submerged sphere obtained from the MSS toolbox [21]. The left-hand-side plots show the fluid memory frequency response data and the response of the identified model. The right-hand-side plots demonstrate the added mass and damping and their re-constructed values from the estimated model. The hat sign, ‘ $\hat{\cdot}$ ’, represents the approximation given by the state-space model.

properties of the fluid and relative velocity between fluid and body (e.g. Reynolds number, flow regime). Equation (3.57) should be applied to all modes j in order to obtain $\mathbf{F}_{visc}(t) \in \mathbb{R}^{6 \times 1}$.

3.4.4 Buoyancy force

For fully submerged wave energy converters, the buoyancy force is constant and can be written as:

$$F_{buoy,3}(t) = g(m_w - m_b), \quad (3.58)$$

where m_b is the mass of the submerged body, and $m_w = \rho V$ is the mass of the displaced water.

However, for floating devices, the buoyancy force, or sometimes called the hydrostatic restoring force, behaves like a spring that brings the oscillating body to its equilibrium position [7]:

$$F_{buoy,3}(t) = -\rho g S_w x_3(t), \quad (3.59)$$

where S_w is the (mean) water plane area of the body, and $x_3(t)$ corresponds to the vertical displacement. This formulation is only valid within linear wave theory assuming small amplitude oscillations. For the axisymmetric body, the non-zero restoring forces act in heave, pitch and roll directions.

The hydrostatic stiffness ($K_s = \rho g S_w$) is an important property of a wave energy converter as it determines the resonance frequency of the device in heave. Moreover, its presence or absence is one of the key differences between floating and fully submerged converters that drives the requirements to the control system of both types of converters as it will be explained in Chapter 4.

3.5 Frequency-domain modelling

Applying the Fourier transform to Equation (3.41) with zero initial conditions, and disregarding the non-linear viscous force, the motion of the wave energy converter in the frequency domain can be written as:

$$\mathbf{M}\hat{\mathbf{x}}(\omega) = \hat{\mathbf{F}}_{exc}(\omega) + \underbrace{[-\mathbf{A}(\omega)\hat{\mathbf{x}}(\omega) - \mathbf{B}(\omega)\hat{\mathbf{x}}(\omega)]}_{\hat{\mathbf{F}}_{rad}(\omega)} + \underbrace{[-\mathbf{K}_s\mathbf{x}(\omega)]}_{\hat{\mathbf{F}}_{buoy}(\omega)} + \hat{\mathbf{F}}_u(\omega), \quad (3.60)$$

where $\mathbf{K}_s$ is a hydrostatic stiffness matrix (for submerged systems $\mathbf{K}_s = \mathbf{0}$).

Using $\hat{\mathbf{x}}(\omega) = i\omega\hat{\mathbf{u}}(\omega) = -\omega^2\hat{\mathbf{x}}(\omega)$, Equation (3.60) takes a form:

$$\{-\omega^2(\mathbf{M} + \mathbf{A}(\omega)) + i\omega\mathbf{B}(\omega) + \mathbf{K}_s\}\hat{\mathbf{x}}(\omega) = \hat{\mathbf{F}}_{exc}(\omega) + \hat{\mathbf{F}}_u(\omega). \quad (3.61)$$

Sometimes it is more convenient to write Equation (3.61) in terms of the intrinsic mechanical impedance $\mathbf{Z}_i(\omega)$ of the oscillating wave energy converter [7]:

$$\left\{ i\omega(\mathbf{M} + \mathbf{A}(\omega)) + \mathbf{B}(\omega) - i\frac{\mathbf{K}_s}{\omega} \right\} \hat{\mathbf{u}}(\omega) \equiv \mathbf{Z}_i(\omega)\hat{\mathbf{u}}(\omega) = \hat{\mathbf{F}}_{exc}(\omega) + \hat{\mathbf{F}}_u(\omega), \quad (3.62)$$

where $\hat{\mathbf{u}}(\omega)$ is used to denote the converter velocity $\hat{\mathbf{x}}(\omega)$.

As a result, Equation (3.62) shows the velocity response of the oscillating system subject to external (wave excitation and control) forces. Despite its relative simplicity, the frequency domain model can provide valuable insights on the system performance (expected motion amplitudes, forces, absorbed power, etc.), and can assist with a preliminary design of the wave energy converter.

3.6 Concluding remarks

This chapter provides an overview of common modelling techniques used in the wave energy sector depending on the available computational resources, required accuracy and potential applications. It was demonstrated that partially or weakly non-linear methods are adequate when there is a need to assess annual power output of the wave energy converter or to optimise its design and configuration. Therefore, it can be concluded that the frequency and time domain models based on linear wave theory, included nonlinear effects due to the form drag, are sufficient to investigate the power absorption performance of the three-tether system, to optimise its design and demonstrate the potential of this WEC to the wave energy community.

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Chapter 4

Floating and fully submerged wave energy converters: difference in performance

The focus of this thesis is on wave energy conversion systems that are fully submerged and operate under the water surface. Such systems have distinctive hydrodynamic and wave energy absorption properties as compared to the extensively studied floating structures. There are general guidelines and ‘rules of thumb’ for the choice of shape and dimensions of the floating point absorber, and considerations for the control system design and performance of WEC arrays. In contrast, fully submerged systems have received limited attention from the research community and some features of these converters remain unexplored.

As a result, this chapter presents insights into the performance of the fully submerged wave energy converters, their advantages and disadvantages compared to the floating systems, while providing an answer to the following research question: *What are the distinctive features of the fully submerged axisymmetric WECs which can be utilised to increase their power absorption efficiency?*

This chapter consists of the published journal article:

Sergiienko, N. Y., Cazzolato, B. S., Ding, B., Hardy, P. and Arjomandi, M. (2017). “Performance comparison of the floating and fully submerged quasi-point absorber wave energy converters”. *Renewable Energy* 108, pp. 425–437.

The article in its published format is available at <https://doi.org/10.1016/j.renene.2017.03.002>.

Statement of Authorship

Title of Paper	Performance comparison of the floating and fully submerged quasi-point absorber wave energy converters
Publication Status	☒ Published ☐ Accepted for Publication ☐ Submitted for Publication ☐ Unpublished and Unsubmitted work written in manuscript style
Publication Details	Sergiienko, N. Y., Cazzolato, B. S., Ding, B., Hardy, P. and Arjomandi, M. (2017). "Performance comparison of the floating and fully submerged quasi-point absorber wave energy converters". <i>Renewable Energy</i> 108, pp. 425–437.

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Overall percentage (%)	80%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
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Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

- the candidate's stated contribution to the publication is accurate (as detailed above);
- permission is granted for the candidate to include the publication in the thesis; and
- the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

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Performance comparison of the floating and fully submerged quasi-point absorber wave energy converters

N. Y. Sergiienko, B. S. Cazzolato, B. Ding, P. Hardy, M. Arjomandi

Abstract

Axisymmetric point absorbers are mostly designed as floating buoys that extract power from heave motion. Power absorption limits of such wave energy converters (WECs) are governed by the displaced volume of the buoy and its ability to radiate waves. In the case of fully submerged WECs, the power performance becomes a function of additional variables including the proximity to the mean surface level of the water, body shape and the maximum stroke length of the power take-off system. Placing the body below the water surface increases its survivability in storm conditions but changes the hydrodynamic properties of the WEC including maximum absorbed power. This paper investigates the differences between floating and fully submerged point absorber converters from the number of perspectives including energy extraction, bandwidth, and optimal size for a particular wave climate. The results show that when compared with floating converters, fully submerged buoys: (i) generally absorb less power at longer wavelengths, (ii) have narrower bandwidth, (iii) cannot be replaced by smaller units of the same total volume without a significant loss of power, and (iv) have a significant advantage as they can effectively utilise several modes of motion (e.g. surge and heave) in order to increase power generation.

4.1 Introduction

Intensive research on extraction energy from ocean waves started in the 1970s [18]. Initially, attention was paid to the terminator-type converters which were studied as two-dimensional devices with an infinitely long body extension perpendicular to the wave front (e.g. Salter's duck [33]). However, due to the sensitivity of such prototypes to the direction of wave propagation, researchers focussed on the concept of a point-absorbing wave energy converter (WEC) [5] whose performance does not depend on the angle of wave incidence. Thereafter, point absorbers (PA) have become one of the most studied WECs, making up a large part of existing full-scale prototypes.

Generally, PAs are designed to operate on or just below the water surface, extracting wave power from the heaving motion. As opposed to submerged buoys, floating converters require less installation and maintenance work under water. However, there could be several very important reasons to keep the WEC fully submerged (see Figure 4.1):

- (i) to increase the survivability of the converter during storms with large wave conditions;
- (ii) when there is an unconditional requirement from the public authorities to minimise visual impact of the wave power generator, e.g. the buoy must not be visible from the shore.

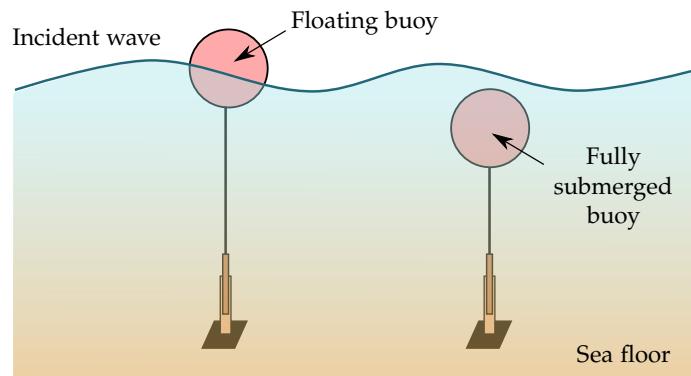


Figure 4.1: Schematic representation of the floating and fully submerged WECs that extract energy from oscillations in heave.

Based on the fundamental equations of maximum power absorption for axisymmetric bodies, floating and submerged WECs are able to extract the same amount of wave power provided unconstrained motion amplitudes [5, 13]. Thus, under this condition the maximum capture width of the oscillating body does not depend on its size, shape or submergence depth, but is governed by the mode of motion [10, 16]. According to these findings, the body that moves in surge and heave simultaneously can absorb three times more power than a heaving buoy.

In practice, WEC motion should be constrained during large waves, hence power absorption becomes dependent on the maximum allowed oscillation amplitude and the wave excitation force exerted on the converter [14, 31]. As the latter is determined by the shape, size and submergence depth of the WEC, it becomes apparent that identical fully submerged and floating buoys cannot capture the same amount of wave energy. It has been observed [18] that submerged converters are poorer wave absorbers as compared to

the floating heaving buoys because their upper and lower portions of the swept volume have different polarities during the oscillatory motion. In addition, floating and fully submerged WECs have distinctive low-frequency limits of the heave excitation forces. As the wave frequency tends to zero, the amplitude of the heave force on the floating body is limited by the hydrostatic stiffness coefficient, whereas for the fully submerged converter the excitation force approaches zero due to the diminishing water plane area [16]. Subsequently, based on these findings and also taking into account the swept volume of the body, Budal [6] was able to formulate power absorption bounds for floating WECs that oscillate in heave. This approach has been extended to the fully submerged buoys where the expressions of the power limits for several basic geometries are derived [39]. However, it may be concluded that in the case of point-absorbing WECs, the main research focus has been drawn to the floating buoys, while some features of submerged converters still remain unclear or have not been sufficiently explored.

The current paper provides a systematic comparison between floating and submerged PAs by generalising existing knowledge and providing an in-depth analysis. All results are based on the linear wave theory assuming regular and irregular wave conditions and infinite water depth. Background information and power absorption limits of heaving PA systems are presented in Section 4.2. Key features of different control strategies are discussed in Section 4.3, followed by the methods of selecting the correct size of the converter in Sections 4.4–4.5. Finally, the possibility of extracting power from additional modes of motion is reported in Section 4.6.

4.2 Power limits for regular waves

A body placed in water captures wave energy only when it moves in an oscillatory manner and radiates waves in order to counteract the incident wave front. Thus, the maximum amount of power that can be removed from waves is defined by the radiating ability of the body. This limit has been derived in [5, 13, 29] and differs for motion modes. A well known equation characterising the maximum absorbed power by an axisymmetric body in monochromatic waves is [16]:

$$P_{\max} = \alpha \frac{J}{k}, \quad (4.1)$$

where $J = \rho g^2 D(kh) A^2 / (4\omega)$ is the wave-energy transport per unit frontage of the incident wave, α is a coefficient that depends on the motion oscillation mode ($\alpha = 1$ for heave, $\alpha = 2$ for surge or pitch, and $\alpha = 3$ when the body

oscillates in heave, surge and pitch simultaneously), k is the wavenumber, A is the wave amplitude, ρ is water density, ω is the wave frequency and $D(kh)$ is the depth function which is equal to 1 for deep water.

Maximum power in Equation (4.1) is obtained when the body velocity is [16]:

$$\hat{u}_{j,opt}(\omega) = \frac{\hat{F}_{j,exc}(\omega)}{2B_{jj}(\omega)}, \quad (4.2)$$

where $\hat{F}_{j,exc}$ is the wave excitation force on the body in mode j , and B_{jj} is the radiation damping coefficient in mode j . However, the amount of power in waves with long period is very high and in order to absorb the absolute maximum, the body should move with large amplitudes at high velocities which is not achievable in practice. Thus, if $|\hat{u}_j| < |\hat{u}_{j,opt}|$, the amount of radiated power (P_r) will be much smaller than the excitation power (P_e) and the absorbed power will be limited by the latter:

$$P = P_e - P_r \leq \frac{1}{2} |\hat{F}_{j,exc} \hat{u}_j|. \quad (4.3)$$

According to Equation (4.3), Budal (as cited in [18]) showed that the power extraction at low frequencies is limited by the swept volume of the body, which is a collective term for the body physical volume and the maximum motion amplitude. Thus, for the floating body, the motion amplitude in heave is constrained by its vertical dimension, such that $|\hat{s}_3| < V/(2S_w)$, where V is the body volume, S_w is the water-plane area of the body, and the subscript $j = 3$ corresponds to the heave motion. Therefore, the maximum velocity in heave cannot be larger than $|\hat{u}_3| < \omega V/(2S_w)$. Furthermore, the heave excitation force is bounded by the integrated pressure force over the water-plane area of the body, which is $|\hat{F}_{3,exc}| < \rho g S_w A$. As a result, the power absorption of the floating heaving buoy has two boundaries:

- (i) a high-frequency limit P_A defined by the body's ability to radiate waves (from Equation (4.1) assuming deep water conditions $\omega^2 = kg$):

$$P_A = \frac{J}{k} = \frac{\rho g^2 A^2}{4\omega k} = \frac{\rho g^3 \left(\frac{H}{2}\right)^2}{4\omega^3} = c_\infty T^3 H^2, \quad (4.4)$$

where $c_\infty = \rho(g/\pi)^3/128$, $H = 2A$ is the wave height, $T = 2\pi/\omega$ is the wave period;

- (ii) a low-frequency limit P_B defined by the maximum swept volume of the body, which applies when the velocity of the converter is smaller than the optimal value due to physical constraints:

$$P_{B,f} = \frac{1}{2} |\hat{F}_{3,exc} \hat{u}_3| = \frac{\rho g \omega V A}{4} = \frac{c_0 V H}{T}, \quad (4.5)$$

where $c_0 = (\pi/4)\rho g$ and the subscript f corresponds to the floating case.

These boundaries have been derived for floating bodies that move in heave only regardless of shape. In general, the P_A -limit depends only on the mode of motion and has the same expression for submerged and floating bodies. With regard to the P_B curve, the power absorption limit of the fully submerged converter is strongly dependent on shape and should be derived for each case under consideration independently. Thus, for a spherical body with its centre placed d_s below the water surface, the P_B -limit can be expressed as [39]:

$$P_{B,s} = 4\pi^3 \rho e^{-kd_s} s_{3,\max} \frac{VH}{T^3}, \quad (4.6)$$

where the subscript s corresponds to the submerged case, and $s_{3,\max}$ is the maximum displacement of the sphere in heave.

To demonstrate the comparison between power limits for the floating and submerged WECs, three spherical bodies with different submergence depths have been chosen for the analysis as indicated in Figure 4.2. All spheres have the same physical volume of 524 m^3 (radius is $a = 5 \text{ m}$) and the motion amplitudes are constrained by $s_{3,\max} = 0.67a = 3.3 \text{ m}$. Regular waves of height $H = 2 \text{ m}$ are considered. Hydrodynamic coefficients of all buoys have been obtained using WAMIT [26]. It should be noted that a sphere which is located in close proximity to the mean water level ($d_s = 6 \text{ m}$) may breach the surface of the water during operation. In this case, the linear wave theory breaks down and thus provides only a rough approximation of the wave-body interaction. Also, in this work the wave parameters and dimensions of all WECs are selected such that the Keulegan-Carpenter (KC) number does not exceed π , thus avoiding excessive viscous losses in the system and ensuring that linear potential theory is considered valid [8].

The power extracted by the spherical WECs over the range of wave periods, assuming optimal reactive control, is displayed in Figure 4.3. The most important difference between power absorption of floating and submerged heaving systems is that the latter has a faster decay rate at the low frequency range. Comparing Equations (4.5) and (4.6), it is obvious that $P_{B,f} = \mathcal{O}(T^{-1})$, while $P_{B,s} = \mathcal{O}(T^{-3})$, which leads to a decrease in power absorption at longer wavelengths. Moreover, due to the fact that the hydrodynamic pressure on the body surface decays exponentially with depth, the presence of $\exp(-kd_s)$ in Equation (4.6) shows a reduction of power for deeper submergences. Consequently, the sphere submerged to $2a = 10 \text{ m}$ extracts less power than that submerged to $1.2a = 6 \text{ m}$. Whilst smaller excitation forces from low-

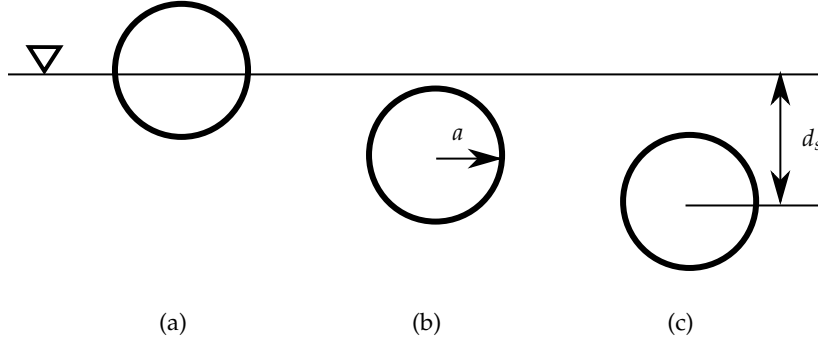


Figure 4.2: Schematic representation of the floating and submerged spheres of radius $a = 5$ m: (a) $d_s = 0$, (b) $d_s = 1.2a = 6$ m and (c) $d_s = 2a = 10$ m.

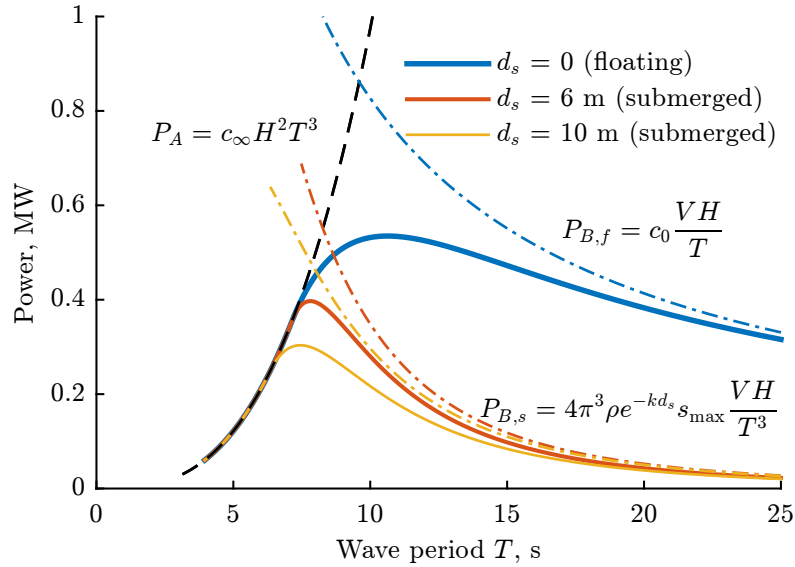


Figure 4.3: Power absorbed by the floating and submerged spheres in regular waves vs. wave period. Sphere radius is $a = 5$ m, displacement in heave is constrained to $0.6a$, wave height is $H = 2$ m. The dashed curve corresponds to the P_A limit from Equation (4.4), and three dash-dotted curves show the P_B bounds from Equations (4.5) and (4.6).

frequency waves increase survivability of the system under storm conditions it comes at the expense of power generation.

The analysis will now be extended to cylindrical WECs with volume of 524 m^3 , which is the same as for the spheres considered. The height to radius ratio of each cylinder is set to 1 leading to $h_c = a = 5.5$ m. Three different submergence depths are examined as shown in Figure 4.4: one floating case, $d_s = 0$, and two submerged cases with $d_s = 0.5h_c + 1 = 0.68a = 3.75$ m

and $d_s = h_c + 1 = 1.18a = 6.5$ m. The volume stroke in heave is set to be equal to the structural body volume leading to the motion constraints of $s_{3,\max} = 0.5h_c = 2.75$ m.

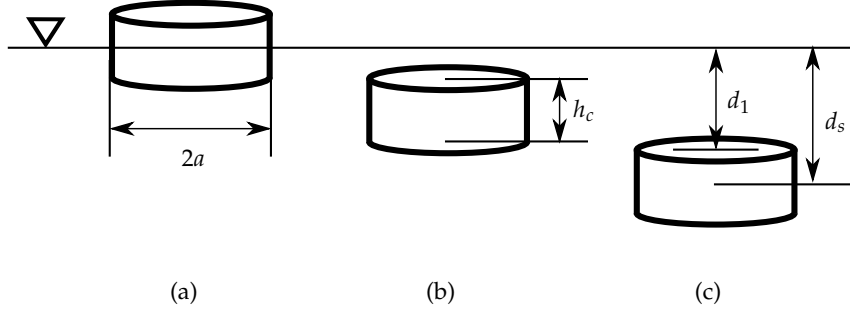


Figure 4.4: Schematic representation of the floating and submerged truncated vertical cylinders, $h_c = a = 5.5$ m: (a) $d_s = 0$, (b) $d_s = 0.5h_c + 1 = 0.68a = 3.75$ m, (c) $d_s = h_c + 1 = 1.18a = 6.5$ m.

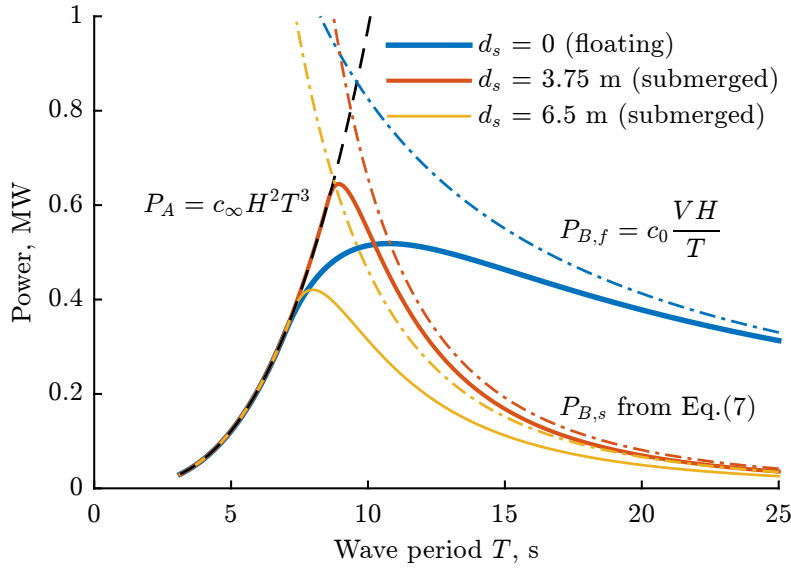


Figure 4.5: Power absorbed by the floating and submerged truncated vertical cylinders in regular waves vs. wave period. Cylinder radius and height are $h_c = a = 5.5$ m, heave displacement is constrained to $0.5h_c$, wave height is $H = 2$ m. The dashed curve corresponds to the P_A limit from Equation (4.4), and three dash-dotted curves show the P_B bounds from Equations (4.5) and (4.7).

In contrast to the sphere, a vertical cylinder has a non-convex shape and if placed very close to the mean water level it will experience resonant motion of the fluid above its flat top surface [28]. This phenomenon causes a rapid

change in the added mass, damping coefficients and excitation forces at the restricted frequency range [27]. Therefore, the P_B -bound for the submerged cylinder has a more complicated expression than for the spherical case (for full derivation see 4.A):

$$P_{B,s} = \frac{\pi^2 a \rho g H}{T} s_{3,\max} \left(\frac{2J_1(\kappa a)}{\kappa J_0(\kappa a) \cosh(\kappa d_1)} - a e^{-\kappa d_2} \right), \quad (4.7)$$

where $d_2 = d_1 + h_c$, J_0 and J_1 are Bessel functions of the first kind of order 0 and 1 respectively, κ is the real solution of the dispersion equation $\omega^2 = g\kappa \tanh(\kappa d_1)$, which can be approximated by $\omega^2 = \kappa^2 g d_1$ using shallow water conditions for the water domain above the cylinder.

The variation of absorbed power over the range of wave periods for the three cylinders with different submergence depth is demonstrated in Figure 4.5. Similar to the spherical case, the absorbed power of the submerged cylinders decays faster at longer wavelengths even though it cannot be clearly seen from Equation (4.7). Comparing power for spherical and cylindrical bodies, it is interesting to note that the performance of the cylinder placed close to the mean water level ($d_s = 3.75$ m) is better than that of the floating one within a particular range of wave periods ($7 \text{ s} < T < 10 \text{ s}$). However, if the cylinder is placed deep enough below the water surface, the power absorption is poorer for the submerged bodies across the entire frequency range as is shown for the $d_s = 6.5$ m case on Figure 4.5.

4.3 Control performance

In this section, differences between floating and submerged converters are analysed from the control point view. Also, frequency domain analysis of power production is extended to the irregular wave conditions in the time domain.

4.3.1 Optimal reactive control

Power levels presented in Figures 4.3 and 4.5 show the average absorbed power from reactively controlled WECs. In the frequency domain, the maximum power output is achieved by applying complex conjugate, or impedance matching, control [32]. The main idea that underlies this control strategy is to tune the resonance frequency of the system to the frequency of the incident wave by means of the load (control) force exerted on the buoy. However, despite the same control strategy applied to floating and submerged converters, there is a principal difference between their implementations

which is governed by the presence (floating) or absence (fully submerged) of the hydrostatic restoring force.

The natural frequency of floating converters is defined by the hydrostatic stiffness that can bring a disturbed system to its equilibrium position. Thus, it can be calculated utilising a simple equation for the mass-spring-damper model:

$$\omega_0 = \sqrt{\frac{C}{m + A(\omega_0)}}, \quad (4.8)$$

where C is the hydrostatic stiffness, m is the mass of the buoy and $A(\omega_0)$ is the value of the buoy added mass at the natural frequency. However, in the case of fully submerged buoys the hydrostatic stiffness is absent and there is no restoring force that would keep the body submerged all the time (if body is lighter than water). Therefore, the natural frequency of submerged heaving WECs approaches 0 ($\omega_0 \rightarrow 0$) unless an external restoring force (spring) is applied to the system. This also relates to the floating and submerged bodies that move in surge as their natural frequency $\omega_0 \rightarrow 0$.

In addition to the natural frequency, the hydrostatic stiffness affects the optimal control force that is required for maximum power absorption. If the buoy is constrained to move in heave only, its motion in frequency domain can be described as [16]:

$$\hat{Z}_i(\omega)\hat{u}(\omega) = \hat{F}_{exc}(\omega) + \hat{F}_{pto}(\omega), \quad (4.9)$$

where $\hat{F}_{pto}(\omega)$ is the control (power take-off) force applied to the buoy and the intrinsic mechanical impedance of the system $\hat{Z}_i(\omega)$ has a form [16]:

$$\hat{Z}_i(\omega) = B(\omega) + j\omega \left(m + A(\omega) - \frac{C}{\omega^2} \right). \quad (4.10)$$

In order to absorb maximum power, the control force $\hat{F}_{pto}(\omega) = -\hat{Z}_{pto}(\omega)\hat{u}(\omega)$ should satisfy the optimal condition [16]:

$$\hat{Z}_{pto}(\omega) = \hat{Z}_i^*(\omega), \quad (4.11)$$

where $*$ denotes the complex conjugate. Assuming that the power take-off system has a linear behaviour, where the machinery force is proportional to the instantaneous position and velocity of the buoy, the load impedance may be written as:

$$\hat{Z}_{pto}(\omega) = B_{pto}(\omega) - j\frac{K_{pto}(\omega)}{\omega}, \quad (4.12)$$

where K_{pto} and B_{pto} are the stiffness and damping coefficients of the PTO system respectively. Thus, substituting Equations (4.9) and (4.12) into Equation (4.11), optimal values of the control parameters are:

$$B_{pto}(\omega) = B(\omega), \quad K_{pto}(\omega) = \omega^2 (m + A(\omega)) - C. \quad (4.13)$$

It can be seen from Equation (4.13) that for the floating WECs, the desired value of the PTO stiffness can take negative values $K_{pto}(\omega) < 0$ for the range of wave frequencies when $C > \omega^2 (m + A(\omega))$. However, for the submerged buoys the PTO stiffness is always positive as $C = 0$. This is a very important feature, as the positive K_{pto} can be easily achieved by using a physical spring component, while implementation of the negative stiffness requires the reactive power flow through the PTO system which is much more difficult to achieve in practice.

4.3.2 Power output in irregular waves

Regular wave results presented in Section 4.2 demonstrate the fundamental differences between floating and submerged WECs. However, the comparison of these systems should be conducted under irregular wave time-series that more accurately represent real sea states. Assuming that the PTO system comprises spring and damping effects, the time-varying load force can be modelled similarly to Equation (4.12):

$$F_{pto}(t) = -B_{pto}\dot{z}(t) - K_{pto}z(t), \quad (4.14)$$

where $z(t)$ and $\dot{z}(t)$ are the displacement and velocity of the buoy in heave.

Depending on the control strategy, K_{pto} and B_{pto} can be tuned on a sea state basis [12] or optimised in real time on a wave by wave basis [22]. The latter strategy comes closest to the optimal reactive control in terms of the power output [23], but it requires an accurate plant model and future knowledge of the wave excitation forces. Therefore, in this work it is assumed that PTO parameters are tuned (optimised) for each sea state, which is easier to implement in practice but leads to a sub-optimal control method. Optimisation of PTO parameters is made using an exhaustive search while allowing negative values of K_{pto} for floating converters. A full description of the modelling routine and simulation set-up can be found in 4.B.

The performance of the spherical and cylindrical WECs over the range of irregular sea states are shown in Figures 4.6 and 4.7 respectively. The power absorption is represented in terms of the relative capture width (a ratio of the absorbed power to the power that is contained in the incident wavefront of a width equal to the characteristic length of the converter). The overall trend is consistent with regular wave results: floating WECs demonstrate better performance across all sea states except several cases with $T_p < 6$ s and $H_s = 2$ and 3 m, where motion of the floating converters has already reached constraints. Also, previous findings, that the cylinder placed closer to the mean water level ($d_s = 3.75$ m) can generate more power than its floating

counterpart at a range of wave periods, have not been confirmed in irregular waves.

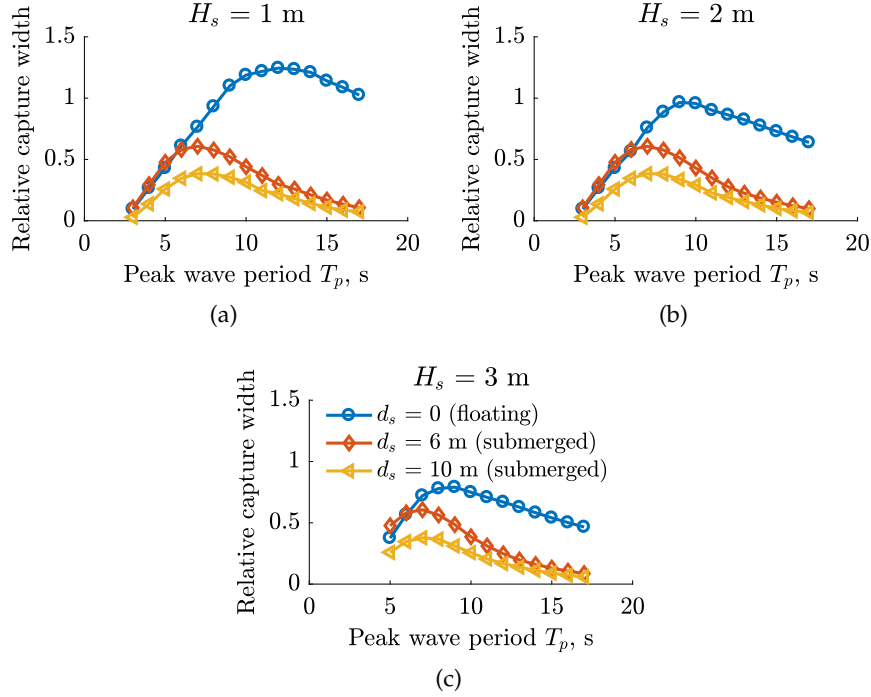


Figure 4.6: Relative capture width of the floating and submerged spherical WECs in irregular waves with significant wave heights of (a) $H_s = 1$ m, (b) $H_s = 2$ m, and (c) $H_s = 3$ m over the range of peak wave periods. The sea states have been generated using a Pierson-Moskowitz wave spectrum.

Another interesting observation is that a sphere submerged deeper ($d_s = 10$ m) shows poorer performance across all sea states, even though according to the regular wave results all buoys should absorb the same amount of power up to wave periods of 7 s (see Figure 4.3). This may be caused by the control strategy applied to all WECs which is not optimal a priori. So the performance of the system with fixed control parameters (even if they are tuned for each sea state) can be highly dependent on the WEC bandwidth.

4.3.3 Resonance bandwidth

Resonance (or absorption) bandwidth of the converter corresponds to the frequency range where the absorbed power stays within 50% of its maximum value. Thus, the broader the bandwidth, the less need for control. Large structures such as terminators or attenuators have broader bandwidth than

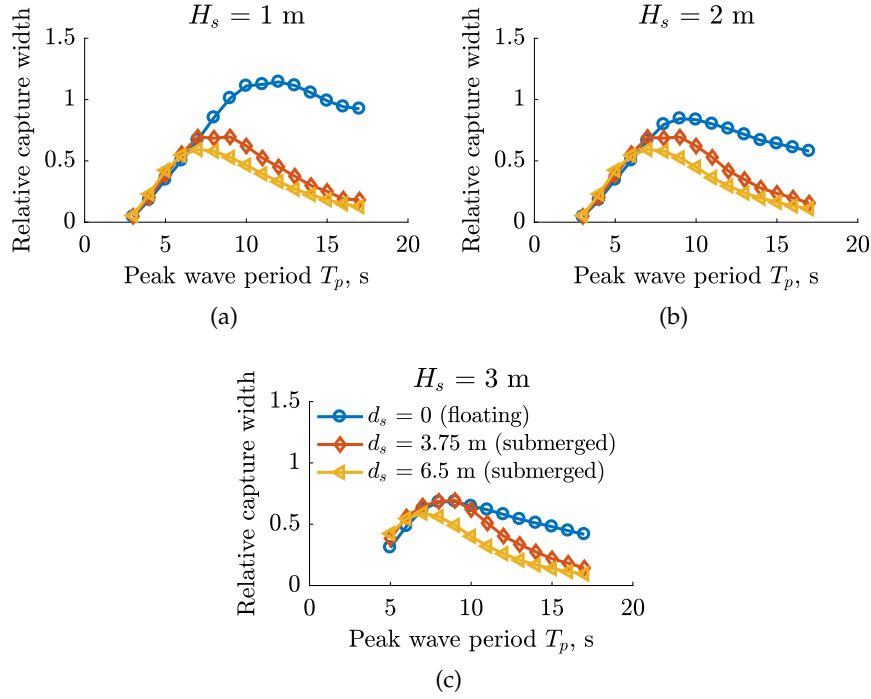


Figure 4.7: Relative capture width of the floating and submerged cylindrical WECs in irregular waves with significant wave heights of (a) $H_s = 1$ m, (b) $H_s = 2$ m, and (c) $H_s = 3$ m over the range of peak wave periods. The sea states have been generated using a Pierson-Moskowitz wave spectrum.

point absorbers, and for the latter, increase in size leads to the bandwidth extension [19].

For comparison between floating and submerged WECs, it is necessary to understand the impact of submergence depth on the system bandwidth. Consider the response of converters when its resonance is tuned to only one wave frequency from the spectrum as in Section 4.3.2. Figure 4.8 shows the non-dimensional power absorption of spherical bodies of $a = 5$ m radius with different submergence depths ($d_s = 0, 6$ m and 10 m) in regular waves of $H = 1$ m height with the mass of all buoys kept constant at $m = 0.5\rho V$. The power take-off damping and spring coefficients are chosen such that each system reaches resonance at the wave period of $T_0 = 8.5$ s ($\omega_0 = 0.74$ rad/s), allowing negative PTO stiffness for the floating case. No motion constraints are considered. The power absorption is presented as $\frac{P_k}{f}$ with a maximum possible value of 1 for the heaving body. It can be seen that the resonance bandwidth $\left(\frac{\Delta\omega_{res}}{\omega_0}\right)$ of the floating sphere is about 4 times wider than that of the submerged one. Moreover, the deeper the body is submerged, the

narrower the resonance bandwidth becomes. This can be explained by the fact that the resistance (radiation damping) of the converter decreases as the immersion depth increases, which leads to a narrow bandwidth of fully submerged buoys.

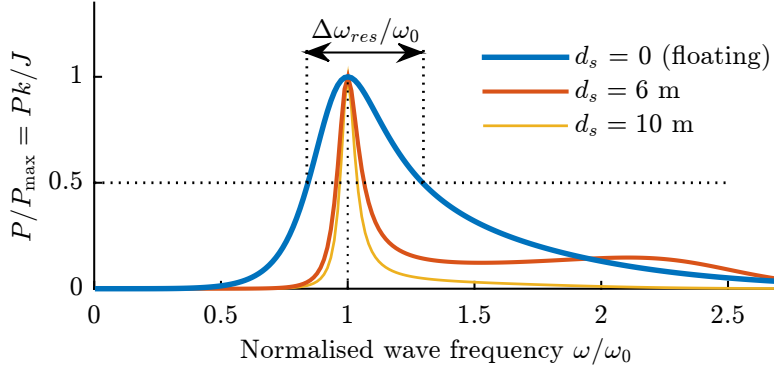


Figure 4.8: Resonance bandwidth of the spherical WECs in regular waves of $H = 1$ m height: non-dimensional absorbed power vs. normalised wave frequency. Converters of 5 m radius are tuned to reach resonance at the wave frequency of $\omega_0 = 0.74$ rad/s.

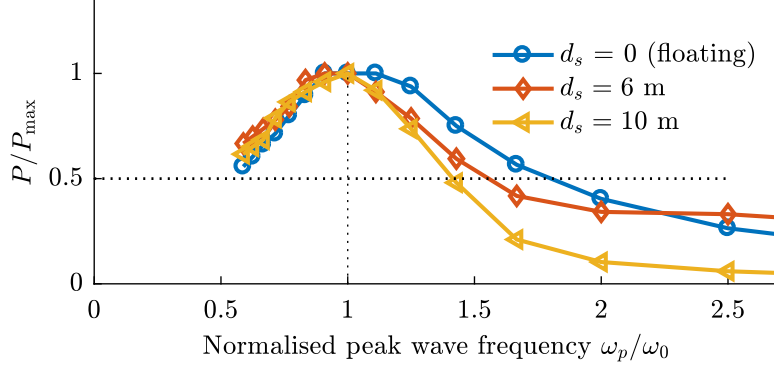


Figure 4.9: Resonance bandwidth of the spherical WECs in irregular waves of $H_s = 1$ m significant wave height: non-dimensional absorbed power vs. normalised peak wave frequency. Converters of 5 m radius are tuned to the sea state with a significant wave height of $H_s = 1$ m and a peak wave period of $T_p = \frac{2\pi}{\omega_p} = 10$ s. All irregular wave time-series have been generated using a Pierson-Moskowitz wave spectrum.

Results in Figure 4.8 show how the performance of the WECs deteriorates when the converter resonance is tuned to only one frequency from the spectrum. Similarly, consider a behaviour of the same converters in irregular waves when fixed controller parameters are chosen to match only one sea

state from the site wave climate. Thus, the non-dimensional power absorption of spherical WECs in irregular waves of $H_s = 1$ m significant wave height is shown in Figure 4.9. The fixed PTO stiffness (K_{pto}) and damping (B_{pto}) coefficients are set such that each converter has maximum power output at the sea state of $H_s = 1$ m and $T_p = 10$ s. So values of P in the figure correspond to the absorbed power when one set of control gains is used across all sea states, while for P_{max} PTO parameters are optimised for each sea state. These results confirm frequency response findings: changes in the sea conditions lead to a more noticeable decrease in energy harvesting for submerged converters than for the floating one. A difference in passband width in Figures 4.8 and 4.9 is due to the fact that bandwidth in regular waves characterises deviations from optimal control, while its value in irregular waves shows the sensitivity of the averaged absorbed power to changes in the sub-optimal controller (even with optimised parameters).

Overall, it is essential to apply optimal control to point absorbers due to their narrow bandwidth, which is even narrower for submerged cases as the bandwidth decreases with increased immersion of the buoy.

4.3.4 Considerations for passive phase control

Optimal control uses bi-directional power flow to manipulate the resonance frequency of the WEC, whereas there are other control strategies that can improve the power output of the system without the need for reactive power flow through the machinery. Phase control, mostly represented by latching [1, 24] and declutching [3], achieves the optimal phase condition between the buoy velocity and the wave excitation force by locking or unlocking the buoy motion during parts of the oscillation cycle. Thus, latching and declutching controls refer to the ‘bang-bang’ strategies where the machinery force is switching between some constant and a very large value for latching and between zero and a constant value for declutching.

Latching control shows the best performance when the incident wave frequency is lower than the resonance frequency of the device ($\omega < \omega_0$) [1]. In this case, the buoy velocity leads the excitation force (see Figure 4.10a) and there is only a small part of the cycle when the buoy should be kept stationary to achieve an optimal phase condition. However, when $\omega > \omega_0$, the buoy velocity lags the excitation force (see Figure 4.10b) meaning that the buoy motion should be locked for at least half of the cycle which is not practical from the power absorption point of view. In contrast, the declutching control strategy performs better with systems where the natural frequency of the WEC is higher than the wave frequency ($\omega > \omega_0$) [3].

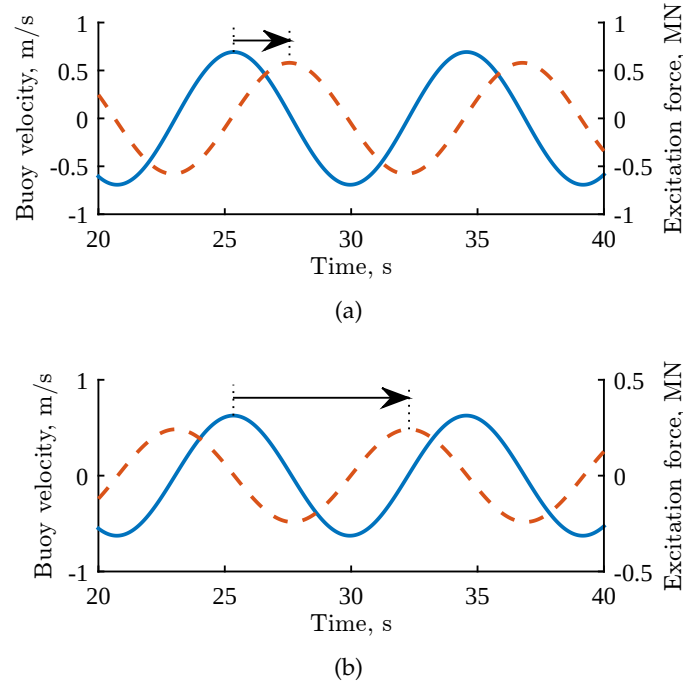


Figure 4.10: Velocity (blue solid line) and excitation force (red dashed line) time series for the spherical WECs of 5 m radius under the regular wave of $H = 2$ m height and period of $T = 9$ s without any control: (a) floating and (b) submerged ($d_s = 10$ m).

As already noted, the natural frequency of submerged converters is always lower than the incident wave frequency, whereas for the floating point absorbers its value lies at higher frequency range. Therefore, it may be concluded, that the latching control is more beneficial for the floating converter while declutching is more suitable for its submerged counterpart. However, the latter may also take advantage of the optimal latching control if to shift its resonance to the higher frequency range by an additional physical spring. These results are graphically shown on Figure 4.11 along with findings on reactive control. Thus, choosing among different control strategies for submerged converters it is necessary to keep in mind their distinctive features from floating ones.

4.4 Choice of a reasonable WEC size

In this section, appropriate sizes of the floating and submerged WECs are studied for several generic body shapes assuming that all buoys are optimally controlled at each wave frequency. Then, the obtained results are examined

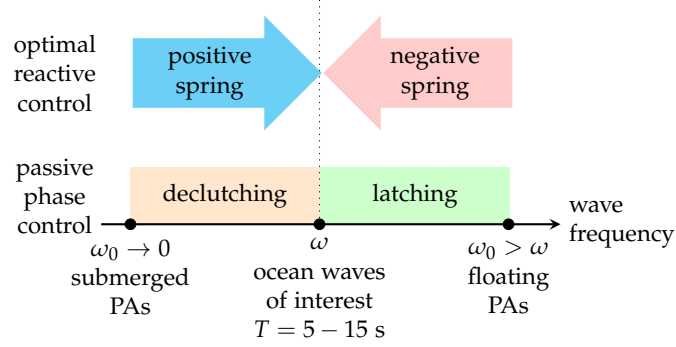


Figure 4.11: The generalised diagram of the wave energy converter control depending on the location of its natural frequency with respect to the incident wave frequency.

under irregular wave conditions.

According to the results presented in Section 4.2, floating and submerged converters should be of different sizes in order to capture the same amount of wave energy. Thus, using the upper power absorption bounds P_A and P_B , it is possible to choose an appropriate size of the WEC for a particular sea site. Falnes [18] proposed a methodology of selection of the WEC size and power take-off capacity according to the following steps:

Sea site $\rightarrow J_T$. Choose an appropriate sea site location and determine a wave power threshold J_T (kW/m) which is being exceeded only one third of the year.

Spectrum $\rightarrow T_p(T_e)$. Find the peak period, or the peak energy period of the most frequent waves according to the sea site probability data (T_p or T_e).

$J_T, T_p(T_e) \rightarrow T, H$. Relate J_T and T_p to the regular wave of period $T = T_e \approx 0.858T_p$ (for the fully developed uni-modal sea) [38] with the same wave power level. Determine the wave height of the corresponding regular wave using the equation $J_T = \frac{\rho g^2 H^2 T}{32\pi}$.

$T, H \rightarrow V$. Calculate the body volume solving the equation $P_A = P_B$ that results in:

$$V = \frac{c_\infty}{c_0} H T^4. \quad (4.15)$$

The body chosen according to this approach will operate at full capacity for at least one third of the year.

Now consider an example assuming that there is a need to select a wave energy converter for the site where wave power level exceeds 34 kW/m about one-third of the year, a significant wave height is $H_s = 2.5\text{--}3$ m and the peak wave period is $T_p = 10$ s. This corresponds to the regular wave of $H = 2$ m and $T = 8.5$ s. Using Equation (4.15) it may be calculated that the volume of the floating converter regardless of shape should be 322 m^3 . In the case of the submerged converters, the same methodology can be applied but using different P_B -bounds and expressing all parameters in terms of the body radius. For the submerged sphere setting the maximum displacement in heave as $s_{3,\max} = 0.67a$, the submergence depth of $d_s = 1.87a$ should ensure the operation of the body remains under water at all times. As a result, solving the equation $P_A = P_{B,s}$ (Equation (4.4) = Equation (4.6)) numerically with one unknown a , the size of the submerged sphere should be 696 m^3 for the same sea site, which is more than twice the required volume of the floating converter. Similarly, the volume of the cylindrical WECs should be 448 m^3 using Equations (4.4) and (4.7), and setting $h_c = a$, $d_s = 1.2a$, $s_{3,\max} = 0.5a$.

To demonstrate the effect of the body size on power efficiency, the power absorption curves for floating and submerged spherical bodies of six different radii (3–8 m) are shown on Figures 4.12a and 4.12b respectively, where the displacement amplitude of all converters is constrained by $0.67a$ and the d_s distance for submerged WECs is $1.87a$. The dotted vertical line and points on curves correspond to the targeted wave period of $T = 8.5$ s.

The data on Figure 4.12 can also be represented in terms of the Power-Volume correlation for the fixed wave period of interest. Thus, in order to make the analysis more generic, the spherical case shown on Figure 4.12 has been complemented by three other body shapes including a cylinder, an ellipsoid (oblate spheroid) and a chamfered cylinder (see Figure 4.13). Table 1 shows parameters of all systems and the correlation between them. The maximum motion of all bodies is chosen in a way that $s_{3,\max} \approx V/(2S_{xy}) = V/(2\pi a^2)$ meaning that all converters have the same volume stroke, where S_{xy} is the cross-section area of the buoy in the horizontal xy plane. The submergence depth is set such that the body of 524 m^3 volume at the maximally extended stroke has 1 m distance from its top surface to the mean water level, for other sizes this distance is scaled down or up according to the body radius. Hydrodynamic parameters of mentioned geometries have been obtained using WAMIT [26]. Mesh convergence has been checked and typical models composed of approximately 2000 panels.

The dependence of the maximum captured power on the structural volume of four bodies for the sea state of interest with $H = 2$ m, $T = 8.5$ s

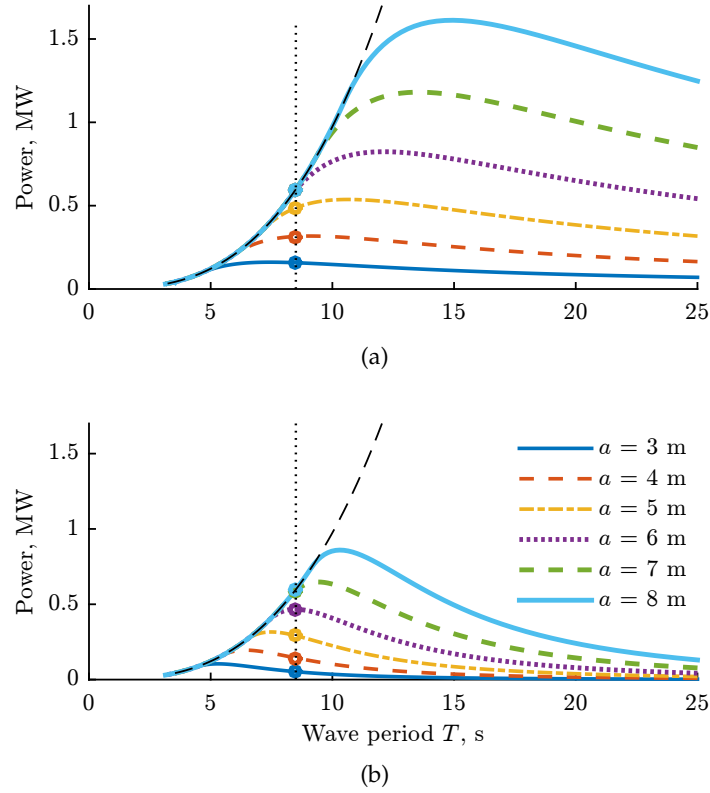


Figure 4.12: Power absorbed by the (a) floating ($d_s = 0$) and (b) submerged ($d_s = 1.87a$) spherical WECs of different radii vs. wave period. Wave height is $H = 2$ m, motion of all buoys is constrained by $0.67a$. The black dashed curve corresponds to the P_A limit from Equation (4.4).

is demonstrated on Figure 4.14. The upper limit of 0.6 MW corresponds to the maximum of P_A curve at $T = 8.5$ s, and markers on each curve show the body volume chosen according to Falnes' methodology. Thus, the size of submerged buoys designed for the same sea site should be 1.4 – 2.2 larger than the floating one with approximately the same power capacity. Interestingly, that among all submerged cases, a body with a cylindrical shape should be the smallest while the spherical buoy should be the largest to generate the same amount of power. The desired volumes of the ellipsoid and chamfered cylinder are estimated to be somewhere in-between.

In addition to the absolute values of power and volume, the power-to-volume ratio is also of interest as it can be indirectly related to the estimation of the converter cost [4]. It has been observed that for the heaving point absorbers the smaller the physical volume of the body, the larger the power-to-volume ratio [17]. To demonstrate the impact of this relationship, it has

Table 4.1: Parameters of the WECs from Figure 4.13.

Parameter	Notation	Sphere	Cylinder	Ellipsoid	Chamfered cylinder
Radius	a	r	$1.1r$	$r\sqrt[3]{2} \approx 1.26r$	$r\sqrt[3]{1.5} \approx 1.14r$
Height (vertical dimension)	h	$2a$	a	a	a
Volume	V	$4\pi a^3/3$	πa^3	$2\pi a^3/3$	$85\pi a^3/96$
Submergence depth	d_s	$1.87a$	$1.2a$	a	$1.1a$
Motion constraints	$s_{3,\max}$	$2a/3$	$0.5a$	$a/3$	$0.45a$

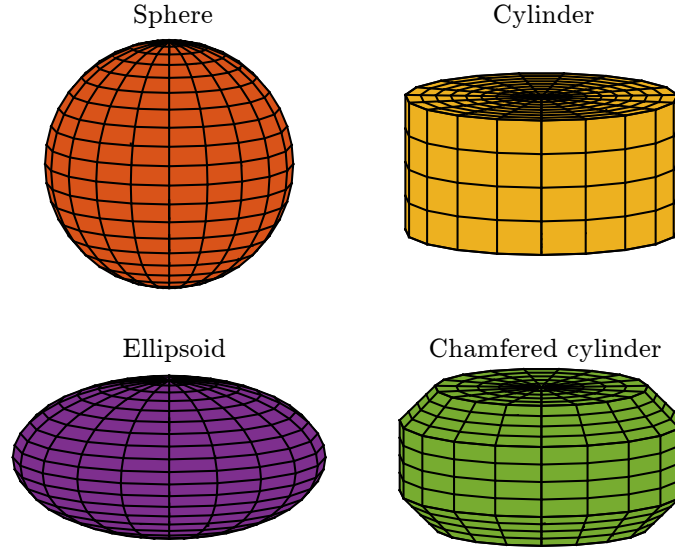


Figure 4.13: Shapes of WECs used in the Power-Volume analysis.

been calculated for five different systems based on the information from Figure 4.14. Power has been normalised according to the Froude scaling law choosing the wavelength $\lambda = 2\pi/k = g/(2\pi)T^2$ as a length parameter:

$$P_n = \frac{P}{\lambda^{3.5}}, V_n = \frac{V}{\lambda^3}, \quad (4.16)$$

and is presented in Figure 4.15. Thus, for the floating converters the power-to-volume ratio decreases with an increase in WEC volume. However, for submerged bodies there is a range of volumes where this ratio takes the maximum value which shows the importance of proper selection of the buoy size. Therefore, based on the maximum value of the power-to-volume ratio, the size of the submerged cylinder should be approximately 420 m³, which

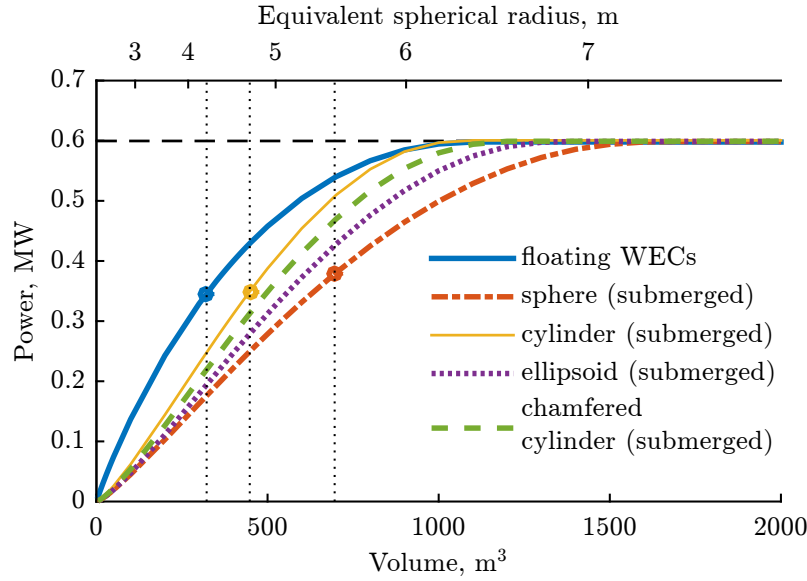


Figure 4.14: Dependence of the absorbed power on the volume of WECs for the regular wave of $H = 2$ m, $T = 8.5$ s. Parameters of all buoys are taken from Table 4.1. The horizontal dashed line corresponds to the maximum power that can be captured from this regular wave by an oscillating axisymmetric body. Markers show optimal volumes of WECs chosen according to Falnes' methodology based on power capacity.

is very close to 448 m^3 found using Falnes' methodology.

In order to investigate whether converters of the volumes found above have similar power absorption potential, their performance is investigated in the sea state with a significant wave height $H_s = H\sqrt{2} = 2.83$ m and a peak wave period $T_p = T_e/0.858 = 10$ s. Thus, the following buoy geometries are included in the irregular wave analysis: a floating cylinder of 322 m^3 , a submerged sphere of 696 m^3 , a submerged cylinder of 448 m^3 , a submerged ellipsoid of 600 m^3 and a submerged chamfered cylinder of 510 m^3 . All other geometric parameters can be calculated using Table 4.1. It is assumed that control gains are optimised on the sea state basis (see Equation 4.14) which are optimised to provide maximum power. As a result, an averaged absorbed power and a relative capture width are presented in Figure 4.16. Colour bars on the left show the averaged absorbed power, while dark blue bars on the right correspond to the relative capture width of each converter.

Despite the fact that all converters have been designed for this particular sea site, they demonstrate power production levels lower than expected from the regular wave analysis. This is due to sub-optimal control applied to all

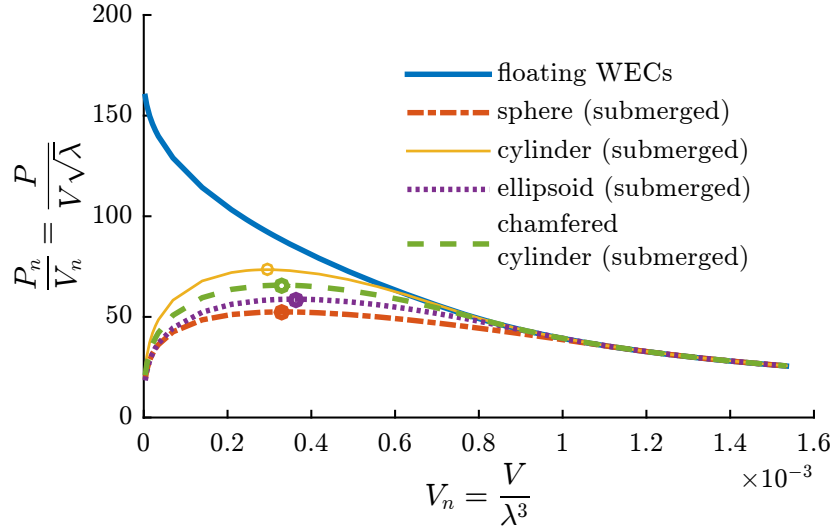


Figure 4.15: Power-to-volume ratios vs. buoy volume normalised to the wavelength for floating and submerged WECs.

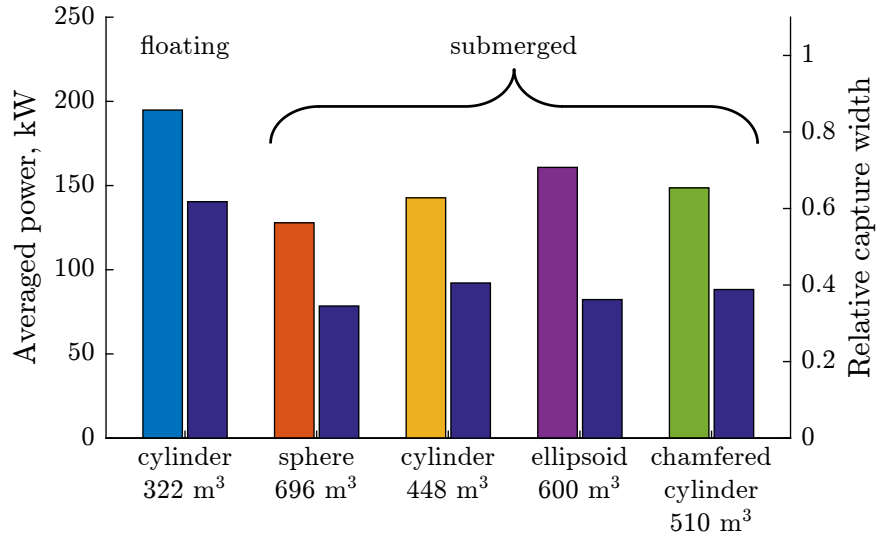


Figure 4.16: Levels of the averaged absorbed power (colour bars on the left) and relative capture width (dark blue bars on the right) of the floating and submerged converters at the irregular wave time-series of $H_s = 2.83$ m and $T_p = 10$ s. Parameters of all buoys are taken from Table 4.1.

cases, which once again demonstrates the importance of the control strategy for the WEC development. In addition, although all submerged converters have larger volumes than their floating counterpart, their power absorption is still around 1.2-1.5 times lower than that of the floating cylinder. This means that fully submerged buoys should be even larger than shown in Figure 4.14 in order to match the total power of the floating converter.

Overall, for the same power output submerged WECs should be at least 1.5 times larger than their floating counterparts where the exact volume ratio is the subject to control, shape, submergence depth and other parameters.

4.5 Array of small WECs vs. a large buoy of equal volume

According to the findings presented in Section 4.4, in particular Figure 4.15, floating wave energy converters of small size are more beneficial when employed for wave power generation in comparison with large ones in terms of the power-to-volume ratio. According to Budal's diagram (as cited in [18]), the volume in Equation (4.5) does not necessary represent the size of one unit, and it can be interpreted as a total volume of all converters within the wave energy array. It has been shown that a compact array of small buoys can capture much more power than a single WEC of equal volume, with the advantage of having wider bandwidth [20], however, this comparison was based on the floating truncated cylinders that are tuned to their natural frequency of oscillation, not optimally controlled.

A similar analysis for the fully submerged converters is performed here but applying optimal control across the entire range of wave periods. One relatively large spherical buoy of $a = 5$ m radius has a displaced volume of $V = 524 \text{ m}^3$. This volume alternatively can be formed using an array of smaller converters, e.g. 5 units of 3m-buoys, 125 units of 1m-buoys or 1000 units of 0.5m-buoys. Of course, the same volume does not guarantee the same manufacturing cost given the different surface areas, but production of buoys in large quantities will inevitably lead to lower unit manufacturing costs.

The amount of power extracted by each array in regular waves of 2 m height is presented in Figure 4.17 for the floating ($d_s = 0$) and fully submerged ($d_s = 1.87a$) cases of WECs, where all spheres have a maximum stroke of $0.67a$. The energy absorbed by each array has been calculated by multiplying the power from an individual WEC by the number of units within the array but neglecting to take into account possible hydrodynamic

4.5 Array of small WECs vs. a large buoy of equal volume

interaction between buoys. As a result, in the case of floating systems, smaller units have higher power absorption than a single large buoy across the entire range of wave periods.

This conclusion, however, is not applicable to the fully submerged WECs as the performance of the array drops dramatically with a decrease of the buoy size at long wavelengths. It seems that the “small is beautiful” adage of Falnes [18] only applies to floating WECs, not submerged. As a result, for sites with low frequency waves using submerged converters it would be more beneficial to design one large buoy that will have the best performance at the targeted sea site than to consider smaller units of equal volume.

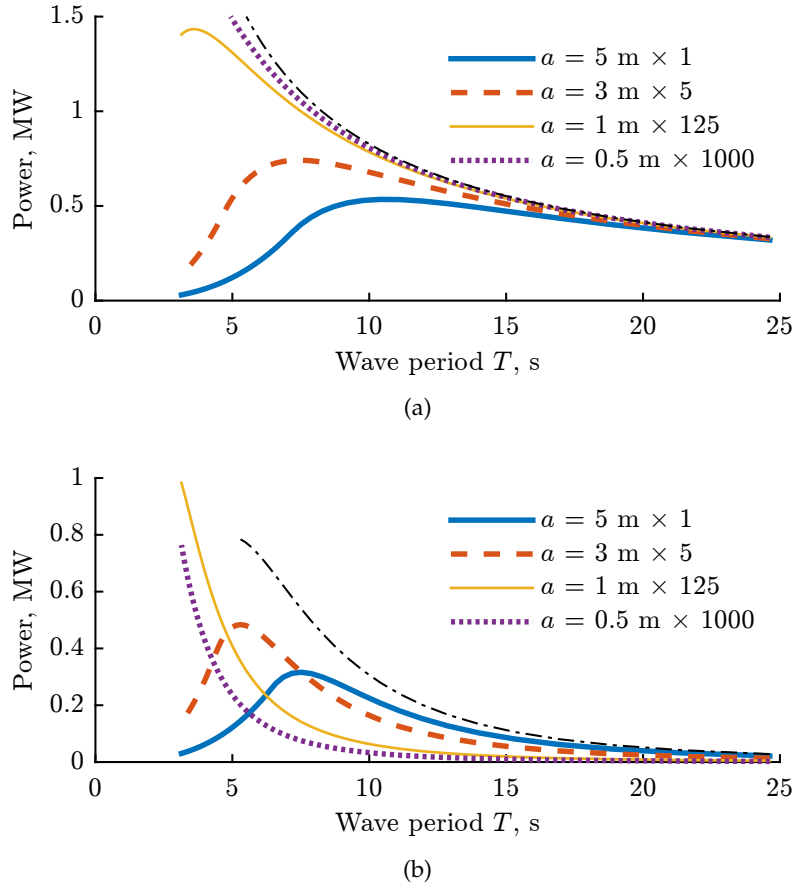


Figure 4.17: Power absorbed by (a) floating ($d_s = 0$) and (b) submerged ($d_s = 1.87a$) wave arrays of the same total volume $V \approx 524 \text{ m}^3$ in regular waves of 2 m height. The radius and number of units within the array is specified in the legend. Dash-dotted curves correspond to the P_B limit of the 5m-buoy. The displacement in heave of all converters is constrained by $0.67a$.

WECs chosen according to Falnes' methodology are designed for one

dominant wave of the particular sea site. However, due to the irregular nature of ocean waves and due to the fact that submerged converters are effective power absorbers only within a specific range of wave periods, it could be advantageous to form an array from the buoys of several different sizes, similar to [21, 41]. Thereby each converter will target a particular wave frequency from the spectrum while maximising the power absorption of the entire farm.

4.6 Combination of modes

The combination of motion modes in wave power absorption is attractive due to increased efficiency and bandwidth. For example, Salter's duck [33] utilises surge, heave and pitch oscillations in order to capture the maximum power available in the wave. Another solution has been offered in [34, 35, 36] introducing three cables connected to the spherical buoy in order to make the surge mode controllable by the power take-off system. Therefore, the power efficiency of submerged and floating WECs with different motion modes is compared in this section.

Surging and heaving floating converters radiate different types of waves that lead to different power absorption. According to Equation (4.1) the P_A -bound of the surging body is twice as high as that of the converter that moves in heave only [16]. The low frequency limits P_B are also different for these motion modes where Equation (4.6) describes heave oscillation, while an expression for the surging floating sphere has a form of [39]:

$$P_{B,f}^{surge} = 2\pi^3 \rho s_{1,\max} \frac{VH}{T^3}. \quad (4.17)$$

Analysing Equations (4.5) and (4.17), it is clear, that the P_B -bound for the surging body is $\mathcal{O}(T^{-3})$, while for the heaving body this bound has a smaller decay rate and is $\mathcal{O}(T^{-1})$. These results are very similar to the previous comparison of floating and submerged heaving bodies meaning that the surging floating sphere is a poorer power absorber at long wavelengths than the same body that oscillates in heave. Comparing motion modes of the floating sphere the following features should be outlined:

- $P_A^{surge} = 2P_A^{heave}$;
- $P_{B,f}^{surge} = \mathcal{O}(T^{-3})$, while $P_{B,f}^{heave} = \mathcal{O}(T^{-1})$.

Unlike floating converters, fully submerged buoys have almost the same power efficiency from oscillations in heave or surge. Thus, the P_B -bounds

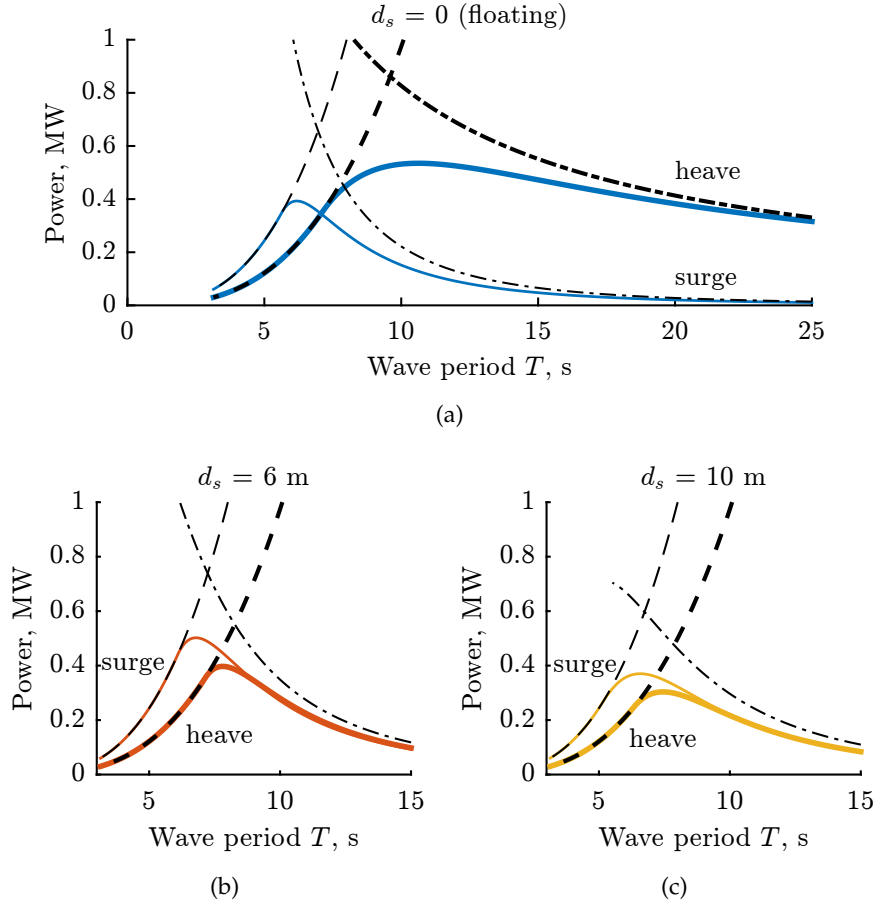


Figure 4.18: The power absorbed by the surging and heaving spherical WECs of 5 m radius with different submergence depth: (a) floating $d_s = 0$ and submerged (b) $d_s = 6$ m, (c) $d_s = 10$ m. Motion amplitudes in heave and surge are constrained by $s_{1,\max} = s_{3,\max} = 0.67a = 3.3$ m. Wave height is set to $H = 2$ m. The dashed curves correspond to the P_A limit from Equation (4.4), and dash-dotted curves show the P_B bounds from Equation (4.5) and (4.6).

for the surging and heaving submerged spheres have the same expressions described by Equation (4.6) which has been shown in [39]. Hence, for the fully submerged spherical WEC:

- $P_A^{\text{surge}} = 2P_A^{\text{heave}},$
- $P_{B,s}^{\text{surge}} = P_{B,s}^{\text{heave}} = \mathcal{O}(T^{-3}).$

The difference in power efficiency between surging and heaving spheres is demonstrated in Figure 4.18. The sphere radius is $a = 5$ m, motion in surge (mode 1) and heave (mode 3) is constrained by $s_{1,\max} = s_{3,\max} = 0.67a =$

3.3 m, submergence depths are $d_s = 0$ m, $1.2a = 6$ m and $2a = 10$ m, wave height is taken as $H = 2$ m. It can be seen, that at longer wave periods heave motion is dominant for floating converters showing that the power contribution from the surge mode may be marginal for floating systems. In contrast, a submerged sphere that oscillates in surge is more efficient across the entire frequency range. Therefore, the power efficiency of the submerged system may increase by two to three times due to the additional controllable degree of freedom. Also, the ratio between power levels from surge and heave does not change with a submergence depth as shown in Figures 4.18b and 4.18c. It should be noted that the surging floating sphere utilises only half of its volume to couple with the fluid (at nominal depth), while for a submerged sphere the total volume is involved in power absorption. This explains why the power level of a surging floating sphere is lower than that of a fully submerged one.

For comparison, similar plots are presented for the vertical cylinder of $h_c = a = 5.5$ m on Figure 4.19. Motion constraints in each mode are calculated as $s_{1,\max} = V/(2S_{xz}) = \pi a^2 h_c / (4ah_c) = \pi a / 4 = 4.32$ m and $s_{3,\max} = V/(2S_{xy}) = \pi a^2 h_c / (2\pi a^2) = a/2 = 2.75$ m in order to have equal volume stroke [39] in surge and heave. Interestingly, that for the vertical cylinder placed closer to the water surface ($d_s = 3.75$ m) at higher wave frequencies surge is dominant, while for low-frequency waves more power can be absorbed from heave. When the cylinder is submerged deeper ($d_s = 6.5$ m), the situation is closer to the spherical case, where the surging body captures more power across entire range of wave periods. As a result, the power distribution between motion modes for the fully submerged bodies depends on the submergence depth and the aspect ratio of the converter and a clear trend cannot be identified, as in the case of floating systems.

Based on this analysis, it may be concluded that employment of several motion modes in power generation is more advantageous for the fully submerged converters, while such a benefit for the floating counterparts will be marginal.

4.7 Conclusion

The comparison between floating and fully submerged WECs has been performed in order to identify main distinctive features between the systems. The analysis has been carried out in regular and irregular waves using a linear wave theory approximation for axisymmetric point absorbers that extract wave energy from heave or surge motion, or both.

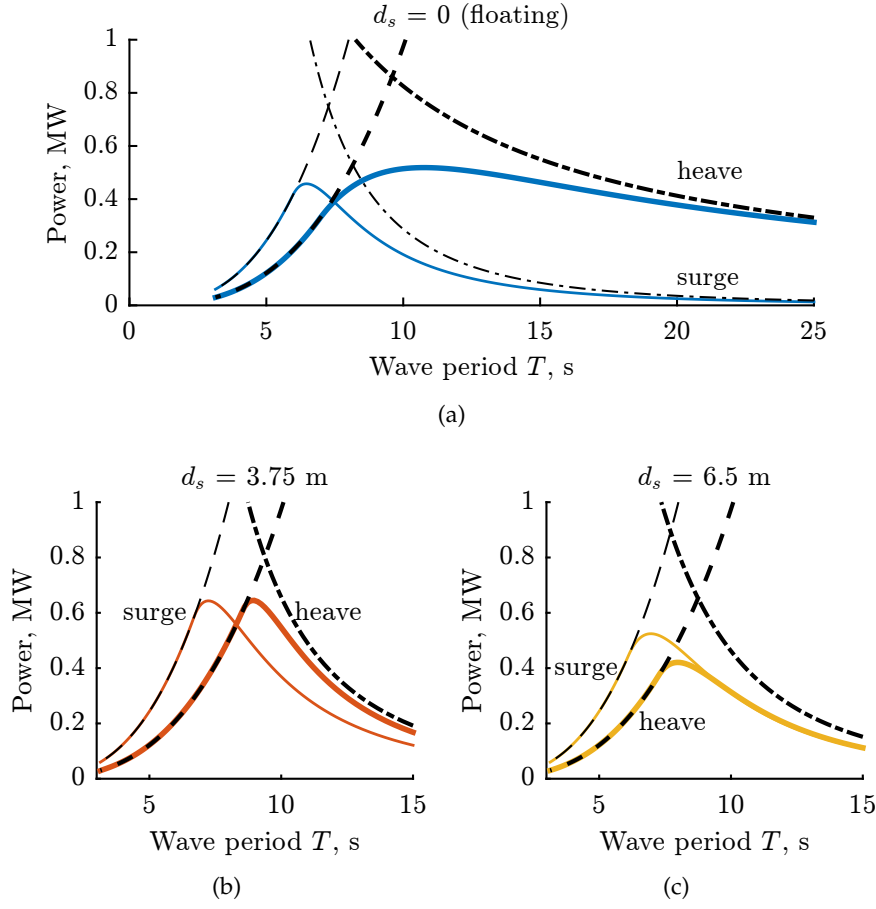


Figure 4.19: The power absorbed by the surging and heaving vertical cylinders ($h_c = a = 5.5$ m) with different submergence depth: (a) floating $d_s = 0$ and submerged (b) $d_s = 3.75$ m, (c) $d_s = 6.5$ m. Displacements in surge and heave are constrained by $s_{1,\max} = \pi a/4 = 4.32$ m and $s_{3,\max} = a/2 = 2.75$ m respectively. Regular wave height is set to $H = 2$ m.

Examples of two generic shapes (sphere and vertical cylinder) have shown that the efficiency of submerged converters is poorer than that of the floating ones at long wavelengths, while there is a narrow range of wave periods where the performance of submerged cylindrical buoys could be superior than that of its floating counterpart. In addition, WEC bandwidth decreases as the submergence depth of the buoy increases indicating the need for reactive control for the fully submerged converters. The absence of the hydrostatic restoring force for the submerged buoys affects the implementation of control strategies for this converter type. Thus, floating converters may benefit from latching phase control, whereas declutching is more suitable for submerged

systems. To achieve the same level of generated power, buoys placed under water should be 1.4–2.2 times larger than those that operate on the water surface. Thus, the size of the fully submerged WECs should be chosen according to the targeted wave climate and cannot be replaced by the array of smaller converters of equal volume. Finally, submerged buoys would benefit more from multiple degrees of freedom than their floating counterparts.

The above analysis may give the impression that submerged buoys are less favourable energy converters. It should be noted, that the main objective of this study is not to show which WEC is better, but to clarify some differences in performance and design criteria. While the current paper is restricted to axisymmetric buoy shapes, there are some examples when submerged converters demonstrated good power absorption abilities. Thus, several solutions of maximising energy harvesting of submerged WECs have been offered so far: (i) to use a device with a dynamically changing volume (e.g. Archimedes Wave Swing device [40]); (ii) to keep the body submerged as close as possible to the mean water level (e.g. CETO system [7]); or (iii) to use a terminator body which inherently has a broad resonance bandwidth (e.g. the Bristol cylinder [9, 15]). In addition, as previously mentioned, submerged converters have a number of advantages which may be essential from the economic perspective.

Appendix 4.A Estimation of the upper bound of the absorbed power for the submerged cylinder that oscillates in heave

The average power absorbed by the oscillating body can be calculated as [16]:

$$P = \frac{1}{2} |\hat{F}_{exc}| |\hat{u}| \cos \phi - \frac{1}{2} B |\hat{u}|^2 \leq \frac{1}{2} |\hat{F}_{exc}| |\hat{u}|, \quad (4.A.1)$$

where $\hat{F}_{exc}$ is the excitation force amplitude, $\hat{u}$ is the amplitude of the body velocity, ϕ is the phase angle between the excitation force and the buoy velocity and B is the hydrodynamic damping coefficient.

The excitation force exerted on the body in the j -direction is:

$$\hat{F}_{exc,j} = - \iint_S \hat{p}(x, y, z) n_j dS, \quad (4.A.2)$$

where $\hat{p}(x, y, z)$ is the complex amplitude of the hydrodynamic pressure on the body surface, n_j is the unit normal to the body surface pointing inside the water domain and dS is the surface element of wet surface S .

4.A Estimation of the upper bound of the absorbed power for the submerged cylinder that oscillates in heave

Hals [39] showed, that the hydrodynamic pressure $\hat{p}$ on the body surface will never be larger than $\sigma \hat{p}_0$, where $\sigma = 2$, $\hat{p}_0 = p_0 e^{i(-kx + \phi_\zeta)} e^{kz}$ is the pressure amplitude of the undisturbed incident wave, $p_0 = \rho g \zeta_0 = \rho g A$ and ϕ_ζ is the phase angle of the incident wave. As a result:

$$\hat{p}(x, y, z) \leq \sigma \hat{p}_0 = \sigma \rho g A e^{i(-kx + \phi_\zeta)} e^{kz}. \quad (4.A.3)$$

In order to derive the simplified equation of the heave excitation force for the submerged cylinder, cylindrical coordinates (r, θ, z) are introduced leading to $dS = r dr d\theta$, where $0 \leq r \leq a$ and $0 \leq \theta \leq 2\pi$. Also, pressure in Equation (4.A.2) should be integrated over two surface areas: top ($\hat{p}_t$) and bottom ($\hat{p}_b$) faces of the truncated cylinder. Thus,

$$\hat{F}_{exc,3} = \int_0^a \int_0^{2\pi} (\hat{p}_t(r, \theta, -d_1) - \hat{p}_b(r, \theta, -d_2)) r dr d\theta. \quad (4.A.4)$$

The hydrodynamic pressure on the cylinder bottom can be described by Equation (4.A.3):

$$\hat{p}_b(r, \theta, -d_2) \leq \sigma \rho g A e^{i(-kr \cos \theta + \phi_\zeta)} e^{-kd_2}. \quad (4.A.5)$$

However, for the cylinder top free-surface effects are more significant due to the possible resonance amplification of waves in the water domain above the cylinder and, therefore, the hydrodynamic pressure cannot be simplified using e^{kz} function. Thus, taking the first order approximation for the upper domain from [25], the hydrodynamic pressure on the cylinder top can be expressed as:

$$\hat{p}_t(r, \theta, -d_1) \approx \rho g A e^{i(-kr \cos \theta + \phi_\zeta)} \frac{J_0(\kappa r)}{J_0(\kappa a) \cosh \kappa d_1}, \quad (4.A.6)$$

where J_0 and J_1 are Bessel functions of the first kind of order 0 and 1 respectively, κ is the real solution of the dispersion equation $\omega^2 = g\kappa \tanh(\kappa d_1)$, which can be approximated by $\omega^2 = \kappa^2 g d_1$ using shallow water conditions for the water domain above the cylinder.

Inserting Equations (4.A.5)–(4.A.6) into (4.A.4) and using $e^{-ikr \cos \theta} = \cos(-kr \cos \theta) + i \sin(-kr \cos \theta)$, $|\sin(-kr \cos \theta)| \leq |kr \cos \theta|$, $|\cos(-kr \cos \theta)| < 1$, the approximate expression of the excitation force is:

$$\begin{aligned} |\hat{F}_{exc,3}| &\leq \left| \sigma \rho g A \int_0^a \int_0^{2\pi} \left(\frac{J_0(\kappa r)}{J_0(\kappa a) \cosh \kappa d_1} - e^{-kd_2} \right) r dr d\theta \right| \\ &= \left| \sigma \rho g A \pi \left(\frac{2a J_1(\kappa a)}{\kappa J_0(\kappa a) \cosh(\kappa d_1)} - a^2 e^{-kd_2} \right) \right|. \end{aligned} \quad (4.A.7)$$

For the deeper submerged bodies and if $\kappa a \rightarrow 0$, then $J_0(\kappa a) \rightarrow 1$ and $J_1(\kappa a) \rightarrow \frac{\kappa a}{2}$ and Equation (4.A.7) can be further simplified to:

$$|\hat{F}_{exc,3}| \leq \left| \sigma \rho g A \pi a^2 \left(\frac{1}{\cosh(\kappa d_1)} - e^{-\kappa d_2} \right) \right|. \quad (4.A.8)$$

Finally, given the expression for the excitation force and setting $|\hat{u}_3| \leq \omega s_{3,\max}$, the upper power bound for the heaving submerged cylinder is:

$$\begin{aligned} P &\leq \frac{1}{2} |\hat{F}_{exc,3}| |\hat{u}_3| \\ &\leq \left| \sigma \rho g A \pi \left(\frac{2a J_1(\kappa a)}{\kappa J_0(\kappa a) \cosh(\kappa d_1)} - a^2 e^{-\kappa d_2} \right) \right| |\hat{u}_3| \\ &= \frac{\pi^2 a \rho g H}{T} s_{3,\max} \left(\frac{2 J_1(\kappa a)}{\kappa J_0(\kappa a) \cosh(\kappa d_1)} - a e^{-\kappa d_2} \right). \end{aligned} \quad (4.A.9)$$

Appendix 4.B Time-domain model of the wave energy converter

The most common mathematical model that describes a time-domain response of the wave energy converter in waves is the Cummins equation [11]:

$$(m + A_\infty) \ddot{z} + \int_0^t K_{rad}(t - \tau) \dot{z}(\tau) d\tau + Cz = F_{exc} + F_{pto} + F_{hs}, \quad (4.B.1)$$

where m is a buoy mass, A_∞ is the infinite-frequency added mass coefficient, C is the hydrostatic stiffness, $K_{rad}(t)$ is the radiation impulse response function, F_{exc} is the wave excitation force, F_{pto} is the load force exerted on the buoy from the power take-off system, and F_{hs} is the additional force that keeps the body motion within allowed boundaries similar to the physical hard stop mechanism.

The load force is modelled as a linear spring-damper system:

$$F_{pto} = -B_{pto} \dot{z} - K_{pto} z, \quad (4.B.2)$$

where K_{pto} and B_{pto} are the PTO stiffness and damping coefficients (control parameters). To constrain the motion of the buoy, the hard stop system is modelled by a repulsive energy potential [2]:

$$F_{hs} = -K_{hs,\min}(z - z_{\min})u(z_{\min} - z) - K_{hs,\max}(z - z_{\max})u(z - z_{\max}), \quad (4.B.3)$$

where $u(\cdot)$ is Heaviside step function, $K_{hs,\min}$ and $K_{hs,\max}$ are the hard stop spring coefficients, $z_{\min}$ and $z_{\max}$ are the stroke limits relative to the nominal

position of the converter. The effect of this force is not taken into account while calculating useful absorbed energy.

Equation (4.B.1) has been implemented in Simulink/MATLAB [37] with a time step of 0.01 seconds using the ode23s solver. The duration of all simulation runs has been set to $300 \times T_p$ but not less than 1200 seconds and the first $15 \times T_p$ have not been included in the analysis due to the initial transient state. Hydrodynamic (excitation and radiation) forces have been calculated using WAMIT [26]. The convolution integral in Equation (4.B.1) has been replaced by the state-space model using the Marine System Simulator toolbox [30]. The irregular wave time-series have been implemented using the Pierson-Moskowitz wave spectrum [38].

The mass of all floating and submerged buoys is kept as $m = 0.5\rho V$. Values of motion constraints are specified in Table 4.1 for each buoy geometry under consideration. The hard stop spring coefficient is set to $K_{hs,\min} = K_{hs,\max} = 10^8$ N/m. Unless otherwise stated, the PTO control parameters K_{pto}, B_{pto} are optimised for each sea state using brute-force search.

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Chapter 5

The effect of geometry on the performance of the three-tether wave energy converter

As discussed in the previous chapter, it is more advantageous to employ multiple modes of motion if the wave energy converter is fully submerged than if it operates on the water surface. This confirms the rationality of using a three-tether mooring configuration for the axisymmetric submerged systems similar to CETO.

The hydrodynamic properties of the three-tether WEC and its potential power output are highly dependent on the shape and dimensions of the absorbing body (buoy), and on the arrangement of mooring lines that governs the coupling between PTO units and the buoy motion. Moreover, as energy is harvested from both heave and surge modes, the aspect ratio of the buoy is one of the key factors that determines the contribution of each mode to the total power efficiency while imposing requirements on the mooring and foundation design. Therefore, this chapter is designed to provide an answer to the following research question: *How geometric parameters of a fully submerged three-tether converter, such as the tether arrangement, shape, and aspect ratio affect the system performance?*

This chapter consists of two sections composed of one published journal article and one published conference paper: the arrangement of mooring lines that maximises power absorption of a WEC is investigated in Section 5.1 based on the frequency domain model, and the effect of the shape and mass on system performance is analysed in the time domain in Section 5.2.

5.1 Arrangement of mooring lines

This section consists of the published journal article:

Sergiienko, N. Y., Cazzolato, B. S., Ding, B. and Arjomandi, M. (2016). “An optimal arrangement of mooring lines for the three-tether submerged point-absorbing wave energy converter”. *Renewable Energy* 93, pp. 27–37.

The article in its published format is available at <https://doi.org/10.1016/j.renene.2016.02.048>.

Statement of Authorship

Title of Paper	An optimal arrangement of mooring lines for the three-tether submerged point-absorbing wave energy converter
Publication Status	☒ Published ☐ Accepted for Publication ☐ Submitted for Publication ☐ Unpublished and Unsubmitted work written in manuscript style
Publication Details	Sergiienko, N. Y., Cazzolato, B. S., Ding, B. and Arjomandi, M. (2016). "An optimal arrangement of mooring lines for the three-tether submerged point-absorbing wave energy converter". <i>Renewable Energy</i> 93, pp. 27–37.

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Name of Principal Author (Candidate)	Nataliia Y. Sergiienko		
Contribution to the Paper	Developed the ideas and concepts, developed the mathematical model, wrote the code for the simulations and optimisation routines, interpreted results, wrote the manuscript, and acted as a corresponding author.		
Overall percentage (%)	75%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
Signature		Date	05/02/2018

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

- the candidate's stated contribution to the publication is accurate (as detailed above);
- permission is granted for the candidate to include the publication in the thesis; and
- the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

Name of Co-Author	Benjamin S. Cazzolato		
Contribution to the Paper	Encouraged to perform the kinematic analysis of the mechanism, participated in developing ideas and concepts, supervised the work, helped in interpretation of obtained results, and provided a critical revision of the manuscript.		
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An optimal arrangement of mooring lines for the three-tether submerged point-absorbing wave energy converter

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Abstract

Point-absorbing wave energy converters (WECs) with a single-tether mooring are capable of extracting power from heave motion, but they do not utilise the full energy harvesting potential. One of the possible ways to increase the total power absorption is to add another controllable degree of freedom. These can be achieved by using a so-called ‘tripod’ configuration when the body is tied to three tethers attached to the power take-off systems at the sea floor. This paper investigates the optimal inclination of tethers considering two different approaches: a purely kinematic analysis, not taking into account the shape of the buoy and a dynamic analysis of spherical and cylindrical WECs, using a linear frequency-domain method. The results show that for a submerged sphere and for a submerged vertical cylinder with an aspect ratio of one, tethers should be orthogonal to each other, forming edges of the cuboidal vertex. Such a configuration of tethers provides for uniform performance of the WEC in all directions of motion. However, for the cylinders with an aspect ratio other than one, an optimal angle between the tethers depends greatly on the ratio between the cylinder height and diameter.

5.1.1 Introduction

Ocean waves are a huge resource of renewable energy with great potential to be captured and employed for electricity generation and water desalination. Many concepts for extracting energy from surface waves have been realised, leading to more than 200 different wave energy converters (WECs) in various stages of development [16]. Despite the wide variety of WECs technologies on offer, floating and fully submerged point absorbers comprise a great proportion of existing full-scale prototypes of WECs, which typically operate in deep water waves with high energy content [9]. In most cases, a point absorber, whose dimensions are smaller than a wavelength, is designed as an axisymmetric buoy with the main advantage being insensitive to wave direction [8].

Nomenclature

A	amplitude of the incident wave
$\mathbf{A}, \mathbf{B}$	hydrodynamic added mass and damping coefficient matrix
$\mathbf{F}_{exc}$	incident wave excitation force
$\mathbf{F}_{pto}$	force exerted on the WEC due to the power take-off system
$\mathbf{F}_{rad}$	hydrodynamic radiation force
G	buoy centre of mass
H	WEC (cylinder) height
$\mathbf{I}_3$	identity matrix of size 3×3
$\mathbf{J}^{-1}, \tilde{\mathbf{J}}^{-1}$	inverse kinematic Jacobian of the WEC: conventional and dimensionally homogeneous
N_i, L_i	attachment points of the i -th tether to the buoy hull and sea floor respectively
$\mathbf{N}_0, \mathbf{S}_0$	skew-symmetric matrices that represent $[\mathbf{n}_0]_{\times}^T$ and $[\mathbf{s}_0]_{\times}^T$ respectively
P	WEC power absorbed
T	dynamic tension in the tether
$\mathbf{Z}_{buoy}, \mathbf{Z}_{pto}$	hydrodynamics impedance of the buoy and impedance of the PTO system
a	WEC radius
$c_t, \mathbf{C}_t, \tilde{\mathbf{C}}_t$	power take-off damping: coefficient, matrix for one tether and a collective matrix for all tethers
$\mathbf{e}_{si}$	unit-vector directed along i -th tether
$\mathbf{f}_t, \boldsymbol{\tau}_t$	force and torque applied from the tether to the buoy
d_s	submergence depth of the WEC (distance from the sea water level to the WEC centre of mass)
g	gravitational acceleration
h	water depth
k	wavenumber ($\omega^2 = gk \tanh kh$)
$k_t, \mathbf{K}_t, \tilde{\mathbf{K}}_t$	power take-off stiffness: coefficient, matrix for one tether and a collective matrix for all tethers
l_i	dynamic length of the i -th mooring line
$m_b, \mathbf{M}$	WEC mass and mass matrix
m_w	mass of water displaced by the WEC
$\mathbf{n}_i$	position vector of the i -th tether attachment point relative to the buoy centre of mass
$\mathbf{q}$	vector of three leg length variables
$\mathbf{r}$	vector of linear displacements of the WEC
$\mathbf{s}_i$	vector directed along i -th tether
$\mathbf{x}$	6 DoF WEC position vector
α	inclination angle of each tether to the vertical
$\delta(\cdot)$	change in vector or scalar from nominal position
γ_0	distribution of the tether tension force over its length
κ	condition number
ω	radial wave frequency
ρ	water density
θ	angle between two tethers in the plane that they form
ϑ	vector of angular displacements of the WEC

An axisymmetric body as a prospective WEC has been thoroughly studied in [5, 10, 27], showing that its maximum power absorption is independent of the scale of the device and is a function of the wavelength of an incident wave and oscillatory modes of the body. The majority of existing point-absorbing WECs operate only in the heave mode, limiting energy extraction to approximately one third of the available energy [12]. Such considerable attention to the heaving buoys can be explained by the relative simplicity of the design and the lower capital cost as compared with WECs with multiple degrees of freedom. However, a point absorber that oscillates in two modes (heave/surge or heave/pitch) with optimal control parameters can theoretically capture three times more power than a heaving device, achieving the maximum power absorption for such types of WECs [12].

Depending on the power take-off (PTO) operating principle and type of mooring configuration, existing WECs with multiple degrees of freedom can be divided into two categories (similar to the classification in [15]):

- (i) WECs that utilise slack mooring lines just to keep a body on site (Figure 5.1a). Such mooring configurations are not involved in power generation and have been applied to the floating WECs, such as SEAREV [1] or Pelamis [8]. The power take-off system of these devices is located inside the buoy's hull, that can impose constraints on the size of the system.
- (ii) Floating or fully submerged energy converters where the mooring legs are always under tension (Figure 5.1b). In this configuration, tethers can be attached through spools to the electrical generators, as is implemented in the 3D-WEC that is under development by Resolute Marine Energy, Inc. [30], or to a piston of a hydraulic PTO system. A solo-duck WEC, developed by The University of Edinburgh [29], can also be classified in this category.

Although the effect of slack mooring lines on the performance of the floating WECs has been assessed in [4, 14, 25], a mooring configuration with tethers under tension is of more practical interest for submerged buoys due to the required positive buoyancy of the WEC.

An axisymmetric body needs to oscillate in two modes (radiating symmetric and antisymmetric waves) to absorb the maximum available energy [12]. Even though a body is symmetrical about a vertical axis, the addition of mooring lines makes the whole system asymmetric and therefore sensitive to the direction of wave propagation. This means that a WEC should oscillate along the vertical axis (heave) radiating symmetric waves, and in a horizontal

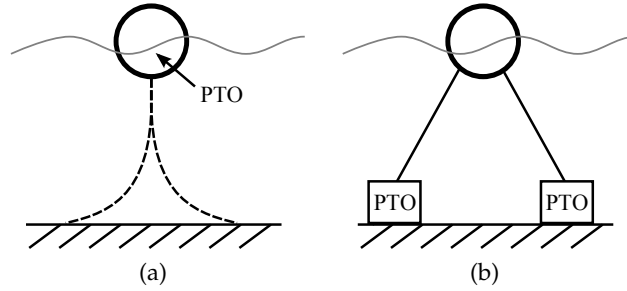


Figure 5.1: A schematic representation of mooring configurations: (a) slack mooring lines, (b) tension leg moorings.

plane (surge-sway or pitch-roll) radiating antisymmetric waves along the axis aligned with the propagation of incident waves. Having three motion modes that need to be controlled, intuitively, the minimum number of tethers in a mooring configuration is also three. Moreover, hydrodynamic forces that act on the axisymmetric body are uncoupled for surge, sway and heave, but are coupled for surge/pitch and sway/roll motions [12]. Therefore, ideally, the mooring design should not impose additional coupling between modes.

A system with a three-cable mooring of a submerged sphere was first proposed in [31]. The configuration, where three cables are equally inclined to the vertical and situated symmetrically around a sphere, provides an independence of surge, sway and heave motions, while all three modes are dependent on the parameters of a power take-off system and an inclination angle of cables [31]. Wave-to-wave tuning of the PTO parameters is an objective of a WEC control system, whereas the inclination angle of the cables cannot be changed during the life-span of the device and should be optimised during the design stage to maximise energy harvesting. However, the system in [31] was explored with only two inclination angles of the cables: 45 and 60 degrees, and the dependence of the power absorption on the angle has not yet been explored. A similar tripod WEC design has been used as a benchmark device in [20, 30] to test a developed optimal causal control system. This WEC consists of a floating cylinder attached to three tethers that are inclined to the vertical by 63.5 degrees, which seems to have been chosen arbitrarily by the researchers. To the best knowledge of the authors, these studies are the only ones that have considered a WEC with multiple degrees of freedom using a three-leg mooring. Consequently, the question of an optimal mooring configuration that maximises absorbed power remains open and is discussed in this paper.

In this research, the tripod system is investigated for a fully submerged

point-absorbing WEC, similar to the CETO system developed by Carnegie Wave Energy Limited [6]. Section 5.1.2 studies the problem from the kinematics point of view without taking into account any hydrodynamic properties of the WEC. Next, an optimal mooring configuration is examined for two generic buoy shapes: a sphere and a vertical cylinder, employing hydrodynamic models that are derived by using a linear wave theory approximation. The dynamic analysis in Section 5.1.3 is performed in the frequency domain, assuming a linear time-invariant system, an ideal control system and small displacements of the buoy, as compared with the length of the mooring lines. Results in Section 5.1.3 demonstrate the sensitivity of the optimal tether inclination angle to the number of parameters, such as the submergence depth of the WEC, sea site water depth, mass of the buoy, wave direction, body aspect ratio and a set of control parameters. Finally, the effect of the tether configuration on power generated by the WEC is investigated in Section 5.1.4.

5.1.2 Kinematic analysis

In this section, the kinematics of the system is studied to provide an understanding of the optimal arrangement of tethers from a kinetic energy transmission and controllability point of view.

A thorough analysis of any mechanism, including wave energy converters, starts from kinematics [2]. For the analysis of existing WEC devices, the kinematics only shows the relationship in coordinates and velocities between moving and fixed parts of the system. However, in the conceptual design stage, a kinetostatic analysis answers several important questions, such as: (i) how the small changes in the buoy position (velocity) relate to changes in the tether length (velocity) and vice versa [32]; (ii) how force and torque loads on the body affect the tension forces in the mooring lines and how these forces are distributed between all tethers; (iii) how many motion modes can be controlled using a specific mooring configuration (controllability analysis) [7].

From the kinematic point of view, the current WEC system shares similarities with parallel mechanisms, also referred to as parallel robots (Figure 5.2), where the buoy acts as an end-effector and tethers play the role of actuated joints/legs (Figure 5.3). It is assumed that tethers are connected to the ocean floor and to the buoy hull through the spherical joints. Such a configuration, 3-SPS (spherical-prismatic-spherical), has 6 degrees of freedom [22] and 3 actuators, meaning that this system is under-actuated.

Optimal design methodologies of parallel robots rely on the forward and inverse kinematic analysis of mechanisms using Jacobian and inverse Jac-

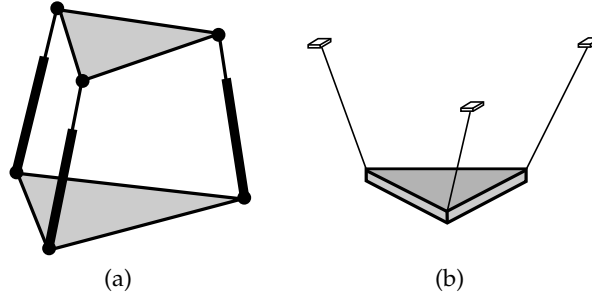


Figure 5.2: A schematic representation of parallel robots: (a) 3-SPS parallel mechanism; (b) cable driven parallel robot with three cables.

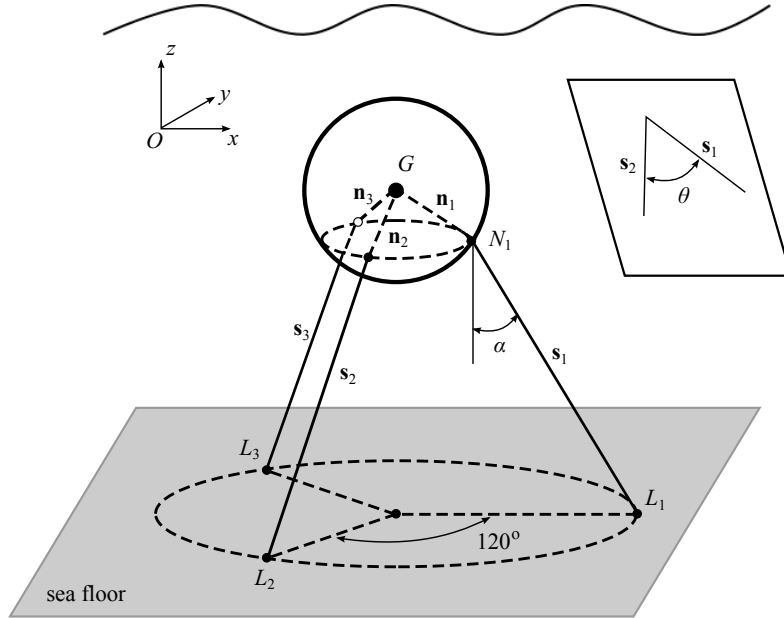


Figure 5.3: The arrangement of mooring lines around a submerged WEC. Adapted from [30].

obian matrices [22]. The kinematic Jacobian provides mapping from actuated joint velocities to the end-effector velocities in a Cartesian coordinate frame. However, for parallel robots it is easier to derive an analytical form of the inverse kinematic Jacobian [24], which is:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{x}}, \quad (5.1)$$

where $\mathbf{x} = [\mathbf{r} \ \vartheta]^\top$ is an end-effector location vector with three translational and three rotational motions, $\mathbf{q} = [l_1 \ l_2 \ l_3]^\top$ is a vector of three leg length

variables and an inverse kinematic Jacobian $\mathbf{J}^{-1}$ can be obtained as

$$\mathbf{J}^{-1} = \begin{pmatrix} \mathbf{e}_{s1}^\top & (\mathbf{n}_1 \times \mathbf{e}_{s1})^\top \\ \mathbf{e}_{s2}^\top & (\mathbf{n}_2 \times \mathbf{e}_{s2})^\top \\ \mathbf{e}_{s3}^\top & (\mathbf{n}_3 \times \mathbf{e}_{s3})^\top \end{pmatrix}, \quad (5.2)$$

where with reference to Figure 5.3, $\mathbf{e}_{si} = L_i \mathbf{N}_i / l_i$, ($i = 1, 2, 3$) is a unit-vector along the mooring line i , that points from the sea floor anchorage point L_i to the attachment point of the tether N_i ; $\mathbf{n}_i$ represents the position of the tether attachment point N_i relative to the buoy centre of mass G , which is also the centre of the system rotation.

The inverse kinematic Jacobian is assessed for the nominal position of the buoy, assuming that the tethers are equally distributed around the buoy and inclined to the vertical at angle α , as shown in Figure 5.3. This arrangement of mooring lines, when they are separated by 120 degrees in the horizontal plane, provides an independence of surge, sway and heave motions that is beneficial for an axysimmetric WEC, as has been shown in [31]. Moreover, only the equilibrium position case should be considered, as, in reality, a buoy will have motion amplitude constraints, leading to considerably small translational and rotational displacements, compared with the length of the mooring lines. Equation (5.2) can be expanded as:

$$\mathbf{J}^{-1} = \begin{pmatrix} e_{s1}^x & e_{s1}^y & e_{s1}^z & (n_1^y e_{s1}^z - n_1^z e_{s1}^y) & (n_1^z e_{s1}^x - n_1^x e_{s1}^z) & (n_1^x e_{s1}^y - n_1^y e_{s1}^x) \\ e_{s2}^x & e_{s2}^y & e_{s2}^z & (n_2^y e_{s2}^z - n_2^z e_{s2}^y) & (n_2^z e_{s2}^x - n_2^x e_{s2}^z) & (n_2^x e_{s2}^y - n_2^y e_{s2}^x) \\ e_{s3}^x & e_{s3}^y & e_{s3}^z & (n_3^y e_{s3}^z - n_3^z e_{s3}^y) & (n_3^z e_{s3}^x - n_3^x e_{s3}^z) & (n_3^x e_{s3}^y - n_3^y e_{s3}^x) \end{pmatrix} \quad (5.3)$$

Assuming that the water depth is constant for all anchorage points L_i , and the z-components of all vectors $\mathbf{e}_{si}$ are equal to $e_{s1}^z = e_{s2}^z = e_{s3}^z$ and all tether attachment points N_i lie in a horizontal plane parallel to the sea floor, so $n_1^z = n_2^z = n_3^z$. Furthermore, as the mass of the tethers are negligibly small when compared with the mass of the buoy, corresponding vectors $\mathbf{n}_i$ and $\mathbf{s}_i$ will always lie in the same vertical plane, so $e_{s1}^x \sim n_1^x$, $e_{s2}^x \sim n_2^x$, $e_{s3}^x \sim n_3^x$ and $e_{s1}^y \sim n_1^y$, $e_{s2}^y \sim n_2^y$, $e_{s3}^y \sim n_3^y$ ($\sim$ is used as a linear dependence operator). Thus, it is obvious from Equation (5.3), that the second and fourth columns of the inverse Jacobian are linearly dependent, likewise the first and fifth ones. This means that there are only three buoy motions that can be independently controlled through three tethers: coupled surge/pitch, coupled sway/roll and heave. As the sixth column of the inverse Jacobian always goes to zero in this configuration, the yaw angle remains uncontrollable, however this is not critical for the axysimmetric bodies because they cannot be excited by waves in yaw in any case.

An inverse Jacobian matrix of parallel mechanisms is conventionally used for the derivation of various dexterity indexes, such as kinematic manipulability [32], or the condition number [24]. In particular the condition number is widely used to determine an optimal design for the robot, considering all the given requirements. For the mechanisms with translational and rotational movement capabilities, the Jacobian matrix that is used to calculate the condition number should be consistent in physical units. One way to derive dimensionally homogeneous inverse Jacobian is to normalise it by dividing its rotational elements by the nominal length of the link [19], so from Equation (5.2):

$$\bar{\mathbf{J}}^{-1} = \begin{pmatrix} \mathbf{e}_{s1}^T & (\mathbf{n}_1 \times \mathbf{e}_{s1})^T / l_{10} \\ \mathbf{e}_{s2}^T & (\mathbf{n}_2 \times \mathbf{e}_{s2})^T / l_{20} \\ \mathbf{e}_{s3}^T & (\mathbf{n}_3 \times \mathbf{e}_{s3})^T / l_{30} \end{pmatrix}. \quad (5.4)$$

Assuming that all tethers point towards the centre of the body ($\mathbf{s}_i$ and $\mathbf{n}_i$ are collinear), it is obvious from Equation (5.4) that the inverse kinematic Jacobian depends only on the inclination angle of tethers included in $\mathbf{e}_{si}$ and $\mathbf{n}_i$, while being independent of water depth and submergence depth of the WEC. The optimal value of this angle can be found using the condition number index, which is calculated as [24]:

$$\kappa(\bar{\mathbf{J}}) = \kappa(\bar{\mathbf{J}}^{-1}) = \|\bar{\mathbf{J}}^{-1}\| \cdot \|\bar{\mathbf{J}}\|, \quad (5.5)$$

where $\|\cdot\|$ defines the two-norm of a matrix, and $\bar{\mathbf{J}}$ can be obtained from Equation (5.4) using pseudo-inverse. The smallest possible value of the condition number is 1, which relates to the best configuration of mooring lines, where each power take-off system does the same amount of work. Therefore, the objective is to find an inclination angle α which minimises the condition number

$$\min_{\alpha} \kappa(\bar{\mathbf{J}}^{-1}). \quad (5.6)$$

The condition number given by Equation (5.5) of the inverse Jacobian has been calculated for the system, varying an inclination angle of three tethers α from 0 (extracting power from the heave only) to 90 degrees (surge/pitch) and the results are shown in Figure 5.4. In addition, to provide better understanding of the mutual arrangement of the mooring lines, an auxiliary angle (θ) was introduced, that is subtended between two tethers in the plane that they form (θ relates to α as $\cos \theta = 1 - \frac{3}{2} \sin^2 \alpha$). Therefore, Figure 5.4 is supplemented by the upper horizontal scale that demonstrates the related angle between tethers θ . Note that all further figures related to the arrangement of tethers will have two scales for α and θ simultaneously.

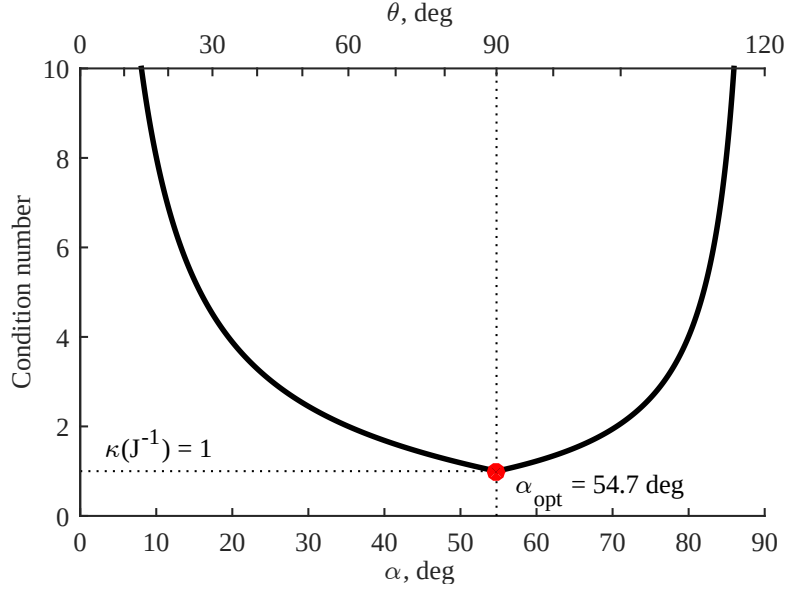


Figure 5.4: The variation of the condition number with the inclination angle of tethers in the 3-tether configuration. α (lower scale) shows the angle between each tether and the vertical, while θ (upper scale) shows the corresponding angle between the tethers in the plane that they form.

The minimum value of $\kappa(\bar{\mathbf{J}}^{-1})$ is reached at an optimal inclination angle α of 54.7 degrees = $\arccos(1/\sqrt{3})$ that corresponds to the orthogonal tethers ($\theta = 90$ degrees) when they form the edges of the cuboidal vertex. Such a configuration, when the condition number κ is equal to 1, is called an isotropic pose [23] indicating that the 3-tether WEC has uniform properties in all directions of motion. As a result, an arrangement of three mooring lines, when they are orthogonal, provides the best translation from the positions and velocities of the buoy to the length and change in length of tethers. Moreover, different locations of tether attachment points to the hull were considered, as well as different aspect ratios of the buoy dimensions to the tether length. It was found that the most optimal arrangement is achieved when all tethers point toward the geometrical centre of the WEC.

5.1.3 Dynamic analysis

In this section an optimal arrangement of mooring lines is studied for two generic shapes of WECs: a sphere and a cylinder.

Firstly, the dynamic equations of the WEC are considered, regardless of its shape (the equations have been partially adapted from [30]). With

reference to Figure 5.3, the position of the buoy centre of mass G , relative to the reference coordinate frame $Oxyz$, is denoted by the vector $\mathbf{r}$. In the undisturbed (equilibrium) position, the centre of mass G coincides with the origin O , so $\mathbf{r} = 0$. A vector $\mathbf{s}_i$ is directed along each tether i from the sea floor to the attachment point on the buoy hull. The relative position of the attachment point of the tether to the centre of mass G is denoted by $\mathbf{n}_i$.

Next, the interaction of only one tether with a body is analysed, so the subscript of the tether number $i = 1, 2, 3$ will be eliminated temporarily. As a buoy has positive buoyancy, the tension in each tether is always positive ($T > 0$). The applied force vector of the tether on the buoy is

$$\mathbf{f}_t = -T\mathbf{e}_s, \quad (5.7)$$

where $\mathbf{e}_s$ is a unit vector in the same direction as $\mathbf{s}$.

The vector of angular displacements of the body $\boldsymbol{\vartheta} = (\vartheta_x, \vartheta_y, \vartheta_z)^\top$ is taken to be relative to the inertial coordinate frame $Oxyz$, and these angles are zero in the undisturbed position. Considering the ‘small angle’ approximation, angular rotations are assumed to be independent and a change in the vector $\mathbf{n}$ from $\mathbf{n}_0$ is

$$\delta\mathbf{n} \approx \boldsymbol{\vartheta} \times \mathbf{n}_0 = \mathbf{N}_0\boldsymbol{\vartheta}, \quad (5.8)$$

where

$$\mathbf{N}_0 = \begin{bmatrix} 0 & n_{0z} & -n_{0y} \\ -n_{0z} & 0 & n_{0x} \\ n_{0y} & -n_{0x} & 0 \end{bmatrix} \quad (5.9)$$

and a subscript ‘0’ stands for the equilibrium position.

The change in the vector $\mathbf{s}$ from the undisturbed position $\mathbf{s}_0$ is

$$\delta\mathbf{s} = \mathbf{r} + \mathbf{N}_0\boldsymbol{\vartheta} \quad (5.10)$$

and the linearised change in the tether length is

$$\delta\|\mathbf{s}\| \approx \mathbf{e}_{s0}^\top \delta\mathbf{s}, \quad (5.11)$$

where $\|\cdot\|$ is the Euclidean norm and $\mathbf{e}_{s0}$ is a unit vector in the direction of $\mathbf{s}_0$. The change in the force vector $\mathbf{f}_t$ is

$$\delta\mathbf{f}_t = \delta\left(-\frac{T}{\|\mathbf{s}\|}\mathbf{s}\right) \approx -\mathbf{e}_{s0}\delta T - \frac{T_0}{\|\mathbf{s}_0\|}(\mathbf{I}_3 - \mathbf{e}_{s0}\mathbf{e}_{s0}^\top)(\mathbf{r} + \mathbf{N}_0\boldsymbol{\vartheta}). \quad (5.12)$$

It is assumed that the change in the tether tension is caused by the reaction of the power-take off system, which can be modelled as a linear spring and damper with variables k_t and c_t , such that

$$\delta T = k_t\delta\|\mathbf{s}\| + c_t\frac{d}{dt}\delta\|\mathbf{s}\| = \mathbf{e}_{s0}^\top(k_t\mathbf{r} + c_t\dot{\mathbf{r}} + \mathbf{N}_0(k_t\boldsymbol{\vartheta} + c_t\dot{\boldsymbol{\vartheta}})). \quad (5.13)$$

Substituting Equation (5.13) into (5.12), the change in the tether force can be expressed as:

$$\delta \mathbf{f}_t = -\frac{T_0}{\|\mathbf{s}_0\|} (\mathbf{I}_3 - \mathbf{e}_{s0} \mathbf{e}_{s0}^T) (\mathbf{r} + \mathbf{N}_0 \vartheta) - \mathbf{e}_{s0} \mathbf{e}_{s0}^T (k_t \mathbf{r} + c_t \dot{\mathbf{r}} + \mathbf{N}_0 (k_t \vartheta + c_t \dot{\vartheta})). \quad (5.14)$$

The change in the torque $\boldsymbol{\tau}_t$ that acts on the rigid body about its centre of mass due to the tether force is:

$$\delta \boldsymbol{\tau}_t = \delta (\mathbf{n} \times \mathbf{f}_t) \approx \mathbf{n}_0 \times \delta \mathbf{f}_t - \mathbf{f}_{t0} \times \delta \mathbf{n}, \quad (5.15)$$

where $\mathbf{f}_{t0}$ is a tether force due to the positive buoyancy of the body.

$\mathbf{S}_0$ is defined in a similar way to $\mathbf{N}_0$ and Equations (5.8) and (5.14) are substituted in Equation (5.15), so the change in the torque is calculated as:

$$\begin{aligned} \delta \boldsymbol{\tau}_t = & \frac{T_0}{\|\mathbf{s}_0\|} \mathbf{N}_0 (\mathbf{I}_3 - \mathbf{e}_{s0} \mathbf{e}_{s0}^T) (\mathbf{r} + \mathbf{N}_0 \vartheta) - \frac{T_0}{\|\mathbf{s}_0\|} \mathbf{S}_0 \mathbf{N}_0 \vartheta \\ & + \mathbf{N}_0 \mathbf{e}_{s0} \mathbf{e}_{s0}^T (k_t \mathbf{r} + c_t \dot{\mathbf{r}} + \mathbf{N}_0 (k_t \vartheta + c_t \dot{\vartheta})). \end{aligned} \quad (5.16)$$

So $\delta \mathbf{f}_t$ and $\delta \boldsymbol{\tau}_t$ can be rewritten in a matrix form:

$$\begin{bmatrix} \delta \mathbf{f}_t \\ \delta \boldsymbol{\tau}_t \end{bmatrix} = -\mathbf{K}_t \begin{bmatrix} \mathbf{r} \\ \vartheta \end{bmatrix} - \mathbf{C}_t \begin{bmatrix} \dot{\mathbf{r}} \\ \dot{\vartheta} \end{bmatrix}, \quad (5.17)$$

when using the fact that $\mathbf{N}_0 = -\mathbf{N}_0^T$,

$$\mathbf{C}_t = c_t \mathbf{G}_t \mathbf{G}_t^T \quad (5.18)$$

and

$$\mathbf{K}_t = (k_t - \gamma_0) \mathbf{G}_t \mathbf{G}_t^T + \gamma_0 \begin{bmatrix} \mathbf{I}_3 \\ \mathbf{N}_0^T \end{bmatrix} \begin{bmatrix} \mathbf{I}_3 & \mathbf{N}_0 \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \gamma_0 \mathbf{S}_0 \mathbf{N}_0 \end{bmatrix}, \quad (5.19)$$

where $\mathbf{G}_t = -\begin{bmatrix} \mathbf{I}_3 \\ \mathbf{N}_0^T \end{bmatrix} \mathbf{e}_{s0}$ and $\gamma_0 = T_0 / \|\mathbf{s}_0\|$.

In the case when all tethers point toward the centre of mass of the buoy, as shown in Figure 5.5, $\mathbf{n}_0$ and $\mathbf{s}_0$ become collinear and it can be shown that $\mathbf{N}_0 \mathbf{e}_{s0} \mathbf{e}_{s0}^T = \mathbf{e}_{s0} \mathbf{e}_{s0}^T \mathbf{N}_0 = \mathbf{0}_{3 \times 3}$. Therefore, $\mathbf{K}_t$ and $\mathbf{C}_t$ are:

$$\mathbf{K}_t = \begin{bmatrix} (k_t - \gamma_0) \mathbf{e}_{s0} \mathbf{e}_{s0}^T + \gamma_0 \mathbf{I}_3 & \gamma_0 \mathbf{N}_0 \\ \gamma_0 \mathbf{N}_0^T & \gamma_0 (\mathbf{N}_0^T \mathbf{N}_0 + \mathbf{S}_0 \mathbf{N}_0) \end{bmatrix}, \quad (5.20)$$

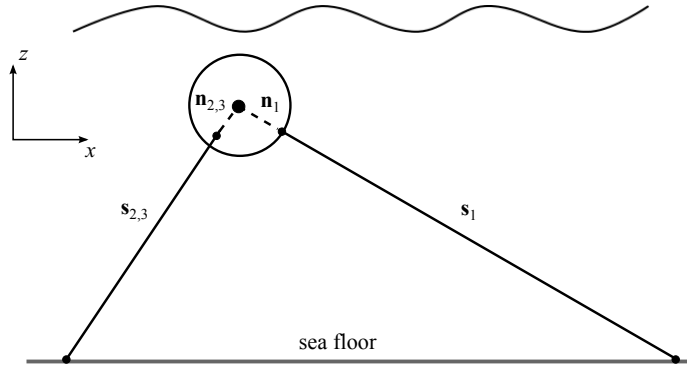


Figure 5.5: The mooring configuration when all three tethers point toward the centre of mass (plane view).

$$\mathbf{C}_t = \begin{bmatrix} \mathbf{e}_{s0} \mathbf{e}_{s0}^T c_t & \mathbf{0}_{3 \times 3} \\ \mathbf{0}_{3 \times 3} & \mathbf{0}_{3 \times 3} \end{bmatrix}. \quad (5.21)$$

Expanding to the case with three tethers, the total generalised force that acts on the body through the PTO system is:

$$\mathbf{F}_{pto} = \begin{bmatrix} \delta \mathbf{f}_{t1} + \delta \mathbf{f}_{t2} + \delta \mathbf{f}_{t3} \\ \delta \boldsymbol{\tau}_{t1} + \delta \boldsymbol{\tau}_{t2} + \delta \boldsymbol{\tau}_{t3} \end{bmatrix} = -\tilde{\mathbf{K}}_t \mathbf{x} - \tilde{\mathbf{C}}_t \dot{\mathbf{x}}, \quad (5.22)$$

where $\tilde{\mathbf{K}}_t = \sum_{i=1}^3 \mathbf{K}_{ti}$ and $\tilde{\mathbf{C}}_t = \sum_{i=1}^3 \mathbf{C}_{ti}$, $\mathbf{x} = [\mathbf{r} \ \vartheta]^T$.

The dynamic equation of the submerged buoy motion is [12]:

$$\mathbf{M} \ddot{\mathbf{x}} = \mathbf{F}_{exc} + \mathbf{F}_{rad} + \mathbf{F}_{pto}, \quad (5.23)$$

where $\mathbf{M}$ is a mass matrix of the buoy, $\mathbf{F}_{exc}$ is the excitation force vector due to the incident wave, $\mathbf{F}_{rad}$ is the hydrodynamic radiation force vector due to the added mass and damping.

Additional forces acting on the WEC, such as mooring line forces, viscous drag and static drift forces, are not considered in the present analysis. The viscous and inertial mooring line forces are neglected in the model as they are significantly smaller than the wave excitation force for relatively small displacements at typical frequencies [3, 4]. In case when mooring lines are just used to keep body on site and are not involved in power generation, a stiffness of the mooring system can significantly affect the dynamic behaviour of the buoy [26], especially motion in surge. For this kind of problems, an additional tension from the catenary cable or tether is expressed in terms of the supplementary mooring stiffness matrix [33]. However, for the current WEC, where all tethers are connected to individual PTO systems, the behaviour

of the mooring system is driven by the PTO stiffness and damping. This is similar to the mechanism of two springs connected in series, where stiffness of the mooring lines is several orders of magnitude higher than that of the PTO system. Thus, considering values used in [26] for the low rotation steel rope, the tether stiffness is around 5.6 MN/m, while a range of the required PTO stiffness should be $\approx 0.1 \dots 1$ MN/m for a similar WEC size, which supports the assumption made earlier. In case of the static drift force, similar to the net buoyancy force, it can be overcome by initial tension in tethers T_0 . Thus, pretension in each tether will be different depending on the wave direction. As a linearised model in Equation (5.23) allows to incorporate only waves of small amplitudes, the corresponding drift force has a negligible effect on the body dynamics. In contrast, when considering larger waves in the time domain, inclusion of the mean drift force may be necessary.

Fourier transforms of all forces and dynamic states of the buoy are introduced as $\mathcal{F}(\mathbf{r}) = \hat{\mathbf{r}}(j\omega)$; therefore, the hydrodynamic radiation force in the frequency domain can be expressed as:

$$\hat{\mathbf{F}}_{rad} = -(j\omega\mathbf{A}(\omega) + \mathbf{B}(\omega)) \hat{\mathbf{x}}, \quad (5.24)$$

where $\mathbf{A}(\omega)$ is a hydrodynamic added mass matrix and $\mathbf{B}(\omega)$ is a matrix of damping coefficients. The PTO force in the frequency domain can be expressed as:

$$\hat{\mathbf{F}}_{pto} = -\left(j\frac{\tilde{\mathbf{K}}_t}{\omega} - \tilde{\mathbf{C}}_t\right) \hat{\mathbf{x}}, \quad (5.25)$$

and the dynamic equation (5.23) can be rewritten as:

$$(\mathbf{Z}_{pto}(\omega) + \mathbf{Z}_{buoy}(\omega)) \hat{\mathbf{x}} = \hat{\mathbf{F}}_{exc}, \quad (5.26)$$

where the hydrodynamic impedance of the submerged body is $\mathbf{Z}_{buoy}(\omega) = \mathbf{B}(\omega) + j\omega(\mathbf{M} + \mathbf{A}(\omega))$ and the impedance of the PTO system is $\mathbf{Z}_{pto}(\omega) = \tilde{\mathbf{C}}_t - j\frac{\tilde{\mathbf{K}}_t}{\omega}$.

Let us denote the complex velocity amplitude as $\hat{\mathbf{u}} = \hat{\mathbf{x}}$. The averaged power absorbed by the system is [11]:

$$P(\hat{\mathbf{u}}) = \frac{1}{4}(\hat{\mathbf{F}}_{exc}^* \hat{\mathbf{u}} + \hat{\mathbf{u}}^* \hat{\mathbf{F}}_{exc}) - \frac{1}{2} \hat{\mathbf{u}}^* \mathbf{B} \hat{\mathbf{u}}, \quad (5.27)$$

where $*$ denotes the conjugate transpose.

The maximum power can be absorbed when the velocity of the system is unconstrained and equal to $\hat{\mathbf{u}}_0 = \frac{1}{2} \mathbf{B}^{-1} \hat{\mathbf{F}}_{exc}$ [12]. However, in reality the velocity value is limited due to the capacity of the components. For example, the stroke of the PTO hydraulic piston limits the maximum displacement of

the buoy, which in turn determines the maximum velocity of the WEC as $\hat{\mathbf{u}} \leq \hat{\mathbf{u}}_{\max} = \omega \hat{\mathbf{x}}_{\max}$. In addition, the velocity of the system is not only a function of k_t and c_t control parameters, but also depends on the tether inclination angle α . Therefore, the following optimisation procedure is performed to find an optimal arrangement of the mooring lines:

- (i) calculate the hydrodynamic parameters of the WEC ($\mathbf{A}(\omega)$, $\mathbf{B}(\omega)$, $\hat{\mathbf{F}}_{exc}(\omega)$) for a particular buoy configuration (e.g. shape, submergence depth, ocean depth);
- (ii) express the complex velocity $\hat{\mathbf{u}}$ as a function of k_t , c_t and α

$$\hat{\mathbf{u}}(k_t, c_t, \alpha) = (\mathbf{Z}_{pto} + \mathbf{Z}_{buoy})^{-1} \hat{\mathbf{F}}_{exc}; \quad (5.28)$$

- (iii) maximise the average power of the system using Equation (5.27), so

$$\max_{\alpha, k_t, c_t} P(\hat{\mathbf{u}}) \quad \text{subject to } \hat{\mathbf{u}} \leq \hat{\mathbf{u}}_{\max}. \quad (5.29)$$

A rigid body has different resonant frequencies in each uncoupled mode of oscillation. For example, the natural frequency of the surging WEC approaches zero [28], while the natural period for heave resonance is short compared with the dominant wave periods [13]. Therefore, the physical meaning of this optimisation procedure is to find such a configuration of mooring lines where the resonant frequencies of heave and surge/pitch motion modes coincide, leading to maximum power absorption.

Two distinct shapes will be considered for optimisation purposes: a sphere and a vertical cylinder. There are several reasons to explain the choice of two bodies. First of all, a cylinder represents a simple classical example of the axisymmetric body that can be excited by ocean waves in heave, surge and pitch modes, meanwhile, a sphere has a unique feature in not being able to radiate waves from any angular motion. Furthermore, the hydrodynamic models of these two bodies can be found analytically and are studied extensively in the literature.

Due to the fact that WECs may have different design properties and operate under diverse sea site conditions, the effect of various parameters on the optimal solution is covered in Sections 5.1.3.1–5.1.3.6. All sensitivity studies are performed for the spherical body except Section 5.1.3.5, where vertical cylinders with different aspect ratios are taken into consideration. It should be noted, that all trends presented for the spherical body are applicable for the cylinder. All results in the following sections have been found limiting displacements of the WEC in heave and surge to $0.5a$ and taking a regular

wave amplitude as $A = 0.2a$. Parameters used in the following analysis are listed in Table 5.1, where the sensitivity study is based on the column ‘Range for sensitivity analysis’ while other WEC parameters are set according to the ‘Fixed value’ column. ‘Body shape’ shows what WEC device is used in the corresponding section.

Table 5.1: WEC parameters used in optimisation and sensitivity studies.

Parameter	Notation	Body shape	Fixed value	Range for sensitivity analysis
Submergence depth	d_s/a	sphere	1.75	1.25, 1.5, 1.75, 2, 3
Water depth	h/a	sphere	10	5, 6, 7, 8, 9, 10
Mass ratio	m_b/m_w	sphere	0.85	0.15, 0.3, 0.5, 0.7, 0.85
Wave direction		sphere	0 deg	-30, -20, -10, 0, +10, +20, +30 deg
Body height	H/a	cylinder		0.5, 1, 1.5, 2, 2.5, 3
Number of control parameters		sphere	3	3, 4, 5
Wave height	A/a		0.2	
Motion constraint			$0.5a$	

5.1.3.1 Sensitivity to the submergence depth

The hydrodynamic (radiation and diffraction) parameters of the spherical body in finite depth are obtained using the analytical model presented in [21]. This model utilises the multi-pole expansion method with the linear wave theory approximation, where the fluid is assumed to be inviscid, incompressible and irrotational [12]. Due to its symmetrical shape, the sphere is excited by ocean waves in heave and surge modes that are hydrodynamically uncoupled.

The optimisation procedure according to Equation (5.29) is obtained using the MATLAB Optimization Toolbox. Figure 5.6 demonstrates the dependence of the optimal inclination angle on the non-dimensional wavenumber ka for various submergence depths of the sphere $d_s = 1.25a, 1.5a, 1.75a, 2a$ and $3a$. The ocean depth is taken as $h = 10a$. The range of ka is chosen on the basis of existing WEC prototypes (e.g. Carnegie CETO system). Thus, ka takes values between 0.1 and 2, which covers the region of wave periods from 5 to 18 sec with a radius of the device $a = 10$ m ($1.61 > ka = a\omega^2/g > 0.12$). As the sphere should be positively buoyant, the value of the mass is chosen as $m_b/m_w = 0.85$, where m_w is the mass of water displaced by the buoy.

As shown in Figure 5.6, all curves lie around an optimal inclination angle of 54.7 degrees within a range of ± 1.5 degrees. These results are in a good

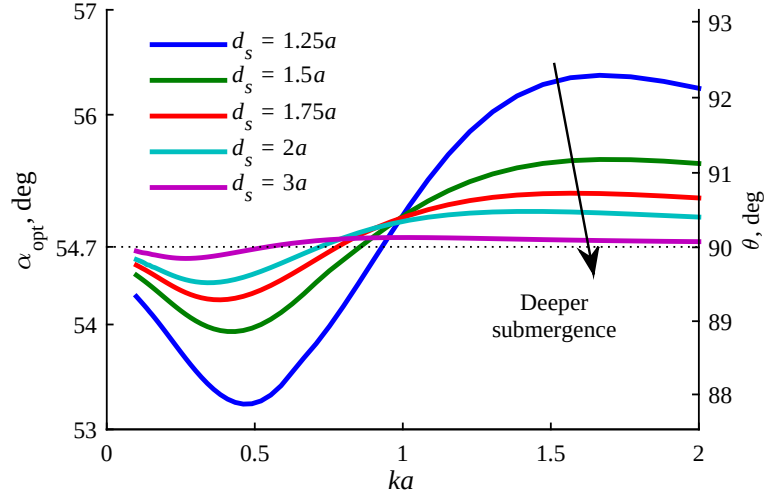


Figure 5.6: The optimal inclination angle of mooring lines of the sphere vs. non-dimensional wavenumber ka for different submergence depth d_s , water depth is $h = 10a$.

agreement with the kinematic analysis performed in Section 5.1.2. Moreover, the deeper the body is submerged, the less sensitive is the optimal angle to the wave frequency.

5.1.3.2 Sensitivity to the water depth

The range of water depth in the current analysis is chosen to represent shallow, intermediate and deep water, such as $h = 5a, 6a, 7a, 8a, 9a$ and $10a$ (Figure 5.7). The lowest value of $h = 5a$ is dictated by limitations of a linearised frequency-domain model, while the maximum value of $h = 10a$ is chosen to represent deep water because hydrodynamic coefficients of the sphere at $h = 10a$ are very close to the infinite water depth results [21].

As shown in Figure 5.7, the ocean depth has a negligible effect on the values of the optimal inclination angle, even though shallower water increases the excitation force in surge and decreases the force in heave. The maximum difference between angles for the shallow and deep water is less than 0.2 degrees over the range of wave frequencies.

5.1.3.3 Sensitivity to the mass of the buoy

Mass plays an important role in the WEC design as it determines the net buoyancy force and initial tension in all tethers. Also when a buoy is connected to the sea floor through flexible tethers (not rigid rods), they may become

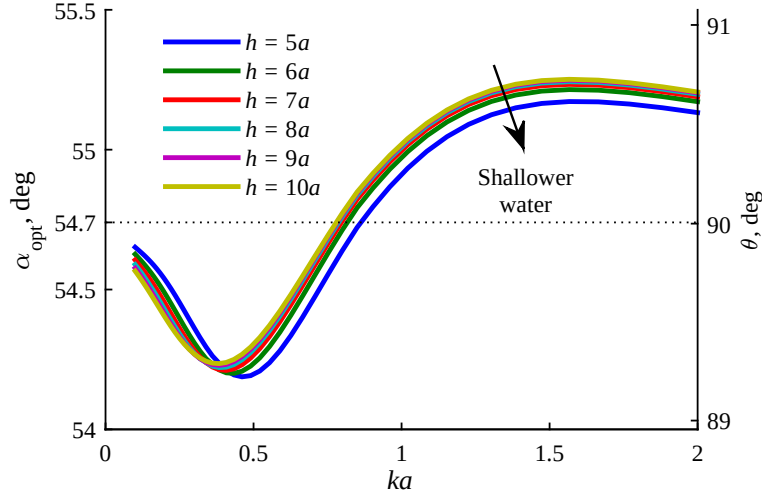


Figure 5.7: The optimal inclination angle of mooring lines of the sphere vs. non-dimensional wavenumber ka for different water depth h , submergence depth is $d_s = 1.75a$.

slack if the required PTO force is larger than the pretension force. Therefore, the device should be light enough to guarantee taut tethers all the time.

The range of masses for the analysis is chosen from similar prototypes [30, 31] and covers values from 0.15 to $0.85m_w$. Figure 5.8 shows that the heavier buoy is, the less sensitive is the optimal angle to the wave frequency.

The occurrence of tether slackness is directly dependent on the wave amplitude and frequency. Thus, the optimally controlled buoy with a mass of $m_b/m_w = 0.85$ at regular waves of 1 m amplitude will experience slackness of tethers at frequencies $ka > 0.16$, almost every cycle of motion. This reduces the total power absorption of the WEC, as power take-off system is not able to extract energy when a tether is slack. In contrast, for $m_b/m_w = 0.5$ tethers become loose only at $ka > 1.2$, which is more suitable for power conversion purposes.

In addition, the buoy mass (through the net buoyancy force and moment of inertia) determines the resonance frequency of the WEC in pitch and roll. It can be seen from Equation (5.20) that the frequency response of surge, sway and heave modes can be changed using a spring control parameter k_t . In contrast, the natural frequencies of angular modes are predominantly dependent on the initial tension and length of tethers through γ_0 -parameter and cannot be directly controlled. Thus, for the spherical buoy with a mass ratio of $m_b/m_w = 0.85$, which is submerged to $d_s = 1.75a$ at water depth of $h = 10a$, the natural frequency of pitch mode occurs around $ka = 0.3$. This is

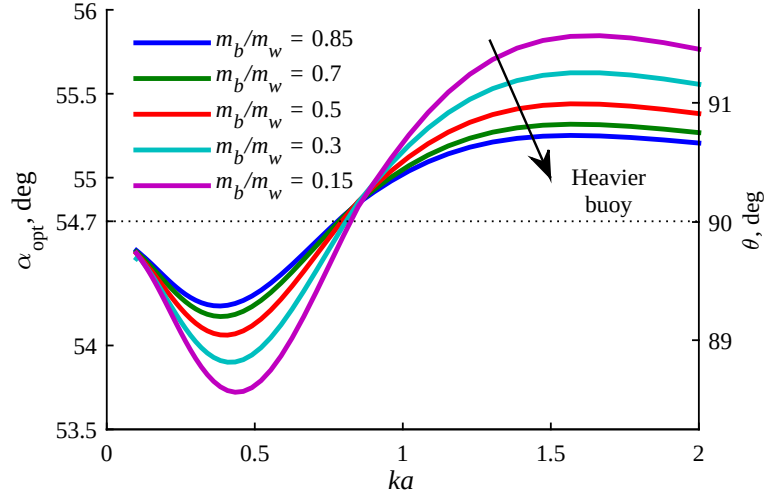


Figure 5.8: The optimal inclination angle of mooring lines of the sphere vs. non-dimensional wavenumber ka for different ratios of the buoy mass to mass of displaced water m_b/m_w .

within the range of operational sea sites frequencies, which is not desirable for the current WEC design. A reduction in mass leads to an increase in the pitch natural frequency, and can be used to move the pitch response outside the frequency range of interest.

5.1.3.4 Sensitivity to the wave direction

The direction of wave propagation may vary greatly depending on the sea site. Consequently an optimisation has been performed for the range of wave angles from -30 to 30 degrees with a 10-degree increment, which covers all possible wave direction options due the cyclic axisymmetry of the tripod configuration. In the investigation it was found that wave direction does not affect the optimal parameters of the PTO and a tether inclination angle have the same dependence on the frequency for all incident wave angles. The only difference in the WEC performance is that the distribution of power absorption between three tethers varies depending on the wave direction.

5.1.3.5 Sensitivity to aspect ratio of the body

A spherical body has an aspect ratio of 1 as its vertical and horizontal dimensions are equal. In case of a cylinder, the aspect ratio may be different varying from a flat, disk-shaped body to the infinitely long cylinder. The dimensions of a WEC determine the ratio between horizontal and vertical

excitation forces that act on the body. This in turn affects the distribution of power absorption between heave and surge motion modes, that will influence the optimal inclination angle of the three-tether mooring system.

An analytical model of all hydrodynamic parameters of a submerged vertical cylinder in finite depth is obtained from [17, 18]. Unlike the spherical case, the cylinder is excited in surge, heave and pitch modes with a coupling in surge/pitch. It means that the added mass matrix $\mathbf{A}(\omega)$ and the matrix of damping coefficients $\mathbf{B}(\omega)$ have off-diagonal elements.

Figure 5.9 demonstrates the comparison of the optimal configuration of tethers for the equally submerged ($d_s = 2a$) vertical cylinders with various aspect ratios ($H = 0.5a, 1a, 1.5a, 2a, 2.5a$ and $3a$). For the cylinder with an aspect ratio of one ($H = 2a$), the curve lies around an optimal inclination angle of 54.7 degrees (orthogonal tethers), but with a greater variation around an optimal point compared with the spherical case. For other cylinder geometries the tether arrangement is highly dependent on the body aspect ratio.

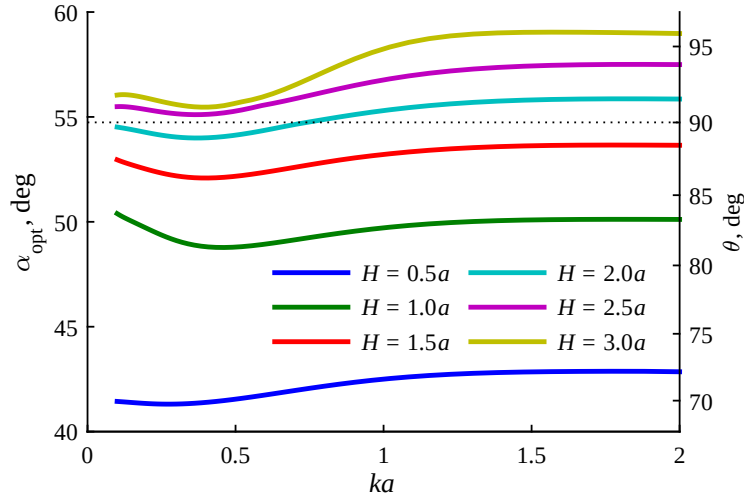


Figure 5.9: A comparison of an optimal inclination angle of mooring lines for vertical cylinders with different aspect ratios ($H = 0.5a, a, 1.5a, 2a, 2.5a, 3a$). All bodies are submerged to the same depth of $d_s = 2a$, the ocean depth is $h = 10a$, mass of cylinders is $m_b/m_w = 0.85$.

It is apparent from Figure 5.9, that flat cylinders ($H < 2a$) require the tether angle to be smaller than the optimal 54.7 degrees as they rely more on heave, thus the mooring lines are closer to the vertical. This is opposed to the slender cylinders ($H > 2a$), where the surge mode is dominant and tethers should have a larger inclination angle.

5.1.3.6 Sensitivity to the number of control parameters

Results in Sections 5.1.3.1–5.1.3.5 are obtained assuming that all three power take-off systems have identical values of control stiffness and damping coefficients ($k_{t1,2,3} = k_t$, $c_{t1,2,3} = c_t$). This makes the WEC insensitive to the direction of wave propagation, where optimal control parameters can be detected for different sea states and then applied regardless of the incoming wave angle.

To investigate the effect of the number of control parameters on the optimal tether configuration, a spherical WEC is considered. Taking into account that the system is symmetric about xz -coordinate plane, tethers 2 and 3 will require identical PTO settings if wave propagates along the x -axis. Thus, four different combinations of independent control parameters are introduced:

- ‘3’ – all tethers have identical values of stiffness and damping coefficients ($k_{t1,2,3} = k_t$, $c_{t1,2,3} = c_t$), therefore three unknowns, such as α, k_t, c_t are included in the optimisation;
- ‘4k’ – all tethers have individual values of spring stiffness while damping coefficients will remain the same for all PTOs. Therefore, the optimisation procedure will include four unknowns, such as $\alpha, k_{t1}, k_{t2,3}, c_t$;
- ‘4c’ – all tethers have individual damping while stiffness coefficients will remain the same for all PTOs resulting in four unknowns, such as $\alpha, k_t, c_{t1}, c_{t2,3}$;
- ‘5’ – all tethers have individual control parameters leading to five optimisation parameters $\alpha, k_{t1}, k_{t2,3}, c_{t1}, c_{t2,3}$.

As a result, the objective function in Equation (5.29) is modified according to the above four cases. Hereinafter, each of the proposed combinations will be referred to as ‘3’, ‘4k’, ‘4c’ and ‘5’-system.

Figure 5.10 shows a variation in the optimisation results between systems for the test case of a fully submerged spherical body (submergence depth is $d_s = 1.75a$, water depth is $h = 10a$, wave amplitude is $A = 0.2a$, motion is constrained to $0.5a$ in heave and surge, mass of the buoy is $m_b/m_w = 0.85$).

From Figure 5.10b it can be seen, that ‘3’-system absorbs slightly less than the maximum power, while ‘4k’, ‘4c’ and ‘5’ cases approach the theoretical maximum. Also, there is no need to use independent parameters for all tethers (as in case ‘5’) since ‘4k’ and ‘4c’-systems can provide the maximum power output (lines on the plot overlap). The data for the inclination angle of

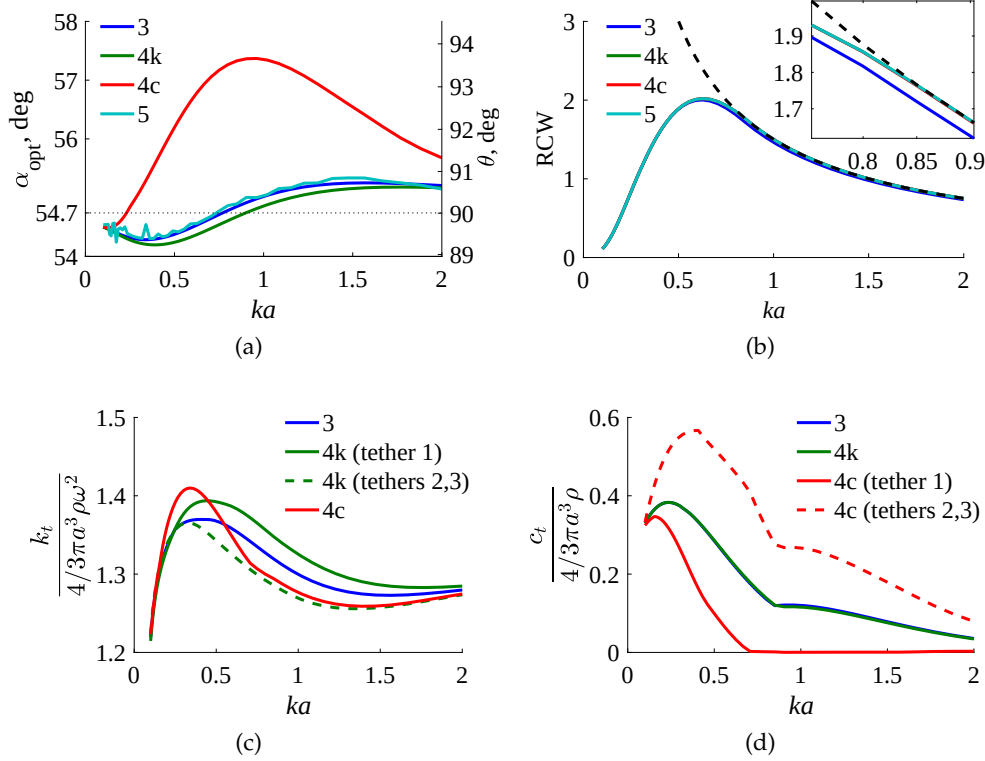


Figure 5.10: Comparison in results for four optimisation procedures (the black dashed line corresponds to the theoretical maximum): (a) inclination angle of tethers; (b) relative capture width; (c) non-dimensional PTO stiffness; (d) non-dimensional PTO damping.

the 5-parameter system (Figure 5.10a) is not smooth indicating that a system is poorly conditioned (overdetermined) and there are many combinations of PTO parameters which achieve maximum power absorption. Since the spring and damper coefficients for this system have the same ‘noisy’ behaviour, they are not displayed on Figures 5.10c and 5.10d.

The main difference in results is associated with the damping coefficient (c_t) shown on Figure 5.10d. It is interesting to note that in case of ‘4c’ optimisation, the values of the damping coefficient for tethers 2 and 3 are several times larger than that for the tether 1; whereas the spring coefficients differ slightly for all optimisation procedures (Figure 5.10c). Also, systems with ‘3’ and ‘4k’ parameters are very close in terms of the optimal angle, while a curve for the 5-parameter system oscillates around those values. The WEC with the independent damping control over each tether requires a larger angle between mooring lines as compared to other systems.

However, despite these obvious discrepancies in results, the normalised power absorption of the system with identical control parameters for each tether is only 2.7% less than for the WEC with individual control over all PTOs. Nonetheless, due to the uneven distribution of damping or stiffness coefficients over all tethers, tether 1 for the 4- and 5-parameter systems experiences 10.8% higher load as compared to the 3-parameter counterpart. This is undesirable from an engineering perspective since it increases the capital cost of the system considerably for only a marginal increase in power production. Another issue associated with independent control parameters is that the optimal damping of the first tether is found to be 0 for the '4c' case. This means that the first PTO system will not absorb any power at frequencies $ka > 0.6$.

Therefore, the use of individual control parameters for all PTO systems does not have a huge impact on the optimal inclination angle of tethers, but may improve the relative capture width (RCW) of the WEC. Such a rise in power absorption is accompanied by a significant increase in the load on tether 1, which may be critical for the WEC design. A corresponding compromise between power and tension force makes it necessary to conduct a techno-economic analysis in order to assess systems with different sets of control parameters, which goes beyond the scope of this article.

Moreover, it should be pointed out that individual tuning of all PTO systems introduces additional complexity of the control system design. Since the improvement in the WEC performance is very minor with respect to the difficulty of the problem; the same control parameters have been considered in Sections 5.1.3.1–5.1.3.5.

5.1.4 Effect of the tether configuration on power absorption

Sections 5.1.2 and 5.1.3 demonstrated how the optimal angle of tethers is sensitive to various parameters, such as a submergence depth of the buoy or its mass. However, the most important question is how this angle affects the total power absorption of the WEC assuming optimal settings of the PTO system, since it is not always possible to achieve the optimal configuration of tethers in practice, e.g. due to geological features of the sea site.

Depending on the parameters that define the system, optimal values of the inclination angle may be almost insensitive to the wave frequency or may vary greatly across the entire frequency range. Therefore, WECs that represent these two cases will be chosen for the analysis: case 1 shows a sensitive system, where a WEC is located close to the water level with a low mass ratio, and case 2 relates to the insensitive system (a heavy buoy that is

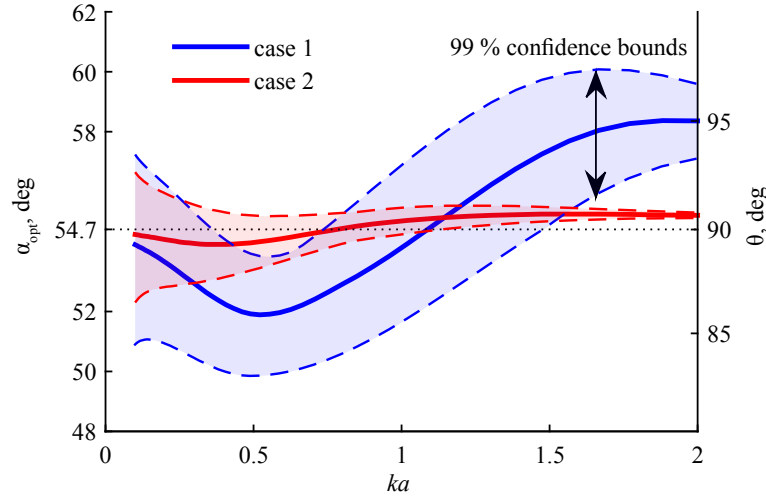


Figure 5.11: The influence of the tether inclination angle on the relative capture width of the system. Case 1 (blue) corresponds to a system where the optimal solution of the tether inclination angle is very sensitive to the wave frequency (a sphere with $d_s = 1.25a$ and $m_b/m_w = 0.15$); while case 2 (red) corresponds to a system, where the optimal tether angle is almost insensitive to the wave frequency (a sphere with $d_s = 1.75a$ and $m_b/m_w = 0.85$). Solid lines show the optimum values that maximise the output power, while broken lines limit the area where RCW falls within 99% of its maximum value.

deeply submerged).

Figure 5.11 demonstrates the influence of the inclination angle of tethers on the power output (relative capture width) for these two cases. The solid lines correspond to the optimal solution that maximises the relative capture width, while shaded areas show the range of inclination angles in which RCW falls within 99% of its maximum value. Thus, for the first case, where the optimal angle is quite sensitive to the wave frequency, the deviation from the optimum in few degrees has a minor impact on the power absorption. Interestingly, the trend is opposite for the second case, where the optimal solution is almost insensitive to the wave frequency, but the power output is highly dependant on the inclination angle. To give more insight to the second case, the corresponding RCW for various inclination angles of tethers $\alpha = 50 \dots 60$ degrees is depicted on Figure 5.12. Thus, for long waves ($ka < 0.5$), a change in the angle by 1-2 degrees causes the reduction of power absorption by no more than 1%, while for short waves ($ka > 1$) even small variations from the optimal solution can involve a sudden drop in the WEC efficiency. However, as the majority of devices target long waves, an inaccuracy of

several degrees in the mooring lines installation is acceptable for the three-tether WEC.

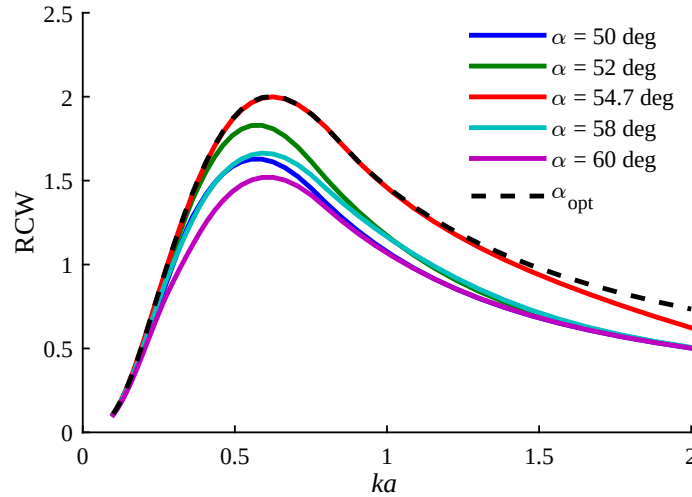


Figure 5.12: The sensitivity of the relative capture width to a variation of the tether inclination angle (corresponds to the case 2 from Figure 5.11). A black broken line shows RCW when the inclination angle is optimal for each frequency.

In summary, whilst for low frequencies a small error in the tether angle has a negligible effect on the power output of the system, at high frequencies this is no longer the case. Therefore, in some cases it may be necessary to select the arrangement of the tethers such that the system targets a particular sea site.

5.1.5 Discussion

Despite the fact that the current study is based on frequency domain analysis utilising linear wave theory and idealised power take-off systems, it should be used as a reference for further investigation of the optimal mooring configurations, taking into account site-specific features, such as typical spectra of the sea state and WEC design limitations. Moreover, non-linear effects such as viscous losses and non-linear dynamics of the hydraulic power take-off system may influence the results presented in this article. At the same time, the three-tether mooring configuration where all tethers are perpendicular to each other (forming the edges of a cuboidal vertex) and point towards the centre of mass of the buoy should be used as a starting point for further design optimisation and control system development. Current

analysis will be extended to irregular wave conditions using a time-domain model in order to validate the main findings of this paper. Also future work involves validation of the concept using a scale-model experiment.

5.1.6 Conclusions

An optimal arrangement of the 3-tether mooring configuration has been investigated for two generic shapes of submerged point-absorbing WECs in frequency domain using a linear wave theory approximation and a linear power take-off system. Since such a configuration allows the extraction of power from surge, heave and pitch motions, the relative contribution from each mode in the absorption of power is different and depends on the inclination angle of mooring lines. Assuming that all tethers are equally distributed around a buoy (120 degrees between tethers in a horizontal plane), the objective of this study is to optimise the inclination angle of tethers in order to reach maximum power absorption. The kinematic study of the problem has shown that all tethers should be orthogonal to each other, allowing all power take-off systems to do the same amount of work. This arrangement turned out to be optimal also for the submerged spherical WEC and for the submerged vertical cylinder with an aspect ratio of one. The analysis of the cylinders with other aspect ratios revealed that more slender cylinders require a larger inclination angle of tethers to the vertical. This can be explained by the fact that the excitation force in surge increases with the height of the cylinder and therefore more energy can be extracted from the surge oscillation if the same motion constraints are applied. Since the optimal inclination angle of tethers to the vertical for the heaving device is 0 degrees (vertical) and for the surging device is 90 degrees (horizontal), the increase in the height of the cylinder implies a larger inclination angle. It should be noted that with the change of the submergence depth, the dependence of the optimal angle on the incoming wave frequency decreases. In addition, it has been revealed that WECs that have an individual control laws for each tether may require slightly different inclination of mooring lines for optimal power absorption. The minor gains in power absorption come at a cost of increased dynamic loads on the PTOs and added complexity.

Acknowledgements

The authors would like to thank Prof. M. A. Srokosz and Prof. I. V. Sturova for their assistance with the modelling.

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5.2 Buoy shape and mass

This section consists of the published conference paper:

Sergiienko, N. Y., Cazzolato, B. S., Ding, B. and Arjomandi, M. (2016).
“Three-tether axisymmetric wave energy converter: Estimation of energy delivery”. In: *Proceedings of the 3rd Asian Wave and Tidal Energy Conference*. Vol. 1. Singapore, pp. 163–171.

The paper in its published format is available at <https://mecheng.adelaide.edu.au/avc/publications/abstract.php?abstract=1295>.

Statement of Authorship

Title of Paper	Three-tether axisymmetric wave energy converter: Estimation of energy delivery
Publication Status	☒ Published ☐ Accepted for Publication ☐ Submitted for Publication ☐ Unpublished and Unsubmitted work written in manuscript style
Publication Details	Sergiienko, N. Y., Cazzolato, B. S., Ding, B. and Arjomandi, M. (2016). "Three-tether axisymmetric wave energy converter: Estimation of energy delivery". <i>Proceedings of the 3rd Asian Wave and Tidal Energy Conference</i> . Vol. 1. Singapore, pp. 163–171.

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Contribution to the Paper	Conceived the original idea, undertook mathematical modelling, developed numerical code, planned and carried out numerical simulations, performed the analysis, drafted the manuscript, produced all figures, and acted as a corresponding author.
Overall percentage (%)	80%
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.
Signature	Date 05/02/2018

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

- i. the candidate's stated contribution to the publication is accurate (as detailed above);
- ii. permission is granted for the candidate to include the publication in the thesis; and
- iii. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

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Signature	Date 5/2/2018

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Contribution to the Paper	Supervised the work, helped in developing ideas, evaluated manuscript.		
Signature		Date	6/2/18

Three-Tether Axisymmetric Wave Energy Converter: Estimation of Energy Delivery

N. Y. Sergiienko, B. S. Cazzolato, B. Ding, M. Arjomandi

Abstract

There are numerous designs and concepts that have been offered to extract energy from ocean waves. A heaving buoy is distinguished as the most popular prototype which predominantly harnesses energy from the vertical motion in waves. In contrast to such devices, a three-tether wave energy converter (WEC) utilises heave, surge and pitch motion modes to increase the total power absorption of the system. The current paper investigates the performance of the three-tether WEC under irregular wave conditions considering various criteria, such as the annual mean power, characteristic mass, wetted surface and significant power take-off (PTO) force. A comprehensive analysis of buoys with different shapes, aspect ratios and masses is included in this study. Thus, the trade-off between different cost-related performance measures gives a clear comparison of this WEC against other existing prototypes.

5.2.1 Introduction

Intensive research on extracting energy from ocean waves started in 1970s leading to a large number of WEC units with various working principles. However, despite a wide range of WECs technologies on offer, none of the existing prototypes has reached the mass production stage still remaining at the proof-of-concept development phase. Therefore, researchers and engineers continue to offer new designs and technologies in order to make wave power conversion more competitive relative to other sources of renewable energy. One such idea is the submerged point absorber which is connected to three cables in order to control more degrees of freedom (heave, surge and pitch) as compared to the conventional heaving buoy. Consequently, this allows the absorption of more power from one WEC unit for the same buoy.

The concept of the three-tether WEC (see Figure 5.13) was introduced in [18], where the system under consideration consists of a submerged spherical buoy, three tethers that are equally distributed around the buoy and each tether is connected to the individual power take-off mechanism at the sea floor. Later, the three-tether converter with a floating cylindrical

buoy was used as a prototype for the control system design [16], [11]. Further analysis has been focused more on the design features of the three-tether WEC trying to identify an optimal arrangement of tethers that will provide the maximum power output of the converter [17]. In addition, there is an interest in developing wave energy arrays consisting of multi-tether buoys [20], [1]. However, the power absorption potential of this WEC type has not been estimated in real wave conditions, and hence its performance cannot be compared against other prototypes.

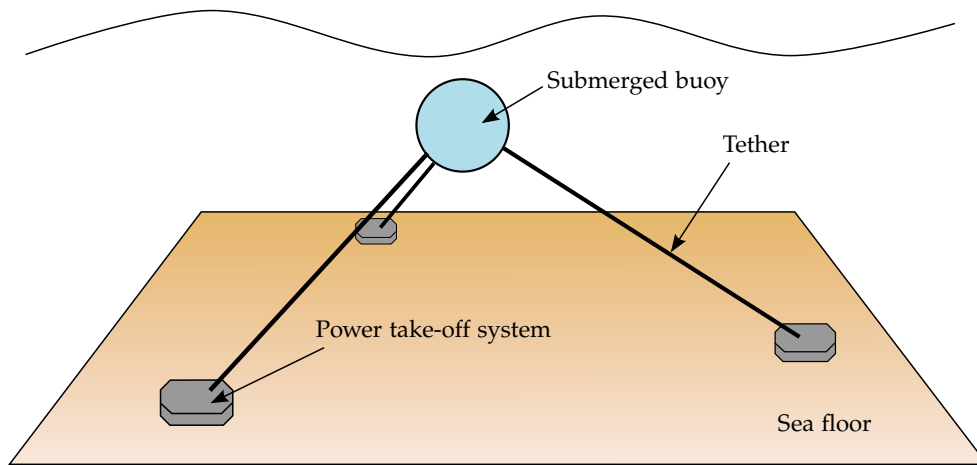


Figure 5.13: Schematic representation of a three-tether WEC (adapted from [6]).

As a result, the current paper is the first attempt to understand the behaviour of the three-tether WEC in irregular wave conditions. Converters of different shapes, aspect ratios and masses are used in this study in order to identify their influence on the range of performance measures. Finally, the results are presented as a comparison between one-tether and multi-tether power take-off arrangements.

5.2.2 System parameters

Currently there is no existing prototype of the three-tether WEC which could be used in the present study. Therefore, the starting point would be to choose the expected location of the operational sea site and identify an appropriate size of the buoy. Thus, the site on the east coast of Australia has been selected for this analysis (see Table 5.2 and Figure 5.14 for the wave data and location).

Table 5.2: Australia/New South Wales (NSW) test site.

Location:	34°00′00.0″S, 152°30′00.0″E
Water depth:	deep water
Mean wave power:	23.2 kW/m (Pierson-Moskowitz)
Type of data:	hindcast (Australian Wave Energy Atlas) [2]

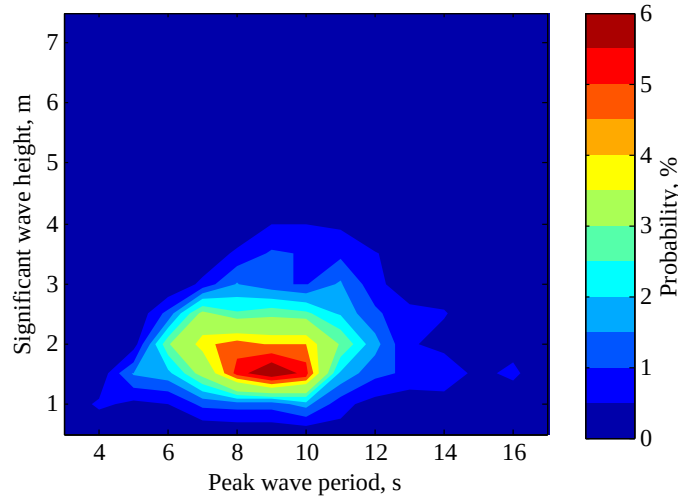


Figure 5.14: Wave data statistics for NSW sea site detailed in Table 5.2.

5.2.2.1 Buoy size and shape

According to the methodology presented in [8], the volume of the WEC should be chosen to guarantee the maximum efficiency of the system at least one-third of the year. For the chosen NSW site the most common sea states have a significant wave height of $H_s = 1.5 - 2$ m, peak wave period of $T_p = 8 - 10$ sec, and the wave power level exceeds 19.6 kW/m about one-third of the year which corresponds to the power in a regular wave of $T \approx 8$ sec and $H \approx 1.6$ m [7]. In contrast to the floating converters, an optimal volume of the submerged buoy depends not only on the wave conditions, but also on the body shape, submergence depth and volume stroke [19]. Thus, a spherical converter of $a = 5$ m radius with its centre placed $1.75a = 8.75$ m below the mean water level will absorb the maximum power at the regular wave of $T = 8$ sec and $H = 1.6$ m. The geometric parameters of the spherical buoy investigated in this study are listed in Table 5.3.

To understand the effect of the buoy geometry on the performance of the 3-tether WEC, other body shapes are investigated keeping the same: (i) volume of the buoy 524 m^3 , (ii) distance from the top of the buoy to the

Table 5.3: Parameters of the benchmark WEC.

Parameter	Unit	Value
Shape		sphere
Radius, a	m	5
Water depth	m	50
Submersion (top of the buoy)	m	3.75
Submergence depth, d_s (centre of the buoy)	m	8.75
Volume, V	m ³	524
Stroke length, $\Delta l_{\max} - \Delta l_{\min}$	m	6 (± 3)

water surface of 3.75 m, and (iii) the maximum tether stroke of 3 m. As a result, four cylindrical buoys with different aspect ratios have been added to the investigation (see Table 5.4). All geometries used in this paper are shown on Figure 5.15.

Table 5.4: Parameters of the cylindrical WECs with various aspect ratios.

Parameter	Unit	Height to radius ratio h_c / a			
		0.5	1	2	3
Radius, a	m	6.93	5.50	4.37	3.82
Height, h_c	m	3.47	5.50	8.74	11.45
Submergence depth, d_s (centre of the buoy)	m	5.48	6.50	8.12	9.47
Volume, V	m ³	524			

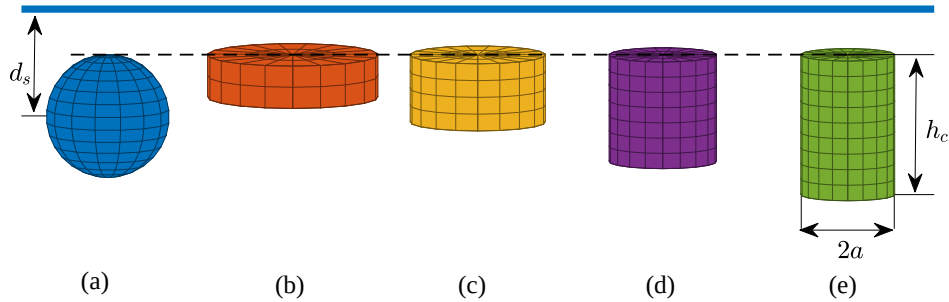


Figure 5.15: Shapes of WECs: (a) sphere, (b) cylinder $h_c = 0.5a$, (c) cylinder $h_c = 1a$, (d) cylinder $h_c = 2a$, (e) cylinder $h_c = 3a$. All buoys have the same displaced volume and the same distance between their top surface and the mean water level.

Other important parameters of the WEC, such as the mass of the buoy and the inclination angle of tethers are considered as variables throughout this paper.

5.2.2.2 Power take-off

For the current study it is assumed that each tether is connected to an individual power take-off mechanism which can be designed as an electric generator [16] or a hydraulic circuit [5]. The load force exerted on the buoy from each PTO system is presumed to have linear spring and damper effects being proportional to the tether extension and the rate of change of the tether extension.

5.2.3 Equation of motion

The wave-to-wire model is based on the linear wave theory assuming small motion amplitudes of the buoy as compared to the length of the mooring lines (tethers). The only second order hydrodynamic effect included in the model is a viscous drag force which is proportional to the square of the body velocity relative to the fluid.

5.2.3.1 Kinematics

A schematic of a three-tether WEC is shown on Figure 5.16. The spatial arrangement of all tethers is defined by $\mathbf{s}_i$:

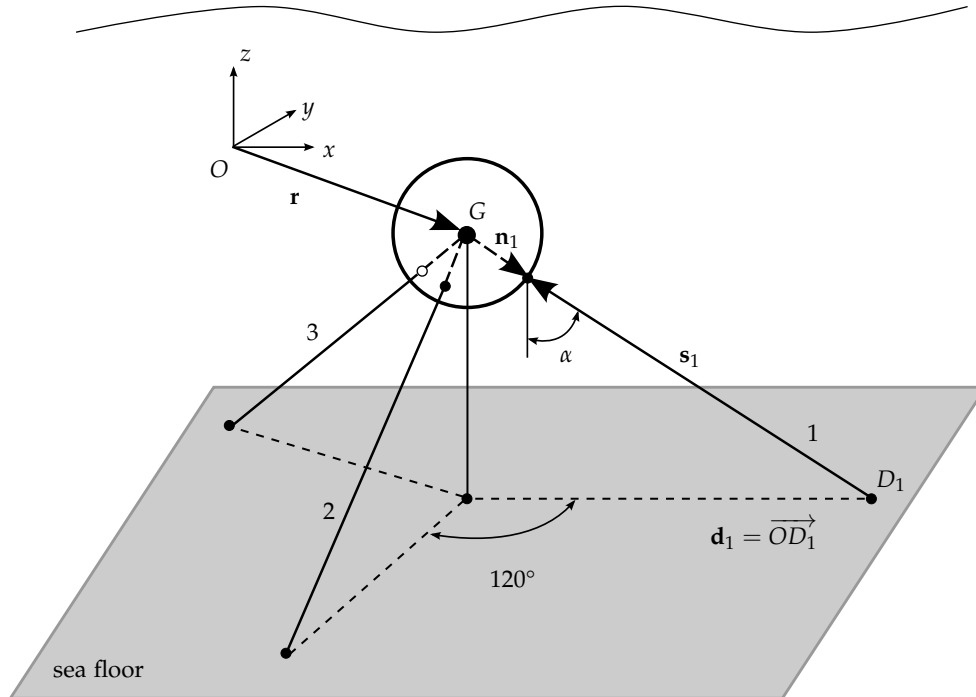


Figure 5.16: Schematic of the three-tether wave energy converter.

$$\mathbf{s}_i = \mathbf{r} + \mathbf{R}\mathbf{n}_i - \mathbf{d}_i, \quad i = 1 \dots 3, \quad (5.30)$$

where $\mathbf{r}$ is the position vector of a buoy in the reference coordinate frame $Oxyz$, $\mathbf{R}$ is the rotation matrix of the buoy with respect to the reference frame, $\mathbf{n}_i$ denotes the position vector of the anchor point of the i -th tether on the hull relative to the buoy centre of mass G , $\mathbf{d}_i$ is the position vector of the anchor point of the i -th tether on the sea floor in the $Oxyz$ coordinate frame. The instantaneous tether length is:

$$l_i = \|\mathbf{s}_i\| = \sqrt{\mathbf{s}_i^T \mathbf{s}_i}, \quad i = 1 \dots 3, \quad (5.31)$$

and the change in the tether length is $\Delta l_i = l_i - l_0$.

Mapping from the buoy velocities in a Cartesian coordinate frame to the rate of change of the tether length is provided by the inverse kinematic Jacobian:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{x}}, \quad (5.32)$$

where $\mathbf{x} = [\mathbf{r} \ \boldsymbol{\theta}]^T$ is the buoy location vector with three translational and three rotational motions, $\mathbf{q} = [l_1 \ l_2 \ l_3]^T$ is a vector of three tether length variables and an inverse kinematic Jacobian $\mathbf{J}^{-1}$ can be obtained as:

$$\mathbf{J}^{-1} = \begin{pmatrix} \mathbf{e}_{s1}^T & (\mathbf{R}\mathbf{n}_1 \times \mathbf{e}_{s1})^T \\ \mathbf{e}_{s2}^T & (\mathbf{R}\mathbf{n}_2 \times \mathbf{e}_{s2})^T \\ \mathbf{e}_{s3}^T & (\mathbf{R}\mathbf{n}_3 \times \mathbf{e}_{s3})^T \end{pmatrix}, \quad (5.33)$$

where $\mathbf{e}_{si}$ is a unit-vector along the tether i , so $\mathbf{e}_{si} = \frac{\mathbf{s}_i}{\|\mathbf{s}_i\|}$.

5.2.3.2 Forces

The generalised forces that act on the body are expressed in the Cartesian coordinate frame $Oxyz$ as vectors of six elements including horizontal and vertical forces and rotational moments.

5.2.3.3 Hydrostatic force

As the buoy is fully submerged, the generalised hydrostatic force is $\mathbf{F}_{buoy} = [0 \ 0 \ (m_w - m_b)g \ 0 \ 0 \ 0]^T$, where $m_w = \rho V$ is the mass of the displaced water, ρ is water density, V is the displaced volume of the buoy and m_b is the mass of the WEC.

5.2.3.4 Power take-off forces

A behaviour of the linear PTO system is modelled as [3]:

$$F_{pto,i} = (C_{pto} - B_{pto}\dot{\Delta l}_i - K_{pto}\Delta l_i), \quad i = 1 \dots 3, \quad (5.34)$$

where K_{pto} and B_{pto} are the PTO stiffness and damping coefficients (control parameters), and

$$C_{pto} = -\frac{(m_w - m_b)g}{3 \cos \alpha} \quad (5.35)$$

is a force that counteracts the hydrostatic force in an undisturbed position, α is the tether angle relative to the vertical.

5.2.3.5 Hard stop forces

As each PTO system has a limited stroke, an additional force from the hard stop mechanism is exerted on the body to constrain its motion. As a result, the hard stop system is modelled by a repulsive energy potential [3]:

$$\begin{aligned} F_{hs,i} = & -K_{hs,\min}(\Delta l_i - \Delta l_{\min})u(\Delta l_{\min} - \Delta l_i) \\ & - K_{hs,\max}(\Delta l_i - \Delta l_{\max})u(\Delta l_i - \Delta l_{\max}), \end{aligned} \quad (5.36)$$

$i = 1 \dots 3$, $u(\cdot)$ is Heaviside step function, $K_{hs,\min}$ and $K_{hs,\max}$ are the hard stop spring coefficients, $\Delta l_{\min}$ and $\Delta l_{\max}$ are the stroke limits relatively to the nominal position.

5.2.3.6 Tension in tether

A linear superposition of the PTO and hard stop forces governs the tension in each tether. In addition, the tethers should be always under tension in order to transmit forces from the machinery:

$$F_{t,i} = \min(0, F_{pto,i} + F_{hs,i}), \quad i = 1 \dots 3. \quad (5.37)$$

Noting that $F_{pto,i}$, $F_{hs,i}$ and $F_{t,i}$ are the forces that act along the mooring line i , the generalised tether force in the Cartesian coordinate frame is:

$$\mathbf{F}_{tens} = \mathbf{J}^{-\top} \mathbf{F}_t, \quad (5.38)$$

where $\mathbf{F}_t = [F_{t,1} \ F_{t,2} \ F_{t,3}]^\top$ and $\mathbf{J}^{-\top}$ is the transposed inverse kinematic Jacobian.

5.2.3.7 Viscous damping forces

The viscous damping force is modelled according to the Morison's equation [3],[14]:

$$\mathbf{F}_{visc} = \begin{pmatrix} -\frac{1}{2}\rho\mathbf{C}_d\mathbf{A}_d\|\mathbf{V}_b - \mathbf{V}_f\|(\mathbf{V}_b - \mathbf{V}_f) \\ -\frac{1}{2}\rho\mathbf{b}_Q D^4 D\|\dot{\boldsymbol{\theta}}\|\dot{\boldsymbol{\theta}} \end{pmatrix}, \quad (5.39)$$

where $\mathbf{V}_b = \dot{\mathbf{r}}$ is the buoy velocity, $\mathbf{V}_f$ is the fluid particle velocity at the position of the centre of mass of the buoy,

$$\mathbf{C}_d = \begin{pmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{pmatrix}, \quad \mathbf{A}_d = \begin{pmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{pmatrix} \quad (5.40)$$

are matrices of drag coefficients and the cross-section areas of the buoy perpendicular to the direction of motion respectively,

$$\mathbf{b}_Q = \begin{pmatrix} b_{yz} & 0 & 0 \\ 0 & b_{xz} & 0 \\ 0 & 0 & b_{xy} \end{pmatrix} \quad (5.41)$$

is the matrix of quadratic angular damping coefficients, D is the buoy diameter and $\boldsymbol{\theta}$ is the vector of angular velocities of the buoy.

5.2.3.8 Time-domain model

The buoy motion in time domain can be described using the Cummins equation that include the wave excitation and radiation forces, and other forces specified in Section 5.2.3.2:

$$\begin{aligned} (\mathbf{M} + \mathbf{A}_\infty)\ddot{\mathbf{x}} = & \mathbf{F}_{exc} - \int_0^t \mathbf{K}_{rad}(t - \tau)\dot{\mathbf{x}}(\tau)d\tau \\ & + \mathbf{F}_{buoy} + \mathbf{F}_{visc} + \mathbf{F}_{tens}, \end{aligned} \quad (5.42)$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{A}_\infty$ is the matrix with infinite-frequency added mass coefficients, $\mathbf{F}_{exc}$ is the generalised wave excitation force, and $\mathbf{K}_{rad}(t)$ is a retardation function.

5.2.3.9 Power absorption

The energy can be generated by the PTO machinery only when the tether, attached to it, is under tension. As a result, the total instantaneous power absorbed by the WEC is calculated as:

$$P_\Sigma(t) = \sum_{i=1}^3 P_i(t) = B_{pto} \sum_{i=1}^3 (\Delta \dot{l}_i(t))^2 \quad \text{if } F_{t,i} < 0. \quad (5.43)$$

5.2.4 Cost-related criteria for analysis

The relative capture width (RCW) is one of the most commonly used criterion for the comparison of different wave energy converters. However, this measure is not sufficient to predict whether the WEC would be economically viable and attractive for investors. Moreover, it is hard to give a precise estimation of the WEC cost due to the limited number of full-scale prototypes. Therefore, several indirect cost-related indices have been suggested [4] for the analysis to replace a traditional measure called Levelised Cost of Electricity.

Table 5.5: WEC performance indices.

Parameter	Notation	Comment
Mass of the buoy	m_b	The mass of the buoy hull.
Characteristic mass	m_Σ	The total mass of the buoy and foundation calculated as $m_\Sigma = m_b + 1.5(m_w - m_b)$, where 1.5 is a safety factor.
Wetted surface	A_{wet}	The surface area of the buoy hull.
Wave power	J	
Mean absorbed power	P	
Capture width ratio		$P/(2aJ)$.
Annual energy output	E_y	$E_y = 8760P$, 8760 hours per year.
Significant PTO force	F_{pto}^{RMS}	RMS value of the PTO force including initial pre-tension force.
Tether slackness		The number of times when the tether experienced slackness over one cycle (time equal to the significant wave period T_p).

As a result, the cost-related performance measures used in this study are [4]:

- (i) annual energy output per characteristic mass E_y/m_Σ refers to the cost of the whole WEC structure;
- (ii) annual energy output per characteristic surface area E_y/A_{wet} refers to the cost of the buoy;
- (iii) annual energy output per significant PTO force E_y/F_{pto}^{RMS} refers to the cost of the PTO system.

5.2.5 Model implementation

Hydrodynamic coefficients of all bodies in frequency domain have been obtained using analytical solutions for the sphere [13] and for cylinders [10] and validated against WAMIT [12].

In order to conduct the time domain simulations, Equation (5.42) has been implemented in Simulink/MATLAB with a time step of 0.01 seconds using the ode45 solver. The duration of all simulation runs has been set to $300 \times T_p$ but not less than 1200 seconds and the first $15 \times T_p$ have not been included in the analysis due to the possible transient state. The convolution integral in Equation (5.42) has been replaced by the state-space model using the Marine System Simulator toolbox [15]. The irregular wave time-series have been implemented using the Pierson-Moskowitz wave spectrum over the grid of significant wave heights of $H_s = 0.5 - 7.5$ m and peak wave periods $T_p = 3 - 17$ sec.

Each body geometry has been analysed using a range of tether angles from 30° to 60° with 1° increment and a range of masses from $0.15m_w$ to $0.85m_w$ (0.15, 0.3, 0.5, 0.7, 0.85). Drag coefficients used in this study have been taken from [9] and are listed in Table 5.6.

Table 5.6: Drag coefficients [9].

Parameter	Sphere	Cylinder			
		$h_c = 0.5a$	$h_c = 1a$	$h_c = 2a$	$h_c = 3a$
C_x, C_y	0.5	1	1	1	1
C_z	0.5	1.2	1.1	0.9	0.85
b_{xz}, b_{yz}	0	0.2	0.2	0.2	0.2

The hard stop spring coefficient is set to $K_{hs,min} = K_{hs,max} = 10^8$ N/m and the PTO control parameters K_{pto}, B_{pto} are optimised for each sea state using Simulink Design Optimization toolbox.

5.2.6 Results

Due to the large number of parametric studies undertaken here, results that are similar for all body shapes will be demonstrated by the example of the spherical buoy only.

5.2.6.1 Power matrix

The performance of the 3-tether spherical WEC with the mass of $m_b = 0.5m_w$ and the initial tether angle of $\alpha = 55^\circ$ is shown on Figure 5.17. The average power output at the dominant sea state ($H_s = 1.5$ m and $T_p = 8$ sec) is approximately 65 kW and can reach 800 kW at the strongest sea states. As shown on Figure 5.17c, the level of instantaneous power is approximately 6–10 times higher than the mean power value meaning that the designed capacity

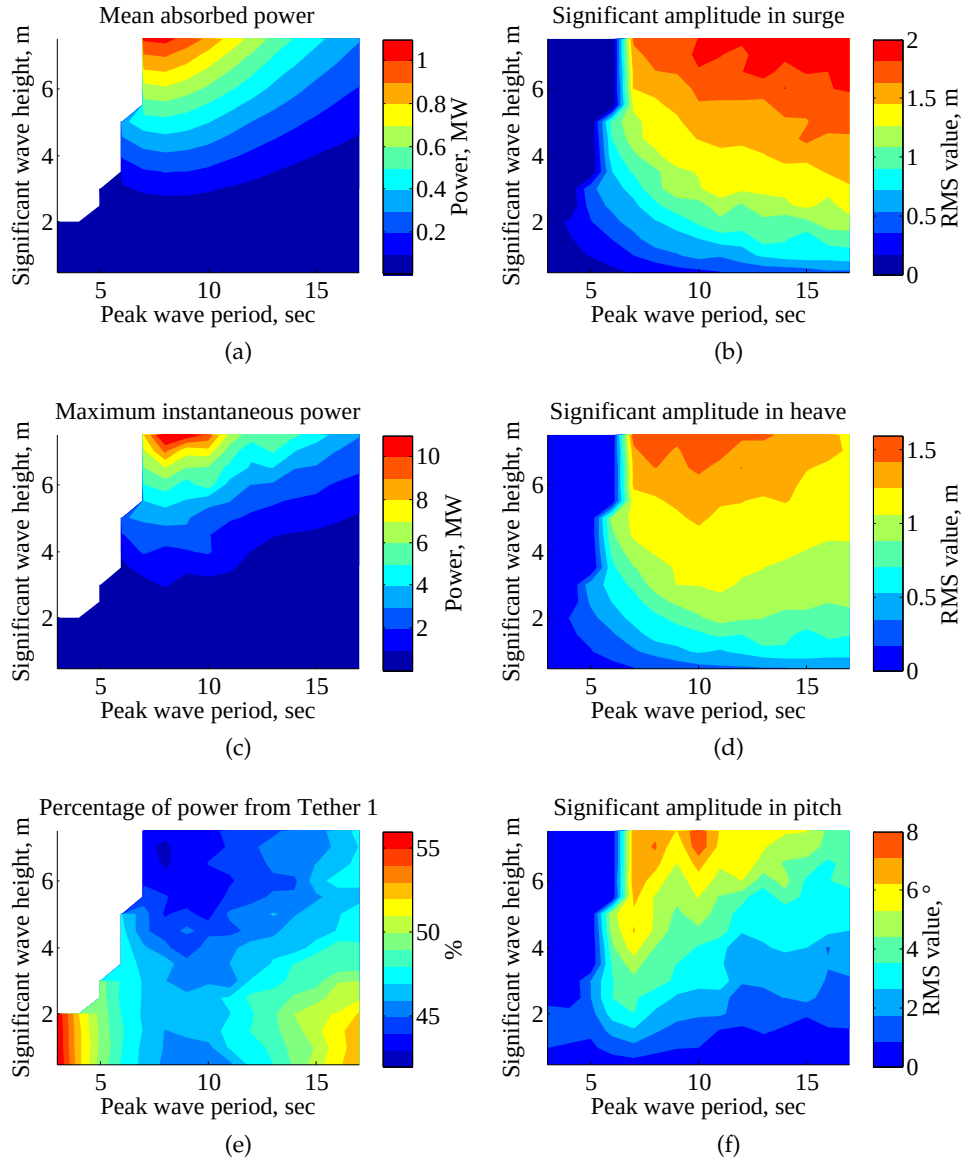


Figure 5.17: Performance of the 3-tether spherical WEC. The mass of the buoy is set $m_b = 0.5m_w$, the tether angle is $\alpha = 55^\circ$, PTO parameters are optimised for each sea state.

of the PTO should be an order of magnitude higher than the expected mean energy output.

The 3-tether WEC consists of three PTO units attached to each tether, and they make different contribution to the total power output. The tether arrangement shown on Figure 5.16 demonstrates only one extreme case

when tether 1 is aligned with the direction of wave propagation. For this configuration, the PTO attached to tether 1 generates approximately a half of the total mean power as shown on Figure 5.17e. The other two PTOs convert the rest of energy with equal share due to the symmetry of the tether arrangement with respect to y -axis. Thus, depending on the wave direction statistics, the first PTO system may be designed as the most powerful of three. Another extreme situation would be when one of tethers operates perpendicular to the wave front with almost zero power output. However, according to the preliminary results in frequency domain, the total power output of the WEC is largely unaffected by wave direction relative to the spatial arrangement of the mooring lines.

RMS values of the buoy motion in surge, heave and pitch are shown on Figure 5.17b – 5.17f. Significant motion amplitudes in all degrees of freedom are proportional to the significant wave height. Thus, the surge RMS is about $1/2$ of the H_s , while heave RMS is $1/4 - 1/3$ of H_s . Interesting, that these values are similar for all body geometries considered in this study. Noting that the RMS of the wave elevation amplitude is equal to $1/4H_s$, which is almost the same as the significant amplitude in heave. This means that the buoy motion is not governed by the maximum PTO stroke but is dependent on the wave amplitude. As a result, the maximum length of the stroke should be chosen according to the targeted sea state.

5.2.6.2 Effect of the tether angle

The inclination angle of tethers (α) affects the power absorption of the WEC, as do the PTO parameters, noting that unlike the control parameters which can be modified to suit sea conditions, the angle cannot be changed during the system lifetime. As a result, the performance of the spherical body has been analysed with a range of tether angles with optimal PTO settings for each sea state and each value of α . Thus, the matrix of the tether angles that correspond to maximum power is shown on Figure 5.18. As a result, for stronger sea states the tether angle should be larger, meaning that more power will be absorbed from surge than heave. According to the sea site considered in this paper, the tether angle for the spherical body should be set to 55° for the maximum mean power.

The sensitivity of the mean absorbed power to the tether installation is shown on Figure 5.19. The change of the angle by $\pm 5^\circ$ from its optimal value will only result in 5% reduction in power.

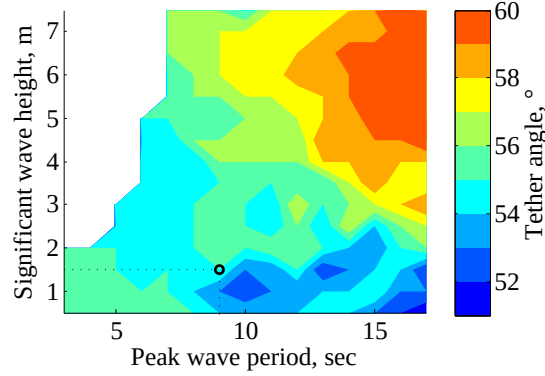


Figure 5.18: Optimal tether angle of the spherical buoy with the mass of $m_b = 0.5m_w$.

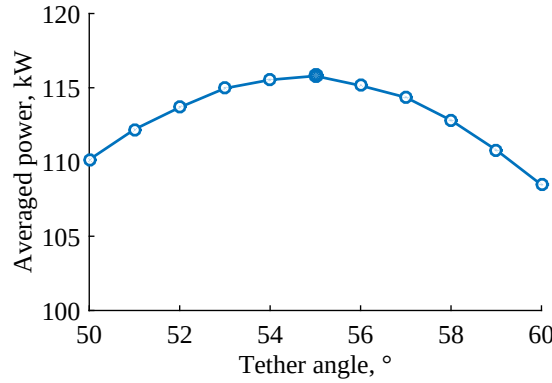


Figure 5.19: The mean absorbed power of the spherical buoy at NSW site vs. the tether inclination angle. The mass of the buoy is set as $m_b = 0.5m_w$.

5.2.6.3 Effect of the mass

The mass ratio of the buoy plays an important role in the design of the bottom-referenced WEC as it governs the weight of the foundation and therefore the total cost of the system. It is known that the mass ratio has an inverse relationship with the weight of the foundation. Thus buoys with a lower mass ratio require a larger concrete ballast on the sea floor. However, it may be argued that the fabrication cost of the metal hull of the buoyant actuator is much higher than that of the foundation concrete and there should be a trade off between mass, capital cost and power output. As a result, the performance of the spherical buoy with a range of different masses has been investigated and the resultant cost-related criteria from Section 5.2.4 are shown in Table 5.7. The tether angle for each mass is set such that the mean

absorbed power is maximum for NSW site, PTO parameters are optimised for each sea state of each mass value.

Table 5.7: Performance of the three-tether spherical buoy with 5 different masses.

Parameters	Unit	Mass ratio m_b/m_w				
		0.15	0.3	0.5	0.7	0.85
Mass of the buoy	t	81	161	268	376	456
Characteristic mass	t	765	725	671	617	577
Wetted surface	m ²			314		
Wave power	kW/m			23.2		
Mean power	kW	135	126	116	107	91
Capture width ratio	%	58	54	50	46	39
E_y/m_b	kWh/kg	14.7	6.9	3.8	2.5	1.75
E_y/m_Σ	kWh/kg	1.55	1.53	1.51	1.51	1.38
E_y/A_{wet}	MWh/m ²	3.77	3.52	3.23	3.98	2.53
E_y/F_{pto}^{RMS}	kWh/N	0.46	0.52	0.66	0.99	1.51
Tether slackness	%	4	3	2	4	30

The main conclusions that can be drawn from Table 5.7 are:

- (i) lighter buoys can produce up to 1.5 more power;
- (ii) lighter buoys require the larger pretension force generated by the PTO system which may affect its cost;
- (iii) the chance of slack tethers is very high for the heavy buoy of $0.85m_w$ mass;
- (iv) lighter buoys more frequently hit hard stop ends (not shown here).

The performance measures from Table 5.7 have been normalised against their maximum values and plotted on Figure 5.20. As a result, the value on the plot equal to 1 means that this particular criterion has the highest value at this mass ratio. Thus, the ratio of the annual energy to the mass of the buoy is maximum for the lightest buoy considered in this study ($m_b = 0.15m_w$). Consequently, light buoys are more attractive in terms of the cost of the buoyant actuator, while heavy buoys are more appealing from the PTO cost standpoint. Therefore, there is a value of the buoy mass which may be optimal from several perspectives. For example, if the cost of the buoy is proportional to its wetted surface, if the cost of the PTO system is proportional to the force RMS and if both these values have approximately the same share in the capital cost of the WEC, then it can be concluded that the buoy mass of around $0.7m_w$ is optimal for the system. However, more information is

required to answer the question of what mass is the most suitable for the converter from the techno-economic point of view.

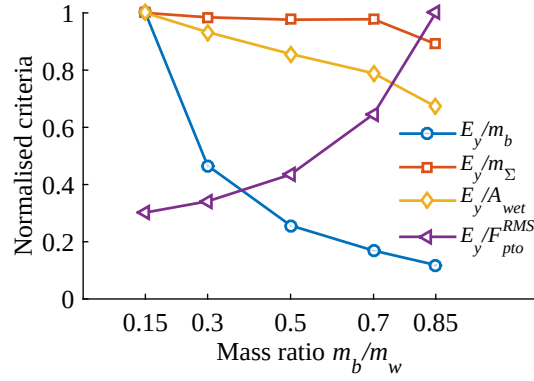


Figure 5.20: Dependence of the cost-related criteria on the mass of the buoy for the 3-tether spherical converter.

5.2.6.4 Different shapes

In this section, the analysis of the three-tether WEC is extended to different body shapes of equal volume that are shown on Figure 5.15. Also, in order to demonstrate how the multi-tether configuration is different from the generic heaving buoy connected to one tether, the same body geometries but with a one-tether PTO configuration are considered for the comparison (see Figure 5.21).

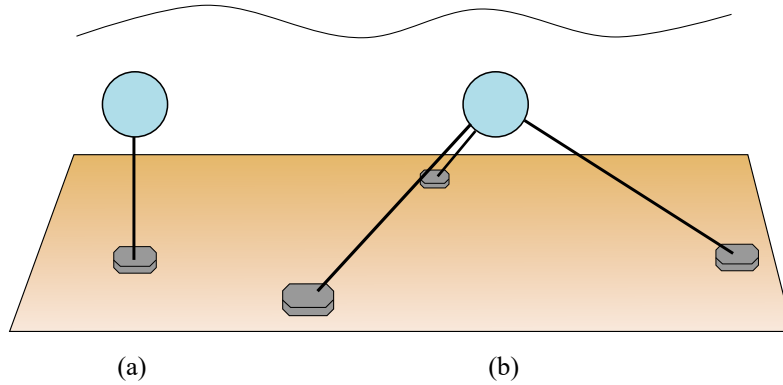


Figure 5.21: Different power take-off configurations for the submerged WECs: (a) the generic heaving buoy connected to one tether; (b) the three-tether system.

The submergence depth and maximum allowed tether extension are kept

the same for all cases. The mass of all buoys in this section is set constant $m_b = 0.5m_w$. The time-domain model of the bottom-referenced submerged WEC with one tether has been adapted from [3]. The parameters of all WECs have been set as close as possible to each other in order to provide a fair comparison between all buoy shapes and tether configurations. However, the main discrepancy in settings refers to the constraints of the buoy motion in the Cartesian space. First of all, WECs of different shapes require different tether configurations in order to absorb maximum power as shown on Figure 5.22 and explained in [17]. This property could be exploited by developers when optimising a site given that the spacing on the sea floor of the moorings is dependent on the optimal angle and water depth.

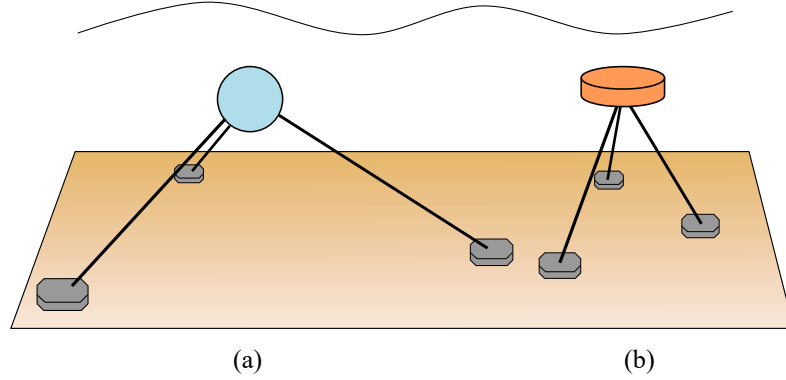


Figure 5.22: An example of the optimal tether configurations for different buoy shapes: (a) the spherical buoy has an optimal tether angle of $\alpha = 55^\circ$, while (b) the flat cylinder of $h_c = 0.5a$ requires about $\alpha = 30^\circ$.

Due to the assumption made in this study, that all converters have the same stroke length in all PTO systems, WECs of different geometries have distinctive motion envelopes depending on the tether angles as shown on Figure 5.23. These motion envelopes are detected from the system kinematics and only show the area where the buoy can operate without reaching hard stop mechanisms.

As shown on Figure 5.23, when tethers are closer to the vertical ($\alpha = 30^\circ$), the converter experiences smaller motion amplitudes in heave as compared to other configurations. The corollary is that surge motion is smaller for larger tether angles. As a result, the comparison between different shapes is made keeping in mind that some of them move in heave or surge more than others and vice versa. This also applies to the comparison between 1-tether and 3-tether WECs. Thus, for all shapes, except the cylinder with $h_c = 0.5a$, converters with the 3-tether PTO configuration are allowed to move in heave more than their 1-tether heaving counterparts.

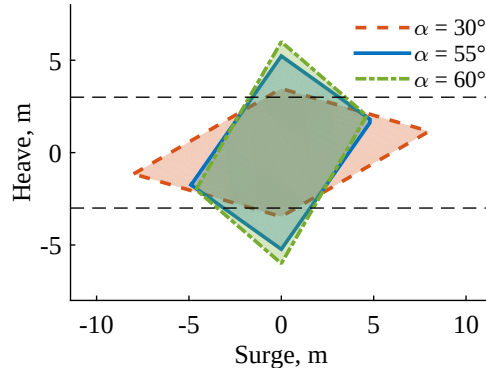


Figure 5.23: A motion envelope in xz -plane of the 3-tether WEC depending on the inclination angle of tethers. The maximum stroke of each tether is 3 m. Horizontal dashed lines show motion constraints of the heaving buoy connected to one tether.

The resultant cost-related criteria for five WECs of different shapes calculated for NSW sea site are demonstrated on Figure 5.24.

The main outcomes from this analysis are as follows:

- (i) among all 3-tether WECs, the cylinder $h_c = 0.5a$ absorbs the most power;
- (ii) among all 1-tether WECs, the cylinder $h_c = 0.5a$ has the highest power output;
- (iii) the 1-tether $h_c = 0.5a$ cylinder absorbs as much power as the 3-tether $h_c = 3a$ cylinder;
- (iv) the 3-tether WECs can absorb 1.6 – 2.3 times more power than the same buoyant actuator connected to one tether;
- (v) the 3-tether cylinders $h_c = 2a$ and $3a$ have the highest capture width ratios;
- (vi) the energy output per wetted surface is the highest for the flat cylinder $h_c = 0.5a$ even though its surface area is 1.5 larger than that of the spherical body;
- (vii) taller cylinders require larger inclination angles of tethers in order to absorb maximum power, which in turn require larger pre-tension forces generated in the PTO. As a result, the flat cylinder $h_c = 0.5a$ with the smallest tether angle is the most attractive from the PTO design point of view.

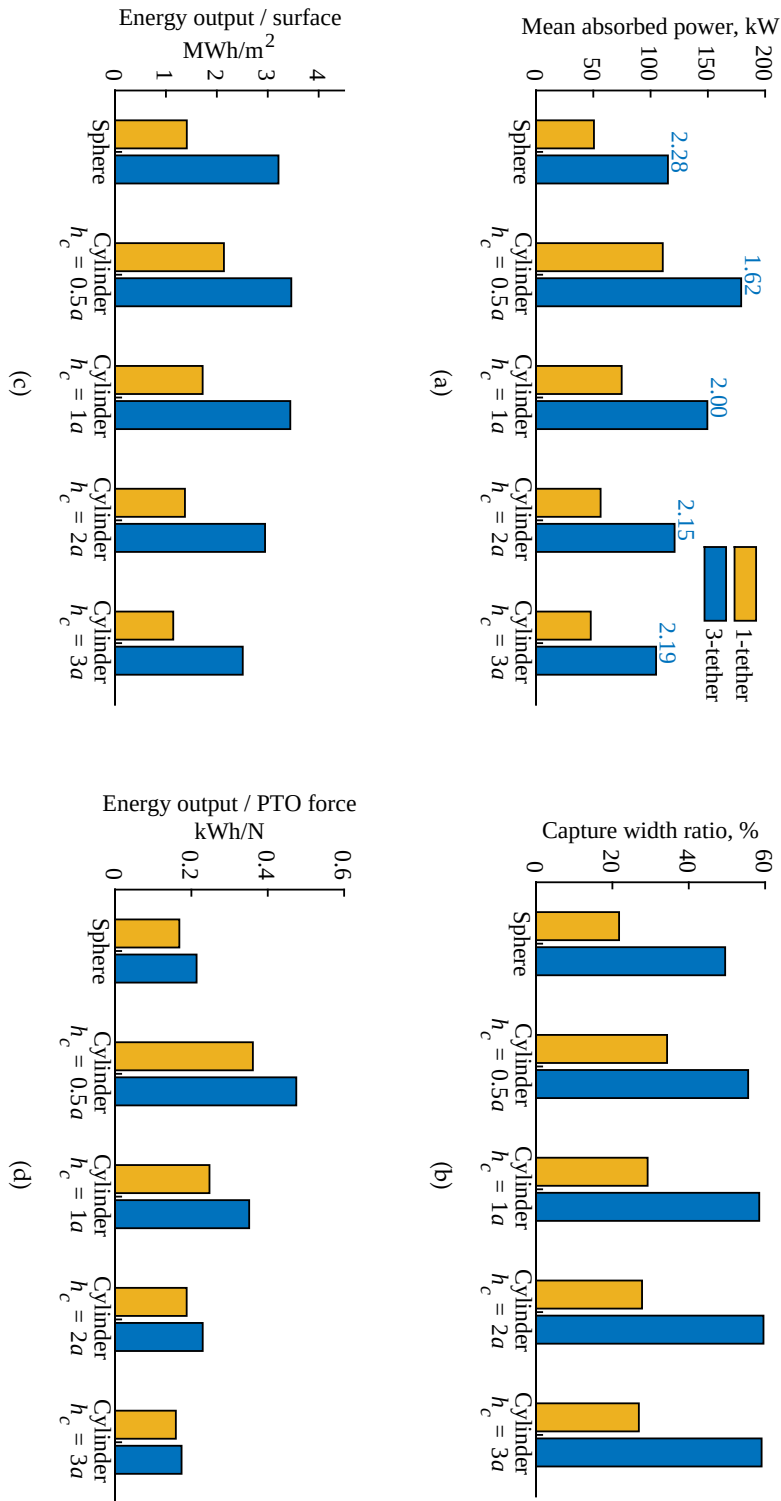


Figure 5.24: Cost-related performance measures for five buoy shapes connected to one and three tethers: (a) mean absorbed power (values on the top of each bar show the ratio between power levels of the three-tether and one-tether units with the same buoy), (b) capture width ratio, (c) annual energy output per wetted surface, (d) annual energy output per significant PTO force (calculated based on the PTO connected to tether 1).

5.2.7 Discussion

There is a number of uncertainties associated with the system analysis in this paper. As has been shown in [3], the power absorption results are greatly dependent on the drag coefficients taken in the model. As a result, it is necessary to assess the performance of the system using different values of viscous coefficients. Another uncertainty refers to the modelling of the PTO system which is assumed to have linear behaviour in this study. In reality, hydraulic circuits have non-linear nature which can be described by the Coulomb damping model with lower power output. Also, advanced control strategies applied to WECs may improve the mean absorbed power up to 4–5 times [8].

5.2.8 Conclusion

The performance of the three-tether WEC has been analysed in irregular waves using the state-of-the-art methods. It has been shown that the buoy mass reduction may lead to an increased power output while requiring more costly PTO system and foundation. The disk-like shape of the buoyant actuator shows the best performance in terms of different cost-related measures including the mean absorbed power, the energy on wetted surface ratio and the energy on the significant PTO force ratio for both one- and multi-tether arrangements. In addition, results demonstrated that the three-tether PTO configuration can provide up to 2.3 times more power as compared to the the same buoy connected to one tether.

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Chapter 6

Feasibility assessment of the three-tether concept

It has been revealed in the previous chapter, that a submerged buoy connected to three tethers has a potential to absorb 1.6–2.3 times more power than its heaving counterpart attached to one tether. However, it is obvious that this efficiency improvement comes at the expense of the increased number of power take-off units and mooring systems, leading to a higher cost of the entire wave energy converter. In order to investigate the technical and economic benefits of employing three tethers instead of one, a comparative study of both technologies is presented in this chapter analysing the system with one buoy shape and considering one deployment site. As a result, this part of the thesis is designed to provide some insights into the following research question: *Does a three-tether concept offer a more cost-effective solution compared to the heaving CETO-like system?*

This chapter consists of the published journal article:

Sergiienko, N. Y., Rafiee, A., Cazzolato, B. S., Ding, B., and Arjomandi, M. (2018). "Feasibility study of the three-tether axisymmetric wave energy converter". *Ocean Engineering* 150, pp. 221–233.

The article in its published format is available at <https://doi.org/10.1016/j.oceaneng.2017.12.055>.

Statement of Authorship

Title of Paper	Feasibility study of the three-tether axisymmetric wave energy converter
Publication Status	☒ Published ☐ Accepted for Publication ☐ Submitted for Publication ☐ Unpublished and Unsubmitted work written in manuscript style
Publication Details	Sergiienko, N. Y., Rafiee, A., Cazzolato, B. S., Ding, B., and Arjomandi, M. (2018). "Feasibility study of the three-tether axisymmetric wave energy converter". <i>Ocean Engineering</i> 150, pp. 221–233.

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Overall percentage (%)	70%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
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Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

- the candidate's stated contribution to the publication is accurate (as detailed above);
- permission is granted for the candidate to include the publication in the thesis; and
- the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

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Feasibility study of the three-tether axisymmetric wave energy converter

N. Y. Sergiienko, A. Rafiee, B. S. Cazzolato, B. Ding, M. Arjomandi

Abstract

There are numerous designs and concepts that have been offered to extract energy from ocean waves. A heaving buoy is distinguished as the most popular device which predominantly harnesses energy from the vertical motion in waves. One such device is the bottom-referenced submerged heaving buoy represented by the Carnegie Clean Energy CETO system. The total power absorption of this converter can be increased by replacing the single-tether power take-off system by a three-tether mooring configuration thereby making motion controllable in heave and surge. The current paper provides a comparative performance analysis of the generic submerged heaving buoy connected to one tether and the three-tether converter in terms of the buoy motion, and design of the power take-off and mooring systems. This is accompanied by a techno-economic analysis of two converters.

6.1 Introduction

Along with wind and solar power, ocean waves are a huge source of sustainable energy that still remain unexploited for electricity generation. Despite more than 40 years of intensive research and more than 200 wave energy converter (WEC) designs, there is no definitive answer to the question of what working principle is more suitable for shallow or deep water, or what scale of the device is more economically viable. As a result, none of the existing designs has reached the commercial-scale stage, still remaining at the proof-of-concept development phase. Therefore, researchers and engineers continue to look for new solutions in order to make wave power competitive with other sources of renewable energy.

Along with various sizes and shapes, WECs differ in operation principle including the mode of motion utilised to convert wave power. Thus, heave, surge, and pitch of the structure are the main modes usually used in practice. Depending on the converter geometry and its location relative to the mean water surface, the oscillatory motion of the WEC in different modes radiates different types of waves leading to the different power absorption levels

[29]. Thus, systems that combine several modes of motions have better hydrodynamic performance and the structure placed in water can be used more efficiently [14]. As an example, bottom-referenced heaving buoys similar to the CETO system [8] are designed to absorb power from the vertical motion in waves (see Figure 6.1a). While heave has a major contribution to the power absorption due to a flexible tether connection between the buoy hull, the power take-off system, and the seabed, the structure can experience movement in all degrees of freedom. In order to increase the efficiency of such WECs, it has been suggested [38] to use a three-cable mooring configuration (see Figure 6.1b) which allows to control motion of the buoy in both heave and surge, and to some extent in pitch. Interestingly, that an added degree of freedom (e.g. surge) can significantly improve the performance of fully submerged quasi-point absorbers, while the efficiency of their floating counterparts will increase only in a limited range of wave periods [37].

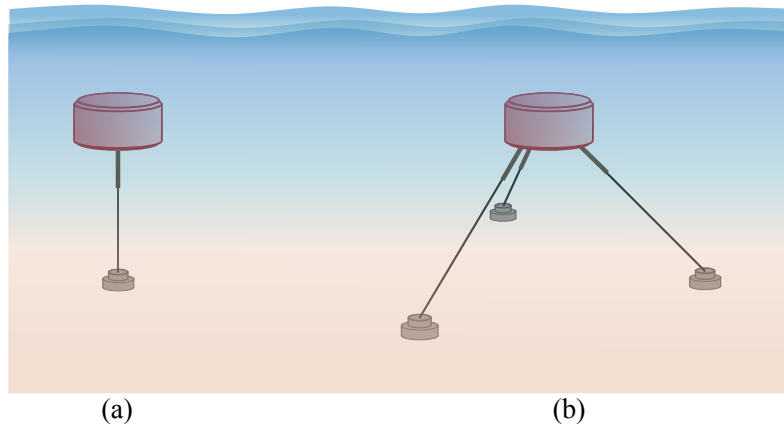


Figure 6.1: Different power take-off configurations for the submerged WECs: (a) a generic heaving buoy connected to one tether; (b) a three-tether system.

The concept of the three-tether WEC was introduced by [38], where a submerged spherical buoy was attached to three cables that were equally spaced around the buoy and each cable was connected to an individual power-take off machinery. The study was conducted in the frequency domain and demonstrated up to a threefold increase in power production as compared to the same buoy connected to only one cable. Later, the three-tether converter with a floating cylindrical buoy was used as a prototype for the control system design [23, 34]. Further analysis has been focused on the design features of the three-tether WEC with an objective to identify the optimal arrangement of tethers which can provide the maximum power output of

the converter [35]. In addition, there is an interest in developing wave energy arrays consisting of multi-tether buoys due to the benefit of shared moorings [1, 43].

At the initial stage of any prototype development, the main research focus is given to the technical side of the problem: efficiency of the WEC under regular waves with idealised control, a dependence of the device performance on the geometry and constraints, etc. However, when it comes to the commercialisation of a particular converter, other factors become significant for investors. Thus, the system with the best hydrodynamic performance does not necessarily guarantee the cheapest electricity production [29]. Therefore, recently, considerable attention has been given to the techno-economic assessments of the WEC development. Since only a few devices have been built in full-scale and undertaken in-ocean testing, the amount of information on levelised costs or capital/operational expenditures of WECs is limited. Therefore, a number of indirect cost-related criteria has been developed for converters at low technology readiness level [4, 5]. Regarding the three-tether WEC, the preliminary results of its performance in irregular waves have been presented in [36] including the estimation of energy delivery and other indirect techno-economic indices.

The current paper demonstrates a feasibility study of the three-tether wave energy converter with one shape of the buoy hull. Descriptions of the system and modelling routine are specified in Sections 6.2 and 6.3 respectively. The performance of the WEC and expected power output are presented in the form of comparative analysis between the single-tether and three-tether WECs in Section 6.4. Uncertainties associated with the modelling assumptions are quantified in Sections 6.5 and 6.6. Finally, the techno-economic analysis of the three-tether WEC is presented in Section 6.7.

6.2 Description of the system

6.2.1 Buoy

Currently, there is no physical prototype for a three-tether wave energy converter which can be used for analysis in the current study. Therefore, it has been decided to utilise a generic shape, namely a vertical cylinder, whose hydrodynamic behaviour has been thoroughly studied [22, 44]. In practice, corners of the buoy should be rounded to reduce drag and viscous losses [29], but for the numerical modelling in this study only the aspect ratio of the device is important. The geometrical dimensions of the buoy hull are selected to be similar to the CETO-5 system as specified in Table 6.1.

Table 6.1: Parameters of the WEC.

Parameter	Value
Shape	vertical cylinder
Radius of the cylinder, a	5.5 m
Height of the cylinder, h_c	5.5 m
Water depth	50 m
Submersion (top of the buoy)*	3.75 m
Submergence depth, d_s (centre of the buoy)	6.50 m
Volume*, V	524 m ³
Surface area*	380 m ²
Mass of the buoy, m_b	268 t
Displaced mass of fluid*, m_w	537 t
Stroke length, $\Delta l_{\max} - \Delta l_{\min}$	6 (± 3) m
Tether inclination angle from the vertical, α	44°
Initial tether length*, l_0	56.6 m
Pretension force in each tether*	1.2MN

* not independent parameters.

6.2.2 Power take-off configuration

In the current study it is assumed that each tether is connected to an individual power take-off (PTO) mechanism, which can be implemented as an electric generator [34] or a hydraulic circuit [11]. For the three-tether WEC it is possible to place the PTO system inside the buoy hull, where all three hydraulic cylinders drive the same power generator [18]. The behaviour of the hydraulic system is usually described by the Coulomb damping force [4]. However, in order to exclude uncertainties associated with a specific PTO design, it is presumed that the machinery force has linear spring and damper effects proportional to the tether extension and the rate of change of the tether extension, respectively.

6.2.3 Sea site

There is a range of sea site locations that can be used to assess the overall performance of the WECs including Australia [2], Europe [4] or the USA. However, Yeu island located in France has been extensively used as a benchmark site for the comparison of various WEC prototypes [5, 10] and therefore will be considered in this study. The wave data statistics and parameters of the site are specified in Figure 6.2.

Location:	46°40'00.0"N, 2°25'00.0"W
Water depth:	48 m
Mean wave power:	25.5 kW/m (Pierson-Moskowitz)
Type of data:	Real sea measurement (as cited in [5]).

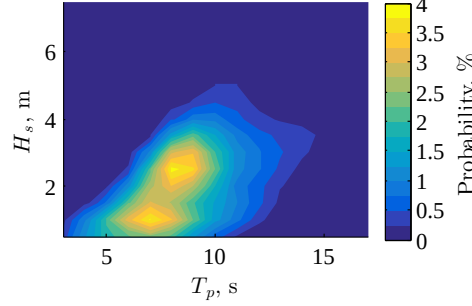


Figure 6.2: Wave data statistics for France/Yeu island site. Source of data is [5].

6.3 Equation of motion

The wave-to-wire model employed in this study is based on linear wave theory, assuming small motion amplitudes of the buoy as compared to the length of the mooring lines (tethers). The only second-order hydrodynamic effect included in the model is a viscous drag force which is proportional to the square of the body velocity relative to the fluid.

6.3.1 Kinematics

A schematic of a three-tether WEC is shown on Figure 6.3. The spatial arrangement of all tethers is defined by $\mathbf{s}_i$:

$$\mathbf{s}_i = \mathbf{r} + \mathbf{R}\mathbf{n}_i - \mathbf{d}_i, \quad i = 1 \dots 3, \quad (6.1)$$

where $\mathbf{r}$ is the position vector of a buoy in the reference coordinate frame $Oxyz$, $\mathbf{R}$ is the rotation matrix of the buoy with respect to the reference frame, $\mathbf{n}_i$ denotes the position vector of the anchor point of the i -th tether on the hull relative to the buoy centre of mass G , $\mathbf{d}_i$ is the position vector of the anchor point of the i -th tether on the sea floor in the $Oxyz$ coordinate frame. The instantaneous tether length is:

$$l_i = \|\mathbf{s}_i\| = \sqrt{\mathbf{s}_i^T \mathbf{s}_i}, \quad i = 1 \dots 3, \quad (6.2)$$

and the change in length of the i -th tether is $\Delta l_i = l_i - l_0$.

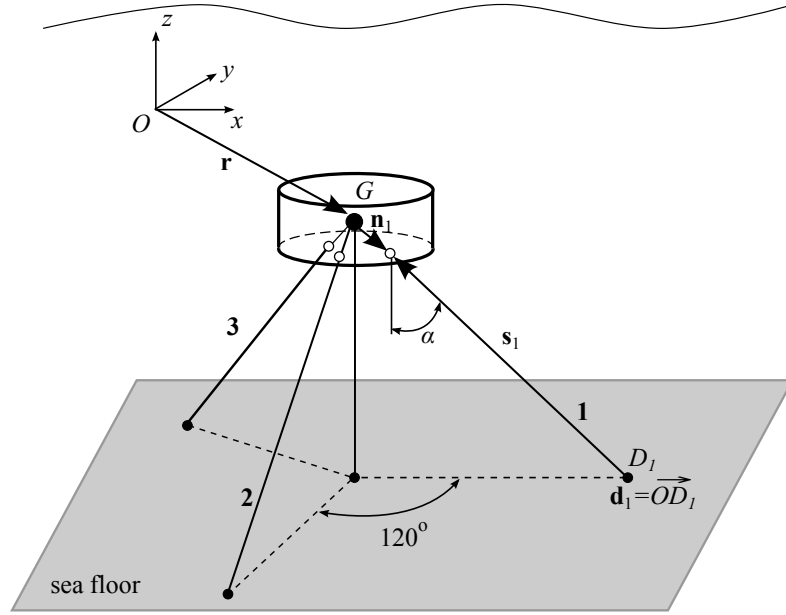


Figure 6.3: Schematic of the three-tether wave energy converter.

Mapping from the buoy velocities in a Cartesian coordinate frame to the rate of change of the tether length is provided by the inverse kinematic Jacobian:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{x}}, \quad (6.3)$$

where $\mathbf{x} = [\mathbf{r} \ \boldsymbol{\theta}]^T$ is the buoy location (pose) vector with three translational and three rotational motions, $\mathbf{q} = [l_1 \ l_2 \ l_3]^T$ is a vector of three tether length variables and the inverse kinematic Jacobian $\mathbf{J}^{-1}$ can be obtained as:

$$\mathbf{J}^{-1} = \begin{pmatrix} \mathbf{e}_{s1}^T & (\mathbf{R}\mathbf{n}_1 \times \mathbf{e}_{s1})^T \\ \mathbf{e}_{s2}^T & (\mathbf{R}\mathbf{n}_2 \times \mathbf{e}_{s2})^T \\ \mathbf{e}_{s3}^T & (\mathbf{R}\mathbf{n}_3 \times \mathbf{e}_{s3})^T \end{pmatrix}, \quad (6.4)$$

where $\mathbf{e}_{si}$ is a unit-vector along the tether i , so $\mathbf{e}_{si} = \frac{\mathbf{s}_i}{\|\mathbf{s}_i\|}$.

6.3.2 Forces

The generalised forces that act on the body are expressed in the Cartesian coordinate frame $Oxyz$ as vectors of six elements including horizontal and vertical forces and rotational moments.

6.3.2.1 Hydrostatic force

As the buoy is fully submerged, the generalised hydrostatic force is $\mathbf{F}_{buoy} = [0 \ 0 \ (m_w - m_b)g \ 0 \ 0 \ 0]^T$, where $m_w = \rho V$ is the mass of the displaced water, ρ is water density, V is the displaced volume of the buoy and m_b is the mass of the WEC.

6.3.2.2 Power take-off forces

A behaviour of the linear PTO system is modelled as [4]:

$$F_{pto,i} = (C_{pto} - B_{pto}\Delta\dot{l}_i - K_{pto}\Delta l_i), \quad i = 1 \dots 3, \quad (6.5)$$

where K_{pto} and B_{pto} are the PTO stiffness and damping coefficients (control parameters) respectively, and

$$C_{pto} = -\frac{(m_w - m_b)g}{3 \cos \alpha} \quad (6.6)$$

is the force that counteracts the hydrostatic force in an undisturbed position, and α is the tether angle relative to the vertical. It is assumed that all PTO machineries have the same stiffness and damping coefficients.

6.3.2.3 Hard stop forces

As each PTO system has a limited stroke, an additional force from the hard stop mechanism is exerted on the body to constrain its motion. As a result, the hard stop system is modelled by a repulsive energy potential [4]:

$$\begin{aligned} F_{hs,i} = & -K_{hs,\min}(\Delta l_i - \Delta l_{\min})u(\Delta l_{\min} - \Delta l_i) \\ & -K_{hs,\max}(\Delta l_i - \Delta l_{\max})u(\Delta l_i - \Delta l_{\max}), \end{aligned} \quad (6.7)$$

$i = 1 \dots 3$, $u(\cdot)$ is the Heaviside step function, $K_{hs,\min}$ and $K_{hs,\max}$ are the hard stop spring coefficients, $\Delta l_{\min}$ and $\Delta l_{\max}$ are the stroke limits relative to the nominal position.

6.3.2.4 Tension in tether

A linear superposition of the PTO and hard stop forces governs the tension in each tether. In addition, the tethers should be always under tension in order to transmit forces to the machinery:

$$F_{t,i} = \min(0, F_{pto,i} + F_{hs,i}), \quad i = 1 \dots 3. \quad (6.8)$$

Noting that $F_{pto,i}$, $F_{hs,i}$ and $F_{t,i}$ are the forces that act along the mooring line i , the generalised tether force in the Cartesian coordinate frame is:

$$\mathbf{F}_{tens} = \mathbf{J}^{-\top} \mathbf{F}_t, \quad (6.9)$$

where $\mathbf{F}_t = [F_{t,1} \ F_{t,2} \ F_{t,3}]^\top$ and $\mathbf{J}^{-\top}$ is the transposed inverse kinematic Jacobian.

6.3.2.5 Viscous damping forces

The viscous damping force is modelled according to the Morison equation [4, 25]:

$$\mathbf{F}_{visc} = \begin{pmatrix} -\frac{1}{2}\rho\mathbf{C}_d\mathbf{A}_d\|\mathbf{V}_b - \mathbf{V}_f\|(\mathbf{V}_b - \mathbf{V}_f) \\ -\frac{1}{2}\rho\mathbf{b}_Q D^4 D\|\dot{\boldsymbol{\theta}}\|\dot{\boldsymbol{\theta}} \end{pmatrix}, \quad (6.10)$$

where $\mathbf{V}_b = \dot{\mathbf{r}}$ is the buoy velocity, $\mathbf{V}_f$ is the fluid particle velocity at the position of the centre of mass of the buoy, and

$$\mathbf{C}_d = \begin{pmatrix} C_x & 0 & 0 \\ 0 & C_y & 0 \\ 0 & 0 & C_z \end{pmatrix}, \quad \mathbf{A}_d = \begin{pmatrix} A_x & 0 & 0 \\ 0 & A_y & 0 \\ 0 & 0 & A_z \end{pmatrix} \quad (6.11)$$

are matrices of drag coefficients and the cross-section areas of the buoy perpendicular to the direction of motion respectively,

$$\mathbf{b}_Q = \begin{pmatrix} b_{yz} & 0 & 0 \\ 0 & b_{xz} & 0 \\ 0 & 0 & b_{xy} \end{pmatrix} \quad (6.12)$$

is the matrix of quadratic angular damping coefficients, D is the buoy diameter and $\dot{\boldsymbol{\theta}}$ is the vector of angular velocities of the buoy.

6.3.3 Time-domain model

The buoy motion in time domain can be described using the Cummins equation [9] that include the wave excitation and radiation forces, and other forces specified in Section 6.3.2:

$$(\mathbf{M} + \mathbf{A}_\infty)\ddot{\mathbf{x}} = \mathbf{F}_{exc} - \int_0^t \mathbf{K}_{rad}(t - \tau)\dot{\mathbf{x}}(\tau)d\tau + \mathbf{F}_{buoy} + \mathbf{F}_{visc} + \mathbf{F}_{tens}, \quad (6.13)$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{A}_\infty$ is the matrix with infinite-frequency added mass coefficients, $\mathbf{F}_{exc}$ is the generalised wave excitation force, and $\mathbf{K}_{rad}(t)$ is a retardation function.

The wave drift forces and ocean current effects are not considered in the model.

6.3.4 Frequency domain

Equation (6.13) can be linearised assuming small angular motion of the system and mapping the change in the tether length to the Cartesian coordinates of the body. Thus, the linearised frequency domain model of the generic point absorber can be written as [4] where only surge, heave and pitch modes are considered:

$$(\mathbf{M} + \mathbf{A}(\omega)) \hat{\mathbf{x}} + (\mathbf{B}_{pto} + \mathbf{B}(\omega)) \hat{\mathbf{x}} + \mathbf{K}_{pto} \hat{\mathbf{x}} = \hat{\mathbf{F}}_{exc}, \quad (6.14)$$

where

$$\mathbf{M} = \begin{pmatrix} m_b & 0 & 0 \\ 0 & m_b & 0 \\ 0 & 0 & I_y \end{pmatrix}, \quad \mathbf{A}(\omega) = \begin{pmatrix} a_{11} & 0 & a_{15} \\ 0 & a_{33} & 0 \\ a_{51} & 0 & a_{55} \end{pmatrix}, \quad (6.15)$$

$$\mathbf{B}(\omega) = \begin{pmatrix} b_{11} & 0 & b_{15} \\ 0 & b_{33} & 0 \\ b_{51} & 0 & b_{55} \end{pmatrix}, \quad (6.16)$$

$$\mathbf{B}_{pto} = \begin{pmatrix} \frac{3}{2} B_{pto} \sin^2 \alpha & 0 & 0 \\ 0 & 3 B_{pto} \cos^2 \alpha & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad (6.17)$$

$$\mathbf{K}_{pto} = \begin{pmatrix} K_{pto,11} & 0 & K_{pto,15} \\ 0 & K_{pto,33} & 0 \\ K_{pto,51} & 0 & K_{pto,55} \end{pmatrix}, \quad (6.18)$$

$$K_{pto,11} = \frac{3 \sin^2 \alpha}{2} \left(K_{pto} + \frac{C_{pto}}{3 l_0 \cos \alpha} \right) - \frac{C_{pto}}{l_0 \cos \alpha},$$

$$K_{pto,15} = K_{pto,51} = -\frac{C_{pto} h_c}{2 l_0 \cos \alpha},$$

$$K_{pto,33} = 3 \cos^2 \alpha \left(K_{pto} + \frac{C_{pto}}{3 l_0 \cos \alpha} \right) - \frac{C_{pto}}{l_0 \cos \alpha},$$

$$K_{pto,55} = -\frac{C_{pto} h_c (h_c + 2 l_0 \cos \alpha) (\cos^2 \alpha + 1)}{8 l_0 \cos^3 \alpha},$$

where l_0 denotes the initial tether length.

The above equations illustrate the cross-coupling between modes and are used to calculate the response amplitude operators presented in Section 6.4.1.1.

6.3.5 Power absorption

Energy can be generated by the PTO machinery only when the tether, attached to it, is under tension. As a result, the total instantaneous power absorbed by the WEC is calculated as:

$$P_{\Sigma}(t) = \sum_{i=1}^3 P_i(t) = \sum_{i=1}^3 F_{t,i}(t) \Delta \dot{l}_i(t) \quad \text{if } F_{t,i} < 0. \quad (6.19)$$

6.3.6 Model implementation

Hydrodynamic coefficients of the WEC in the frequency domain have been obtained using an analytical solution [22] and verified against WAMIT [24] (see Figure 6.4).

In order to conduct the time domain simulations, Equation (6.13) has been implemented in Simulink/MATLAB with a time step of 0.01 seconds using the ode45 solver. The duration of all simulation runs has been set to $300 \times T_p$ but not less than 1200 seconds and the first $15 \times T_p$ have not been included in the analysis due to initial transient states. The convolution integral in Equation (6.13) has been replaced by a state-space model using the Marine System Simulator toolbox [30]. The irregular wave time-series have been implemented using the Pierson-Moskowitz wave spectrum [41] over the grid of significant wave heights of $H_s = 0.5 - 7.5$ m and peak wave periods $T_p = 3 - 17$ sec. In all simulations, waves propagate along the x -axis making the first tether aligned with the direction of the incident wave. Drag coefficients have been taken from [21] and set to $C_x = C_y = 1$, $C_z = 1.1$ and $b_{xz} = b_{yz} = 0.2$. The hard stop spring coefficient is set to $K_{hs,\min} = K_{hs,\max} = 10^8$ N/m. The PTO control parameters K_{pto} , B_{pto} are simultaneously optimised for each sea state using Simulink Design Optimization toolbox with an objective function to maximise the mean value of the generated power over the duration of the simulation: $\max_{K_{pto}, B_{pto}} \bar{P}_{\Sigma}(t)$.

The model of the single-tether WEC is adapted from [4] and will not be repeated in this paper. All simulation parameters are consistent with the three-tether case except the PTO control settings which are optimised separately for the single-tether system.

Optimal values of the PTO control parameters for the single- and three-tether WECs are shown in Figure 6.5. These PTO settings are used to generate results in Section 6.4.

6.4 Comparison of performance between single- and three-tether buoys

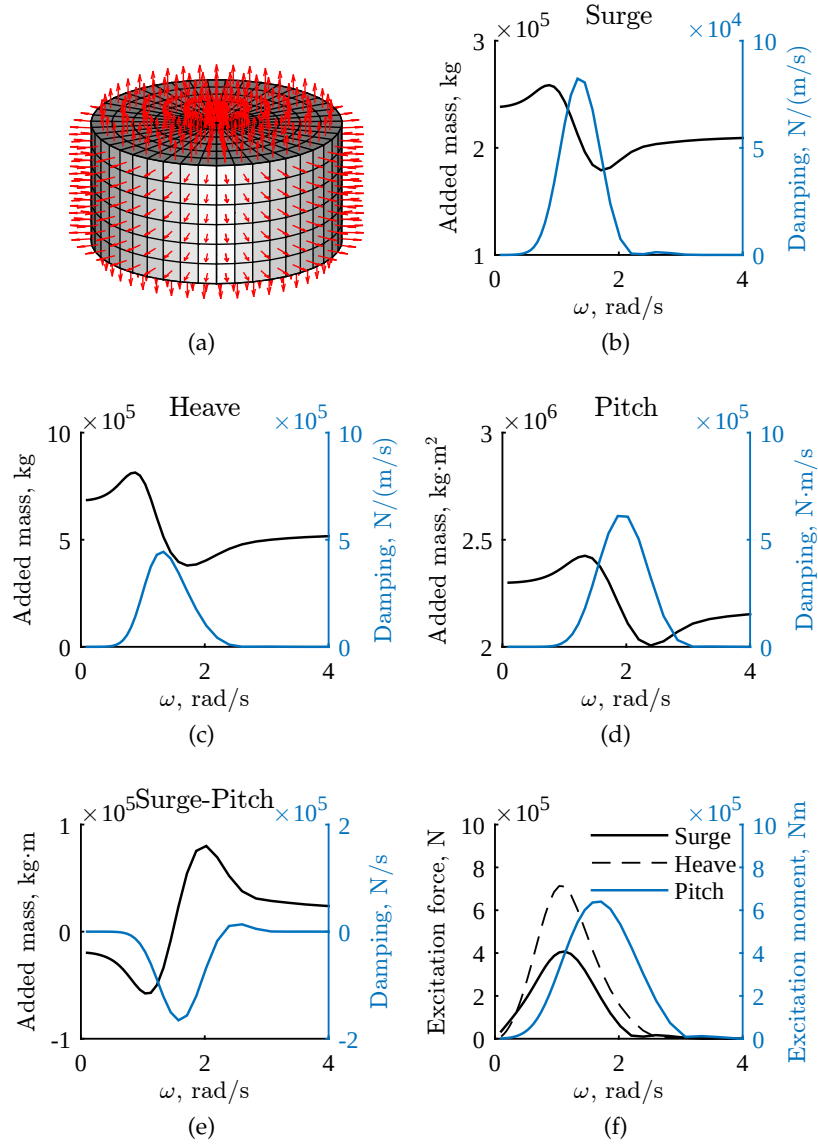


Figure 6.4: Frequency dependent hydrodynamic parameters of the submerged cylindrical buoy obtained from WAMIT [24].

6.4 Comparison of performance between single- and three-tether buoys

The main distinctive features of the three-tether WEC are demonstrated in this section including motion amplitudes, acting forces and power output.

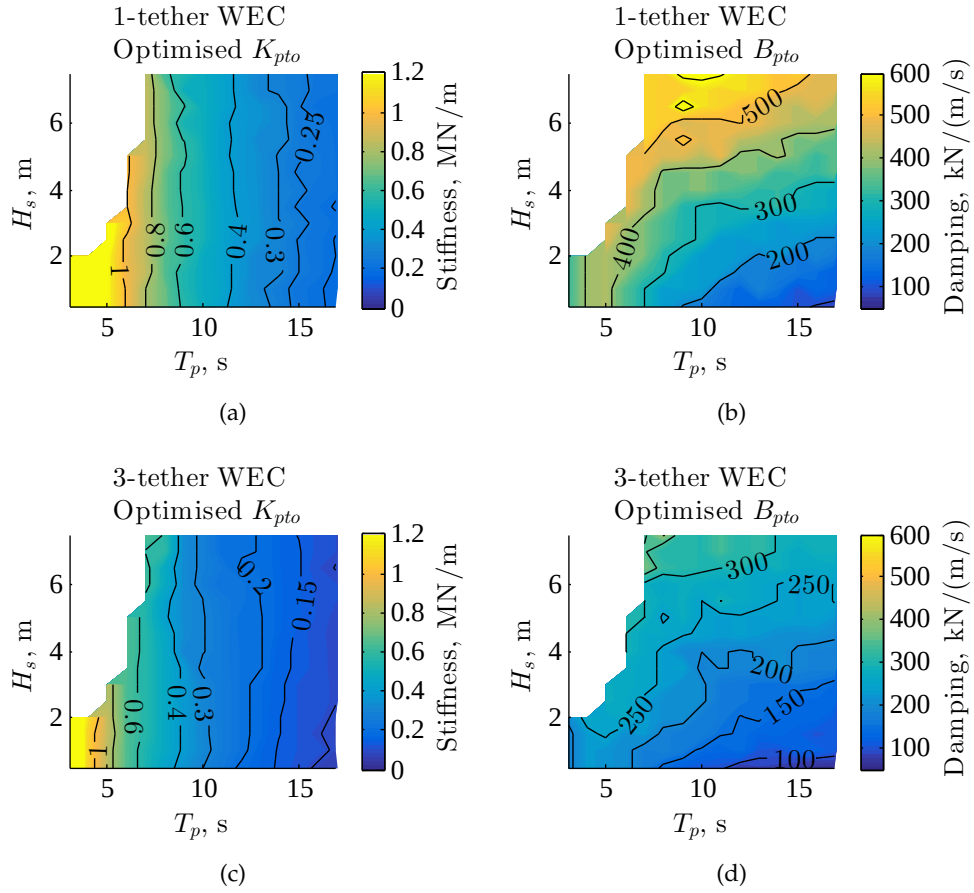


Figure 6.5: Optimal values of the PTO control parameters for (a,b) the single-tether WEC and (c,d) the three-tether WEC used to generate results presented in Section 6.4.

6.4.1 Buoy motion

6.4.1.1 Resonance frequency

In the absence of a power take-off and mooring systems, the natural frequency of neutrally buoyant submerged WECs approaches zero due to the absence of the hydrostatic stiffness in all degrees of freedom (DOF). For submerged buoys, the mooring lines and the PTO system provide the only stiffness to the plant, which controls its resonance behaviour and mode shapes. The conventional motion modes such as surge, heave or pitch become coupled not only hydrodynamically, but also through the tether arrangement, which is indicated by the presence of the off-diagonal terms in the $\mathbf{K}_{pto}$ matrix (see Equation (6.18)). The behaviour of such coupled multi-DOF systems can

6.4 Comparison of performance between single- and three-tether buoys

be investigated using eigenvectors and eigenvalues adapted from vibration analysis (for details see [12]). As a result, resonance frequencies of the WEC described by Equation (6.14) can be found solving the eigenvalue problem:

$$(\mathbf{M} + \mathbf{A})^{-1} \mathbf{K}_{pto} - \omega_0^2 \mathbf{I} = \mathbf{0}, \quad (6.20)$$

where $\mathbf{I}$ is the identity matrix.

For the single-tether WEC, the exact solution of Equation (6.20) in heave is:

$$\omega_0^{heave} = \sqrt{\frac{K_{pto}}{m_b + a_{33}(\omega_0)}}, \quad (6.21)$$

while for surge/pitch coupled modes the expressions are more complex. However, by drawing an analogy of the bottom-referenced heaving buoy with an inverted pendulum, the resonance frequency of a single-tether buoy in surge/pitch can be approximated by:

$$\omega_0^{surge} \approx \sqrt{\frac{g(m_w - m_b)}{l_0(m_b + a_{11}(\omega_0))}}. \quad (6.22)$$

Thus, while the resonance of the single-tether WEC in heave can be controlled by changing the PTO stiffness K_{pto} , the buoy behaviour in surge is only dependant on such parameters as the mass ratio (m_b/m_w) and the initial tether length l_0 . In contrast, the PTO configuration of the three-tether WEC allows the control of both heave and surge due to the additional coupling through inclined mooring lines. To demonstrate the effect of the resonance frequency on the WEC's performance, the response amplitude operators (RAOs) of the single- and three-tether converters with fixed control parameters ($K_{pto} = 10^5$ N/m and $B_{pto} = 10^5$ N/(m/s)) are shown in Figures 6.6 and 6.7 respectively. Dashed lines correspond to exact solutions of Equation (6.20) for each WEC.

It is clear from the plots that none of these mooring configurations can effectively control angular rotation of the buoy, which is evident from very large pitch amplitudes at higher frequencies. However, pitch motion of the single-tether WEC has two resonance peaks due to the undamped coupling with surge, while there is only one peak for the three-tether case. Also, the latter configuration demonstrates effective coupling of heave and surge leading to the same (degenerate) resonance frequency.

6.4.1.2 Motion envelope

Due to the fact that the surge motion of the single-tether WEC cannot be controlled by the PTO machinery, the buoy may experience large displacements in the horizontal direction depending on the sea state conditions.

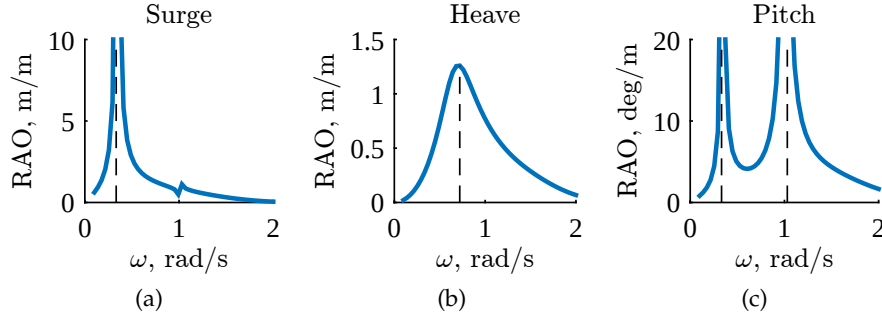


Figure 6.6: RAOs and resonance frequencies of the single-tether WEC in (a) surge, (b) heave and (c) pitch. PTO parameters are taken as $K_{pto} = 10^5$ N/m and $B_{pto} = 10^5$ N/(m/s). Dashed lines correspond to exact solutions of Equation (6.20) for the single-tether converter.

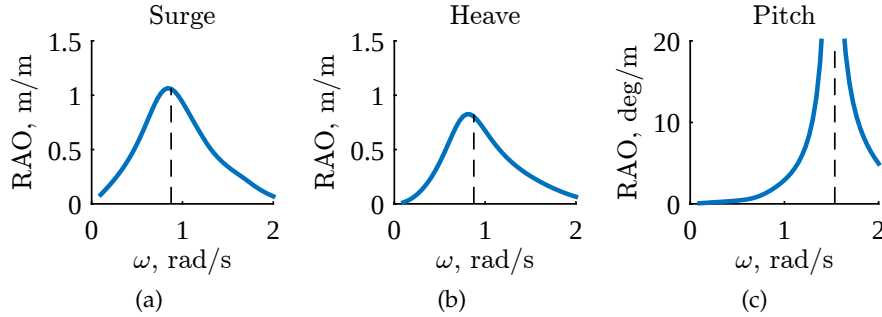


Figure 6.7: RAOs and resonance frequencies of the three-tether WEC in (a) surge, (b) heave and (c) pitch. PTO parameters are taken as $K_{pto} = 10^5$ N/m, $B_{pto} = 10^5$ N/(m/s) and $\alpha = 44$ deg. Dashed lines correspond to exact solutions of Equation (6.20) for the three-tether converter.

Moreover, the nominal operation position of such WECs can vary with time due the presence of ocean currents and wave drift forces. In the case of the three-tether WEC, the buoy motion is constrained and controlled in all translational degrees of freedom; and the motion envelope of the structure is governed by the maximum stroke lengths and the tether inclination angle. For the demonstration purposes, motion envelopes of the single-tether and three-tether WECs in irregular waves of $H_s = 3$ m and $T_p = 14$ sec are shown in Figure 6.8. Data is represented in terms of the frequency of the buoy position occurrence over 4200 sec. The white borders outline the area in the xz -plane where the buoy can operate without reaching hard stop mechanisms. As expected, the three-tether WEC motion has a circular nature due to the arrangements of mooring lines, while the single-tether converter

6.4 Comparison of performance between single- and three-tether buoys

swings a lot in surge, similar to the inverted pendulum mechanism.

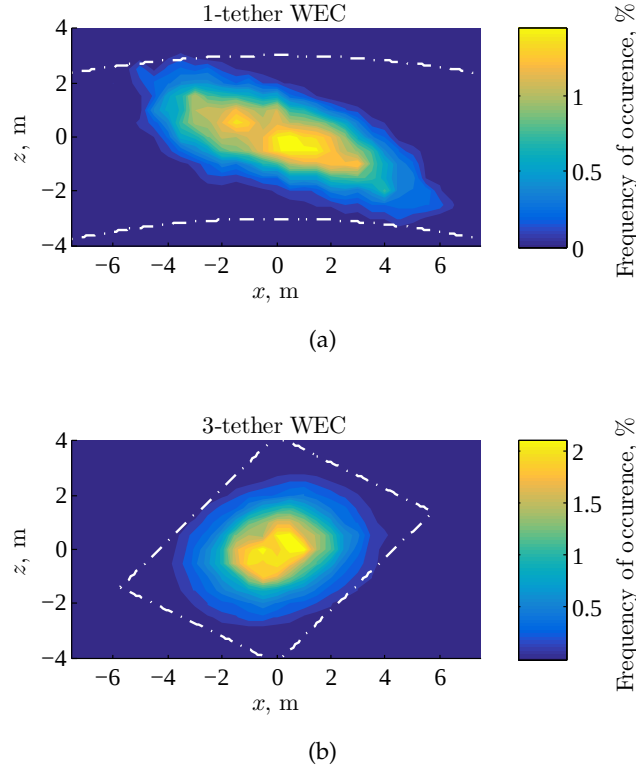


Figure 6.8: Motion envelopes of the WEC in irregular wave of $H_s = 3$ m, $T_p = 14$ s with various arrangements of mooring lines: (a) 1-tether and (b) 3-tether. Dashed white lines represent kinematic limits.

There is another criterion commonly used to analyse the motion of the surface buoys and vessels which is called ‘watch circle’ [39]. It indicates the area (diameter) on the water surface within which the moored structure can move. A similar performance-related measure has been introduced for the comparison of various WECs that participated in the recent Wave Energy Prize competition arranged by the [42]. Thus, the statistical peak (not the maximum value) of the mooring watch circle has been calculated based on the major axis of the watch circle.

As fully submerged WECs operate under water all the time, it is possible to assume that the watch circle is the area in the horizontal plane formed by the buoy motion in surge and sway. In our study, the ‘statistical peak’ refers to the 99% confidence level of the buoy displacement in surge. A comparison between statistical peaks of the mooring watch circle for the single- and three-tether WECs over the range of sea states is demonstrated in Figure 6.9.

For both tether configurations the buoy displacement in surge increases with the wave height and the peak wave period. However, for the single-tether case, the mooring watch circle can reach up to 10 m in energetic sea states, while for the three-tether WEC the motion is constrained within the 4.5 m circle.

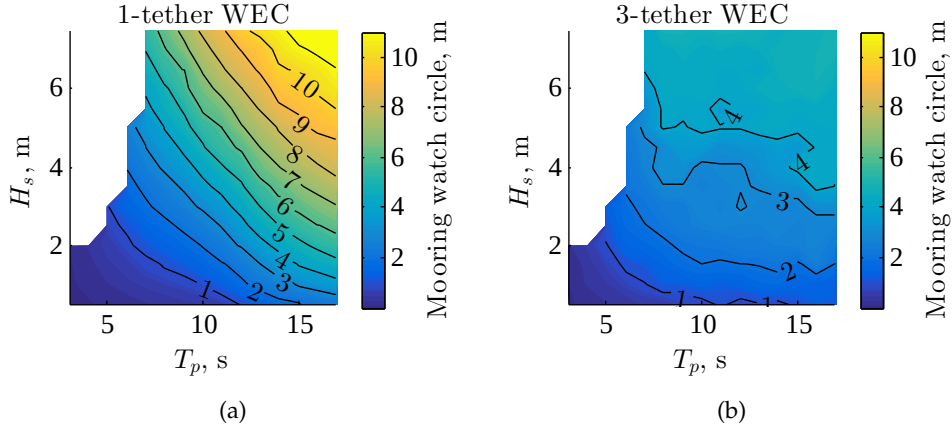


Figure 6.9: Statistical peak of the mooring watch circle diameter for the (a) single-tether and (b) three-tether WECs.

6.4.1.3 Pitch

The angular motion of both single- and three-tether WECs in pitch cannot be directly controlled by the PTO system but it affects the total performance of the WEC through coupled dynamics. Also, if significant pitching occurs, the buoy may experience surface wave breaking and undesired additional loads on the system. The comparison of the statistical peak values of the pitch angle of the single- and three-tether WECs are shown in Figure 6.10. As a result, angular rotation of the three-tether buoy remains less than 30 deg even for waves with high amplitudes, while pitch angle of the single-tether WEC can reach 44 deg. However, these peak values represent mostly extreme operational conditions for each system, while the significant pitch angle calculated as the root mean square over the time series shows that the angular rotation of both converters will be less than 10 deg on average.

6.4.2 Mooring system

One of the biggest differences between single- and three-tether configurations is a mooring system design. First of all, the footprint of the multi-mooring

6.4 Comparison of performance between single- and three-tether buoys

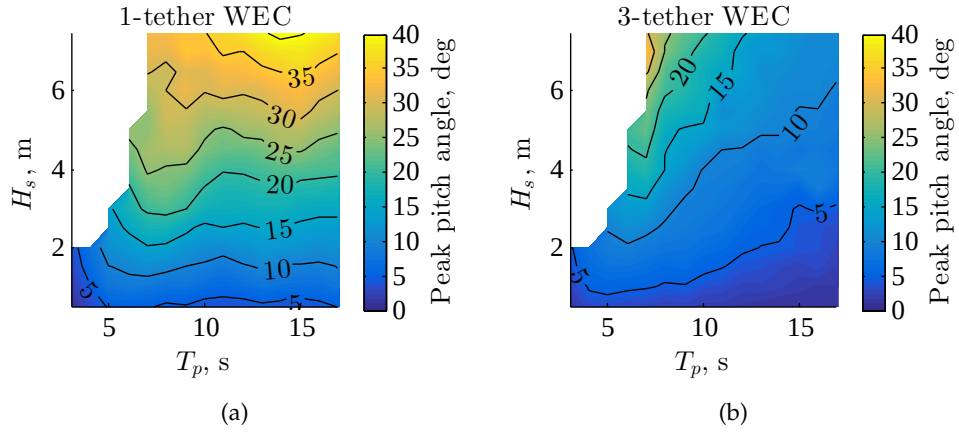


Figure 6.10: Statistical peak values of the pitch angle of the (a) 1-tether and (b) 3-tether WECs.

WEC may reach 100 m in diameter in 50 m water depth, while the single-point mooring is limited by the dimensions of its anchorage. This fact can be regarded as a drawback of the three-tether system, causing difficulties for the WEC installation due to the uneven surface of the seabed. However, in the long term perspective, wave energy converters will operate in groups forming wave energy farms [3] and the three-tether buoys have a potential to share mooring points with adjacent devices [16] as shown in Figure 6.11.

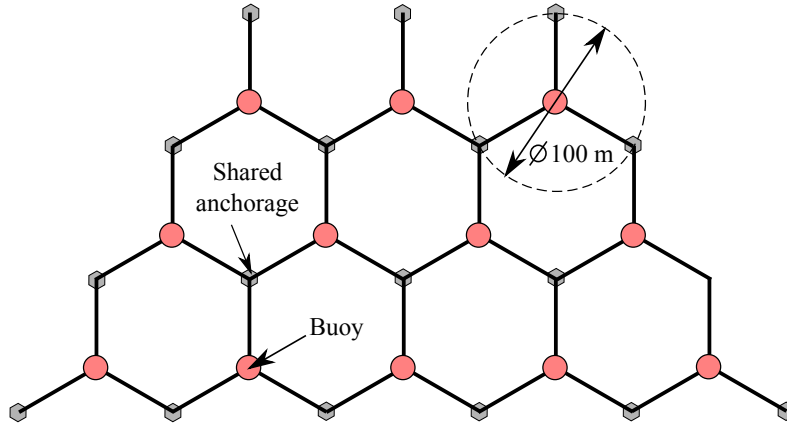


Figure 6.11: A possible arrangement of the three-tether WEC array with shared mooring points.

Another distinctive feature of the mooring systems is the direction of the cyclic loading applied from the buoy. The taut-moored systems with a single line should withstand large vertical forces and a common type of the mooring

arrangement for such systems is a gravity anchor or a piled foundation [19]. For the three-tether WEC, the mooring system experiences inclined loading, with a lower magnitude as the total acting force is distributed between three mooring points. Moreover, the resistance of a pile to inclined loading is much higher as compared to the vertical loading of the same amplitude [20]. Therefore, each pile for the three-tether configuration is potentially smaller in diameter and in length than that for the single-tether WEC. For comparison, peak tether forces experienced by the single- and three-tether WECs are demonstrated in Figure 6.12.

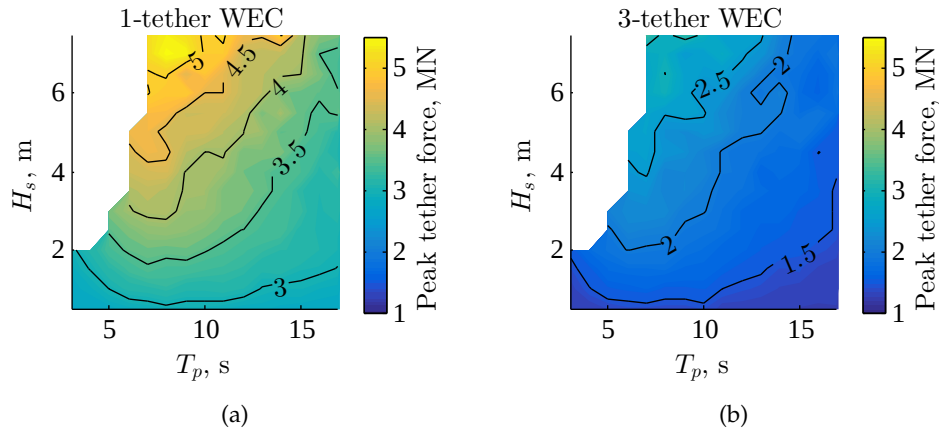


Figure 6.12: Statistical peak tether force of the (a) 1-tether and (b) 3-tether WECs. For the latter case, data is shown for the tether aligned with the wave front as it experiences the highest peak force.

As a result, loading on each mooring point from the three-tether WEC is half that of the single-tether device. However, wave tank experiments should be conducted in order to identify extreme (not statistical peak) forces acting on the structure [33].

6.4.3 End/hard stop and tether slackness

Each WEC should be equipped with an end stop mechanism in order to keep the buoy motion within desired boundaries. The behaviour of the hard stop system is governed by Equation (6.7). As shown in Figure 6.8, displacements of the three-tether converter are constrained more as compared to the single-tether WEC which leads to the greater chance of hitting end stop limits. The occurrence of the hard stop events for both converters is demonstrated in Figure 6.13. As a result, for significant wave heights less than 2.5 m, motion of buoys stays within set boundaries. However, for higher waves, impact events

6.4 Comparison of performance between single- and three-tether buoys

occur twice as often for the three-tether WEC than for the heaving buoy. This situation is not desirable for the wave converter and can be avoided using longer strokes or advanced control algorithms [17], for example increasing the PTO damping or detuning the WEC under high sea states.

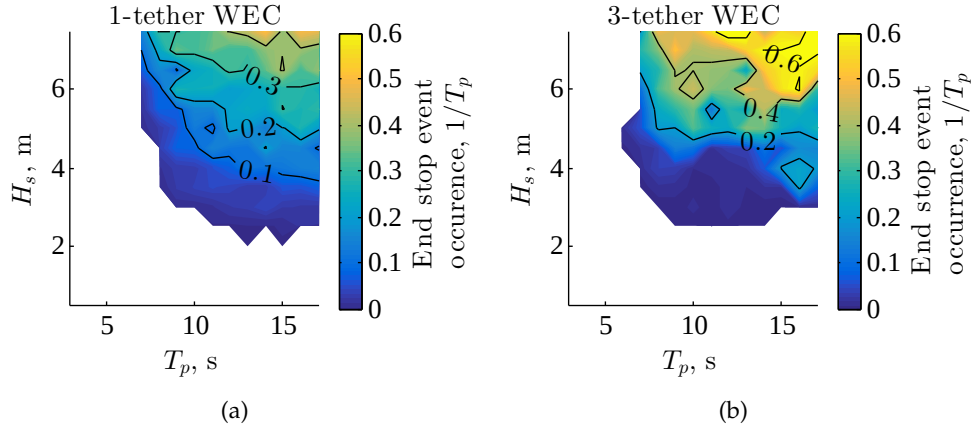


Figure 6.13: Occurrence of the end stop events for the (a) single-tether and (b) three-tether WECs. For the latter case, data is shown for the tether aligned with the wave front as it is more exposed to the end-stop events.

Another situation that should be considered with caution is a slackness of tethers. Due to the positive buoyancy of the submerged converters, tethers should be under tension all the time in order to effectively transmit the buoy motion in waves into the translational or rotational motion of the PTO system. However, mooring lines can become slack because of the large wave excitation forces or additional forces exerted from the hard stop mechanisms. When the tether become loose and then re-tighten, the WEC experiences high snapping loads which may cause damage to the PTO and mooring systems. As can be seen from Figure 6.14, the three-tether converters are more exposed to such events due to the lower initial tension in each tether. So the mass-to-buoyancy ratio of the three-tether WEC should be chosen in such a way as to provide a required pre-tension force in all mooring lines.

6.4.4 Power output

The power output of both converters has been analysed in terms of the peak-to-average ratio, the contribution of individual PTO systems and different motion modes to the averaged absorbed power.

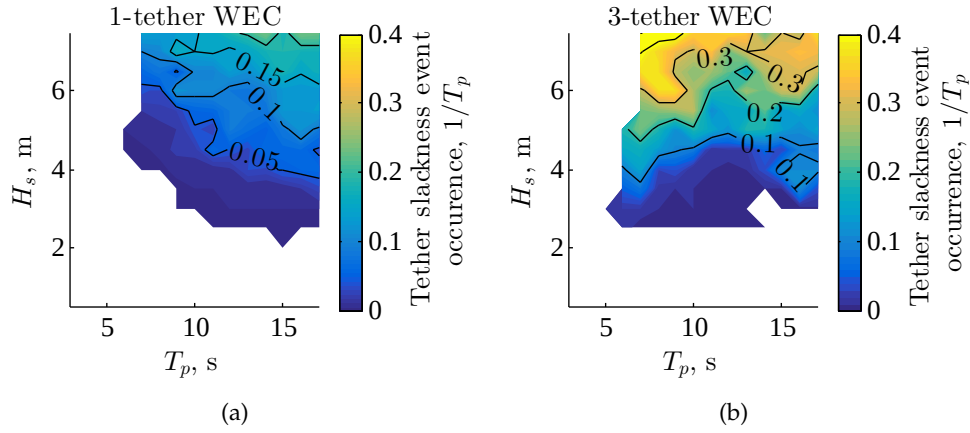


Figure 6.14: Occurrence of the tether slackness events for the (a) single-tether and (b) three-tether WECs. For the latter case, data is shown for the tether aligned with the wave front as it is more exposed to the slackness.

6.4.4.1 Power matrix

The averaged absorbed power and also statistical peak power for the single-tether and three-tether WECs are shown in Figure 6.15, where control parameters have been optimised for each sea state. The power output of the WEC attached to three tethers is approximately two times higher than that of the single-tether device across all sea states. For example, at the wave climate with $H_s = 2$ m and $T_p = 9$ sec, the power output of the converter with multiple mooring lines reaches 82 kW, while the same buoy with a single tether shows 41 kW of absorbed power. However, this factor of 2 is a feature of the particular buoy geometry considered in this study. For other shapes and aspect ratios the increase in power production can be in the range of 1.6–2.5 times [36]. The level of instantaneous power for both converters is approximately 6–10 times higher than the mean power value, meaning that the designed capacity of the PTO should be an order of magnitude higher than the expected average energy output.

6.4.4.2 Power per PTO system

As stated, the power take-off for the three-tether WEC can be implemented in several ways. For the case of individual PTO units for each mooring line, they make different contribution to the total power output. The tether arrangement shown in Figure 6.3 demonstrates only one extreme case when tether 1 is aligned with the direction of wave propagation. For this config-

6.4 Comparison of performance between single- and three-tether buoys

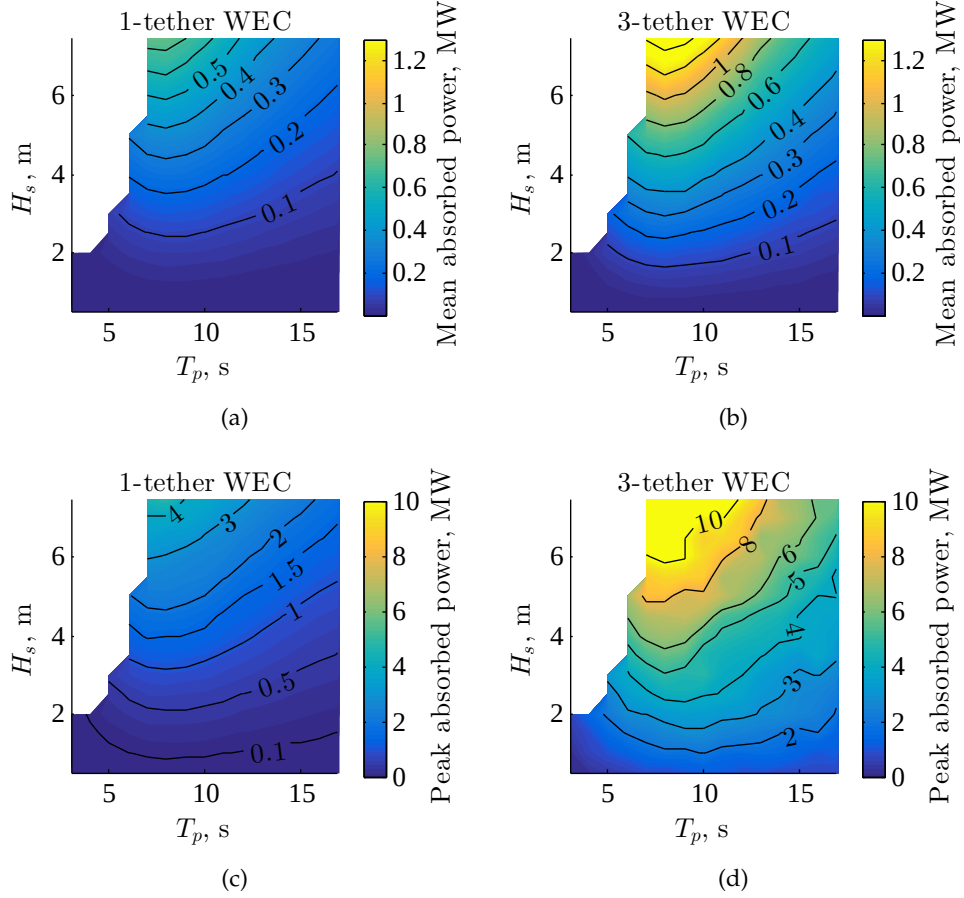


Figure 6.15: Power matrices for the single-tether and three-tether WECs: top plots correspond to the mean absorbed power, while plots on the bottom show the statistical peak values. The PTO parameters have been optimised for each sea state.

uration, the PTO attached to tether 1 generates approximately 45% of the total mean power for the most sea states as shown on Figure 6.16a. The other two PTOs convert the rest of the energy with an equal share due to the symmetrical arrangement of the tethers arrangement with respect to y -axis. Thus, depending on the wave direction statistics, the first PTO system may be designed to be larger than the other two. Another extreme situation is when one of tethers operates perpendicular to the wave front with almost zero power output. However, according to the preliminary results in frequency domain [35], the total power output of the WEC is largely unaffected by wave direction relative to the arrangement of the mooring lines.

The comparison of power absorption between both converters is demon-

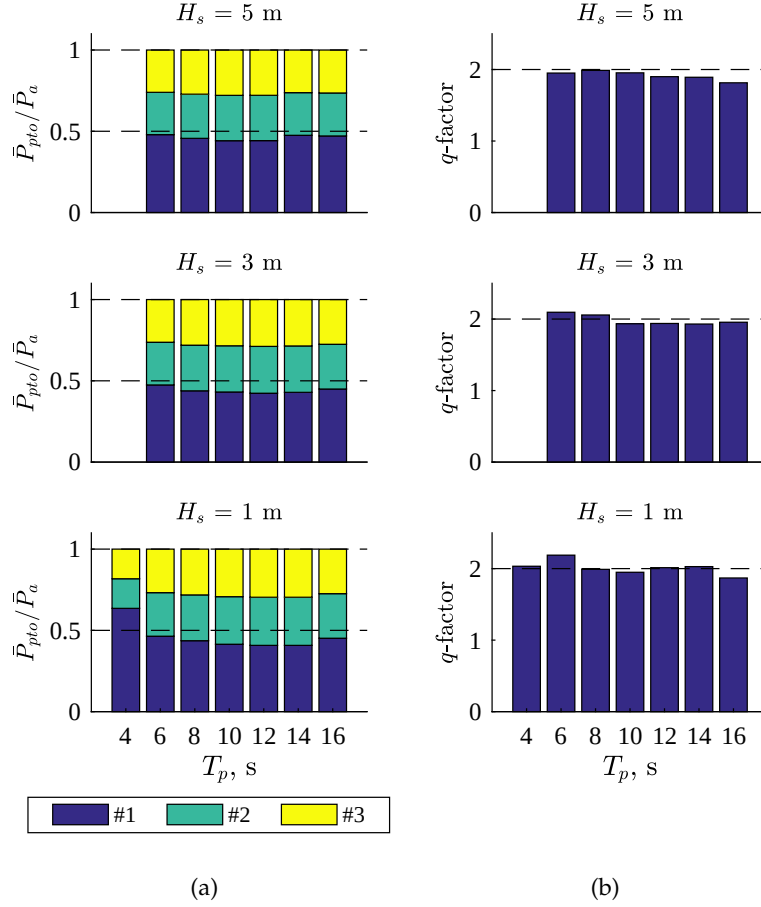


Figure 6.16: (a) Contribution of each power take-off system ($\bar{P}_{pto}$) to the total power absorption ($\bar{P}_a$) of the three-tether WEC for the selection of sea states; (b) q -factor as an efficiency improvement ratio between power absorption from the three-tether and the single-tether WECs.

strated in Figure 6.16b, showing that the three-tether WEC can generate approximately 1.8–2.2 times more power than its single-tether counterpart. To explain the main reason why the improvement is only twofold and not three times as derived in [27], the Budal diagram (as cited in [15]) for the single- and three-tether WECs is shown in Figure 6.17a assuming optimal control at each wave period. The blue area on the plot corresponds to the radiation limit where buoy displacements are not constrained and power production is only governed by the mode of oscillation. As a result, the maximum increase in the WEC efficiency can be up to three times if both heave and surge contribute to the power take-off. The red area in Figure 6.17a shows the

6.4 Comparison of performance between single- and three-tether buoys

range of wave periods where the buoy displacements reach constraints and power absorption limits depend on the buoy shape and maximum allowed displacements. The q -factor in Figure 6.17b corresponds to the increase of the power production for a three-tether case when compared with its single tether counterpart. Thus, its values vary between 1.85 and 2.7 depending on the wave period. As the irregular waves are composed from the number of waves with different frequencies, the overall effect of having three tethers (quality factor) is averaged to the level of 2–2.2 as shown in Figure 6.16.

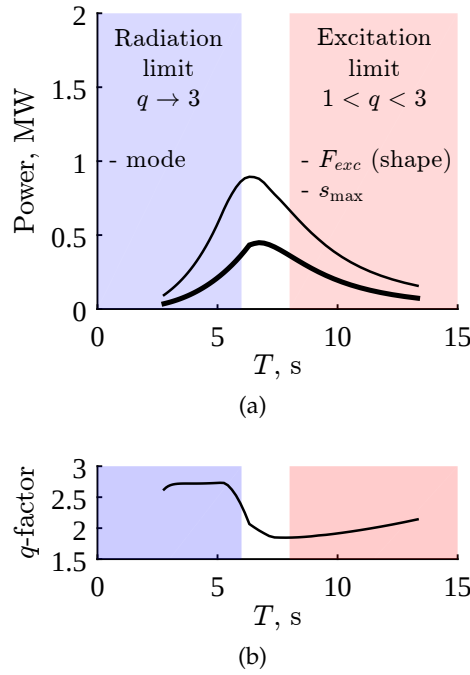


Figure 6.17: Comparison of power absorption between a single-tether and three-tether WECs in regular waves of 1 m amplitude assuming optimal control: (a) power levels (b) q -factor as a ratio between power absorption from the three-tether and the single-tether WECs.

6.4.4.3 Power per mode

Since the main idea of the application of three mooring lines is to capture power from the buoy motion in surge, it is important to show how much energy is converted from each motion mode. The power contribution from one mode is calculated as the difference between excitation power, radiated power and viscous losses:

$$\bar{P}_{mode} = \bar{P}_{exc} - \bar{P}_{rad} - \bar{P}_{vis}, \quad (6.23)$$

where $\bar{P}$ denotes the time average power.

The relative contribution from surge, heave and pitch modes is shown in Figure 6.18 for the range of sea states. As expected for the single-tether WEC, the heave motion provides around 90–95% of the total power, while the remaining 5% are produced by surge. Interestingly, the buoy oscillations in pitch have a negative impact on the power production, which is indicated by the bars placed above 1 on the plot. This loss is presumably due to extremely poor coupling between pitch motion of the buoy and the PTO, so useful work is minimal, whereas losses due to radiation and drag remain high. In contrast, in the three-tether configuration pitch has a negligible effect on the WEC performance in terms of the absorbed power. In general, slightly more than half of the energy (50–55%) is generated from the heave while the remaining 45–50% are produced from surge. The contribution from surge decreases by several percent with an increasing wave height. Interestingly, the three-tether WEC demonstrates slightly higher (by 5%) efficiency of the heave mode as compared to the single-tether device which is indicated by the numbers on bars. This may be caused by the different arrangements of motion envelopes as shown in Figure 6.8.

6.5 System sensitivity

In this section the sensitivity of the power output to changes of some geometric parameters is investigated for the three-tether WEC for the Yeu site.

6.5.1 Buoy mass

The mass ratio of the buoy plays an important role in the design of the bottom-referenced WEC as it governs the loading on the mooring system affecting dimensions of the foundation and, therefore, the total cost of the converter. Thus, the sensitivity of the power output to the mass of the three-tether WEC is demonstrated in Figure 6.19a. Control parameters have been optimised for each mass ratio presented.

As a result, more energy can be generated using lighter buoy hulls, while the variation in mass by $\pm 40\%$ from the default value of $m_b/m_w = 0.5$ results in $[-10, +5]\%$ change in power absorption. However, such relatively small gain (5%) in the averaged power is also associated with a significant increase in loading on the mooring system by 1.4 times.

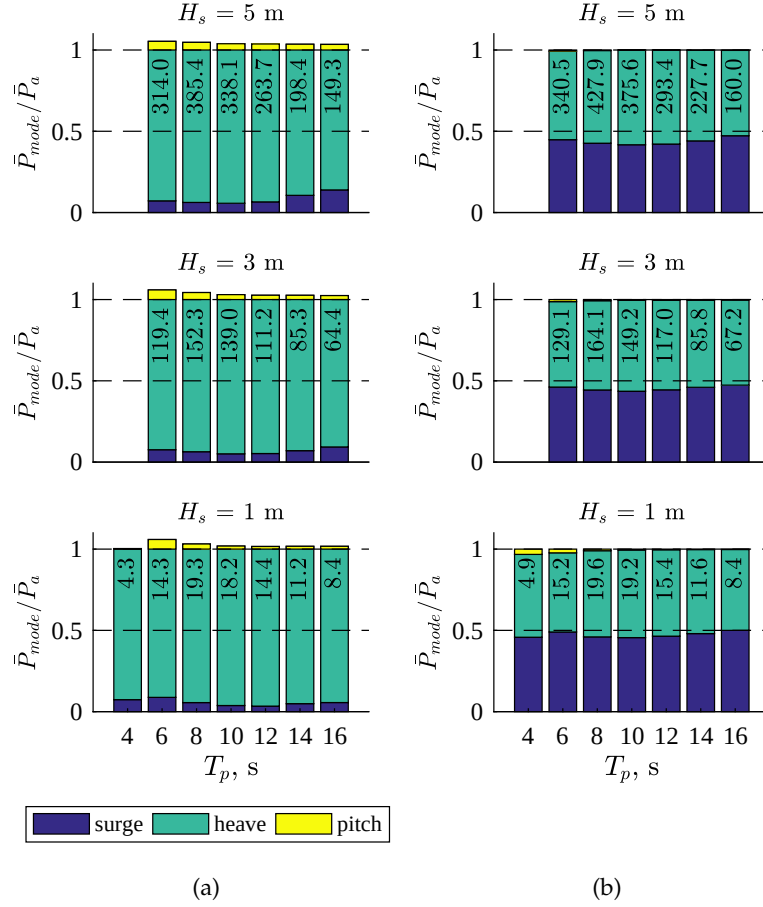


Figure 6.18: Contribution of modes of oscillation to the total power absorption of the (a) single-tether (b) three-tether WECs. Numbers on bars indicate the absolute value of absorbed power by heave only.

6.5.2 Tether stroke

The length of the tether stroke governs the motion envelope of the converter and the frequency of contacting end stop mechanisms. An influence of the tether stroke length on the power absorption abilities of the three-tether WEC is shown in Figure 6.19b. Control parameters have been optimised for each value of the tether stroke over the range of sea states. It is interesting to note, that a change in the stroke by several metres corresponds to a marginal increase in the averaged power (only 2%). However, it significantly affects the frequency of the end stop events and the value of 8 m (± 4 m) seems to be the most appropriate for the current buoy design. However, the choice of the

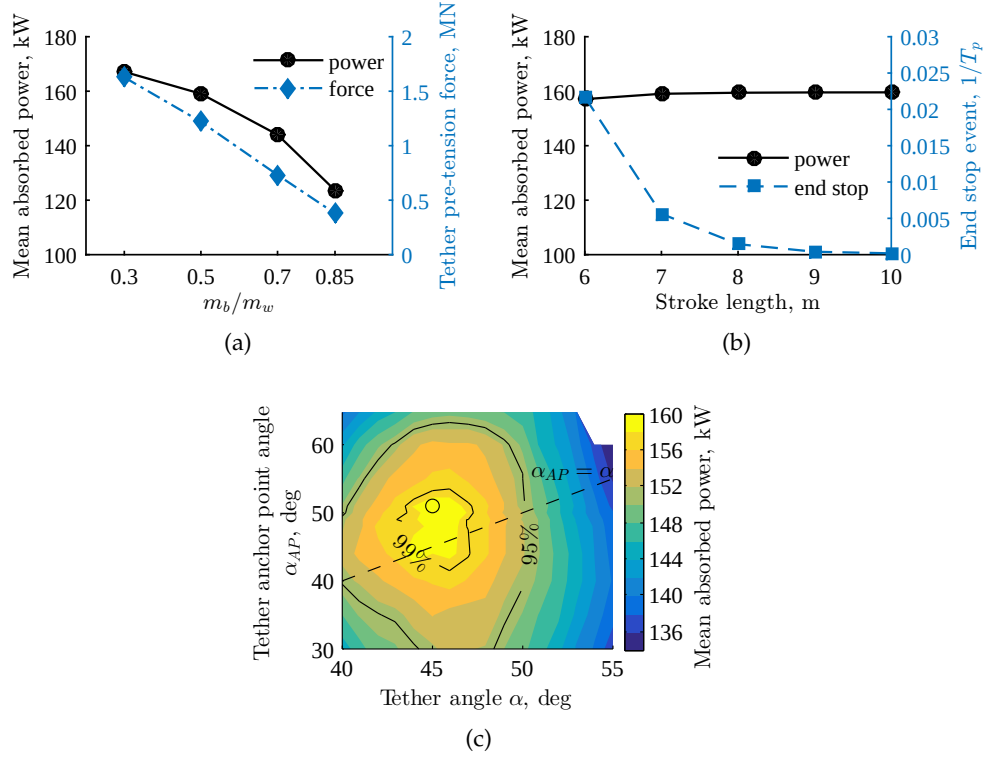


Figure 6.19: Sensitivity of the three-tether WEC power output to: (a) the mass of the buoy hull, (b) the length of the tether stroke, and (c) the tether arrangement (the black contour lines show 95% and 99% confidence intervals and the dashed line corresponds to the case when $\alpha_{AP} = \alpha$).

maximum tether stroke is generally governed not by the operational sea states, but by the extreme wave conditions and requires further investigation using more accurate numerical or experimental methods [31]. Another approach is to employ control algorithms that keep displacements of the buoy within pre-set boundary values without engagement with end-stops [17].

6.5.3 Tether angle

The spatial arrangement of tethers is determined by (i) their inclination angle to the vertical, and (ii) the anchor point of each tether to the buoy hull. In this work it is assumed that all tethers point towards the geometric centre of the buoy as shown in Figure 6.20a. Such configuration minimises an influence of the power take-off system on the angular motion of the WEC. However, when the mooring lines are attached at the larger radius as shown in Figure 6.20b, the kinematic coupling between the angular motion of the buoy and change

in the tether length increases.

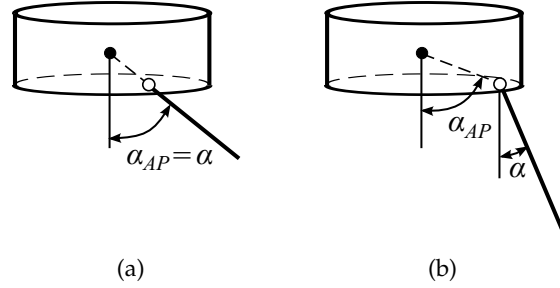


Figure 6.20: Examples of the tether attachment points on the buoy hull: (a) tethers point towards the centre of mass of the buoy, (b) tethers are attached at the larger radius. α corresponds to the tether inclination angle to the vertical, α_{AP} is the angle of the tether anchor point relative to the geometric centre of the buoy.

The performance of the three-tether WEC has been analysed with a range of tether angles ($\alpha = 40 - 50^\circ$) and the tether anchor point angles ($\alpha_{AP} = 30 - 65^\circ$) with optimal PTO settings for each sea state. The sensitivity of the mean absorbed power to the tether arrangement is shown in Figure 6.19c, where the black dot corresponds to the maximum power output at the Yeu site, black contour lines show 95% and 99% confidence intervals and the dashed line demonstrates the case when $\alpha_{AP} = \alpha$. It can be seen from the plot that it is not necessary to have all tethers point towards the geometric centre of the buoy, and a slight offset of the anchor point to a larger radius on the buoy hull increases power generation of the three-tether WEC. Thus, the angles that maximise the absorbed power are $\alpha = 45^\circ$ and $\alpha_{AP} = 51^\circ$. However, the change of both angles by $\pm 5^\circ$ from its optimal value will only result in 5% reduction in power.

6.6 Modelling uncertainties

The numerical modelling of the WEC in this study is associated with a number of assumptions including:

- (i) hydrodynamic parameters are obtained using linear wave theory;
- (ii) viscous drag coefficients are taken from published literature;
- (iii) control parameters are optimised for each sea state.

Uncertainties associated with the first assumption can be quantified only conducting an experimental study or using numerical results from the computational fluid dynamics simulations [32], while the last two can be evaluated by varying the default value of a particular parameter.

6.6.1 Viscous losses

The averaged absorbed power of the three-tether WEC has been calculated for the range of viscous drag coefficients and presented in Figure 6.21. The horizontal axis corresponds to the change of the drag coefficients relative to its nominal (default) value used in this study. Thus, the matrices $\mathbf{C}_d$ and $\mathbf{b}_Q$ from Equations (6.11) and (6.12) have been multiplied by the corresponding scale factor. Control parameters have been optimised for each value of the drag coefficient.

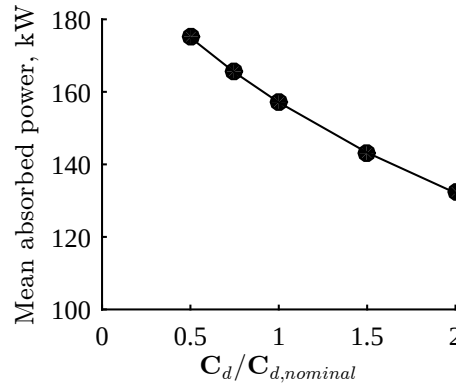


Figure 6.21: The averaged absorbed power of the three-tether WEC depending on the modelling of the viscous drag coefficient.

As a result, uncertainties of 50% associated with a modelling of viscous losses result in power production uncertainties of $[-16, +11]\%$.

6.6.2 Control

In this work it has been assumed that the PTO systems have a linear behaviour and control parameters have been optimised for each sea state. In order to quantify uncertainties associated with this, the performance of the WEC has been evaluated using different sets of PTO parameters and presented in Figure 6.22.

As a result, uncertainties associated with the implementation of control are $[-22, 0]\%$.

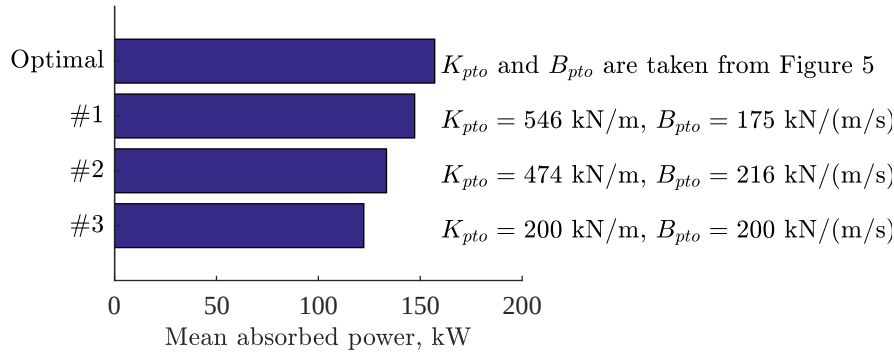


Figure 6.22: The averaged absorbed power of the three-tether WEC depending on the settings of control parameters. Set #1 corresponds to the parameters that are optimal for $H_s = 1$ m and $T_p = 7$ s; set #2 is optimal for $H_s = 2.5$ m and $T_p = 8$ s; and set #3 represent randomly chosen parameters within the range of operational values.

The overall uncertainty caused by the modelling assumptions is $[-27, 11]\%$. However, for the WEC at such a low development stage it is more common to assume errors within $[-40, 40]\%$.

6.7 Economic assessment

In this section two different approaches are used to assess the practicality of the three-tether WEC.

6.7.1 Cost breakdown and scaling to the three-tether WEC

Whilst there is an increase in the power production from the three-tether WEC, there is also an increase in the total capital cost of the converter due to the additional power take-off machinery and mooring systems. Thus, using the cost breakdown of the submerged heaving buoy, each individual component should be scaled accordingly to the 3-tether WEC, leading to the total capital cost scaling factor. Despite the fact that wave companies are reluctant to share detailed information associated with a breakdown of costs, there is a number of publications which can be used as a basis for capital expenditure (CapEx) breakdown of the wave energy converters. The data is summarised in Table 6.2 specifying the source of information, date of publication and the prototype considered. For the cases, where the levelised cost of electricity (LCoE) has been presented including CapEx and OpEx

(operating expenditure), the components of CapEx have been isolated and their contribution re-calculated in order to match the items in Table 6.2.

Table 6.2: Review of the existing CapEx cost breakdown models.

	[6]	[13]	[40]	[28]	[26]	[26]
	Farm	Farm	Farm	1 unit	1 unit	Farm
	Pelamis	N/A	Two-body FPA	N/A	Two-body FPA	Two-body FPA
Device structural components	80	85	100	90.3	32.9	60.2
Structure	46.6	27	30.5	50	21.4	36.4
Foundation/mooring	26.7	5	24	25	2.9	7.7
Control/instrument	5	-	23	-	-	-
Power take-off	-	49	22.5	15.3	2.9	8.1
Grid connection	1.7	4	-	-	5.7	7.9
Installation	5	13	-	9.7	35.7	15.3
Installation surveys	-	-	-	-	1.4	0.5
Installation of structure	-	-	-	-	1.6	0.7
Installation of mooring	-	-	-	-	19.5	8.4
Installation of grid connection	-	-	-	-	13.3	5.7
Commissioning	-	-	-	-	1.4	4.2
Management and others	15	2	-	-	30	20.3
Total	100	100	100	100	100	100
Capital Cost Economic Scaling	1.55	2.32	1.93	1.95	1.56	1.56
Relative Efficiency	1.29	0.86	1.04	1.02	1.28	1.28

Despite the fact that CapEx models presented in Table 6.2 vary significantly, the relative efficiency of the three-tether WEC is calculated assuming that each cost-breakdown represents the single-tether case. The following steps are undertaken in order to estimate how the cost of the three-tether WEC differs from the single-tether device due to the additional power take-off mechanisms and mooring points:

- (i) apply a CapEx model;
- (ii) scale each item for the three-tether WEC according to Table 6.3. Scaling factors are chosen such that in the most unfavourable case the cost of PTO, foundation and its installation are tripled, while the grid connection has a factor of 2 due to the doubling of the power output;
- (iii) calculate the total capital cost economic scaling factor;
- (iv) estimate the relative efficiency of the three-tether converter as a ratio between the power production gain and the associated increase in the WEC cost.

Table 6.3: Example of scaling CapEx components for the three-tether WEC using data from [26] for 1 unit.

CapEx components	Weighting of total (%)	3-tether WEC scaling	Weighted scaling factor
Structure	21.4	1	0.21
Foundation/mooring	2.9	3	0.09
Control/instrument	-	1	0.00
Power take-off	2.9	3	0.09
Grid connection	5.7	2	0.11
Installation surveys	1.4	1	0.01
Installation of structure	1.6	1	0.02
Installation of mooring	19.5	3	0.58
Installation of grid	13.3	1	0.13
Commissioning	1.4	1	0.01
Management	30	1	0.30
Capital Cost Economic Scaling			1.56
Relative efficiency = $\frac{\text{Performance factor}}{\text{Capital cost factor}} = \frac{2}{1.56}$			1.28

The resultant data for the three-tether WEC using each CapEx model are added to the bottom of Table 6.2.

Three out of six models [6, 26] show that the three-tether WEC is more beneficial from the economic point of view as compared to its single-tether counterpart. Another two [28, 40] demonstrate that there is only a marginal advantage of having multiple mooring lines when using the conservative scaling factor in Table 6.3. According to the cost breakdown presented by Carbon Trust [13], it seems unprofitable to invest in the three-tether WECs. This negative result is associated with a fairly large proportion of the PTO system (49%) in the total cost of the converter. A simple calculation shows that if the power take-off and mooring systems and their installation make up more than half the total cost of the WEC with one tether, then the three-tether converter would become uneconomical to manufacture.

As the CapEx breakdown models that are publicly available differ greatly from case to case, from country to country and also depending on the number of WECs involved in the farm, it is very difficult to draw a definitive conclusion on the three-tether WEC. However, it is important to note that even with very conservative scaling factors, the three-tether WEC appears to outperform the single-tether WEC.

6.7.2 Cost-related metrics

Data in Table 6.2 clearly show that there is a lot of unknowns and uncertainties associated with the financial side of the wave energy production. Therefore, to simplify the analysis of any WEC it has been suggested to use several cost-related metrics [5], arguing that the significant mass or wetted surface can indirectly represent the total cost of the WEC. These parameters take into account the buoy hull and the foundation for the WEC installation. The most important criteria are:

- (i) relative capture width (hydrodynamic efficiency);
- (ii) annual energy output per significant mass;
- (iii) annual energy output per significant wetted surface;
- (iv) annual energy output per significant tether force.

Following a similar approach and assuming that the three-tether and the single-tether converters have piled foundations, the comparison of performance measures is demonstrated in Table 6.4. Furthermore, other parameters specific for the taut-moored WECs, such as the end stop impact events and also slack tether events, have been added to the analysis.

In order to calculate the significant mass and wetted surface it has been assumed that the single-tether WEC has the same foundation as the CETO-5 system. According to images found in [7], the parameters of the pile used for CETO-5 are: length is 20 m, diameter is 2 m and the wall thickness is approximately 0.15 m. Taking these dimensions as a reference and assuming that piles are manufactured from steel, the mass of foundation for the single-tether WEC can be estimated to be around 150 t and the surface area to be 125 m² leading to the total significant mass of 420 t and the total wetted surface of around 505 m². As stated in Section 6.4.2, piles for the three-tether WEC should be smaller (per unit force) due to the distributed loading between three mooring points. Thus, it is assumed that the mass and wetted surface of each pile for the multi-tether system is half of those manufactured for CETO-5. Therefore, the significant mass of the three-tether WEC is assumed to be 496 t and the total wetted surface is 569 m³.

According to data presented in Table 6.4 the only drawback associated with the three-tether WEC is the frequency of the end stop impact events and chances of slack tethers. This issue may be addressed extending the tether stroke as shown in Section 6.5.2 and applying advanced control strategies in order to keep the motion of the buoy within desired boundaries. Other

Table 6.4: Cost-related indices for the single-tether and three-tether WECs calculated for the Yeu sea site.

Parameter	Notation	1-tether WEC	3-tether WEC
Significant mass, t	m_{Σ}	420	496
Significant wetted surface, m ²	A_{wet}	505	569
Mean power, kW		79.6	159.1
Capture width, m		3.12	6.24
Relative capture width		0.28	0.57
Mooring watch circle, m		1.89	1.93
Peak tether force, MN		3.49	1.81
Significant tether force, MN	F_t	2.67	1.25
End stop impact events		0.009	0.022
Slack tether events		0.007	0.014
E_y/m_{Σ} , kWh/kg		1.66	2.81
E_y/A_{wet} , kWh/m ²		1.38	2.45
E_y/F_t , kWh/N		0.26	1.11

techno-economic metrics indicate that the three-tether WEC has a potential to double the power output as well as to produce cheaper electricity as compared to the same buoy with a single-tether configuration.

6.7.3 ACE

Another complex metric called Average Climate Capture Width per Characteristic Capital Expenditure (ACE) has been offered recently for assessing WECs at a low technology readiness level [42]. Characteristic capital expenditure can be calculated using the following formula:

$$\begin{aligned} \text{Characteristic Capital Expenditure}(\$) = & \\ & \text{Total Surface Area (m}^2\text{)} \times \text{Structural thickness (m)} \times \\ & \text{Material Density (kg/m}^3\text{)} \times \text{Cost of Material (\$/kg)}. \end{aligned} \quad (6.24)$$

In [10], the ACE index is calculated for the range of WECs using the price of steel as 0.615€/kg and the density of steel as 7900 kg/m³. In order to be consistent with [10] and to make a comparison of the three-tether device with other WEC prototypes, the same values are used in this paper. Taking data from Table 6.4, ACE can be calculated as:

$$\begin{aligned} ACE_{1tether} &= \frac{3.12 \text{ m}}{420 \cdot 10^3 \text{ kg} \times 0.615\text{€/kg}} = 12.08\text{m/M€} \\ ACE_{3tether} &= \frac{6.24 \text{ m}}{496 \cdot 10^3 \text{ kg} \times 0.615\text{€/kg}} = 20.46\text{m/M€} \end{aligned}$$

As ACE is correlated with the performance index ‘annual power output per significant mass’, the higher ACE is obtained for the three-tether WEC. Moreover, the comparison of this metric has been performed for the range of converters (refer to [10]), and it appears that the three-tether WEC demonstrates the second best ACE out of 9 WECs included in the study in [10].

6.8 Conclusion

The performance of the submerged three-tether WEC has been analysed in irregular waves using the state-of-the-art methods. The submerged three-tether WEC has a potential to absorb twice as much power as the submerged heaving buoy while reducing load on the mooring system. Due to the additional constraints from the multi-tether configuration, the buoy experiences much less displacement in the horizontal direction. Moreover, surge is effectively coupled to the power take-off system and contributes the same amount of power as heave. The economic assessment also demonstrated that despite doubling the power, the three-tether WEC can produce cheaper electricity as compared to the same buoy with a single-tether configuration.

Acknowledgement

This research has been supported by the Australian Government Research Training Program Scholarship and by the Australian Research Council (Linkage Project LP130100117). This work has been conducted using supercomputing resources provided by the Phoenix HPC service at the University of Adelaide.

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Chapter 7

Control of the three-tether converter

To this point, all calculations and analysis of a three-tether WEC have been performed assuming a linear behaviour of the power take-off machinery and sea-state-based tuning of control parameters (the stiffness and damping coefficients of the control force). This choice of PTO model and tuning method allows one to abstract from the performance of the control system and focus on the hydrodynamic properties of the wave energy converter. However, the power absorption of the converter can be significantly increased when more efficient control algorithms are implemented in the power take-off machinery. As a result, the main purpose of this chapter is to explore ways to implement the effective causal controller [1], developed for the floating heaving WEC, to the multi-mode converter, while addressing the following research question: *What factors influence the practical implementation of control strategies on a fully submerged three-tether WEC?*

The chapter consists of two sections composed of one published conference paper and one journal article submitted for publication. The main focus of Section 7.1 is given to the design of the low-level control loop and to the realisation of motion constraints. The sensitivity of the controller to the modelling uncertainties and challenges associated with its practical applications are investigated in Section 7.2.

7.1 IMC-based velocity tracking control

This section consists of the published conference paper:

Sergiienko, N. Y., Cazzolato, B. S., Hardy, P., Arjomandi, M. and Ding, B. (2017). “Internal-model-based velocity tracking control of a submerged three-tether wave energy converter”. In: *Proceedings of the 12th European Wave and Tidal Energy Conference*. Ed. by A. Lewis. Cork, Ireland, pp. 1126-1-1126-8.

The paper in its published format is available at <https://mecheng.adelaide.edu.au/avc/publications/abstract.php?abstract=1312>.

Statement of Authorship

Title of Paper	Internal-model-based velocity tracking control of a submerged three-tether wave energy converter
Publication Status	☒ Published ☐ Accepted for Publication ☐ Submitted for Publication ☐ Unpublished and Unsubmitted work written in manuscript style
Publication Details	Sergiienko, N. Y., Cazzolato, B. S., Hardy, P., Arjomandi, M. and Ding, B. (2017). "Internal-model-based velocity tracking control of a submerged three-tether wave energy converter", in A Lewis (ed.), <i>Proceedings of the 12th European Wave and Tidal Energy Conference</i> , Cork, pp. 1126-1–1126-8.

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Overall percentage (%)	80%		
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.		
Signature		Date	05/02/2018

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

- i. the candidate's stated contribution to the publication is accurate (as detailed above);
- ii. permission is granted for the candidate to include the publication in the thesis; and
- iii. the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

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Internal-model-based velocity tracking control of a submerged three-tether wave energy converter

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Abstract

Control algorithms play an important role in the design of wave farms and individual wave energy converters (WECs). Advanced control systems can significantly increase the performance of any oscillating WEC, while also improving the investment attractiveness of the entire wave energy industry. Whilst the majority of effective controllers have been tested in simulations, many cannot be implemented in real life as they heavily rely on future (predicted) knowledge of the incident wavefront. In order to overcome this problem, several causal control strategies have been developed to maximise power output of the WEC. One such example is a simple and effective real-time controller based on the concept of the internal model control (IMC), where the performance of the WEC is tuned to the dominant wave frequency from the spectrum. This controller has been previously applied to the floating heaving buoy with one degree of freedom. In the current study, the IMC approach is extended to the three-tether submerged WEC, where several motion modes contribute to power generation. The performance of the controller is compared with another existing acausal solution in order to quantify its effectiveness.

7.1.1 Introduction

A control system is an integral part of any wave energy converter with the main objective to achieve the desired motion of the buoy in response to the incident wavefront. Control is carried out through the power take-off (PTO) machinery which exerts the load force on the WEC. Such important parameters of the WEC as its size, shape and capacity of the PTO system directly depend on the selected control strategy [6, 11]. In addition, advanced control solutions can significantly increase the performance of any oscillating WEC [5], while improving the investment attractiveness for the entire wave energy industry.

In order to maximise the amount of absorbed power, the converter velocity should satisfy two optimality conditions: (i) to be in phase with a

wave excitation force and (ii) to have an optimal amplitude [5]. The control algorithm that demonstrates the performance closest to optimal is a model-predictive control (MPC) suggested in [14, 19]. The main difficulty of optimal control solutions for WECs is their non-causality where the current value of the required load force depends on the future knowledge of the buoy behaviour (velocities, forces). In addition, the MPC relies heavily on the WEC model and its efficiency deteriorates significantly in the presence of modelling uncertainties [9].

As a result, it is desirable to design a controller which is causal (independent of future inputs), robust to uncertainty (insensitive to the modelling errors) and close to optimal. The simplest control strategy which satisfies first two conditions is a spring-damper control, sometimes referred to as an approximate complex conjugate control [13]. The load force, in this case, is set proportional to the displacement and velocity of the buoy where two proportionality constants act as control parameters. The tuning of such controller is mostly performed off-line and parameters can be changed depending on the current sea state. The spring-damper controller is easy to implement in practice, but its performance is suboptimal in irregular waves.

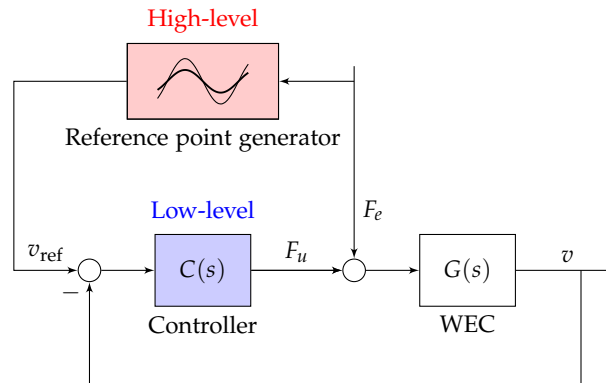


Figure 7.1: Block-diagram of the velocity tracking control, where v_{ref} and v are the reference and actual buoy velocities respectively, F_u is a control force and F_e is a wave excitation force.

Another example of a causal and robust strategy is a velocity control which operates on the wave-by-wave basis, with the main objective to keep the buoy velocity in phase with the wave excitation force. This approach does not require a prediction of the future wave force acting on the WEC, but the knowledge of its current value is essential. As shown in Figure 7.1, the velocity tracking controller usually consists of two loops: (i) a high level sets the reference (desired) velocity of the buoy and (ii) a low level provides the

necessary machinery (power take-off) force to achieve this velocity.

The implementation of the high-level control is mostly limited to the choice of the transfer function between the estimated wave excitation force and the reference buoy velocity. Thus, a fixed value of the transfer function has been used in [13] and [17], while a frequency-dependent gain has been suggested in [9]. The latter case allows satisfying both optimal phase and amplitude conditions, while the fixed force-to-velocity function fails to reach optimal velocity amplitude [13]. A velocity tracking control loop (low level) can be designed using established approaches from the classical control theory. Thus, proportional and proportional-integral (PI) controllers were considered in [17] and [13] respectively, while the internal model control (IMC) has been implemented in [3]. The IMC has proven its robustness and applicability to the multivariable problems and is frequently used to tune PI and PID controllers [16]. In addition, it can utilise adaptive algorithms if the system model is unknown [21].

The velocity tracking controller based on the IMC has demonstrated its simplicity and effectiveness [9] in power absorption, but it has been only tested on the floating buoys with a constrained motion in heave. However, the submerged three-tether WEC shown in Figure 7.2 should be controlled in surge, sway and heave in order to achieve a maximum power absorption. Therefore, in this study, the IMC concept is extended to the WEC with multiple degrees of freedom using the multiloop approach. This paper is based on linear wave theory assumptions where buoy displacements are of small amplitude. Even though the performance of this type of WEC may depend on the direction of wave propagation, only one wave angle is considered for now.

The WEC model used for numerical simulations is derived in Section 7.1.2. The controller is designed in Section 7.1.3 including the high- and low-level control loops and constraints handling. The performance of the suggested control strategy is tested and compared against the spring-damper controller in Section 7.1.4.

7.1.2 Wave energy converter model

7.1.2.1 System parameters

The WEC considered in this study is a submerged cylindrical buoy connected to three power generators by flexible tethers (see Figure 7.2). Such a configuration of the PTO allows the system to extract power from the buoy oscillations in heave, surge and sway motions simultaneously. Parameters of this WEC are specified in Table 7.1.

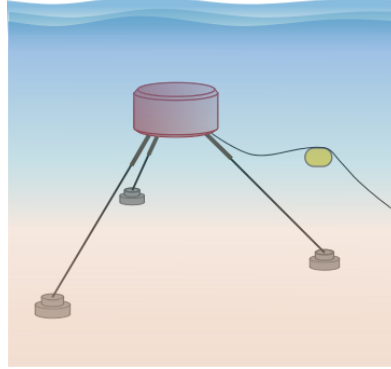


Figure 7.2: Schematic of the three-tether WEC.

Table 7.1: Parameters of the WEC.

Parameter	Value
Shape	vertical cylinder
Radius of the cylinder, a	5.5 m
Height of the cylinder, h_c	5.5 m
Water depth	50 m
Submergence depth, d_s (centre of the buoy)	6.5 m
Mass of the buoy, m_b	268 t
Moments of inertia:	
$I_x = I_y$	2709 t·m ²
I_z	4064 t·m ²
Tether inclination angle from the vertical, α	45°

7.1.2.2 Equation of motion

The wave-to-wire model employed in this study is based on linear wave theory, assuming small motion amplitudes of the buoy as compared to the length of the mooring lines (tethers). The only second-order hydrodynamic effect included in the model is a viscous drag force which is proportional to the square of the body velocity relative to the fluid. The full derivation of the equation of motion can be found in [24, 26], and only key points will be outlined in this paper.

The location of the buoy in a reference (Cartesian) coordinate frame can be defined by the position vector $\mathbf{x} \in \mathbb{R}^{6 \times 1}$. The buoy is connected to three tethers that are represented by a vector of tether length variables $\mathbf{q} = [l_1 \ l_2 \ l_3]^T$. The relationship between the buoy velocities and the rate of change of the tether length can be described by:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{x}}, \quad (7.1)$$

where $\mathbf{J}^{-1}$ is the inverse kinematic Jacobian [25]. Equation (7.1) represents

the kinematics equation of the three-tether WEC.

To derive the full nonlinear dynamic equation of the converter, the following forces are considered in this study:

- (i) the buoy inertia force $\mathbf{M}\ddot{\mathbf{x}}$, where a mass matrix is $\mathbf{M} = \text{diag}([m_b \ m_b \ m_b \ I_x \ I_y \ I_z])$;
- (ii) the buoyancy force $\mathbf{F}_{buoy} = [0 \ 0 \ (m_w - m_b)g \ 0 \ 0 \ 0]^T$, where m_w is the mass of the displaced water and m_b is the mass of the WEC;
- (iii) the tether force is modelled as a linear superposition of:
 - initial pretension force $F_{t0} = -\frac{m_w - m_b}{3 \cos \alpha}$,
 - control force $F_{u,i}$,
 - artificially introduced spring force that tunes the buoy resonance frequency $F_{s,i} = -K_t \Delta l_i$, $K_t = 500$ kN/m. This force can be implemented as a physical spring with a corresponding stiffness coefficient.

As a result, the tension in each tether is:

$$F_{t,i} = F_{u,i} + F_{t0} + F_{s,i}, \quad i = 1 \dots 3, \quad (7.2)$$

and the generalised tether force in the Cartesian coordinate frame is:

$$\mathbf{F}_{tens} = \mathbf{J}^{-T} \mathbf{F}_t, \quad (7.3)$$

where $\mathbf{F}_t = [F_{t,1} \ F_{t,2} \ F_{t,3}]^T$ and $\mathbf{J}^{-T}$ is the transposed inverse kinematic Jacobian.

- (iv) the wave excitation force $\mathbf{F}_{exc}$. The frequency dependent force amplitude and phase are obtained using WAMIT [18] and the force time series is generated using the wave elevation spectrum (see [12]);
- (v) radiation force $\mathbf{F}_{rad}$. The frequency dependent force amplitude and phase are obtained using WAMIT [18] and the time domain representation has a form:

$$\mathbf{F}_{rad} = -\mathbf{A}_\infty \ddot{\mathbf{x}} - \int_0^t \mathbf{K}_{rad}(t - \tau) \dot{\mathbf{x}}(\tau) d\tau \equiv -\mathbf{A}_\infty \ddot{\mathbf{x}} - \mathbf{F}_r, \quad (7.4)$$

where $\mathbf{A}_\infty$ is the matrix with infinite frequency added mass coefficients, and $\mathbf{K}_{rad}(t)$ is a retardation function. For the cylindrical WEC that moves in all six degrees of freedom, $\mathbf{K}_{rad}(t) \in R^{6 \times 6}$ comprises nine non-zero elements, which correspond to the hydrodynamic coupling between modes. Each element of the matrix convolution integral in Equation (7.4) can be replaced by the state-space model using the Marine System Simulator toolbox [22], where a reasonable approximation of the force can be achieved by a state-space system of order four. Consequently, the augmented state-space model that describes the radiation force component $\mathbf{F}_r$ is:

$$\dot{\mathbf{p}}_r = \mathbf{A}_r \mathbf{p}_r + \mathbf{B}_r \dot{\mathbf{x}}, \quad (7.5)$$

$$\mathbf{F}_r = \mathbf{C}_r \mathbf{p}_r. \quad (7.6)$$

where $\mathbf{p}_r \in R^{36 \times 1}$ is an auxiliary vector without any physical meaning, $\mathbf{A}_r \in R^{36 \times 36}$, $\mathbf{B}_r \in R^{36 \times 6}$, and $\mathbf{C}_r \in R^{6 \times 36}$ are the state space matrices.

(vi) viscous damping force given by

$$\mathbf{F}_{visc} = -\frac{1}{2} \rho \mathbf{C}_d \mathbf{A}_d \|\dot{\mathbf{x}}\| \dot{\mathbf{x}}, \quad (7.7)$$

where $\mathbf{C}_d$ and $\mathbf{A}_d$ are the matrices of the drag coefficients and the cross-section areas of the buoy perpendicular to the direction of motion respectively. Drag coefficients are taken as $C_{d,x} = C_{d,y} = 1.1$, $C_{d,z} = 1$ according to [15].

As a result, the nonlinear equation that describes the motion of the 3-tether WEC in waves is based on the Cummins equation [2]:

$$(\mathbf{M} + \mathbf{A}_\infty) \ddot{\mathbf{x}} = \mathbf{F}_{exc} - \mathbf{F}_r + \mathbf{F}_{buoy} + \mathbf{F}_{visc} + \mathbf{F}_{tens}. \quad (7.8)$$

For the control system development, Equation (7.8) should be represented in a state-space form and, therefore, linearised.

7.1.2.3 Linearised state-space representation

Two non-linear terms are present in Equation (7.8), namely $\mathbf{F}_{visc}$ and $\mathbf{F}_{tens}$. The approximation of the viscous damping force by its linear analogue has been suggested in [7] using a Lorenz linearisation approach. Thus, $\mathbf{F}_{visc} \approx -\mathbf{B}_v \dot{\mathbf{x}}$, where the diagonal elements of the linearised damping matrix $\mathbf{B}_v \in R^{6 \times 6}$ are obtained using the iteration procedure described in [1]. However, these coefficients depend on the buoy velocity and for different wave conditions

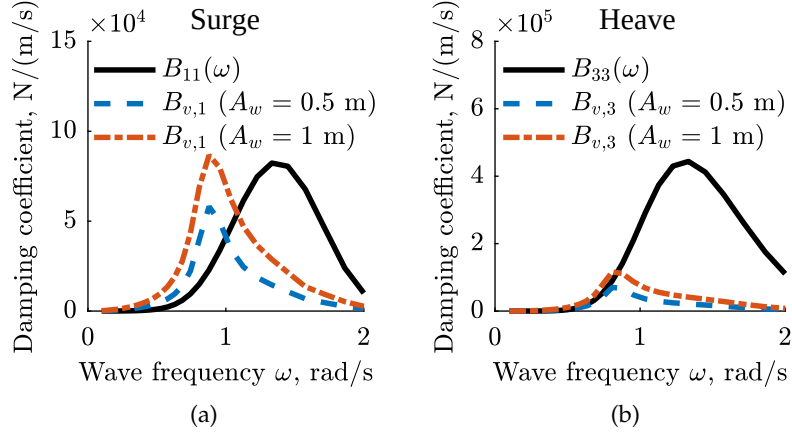


Figure 7.3: Comparison of the linearised viscous damping coefficient B_v vs. the radiation damping $B(\omega)$ for (a) surge and (b) heave motion modes of the open-loop WEC model. The linearised viscous damping is calculated for two wave amplitudes of $A_w = 0.5$ and 1 m.

their values vary. For comparison, the linearised viscous damping coefficients are obtained for the regular waves of two different amplitudes $A_w = 0.5$ and 1 m and plotted versus the radiation damping coefficient in Figure 7.3.

Even though the linearised viscous coefficient is velocity-dependant, only one value for each motion mode should be chosen depending on the range of operational velocities. Thus, $B_{v,1} = B_{v,2} = 52.5$ kN/(m/s) and $B_{v,3} = 87$ kN/(m/s) corresponding to a buoy velocity of around 2 m/s are used throughout this paper.

Nonlinearities in the generalised tether force are associated with the inverse kinematic Jacobian, which is position-dependent. So, the approximated tether force is:

$$\mathbf{F}_{tens} \approx -\mathbf{K}_t \mathbf{x} + \mathbf{J}_0^{-T} \mathbf{F}_u, \quad (7.9)$$

where $\mathbf{K}_t$ can be found in [24] and $\mathbf{J}_0^{-1}$ is the inverse kinematic Jacobian at the nominal position of the buoy.

$$\begin{aligned}
 \dot{\mathbf{X}} &= \begin{bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \\ \dot{\mathbf{p}}_r \end{bmatrix} = \mathbf{A}_{\text{WEC}} \mathbf{X} + \mathbf{B}_{\text{WEC}} \mathbf{U} \\
 &= \left[\begin{array}{cc|c} \mathbf{0}_{6 \times 6} & \mathbf{I}_{6 \times 6} & \mathbf{0}_{6 \times 36} \\ -(\mathbf{M} + \mathbf{A}_\infty)^{-1} \mathbf{K}_t & -(\mathbf{M} + \mathbf{A}_\infty)^{-1} \mathbf{B}_v & -(\mathbf{M} + \mathbf{A}_\infty)^{-1} \mathbf{C}_r \\ \hline \mathbf{0}_{36 \times 6} & \mathbf{B}_r & \mathbf{A}_r \end{array} \right] \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \\ \mathbf{p}_r \end{bmatrix} \\
 &\quad + \left[\begin{array}{c} \mathbf{0}_{6 \times 6} \\ (\mathbf{M} + \mathbf{A}_\infty)^{-1} \\ \hline \mathbf{0}_{36 \times 6} \end{array} \right] \left(\mathbf{F}_{exc} + \mathbf{J}_0^{-\top} \mathbf{F}_u \right), \\
 \mathbf{Y} = \dot{\mathbf{x}} &= \left[\begin{array}{cc|c} \mathbf{0}_{6 \times 6} & \mathbf{I}_{6 \times 6} & \mathbf{0}_{6 \times 36} \end{array} \right] \mathbf{X} = \mathbf{C}_{\text{WEC}} \mathbf{X}.
 \end{aligned} \tag{7.10}$$

The linearised state-space model of the WEC is specified in Equation (7.10), while the transfer function that corresponds to this model is:

$$\mathbf{G}(s) = \mathbf{C}_{\text{WEC}}(s\mathbf{I} - \mathbf{A}_{\text{WEC}})^{-1} \mathbf{B}_{\text{WEC}}. \tag{7.11}$$

7.1.3 Controller

7.1.3.1 High level

As stated above, the main objective of the high-level controller is to set the reference buoy velocity having measured or estimated the wave excitation force. In [17], an observer has been designed to estimate the excitation force of the floating device based on the real-time measurements of its position, velocity, acceleration and control force. In the current study, it is assumed that the wave force is ideally estimated.

In general, the optimal buoy velocity should satisfy the following equation:

$$\hat{v}_{\text{opt}}(\omega) = \frac{\hat{F}_{exc}(\omega)}{2(B(\omega) + B_v)}, \tag{7.12}$$

where $B(\omega)$ is a radiation damping coefficient. Even though the time-dependent excitation force contains the range of wave frequencies, in [9] it has been suggested to assume that the wave force is narrow-banded and can be approximated by the harmonic process with one dominant frequency:

$$F_{exc}(t) \approx f_{exc}(t) \cos(\omega(t)t + \phi(t)), \tag{7.13}$$

where $f_{exc}(t)$, $\omega(t)$ and $\phi(t)$ are the time-variant amplitude, frequency and phase respectively.

In this way it is assumed that the reference buoy velocity is [10]:

$$v_{\text{ref}}(t) = \frac{F_{\text{exc}}(t)}{H(t)}, \quad H(t) = 2(B(\omega) + B_v), \quad (7.14)$$

where ω is estimated in real time and the corresponding value of $B(\omega)$ is set using gain scheduling.

7.1.3.2 Dominant frequency identification

Identification of the dominant frequency component in the excitation force signal is obtained using the Extended Kalman filter (EKF) suggested in [8].

It is worth noting that for submerged WECs the dominant frequency of the excitation force signal does not coincide with a dominant frequency present in the wave elevation. This is due to the fact that submerged WECs act as a bandpass filter, as they are excited only by incoming waves in a limited range of frequencies, unlike floating WECs with low-pass filter properties. The bandpass characteristics of the submerged cylindrical converter are demonstrated in Figure 7.4 for the sea state of $H_s = 1$ m, $T_p = 12$ s, where $S_\eta(\omega)$ is a Pierson-Moskowitz spectrum [27] of the wave elevation (η), $|H_{Fe,\eta}(\omega)|$ is a magnitude of the transfer function from wave elevation to the heave excitation force, and $S_{Fe}(\omega) = |H_{Fe,\eta}(\omega)|^2 S_\eta(\omega)$ shows the resultant spectrum of the excitation force. It is clear from the plot that the peak frequency of the wave elevation is $\omega_p^\eta = 2\pi/12$ s = 0.52 rad/s, while the peak frequency of the excitation force is $\omega_p^{Fe} \approx 0.68$ rad/s (≈ 9 s). However, the value estimated by the EKF from the excitation force time series is 0.79 rad/s (8 s), that approximately corresponds to the energy frequency of $S_{Fe}(\omega)$ and equal to $\hat{\omega} \approx \omega_e^{Fe} = 0.858\omega_p^{Fe}$ according to [27]. The main outcome of these findings is that the buoy oscillations will always occur at the higher frequency range in relation to the incoming wavefront.

An example of the real-time estimation of the dominant wave frequency and amplitude of the heave excitation force using the EKF is shown in Figure 7.5. As explained above, the estimated frequency converges to the energy period of the excitation force spectrum.

7.1.3.3 Constraints

As shown in [9], the current controller is able to handle constraints at the stage of setting the reference velocity. Due to the limited stroke, the constraints should be imposed on the maximum elongation of each tether such as $|\Delta l_i| \leq \Delta l_{\text{max}}, \forall i$. However, the reference buoy velocities are set in the Cartesian coordinate frame and the constraints should be mapped from the

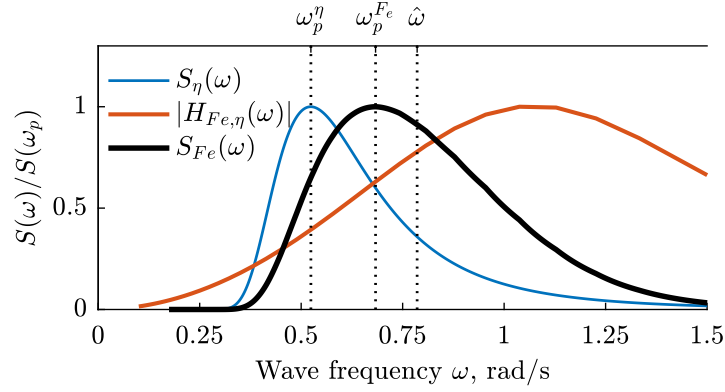


Figure 7.4: Normalised Pierson-Moskowitz [27] wave spectrum ($S_\eta(\omega)$ for $H_s = 1$ m, $T_p = 12$ s) and the corresponding heave excitation force spectrum ($S_{Fe}(\omega)$) for the submerged cylinder. $|H_{Fe,\eta}(\omega)|$ represents the excitation force transfer function, ω_p^η is a peak wave frequency, ω_p^{Fe} is a peak frequency of $S_{Fe}(\omega)$ and $\hat{\omega}$ shows the estimated dominant frequency by the EKF.

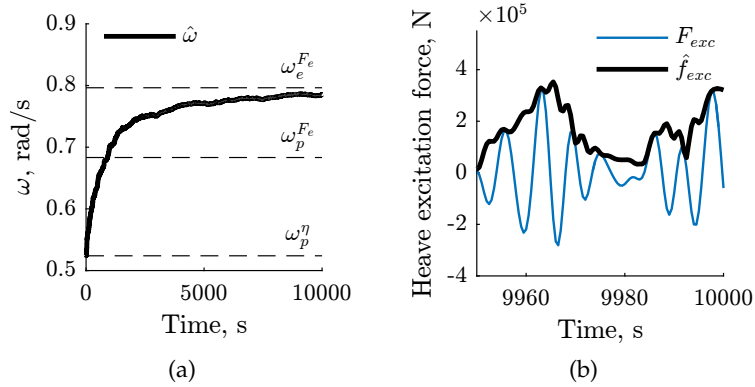


Figure 7.5: Real-time estimation of (a) the dominant frequency and (b) amplitude of the heave excitation force for the sea state of $H_s = 1$ m, $T_p = 12$ s. The convergence speed of the estimated frequency depends on the initial conditions of the EKF.

tether space to the Cartesian space. So the realistic engineering constraint for the three-tether WEC is:

$$|\mathbf{q}| \leq \Delta \mathbf{I}_{\max}. \quad (7.15)$$

Assuming the narrow-banded process with one dominant frequency, the position constraints can be re-written in terms of the velocity constraint:

$$|\dot{\mathbf{q}}| \leq \omega \Delta \mathbf{I}_{\max}. \quad (7.16)$$

A stronger global constraint can be imposed on $\dot{\mathbf{q}}$ given by [23]:

$$\dot{\mathbf{q}}^T \mathbf{\Lambda}^{-2} \dot{\mathbf{q}} / \omega^2 \leq 1, \quad (7.17)$$

where $\mathbf{\Lambda}$ is a diagonal matrix with elements $\Delta l_{\max}$. Using Equation (7.1), the approximate limits of the buoy velocities can be calculated as:

$$\dot{\mathbf{x}}^T \mathbf{J}^{-T} \mathbf{\Lambda}^{-2} \mathbf{J}^{-1} \dot{\mathbf{x}} / \omega^2 \equiv \dot{\mathbf{x}}^T \mathbf{\Gamma}^{-2} \dot{\mathbf{x}} / \omega^2 \leq 1, \quad (7.18)$$

where diagonal elements of $\mathbf{\Gamma}$ correspond to the maximum allowed displacements of the buoy in the Cartesian space $\mathbf{\Gamma} \equiv \mathbf{x}_{\max} \mathbf{I}$. The difference between the kinematic condition described by Equation (7.16) and the global constraint from Equation (7.18) is demonstrated in Figure 7.6 for the 2D case. Thus, the global constraint imposes stricter limits on the motion of the system, where the WEC workspace is bounded by an ellipsoid with semi-principal axis equal to $x_{i,\max}$, $i = 1 \dots 3$. In this way, the multi-dimensional constrained problem is reduced to a single equation which is easier to manipulate in practice.

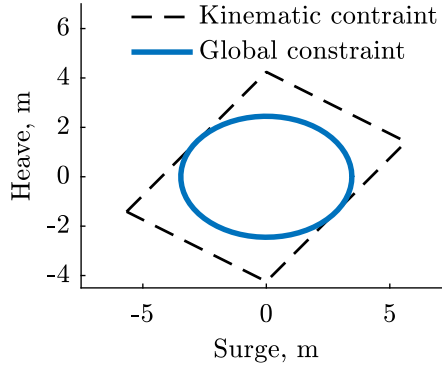


Figure 7.6: Visual representation of the global constraint from Equation 7.18 vs. the kinematic constraint from Equation (7.16) for $\Delta l_{\max} = 3$ m.

For the planar motion, where only heave, surge and pitch modes are considered, Equation (7.18) can be re-written as:

$$\frac{v_{1,\text{opt}}^2}{(v_{1,\max})^2} + \frac{v_{3,\text{opt}}^2}{(v_{3,\max})^2} = k^2. \quad (7.19)$$

So if $k \leq 1$, the buoy motion stays within the limits, and the reference buoy velocity should be set according to Equation (7.14). However, if $k > 1$, more damping should be added to the system in order to decrease buoy oscillations in all degrees of freedom. So the following procedure should be taken for the higher level control at each time step:

1. Pre-calculate matrix Γ (or $\mathbf{x}_{\max}$) using Equation (7.18) based on the given $\Delta l_{\max}$ and the inverse kinematic Jacobian at the nominal buoy pose $\mathbf{J}_0^{-1}$.
2. Measure or estimate the current value of the wave excitation force $\mathbf{F}_{exc}$.
3. Apply the EKF to $\mathbf{F}_{exc}$ in order to estimate its current amplitude ($\hat{\mathbf{f}}_{exc}$) and dominant frequency $\hat{\omega}$. Note that each degree of freedom is processed separately.
4. Calculate the optimal buoy velocity for the current dominant frequency $v_{i,\text{opt}} = \frac{\hat{f}_{exc,i}}{2(B_{ii}(\hat{\omega}_i) + B_{v,i})}$, where $i = 1 \dots 3$.
5. Calculate the maximum allowed buoy velocity for the current dominant frequency $v_{i,\max} = \hat{\omega}_i x_{i,\max}$.
6. Calculate gain k from Equation (7.19) and according to its value, set:

– if $k \leq 1$ then

$$H_{jj}(t) = 2(B_{ii}(\hat{\omega}_i) + B_{v,i}), \quad i = 1 \dots 3, \quad (7.20)$$

– if $k > 1$ then

$$H_{ii}(t) = 2k(B_{ii}(\hat{\omega}_i) + B_{v,i}), \quad i = 1 \dots 3. \quad (7.21)$$

7. Set the reference buoy velocity as:

$$v_{i,\text{ref}} = \frac{F_{exc,i}}{H_{ii}(t)}, \quad i = 1 \dots 3. \quad (7.22)$$

7.1.3.4 Low level

The main purpose of the low-level controller is to find a machinery (control) force that will allow a desired velocity of the WEC to be achieved. In [9] it has been suggested to use an internal model control strategy for this purpose. The IMC structure for the single-input-single-output (SISO) system is shown in Figure 7.7 and has two main components: a controller $Q(s)$ and a plant model $\tilde{G}(s)$. Ideally, the model should exactly represent the dynamic behaviour of the plant $\tilde{G}(s) = G(s)$ in order to estimate an influence of disturbances acting on the system. However, in practice, the model is just a linear approximation of the process and the controller should be designed

robust in order to accommodate modelling uncertainties. The established way to design $Q(s)$ is [20]:

$$Q(s) = F(s)\tilde{G}^{-1}(s), \quad (7.23)$$

where $F(s)$ is a filter that is added to make $Q(s)$ proper and improve robustness of the control system.

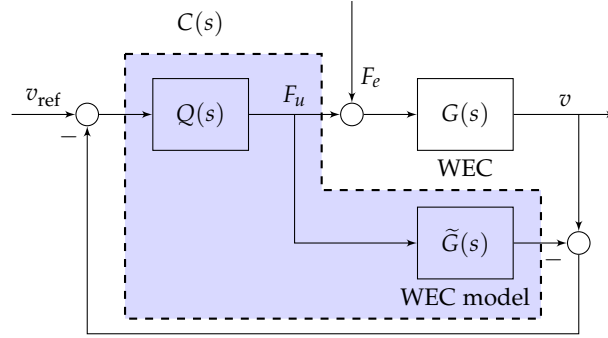


Figure 7.7: Block diagram of the IMC strategy.

Despite the fact that the studied WEC has three control inputs through the tethers, it is more convenient to consider control in the Cartesian coordinate frame and then map all required forces to the related tether force using $\mathbf{J}^T \mathbf{F}_u$. Thus, the three-tether WEC refers to the multiple-input-multiple-output systems (MIMO), where the matrix transfer function $\mathbf{G}(s)$ contains non-diagonal elements, which complicate implementation of the IMC. However, the WEC has a poor controllability over a rotational motion of the buoy, so the MIMO system $\mathbf{G}(s)$ can be decoupled employing a series of SISO controllers as shown in Figure 7.8 (multiloop method).

Therefore, the multiloop IMC model is:

$$\tilde{\mathbf{G}}(s) = \begin{pmatrix} \tilde{G}_{11}(s) & 0 & 0 \\ 0 & \tilde{G}_{22}(s) & 0 \\ 0 & 0 & \tilde{G}_{33}(s) \end{pmatrix} \quad (7.24)$$

and the multiloop IMC controller is:

$$\mathbf{Q}(s) = \begin{pmatrix} Q_{11}(s) & 0 & 0 \\ 0 & Q_{22}(s) & 0 \\ 0 & 0 & Q_{33}(s) \end{pmatrix}, \quad (7.25)$$

where each element is designed as:

$$Q_{ii}(s) = F_i(s)\tilde{G}_{ii}^{-1}(s). \quad (7.26)$$

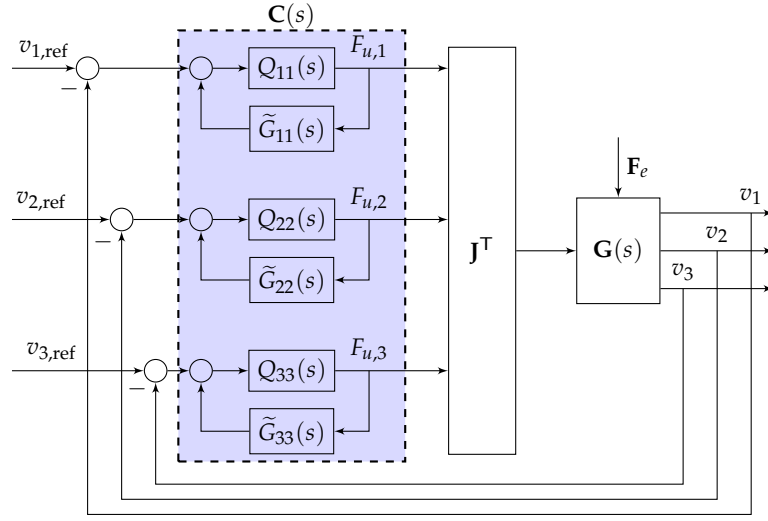


Figure 7.8: Block-diagram of the decoupled multi-loop IMC.

For the system considered here:

$$\begin{aligned}\tilde{G}_{11}(s) &= \tilde{G}_{22}(s) = \\ &= \frac{2.1 \times 10^{-6} s(s^2 + 0.8s + 1.1)(s^2 + 0.8s + 2.6)}{(s^2 + 0.1s + 0.8)(s^2 + 0.9s + 1.3)(s^2 + 0.7s + 2.7)}, \\ \tilde{G}_{33}(s) &= \frac{1.2 \times 10^{-6} s(s^2 + 0.8s + 1.2)(s^2 + 0.2s + 2.8)}{(s^2 + 0.1s + 0.7)(s^2 + 1.2s + 1.6)(s^2 + 0.7s + 2.9)}.\end{aligned}\quad (7.27)$$

It is important to note that by disregarding the off-diagonal elements, ‘known’ modelling errors are introduced to the system. In order to assess the significance of the omitted elements on the performance of the control system, several interaction measures have been offered in [4]:

$$R_i(i\omega) = \frac{\sum_{j \neq i} |G_{ij}(i\omega)|}{\sum_j |G_{ij}(i\omega)|}, \quad 0 \leq \omega < \infty, \quad (7.28)$$

$$C_i(i\omega) = \frac{\sum_{j \neq i} |G_{ji}(i\omega)|}{\sum_j |G_{ji}(i\omega)|}, \quad 0 \leq \omega < \infty, \quad (7.29)$$

where R_i relates to the Row IMC interaction and shows how other control inputs except the i -th may affect the i -th output, C_i is a Column interaction measure showing how i -th control input may affect other system outputs with

the exception of the i -th. The main concern in the current WEC design is a coupling between surge and pitch modes, therefore, the interaction measures for surge ($i = 1$) are shown in Figure 7.9. As a result, the buoy response in surge is mostly affected by the horizontal control force (see R_1), and this force does have an impact on the pitch motion with the strongest coupling at the frequency around 0.7 rad/s. These results demonstrate that the chosen SISO controllers can effectively control the WEC in all translational degrees of freedom.

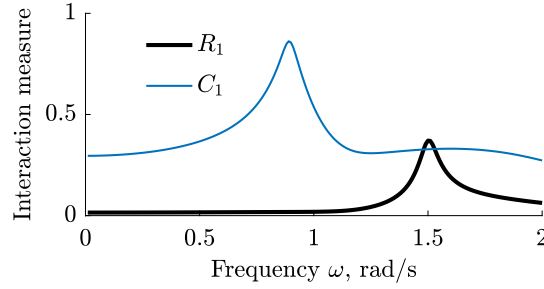


Figure 7.9: IMC interaction measures for the three-tether WEC.

Finally, the filter design $F_i(s)$ is adapted from [9], where it is implemented as a bandpass filter:

$$F_i(s) = \frac{5s}{(5s + 1)(0.2s + 1)}, \quad i = 1 \dots 3. \quad (7.30)$$

7.1.4 Results

7.1.4.1 Controller performance

The proposed controller satisfies the phase optimality condition which is demonstrated in Figure 7.10. The buoy velocity in both heave and surge has the same phase as the corresponding wave excitation force.

The velocity tracking performance of the multiloop IMC controller is shown in Figure 7.11 for the heave mode only. The actual buoy velocity follows the reference point quite well with a minimal error between two signals.

As one of the important parts of the current control strategy is to keep the buoy motion within set boundaries, the probability of the buoy position and also tether elongations over simulation time of 5000 s for the sea state of $H_s = 3$ m and $T_p = 12$ s is shown in Figure 7.12. Predominantly the elongation of the first tether slightly exceeds the physical limits ($\Delta l_{\max}$) which may be caused by the fact that the nominal buoy position ($\mathbf{J}_0^{-T}$) is used to

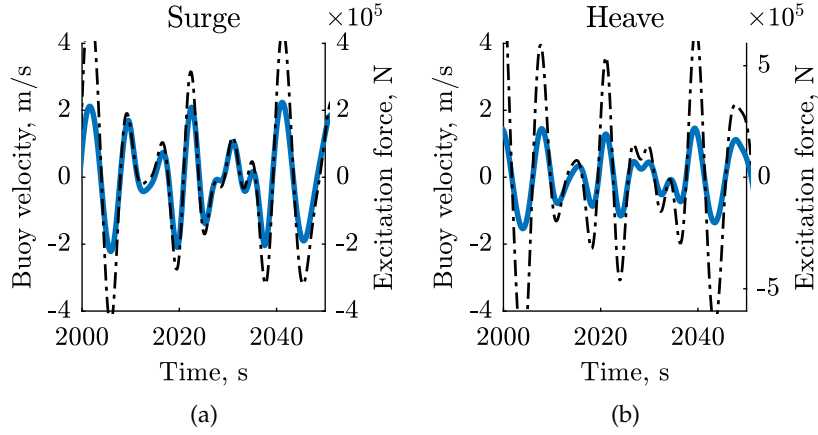


Figure 7.10: Time series of the buoy velocity (blue line) and the wave excitation force (black dash-dot-line) for the sea state of $H_s = 3$ m and $T_p = 12$ s: (a) surge and (b) heave modes.

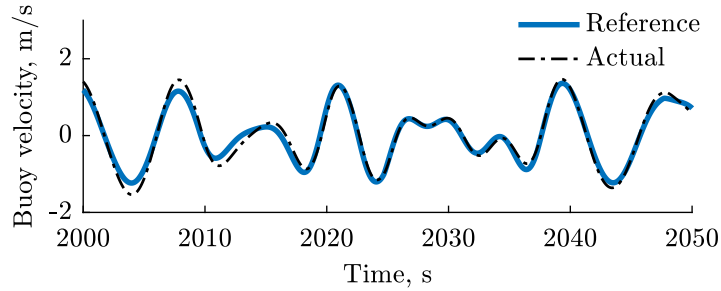


Figure 7.11: Tracking of the reference buoy velocity in heave by the multiloop IMC controller.

calculate boundaries in the Cartesian space. Moreover, modelling errors can affect this undesirable behaviour which can be overcome employing more conservative constraints.

7.1.4.2 Comparison with a spring-damper control

The performance of the proposed multiloop controller is compared with a spring-damper strategy, where the control force for the latter is: $F_{u,i}(t) = -K_{pto}\Delta l_i(t) - B_{pto}\Delta \dot{l}_i(t)$, $i = 1 \dots 3$. The control parameters K_{pto} and B_{pto} are optimised to maximise power output for each sea state using exhaustive search.

The performance of both controllers is presented in Figure 7.13 for the range of sea states using JONSWAP wave spectra with different sharpness.

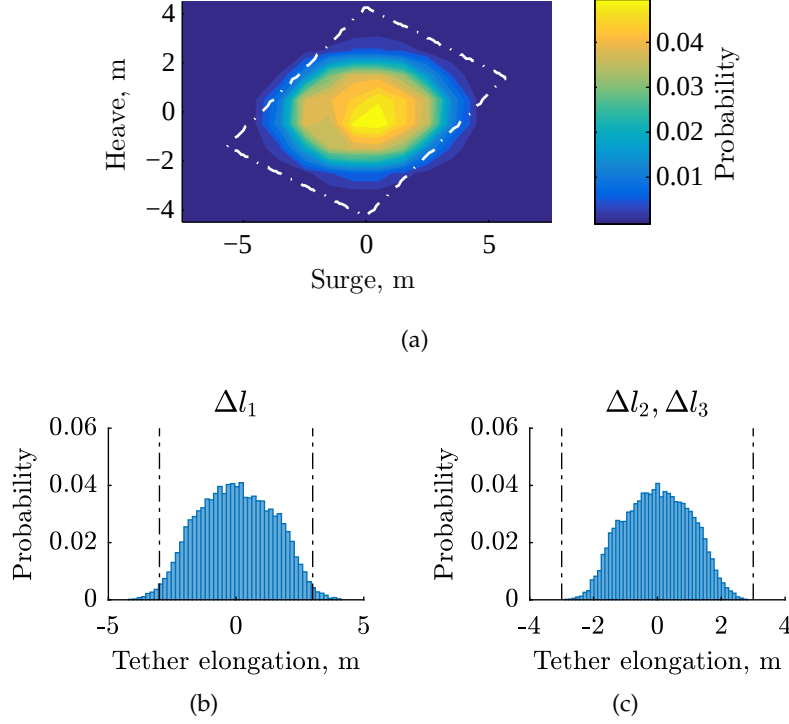


Figure 7.12: The probability that the buoy motion is within the set boundaries for the sea state of $H_s = 3$ m and $T_p = 12$ s: (a) 2D motion in Cartesian coordinates, (b) and (c) elongations of the first and second tethers respectively. Maximum tether elongation is set to $\Delta l_{\max} = 3$ m. Dash-dot lines correspond to the kinematic limits.

The proposed IMC-based velocity tracking controller (VTC) outperforms the spring-damper (SDC) system for all sea states considered in this study. As expected, the improvement is more significant for the wide-banded spectrum ($\gamma = 1$) as the spring-damper control is tuned to only one frequency from the range and its performance deteriorates as the spectrum sharpness decreases.

The efficiency improvement ratio for the VTC over the SDC is shown in Figure 7.14 based on data from Figure 7.13. Overall, it is possible to achieve an increase in power generation by approximately 10-35% from the VTC for the wide-banded spectra, while for the narrow-banded sea states the improvement is 5-20%.

Further investigations are required to identify the sensitivity of the controller to modelling parameters including the linearised viscous damping and the filter design. Moreover, it is critical to assess the controller perform-

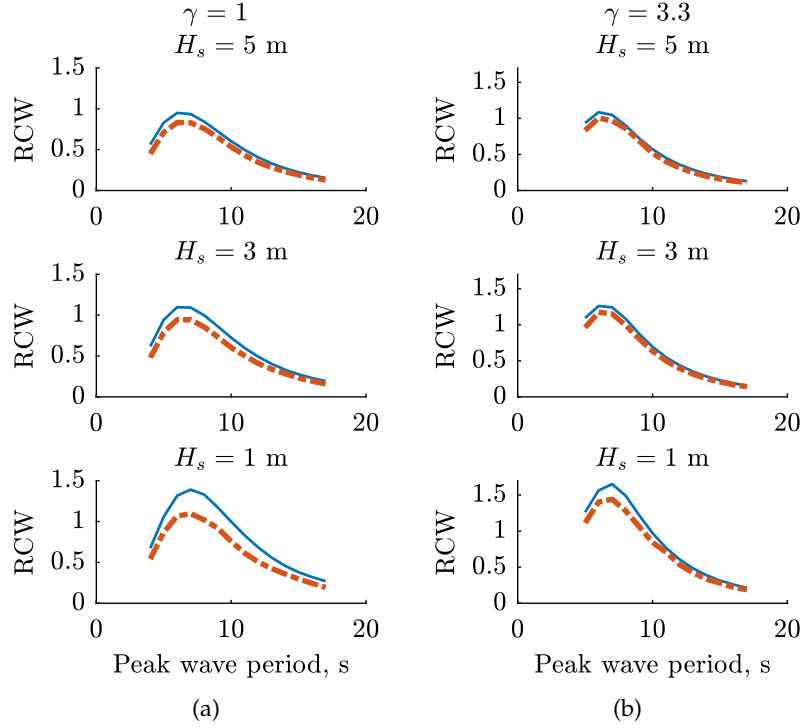


Figure 7.13: Performance comparison of the IMC-based velocity tracking control (blue solid line) vs. the spring-damper control (red dash-dot line) for the range of sea states generated using JONSWAP spectrum: (a) $\gamma = 1$ (wide-banded) and (b) $\gamma = 3.3$ (narrow-banded).

ance over the range of wave directions as well as degraded excitation force estimates.

7.1.5 Conclusion

The IMC-based velocity tracking control has been applied to the three-tether WEC using the multiloop design. The benefits of this controller are that it does not require the prediction of the wave forces and allows the inclusion of motion constraints. The high-level controller sets the reference buoy velocity based on the harmonic assumption of the wave force. Therefore, it is shown that for the submerged WECs it is very important to identify the targeted dominant frequency from the excitation force signal and not from the wave elevation as they may be very different due to the bandpass properties of submerged converters. Despite the fact that the three-tether WEC is a MIMO system where a multi-variable control strategy would normally be applied, it is demonstrated that the system of SISO controllers is able to control the

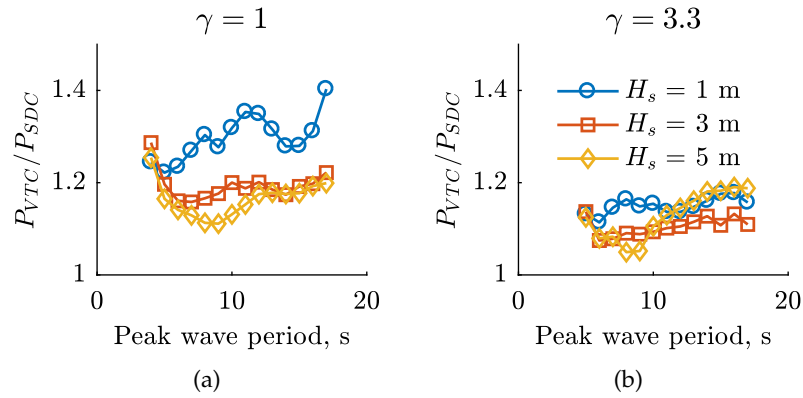


Figure 7.14: Efficiency improvement ratio between the IMC-based velocity tracking control (VTC) over the spring-damper control (SDC) for the range of sea states generated using JONSWAP spectrum: (a) $\gamma = 1$ (wide-banded) and (b) $\gamma = 3.3$ (narrow-banded).

plant over a desired range of frequencies. The performance of the proposed controller is compared with a quasi-standard spring-damper approach where an increase in energy extraction of around 10-35% is achieved for the wide range of considered sea states.

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7.2 Considerations on the control design

This section consists of the journal article submitted for publication:

Sergiienko, N. Y., Cazzolato, B. S., Arjomandi, M. and Ding, B. (2018). “Considerations on the control design for a three-tether wave energy converter”. Submitted to IEEE Transactions on Sustainable Energy.

Statement of Authorship

Title of Paper	Considerations on the optimal control of the three-tether wave energy converter
Publication Status	☐ Published ☐ Accepted for Publication ☒ Submitted for Publication ☐ Unpublished and Unsubmitted work written in manuscript style
Publication Details	Sergiienko, N. Y., Cazzolato, B. S., Arjomandi, M. and Ding, B. (2018). "Considerations on the control design of a three-tether wave energy converter". Submitted to IEEE Transactions on Sustainable Energy.

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Contribution to the Paper	Developed the ideas and concepts, developed modelling routines, interpreted results, wrote the manuscript, and acted as a corresponding author.
Overall percentage (%)	85%
Certification:	This paper reports on original research I conducted during the period of my Higher Degree by Research candidature and is not subject to any obligations or contractual agreements with a third party that would constrain its inclusion in this thesis. I am the primary author of this paper.
Signature	Date 05/02/2018

Co-Author Contributions

By signing the Statement of Authorship, each author certifies that:

- the candidate's stated contribution to the publication is accurate (as detailed above);
- permission is granted for the candidate to include the publication in the thesis; and
- the sum of all co-author contributions is equal to 100% less the candidate's stated contribution.

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Considerations on the control design for a three-tether wave energy converter

N. Y. Sergiienko, B. S. Cazzolato, M. Arjomandi, B. Ding

Abstract

Wave energy converters (WECs) capable of extracting power in multiple degrees of freedom require a special attention from control engineers as the control problem becomes multivariable involving highly coupled dynamics of the plant. Taking the three-tether submerged buoy as an example of a multi-degree-of-freedom WEC, this paper presents three main steps that should be taken during control system development. Firstly, an understanding of the system dynamics, its rigid body modes of vibration and input/output controllability is built using the singular value decomposition approach. Then, a causal close-to-optimal controller developed for the single-tether heaving WEC is extended to the multivariable control problem, demonstrating a significant increase in the power output as compared to the simple spring-damper approach. At the final stage, technical requirements imposed by this controller on the power take-off (PTO) machinery are investigated showing that, in order to achieve a 15%-improvement in power absorption compared to a quasi-standard spring-damper control, the amount of reactive power should be increased by 50%, forcing one PTO unit to operate as an actuator all the time.

7.2.1 Introduction

The power production of an oscillating wave energy converter is directly dependent on its hydrodynamic properties and its interaction with an incoming wave front. The latter is determined by the control system design which defines whether the buoy passively follows waves or it is actively controlled to harvest more energy. Therefore, a properly designed control system of the WEC can increase the productivity several times [17] in comparison to a passive control load.

The majority of proposed control algorithms developed for WECs are for systems that operate in one degree of freedom and are considered as single-input-single-output (SISO) systems. However, more energy can be harvested when several degrees of freedom are involved and coupled to the power take-off unit [3]. It is generally assumed that any SISO controller

can be easily extended and applied to the multivariable WEC, which is true when there is an independent control effort over each mode of oscillation as shown in [1, 18]. In the case when the PTO system has an operational space different from the wave-induced motion of the WEC (e.g. NEMOS [8]), all degrees-of-freedom become coupled through the PTO and mooring system. Such WECs require more complex multiple-input-multiple-output (MIMO) control algorithms which take into account interactions in the system dynamics.

Regardless of whether the converter operates in one or several degrees of freedom, its control is generally considered as an optimisation problem with the objective to maximise power generation subject to displacement and/or load force constraints [11]. However, the design of the WEC is not only driven by the maximisation of the energy production but takes into consideration the cost of the delivered electricity accounting for the capital and operational expenditures of the system. Therefore, improved performance of the WEC from an advanced control law is almost always associated with a more complex design of the power take-off machinery. For example, the model predictive control outperforms other controllers in terms of the absorbed power, but it requires the highest peak-to-average ratio of the power flowing through the PTO [6].

Therefore, recently, more attention has been brought to the practical side of the development and implementation of the WEC control systems. Sandia National Laboratories [17] have performed extensive research on the assessment of the most common control strategies, where the comparison has been done not only in terms of the power output but also involving other performance indices related to the PTO capacity requirements. The results have demonstrated that PTO specifications, such as the required energy storage, the maximum slew rate of the load force, etc., significantly vary depending on the control strategy. Furthermore, the implemented controller should account for practical limitations of the power take-off as demonstrated in [9], where the transmission energy losses and efficiency of the machinery are included in the formulation of the control law. Therefore, at the early stage of the WEC development, the PTO machinery should be designed according to the requirements imposed by the selected control strategy, and if not possible, the controller should take into account technical constraints of the system.

The current paper addresses two research questions: (i) how to modify the SISO controller for the WEC that operates with multiple degrees of freedom? and (ii) what challenges may arise when it is required to implement the close-to-optimal controller in practice? The description of the converter, its

numerical model and the controllability analysis are presented in Section 7.2.2. The design of the high- and low-level control loops and the performance of the suggested control strategy are shown in Section 7.2.3. The possibility of practical application of optimal control and associated specifications and requirements for the power take-off machinery are discussed in Section 7.2.4.

7.2.2 Wave energy converter

7.2.2.1 System description

The WEC considered in this study is a fully submerged disk-like buoy connected to three power take-off units by means of the flexible tethers as shown in Figure 7.15. Such configuration of the PTO allows the extraction of power from surge, heave and pitch motions simultaneously. All parameters and dimensions of the converter have been selected such that the cost of electricity is minimised considering that the PTO is capable of generating a force proportional to the tether displacement and velocity (spring-damper control).

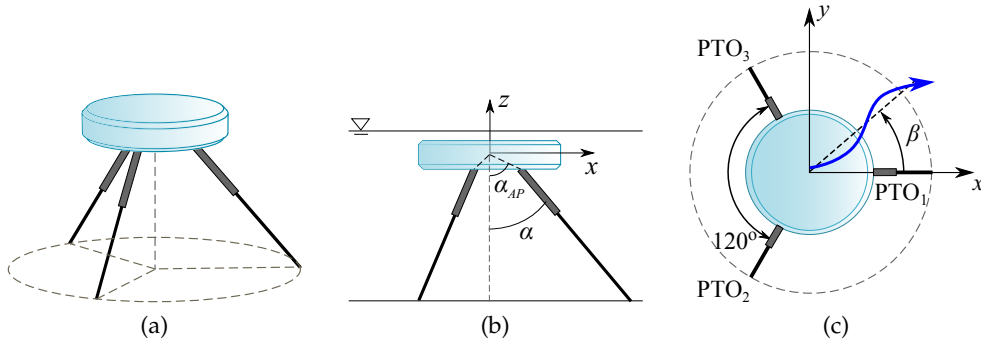


Figure 7.15: Geometry and parameters of the three-tether wave energy converter: (a) 3D view, (b) front view, (c) top view. The angle β shows the direction of the wave propagation.

7.2.2.2 Dynamic model

A detailed description of the non-linear model of the three-tether WEC has been documented in [12, 13, 14] and only key points will be outlined in this paper.

The motion of the WEC in six degrees-of-freedom is described by the position vector $\mathbf{x} \in \mathbb{R}^{6 \times 1}$ (x_1 surge, x_2 sway, x_3 heave, x_4 roll, x_5 pitch, x_6 yaw) in the reference (Cartesian) coordinate frame $Oxyz$ according to the following

Table 7.2: WEC parameters.

Parameter	Value
WEC radius	12.5 m
WEC height	5 m
Submergence depth	2 m
Water depth	30 m
Mass	1.99×10^6 kg
Moments of inertia	
$I_{xx} = I_{yy}$	8.18×10^7 kg·m ²
I_{zz}	1.55×10^8 kg·m ²
Tether angles	
α_{AP}	65°
α	40°

equation:

$$\mathbf{M}\ddot{\mathbf{x}} = \mathbf{F}_{exc} + \mathbf{F}_{rad} + \mathbf{F}_{visc} + \mathbf{F}_{buoy} + \mathbf{F}_{tens}, \quad (7.31)$$

where $\mathbf{M}$ is a mass matrix, $\mathbf{F}_{exc}$ is the wave excitation force, $\mathbf{F}_{rad}$ is the wave radiation force, $\mathbf{F}_{visc}$ is the viscous damping force, $\mathbf{F}_{buoy}$ is the buoyancy force, $\mathbf{F}_{tens}$ is the generalised tether force. The nonlinear (due to the presence of $\mathbf{F}_{visc}$ and $\mathbf{F}_{tens}$) model in Equation (7.31) is used to assess the controller performance.

For the controller design, Equation (7.31) is reduced to the classic state-space representation $\dot{\mathbf{X}}_{WEC} = \mathbf{A}_{WEC}\mathbf{X}_{WEC} + \mathbf{B}_{WEC}\mathbf{U}_{WEC}$; $\mathbf{Y}_{WEC} = \mathbf{C}_{WEC}\mathbf{X}_{WEC}$ detailed in [13]. The transfer function that corresponds to this linearised model is:

$$\mathbf{G}(s) = \mathbf{C}_{WEC}(s\mathbf{I} - \mathbf{A}_{WEC})^{-1}\mathbf{B}_{WEC}. \quad (7.32)$$

7.2.2.3 Controllability

The converter shown in Figure 7.15 belongs to the class of underactuated mechanisms as it has six degrees-of-freedom (DOF) but only three independent control inputs (non-square plant). In order to identify what DOFs can be controlled and quantify interactions in this MIMO system, the transfer function of the WEC $\mathbf{G}(j\omega) \in R^{6 \times 3}$ should be decomposed using singular value decomposition (SVD):

$$\mathbf{G}(j\omega) = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H, \quad (7.33)$$

where $\mathbf{U} \in R^{6 \times 6}$ and $\mathbf{V} \in R^{3 \times 3}$ are the singular vectors that form orthonormal bases for the output and input space, correspondingly, and $\mathbf{\Sigma} \in R^{6 \times 3}$ is a matrix with three non-negative singular values $\bar{\sigma} \triangleq \sigma_1 \geq \sigma_2 \geq \sigma_3 \triangleq \underline{\sigma}$.

Singular vectors correspond to the six rigid body modes of vibration (not to be confused with hydrodynamic modes) each of which has its own natural frequency and mode shape [4]. The graphical representation of the mode shapes that correspond to U_i ($i = 1 \dots 6$) are shown in Figure 7.16. Since the system has three control inputs, only the first three modes out of the six, namely U_1 , U_2 , and U_3 can be actively controlled by the power take-off machinery. It can be seen that both output modes U_1 and U_4 involve buoy motion in surge and pitch, but: (i) these DOFs are included in U_1 and U_4 with different phases and amplitudes; (ii) U_1 and U_4 have different resonant frequencies, so the modes primarily responsible for buoy motion depend on the period of the incoming wave; and (iii) U_1 is controllable while U_4 is not.

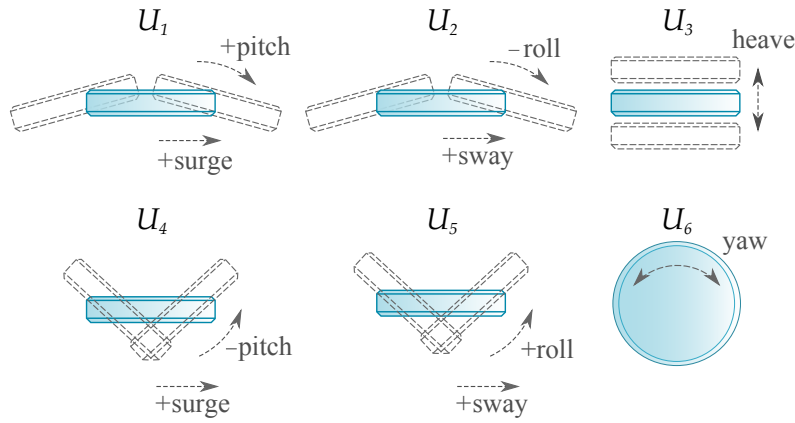


Figure 7.16: Graphical representation of the rigid body modes of the three-tether WEC.

In control theory, singular values show the output directions in which the system inputs are most effective [15]. The frequency-dependent singular values of the WEC which correspond to the controllable modes U_i ($i = 1 \dots 3$) are demonstrated in Figure 7.17a. It is clear from the plot that in the frequency range of $0.5 - 0.75$ rad/s (wave period $8 - 12$ s) the coupled surge/pitch U_1 and sway/roll U_2 modes have singular values an order of magnitude higher than these for heaving mode U_3 . This implies that less effort is required to control the buoy in surge and sway than in heave at this range of frequencies. Moreover, if the ratio between the maximum and minimum singular values (condition number) is larger than 10, inverse-based controllers may be sensitive to ‘unstructured’ input uncertainty which is undesirable for practical applications [15].

Another measure which is widely used to identify control properties of the plant is the Relative Gain Array $\text{RGA}(G) = \Lambda(G) \triangleq G \circ (G^{-1})^T$, where

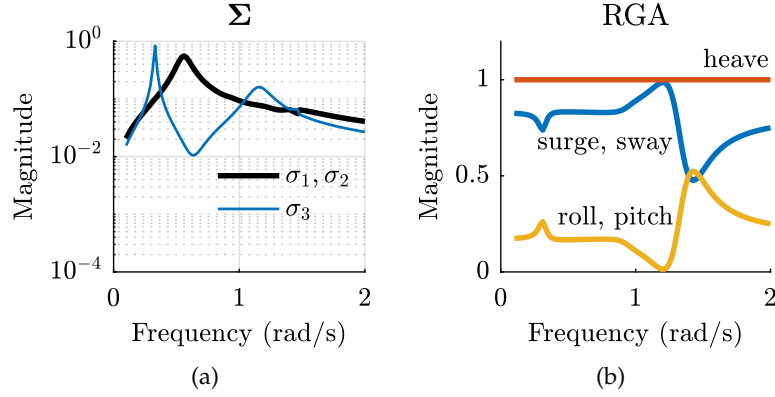


Figure 7.17: Controllability measures of the three-tether WEC: (a) singular values, and (b) Relative Gain Array.

$\circ$ denotes the Hadamard product (element-by-element multiplication). For non-square plants, small elements ($\ll 1$) in a row of RGA correspond to the output which cannot be controlled. The frequency dependent sum of each row of RGA for the three-tether WEC shown in Figure 7.17b indicates that the PTO system has a strong control authority in surge, sway and heave, but poor controllability over pitch and roll. However, it should be noted that this analysis has been performed for the nominal position of the buoy $\mathbf{x} = \mathbf{0}_{6 \times 1}$; and the control property of this WEC is subject to the buoy orientation.

7.2.3 Controller design and performance

In this section, the ‘simple but effective’ velocity tracking controller developed by Fusco and Ringwood [5] is extended to the multivariable system. As shown in Figure 7.18, the control structure has two major loops: (i) a high level loop sets the reference (desired) velocity of the buoy, and (ii) a low level loop provides the required machinery (power take-off) force to achieve this velocity.

7.2.3.1 High-level control

The main objective of the high-level controller is to set the reference buoy velocity, $\dot{\mathbf{x}}_{\text{ref}}$, having measured or estimated the wave excitation force, $\mathbf{F}_{\text{exc}}$. In [7], an observer has been designed to estimate the excitation force of a floating device based on the real-time measurements of its position, velocity, acceleration and control force. In the current study, it is assumed that the wave force is ideally estimated $\hat{\mathbf{F}}_{\text{exc}} = \mathbf{F}_{\text{exc}}^{\text{ideal}}$.

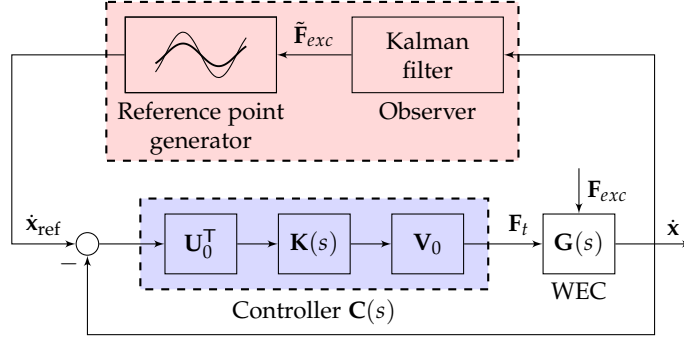


Figure 7.18: The block-diagram of the velocity tracking control designed for the three-tether WEC. In this paper, the ‘tilde’ symbol, ‘~’, denotes an estimated value of the variable.

7.2.3.2 Surge-pitch compromise

According to the linear potential theory, the optimal velocity of the WEC that is able to absorb power from several degrees-of-freedom should satisfy the following equation [3]:

$$\mathbf{B}(\omega)\hat{\mathbf{x}}_{\text{opt}}(\omega) = \frac{1}{2}\hat{\mathbf{F}}_{exc}(\omega), \quad (7.34)$$

where the ‘hat’ symbol, ‘^’, denotes the complex amplitude, $\mathbf{B}(\omega) \in R^{6 \times 6}$ is the matrix of radiation damping coefficients, $\hat{\mathbf{x}}_{\text{opt}}(\omega) \in R^{6 \times 1}$ is the vector of optimal velocity, and $\hat{\mathbf{F}}_{exc}(\omega) \in R^{6 \times 1}$ is the excitation force vector. If $\mathbf{B}(\omega)$ is non-singular, then Equation (7.34) has a unique solution $\hat{\mathbf{x}}_{\text{opt}}(\omega) = \frac{1}{2}\mathbf{B}^{-1}(\omega)\hat{\mathbf{F}}_{exc}(\omega)$. However, $\mathbf{B}^{-1}(\omega)$ does not exist for the axisymmetric body if surge and pitch are involved in power absorption, and Equation (7.34) cannot be solved simultaneously for $\hat{x}_{1,\text{opt}}$ and $\hat{x}_{5,\text{opt}}$. Therefore, if one of the velocities is known, the other one can be found from the following equation:

$$\begin{bmatrix} B_{11} & B_{15} \\ B_{51} & B_{55} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_5 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \hat{F}_{exc,1} \\ \hat{F}_{exc,5} \end{bmatrix}. \quad (7.35)$$

For instance, for the known value of the pitch velocity $\hat{x}_5$, the optimal buoy velocity in surge can be calculated as [3]:

$$\hat{x}_{1,\text{opt}} = \frac{\hat{F}_{exc,5} - 2B_{55}\hat{x}_5}{2B_{51}} = \frac{\hat{F}_{exc,1} - 2B_{15}\hat{x}_5}{2B_{11}}, \quad (7.36)$$

and vice-versa. So according to Equation (7.36), the requirement that the buoy velocity should be in phase with the excitation force does not apply if both surge and pitch modes are involved in power generation. For the

number of cases considered in this paper, it is found that $\hat{F}_{exc,1} \gg 2B_{15}\hat{x}_5$, and the coupling between surge and pitch is neglected in the design of the controller. In addition, due to the presence of the viscous damping force, the formulation of the optimal buoy velocity is modified as [5]:

$$\hat{x}_{1,opt} = \frac{\hat{F}_{exc,1}}{2(B_{11} + B_{v,1})}, \quad \hat{x}_{3,opt} = \frac{\hat{F}_{exc,3}}{2(B_{33} + B_{v,3})}, \quad (7.37)$$

where $B_{v,1}$ and $B_{v,3}$ are linearised damping coefficients in surge and heave modes, respectively.

7.2.3.3 Causality assumption

The ‘simple and effective’ controller is based on the assumption that the wave excitation force has a narrow bandwidth which allows one to eliminate the complexity of the noncausal WEC control problem. Thus, the excitation force can be represented as a harmonic process with one dominant frequency:

$$F_{exc,i}(t) \approx f_{exc,i}(t) \cos(\omega_i(t)t + \phi_i(t)), \quad (7.38)$$

where $f_{exc,i}(t)$, $\omega_i(t)$ and $\phi_i(t)$ are the time-variant amplitude, frequency and phase of the excitation force, respectively.

This assumption allows the reference velocity of the WEC in each mode i to be set as:

$$\dot{x}_{i,ref} = \frac{F_{exc,i}(t)}{2(B_{ii}(\tilde{\omega}_i) + B_{v,i})}, \quad (7.39)$$

where the estimated frequency $\tilde{\omega}_i$ is obtained in real time using the Extended Kalman filter (EKF) for each mode i separately, and the corresponding value of $B_{ii}(\tilde{\omega}_i)$ is set using gain scheduling.

The unique feature of the fully submerged WECs is that they act as a bandpass filter being excited by the ocean waves in a limited range of wave frequencies. Therefore, the dominant frequency of the excitation force is different from the peak frequency of the incoming wave front. The bandpass characteristics of the disk-like WEC considered in this study are shown in Figure 7.19a for the irregular wave of $H_s = 1$ m and $T_p = 14$ s. $S_\eta(\omega)$ corresponds to the Bretschneider wave spectrum [16], $|H_{Fe,\eta}(\omega)|$ shows the magnitude of the transfer function from the wave elevation (η) to the heave excitation force, and $S_{Fe}(\omega) = S_\eta(\omega)|H_{Fe,\eta}(\omega)|^2$ demonstrates the resultant spectrum of the wave excitation force. So despite the fact that the incident waves have a peak frequency $\omega_p^\eta = 0.45$ rad/s (14 s), the peak excitation forces occur at a frequency of 0.6 rad/s (10.5 s). $\tilde{\omega}$ corresponds to the value estimated by the EKF.

The bandpass effect of the submerged WEC is clearly demonstrated in Figure 7.19b, where the horizontal axis shows to the peak wave period of the sea states of $H_s = 1$ m and the vertical axis shows the corresponding dominant period in the excitation force signal. Thus, for all the sea states with $9 \text{ s} < T_p < 17 \text{ s}$, the buoy oscillations in heave will be dominated by a period of 10 s.

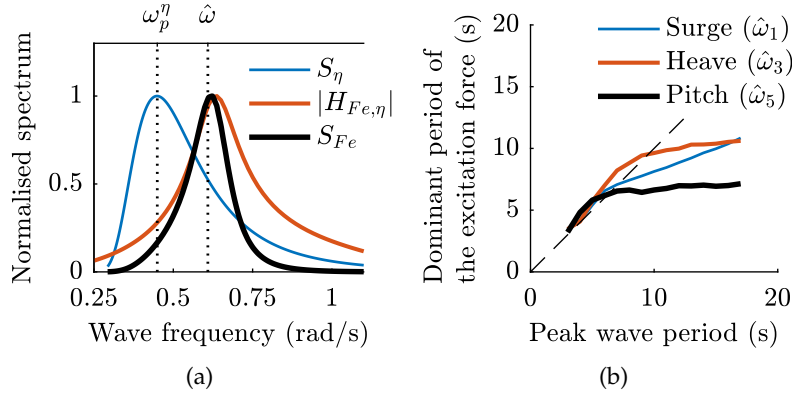


Figure 7.19: Bandpass properties of the three-tether WEC: (a) one sea state of $H_s = 1$ m and $T_p = 14$ s, (b) range of sea states with $H_s = 1$ m. Further details can be found in text.

7.2.3.4 Low-level control

As shown in Section 7.2.2.3, the three-tether WEC with a buoy shape considered in this study is a bad candidate for the inverse-based controllers due to the large condition number at the frequency range of interest. Therefore, as opposed to the internal-model approach implemented in [5, 13], it was decided to use the SVD-controller for the tracking control loop (see Figure 7.18). This controller is proven to be robust [15] and allows one to decouple the control problem of the multivariable system:

$$\mathbf{C}(s) = \mathbf{V}_0 \mathbf{K}(s) \mathbf{U}_0^T, \quad (7.40)$$

where $\mathbf{V}_0$ and $\mathbf{U}_0$ are obtained from the singular value decomposition of $\mathbf{G}_0 = \mathbf{U}_0 \mathbf{\Sigma}_0 \mathbf{V}_0^T$, $\mathbf{G}_0$ is a real approximation of $\mathbf{G}(j\omega_0)$ at a given frequency ω_0 , and $\mathbf{K}(s) = K(s) \mathbf{\Sigma}_0^{-1}$. This controller only requires the design of $K(s)$ which is chosen as a simple PI controller: $K(s) = k_p + \frac{k_i}{s}$. In this study, $\omega_0 = 0.63 \text{ rad/s}$, $k_p = 5$ and $k_i = 0$.

7.2.3.5 Controller performance

With the control structure defined, the performance of the SVD-based velocity tracking controller was investigated in terms of the reference tracking and power absorption properties.

7.2.3.6 Tracking

As demonstrated in Figure 7.20a–7.20b, the buoy velocity in both heave and surge has the same phase as the corresponding wave excitation force. As the pitch mode is not taken into account in the current controller design, it does not satisfy the phase optimality condition which is shown in Figure 7.20c.

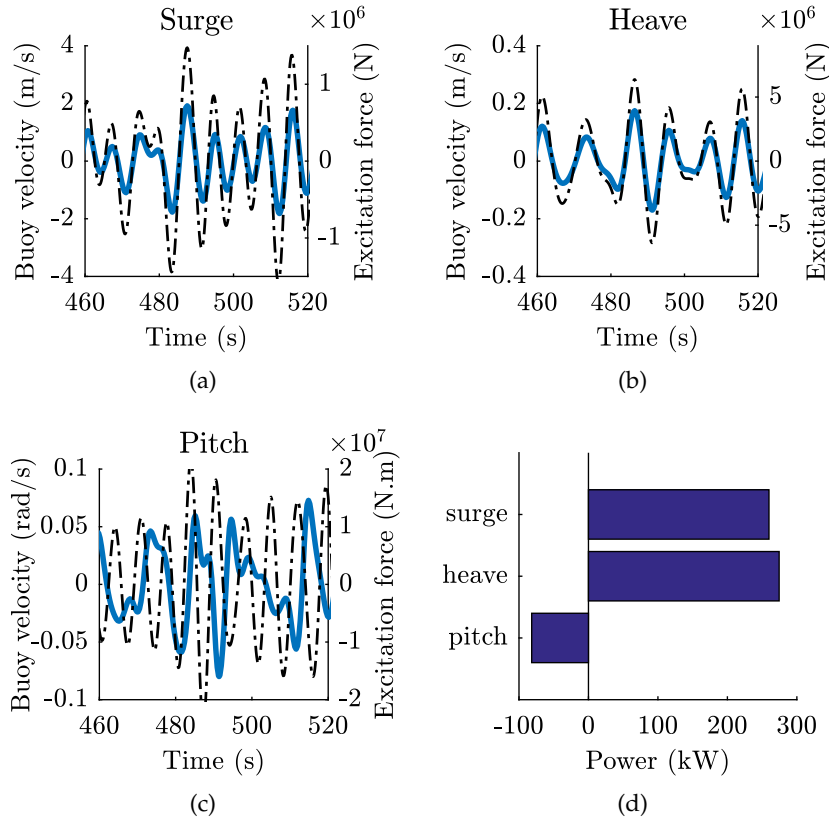


Figure 7.20: Time series of the buoy velocity (blue line) and the wave excitation force (black dash-dot-line) for the sea state of $H_s = 2$ m, $T_p = 12$ s: (a) surge, (b) heave, and (c) pitch modes. (d) The contribution of each hydrodynamic mode into the total power output of the three-tether WEC.

Moreover, the power losses associated with the buoy oscillations in pitch due to the radiation and drag forces constitute approximately 15% of the

total amount of absorbed power (refer to Figure 7.20d).

7.2.3.7 Comparison with a spring-damper control strategy

The performance of the proposed controller is compared with the commonly implemented spring-damper control (SDC) strategy, where the load force for the latter is defined as: $F_{u,i}(t) = -K_{pto}\Delta l_i(t) - B_{pto}\Delta \dot{l}_i(t)$, $i = 1 \dots 3$. The control parameters K_{pto} and B_{pto} are optimised to maximise power output for each sea state using an exhaustive search.

The performance of both controllers is presented in Figure 7.21 for the range of sea states using Bretschneider wave spectra. The chosen SVD-based velocity tracking controller (VTC) is capable of increasing the power output of the three-tether WEC by up to 40% for $T_p > 8$ s, while for the wave periods of $T_p < 8$ s the spring-damper controller demonstrates slightly better results. This is mainly due to the linearisation of the SVD controller around ω_0 ($T_0 = 10$ s) frequency.

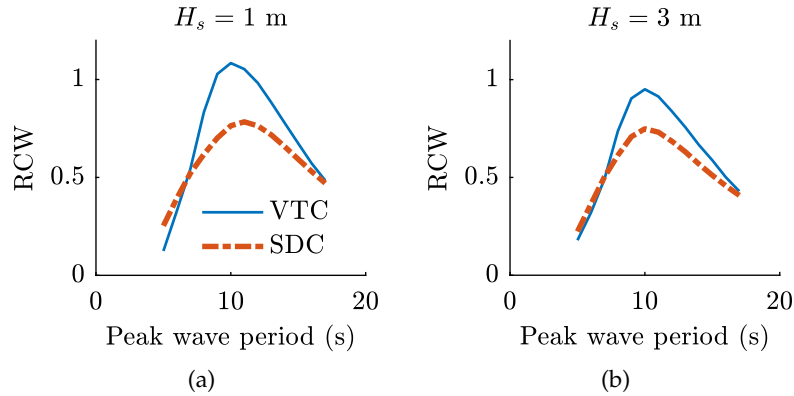


Figure 7.21: Performance comparison of the SVD-based velocity tracking control (blue solid line) vs. the spring-damper control (red dash-dot line) for the range of sea states generated using Bretschneider spectrum: (a) $H_s = 1$ m and (b) $H_s = 3$ m.

7.2.3.8 Sensitivity to the wave force estimation

WEC control systems aim to keep the buoy velocity in phase with the wave force, and modelling uncertainties and input/output disturbances may introduce errors in the reference and tracking loops leading to a significant degradation of the converter efficiency [10]. For the controller shown in Figure 7.18, the main sources of the modelling errors and their effect on the buoy

performance are listed in Table 7.3. As it has been assumed that the perfect estimation of the wave excitation force is available, the results presented in Figure 7.21 demonstrate the maximum capability of the velocity-tracking controller for the three-tether WEC. However, the information provided by the observer plays an essential role in this control strategy, and it is important to understand the effect of imperfect wave force estimation on the power output of the WEC.

Table 7.3: ‘Known’ modelling errors of the control structure shown in Figure 7.18.

Controller sub-system	Source of error	Buoy velocity	
		Amplitude	Phase
Excitation force observer	WEC model	x	x
Reference point generator	$B_{ii}(\omega), B_{v,i}$	x	
Velocity tracking loop	$K(s), \Sigma_0$	x	x

The sensitivity of the controller to the phase and amplitude errors of the estimated wave force is presented in Figure 7.22. To generate these results, it has been assumed that the force value used by the reference point generator has a time delay $\tilde{\mathbf{F}}_{exc}(t) = \mathbf{F}_{exc}^{\text{ideal}}(t - \tau)$ and the wave force amplitude error is modelled separately as $\tilde{\mathbf{F}}_{exc}(t) = (1 + \varepsilon)\mathbf{F}_{exc}^{\text{ideal}}(t)$.

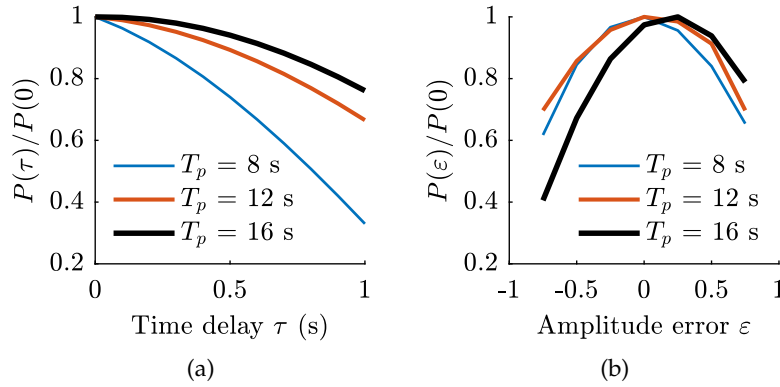


Figure 7.22: Sensitivity of the power output of the three-tether WEC to the imperfect estimation of the wave excitation force: (a) time delay, and (b) amplitude error.

As expected, the accurate estimation of the wave force phase is very important and the time delays of 0.3–0.7 s may result in power reduction of 20%. However, the system is more robust to the amplitude errors where uncertainties of $\pm 50\%$ in the estimation of the excitation force amplitude

lead to a decrease in the generated power by only 20%. As a result, it should be realised that no matter how robust the velocity tracking loop is, the reference point generator is of greater importance, and to provide an accurate estimation of the wave forces it requires a high-fidelity model of the plant.

7.2.4 Considerations on practical application of the optimal control

The SVD-based velocity tracking control provides a significant increase in power absorption of the WEC. However, this efficiency comes at a cost of the PTO machinery since its specifications, such as the peak force, slew rate, energy storage, etc., directly depend on the implemented control law. This section will specify what requirements the power take-off should meet in order to accommodate the close-to-optimal control strategy for the three-tether WEC.

7.2.4.1 Equivalent spring-damper control

The spring-damper controller is usually designed to operate on the sea-state basis, while the velocity tracking is a wave-by-wave control strategy. Thus, the question arises, assuming that the PTO can only operate as a spring-damper system, is it possible to implement the velocity-tracking control on such machinery using time-varying parameters $K_{pto}(t)$ and $B_{pto}(t)$? The results obtained for the regular wave of $H_w = 1$ m and $T_p = 10$ sec are shown in Figure 7.23, where the required control force for each PTO unit of the three-tether WEC is calculated based on the VTC and then equivalent spring-damper parameters are estimated using the least-squares method:

$$K_{pto,i}^{eq}(t)\Delta l_i(t) + B_{pto,i}^{eq}(t)\Delta \dot{l}_i(t) := F_{u,i}^{VTC}(t), \quad i = 1, 2, 3. \quad (7.41)$$

The analysis is done assuming that the wave propagates along the x -axis ($\beta = 0^\circ$) making the first tether aligned with an incoming wave front (refer to Figure 7.15).

The results imply that:

- (i) in order to implement an optimal control on the three-tether WEC, all PTO units should have different settings $K_{pto,1}^{eq} \neq K_{pto,2,3}^{eq}$ and $B_{pto,1}^{eq} \neq B_{pto,2,3}^{eq}$;
- (ii) the PTO attached to the first tether should act as an *actuator* not generating any power which is indicated by the negative value of the damping $B_{pto,1}^{eq} < 0$.

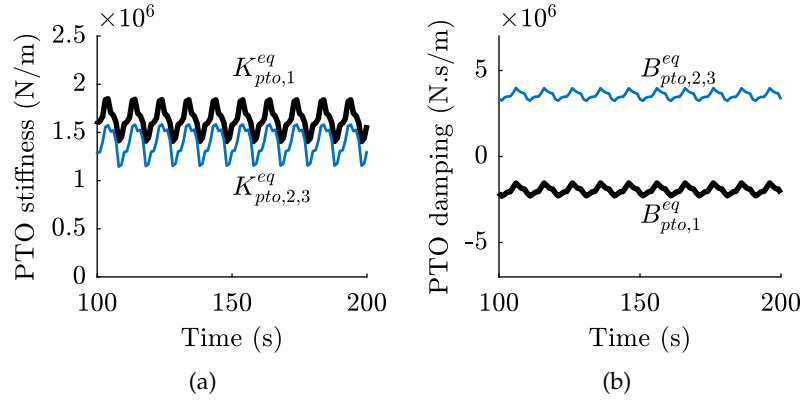


Figure 7.23: The time-dependent (a) stiffness and (b) damping coefficients of the equivalent spring-damper control for the regular wave of $H_w = 1$ m, $T_w = 10$ s.

These findings are shown more clearly in Figure 7.24 for the range of irregular waves of $H_s = 1$ m. The contribution of each power take-off unit to the total power output of the three-tether WEC is shown for two different control strategies: velocity-tracking (Figure 7.24b) and spring-damper (Figure 7.24a). Power factor is calculated as the ratio of the power absorbed by one PTO to the total power absorbed by all three PTO systems ($\bar{P}_i / \sum \bar{P}_i$).

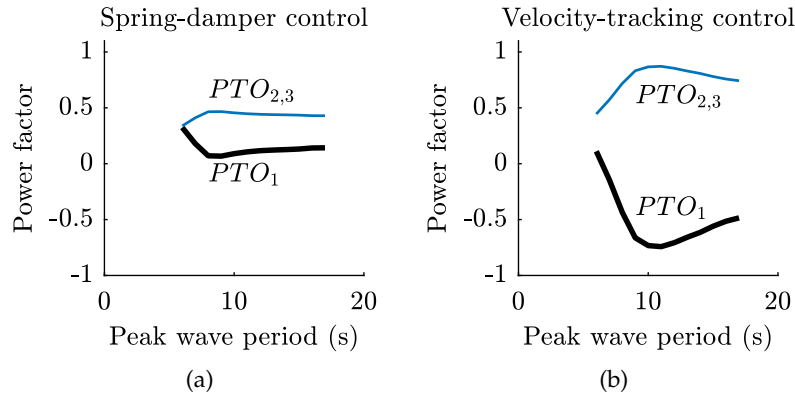


Figure 7.24: The contribution of each PTO unit to the total power absorption of the three-tether WEC depending on the sea state for two control strategies (a) spring-damper and (b) velocity tracking.

In the case of the spring-damper control considered here, all tethers have the same settings of the stiffness and damping, and these parameters are positive definite. As a result, all PTOs contribute to the power output, however

the first unit generates much less energy than the other two as shown in Figure 7.24a. This is opposed to the velocity tracking control which requires the first PTO to act as an actuator across all sea states, while the power to the grid is delivered by the second and third PTOs. Similar observations have been reported in [12] for the optimal causal control of the floating three-tether WEC. Moreover, this principle is also used in NEMOS [8] wave energy converter, where one ‘leg’ is actuating the buoy motion according to the prescribed trajectory.

7.2.4.2 Wave direction

The results shown in Section 7.2.4.1 cover the scenario when the first tether is aligned with the wave propagation. However, the operational behaviour of each PTO unit (whether it operates as a generator or as an actuator) is highly dependent on the direction of the incoming wave. The effect of the wave angle β on the power factor of each PTO is demonstrated in Figure 7.25a for two different sea states. Thus, the operational behaviour of each PTO changes when waves come from the opposite direction ($\beta = 180^\circ$): the first unit starts delivering power to the grid while the second and third PTOs serve as WEC actuators. Also it is interesting that for some wave angles (e.g. $\beta = 60^\circ$), a single PTO needs to generate more power (power factor > 1) than the net power produced by the WEC.

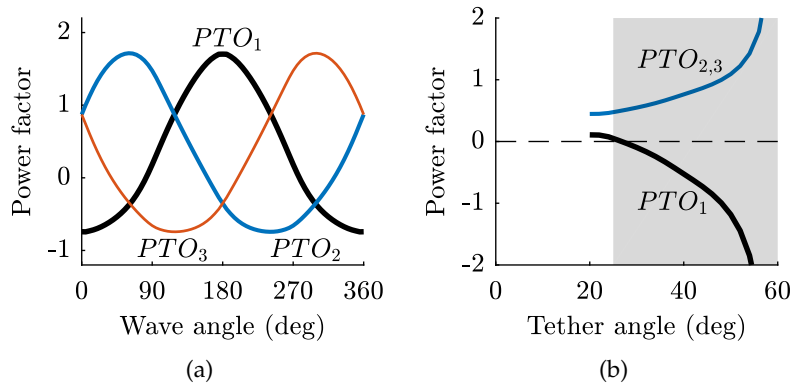


Figure 7.25: The effect of (a) the wave direction and (b) the tether inclination angle on the power factor of each PTO unit with implemented velocity tracking control for the sea state of $H_s = 1$ m, $T_p = 12$ s. Default settings of the WEC are specified in Figure 7.15.

7.2.4.3 Tether arrangement

The arrangement of tethers, including their inclination angles and attachment points, affect the power take-off capability of this WEC. As already mentioned in Section 7.2.2.1, the choice of the converter geometry and other parameters has been driven by minimisation of the levelised cost of electricity provided that the spring-damper control is implemented. Therefore, it is possible that the design of this WEC, in particular the tether arrangement, does not meet the requirements of the velocity-tracking controller. The effect of the tether angle α on the power factor of each PTO is demonstrated in Figure 7.25b for the sea state of $H_s = 1$ m and $T_p = 12$ s. It is clear that to be able to implement the velocity-tracking control on the three-tether WEC and to avoid the situation when one of the PTOs is working as an actuator, the tether angle should be reduced to approximately 25° . This can be explained by the fact that tethers and the PTO are trying to provide the required optimal motion of the buoy; and if the tether arrangement does not allow to achieve this trajectory with all PTOs generating power, then the optimal control pushes one or several PTO units to actuate the WEC instead of harvesting power.

More clearly the importance of the tether arrangement on the implementation of the optimal control can be shown by an example of the three-tether WEC with a different shape of the buoy (vertical cylinder with a radius of 3.8 m, height of 11.4 m, submergence depth of 3.75 m and water depth of 50 m). The new WEC and the power factors of its PTOs are demonstrated in Figure 7.26 for one sea state of $H_s = 1$ m and $T_p = 12$ s. It is obvious that for the tether angle close to $58 - 60^\circ$, all three PTOs are generating useful power, while for the angles $< 55^\circ$, the second and third PTOs are actuating the buoy motion (power factor is < 0). However, for larger tether angles the situation is completely opposite and to achieve an optimal trajectory of the buoy, the first PTO behaves as an actuator, whereas the second and third deliver electricity to the grid. It is important to note that only an axisymmetric arrangement of tethers is considered in this case study (all tethers have the same inclination angle to the vertical), while the possibility of other arrangements should be further investigated from a controllability point of view.

As a result, the arrangement of tethers should be considered as an integral part of the control strategy and if it is not possible to install them according to the requirements, then the design of the PTO system should be modified appropriately.

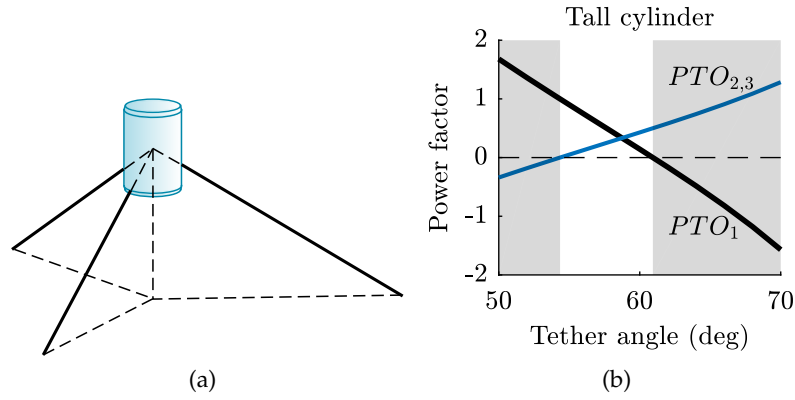


Figure 7.26: For demonstration purposes only: (a) a 3D view of the three-tether WEC with a cylindrical buoy of 3.8 m radius, 11.4 m height and submergence depth of 3.75 m; (b) power factor of each PTO depending on the tether arrangement assuming that $\alpha = \alpha_{AP}$ for the sea state of $H_s = 1$ m and $T_p = 12$ s.

7.2.4.4 Metrics

In order to implement a particular control strategy for the WEC, the PTO machinery should meet certain requirements. The analysis of the two control strategies is completed assuming a deployment site of Albany, Western Australia with a wave climate shown in Figure 7.27. The quantities used to assess PTO specifications are adopted from [17] and specified in Table 7.4. An explanation of notations can be found in Figure 7.28.

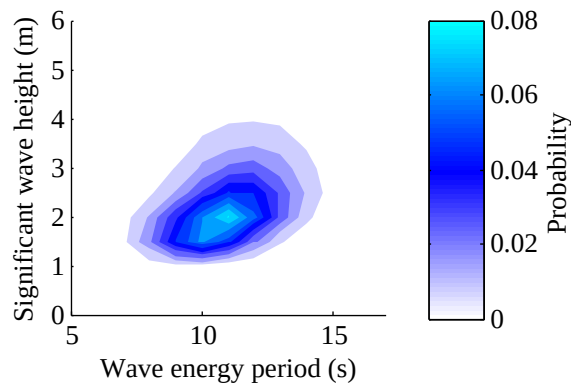
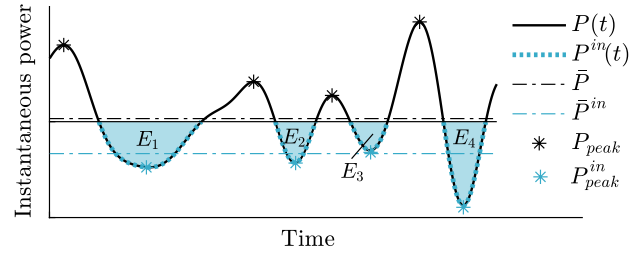


Figure 7.27: The wave climate at the Albany deployment site located in Western Australia (117.7547°E, 35.1081°S, 30 m water depth) [2].

Data in Table 7.4 demonstrate that the annual power production for

Table 7.4: Comparison of power production characteristics of two control strategies implemented on the three-tether WEC depicted in Figure 7.15.

Performance index		Spring-damper control	Velocity tracking control
Average power-net, kW	$\frac{1}{T} \int_T P(t) dt$	496.0	567.4
PTO 1		46.7	-171.2
PTO 2,3		224.7	369.3
Average power-in, kW	$\frac{1}{T^{in}} \int_{T^{in}} P^{in}(t) dt$	480.2	653.3
PTO 1		122.3	1488.5
PTO 2,3		203.0	393.3
Average energy stored, kJ	$\frac{1}{N} \sum_{i=1}^N E_i$	2393.3	1257.6
PTO 1		655.4	5147.0
PTO 2,3		936.6	1561.0
Power-in, peak-to-average		3.9	5.6
Power-net, peak-to-average		14.2	10.8
Total absolute power flow	$\frac{1}{T} \int_T P(t) dt$	938.3	965.1


 Figure 7.28: An example of calculating the power-production metrics for one time series. $P(t)$ is an instantaneous power, $P^{in}(t)$ is the input power used to actuate the WEC, the ‘bar’ symbol $\bar{}$ denotes the average value over time, the subscript ‘peak’ corresponds to the peak values of the signal.

the velocity-tracking control is 15% higher than that for the spring-damper strategy. However, the negative value of the average net power for the PTO 1 confirms that this machinery should be designed to actuate the WEC all the time. The amount of energy required to provide the reactive power flow without using power from external sources is shown as ‘average energy stored’. When all three PTO units have independent circuits (e.g. hydraulic), the VTC strategy involves much higher energy storage for each machinery, especially for PTO 1. However, it is more beneficial to design shared machinery, where power can flow between three PTO systems (e.g. inside the buoy), as

this scenario allows the velocity-tracking controller to be implemented to the three-tether WEC with a reduction in energy storage costs as compared to the spring-damper control (1257.6 kJ vs. 2393.3 kJ).

7.2.5 Conclusion

The SVD-based velocity tracking control has been applied to the submerged multi-degree-of-freedom WEC. This controller is causal, robust, and achieves the optimal amplitude and phase between the buoy velocity and the excitation force in both heave and surge modes. The analysis of the controller sensitivity has revealed that while the low-level control loop is robust, the high-level reference point generator requires a high-fidelity model of the WEC in order to satisfy the optimal phasing condition. The performance of the proposed controller has been compared with a quasi-standard spring-damper approach, where a 15% increase in power production is achieved for the deployment site considered. It has been found that the velocity-tracking control implemented on the three-tether converter requires one of the PTO units to operate as an actuator all the time. However, this drawback can be compensated by the shared power take-off machinery across all the tethers.

7.2.6 Acknowledgement

This research has been supported by the Australian Government Research Training Program Scholarship. The authors would like to thank Dr Ashkan Rafiee for providing the parameters of the WEC and the discussions of the results. Also, the assistance of Dr Giorgio Bacelli in calculating the power production measures is gratefully acknowledged.

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Chapter 8

Conclusion

8.1 Overall conclusions

The research work presented in this thesis is devoted to the study of the wave energy converter which is bottom-referenced, fully submerged, has an axisymmetric shape and attached to three power take-off units through flexible tethers. This WEC concept benefits from multi-degree-of-freedom control of the absorbing body that leads to a higher power production. The performance and behaviour of the three-tether WEC were analysed using frequency and time domain models based on linear wave theory with included second-order effects due to the viscous drag force. Each power take-off unit is modelled as a linear spring-damper system with tunable control parameters. The main attention of this study was given to the power production mode of the wave energy converter in order to introduce the concept, estimate the annual power output, and assess the impact of design variables on the WEC performance.

As a starting point of this work, it was necessary to determine the characteristics that distinguish fully submerged converters from the floating ones. It was found that fully submerged WECs are excited by ocean waves in the limited range of wave frequencies, acting as a bandpass filter, in contrast to the floating systems with the properties of a low-pass filter. This difference in hydrodynamics of both technologies affects the controllability, scalability and overall power performance of each system. As a result, the correct choice of the size and shape of the absorbing buoy is more critical for the fully submerged converters. Moreover, in order to generate the same amount of energy, submerged WECs should have a larger volume than their floating counterparts of the same shape. It is generally assumed that oscillating bodies controlled in surge, heave and pitch modes are able to absorb three times

more power than just heaving devices. However, it turned out that this assertion is true for both submerged and floating converters but only at relatively short wave periods. In ocean waves with longer wavelengths, the contribution of the surge mode to the power absorption of floating systems is marginal, while its effect is still significant for the fully submerged WECs.

Investigating the effect of design parameters on the performance of the three-tether WEC, it was demonstrated that a cylindrical buoy with a relatively small (<0.25) height to diameter ratio has the highest power output and best economic measures as compared to other shapes and aspect ratios. A comparison was also made between the single-tether and three-tether technologies using data from six publicly available cost models in order to analyse the economic efficiency of the latter. As a result, despite the increased number of power take-off units and mooring points, the three-tether concept has the potential to produce cheaper electricity while doubling the power output.

The focus of the last part of this research was on the design of the control system for the three-tether WEC to maximise its energy production. Using one causal controller as an example, it was demonstrated how to extend the control approach developed for a single-degree-of-freedom system to a three-tether converter. Moreover, it was found that the control strategy which aims to achieve an optimal phase condition between the buoy velocity and the wave excitation force in both heave and surge modes requires one out of three PTO units to operate as an actuator while not generating any power and using energy from the local storage or electrical network.

More detailed conclusions are provided in each article included in this thesis.

8.2 Original contributions

The author considers that the main contributions of this thesis to the field of wave energy are as follows:

1. demonstrating the significance of the buoy dimensions and submergence depth on the power absorption ability of the fully submerged converters;
2. comparing, based on numerical simulations, the performance and power output of the floating and fully submerged converters with the same buoyant actuator;

3. identifying the advantage of controlling the submerged system in multiple hydrodynamic modes;
4. applying a numerical method for determining the optimal arrangement of mooring lines for a three-tether system in regular and irregular waves;
5. identifying the optimal buoy shape and aspect ratio of the three-tether system based on the techno-economic performance measures;
6. demonstrating the economic benefits of employing three tethers in comparison with using one tether for wave power harvesting;
7. highlighting the importance of tuning the control system parameters to the energy period of the excitation force instead of the peak frequency of the wave elevation;
8. investigating and developing several techniques for decoupling the multi-degree-of-freedom control system of a three-tether converter;
9. quantifying the challenges associated with the implementation of the close-to-optimal control strategy on a three-tether system;
10. demonstrating that application of an advanced control strategy can result in forcing one of three power take-off units to operate as an actuator all the time.

8.3 Recommendations for future work

The work presented in this thesis is focused on the initial development stage of a submerged bottom-referenced wave energy converter connected to a three tether system. This covers the first two out of nine technology readiness levels proposed to assess the maturity and economic viability of a new technology.

In order to validate the conceptual model and go from idea to commercialisation, several areas for further research are identified throughout this thesis.

High fidelity numerical models

The main assumption in this thesis is related to the small motion amplitude and use of linear wave theory. However, a submerged WEC operating in close proximity to the water surface can be subjected to nonlinear forces and large displacements discussed in Section 3.2. Therefore, high fidelity non-linear

numerical models are required in order to identify the limits of linear wave theory and develop new knowledge about the hydrodynamics of subsurface converters.

Laboratory scale experiments

The laboratory testing of different scale prototypes (1:100–1:20) should be conducted to validate: (i) the hydrodynamic models of the WECs used in this thesis (submerged sphere and submerged cylinder); (ii) the optimal arrangement of mooring lines for different submergence depth, buoy shape and mass (outcomes of Chapter 5); and (iii) the performance of the control system (outcomes of Chapter 7). In addition, there is a need for experimental studies to estimate the mooring and structural loads on the system in extreme sea states.

The preliminary experiments carried out in the wave flume at the University of Adelaide, Australia, are demonstrated in Appendices A and B. The testing was performed for the submerged spherical single-tether system (scale 1:73.5) using a servo-motor as a virtual power take-off machinery. These experiments allowed one to identify the limitations of the proposed PTO assembly, and to determine the relationship between motor size and parasitic losses in the set-up. These results can be used when choosing the appropriate hardware for the three-tether test rig which requires the simultaneous use of three servo-motors.

Parametric excitation

While working on the controller for the three-tether WEC, it was found that the converter may experience parametric excitation between heave and pitch modes due to the symmetry of the system. This phenomenon has to be studied further in order to identify regimes and frequencies of instability. Moreover, it should be investigated whether the parametric excitation can be exploited to harvest more power, or to decrease structural loads on the buoy in strong sea states.

Control system development

In this thesis, only one control strategy was implemented for a three-tether WEC. It is necessary to assess the performance of other effective control approaches and estimate their requirements for the power take-off machinery. Moreover, it is required to study and compare two different configurations of the PTO system: (i) when all tethers are attached to individual generators

(used in this thesis); and (ii) when the tethers are connected to one generator located inside the buoy.

Array performance

The final development stage of any wave energy converter is related to the performance of a wave farm. The development of the array of three-tether WECs raises a number of research questions including: how the arrangement and distance between WECs that operate in surge and heave modes affect the performance of the array? how to decrease the loading on the foundation when three-tether WECs share mooring points in a staggered array? how to design a global (centralised) control strategy which maximises the power output of the array?

During the course of this thesis, the frequency domain model of the array of three-tether WECs with submerged spherical buoys was developed based on linear wave theory. This model was used to identify the effect of the buoy radius on the power output of a staggered array and apply some single- (Appendix C) and multi-objective (Appendix D) optimisation procedures which can be potentially used in the wave energy sector.

Appendix A

Study of fully submerged point absorber wave energy converter – modelling, simulation and scaled experiment

Ding, B., da Silva, L. S. P., Sergiienko, N., Meng, F., Piper, J. D., Bennetts, L., Wagner, M., Cazzolato, B. and Arjomandi, M. (2017). “Study of fully submerged point absorber wave energy converter-modelling, simulation and scaled experiment”. In: *The 32nd International Workshop on Water Waves and Floating Bodies (IWWWFB)*. Dalian, China.

Statement of Authorship

Publication Status	Published
Name of Co-author	Nataliia Y. Sergiienko
Contribution to the Paper	Provided technical support for the experimental study presented in the paper: developed MATLAB/Simulink code to be executed on dSpace, calibrated the PTO assembly and quantified the parasitic losses in a set-up.
Overall Percentage (%)	5%

Study of fully submerged point absorber wave energy converter - modelling, simulation and scaled experiment

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1. INTRODUCTION

Ocean waves are a huge resource of renewable energy. Wave energy converter (WEC) devices are being developed to optimise capture of this energy. A point absorber (PA), whose dimensions are much smaller than a wavelength, is a typical type of WEC with the main advantage being its insensitivity to wave direction. Examples of operational PA devices are the Carnegie CETO and Wavebob. PAs normally float on the water surface since wave power decreases with increasing submergence depth of the device (Falnes, 2002). However, there is an increasing trend to keep PAs fully submerged in order to increase the survivability of the device under extreme weather conditions. Sergiienko et al. (2017) undertook a systematic comparison study on floating and fully submerged PAs based on numerical simulations, which demonstrates that fully submerged PAs have the potential to capture a large amount of power from surge motion of the buoy component, twice as much as the power captured from heave motion of the buoy, whereas floating PAs capture power purely from heave motion of the buoy. This unique characteristic allows fully submerged PAs to capture the equivalent amount of wave energy to floating devices while maintaining high survivability.

High expenditures arise from manufacturing, installation, maintenance, and testing of the full-scale WECs in the open sea. Therefore, prior to the design and commissioning of the full-scale devices, numerical simulations and scaled experiments in wave tanks are required to prove design and control concepts, as well as to estimate their economic efficiencies. Modelling of hydrodynamics resulting from wave-body interaction is fundamental for the numerical simulation of point absorber WECs. Linear wave theory, based on the assumption of small wave amplitudes, is the most popular method to model hydrodynamics due to its high computational efficiency. The computed hydrodynamics of the PA can then be assembled in the frequency-domain. The resulting frequency-domain model can only be used to simulate the behaviour of the PA under regular wave conditions and does not have the capability to model nonlinearities such as drag forces and nonlinear power-take-off (PTO) forces. Alternatively, the hydrodynamics can be substituted into the Cummins equation (Cummins, 1962), a deterministic solution originally developed to investigate ship dynamics in the time-domain. The resulting time-domain model is able to include nonlinear forces and can be used for simulation under both regular and irregular wave conditions. However, it is more computationally expensive than the frequency-domain model. A spectral-domain model has recently been applied to WECs due to its high efficiency in solving nonlinear hydrodynamic problems. It is a probabilistic model of the system dynamics, which uses a statistical representation of the waves, and when passed through an appropriate transformation function produces a probabilistic estimate of the WEC response (Folley, 2010). The principle advantage of using a spectral-domain model over the conventional Cummins models is that for complex nonlinear systems it is computationally more efficient at providing estimates of power absorption under irregular waves.

In addition to numerical simulation, scaled experiments are also used to quantify the performance of WECs. There are few world-class wave tank testing facilities for WECs around the world. Examples are FloWave at the University of Edinburgh, Scotland; Model Test Basin at the Australian Maritime College, Australia; Ocean Engineering Water Tank at Shanghai Jiao Tong University, China; and Wave Basin at Indian Institute of Technology Madras, India. These facilities are normally developed for testing ship dynamics and offshore structures, and therefore are not always available for the testing of WECs. In addition, these facilities are often expensive to access, which is not affordable for many WEC research groups.

Besides discussing and benchmarking the typical tools for the study of fully submerged point absorber WECs, this paper has two main original contributions. Firstly, the spectral-domain model is applied to a fully submerged PA with numerical validation. Secondly, cost-effective scaled experiments are proposed and investigated for a fully submerged PA within a standard wave flume facility. The methods developed in this work could be applied to study floating PAs and even other types of WECs.

2. MATHEMATICAL MODELLING

In this study, a fully submerged PA is considered and the nonlinear force considered in modelling is the quadratic drag force. The PA dynamic equation takes the following form in the frequency-domain:

$$(j\omega(\mathbf{M} + \mathbf{A}(\omega)) + \mathbf{B}(\omega))\hat{\mathbf{x}} = \hat{\mathbf{F}}_e(\omega) + \hat{\mathbf{F}}_b + \hat{\mathbf{F}}_{pto}, \quad (1)$$

where $\mathbf{M}$ is the mass matrix of the buoy; $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are the hydrodynamic added mass and radiation damping matrices, which vary with wave frequency ω ; $\hat{\mathbf{F}}_e(\omega)$ is the wave excitation force, which is also frequency dependent; $\hat{\mathbf{F}}_b$ is the net buoyancy force; $\hat{\mathbf{F}}_{pto}$ is the PTO control force; $\hat{\mathbf{x}}$ is the buoy velocities in heave, surge, and pitch under the assumption of plane incident waves. The hydrodynamic terms $\mathbf{A}(\omega)$, $\mathbf{B}(\omega)$ and $\hat{\mathbf{F}}_e(\omega)$ can be solved using boundary element solvers (e.g. WAMIT, ANSYS AQWA, NEMOH) for specific buoy shapes. For generic buoy shapes (e.g. sphere and cylinder), the coefficients can be solved using an analytical model developed by Srokosz (1979) and Jiang et al. (2014).

The Cummins equation takes the following form in the time-domain:

$$(\mathbf{M} + \mathbf{A}(\infty))\ddot{\mathbf{x}}(t) + \int_0^t \mathbf{B}(t - \tau) \dot{\mathbf{x}}(\tau) d\tau = \mathbf{F}_e(t) + \mathbf{F}_b(t) + \mathbf{F}_{pto}(t) + \mathbf{F}_d(t), \quad (2)$$

where $\mathbf{A}(\infty)$ is the infinite-frequency added mass matrix ($\mathbf{A}(\omega)$ for $\omega = \infty$); $\int_0^t \mathbf{B}(t - \tau) \dot{\mathbf{x}}(\tau) d\tau$ represents the memory effect of the radiation force, which can be approximated as a transfer function and solved by using a Matlab toolbox developed by Perez and Fossen (2009). The wave excitation force time series $\mathbf{F}_e(t)$ can be calculated for both regular waves at a single frequency ω and irregular waves based on wave spectra models. Typical wave spectra models are JONSWAP, Bretschneider and Pierson-Moskowitz; $\mathbf{F}_d(t)$ is the quadratic nonlinear drag force that takes the form $\frac{1}{2}\rho C_d A |\dot{\mathbf{x}}| \dot{\mathbf{x}}$ on each dimension, where ρ is the density of seawater, C_d is the drag coefficient usually determined from experiments for particular buoy shapes, and A is the cross-sectional area of the buoy perpendicular to the motion direction.

The spectral-domain model is formulated using the same construction as the frequency-domain model and the response of the PA at each wave frequency can be obtained from (Folley, 2010)

$$(j\omega_i(\mathbf{M} + \mathbf{A}(\omega_i)) + (\mathbf{B}(\omega_i) + \mathbf{G}(\omega_i)))\hat{\mathbf{x}}_i = \hat{\mathbf{F}}_e(\omega_i) + \hat{\mathbf{F}}_b + \hat{\mathbf{F}}_{pto}, \quad (3)$$

where $\mathbf{G}(\omega_i)$ is the quasilinear drag coefficients linearised from the quadratic nonlinear drag force and takes the form $2C_d \sqrt{\frac{1}{\pi} \sum_i \omega_i^2 |x_i|^2}$ on each dimension. The response of the PA at each frequency is dependent not only on hydrodynamic coefficients at that frequency, but also the response of the PA at all frequencies due to the nonlinear force. Total response under irregular waves can then be calculated by superposition of all the wave components. There is no known analytical solution to the spectral-domain model, so an iterative solver has to be used to estimate the response of the PA for the estimation the quasilinear drag coefficients.

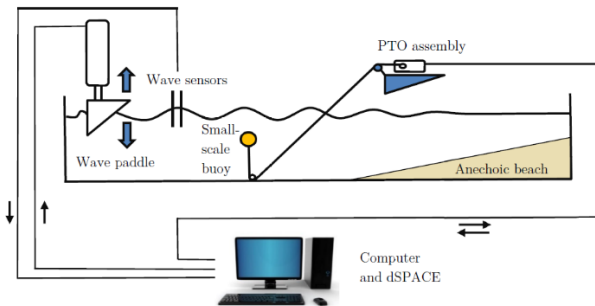


Figure 1. Scaled experiment set-up in the wave flume



Figure 2. Buoy assembly (0.136m diameter sphere)



Figure 3. PTO assembly (from left to right: rope spool, shaft coupling, motor-encoder-housing, torque sensor)

3 SMALL-SCALE EXPERIMENT

Figure 1 shows the set-up of the scaled experiment in a standard wave flume facility located in the Chapman lab within the University of Adelaide, which is 32m long, 1.3m wide and 1m deep. A piston-type hydraulic-driven wave paddle is placed at the upstream end of the flume. The up and down motion of the wave paddle generates propagating waves. A perforated anechoic beach sits at the downstream end of the flume, which is used to prevent waves from reflecting back. A fully submerged PA is placed at 5m downstream of the wave paddle, where the generated waves demonstrate the desired regularity. A small boat pulley is placed at the bottom of the flume forming a mooring point. The PA is anchored via a high strength fishing line, which goes through the pulley and rolls on the rope spool within a custom-built PTO system. Six custom-built wave probes are placed around the PA to monitor surrounding wave conditions, two upstream, two downstream, and two at the sides. A DS1104 dSPACE controller is coded via Matlab to control the wave conditions generated by the paddle based on the wave probe readings, as well as control the PTO system to apply desired force on the PA. It also collects data from wave probes, IMU, and motor for post-processing.

Figure 2 shows the buoy assembly of the PA, whose shell is 3D printed from VisiJet M3. The inner space of the buoy shell is used to place an IMU (LORD MicroStrain 3DM-GX4-25) that measures buoy movements and additional weights that allow the change of the net buoyancy and the centre of gravity of the buoy. The shell of the buoy is closed via screw fixation and sealed by double O-rings. The IMU cable exits the buoy shell via a cable gland fixed at the top of the shell. A U tunnel profile is printed at the bottom of the buoy shell, forming an anchoring point for the tether. The dimension of the 3-D printed buoy (0.136m diameter) is 1/73.5 of the full-scale buoy (10m diameter) so that the blockage ratio of the buoy to the flume width is less than 12%. A higher blockage ratio has the risk of modifying the scattering force on the buoy. The ratio (1/73.5) was also chosen considering the similarity of drag coefficients between small/full scales.

Figure 3 shows the PTO assembly that is used to emulate any PTO behaviour. A Maxon RE50 motor drives a rope spool via a shaft coupling and consequently applies PTO force to the PA via pulley-tether coupling. The shafts of the rope spool and motor sit on three rolling-element bearings fixed to the base. The housing of the motor is attached to the base via an aluminium shell and a Lorenz Messtechnik GmbH D-2209 torque sensor. The torque sensor measures the torque applied by the motor. A motor encoder measures the tether displacement. The motor is selected to generate sufficient PTO force to overcome the buoy net buoyancy force and the Coulomb friction force within the PTO assembly, as well as to generate the desired PTO force. Assuming generic linear spring-damper control, the torque command for the motor takes the following form:

$$T_m = -(\rho V - m)g \cdot r + \text{sign}(\dot{\theta}) \cdot c_c + k_{pto} \cdot \theta \cdot r^2 + c_{pto} \cdot \dot{\theta} \cdot r^2, \quad (4)$$

where V is the displaced water due to the buoy; m is the buoy mass; g is the gravitational acceleration; r is the radius of the rope spool; θ is the angular displacement of the motor; c_c is the Coulomb friction coefficient of the PTO assembly; k_{pto} is the scaled PTO spring stiffness; c_{pto} is the scaled PTO damping coefficient. From Eq. 4, it can be seen that the selection of the rope spool radius r compromises the maximum required motor torque and the magnitude of the Coulomb friction torque within the total required motor torque. A current feedback control is applied to track the desired motor torque command calculated from Eq. 4, in order to achieve a desired spring-damper behaviour. Since there is noise within $\dot{\theta}$ arising from differentiating the encoder reading, $\text{sign}(\dot{\theta}) \cdot c_c$ is replaced by a relay function and a low pass filter is applied to remove noise within $\dot{\theta}$ for PTO damping control. The selection of the filter cut-off frequency compromises the phase delay introduced into damping control and the noise attenuation capability.

4. RESULTS

A benchmark study has been undertaken between the modelling methods and the scaled experiment under both regular and irregular wave conditions. Table 1. summarises the full-scale PA and scaled PTO properties.

Table 1. Configuration for the benchmark study

PA property (sphere buoy)	Values	Wave property (Pierson-Moskowitz spectra used)	Values	PTO property (for 1/73.5 scale ratio)	Values
Water depth	48m	Wave amp (regular)	1.5m	Filter cut-off frequency	50Hz
Buoy radius	5m	Wave period	7, 9, 12s	Relay threshold	eps in Matlab
Submergence depth	8m	Wave height (irregular)	3m	r	15mm
Mass to buoyancy ratio	0.5	Energy wave period	7, 9, 12s	c_c	0.012Nm

Before conducting testing in the wave flume, the buoy was attached to the PTO system without interacting with water to validate the control performance of the PTO system. A decay testing was undertaken by realising the buoy 50mm away from its equilibrium position under spring-damper control. It was evident that the buoy oscillates around its natural frequency $\sqrt{k_{pto}/m}$ and the oscillation decays exponentially almost under the defined PTO damping, c_{pto} . In addition, the hydrodynamic coefficients of the scaled buoy were measured from wave flume experiment, which are similar to the ones computed from linear wave theory, with errors within 20%.

Figure 4 shows the power outputs of the PA resulting from the frequency-domain model (Eq. 1), the time-domain model (Eq. 2) and the scaled experiment under regular wave conditions (power averaged over 16 runs for each wave period, where power in each run averaged over 30 wave periods). The PTO spring and damper were optimised for each wave period subjected to a buoy motion constraint of 3m using the frequency-domain model. The frequency-domain model overestimates the power absorption efficiency of the PA because the nonlinear drag force is not considered. The time-domain model gives much closer results to the experiments, with errors within 10%. Figure 5 shows the power outputs of the PA resulting from the time-domain model (Eq. 2) and the spectral-domain model (Eq. 3) under irregular wave conditions, averaged over 300 peak wave periods (experimental data under irregular waves are still under investigation). The PTO spring and damper were optimised for each energy wave period. The spectral-domain model results match the time-domain model results reasonably well, with errors within 10%.

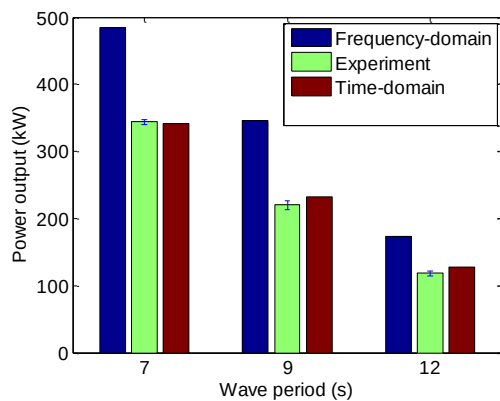


Figure 4. Power outputs under regular waves

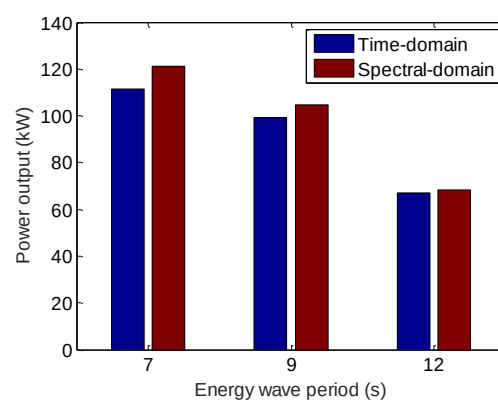


Figure 5. Power outputs under irregular waves

5. CONCLUSION

- We successfully applied a spectral-domain model to fully submerged PA WECs with numerical validation against the conventional time-domain nonlinear model.
- The proposed cost-effective scaled experiment demonstrated high fidelity against the time-domain model, with errors increasing with the increase of wave period. Under long waves, high PTO damping was required to keep the buoy fully submerged, leading to slow tether motion, where the parasitic loss in the PTO assembly cannot be properly compensated.
- More experimental studies are being undertaken to understand the optimal control solution to compensate for parasitic loss within the set-up, especially when the tether motion is slow. One potential solution that will be investigated is the use of dither signals.

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Appendix B

Power take-off control in a scaled experiment of a point absorber wave energy converter

Ding, B., Sergiienko, N., Bleckly, B., da Silva, L. S. P., Cazzolato, B., and Arjomandi, M. (2017). "Power-take-off control in a scaled experiment of a point absorber wave energy converter". In: *Proceedings of the 12th European Wave and Tidal Energy Conference*. Ed. by A. Lewis. Cork, Ireland, pp. 1077-1–1077-7.

Statement of Authorship

Publication Status	Published
Name of Co-author	Nataliia Y. Sergiienko
Contribution to the Paper	Developed MATLAB/Simulink code to be executed on dSpace, calibrated the PTO assembly and quantified the parasitic losses in a set-up, provided the data and plot for Section IV.A, helped in interpretation of obtained results, edited the manuscript.
Overall Percentage (%)	10%

Power-take-off Control in a Scaled Experiment of a Point Absorber Wave Energy Converter

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Abstract— This paper investigated the design of a power-take-off control system for the study of a model scaled point absorber wave energy converter in a wave flume experiment, where the scale of the experiment must be kept low to reduce the blockage effects of the buoy in the wave flume. In keeping the model small, the parasitic loss in the experimental set-up due to friction in the power-take-off was significant in comparison to the power absorption capability of the model scaled wave energy converter, and consequently, the power absorbed by the wave energy converter was significantly underestimated. To address this major limitation, a simple friction compensation algorithm was tested in the power-take-off control. Experimental results were benchmarked against those obtained from potential wave theory solvers for the understanding of the efficacy of the proposed method. It was evident that the proposed power-take-off control provided high fidelity results under both regular and irregular incident waves as long as the dynamic power-take-off control force was able to overcome the friction force in the experimental set-up.

Keywords— Point absorber, model-scale experiment, wave flume, power-take-off control, parasitic loss compensation

I. INTRODUCTION

A point absorber (PA) wave energy converter (WEC) is a typical type of WEC with the main advantage being its insensitivity to wave direction [1]. Examples of operational PA devices are the Carnegie CETO and Wavebob. Prior to the manufacturing and commissioning of full-scale PA WECs in the open sea, model scaled experiments in wave tanks are required to fully understand the efficiency of the design as well as its survivability [2]. There are some world-class wave tank testing facilities around the world such as Flowave at the University of Edinburgh, Model Test Basin at the Australian Maritime College, and the Ocean Engineering Water Tank at Shanghai Jiao Tong University. A general guide for conducting model scaled experiments on WECs is presented in [3], under the assumption that a dedicated wave tank testing facility is available, which is however not always the case.

This work aims to address the challenges of undertaking model scaled experiments on point absorber WECs [4, 5] in a standard wave flume that many universities have. The main challenge arises from the scale of the experimental set-up in the wave flume. In order to minimise the effects of side wall reflection on the PA, the scale of the buoy must be sufficiently

small so that the blockage ratio of the buoy in the flume is lower than 15% [6]. As the scale of the experiment is reduced, the parasitic losses in the experimental set-up (e.g. motor friction) become significant in comparison to the desired power-take-off (PTO) control force, affecting the total control force applied to the PA and therefore leading to significantly less power absorbed by the PA. Although friction models and their compensation methods have been widely studied [7], there is no evidence of its application to the research field of WEC PTO control.

This paper investigated the design of model scaled experiments on a fully submerged point absorber WECs within a standard wave flume facility (32x1.3x1m) located at the University of Adelaide, with an emphasis on the methodology of mitigating the influence of parasitic loss in the model scaled experiments. The method was briefly introduced in [8], however more details and new results are discussed in the current paper. The arrangement of the paper is as follows. Section II presents the potential wave theory used for the benchmark of the experimental results. Section III discusses the experimental set-up in the wave flume, in particular the design of the power-take-off control system for addressing the parasitic loss issue. Results and conclusions are discussed in Section IV and Section V respectively.

II. POTENTIAL WAVE THEORY

The hydrodynamics of the fully submerged point absorber WECs can be solved using potential wave theory, assuming the fluid is incompressible and inviscid, which is reliable as long as the incident waves are small [9]. The computed hydrodynamics of the PA can then be assembled in the frequency-domain. The resulting frequency-domain model can only be used to simulate the behaviour of the PA under regular wave conditions and does not have the capability to model nonlinearities such as drag forces and nonlinear PTO forces. Alternatively, the hydrodynamics can be substituted into the Cummins equation [10], a deterministic solution originally developed to investigate ship dynamics in the time-domain. The resulting time-domain model is able to include nonlinear forces and can be used for simulation under both regular and irregular wave conditions.

A. Frequency-domain Model

The PA dynamic equation takes the following form in the frequency-domain:

$$(j\omega(\mathbf{M} + \mathbf{A}(\omega)) + \mathbf{B}(\omega))\hat{\mathbf{x}} = \hat{\mathbf{F}}_e(\omega) + \hat{\mathbf{F}}_b + \hat{\mathbf{F}}_{pto}, \quad (1)$$

where $\mathbf{M}$ is the mass matrix of the buoy; $\mathbf{A}(\omega)$ and $\mathbf{B}(\omega)$ are the hydrodynamic added mass and radiation damping matrices, which vary with wave frequency ω ; $\hat{\mathbf{F}}_e(\omega)$ is the wave excitation force, which is also frequency dependent; $\hat{\mathbf{F}}_b$ is the net buoyancy force; $\hat{\mathbf{F}}_{pto}$ is the PTO control force; $\hat{\mathbf{x}}$ is the buoy velocities in heave, surge, and pitch under the assumption of plane incident waves. The hydrodynamic terms $\mathbf{A}(\omega)$, $\mathbf{B}(\omega)$ and $\hat{\mathbf{F}}_e(\omega)$ can be solved using boundary element solvers (e.g. WAMIT, ANSYS AQWA, NEMOH) for specific buoy shapes. For generic buoy shapes (e.g. sphere and cylinder), the coefficients can be solved using an analytical model developed by [11] and [12].

B. Time-domain Model

The Cummins equation takes the following form in the time-domain:

$$(\mathbf{M} + \mathbf{A}(\infty))\dot{\mathbf{x}}(t) + \int_0^t \mathbf{B}(t - \tau) \dot{\mathbf{x}}(\tau) d\tau = \mathbf{F}_e(t) + \mathbf{F}_b(t) + \mathbf{F}_{pto}(t) + \mathbf{F}_d(t), \quad (2)$$

where $\mathbf{A}(\infty)$ is the infinite-frequency added mass matrix ($\mathbf{A}(\omega)$ for $\omega = \infty$); $\int_0^t \mathbf{B}(t - \tau) \dot{\mathbf{x}}(\tau) d\tau$ represents the memory effect of the radiation force, which can be approximated as a transfer function. The wave excitation force time series $\mathbf{F}_e(t)$ can be calculated for both regular waves at a single frequency ω and irregular waves based on wave spectra models. Typical wave spectra models are JONSWAP, Bretschneider and Pierson-Moskowitz; $\mathbf{F}_d(t)$ is the quadratic nonlinear drag force that takes the form $\frac{1}{2}\rho C_d A |\dot{\mathbf{x}}| \dot{\mathbf{x}}$ on each dimension, where ρ is the density of seawater, C_d is the drag coefficient usually determined from experiments for particular buoy shapes, and A is the cross-sectional area of the buoy perpendicular to the motion direction.

III. EXPERIMENTS

Fig. 1 shows the set-up of the scaled experiment in a standard wave flume facility located in the Chapman Laboratory within the University of Adelaide, which is 32m long, 1.3m wide and 1m deep. A piston-type hydraulically-driven wave paddle is placed at the upstream end of the flume. The up and down motion of the wave paddle generates propagating waves. A perforated anechoic beach sits at the downstream end of the flume, which is used to prevent waves from reflecting back. A fully submerged PA is placed 5m downstream of the wave paddle. A small pulley is placed at the bottom of the flume forming a mooring point. The PA is anchored via a high strength non-compliant fishing line, which passes through the pulley and winds onto the rope spool within a custom-built PTO system. Six custom-built wave probes are placed around the PA to monitor surrounding wave conditions, two upstream, two downstream, and two at the sides. A DS1104 rapid prototyping

dSPACE was coded via Matlab to control the wave conditions generated by the paddle based on the wave probe readings, as well as control the PTO system to apply desired PTO control force on the PA. It also collects data from wave probes and PTO unit for post-processing.

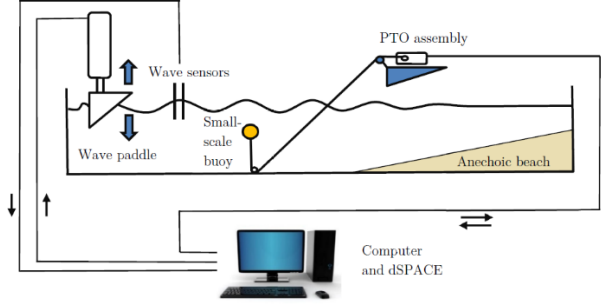


Fig. 1. Layout of the experimental set-up in the wave flume

A. Design of PA Buoy Assembly

In this study, a spherical fully submerged buoy was used due to its simple hydrodynamics. Fig. 2 shows the buoy assembly of the PA, whose shell was 3D printed from VisiJet M3. The inner space of the buoy shell is used to place an IMU (LORD MicroStrain 3DM-GX4-25) that measures buoy movements and additional weights that allow the change of the net buoyancy and the centre of gravity of the buoy. The shell of the buoy is fixed via screws and sealed by double O-rings. The IMU cable exits the buoy shell via a cable gland fixed at the top of the shell. A U channel profile is printed at the bottom of the buoy shell, forming an anchoring point for the tether. The dimension of the 3-D printed buoy (0.136m diameter) is 1/73.5 of the full-scale buoy (10m diameter) so that the blockage ratio of the buoy to the flume width is 11%. A higher blockage ratio has the risk of modifying the scattering force on the buoy. The ratio (1/73.5) was also chosen considering the similarity of drag coefficients between small/full scales.

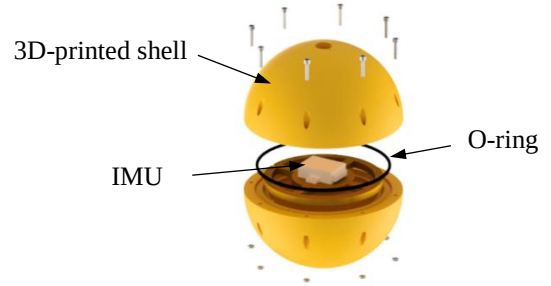


Fig. 2. Spherical buoy assembly

B. Design of Power-take-off Control System

Fig. 3 shows the PTO assembly design that is used to actively generate any desired PTO control force on the PA and therefore to emulate any PTO behaviour. A Maxon RE50 motor drives a rope spool via a shaft coupling and consequently applies a PTO force to the PA via pulley-tether coupling. The shafts of the rope spool and motor are held in position by three rolling-

element bearings fixed to the base. The housing of the motor is attached to the base via an aluminium shell and a Lorenz Messtechnik GmbH D-2209 torque sensor. The torque sensor measures the torque applied by the motor and therefore measures the total PTO control force. A motor encoder measures the motor/tether displacement, and therefore measures the displacement of the emulated PTO.

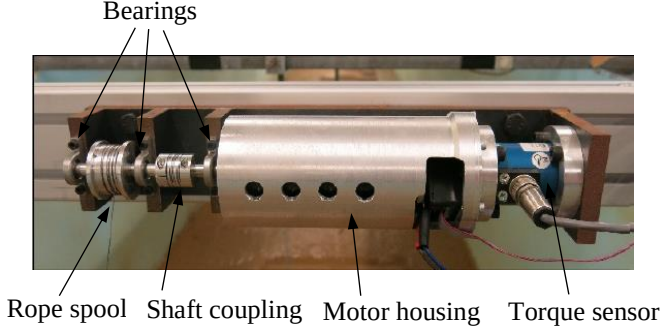


Fig. 3. PTO assembly design

The linear spring-damper is the most generic PTO design, and therefore is used as a typical example for the tests of the proposed PTO control system in this paper. Therefore, the desired PTO control force along the tether is given by

$$f_{pto} = -(\rho V - m)g + k_{pto} \cdot l_d + c_{pto} \cdot \dot{l}_d, \quad (3)$$

where V is the displaced water when the buoy is fully submerged; m is the buoy mass; g is the gravitational acceleration; k_{pto} is the model scaled PTO spring stiffness; c_{pto} is the model scaled PTO damping coefficient; and l_d is the displacement of the tether. This PTO control force along the tether can be mapped to a torque generated by the motor in the PTO assembly:

$$T_m = -(\rho V - m)g \cdot r + k_{pto} \cdot \theta \cdot r^2 + c_{pto} \cdot \dot{\theta} \cdot r^2, \quad (4)$$

where r is the radius of the rope spool; θ is the instantaneous angular displacement of the motor that can be measured by the motor encoder. This motor control torque is computed in the dSPACE controller and is converted to a motor current command in real-time. The motor current command is input into a Maxon ESCON 70/10 controller via an analog signal. The Maxon ESCON controller operates a current control loop that continuously applies the desired control torque to the motor.

As the scale of the experiment is small, the Coulomb friction within the PTO assembly is significant in comparison to the desired PTO control force, and therefore must be compensated in the motor torque control:

$$T_m = -(\rho V - m)g \cdot r + \text{sign}(\dot{\theta}) \cdot c_c + k_{pto} \cdot \theta \cdot r^2 + c_{pto} \cdot \dot{\theta} \cdot r^2, \quad (5)$$

where c_c is the Coulomb friction coefficient of the PTO assembly. Since there is noise within $\dot{\theta}$ arising from differentiating the encoder reading, $\text{sign}(\dot{\theta}) \cdot c_c$ is replaced by a relay function and a low pass filter is applied to remove noise within $\dot{\theta}$ for the PTO damping control. The cut-off frequency of the low-pass filter must be sufficiently high ($>50\text{Hz}$) so that the phase of the PTO damping force is not largely affected, which is important from the power absorption point of view.

By observing Eqs (3-5), the following statements can be made for the design of the PTO assembly:

- 1) The motor needs to be selected to generate sufficient PTO control force including the static pretension force that overcomes the net buoyancy force of the buoy as well as the dynamic force arising from the desired characteristics of the PTO system.
- 2) The selection of the rope spool radius r compromises the maximum PTO control force that can be generated by the motor and the magnitude of the Coulomb friction within the total motor control torque. An optimal value of r exists depending on the characteristics of the motor and the required PTO control force.
- 3) The tether can become slack if the viscous force exceeds the spring (and pretension) force when the tether is retracting. This can only happen under excessively high sea states when both the damping constant the tether velocity are high. The slackness of the tether will not only compromise the power absorption capability of the point absorber but also degrade the survivability of the device in practice. Therefore, to prevent the tether from slack in a real tether-moored fully submerged point absorber system, either the pretension force needs to be sufficiently high by reducing the buoy mass [13], or the device needs to be switched from power generation mode to survival mode by using detuned PTO control.

The instantaneous power absorbed by the linear spring-damper PTO system can be estimated by

$$P_{inst} = c_{pto} \cdot \dot{\theta}^2 \cdot r^2. \quad (6)$$

The mean absorbed power can then be obtained by averaging the instantaneous power over a certain period.

TABLE I
EXPERIMENTAL CONFIGURATION

PA property	Values in both full-scale and (small-scale)	PTO property	Values
Water depth	48m (0.65m)	Rope spool radius	15mm
Buoy radius	5m (0.068m)	Rope spool & motor shaft inertia	0.273kg
Submergence depth (buoy centre to water surface)	8m (0.109m)	Coulomb friction	0.012Nm
Buoy mass	264048kg (0.665kg)	Relay threshold	ϵps
Buoy mass to buoyance ratio	0.5	Cut-off frequency of the low-pass filter	50Hz

IV. RESULTS

Table I illustrates the configuration of the experiments for the assessment of the proposed PTO control system. The parameters of the full scale system are typical for a fully submerged point absorber WEC. Rope spool radius and inertia are 15mm and 0.273kg (along the tether) respectively. The total Coulomb friction coefficient is 0.012Nm, the identification of which is described in Section IV-A. The relay function of the friction compensation term $\text{sign}(\dot{\theta}) \cdot c_c$ is defined to have a threshold of *eps* in Matlab. The cut-off frequency of the low-pass filter on the motor encoder velocity output was set to be 50Hz.

A. Identification of Friction in the PTO Assembly

Free rotation tests were implemented on the PTO assembly for the identification of the friction components in the PTO assembly. The motor was run at various speeds, with the equivalent motor torques measured by the torque sensor. At each motor speed, when the motor reached steady state and therefore the torque balance was achieved in the PTO assembly, the motor torque was equal to the total friction torque in the PTO assembly. Fig. 4 shows the test results. The PTO assembly has a stiction of 11mNm, a Coulomb friction of 6mNm, and a viscous friction coefficient of 0.018mNm.s/rad. The viscous friction is negligible for our experiments because the motor velocity is within 6rad/s. It is difficult to compensate for the stiction given its complex dynamic behaviour. In addition, the system is not going to experience stiction as long as it is moving. Therefore, only the Coulomb friction force is compensated in the PTO control algorithm as shown in Eq. (5), where c_c is set to be 12mNm consisting of the Coulomb friction within the PTO assembly (6mNm) and the Coulomb friction within the pulley placed at the bottom of the wave flume that the tether goes through (6mNm).

It was assumed that the variation of the Coulomb friction subjected to tether tension change is negligible. This was because of the small changes in the tether tension force during standard PTO control (3-10N; 0.6-1.8% of the radial capacity of the rotary bearing), as a result of the small model scale used in the experiments. Experiments demonstrated that the Coulomb friction within the system only increased by 10% from zero tether tension to the nominal tether tension.

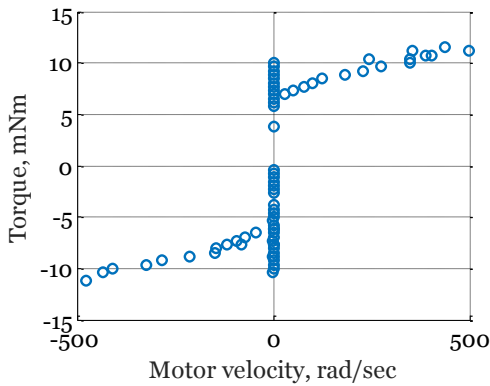


Fig. 4. Motor torque vs motor velocity

B. Dry Tests for the Validation of PTO Control

Before conducting testing in the wave flume, tests were conducted in air where the buoy was attached to the PTO system via the fishing line without interacting with water in order to validate the PTO control algorithm as shown in Eq. (5). The virtual spring k_{pto} was set to the optimal values for various regular wave conditions, which were determined from time-domain simulations. The virtual damper c_{pto} was set to be zero so that if the friction compensation is successful, the system will undergo an undamped oscillation. Free oscillation tests were undertaken by releasing the buoy 30mm away from its equilibrium position under spring-damper and friction compensation control. Results are shown in Table II, where the analytical natural frequency was calculated from $\sqrt{k_{pto}/m}$ and both the measured natural frequency and damper were measured from the free oscillation histories of the buoy. It was evident that under the proposed PTO control, the buoy oscillated around its analytical natural frequency and the damping residue in the system after friction compensation was negligible in comparison to the optimal damping to be applied in the wave flume experiments (usually higher than 3 N.s/m).

TABLE II
COMPARISON OF THE NATURAL FREQUENCY AND DAMPING RESIDUE

Spring value N/m	Damper value N.s/m	Analytical natural frequency Hz	Measured natural frequency Hz	Measured damping residue N.s/m
90	0	1.559	1.561	0.0743
60	0	1.273	1.279	0.1227
30	0	0.900	0.901	0.0924

C. Tests in the Wave Flume

The optimal PTO settings for six typical regular and irregular wave conditions were determined from time-domain simulations subjected to a buoy heave motion constraint of 3m. The 3m constraint was set to prevent the buoy from moving out of the water and therefore avoid the occurrence of higher order nonlinear forces in the experiments. Table III shows the six wave conditions (e.g. three regular wave conditions and three irregular wave conditions) used in simulations and experiments, as well as the corresponding optimal PTO settings applied.

TABLE III
OPTIMAL PTO SETTINGS AT VARIOUS WAVE CONDITIONS (IN SMALL SCALE)

Wave amplitude or (height) (m)	Wave period (s) *energy period for irregular wave	Regular waves		Irregular waves	
		k_{pto} N/m	c_{pto} N.s/m	k_{pto} N/m	c_{pto} N.s/m
1.5 (3)	7	89.6	4.1	71.1	3.1
1.5 (3)	9	54	4.8	53.7	2.9
1.5 (3)	12	29.7	4.3	35.7	4.3

Fig. 5 shows the power output of the PA resulting from the frequency-domain simulation (Eq. (1)), the time-domain simulation (Eq. (2)), and the scaled experiments with friction compensation (Eq. (5)) and without friction compensation (Eq. (4)) under three regular wave conditions. For experiments, at each wave period, the mean power and the standard deviation was calculated from eight experimental runs in the wave flume, where the mean power in each run was averaged over 60 wave periods. As evident in Fig. 5, the frequency-domain simulation overestimated the power absorption efficiency of the PA because the nonlinear drag force was not considered. The experiments without friction compensation significantly underestimated the power absorption capability of the PA because the Coulomb friction within the PTO assembly absorbs the majority of the power arising from the waves. The experiments with friction compensation provided results similar to the ones obtained from time-domain simulation for wave periods of 7s and 9s, with errors within 10%. Even with friction compensation, the experiments at 12s wave period significantly underestimated the power. In addition, there was a large deviation between runs.

To understand the cause of this issue, the time histories of the control torque components on the motor under regular waves of 7s and 12s periods (Fig. 6 and Fig. 7) are analysed. For the 7s wave period case, the magnitude of the dynamic (e.g. spring + damper) control torque was much higher than the magnitude of the stiction torque in the PTO assembly (e.g. 11mNm) and friction compensation worked properly when the buoy changed its motion direction (e.g. the compensation torque always had the sign that was opposite to the Coulomb friction). However, for the 12s wave period case, the magnitude of the dynamic control torque was about the same as the stiction torque in the PTO assembly. Furthermore, the dynamic PTO control torque reached zero during the direction change of the buoy motion (e.g. at 51.3s, 52.7s and 54.1s), in which circumstance the motion of the buoy had a high risk to be caught by the motor stiction (e.g. 54.1s onwards). When the buoy was caught by stiction, no power was generated in the PTO until the buoy was forced to move again. Therefore, power was lost at partial cycles and the number of cycles caught by stiction were random due to the complex dynamics of stiction.

To further validate the conclusion drawn from the previous analysis, additional experimental tests were undertaken for the 12s wave period case, however, with a higher wave amplitude (2m) that resulted in higher wave excitation. Fig. 8 shows the power output of the PA under regular waves of 12s period and 1.5m and 2m amplitudes respectively. It was evident that at wave amplitude of 2m, the experiments provided similar results to the ones obtained from time-domain simulations. The time histories of the control torque components on the motor at regular wave of 12s period and 2m amplitude are shown in Fig. 9. It was evident that the dynamic PTO control torque under 2m amplitude wave was no longer dominated by motor stiction. In addition, the dynamic PTO control torque is non-zero during the direction change of the buoy motion as highlighted in Fig. 9.

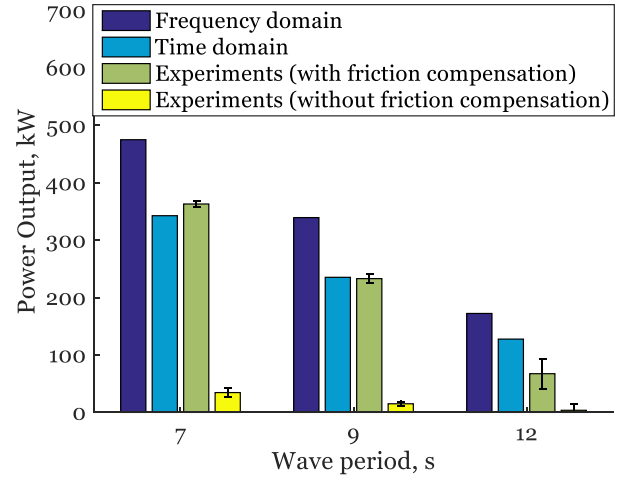


Fig. 5. Comparison of absorbed power (in full-scale) under regular waves with optimal PTO settings shown in Table III. Error bars show standard deviation in experiments.

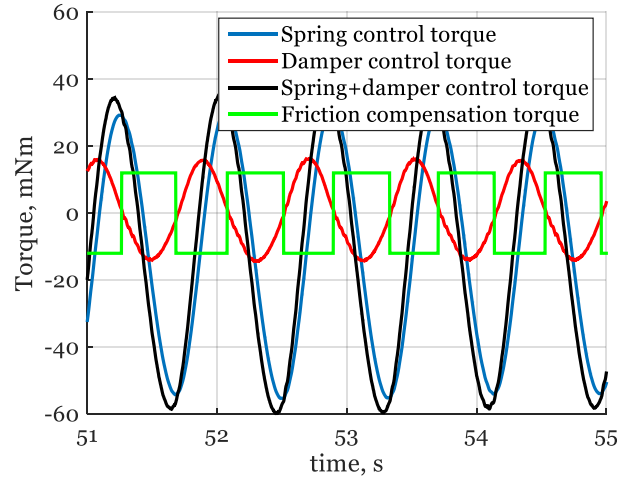


Fig. 6. Time history of control torque components on the motor under regular wave of 7s period and 1.5m amplitude

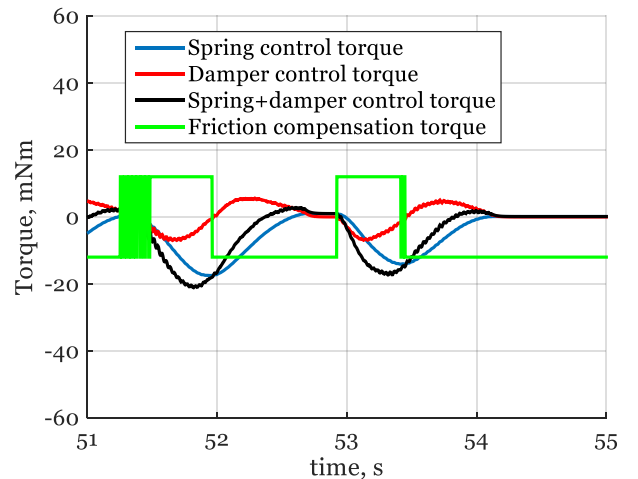


Fig. 7. Time history of control torque components on the motor under regular wave of 12s period and 1.5m amplitude

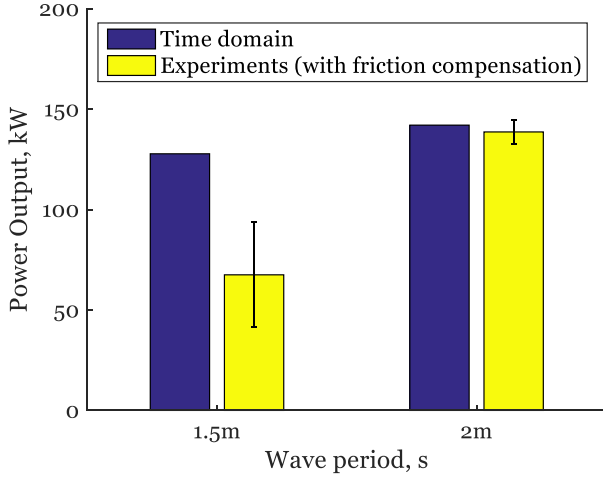


Fig. 8. Comparison of absorbed power (in full-scale) under regular wave of 12s period and different amplitudes. Error bars show standard deviation in experiments.

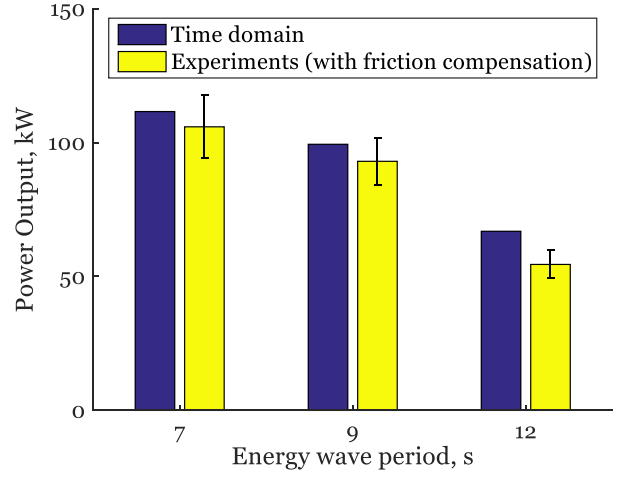


Fig. 10. Comparison of absorbed power under irregular waves with optimal PTO settings shown in Table III. Error bars show standard deviation in experiments.

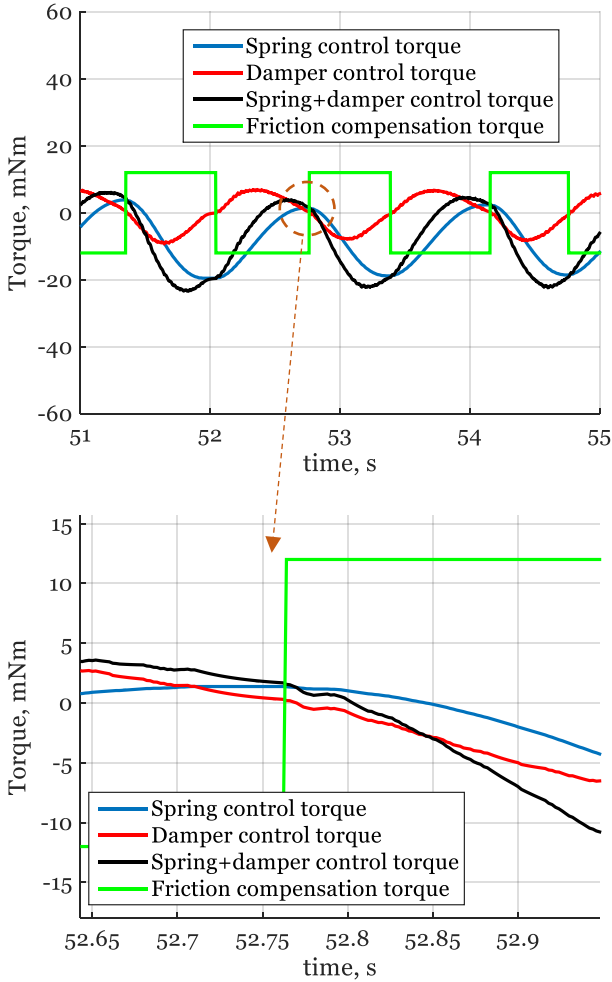


Fig. 9. Time history of control torque components on the motor under regular wave of 12s period and 2m amplitude

Experimental validation was also implemented for the three irregular wave conditions as shown in Table III, with results shown in Fig. 10. At each energy wave period, the mean power and the standard deviation was calculated from eight experimental runs in the wave flume, where the mean power in each run was averaged over 300 wave periods. It was evident that at all the wave periods, the mean power output of the PA obtained from experiments were slightly lower than those obtained from the time-domain simulations. In addition, the standard deviation in the power output between experimental runs were significant. This was due to the low amplitude components at the bounds of the spectrum (Pierson-Moskowitz) of the irregular wave, resulting in some small waves in the whole wave series that were not able to generate excitation force higher than the motor stiction. Therefore, the power output was sensitive to the stiction dynamics for these small waves.

V. CONCLUSIONS

A PTO control system has been designed for the model scaled experimental study on point absorber WECs. A simple friction compensation algorithm has been tested in both regular and irregular wave experiments, with results compared to those obtained from numerical solvers. In general, the PTO control system worked well and provided good estimation to the full-scale system. However, in particular cases, where the dynamic PTO control torque is close to the magnitude of the stiction torque in the PTO assembly, the experiments have the risk to significantly underestimate the power absorption capability of the PA. A potential solution for this problem is to add a high frequency dither signal to the PTO control signal [7], which is currently under investigation by the authors. The authors of this paper are in the process of determining the optimal scale size of the point absorber WEC for the wave flume experiments taking into consideration both the variations and effects of the parasitic loss in the PTO assembly, as well as the effects of

buoy to flume blockage ratio. In addition, a better PTO assembly design is also under investigation, with a physical torsional spring in parallel to the motor to support the static PTO force required to overcome the net buoyancy force and also provide the majority of the PTO spring force so that a smaller motor with less stiction and parasitic loss can be chosen. Follow-up findings will be presented at 2017 EWTEC.

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Appendix C

Fast and effective optimisation of arrays of submerged wave energy converters

Wu, J., Shekh, S., Sergiienko, N. Y., Cazzolato, B. S., Ding, B., Neumann, F., and Wagner, M. (2016). Fast and effective optimisation of arrays of submerged wave energy converters. In: *Proceedings of the 2016 on Genetic and Evolutionary Computation Conference*. Denver, CO, USA, pp. 1045–1052.

Statement of Authorship

Publication Status	Published
Name of Co-author	Nataliia Y. Sergiienko
Contribution to the Paper	Wrote Section 2 of the manuscript, developed the mathematical model of the three-tether WEC array used by the optimisation algorithm, helped in interpretation of obtained results (Figure 9).
Overall Percentage (%)	15%

Fast and Effective Optimisation of Arrays of Submerged Wave Energy Converters

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ABSTRACT

Renewable forms of energy are becoming increasingly important to consider, as the global energy demand continues to grow. Wave energy is one of these widely available forms, but it is largely unexploited. A common design for a wave energy converter is called a point absorber or buoy. The buoy typically floats on the surface or just below the surface of the water, and captures energy from the movement of the waves. It can use the motion of the waves to drive a pump to generate electricity and to create potable water. Since a single buoy can only capture a limited amount of energy, large-scale wave energy production necessitates the deployment of buoys in large numbers called arrays. However, the efficiency of arrays of buoys is affected by highly complex intra-buoy interactions.

The contributions of this article are two-fold. First, we present an approximation of the buoy interactions model that results in a 350-fold computational speed-up to enable the use inside of iterative optimisation algorithms. Second, we study arrays of fully submerged three-tether buoys, with and without shared mooring points.

CCS Concepts

•Computing methodologies → Heuristic function construction; Randomized search;

Keywords

Renewable energy; evolutionary algorithm; wave energy

1. INTRODUCTION

Global energy demand is on the rise, and finite reserves of fossil fuels, renewable forms of energy are playing a more and more important role in our energy supply [20]. Wave energy is a widely available but largely unexploited source of renewable energy with the potential to make a substantial contribution to future energy production [8, 16]. The idea of harnessing wave energy has been around for at least two

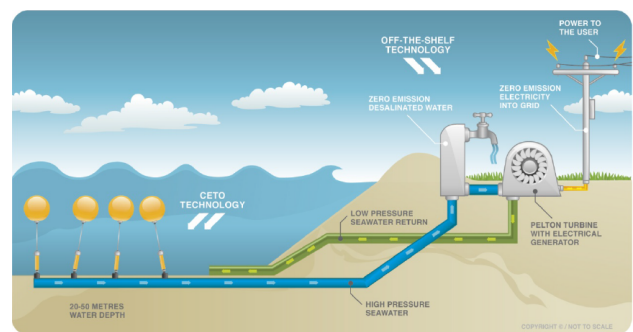


Figure 1: Operation of the CETO system [22].

centuries, with the first patent for a wave energy device being filed in 1799 [9]. However, it was not until the oil crisis of the 1970s and the publication of Stephen Salter’s iconic paper in Nature [26] that interest in wave energy truly began to surge. Since that time, the utilisation of wave energy has continued to be a very active research area. There are currently dozens of ongoing wave energy projects at various stages of development, exploring a variety of techniques [8, 9, 16, 19].

A device that captures and converts wave energy to electricity is often referred to as a wave energy device or wave energy converter (WEC). One common WEC design is called a point absorber or buoy. The buoy typically floats on the surface or just below the surface of the water, and captures energy from the movement of the waves [16]. An example of a point absorber is the CETO wave energy converter, developed by Carnegie Wave Energy and named after the Greek sea goddess Ceto [21]. The CETO system consists of one or more fully submerged buoys that are tethered to the seabed in an offshore location, as shown in Figure 1. These buoys use the motion of the waves to drive a hermetically sealed hydraulic line to drive hydroelectric turbines to generate electricity, or to power a reverse osmosis desalination plant to create potable water [22].

One of the central goals in designing and operating a wave energy device is to maximise its overall energy absorption. As a result, the optimisation of various aspects of wave energy converters is an important and active area of research. Three key aspects that are often optimised are geometry, control, and positioning. Geometric optimisation seeks to improve the shape and/or dimensions of a wave energy converter (or some part of it) with the objective of maximising

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GECCO '16, July 20-24, 2016, Denver, CO, USA

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DOI: <http://dx.doi.org/10.1145/2908812.2908844>

energy capture [23, 24]. On the other hand, the optimisation of control is concerned with finding good strategies for actively controlling a WEC [25]. A suitable control strategy is needed for achieving high WEC performance in real seas and oceans, due to the presence of irregular waves [13]. In this article we focus on the third aspect, namely the positioning of wave energy converters.

A single wave energy converter can only capture a limited amount of energy alone. For large-scale wave energy production and in order to make any significant contribution to addressing global energy demand, it is essential to deploy wave energy devices in large numbers. A group of wave energy devices working in close proximity to one another is referred to as a wave energy farm or array [7]. Just as the optimisation of individual wave energy devices is an area of research, so is the optimisation of arrays of such devices. In the case of arrays, the aspects that are typically optimised include the layout or configuration of the array [6] and active control of individual devices [12].

In the current body of research on wave energy converter arrays and their optimisation, many of the devices under consideration are semi-submerged or floating [4, 6, 11]. In contrast, the CETO WEC is fully submerged beneath the ocean surface [22], as this increases the survivability in high sea states and it has almost no visual impact. There is very limited research into fully submerged wave energy converters. In particular, we are not aware of any research into optimising the placement or configuration of arrays of fully submerged wave energy converters. With this article, we are addressing this issue.

A technological alternative to single-tether CETO WECs are three-tether WECs as shown in Figure 2. The capital cost of such devices are higher than of conventional single-tether heaving buoys due to the increased number of separate power take-off systems for each tether. The total cost of the three-tether WEC array can be reduced significantly, if the layout allows adjacent devices to share the same mooring points (see Figure 3). In this article, we will investigate array layouts for shared mooring points and layouts without shared mooring points.

In order to evaluate arrays, we use a recently developed frequency domain model for arrays of fully submerged three-tether WECs. This model allows us to investigate different parameters, such as number of devices, array layout and buoy size. The ideal choice of parameters leads to an optimisation problem: what is the best combination of buoy radii and their locations for different array sizes?

The structure of the paper is as follows. We introduce the model of interacting three-tether buoys in Section 2 that we base our investigations on. In Section 3, we describe our speed-ups of the original model, as it is computationally prohibitively expensive for the use in iterative optimisation approaches. Then, we present our experimental results in Section 4, and finish with some concluding remarks.

2. MODEL OF THE THREE-TETHER WEC ARRAY

2.1 System description

The WEC design that we consider is a fully submerged spherical body connected to three tethers that are equally distributed around the buoy hull (Figure 2). Each tether is

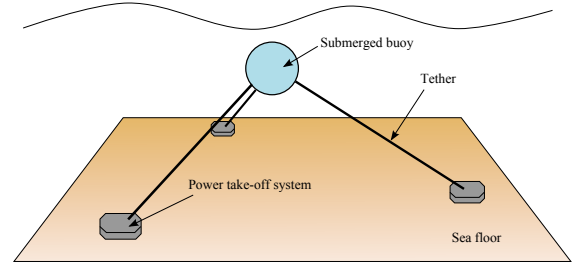


Figure 2: Schematic representation of a three-tether WEC (adapted from [5]).

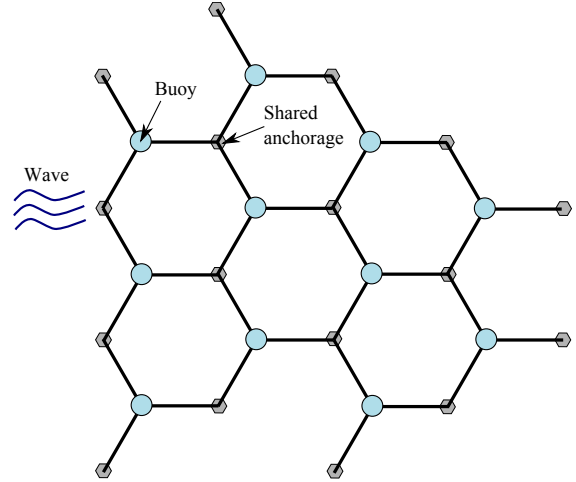


Figure 3: Top view on the array of WECs with shared mooring points.

connected to the individual power generator at the sea floor, which allows to extract power from surge and heave motions simultaneously [27].

The arrangement of a three-tether WEC array may be considered in two different ways:

- (i) In arrays where all adjacent devices share common anchorage points and/or power take-off system (see Figure 3). The main benefit of this layout is a significant reduction in the capital cost of the array due to the smaller number of mooring points as compared to the separately placed WECs. At the same time, the optimal buoy placement in such arrays is fixed and depends only on the ocean depth at the particular sea site and desired submergence depth of the buoy [28];
- (ii) In arrays where all devices are placed separately (see Figure 2). This layout does not have any constraints on the farm geometry and a buoy placement can be chosen considering various optimisation procedures.

2.2 System dynamics

In the following, we briefly outline the model of this kind of WECs arrays as it was derived by Sergiienko et al. [28].

The dynamic equation of the WECs array is derived in the frequency domain using linear wave theory, where a fluid is inviscid, irrotational and incompressible [10]. This model considers three dominant forces that act on the WECs:

- (i) excitation force includes incident and diffracted wave forces when all bodies are assumed to be fixed;
- (ii) radiation force acts on the oscillating body due to its own motion in the absence of incident waves;
- (iii) control, or power take-off force, that exerts on the WEC from machinery through tethers.

The key point in the array performance is the hydrodynamic interaction between buoys that can be constructive or destructive depending on the array size and geometry.

A spherical body is excited by ocean waves in surge, sway and heave only [18, 29]. However, a geometrical arrangement of a WEC with three tethers induces small angular motions of the body that do not contribute to the power absorption. Therefore, only translational motion of each body is included in the dynamic equation of the system.

Assuming that the total number of devices in the array is N and p is the body number, then the dynamics of the p -th WEC in time domain is described as:

$$\mathbf{M}_p \ddot{\mathbf{x}}_p(t) = \mathbf{F}_{exc,p}(t) + \mathbf{F}_{rad,p}(t) + \mathbf{F}_{pto,p}(t), \quad (1)$$

where $\mathbf{M}_p$ is a mass matrix of the p -th buoy, $\ddot{\mathbf{x}}_p(t)$ is a body acceleration vector in surge, sway and heave, $\mathbf{F}_{exc,p}(t)$, $\mathbf{F}_{rad,p}(t)$, $\mathbf{F}_{pto,p}(t)$ are excitation, radiation and PTO forces respectively. The power take-off system is modelled as a linear spring and damper for each mooring line with two control parameters, such as stiffness K_{pto} and damping coefficient B_{pto} .

In case of multiple bodies, where $p = 1 \dots N$, Equation (1) can be extended to include all WECs and expressed in frequency domain:

$$\left((\mathbf{M}_\Sigma + \mathbf{A}_\Sigma(\omega)) j\omega + \mathbf{B}_\Sigma(\omega) - \frac{\mathbf{K}_{pto,\Sigma}}{\omega} j + \mathbf{B}_{pto,\Sigma} \right) \hat{\mathbf{x}}_\Sigma = \hat{\mathbf{F}}_{exc,\Sigma}, \quad (2)$$

where subscript Σ indicates a generalised vector/matrix for the array of N bodies, $\mathbf{A}_\Sigma(\omega)$ and $\mathbf{B}_\Sigma(\omega)$ are radiation added mass and damping coefficient matrices that include hydrodynamic interaction between buoys, $\mathbf{K}_{pto,\Sigma}$, $\mathbf{B}_{pto,\Sigma}$ are the stiffness and damping block-matrices of the PTO system.

2.3 Performance index

The total power absorbed by the array of WECs can be calculated as:

$$P_\Sigma = \frac{1}{4} (\hat{\mathbf{F}}_{exc,\Sigma}^* \hat{\mathbf{x}}_\Sigma + \hat{\mathbf{x}}_\Sigma^* \hat{\mathbf{F}}_{exc,\Sigma}) - \frac{1}{2} \hat{\mathbf{x}}_\Sigma^* \mathbf{B} \hat{\mathbf{x}}_\Sigma, \quad (3)$$

where $*$ denotes the conjugate transpose.

The performance of an array of N WECs is usually summarised using the so-called q-factor:

$$q = \frac{P_\Sigma}{N \cdot P_0}, \quad (4)$$

where P_0 is the power absorption of a single device in isolation. The q-factor is the ratio of the power absorption of an array of WECs compared to the power absorption of those same converters in isolation. A q-factor greater than one indicates the presence of constructive interference in the array, as the array of devices is producing more energy than the devices would individually. Conversely, a q-factor less than one is a sign of destructive interference, which may be detrimental to the performance of the array.

Lastly, for the fair analysis of layouts that involve WECs of different sizes, we choose the *relative capture width* (RCW) to be a non-dimensional index of power absorption:

$$RCW = \frac{P_\Sigma}{P_w \left(2 \sum_{p=1}^N a_p \right)}, \quad (5)$$

where P_w is the incident wave-energy transport per unit frontage, a_p is a radius of the p -th body. RCW shows the fraction of power extracted from the wave per unit length of the device. RCW from Equation (5) is frequency dependent, therefore, for the particular sea site location, the RCW should be weighted according to the sea state probability data. Thus,

$$\overline{RCW} = \frac{\sum_i n_i \cdot RCW(\omega_i)}{\sum_i n_i}, \quad (6)$$

where n_i is an occurrence probability of waves at particular frequency.

2.4 Model specification

In Table 1 we provide the dimensions of the WECs used in the remainder of this article. We choose constant power take-off coefficients to give optimal power for the regular wave of 1m amplitude and 9-second period. The mass of each buoy is equal to 0.85 times the mass of the displaced water. Ocean depth is chosen to be 50m and all WECs are submerged 6m to centre of buoy.

Table 1: Specification of WECs used in array optimisation.

Buoy radius a , m	5	4	3.2	2.5	2
PTO spring coefficient	387	185	92	43	22
K_{pto} , kN/m					
PTO damping coefficient	161	76	38	18	8.9
B_{pto} , kN/(m/sec)					

We calculate the hydrodynamic parameters of the WEC array (excitation force, added mass and damping coefficients) based on the algorithm presented by Wu [30]. The results of various array layouts and buoy sizes have been validated against WAMIT [17], which is a computer program for computing wave loads and motions of offshore structures in waves.

3. ARRAY OPTIMISATION

In this section we present our approaches used to speed-up the simulations of the WEC arrays. The techniques include approximations and caching. For an array of 50 WECs, the eventual speed-up is 350-fold, i.e., from approximately 2100 minutes down to six minutes.

3.1 Model Approximation

The model approximation $\mathcal{M}'$ is a substitute of the three-tether model $\mathcal{M}$ with significantly reduced computational cost and acceptable error of accuracy. In terms of accuracy, we create a function p to compare the two models only based on the agreement of their trends. In other words, if the benefit is increasing/decreasing in $\mathcal{M}$ when changing from layout l_1 to layout l_2 , we compare whether the same trend

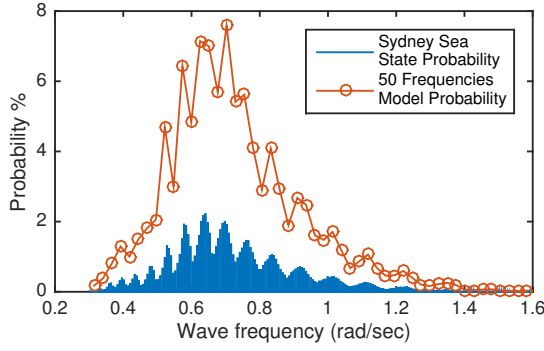


Figure 4: Sydney sea state: historic distribution and 50 samples

takes place in the approximation $\mathcal{M}'$. Function p is defined as:

$$p(f(x), x_1, x_2) = \begin{cases} \frac{f(x_1) - f(x_2)}{|f(x_1) - f(x_2)|} & f(x_1) \neq f(x_2) \\ 0 & f(x_1) = f(x_2) \end{cases}$$

Based on the function, a standard binary test is introduced according to the rule that $p(\mathcal{M}', l_1, l_2) = p(\mathcal{M}, l_1, l_2)$ means *positive* and the contrary means *negative*. This way we can compute the accuracy with regard to True Positive, True Negative, False Positive and False Negative.

In order to reduce the computational cost, we consider to reduce the sampling of frequencies. The original variant of the three-tether model utilises 50 sample frequencies to simulate the probability of wave frequencies in reality. In Figure 4, the blue histogram shows the records of different wave frequencies with their probabilities taking place in a sea area close to Sydney [15], and in red we illustrate the 50 evenly chosen frequencies. Each point represents a certain small range of wave frequency and its probability is the sum of the probability of this range. Therefore the total probability of 50 frequencies still sums up to 1, so that the approximate power absorbed by WECs can be calculated by using this simplified version. However, the computation of total power is still costly. The calculation of an array with 50 WECs takes around 35 hours on one core of an Intel Core i5-4250U processor. Since the computation time in linear in the number of considered frequencies, a natural way to the reduce computational cost is to approximate the accurate model with fewer sample frequencies. Our goal is to reduce computation time while keeping the accuracy above 80%.

To achieve this, we create the model approximations with the numbers of sample frequencies to be 10, 5, 4, 3, 2 and 1. Each sample frequency represents a range of actually occurring wave frequencies, and for each approximation we distribute them equally over the spectrum. For the single frequency, however, we select the most likely occurring frequency: 0.7 rad/s. Figure 5 illustrates all the probabilities of frequencies used in the six approximation models compared with the probabilities in the original three-tether model.

We investigate the six approximations in two specific scenarios: 1) arbitrary layouts and 2) evolving layouts. Both of the scenarios are typical in optimisation, especially for evolutionary algorithms. We study layouts with 50 WECs in a one square kilometre rectangular area with a safety constraint that the minimal distance between each pair of WECs must be 50 meters.

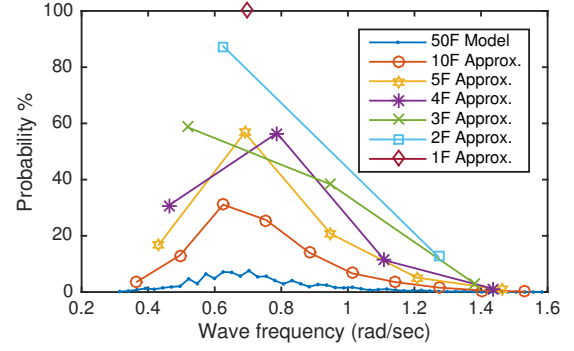


Figure 5: Probabilities of frequencies in six approximation models and the original three-tether model

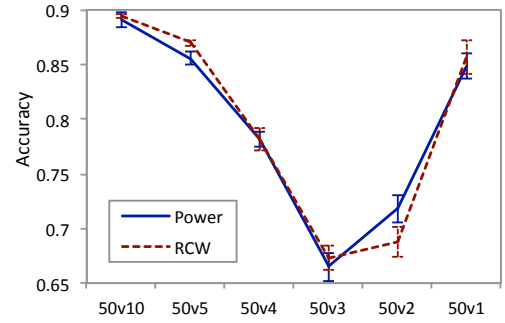


Figure 6: Accuracy of six approximation models for arbitrary layouts. Shown are the results when 10, 5, 4, 3, 2, and only 1 (of 50) frequencies are used.

Arbitrary Layouts

In this scenario, we randomly generate 100 valid layouts and divide them into five groups. For each group, we calculate the accuracy between the three-tether model and each of six approximations. Then we plot the averages and standard deviations of the groups of data in Figure 6. As we can see, the two- and three-frequency approximations are the least accurate ones. The fastest model that considers only the prevailing frequency is comparable in accuracy with the one that uses five frequencies, however, the latter takes five times as long to compute.

Evolving Layouts

In this scenario we use a simple evolutionary algorithm called (1+1)-EA to study the optimisation using the approximating three-tether model. This algorithm is a hill-climber where new solutions are created based on the best-so-far encountered. If the new solution provides a higher score, then it replaces the best-so-far, otherwise the new solution is discarded; this is repeated until the total time budget is used up. We run 400 generations of the algorithm with a simple mutation which randomly chooses and moves only one WEC in a layout. This optimisation results in an increase of the power output by around 5% (as shown in Figure 7), and it also generates 401 layouts including an initial random layout. We then calculate the accuracy between the original three-tether model and each of six approximations based on the layouts by using the same approach as for arbitrary layouts. The results are again shown in Figure 8.

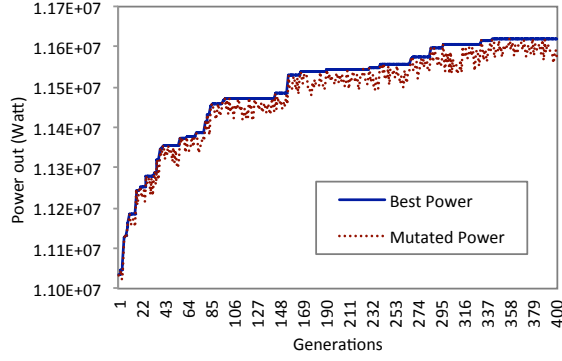


Figure 7: Optimisation results of (1+1)-EA with a simple mutation

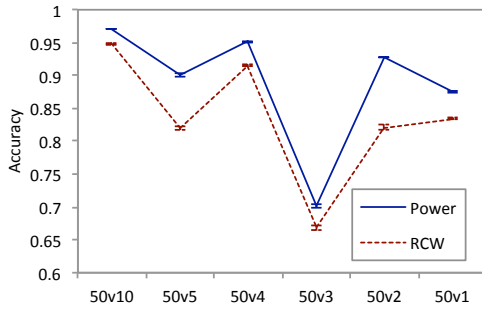


Figure 8: Accuracy of six approximation models for evolving layouts. Shown are the results when 10, 5, 4, 3, 2, and only 1 (of 50) frequencies are used.

The results of both scenarios largely agree. The 10 and 5 frequencies approximations provide the best accuracy and precision in both scenarios. However, we do not choose them due to their relatively higher cost compared with the single frequency approximation. The single frequency (i.e., the prevailing frequency) approximation provides acceptable accuracy and precision with minimal cost, which makes it the ideal trade-off in our case. With incorporating the approximation, the cost of computation is reduced by around 98%, i.e., from around 2,100 minutes to be around 42 minutes for calculating one layout of a 50 WECs array.

3.2 Model Speed-Up Through Caching

Another approach that we introduce along with the single frequency approximation in order to reduce the computational cost is ‘caching’, which is a technique widely used in software engineering for improving performance. In our particular model, the most frequently used calculations in our MATLAB model are `integral`, `factorial`, and `bessel`. The time spent with such calculations is significant, and a number of them are duplicated during the power computations for a single layout. For instance, in order to calculate the power output of a 50 WECs array, one million calls of `integral` are made, while around 89.5% of them are duplicates. Therefore we cache the results of such calculations into several hash-maps with their parameters hashed to be the corresponding keys. This way, subsequent calls can query the results with their parameters instead of recalculating them. By implementing this technique, the computational cost can be reduced by around 85% without influencing the accuracy.

For calculating one layout of a 50 WECs array mentioned in the previous section, the cost is decreased further from about 42 minutes to about 6 minutes.

4. COMPUTATIONAL STUDY

In this section we report on our layout investigations of submerged wave energy converters. In the first set of experiments, we consider WECs arranged in a grid-based layout. There, the devices can share mooring points and/or power take-off systems, which results in a significant reduction in capital cost. In the second part, we relax this constraint to investigate layouts where the buoys can be placed arbitrarily, as long as the minimum safety distance is maintained.

4.1 Radii Optimisation

We conduct a range of experiments for optimising the radii of buoys in a staggered array as shown in the introductory Figure 3. In this array, the columns of buoys are spaced 93.33m apart and the rows 107.77m, due to technical reasons. Each buoy in the array can have a different radius of either 2m, 2.5m, 3.2m, 4m or 5m. This quantisation is necessary for both optimisation and also in practice, in order to reduce the number of buoy variants. q-factor is primarily used as the optimisation criterion, although some experiments also consider the relative capture width (Equation 6).

For small array sizes, including 1x1, 1x2, 2x1 and 2x2, it is feasible to use brute force search (BFS) to explore the entire solution space and find the optimal solution. For example, the largest of these small arrays is the 2x2 configuration, which has 625 possible solutions and takes 10 hours to evaluate them all. The best 2x2 configuration has a q-factor of 0.9990 (with a corresponding $\overline{RCW}$ value of 0.6453), which is a layout comprising of two 2m buoys and two 5m buoys. Interestingly, the best-performing 2x2 layout in terms of $\overline{RCW}$ achieved a significantly higher value of 0.7988 (a layout with four 5m buoys), while the q-factor value decreased slightly to 0.9658, see Figure 9. However, this is actually not surprising since the q-factor and $\overline{RCW}$ are two different measures: while $\overline{RCW}$ refers to the maximum power, the q-factor shows to the maximum efficiency of the array in comparison to individual devices. In the model, buoys of different sizes are submerged to the same depth due to constraints of the staggered layout, which affects the efficiency of smaller buoys. Thus, 5m devices submerged to 6 m are more productive in terms of power than 2m buoys submerged to the same depth. As a result, the optimisation using $\overline{RCW}$ ends up with larger WECs. On the other hand, for the q-factor optimisation it is more important to have a constructive hydrodynamic interaction between buoys in the array. Taking into account that in a staggered layout distances between devices are around 100m, values of q-factor are much higher for smaller buoys as at such distances interaction is reduced to a minimum.

The 3x3 array configuration has almost 2 million solutions, meaning that a brute force search is no longer feasible due to the simulation times needed. Yet for smaller arrays sizes, the optimal configuration is found to only consist of buoys with a radius of either 2m or 5m. Using this insight, we are able to conduct a partial BFS of the 3x3 array by examining only those solutions containing 2m and 5m buoys. This partial BFS takes approximately 2 days to complete, but the result is a solution with a q-factor of 0.9956 (see Figure 10), which is comparable to the 2x2 optimal configuration, even though

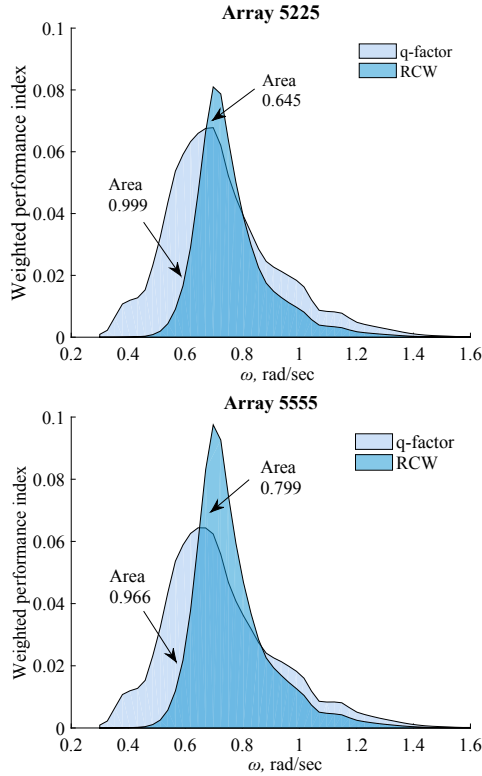


Figure 9: Comparison of the best performing layouts 5225 and 5555. Shown is the performance for different wave frequencies. The area under the curve corresponds to the expected performance.

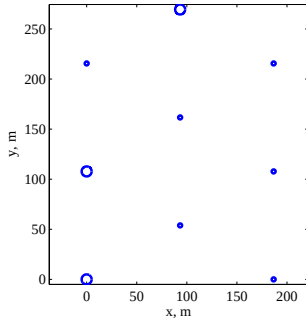


Figure 10: Best solution found for the 3x3 staggered array. The direction of wave propagation is from left to right. All buoys have diameters of 2m or 5m. q -factor value = 0.9956. RCW value = 0.5303. The large 5m buoys make the most of the incoming waves, and the small 2m buoys are most efficient when placed behind the 5m buoys.

the search was not completely exhaustive in this case.

Since this 3x3 solution found by the partial BFS is not necessarily optimal, we tried using several variants of randomised local search. This did not yield a better 3x3 configuration. An exhaustive evaluation of the local neighbourhood further revealed that this all 2m buoy solution was indeed a local optimum for single changes in the buoy diameters.

We also briefly consider the 4x4 and 5x5 configurations. As BFS has proved to be inefficient, we simply generate all

2m buoy solutions for 4x4 and 5x5, and all of them proved to have q -factors of approximately 0.99. Although these are unlikely to be optimal, the relatively high q -factors show that all 2m buoy solutions may provide configurations with relatively high q -factors for even larger arrays. A similar local neighbourhood check for these 4x4 and 5x5 solutions shows that they are indeed local optima for performing changes to single buoys. This proves that a q -factor is not suitable for the buoy size optimisation at the fixed layout and another performance index should be developed for such a task.

4.2 Placement Optimisation

4.2.1 Experimental Setup

In the following experiments we no longer enforce the grid-like layout from before. We employ two different algorithms to optimise the layouts. The first one is the (1+1)-EA (as used in Section 3), which randomly chooses and moves only one WEC in a layout to a new feasible location. The second algorithm is the Covariance Matrix Adaptation based Evolutionary Strategy (CMA-ES) [14]. CMA-ES self-adapts the covariance matrix of a multivariate normal distribution. This normal distribution is then used to sample from the multidimensional search space where each variate is a search variable. The covariance matrix allows the algorithm to respect the correlations between the variables making it a powerful (and popular) heuristic search algorithm.

Initially, both algorithms place the N buoys randomly in the provided area. In preliminary experiments we found that the regular grid initialisation with maximal distances in the rows and columns to perform similar to the random one. While the grid minimises the interactions by maximising the intra-buoy distance, interestingly the positive interferences appear to outweigh what would intuitively be considered a disadvantage.

Both algorithms take care of the constraints in the following ways. When a layout has buoys which violate the proximity constraint or if a buoy is located outside the allowed area, we resample a new solution in (1+1)-EA and CMA-ES, before invoking the time-consuming simulations. For boundary constraints, CMA-ES rounds the coordinates to the nearest boundary value.

The CMA-ES configuration we use here is as follows. We use a population size of two, which is used to generate two new solutions. We run this (2+2)-CMA-ES for 200 generations, and with an initial standard deviation for each decision variable of 20, based on preliminary experiments. The second algorithm, (1+1)-EA, we run with the same total evaluation budget of 400 evaluations.

4.2.2 Observations

As we now focus on larger arrays, we use the approximate model from Section 3, where only a single frequency is considered. Under the provided conditions, a single isolated 5m buoy has a power output of $5.547e+5$ Watts.

Figure 11 shows the results from our optimisation of both the simple (1+1)-EA and the CMA-ES. In both cases, the former produces layouts with significantly higher outputs. Interestingly, this simple algorithm outperforms CMA-ES, even though the latter can adapt itself to the problem. It appears that the 200 generations given to CMA-ES are not enough time. To a limited degree this is supported by our observation that CMA-ES begins to converge at the end

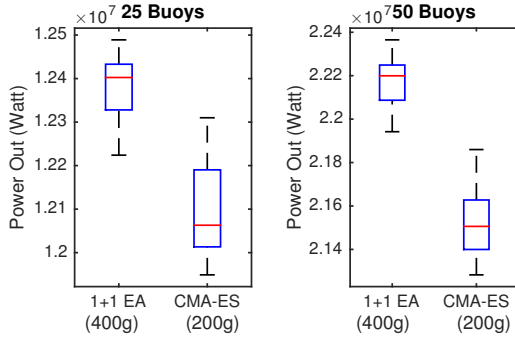


Figure 11: Optimisation results from our 25 and 50 buoys study. Shown are the results of 20 independent runs. For 25 buoys, the initial average q-factor is 0.8123, and the final q-Factors for (1+1)-EA and CMA-ES are 0.8930 and 0.8723. For 50 buoys, the initial average is 0.7267, and the final ones are 0.7995 for (1+1)-EA and 0.7760 for CMA-ES.

of the computation budget provided. By then, the average standard deviation decreases to values of about 4 to 8, which means that large changes to the layouts become increasingly unlikely.

As (1+1)-EA is not able to fine-tune a solution, we take a solution found for 25 buoys and give it to CMA-ES for fine-tuning, with $\sigma = 1.0$ for 200 generations. The resulting layout is shown in Figure 12 and its power output increased by 1.1%. This means that while CMA-ES experiences difficulties in creating good layouts from scratch, it can still be used to tune existing solutions.

In this layout, it is not very surprising that the buoys facing the incoming waves have the highest power output. Further into the farm, the output decreases quickly, because the interactions become increasingly important with increasing number of columns [1]. This in turn shows the fidelity of our optimisation results. The optimisation considers this indirectly, as the density of the buoys on the left hand side of the final layout is significantly higher than the density of buoys in the right hand side. Interestingly, constructive interferences result at times in individual WECs having an above-average output (greater than 5.547×10^5 Watts) at certain locations, e.g. the buoy located at (360, 680).

Finally, we briefly demonstrate the applicability of our approach to a very large array. In the single run that we perform (1+1)-EA (again using 400 generations) increases the q-factor significantly by 10.4% over the initial layout. Note that the optimisation with the original model would have taken about 2750 days. The actual optimisation using our speed-ups presented in Section 3 took only 8.3 days, which corresponds to a speed-up by a factor of 330.

We show the final layout in Figure 13. Just as before, the buoys facing the incoming waves have the highest power output. Further into the farm, the output decreases quickly, however, at times positive interferences result in individual turbines having an above-average output.

5. CONCLUSIONS

This study provides first insights into the layout problem of submerged wave energy converters. It is also the first time multiple three tether buoys have ever been investigated.

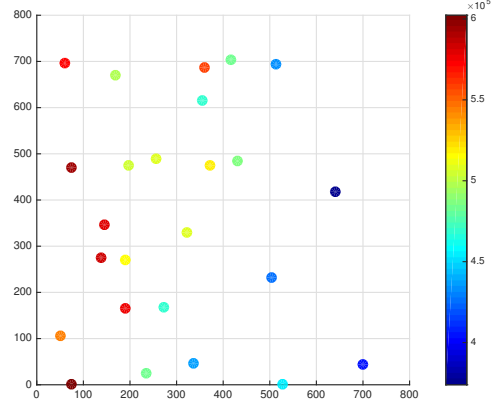


Figure 12: Best layout found for the 25 buoy study. The direction of wave propagation is from left to right. All buoys have diameter 5m, and the area is $0.707 \cdot 0.707 \text{ km}^2$. The overall power out is 1.257×10^7 Watts. The q-factor value is 0.9063 (initially 0.8964) and the $\overline{RCW}$ value is 1.434 (initially 1.252). The colours indicate the power generated by each buoy.

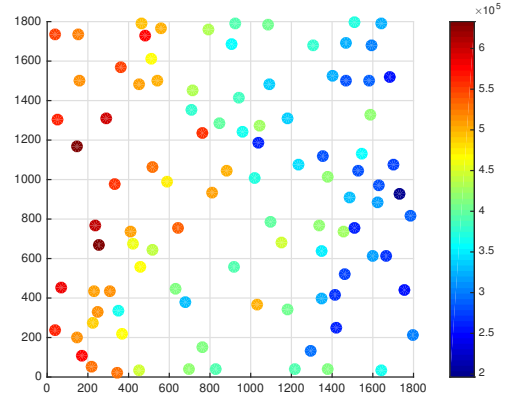


Figure 13: Best layout found for the 100 buoy study. The direction of wave propagation is from left to right. All buoys have diameter 5m, and the area is $1.8 \cdot 1.8 \text{ km}^2$. The overall power out is 4.147×10^7 Watts. The q-factor value is 0.7476 (initially 0.6769) and the $\overline{RCW}$ value is 1.183 (initially 1.071). The colours indicate the power generated by each buoy.

The first simulations of the buoy interactions were computationally prohibitively expensive, taking hours or even days. Through model approximations and caching, we achieved up to 350-fold speed-ups in the simulation times needed. This in turn allowed us to iteratively optimise the interactions in WEC arrays.

Among others, we have made two high-level observations that add to the knowledge of designing such arrays. First, we have discovered a potential design flaw, i.e., buoys of different diameters should not be submerged at the same depth, but the top surface of all buoys should be same distance to the sea surface. Second, we have learned that positive interference can result in higher than normal power outputs for individual buoys. This is surprising, since such effects are hardly ever heard of. For example in wind energy related

research, wake effects and turbulences with their negative effects are well-known, but positive effects are not.

Our next steps include the refinement of the interaction model to allow varying submergence depths. Also, we will revisit the objective function to include economic aspects of the array construction and operation. For example, cost related measures such as the characteristic mass of the buoy and the significant power-take off force relative to the yearly power production can be good indirect cost estimators [2, 3].

In the long-term, we will further speed-up the software side in MATLAB, we will consider additional objectives such as stress on the mooring points, which drives capital cost, and we will increase the realism by considering additional incident wave directions. At present, the latter will pose the biggest challenge, as it will come at a significant increase in simulation time.

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Appendix D

Fast and effective multi-objective optimisation of submerged wave energy converters

Arbonès D.R., Ding B., Sergiienko N.Y., and Wagner M. (2016) Fast and effective multi-objective optimisation of submerged wave energy converters. In: *Proceedings on 14th International Conference on Parallel Problem Solving from Nature*. Springer, Cham, pp. 675–685.

Statement of Authorship

Publication Status	Published
Name of Co-author	Nataliia Y. Sergiienko
Contribution to the Paper	Wrote Section 2.1 of the manuscript, developed the mathematical model of the three-tether WEC array used by the optimisation algorithm, helped in interpretation of obtained results.
Overall Percentage (%)	15%

Fast and Effective Multi-Objective Optimisation of Submerged Wave Energy Converters

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Abstract. Despite its considerable potential, wave energy has not yet reached full commercial development. Currently, dozens of wave energy projects are exploring a variety of techniques to produce wave energy efficiently. A common design for a wave energy converter is called a buoy. A buoy typically floats on the surface or just below the surface of the water, and captures energy from the movement of the waves.

In this article, we tackle the multi-objective variant of this problem: we are taking into account the highly complex interactions of the buoys, while optimising the energy yield, the necessary area, and the cable length needed to connect all buoys. We employ caching-techniques and problem-specific variation operators to make this problem computationally feasible. This is the first time the interactions between wave energy resource and array configuration are studied in a multi-objective way.

Keywords: Wave energy; multi-objective optimisation; simulation speed-up

1 Introduction

Global energy demand is on the rise, and finite reserves of fossil fuels, renewable forms of energy are playing a more and more important role in our energy supply [11]. Wave energy is a widely available but largely unexploited source of renewable energy with the potential to make a substantial contribution to future energy production [3, 9]. There are currently dozens of ongoing wave energy projects at various stages of development, exploring a variety of techniques [10].

A device that captures and converts wave energy to electricity is often referred to as a wave energy device or wave energy converter (WEC). One common WEC design is called a point absorber or buoy. A buoy typically floats on the surface or just below the surface of the water, and it captures energy from the movement of the waves [9]. In our research, we consider three-tether WECs (Figure 1) as a technological alternative to the common single-tether WECs. While their capital cost are higher than of conventional single-tether heaving buoys, they can extract significantly more energy from the waves [16]. In our case, the buoys are fully submerged and tethered to the seabed in an offshore location. They use the motion of the waves to drive a hermetically sealed hydraulic line to

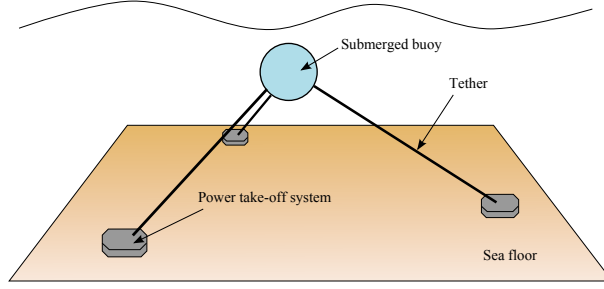


Fig. 1. Schematic representation of a three-tether WEC [18].

drive hydroelectric turbines to generate electricity, or to power a reverse osmosis desalination plant to create potable water.

A single wave energy converter can only capture a limited amount of energy alone, which is why it is essential to deploy wave energy devices in large numbers. A group of wave energy devices is commonly referred to as a wave energy farm or array [2]. In order to evaluate our arrays, we use a recently developed frequency domain model for arrays of fully submerged three-tether WECs [15]. This model allows us to investigate different parameters, such as number of devices and array layout. In addition to the objective of producing energy, we consider the following two objectives: the cable length needed to connect all buoys as given by the minimum spanning tree, and the area of the convex hull needed to place all buoys. The ideal choice of parameters leads to an optimisation problem: what are the best trade-offs of the buoys' locations, the area needed, and the cable length needed? To the best of our knowledge, this study is the first to investigate this question to reduce costs and to increase efficiency.

We proceed as follows. In Section 2, we introduce the multi-objective buoy placement problem and the different objectives that are subject to our investigations. Then, we present in Section 3 our speed-ups, the operators, and the constraint handling used. We report on our computational study in Section 4 before we conclude with a summary.

2 Preliminaries

In the following, we outline the different objectives and constraints that we consider for the WEC array optimisation.

Let $X = \{x_1, \dots, x_n\}$ and $Y = \{y_1, \dots, y_n\}$ be the set of x and y coordinates of n WECs in the plane. The goal is to find a set of coordinates such that the energy output of the whole wave farm is maximised. At the same time, the total length of the cable or pipe necessary to interconnect the buoys, as well as the area necessary for the wave farm, should be minimised.

The WEC design that we consider is a fully submerged spherical body connected to three tethers that are equally distributed around the buoy hull (Figure 1). Each tether is connected to the individual power generator at the sea floor, which allows to extract power from surge and heave motions simultaneously [14].

2.1 Power Output Prediction

In the following, we briefly outline the model of this kind of WECs arrays as it was derived by Sergiienko et al. [15] and used by Wu et al. [18].

The dynamic equation of the WECs array is derived in the frequency domain using linear wave theory, where a fluid is inviscid, irrotational and incompressible [4]. This model considers three dominant forces that act on the WECs:

- (i) excitation force includes incident and diffracted wave forces;
- (ii) radiation force acts on the oscillating body due to its own motion;
- (iii) power take-off force that exerts on the WEC from machinery through tethers.

The key point in the array performance is the hydrodynamic interaction between buoys that can be constructive or destructive depending on the array size and geometry.

Assuming that the total number of devices in the array is n and p is the body number, then the dynamics of the p -th WEC in time domain is described as:

$$\mathbf{M}_p \ddot{\mathbf{x}}_p(t) = \mathbf{F}_{exc,p}(t) + \mathbf{F}_{rad,p}(t) + \mathbf{F}_{pto,p}(t), \quad (1)$$

where $\mathbf{M}_p$ is a mass matrix of the p -th buoy, $\ddot{\mathbf{x}}_p(t)$ is a body acceleration vector in surge, sway and heave, $\mathbf{F}_{exc,p}(t)$, $\mathbf{F}_{rad,p}(t)$, $\mathbf{F}_{pto,p}(t)$ are excitation, radiation and power take-off (PTO) forces respectively. The power take-off system is modelled as a linear spring and damper for each mooring line with two control parameters, such as stiffness K_{pto} and damping coefficient B_{pto} .

In case of multiple bodies, where $p = 1 \dots n$, Equation (1) can be extended to include all WECs and expressed in frequency domain:

$$\left((\mathbf{M}_\Sigma + \mathbf{A}_\Sigma(\omega)) j\omega + \mathbf{B}_\Sigma(\omega) - \frac{\mathbf{K}_{pto,\Sigma}}{\omega} j + \mathbf{B}_{pto,\Sigma} \right) \hat{\mathbf{x}}_\Sigma = \hat{\mathbf{F}}_{exc,\Sigma}, \quad (2)$$

where subscript Σ indicates a generalised vector/matrix for the array of N bodies, $\mathbf{A}_\Sigma(\omega)$ and $\mathbf{B}_\Sigma(\omega)$ are radiation added mass and damping coefficient matrices that include hydrodynamic interaction between buoys, $\mathbf{K}_{pto,\Sigma}$, $\mathbf{B}_{pto,\Sigma}$ are the stiffness and damping block-matrices of the PTO system.

The total power absorbed by the array of WECs can be calculated as:

$$P_\Sigma = \frac{1}{4} (\hat{\mathbf{F}}_{exc,\Sigma}^* \hat{\mathbf{x}}_\Sigma + \hat{\mathbf{x}}_\Sigma^* \hat{\mathbf{F}}_{exc,\Sigma}) - \frac{1}{2} \hat{\mathbf{x}}_\Sigma^* \mathbf{B} \hat{\mathbf{x}}_\Sigma, \quad (3)$$

where $*$ denotes the conjugate transpose.

For more details on the model, we refer the interested reader to [15, 18].

2.2 Constraints and Assumptions

We have the following constraints placed on our optimisation. The first one enforces an upper bound on the area of the farm. This constraint ensures that we can only place a buoy i within a certain area, which is a realistic constraint

for most layout problems. For a rectangular wave farm with length l and width w , this constraint is satisfied *iff*

$$0 \leq x_i \leq l \text{ and } 0 \leq y_i \leq w, 1 \leq i \leq n. \quad (4)$$

Because buoys can damage each other if they get too close, and also maintenance ships need to be able to navigate between them, the second constraint regulates the spacial proximity. It is satisfied *iff*

$$\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \geq 50 \text{ meters}. \quad (5)$$

In addition to the above constraints, we assume that all WECs have the same power take-off characteristics.

2.3 Euclidean Minimum Spanning Tree

We use the Euclidean minimum spanning tree (MST) to calculate the minimum length of cable or pipe required to connect all buoys in a particular array configuration. It is computed by first constructing the complete graph on the set of points that represent the buoys and edge costs given by the Euclidean distance between any pair of buoys. Then, the minimum spanning tree for this graph is computed and used as an objective representing the costs of the cable or pipe length. Figure 2 displays an array layout as, as well as the minimum spanning tree, represented by lines joining each buoy.

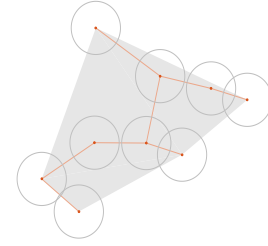


Fig. 2. An example WEC array. The circles visualise the safety distance.

2.4 (Cost of the) Convex Hull

In our study, the cost of the convex hull is defined as the area contained by the set of points forming the convex hull. This value is the minimum land area that is required for a wave farm layout. Figure 2 displays a buoy layout, as well as the area (cost) of the convex hull shaded in grey.

3 Computational speed-up, operators, and constraint handling

3.1 Speed-up of Simulation

In order to make the simulations computationally feasible, and to make the best use of the available hardware, we reimplemented the PTO system in C++ and parallelised it with OpenMP [13]. Because the system is defined as a series, it is inherently parallelisable and a linear speed-up possible. Furthermore, OpenMP's framework allows for nested parallelisation, which further decreases the overall running time. The integrals are calculated with the GNU Scientific Library [5], which has support for integrals with singularities. Due to the lack of a proper

Table 1. Runtime per evaluation in seconds (median of 20 runs). Ω is the set of frequencies ω used. In each cell, single-thread results are on the left and multi-thread ones on the right. *Laptop* denotes a computer with a Intel Core i7-4910MQ CPU (up to 3.9GHz, used with 4 threads) and 32GB RAM. *Server* is a compute server with four AMD Opteron 6348 CPUs (up to 3.4GHz, used with 48 threads) and 128GB RAM.

n	Ω	MATLAB		C++	
		Laptop	Server	Laptop	Server
4	50	29.28 / 11.64	56.15 / 3.54	2.83 / 0.98	5.48 / 0.38
9	25	80.96 / 31.14	153.75 / 9.48	8.32 / 3.05	16.28 / 0.98
16	25	262.86 / 97.58	508.63 / 29.88	29.31 / 9.94	55.42 / 3.23
25	25	658.21 / 239.37	1265.97 / 72.46	71.92 / 26.20	141.16 / 8.95

linear algebra library with complex number support, the result from Equation (2) is obtained from MATLAB [12]. Note that the evaluation of Equation (3) can only be parallelised for each of the frequencies ω considered.

The evaluation of a WEC array is time consuming even with parallelisation. The integral calculation is the bottleneck consuming upwards of 95% of the running time. Wu et al. [18] used caching of integral computations because a large portion of these integrals are repeated, achieving a factor 7 speed-up in running time. We use the caching approach even more comprehensively, by caching results not only within a single layout evaluation like Wu et al., but by reusing them across multiple evaluations. This additional improvement can help if an optimisation algorithm modifies only part of a solution at each iteration, therefore reusing integrals computed in previous iterations.

In Table 1 we list the achieved time needed to compute the intra-buoy interactions. As we can see, the speed-ups (up to 142-fold) allow us to run significantly more evaluations if the overall available time is limited.

3.2 Problem-Specific Operators

As the problem is highly constrained due to a large number of buoys and the given safety margin around each buoy, the operators have to ensure that feasible placements are produced. We investigate the benefit of the two variation operators MOVEMENTMUTATION and BLOCKSWAPCROSSOVER by Tran et al. [17] over the commonly used Polynomial Mutation and Simulated Binary Crossover. The former pair was designed for wind turbine placement optimisation, where safety distance constraints and area constraints also need to be considered.

MOVEMENTMUTATION is an operator that does a local change to the current solution. For a randomly picked WEC, MOVEMENTMUTATION moves it to a randomly selected spot along a selected direction to a feasible location.

BLOCKSWAPCROSSOVER is designed to implant a randomly selected rectangular “block” of WECs from each of the two parents to produce two children, each with a varying degree of information from each parent. A repair operator is applied in case the number of WECs does not match to the target number.

Note that the fundamental difference between both the WEC positioning and the wind turbine positioning is that “shading” is the primary inter-turbine effect,

while the primary inter-buoy effect is “phase shifting”. However, as the operators do not consider these effects directly, we can apply them to our problem as well.

3.3 Constraint Handling

As described in Section 2.2, we consider area constraints and safety distance constraints in this study.

The area (box) constraint is enforced by applying a sinusoidal-shaped function that maps any value to a closed range. The function used has the form [6]:

$$x = a + (b - a) * (1 + \cos(\pi * x / (b - a) - \pi)) / 2 \quad (6)$$

The advantage of this function is twofold. First, the boundaries of the region are automatically enforced without the need for a check on each iteration. Second, it provides a smooth transition of the movements of buoys close to the boundaries, contributing to the performance of the optimisation algorithm.

The inter-buoy distance is enforced by applying a penalty to the objectives. This penalty is proportional to the distance that the buoys lie outside the safety margin. The resulting objectives O' are used in the optimisation process:

$$O' = O \left(1 - K \sum_i^n \sum_{j \neq i}^n \text{MAX}(M - d(i, j), 0) \right) \quad (7)$$

where n is the number of buoys, M is the safety distance to keep between buoys, $d(i, j)$ is the Euclidean distance between the buoys, and $K \in \mathbb{R}^+$ is the penalty regularisation parameter. This parameter is meant to control the slope of the penalty applied, acting as a trade-off between discouraging solutions that lie far into the infeasible region, and allowing the exploration of boundary regions.

4 Experimental Study

In this section, we describe our experimental setup and report on the results of different multi-objective evolutionary algorithms using our speed-ups and variation operators for the multi-objective buoy placement problem.

4.1 Experimental Setup

For the basis of our study, we utilise the algorithms SMS-EMOA [1] and MO-CMA-ES [7], as implemented in the optimisation framework Shark 3.0 [8]. We use SMS-EMOA in two variants: (i) the default SMS-EMOA with Polynomial Mutation and Simulated Binary Crossover, and (ii) the problem-specific SMS-EMOA* with MOVEMENTMUTATION and BLOCKSWAPCROSSOVER (see Section 3.2).

We use a population of size $\mu = 50$ for all experiments, and the evaluation budget for each run is 6000 evaluations. All other parameters are used with their default values in the Shark library. Unless stated otherwise, we report the results of 20 independent runs.

For all runs, we initialise the first population with regular grids that are scaled from the tightest grid to the most generous one where buoys are placed on the boundaries as well. In Figure 3 we show an example, which also shows the non-linear effect that arises from the constraint handling of the box constraints (Equation 4). The side-effect of this initialisation is that we already achieve right from the beginning a population of solutions that is guaranteed to be diverse in the size of the convex hull and in the length of the minimum spanning tree. In preliminary experiments, we observed that this approach performed better than one with random initial layouts.

The scenarios are defined as follows. The goal is to place 4, 9, 16, and 25 buoys subject to the three objectives in a quadratic area. We scale the area available with the number (considering an area of $20,000\text{m}^2$ per buoy), which results in squares with sides of length 283m, 424m, 566m, and 707m respectively.

To compare the performance of the different setups, we inspect the sets of trade-offs visually, and we employ the hypervolume indicator. To compute the latter, we rescale the final solution set into the unit cube that is defined by the extreme values (of feasible layouts) observed for each scenario.

In each scenario, we use the sea state, i.e. the wave frequency distribution, and the features of the buoys as defined in the single-objective investigations in [18], which allows us to compare results for 25 buoys. The WEC radius is $a = 5$ meters, and their power take-off characteristics are kept static. The mass of each buoy is equal to 0.85 times the mass of the displaced water. Ocean depth is chosen to be 30m and all WECs are submerged 6m to the centre of buoy.

To compute the power output of a solution, we need to choose a number of discrete wave frequencies from the wave spectrum. While Wu et al. [18] observed that a single frequency from the entire spectrum of waves can be used with reasonable accuracy during buoy placement optimisation, we prefer to use a significantly more time-consuming approach with 25 or 50 wave frequencies. This provides us with very accurate power output predictions. Also, this greatly reduces the risk of unrealistic exploitation of local optima due to ill-conditioned scenarios, which we have observed in the one-frequency case.

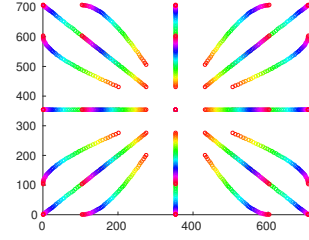


Fig. 3. Initial population in the 25-buoy scenario. The $\mu = 50$ layouts are shown in different colours.

4.2 Experimental Results

In the following, we compare the performance of the different multi-objective approaches. Figure 4 summarises the hypervolumes achieved by the final populations for the different scenarios. While the standard version of MO-CMA-ES outperforms the standard version of SMS-EMOA, both are easily outperformed (in terms of achieved hypervolume) by SMS-EMOA*. It appears that even though the latter employs operators previously used in wind turbine placement optimisation, they are also beneficial in our case.

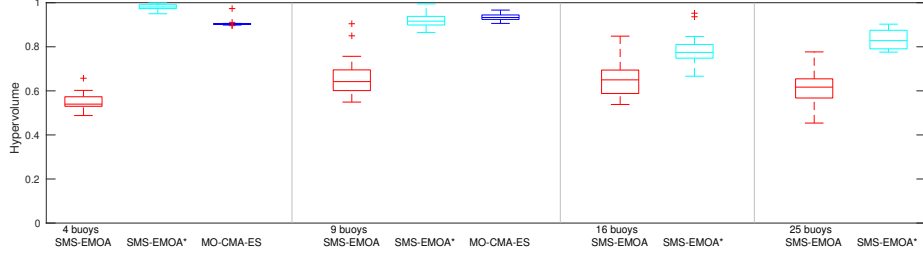


Fig. 4. Hypervolumes covered by the final populations. MO-CMA-ES results are missing for 16 and 25 buoys due to the unacceptable run-times.

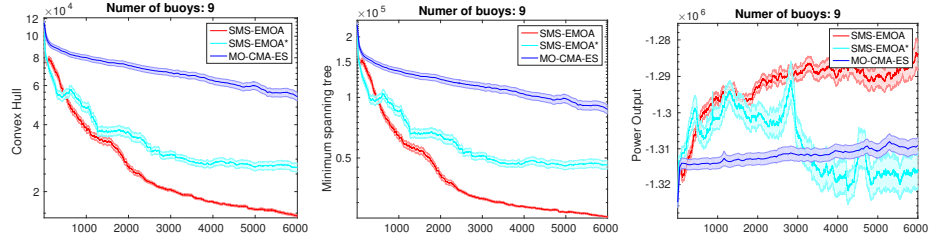


Fig. 5. Evolution of objective scores over time in case of the 9-buoy scenario. Shown are the averages and the 95% confidence intervals. Note that we are minimising the negative power output, meaning that smaller values are better.

Note that MO-CMA-ES hardly benefits from the caching of simulation results, as it tends to sample new coordinates for all buoys every time. While this behaviour is typically an advantage, it rendered it impossible for us to apply MO-CMA-ES to the optimisation of the larger scenarios.

Exemplarily, we show in Figure 5 how the average objective scores across the populations as they evolve over time. Interestingly, SMS-EMOA and SMS-EMOA* behave quite differently, even though they differ only in their variation operators. For example, the standard SMS-EMOA performs best in terms of convex hull and length of the minimum spanning tree, but it produces on average the layouts with the worst power output. It appears that all three approaches explore quite different parts of the objective space across all runs. This is not only reflected by the final mean values, but also by the spread across different runs by the same algorithm. In summary, this shows to us that a decision maker should not blindly trust multi-objective performance indicators, but inspect the solution sets as well. In practice, a single trade-off layout has to be chosen for implementation, and even though the nature of the problem is multi-objective, the decision eventually boils down to hidden preferences or economic factors.

Lastly, we briefly compare our results for 25 buoys with the ones by [18]. We take their best result and recompute the power prediction more accurately using 25 frequencies instead of the single frequency they used. As a result, their best layout has a predicted power output of 3.459 MW. Compared to this, our best layout (Figure 6) has an output of 3.590 MW, which is 3.8% better.

To conclude, we can see that the multi-objective optimisation of arrays of wave energy converters is feasible, if software engineering tricks are employed to

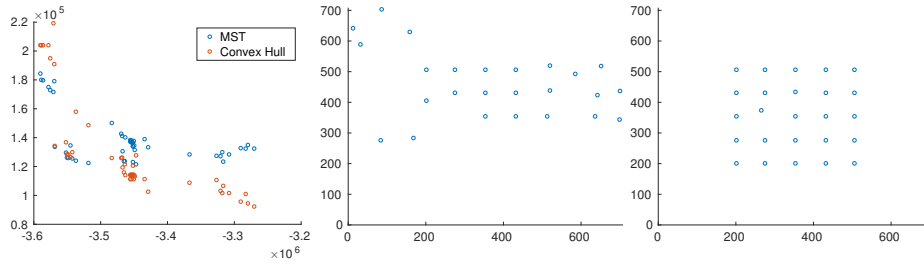


Fig. 6. Population with the highest power output layout for 25 buoys. The population computed by SMS-EMOA* is shown on the left, with the three-dimensional space being projected twice into the two-dimensional space. The layout with the highest power output (3.590 MW) is shown in the middle, and it uses more than twice the area of the layout with the lowest output (3.270 MW) in this population. Note that the waves arrive from the top of the layouts. The circles in the middle and in the right show the 5m-buoys (to scale), and the safety distance is 50m (not shown).

speed-up the simulations, and if problem-specific variation operators are used. Also, it is important for engineers to explore different algorithms as these explore the objective spaces with different biases—and the consequence of this bias might matter to the decision maker.

5 Conclusions

Wave energy plays an increasing role in the energy supply world-wide. We have investigated the problem of placing wave energy converters on a given offshore area using different conflicting objective functions.

In a first step, we speeded up and parallelised the computations of the buoys' interactions, which resulted in a speed-up by a factor of up to 142 for 25 buoys. In order to improve the actual optimisation, we employed variation operators from the loosely related wind turbine problem. Interestingly, these problem-specific operators proved to be effective in our setting as well, despite the intra-device interactions being fundamentally different.

The computational study shows that multi-objective evolutionary algorithms can be used for the multi-objective buoy placement problem; in particular, our best performing configuration even improves the power output upon a previous single-objective results by 3.8%. This improvement can translate into millions of additional dollars of income per year during the lifetime of the wave farm.

In the future, we will extend our research to optimising the individual power take-off characteristics of the buoys in addition to their position, as the effective sea state within the WEC arrays differs significantly from the state outside.

Acknowledgments. This work has been supported by the ARC Discovery Early Career Researcher Award DE160100850.

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