Reliable Statistical Methods and their Applications for Testing Incomplete Multidisciplinary Data

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Contents

Abstract

Recently, left-truncated distributions have proved to be of use in modelling a range of phenomena in fields as diverse as finance, insurance, medicine, earthquake prediction and wind power. In this thesis, we present a comprehensive analysis of the left-truncated Weibull, loglogistic, lognormal and Pareto distributions in cases where the scale, shape or both parameters are unknown and estimated from the data with the maximum likelihood estimator. We define criteria which ensure that the maximum likelihood equations have a unique solution. We determine the critical values of the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests when the parameters are unknown for all of the left-truncated distributions via quantile analysis. In this work, these critical values are coupled with a rigorous point estimation and uncertainty analysis, and compared to the critical values of the complete (untruncated) distributions in the literature. We find strong agreement between our results and the most recent additions to the literature.

Analytically, we provide evidence that the critical values are parameter independent for all of the left-truncated distributions and goodness-of-fit tests. This result is verified by determining the critical values via Monte Carlo simulations for a range of parameter values. We find that the critical values are dependent upon sample size and truncation level (as percentage of the complete distribution), and determine suitable models to describe this behaviour. We modelled these critical values successfully for each of the three fitting scenarios (i) truncation level dependence, (ii) sample size dependence and (iii) truncation level and sample size dependence, which describes the behaviour for the critical values of all goodness-of-fit tests, left-truncated distributions and significance levels. The fact that one functional form describes the critical values for all different goodness-of-fit tests and distributions is a very useful and interesting result. The models are validated through an exhaustive power testing procedure, which also serves to compare the discriminatory power the four tests. We find the Anderson-Darling test has marginally better statistical power than the others in every situation and that the discrimantory power of all tests is weak for small sample sizes.

We conclude the work by applying all these statistical methods to analysing the inter-arrival times of market orders on the London Stock Exchange for a range truncation values and sample sizes. We find that the left-truncated Weibull distribution most accurately describes this data and that increasing the truncation level significantly increases the pass rates.

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Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

I give permission for the digital version of my thesis to be made available on the web, via the University's digital research repository, the Library Search and also through web search engines, unless permission has been granted by the University to restrict access for a period of time.

I acknowledge the support I have received for my research through the provision of an Australian Government Research Training Program Scholarship.

Signature: .... Date: ....

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Chapter 1

Introduction

Scientists have always endeavored to enhance their understanding of the world around them. Through experiment and observation, our comprehension has grown at a truly astonishing rate. The phenomena we observe can be loosely grouped into two categories, deterministic and stochastic. Deterministic processes can be predicted with absolute certainty, if sufficient information is gathered about the system, i.e. the state of the system is completely specified. A deterministic process in a completely specified system will consistently yield the same outcome if all of the relevant variables in the system are held constant. Consider the time it takes a ball to fall a certain distance within a vacuum, the distance is the only variable relevant to the fall time. Hence, if the distance the ball falls is unchanged, we expect a significant overlap of uncertainty margins for the times measured in each repetition of the experiment. By contrast, a stochastic process is one in which the outcome cannot be predicted with absolute certainty, regardless of how much information is collected about the system. One can only determine the probability of an outcome or set of outcomes. The time taken for a uranium-235 nucleus to decay, or which number will land face up after rolling a six-sided die are examples of stochastic processes. In both of these situations, the experiment will not consistently yield the same outcome. Stochastic processes are prolific throughout our world and form the foundation of the field of statistics in which this thesis seeks to contribute.

Statistics is a division of mathematics that primarily deals with the collection, analysis and interpretation of data. In the context of this thesis, we define "data" (or "observations") in the following way:
Consider a random experiment with results that are constrained to sample space, Ω. A random vector $\boldsymbol{X} = (x_1, x_2, ..., x_n)$ is defined on this sample space. The "observations" or "data" of a particular experiment are given by $\boldsymbol{X}(\omega)$, where ω is the result of that experiment.
Observations can be made in nearly every field of interest and subjecting those observations to statistical analysis often allows us to better understand the underlying phenomena and to make more informed decisions regarding our interaction with said phenomena. The value of statistical analysis has not gone unnoticed by industry nor scientists and practitioners, hence, it is employed in an extremely diverse array of applications. Some fields which have come to rely heavily upon statistical analysis include:

- **Insurance:** Fundamentally, an insurance policy is an agreement between an insurance agency and a policyholder in which the agency accepts some defined risk of the policyholder's in return for a premium. Thus, in order to decide upon an appropriate premium it is necessary for the agency to understand the risk which it is acquiring. Statistical analysis allows the insurer to determine the total risk resulting from the large number of underwritten policies, the total premium for this risk can then be distributed among the many policyholders [1].

- **Medicine:** Evidence based medicine is the basis upon which the modern health care system is built. People suffering from (what appears to be) the same ailment often experience different reactions to a given medicinal treatment, whether that be pharmaceutical or otherwise. For a given treatment to become an accepted part of modern medicine, it must demonstrate statistically significant results. That is, it must produce positive results frequently enough that the performance cannot reasonably be explained by the random variation in each patient's reaction [2].

- **Finance:** The price of different financial products-such as stocks-are subject to frequent variation. For a superannuation or retail fund to make an informed decision regarding a particular product they must understand the price fluctuations which they can reasonably expect over a relevant time horizon. People close to retirement for example, often require slow, steady growth of their portfolio, hence, selecting investment vehicles with low volatility is of the utmost importance. Statistical modelling allows us to understand the distribution of possible financial returns and choose products which are in line with our wants and needs.

- **Biology:** Often, different values of key characteristics are observed in different individuals of a particular genus. McDonald [3] gives the example of measuring chickens feet. If we measure the size of ten female and male chicken's feet, due to random fluctuations we expect distinct values for each measurement. If we observe different mean foot sizes in the two groups then we must answer the following question *Can the difference in average foot size be explained by random fluctuations?* If the answer is yes, we cannot determine whether there is a foot size difference. However, if the answer is no, we can declare a statistically significant difference which would lead us to conduct further research to understand why this may be the case. Statistical analysis is required to understand whether noise or underlying phenomena is responsible for the interesting aspects of our data.

There are many other cases in which statistics plays a crucial role, indeed the examples given above merely scratch the surface of statistical analysis within their respective fields. In finance for example, Steland [4] discusses the importance of statistical analysis in accurately pricing derivatives, for which there is an international market that exceeds $200 trillion [5]. In summary, statistical analysis allows us to make sense of the observations we have made and to disentangle interesting attributes from those that arise due to chance. This has proven to be extremely powerful in nearly all disciplines in which measurements are taken or observations are made.

1.1 Discrete and Continuous Events

Modelling the probability distribution of outcomes from stochastic processes is a field which can significantly enhance our understanding of a wide range of phenomena. Generally speaking, we can classify stochastic processes as having either (i) discrete or (ii) continuous outcomes.

(i) **Processes with discrete outcomes** can only produce a finite number of possible results with non-zero probability, i.e. they have a discrete sample space, Ω. Repeating the process-or experiment-may yield the same or different outcomes, however, more probable outcomes-by definition-occur with higher frequency. Thus, we can describe the distribution of outcomes probabilistically. The earlier example of rolling a six-sided die is a classic example of a stochastic process with a discrete outcome, X. There are only six possible outcomes of this process. The probability of rolling a particular number, x, is given by evaluating the probability mass function, f_{pm}, at x

$$f_{pm}(x) = \mathbb{P}(X = x) \quad \forall \quad x \in \Omega = \{1, 2, 3, 4, 5, 6\} \quad \text{where} \quad \sum_{\Omega} f_{pm}(x) = 1 \ . \tag{1.1}$$

Figure 1.1 displays the probability mass function under the assumption that the die is fair, so that each of the outcomes are equally probable. We note that $f_{pm}(x)$ is only defined for the values one, two, three, four, five or six, as these are the only valid outcomes of the die roll. Nothing can be returned for any other value that is passed to $f_{pm}(x)$.

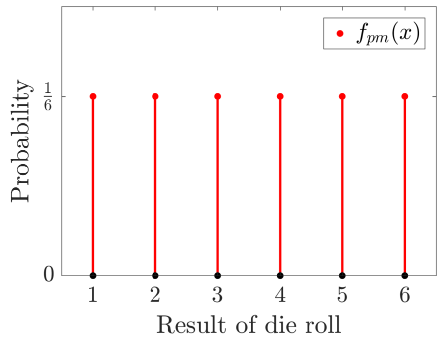


Figure 1.1: Probability mass function of a fair, six-sided die

Another area in which processes with discrete outcomes play a very important role is in modern experimental particle physics. Particles can take a continuous range of energy and momenta as they pass through a detector, however, all detectors have a limit on energy resolution. In practice, observations of particle energy are necessarily grouped into a finite number of energy bins. In cases where the total number of particles detected is small, binning the data may be necessary to obtain a statistically significant number of counts in each bin. The probability mass function that describes the distribution of energy can play a key role in determining whether a particular theory is more probable than another.

(ii) **Processes with continuous outcomes** can produce an uncountably infinite number of possible values with non-zero probability, i.e. they have a continuous sample space, Ω. The distribution of probability in these processes is typically defined by the probability density function, pdf, which is denoted $f(x)$. As this is a probability density function, rather than a probability mass function (as used in the discrete case) we must integrate $f(x)$ between two points a and b $(a \leq b)$ to determine the probability that the outcome will fall within the interval $[a, b]$,

$$\mathbb{P}(a \leq x \leq b) = \int_a^b f(x)dx \ . \tag{1.2}$$

The product of the pdf at a point x, and an infinitesimal interval, dx, gives the probability that an observation falls within the infinitesimal interval $[x, x + dx]$, hence, $f(x) \in \mathbb{R}^+$. Normalisation requires that the result of the integral in Eq. (1.2) be one, if the integration limits are extended to cover the complete range of possible values, i.e. the probability of an observation taking a value in the sample space, Ω, is 100%.

$$\int_\Omega f(x)dx = 1 \quad \text{and} \quad \Omega = (X_l, X_r) \ , \tag{1.3}$$

where X_l and X_r are the left and right limits of the sample space respectively. The time taken for a uranium-235 nucleus to decay is one example of such a process, it can take any value above 0, hence, $X_l = 0$ and $X_r = \infty$. The pdf of this process is

$$f(t) = \frac{\log(2)}{t_{\frac{1}{2}}} \exp\left[-\log(2)\frac{t}{t_{\frac{1}{2}}}\right] \ , \tag{1.4}$$

where t is the time and $t_{\frac{1}{2}}$ is the half-life of uranium-235, which is 700 million years. Figure 1.2 gives a visual representation of Eq. (1.4).

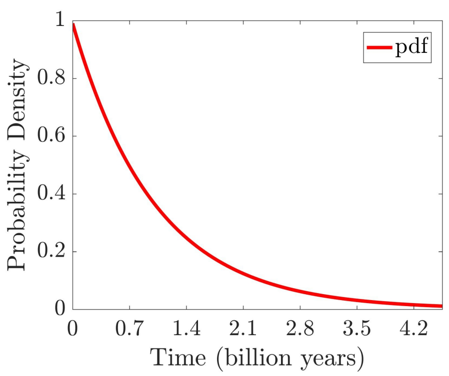


Figure 1.2: Probability density function of decay for a uranium-235 nucleus

In this work, we focus our attention upon processes with continuous sample spaces. More specifically, we restrict ourselves to processes for which the range of outcomes is necessarily non-negative and unbounded from above $(\Omega = (0, \infty))$ such as the time taken for a uranium-235 nucleus to decay. In these cases the normalisation condition in Eq. (1.3) can be reduced to,

$$\int_0^\infty f(x)dx = 1 \ . \tag{1.5}$$

The probability of X taking any value less than or equal to x is given by the cumulative distribution function, cdf, denoted $F(x)$. The pdf and cdf have the same support, and are related through

$$F(x) = \int_{X_l}^x f(t)dt = \mathbb{P}\left(X \leq x\right) \quad \forall \quad x \in \Omega \ . \tag{1.6}$$

Eq. (1.6) reveals some noteworthy features of the cdf,

$$
\begin{aligned}
\int_{X_l}^{X_r} f(t)dt = 1 \quad &\Rightarrow \quad \lim_{x \to X_r} F(x) = 1 \quad \text{and} \quad \lim_{x \to X_l} F(x) = 0 \\
f(x) \in \mathbb{R}^+ \quad &\Rightarrow \quad F(x) \quad \text{is a non-decreasing function of } x \\
&\Rightarrow \quad F(x) \in [0, 1) \,.
\end{aligned}
$$

The discussion thus far has largely been directed toward narrowing our field of focus. We have discussed the differences between deterministic and stochastic processes and restricted our study to stochastic phenomena. These were further separated into processes which yield either discrete or continuous outcomes and we elected to focus upon the continuous case. Figure 1.3 visually represents that **our scope is stochastic processes with continuous outcomes**.

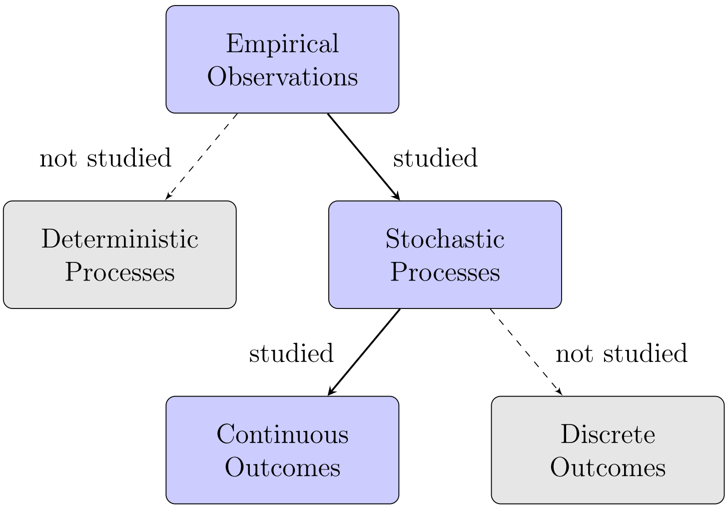


Figure 1.3: Flow chart of topics studied in this thesis

1.2 Probability Distributions with Parameters

A large number of probability distributions are parametrised, that is they contain some parameters which allow them to be more flexible. The m parameters of a general distribution are denoted $\boldsymbol{\theta} = \{\theta_1, \theta_2, ..., \theta_m\}$, and are confined to a known *parameter space*, $\boldsymbol{\Theta}$ ($\boldsymbol{\theta} \in \boldsymbol{\Theta}$) which is restricted to a subset of m-dimensional space. The pdf that corresponds to $\boldsymbol{\theta}$ is denoted $f(x|\boldsymbol{\theta})$, however, each parameter set does not necessarily have a one-to-one relationship with the pdf. Therefore, we cannot always identify the parameters from observations drawn from the pdf:

$$
\begin{array}{lllll}
\text{if} \quad \boldsymbol{\theta^1} \neq \boldsymbol{\theta^2} & \text{implies} & f(x|\boldsymbol{\theta^1}) \neq f(x|\boldsymbol{\theta^2}) & \forall\, x & \text{the parameters are identifiable} \\
\text{if} \quad \boldsymbol{\theta^1} \neq \boldsymbol{\theta^2} & \text{does not necessarily imply} & f(x|\boldsymbol{\theta^1}) \neq f(x|\boldsymbol{\theta^2}) & \forall\, x & \text{the parameters are unidentifiable} \,.
\end{array}
$$

The work in this thesis requires the ability to obtain parameter estimates from observations, hence, we restrict our attention to cases in which the parameters are identifiable. The normal distribution is one such example, its parameters are the mean, μ, and standard deviation, σ, hence $\boldsymbol{\theta} = \{\mu, \sigma\}$. The pdf is given by

$$
f_n(x|\mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right] . \tag{1.7}
$$

The support, mean and standard deviation are restricted to

Support	$0 < x < \infty$
Scale parameter	$-\infty < \mu < \infty$
Shape parameter	$-\infty < \beta < \infty.$

Clearly, the probability distribution specified by Eq. (1.7) is sensitive to changes in μ and σ. Thus, for given values of μ and σ, one can deduce the relative likelihood of making an observation of value x with Eq. (1.7). However, in many contexts we do not know the true values of the parameters in advance. In these cases we seek to formulate estimates of the distribution parameters. Throughout this work, we will make repeated reference to the true and estimated parameters of probability distributions. These are distinct entities and thus it is necessary to employ notation which allows us differentiate between the two. Table 1.1 summarises the notation we will use to make this distinction.

$\boldsymbol{\theta}^0$	known *a priori*
$\hat{\boldsymbol{\theta}}$	estimated from data

Table 1.1: Summary of parameter notation

Deciding how to use the data and finding a suitable model to describe the data are often related problems. One of the difficulties that arises from structuring the model around the data is that it becomes very challenging to ascertain the accuracy of the estimates, or probability of reaching correct conclusions. This is going to be discussed in the later chapters. The models studied in this work are discussed in the next section.

1.3 Studied Probability Distributions

The research we conducted was restricted to the Weibull, loglogistic, lognormal and Pareto distributions. In this section we simply introduce the pdf, cdf and applicable parameter ranges for each distribution, a detailed discussion of the distributions will be given in their respective chapters. The majority of distributions in this study have two parameters which have been specified as either a 'shape' or 'scale' parameter. The results of this study are intended to be applied in fields for which accurately determining the probability density in the tail region is of the most importance.

- **Weibull Distribution:** The pdf, $f(x)$, and cdf, $F(x)$, of the Weibull distribution are defined as,

$$\boldsymbol{f(x|\alpha,\beta)} = \frac{\boldsymbol{\beta}}{\boldsymbol{\alpha}}\left(\frac{\boldsymbol{x}}{\boldsymbol{\alpha}}\right)^{\boldsymbol{\beta-1}} \mathbf{exp}\left[-\left(\frac{\boldsymbol{x}}{\boldsymbol{\alpha}}\right)^{\beta}\right], \tag{1.8}$$

$$\boldsymbol{F(x|\alpha,\beta)} = \mathbf{1} - \mathbf{exp}\left[-\left(\frac{\boldsymbol{x}}{\boldsymbol{\alpha}}\right)^{\beta}\right]. \tag{1.9}$$

The support, scale parameter, α, and shape parameter, β, are restricted to,

Support	$0 \leq x < \infty$
Scale parameter	$0 < \alpha < \infty$
Shape parameter	$0 < \beta < \infty$.

The Weibull distribution is the subject of a thorough discussion in Chapter 3.

- **Loglogistic Distribution:** The pdf, $f(x)$, and cdf, $F(x)$, of the loglogistic distribution are defined as,

$$\boldsymbol{f(x|\phi,\rho) = \frac{\rho}{\phi^\rho}\frac{x^{\rho-1}}{\left[1+\left(\frac{x}{\phi}\right)^\rho\right]^2}} \ , \tag{1.10}$$

$$\boldsymbol{F(x|\phi,\rho) = \frac{1}{1+\left(\frac{x}{\phi}\right)^{-\rho}}} \ . \tag{1.11}$$

The support, scale parameter, ϕ, and shape parameter, ρ, are restricted to,

Support	$0 \leq x < \infty$
Scale parameter	$0 < \phi < \infty$
Shape parameter	$0 < \rho < \infty$.

The loglogistic distribution is the subject of a thorough discussion in Chapter 4.

- **Lognormal Distribution:** The pdf, $f(x)$, and cdf, $F(x)$, of the lognormal distribution are defined as,

$$\boldsymbol{f(x|\mu,\sigma) = \frac{1}{\sigma x\sqrt{2\pi}}\exp\left[-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right]} \ , \tag{1.12}$$

$$\boldsymbol{F(x|\alpha,\beta) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\log(x)-\mu}{\sigma\sqrt{2}}\right)\right]} \ . \tag{1.13}$$

The mean, μ, and standard deviation, σ, have been classified as the scale and shape parameters respectively for consistency with the other distributions. The support, scale parameter, μ, and shape parameter, σ, are restricted to,

Support	$0 \leq x < \infty$
Scale parameter	$-\infty < \mu < \infty$
Shape parameter	$0 < \sigma < \infty$.

The lognormal distribution is the subject of a thorough discussion in Chapter 5.

- **Pareto Distribution:** The pdf, $f(x)$, and cdf, $F(x)$, of the Pareto distribution are defined as,

$$\boldsymbol{f_T(x|k) = \frac{k\tau_l^k}{x^{k+1}}} \ , \tag{1.14}$$

$$\boldsymbol{F_T(x|k) = 1-\left(\frac{\tau_l}{x}\right)^k} \ . \tag{1.15}$$

The support and scale parameter, k, are restricted to,

Support	$\tau_l \leq x < \infty$
Scale parameter	$0 < k < \infty$.

The Pareto distribution is the subject of a thorough discussion in Chapter 6.

1.4 Likelihood Function

More often than not, multiple observations of the variable of interest are recorded within a data set. To develop an understanding of the phenomena of interest, it is necessary to construct a probability distribution that accounts for the complete set of observations. Suppose $J(\boldsymbol{x}, \boldsymbol{\theta})$ is the density function of $\boldsymbol{x}$, where $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$ is the set of n observations. $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_m)$, hence, the $\boldsymbol{\Theta}$ is a subset of m-dimensional space. Now, consider $J(\boldsymbol{x}, \boldsymbol{\theta})$ as a function of $\boldsymbol{\theta}$ for fixed $\boldsymbol{x}$. With this interpretation we can write

$$J(\boldsymbol{x}, \boldsymbol{\theta}) = L(\boldsymbol{\theta}|\boldsymbol{x}) \tag{1.16}$$

and refer to $L(\boldsymbol{\theta}|\boldsymbol{x})$ as the "likelihood function". Here $\boldsymbol{x}$ is thought of as the set of values that were observed in an experiment. Thus, we can think of $L(\boldsymbol{\theta}|\boldsymbol{x})$ as a measure of how "likely" $\boldsymbol{\theta}$ is to have produced $\boldsymbol{x}$. In this work we restrict ourselves to the study of phenomena for which each measurement is independent and identically distributed (i.i.d.) to each of the other measurements, hence, $J(\boldsymbol{x}, \boldsymbol{\theta})$ is defined as the product of the pdf evaluated at each observation

$$L(\boldsymbol{\theta}|\boldsymbol{x}) = J(\boldsymbol{x}, \boldsymbol{\theta}) = \prod_{i=1}^{n} f(x_i|\boldsymbol{\theta}) \ . \tag{1.17}$$

In statistics, the term 'likelihood' is not synonymous with probability, indeed the term is void of a rigorous definition. It is merely the case that the parameter values $\boldsymbol{\theta_1}$ are more likely than $\boldsymbol{\theta_2}$ given observations $\boldsymbol{x}$, if $L(\boldsymbol{\theta_1}|\boldsymbol{x}) > L(\boldsymbol{\theta_2}|\boldsymbol{x})$.

1.5 Maximum Likelihood Estimation

As discussed previously, it is not always the case that the true parameters, $\boldsymbol{\theta^0}$, of a distribution are known *a priori*. Thus, we must construct estimates of these parameters, $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$, from a set of observations, $\boldsymbol{x}$. For example, one may anticipate that the heights of 30 year old women in Australia are normally distributed. However, it is not clear what the mean and standard deviation of this distribution should be. Therefore, in order to construct an accurate probability distribution, we must estimate the parameters from a sample (set of observations) of height measurements from the relevant demographic. There are a number of methods by which we can construct these parameter estimates, some examples are the "moment estimator", "Cramér-Rao estimator", "Linear unbiased estimator", "Maximum likelihood estimator" and "'Bayes estimator". For any reasonably accurate parameter estimation method, we must employ modern computers to calculate the estimates. In this thesis, we use the "Maximum Likelihood Method" of parameter estimation exclusively, because of its prolific position within the literature and numerous favourable attributes (which are discussed later in this section) [6–15].

The method of maximum likelihood estimation is based on the principle that the parameters, $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$, which maximise the likelihood function are most likely to have produced the data. Therefore, the problem of parameter estimation is reduced to finding the parameter values which maximise the likelihood function, i.e.

$$\hat{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\boldsymbol{\theta}\in\Theta}{\arg\max} \quad L(\boldsymbol{\theta}|\boldsymbol{x}) \quad \text{where } \Theta \text{ is the entire parameter space.} \tag{1.18}$$

$L(\boldsymbol{\theta}|\boldsymbol{x})$ is the product of n terms, and thus is quite complicated and expensive to compute for large samples. It is often simpler to deal with the loglikelihood function, $l(\boldsymbol{\theta}|\boldsymbol{x})$

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = \log\left\{L(\boldsymbol{\theta}|\boldsymbol{x})\right\} = \log\left\{\prod_{i=1}^{n} f(x_i|\boldsymbol{\theta})\right\} = \sum_{i=1}^{n} \log\left(f(x_i|\boldsymbol{\theta})\right) \ . \tag{1.19}$$

The natural logarithm is a monotonically increasing function, hence, the $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$ which maximise the likelihood function also maximise the loglikelihood function. Therefore, our parameter estimation problem becomes

$$\hat{\boldsymbol{\theta}}(\boldsymbol{x}) = \underset{\boldsymbol{\theta}\in\Theta}{\arg\max} \quad l(\boldsymbol{\theta}|\boldsymbol{x}) \quad \text{where } \Theta \text{ is the entire parameter space.} \tag{1.20}$$

At $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$, the first partial derivative of $l(\boldsymbol{\theta}|\boldsymbol{x})$ with respect to each of $\{\theta_1, \theta_2, ..., \theta_m\}$ vanishes. Therefore, the parameter estimates $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$ are solutions to the MLE equations

$$\frac{\partial}{\partial\theta_j} l(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{\partial}{\partial\theta_j}\sum_{i=1}^{n} \log\left(f(x_i|\boldsymbol{\theta})\right) = 0 \quad \forall \quad j = 1, 2, ..., m \ . \tag{1.21}$$

The MLE parameter estimates are simultaneous solutions to this set of nonlinear equations, however, we also need to satisfy conditions on the second-order derivatives of the loglikelihood function to ensure that the solutions maximise likelihood. If these conditions are not easily satisfied, we can maximise the loglikelihood function numerically. Computationally, this can be done with an iterative scheme, a direct search for global maxima or a multi-dimensional root finding algorithm such as the Newton-Raphson method.

The maximum likelihood estimator has achieved pre-eminence because it has many desirable properties, such as the ease with which the covariance matrix of estimates can be obtained. Three of the most important properties are proven by Lehmann and Casella [16] in Theorem 1.5 (Thm. 5.1 of section 6.5 p.463 of their book 'Theory of Point Estimation'). This theorem denotes the parameter estimates from a sample of size n, as $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$, which is slightly different from the $\hat{\boldsymbol{\theta}}$ we have used to denote all parameter estimates thus far. This theorem applies when the conditions (A0)–(A2) and assumptions (A)–(D) are true.

Conditions:

- A0: The probability distributions of the observations are distinct, i.e.

$$\text{For } \boldsymbol{\theta} \neq \boldsymbol{\theta}',\ f(x|\boldsymbol{\theta}) \neq f(x|\boldsymbol{\theta}') \ \forall \ x \in [0, \infty) \ .$$

- A1: The probability distributions have common support, i.e.

$$\text{if } f(\boldsymbol{\theta})\colon X \to Y \text{ and } f(\boldsymbol{\theta}')\colon X' \to Y' \text{ then } X = X' \ \forall \ \boldsymbol{\theta}, \boldsymbol{\theta}' \ .$$

- A2: The observations $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$ are i.i.d. with a probability density function $f(x|\boldsymbol{\theta}^0)$

Assumptions:

- A: There exists an open subset ω of Ω containing the true parameter point $\boldsymbol{\theta}^0$ such that for almost all x, the probability density function, $f(x|\boldsymbol{\theta}^0)$, admits all third derivatives,

$$\frac{\partial^3}{\partial\theta_i\partial\theta_j\partial\theta_k} f(x|\boldsymbol{\theta}^0) \quad \forall \quad \boldsymbol{\theta} \in \omega \ ,$$

- B: The first and second logarithmic derivatives of f satisfy the equations,

$$\mathbb{E}_\theta \left[\frac{\partial}{\partial\theta_j} \log f(x|\boldsymbol{\theta}) \right] = 0 \ \forall \ j \in \{1, 2, ..., m\} \ ,$$

and

$$\begin{aligned} I_{jk}(\boldsymbol{\theta}) &= \mathbb{E}_\theta \left[\frac{\partial}{\partial\theta_j} \log f(x|\boldsymbol{\theta}) . \frac{\partial}{\partial\theta_k} \log f(x|\boldsymbol{\theta}) \right] \\ &= \mathbb{E}_\theta \left[-\frac{\partial^2}{\partial\theta_j\partial\theta_k} \log f(x|\boldsymbol{\theta}) \right] \quad \forall \ j, k \in \{1, 2, ..., m\} \ . \end{aligned}$$

- C: The matrix $I(\boldsymbol{\theta})$ is finite and positive definite $\forall \ \boldsymbol{\theta} \in \omega$.

- D: There exist functions $M_{jkl}(x)$ such that,

$$\left| \frac{\partial^3}{\partial\theta_i\partial\theta_j\partial\theta_k} \log f(x|\boldsymbol{\theta}) \right| \leq M_{jkl}(x) \ \forall \ \boldsymbol{\theta} \in \omega$$
$$\text{where } \mathbb{E}_{\theta^0} \left[M_{jkl}(x) \right] < \infty \ \forall \ j, k, l \ .$$

Theorem 1.5. Let $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$ be i.i.d. each with a probability density function $f(x|\boldsymbol{\theta^0})$ which satisfies conditions (A0)–(A2) and assumptions (A)–(D). Then with probability tending to 1 as $n \to \infty$, there exist solutions $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}} = \hat{\boldsymbol{\theta}}_{\boldsymbol{n}}(\boldsymbol{x})$ of the likelihood equations such that

- The vector $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$ is consistent for estimating $\boldsymbol{\theta^0}$.

> Explanation: As $n \to \infty$ the parameter estimates produced by the MLE, $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$, tend to the true parameter values, $\boldsymbol{\theta^0}$. Mathematically this is stated as
>
> $$\lim_{n\to\infty} \hat{\boldsymbol{\theta}}_{\boldsymbol{n}} = \boldsymbol{\theta^0}. \tag{1.22}$$

- $\sqrt{n}(\hat{\boldsymbol{\theta}}_{\boldsymbol{n}} - \boldsymbol{\theta^0})$ is asymptotically normal with vector mean zero and covariance matrix $[I(\boldsymbol{\theta^0})]^{-1}$ being the inverse of the Fisher information matrix,

$$I(\boldsymbol{\theta^0})_{j,k} = -\mathbb{E} \left(\frac{\partial^2}{\partial\theta_j\partial\theta_k} \log \left(f(x|\boldsymbol{\theta^0}) \right) \right) \ ,$$
where $\quad j, k \in (1, 2, ..., m) \quad$ and $\mathbb{E}$ denotes the expected value operator.

> Explanation: As $n \to \infty$ the parameter estimates produced by the MLE, $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$, tend to a multivariate Gaussian distribution with mean $\boldsymbol{\theta^0}$ and covariance matrix $\frac{1}{\sqrt{n}} \left[I(\boldsymbol{\theta^0}) \right]^{-1}$. The Fisher information matrix is denoted $I(\boldsymbol{\theta^0})$ and its elements are specified above. A random vector follows a k-variate Gaussian distribution if every linear combination of its k components has a univariate Gaussian distribution. The covariance matrix, Σ, is the matrix whose (i, j) entry is the covariance
>
> $$\Sigma_{i,j} = \text{cov}\,(X_i, X_j) = \mathbb{E}[(X_i - \mathbb{E}(X_i))(X_j - \mathbb{E}(X_j))].$$

- $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$ is asymptotically efficient in the sense that as $n \to \infty$,

$$\sqrt{n}(\hat{\theta}_{n,j} - \theta_j^0) \xrightarrow{\mathcal{L}} \mathcal{N}\left(0, \left[I(\boldsymbol{\theta^0})\right]_{j,j}^{-1}\right)^{1} \quad \text{where} \quad j \in (1, 2, ..., m),$$

  and $\xrightarrow{\mathcal{L}}$ denotes convergence in distribution.

> Explanation: As $n \to \infty$ the MLE achieves the Cramér-Rao lower bound, which restricts the covariance that any estimator can attain. For unbiased estimators, the Cramer-Rao lower bound states that the covariance matrix of the estimated parameters is not less than the inverse of the Fisher information matrix, in the sense that their difference is a positive definite matrix. In the case of the MLE, the covariance matrix tends toward the inverse of the Fisher information matrix as the sample size increases, therefore the MLE is said to be 'efficient' [17–20].

The properties that result from Theorem 1.5 are important because they confirm that the maximum likelihood estimator tends toward the true parameters efficiently for large sample sizes. Hence, it would be beneficial to show that conditions (A0)–(A2) and assumptions (A)–(D) hold for all of the distributions studied in this thesis. This is out of the scope of this thesis, hence, in a similar manner to Kreer et al. [11], we give plausible arguments to support our assertion that the conditions and assumptions hold. We must keep in mind that these attributes are only true asymptotically, i.e. we cannot expect the MLE to produce estimates that are distributed as a multivariate Gaussian centered around the true parameter values for small n. Indeed, it is the case that for finite n there exists a non-zero bias vector, $\boldsymbol{B}$, associated with the MLE parameter estimates [21, 22]. The bias vector is defined as,

$$\boldsymbol{B} = \mathbb{E}\left[\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}\right] - \boldsymbol{\theta^0}. \tag{1.23}$$

The magnitude of this bias vector is of order $\mathcal{O}\left(\frac{1}{n}\right)$, hence, as $n \to \infty$ we achieve the behaviour specified by Theorem 1.5. Determining what n value is large enough for the bias to become negligible is an important question that deserves the attention of future studies.

1.6 Hypothesis Testing

In areas such as science, industry and medicine, people often strive to find yes or no answers to important questions. For example Louis Pasteur asked "Can life be spontaneously generated from inert matter?", a cancer researcher may ask "Does this drug reduce the tumour size?", an actuary may ask "Is one type of car safer than another?" and a manufacturer may ask "Do our products exhibit more than the acceptable level of defects?". To try to answer these questions, we construct experiments whose outcomes have some bearing on the question of interest. The process of determining whether the outcome points to a yes or no is called *hypothesis testing*. The nuances of hypothesis testing are discussed Chapter 2. In the current section we give a brief outline of how it works and why it is important to our research.

Hypothesis testing is a statistical technique employed to determine whether a null hypothesis, H_0, can be rejected given a set of n observations $\boldsymbol{x} = (x_1, x_2, ..., x_n)$. In the context of this thesis, the null hypothesis is that the observations come from a population

[1] $\mathcal{N}(\mu, \sigma^2)$ denotes the normal distribution with mean μ and variance σ^2.

with cumulative distribution function (cdf), $F(x)$. Hypothesis testing allows us to ascertain a level of confidence about whether observations were drawn from the distribution specified in the null hypothesis. This is important because it enables us to verify theoretical predictions and develop models that describe the empirical observations we make. These capabilities play a fundamental role in the continuing development of science and technology. For example, unstable quantum mechanical particles exist for some finite lifetime that is described by a continuous probability distribution. Determining the degree to which observations of this lifetime agree with various theories plays a crucial role in advancing our understanding of the universe.

In practice, we employ a goodness-of-fit test as a criterion that allows us to decide whether or not to reject H_0 given $\boldsymbol{x}$. Goodness-of-fit tests produce a real, scalar measure of how well the observation set, $\boldsymbol{x}$, agrees with the null hypothesis. **This measure is called the test statistic**. In general, a smaller test statistic implies stronger agreement between the observations and the predictions of the null hypothesis. To determine whether or not to reject the null hypothesis one must compare the test statistic to a previously known **critical value**. If the test statistic is lower than the critical value, then the null hypothesis cannot be rejected, i.e.

$$\text{if} \quad \begin{cases} T(\boldsymbol{x}) \geq c & : \quad H_0 \text{ is rejected} \\ T(\boldsymbol{x}) < c & : \quad H_0 \text{ cannot be rejected} \end{cases} \tag{1.24}$$

where $T(\boldsymbol{x})$ is the test statistic and c is the critical value. Prior knowledge of the relevant critical values is required in order to conduct hypothesis testing. One of the main objectives of this thesis is to determine a set of formerly unknown critical values and thus allow others to conduct hypothesis testing in a new array of situations. Critical values are dependent upon a number of variables including the sample size, n, the cdf, $F(x)$ (specified by H_0) and which parameters are estimated from the sample[2]. A more in-depth description of critical values and how we calculate them is given in 2.1.

1.6.1 Random Generation Formula

In order to determine critical values via Monte Carlo methods, it is necessary to randomly generate many observations taken from a known distribution (details are given in section 2.1). Here we briefly describe the process of generating numbers from an arbitrary distribution specified by the cdf, $F(x|\boldsymbol{\theta^0})$. Evaluating the inverse cdf, denoted $F^{-1}(u|\boldsymbol{\theta^0})$ where $u \in (0,1)$, at the point $F(x|\boldsymbol{\theta^0})$ will yield the value x,

$$F^{-1}(F(x|\boldsymbol{\theta^0})|\boldsymbol{\theta^0}) = x. \tag{1.25}$$

The range of the cdf is $(0,1)$, hence, generating a set of uniformly distributed random numbers $u_i \in (0,1)$ and passing them to the inverse cdf produces a set of randomly generated data points, x_i, from the distribution defined by $F(x|\boldsymbol{\theta^0})$

$$F^{-1}(u_i|\boldsymbol{\theta^0}) = x_i. \tag{1.26}$$

This set of data points can then be used to test the behaviour of observations that originate from the distribution specified by $F(x|\boldsymbol{\theta^0})$. Also, we define the array of uniform random variates $\boldsymbol{u}$,

$$\boldsymbol{u} = (u_1, u_2, ..., u_n) \quad \text{where} \quad u_i \in (0,1) \quad \forall \quad i = (1, 2, ..., n) \;, \tag{1.27}$$

that produces the observations, x.

[2]Critical values are also dependent upon truncation level which is introduced in 1.9

1.7 Nomenclature of Parameter Estimation

One of the main objectives of this thesis is to determine and model new critical values. They will be dependent upon the number and type of parameters that were estimated from the sample, hence, it is necessary to distinguish the different parameter estimation cases. The notation we have employed has been taken from Kizilersü et al. [10] and is given in Table 1.2. As the distributions considered in this study have at most two parameters, there are four distinct cases of parameter estimation.

Case	Scale Parameter	Shape Parameter
I	known *a priori*	known *a priori*
II	acquired from data	acquired from data
IIIa	acquired from data	known *a priori*
IIIb	known *a priori*	acquired from data

Table 1.2: Cases of parameter estimation

1.8 Frequentist and Bayesian Statistics

So far we have exclusively discussed the frequentist approach to statistical analysis because we intend to utilise it throughout this thesis. However, it would be remiss not to acknowledge the existence of the Bayesian approach. In this section we compare the two methodologies and discuss the situations in which each method performs best. The frequentist framework is the traditional way to approach statistical problems, however, neither method is objectively better. Each has a distinct way of interpreting probability, and this distinction underpins the difference between the two.

- **Frequentist Approach:** In the frequentist school of thought, probability (p) is the limiting value of the number of successes in a sequence of trials

$$p = \lim_{N\to\infty} \frac{k}{N} \quad \text{where } k \text{ is the number of successes in } N \text{ trials.} \tag{1.28}$$

  Frequentists think of the observations as being drawn from some absolutely defined probability distribution, specified by the pdf $f(x|\boldsymbol{\theta^0})$, where $\boldsymbol{\theta^0}$ are the true parameter values. The true parameter values are fixed, hence, it does not make sense to talk about the probability distribution of the true parameters. When the true parameters are not known *a priori*, it is necessary to construct parameter estimates[3], denoted $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$, based upon a set of observations $\boldsymbol{x}$. There is an uncertainty associated with these parameter estimates called the confidence interval, and it is denoted $\boldsymbol{I_F}$. The confidence interval must be defined at some arbitrarily selected confidence level, with 95% being the most common choice and the one we will apply in this thesis. Given this specification, $\boldsymbol{I_F}$, is to be interpreted in the following way: If the experiment is repeated N times, each producing a distinct estimate of parameters and corresponding confidence interval, then in the limit as $N \to \infty$ the true parameter values will fall within the confidence interval for 95% of the repetitions. Hypothesis testing is the most common method used to determine whether a model describes our observations, or to select the optimal model from a range of viable options.

- **Bayesian Approach:** Under the Bayesian approach, probability quantifies our uncertainty about the world. Observations are objectively true, and are not considered

[3] Frequentist parameter estimation is discussed in 1.5

to come from a probability distribution. The parameters, on the other hand, are treated as random variables which can be described with a probability distribution. The Bayesian methodology allows us to evolve our prior knowledge of the parameters toward a more accurate probability distribution by incorporating additional observations. Consider an experiment from which we obtain a set of observations, $\boldsymbol{x}$. Before the experiment, we described the parameters with a probability distribution, $\mathbb{P}(\boldsymbol{\theta})$, called the prior probability. After the experiment, we can update our probability distribution to $\mathbb{P}(\boldsymbol{\theta}|\boldsymbol{x})$ which is called the posterior probability,

$$\mathbb{P}(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{L(\boldsymbol{\theta}|\boldsymbol{x})\mathbb{P}(\boldsymbol{\theta})}{\int_{\boldsymbol{\Theta}} L(\boldsymbol{\theta}|\boldsymbol{x})\mathbb{P}(\boldsymbol{\theta})d\boldsymbol{\theta}} \ , \tag{1.29}$$

where Θ is the entire parameter space. $L(\boldsymbol{\theta}|\boldsymbol{x})$ is defined by Eq. (1.17), which is the same as in the frequentist approach. When we are conducting our first experiment we are faced with the problem of having to use a prior probability distribution, $\mathbb{P}(\boldsymbol{\theta})$, despite having no understanding of how the parameters are distributed. We are free to select any distribution which is normalised over Θ, however, some subjectivity is necessarily introduced into the posterior probability. Commonly a uniform prior is used, however, normalisation requires us to restrict the parameter space to some finite region $\omega \in \Theta$. After a large number of experiments, the effect of an initial prior is minimal. Given that the parameters are described by a probability distribution, we can define an uncertainty margin analogous to the confidence interval, called the credible interval, $\boldsymbol{I_B}$. Assuming that the credible interval is defined at the 95% confidence level, we interpret it in the following way: $\boldsymbol{I_B}$ is the densest region of probability such that

$$\int_{\boldsymbol{I_B}} \mathbb{P}(\boldsymbol{\theta}|\boldsymbol{x})d\boldsymbol{\theta} = 0.95 \ . \tag{1.30}$$

There is no concept of hypothesis testing in the Bayesian framework, instead we select optimal models by employing the Bayes factor, K

$$K = \frac{\int_{\boldsymbol{\Theta}} \mathbb{P}_A(\boldsymbol{\theta}|\boldsymbol{x})d\boldsymbol{\theta}}{\int_{\boldsymbol{\Theta}} \mathbb{P}_B(\boldsymbol{\theta}|\boldsymbol{x})d\boldsymbol{\theta}} \ , \tag{1.31}$$

where $\mathbb{P}_A(\boldsymbol{\theta}|\boldsymbol{x})$ and $\mathbb{P}_B(\boldsymbol{\theta}|\boldsymbol{x})$ are the posterior probabilities of two competing models, A and B. If K takes a larger value then there is more support for A than for B and visa-versa for a low K. The level of confidence associated with 'high' and 'low' values is defined by the Jeffreys scale [23].

The comparison we have given is extremely brief, however, it does highlight the key differences between the two methods. The ability to utilise the knowledge we have prior to an experiment is a clear point of difference between the frequentist and Bayesian approaches. There is no mechanism by which we can include results from previous experiments in the frequentist framework, this is a limitation. However, in the Bayesian framework one must select an initial prior despite having no information to guide this decision. Hence, subjectivity is introduced into the probability distribution describing the parameters, which is not ideal. For cases in which many experiments have been conducted to study a fixed parameter distribution, the effect of a subjective prior is reasonably meager, and the ability to include all parameter estimates is of great value, hence, the Bayesian methodology is preferable. However, if one has access to only one data set, incorporating previous knowledge is of no use, and subjective priors introduce an unacceptable level of subjectivity into the analysis. Thus, the frequentist approach is the most useful in these situations. We intend to apply our work to data drawn from distributions which change with respect to time, therefore, we can only ever have one data set from a particular distribution. The frequentist framework is best suited to the type of problem we wish to address, thus, we have used it throughout this thesis.

1.9 Truncation

There is an abundance of situations in which it is not possible to make reliable measurements for some region of a randomly distributed variable of interest. When analysing these situations, one must only consider observations that occur within the reliable region. Restricting the support in this way, and adjusting the probability distribution accordingly, is known as truncation. Probability distributions with a restricted support are dubbed truncated distributions, and those with an unrestricted support are called complete (or untruncated). Failure to account for the restricted support (by employing a complete distribution) will lead to incorrect conclusions. Often, truncation is either conducted from the left, right or a combination of the two. In right truncation, measurements made above the right truncation limit, τ_r, are not to be trusted and in left-truncation, measurements below the left-truncation limit, τ_l, are unreliable or do not exist. It is possible to impose both left and right truncation simultaneously, so that the support is restricted to a region of width $\tau_r - \tau_l$, this is called two-sided truncation. Note that left-truncation is essentially two-sided truncation in the special case that $\tau_r = \infty$, and right truncation is when $\tau_l = 0$. Additionally, one can remove a segment(s) of the support, however, to our knowledge this form of truncation is of extremely limited use. **In this thesis, we focus our attention upon left-truncation** because it can be easily applied to real world data-to which we have been granted access-in the fields of insurance, medicine, finance and energy. Some other examples of situations in which truncated distributions are required for accurate modelling include:

- **Optimal tolerances in a manufacturing assembly**: Nadarajah et al. [24] summarises the use of truncated distributions in manufacturing assembly as largely falling into two main categories:

  (i) Often, products undertake a screening test before leaving the factory floor, and they must demonstrate performance within tolerance limits in order to pass. Units which fail to demonstrate the required performance are not delivered to customers. Therefore, the performance of products that customers interact with is governed by a truncated distribution.

  (ii) Many modern manufacturing processes have multiple production stages, each of which has its own testing and rejection procedure. Only units that perform within a restricted support are passed onto subsequent tests. As a result, a truncated distribution is required to describe the performance of products in the latter stages of production.

  The summary given by Nadarajah et al. [24] is an overview of several studies that sought to optimise tolerances and improve operational efficiency [25–31].

- **Swiss insurance automotive claims**: Kreer et al. [11] studied automotive claims with an excess loss treaty of CHF 100,000. In this case, claims under CHF 100,000 are not recorded because it is cheaper for the concerned party to settle the payment themselves. The lack of information for claims under CHF 100,000 necessitates the use of left-truncation (with $\tau_l = 100,000$) to describe the probability distribution of claim values.

- **High-performance Ethernet**: Jobs arrive at a server according to a Poisson process and they are removed in blocks (Ethernet frames). Between each block, the server waits for a short period of time (inter-frame gap). The inter-frame gap has a minimum value, I, however if the server is idle this gap is extended. Therefore, the inter-frame gaps are distributed according to a left-truncated distribution with $\tau_l = I$. Field et al. [32] modelled network traffic at various parts of the computing department of Imperial College London, and found that a truncated Cauchy distribution was a much better fit than the prevailing model at the time.

- **Inter-arrival times of orders on the LSE**: The time between successive orders placed on the London Stock Exchange (LSE) is of great interest to financial institutions, as it allows them to infer future demand and price fluctuations for a given stock. However, ultra-high frequency manipulation causes a huge number of orders to be placed that are not intended to be executed. If one neglects the effect of these orders, the resulting demand predictions will be insufficiently accurate. Kizilersü et al. [33] found that removing orders with a time separation of less than 10ms is a pragmatic way of dealing with ultra-high frequency manipulation. They proposed, and verified the left-truncated Weibull distribution as a model of the inter-arrival times of orders on the LSE.

- **Remaining life of power transformers**: Hong et al. [34] were tasked with predicting the remaining life of high-voltage power transformers for an energy company. The company started keeping records on the 1^{st} of January 1980 and had complete records of all transformers that were installed and failed after that date. However, there were a number of transformers that were installed before 1980, some of these had failed, but the relevant dates were not recorded. Therefore, Hong et al. [34] employed a left-truncated probability distribution to model the life time of the remaining transformers that were installed before records were kept.

- **Earthquake inter-arrival times in California**: Kreer et al. [11] studied the inter arrival times between subsequent earthquakes in California from the 25^{th} of March, 1806 until the 29^{th} of March, 2014 from the perspective of a reinsurance agency. An insurance agency may take out a policy with a reinsurer so that in the case of a catastrophic event, i.e. a large earthquake, they have access to sufficient funds to remunerate their policy holders in a timely fashion. For the reinsurer to pay, claims associated with a particular earthquake and its aftershocks must exceed a predetermined threshold. Geologically, aftershocks can continue months or even years after the main earthquake, however, it is hardly tenable for a reinsurance agency to wait this long before paying out on a claim. As a result, only earthquakes which occur within a certain number of hours (often 72 or 168) after the main earthquake are counted as aftershocks, everything else is treated as an 'independent' earthquake. Therefore, Kreer et al. [11] successfully modelled the inter arrival times between 'independent' earthquakes with a left-truncated probability distribution (where τ_l is often 72 or 168 hours).

There are many other examples of cases in which truncated probability distributions are required, however, we deemed the above list sufficiently extensive and diverse to motivate the study of left-truncated distributions in this thesis. Previously, there has been a limited amount of research into goodness-of-testing with truncated distributions, however, a reasonable amount has been conducted on goodness-of-testing with censored distributions [35–37].

1.9.1 Truncation and Censoring

Truncation and censoring are two closely related but distinct methods of dealing with an incomplete data set [38]. To highlight the differences, we consider the example from Kendall and Stuart [38] (32.15 page 522) concerning a game of darts (with reasonably inexperienced players). Here, we wish to model the distribution of distances, x, between a dart's location on the board and the centre of the dartboard. The measurements can only be made after all n of the darts have been thrown at the board with radius R. Inexperienced players only manage to hit the board m $(m < n)$ times, hence, it is not possible to get a measurement of x for each of the darts. For $r = n - m$ of the darts, we only know that $x > R$. We can deal with this situation through truncation or censoring.

- **Truncation**: For truncation to be employed, the r darts which did not hit the target are disregarded. Right truncation is employed with a limit of $\tau_r = R$. In this situation, we are modelling the distribution of distances for **darts that hit the dartboard**, we make no attempt to account for those which did not hit. The total number of darts thrown, n, is irrelevant, we are only concerned with the m darts that hit the board. The resulting model only describes the spread of darts on the dartboard, however, it is reasonable to expect the model to work outside of the board in the absence of phenomena that only effect darts that do not land on the board.

- **Censoring**: Censoring uses the information about all of the n darts thrown. We have m complete observations of x from the darts that hit the dartboard, however, we also know that r darts landed a distance $x > R$ away from the centre. Our model includes all of this information and as a result, we are modelling the distribution of distances for **all of the darts thrown**. In this way, we employ 'right censoring' with a right censoring limit of R. The resulting model describes the spread of all the darts thrown.

Whether one employs truncation or censoring is dependent upon what property one is trying to model, and what observations are available. In the above example, truncation is used to model the distances for darts on the board, and censoring is used to describe the distances for all darts. Thus far, we have treated all censoring as being equal, however, there is actually two distinct types:

- **Type I**: Type I censoring occurs when the censoring limits are fixed. Consider the right censoring limit, R, in the aforementioned game of darts. This limit was fixed, hence, this case is an example of type I right censoring. Another example occurs when a series of voltages are measured with an analogue voltmeter. If the needle reaches the upper most value, V_{max}, then all we know is that the voltage is at least V_{max}. In this case, the data is type I right censored with limit V_{max}.

- **Type II**: Type II censoring occurs when a *fixed proportion of the sample size, n,* is censored [38]. An example of this is measuring the failure time of manufacturing equipment, where the experiment is stopped after the first m of the n pieces of equipment fail. The proportion of censored values $\frac{n-m}{n}$ is fixed, hence, this is a case of type II right censoring.

Kizilersü et al. [10] noted that in the literature, type I censoring is sometimes referred to as 'truncation' and type II censoring is occasionally called simply 'censoring' [39, 40]. Needless to say, this confusion is counter productive. All the examples we have given for censoring have been of right censoring. However, utilising left censoring or a combination of left and right censoring is a perfectly legitimate statistical practice. Indeed it is possible to combine censoring with truncation for particular problems. Consider the example of

‘Remaining life of power transformers’ in section 1.9, recall that Hong et al. [34] used left-truncation to account for the transformers that were installed before 1980. Right truncation was also used in that study, to accommodate the transformers which had not yet failed. Balakrishnan et al. [41] also discussed the combination of left-truncation and right censoring. We reiterate that **in this thesis we exclusively study left-truncation**. Censoring and other forms of truncation were included merely for the sake of completeness.

1.9.2 Mathematical Definition of Truncated Distribution

In the case of two-sided truncation, we restrict the support, such that $x \in [\tau_l, \tau_r]$ where $\tau_l, \tau_r \in [0, \infty)$ and $\tau_l \leq \tau_r$. The truncated pdf, $f_T(x|\boldsymbol{\theta})$, is proportional to the untruncated pdf on $[\tau_l, \tau_r]$. Therefore,

$$f_T(x) = \lambda f(x) \ , \tag{1.32}$$

where λ is the constant of proportionality. The value taken by λ is defined by the normalisation condition of the truncated distribution,

$$\begin{aligned} 1 &= \int_{\tau_l}^{\tau_r} f_T(x)dx \\ \Rightarrow \frac{1}{\lambda} &= \int_{\tau_l}^{\tau_r} f(x)dx \ . \end{aligned} \tag{1.33}$$

In this work, we are only interested in left-truncation, hence, we set $\tau_r = \infty$ and the normalisation condition becomes

$$\Rightarrow \frac{1}{\lambda} = \int_{\tau_l}^{\infty} f(x)dx \ . \tag{1.34}$$

$F(x)$ and $f(x)$ are related by equation Eq. (1.6), and λ is subject to the restriction that $\lambda \neq \lambda(x)$, therefore the left-truncated cdf, $F_T(x|\boldsymbol{\theta})$, and be evaluated through

$$F_T(x) = \int_{\tau_l}^{x} f_T(t)dt = \lambda \int_{\tau_l}^{x} f(t)dt \ , \tag{1.35}$$

where $\tau_l \leq x$. The probability, p, that a random data point from the complete distribution is less than τ_l, is given by the complete cdf evaluated at the truncation point τ_L,

$$p = F(\tau_L) \ . \tag{1.36}$$

In general, λ will take a value that is dependent upon the parameters of the distribution, i.e. $\lambda = \lambda(\boldsymbol{\theta})$. As a result, the MLE equations specified in Eq. (1.21) take a different form for the left-truncated distribution. Kreer et al. [11] showed that there is not always a unique solution to the MLE equations of the left-truncated Weibull distribution. They derived a non-trivial criterion for determining whether a unique solution exists for a given data set. More recently, Kreer et al. [42] showed that this is also the case for the left-truncated lognormal and loglogistic distributions. Therefore, we cannot assume that there will always be a unique solution to these equations, hence, we may not be able to obtain parameter estimates for all left-truncated data sets.

1.10 Thesis Objectives and Outline

Kizilersü et al. [10] showed that the critical values of the left-truncated Weibull distribution are dependent upon the truncation level for a specific goodness-of-fit test[4]. In

[4]It was the Kolmogorov-Smirnov Test

this thesis we seek to verify and extend that work by determining the critical values associated with observations drawn from three additional left-truncated distributions. The additional distributions we have elected to study are the lognormal, loglogistic and Pareto, all of which were introduced in section 1.3. The conversion from their complete to left-truncated versions is conducted in the relevant distribution specific chapters, e.g. Chapter 5 for the lognormal distribution. Our work will also consider the Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests, which are defined and discussed in sections 2.4 and 2.5.

Obtaining the correct conclusion from a goodness-of-fit test requires that the appropriate critical values be employed. For example, the critical values of the Kolmogorov-Smirnov test (see subsection 2.4.1) for observations taken from the Weibull distribution vary greatly depending on how many parameters are estimated from the data. If the untruncated distribution is completely specified with a sample size of $n = 30$, the relevant critical value at the 95% significance level (see section 2.1) is 1.3237, however, if both parameters are estimated from the observations, the relevant critical value is 0.8581. There is a significant difference between the two critical values. If we were to employ the critical value for the completely specified distribution to a situation in which both parameters were estimated, we would believe the model had performed far better than it actually had. This could lead us to apply the inaccurate Weibull distribution to situations in which under-performance is of grave consequence. If this distribution had been used to model the rate at which a drug leaves the body, the inaccuracies could cause medical staff to administer too much of the drug, potentially inducing an overdose. In some situations, accurate modelling is the difference between life and death, for this reason it is essential to use the correct critical values at all times.

Critical values are only common in the literature for data drawn from complete distributions, however, there is a huge amount of truncated data in the real world, as discussed in section 1.9. It is often necessary to estimate the parameters of a truncated distribution, and this procedure drastically changes the critical values and thus the results of hypothesis testing. In order to draw correct conclusions from these tests, we must employ the correct critical values, which are specific to the truncated distributions. Therefore, finding the relevant critical values for truncated distributions-which is the main objective of this thesis-is of the utmost importance.

Amongst other things, the critical values are generally dependent upon truncation level and sample size, thus, we evaluate the critical values for a range of combinations of these two variables. This allows rigorous statistical tests to take place at the studied left-truncation points, and n values, however, in practice one can seldom achieve both of these conditions consistently. For example, it is unlikely that someone measuring radiation with a Geiger counter will be able to control the number particle detections (n) or the dead time of the detector (τ_l). In order for our results to be usable in the real world it is necessary to interpolate the critical values between the left-truncation points and n values for which we have determined the critical values. In Chapter 9, we discuss the modelling procedure and articulate our models for use in other works.

Ultimately, the value of hypothesis testing is its power to reject H_0 when it is false and not reject H_0 when it is true. In Chapter 10 we evaluate this power for the goodness-of-fit tests we have studied. We utilise both the raw critical values and the their respective models to showcase the performance of the models for comparison purposes. Finally in Chapter 11 we apply our critical values to some financial data from the London stock exchange and determine a suitable model for the arrival times of executed orders.

Armed with these new critical values, statisticians will be able to conduct rigorous hypothesis tests upon several left-truncated distributions for the first time. Without hypothesis testing, one can never be certain how well a proposed model performs, or to what degree a theory describes observations. Our work will add another string to the bow of analysts who work in one of the numerous areas in which incomplete data sets proliferate. Figure 1.4 gives an outline of this thesis, and describes how we will determine the aforementioned critical values.

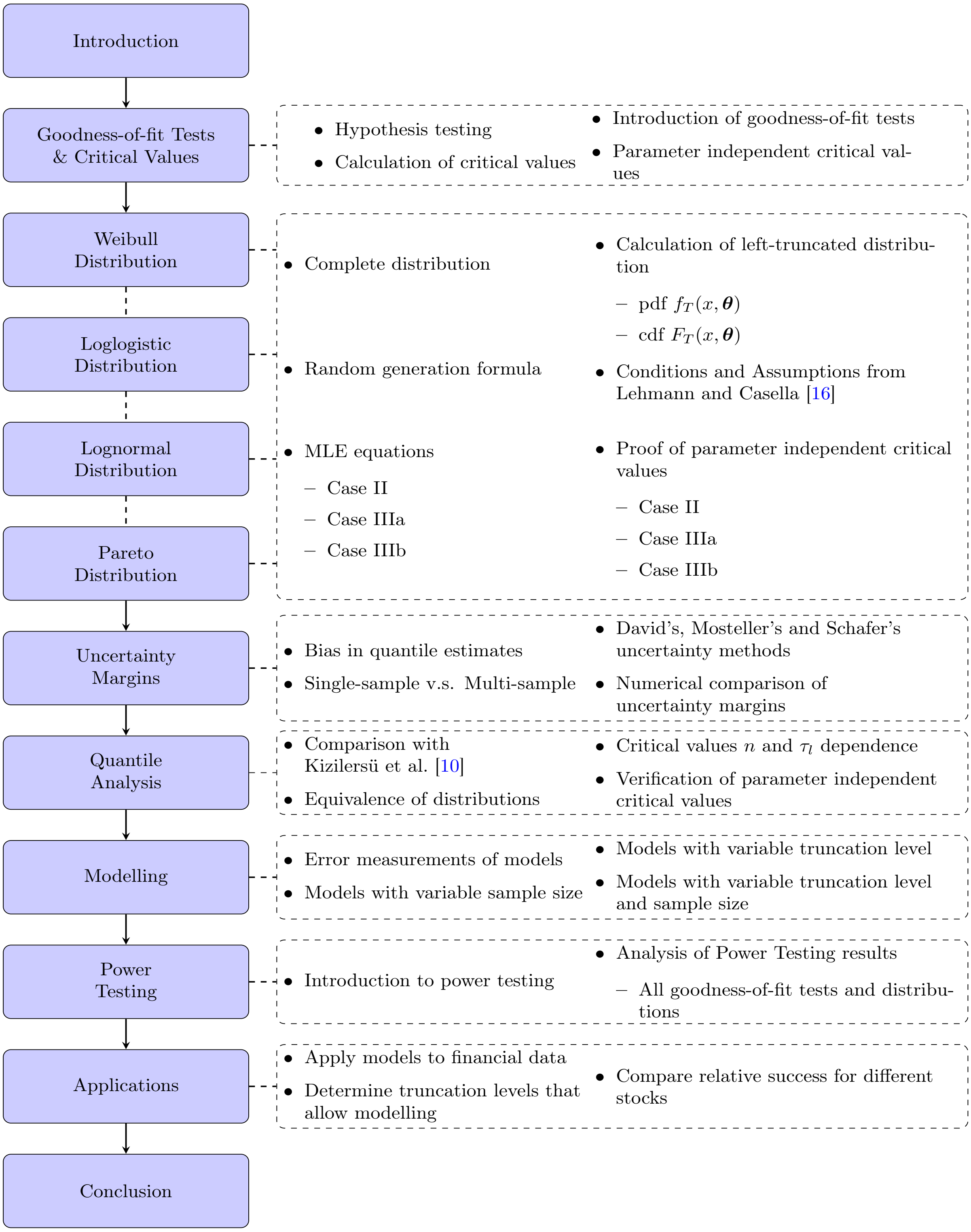


Figure 1.4: Thesis Outline

Chapter 2

Goodness-of-fit Tests and Critical Values

Hypothesis testing is a statistical technique employed to determine whether a null hypothesis, H_0, can be rejected given a set of n observations $\boldsymbol{x} = (x_1, x_2, ..., x_n)$. In the context of this thesis, the null hypothesis is that the observations come from a population with cumulative distribution function, $F(x)$. A goodness-of-fit test is a criterion that allows one to decide whether or not to reject H_0 given $\boldsymbol{x}$. The quality of a goodness-of-fit test is assessed by the prevalence of type 1 and type 2 errors.

- Type I Error: H_0 is true, but rejected

  The significance level, α, is specified by the probability of rejecting the null hypothesis given that it is true,

$$\alpha = \mathbb{P}\left(\text{reject } H_0 | H_0 \text{ is true}\right) \ . \tag{2.1}$$

- Type II Error: H_0 is false, but not rejected

  $1 - \beta$ is the probability of not rejecting the null hypothesis given that it is false,

$$1 - \beta = \mathbb{P}\left(\text{do not reject } H_0 | H_0 \text{ is false}\right) \ , \tag{2.2}$$

  where β is the statistical power.

If both α and $1 - \beta$ are small, the goodness-of-fit test can accurately determine whether the null hypothesis should be rejected. In practice, decreasing the probability of a type 1 error usually increases the probability of a type 2 error, therefore some compromise is necessary. The observations, $\boldsymbol{x}$, can be either continuous or discrete. For a discussion comparing the two, please consult section 1.1. It is always possible to discretise continuous data by grouping it into bins, however, this process involves a loss of information and hence, is not reversible. Additionally, some subjectivity is required to determine how the data should be binned. As a result, we restrict our attention to the study of continuous data.

For an example of hypothesis testing in science, we recall that prior to 2012 there was a popular theory that there exists a particle (the Higgs boson) which gives mass to fundamental particles. Many physicists devoted an enormous amount of time and energy into testing whether or not said particle existed. In this situation, researchers adopted a null hypothesis that the particle did not exist and then found a test (or series of tests) which could potentially reject this hypothesis. The vast amounts of data collected at the LHC were then passed to said test, which was able to reject to null hypothesis at the 5σ significance level, which is considered to be the gold standard of particle discovery [43].

Therefore, it was concluded that the Higgs boson must exist in some form, the specifics of which are actively being researched.

Goodness-of-fit tests typically produce a real, scalar measure of how well the observation set, $\boldsymbol{x}$, agrees with the null hypothesis. This measure is called the *test statistic*, a smaller test statistic implies stronger agreement between the observations and the predictions of the null hypothesis. Different tests are distinguished by the way in which their test statistics are formulated. To determine whether or not to reject the null hypothesis, one must compare this test statistic to a previously known *critical value*. **If the test statistic is lower than the critical value, then the null hypothesis cannot be rejected**. The definition and determination of critical values is discussed in section 2.1.

2.1 Critical Values

The critical value at significance level, α, is the value such that the lowest $100p\%$ of the test statistics (generated when the null hypothesis is true) fall below it, where $p = (1-\alpha)$. For example, if the critical value is selected such that it is above 95% of the tests statistics $\alpha = 1 - 0.95 = 0.05$. In effect, the critical value at p is the $100p\%$ percentile estimate of the test statistics, ξ_p. Figure 2.1 displays how the critical value at the 95% significance level would be selected; the hatched area is the probability that a random test statistic is less than the critical value, $\xi_{0.95}$. **Note:** The test statistics are not necessarily normally distributed (see section 2.7) despite the way Figure 2.1 depicts them. To determine the critical values, it is necessary to understand how the test statistics are distributed when the null hypothesis is true. In the early days of hypothesis testing, statisticians were restricted to using goodness-of fit tests for which the distribution of test statistics could be found analytically when the null hypothesis was true [44–47]. With the explosion in computational power, it has become possible to determine the distribution of test statistics numerically via Monte Carlo methods.

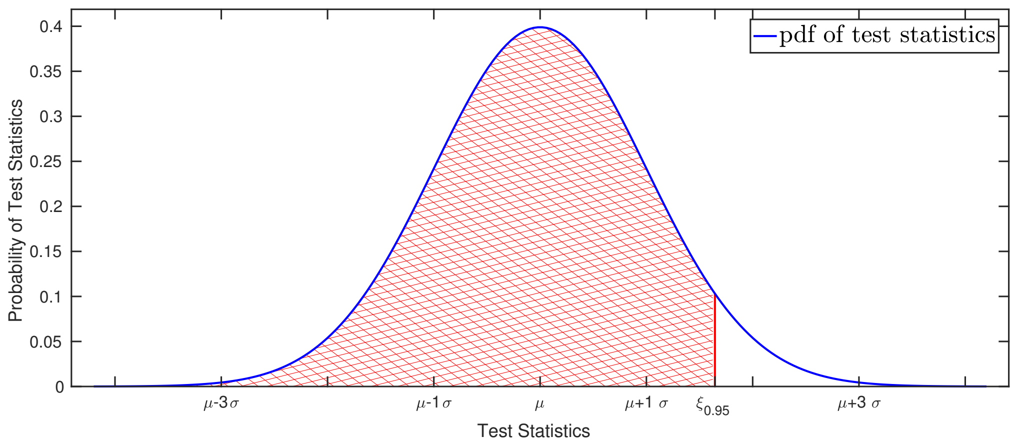


Figure 2.1: Selection of the critical value at the 95% significance level ($\xi_{0.95}$)
Hatched area $= \mathbb{P}(\text{random test statistic} < \xi_{0.95}) = 0.95$

Each distinct α value corresponds to a unique critical value, hence, rejecting (or not rejecting) H_0 is done with respect to a particular significance level. When we do not reject H_0 for a sample of data with test statistic, T, at significance level α, we are really making the following statement: T is less than the highest $100\alpha\%$ of the test statistics that are produced when H_0 is true. If a sample, $\boldsymbol{x} = (x_1, x_2, ..., x_n)$, passes a goodness-of-fit test at the 95% significance level, we do **not** have 95% confidence that the null hypothesis is true. We can merely say that the sample has performed better (under the given test) than 5% of the samples that were drawn when the null hypothesis was true. This is an important distinction, and one that is often not understood. The test becomes more difficult to pass for higher α values. As $p = (1 - \alpha)$, a higher p value actually corresponds to a less stringent test; it is important to keep this in mind throughout the rest of this thesis. The goodness-of-fit tests at the 85% significance level, are more difficult to pass than those at 99%, and if a sample of observations passes the test at the 85% significance level we have **more** confidence that the null hypothesis is true than if it passed the test at 99%. **Note:** As the 99% critical value for a goodness-of-fit test is necessarily higher than the 85% critical value, if a sample passes the test at the 85% significance level it will also pass at 99% (and indeed at any percentage above 85%).

It is often the case that some parameters of the null hypothesis are estimated from the sample. For example, one may have the null hypothesis that the heights of year 10 students in Australia are normally distributed, in which case the mean and variance would need to be extracted from the data. When this occurs, the difference between the observed and expected values of the model will be reduced, as we have selected the parameters which minimize the difference. This reduction will cause the corresponding test statistics to decrease, hence, the distribution of test statistics is in general lower when parameters are estimated from the sample. For distributions with multiple parameters, as the number of estimated parameters increases we expect the critical values to decrease, however there is no guarantee that estimating different parameters will affect the test statistic in the same way. Therefore, the relevant critical values will change depending on how many and which parameters are estimated from the data. In the above example concerning Australian year 10 students, it is not necessarily true that the test statistics generated when the mean is known and the variance is estimated will be the same as when the variance is known and the mean is estimated. Hence, to understand how the test statistics are distributed in each of these cases, it is necessary to conduct independent studies into both. In this thesis we have evaluated the critical values for each case separately, and thus our results are defensible.

2.2 Empirical Distribution Function

The empirical distribution function (edf) is the cumulative distribution function of a set of observations, $\boldsymbol{x} = (x_1, x_2, ..., x_n)$. The edf is a step function that increases by $\frac{1}{n}$ at each of the n observations within a sample. Denoted $F_n(t)$, the value of the edf at a point t is the proportion of observations in the sample that are less than or equal to t. Mathematically, $F_n(t)$ is defined as

$$F_n(t) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}_{x_i \leq t} \text{ where } \mathbb{1}_{x_i \leq t} = \begin{cases} 1 \text{ if } x_i \leq \text{t} \\ 0 \text{ if } x_i > \text{t} \end{cases} . \tag{2.3}$$

Figure 2.2 displays cdf for the complete case I Weibull distribution (with $\alpha = 1$ and $\beta = 1$) and the edf of 30 observations. We observe that the edf is a step function and that the cdf is continuous. In edf based statistical tests, one determines the test statistic from the difference between the edf and the cdf. There are a number of ways this can be done, such as taking the maximum difference between the two (Kolmogorov-Smirnov Test, section 2.4.1) or the sum of the square differences (Cramér-von Mises Test, section 2.5.1). All of the goodness-of-fit tests studied in this thesis are edf based because of their high power and widespread use in the literature [12, 13, 45–47]. The tests we consider can be defined as being either supremum class tests or quadratic class tests, further discussion on how these categories are defined is available in sections 2.4 and 2.5 respectively [48].

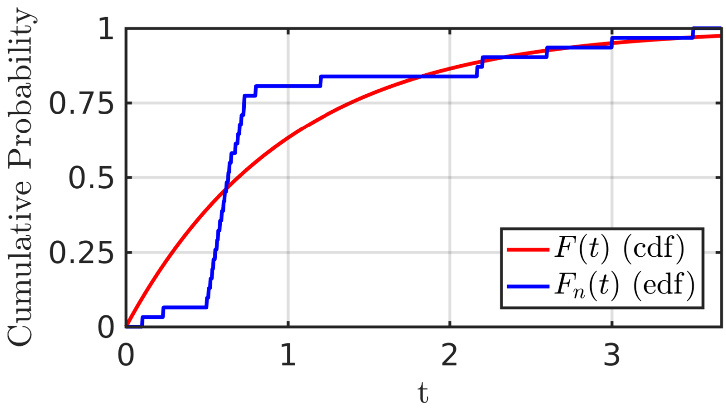


Figure 2.2: Comparison of complete case I Weibull distribution ($\alpha = 1$, $\beta = 1$) cdf and edf

2.3 Pearson's χ^2 Test

Pearson's χ^2 test is the classical approach for goodness of fit problems [12, 44]. This test is **not studied extensively in this thesis**. However, as it is perhaps the most commonly used goodness-of-fit test we thought it was necessary to discuss it to some degree. Stephens [12] lists the advantages of this test as (i) it is well adapted for discrete distributions and (ii) it is known to a good approximation how to adapt the statistic when the parameters of $F(x)$ must be estimated from the data. Indeed, before Monte Carlo simulations were a viable way of producing critical values, Pearson's χ^2 test was one of the only tests for which the critical values could be determined when the parameters were estimated. Pearson's χ^2 test is not necessarily edf based, however, as the other goodness-of-fit tests we will study are edf based, we define the test statistic, χ^2, in the form that can most easily make use of the edf, i.e.

$$\chi^2 = \sum_{i=1}^{n} \frac{(\frac{i}{n} - F(x_i))^2}{F(x_i)} \;, \tag{2.4}$$

where n is the sample size and $F(x)$ is the cdf of the distribution from H_0. **Note:** We have replaced the explicit edf dependence with $\frac{i}{n}$ for simplicity. Pearson [44] showed that if the null hypothesis is true and the number of data points is large, the χ^2 distribution will describe the distribution of the χ^2 test statistics. The number of degrees of freedom, ν, in the χ^2 distribution is given by $\nu = n - 1 - d$, where d is the number of model parameters which are estimated from the data. The χ^2 distribution with ν degrees of freedom [49] has the cdf,

$$F(x|\nu) = \frac{1}{2^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)} \int_0^x t^{\frac{\nu}{2}-1} e^{-\frac{t}{2}} dt \;, \tag{2.5}$$

where $\Gamma(x)$ is the Gamma function,

$$\Gamma(x) = \int_0^\infty z^{x-1} e^{-z} dz \;. \tag{2.6}$$

While Pearson's χ^2 test is very easy to use, a number of studies have shown that under certain circumstances edf based statistical tests are more powerful [12, 50–52].

2.4 Supremum Class Tests

Supremum class statistics are related to the maximum (or supreme) difference between the cdf and edf. The maximum value of the cdf subtracted from the edf is denoted D^+

and maximum value of the edf subtracted from the cdf is denoted D^-. Mathematically, D^+ and D^- are defined as

$$\begin{aligned} D^+ &= \sup_{\tau_l \le x \le \infty} [F_n(x) - F(x)] = \max_{1 \le i \le n} \left[\frac{i}{n} - F(x_i) \right] , \\ D^- &= \sup_{\tau_l \le x \le \infty} [F(x) - F_n(x)] = \max_{1 \le i \le n} \left[F(x_i) - \frac{i-1}{n} \right] , \end{aligned} \tag{2.7}$$

where $\boldsymbol{x} = (x_1, x_2, ..., x_n)$ are the observations that define the edf. Figure 2.3 gives a visual representation of how D^+ and D^- are defined for the edf and cdf displayed in Figure 2.2. The supremum class tests considered in this thesis are the Kolmogorov-Smirnov test and Kuiper's test, which are discussed in sections 2.4.1 and 2.4.2 respectively.

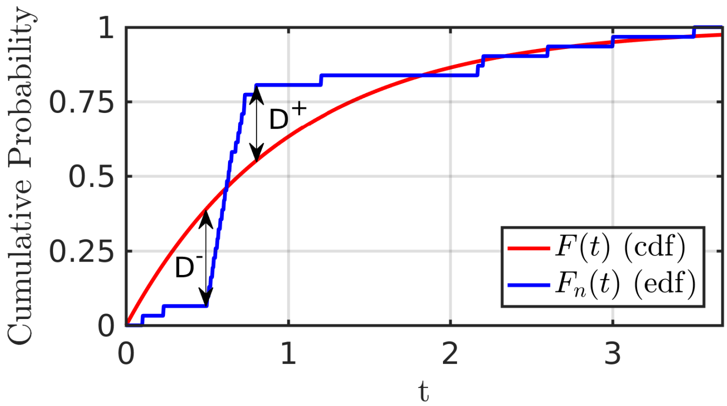


Figure 2.3: D^+ and D^- for the complete case I Weibull distribution cdf and edf of 30 observations

2.4.1 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov (KS) test was proposed by Kolmogorov [53] and Smirnov [46] in the 1930's. The KS distance, D, is defined as being the maximum value of the absolute difference between the edf and cdf,

$$\begin{aligned} D &= \sup_{\tau_l \le x \le \infty} |F_n(x) - F(x)| \\ &= \max_{1 \le i \le n} \left[\frac{i}{n} - F(x_i), F(x_i) - \frac{i-1}{n} \right] \\ &= \max \left[D^+, D^- \right] . \end{aligned} \tag{2.8}$$

There is a lack of consensus in the literature [11,12,54,55] as to whether D or $\sqrt{n}D$ should be the test statistic for the Kolmogorov-Smirnov test. In this work we follow the lead of Kizilersü et al. [10] and employ $\sqrt{n}D$. This is of no consequence however, as the test performs identically with either test statistics if the correct critical values are employed. As both the cdf and edf are monotonically increasing functions, the Kolmogorov-Smirnov test is generally more sensitive to fluctuations near the median value of the sample than the tails. Additionally, it has been shown to have greater discriminatory power than Pearson's χ^2 test under some circumstances [50] [52].

The Kolmogorov-Smirnov test was one of the first edf based goodness-of-fit tests and has become useful and very common to use. Kolmogorov [53] was able to find the asymptotic ($n \to \infty$) distribution for case I in 1933. The usefulness of this test comes from the fact that it is invariant under reparametrisation of t; one can locally slide or stretch the t axis in Figure 2.3 retain the same D^+ and D^- values. The critical values can be found from this distribution very easily and are applicable to all distributions [38]. However, they require the null hypothesis to be completely specified (no estimated parameters) and a large sample size (n) to be valid. Several such studies [12, 56, 57] have estimated the critical values for completely specified distributions for a range of finite n values. For cases in which parameters must be estimated from the data, the critical values are dependent upon the distribution from which the data was drawn. There have been a number of studies that assess the critical values for some complete distributions, e.g. normal [12] and Weibull [6]. However, to our knowledge there are very few studies where the critical values have been calculated for the cases in which the parameters were estimated from the data and observations were drawn from a left truncated distribution [10]. In this thesis we have determined the critical values of the left-truncated Weibull, lognormal, loglogistic and Pareto distributions with parameter estimation. These were previously unknown in the literature.

This test was selected for study because (i) it is very common and has an established place in the literature (ii) it has greater discriminatory power than Pearson's χ^2 test under some circumstances, and (iii) the critical values we produce were not previously available in the literature.

2.4.2 Kuiper's Test

Kuiper's test [47] was proposed in 1960 by Nicolaas Kuiper as an altered version of the Kolmogorov-Smirnov test, that makes it invariant under cyclic transformations of the independent variable. In his seminal work on the test [47], Kuiper discusses the situation in which the measurements, $(x_1, x_2, ..., x_n)$, are the directions in which n groups of birds have been observed flying from a given point on Earth. The Kolmogorov-Smirnov test cannot be used to test the null hypothesis that the directions are uniformly distributed, as the test statistic is dependent on the (arbitrarily chosen) position of zero angle. However, Kuiper's test statistic is independent of the zero angle position, and thus can be used to address the problem. In Kuiper's test, V is analogous to D and is defined as being the sum of D^+ and D^-

$$\begin{aligned} V &= \sup_{\tau_l \le x \le \infty} \Big[F_n(x) - F(x)\Big] + \sup_{\tau_l \le x \le \infty} \Big[F(x) - F_n(x)\Big] \\ &= \max_{1 \le i \le n} \left[\frac{i}{n} - F(x_i)\right] + \max_{1 \le i \le n} \left[F(x_i) - \frac{i-1}{n}\right] \\ &= D^+ + D^-. \end{aligned} \tag{2.9}$$

Again, we must make an arbitrary decision of whether V or $\sqrt{n}V$ is the best test statistic to use. For the sake of consistency with the Kolmogorov-Smirnov test we have elected to use $\sqrt{n}V$ in this work. By summing D^+ and D^-, Kuiper's test becomes equally sensitive to fluctuations everywhere in the sample, which is a very useful attribute of a goodness-of-fit test. The distribution of Kuiper's test statistics for all completely specified distributions was also given in his original 1960 work [47], thus allowing for easy computation of the critical values. This distribution suffers from the same problems as that given by Kolmogorov [53], in that it does not hold for cases in which the parameters must be estimated from data. When this is the case, the critical values are dependent upon the null hypothesis and a number of previous studies have determined the critical values for common untruncated distributions [13] [9]. To the best of our knowledge, critical values of left-truncated distributions under Kuiper's test are unknown. In this thesis we have determined the critical values of the left-truncated Weibull, lognormal, loglogistic and

Pareto distributions with parameter estimation.

This test was selected for study because (i) it is closely related to the popular Kolmogorov-Smirnov test (and we were interested in the comparison), (ii) it is equally sensitive to fluctuations everywhere in the sample and (iii) the critical values we produce were not previously available in the literature.

2.5 Quadratic Class Tests

Quadratic class test statistics, Q, are related to the integral of squared differences between the edf and cdf. The statistics themselves have the form,

$$Q = n \int_{-\infty}^{\infty} \left[F_n(x) - F(x)\right]^2 \psi\left\{F(x)\right\} dF(x), \tag{2.10}$$

where $\psi\left\{F(x)\right\}$ is the weight function, which must be non-negative. The quadratic class tests considered in this thesis are the Cramér-von Mises test and the Anderson-Darling test, which are discussed in sections 2.5.1 and 2.5.2 respectively. For these tests, the integral can be evaluated explicitly and expressed as a sum over the observations, $(x_1, x_2, ..., x_n)$, which significantly reduces the computational expense of calculating these statistics.

2.5.1 Cramér-von Mises Test

The Cramér-von Mises test was first proposed by Cramér [58] and von Mises [59] in 1928, making it one of the oldest edf based goodness-of-fit tests. The test statistic results from setting the weight function in Eq. (2.10) to one, i.e. $\psi\left\{F(x)\right\} = 1$. It is perhaps the most intuitive of test statistics, given that it is plainly the integral of the squared difference between the cdf and edf. Additionally, it has been shown to have greater discriminatory power than Pearson's χ^2 test, under some circumstances [50, 52]. The test statistic, W^2, can be defined as

$$W^2 = n \int_{-\infty}^{\infty} \left[F_n(x) - F(x)\right]^2 dF(x) \; . \tag{2.11}$$

Anderson and Darling [45] state that Eq. (2.11) can simplified to a sum over the observations within a sample

$$W^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left(\frac{2i-1}{2n} - F(x_i)\right)^2 \; . \tag{2.12}$$

The asymptotic ($n \to \infty$) distribution of test statistics for the Cramér-von Mises test was determined by Anderson and Darling [60] for a completely specified (no estimated parameters) distribution, which allows for easy calculation of the critical values. Again, for cases in which parameters are estimated from the data, the critical values are dependent upon the null hypothesis. Studies [13, 61] have been conducted to find the critical values for common untruncated distributions with estimated parameters. In this thesis, we have determined the critical values of the left-truncated Weibull, lognormal, loglogistic and Pareto distributions with parameter estimation. These were previously unknown.

This test was selected for study because (i) it has an established place in the literature and has remained in continual use for nine decades, (ii) it is similar to the Anderson-Darling test which is very widely used and (iii) the critical values we produce were not previously available in the literature.

2.5.2 Anderson-Darling Test

The Anderson-Darling test was first proposed in 1954 by Anderson and Darling [45] as a goodness-of-fit test that was more sensitive to fluctuations in the tails than the Cramér-von Mises test. Increased sensitivity was achieved by selecting a weight function that increased significantly near the end points. The function they decided upon is

$$\psi\left\{F(x)\right\} = \frac{1}{F(x)\left[1 - F(x)\right]} \ . \tag{2.13}$$

Therefore, the test statistic A^2 is given by,

$$A^2 = n\int_{-\infty}^{\infty} \frac{\left[F_n(x) - F(x)\right]^2}{F(x)\left[1 - F(x)\right]} dF(x) \ . \tag{2.14}$$

Anderson and Darling [45] show that Eq. (2.14) can simplified to a sum over the observations within a sample,

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}\{2i - 1\}\left\{\log[F(x_i)] + \log[1 - F(x_{n-i+1})]\right\} \ . \tag{2.15}$$

It has been shown that the Anderson-Darling test has greater discriminatory power than Pearson's χ^2 test under some circumstances [50, 52]. In the same 1954 paper as the test was proposed, Anderson and Darling derived the asymptotic ($n \to \infty$) distribution of test statistics for a completely specified (no estimated parameters) distribution, which allows for easy calculation of the critical values. For cases in which parameters are estimated from the data, the critical values are dependent upon the null hypothesis. Studies [6, 8] have been conducted to find the critical values for common untruncated distributions with estimated parameters. In this thesis we have determined the critical values of the left-truncated Weibull, lognormal, loglogistic and Pareto distributions with parameter estimation, these were previously unknown.

This test was selected for study because (i) it is very common and has an established place in the literature, (ii) it is more sensitive to fluctuations in the tails of a distribution than the Cramér-von Mises test and (iii) the critical values we produce were not previously available in the literature.

2.6 Parameter Independent Critical Values

When the true values of the parameters are known *a priori* the distribution is completely specified. *All completely specified distributions have the same critical values for a given goodness-of-fit test, sample size and significance level.* Therefore, the critical values are independent of the true parameter values when the parameters are known *a priori* [12]. As discussed in section 2.1, the critical values are reduced for cases in which some of the parameters must be estimated from the data. The magnitude of this reduction is dependent upon the distribution, and the parameters which are estimated from the data. Additionally, **the critical values are not necessarily independent of the parameters for cases in which the parameters are estimated** from the observations. Therefore, if we wish to use the critical values in practice, we must have access to the critical values for all the parameter estimates which may result from the MLE. The solution space of the MLE equations is continuous, i.e. uncountably infinite, hence we cannot always have access to the required critical values. Potentially, we could evaluate the critical values for a range of parameter estimates and interpolate the result over the parameter space. This procedure would solve the issue of having an uncountably infinite number of solutions, however, it is still extremely time consuming for even the simplest cases. Alternatively, we can look for cases in which the critical values are parameter independent. In these situations, we are only required to determine the critical values for one set of parameters and we can apply the result universally. We elect to use this approach because it requires the least work and offers the most general result. In this section, we outline a procedure which allows us to show whether the critical values are parameter independent. We give an example with the untruncated normal distribution.

Recall the structure of the goodness-of-fit tests given in this chapter. The only inclusion of the distribution (and hence its parameters) we test the data against in the goodness-of-fit test statistics is through the cdf, $F(x)$. Thus if we can show that there exists a function, $\widetilde{F}(u)$, such that,

$$F(x_i) = \widetilde{F}(u_i) \ \forall \ x_i \ , \tag{2.16}$$

where $\widetilde{F}(u_i)$ is parameter independent, the goodness-of-fit test statistics will also be parameter independent. By the nature of critical values (as described in section 2.1), they too will be parameter independent. For a detailed discussion on how this is possible, please read section 2.6.1. To achieve this, we define $\widetilde{F}(u)$ as $F(x|\hat{\boldsymbol{\theta}})$ where the x dependence has been substituted for the random generation formula introduced in Eq. (1.26). In the case of the complete normal distribution, $F(x|\hat{\boldsymbol{\theta}})$, takes the form,

$$F_N(x|\hat{\boldsymbol{\theta}}) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x-\hat{\mu}}{\sqrt{2}\hat{\sigma}}\right)\right] \ . \tag{2.17}$$

Application of the general random generation formula specified in Eq. (1.26) to the complete normal distribution yields,

$$\begin{aligned} F_N(x|\boldsymbol{\theta^0}) = u = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x-\mu^0}{\sqrt{2}\sigma^0}\right)\right] \\ \Rightarrow \mathrm{erf}^{-1}(2u-1) = \frac{x-\mu^0}{\sqrt{2}\sigma^0} \\ \Rightarrow x = \mu^0 + \sqrt{2}\sigma^0 \mathrm{erf}^{-1}(2u-1) \ . \end{aligned} \tag{2.18}$$

We can now substitute this expression into Eq. (2.17) to define $\widetilde{F_N}(u)$ as,

$$\begin{aligned}\widetilde{F_N}(u|\hat{\boldsymbol{\theta}}, \boldsymbol{\theta^0}) &= \frac{1}{2}\left[1 + \text{erf}\left(\frac{\mu^0 + \sqrt{2}\sigma^0\text{erf}^{-1}(2u-1) - \hat{\mu}}{\sqrt{2}\hat{\sigma}}\right)\right] \\ &= \frac{1}{2}\left[1 + \text{erf}\left(\frac{1}{\sqrt{2}}\frac{\mu^0 - \hat{\mu}}{\hat{\sigma}} + \frac{\sigma^0}{\hat{\sigma}}\text{erf}^{-1}(2u-1)\right)\right] \ . \end{aligned} \tag{2.19}$$

Now we introduce the pivotal functions, γ and ξ, with the aim of containing all of the $\boldsymbol{\theta^0}$ and $\hat{\boldsymbol{\theta}}$ dependence in the cdf within them. We define γ and ξ as,

$$\begin{aligned}\gamma &= \frac{\mu^0 - \hat{\mu}}{\hat{\sigma}} \ , \\ \xi &= \frac{\sigma^0}{\hat{\sigma}} \ . \end{aligned} \tag{2.20}$$

Therefore we can write Eq. (2.19) as

$$\widetilde{F_N}(u|\hat{\boldsymbol{\theta}}, \boldsymbol{\theta^0}) = \widetilde{F_N}(u|\gamma, \xi) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{1}{\sqrt{2}}\gamma + \xi\text{erf}^{-1}(2u-1)\right)\right] . \tag{2.21}$$

All of the parameter dependence of $\widetilde{F_N}(u)$ has been contained within the pivotal functions $\gamma = \gamma(\mu^0, \hat{\mu}, \hat{\sigma})$ and $\xi = \xi(\sigma^0, \hat{\sigma})$. Therefore, if the pivotal functions are parameter independent, the same will be true of $\widetilde{F_N}(u)$, and the critical values. We now direct our attention to showing that this is the case. We begin by substituting the random generation formula for the x dependence in the MLE equations and containing the parameter dependence within the pivotal functions. Next, we aim to show that there exists a solution to these modified MLE equations. If this is the case, the pivotal functions can be expressed as functions of the other variables in the equations, e.g. $\boldsymbol{u}$, and are therefore parameter independent. For the complete normal distribution the MLE equations have the form,

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i \ , \tag{2.22}$$

$$\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2 \ . \tag{2.23}$$

Substituting the random generation formula into Eq. (2.22) yields,

$$\begin{aligned}\hat{\mu} &= \frac{1}{n}\sum_{i=1}^{n}\left[\mu^0 + \sqrt{2}\sigma^0\text{erf}^{-1}(2u_i - 1)\right] \\ \Rightarrow \frac{\hat{\mu} - \mu^0}{\sigma^0} &= \frac{\sqrt{2}}{n}\sum_{i=1}^{n}\left[\text{erf}^{-1}(2u_i - 1)\right] \\ \Rightarrow \frac{\hat{\mu} - \mu^0}{\hat{\sigma}}\frac{\hat{\sigma}}{\sigma^0} &= \frac{\sqrt{2}}{n}\sum_{i=1}^{n}\left[\text{erf}^{-1}(2u_i - 1)\right] \\ \Rightarrow \frac{\gamma}{\xi} &= \frac{\sqrt{2}}{n}\sum_{i=1}^{n}\left[\text{erf}^{-1}(2u_i - 1)\right] \\ \Rightarrow \gamma &= \xi\frac{\sqrt{2}}{n}\sum_{i=1}^{n}\left[\text{erf}^{-1}(2u_i - 1)\right] . \end{aligned} \tag{2.24}$$

All of the parameter dependence within Eq. (2.22) has been contained within the pivotal functions, and we have managed to express γ as a function of ξ. Next, we substitute the random generation formula into Eq. (2.23) to produce

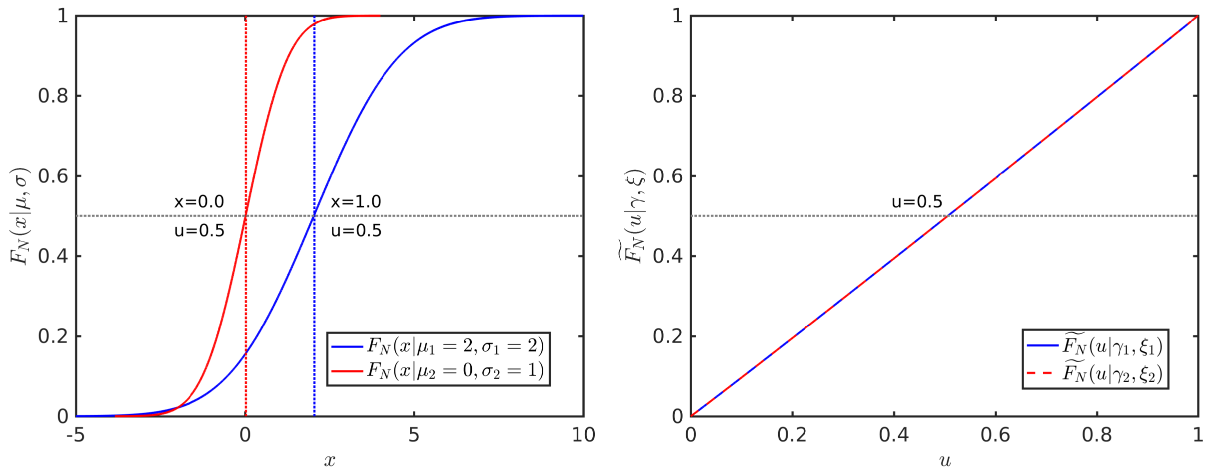
$$
\begin{aligned}
&\hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu})^2 \\
&\Rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left(\mu^0 + \sqrt{2}\sigma^0 \mathrm{erf}^{-1}(2u_i - 1) - \hat{\mu}\right)^2 \\
&\Rightarrow \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}\left[(\mu^0 - \hat{\mu})^2 + 2\sqrt{2}(\mu^0 - \hat{\mu})\sigma^0 \mathrm{erf}^{-1}(2u_i - 1) + 2(\sigma^0)^2\left(\mathrm{erf}^{-1}(2u_i - 1)\right)^2\right] \\
&\quad\Rightarrow 1 = \frac{(\mu^0 - \hat{\mu})^2}{\hat{\sigma}^2} + \frac{2\sqrt{2}}{n}\frac{(\mu^0 - \hat{\mu})}{\hat{\sigma}}\frac{\sigma^0}{\hat{\sigma}}\sum_{i=1}^{n}\left[\mathrm{erf}^{-1}(2u_i - 1)\right] + \frac{2}{n}\left(\frac{\sigma^0}{\hat{\sigma}}\right)^2\sum_{i=1}^{n}\left[\mathrm{erf}^{-1}(2u_i - 1)\right]^2 \\
&\quad\Rightarrow 1 = \gamma^2 + \frac{2\sqrt{2}}{n}\gamma\xi\sum_{i=1}^{n}\left[\mathrm{erf}^{-1}(2u_i - 1)\right] + \frac{2}{n}\xi^2\sum_{i=1}^{n}\left[\mathrm{erf}^{-1}(2u_i - 1)\right]^2 .
\end{aligned}
\tag{2.25}
$$

Therefore, we have expressed Eq. (2.23) with all of its parameter dependence contained within the pivotal functions. There is a solution to the MLE equations at the parameters which maximise likelihood. Hence, we know that there exists simultaneous solutions to Eq. (2.24) and (2.25) such that the pivotal functions can be defined in terms of the array of uniform random variates, $\boldsymbol{u}$ (specified in Eq. (1.27)), and not the parameters. We conclude that the pivotal functions are parameter independent. Therefore, the critical values are independent of both the true and estimated parameter values. We must highlight that is only true for goodness-of-fit tests for which all the parameter dependence in the test statistic is contained within the cdf, $F(x)$. The above procedure is used in all of the distribution specific chapters.

2.6.1 How can $\widetilde{F_N}(u)$ be parameter independent?

Consider the case in which n observations are drawn from the uniform distribution between 0 and 1, i.e. $u_i \in (0, 1)$. These can then be converted to observations, x_i, drawn from the normal distribution with mean, μ, and standard deviation, σ via Eq. (2.18). Different combinations of μ and σ, will cause each x_i to take a different value. For example, from the same uniform variate, u_i, one set of parameters, $\boldsymbol{\theta}_\mathbf{1}^\mathbf{0} = (\mu_1^0, \sigma_1^0)$, will produce $x_{1,i}$, and another set of parameters, $\boldsymbol{\theta}_\mathbf{2}^\mathbf{0} = (\mu_2^0, \sigma_2^0)$, will produce $x_{2,i}$. Repeating this procedure for each u_i allows us to produce two distinct data sets from which we can estimate the parameters (from Eq. (2.22) and (2.23)), $\hat{\boldsymbol{\theta}}_\mathbf{1} = (\hat{\mu_1}, \hat{\sigma_1})$ and $\hat{\boldsymbol{\theta}}_\mathbf{2} = (\hat{\mu_2}, \hat{\sigma_2})$. Substituting the relevant values into Eq. (2.17) yields two distinct cdfs, as shown in Figure 2.4a. However, if we plot Eq. (2.19) in the asymptotic limit, as in Figure 2.4b, for each of the parameters sets, both versions of $\widetilde{F_N}(u|\gamma, \xi)$ are identical. This is because the pivotal functions are parameter independent, i.e. $\gamma_1 = \gamma_2 = \gamma$ and $\xi_1 = \xi_2 = \xi$ [62]. We note that $x_{1,i}$ and $x_{2,i}$ yield the same value in the their respective cdfs, and that u_i reproduces this value in $\widetilde{F_N}(u|\gamma, \xi)$, i.e.,

$$F_N(x_{1,i}|\hat{\boldsymbol{\theta}}_\mathbf{1}) = F_N(x_{2,i}|\hat{\boldsymbol{\theta}}_\mathbf{2}) = \widetilde{F_N}(u|\gamma, \xi)\ . \tag{2.26}$$



(a) $F_N(x|\boldsymbol{\theta})$ for two distinct parameter sets

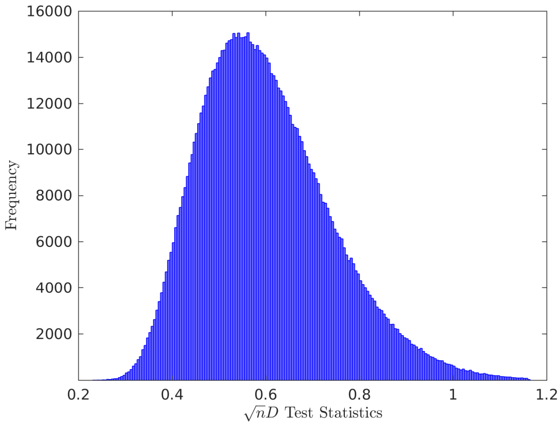
(b) $\widetilde{F_N}(u|\gamma, \xi)$ for two distinct parameter sets

Figure 2.4: $F_N(x|\boldsymbol{\theta})$ and $\widetilde{F_N}(u|\gamma, \xi)$ for two distinct parameter sets

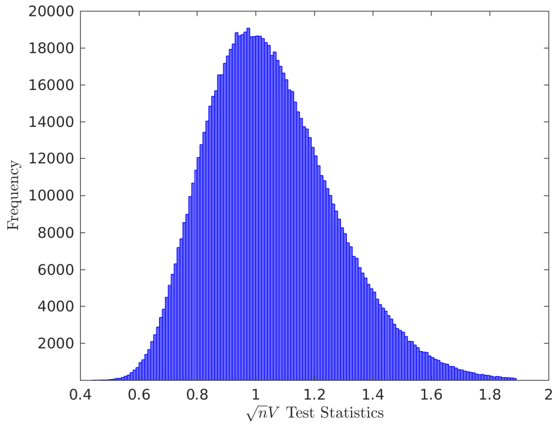
In the real world, we measure a finite number of observations, $\boldsymbol{x}$. Hence, the parameter estimates will never be perfect, and as a result, we will never be able to find the 'true' uniform variates, $\boldsymbol{u}$. The result is that Figure 2.4b will typically yield two slightly different $\widetilde{F_N}(u|\gamma, \xi)$ lines that approach each other as the sample size increases. We can combat this by averaging over a huge number of samples, in this case the averages produced for each $\widetilde{F_N}(u|\gamma, \xi)$ will overlap. This is effectively the procedure that we undertake in the Monte Carlo procedure.

2.7 Distribution of Test Statistics

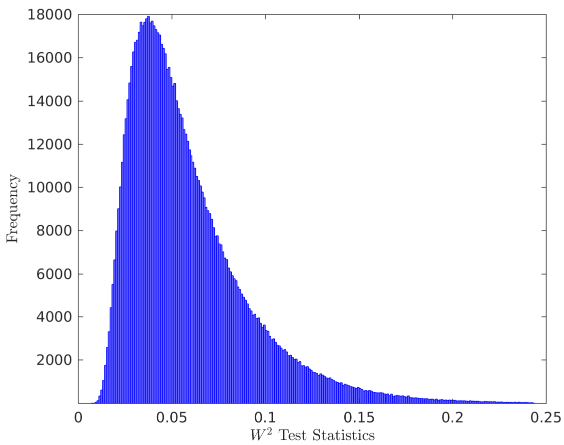
If the test statistics are normally distributed then any percentile estimate can be defined in terms of the mean, μ, and standard deviation, σ, of the distribution. This would greatly simplify the task of estimating the percentiles and hence critical values. Figure 2.5 displays the distribution of $1,000,000$ test statistics for the complete (untruncated) case II Weibull distribution with $n = 30$. Unfortunately, inspection of Figure 2.5 confirms that the test statistics are not normally distributed, however, it appears as though a skew normal distribution may accurately describe the spread of test statistics. **Future studies should assess whether a skew normal distribution adequately models the test statistics**. If this model is successful then one can evaluate all of the percentiles given only the three parameters of the skew normal distribution, which would simplify proceedings greatly. As we did not model the distribution of test statistics we are required to find the percentiles, ξ_p, for all of the significance levels at which we desire critical values.



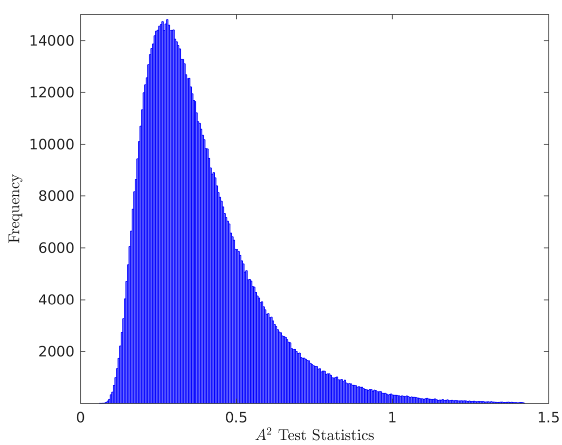
(a) Distribution of Kolmogorov-Smirnov test statistics



(b) Distribution of Kuiper test statistics



(c) Distribution of Cramér-von Mises test statistics



(d) Distribution of Anderson-Darling test statistics

Figure 2.5: Distribution of $1,000,000$ complete, case II Weibull test statistics with $n = 30$

Chapter 3

Weibull Distribution

The Weibull distribution is named after Waloddi Weibull, a Swedish mathematician who first described it in detail in 1951 [63], however, it was first discussed by Maurice Fréchet in 1927 [64]. The distribution has a scale parameter, α, and shape parameter β, and is defined on the support $\mathbb{R}^+ \cup 0$. When the shape parameter is set to $\beta = 1$, the distribution is the same as the exponential distribution and when $\beta = 2$, it is identical to the Rayleigh distribution.

The Weibull distribution is frequently used in life-time analysis for a huge variety of fields. For example, it has been used to describe the survival pattern of cancer patients [65] and the shelf-life of pickles [66]. In life-time analysis, often the data we have access to has been left-truncated (for some examples, please read section 1.9) and the left-truncated Weibull distribution has been employed in these situations. Kizilersü et al. [10] successfully used a Weibull distribution to describe the time difference between consecutive orders placed at the New York Stock Exchange (NYSE), the duration of ethnically mixed marriages in the US, and the time between major terrorist attacks with at least 10 casualties. We have elected to study the Weibull distribution because; (i) it can be applied in a large number of fields and (ii) Kizilersü et al. [10] have previously determined Kolmogorov-Smirnov critical values of the left-truncated distribution.

3.1 Complete Distribution

The pdf, $f(x)$, and cdf, $F(x)$, of the complete Weibull distribution are defined as

$$\boldsymbol{f(x|\alpha,\beta)} = \frac{\boldsymbol{\beta}}{\boldsymbol{\alpha}}\left(\frac{\boldsymbol{x}}{\boldsymbol{\alpha}}\right)^{\boldsymbol{\beta-1}} \mathbf{exp}\left[-\left(\frac{\boldsymbol{x}}{\boldsymbol{\alpha}}\right)^{\boldsymbol{\beta}}\right] \tag{3.1}$$

$$\boldsymbol{F(x|\alpha,\beta)} = \mathbf{1} - \mathbf{exp}\left[-\left(\frac{\boldsymbol{x}}{\boldsymbol{\alpha}}\right)^{\boldsymbol{\beta}}\right]. \tag{3.2}$$

The support, scale parameter, α, and shape parameter, β, are restricted to,

$$\begin{array}{ll} \text{Support} & 0 < x < \infty \\ \text{Scale parameter} & 0 < \alpha < \infty \\ \text{Shape parameter} & 0 < \beta < \infty\ . \end{array}$$

3.2 Truncated Distribution

The truncated pdf, $f_T(x)$, and corresponding λ are defined as

$$f_T(x) = \lambda f(x) \qquad \text{(1.32 revisited)}$$

$$\frac{1}{\lambda} = \int_{\tau_l}^{\tau_r} f(x)dx \ . \qquad \text{(1.33 revisited)}$$

Combining Eq. (3.1) and Eq. (1.33) yields

$$\frac{1}{\lambda} = \int_{\tau_l}^{\tau_r} \frac{\beta}{\alpha}\Big(\frac{x}{\alpha}\Big)^{\beta-1} \exp\Big[-\Big(\frac{x}{\alpha}\Big)^{\beta}\Big]dx \ . \tag{3.3}$$

To aid the evaluation of Eq. (3.3), u is defined as

$$u \equiv \frac{x}{\alpha} \quad \Rightarrow \quad x = u\,\alpha \quad \Rightarrow \quad dx = \alpha\, du \ . \tag{3.4}$$

Substitution of u into Eq. (3.3) reduces the problem to,

$$\begin{aligned}
\frac{1}{\lambda} &= \int_{\frac{\tau_l}{\alpha}}^{\frac{\tau_r}{\alpha}} \beta u^{\beta-1} \exp\Big[-u^{\beta}\Big]du \\
&= -\int_{\frac{\tau_l}{\alpha}}^{\frac{\tau_r}{\alpha}} \frac{d}{du}\Big(\exp\Big[-u^{\beta}\Big]\Big)\, du \\
&= -\exp\Big[-u^{\beta}\Big]\Big|_{\frac{\tau_l}{\alpha}}^{\frac{\tau_r}{\alpha}} \\
\Rightarrow \frac{1}{\lambda} &= \exp\Big[-\Big(\frac{\tau_l}{\alpha}\Big)^{\beta}\Big] - \exp\Big[-\Big(\frac{\tau_r}{\alpha}\Big)^{\beta}\Big] \ .
\end{aligned} \tag{3.5}$$

This study is only concerned with left truncation, hence, the right-truncation limit has been removed, i.e. $\tau_r = \infty$. Therefore Eq. (3.5) reduces to,

$$\begin{aligned}
\frac{1}{\lambda} &= \exp\Big[-\Big(\frac{\tau_l}{\alpha}\Big)^{\beta}\Big] \\
\Rightarrow \boldsymbol{\lambda} &= \mathbf{exp}\Big[\Big(\frac{\boldsymbol{\tau_l}}{\boldsymbol{\alpha}}\Big)^{\boldsymbol{\beta}}\Big] \ .
\end{aligned} \tag{3.6}$$

Therefore, the left-truncated Weibull pdf can be expressed as,

$$\boxed{\boldsymbol{f_T(x|\alpha,\beta) = \frac{\beta}{\alpha}\Big(\frac{x}{\alpha}\Big)^{\beta-1}} \mathbf{exp}\boldsymbol{\Big[\Big(\frac{\tau_l}{\alpha}\Big)^{\beta} - \Big(\frac{x}{\alpha}\Big)^{\beta}\Big]} \ .} \tag{3.7}$$

The truncated cdf, $F_T(x)$, is defined as,

$$F_T(x) = \int_{\tau_l}^{x} f_T(t)dt = \lambda\int_{\tau_l}^{x} f(t)dt \ . \qquad \text{(1.35 revisited)}$$

Substituting Eq. (3.1) into Eq. (1.35) produces,

$$\begin{aligned}
F_T(x|\alpha,\beta) &= \exp\Big[\Big(\frac{\tau_l}{\alpha}\Big)^{\beta}\Big]\int_{\tau_l}^{x} \frac{\beta}{\alpha}\Big(\frac{x}{\alpha}\Big)^{\beta-1} \exp\Big[-\Big(\frac{x}{a}\Big)^{\beta}\Big]dt \text{ taking the result from Eq. (3.5)} \\
&= \exp\Big[\Big(\frac{\tau_l}{\alpha}\Big)^{\beta}\Big]\Big(\exp\Big[-\Big(\frac{\tau_l}{\alpha}\Big)^{\beta}\Big] - \exp\Big[-\Big(\frac{x}{\alpha}\Big)^{\beta}\Big]\Big) \ .
\end{aligned} \tag{3.8}$$

Hence, the left-truncated Weibull cdf can be expressed as,

$$\boxed{\boldsymbol{F_T(x|\alpha,\beta) = 1 -} \mathbf{exp}\boldsymbol{\Big[\Big(\frac{\tau_l}{\alpha}\Big)^{\beta} - \Big(\frac{x}{\alpha}\Big)^{\beta}\Big]} \ .} \tag{3.9}$$

The truncation percentage, p, is defined by,

$$p \equiv F(\tau_L) \; . \qquad \text{(1.36 revisited)}$$

Employing Eq. (3.2) in Eq. (1.36) gives,

$$\boldsymbol{p = p(\tau_l, \alpha, \beta) = 1 - \exp\Big[- \Big(\frac{\tau_l}{\alpha}\Big)^{\beta} \Big]} \; . \tag{3.10}$$

Manipulation of Eq. (3.10) allows one to define τ_l in terms of p, α and β,

$$\boldsymbol{\tau_l = \alpha\Big(- \log(1-p)\Big)^{\frac{1}{\beta}}}. \tag{3.11}$$

3.3 Generation of Random Numbers from the Weibull Distribution

Random generation formulae make use of the true parameters of a distribution. The process of parameter estimation for the left-truncated Weibull distribution is thoroughly discussed in section 3.4. In this section, both the true and estimated parameters are used, hence, it is necessary to introduce notation which distinguishes them from each other. The notation employed in this thesis is discussed in section 1.2 and is revisited in Table 1.1 for a general parameter θ.

θ^0	known *a priori*
$\hat{\theta}$	estimated from data

Table 1.1: Summary of parameter notation

Recall that the general random number generation formula is defined by,

$$F^{-1}(u_i) = x_i \; , \qquad \text{(1.26 revisited)}$$

and that the array of uniform random variates, $\boldsymbol{u}$, is defined as,

$$\boldsymbol{u} = (u_1, u_2, ..., u_n) \quad \text{where} \quad u_i \in (0,1) \quad \forall \quad i = 1, 2, ..., n. \qquad \text{(1.27 revisited)}$$

The random generation formula for the left-truncated Weibull distribution can be obtained by determining the inverse of the cumulative distribution function of the left-truncated Weibull distribution, Eq. (3.9),

$$\begin{aligned} F_T(x_i|\alpha^0, \beta^0) = u_i = 1 - \exp\Big[\Big(\frac{\tau_l}{\alpha^0}\Big)^{\beta^0} - \Big(\frac{x_i}{\alpha^0}\Big)^{\beta^0}\Big] \\ \Rightarrow \log(1-u_i) = \Big(\frac{\tau_l}{\alpha^0}\Big)^{\beta^0} - \Big(\frac{x_i}{\alpha^0}\Big)^{\beta^0} \\ \Rightarrow x_i = \alpha^0 \Big[\Big(\frac{\tau_l}{\alpha^0}\Big)^{\beta^0} - \log(1-u_i)\Big]^{\frac{1}{\beta^0}} = F_T^{-1}(u_i|\alpha^0, \beta^0). \end{aligned} \tag{3.12}$$

Eq. (3.12) can be simplified by defining a quantity, η^0,

$$\boldsymbol{\eta^0 \equiv \Big[\frac{\tau_l}{\alpha^0}\Big]^{\beta^0}} \; . \tag{3.13}$$

Combining Eq. (3.13) with Eq. (3.10), reveals that η^0 can be expressed as a function of the truncation percentage, p,

$$\begin{aligned} p &= 1 - \exp\left[- \left(\frac{\tau_l}{\alpha}\right)^{\beta} \right] \\ &= 1 - \exp\left[- \eta^0 \right] \\ \Rightarrow \boldsymbol{\eta^0} &= \boldsymbol{\eta^0(p) = -\log\left(1 - p\right)} \; . \end{aligned} \tag{3.14}$$

Therefore, random numbers from the left-truncated Weibull distribution can be generated from uniform random variates through,

$$\boxed{\boldsymbol{x_i = \alpha^0 \left[\eta^0 - \log\left(1 - u_i\right)\right]^{\frac{1}{\beta^0}}} \; ,} \tag{3.15}$$

which is a simplification of Eq. (3.12).

Notes

- If one is dealing with the complete (untruncated) Weibull distribution, random data can be generated through,

  $$x_i = \alpha^0 \left[-\log\left(1 - u_i\right)\right]^{\frac{1}{\beta^0}} \; , \tag{3.16}$$

  by substituting,

  $$\begin{aligned} \tau_l &= 0 \; , \\ \Rightarrow p &= 0 \; , \\ \Rightarrow \eta &= 0 \; , \end{aligned} \tag{3.17}$$

  into Eq. (3.15).

- The definition of η^0 in Eq. (3.13) allows one to specify the left-truncation point, τ_l, in terms of η^0,

  $$\tau_l = \alpha^0 (\eta^0)^{\frac{1}{\beta^0}} . \tag{3.18}$$

3.4 Estimation of Parameters

For observations drawn from a population with pdf, $f(x|\boldsymbol{\theta})$, knowledge of $\boldsymbol{\theta}$ estimated from the data helps us to describe the entire population.

Definition 3.4.1. A *point estimator* is any function, $W(\boldsymbol{x})$, of a sample; that is, any statistic is a point estimator.

We must be clear that an *estimator* is a function of the sample, while the corresponding *estimate* is the realised value of that estimator obtained for a particular sample of observations, $\boldsymbol{x}$. As discussed in section 1.5, we will be using the method of *maximum likelihood* as a point estimator estimator for estimating the distribution parameters in this work.

Definition 3.4.2. For each sample, $\boldsymbol{x}$, let $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$ be the set of parameter values at which $L(\boldsymbol{\theta}|\boldsymbol{x})$ attains its maximum as a function of $\boldsymbol{\theta}$. A *maximum likelihood estimator* (MLE) of the true parameters, $\boldsymbol{\theta^0}$, based on any sample, $\boldsymbol{x}$, is $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$.

Table 1.2 summarises the four different cases of parameter estimation studied in this work.

Case	Scale Parameter	Shape Parameter
I	known *a priori*	known *a priori*
II	acquired from data	acquired from data
IIIa	acquired from data	known *a priori*
IIIb	known *a priori*	acquired from data

Table 1.2: Cases of parameter estimation

For the left-truncated Weibull distribution $f_T(x)$ is given by Eq. (3.7), which reduces Eq. (1.17) to

$$L(\boldsymbol{\theta}|\boldsymbol{x}) = \prod_{i=1}^{n} \exp\left[\left(\frac{\tau_l}{\alpha}\right)^{\beta}\right] \frac{\beta}{\alpha^{\beta}} x_i^{(\beta-1)} \exp\left[-\left(\frac{x_i}{\alpha}\right)^{\beta}\right]$$
$$\Rightarrow L(\boldsymbol{\theta}|\boldsymbol{x}) = \left[\exp\left[\left(\frac{\tau_l}{\alpha}\right)^{\beta}\right] \frac{\beta}{\alpha^{\beta}}\right]^{n} \prod_{i=1}^{n} x_i^{(\beta-1)} \exp\left[-\left(\frac{x_i}{\alpha}\right)^{\beta}\right]. \tag{3.19}$$

The loglikelihood function, l, is defined as

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = \log(L(\boldsymbol{\theta}|\boldsymbol{x})) = \sum_{i=1}^{n} \log(f_T(x_i|\boldsymbol{\theta})) \ , \qquad \text{(1.19 revisited)}$$

which reduces to,

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = n\left(\frac{\tau_l}{\alpha}\right)^{\beta} + n\log(\beta) - n\beta\log(\alpha) + \sum_{i=1}^{n}(\beta-1)\log(x_i) - \left(\frac{x_i}{\alpha}\right)^{\beta} \ , \tag{3.20}$$

in the case of the left-truncated Weibull distribution. The most general form of the MLE equations is

$$\frac{\partial}{\partial\theta_j} l(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{\partial}{\partial\theta_j} \sum_{i=1}^{n} \log\left(f(x_i|\boldsymbol{\theta})\right) = 0 \quad \forall \quad j = (1, 2, ..., m) \ , \qquad \text{(1.21 revisited)}$$

where m is the number of parameters in $f_T(x|\boldsymbol{\theta})$.

3.4.1 Case IIIa

In this case the scale parameter, α, must be estimated from the data ($\alpha = \hat{\alpha}$) and the shape parameter, β, is known ($\beta = \beta^0$). For the left-truncated Weibull distribution, the corresponding MLE equation is achieved by substituting $\theta_1 = \alpha$ into Eq. (1.21) to yield,

$$\begin{aligned} 0 &= \frac{dl}{d\alpha} \\ &= -\frac{n\beta}{\alpha} - \frac{n\beta}{\alpha}\left(\frac{\tau_l}{\alpha}\right)^{\beta} + \frac{\beta}{\alpha}\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}. \end{aligned} \tag{3.21}$$

Multiplying both side of Eq. (3.21) by $\frac{\alpha}{n\beta}$ we get,

$$0 = -1 - \left(\frac{\tau_l}{\alpha}\right)^{\beta} + \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^{\beta}$$
$$\boxed{\Rightarrow \boldsymbol{\alpha} = \left(\frac{1}{n}\sum_{i=1}^{n}\left[x_i^{\beta} - \tau_l^{\beta}\right]\right)^{\frac{1}{\beta}}.} \tag{3.22}$$

3.4.2 Case IIIb

In this case the shape parameter, β, must be estimated from the data ($\beta = \hat{\beta}$) and the scale parameter, α, is known ($\alpha = \alpha^0$). For the left-truncated Weibull distribution, the corresponding MLE equation is achieved by substituting $\theta_2 = \beta$ into Eq. (1.21) to yield,

$$\begin{aligned} 0 &= \frac{dl}{d\beta} \\ &= n\log\left(\frac{\tau_l}{\alpha}\right)\left(\frac{\tau_l}{\alpha}\right)^\beta + \frac{n}{\beta} - n\log(\alpha) + \sum_{i=1}^{n}\log(x_i) - \sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^\beta \log\left(\frac{x_i}{\alpha}\right). \end{aligned} \tag{3.23}$$

Noting that $-n\log(\alpha) = -\sum_{i=1}^{n}\log(\alpha)$ reduced Eq. (3.23) to,

$$\boxed{\boldsymbol{0 = \frac{n}{\beta} + \sum_{i=1}^{n}\log\left(\frac{x_i}{\alpha}\right)\left[1 - \left(\frac{x_i}{\alpha}\right)^\beta\right] + n\log\left(\frac{\tau_l}{\alpha}\right)\left(\frac{\tau_l}{\alpha}\right)^\beta.}} \tag{3.24}$$

This is a non-linear equation and is more complicated than Eq. (3.22).

3.4.3 Case II

In this case, both the shape and scale parameters must be estimated from the data. This is achieved by finding simultaneous solutions to Eq. (3.21) and Eq. (3.23). The expression for α given in Eq. (3.22) can be substituted into the loglikelihood function [67] given in Eq. (3.20) to remove the dependence of l on α,

$$\begin{aligned} l &= -n\log(\alpha^\beta) + n\log(\beta) + (\beta-1)\sum_{i=1}^{n}\log(x_i) - \frac{1}{\alpha^\beta}\sum_{i=1}^{n}\left[x_i^\beta - \tau_l^\beta\right] \\ &= -n\log\left(\frac{1}{n}\sum_{i=1}^{n}\left[x_i^\beta - \tau_l^\beta\right]\right) + n\log(\beta) \\ &\quad + (\beta-1)\sum_{i=1}^{n}\log(x_i) - \frac{n}{\sum_{j=1}^{n}\left[x_j^\beta - \tau_l^\beta\right]}\sum_{i=1}^{n}\left[x_i^\beta - \tau_l^\beta\right] \\ &= n\left[\log(n) - 1\right] - n\log\left(\sum_{i=1}^{n}\left[x_i^\beta - \tau_l^\beta\right]\right) + n\log(\beta) + (\beta-1)\sum_{i=1}^{n}\log(x_i)\,. \end{aligned} \tag{3.25}$$

Differentiating Eq. (3.25) with respect to β [10, 67, 68] produces,

$$\begin{aligned} \frac{dl}{d\beta} = 0 &= \frac{n}{\beta} - \sum_{i=1}^{n}\log(x_i) - n\frac{\sum_{i=1}^{n}\left[x_i^\beta\log(x_i) - \tau_l^\beta\log(\tau_l)\right]}{\sum_{i=1}^{n}\left[x_i^\beta - \tau_l^\beta\right]} \\ \Rightarrow 0 &= \frac{n}{\beta} - \sum_{i=1}^{n}\log(x_i) - n\frac{\sum_{i=1}^{n}\left(\frac{x_i}{\alpha}\right)^\beta\log\left(\frac{x_i}{\alpha}\right)}{\sum_{i=1}^{n}\left[\left(\frac{x_i}{\tau_l}\right)^\beta - 1\right]} + n\log(\tau_l) \end{aligned}$$

$$\boxed{\boldsymbol{\Rightarrow 0 = \frac{1}{\beta} - \frac{\sum_{i=1}^{n}\left(\frac{x_i}{\tau_l}\right)^\beta\log\left(\frac{x_i}{\tau_l}\right)}{\sum_{i=1}^{n}\left[\left(\frac{x_i}{\tau_l}\right)^\beta - 1\right]} + \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right).}} \tag{3.26}$$

Solving Eq. (3.26) for β yields the MLE estimate of the shape parameter, $\hat{\beta}$, which can be employed in Eq. (3.22) to determine the corresponding estimate of the scale parameter,

$\hat{\alpha}$. There are a few conditions to this that are addressed in the following paragraphs.

It is guaranteed that the parameters which maximise the likelihood function satisfy the MLE equations, Eq. (1.21), however, it is not necessarily the case that parameters which solve Eq. (1.21) also maximise likelihood; it is a necessary but not sufficient condition. Eq. (1.21) can be satisfied by local maxima/minima or a stationery point in the log-likelihood function. To verify that the solution of the MLE equations also maximises likelihood, we must inspect the second-order partial derivatives of l and ensure that the Hessian is negative definite at $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$. For the left-truncated Weibull distribution Kreer et al. [11,42] and Kizilersü et al. [10] showed that there is always a set of parameters which maximises likelihood. Therefore, if it can be shown that there is a unique solution to the MLE equations, we can say that this solution also maximises likelihood. The alternative is to numerically find the maximum of the loglikelihood function using a direct search or iterative algorithm. There are two inherent drawbacks associated with the general problem of finding the maximum of a function, and hence of conducting maximum likelihood estimation in this manner:

(i) Finding the *global maximum* and verifying that, indeed, a global maxima has been found.

(ii) Numerical sensitivity, that is, how sensitive are the estimates to small changes in the data.

Additionally, it is generally more computationally expensive to find the parameter estimates via a maximisation algorithm. Therefore, we produced the maximum likelihood estimates via the MLE equations when we could ensure that the solution would also maximise likelihood. The solutions of the MLE equations had to be analysed separately for each case:

- **Case IIIa:** Eq. (3.22) has a closed-form and thus, a unique solution. Therefore, in case I there is a unique maximum likelihood estimate, which is the α value which solves Eq. (3.22). In our Monte Carlo simulations we solved Eq. (3.22) to produce our parameter estimates.

- **Case IIIb:** Eq. (3.24) does not necessarily have a unique solution, it may be solved at local maxima/minima or at a stationery point of the loglikelihood function. Thus, to determine $\hat{\beta}$ we numerically found the β value which maximised the loglikelihood function, Eq. (3.20).

- **Case II:** Kreer et al. [11,42] have shown that in case II, the left-truncated Weibull distribution MLE equations have a unique solution if and only if the samples, $\boldsymbol{x}$, satisfy,

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\{\log\left(\frac{x_i}{\tau_l}\right)\right\}^2-\left\{\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right)\right\}^2}<\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right)\,. \tag{3.27}$$

  Therefore, if Eq. (3.27) is satisfied, the parameters which simultaneously solve Eq. (3.22) and (3.26) also maximise likelihood. In our Monte Carlo simulations, if samples did not satisfy Eq. (3.27) they were disregarded and replaced. For the samples which did satisfy this inequality, Brent's method was used to solve Eq. (3.26), yielding the maximum likelihood estimate of the shape parameter, $\hat{\beta}$. This value was then substituted into Eq. (3.22) to produce the corresponding estimate of the scale parameter, $\hat{\alpha}$.

3.4.4 Summary of Applicable MLE Equations

Case	Scale Parameter (α) Equation	Shape Parameter (β) Equation
I	N/A	N/A
II	Eq. (3.22)	Eq. (3.26)
IIIa	Eq. (3.22)	N/A
IIIb	N/A	Eq. (3.24)

Table 3.1: Summary of applicable MLE equations

Recall that we chose to use the maximum likelihood estimator to estimate our parameters because it produces consistent and efficient results. These features are a consequence of Lehmann and Casella's work [16]. In the next section we ensure that those results are valid for observations drawn from the left-truncated Weibull distribution.

3.5 Assumptions and Conditions of Theorem 1.5

The most desirable attributes of the MLE as a parameter estimator are consistency and efficiency, these are a result of Theorem 1.5 (Theorem 5.1 in [16]). As we employ the MLE extensively in this work, we should make some attempt to check that Theorem 1.5 is applicable to the left-truncated Weibull distribution. We have outlined conditions (A0)–(A2) and assumptions (A)–(D) and provide an argument for their plausibility, however, a rigorous proof is beyond the scope of this thesis. This "verification" procedure has largely been taken from Appendix 1 of Kreer et al. [11] and will not be repeated for the other distributions, as the reasoning put forth is valid for (or analogous to that which would be used for) the loglogistic, lognormal, and Pareto distributions.

Theorem 1.5. Let $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$ be i.i.d. each with a probability density function $f(x|\boldsymbol{\theta^0})$ which satisfies conditions (A0)–(A2) and assumptions (A)–(D). Then with probability tending to 1 as $n \to \infty$, there exist solutions $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}} = \hat{\boldsymbol{\theta}}_{\boldsymbol{n}}(\boldsymbol{x})$ of the likelihood equations such that

- The vector $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$ is consistent for estimating $\boldsymbol{\theta^0}$
- $\sqrt{n}(\hat{\boldsymbol{\theta}}_{\boldsymbol{n}} - \boldsymbol{\theta^0})$ is asymptotically normal with vector mean zero and covariance matrix $[I(\boldsymbol{\theta^0})]^{-1}$ being the inverse of the Fisher information matrix

$$I(\boldsymbol{\theta^0})_{j,k} = -\mathbb{E}\left(\frac{\partial^2}{\partial\theta_j \partial\theta k} \log\left(f(x|\boldsymbol{\theta^0})\right)\right) \quad \text{where} \quad j,k \in (1,2,...,m)$$

- $\hat{\boldsymbol{\theta}}_{\boldsymbol{n}}$ is asymptotically efficient in the sense that as $n \to \infty$

$$\sqrt{n}(\hat{\theta}_{n,j} - \theta_j^0) \xrightarrow{\text{distrib.}} \mathrm{N}\left(0, \left[I(\boldsymbol{\theta^0})\right]_{j,j}^{-1}\right) \quad \text{where} \quad j \in (1,2,...,m)$$

Conditions

- A0: The probability distributions of the observations are distinct, i.e.

$$\text{For } \boldsymbol{\theta} \neq \boldsymbol{\theta}',\ f(x|\boldsymbol{\theta}) \neq f(x|\boldsymbol{\theta}')\ \forall\ x \in [0,\infty)$$

This is readily checked because $f(x|\alpha,\beta) \neq f(x|\alpha',\beta')$ for $\alpha \neq \alpha'$ and $\beta \neq \beta'$.

- A1: The probability distributions have common support, i.e.

$$\text{if } f(\boldsymbol{\theta})\colon X \to Y \text{ and } f(\boldsymbol{\theta}')\colon X' \to Y' \text{ then } X = X'\ \forall\ \boldsymbol{\theta}, \boldsymbol{\theta}'$$

This is true because $x \in (\tau_l, \infty)$ for all α and β.

- A2: The observations $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$ are i.i.d. with a probability density function $f(x|\boldsymbol{\theta}^0)$.

This is an explicit assumption we make for all observations.

Assumptions

- A: There exists an open subset ω of Ω containing the true parameter point $\boldsymbol{\theta}^0$ such that for almost all x, the probability density function, $f(x|\boldsymbol{\theta}^0)$, admits all third derivatives

$$\frac{\partial^3}{\partial\theta_i\partial\theta_j\partial\theta_k} f(x|\boldsymbol{\theta}^0) \quad \forall \quad \boldsymbol{\theta} \in \omega$$

This condition is readily checked because the left truncated Weibull distribution is continuously differentiable with respect to its parameters.

- B: The first logarithmic derivatives of f satisfies

$$\mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_j}\log f(x|\boldsymbol{\theta})\right] = 0 \ \forall \ j \in \{1, 2, ..., m\}$$

This assumption can be verified by

$$\begin{aligned}
\mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_j}\log f\right] &= \mathbb{E}_\theta\left[\frac{1}{f}\frac{\partial f}{\partial\theta_j}\right] \\
&= \int_{\tau_l}^{\infty} f\frac{1}{f}\frac{\partial f}{\partial\theta_j}dx \\
&= \int_{\tau_l}^{\infty} \frac{\partial f}{\partial\theta_j}dx \\
&= \frac{\partial}{\partial\theta_j}\left(\int_{\tau_l}^{\infty} f dx\right) \\
&= \frac{\partial}{\partial\theta_j}(1) = 0.
\end{aligned}$$

and the second derivatives of f satisfies

$$\begin{aligned}
I_{jk}(\boldsymbol{\theta}) &= \mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_j}\log f(x|\boldsymbol{\theta}).\frac{\partial}{\partial\theta_k}\log f(x|\boldsymbol{\theta})\right] \\
&= \mathbb{E}_\theta\left[-\frac{\partial^2}{\partial\theta_j\partial\theta_k}\log f(x|\boldsymbol{\theta})\right] \quad \forall \ j,k \in \{1, 2, ..., m\}
\end{aligned}$$

where m is the number of parameters in f.

This assumption holds because

$$\begin{aligned}
\mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_j}\log f.\frac{\partial}{\partial\theta_k}\log f\right] &= -\mathbb{E}_\theta\left[\frac{\partial^2}{\partial\theta_j\partial\theta_k}\log f\right]\\
\mathbb{E}_\theta\left[\frac{1}{f}\frac{\partial f}{\partial\theta_j}.\frac{1}{f}\frac{\partial f}{\partial\theta_k}\right] &= -\mathbb{E}_\theta\left[\frac{\partial}{\partial\theta_j}\left(\frac{1}{f}\frac{\partial f}{\partial\theta_k}\right)\right]\\
\int_{\tau_l}^{\infty} f\frac{1}{f}\frac{\partial f}{\partial\theta_j}.\frac{1}{f}\frac{\partial f}{\partial\theta_k}dx &= -\int_{\tau_l}^{\infty} f\frac{\partial}{\partial\theta_j}\left(\frac{1}{f}\frac{\partial f}{\partial\theta_k}\right)dx\\
\int_{\tau_l}^{\infty} \frac{\partial f}{\partial\theta_j}\frac{1}{f}\frac{\partial f}{\partial\theta_k}dx &= -\int_{\tau_l}^{\infty} f\frac{\partial}{\partial\theta_j}\left(\frac{1}{f}\frac{\partial f}{\partial\theta_k}\right)dx\\
\int_{\tau_l}^{\infty} \frac{\partial f}{\partial\theta_j}\frac{1}{f}\frac{\partial f}{\partial\theta_k}dx + \int_{\tau_l}^{\infty} f\frac{\partial}{\partial\theta_j}\left(\frac{1}{f}\frac{\partial f}{\partial\theta_k}\right)dx &= 0\\
\int_{\tau_l}^{\infty} \frac{\partial}{\partial\theta_j}\left(\left[f\right]\left[\frac{1}{f}\frac{\partial f}{\partial\theta_k}\right]\right)dx &= 0\\
\frac{\partial^2}{\partial\theta_j\partial\theta_k}\int_{\tau_l}^{\infty} f dx &= 0\\
\frac{\partial^2}{\partial\theta_j\partial\theta_k}(1) &= 0 \;\forall\; j,k\in\{1,2,...,m\}.
\end{aligned}$$

- C: The matrix $I(\boldsymbol{\theta})$ is finite and positive definite $\forall\ \boldsymbol{\theta}\in\omega$

The integrals can be computed explicitly and are finite because the asymptotic condition requires that $f(x|\alpha,\beta)\approx\mathcal{O}\left(\exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right]\right)$ as $x\to\infty$ for $\alpha>0$ and $\beta\in(0,\infty)$. Therefore, $I_{jk}(\boldsymbol{\theta})$ is also finite. Thus, $I(\boldsymbol{\theta})$ is well-defined and (as a covariance matrix) is by construction positive definite.

- D: There exists functions $M_{jkl}(x)$ such that

$$\left|\frac{\partial^3}{\partial\theta_i\partial\theta_j\partial\theta_k}\log f(x|\boldsymbol{\theta})\right|\le M_{jkl}(x)\ \forall\ \boldsymbol{\theta}\in\omega$$
$$\text{where } \mathbb{E}_{\theta^0}\left[M_{jkl}(x)\right]<\infty\ \forall\ j,k,l$$

This is a consequence of the fact that $f(x|\alpha,\beta)\approx\mathcal{O}\left(\exp\left[-\left(\frac{x}{\alpha}\right)^\beta\right]\right)$ as $x\to\infty$ for $\alpha>0$ and $\beta\in(0,\infty)$.

3.6 Parameter Independent Pivotal Functions and Critical Values

As discussed in section 2.6, parameter independent pivotal functions are important because they produce parameter independent critical values. Without parameter independence, the critical values would need to be produced for every parameter combination from which observations may be taken. Additionally, in the real world, one usually does not know the true parameters of the distribution from which they are making measurements, hence, selecting the correct critical values becomes an impossible task. Employing incorrect critical values can alter the result of the hypothesis tests, which can have adverse affects if the result of the test affect decisions made in relation to health or safety. Thus, for our results to be of use in the real world, parameter independent critical values are a necessity. It is the objective of this section to show that the left-truncated Weibull distribution has parameter independent critical values. The methodology we will employ is employed in section 2.6.

Expressing the x dependence of the left-truncated Weibull cdf, Eq. (3.9), with the random generation formula, Eq. (3.15), allows us to define a function, $\widetilde{F_T}(u)$ where $x = F_T^{-1}(u)$, such that $\widetilde{F_T}(u) = F_T(x)$, for all x,

$$\begin{aligned} F_T(x_i|\hat{\alpha},\hat{\beta}) &= 1 - \exp\left[\left(\frac{\tau_l}{\hat{\alpha}}\right)^{\hat{\beta}} - \left(\frac{x_i}{\hat{\alpha}}\right)^{\hat{\beta}}\right] \\ \Rightarrow \widetilde{F_T}(u_i|\hat{\alpha},\hat{\beta},\alpha^0,\beta^0) &= 1 - \exp\left[\left(\frac{\alpha^0}{\hat{\alpha}}\eta^{\frac{1}{\beta^0}}\right)^{\hat{\beta}} - \left(\frac{\alpha^0}{\hat{\alpha}}\left[\eta - \log\left(1-u_i\right)\right]^{\frac{1}{\beta^0}}\right)^{\hat{\beta}}\right] \\ &= 1 - \exp\left[\left(\frac{\alpha^0}{\hat{\alpha}}\right)^{\hat{\beta}}\eta^{\frac{\hat{\beta}}{\beta^0}} - \left(\frac{\alpha^0}{\hat{\alpha}}\right)^{\hat{\beta}}\left[\eta - \log\left(1-u_i\right)\right]^{\frac{\hat{\beta}}{\beta^0}}\right]. \end{aligned} \tag{3.28}$$

As discussed in section 2.6, the pivotal functions γ and ξ are introduced to simplify Eq. (3.28),

$$\boldsymbol{\gamma} = \left(\frac{\boldsymbol{\alpha^0}}{\hat{\boldsymbol{\alpha}}}\right)^{\hat{\boldsymbol{\beta}}} \tag{3.29}$$

$$\boldsymbol{\xi} = \frac{\hat{\boldsymbol{\beta}}}{\boldsymbol{\beta^0}}\,. \tag{3.30}$$

Substituting Eq. (3.29) and Eq. (3.30) into Eq. (3.28) reduces $\widetilde{F_T}(u)$ to,

$$\widetilde{F_T}(u_i|\hat{\alpha},\hat{\beta},\alpha^0,\beta^0) = \widetilde{F_T}(u_i|\gamma,\xi,\eta) = 1 - \exp\left[\gamma\eta^{\xi} - \gamma\left[\eta - \log\left(1-u_i\right)\right]^{\xi}\right]. \tag{3.31}$$

All parameter dependence in $\widetilde{F_T}(u)$ is completely contained within the pivotal functions $\gamma = \gamma(\alpha^0,\hat{\alpha},\hat{\beta})$ and $\xi = \xi(\beta^0,\hat{\beta})$. In this section we will show that these pivotal functions are distributed independently of α^0 and β^0, therefore, proving that the critical values are parameter independent.

The random number generation formula, Eq. (3.15), can be substituted into the MLE equations summarised in Table 3.1. Manipulations of the resulting equations can contain their parameter dependence within the pivotal functions γ and ξ. Kreer et al. [11, 42] and Kizilersü et al. [10] showed that these equations are solved at the parameter values which maximise likelihood. Hence, these pivotal functions can be expressed in terms of the other variables in the equations, i.e. η and $\boldsymbol{u}$, and not the parameters, α^0 and β^0. This method is outlined in dot points below.

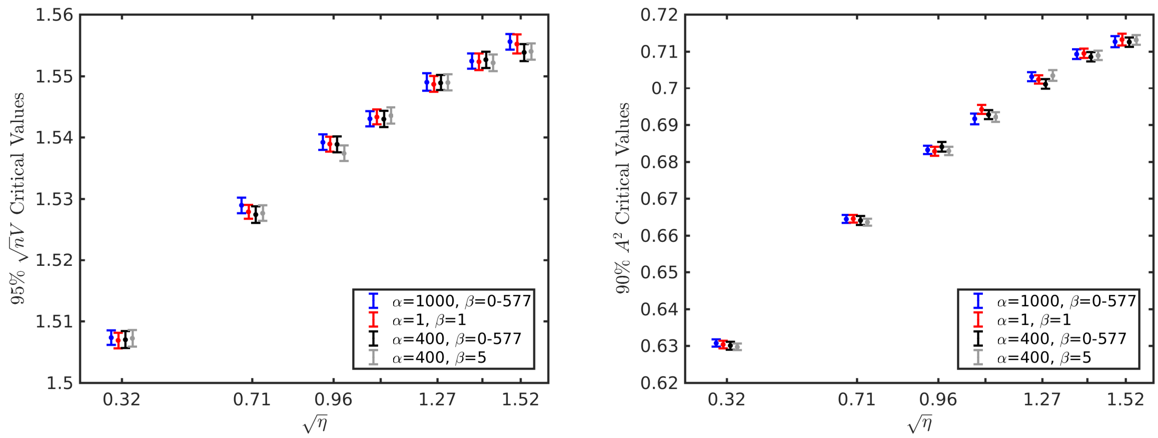
- Substitute the random generation formula into the MLE equations
- Manipulate the MLE equations to contain all parameter dependence in the pivotal functions

$\Rightarrow$ Pivotal functions are distributed independently of parameters

$\Rightarrow$ $\widetilde{F_T}(u)$ is parameter independent

$\Rightarrow$ Critical values are parameter independent

In section 8.1 we have numerically verified that the critical values are parameter independent for all of the distributions we studied. Figures 3.1a and 3.1b display the Kuiper and Anderson-Darling critical values at the 95% and 90% significance levels with samples of size $n = 1,000$ and $n = 50$ (respectively) drawn from the case II Weibull distribution for a range of parameter values. This figure shows an example supporting the assertion that the critical values are parameter independent. A more detailed discussion is included in section 8.1.



(a) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl. for $n = 1000$ observations

(b) Anderson-Darling (A^2) crit. vals. at 90% sig. lvl. for $n = 50$ observations

Figure 3.1: Case II Weibull critical values
Critical values from different parameter values are staggered on the $\sqrt{\eta}$ axis for clarity

3.6.1 Case IIIa

In this case, α is to be estimated from the data and β is known *a priori*, therefore

$$\begin{aligned}
\hat{\beta} &= \beta^0 \\
\Rightarrow \xi &= 1 \\
\Rightarrow \widetilde{F_T}(u_i|\gamma,\eta) &= 1 - \exp\left[\gamma\eta - \gamma\left[\eta - \log\left(1-u_i\right)\right]\right] \\
&= 1 - \exp\left[\gamma \log\left(1-u_i\right)\right] \\
&= 1 - \left(1-u_i\right)\exp(\gamma)\ .
\end{aligned} \tag{3.32}$$

Additionally the remaining pivotal function γ becomes

$$\gamma = \left(\frac{\alpha^0}{\hat{\alpha}}\right)^{\beta^0} . \tag{3.33}$$

The random number generation formula Eq. (3.15), is inserted into Eq. (3.22), and reduced to the form,

$$\begin{aligned}
\hat{\alpha} &= \left(\frac{1}{n}\sum_{i=1}^{n}\left[x_i^{\beta^0} - \tau_l^{\beta^0}\right]\right)^{\frac{1}{\beta^0}} \\
\hat{\alpha}^{\beta^0} &= \frac{1}{n}\sum_{i=1}^{n}\left[\left(\alpha^0\left[\eta - \log\left(1-u_i\right)\right]^{\frac{1}{\beta^0}}\right)^{\beta^0} - \left(\alpha_0\eta^{\frac{1}{\beta_0}}\right)^{\beta^0}\right] \\
\left(\frac{\hat{\alpha}}{\alpha^0}\right)^{\beta^0} &= -\frac{1}{n}\sum_{i=1}^{n}\log\left(1-u_i\right) \\
\gamma = \gamma(\boldsymbol{u}) &= \frac{-n}{\sum_{i=1}^{n}\log\left(1-u_i\right)} \, .
\end{aligned} \tag{3.34}$$

One can then substitute Eq. (3.34) into Eq. (3.32) to remove the parameter dependence of the truncated cdf, $\widetilde{F_T}(u_i)$,

$$\Rightarrow \widetilde{F_T}(u_i|\gamma(\boldsymbol{u}),\eta) = \widetilde{F_T}(u_i|\boldsymbol{u},\eta) \, . \tag{3.35}$$

Therefore, $\widetilde{F_T}(u_i)$, is distributed independently of α^0 and β^0, hence, we have parameter independent critical values.

3.6.2 Case IIIb

In this case, β is to be estimated and α is known *a priori*, therefore the truncated cdf can be reduced to

$$\begin{aligned}
\hat{\alpha} &= \alpha^0 \\
\Rightarrow \gamma &= 1 \\
\Rightarrow \widetilde{F_T}(u_i|\xi,\eta) &= 1 - \exp\left[\eta^{\xi} - \left[\eta - \log\left(1-u_i\right)\right]^{\xi}\right] \, .
\end{aligned} \tag{3.36}$$

The random number generation formula Eq. (3.15), is inserted into Eq. (3.24), and reduced to the form

$$\begin{aligned}
0 &= \frac{n}{\hat{\beta}} + \sum_{i=1}^{n}\log\left(\frac{x_i}{\alpha^0}\right)\left[1 - \left(\frac{x_i}{\alpha^0}\right)^{\hat{\beta}}\right] + n\log\left(\frac{\tau_l}{\alpha^0}\right)\left(\frac{\tau_l}{\alpha^0}\right)^{\hat{\beta}} \\
&= \frac{n}{\hat{\beta}} + \sum_{i=1}^{n}\log\left(\left[\eta - \log\left(1-u_i\right)\right]^{\frac{1}{\beta^0}}\right)\left[1 - \left(\left[\eta - \log\left(1-u_i\right)\right]^{\frac{1}{\beta^0}}\right)^{\hat{\beta}}\right] + n\log\left(\eta^{\frac{1}{\beta_0}}\right)\left(\eta^{\frac{1}{\beta_0}}\right)^{\hat{\beta}} \\
&= \frac{n}{\hat{\beta}} + \frac{1}{\beta^0}\sum_{i=1}^{n}\log\left(\eta - \log\left(1-u_i\right)\right)\left[1 - \left[\eta - \log\left(1-u_i\right)\right]^{\xi}\right] + \frac{n}{\beta^0}\log\left(\eta\right)\eta^{\xi} \\
0 &= \frac{1}{\xi} + \frac{1}{n}\sum_{i=1}^{n}\log\left(\eta - \log\left(1-u_i\right)\right)\left[1 - \left[\eta - \log\left(1-u_i\right)\right]^{\xi}\right] + \log\left(\eta\right)\eta^{\xi}.
\end{aligned} \tag{3.37}$$

Kreer et al. [11, 42] and Kizilersü et al. [10] showed that there is always a set of parameters which maximise likelihood and thus solve the MLE equations. Therefore, we are guaranteed that Eq. (3.37) has a solution. Therefore, ξ can be expressed in terms of the other variables, i.e. $\xi = \xi(\eta, \boldsymbol{u})$. From this expression of ξ, it is possible to remove the parameter dependence of $\widetilde{F_T}(u_i)$,

$$\begin{aligned}
\xi &= \xi(\eta, \boldsymbol{u}) \\
\Rightarrow \widetilde{F_T}(u_i|\xi(\boldsymbol{u}),\eta) &= \widetilde{F_T}(u_i|\boldsymbol{u},\eta).
\end{aligned} \tag{3.38}$$

Therefore, $\widetilde{F_T}(u_i)$, is independent of α^0 and β^0, hence, we have parameter independent critical values.

3.6.3 Case II

In this case the objective is to show that parameter independent solutions for γ and ξ exist. Initially one inserts the random generation formula Eq. (3.15), into Eq. (3.26),

$$\begin{aligned}
\hat{\alpha} &= \left(\frac{1}{n}\sum_{i=1}^{n}\left[x_i^{\hat{\beta}} - \tau_l^{\hat{\beta}}\right]\right)^{\frac{1}{\hat{\beta}}} \\
\hat{\alpha}^{\hat{\beta}} &= \frac{1}{n}\sum_{i=1}^{n}\left[\left(\alpha^0\left[\eta - \log\left(1-u_i\right)\right]^{\frac{1}{\beta^0}}\right)^{\hat{\beta}} - \left(\alpha_0 \eta^{\frac{1}{\beta_0}}\right)^{\hat{\beta}}\right] \\
\left(\frac{\hat{\alpha}}{\alpha^0}\right)^{\hat{\beta}} &= \frac{1}{n}\sum_{i=1}^{n}\left[\left(\eta - \log\left(1-u_i\right)\right)^{\xi} - \eta^{\xi}\right] \\
\gamma = \gamma(\xi,\eta,\boldsymbol{u}) &= \frac{-n}{\sum_{i=1}^{n}\left[\left(\eta - \log\left(1-u_i\right)\right)^{\xi} - \eta^{\xi}\right]} \; .
\end{aligned} \tag{3.39}$$

Eq. (22) in the work of Kreer et al. [11] shows that algebraic manipulations can be performed to contain all parameter dependence of Eq. (3.26) within the pivotal function ξ,

$$\begin{aligned}
&\frac{1}{n}\sum_{i=1}^{n}\log(\eta+\log(\frac{1}{u_i}))^{\xi} = \frac{1}{n}\sum_{i=1}^{n}\frac{(\eta+\log(\frac{1}{u_i}))^{\xi}}{\frac{1}{n}\sum_{i=1}^{n}(\eta+\log(\frac{1}{u_i}))^{\xi}-\eta^{\xi}}\log\left(\frac{(\eta+\log(\frac{1}{u_i}))^{\xi}}{\frac{1}{n}\sum_{i=1}^{n}(\eta+\log(\frac{1}{u_i}))^{\xi}-\eta^{\xi}}\right) + \\
&\log\left(\frac{1}{n}\sum_{i=1}^{n}(\eta+\log(\frac{1}{u_i}))^{\xi}-\eta^{\xi}\right) - 1 + \frac{\eta^{\xi}}{\frac{1}{n}\sum_{i=1}^{n}(\eta+\log(\frac{1}{u_i}))^{\xi}-\eta^{\xi}}\log\left(\frac{\eta^{\xi}}{\frac{1}{n}\sum_{i=1}^{n}(\eta+\log(\frac{1}{u_i}))^{\xi}-\eta^{\xi}}\right).
\end{aligned} \tag{3.40}$$

As discussed in section 3.4, Kreer et al. [11, 42] and Kizilersü et al. [10] showed that the MLE equations of the case II, left-truncated Weibull distribution have a unique solution if

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\{\log\left(\frac{x_i}{\tau_l}\right)\right\}^2 - \left\{\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right)\right\}^2} < \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right) \; . \qquad \text{(3.27 revisited)}$$

Thus, if Eq. (3.27) is satisfied, we know that there is a solution to Eq. (3.40), hence, ξ can be expressed in terms of the other variables in Eq. (3.40), i.e. $\xi = \xi(\eta, \boldsymbol{u})$. Hence, we can remove the parameter dependence in $\widetilde{F_T}(u_i)$ (Eq. (3.31)),

$$\begin{aligned}
&\xi = \xi(\boldsymbol{u},\eta) \\
&\Rightarrow \gamma = \gamma(\xi(\boldsymbol{u},\eta),\boldsymbol{u},\eta) = \gamma(\boldsymbol{u},\eta) \\
&\Rightarrow \widetilde{F_T}(u_i|\gamma,\xi) = \widetilde{F_T}(u_i|\gamma(\boldsymbol{u},\eta),\xi(\boldsymbol{u},\eta)) = \widetilde{F_T}(u_i|\boldsymbol{u},\eta).
\end{aligned} \tag{3.41}$$

Hence, $\widetilde{F_T}(u_i)$, is independent of α^0 and β^0, thus, we have parameter independent critical values.

Chapter 4

Loglogistic Distribution

The loglogistic distribution is based upon the more widely known logistic distribution, whose cdf is part of the logistic family of functions, hence the name. If the probability distribution of a random variable is given by the loglogistic distribution, its logarithm is described by the logistic distribution. The loglogistic distribution has scale parameter, ϕ, and shape parameter, ρ, and is defined on the support $\mathbb{R}^+ \cup 0$; correspondingly, the logistic distribution has the support, $\mathbb{R}$.

In the field of hydrology, the loglogistic distribution has been employed to describe steam flow rates and precipitation levels [69, 70]. In economics, where it is referred to as the Fisk distribution, the loglogistic distribution has been used to model the distribution of incomes [71]. In this work, we are most interested in its application to life-time analysis [72], as this is directly comparable to the most common application of the Weibull distribution. We have elected to study the loglogistic distribution because; (i) it has a similar shape to the Weibull distribution and (ii) we wanted to determine whether the left-truncated version is a better model for the financial data that Kizilersü et al. [33] described with the left-truncated Weibull distribution.

4.1 Complete Distribution

The pdf, $f(x)$, and cdf, $F(x)$, of the complete loglogistic distribution are defined as,

$$\boldsymbol{f(x|\phi,\rho) = \frac{\rho}{\phi^\rho}\frac{x^{\rho-1}}{\left[1+\left(\frac{x}{\phi}\right)^\rho\right]^2}} \tag{4.1}$$

$$\boldsymbol{F(x|\phi,\rho) = \frac{1}{1+\left(\frac{x}{\phi}\right)^{-\rho}}} \quad . \tag{4.2}$$

The support, scale parameter, ϕ, and shape parameter, ρ, are restricted to,

$$\begin{array}{ll} \text{Support} & 0 < x < \infty \\ \text{Scale parameter} & 0 < \phi < \infty \\ \text{Shape parameter} & 0 < \rho < \infty \ . \end{array}$$

4.2 Truncated Distribution

The truncated pdf, $f_T(x)$, and corresponding λ are defined as,

$$f_T(x) = \lambda f(x) \qquad \text{(1.32 revisited)}$$

$$\frac{1}{\lambda} = \int_{\tau_l}^{\tau_r} f(x)dx \ . \tag{1.33 revisited}$$

Combining Eq. (4.1) and Eq. (1.33) yields,

$$\frac{1}{\lambda_{Ll}} = \int_{\tau_l}^{\tau_r} \frac{\rho}{\phi^\rho} \frac{x^{\rho-1}}{\left[1+\left(\frac{x}{\phi}\right)^\rho\right]^2} dx \ . \tag{4.3}$$

To aid the evaluation of Eq. (4.3), u is defined as,

$$u \equiv \left(\frac{x}{\phi}\right)^\rho \quad \Rightarrow \quad x = \phi u^{\frac{1}{\rho}} \quad \Rightarrow \quad dx = \frac{\phi u^{\left(\frac{1}{\rho}-1\right)}}{\rho} du \ . \tag{4.4}$$

Substitution of u into Eq. (4.3) reduces the problem to,

$$\begin{aligned}
\Rightarrow \frac{1}{\lambda_{Ll}} &= \frac{\rho}{\phi^\rho} \int_{\left(\frac{\tau_l}{\phi}\right)^\rho}^{\left(\frac{\tau_r}{\phi}\right)^\rho} \phi^{\rho-1} \frac{u^{\frac{1}{\rho}(\rho-1)}}{(1+u)^2} \frac{\phi}{\rho} u^{\left(\frac{1}{\rho}-1\right)} du \\
&= \int_{\left(\frac{\tau_l}{\phi}\right)^\rho}^{\left(\frac{\tau_r}{\phi}\right)^\rho} \frac{1}{(1+u)^2} du \\
&= \left.\frac{-1}{1+u}\right|_{\left(\frac{\tau_l}{\phi}\right)^\rho}^{\left(\frac{\tau_r}{\phi}\right)^\rho} \\
\Rightarrow \frac{1}{\lambda} &= \frac{1}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} - \frac{1}{1+\left(\frac{\tau_r}{\phi}\right)^\rho} \ .
\end{aligned} \tag{4.5}$$

This study is only concerned with left truncation, hence, the right-truncation limit has been removed, i.e. $\tau_r = \infty$. Therefore Eq. (4.5) reduces to,

$$\begin{aligned}
\frac{1}{\lambda} &= \frac{1}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} \\
\Rightarrow \boldsymbol{\lambda} &= \mathbf{1} + \left(\frac{\boldsymbol{\tau_l}}{\boldsymbol{\phi}}\right)^{\boldsymbol{\rho}} \ .
\end{aligned} \tag{4.6}$$

The left-truncated loglogistic pdf can be expressed as,

$$\boxed{\boldsymbol{f_T(x|\alpha,\beta) = \left[1+\left(\frac{\tau_l}{\phi}\right)^\rho\right] \frac{\rho}{\phi^\rho} \frac{x^{\rho-1}}{\left[1+\left(\frac{x}{\phi}\right)^\rho\right]^2}} \ .} \tag{4.7}$$

The left-truncated cdf, $F_T(x)$, is defined as,

$$F_T(x) = \int_{\tau_l}^{x} f_T(t)dt = \lambda \int_{\tau_l}^{x} f(t)dt \ . \tag{1.35 revisited}$$

Substituting Eq. (4.1) into Eq. (1.35) produces

$$\begin{aligned}
F_T(x|\alpha,\beta) & \left[1+\left(\frac{\tau_l}{\phi}\right)^\rho\right] \int_{\tau_l}^{x} \frac{\rho}{\phi^\rho} \frac{x^{\rho-1}}{\left[1+\left(\frac{x}{\phi}\right)^\rho\right]^2} dt \text{ taking the result from Eq. (4.5)} \\
&= \left(1+\left(\frac{\tau_l}{\phi}\right)^\rho\right) \left[\frac{1}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} - \frac{1}{1+\left(\frac{x}{\phi}\right)^\rho}\right] \ .
\end{aligned} \tag{4.8}$$

Hence, the left-truncated loglogistic cdf can be expressed as,

$$\boxed{\boldsymbol{F_T(x|\alpha,\beta) = \frac{\left(\frac{x}{\phi}\right)^{\rho} - \left(\frac{\tau_l}{\phi}\right)^{\rho}}{1+\left(\frac{x}{\phi}\right)^{\rho}}}} \ . \tag{4.9}$$

The truncation percentage, p, is defined as,

$$p = F(\tau_L) \ . \qquad \text{(1.36 revisited)}$$

Employing Eq. (4.2) in Eq. (1.36) gives,

$$\boldsymbol{p = p(\tau_l, \phi, \rho) = \frac{1}{1+\left(\frac{\tau_l}{\phi}\right)^{-\rho}}} \ . \tag{4.10}$$

Manipulation of Eq. (4.10) allows one to define τ_l in terms of p, ϕ and ρ,

$$\boldsymbol{\tau_l = \tau_l(p, \phi, \rho) = \phi \left(\frac{1}{p} - 1\right)^{-\frac{1}{\rho}}} \ . \tag{4.11}$$

4.3 Generation of Random Numbers from the Loglogistic Distribution

Recall that the general random number generation formula is defined by

$$F^{-1}(u_i) = x_i \ , \qquad \text{(1.26 revisited)}$$

and that the array of uniform random variates, $\boldsymbol{u}$, is defined as

$$\boldsymbol{u} = (u_1, u_2, ..., u_n) \quad \text{where } \ u_i \in (0,1) \ \forall \, i = 1,2,...,n \ . \qquad \text{(1.27 revisited)}$$

The random generation formula for the left-truncated loglogistic distribution can be obtained by determining the inverse of the cumulative distribution function of the left-truncated loglogistic distribution, Eq. (4.9),

$$\begin{aligned} F_T(x|\phi^0, \rho^0) = u_i &= \frac{\left(\frac{x_i}{\phi^0}\right)^{\rho^0} - \left(\frac{\tau_l}{\phi^0}\right)^{\rho^0}}{1+\left(\frac{x_i}{\phi^0}\right)^{\rho^0}} \\ \Rightarrow \left(\frac{x_i}{\phi^0}\right)^{\rho^0} &= \frac{u_i + \left(\frac{\tau_l}{\phi^0}\right)^{\rho^0}}{1-u_i} \\ \Rightarrow x_i = \phi^0 &\left(\frac{1+\left(\frac{\tau_l}{\phi^0}\right)^{\rho^0}}{1-u_i} - 1\right)^{\frac{1}{\rho^0}} \ . \end{aligned} \tag{4.12}$$

Eq. (4.12) can be simplified by defining a quantity, η^0,

$$\boldsymbol{\eta^0 \equiv \left[\frac{\tau_l}{\phi^0}\right]^{\rho^0}} \ . \tag{4.13}$$

Combining Eq. (4.13) with Eq. (4.10) reveals that η^0 can be expressed as a function of the truncation percentage, p,

$$p = \frac{1}{1 + \left(\frac{\tau_l}{\phi}\right)^{-\rho}}$$

$$p = \frac{1}{1 + \frac{1}{\eta^0}} = \frac{\eta^0}{1 + \eta^0}$$

$$\Rightarrow \boldsymbol{\eta^0 = \eta^0(p) = \frac{p}{1-p}} \;. \tag{4.14}$$

Therefore, random numbers from the left-truncated loglogistic distribution can be generated from uniform random variates through,

$$\boxed{\boldsymbol{x_i = \phi^0 \left(\frac{1+\eta^0}{1-u_i} - 1\right)^{\frac{1}{\rho^0}}} \;.} \tag{4.15}$$

Notes

- If one is dealing with the complete (untruncated) loglogistic distribution, random data can be generated through,

$$x_i = \phi^0 \left(\frac{1}{u_i} - 1\right)^{-\frac{1}{\rho^0}} \;. \tag{4.16}$$

  by substituting,

$$\begin{aligned} \tau_l &= 0 \;, \\ \Rightarrow p &= 0 \;, \\ \Rightarrow \eta &= 0 \;, \end{aligned} \tag{4.17}$$

  into Eq. (4.16).

- The definition of η^0 in Eq. (4.13) allows one to specify the left-truncation point, τ_l, in terms of η^0,

$$\tau_l = \phi^0 \left(\eta^0\right)^{\frac{1}{\rho^0}} \;. \tag{4.18}$$

4.4 Estimation of Parameters

In section 3.4 we defined the terms *point estimator* and *maximum likelihood estimator*, hence, we will refrain from repeating that discussion here. Suffice to say that we will employ the maximum likelihood estimator to produce our parameter estimates, $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$. These estimates maximise the likelihood function, $L(\boldsymbol{\theta}|\boldsymbol{x})$, for a set of observations, $\boldsymbol{x}$. Table 1.2 summarises the four different cases of parameter estimation studied in this work.

Case	Scale Parameter	Shape Parameter
I	known *a priori*	known *a priori*
II	acquired from data	acquired from data
IIIa	acquired from data	known *a priori*
IIIb	known *a priori*	acquired from data

Table 1.2: Cases of parameter estimation

For the left-truncated loglogistic distribution $f_T(x)$ is given by Eq. (4.7), which reduces Eq. (1.17) to,

$$L(\boldsymbol{\theta}|\boldsymbol{x}) = \prod_{i=1}^{n} \left[1+\left(\frac{\tau_l}{\phi}\right)^{\rho}\right] \frac{\rho}{\phi^{\rho}} \frac{x_i^{\rho-1}}{\left[1+\left(\frac{x_i}{\phi}\right)^{\rho}\right]^2}$$

$$\Rightarrow L(\boldsymbol{\theta}|\boldsymbol{x}) = \left[1+\left(\frac{\tau_l}{\phi}\right)^{\rho}\right]^n \left(\frac{\rho}{\phi^{\rho}}\right)^n \prod_{i=1}^{n} \frac{x_i^{\rho-1}}{\left[1+\left(\frac{x_i}{\phi}\right)^{\rho}\right]^2}. \tag{4.19}$$

The loglikelihood function, l, is defined by,

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = \log(L(\boldsymbol{\theta}|\boldsymbol{x})) = \sum_{i=1}^{n} \log(f_T(x_i|\boldsymbol{\theta})) \ , \qquad \text{(1.19 revisited)}$$

which reduces to,

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = n\log\left(1+\left(\frac{\tau_l}{\phi}\right)^{\rho}\right)+n\log(\rho)-n\rho\log(\phi)+\sum_{i=1}^{n}(\rho-1)\log(x_i)-2\log\left(1+\left(\frac{x_i}{\phi}\right)^{\rho}\right) , \tag{4.20}$$

in the case of the left-truncated loglogistic distribution. The most general form of the MLE equations is

$$\frac{\partial}{\partial\theta_j} l(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{\partial}{\partial\theta_j} \sum_{i=1}^{n} \log\left(f(x_i|\boldsymbol{\theta})\right) = 0 \quad \forall \quad j=(1,2,...,m) \ , \qquad \text{(1.21 revisited)}$$

where m is the number of parameters in $f_T(x|\boldsymbol{\theta})$.

4.4.1 Case IIIa

In this case, the scale parameter, ϕ, must be estimated from the data ($\phi = \hat{\phi}$) and the shape parameter, ρ, is known ($\rho = \rho^0$). For the left-truncated loglogistic distribution, the corresponding MLE equation is achieved by substituting $\theta_1 = \phi$ into Eq. (1.21) to yield,

$$\begin{aligned} 0 &= \frac{dl}{d\phi} \ , \\ &= -n\frac{\rho}{\phi}\frac{\left(\frac{\tau_l}{\phi}\right)^{\rho}}{1+\left(\frac{\tau_l}{\phi}\right)^{\rho}} - n\frac{\rho}{\phi} + 2\frac{\rho}{\phi}\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\phi}\right)^{\rho}}{1+\left(\frac{x_i}{\phi}\right)^{\rho}} \ . \end{aligned} \tag{4.21}$$

Multiplying both side of Eq. (4.21) by $\frac{\phi}{n\rho}$ and pulling the first term to the left hand side we get,

$$\boxed{\frac{\left(\frac{\tau_l}{\phi}\right)^\rho}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} = \frac{2}{n}\sum_{i=1}^{n}\frac{\left(\frac{x_i}{\phi}\right)^\rho}{1+\left(\frac{x_i}{\phi}\right)^\rho} - 1.} \tag{4.22}$$

4.4.2 Case IIIb

In this case the shape parameter, ρ, must be estimated from the data ($\rho = \hat{\rho}$) and the scale parameter, ϕ, is known ($\phi = \phi^0$). For the left-truncated loglogistic distribution, the corresponding MLE equation is achieved by substituting $\theta_2 = \rho$ into Eq. (1.21) to yield,

$$\begin{aligned} 0 &= \frac{dl}{d\rho}\,, \\ &= \frac{n\log(\frac{\tau_l}{\phi})\left(\frac{\tau_l}{\phi}\right)^\rho}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} + \frac{n}{\rho} - n\log(\phi) + \sum_{i=1}^{n}\left[\log(x_i) - 2\frac{\log(\frac{x_i}{\phi})\left(\frac{x_i}{\phi}\right)^\rho}{1+\left(\frac{x_i}{\phi}\right)^\rho}\right] \;. \end{aligned} \tag{4.23}$$

Incorporating $n\log(\phi)$ into the first term of the sum, and multiplying both sides of Eq. (4.23) by $\frac{\rho}{n}$ we get,

$$0 = \frac{\rho\log(\frac{\tau_l}{\phi})\left(\frac{\tau_l}{\phi}\right)^\rho}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} + 1 + \frac{1}{n}\sum_{i=1}^{n}\left[\rho\log\left(\frac{x_i}{\phi}\right) - 2\frac{\rho\log\left(\frac{x_i}{\phi}\right)\left(\frac{x_i}{\phi}\right)^\rho}{1+\left(\frac{x_i}{\phi}\right)^\rho}\right] \,,$$

$$\Rightarrow 1 + \rho\log\left(\frac{\tau_l}{\phi}\right)\frac{\left(\frac{\tau_l}{\phi}\right)^\rho}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} = \frac{\rho}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\phi}\right)\frac{\left(\frac{x}{\phi}\right)^\rho - 1}{\left(\frac{x_i}{\phi}\right)^\rho + 1} \,.$$

$$\boxed{\Rightarrow 1 + \rho\log\left(\frac{\tau_l}{\phi}\right)\frac{\left(\frac{\tau_l}{\phi}\right)^\rho}{1+\left(\frac{\tau_l}{\phi}\right)^\rho} = \frac{\rho}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\phi}\right)\tanh\left[\frac{\rho}{2}\log\left(\frac{x_i}{\phi}\right)\right] \,.} \tag{4.24}$$

4.4.3 Case II

In this case both the shape and scale parameters must be estimated from the data. This is achieved by finding simultaneous solutions to Eq. (4.22) and (4.24), we are not able to disentangle the equations for the scale and shape parameters as we did for the case II Weibull distribution.

Eq. (4.22) and (4.24) are satisfied by the parameters that maximise the likelihood function, however, the parameters which solve Eq. (4.22) and (4.24) do not necessarily maximise likelihood. A more in-depth discussion of why this is the case is available in section 3.4. The key result is that we need to ensure that the Hessian of the loglikelihood function is negative definite at the parameters which solve Eq. (4.22) and (4.24). Fortunately, Kreer et al. [42] showed that there is always a set of parameters which maximise the loglikelihood function, and solve Eq. (4.22) and (4.24) simultaneously. If it is not possible to show that the relevant MLE equations have a unique solution, we must estimate the parameters through an algorithm that numerically maximises the loglikelihood function. Again, more detail is given in section 3.4. Each parameter case is different, hence we must analyse them separately:

- **Case IIIa:** Eq. (4.22) does not necessarily have a unique solution. Thus, to determine $\hat{\phi}$ we numerically found the ϕ value which maximised the loglikelihood function, Eq. (4.20).

- **Case IIIb:** Eq. (4.24) does not necessarily have a unique solution, hence, we used a maximisation algorithm to find the ρ value which maximised the loglikelihood function, Eq. (4.20). This ρ was our parameter estimate $\hat{\rho}$.

- **Case II:** Kreer et al. [42] have showed that in case II, the left-truncated loglogistic distribution MLE equations have a unique solution if and only if the samples, $\boldsymbol{x}$, satisfy,

$$\frac{1}{\tau_l^\beta} < \frac{2}{n}\sum_{i=1}^{n}\frac{1}{x_i^\beta} \ ,$$
$$\text{where} \quad \frac{1}{\beta} = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right) \ . \tag{4.25}$$

Therefore, if Eq. (4.25) is satisfied, the parameters which simultaneously solve Eq. (4.22) and (4.24) also maximise likelihood. In our Monte Carlo simulations, if samples did not satisfy Eq. (4.25) they were disregarded and replaced. For the samples which did satisfy this inequality, we employed an algorithm that maximised the loglikelihood function, regardless of the fact that we knew that the solutions of Eq. (4.22) and (4.24) maximised likelihood. This was because we found that results between the two methods were very similar, however, the maximisation algorithm was less computationally expensive.

4.4.4 Summary of Applicable MLE Equations

Case	Scale Parameter Equation	Shape Parameter Equation
I	N/A	N/A
II	Eq. (4.22)	Eq. (4.24)
IIIa	Eq. (4.22)	N/A
IIIb	N/A	Eq. (4.24)

Table 4.1: Summary of applicable MLE equations

Recall that we chose to use the maximum likelihood estimator to estimate our parameters because it produces consistent and efficient results. These features are a consequence of Lehmann and Casella's work [16]. In section 3.5, we outlined the reasoning behind why we believe the conditions (A0)–(A2) and assumptions (A)–(D) (necessary for Theorem 1.5, Theorem 5.1 in [16]) are valid for the left-truncated Weibull distribution. The reasoning given in that section would remain largely unchanged for the left-truncated loglogistic distribution, hence we have not repeated that argument.

4.5 Parameter Independent Pivotal Functions and Critical Values

As discussed in section 2.6, parameter independent pivotal functions are important because they produce parameter independent critical values. It is the objective of this section to show that the left-truncated loglogistic distribution has parameter independent critical values. The methodology we will employ is employed in section 2.6.

Expressing the x dependence of the left-truncated loglogistic cdf, Eq. (4.9), with the random generation formula, Eq. (4.15), allows us to define a function, $\widetilde{F_T}(u)$ where $x = F_T^{-1}(u)$, such that $\widetilde{F_T}(u) = F_T(x)$, for all x,

$$\begin{aligned} F_T &= \frac{\left(\frac{x_i}{\hat{\phi}}\right)^{\hat{\rho}} - \left(\frac{\tau_l}{\hat{\phi}}\right)^{\hat{\rho}}}{1 + \left(\frac{x_i}{\hat{\phi}}\right)^{\hat{\rho}}} \\ \Rightarrow \widetilde{F_T}(u_i|\hat{\phi}, \hat{\rho}, \phi^0, \rho^0) &= \frac{\left(\frac{\phi_0}{\hat{\phi}}\right)^{\hat{\rho}} \left(\frac{1+\eta}{1-u_i} + 1\right)^{\frac{\hat{\rho}}{\rho_0}} - \left(\frac{\phi_0}{\hat{\phi}}\right)^{\hat{\rho}} \eta^{\frac{\hat{\rho}}{\rho_0}}}{1 + \left(\frac{\phi_0}{\hat{\phi}}\right)^{\hat{\rho}} \left(\frac{1+\eta}{1-u_i} + 1\right)^{\frac{\hat{\rho}}{\rho_0}}}. \end{aligned} \tag{4.26}$$

As discussed in section 2.6 the pivotal functions γ and ξ are introduced to simplify Eq. (4.26),

$$\boldsymbol{\gamma} = \left(\frac{\boldsymbol{\phi^0}}{\boldsymbol{\hat{\phi}}}\right)^{\boldsymbol{\hat{\rho}}} \tag{4.27}$$

$$\boldsymbol{\xi} = \frac{\boldsymbol{\hat{\rho}}}{\boldsymbol{\rho^0}} \ . \tag{4.28}$$

Substituting Eq. (4.27) and Eq. (4.28) into Eq. (4.26) reduces $\widetilde{F_T}(u)$ to

$$\widetilde{F_T}(u_i|\hat{\phi}, \hat{\rho}, \phi^0, \rho^0) = \widetilde{F_T}(u_i|\gamma, \xi, \eta) = \frac{\gamma \left(\frac{1+\eta}{1-u_i} + 1\right)^{\xi} - \gamma\eta^{\xi}}{1 + \gamma \left(\frac{1+\eta}{1-u_i} + 1\right)^{\xi}} \ . \tag{4.29}$$

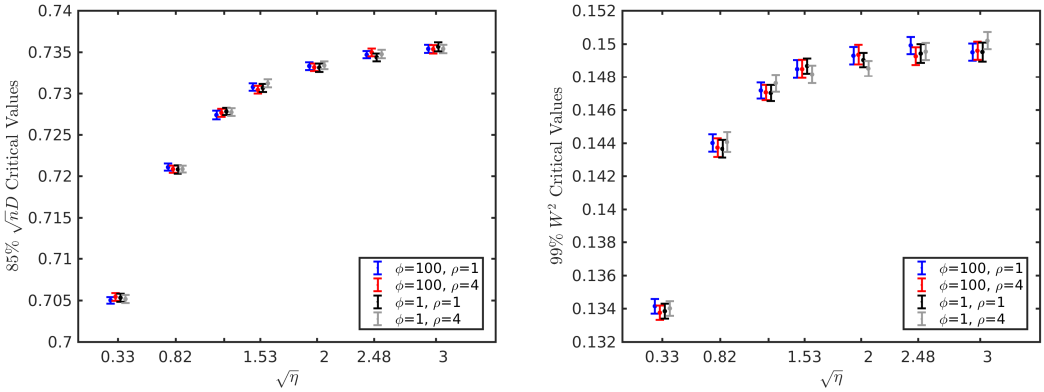
All parameter dependence in $\widetilde{F_T}(u)$ is completely contained within the pivotal functions $\gamma = \gamma(\phi^0, \hat{\phi}, \hat{\rho})$ and $\xi = \xi(\rho^0, \hat{\rho})$. In this section we will show that these pivotal functions are distributed independently of ϕ^0 and ρ^0, therefore, proving that the critical values are parameter independent. The approach we will use to achieve this is given in sections 2.6 and 3.6.

In section 8.1 we have numerically verified that critical values are parameter independent for all of the distributions we studied. Figures 4.1a and 4.1b display the Kolmogorov-Smirnov and Cramér-von Mises critical values at the 85% and 99% significance levels with samples of size $n = 10,000$ and $n = 30$ (respectively) drawn from the case II loglogistic distribution for a range of parameter values. This figure shows an example supporting the assertion that the critical values are parameter independent. A more detailed discussion is included in section 8.1.

4.5.1 Case IIIa

In this case, ϕ is to be estimated from the data and ρ is known *a priori*, therefore,

$$\begin{aligned} \hat{\rho} &= \rho^0 \\ \Rightarrow \xi &= 1 \\ \Rightarrow \widetilde{F_T}(u_i|\gamma, \eta) &= \frac{\gamma \left(\frac{1+\eta}{1-u_i} + 1\right) - \gamma\eta}{1 + \gamma \left(\frac{1+\eta}{1-u_i} + 1\right)} \ . \end{aligned} \tag{4.30}$$



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 85% sig. lvl. for $n = 10,000$ observations

(b) Cramér-von Mises (W^2) crit. vals. at 99% sig. lvl. for $n = 30$ observations

Figure 4.1: Case II Loglogistic critical values
Critical values from different parameter values are staggered on the $\sqrt{\eta}$ axis for clarity

Additionally, the remaining pivotal function γ is altered

$$\gamma = \left(\frac{\phi^0}{\hat{\phi}}\right)^{\rho^0} . \tag{4.31}$$

The random number generation formula Eq. (4.15), is inserted into Eq. (4.22), and reduced to the form,

$$\begin{aligned}
\frac{\left(\frac{\tau_l}{\hat{\phi}}\right)^{\rho^0}}{1+\left(\frac{\tau_l}{\hat{\phi}}\right)^{\rho^0}} &= \frac{2}{n}\sum_{i=1}^{n} \frac{\left(\frac{x_i}{\hat{\phi}}\right)^{\rho^0}}{1+\left(\frac{x_i}{\hat{\phi}}\right)^{\rho^0}} - 1\\
\frac{\left(\frac{\phi^0}{\hat{\phi}}\right)^{\rho^0}\eta}{1+\left(\frac{\phi^0}{\hat{\phi}}\right)^{\rho^0}\eta} &= \frac{2}{n}\sum_{i=1}^{n} \frac{\left(\frac{\phi^0}{\hat{\phi}}\right)^{\rho^0}\left(\frac{1+\eta}{1-u_i}+1\right)}{1+\left(\frac{\phi^0}{\hat{\phi}}\right)^{\rho^0}\left(\frac{1+\eta}{1-u_i}+1\right)} - 1\\
\frac{\gamma\eta}{1+\gamma\eta} &= \frac{2}{n}\sum_{i=1}^{n} \frac{\gamma\left(\frac{1+\eta}{1-u_i}+1\right)}{1+\gamma\left(\frac{1+\eta}{1-u_i}+1\right)} - 1\\
\frac{\gamma\eta}{1+\gamma\eta} &= \frac{1}{n}\sum_{i=1}^{n} \frac{\gamma\left(\frac{1+\eta}{1-u_i}+1\right)-1}{\gamma\left(\frac{1+\eta}{1-u_i}+1\right)+1} .
\end{aligned} \tag{4.32}$$

Kreer et al. [42] showed that there is always a set of parameters which maximise likelihood, and thus solve the MLE equations. Hence, we know that Eq. (4.32) has at least one solution. Therefore, γ can be expressed in terms of the other variables, i.e. $\gamma = \gamma(\eta, \boldsymbol{u})$. From this expression of γ, it is possible to remove the parameter dependence of $\widetilde{F_T}(u_i)$,

$$\Rightarrow \widetilde{F_T}(u_i|\gamma,\eta) = \widetilde{F_T}(u_i|\gamma(\boldsymbol{u}),\eta) = \widetilde{F_T}(u_i|\boldsymbol{u},\eta) . \tag{4.33}$$

Therefore, $\widetilde{F_T}(u_i)$, is distributed independently of ϕ^0 and ρ^0, hence, we have parameter independent critical values.

4.5.2 Case IIIb

In this case, ρ is to be estimated from the data and ϕ is known *a priori*, therefore,

$$\begin{aligned}
\hat{\phi} &= \phi^0 \\
\Rightarrow \gamma &= 1 \\
\Rightarrow \widetilde{F_T}(u_i|\xi,\eta) &= \frac{\left(\frac{1+\eta}{1-u_i}+1\right)^{\xi} - \eta^{\xi}}{1+\left(\frac{1+\eta}{1-u_i}+1\right)^{\xi}} \; .
\end{aligned} \tag{4.34}$$

The random number generation formula Eq. (4.15), is inserted into Eq. (4.24), and reduced to the form

$$\begin{aligned}
1+\rho^0 \log\left(\frac{\tau_l}{\phi^0}\right)\frac{\left(\frac{\tau_l}{\phi^0}\right)^{\hat{\rho}}}{1+\left(\frac{\tau_l}{\phi^0}\right)^{\hat{\rho}}} &= \frac{\rho^0}{n}\sum_{i=1}^{n}\left[\log\left(\frac{x_i}{\phi^0}\right)\frac{\left(\frac{x_i}{\phi^0}\right)^{\hat{\rho}}-1}{\left(\frac{x_i}{\phi^0}\right)^{\hat{\rho}}+1}\right] \\
1+\log\left(\eta^{\frac{\hat{\rho}}{\rho^0}}\right)\frac{\eta^{\frac{\hat{\rho}}{\rho^0}}}{1+\eta^{\frac{\hat{\rho}}{\rho^0}}} &= \frac{1}{n}\sum_{i=1}^{n}\left[\log\left(\left(\frac{1+\eta}{1-u_i}+1\right)^{\frac{\hat{\rho}}{\rho^0}}\right)\frac{\left(\frac{1+\eta}{1-u_i}+1\right)^{\frac{\hat{\rho}}{\rho^0}}-1}{\left(\frac{1+\eta}{1-u_i}+1\right)^{\frac{\hat{\rho}}{\rho^0}}+1}\right] \\
1+\xi\log\left(\eta\right)\frac{\eta^{\xi}}{1+\eta^{\xi}} &= \frac{\xi}{n}\sum_{i=1}^{n}\log\left(\frac{1+\eta}{1-u_i}+1\right)\frac{\left(\frac{1+\eta}{1-u_i}+1\right)^{\xi}-1}{\left(\frac{1+\eta}{1-u_i}+1\right)^{\xi}+1} \\
1+\xi\log\left(\eta\right)\frac{\eta^{\xi}}{1+\eta^{\xi}} &= \frac{\xi}{n}\sum_{i=1}^{n}\log\left(\frac{1+\eta}{1-u_i}+1\right)\tanh\left(\frac{\xi}{2}\log\left(\frac{1+\eta}{1-u_i}+1\right)\right) .
\end{aligned} \tag{4.35}$$

Kreer et al. [42] showed that there is always a set of parameters which maximise likelihood, and thus solve the MLE equations. Hence, we know that Eq. (4.35) has at least one solution. Therefore, ξ can be expressed in terms of the other variables, i.e. $\xi = \xi(\eta, \boldsymbol{u})$. From this expression of ξ, it is possible to remove the parameter dependence of $\widetilde{F_T}(u_i)$,

$$\Rightarrow \widetilde{F_T}(u_i|\xi,\eta) = \widetilde{F_T}(u_i|\xi(\boldsymbol{u}),\eta) = \widetilde{F_T}(u_i|\boldsymbol{u},\eta) \; . \tag{4.36}$$

Therefore, $\widetilde{F_T}(u_i)$, is distributed independently of ϕ^0 and ρ^0, hence, we have parameter independent critical values.

4.5.3 Case II

In this case the objective is to show that parameter independent solutions for γ and ξ exist. In Eq. (4.32) and (4.35) we have shown that the parameter dependence of Eq. (4.22) and (4.24) can be contained within the pivotal functions. As discussed in section 4.4, Kreer et al. [42] showed that the MLE equations of the case II, left-truncated loglogistic distribution have a unique solution if

$$\begin{aligned}
\frac{1}{\tau_l^{\beta}} &< \frac{2}{n}\sum_{i=1}^{n}\frac{1}{x_i^{\beta}} \; , \\
\text{where} \quad \frac{1}{\beta} &= \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right) \; .
\end{aligned} \tag{4.25 revisited}$$

Thus, if Eq. (4.25) is satisfied, we know that there is a unique, simultaneous solution to Eq. (4.22) and (4.24). As the samples that did not satisfy Eq. (4.25) were rejected, we have a unique, simultaneous solution to Eq. (4.22) and (4.24) for all the samples that contributed to the critical values. As a result, we can conclude that there is an expression for both γ and ξ in terms of the other variables, i.e.$\gamma = \gamma(\eta, \boldsymbol{u})$ and $\xi = \xi(\eta, \boldsymbol{u})$. Hence, we can remove the parameter dependence in $\widetilde{F_T}(u_i)$ (Eq. (4.29)),

$$\begin{aligned} \gamma = \gamma(\boldsymbol{u}, \eta) \quad \text{and} \quad \xi = \xi(\boldsymbol{u}, \eta) \ , \\ \Rightarrow \widetilde{F_T}(u_i|\gamma, \xi) = \widetilde{F_T}(u_i|\gamma(\boldsymbol{u}, \eta), \xi(\boldsymbol{u}, \eta)) = \widetilde{F_T}(u_i|\boldsymbol{u}, \eta). \end{aligned} \tag{4.37}$$

Hence, $\widetilde{F_T}(u_i)$, is independent of ϕ^0 and ρ^0, thus, we have parameter independent critical values.

Chapter 5

Lognormal Distribution

The lognormal distribution is based upon the extremely common normal distribution, the pdf of the normal distribution is given by a univariate Gaussian. The normal distribution is informally called the 'bell curve' and has been used to model everything from the heights of students, to the rate of defects in manufacturing processes. The lognormal distribution is the probability distribution of a random variable whose logarithm is normally distributed. The lognormal distribution has two parameters, the mean, μ, and standard deviation, σ; for simplicity, in this thesis we shall refer to, μ, as the scale parameter and, σ, as the shape parameter. The distribution is defined on the support $\mathbb{R}^+ \cup 0$; correspondingly, the normal distribution has the support, $\mathbb{R}$.

In hydrology, the lognormal distribution is often used to model extreme values, such as maximum one-day rainfall [73]. Additionally, the lognormal distribution describes the file sizes of publicly available audio and video files over five orders of magnitude [74]. In this work, we focus upon the application of the lognormal distribution to life-time analysis, as this is directly comparable to the Weibull distribution [75]. To our knowledge, there has been no work analysing the critical values of the left-truncated lognormal distribution. We have elected to study the lognormal distribution because; (i) it has a similar shape to the Weibull distribution and (ii) we wanted to determine whether the left-truncated version is a better model for the financial data that Kizilersü et al. [33] described with the left-truncated Weibull distribution.

5.1 Complete Distribution

The pdf, $f(x)$, and cdf, $F(x)$, of the complete lognormal distribution are defined as,

$$\boldsymbol{f(x|\mu,\sigma) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right]} \tag{5.1}$$

$$\boldsymbol{F(x|\alpha,\beta)} = \boldsymbol{\frac{1}{2}\left[1+\mathrm{erf}\left(\frac{\log(x)-\mu}{\sigma\sqrt{2}}\right)\right]}\ . \tag{5.2}$$

The support, scale parameter, μ, and shape parameter, σ, are restricted to,

Support	$0 < x < \infty$
Scale parameter	$-\infty < \mu < \infty$
Shape parameter	$0 < \sigma < \infty$.

In the truncated cdf, Eq. (5.2), we make use of the error function, $\mathrm{erf}(x)$, which is defined as,

$$\mathrm{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-t^2} dt\ . \tag{5.3}$$

Throughout this chapter, we will also use the complementary error function, erfc(x),

$$\text{erfc}(x) = 1 - \text{erf}(x) \ , \tag{5.4}$$

and scaled complementary error function, erfcx(x),

$$\text{erfcx}(x) = e^{x^2}\text{erfc}(x) \ . \tag{5.5}$$

5.2 Truncated Distribution

The truncated pdf, $f_T(x)$, and corresponding λ are defined as,

$$f_T(x) = \lambda f(x) \qquad \text{(1.32 revisited)}$$

$$\frac{1}{\lambda} = \int_{\tau_l}^{\tau_r} f(x)dx \ . \qquad \text{(1.33 revisited)}$$

Combining Eq. (5.1) and Eq. (1.33) yields,

$$\frac{1}{\lambda} = \int_{\tau_l}^{\tau_r} \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[-\frac{\left(\log(x)-\mu\right)^2}{2\sigma^2}\right] dx \ . \tag{5.6}$$

To aid the evaluation of Eq. (5.6), u is defined as,

$$u \equiv \log(x) - \mu \quad \Rightarrow \quad x = \exp(u+\mu)\,\alpha \quad \Rightarrow \quad dx = \exp(u+\mu)\,du \ . \tag{5.7}$$

Substitution of u into Eq. (5.6) reduces the problem to,

$$\begin{aligned}
\frac{1}{\lambda} &= \frac{1}{\sigma\sqrt{2\pi}} \int_{\log(\tau_l)-\mu}^{\log(\tau_r)-\mu} \frac{1}{\exp(u+\mu)} \exp\left(\frac{-u^2}{2\sigma^2}\right) \exp(u+\mu)du \\
&= \frac{1}{\sigma\sqrt{2\pi}} \int_{\log(\tau_l)-\mu}^{\log(\tau_r)-\mu} \exp\left(\frac{-u^2}{2\sigma^2}\right)du \\
&= \frac{1}{2}\text{erf}\left[\frac{u}{\sigma\sqrt{2}}\right]\Bigg|_{\log(\tau_l)-\mu}^{\log(\tau_r)-\mu} \\
&= \frac{1}{2}\left(\text{erf}\left[\frac{\log(\tau_r)-\mu}{\sigma\sqrt{2}}\right] - \text{erf}\left[\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right]\right) \ .
\end{aligned} \tag{5.8}$$

This study is only concerned with left truncation, hence, the right-truncation limit has been removed, i.e. $\tau_r = \infty$. Therefore Eq. (5.8) reduces to,

$$\begin{aligned}
\frac{1}{\lambda} &= \frac{1}{2}\left(\text{erfc}\left[\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right]\right) \\
\Rightarrow \boldsymbol{\lambda} &= \frac{\mathbf{2}}{\textbf{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} \ .
\end{aligned} \tag{5.9}$$

The left-truncated lognormal pdf can be expressed as,

$$\boxed{\boldsymbol{f_T(x|\mu,\sigma) = \frac{2}{\sigma x\sqrt{2\pi}} \frac{\exp\left[-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right]}{\textbf{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)}}} \ . \tag{5.10}$$

The left-truncated lognormal cdf, $F_T(x)$, is defined as,

$$F_T(x) = \int_{\tau_l}^{x} f_T(t)dt = \lambda \int_{\tau_l}^{x} f(t)dt \ . \qquad \text{(1.35 revisited)}$$

Substituting Eq. (5.1) into Eq. (1.35) produces,

$$\begin{aligned} F_T(x|\alpha,\beta) &= \frac{2}{\mathrm{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} \int_{\tau_l}^{x} \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{\left(\log(t)-\mu\right)^2}{2\sigma^2}\right] dt \ , \text{ which yields (Eq. (5.8))} \\ &= \frac{2}{\mathrm{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} \frac{1}{2} \left(\mathrm{erf}\left[\frac{\log(x)-\mu}{\sigma\sqrt{2}}\right] - \mathrm{erf}\left[\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right]\right) \\ &= \frac{1}{\mathrm{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} \left(\mathrm{erfc}\left[\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right] - \mathrm{erfc}\left[\frac{\log(x)-\mu}{\sigma\sqrt{2}}\right]\right) \ . \end{aligned} \qquad (5.11)$$

Hence, the left-truncated lognormal cdf can be expressed as

$$\boxed{\boldsymbol{F_T(x|\alpha,\beta) = 1 - \frac{\mathrm{erfc}\left(\frac{\log(x)-\mu}{\sigma\sqrt{2}}\right)}{\mathrm{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)}}} \ . \qquad (5.12)$$

The truncation percentage, p, is defined as

$$p \equiv F(\tau_l) \ . \qquad \text{(1.36 revisited)}$$

Employing Eq. (5.2) in Eq. (1.36) gives

$$\boldsymbol{p = p(\tau_l,\mu,\sigma) = \frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)\right]} \ . \qquad (5.13)$$

Manipulation of Eq. (3.10) allows one to define τ_l in terms of p, μ and σ,

$$\boldsymbol{\tau_l = \tau_l(p,\mu,\sigma) = \exp\left[\mu + \sqrt{2}\sigma\mathrm{erf}^{-1}\left(2p-1\right)\right]} \ . \qquad (5.14)$$

5.3 Generation of Random Numbers from the Lognormal Distribution

Recall that the general random number generation formula is defined by,

$$F^{-1}(u_i) = x_i \ , \qquad \text{(1.26 revisited)}$$

and that the array of uniform random variates, $\boldsymbol{u}$, is defined as

$$\boldsymbol{u} = (u_1, u_2, ..., u_n) \quad \text{where} \quad u_i \in (0,1) \quad \forall \quad i = 1,2,...,n \ . \qquad \text{(1.27 revisited)}$$

The random number generation formula for the left-truncated lognormal distribution can be obtained by determining the inverse of the cumulative distribution function of the left-truncated lognormal distribution, Eq. (5.12),

$$\begin{aligned} F_T(x_i|\mu^0,\sigma^0) = u_i = 1 - \frac{\mathrm{erfc}\left(\frac{\log(x_i)-\mu^0}{\sigma^0\sqrt{2}}\right)}{\mathrm{erfc}\left(\frac{\log(\tau_l)-\mu^0}{\sigma^0\sqrt{2}}\right)} \\ \Rightarrow \mathrm{erfc}\left(\frac{\log(\tau_l)-\mu^0}{\sigma^0\sqrt{2}}\right)(1-u_i) = \mathrm{erfc}\left(\frac{\log(x_i)-\mu^0}{\sigma^0\sqrt{2}}\right) \\ \Rightarrow x_i = \exp\left[\mu^0 + \sigma^0\sqrt{2}\mathrm{erfc}^{-1}\left\{\mathrm{erfc}\left(\frac{\log(\tau_l)-\mu^0}{\sigma^0\sqrt{2}}\right)(1-u_i)\right\}\right] . \end{aligned} \qquad (5.15)$$

Eq. (5.15) can be simplified by defining a quantity, η^0,

$$\boldsymbol{\eta^0 = \frac{1}{\text{erfc}\left(\frac{\log(\tau_l)-\mu^0}{\sigma^0\sqrt{2}}\right)}} \quad . \tag{5.16}$$

Combining Eq. (5.16) with Eq. (5.13) reveals that η^0 can be expressed as a function of the truncation percentage, p,

$$\begin{aligned} p &= \frac{1}{2}\left[1+\text{erf}\left(\frac{\log(\tau_l)-\mu^0}{\sigma^0\sqrt{2}}\right)\right] \\ &= \frac{1}{2}\left[2-\text{erfc}\left(\frac{\log(\tau_l)-\mu^0}{\sigma^0\sqrt{2}}\right)\right] \\ &= 1-\frac{1}{2\eta^0} \\ \Rightarrow \boldsymbol{\eta^0} &\boldsymbol{= \eta^0(p) = \frac{1}{2\,(1-p)}} \ . \end{aligned} \tag{5.17}$$

Therefore, random numbers from the left-truncated lognormal distribution can be generated from uniform random variates through,

$$\boxed{\boldsymbol{x_i = \exp\left[\mu^0+\sigma^0\sqrt{2}\text{erfc}^{-1}\left\{\frac{1-u_i}{\eta^0}\right\}\right]} \ .} \tag{5.18}$$

Notes

- If one is dealing with the complete (untruncated) lognormal distribution, random data can be generated through,

$$x_i = \exp\left[\mu^0-\sigma^0\sqrt{2}\text{erfc}^{-1}\left\{2u_i\right\}\right] , \tag{5.19}$$

  by substituting,

$$\begin{aligned} \tau_l &= 0 \ , \\ \Rightarrow p &= 0 \ , \\ \Rightarrow \eta &= \frac{1}{2} \ , \end{aligned} \tag{5.20}$$

  into Eq. (5.18).

- The definition of η^0 in Eq. (5.16) allows one to specify the left-truncation point, τ_l, in terms of η^0,

$$\tau_l = \exp\left[\mu^0+\sigma^0\sqrt{2}\text{erfc}^{-1}\left(\frac{1}{\eta^0}\right)\right] \ . \tag{5.21}$$

The alternate definition of x_i in Eq. (5.19) is possible because,

$$\begin{aligned} \text{erfc}^{-1}(2-x) &= g \\ \Rightarrow 2-x &= \text{erfc}(g) \\ \Rightarrow 2-x &= 1-\text{erf}(g) \\ \Rightarrow 1-x &= \text{erf}(-g) \\ \Rightarrow 1-\text{erf}(-g) &= x \\ \Rightarrow \text{erfc}(-g) &= x \\ \Rightarrow g &= -\text{erfc}^{-1}(x) \\ &\boxed{\Rightarrow \text{erfc}^{-1}(2-x) = -\text{erfc}^{-1}(x) \ .} \end{aligned} \tag{5.22}$$

5.4 Estimation of Parameters

In section 3.4 we defined the terms *point estimator* and *maximum likelihood estimator*, hence, we will refrain from repeating that discussion here. Suffice to say that we will employ the maximum likelihood estimator to produce our parameter estimates, $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$. These estimates maximise the likelihood function, $L(\boldsymbol{\theta}|\boldsymbol{x})$, for a set of observations, $\boldsymbol{x}$. Table 1.2 summarises the four different cases of parameter estimation studied in this work.

Case	Scale Parameter	Shape Parameter
I	known *a priori*	known *a priori*
II	acquired from data	acquired from data
IIIa	acquired from data	known *a priori*
IIIb	known *a priori*	acquired from data

Table 1.2: Cases of parameter estimation

For the left-truncated lognormal distribution, $f_T(x)$ is given by Eq. (5.10), which reduces Eq. (1.17) to,

$$L(\boldsymbol{\theta}|\boldsymbol{x}) = \prod_{i=1}^{n} \frac{2}{\sigma x_i \sqrt{2\pi}} \frac{\exp\left[-\frac{(\log(x_i)-\mu)^2}{2\sigma^2}\right]}{\operatorname{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} ,$$

$$\Rightarrow L(\boldsymbol{\theta}|\boldsymbol{x}) = \left(\frac{2}{\sigma\sqrt{2\pi}}\right)^n \left(\frac{1}{\operatorname{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)}\right)^n \prod_{i=1}^{n} \frac{1}{x_i} \exp\left[-\frac{(\log(x_i)-\mu)^2}{2\sigma^2}\right] . \tag{5.23}$$

The loglikelihood function, l, is defined by,

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = \log(L(\boldsymbol{\theta}|\boldsymbol{x})) = \sum_{i=1}^{n} \log(f(x_i|\boldsymbol{\theta})) , \qquad \text{(1.19 revisited)}$$

which reduces to,

$$\begin{aligned} l(\boldsymbol{\theta}|\boldsymbol{x}) = n\log(2) - n\log(\sigma) - \frac{n\log(2\pi)}{2} - n\log\left(\operatorname{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)\right) \\ - \sum_{i=1}^{n}\left(\log(x_i) + \frac{(\log(x_i)-\mu)^2}{2\sigma^2}\right) , \end{aligned} \tag{5.24}$$

in the case of the left-truncated lognormal distribution. The most general form of the MLE equations is,

$$\frac{\partial}{\partial\theta_j} l(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{\partial}{\partial\theta_j} \sum_{i=1}^{n} \log\left(f(x_i|\boldsymbol{\theta})\right) = 0 \quad \forall \quad j = (1, 2, ..., m) , \qquad \text{(1.21 revisited)}$$

where m is the number of parameters in $f_T(x|\boldsymbol{\theta})$.

5.4.1 Case IIIa

In this case, the scale parameter or mean, μ, must be estimated from the data ($\mu = \hat{\mu}$) and the shape parameter or standard deviation, σ, is known ($\sigma = \sigma^0$). For the left-truncated lognormal distribution, the corresponding MLE equation is achieved by substituting $\theta_1 = \mu$ into Eq. (1.21) to yield,

$$
\begin{aligned}
0 &= \frac{dl}{d\mu} \\
&= \frac{-n}{\sigma}\sqrt{\frac{2}{\pi}}\frac{\exp\left(-\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)^2\right)}{\operatorname{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} + \sum_{i=1}^{n}\frac{\log(x_i)-\mu}{(\sigma)^2} \ .
\end{aligned}
\tag{5.25}
$$

Multiplying both sides of Eq. (5.25) by $\frac{1}{n}$, and pulling the first term over to the left hand side and we get,

$$
\boxed{\boldsymbol{\frac{1}{\sigma}\sqrt{\frac{2}{\pi}}\frac{1}{\operatorname{erfcx}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} = \frac{1}{n(\sigma)^2}\sum_{i=1}^{n}\left(\log(x_i)-\mu\right)} \ .}
\tag{5.26}
$$

5.4.2 Case IIIb

In this case the shape parameter or standard deviation, σ, must be estimated from the data ($\sigma = \hat{\sigma}$) and the scale parameter or mean, μ, is known ($\mu = \mu^0$). For the left-truncated lognormal distribution, the corresponding MLE equation is achieved by substituting $\theta_2 = \sigma$ into Eq. (1.21) to yield,

$$
\begin{aligned}
0 &= \frac{dl}{d\sigma} \\
&= -\frac{n}{\sigma} + \frac{-n}{\sigma^2}\sqrt{\frac{2}{\pi}}\left(\log(\tau_L)-\mu\right)\frac{\exp\left(-\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)^2\right)}{\operatorname{erfc}\left(\frac{\log(\tau_l)-\mu}{\sigma\sqrt{2}}\right)} + \sum_{i=1}^{n}\frac{(\log(x_i)-\mu)^2}{\sigma^3} \ .
\end{aligned}
\tag{5.27}
$$

Multiplying both sides of Eq. (5.27) by $\frac{\sigma}{n}$, and pulling the second term over to the left hand side and we get,

$$
\boxed{\boldsymbol{\frac{1}{\sigma}\sqrt{\frac{2}{\pi}}\frac{\log(\tau_L)-\mu}{\operatorname{erfcx}\left(\frac{\log(\tau_L)-\mu}{\sigma\sqrt{2}}\right)} = \frac{1}{n}\frac{1}{\sigma^2}\sum_{i=1}^{n}\left[\log(x_i)-\mu\right]^2 - 1} \ .}
\tag{5.28}
$$

5.4.3 Case II

In 1994 Castillo [76] published a work which detailed a method for determining the parameters which maximise the loglikelihood function of the left-truncated normal distribution much more efficiently than a maximisation algorithm. This method begins by translating all of the data points, $\boldsymbol{z} = (z_1, z_2, ..., z_n)$, to the left by the left-truncation limit, τ_l',

$$
\widetilde{x}_i \equiv z_i - \tau_l' \quad \forall \quad i = 1, ..., n \ .
\tag{5.29}
$$

Castillo's method can only be employed if the observations satisfy,

$$
c = \frac{s}{t_1} < 1 \ ,
\tag{5.30}
$$

where,

$$
\begin{aligned}
\widetilde{t}_1 \equiv \frac{1}{n}\sum_{i=1}^{n}\widetilde{x}_i \ , \quad \widetilde{t}_2 \equiv -\frac{1}{n}\sum_{i=1}^{n}\widetilde{x}_i^{\,2} \ , \\
\text{and} \quad s \equiv -(\widetilde{t}_2 + \widetilde{t}_1^{\,2}) \ .
\end{aligned}
\tag{5.31}
$$

For samples which satisfy Eq. (5.30), the parameters of the left-truncated normal distribution can be estimated through,

$$\boxed{\hat{\boldsymbol{\mu}} = \frac{2\widetilde{\boldsymbol{t}_1}\boldsymbol{x}^*}{\boldsymbol{k}(\boldsymbol{x}^*)} + \boldsymbol{\tau}_l' \ ,} \tag{5.32}$$

and

$$\boxed{\hat{\boldsymbol{\sigma}} = \frac{\sqrt{2}\widetilde{\boldsymbol{t}_1}}{\boldsymbol{k}(\boldsymbol{x}^*)} \ ,} \tag{5.33}$$

where x^* is the x value which solves,

$$k(x) = 2x + \frac{2}{\sqrt{\pi}} \frac{\exp\left[-x^2\right]}{1 + \text{erf}\,(x)} = \frac{x + \sqrt{x^2 + 2(1 + c^2)}}{1 + c^2} \ .$$

To apply these simplifications to observations from the lognormal distribution, we have to transform the observations and left-truncation point from the lognormal distribution's support to the normal distribution's support. For a set of lognormal observations, $\boldsymbol{x} = (x_1, x_2, ..., x_n)$, with left-truncation point, τ_l, the equivalent observations from the normal distribution are $\boldsymbol{z} = (z_1, z_2, ..., z_n)$,

$$\boldsymbol{z_i} = \log(\boldsymbol{x_i}) \quad \forall \quad \boldsymbol{i} = \mathbf{1}, ..., \boldsymbol{n} \ , \tag{5.34}$$

and the equivalent left-truncation point is τ_l',

$$\boldsymbol{\tau}_l' = \log(\boldsymbol{\tau}_l) \ . \tag{5.35}$$

A more rigorous definition of the equivalence between the lognormal and normal distributions is given in section 8.2. We note that the parameter estimates produced by Eq. (5.32) and (5.33) are the solutions of the MLE equations which maximise the likelihood function. The method introduced by Castillo [76] does not change the solutions, it just allows us to calculate them much more accurately and rapidly.

Eq. (5.26) and (5.28) are satisfied by the parameters that maximise the likelihood function, however, the parameters which solve Eq. (5.26) and (5.28) do not necessarily maximise likelihood. A more in-depth discussion of why this is the case is available in section 3.4. The key result is that we need to ensure that the Hessian of the loglikelihood function is negative definite at the parameters which solve Eq. (5.26) and (5.28) or that the loglikelihood function is bounded and that Eq. (5.26) and (5.28) have a unique solution. Fortunately, Castillo [76] showed that there is always a set of parameters which maximise likelihood and simultaneously solve Eq. (5.26) and (5.28). If it is not possible to show that the relevant MLE equations have a unique solution, we can estimate the parameters through an algorithm that numerically maximises the loglikelihood function. Again, more detail is given in section 3.4. Each parameter case is different, hence we must analyse them separately:

- **Case IIIa:** Eq. (5.26) does not necessarily have a unique solution. Thus, to determine $\hat{\mu}$ we numerically found the μ value which maximised the loglikelihood function, Eq. (5.24).

- **Case IIIb:** Eq. (5.28) does not necessarily have a unique solution, hence, we used a maximisation algorithm to find the σ value which maximised the loglikelihood function, Eq. (5.24).

- **Case II:** Kreer et al. [42] have showed that in case II, the left-truncated lognormal distribution MLE equations have a unique solution if and only if the sample, $\boldsymbol{x}$, satisfies, Eq. (5.30) after they have been transformed to the normal distribution's support. For lognormal observations, Kreer et al. [42] found that this criteria can be simplified to,

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\{\log\left(\frac{x_i}{\tau_l}\right)\right\}^2 - \left\{\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right)\right\}^2} < \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right) . \tag{5.36}$$

In our Monte Carlo simulations, if samples did not satisfy Eq. (5.30) they were disregarded and replaced. For the samples which did satisfy this inequality, we employed Eq. (5.32) and (5.33) to the correctly translated data points in order to produce the parameter estimates.

5.4.4 Summary of Applicable MLE Equations

Case	Scale Parameter Equation	Shape Parameter Equation
I	N/A	N/A
II	Eq. (5.32)	Eq. (5.33)
IIIa	Eq. (5.26)	N/A
IIIb	N/A	Eq. (5.28)

Table 5.1: Summary of applicable MLE equations

Recall that we chose to use the maximum likelihood estimator to estimate our parameters because it produces consistent and efficient results. These features are a consequence of Lehmann and Casella's work [16]. In section 3.5, we outlined the reasoning behind why we believe the conditions (A0)–(A2) and assumptions (A)–(D) (necessary for Theorem 1.5, Theorem 5.1 in [16]) are valid for the left-truncated Weibull distribution. The reasoning given in that section would remain largely unchanged for the left-truncated lognormal distribution, hence we have not repeated that argument.

5.5 Parameter Independent Pivotal Functions and Critical Values

As discussed in section 2.6, parameter independent pivotal functions are important because they produce parameter independent critical values. It is the objective of this section to show that the left-truncated lognormal distribution has parameter independent critical values. The methodology we will employ is employed in section 2.6.

Expressing the x dependence of the left-truncated lognormal cdf, Eq. (5.12), with the random number generation formula, Eq. (5.18), allows us to define a function, $\widetilde{F_T}(u)$

where $x = F_T^{-1}(u)$, such that $\widetilde{F_T}(u) = F_T(x)$, for all x,

$$
\begin{aligned}
F_T &= 1 - \frac{\operatorname{erfc}\left(\frac{\log(x)-\hat{\mu}}{\hat{\sigma}\sqrt{2}}\right)}{\operatorname{erfc}\left(\frac{\log(\tau_l)-\hat{\mu}}{\hat{\sigma}\sqrt{2}}\right)} \\
\Rightarrow \widetilde{F_T}(u_i|\hat{\mu},\hat{\sigma},\mu^0,\sigma^0) &= 1 - \frac{\operatorname{erfc}\left(\frac{\mu^0+\sigma^0\sqrt{2}\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)-\hat{\mu}}{\hat{\sigma}\sqrt{2}}\right)}{\operatorname{erfc}\left(\frac{\mu^0+\sigma^0\sqrt{2}\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)-\hat{\mu}}{\hat{\sigma}\sqrt{2}}\right)} \\
&= 1 - \frac{\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\frac{\mu^0-\hat{\mu}}{\hat{\sigma}} + \frac{\sigma^0}{\hat{\sigma}}\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)\right)}{\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\frac{\mu^0-\hat{\mu}}{\hat{\sigma}} + \frac{\sigma^0}{\hat{\sigma}}\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)\right)} \, .
\end{aligned}
\tag{5.37}
$$

As discussed in section 2.6 the pivotal functions γ and ξ are introduced to simplify Eq. (5.37),

$$
\boldsymbol{\gamma} = \frac{\boldsymbol{\mu^0} - \boldsymbol{\hat{\mu}}}{\boldsymbol{\hat{\sigma}}} \tag{5.38}
$$

$$
\boldsymbol{\xi} = \frac{\boldsymbol{\sigma^0}}{\boldsymbol{\hat{\sigma}}} \, . \tag{5.39}
$$

Substituting Eq. (5.38) and Eq. (5.39) into Eq. (5.37) reduces $\widetilde{F_T}(u)$ to,

$$
\widetilde{F_T}(u_i|\hat{\mu},\hat{\sigma},\mu^0,\sigma^0) = \widetilde{F_T}(u_i|\gamma,\xi,\eta) = 1 - \frac{\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\gamma + \operatorname{erfc}^{-1}(u_i\eta)\,\xi\right)}{\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\gamma + \operatorname{erfc}^{-1}(\eta)\,\xi\right)} \, . \tag{5.40}
$$

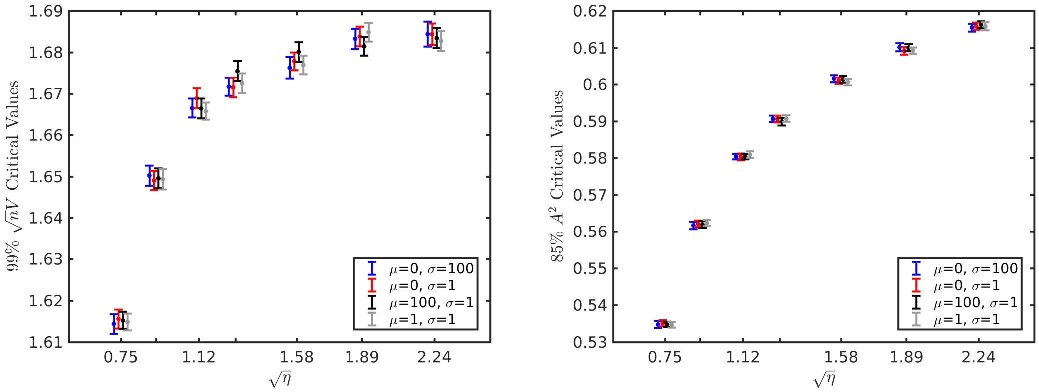
All parameter dependence in $\widetilde{F_T}(u)$ is completely contained within the pivotal functions $\gamma = \gamma(\mu^0, \hat{\mu}, \hat{\sigma})$ and $\xi = \xi(\sigma^0, \hat{\sigma})$. In this section we will show that these pivotal functions are distributed independently of μ^0 and σ^0, therefore, proving that the critical values are parameter independent. The approach we will use to achieve this is given in sections 2.6 and 3.6.

In section 8.1 we have numerically verified that critical values are parameter independent for all of the distributions we studied. Figures 5.1a and 5.1b display the Kuiper and Anderson-Darling critical values at the 99% and 85% significance levels with samples of size $n = 30$ and $n = 10,000$ (respectively) drawn from the case II lognormal distribution for a range of parameter values. This figure shows an example supporting the assertion that the critical values are parameter independent. A more detailed discussion is included in section 8.1.

5.5.1 Case IIIa

In this case, μ is to be estimated and σ is known *a priori*, therefore,

$$
\begin{aligned}
\hat{\sigma} &= \sigma^0 \\
\Rightarrow \xi &= 1 \\
\Rightarrow \widetilde{F_T}(u_i|\gamma,\eta) &= 1 - \frac{\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\gamma + \operatorname{erfc}^{-1}(u_i\eta)\right)}{\operatorname{erfc}\left(\frac{1}{\sqrt{2}}\gamma + \operatorname{erfc}^{-1}(\eta)\right)} \, .
\end{aligned}
\tag{5.41}
$$



(a) Kuiper ($\sqrt{n}V$) crit. vals. at 99% sig. lvl. for $n = 30$ observations

(b) Anderson-Darling (A^2) crit. vals. at 85% sig. lvl. for $n = 10,000$ observations

Figure 5.1: Case II Lognormal critical values
Critical values from different parameter values are staggered on the $\sqrt{\eta}$ axis for clarity

Additionally, the remaining pivotal function γ becomes,

$$\gamma = \frac{\mu^0 - \hat{\mu}}{\sigma^0} \; . \tag{5.42}$$

The random number generation formula Eq. (5.18), is inserted into Eq. (5.26), and reduced to the form,

$$\begin{aligned}
\frac{1}{\sigma^0}\sqrt{\frac{2}{\pi}}\frac{1}{\operatorname{erfcx}\left(\frac{\log(\tau_l)-\hat{\mu}}{\sigma^0\sqrt{2}}\right)} &= \frac{1}{n(\sigma^0)^2}\sum_{i=1}^{n}\left(\log(x_i)-\hat{\mu}\right) . \\
\sqrt{\frac{2}{\pi}}\frac{1}{\operatorname{erfcx}\left(\frac{\mu^0+\sigma^0\sqrt{2}\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)-\hat{\mu}}{\sigma^0\sqrt{2}}\right)} &= \frac{1}{\sigma^0 n}\sum_{i=1}^{n}\left(\mu^0+\sigma^0\sqrt{2}\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)-\hat{\mu}\right) \\
\sqrt{\frac{2}{\pi}}\frac{1}{\operatorname{erfcx}\left(\frac{\mu^0-\hat{\mu}}{\sigma^0}+\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)\right)} &= \frac{\sqrt{2}}{n}\sum_{i=1}^{n}\left(\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)\right)+\frac{\mu^0-\hat{\mu}}{\sigma^0} \\
\sqrt{\frac{2}{\pi}}\frac{1}{\operatorname{erfcx}\left(\gamma+\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)\right)}-\gamma &= \frac{\sqrt{2}}{n}\sum_{i=1}^{n}\left(\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)\right) .
\end{aligned} \tag{5.43}$$

Castillo [76] showed that there is always a set of parameters which maximise likelihood, and thus solves the MLE equations. Hence, Eq. (5.43) always has a solution. Therefore, γ can be expressed in terms of the other variables, i.e. $\gamma = \underset{\sim}{\gamma}(\eta, \boldsymbol{u})$. From this expression of γ, it is possible to remove the parameter dependence of $\widetilde{F_T}(u_i)$,

$$\Rightarrow \widetilde{F_T}(u_i|\gamma,\eta) = \widetilde{F_T}(u_i|\gamma(\boldsymbol{u}),\eta) = \widetilde{F_T}(u_i|\boldsymbol{u},\eta) \; . \tag{5.44}$$

Therefore, $\widetilde{F_T}(u_i)$, is distributed independently of μ^0 and σ^0, hence, we have parameter independent critical values.

5.5.2 Case IIIb

In this case, σ is to be estimated and μ is known *a priori*, therefore,

$$\begin{aligned} \hat{\mu} &= \mu^0 \\ \Rightarrow \gamma &= 1 \\ \Rightarrow \widetilde{F_T}(u_i|\xi,\eta) 1 &- \frac{\operatorname{erfc}\left(\frac{1}{\sqrt{2}} + \operatorname{erfc}^{-1}(u_i\eta)\,\xi\right)}{\operatorname{erfc}\left(\frac{1}{\sqrt{2}} + \operatorname{erfc}^{-1}(\eta)\,\xi\right)} \ . \end{aligned} \tag{5.45}$$

The random number generation formula Eq. (5.18), is inserted into Eq. (5.28), and reduced to the form,

$$\begin{aligned} \frac{1}{n}\frac{1}{\hat{\sigma}^2}\sum_{i=1}^{n}\left[\log(x_i)-\mu^0\right]^2 - 1 &= \frac{1}{\hat{\sigma}}\sqrt{\frac{2}{\pi}}\frac{\log(\tau_L)-\mu^0}{\operatorname{erfcx}\left(\frac{\log(\tau_L)-\mu^0}{\hat{\sigma}\sqrt{2}}\right)} \\ \frac{2}{n}\left(\frac{\sigma^0}{\hat{\sigma}}\right)^2\sum_{i=1}^{n}\left[\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)\right]^2 - 1 &= \frac{\sigma^0}{\hat{\sigma}}\frac{2}{\sqrt{\pi}}\frac{\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)}{\operatorname{erfcx}\left(\frac{\sigma^0}{\hat{\sigma}}\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)\right)} \\ \frac{2}{n}\xi^2\sum_{i=1}^{n}\left[\operatorname{erfc}^{-1}\left(\frac{1-u_i}{\eta}\right)\right]^2 - 1 &= \frac{2}{\sqrt{\pi}}\frac{\xi\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)}{\operatorname{erfcx}\left(\xi\operatorname{erfc}^{-1}\left(\frac{1}{\eta}\right)\right)} \ . \end{aligned} \tag{5.46}$$

Castillo [76] showed that there is always a set of parameters which maximise likelihood, and thus solves the MLE equations. Hence, Eq. (5.46) always has a solution. Therefore, ξ can be expressed in terms of the other variables, i.e. $\xi = \xi(\eta, \boldsymbol{u})$. From this expression of ξ, it is possible to remove the parameter dependence of $\widetilde{F_T}(u_i)$,

$$\Rightarrow \widetilde{F_T}(u_i|\xi,\eta) = \widetilde{F_T}(u_i|\xi(\boldsymbol{u}),\eta) = \widetilde{F_T}(u_i|\boldsymbol{u},\eta) \ . \tag{5.47}$$

Therefore, $\widetilde{F_T}(u_i)$, is distributed independently of μ^0 and σ^0, hence, we have parameter independent critical values.

5.5.3 Case II

In this case, the objective is to show that parameter independent solutions for γ and ξ exist. In Eq. (5.38) and (5.39) we have shown that the parameter dependence of Eq. (5.26) and (5.28) can be contained within the pivotal functions. Castillo [76] and Kreer et al. [42][1] showed that the MLE equations of the case II, left-truncated lognormal distribution have a unique solution if

$$\sqrt{\frac{1}{n}\sum_{i=1}^{n}\left\{\log\left(\frac{x_i}{\tau_l}\right)\right\}^2 - \left\{\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right)\right\}^2} < \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{x_i}{\tau_l}\right) \ . \tag{5.30 revisited}$$

Thus, if Eq. (5.30) is satisfied, we know that there is a unique, simultaneous solution to Eq. (5.26) and (5.28). As the samples that did not satisfy Eq. (5.30) were rejected, we have a unique, simultaneous solution to Eq. (5.26) and (5.28) for all the samples that contributed to the critical values. As a result, we can conclude that there is an expression for both γ and ξ in terms of the other variables, i.e.$\gamma = \gamma(\eta, \boldsymbol{u})$ and $\xi = \xi(\eta, \boldsymbol{u})$. Hence, we can remove the parameter dependence in $\widetilde{F_T}(u_i)$ (Eq. (5.40)),

$$\begin{aligned} \gamma = \gamma(\boldsymbol{u},\eta) \quad &\text{and} \quad \xi = \xi(\boldsymbol{u},\eta) \ , \\ \Rightarrow \widetilde{F_T}(u_i|\gamma,\xi) = \widetilde{F_T}(u_i|\gamma(\boldsymbol{u},\eta),\xi(\boldsymbol{u},\eta)) &= \widetilde{F_T}(u_i|\boldsymbol{u},\eta). \end{aligned} \tag{5.48}$$

[1]The work of Kreer et al. [42] was based on that of Castillo [76].

Hence, $\widetilde{F_T}(u_i)$, is independent of μ^0 and σ^0, thus, we have parameter independent critical values.

Chapter 6

Pareto Distribution

Observations which are drawn from a Pareto distribution are commonly said to follow a power law probability distribution. The distribution is named after Vilfredo Federico Damaso Pareto who discovered that incomes are approximately distributed by a power law. The distribution only has one parameter, k, which we shall call the scale parameter, however, one is also required to specify a τ_l such that $\tau_l \leq x$. In this work we do not consider τ_l to be a parameter as we use it to specify the left truncation limit. The Pareto distribution has been generalised into a range of forms, some of which remove the restriction on the support ($\tau_l \leq x$), however, we shall use the form specified in Eq. (6.1).

The Pareto distribution has been used to describe a wide range of phenomena such as the size of human settlements and standardised returns on individual stocks [77]. The latter is of more interest to us, as we study the Pareto distribution primarily to determine whether it is a better model for the financial data that Kizilersü et al. [33] described with the left-truncated Weibull distribution.

6.1 Truncated Distribution

The Pareto distribution is intrinsically left-truncated, i.e. $x > \tau_L$, therefore the truncated pdf, $f_T(x)$, and cdf, $F_T(x)$, are defined as,

$$\boxed{\boldsymbol{f_T(x|k) = \frac{k\tau_l^k}{x^{k+1}}}} \tag{6.1}$$

$$\boxed{\boldsymbol{F_T(x|k) = 1 - \left(\frac{\tau_l}{x}\right)^k} \ .} \tag{6.2}$$

The support and scale parameter, k, are restricted to

Support	$\tau_l < x < \infty$
Scale parameter	$0 < k < \infty$.

6.2 Generation of Random Numbers from the Pareto Distribution

Recall that the general random generation formula is defined by

$$F^{-1}(u_i) = x_i \ , \tag{1.26 revisited}$$

and that the array of uniform random variates, $\boldsymbol{u}$, is defined as,

$$\boldsymbol{u} = (u_1, u_2, ..., u_n) \quad \text{where } u_i \in (0,1) \ \forall\, i = 1, 2, ..., n. \tag{1.27 revisited}$$

The random generation formula for the Pareto distribution can be obtained by determining the inverse of the cumulative distribution function,

$$F_T(x_i|k^0) = u_i = 1 - \left(\frac{\tau_l}{x_i}\right)^{k^0}$$

$$\Rightarrow \left(\frac{\tau_l}{x_i}\right)^{k^0} = 1 - u_i$$

$$\boxed{\Rightarrow \boldsymbol{x_i} = \frac{\boldsymbol{\tau_l}}{(\mathbf{1} - \boldsymbol{u_i})^{\frac{1}{k^0}}} \ .} \tag{6.3}$$

6.3 Estimation of Parameters

In section 3.4 we defined the terms *point estimator* and *maximum likelihood estimator*, hence, we will refrain from repeating that . Suffice to say that we will employ the maximum likelihood estimator to produce our parameter estimates, $\hat{\boldsymbol{\theta}}(\boldsymbol{x})$. These estimates maximise the likelihood function, $L(\boldsymbol{\theta}|\boldsymbol{x})$, for a set of observations, $\boldsymbol{x}$. Table 1.2 summarises the four different cases of parameter estimation studied in this work, however, as the Pareto distribution has only a scale parameter, only case I and case IIIa are applicable.

Case	Scale Parameter	Shape Parameter
I	known *a priori*	known *a priori*
II	acquired from data	acquired from data
IIIa	acquired from data	known *a priori*
IIIb	known *a priori*	acquired from data

Table 1.2: Cases of parameter estimation

For the Pareto distribution, $f_T(x)$ is given by Eq. (6.1), which reduces Eq. (1.17) to,

$$L(\boldsymbol{\theta}|\boldsymbol{x}) = \prod_{i=1}^{n} \frac{k\tau_l^k}{x_i^{k+1}}$$

$$\Rightarrow L(\boldsymbol{\theta}|\boldsymbol{x}) = [k\tau_l^k]^n \prod_{i=1}^{n} x_i^{-(k+1)} \ . \tag{6.4}$$

The loglikelihood function, l, is defined by,

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = \log(L(\boldsymbol{\theta}|\boldsymbol{x})) = \sum_{i=1}^{n} \log(f(x_i|\boldsymbol{\theta})), \tag{1.19 revisited}$$

which reduces to,

$$l(\boldsymbol{\theta}|\boldsymbol{x}) = n\log(k) + kn\log(\tau_l) - \sum_{i=1}^{n}(k+1)\log(x_i) \ , \tag{6.5}$$

in the case of the Pareto distribution. The most general form of the MLE equations is,

$$\frac{\partial}{\partial\theta_j} l(\boldsymbol{\theta}|\boldsymbol{x}) = \frac{\partial}{\partial\theta_j} \sum_{i=1}^{n} \log\left(f(x_i|\boldsymbol{\theta})\right) = 0 \quad \forall \quad j = (1, 2, ..., m) \ , \tag{1.21 revisited}$$

where m is the number of parameters in $f_T(x|\boldsymbol{\theta})$.

6.3.1 Case IIIa

In this case the scale parameter, k, must be estimated from the data ($k = \hat{k}$). For the Pareto distribution, the corresponding MLE equation is achieved by substituting $\theta_1 = k$ into Eq. (1.21) to yield,

$$\begin{aligned} 0 =& \frac{dl}{dk} \\ =& \frac{n}{k} + n\log(\tau_l) - \sum_{i=1}^{n} log(x_i) \ . \end{aligned} \tag{6.6}$$

Pulling the second and third terms to the left hand side reduces Eq. (6.6) to,

$$\sum_{i=1}^{n} \left[\log(x_i)\right] - n\log(\tau_l) = \frac{n}{k}$$

$$\boxed{\Rightarrow \boldsymbol{k} = \left[\frac{\mathbf{1}}{\boldsymbol{n}} \sum_{\boldsymbol{i}=\mathbf{1}}^{\boldsymbol{n}} \left[\mathbf{log}(\boldsymbol{x_i})\right] - \mathbf{log}(\boldsymbol{\tau_l})\right]^{-\mathbf{1}} .} \tag{6.7}$$

Eq. (6.7) has a closed form, thus, it produces a unique estimate for k. As discussed in section 3.4, we need to ensure that the value of k which solves Eq. (6.7) also maximises the loglikelihood function. To do this, we must verify that the Hessian of the loglikelihood function is negative definite at the k value which solves Eq. (6.7). For the Pareto distribution, in which there is only one parameter, this is tantamount to showing that the second-order derivative of the loglikelihood function with respect to k is negative. This derivative is given by,

$$\frac{d^2l}{dk^2} = \frac{d}{dk}\left(\frac{n}{k} + n\log(\tau_l) - \sum_{i=1}^{n} log(x_i)\right) = -\frac{n}{k^2} \ . \tag{6.8}$$

Both n and k are necessarily positive, hence, Eq. (6.8) is always negative, in particular it is negative at the k value which solves Eq. (6.7). Therefore, the solution to Eq. (6.7) always maximises the loglikelihood function. Hence, in our Monte Carlo procedure we used the solution of Eq. (6.7) as our parameter estimate.

6.3.2 Summary of Applicable MLE Equations

Case	Scale Parameter Equation
I	N/A
IIIa	Eq. (6.7)

Table 6.1: Summary of applicable MLE equations

Recall that we chose to use the maximum likelihood estimator to estimate our parameters because it produces consistent and efficient results. These features are a consequence of Lehmann and Casella's work [16]. In section 3.5, we outlined the reasoning behind why we believe the conditions (A0)–(A2) and assumptions (A)–(D) (necessary for Theorem 1.5, Theorem 5.1 in [16]) are valid for the left-truncated Weibull distribution. The reasoning given in that section would remain largely unchanged for the Pareto distribution, hence we have not repeated that argument.

6.4 Parameter Independent Pivotal Functions and Critical Values

As discussed in section 2.6, parameter independent pivotal functions are important because they produce parameter independent critical values. It is the objective of this section to show that the Pareto distribution has parameter independent critical values. The methodology we will employ is employed in section 2.6.

Expressing the x dependence of the Pareto cdf, Eq. (6.2), with the random generation formula, Eq. (6.3), allows us to define a function, $\widetilde{F_T}(u)$ where $x = F_T^{-1}(u)$, such that $\widetilde{F_T}(u) = F_T(x)$, for all x,

$$\begin{aligned} F_T = u_i &= 1 - \left(\frac{\tau_l}{x_i}\right)^{\hat{k}} \\ \Rightarrow \widetilde{F_T}(u_i|\hat{k}, k^0) = u_i &= 1 - \left(\frac{\tau_l}{\frac{\tau_l}{(1-u_i)^{\frac{1}{k^0}}}}\right)^{\hat{k}} \\ = u_i &= 1 - (1-u_i)^{\frac{\hat{k}}{k^0}}\,. \end{aligned} \tag{6.9}$$

As discussed in section 2.6 the pivotal function, γ, is introduced to simplify Eq. (6.9),

$$\boldsymbol{\gamma = \frac{\hat{k}}{k^0}}\;. \tag{6.10}$$

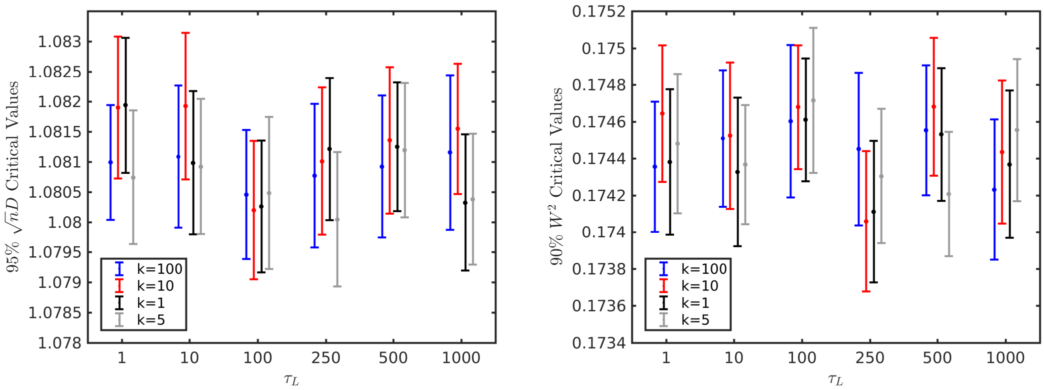
$$\tag{6.11}$$

Substituting Eq. (6.10) into Eq. (6.9) reduces $\widetilde{F_T}(u)$ to,

$$\widetilde{F_T}(u_i|\hat{k}, k^0) = \widetilde{F_T}(u_i|\gamma) = 1 - (1-u_i)^{\gamma}\,. \tag{6.12}$$

All parameter dependence in $\widetilde{F_T}(u)$ is completely contained within the pivotal function $\gamma = \gamma(\alpha^0, \hat{\alpha}, \hat{\beta})$. In this section we will show that this pivotal function is distributed independently of k^0, therefore, proving that the critical values are parameter independent. The approach we will use to achieve this is given in sections 2.6 and 3.6.

In section 8.1 we have numerically verified that critical values are parameter independent for all of the distributions we studied. Figures 6.1a and 6.1b display the Kolmogorov-Smirnov and Cramér-von Mises critical values at the 95% and 90% significance levels with samples of size $n = 200$ and $n = 500$ (respectively) drawn from the case IIIa Pareto distribution for a range of parameter values. This figure shows an example supporting the assertion that the critical values are parameter independent. A more detailed discussion is included in section 8.1.



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl. for $n = 200$ observations

(b) Cramér-von Mises (W^2) crit. vals. at 90% sig. lvl. for $n = 500$ observations

Figure 6.1: Case IIIa Pareto critical values
Critical values from different parameter values are staggered on the τ_l axis for clarity

6.4.1 Case IIIa

In this case the objective is to show that a parameter independent solution for γ exists. Initially one inserts the random generation formula Eq. (6.3), into Eq. (6.7),

$$\begin{aligned}
\hat{k} &= \left[\frac{1}{n}\sum_{i=1}^{n}\left[\log\left(\frac{\tau_L}{(1-u_i)^{\frac{1}{k^0}}}\right)\right] - \log(\tau_l)\right]^{-1} \\
\frac{1}{\hat{k}} &= \frac{-1}{n}\sum_{i=1}^{n}\frac{1}{k^0}\log\left(1-u_i\right) \\
\frac{k^0}{\hat{k}} &= \frac{-1}{n}\sum_{i=1}^{n}\log\left(1-u_i\right) \\
\gamma = \gamma(\boldsymbol{u}) &= \frac{-n}{\sum_{i=1}^{n}\log\left(1-u_i\right)}. \qquad (6.13)
\end{aligned}$$

One can then substitute Eq. (6.13) into Eq. (6.12) to remove the parameter dependence of, $\widetilde{F_T}(u_i)$,

$$\Rightarrow \widetilde{F_T}(u_i|\gamma) = \widetilde{F_T}(u_i|\gamma(\boldsymbol{u})) = \widetilde{F_T}(u_i|\boldsymbol{u})\ . \qquad (6.14)$$

Therefore, $\widetilde{F_T}(u_i)$, is distributed independently of k^0, hence, we have parameter independent critical values.

Chapter 7

Uncertainty Margins

When critical values of hypothesis tests are determined via Monte Carlo methods, the question is raised whether it is better to conduct one large run of size N (single-sample) or to take several (C) smaller runs of size M (multi-sample, $N = C.M$) and average the results. This chapter will discuss the advantages and disadvantages of each method [78, 79]. Three methods of calculating the uncertainty will be discussed here, two for the single-sample method and one for the multi-sample method. As early as 1946 Mosteller [80] proposed a single-sample method for estimating the uncertainty, later the proof of this method was modernised by Walker [81]. David [82] introduced an another single-sample method of determining the uncertainty margin which was expanded upon by Juritz et al. [78]. Schafer [83] proposed a method for determining uncertainty from multi-sampled data.

Single-sample:

- David's Method [78, 82]
- Mosteller's Method [80, 81]

Multi-sample:

- Schafer's Method [78, 83]

In this chapter the mathematical reasoning behind these methods is articulated and a numerical comparison follows. Additionally the concept of bias will be introduced and explored. Throughout this section the k^{th} order statistic (k^{th} highest test statistic) will be denoted $T_{(k)}$ and the cdf of the normal distribution with mean μ and variance σ^2 will be denoted $F_N(x|\mu, \sigma^2)$,

$$F_N(x|\mu, \sigma^2) = \Phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{x-\mu}{\sqrt{2}\sigma}\right)\right]. \qquad \text{(2.17 revisited)}$$

7.1 David's Method

In this method one has N test statistics and wishes to determine the $100p$ percentile, ξ_p. The estimator for this percentile, $\hat{\xi}_p$, and its corresponding confidence interval, I_s, are defined as

$$\begin{aligned} &\hat{\xi}_p = T_{(k)} \quad \text{where } \ k = [Np] + 1 \ \text{ and } \ [x] \ \text{ denotes the integer component (floor) of } x, \\ &I_s = (T_{(r)}, T_{(s)}) \ \text{ where } \ r, s \in (1, 2, ..., N), r < s. \end{aligned} \tag{7.1}$$

The integers r, s are determined by requiring the probability that ξ_p is inside I_s is equal to the confidence level, $1 - \alpha$,

$$1 - \alpha = \mathbb{P}\left(T_{(r)} \le \xi_p \le T_{(s)}\right) = \mathbb{P}\left(\xi_p \le T_{(s)}\right) - \mathbb{P}\left(\xi_p < T_{(r)}\right) \ . \tag{7.2}$$

The probability of ξ_p being less than a random $T_{(i)}, i \in (1, ..., n)$ is p, and all $T_{(i)}$ are i.i.d., therefore the probability of ξ_p falling between $T_{(i)}$ and $T_{(i+1)}$ can be expressed by the binomial distribution, $B(N, p)$. As a result, the last two terms of Eq. (7.2) are equivalent to the binomial probabilities, $\mathbb{P}_B\left(T_{(s)} \leq \xi_p\right)$ and $\mathbb{P}_B\left(T_{(r)} < \xi_p\right)$ [78, 82],

$$
\begin{aligned}
\mathbb{P}_B\left(\xi_p \leq T_{(s)}\right) &= \sum_{i=0}^{s} \binom{N}{i} p^i (1-p)^{N-i} \ , \\
\mathbb{P}_B\left(\xi_p < T_{(r)}\right) &= \sum_{i=0}^{r-1} \binom{N}{i} p^i (1-p)^{N-i} \ .
\end{aligned}
\tag{7.3}
$$

Employing Eq. (7.3) reduces Eq. (7.2) to [78, 84] [1],

$$
\begin{aligned}
1-\alpha = \mathbb{P}_B\left(\xi_p \leq T_{(s)}\right) - \mathbb{P}_B\left(\xi_p < T_{(r)}\right) &= \sum_{i=0}^{s} \binom{N}{i} p^i (1-p)^{N-i} - \sum_{i=0}^{r-1} \binom{N}{i} p^i (1-p)^{N-i} \\
&= \sum_{i=r}^{s} \binom{N}{i} p^i (1-p)^{N-i} \ .
\end{aligned}
\tag{7.4}
$$

For a "large" number of samples, N, the binomial distribution, $B(N, p)$, can be approximated by the normal distribution, $\mathcal{N}(Np, Np(1-p))$, with mean, $\mu = Np$, variance, $\sigma^2 = Np(1-p)$. A discussion of how large N must be to use this approximation can be found in 7.1.1. Because $B(N, p)$ is discrete and $\mathcal{N}(Np, Np(1-p))$ is continuous, one must include a continuity correction factor, CCF, when making this approximation. A detailed argument is included in 7.1.2. The $\mathbb{P}_B$ notation is used for probabilities expressed from the discrete binomial distribution, and the $\mathbb{P}_N$ notation is used for probabilities expressed from the continuous normal distribution.

$$
\begin{aligned}
\mathbb{P}_B\left(T_{(s)} \leq \xi_p\right) &\mapsto \mathbb{P}_N\left(T_{(s)} + \frac{1}{2}(T_{(s+1)} - T_{(s)}) < \xi_p\right) = \mathbb{P}_N\left(\frac{1}{2}(T_{(s)} + T_{(s+1)}) < \xi_p\right) \\
\mathbb{P}_B\left(T_{(r)} < \xi_p\right) &\mapsto \mathbb{P}_N\left(T_{(r)} - \frac{1}{2}(T_{(r)} - T_{(r-1)}) < \xi_p\right) = \mathbb{P}_N\left(\frac{1}{2}(T_{(r-1)} + T_{(r)}) < \xi_p\right)
\end{aligned}
\tag{7.5}
$$

[1] The sum in (2.1) of [78] is supposed to run from r to $s-1$ not from 1 to $s-1$. This is made clear in the following column in the work, and is not present in an earlier version submitted as a technical report to the Department of Statistics at Stanford University [84]

In the normal approximation for large N, Eq. (7.2) reduces to,

$$
\begin{aligned}
\mathbb{P}\left(T_{(r)} \le \xi_p \le T_{(s)}\right) &= \mathbb{P}_N(\frac{1}{2}(T_{(s)} + T_{(s+1)}) < \xi_p) - \mathbb{P}_N\left(\frac{1}{2}(T_{(r-1)} + T_{(r)}) < \xi_p\right) \\
&= \mathbb{P}_N\left(NF\left(\frac{1}{2}(T_{(s)} + T_{(s+1)})\right) < NF\left(\xi_p\right)\right) \\
&\qquad - \mathbb{P}_N\left(NF\left(\frac{1}{2}(T_{(r-1)} + T_{(r)}) < \xi_p\right) < NF(\xi_p)\right) \\
&\qquad \text{where F is cdf of test statistics} \\
&= \mathbb{P}_N\left(s + \frac{1}{2} < Np\right) - \mathbb{P}_N\left(r - \frac{1}{2} < Np\right) \\
&= F_N\left(s + \frac{1}{2}|Np, Np(1-p)\right) - F_N\left(r - \frac{1}{2}|Np, Np(1-p)\right) \\
&\qquad \text{where } F_N \text{ is the normal cdf} \\
&= \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{s + \frac{1}{2} - Np}{\sqrt{2}\sqrt{Np(1-p)}}\right)\right] \\
&\qquad - \frac{1}{2}\left[1 + \operatorname{erf}\left(\frac{r - \frac{1}{2} - Np}{\sqrt{2}\sqrt{Np(1-p)}}\right)\right] \text{ by Eq. (2.17)} \\
&= \frac{1}{2}\left[\operatorname{erf}\left(\frac{s - Np + \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}}\right) - \operatorname{erf}\left(\frac{r - Np - \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}}\right)\right] . \qquad (7.6)
\end{aligned}
$$

For the probability region to be symmetric about the mean, Np,

$$
\begin{aligned}
Np - r &= s - Np \\
\Rightarrow r - Np &= -s + Np . \qquad (7.7)
\end{aligned}
$$

Therefore, Eq. (7.6) can be simplified to,

$$
\begin{aligned}
1 - \alpha &= \frac{1}{2}\left[\operatorname{erf}\left(\frac{s - Np + \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}}\right) - \operatorname{erf}\left(\frac{-s + Np - \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}}\right)\right] \\
&= \frac{1}{2}\left[\operatorname{erf}\left(\frac{s - Np + \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}}\right) - \operatorname{erf}\left(\frac{-(s - Np + \frac{1}{2})}{\sqrt{2}\sqrt{Np(1-p)}}\right)\right] \\
&= \operatorname{erf}\left(\frac{s - Np + \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}}\right) . \qquad (7.8)
\end{aligned}
$$

Rearranging Eq. (7.8);

$$
\begin{aligned}
\operatorname{erf}^{-1}(1-\alpha) &= \frac{s - Np + \frac{1}{2}}{\sqrt{2}\sqrt{Np(1-p)}} \\
\sqrt{Np(1-p)}\sqrt{2}\operatorname{erf}^{-1}(1-\alpha) - \frac{1}{2} &= s - Np \\
s &= Np + \sqrt{Np(1-p)}\sqrt{2}\operatorname{erf}^{-1}(1-\alpha) - \frac{1}{2} . \qquad (7.9)
\end{aligned}
$$

Finally, Eq. (7.8) and Eq. (7.7) can be combined to yield, an equivalent expression for r,

$$
\begin{aligned}
-(r - Np) &= \sqrt{Np(1-p)}\sqrt{2}\operatorname{erf}^{-1}(1-\alpha) - \frac{1}{2} \\
r &= Np - \sqrt{Np(1-p)}\sqrt{2}\operatorname{erf}^{-1}(1-\alpha) + \frac{1}{2} . \qquad (7.10)
\end{aligned}
$$

The indices s and r are integers, however the expressions Eq. (7.8) and Eq. (7.10) do not necessarily yield integer values, hence a rounding convention must be employed. Juritz et. al. [78] round both s and r in the same direction (they arbitrarily select up) to ensure that $\mathbb{P}\left(T_{(r)} \leq \xi_p \leq T_{(s)}\right)$ is close to $1-\alpha$. In the present work a more conservative approach is implemented, the lower limit, r, is rounded down and the upper limit, s, is rounded up.

7.1.1 What is a "large" N?

The binomial distribution, $B(N,p)$, can be approximated by the normal distribution, $\mathcal{N}(Np, Np(1-p))$, for a "large" enough N. Only integers between 0 and N are accessible to the binomial distribution, hence any probability density of the normal distribution outside of $(0,N)$ induces a discrepancy between $B(N,p)$ and $\mathcal{N}(Np, Np(1-p))$. As a result, N is considered "large", if samples drawn from $\mathcal{N}(Np, Np(1-p))$ have negligible probability of being outside $(0,N)$. A rule of thumb is to require that three standard deviations of the normal distribution are inside this region, the restrictions this places on N are evaluated as

$$\begin{aligned} Np - 3\sqrt{Np(1-p)} &> 0 \\ Np &> 3\sqrt{Np(1-p)} \\ N^2p^2 &> 9Np(1-p) \\ N &> 9\frac{1-p}{p} \end{aligned} \tag{7.11}$$

$$\begin{aligned} N &> Np + 3\sqrt{Np(1-p)} \\ N(1-p) &> 3\sqrt{Np(1-p)} \\ N^2(1-p)^2 &> 9Np(1-p) \\ N &> 9\frac{p}{1-p}. \end{aligned} \tag{7.12}$$

Eq. (7.11) is strictest when p is minimised, i.e. $p = p_{min} = 0.85$, and Eq. (7.12) is strictest when p is maximised, i.e. $p = p_{max} = 0.99$, these conditions are evaluated at their strictest in Eq. (7.13) and Eq. (7.14) respectively.

$$\begin{aligned} N &> 9\frac{1-p_{min}}{p_{min}} \\ N &> 9\frac{1-0.85}{0.85} \\ N &> 1.588 \end{aligned} \tag{7.13}$$

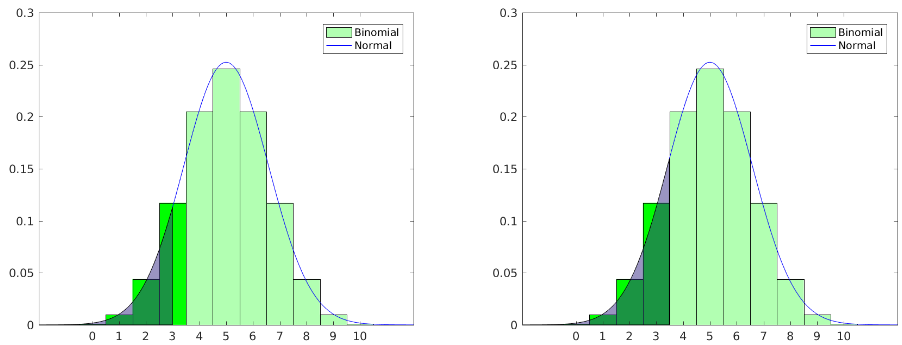
$$\begin{aligned} N &> 9\frac{p_{max}}{1-p_{max}} \\ N &> 9\frac{0.99}{1-0.99} \\ N &> 891 \end{aligned} \tag{7.14}$$

The smallest N considered in this study is $10,000$, hence even the stricter condition Eq. (7.14) is passed without challenge. Indeed the sample size employed in this study is large enough such that ten standard deviations of the normal distribution are inside the allowed range. Therefore it is acceptable to use the normal approximation to the binomial distribution throughout this study.

The condition Eq. (7.13) places upon N is very meager. This is because it requires the normal distribution, $\mathcal{N}(0.85N, 0.85(1-0.85)N)$, to have at least three standard deviations between $0.85N$ and 0. A very small N is required to ensure this because 0.85 is far enough above 0 for an insignificant amount of probability under the normal distribution to fall below 0.

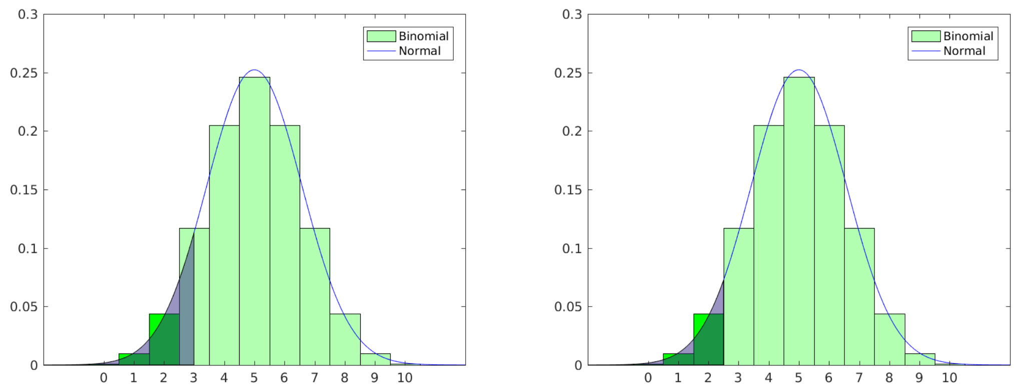
7.1.2 Continuity Correction Factor

When one approximates a discrete distribution with a continuous one, a continuity correction factor (CCF) must be included. Consider the following scenario, one is trying to approximate the binomial probability $\mathbb{P}_B(x \leq 3)$ with the normal probability $\mathbb{P}_N(x < 3)$, without the correction only half of the largest 'bin' is included. The most accurate approximation is given by $\mathbb{P}_N\left(x < 3 + \frac{1}{2}\right)$, i.e. the $CCF = +\frac{1}{2}$. These two scenarios are displayed in Figure 7.1a and Figure 7.1b respectively, where the solid green represents $\mathbb{P}_B(x \leq 3)$ and the shaded blue region represents $\mathbb{P}_N(x < 3)$ or $\mathbb{P}_N\left(x < 3 + \frac{1}{2}\right)$. Now consider the situation in which one is trying to approximate the binomial probability $\mathbb{P}_B(x < 3)$ with the normal probability $\mathbb{P}_N(x < 3)$. The normal approximation is now too large, the most accurate approximation is given by $\mathbb{P}_N\left(x < 3 - \frac{1}{2}\right)$, i.e. $CCF = -\frac{1}{2}$, as displayed in Figure 7.2a and Figure 7.2b respectively. In general the CCF is summarised by Table 7.1.



(a) Shaded blue is $\mathbb{P}_N(x \leq 3)$ (b) Shaded blue is $\mathbb{P}_N\left(x < 3 + \frac{1}{2}\right)$

Figure 7.1: Continuity correction factor of $+\frac{1}{2}$



(a) Shaded blue is $\mathbb{P}_N(x < 3)$ (b) Shaded blue is $\mathbb{P}_N\left(x < 3 - \frac{1}{2}\right)$

Figure 7.2: Continuity correction factor of $-\frac{1}{2}$

Binomial Probability	Normal Probability	CCF
$\mathbb{P}_B(x < N)$	$\mathbb{P}_N\left(x < N - \frac{1}{2}\right)$	$-\frac{1}{2}$
$\mathbb{P}_B(x \leq N)$	$\mathbb{P}_N\left(x < N + \frac{1}{2}\right)$	$+\frac{1}{2}$
$\mathbb{P}_B(x > N)$	$\mathbb{P}_N\left(x > N + \frac{1}{2}\right)$	$+\frac{1}{2}$
$\mathbb{P}_B(x \geq N)$	$\mathbb{P}_N\left(x > N - \frac{1}{2}\right)$	$-\frac{1}{2}$

Table 7.1: Summary of CCF

7.2 Mosteller's Method

In this method one has N test values and wishes to determine the $100p$ percentile, ξ_p. The following discussion is taken from Walker's 1968 paper [81] with some additional intermediate steps included.

Theorem 7.2. Let $T_{(1)} \leq T_{(2)} \leq ... \leq T_{(N)}$ be N ordered test statistics drawn from a cdf, $F(x)$, which is absolutely continuous and monotone increasing for $0 < F(x) < 1$. Let $T_{(r(N))} = \hat{\xi}_p$, where $r(N) = [Np] + 1$, hence $r(N)$ are a sequence of integers such that $\frac{r(N)-Np}{\sqrt{N}}$ tends to 0 as $N \to \infty$, where $0 < p < 1$ and let $\xi_p = F^{-1}(p)$. If $F'(\xi_p) = f(\xi_p)$ exists and is positive then $\hat{\xi}_p$ is normally distributed with mean ξ_p and variance $\frac{p(1-p)}{Nf(\xi_p)^2}$,

$$\hat{\xi}_p \sim \mathcal{N}\left(\xi_p, \frac{p(1-p)}{N\left[f(\xi_p)\right]^2}\right). \tag{7.15}$$

Proof: Our proof follows the same strategy as the standard proof of the central limit theorem using characteristic functions [81]. Let $\mathbb{P}_N(z)$ be defined as,

$$\mathbb{P}_N(z) = \mathbb{P}\left\{\hat{\xi}_p \leq \xi_p + \frac{z}{\sqrt{N}f(\xi_p)}\right\} . \tag{7.16}$$

As $F(x)$ is monotonically increasing, define $\hat{p} = F(\hat{\xi}_p)$, thus Eq. (7.16) becomes,

$$\mathbb{P}_N(z) = \mathbb{P}\left\{\hat{p} \leq F\left(\xi_p + \frac{z}{\sqrt{N}f(\xi_p)}\right)\right\} . \tag{7.17}$$

As F is absolutely continuous, the derivative $F' = f$ exists almost everywhere, therefore we can expand it to at least first order at $x = \xi_p + \frac{z}{\sqrt{N}f(\xi_p)}$,

$$\begin{aligned}
F\left(\xi_p + \frac{z}{\sqrt{N}f(\xi_p)}\right) &= F(\xi_p) + F'(\xi_p)\frac{z}{\sqrt{N}f(\xi_p)} + \mathcal{O}\left(\frac{1}{N}\right)\\
F\left(\xi_p + \frac{z}{\sqrt{N}f(\xi_p)}\right) &= p + f(\xi_p)\frac{z}{\sqrt{N}f(\xi_p)} + \mathcal{O}\left(\frac{1}{N}\right)\\
F\left(\xi_p + \frac{z}{\sqrt{N}f(\xi_p)}\right) &= p + \frac{z + \mathcal{O}(N^{-\frac{1}{2}})}{\sqrt{N}} .
\end{aligned} \tag{7.18}$$

Substituting Eq. (7.18) into Eq. (7.17) yields,

$$
\begin{aligned}
\mathbb{P}_N(z) &= \mathbb{P}\left\{\hat{\xi}_p \le p + \frac{z+\mathcal{O}(N^{-\frac{1}{2}})}{\sqrt{N}}\right\} \\
&= \mathbb{P}\left\{N\hat{\xi}_p \le N\left(p + \frac{z+\mathcal{O}(N^{-\frac{1}{2}})}{\sqrt{N}}\right)\right\} \\
&= \mathbb{P}\left\{N_n \ge r(N)\right\} \ ,
\end{aligned} \tag{7.19}
$$

where N_n denotes the number of observations not exceeding $p + \frac{z+\mathcal{O}(N^{-\frac{1}{2}})}{\sqrt{N}}$ and $r(N)$ is as defined as above ($r(N) = [Np] + 1$). Let $p_n = p + \frac{z+\mathcal{O}(N^{-\frac{1}{2}})}{\sqrt{N}}$ and define W_n,

$$
W_n \equiv \frac{N_n - Np_n}{\sqrt{N}} \ . \tag{7.20}
$$

This definition allows Eq. (7.19) to be rearranged into,

$$
\begin{aligned}
\mathbb{P}_N(z) &= \left\{W_n \ge \frac{r(N) - Np_n}{\sqrt{N}}\right\} \\
&= \left\{W_n \ge \frac{r(N) - Np}{\sqrt{N}} - \left[z + O(N^{-\frac{1}{2}})\right]\right\} \\
&= \left\{W_n \ge \frac{r(N) - Np}{\sqrt{N}} - O(N^{-\frac{1}{2}}) - z\right\} \ .
\end{aligned} \tag{7.21}
$$

To proceed, we determine the characteristic function of W_n, and aim show that it becomes the characteristic function of the normal distribution in the limit as $N \to \infty$. Therefore we can can conclude that W_n is normally distributed in the limit as $N \to \infty$. The characteristic function, $S(t)$ of a distribution s is defined in the usual way,

$$
S(t) = \mathbb{E}\left\{\exp\left[its\right]\right\} \ . \tag{7.22}
$$

The characteristic function of the normal distribution, $\phi(t)$, is given by,

$$
\phi(t) = \exp\left[it\mu - \frac{1}{2}\sigma^2 t^2\right] \ . \tag{7.23}
$$

We elect to deal with the logarithm of the characteristic function [81] of W_n, denoted $\psi_n(t)$,

$$
\begin{aligned}
\psi_n(t) &= \log\left[\mathbb{E}\left\{\exp\left(itW_n\right)\right\}\right] \\
&= \log\left[\mathbb{E}\left\{\exp\left(it\frac{N_n - Np_n}{\sqrt{N}}\right)\right\}\right] \\
&= \log\left[\mathbb{E}\left\{\exp\left(it\frac{N_n}{\sqrt{N}}\right)\right\}\right] - \sqrt{N}p_n it \ .
\end{aligned} \tag{7.24}
$$

As N_n has a binomial distribution, $B(N, p_n)$, one can substitute the characteristic function of the Binomial distribution $\left\{1 + p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right)\right\}^N$ into Eq. (7.24) to yield,

$$
\begin{aligned}
\psi_n(t) &= \log\left[\left\{1 + p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right)\right\}^N\right] - \sqrt{N}p_n it \\
&= N\log\left\{1 + p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right)\right\} - \sqrt{N}p_n it \ .
\end{aligned} \tag{7.25}
$$

To simplify the first term in Eq. (7.25) we introduce the Taylor's series expansion of $\log(1+h)$

$$
\begin{aligned}
\log\{1+h\} &= \log(1) + h\,\frac{d}{dx}\log(x)\bigg|_{x=1} + \frac{h^2}{2}\,\frac{d^2}{dx^2}\log(x)\bigg|_{x=1} + \mathcal{O}(h^3) \\
&= h - \frac{h^2}{2} + \mathcal{O}(h^3)\ .
\end{aligned} \tag{7.26}
$$

Now, to make Eq. (7.26) look like the first term of Eq. (7.25) we define h,

$$
h \equiv p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right). \tag{7.27}
$$

This definition allows us to specify the higher order terms as a function of t, N and p_n,

$$
\begin{aligned}
h &\equiv p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right) = p_n\left(\sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{it}{\sqrt{N}}\right)^j - 1\right) \\
&\equiv p_n\sum_{j=1}^{\infty}\frac{1}{j!}\left(\frac{it}{\sqrt{N}}\right)^j = \frac{p_n t}{\sqrt{N}}\sum_{j=1}^{\infty}\frac{i^j}{j!}\left(\frac{t}{\sqrt{N}}\right)^{j-1} \\
\Rightarrow h &\propto \frac{p_n t}{\sqrt{N}} \Rightarrow \mathcal{O}(h) \approx \mathcal{O}\left(\frac{p_n t}{N}\right).
\end{aligned} \tag{7.28}
$$

Substituting Eq. (7.27) into Eq. (7.26) yields,

$$
\log\{1+h\} = p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right) - \frac{p_n^2}{2}\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right)^2 + \mathcal{O}\left(\frac{p_n^3 t^3}{N^{\frac{3}{2}}}\right). \tag{7.29}
$$

We can then incorporate Eq. (7.29) into our expression of $\psi_n(t)$ to produce,

$$
\begin{aligned}
\psi_n(t) &= N\left[p_n\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right) - \frac{p_n^2}{2}\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right)^2 + \mathcal{O}\left(\frac{p_n^3 t^3}{N^{\frac{3}{2}}}\right)\right] - \sqrt{N}p_n it \\
&= Np_n\left\{\exp\left(\frac{it}{\sqrt{N}}\right) - 1 - \frac{it}{\sqrt{N}}\right\} - \frac{Np_n^2}{2}\left(\exp\left(\frac{it}{\sqrt{N}}\right) - 1\right)^2 + \mathcal{O}\left(\frac{p_n^3 t^3}{\sqrt{N}}\right)\ .
\end{aligned} \tag{7.30}
$$

To further simplify this expression, another Taylor series expansion is undertaken,

$$
\begin{aligned}
\exp\left(\frac{it}{\sqrt{N}}\right) &= \sum_{j=0}^{\infty}\frac{1}{j!}\left(\frac{it}{\sqrt{N}}\right)^j \\
&= 1 + \frac{it}{\sqrt{N}} - \frac{1}{2}\frac{t^2}{N} + \mathcal{O}\left(\frac{t^3}{N^{\frac{3}{2}}}\right)\ .
\end{aligned} \tag{7.31}
$$

Employing Eq. (7.31) in Eq. (7.30) allows one to simplify $\psi_n(t)$ to,

$$
\begin{aligned}
\psi_n(t) &= Np_n\left\{1 + \frac{it}{\sqrt{N}} - \frac{1}{2}\frac{t^2}{N} + \mathcal{O}\left(\frac{t^3}{N^{\frac{3}{2}}}\right) - 1 - \frac{it}{\sqrt{N}}\right\} \\
&\quad - \frac{Np_n^2}{2}\left(1 + \frac{it}{\sqrt{N}} + \mathcal{O}\left(\frac{t^2}{N}\right) - 1\right)^2 + \mathcal{O}\left(\frac{p_n^3 t^3}{\sqrt{N}}\right) \\
&= -\frac{Np_n}{2}\frac{t^2}{N} + \frac{Np_n^2}{2}\frac{t^2}{N} + \mathcal{O}\left(\frac{t^3}{\sqrt{N}}\right) \\
\psi_n(t) &= -\frac{1}{2}p_n\left(1 - p_n\right)t^2 + \mathcal{O}\left(\frac{t^3}{\sqrt{N}}\right)\ .
\end{aligned} \tag{7.32}
$$

In the limit as $N \to \infty$, $p_n \to p$ and $O\left(\frac{t^3}{\sqrt{N}}\right) \to 0$, hence $\psi_n(t)$ is reduced to,

$$\lim_{N\to\infty} \psi_n(t) = -\frac{1}{2}p\,(1-p)\,t^2 \ . \tag{7.33}$$

By comparing Eq. (7.33) to Eq. (7.23) we find that $\lim_{N\to\infty}\psi_n(t)$ is the logarithm of the characteristic function for a normal distribution (i.e. $\lim_{N\to\infty}\psi_n(t) = \log[\phi(t)]$) with mean 0 and variance $p(1-p)$. Therefore we can conclude that W_n is normally distributed in the limit as $N \to \infty$,

$$\lim_{N\to\infty} W_n \sim \mathcal{N}(0, p(1-p)) \ . \tag{7.34}$$

Making use of Eq. (7.21) we can see that for sufficiently large N, and arbitrary ϵ, Eq. (7.21) will lie between,

$$\begin{aligned}
\mathbb{P}(W_n \geq \epsilon - z) &= 1 - F_N\left(\epsilon - z|0, p(1-p)\right) \\
&= 1 - \frac{1}{2}\left[1 + \text{erf}\left(\frac{\epsilon - z}{\sqrt{2p(1-p)}}\right)\right] \\
&= \frac{1}{2}\left[1 - \text{erf}\left(\frac{-(z-\epsilon)}{\sqrt{2p(1-p)}}\right)\right] \\
&= \frac{1}{2}\left[1 + \text{erf}\left(\frac{z-\epsilon}{\sqrt{2p(1-p)}}\right)\right] \\
&= F_N\left(z|\epsilon, p(1-p)\right)
\end{aligned} \tag{7.35}$$

$$\begin{aligned}
\mathbb{P}(W_n \geq -\epsilon - z) &= 1 - F_N\left(-\epsilon - z|0, p(1-p)\right) \\
&= 1 - \frac{1}{2}\left[1 + \text{erf}\left(\frac{-\epsilon - z}{\sqrt{2p(1-p)}}\right)\right] \\
&= \frac{1}{2}\left[1 - \text{erf}\left(\frac{-(z+\epsilon)}{\sqrt{2p(1-p)}}\right)\right] \\
&= \frac{1}{2}\left[1 + \text{erf}\left(\frac{z+\epsilon}{\sqrt{2p(1-p)}}\right)\right] \\
&= F_N\left(z|-\epsilon, p(1-p)\right)
\end{aligned} \tag{7.36}$$

Therefore, for sufficiently large N, $\mathbb{P}_N(z)$ is distributed normally,

$$\begin{aligned}
\lim_{N\to\infty} \mathbb{P}_N(z) &= \frac{1}{2}\left[1 + \text{erf}\left(\frac{z}{\sqrt{2p(1-p)}}\right)\right] \\
&= F_N(z|0, p(1-p)) \ .
\end{aligned} \tag{7.37}$$

Taking the limit of Eq. (7.16) as $N \to \infty$ and using Eq. (7.37) gives,

$$\begin{aligned}
&\lim_{N\to\infty} \mathbb{P}\left\{\hat{\xi}_p \leq \xi_p + \frac{z}{\sqrt{N}f(\xi_p)}\right\} = F_N(z|0, p(1-p)) \\
&\text{Let } z' \equiv \xi_p + \frac{z}{\sqrt{N}f(\xi_p)} \Rightarrow z = \sqrt{N}f(\xi_p)\,(z' - \xi_p) \\
&\Rightarrow \lim_{N\to\infty} \mathbb{P}\left\{\hat{\xi}_p \leq z'\right\} = F_N(\sqrt{N}f(\xi_p)\,(z'-\xi_p)\,|0, p(1-p)) \\
&\qquad = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\sqrt{N}f(\xi_p)\,(z'-\xi_p)}{\sqrt{2}\sqrt{p(1-p)}}\right)\right] \\
&\qquad = \frac{1}{2}\left[1 + \text{erf}\left(\frac{z'-\xi_p}{\sqrt{2}\frac{\sqrt{p(1-p)}}{\sqrt{N}f(\xi_p)}}\right)\right] \\
&\qquad = F_N\left(z'|\xi_p, \frac{p(1-p)}{Nf(\xi_p)^2}\right) \ .
\end{aligned} \tag{7.38}$$

Therefore $\hat{\xi}_p$ is normally distributed with mean ξ_p and variance $\frac{p(1-p)}{Nf(\xi_p)^2}$,

$$\lim_{N\to\infty} \hat{\xi}_p \sim \mathcal{N}\left(\xi_p, \frac{p(1-p)}{Nf(\xi_p)^2}\right) \ . \tag{7.39}$$

Q.E.D.

From this theorem we can conclude that the uncertainty interval, I_u, at the one sigma confidence level is,

$$I_u = \left(\hat{\xi}_p - \sqrt{\frac{p(1-p)}{Nf(\xi_p)^2}}, \hat{\xi}_p + \sqrt{\frac{p(1-p)}{Nf(\xi_p)^2}} \right) \ . \tag{7.40}$$

Note:
To determine the pdf (f) of the test statistics one produces a normalised histogram where the bin-widths, h, are determined by the Freedman-Diaconis rule [85],

$$h = 2\frac{IQR}{N^{\frac{1}{3}}} \ , \tag{7.41}$$

where IQR denotes the interquartile range. The pdf can then be interpolated and evaluated at the estimated percentile, $\hat{\xi}_p$, to yield an estimate of $f(\xi_p)$.

7.3 Schafer's Method

In this method, one has C sets of M i.i.d. test statistics and desires to determine the p percentile, ξ_p. For direct comparison to the single-sample methods, one requires the same number of test statistics be analysed, i.e. $N = CM$. Each set of statistics, $T_{(i)j}, i \in (1, ..., M), j \in (1, ..., C)$, produces an estimator $\hat{\xi}_{p_j}$

$$\hat{\xi}_{p_j} = T_{(k)j} \quad \text{where} \quad k = [Mp] + 1 \ . \tag{7.42}$$

The central limit theorem states that these $\hat{\xi}_{p_j}$ are normally distributed for "large" enough C provided that the third moment is finite (by the Berry-Esseen result [86] [87]). A rule of thumb for determining whether C is "large" enough requires that $C \geq 40$, in this work the multi-sample method is only employed for situations where $C \geq 100$ (and $M \geq 10,000$), therefore the central limit theorem can be applied throughout. The mean of $\hat{\xi}_{p_j}$ becomes the estimator $\hat{\xi}_p$ in this method,

$$\hat{\xi}_p = \frac{1}{C}\sum_{j=1}^{C} \hat{\xi}_{p_j} \ . \tag{7.43}$$

Consider the case in which the true percentile, ξ_p, is known *a priori* and one wishes to determine whether $\hat{\xi}_p$ will converge to ξ_p as $C \to \infty$ or $M \to \infty$. A Student's t test can be conducted to evaluate such a hypothesis, the t-statistic employed in this test is defined as,

$$t = \frac{\hat{\xi}_p - \xi_p}{\frac{s}{\sqrt{C}}} \ , \tag{7.44}$$

where s is the sample standard deviation. In this case, one wishes to evaluate $|\hat{\xi}_p - \xi_p|$, hence a two-sided test is employed and thus the relevant critical values for a significance level α are given by $t_{1-\frac{\alpha}{2}}$ and $t_{\frac{\alpha}{2}}$ for the upper and lower limits respectively. The critical values are defined by,

$$t_y = F^{-1}(y) \quad \text{where} \quad y \in (0, 1) \ , \tag{7.45}$$

where F^{-1} is the inverse cdf of the Student's t distribution with $c-1$ degrees of freedom. The t-statistic is symmetric about ξ_p, hence $t_{\frac{\alpha}{2}} = -t_{1-\frac{\alpha}{2}}$. For $\hat{\xi}_p$ to converge to ξ_p (up to the bias, see section 7.5) at the α significance level one requires $-t_{1-\frac{\alpha}{2}} < t < t_{1-\frac{\alpha}{2}}$, which implies,

$$\begin{aligned} & t < t_{1-\frac{\alpha}{2}} & & -t_{1-\frac{\alpha}{2}} < t \\ \Rightarrow & \frac{\hat{\xi}_p - \xi_p}{\frac{s}{\sqrt{C}}} < t_{1-\frac{\alpha}{2}} & \Rightarrow & -t_{1-\frac{\alpha}{2}} < \frac{\hat{\xi}_p - \xi_p}{\frac{s}{\sqrt{C}}} \\ \Rightarrow & \hat{\xi}_p - \xi_p < t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}} & \Rightarrow & -t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}} < \hat{\xi}_p - \xi_p \\ \Rightarrow & \hat{\xi}_p - t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}} < \xi_p & \Rightarrow & \xi_p < \hat{\xi}_p + t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}} \end{aligned}$$

$$\Rightarrow \hat{\xi}_p - t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}} < \xi_p < \hat{\xi}_p + t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}} \ . \tag{7.46}$$

Therefore by setting a significance level α, one can determine the confidence interval, I_s for the true quantile ξ_p [78, 83] defined as,

$$I_s = (\hat{\xi}_p - t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}}, \hat{\xi}_p + t_{1-\frac{\alpha}{2}}\frac{s}{\sqrt{C}}) \ . \tag{7.47}$$

Note: At the one-sigma confidence level the critical value of the student's t-test is one.

7.4 Numerical Comparison

To compare the three methods of determining uncertainty, both the multi-sample and single-sample methods were carried out. In both cases $1,000,000$ test statistics were collected. For the single sample, this was conducted as one $N = 1,000,000$ run; in the multi-sample case 100 runs, each with $10,000$ test statistics were recorded, i.e. $C = 100, M = 10,000$. The uncertainty margins were determined for a confidence level of one-sigma, i.e. $\alpha = \frac{1}{2}\left[1 - \text{erf}\left(\frac{1}{\sqrt{2}}\right)\right]$, and $t_{1-\frac{\alpha}{2}} = 1$. Table 7.2 displays the confidence interval lengths (CILs) of the Kolmogorov-Smirnov critical values for the case II Weibull at the 95% significance level. **All CILs in Table 7.2 have been multiplied by 10^5.**

Table 7.2 shows that the CILs for each method are very similar, hence, one cannot claim that a particular method produces the smallest confidence intervals consistently. Juritz et al. [78] and Cuddington et al. [79] have previously determined the samples required for a given precision in a number of quantiles of the standard normal distribution. In both cases, it is claimed that a significantly higher number of test statistics is required in order to declare the level of precision achieved in this work. The apparent discrepancy is attributable to the variance of the distribution from which test statistics are drawn. The standard normal distribution has $\sigma = 1$, however the data corresponding to Table 7.2 have a standard deviation of approximately 0.2. As the standard deviation of the normal distribution decreases the pdf changes more rapidly at a given percentile, hence, one anticipates a smaller confidence interval. More accurate estimates for the confidence intervals can be achieved by assuming the test statistics are drawn from a normal distribution with $\sigma = 0.2$ [2]. Table 7.3 shows the anticipated CILs as calculated by David's and Mosteller's methods when the test statistics are drawn from a normal distribution with standard deviation 1 and 0.2.

[2] The normal distribution is not a good approximation for the distribution of the test statistics, however, it should suffice in this case as we are only aiming to ensure that the lengths of the confidence intervals are of the correct order of magnitude.

		p											
n	Uncertainty	0	0.0323	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.8606	0.9
30	Schafer	75	79	72	69	76	72	79	72	80	89	81	81
"	David	84	74	73	74	75	73	73	75	74	78	78	80
"	Mosteller	81	75	74	73	73	74	74	77	76	79	79	80
50	Schafer	87	82	73	73	72	75	82	81	92	83	79	82
"	David	82	81	74	71	77	75	78	81	80	79	81	80
"	Mosteller	82	78	75	75	75	76	77	78	79	81	82	83
100	Schafer	80	79	81	69	66	78	75	75	88	75	84	82
"	David	86	81	79	79	78	77	82	81	82	81	84	84
"	Mosteller	83	80	77	76	77	77	80	80	82	83	83	86
200	Schafer	83	75	73	75	81	79	84	72	88	82	84	73
"	David	83	79	78	78	77	80	83	79	84	86	88	82
"	Mosteller	84	80	76	77	77	78	78	81	82	84	84	86
500	Schafer	72	67	70	71	84	77	74	82	82	80	80	88
"	David	82	81	79	75	79	77	77	81	83	86	88	86
"	Mosteller	84	81	78	76	76	78	80	81	82	85	86	86
1,000	Schafer	83	90	83	70	76	79	91	84	80	85	88	88
"	David	86	80	77	78	78	77	83	81	82	82	86	83
"	Mosteller	84	81	77	77	77	79	80	81	84	85	86	87
10,000	Schafer	93	80	88	70	75	70	77	90	83	87	82	90
"	David	83	81	76	76	77	78	76	83	82	86	86	88
"	Mosteller	84	80	78	77	77	78	78	82	82	85	85	87

Table 7.2: Various uncertainty margins for the Kolmogorov-Smirnov 95% case II Weibull
All CILs have been multiplied by 10^5

N	σ	David ($\mu = 0$)	David ($\mu = 100$)	Mosteller
100,000	1	0.0135±0.0012	0.0135±0.0011	0.0134
179,000	1	0.0100±0.00072	0.0101±0.00071	0.0100
1,000,000	1	0.00424±0.00020	0.00424±0.00021	0.00423
100,000	0.2	0.00269±0.00024	0.00270±0.00022	0.00267
179,000	0.2	0.00200±0.00014	0.00200±0.00014	0.00200
1,000,000	0.2	0.000849±0.000040	0.000848±0.000040	0.000843

Table 7.3: David and Mosteller CILs for the 95^{th} percentile of normally distributed test statistics

David's method of determining the uncertainty margin specifies the indices at the limits of the confidence interval. As the values these indices correspond to are dependent upon the specific test statistics drawn, each data set produces a confidence interval of different length. The David values and uncertainty margins in Table 7.3 are the mean and standard deviation of 1,000 simulated confidence interval lengths respectively. Both the David and Mosteller values support the earlier claim that a lower standard deviation should lead to a smaller confidence interval [3]. Additionally, simulations were conducted for $\mu = 0$ and $\mu = 100$ for David's method to verify that the length of the confidence interval is independent of μ; this proved to be the case as displayed in Table 7.3 (the confidence interval lengths are analytically identical for different values of μ in Mosteller's

[3] It appears as though there is a direct proportionality between the standard deviation and the length of the confidence interval.

method). The David and Mosteller uncertainty margins for $N = 1,000,000$ and $\sigma = 0.2$ in Table 7.3 have strong agreement with the uncertainty margins taken from the Monte Carlo simulations in Table 7.2.

To validate the simulation procedure used to produce Table 7.3, the results were compared to those quoted by Cuddington et. al. [79] and Juritz et. al. [78]. The Mosteller uncertainty regions in Table 7.3 have the same precision as stated in Table 2 of Cuddington et al. [79], a comparison is displayed in Table 7.4. Juritz et al. [78] and Stephens [88] studied the David uncertainty and employed a 95% confidence level in determining the uncertainty region, hence, their CILs are larger than that of the current work. Multiplying the values given in Table 7.3 by 1.96 extends the confidence level to 95%, allowing a direct comparison between Juritz et al. and the present work ("Current Adjusted" column of Table 7.5). In Table 7.4 and Table 7.5 the "Current" column refers to the results of this thesis.

The strong agreement between the present work and that of Cuddington et al. [79] and Juritz et al. [78] provides support for the uncertainty methodology outlined in this section, and gives credibility to the CILs given in Table 7.2. The reader should not be concerned at the high level of precision declared in Table 7.2 as it is concurs with the literature [78, 79, 88].

	Cuddington et al. [79]	Current
N	179,00	179,000
σ	1	1
α	0.683	0.683
CIL.	0.010	0.0100

Table 7.4: Mosteller confidence interval lengths

	Juritz et al. [78]	Current	Current Adjusted
N	100,00	100,000	100,000
σ	1	1	1
α	0.95	0.683	0.95
CIL	0.026	0.0135	0.0264

Table 7.5: David confidence interval lengths

7.5 Bias

The bias of a statistical estimator is the difference between the expected value of the estimator and the true value of the quantity. Unbiased estimators are always preferable, however it is not always possible to employ an unbiased estimator. In this section we determine the bias of the $100p$ percentile estimate, ξ_p, of a set of N test values and discuss the conditions under which the estimator is unbiased. Ultimately, we wish to determine whether the bias provides evidence for selecting the single-sample or multi-sample method of data generation. The bias, B, is given by,

$$\begin{aligned} B &= \mathbb{E}\left[\hat{\xi}_p\right] - \xi_p \\ &= \mathbb{E}\left[T_{(k)}\right] - \xi_p, \ \text{where } k = [Np] + 1 \ ([x] \text{ denotes the floor of } x) \ , \end{aligned} \tag{7.48}$$

and $T_{(k)}$ is the k^{th} highest test value in the set. The probability of a randomly selected test value, $T_{(j)}$ $j \in (1, ..., N)$, being below $\hat{\xi}_p$ is given by,

$$p_k \equiv \frac{k}{N+1} \ . \tag{7.49}$$

The difference between p and p_k is the ultimate source of the bias in ξ_p, and can be written as,

$$p_k - p = \frac{[Np] + 1}{N+1} - p = \frac{1 - p - \epsilon_1}{N+1} \text{ where } \epsilon_1 = Np - [Np] \ . \tag{7.50}$$

We define $q_k \equiv 1 - p_k$ and note that $p_k - p \propto \frac{1}{N+1}$, therefore $p_k - p$ is of order $\frac{1}{N}$ for large N; additionally, ϵ_1 decreases quickly with N. p_k and p are equal only when p is an integer multiple of $\frac{1}{N+1}$, hence, **future studies can achieve an unbiased estimator** through careful selection of N. The test statistics have the cdf, $F(T)$ and pdf, $f(T)$ such that $F'(T) = f(T)$. Additionally, we define u such that $u = F(T)$ and Q such that $F^{-1}(T) = Q(T)$,

$$\hat{\xi}_p = T_{(k)} = F^{-1}(u_k) = Q(u_k) \ . \tag{7.51}$$

We undertake a Taylor's series expansion of the percentile estimate, $T_{(k)}$, about p_k,

$$T_{(k)} = Q(u_k) = Q(p_k) + (u_{(k)} - p_k)Q'(p_k) + \frac{(u_{(k)} - p_k)^2}{2}Q''(p_k) + \mathcal{O}\left((u_{(k)} - p_k)^3\right) \ . \tag{7.52}$$

By neglecting higher order terms we can calculate the expectation value of this expansion,

$$\mathbb{E}\left[T_{(k)}\right] = Q(p_k) + \mathbb{E}\left[(u_{(k)} - p_k)Q'(p_k)\right] + \frac{1}{2}\mathbb{E}\left[(u_{(k)} - p_k)^2 Q''(p_k)\right] \ . \tag{7.53}$$

The second term of Eq. (7.53) vanishes,

$$\begin{aligned}\mathbb{E}\left[(u_{(k)} - p_k)Q'(p_k)\right] &= \left(\mathbb{E}\left[u_{(k)}\right] - p_k\right) Q'(p_k) \\ &= (p_k - p_k)\, Q'(p_k) \\ &= 0 \ ,\end{aligned} \tag{7.54}$$

however, the third term is non-zero,

$$\mathbb{E}\left[(u_{(k)} - p_k)^2 Q''(p_k)\right] = \frac{p_k q_k}{N+2}Q''(p_k) \text{ (David page 36 [82])} \ . \tag{7.55}$$

This reduces Eq. (7.53) to,

$$\mathbb{E}\left[T_{(k)}\right] = Q(p_k) + \frac{1}{2}\frac{p_k q_k}{N+2}Q''(p_k) \ . \tag{7.56}$$

To proceed we expand $Q(p_k)$ about $Q(p)$,

$$Q(p_k) = Q(p) + (p_k - p)Q'(p) + \frac{1}{2}(p_k - p)^2 Q''(p) + \mathcal{O}((p_k - p)^3) \ , \tag{7.57}$$

and $Q''(p_k)$ about $Q''(p)$,

$$Q''(p_k) = Q''(p) + (p_k - p)Q'''(p) + \frac{1}{2}(p_k - p)^2 Q''''(p) + \mathcal{O}((p_k - p)^3) \ . \tag{7.58}$$

Recall that $p_k - p$ is of order $\frac{1}{N}$ for large N. We truncate Eq. (7.57) and (7.58) to terms of order $\frac{1}{N}$ and substitute them into Eq. (7.56) to yield,

$$\mathbb{E}\left[T_{(k)}\right] = Q(p) + (p_k - p)Q'(p) + \frac{1}{2}\frac{p_k q_k}{N+2}Q''(p) \ . \tag{7.59}$$

The third term in Eq. (7.59) can be written as,

$$\frac{1}{2}\frac{p_k q_k}{N+2}Q''(p) = \frac{(p + \{p_k - p\})(1 - p - \{p_k - p\})}{2(N+2)}Q''(p) \ , \tag{7.60}$$

removing higher order terms produces,

$$\frac{1}{2}\frac{p_k q_k}{N+2}Q''(p) \approx \frac{p(1-p)}{2(N+2)}Q''(p) \ . \tag{7.61}$$

The expansion for $p_k - p$, Eq. (7.50), and (7.61) can be substituted in Eq. (7.59) to produce,

$$\mathbb{E}\left[T_{(k)}\right] = Q(p) + \frac{1-p-\epsilon_1}{N+1}Q'(p) + \frac{1}{2}\frac{p(1-p)}{N+2}Q''(p) \ , \tag{7.62}$$

ϵ_1 can also be removed (due to its N dependence) to yield,

$$\mathbb{E}\left[T_{(k)}\right] = Q(p) + \frac{1-p}{N+1}Q'(p) + \frac{1}{2}\frac{p(1-p)}{N+2}Q''(p) \ . \tag{7.63}$$

To further simplify this expression we derive an expression for the derivative of $Q(p)$,

$$Q'(p) = \frac{d}{dp}F^{-1}(p) = \frac{1}{F'\left[F^{-1}(p)\right]} = \frac{1}{F'\left[\xi_p\right]} = \frac{1}{f\left[\xi_p\right]} \ , \tag{7.64}$$

and the second derivative of $Q(p)$,

$$\begin{aligned} Q''(p) &= \frac{d}{dp}\left(\frac{1}{f\left[\xi_p\right]}\right) = \left(\frac{d}{df\left[\xi_p\right]}\frac{1}{f\left[\xi_p\right]}\right)\left(\frac{d}{d\xi_p}f\left[\xi_p\right]\right)\left(\frac{d}{dp}\xi_p\right) \\ &= \left(\frac{-1}{f\left[\xi_p\right]^2}\right)f'\left[\xi_p\right]\left(\frac{d}{dp}F^{-1}(p)\right) \\ &= -\frac{f'\left[\xi_p\right]}{f\left[\xi_p\right]^3} \ . \end{aligned} \tag{7.65}$$

As a result, the expected value of $\hat{\xi}_p$ is given by,

$$\mathbb{E}\left[\hat{\xi}_p\right] = \mathbb{E}\left[T_{(k)}\right] \approx Q(p) + \frac{1-p}{N+1}\frac{1}{f\left[\xi_p\right]} - \frac{1}{2}\frac{p(1-p)}{N+2}\frac{f'\left[\xi_p\right]}{f\left[\xi_p\right]^3} \ , \tag{7.66}$$

and therefore the bias, B, of the estimator is,

$$\begin{aligned} B &= \mathbb{E}\left[\hat{\xi}_p\right] - \xi_p \\ &\approx \xi_p + \frac{1-p}{N+1}\frac{1}{f\left[\xi_p\right]} - \frac{1}{2}\frac{p(1-p)}{N+2}\frac{f'\left[\xi_p\right]}{f\left[\xi_p\right]^3} - \xi_p \\ &\approx \frac{1-p}{N+1}\frac{1}{f\left[\xi_p\right]} - \frac{1}{2}\frac{p(1-p)}{N+2}\frac{f'\left[\xi_p\right]}{f\left[\xi_p\right]^3} \ . \end{aligned} \tag{7.67}$$

We note that B is of order $\frac{1}{N}$ for large values of N, and that the uncertainty margin size is of order $\frac{1}{\sqrt{N}}$. Hence, we expect the uncertainty margin to dominate the bias in the limit as $N \to \infty$. To calculate the bias for the multi-sample method one can employ Eq. (7.67), however N must be replaced with the number of test values per sample, i.e. M. Therefore the bias takes the form,

$$B \approx \frac{1-p}{M+1}\frac{1}{f\left[\xi_p\right]} - \frac{1}{2}\frac{p(1-p)}{M+2}\frac{f'\left[\xi_p\right]}{f\left[\xi_p\right]^3} \ . \tag{7.68}$$

The Biases and uncertainty margins for a selection of the single-sample and multi-sample methods are given in Tables 7.6 and 7.7 respectively. The Bias and David CIL are the average of 100 samples, each with $M = 10,000$ test statistics. The Schafer CIL is the combined uncertainty from $C = 100$ data sets, hence the difference between the CILs is not surprising. The bias is significantly smaller for the single-sample method, however, in both cases the uncertainty is significantly larger than the bias. Therefore, the difference in bias should not make a significant contribution to choosing either the single-sample or multi-sample method. Future studies should seek to employ unbiased estimators by selecting N (or M) such that p is an integer multiple of $\frac{1}{N+1}$ (or $\frac{1}{M+1}$).

N	p	Bias	David CIL 1 set
30	0.9	$8.32e-7$	$8.02e-4$
100	0.3	$8.23e-7$	$7.62e-4$
500	0.7	$8.09e-7$	$8.32e-4$
10,000	0	$8.67e-7$	$8.46e-4$

Table 7.6: Single-sample Biases and David CIL
$N = 1,000,000$ case II Weibull 95% KS

N	p	Bias	David CIL 1 set	Schafer CIL 100 sets
30	0.9	$7.76e-5$	$8.13e-3$	$8.11e-4$
100	0.3	$6.70e-5$	$7.87e-3$	$6.63e-4$
500	0.7	$8.24e-5$	$8.51e-3$	$8.17e-4$
10,000	0	$8.99e-5$	$8.41e-3$	$9.27e-4$

Table 7.7: Multisample Biases, David and Schafer CILs
$C = 100$, $M = 10,000$ case II Weibull 95% KS

7.6 Single-sample v.s. Multi-sample

Juritz (1983) et al. [78] and Stephens [88] discuss the differences between the single-sample and multi-sample methods and conclude that one should utilise the single-sample method because

a Single-sample produces percentile estimates with a smaller bias [88]

b David's uncertainty method can be used to produce an confidence region symmetric in p

c Single-sample produces smaller confidence intervals [88].

In contrast, MacKinnon [89] and Cuddington [79] argue that the multi-sample method is preferable because

A Multi-sample allows for simple determination of the uncertainty margin

B Less memory is required as the program must store M rather than $N = CM$ test statistics

C Sorting M test statistics C times is faster than sorting $N = CM$ test statistics once

D Allows runs to be parallelised extensively.

MacKinnon and Cuddington were more concerned with practical issues associated with computation than the theoretical ones that Juritz et al. and Stephens discuss, this is perhaps no surprise given the astronomical increase in computational power between their publishing dates (2001 and 1983). The aforementioned issues are discussed below.

- (a) The bias is larger for percentile estimates which are taken from a smaller number of test statistics, hence it will indeed be larger for the multi-sample case than the single-sample. However, for the number and distribution of samples taken in the present work, the bias is negligible in comparison with the uncertainty margin, hence the reduction in bias associated with employing the single-sample method does not present a material advantage to the researcher.

- (b) It is preferable to have uncertainty margins that are symmetric in p space rather than $T_{(i)}$ space, however in the situation of interest in this work the two are very similar.

- (c) Juritz et al. and Stephens found that the confidence intervals resulting from the multi-sample method were larger than for the single-sample, however the present work found that the methods were largely equivalent (refer to Table 7.2). The present work used a much large number of samples, $C = 100$ and $M = 10,000$ v.s. $C = 10$ and $M = 500$, hence this point did not induce significant concern [78, 88] [4].

- (A) The Schafer method of determining uncertainty is undoubtedly the most simple of the three proposed, however, a thorough analysis all three methods has already been conducted. Implementing a well understood method of determining uncertainty does not contribute significantly to the study's timeline, hence this point isn't pivotal in the decision making process.

- (B) To conduct our Monte Carlo simulations we used the Phoenix supercomputer which has 4GB of RAM allocated per CPU. This far exceeded the requirements of both the single-sample and multi-sample code developed for this work. Memory may become a limiting factor in the future as the number of samples increases, however, it was not a major concern here.

- (C) Reducing the amount of time required to run the Monte Carlo simulations was a significant part of this study. As a result, the reduction in run time associated with sorting smaller chunks of data (although small) provides support for the multi-sample method.

- (D) Increased parallelisation allows the workload to be distributed amongst many computers and/or nodes of a supercomputer, which reduces the run time. Additionally output can be written more frequently which reduces the amount of data lost due to unforeseen circumstances and allows the researcher to better track the progress of the simulation. This was the dominant factor in deciding between the single-sample and multi-sample methods.

Ultimately for this study the multi-sample method was selected, largely due to reason (D).

7.7 Summary

We elected to use the multi-sample method, primarily because it allowed us to parallelise the data generation procedure to a significant extent. This decreased the run time by a great deal, and reduced the susceptibility of the experiment to power and hardware failures. A number of others factors were considered, however parallelisation was determined to be the most important factor in this particular work. A thorough analysis of David's [82], Mosteller's [80] and Schafer's [83] methods for determining uncertainty was conducted and it was determined that they produce very similar size confidence intervals. Schafer's method is best suited to the multi-sample method, hence it was utilised to determine the uncertainty margin in this work.

[4] The data from Juritz et al. and Stephens was drawn from a standard normal distribution whereas this study utilised and is concerned with test statistics drawn from a different distribution, this may contribute to the discrepancy [78] [88]

Chapter 8

Quantile Analysis

In this chapter we present the results of the Monte Carlo procedure for the cases, distributions, goodness-of-fit tests and significance levels listed below. It is not possible to assess case II or case IIIb for the Pareto distribution as it has no shape parameter, however, every other permutation of the listed options has been studied. For each of these permutations, the number of observations in a data set, n, and the probability of truncation, p took the values specified by Eq. (8.1). A separate set of Monte Carlo simulations was required for every unique combination of n, p, case and distribution. Due to their cumulative volume, it is not possible to display the critical values for every combination in this chapter, hence a representative sample is given. The complete set of critical values is available in Appendix A.

Cases		Distributions:	Goodness-of-fit Tests	Significance Levels
• Case I:	scale: known shape: known	• Weibull	• Kolmogorov-Smirnov	• 85%
• Case II:	scale: acquired shape: acquired	• Loglogistic	• Kuiper	• 90%
• Case IIIa:	scale: known shape: acquired	• Lognormal	• Cramér-von Mises	• 95%
• Case IIIb:	scale: acquired shape: known	• Pareto	• Anderson-Darling	• 99%

$$
\begin{aligned}
n &\in \{30, 50, 100, 200, 500, 1000, 10000\} \\
p &\in \{0, 0.0323, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.8605, 0.9\}
\end{aligned}
\tag{8.1}
$$

All Monte Carlo simulations were conducted using the multi-sample method with 100 repetitions (i.e. $C = 100$) of $10,000$ data sets (i.e. $M = 10,000$) and employed Schafer's method [83] of determining the uncertainty margin (at the 95% confidence level). Using this method of uncertainty determination, it was possible to restrict the confidence interval length to 1% of the corresponding critical value in the worst case scenario, in many instances this length is less than 0.1% of the corresponding critical value. For more information on why the multi-sample method was employed please consult section 7.6.

Kizilersü et al. [10] determined the Kolmogorov-Smirnov critical values of the left-truncated Weibull distribution at the 95% significant level via the multi-sample method with 100 repetitions (i.e. $C = 100$) of $1,000$ data sets (i.e. $M = 1,000$). Tables 8.3, 8.8, 8.21 and 8.38 compare our results with those of Kizilersü et al. [10] for cases I, II, IIIa and IIIb respectively. Kizilersü et al. [10] employed a different method of determining the uncertainty margin, which produces significantly larger confidence interval lengths (CILs). They used the standard deviation of the (100) percentile estimates as their uncertainty margin, which is a factor of $\frac{\sqrt{100}}{1.984}$ [90] larger than the CILs resulting from the method we used (Schafer's method [83]). If we had collected the same amount of data as Kizilersü et al. [10], one would expected their CILs to be a factor of $\frac{\sqrt{100}}{1.984}$ larger than ours, however, we collected ten times as much data. In the large N limit CILs are proportional to $\frac{1}{\sqrt{N}}$, hence, we expect the CILs determined by Kizilersü et al. [10] to be a factor of $\frac{\sqrt{1000}}{1.984} \approx 15.94$ larger than ours. Kizilersü et al. [10] did not state the critical values at $p = 0.0323$ or $p = 0.8605$ for any case, nor did they state them for $n = 10,000$ for cases IIIa and IIIb. Hence, these values have been omitted from the aforementioned tables.

The results from the present work are compared against the available literature in a series of tables in Appendix B. The results of Kizilersü et al. [10], were tabulated separately as our work was based heavily upon theirs[1] and it is the only study (to our knowledge) that estimates the critical values of a left-truncated distribution of interest (Weibull, lognormal or loglogistic). The truncation probability, p, is a function of the distribution parameters $\boldsymbol{\theta}$ and τ_l (the left-truncation point), additionally η is a 1-1 function of p. Both p and η are monotone increasing functions of τ_l, hence, higher values of τ_l, p and η all indicate a greater level of truncation. All three of these are used to describe the level of truncation in this chapter.

The first sections in this chapter discuss the equivalence of different distributions and verify that the critical values are parameter independent, these sections are important for all cases. Following this, case I, II, IIIa and IIIb each have their own section, which is dedicated to displaying some sample results and determining whether the critical values are dependent upon sample size, n and left-truncation level, τ_l.

8.1 Parameter Independent Critical Values

In general, the critical values are dependent on the true parameter values of the distribution from which the observations were measured. Thus, one must repeat the Monte Carlo procedure for every set of parameter values which may be required, this is an extremely computationally expensive process. Additionally, in the real world one seldom knows what the true parameter values are, therefore, even if the required Monte Carlo procedures have been conducted it is often not possible to select the correct critical values. This is clearly not a tenable position for an analyst as it prevents them from having access to, and applying the relevant critical values for the vast majority of realistic scenarios. The solution is to find situations in which the critical values are parameter independent. In these situations, only one set of Monte Carlo simulations need be conducted, and their results are applicable regardless of what the true parameter values are, therefore, one need not know the true parameter values at all.

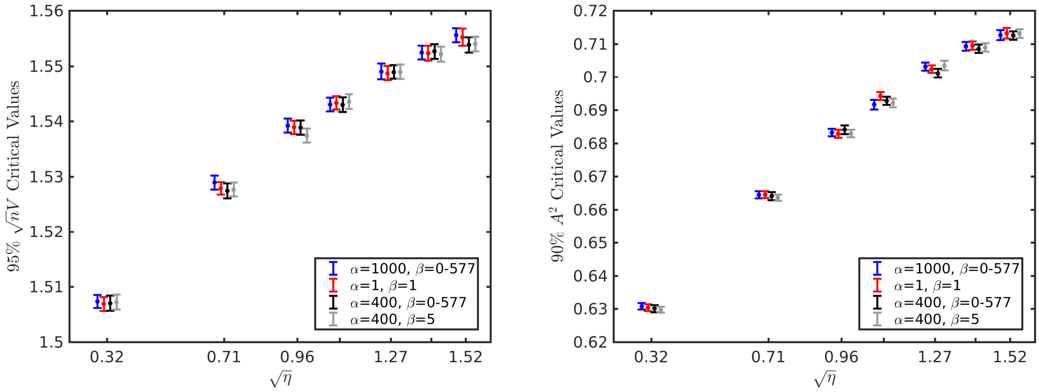
As shown analytically in sections 3.6, 4.5, 5.5 and 6.4, we expect the critical values determined in this work to be parameter independent, i.e. this study has been restricted to parameter independent situations. Without this condition, the critical values apply in such a small subset of scenarios that they are of extremely limited value. In this section, we compare the critical values determined from four distinct parameter sets for every distribution (Weibull, lognormal, loglogistic and Pareto). If the critical values are the same (within uncertainty margins) for all of the parameter sets, our algebraic findings will be supported and we can conclude that the critical values are parameter independent.

The complete set of results includes over 100 graphs for each distribution, hence it is only possible to display a small sample of them, which is done with Figures 8.1, 8.2, 8.3 and 8.4. It was anticipated that the likelihood of parameter dependence was most signif-

[1] Ayşe Kizilersü and Anthony W. Thomas were the supervisors of this thesis

icant for the cases which required the highest number of estimated parameters (i.e. case II for Weibull, loglogistic and lognormal; case IIIa for Pareto). As a result, the numerical search for parameter independence was restricted to these cases. For the sake of clarity, the critical values are only depicted for a subset of the studied range, and the critical values that result from different parameter configurations have been staggered along the $\sqrt{\eta}$ (τ_l for Pareto) axis; this does not imply that the critical were evaluated at distinct truncation levels. The sample size, n, and significance level appear to have a reasonably small effect upon the confidence interval length, it is not clear whether small fluctuations should be attributed to a general trend. As a result, the critical values have been displayed for different combinations of sample size and significance level for each distribution in order to depict the most diverse array of results possible.

8.1.1 Weibull

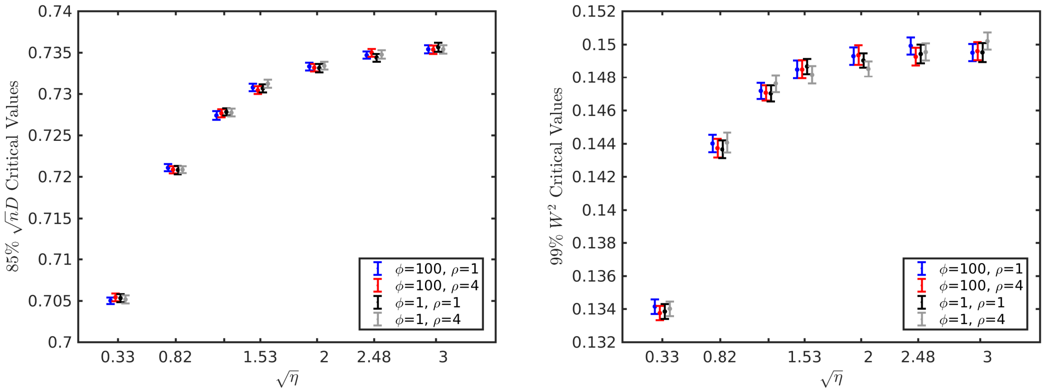


(a) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl. for $n = 1000$ observations

(b) Anderson-Darling (A^2) crit. vals. at 90% sig. lvl. for $n = 50$ observations

Figure 8.1: Case II Weibull critical values
Critical values from different parameter values are staggered on the $\sqrt{\eta}$ axis for clarity

8.1.2 Loglogistic

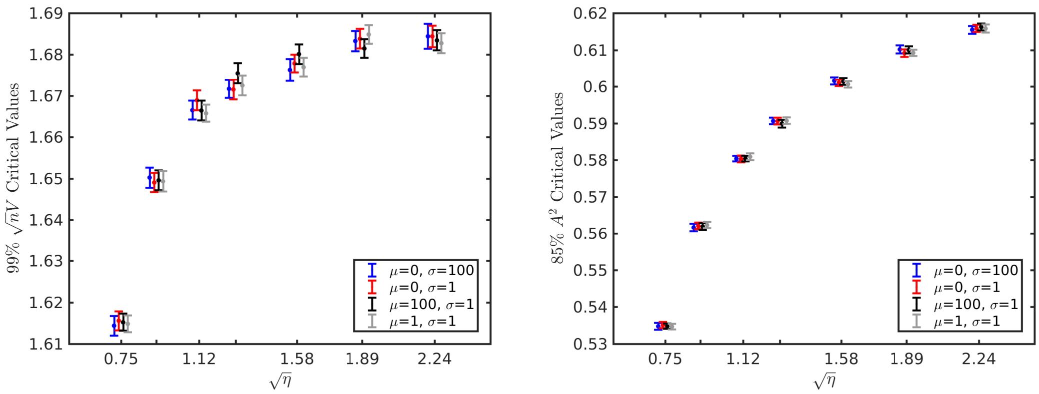


(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 85% sig. lvl. for $n = 10,000$ observations

(b) Cramér-von Mises (W^2) crit. vals. at 99% sig. lvl. for $n = 30$ observations

Figure 8.2: Case II Loglogistic critical values
Critical values from different parameter values are staggered on the $\sqrt{\eta}$ axis for clarity

8.1.3 Lognormal

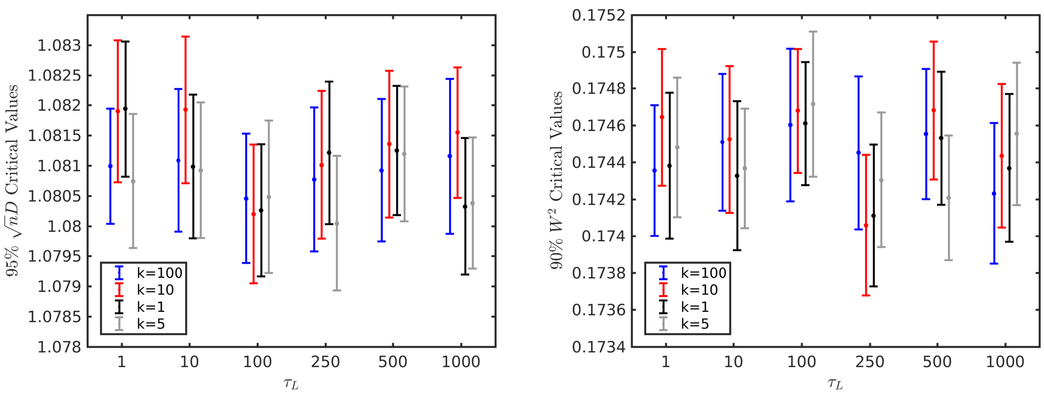


(a) Kuiper ($\sqrt{n}V$) crit. vals. at 99% sig. lvl. for $n = 30$ observations

(b) Anderson-Darling (A^2) crit. vals. at 85% sig. lvl. for $n = 10,000$ observations

Figure 8.3: Case II Lognormal critical values
Critical values from different parameter values are staggered on the $\sqrt{\eta}$ axis for clarity

8.1.4 Pareto



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl. for $n = 200$ observations

(b) Cramér-von Mises (W^2) crit. vals. at 90% sig. lvl. for $n = 500$ observations

Figure 8.4: Case IIIa Pareto critical values
Critical values from different parameter values are staggered on the τ_l axis for clarity

Figures 8.1, 8.2, 8.3 and 8.4 show strong agreement between the critical values from different parameter sets. Therefore, the results of the Monte Carlo procedure provide support for our algebraic findings. We can therefore conclude that the critical values are indeed parameter independent.

8.2 Equivalence of Distributions

There are a large number of extant probability distributions used to model data from a wide variety sources, several of them are identical under particular mappings and parameter redefinitions. Consider a random variable X with cdf $F_X(x)$ $x \in \mathcal{X}$, and another random variable Y with cdf $F_Y(y)$ $y \in \mathcal{Y}$. If there exists a one-to-one and onto map, Φ, such that $\Phi(X) = Y$ (i.e. $\Phi(x) = y$) we consider the two distributions to be equivalent. When this is the case, we can use a simplified form of the working employed by Casella and Berger in page 48, Chapter 2 of their 2002 book 'Statistical Inference' [91] to show that $F_X(x) = F_Y(y)$ under the mapping Φ,

$$\begin{aligned} F_Y(y) &= \mathbb{P}(Y \leq y) \\ &= \mathbb{P}(\Phi(X) \leq y) \\ &= \mathbb{P}(X \leq \Phi^{-1}(y)) \\ &= \mathbb{P}(X \leq x) = F_X(x) \ . \end{aligned} \tag{8.2}$$

A consequence of $\Phi(x) = y$ is that we can define Φ on the empirical distribution function. Recall,

$$F_{(n)X}(t) = \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{x_i \leq t} \text{ where } \mathbb{1}_{x_i \leq t} = \begin{cases} 1 \text{ if } x_i \leq \text{t} \\ 0 \text{ if } x_i > \text{t} \end{cases} . \tag{2.3 revisited}$$

If $\Phi(x)$ is monotone increasing,

$$\begin{aligned} x_i > t &\Rightarrow \Phi(x_i) > \Phi(t) \\ &\Rightarrow y_i > z \ . \end{aligned} \tag{8.3}$$

where $\Phi(t) = z$. Therefore, we can show that the empirical cdf is constant under Φ,

$$\begin{aligned} F_{(n)X}(t) &= \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{x_i \leq t} \\ &= \frac{1}{n}\sum_{i=1}^{n} \mathbb{1}_{y_i \leq z} \\ &= F_{(n)Y}(z) \ . \end{aligned} \tag{8.4}$$

Thus, the difference between the cdf and edf is constant with respect to the mapping, i.e.

$$F_X(x) - F_{(n)X}(x) = F_Y(y) - F_{(n)Y}(y) \quad \forall \quad x \in \mathcal{X} \quad \text{and} \quad \Phi(x) = y \in \mathcal{Y}. \tag{8.5}$$

As $F_X(x) - F_{(n)X}(x)$ determines the value of the test statistic for all of the goodness-of-fit tests considered in this work, the critical values are constant whether they are calculated in $\mathcal{X}$ or $\mathcal{Y}$ space. Therefore **Equivalent distributions have the same critical values**, hence, one can determine the critical values for a particular distribution and then utilise them for all distributions that are equivalent to it. For example, in this section, we show that the Weibull distribution is equivalent to the Extreme value distribution, therefore, the critical values that we produce for the Weibull distribution can be used by anyone using the Extreme value distribution. Additionally, we can directly compare our critical values for the Weibull distribution to the those in the literature that were determined for the Extreme value distribution. In this way, we can considerably extend both the volume of applicable literature and the number of situations in which our results can be used in the real world. We have explicitly studied both the lognormal and loglogistic distributions, in this section we show that they are equivalent to the normal and logistic distributions respectively. The normal and logistic distributions are far more common than the lognormal and loglogistic, hence, showing this equivalence is of great importance. In the remainder of this section the mapping between a number of relevant distributions is articulated.

8.2.1 Weibull and Extreme Value Distributions

Recall the complete Weibull cdf,

$$F_X(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \qquad (3.2 \text{ revisited})$$

Support	$0 < x < \infty$
Scale parameter	$0 < \alpha < \infty$
Shape parameter	$0 < \beta < \infty$.

The mapping Φ is defined such that,

$$\Phi : x \mapsto \exp(y) \ . \tag{8.6}$$

Trivially, this mapping is one-to-one and onto. Redefining the parameters as,

$$\alpha = \exp(\mu) \quad \text{and} \quad \beta = \frac{1}{\gamma} \ , \tag{8.7}$$

produces the cdf of the Extreme value distribution,

$$\begin{aligned} F_Y(y) &= 1 - \exp\left[-\left(\frac{\exp(y)}{\exp(\mu)}\right)^{\frac{1}{\gamma}}\right] \ , \\ &= 1 - \exp\left[-\left(\frac{\exp(y-\mu)}{\gamma}\right)\right] \end{aligned} \tag{8.8}$$

Support	$-\infty < y < \infty$
Scale parameter	$-\infty < \mu < \infty$
Shape parameter	$0 < \gamma < \infty$.

Therefore, Φ maps the cdf of the Weibull distribution to that of the Extreme value distribution. Hence, the distributions are equivalent and the critical values of Weibull distribution are the same as those of the Extreme value distribution.

8.2.2 Loglogistic and Logistic Distributions

Recall the complete loglogistic cdf,

$$F_X(x) = \frac{1}{1 + \left(\frac{x}{\phi}\right)^{-\rho}} \ . \qquad (4.2 \text{ revisited})$$

Support	$0 < x < \infty$
Scale parameter	$0 < \phi < \infty$
Shape parameter	$0 < \rho < \infty$.

The mapping Φ is defined such that,

$$\Phi : x \mapsto \exp(y) \ . \tag{8.9}$$

Trivially, this mapping is one-to-one and onto. Redefining the parameters as,

$$\phi = \exp(\alpha) \quad \text{and} \quad \rho = \frac{1}{\beta} \ , \tag{8.10}$$

produces the cdf of the logistic distribution,

$$\begin{aligned} F_Y(y) &= \frac{1}{1 + \left(\frac{\exp(y)}{\exp(\alpha)}\right)^{-\frac{1}{\beta}}} \ , \\ &= \frac{1}{1 + \exp\left(-\frac{y-\alpha}{\beta}\right)} \end{aligned} \tag{8.11}$$

Support	$-\infty < y < \infty$
Scale parameter	$-\infty < \alpha < \infty$
Shape parameter	$0 < \beta < \infty$.

Therefore, Φ maps the cdf of the loglogistic distribution to that of the logistic distribution. Hence, the distributions are equivalent and the critical values of loglogistic distribution are the same as those of the logistic distribution.

8.2.3 Lognormal and Normal Distributions

Recall the complete lognormal cdf,

$$F_X(x) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\log(x) - \mu}{\sigma\sqrt{2}}\right)\right] \tag{5.2 revisited}$$

Support	$0 < x < \infty$
Scale parameter	$-\infty < \mu < \infty$
Shape parameter	$0 < \sigma < \infty$.

The mapping Φ is defined such that,

$$\Phi : x \mapsto \exp(y) \ . \tag{8.12}$$

Trivially, this mapping is one-to-one and onto, additionally substituting it into Eq. (5.2) produces the cdf of the normal distribution,

$$F_Y(y) = \frac{1}{2}\left[1 + \text{erf}\left(\frac{y - \mu}{\sigma\sqrt{2}}\right)\right] \tag{8.13}$$

Support	$-\infty < y < \infty$
Scale parameter	$-\infty < \mu < \infty$
Shape parameter	$0 < \sigma < \infty$.

Therefore, Φ maps the cdf of the lognormal distribution to that of the normal distribution. Hence, the distributions are equivalent and the critical values of lognormal distribution are the same as those of the normal distribution.

8.3 Reparametrisations of Distributions

In some cases, distributions become identical under a change in the definition of their parameters. As the argument of the cdf is not subject to mapping, distributions which are related in this way are not equivalent, however, they will produce the same test statistics, and thus critical values. Therefore, we can still use these relations to extend the volume of literature we compare our results to. Additionally, others can employ the critical values we have generated to conduct hypothesis testing against a distribution which can be identical to the Weibull, loglogistic, lognormal or Pareto distributions under a reparametrisation. In the remainder of this section, we give two examples of reparametrisations of distributions that we have compared our results to.

8.3.1 Weibull and Exponential Distributions

Recall the complete Weibull cdf

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right] \qquad \text{(3.2 revisited)}$$

$$\begin{array}{ll} \text{Support} & 0 < x < \infty \\ \text{Scale parameter} & 0 < \alpha < \infty \\ \text{Shape parameter} & 0 < \beta < \infty \ . \end{array}$$

If restrict our study to case IIIa, and set the shape parameter to $\beta = 1$ the cdf becomes,

$$F(x) = 1 - \exp\left[-\frac{x}{\alpha}\right] \ . \tag{8.14}$$

Redefining the parameters as

$$\alpha = \frac{1}{\lambda} \ , \tag{8.15}$$

produces the cdf of the exponential distribution,

$$F(x) = 1 - \exp\left[-\lambda x\right] \tag{8.16}$$

$$\begin{array}{ll} \text{Support} & 0 < x < \infty \\ \text{Scale parameter} & 0 < \lambda < \infty \ . \end{array}$$

Therefore, the Weibull distribution with known shape parameter (such that $\beta = 1$ (case IIIa)) is identical to the exponential distribution under the reparametrisation $\alpha = \frac{1}{\lambda}$. As a result, the case IIIa Weibull distribution has the same critical values as the exponential distribution with unknown scale parameter. Due to parameter independence, the case IIIa Weibull distribution with any β value, will produce the same critical values as the exponential distribution. This is a surprising but helpful result.

8.3.2 Weibull and Pareto Distributions

Kreer et al. [42] found that there is L^1-convergence in the sense of Lebesgue between the left-truncated Weibull pdf and the Pareto pdf if the Weibull parameters $(\alpha, \beta) \to (0, 0)$ with $k = \alpha^{-\beta}\beta > 0$ and $\beta \in (0, \frac{k}{2})$. Under this reparametrisation the MLE equation for the scale parameter of Weibull distribution, α, reduces to an equation for the Pareto scale parameter, k. The details of this work are outside of the scope of this thesis, however, as the MLE equation for α reduces to one for k, the critical values for the Pareto distribution are the same as those of the case IIIa Weibull distribution, the case in which α must be estimated.

Distribution	Case(s)	Distribution	Equivalent or Reparametrised
Weibull	All	Extreme value	Equivalent
Loglogistic	All	Logistic	Equivalent
Lognormal	All	Normal	Equivalent
Weibull	IIIa	Exponential	Reparametrised
Weibull	IIIa	Pareto	Reparametrised

Table 8.1: Equivalent and Reparametrised Distributions

8.4 Case I

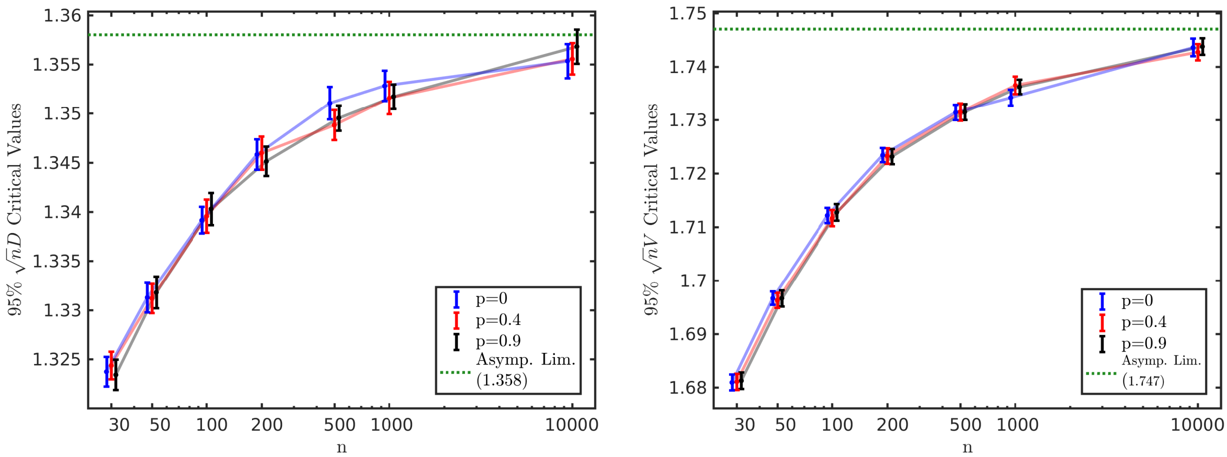
- Weibull
- Loglogistic
- Lognormal
- Pareto

8.4.1 Variable Dependence

In case I all parameters of the distribution are known *a priori*, therefore, the distribution is completely specified. This implies that the critical values are independent of (i) distribution, (ii) parameters and (iii) truncation level [12]. The Kolmogorov-Smirnov and Kuiper critical values were found to have strong dependence upon sample size, n, as displayed in Figure 8.5. The Cramér-von Mises and Anderson-Darling critical values have uncertainty margins which are large enough to prevent a definitive statement about their n dependence from being made, as shown in Figure 8.6. It does appear however, as though very slight n dependence may exist [2]. Stephens found that the Anderson-Darling test was independent of n (at their precision level) for $n \geq 5$, and reported a very weak n dependence for the Cramér-von Mises test, which agrees with the results from our work; it is anticipated that, with more data some n dependence will be resolved for both tests [12]. Table 8.2 summarises the n and τ_l dependence of the case I critical values; a hyphen is used to denote situations in which dependence is suggested, but cannot be confirmed with our data.

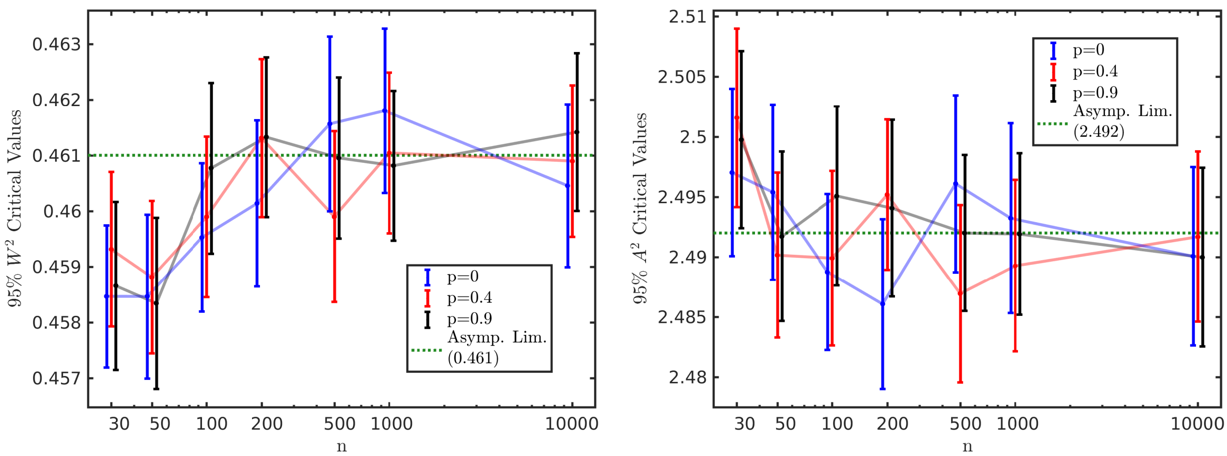
For the sake of clarity, the critical values resulting from different truncation levels (p values) are staggered along the n axis; this does not imply that the critical were evaluated at distinct sample sizes. There is no discernable relation between the confidence interval length and significance level, hence, we have elected to show only the results for one significance level. The 95% significance level has been displayed because it was previously studied by Kizilersü et al. [10] and our work has a lot of similarities with that study. All the uncertainty margins are given at the 95% confidence level (see Chapter 7 for more information).

[2] Increasing the number of repetitions may allow for concrete statements regarding n dependence to be made



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl. (b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

Figure 8.5: Case I Kolmogorov-Smirnov and Kuiper critical values
Critical values from different p values are staggered on the n axis for clarity



(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl. (b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

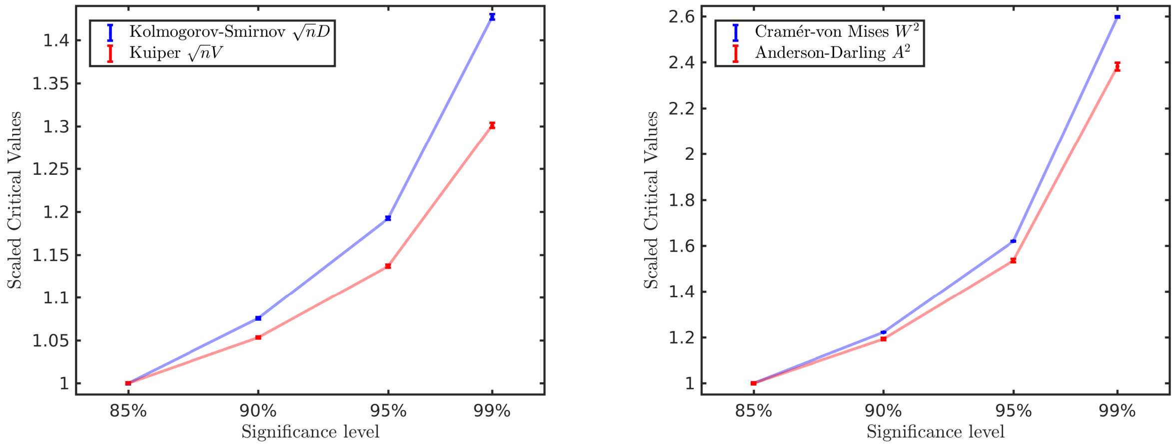
Figure 8.6: Case I Cramér-von Mises and Anderson-Darling critical values
Critical values from different p values are staggered on the n axis for clarity

Test	n dependent	τ_L dependent
KS ($\sqrt{n}D$) & Kuiper ($\sqrt{n}V$)	✓	✗
CvM (W^2) & AD (A^2)	-	✗

Table 8.2: Case I critical value dependences
A hyphen implies dependence is suggested but cannot be confirmed

8.4.2 Effect of Significance Level on Critical Values

Figure 8.7 displays the complete (untruncated) case I critical values at $n = 10,000$ plotted against significance level. All critical values have been divided by their respective 85% critical values so that the change between different goodness-of-tests can be compared with increased clarity. **The critical values are necessarily monotone increasing with significance level** as shown in Figure 8.7. This behaviour is common to all sample sizes and truncation levels, even though only one example has been depicted. **Future studies should model how the critical values change with significance level.**



(a) Kolmogorov-Smirnov and Kuiper Critical Values (b) Cramér-von Mises and Anderson-Darling Critical Values

Figure 8.7: Case I Lognormal distribution p=0, n=10,000 critical values
Scaled to the 85% significance level

8.4.3 Comparison with Kizilersü et al.

Table 8.3 gives the Kolmogorov-Smirnov critical values for case I at the 95% significance level. The results of the present work ($C = 100$ and $M = 10,000$) are displayed in bold below the results of Kizilersü et al. [10] ($C = 100$ and $M = 1,000$). There is strong agreement between the two studies, and the difference between the uncertainty margins is roughly $\frac{\sqrt{1000}}{1.984}$, as expected.

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.322(25) **1.3237(09)**	1.329(24) **1.3332(07)**	1.336(24) **1.3405(08)**	1.343(24) **1.3446(08)**	1.346(24) **1.3500(08)**	1.346(23) **1.3520(07)**	1.354(27) **1.3564(08)**
0.1	0.32	1.321(24) **1.3229(08)**	1.333(23) **1.3307(09)**	1.339(26) **1.3414(08)**	1.345(26) **1.3448(07)**	1.348(27) **1.3504(07)**	1.351(24) **1.3530(09)**	1.352(21) **1.3574(07)**
0.2	0.47	1.321(25) **1.3243(08)**	1.327(24) **1.3316(08)**	1.339(25) **1.3398(08)**	1.345(23) **1.3450(08)**	1.344(25) **1.3502(08)**	1.352(27) **1.3532(09)**	1.351(26) **1.3559(08)**
0.3	0.6	1.322(23) **1.3255(07)**	1.335(28) **1.3331(08)**	1.341(25) **1.3400(08)**	1.349(25) **1.3449(07)**	1.349(24) **1.3488(09)**	1.350(25) **1.3531(07)**	1.359(26) **1.3564(08)**
0.4	0.71	1.319(26) **1.3245(08)**	1.330(27) **1.3319(08)**	1.338(26) **1.3400(08)**	1.345(26) **1.3453(09)**	1.347(24) **1.3497(08)**	1.356(25) **1.3525(09)**	1.352(26) **1.3555(08)**
0.5	0.83	1.322(24) **1.3239(08)**	1.331(24) **1.3325(09)**	1.334(24) **1.3412(09)**	1.345(28) **1.3457(08)**	1.349(22) **1.3499(07)**	1.356(25) **1.3523(08)**	1.353(28) **1.3558(07)**
0.6	0.96	1.322(25) **1.3241(08)**	1.331(24) **1.3321(08)**	1.340(23) **1.3397(08)**	1.343(24) **1.3456(08)**	1.349(28) **1.3499(08)**	1.352(26) **1.3524(08)**	1.357(26) **1.3555(07)**
0.7	1.1	1.322(27) **1.3232(07)**	1.330(23) **1.3332(08)**	1.339(27) **1.3417(09)**	1.345(24) **1.3462(07)**	1.346(23) **1.3479(08)**	1.350(25) **1.3508(07)**	1.359(25) **1.3569(08)**
0.8	1.27	1.319(29) **1.3228(07)**	1.330(25) **1.3314(07)**	1.338(21) **1.3407(09)**	1.345(29) **1.3453(07)**	1.348(24) **1.3497(07)**	1.351(25) **1.3515(09)**	1.355(26) **1.3563(08)**
0.9	1.52	1.323(24) **1.3228(08)**	1.328(23) **1.3332(08)**	1.340(26) **1.3390(09)**	1.348(24) **1.3448(08)**	1.346(23) **1.3511(07)**	1.349(24) **1.3528(09)**	1.354(26) **1.3554(07)**

Table 8.3: Case I Kolmogorov-Smirnov critical values at 95% sig. lvl
Results from our work are **bold**, the others are from Kizilersü et al. [10]

8.4.4 Critical Values

Tables 8.4, 8.5 and 8.6 display the Kuiper, Cramér-von Mises and Anderson-Darling critical values for case I at the 95% significance level. The complete set of case I critical values (found in Appendix A.1) is too extensive to include, hence, we have elected to show only the results for only one significance level. Again, the 95% significance level has been displayed because it was previously studied by Kizilersü et al. [10] and all the uncertainty margins are given at the 95% confidence level.

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.6803(16)	1.6968(15)	1.7119(15)	1.7239(15)	1.7313(16)	1.7361(14)	1.7422(14)
0.0323	0.18	1.6805(14)	1.6972(15)	1.7131(15)	1.7207(15)	1.7324(16)	1.7373(15)	1.7442(14)
0.1	0.32	1.6817(14)	1.6960(16)	1.7122(15)	1.7230(15)	1.7313(15)	1.7357(15)	1.7443(14)
0.2	0.47	1.6807(17)	1.6968(16)	1.7122(14)	1.7225(15)	1.7315(15)	1.7362(16)	1.7445(16)
0.3	0.6	1.6817(13)	1.6976(13)	1.7115(15)	1.7230(11)	1.7305(15)	1.7370(13)	1.7436(16)
0.4	0.71	1.6810(15)	1.6963(16)	1.7110(14)	1.7232(14)	1.7313(16)	1.7366(17)	1.7429(13)
0.5	0.83	1.6815(16)	1.6986(15)	1.7125(17)	1.7224(16)	1.7319(16)	1.7379(14)	1.7437(12)
0.6	0.96	1.6804(15)	1.6943(14)	1.7109(14)	1.7217(14)	1.7327(15)	1.7359(13)	1.7438(14)
0.7	1.1	1.6803(14)	1.6968(15)	1.7119(17)	1.7230(15)	1.7303(15)	1.7354(14)	1.7445(16)
0.8	1.27	1.6804(14)	1.6967(14)	1.7111(16)	1.7234(14)	1.7310(15)	1.7357(15)	1.7430(17)
0.8605	1.4	1.6815(15)	1.6969(15)	1.7117(15)	1.7232(16)	1.7332(15)	1.7365(16)	1.7436(15)
0.9	1.52	1.6805(14)	1.6982(15)	1.7122(14)	1.7233(17)	1.7327(13)	1.7367(16)	1.7426(16)

Table 8.4: Case I Kuiper critical values at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4589(15)	0.4612(14)	0.4610(15)	0.4601(13)	0.4611(13)	0.4612(14)	0.4610(15)
0.0323	0.18	0.4581(14)	0.4594(15)	0.4604(12)	0.4595(15)	0.4608(13)	0.4587(13)	0.4623(15)
0.1	0.32	0.4580(15)	0.4588(14)	0.4610(15)	0.4604(13)	0.4619(15)	0.4615(14)	0.4613(13)
0.2	0.47	0.4600(14)	0.4590(14)	0.4593(15)	0.4604(15)	0.4615(14)	0.4609(16)	0.4612(16)
0.3	0.6	0.4605(16)	0.4595(15)	0.4610(15)	0.4606(14)	0.4606(16)	0.4618(16)	0.4616(15)
0.4	0.71	0.4593(16)	0.4596(15)	0.4608(15)	0.4602(14)	0.4612(14)	0.4611(16)	0.4613(15)
0.5	0.83	0.4590(15)	0.4603(15)	0.4614(16)	0.4607(13)	0.4597(13)	0.4611(12)	0.4607(15)
0.6	0.96	0.4593(15)	0.4609(14)	0.4612(13)	0.4606(14)	0.4608(15)	0.4615(14)	0.4605(14)
0.7	1.1	0.4595(12)	0.4609(15)	0.4607(15)	0.4621(14)	0.4589(14)	0.4607(13)	0.4618(15)
0.8	1.27	0.4579(13)	0.4597(14)	0.4606(16)	0.4608(15)	0.4608(15)	0.4599(15)	0.4612(15)
0.8605	1.4	0.4593(14)	0.4593(14)	0.4600(14)	0.4617(16)	0.4618(14)	0.4617(15)	0.4602(15)
0.9	1.52	0.4585(15)	0.4604(15)	0.4595(14)	0.4614(15)	0.4616(15)	0.4615(15)	0.4609(14)

Table 8.5: Case I Cramér-von Mises critical values at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	2.4976(76)	2.5037(66)	2.4950(74)	2.4879(69)	2.4889(64)	2.4935(67)	2.4913(73)
0.0323	0.18	2.4924(72)	2.4966(78)	2.4931(61)	2.4858(68)	2.4912(69)	2.4830(68)	2.4940(76)
0.1	0.32	2.4948(74)	2.4936(68)	2.4973(77)	2.4894(64)	2.4942(67)	2.4948(64)	2.4930(63)
0.2	0.47	2.5011(74)	2.4931(69)	2.4885(69)	2.4931(72)	2.4918(71)	2.4914(75)	2.4927(72)
0.3	0.6	2.5071(72)	2.4984(74)	2.4947(75)	2.4923(68)	2.4916(76)	2.4924(79)	2.4927(70)
0.4	0.71	2.4980(69)	2.4991(67)	2.4947(75)	2.4903(66)	2.4926(75)	2.4932(75)	2.4908(78)
0.5	0.83	2.5002(72)	2.4961(72)	2.4963(80)	2.4908(71)	2.4839(67)	2.4920(61)	2.4879(71)
0.6	0.96	2.5021(74)	2.5002(68)	2.4935(67)	2.4929(63)	2.4899(77)	2.4912(75)	2.4879(71)
0.7	1.1	2.5009(62)	2.5015(79)	2.4914(71)	2.4990(70)	2.4832(65)	2.4900(62)	2.4911(74)
0.8	1.27	2.4926(66)	2.4963(68)	2.4964(81)	2.4906(68)	2.4918(71)	2.4851(75)	2.4910(72)
0.8605	1.4	2.5013(72)	2.4941(67)	2.4934(62)	2.4954(79)	2.4951(72)	2.4968(75)	2.4890(66)
0.9	1.52	2.4946(79)	2.4990(76)	2.4856(76)	2.4943(68)	2.4947(72)	2.4873(73)	2.4908(69)

Table 8.6: Case I Anderson-Darling critical values at 95% sig. lvl

8.4.5 Comparison with the available literature

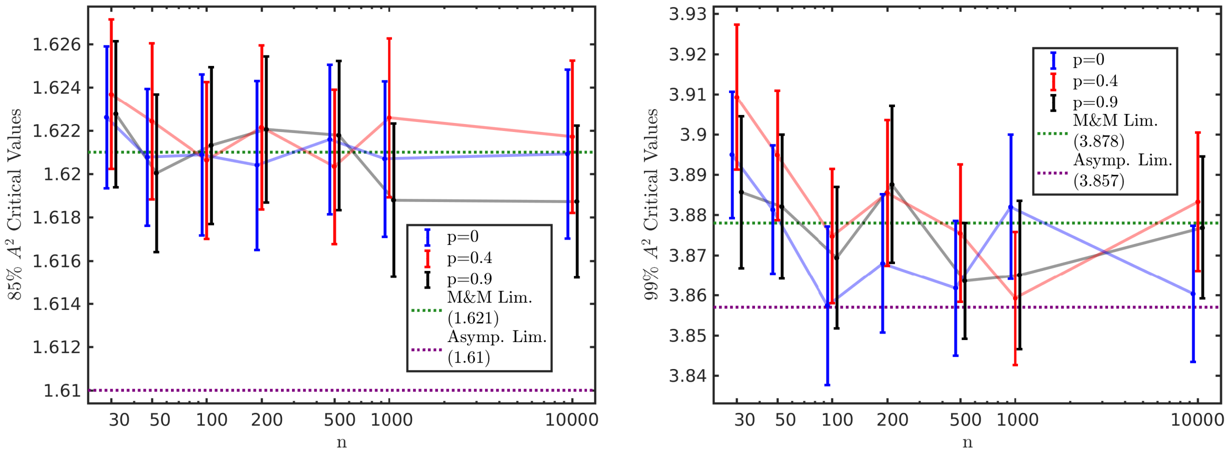
The asymptotic critical values of the test statistics evaluated in our work have been studied extensively for completely specified (case I) complete (untruncated) distributions and are summarised by Stephens [12], Pearson & Hartley [92] and others [45, 47, 54, 60, 93–95]. There is broad agreement among these studies, however, at particular significance levels there are conflicting estimates for the Anderson-Darling critical values. Marsaglia & Marsaglia [93] discuss the discrepancy:

> "*An expression for the limiting distribution of A_n was given by Anderson and Darling [60]. The method was based on a development of Doob for the absorption probability of a diffusion model. They gave*
>
> $$\lim_{n\to\infty} Pr\left(A_n < z\right) = \frac{\sqrt{2\pi}}{z}\sum_{j=0}^{\infty}\binom{-\frac{1}{2}}{j}(4j+1)e^{\left\{-\frac{(4j+1)^2\pi^2}{8z}\right\}}\int_0^{\infty} e^{\left\{\frac{z}{8(1+w^2)}-\frac{w^2(4j+1)^2\pi^2}{8z}\right\}}dw.$$
>
> *This is a strange distribution function. Anderson and Darling [45] used numerical integration to find the 90, 95 and 99 percentiles. (They are reported as* 1.933,2.492 *and* 3.857*; the true values to 20 places are* 1.9329578327415937304, 2.4923671600494096176 *and* 3.8781250216053948842.*) Lewis [95], also using numerical integration, published a table giving lim $Pr(A_n < z)$ with 4-place accuracy for selected z values, as well as the same three percentiles with the wrong* 3.857 *value for* 3.878125.... *Other values have been provided by Sinclair and Spurr [96], (approximate inversion of the characteristic function), and Giles [94], (saddle-point approximations). In all, it seems that relatively few values or percentiles have been provided, all by approximation methods and sometimes giving less than the claimed 3-4 digits of accuracy. Note that Sinclair and Spur report a better value,* 3.880 *as the 99 percentile* 3.878125..., *which Giles disputes in a footnote, sticking to* 3.857 *as the 'true' value, presumably because it was given by both Anderson-Darling [45] and Lewis [95]. We will provide a method for evaluating the above distribution with accuracy limited to the computer's ability to distinguish between floating point numbers, give a C program for implementing it, and also give a quick-and-easy approximation that gives accuracy better than* .000002 *for probabilities less than .9 and* .0000008 *for those beyond.*"

We utilised the C program given by Marsaglia & Marsaglia to determine the Anderson-Darling critical values and found a similar discrepancy at the 85^{th} percentile [93]. Marsaglia & Marsaglia's result has been implemented in R [97] and agrees with the critical values utilised by MATLAB, therefore it was considered the verification benchmark for this work. Figure 8.8 displays the critical values determined by Marsaglia & Marsaglia and Anderson and Darling at the significance levels for which they differ, these are labelled "M&M Lim." and "Asymp. Lim." respectively. There is much stronger agreement between the simulation results and value given by Marsaglia & Marsaglia's method [12, 45, 92–95].

Figure 8.5 shows that the Kolmogorov-Smirnov and Kuiper critical values generated from the present work tend to the asymptotic values from the literature (labelled "Asymp. Lim.") as $n \to \infty$ [12, 45, 47, 54, 60, 92, 94, 95]. At the largest sample size ($n = 10,000$) the Cramér-von Mises and Anderson-Darling critical values are clustered around the asymptotic values (displayed in Figure 8.6); large uncertainty margins prevent one from estimating the limit as $n \to \infty$ however there is agreement between the literature and the present work. Broad agreement between the simulation results and asymptotic critical values was achieved for all goodness-of-fit tests and distributions.

The case I critical values from our work are compared to the literature in Appendix



(a) Anderson-Darling (A^2) crit. vals. at 85% sig. lvl. (b) Anderson-Darling (A^2) crit. vals. at 99% sig. lvl.

Figure 8.8: Case I Lognormal distribution Anderson-Darling critical values
Critical values from different p values are staggered on the n axis for clarity

B.1. Tables B.1, B.2, B.3 and B.4 display the comparison for the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests, respectively. The results from the current work are displayed in bold, below the critical values from other studies. Several of the comparison papers give formulas for the critical values as a function of n. Where this is the case, the values are preceded by an asterisk (*). Additionally, some papers provide C code from which the critical values can be quickly computed; values that were determined via this method are preceded with a dagger (†) in Appendix B.

We found that in the vast majority of studies, critical values were stated without a corresponding confidence interval, even if the critical values were produced through Monte Carlo simulation. Determining critical values in this way requires percentile estimates to be undertaken, and there is an established method of calculating the uncertainty margin associated with this procedure. This procedure is the subject of detailed discussion in Chapter 7. We presume that confidence intervals were not stated because they are not particularly useful when the critical values are used in a goodness-of-fit test, which is their primary employment. The lack of an uncertainty margin, however, does make the task of comparing our results to the literature more difficult.

The critical values we determined achieved broad agreement with the majority of values we consulted. However, there were some cases in which significant differences occured. In these situations, there was usually a limited degree of agreement within the literature prior to our work. For example, Smirnov estimated the case I critical values of the Kolmogorov-Smirnov test with a sample size of $n = 30$ in his 1948 [54] paper. At the 95% significance level, he gives a value of 1.36, our work attained a result 1.3237 ± 0.0018, thus Smirnov's result is far outside of our uncertainty margin. In 1951 Massey [55] calculated a value of 1.31, and in a 1952 paper Birnbaum [98] quoted a value of 1.3238. In 2003 Tsang et al. [57] provided some C code that produced a result of 1.3239, which is well within our confidence interval. This information is displayed in Table B.1. Our results agree with the majority of the literature, but it is not possible to agree with all of it with the confidence interval length we have managed to attain. In this case, we presume the change in critical value over the years is attributable to the advance of technology, that Smirnov was able to attain an estimate at all in 1948 is truly remarkable.

8.5 Case II

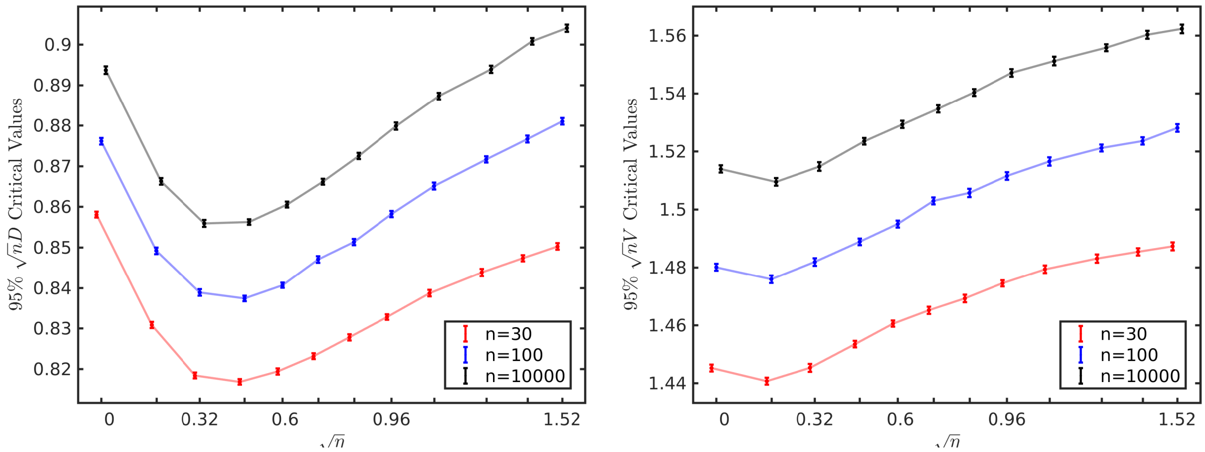
- Weibull
- Loglogistic
- Lognormal

8.5.1 Variable Dependence

In case II, both parameters are unknown and thus must be determined from the data. As the Pareto distribution has just one parameter, only the Weibull, loglogistic and lognormal distributions need to be considered. Figure 8.9 displays the Kolmogorov-Smirnov and Kuiper critical values at the 95% significance level for the case II Weibull distribution, plotted against truncation level ($\sqrt{\eta}$); Figure 8.10 displays the same critical values plotted against sample size (n). The corresponding plots for the Cramér-von Mises and Anderson-Darling tests are shown in Figures 8.11 and 8.12 respectively. Only the Weibull critical values at the 95% significance level are depicted, however, these plots are indicative of the loglogistic and lognormal distributions and all significance levels. All these figures clearly show that critical values are strongly dependent upon sample size, n, and truncation level, $\sqrt{\eta}$. It is interesting to note-but of no consequence in this work-that Figures 8.10a and 8.12a show that the critical values are more heavily dependent upon n for higher levels of truncation.

The critical values depicted in Figures 8.9 and 8.11 resulting from different sample size (n values) are staggered along the $\sqrt{\eta}$ axis; this does not imply that the critical were evaluated at distinct sample sizes. In Figures 8.10 and 8.12 the critical values resulting from different truncation levels (p values) are staggered along the n axis; this does not imply that the critical were evaluated at distinct truncation levels. There is no discernible relation between the confidence interval length and significance level, hence, we have elected to show only the results for the 95% significance level as discussed in section 8.4. Table 8.7 summarises the dependence of the case II critical values on n and τ_l.

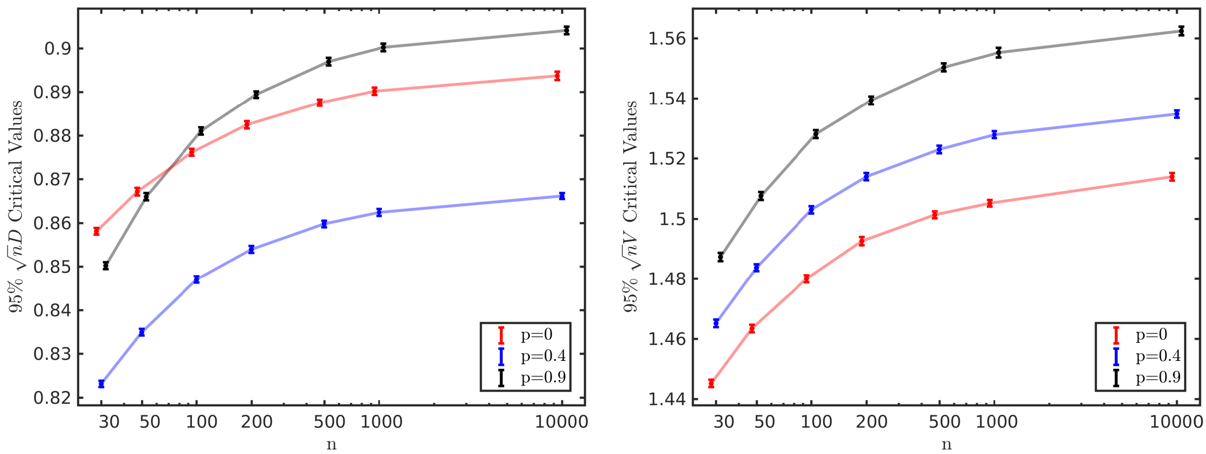
Curiously, there appears to be a non-zero truncation level which minimises the critical values for all sample sizes. We are not sure why this should be the case, and suggest that **future studies should try to understand the underlying mechanism**. Kizilersü et al. [10] first noted this behaviour for the Kolmogorov-Smirnov critical values of the case II Weibull distribution at the 95% significance level, however we can confirm that it is present for the case II Weibull, loglogistic and lognormal distributions in all goodness-of-fit tests and significance levels analysed in our work.



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl.

(b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

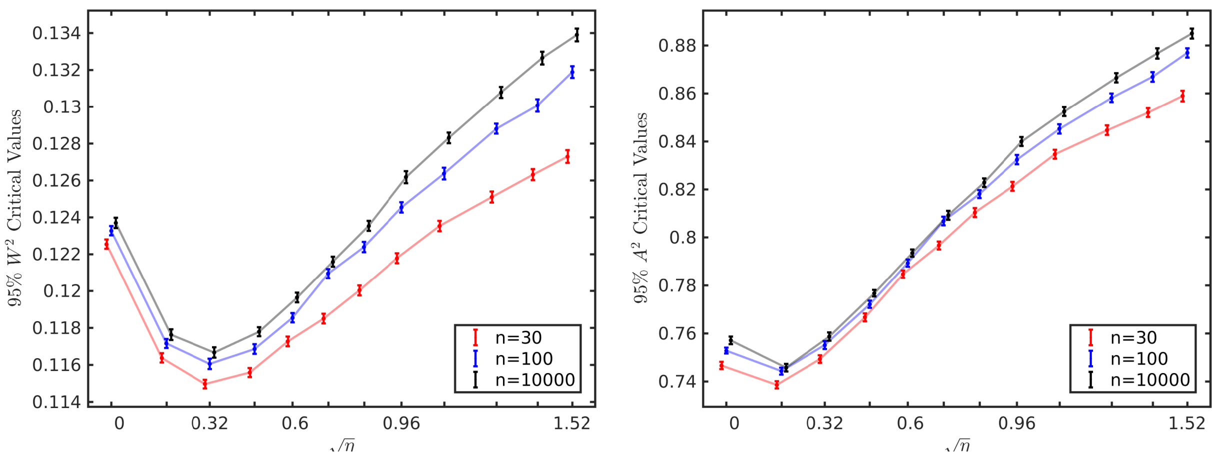
Figure 8.9: Case II Weibull Kolmogorov-Smirnov and Kuiper critical values
Critical values from different n values are staggered on the $\sqrt{\eta}$ axis for clarity



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl.

(b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

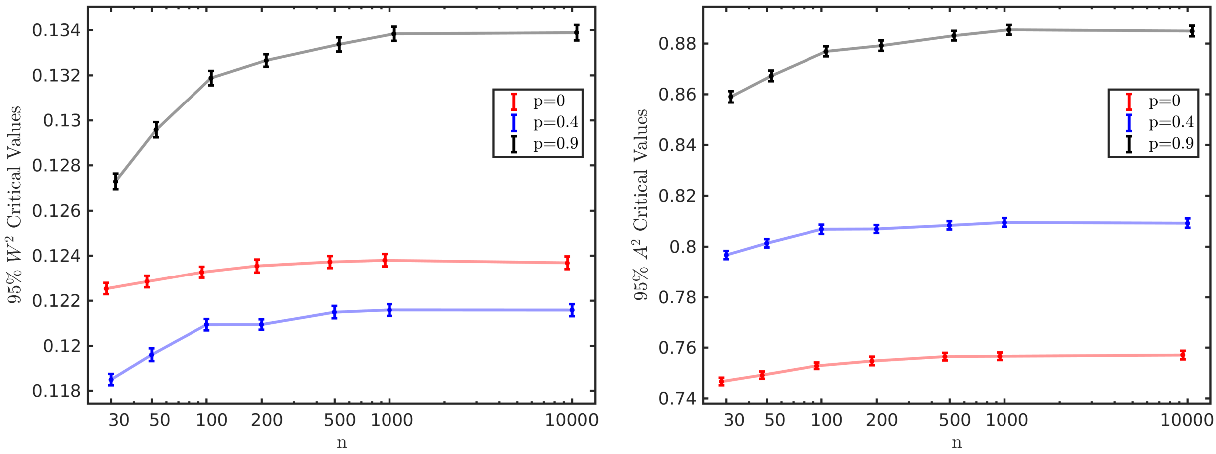
Figure 8.10: Case II Weibull Kolmogorov-Smirnov and Kuiper critical values
Critical values from different p values are staggered on the n axis for clarity



(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl.

(b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

Figure 8.11: Case II Weibull Cramér-von Mises and Anderson-Darling critical values
Critical values from different n values are staggered on the $\sqrt{\eta}$ axis for clarity



(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl.
(b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

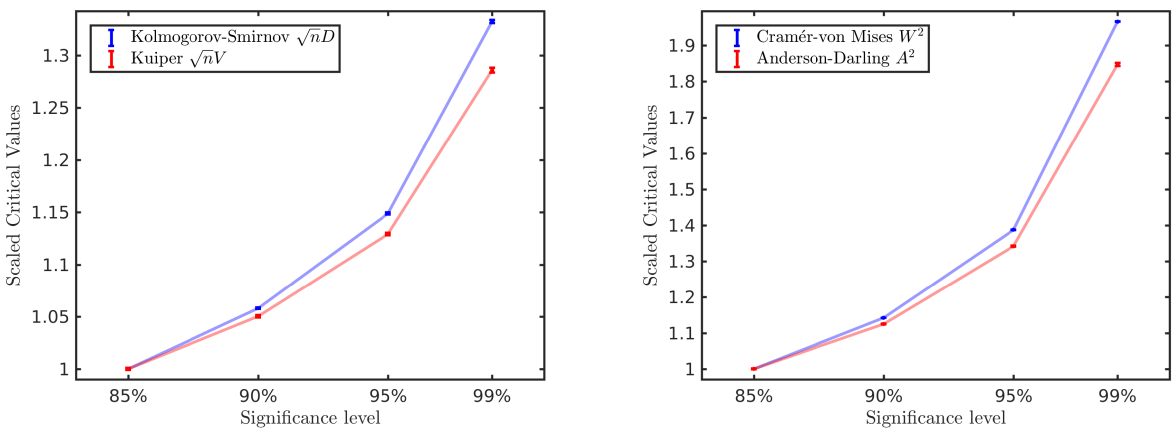
Figure 8.12: Case II Weibull Cramér-von Mises and Anderson-Darling critical values
Critical values from different p values are staggered on the n axis for clarity

Distribution	Test	n dependent	τ_L dependent
Weibull	KS, Kuiper, CvM & AD	✓	✓
Loglogistic	KS, Kuiper, CvM & AD	✓	✓
Lognormal	KS, Kuiper, CvM & AD	✓	✓

Table 8.7: Case II critical value dependences

8.5.2 Effect of Significance Level on Critical Values

Figure 8.13 displays the complete (untruncated) case II critical values at $n = 10,000$ plotted against significance level. All critical values have been divided by their respective 85% critical values so that the change between different goodness-of-tests can be compared with increased clarity. As discussed in subsection 8.4.2, the critical values are necessarily monotone increasing and this behaviour is common to all sample sizes and truncation levels.



(a) Kolmogorov-Smirnov and Kuiper critical values
(b) Cramér-von Mises and Anderson-Darling critical values

Figure 8.13: Case II Weibull distribution p=0, n=10,000 critical value
Scaled to the 85% significance level

8.5.3 Comparison with Kizilersü et al.

Table 8.8 gives the Kolmogorov-Smirnov critical values for case II at the 95% significance level. The results of the present work ($C = 100$ and $M = 10,000$) are displayed in bold below the results of Kizilersü et al. [10] ($C = 100$ and $M = 1,000$). There is strong agreement between the two studies, and the difference between the uncertainty margins is roughly $\frac{\sqrt{1000}}{1.984}$, as expected.

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.858(11) **0.8581(04)**	0.865(12) **0.8671(04)**	0.874(12) **0.8761(04)**	0.881(13) **0.8825(04)**	0.887(12) **0.8875(04)**	0.890(15) **0.8902(04)**	0.893(15) **0.8937(05)**
0.1	0.32	0.817(12) **0.8183(04)**	0.829(11) **0.8294(04)**	0.838(12) **0.8388(04)**	0.843(13) **0.8448(04)**	0.850(13) **0.8499(04)**	0.851(13) **0.8523(04)**	0.857(12) **0.8559(04)**
0.2	0.47	0.815(12) **0.8167(03)**	0.824(11) **0.8276(04)**	0.838(12) **0.8374(03)**	0.842(12) **0.8446(04)**	0.847(13) **0.8494(04)**	0.852(12) **0.8524(04)**	0.856(11) **0.8563(04)**
0.3	0.6	0.818(13) **0.8193(04)**	0.830(12) **0.8302(04)**	0.840(10) **0.8406(03)**	0.848(13) **0.8487(04)**	0.854(13) **0.8532(04)**	0.856(12) **0.8564(04)**	0.859(11) **0.8605(04)**
0.4	0.71	0.821(12) **0.8231(04)**	0.832(11) **0.8349(04)**	0.846(11) **0.8470(04)**	0.853(12) **0.8540(04)**	0.857(12) **0.8598(04)**	0.862(13) **0.8624(04)**	0.866(13) **0.8662(04)**
0.5	0.83	0.842(13) **0.8278(04)**	0.840(13) **0.8401(04)**	0.852(13) **0.8513(04)**	0.860(12) **0.8596(04)**	0.863(11) **0.8660(04)**	0.868(14) **0.8686(05)**	0.872(12) **0.8725(04)**
0.6	0.96	0.830(12) **0.8328(04)**	0.844(13) **0.8457(04)**	0.857(12) **0.8582(04)**	0.866(12) **0.8665(04)**	0.873(11) **0.8730(04)**	0.876(13) **0.8759(04)**	0.881(11) **0.8799(04)**
0.7	1.1	0.835(12) **0.8387(04)**	0.853(12) **0.8522(05)**	0.864(13) **0.8651(04)**	0.873(12) **0.8728(04)**	0.878(13) **0.8791(04)**	0.882(13) **0.8827(04)**	0.888(12) **0.8873(04)**
0.8	1.27	0.839(12) **0.8438(04)**	0.855(12) **0.8580(04)**	0.871(13) **0.8717(04)**	0.880(13) **0.8810(04)**	0.886(14) **0.8870(04)**	0.890(15) **0.8904(04)**	0.894(14) **0.8939(04)**
0.9	1.52	0.843(12) **0.8502(04)**	0.864(11) **0.8660(04)**	0.880(14) **0.8810(04)**	0.890(14) **0.8894(04)**	0.897(14) **0.8970(04)**	0.897(13) **0.9002(04)**	0.904(14) **0.9041(05)**

Table 8.8: Kolmogorov-Smirnov critical values of the case II Weibull distrib. at 95% sig. lvl
Results from our work are **bold**, the others are from Kizilersü et al. [10]

8.5.4 Critical Values

Tables 8.9, 8.10 and 8.11 display the Kuiper, Cramér-von Mises and Anderson-Darling critical values for case II Weibull distribution at the 95% significance level. The critical values are given for the loglogistic and lognormal distributions at the 95% significance level in Tables 8.12-8.15 and 8.16-8.19 respectively. The complete set of case II critical values (found in Appendix A.2) is too extensive to include, hence, we have elected to show only the results for only one significance level. Again, the 95% significance level has been displayed because it was previously studied by Kizilersü et al. [10] and all the uncertainty margins are given at the 95% confidence level.

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4452(12)	1.4634(13)	1.4800(12)	1.4925(14)	1.5013(12)	1.5051(11)	1.5139(12)
0.0323	0.18	1.4406(12)	1.4588(15)	1.4759(13)	1.4870(13)	1.4976(09)	1.5014(12)	1.5095(13)
0.1	0.32	1.4452(14)	1.4640(12)	1.4819(13)	1.4930(12)	1.5023(12)	1.5069(13)	1.5148(15)
0.2	0.47	1.4534(11)	1.4724(10)	1.4889(11)	1.5014(13)	1.5096(12)	1.5148(12)	1.5235(12)
0.3	0.6	1.4605(11)	1.4776(11)	1.4950(12)	1.5077(14)	1.5174(14)	1.5217(13)	1.5293(13)
0.4	0.71	1.4652(13)	1.4837(11)	1.5030(12)	1.5139(12)	1.5230(13)	1.5279(12)	1.5347(12)
0.5	0.83	1.4693(13)	1.4895(12)	1.5057(14)	1.5183(14)	1.5280(12)	1.5328(12)	1.5404(12)
0.6	0.96	1.4745(11)	1.4925(14)	1.5115(13)	1.5237(11)	1.5339(14)	1.5389(12)	1.5472(13)
0.7	1.1	1.4793(13)	1.4982(14)	1.5165(14)	1.5280(14)	1.5381(13)	1.5433(12)	1.5512(15)
0.8	1.27	1.4831(14)	1.5029(14)	1.5212(12)	1.5343(14)	1.5435(12)	1.5487(14)	1.5558(12)
0.8605	1.4	1.4854(13)	1.5058(14)	1.5236(13)	1.5363(13)	1.5472(13)	1.5523(14)	1.5603(14)
0.9	1.52	1.4873(14)	1.5075(13)	1.5281(13)	1.5393(13)	1.5503(13)	1.5552(16)	1.5624(15)

Table 8.9: Kuiper critical values of the case II Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.12253(25)	0.12285(26)	0.12326(25)	0.12354(29)	0.12372(26)	0.12379(27)	0.12368(28)
0.0323	0.18	0.11636(25)	0.11672(27)	0.11715(25)	0.11718(26)	0.11751(26)	0.11743(24)	0.11762(28)
0.1	0.32	0.11495(24)	0.11572(26)	0.11604(28)	0.11608(25)	0.11642(22)	0.11621(27)	0.11665(28)
0.2	0.47	0.11558(24)	0.11655(26)	0.11684(26)	0.11738(28)	0.11740(25)	0.11760(26)	0.11778(24)
0.3	0.6	0.11725(26)	0.11780(24)	0.11853(25)	0.11928(27)	0.11944(29)	0.11950(27)	0.11963(27)
0.4	0.71	0.11849(25)	0.11959(29)	0.12093(26)	0.12093(23)	0.12148(27)	0.12158(26)	0.12158(27)
0.5	0.83	0.12002(27)	0.12152(30)	0.12238(28)	0.12313(31)	0.12358(27)	0.12342(26)	0.12352(27)
0.6	0.96	0.12177(28)	0.12292(30)	0.12452(28)	0.12526(23)	0.12553(30)	0.12596(27)	0.12616(33)
0.7	1.1	0.12352(28)	0.12529(31)	0.12636(32)	0.12730(31)	0.12787(27)	0.12819(32)	0.12830(29)
0.8	1.27	0.12508(30)	0.12723(32)	0.12879(27)	0.12990(31)	0.13056(30)	0.13049(31)	0.13077(29)
0.8605	1.4	0.12629(30)	0.12851(30)	0.13005(33)	0.13136(30)	0.13211(30)	0.13246(30)	0.13263(35)
0.9	1.52	0.12728(34)	0.12957(34)	0.13187(32)	0.13265(28)	0.13337(32)	0.13384(31)	0.13388(35)

Table 8.10: Cramér-von Mises critical values of the case II Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7466(15)	0.7491(15)	0.7528(12)	0.7547(17)	0.7565(15)	0.7566(15)	0.7571(16)
0.0323	0.18	0.7386(15)	0.7410(17)	0.7443(15)	0.7446(14)	0.7459(16)	0.7454(14)	0.7457(16)
0.1	0.32	0.7493(16)	0.7534(15)	0.7551(17)	0.7558(16)	0.7570(14)	0.7565(16)	0.7587(17)
0.2	0.47	0.7666(16)	0.7721(14)	0.7720(15)	0.7753(17)	0.7752(16)	0.7754(16)	0.7766(13)
0.3	0.6	0.7846(16)	0.7867(16)	0.7893(15)	0.7928(16)	0.7936(17)	0.7917(16)	0.7935(15)
0.4	0.71	0.7966(16)	0.8013(16)	0.8068(18)	0.8069(16)	0.8083(16)	0.8095(17)	0.8092(18)
0.5	0.83	0.8103(19)	0.8148(18)	0.8180(17)	0.8203(18)	0.8233(19)	0.8227(18)	0.8228(18)
0.6	0.96	0.8212(19)	0.8260(18)	0.8323(19)	0.8349(16)	0.8362(18)	0.8376(18)	0.8399(19)
0.7	1.1	0.8346(18)	0.8426(22)	0.8452(19)	0.8478(19)	0.8512(18)	0.8514(22)	0.8524(19)
0.8	1.27	0.8446(20)	0.8521(15)	0.8581(18)	0.8645(21)	0.8667(19)	0.8652(19)	0.8665(19)
0.8605	1.4	0.8519(19)	0.8625(19)	0.8669(20)	0.8724(16)	0.8747(20)	0.8775(19)	0.8766(22)
0.9	1.52	0.8589(22)	0.8672(21)	0.8769(20)	0.8791(20)	0.8831(19)	0.8854(20)	0.8849(21)

Table 8.11: Anderson-Darling critical values of the case II Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7661(06)	0.7774(07)	0.7860(07)	0.7916(08)	0.7976(07)	0.8002(07)	0.8039(07)
0.0323	0.18	0.7602(06)	0.7711(07)	0.7799(07)	0.7871(07)	0.7915(06)	0.7942(08)	0.7981(06)
0.1	0.33	0.7640(07)	0.7746(07)	0.7840(07)	0.7898(07)	0.7952(07)	0.7982(07)	0.8017(07)
0.2	0.5	0.7705(06)	0.7809(06)	0.7912(07)	0.7972(06)	0.8017(07)	0.8048(08)	0.8095(08)
0.3	0.65	0.7761(07)	0.7869(07)	0.7975(08)	0.8032(07)	0.8090(06)	0.8117(07)	0.8148(07)
0.4	0.82	0.7795(07)	0.7919(07)	0.8024(08)	0.8094(08)	0.8144(07)	0.8171(08)	0.8211(07)
0.5	1	0.7832(07)	0.7952(08)	0.8071(07)	0.8138(07)	0.8192(07)	0.8210(07)	0.8253(07)
0.6	1.22	0.7843(06)	0.7981(07)	0.8098(07)	0.8164(07)	0.8224(07)	0.8248(07)	0.8292(07)
0.7	1.53	0.7865(07)	0.7991(07)	0.8124(08)	0.8205(07)	0.8258(07)	0.8287(08)	0.8326(07)
0.8	2	0.7880(07)	0.8015(07)	0.8146(08)	0.8215(07)	0.8292(07)	0.8319(07)	0.8352(08)
0.8605	2.48	0.7885(07)	0.8015(08)	0.8144(08)	0.8227(07)	0.8301(07)	0.8328(08)	0.8375(07)
0.9	3	0.7884(07)	0.8028(07)	0.8157(08)	0.8242(08)	0.8310(07)	0.8339(08)	0.8393(07)

Table 8.12: Kolmogorov-Smirnov critical values of the case II loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3634(10)	1.3810(12)	1.3960(10)	1.4064(12)	1.4169(11)	1.4212(10)	1.4290(11)
0.0323	0.18	1.3542(11)	1.3709(11)	1.3852(10)	1.3976(12)	1.4060(10)	1.4109(11)	1.4184(10)
0.1	0.33	1.3632(10)	1.3794(12)	1.3960(12)	1.4059(12)	1.4154(11)	1.4217(11)	1.4281(12)
0.2	0.5	1.3779(11)	1.3945(12)	1.4112(11)	1.4220(11)	1.4308(13)	1.4366(11)	1.4444(12)
0.3	0.65	1.3898(11)	1.4071(12)	1.4242(13)	1.4348(13)	1.4447(10)	1.4500(13)	1.4567(12)
0.4	0.82	1.3972(12)	1.4176(13)	1.4348(13)	1.4459(13)	1.4561(13)	1.4606(13)	1.4685(12)
0.5	1	1.4039(12)	1.4245(15)	1.4436(09)	1.4555(12)	1.4653(12)	1.4684(14)	1.4770(12)
0.6	1.22	1.4076(11)	1.4292(13)	1.4493(12)	1.4622(12)	1.4727(12)	1.4764(14)	1.4847(12)
0.7	1.53	1.4112(10)	1.4335(13)	1.4540(13)	1.4685(13)	1.4786(14)	1.4842(14)	1.4908(12)
0.8	2	1.4135(14)	1.4360(11)	1.4590(12)	1.4710(12)	1.4845(12)	1.4904(13)	1.4972(13)
0.8605	2.48	1.4149(12)	1.4374(13)	1.4593(13)	1.4740(13)	1.4870(13)	1.4923(12)	1.5011(13)
0.9	3	1.4148(13)	1.4392(11)	1.4603(13)	1.4757(14)	1.4885(14)	1.4941(12)	1.5032(12)

Table 8.13: Kuiper critical values of the case II loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.09709(20)	0.09732(23)	0.09748(18)	0.09762(23)	0.09749(20)	0.09771(20)	0.09777(18)
0.0323	0.18	0.09541(18)	0.09554(19)	0.09531(20)	0.09569(22)	0.09573(18)	0.09570(20)	0.09558(20)
0.1	0.33	0.09664(22)	0.09669(20)	0.09699(19)	0.09694(18)	0.09694(20)	0.09708(19)	0.09696(22)
0.2	0.5	0.09912(19)	0.09921(21)	0.09949(22)	0.09949(21)	0.09939(21)	0.09970(20)	0.09969(21)
0.3	0.65	0.10103(20)	0.10158(21)	0.10182(22)	0.10191(24)	0.10208(23)	0.10215(21)	0.10206(22)
0.4	0.82	0.10242(21)	0.10353(23)	0.10392(24)	0.10418(23)	0.10417(24)	0.10423(25)	0.10434(24)
0.5	1	0.10388(22)	0.10488(25)	0.10561(23)	0.10603(23)	0.10612(20)	0.10603(25)	0.10614(25)
0.6	1.22	0.10434(21)	0.10586(23)	0.10703(25)	0.10743(22)	0.10785(23)	0.10773(24)	0.10785(22)
0.7	1.53	0.10536(23)	0.10661(23)	0.10793(23)	0.10892(25)	0.10920(25)	0.10935(25)	0.10929(24)
0.8	2	0.10564(23)	0.10736(23)	0.10901(21)	0.10948(23)	0.11050(25)	0.11084(24)	0.11064(24)
0.8605	2.48	0.10588(22)	0.10755(26)	0.10914(23)	0.11001(26)	0.11106(25)	0.11145(29)	0.11157(24)
0.9	3	0.10587(23)	0.10789(20)	0.10935(27)	0.11054(27)	0.11131(26)	0.11174(23)	0.11206(25)

Table 8.14: Cramér-von Mises critical values of the case II loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.6594(12)	0.6614(14)	0.6632(12)	0.6634(15)	0.6647(13)	0.6653(14)	0.6652(13)
0.0323	0.18	0.6580(12)	0.6589(14)	0.6572(12)	0.6598(13)	0.6592(12)	0.6595(13)	0.6589(13)
0.1	0.33	0.6695(13)	0.6689(14)	0.6707(13)	0.6699(12)	0.6699(13)	0.6705(13)	0.6698(13)
0.2	0.5	0.6857(14)	0.6845(14)	0.6851(13)	0.6855(13)	0.6849(14)	0.6865(14)	0.6869(13)
0.3	0.65	0.6972(12)	0.6987(13)	0.6995(14)	0.6995(15)	0.7006(13)	0.6996(14)	0.6996(15)
0.4	0.82	0.7055(14)	0.7091(15)	0.7108(16)	0.7113(13)	0.7110(15)	0.7114(15)	0.7127(15)
0.5	1	0.7133(14)	0.7166(15)	0.7203(14)	0.7224(15)	0.7217(13)	0.7224(15)	0.7222(16)
0.6	1.22	0.7161(14)	0.7235(16)	0.7280(15)	0.7296(14)	0.7319(14)	0.7312(14)	0.7317(15)
0.7	1.53	0.7219(14)	0.7270(15)	0.7340(15)	0.7376(15)	0.7392(15)	0.7400(15)	0.7388(14)
0.8	2	0.7231(16)	0.7316(16)	0.7394(15)	0.7403(15)	0.7460(16)	0.7479(15)	0.7468(15)
0.8605	2.48	0.7256(15)	0.7314(16)	0.7400(13)	0.7440(16)	0.7496(16)	0.7513(18)	0.7523(15)
0.9	3	0.7252(17)	0.7345(16)	0.7399(17)	0.7469(16)	0.7507(16)	0.7525(14)	0.7541(16)

Table 8.15: Anderson-Darling critical values of the case II loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.8728(10)	0.8820(08)	0.8909(08)	0.8968(08)	0.9017(08)	0.9033(09)	0.9074(10)
0.0323	0.72	0.8443(07)	0.8525(09)	0.8604(09)	0.8655(09)	0.8706(09)	0.8727(09)	0.8758(07)
0.1	0.75	0.8252(07)	0.8342(07)	0.8416(09)	0.8458(08)	0.8513(07)	0.8526(08)	0.8570(09)
0.2	0.79	0.8164(08)	0.8238(08)	0.8319(09)	0.8368(08)	0.8413(08)	0.8436(08)	0.8473(08)
0.3	0.85	0.8147(07)	0.8221(07)	0.8297(08)	0.8343(07)	0.8394(07)	0.8410(08)	0.8450(08)
0.4	0.91	0.8155(08)	0.8243(08)	0.8315(08)	0.8366(08)	0.8411(07)	0.8430(08)	0.8477(07)
0.5	1	0.8185(07)	0.8269(07)	0.8352(08)	0.8405(07)	0.8451(08)	0.8470(07)	0.8513(08)
0.6	1.12	0.8217(07)	0.8321(08)	0.8401(07)	0.8465(09)	0.8511(07)	0.8528(08)	0.8568(07)
0.7	1.29	0.8256(06)	0.8369(08)	0.8468(08)	0.8536(08)	0.8581(08)	0.8604(08)	0.8650(08)
0.8	1.58	0.8299(07)	0.8430(07)	0.8554(08)	0.8617(08)	0.8663(08)	0.8693(07)	0.8727(08)
0.8605	1.89	0.8332(08)	0.8474(08)	0.8607(08)	0.8684(08)	0.8729(09)	0.8761(08)	0.8791(07)
0.9	2.24	0.8347(07)	0.8496(08)	0.8650(10)	0.8732(10)	0.8784(07)	0.8809(08)	0.8851(09)

Table 8.16: Kolmogorov-Smirnov critical values of the case II lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.4361(11)	1.4530(11)	1.4695(10)	1.4810(13)	1.4901(10)	1.4942(12)	1.5013(14)
0.0323	0.72	1.4167(11)	1.4329(12)	1.4479(12)	1.4582(11)	1.4677(12)	1.4722(13)	1.4788(12)
0.1	0.75	1.4178(12)	1.4359(13)	1.4511(13)	1.4613(12)	1.4714(13)	1.4752(11)	1.4834(13)
0.2	0.79	1.4272(12)	1.4436(12)	1.4615(13)	1.4714(13)	1.4812(12)	1.4858(14)	1.4927(14)
0.3	0.85	1.4360(12)	1.4533(13)	1.4700(13)	1.4811(12)	1.4906(13)	1.4932(12)	1.5024(14)
0.4	0.91	1.4428(13)	1.4619(13)	1.4781(15)	1.4890(13)	1.4985(12)	1.5023(12)	1.5112(14)
0.5	1	1.4499(12)	1.4675(12)	1.4853(13)	1.4971(12)	1.5065(13)	1.5097(11)	1.5180(13)
0.6	1.12	1.4559(12)	1.4755(13)	1.4912(13)	1.5041(16)	1.5140(12)	1.5185(13)	1.5252(12)
0.7	1.29	1.4608(13)	1.4803(12)	1.4986(13)	1.5112(14)	1.5204(14)	1.5255(13)	1.5335(14)
0.8	1.58	1.4658(12)	1.4871(13)	1.5056(12)	1.5191(13)	1.5280(13)	1.5341(12)	1.5398(12)
0.8605	1.89	1.4687(13)	1.4911(13)	1.5110(14)	1.5239(15)	1.5319(13)	1.5376(14)	1.5455(12)
0.9	2.24	1.4705(14)	1.4921(12)	1.5138(14)	1.5267(14)	1.5375(14)	1.5415(14)	1.5505(13)

Table 8.17: Kuiper critical values of the case II lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.12507(29)	0.12561(27)	0.12581(26)	0.12609(29)	0.12617(26)	0.12594(28)	0.12587(32)
0.0323	0.72	0.11613(26)	0.11614(26)	0.11602(25)	0.11596(25)	0.11594(24)	0.11579(24)	0.11582(26)
0.1	0.75	0.11274(24)	0.11287(28)	0.11242(27)	0.11245(25)	0.11252(24)	0.11244(24)	0.11247(28)
0.2	0.79	0.11222(22)	0.11196(24)	0.11212(26)	0.11205(22)	0.11199(24)	0.11195(25)	0.11198(25)
0.3	0.85	0.11290(24)	0.11286(24)	0.11291(27)	0.11301(26)	0.11294(27)	0.11276(26)	0.11281(23)
0.4	0.91	0.11406(27)	0.11439(28)	0.11451(28)	0.11434(24)	0.11427(23)	0.11413(24)	0.11446(26)
0.5	1	0.11562(23)	0.11584(23)	0.11614(28)	0.11618(23)	0.11631(27)	0.11628(27)	0.11629(24)
0.6	1.12	0.11722(26)	0.11795(28)	0.11811(25)	0.11837(30)	0.11858(25)	0.11832(26)	0.11822(27)
0.7	1.29	0.11862(29)	0.11960(27)	0.12047(26)	0.12078(33)	0.12066(25)	0.12091(29)	0.12094(27)
0.8	1.58	0.12012(29)	0.12187(24)	0.12290(27)	0.12359(30)	0.12374(28)	0.12394(30)	0.12352(28)
0.8605	1.89	0.12104(31)	0.12327(27)	0.12510(29)	0.12548(30)	0.12571(29)	0.12582(31)	0.12594(26)
0.9	2.24	0.12163(31)	0.12394(29)	0.12626(30)	0.12719(32)	0.12757(30)	0.12742(31)	0.12756(28)

Table 8.18: Cramér-von Mises critical values of the case II lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.7437(15)	0.7468(15)	0.7493(14)	0.7511(16)	0.7518(15)	0.7510(16)	0.7510(17)
0.0323	0.72	0.7172(14)	0.7165(15)	0.7161(15)	0.7159(14)	0.7154(14)	0.7149(13)	0.7150(13)
0.1	0.75	0.7185(13)	0.7190(16)	0.7165(15)	0.7162(14)	0.7160(14)	0.7159(13)	0.7174(15)
0.2	0.79	0.7330(15)	0.7306(15)	0.7318(16)	0.7300(12)	0.7307(16)	0.7299(15)	0.7298(15)
0.3	0.85	0.7471(15)	0.7462(16)	0.7458(15)	0.7465(18)	0.7458(17)	0.7442(16)	0.7449(15)
0.4	0.91	0.7607(17)	0.7622(17)	0.7622(16)	0.7603(16)	0.7598(15)	0.7601(15)	0.7609(18)
0.5	1	0.7767(15)	0.7758(17)	0.7751(17)	0.7752(16)	0.7768(17)	0.7757(16)	0.7767(16)
0.6	1.12	0.7883(19)	0.7921(16)	0.7905(17)	0.7914(20)	0.7923(15)	0.7902(18)	0.7902(17)
0.7	1.29	0.7994(16)	0.8053(18)	0.8071(19)	0.8081(20)	0.8081(17)	0.8085(18)	0.8080(16)
0.8	1.58	0.8125(19)	0.8201(16)	0.8247(17)	0.8271(18)	0.8268(20)	0.8271(17)	0.8242(17)
0.8605	1.89	0.8190(20)	0.8299(19)	0.8364(19)	0.8386(19)	0.8390(19)	0.8394(18)	0.8388(16)
0.9	2.24	0.8237(19)	0.8336(17)	0.8440(20)	0.8489(18)	0.8493(19)	0.8483(18)	0.8483(17)

Table 8.19: Anderson-Darling critical values of the case II lognormal distrib. at 95% sig. lvl

8.5.5 Comparison with the available literature

The case II critical values from our work are compared to the literature in Appendix B.2. Tables B.5, B.6, B.7 and B.8 (cont. in Table B.9) display the comparison for the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests, respectively. As discussed in subsection 8.4.5, the critical values from this work are displayed in bold, below the critical values from other studies. Critical values from the literature that were determined from a formula of n, are preceded by an asterisk (*) and those determined from C code are preceded with a dagger (†). To the best of our knowledge, the work conducted by Kizilersü et al. [10] is the only one that has determined critical values corresponding to one of the case II left-truncated distributions that we studied. Therefore, the comparison to literature in Appendix B.2 is restricted to complete (untruncated) distributions, i.e. $\tau_l = p = \eta = 0$.

Again, our comparison was hampered by the fact that most studies stated critical values without a corresponding confidence interval, even if the critical values were produced through Monte Carlo simulation[3]. The critical values we determined achieved broad agreement with the majority of values we consulted. However, there were some cases in which significant differences occured. In these situations previous studies had usually undertaken a Monte Carlo simulation with far less repetition than ours. Thus, it is more likely that the discrepancy is attributable to a large uncertainty margin in the literature. For example, in his 1974 work Stephens [12] gives a series of formulae for determining the critical values as a function of n for all the goodness-of-fit tests we studied[4]. These formulae were derived based on a theoretical asymptotic point and single-sample Monte Carlo simulations with $N = 10,000$ repetitions, conducted at sample sizes $n = 10, 20, 50, 100$. Consider the Anderson-Darling critical values at the 95% significance level resulting from a sample of size $n = 1000$ drawn from the case II normal/lognormal[5] distribution, we achieved a result of 0.7512 ± 0.0014 and Stephens' formula produced a value of 0.784. In our work, we took 100 times as many samples and conducted them at $n = 1000$, rather interpolating between $n = 100$ and an asymptotic point as Stephens did. Therefore, it is likely that the uncertainty margin on Stephens' result is far larger than ours. If this could be taken into account there may be well be agreement between the two values. Table B.8 displays these results. This example is typical of the reason and magnitude that our results differ from the literature.

[3]Kizilersü et al. [10] is a notable exception.

[4]It is worth noting that we found Stephens' 1974 work [12] extremely helpful.

[5]The normal and lognormal distributions are equivalent and therefore have the same critical values.

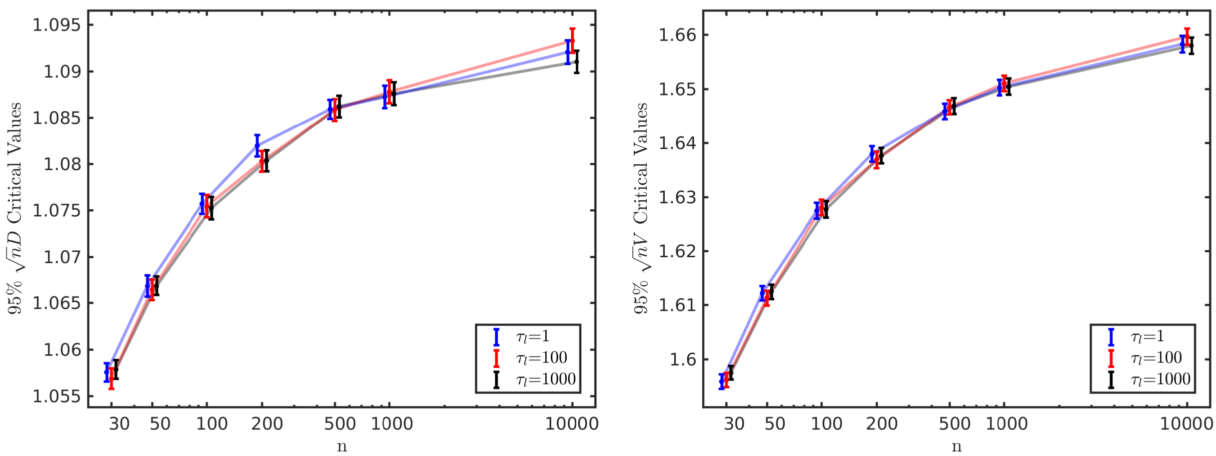
8.6 Case IIIa

- Weibull
- Loglogistic
- Lognormal
- Pareto

8.6.1 Variable Dependence

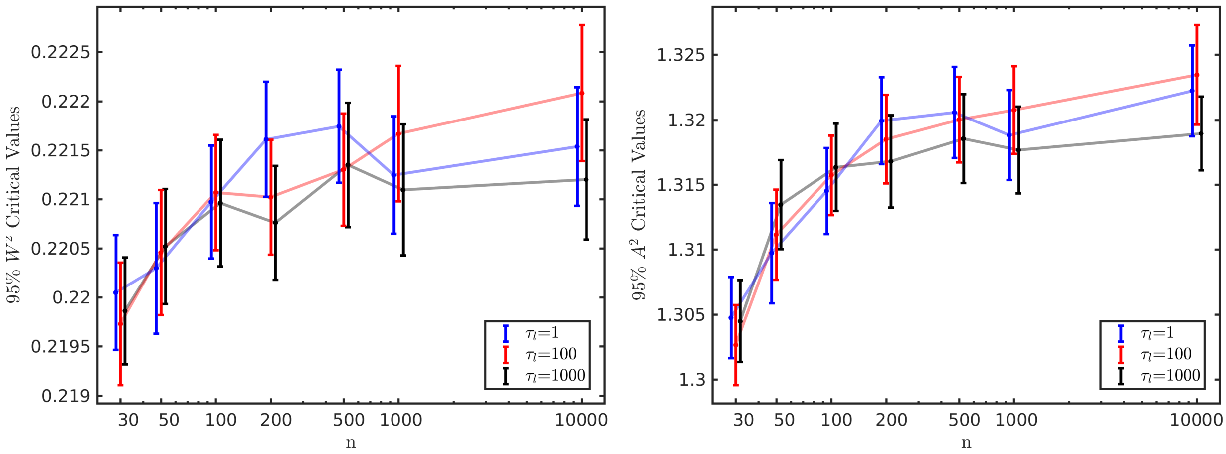
The 95% Kolmogorov-Smirnov and Kuiper critical values for the Pareto distribution are displayed in Figure 8.14, the corresponding values for the Cramér-von Mises and Anderson-Darling tests are shown in Figure 8.15. These plots showcase the characteristics that are common to all confidence levels for the Weibull, loglogistic and Pareto distributions, namely n dependence and a lack of truncation dependence. The Cramér-von Mises and Anderson-Darling critical values have uncertainty margins that are of the same order as the change associated with different n values, however, n dependence is still resolvable. The lognormal distribution produced Kolmogorov-Smirnov and Kuiper critical values with strong n and truncation dependence, as displayed in Figure 8.16. The lognormal Cramér-von Mises and Anderson-Darling critical values are shown in Figure 8.17 and showcase strong truncation dependence; it appears that for high truncation values the Anderson-Darling test is slightly dependent upon n, however more data is required to confirm this.

The critical values depicted in depicted in Figures 8.14 and 8.15 resulting from different truncation levels (τ_l values) are staggered along the n axis; this does not imply that the critical were evaluated at distinct sample sizes. In Figures 8.16 and 8.17 the critical values resulting from different sample sizes (n values) are staggered along the $\sqrt{\eta}$ axis; this does not imply that the critical were evaluated at distinct truncation levels. There is no discernible relation between the confidence interval length and significance level, hence, we have elected to show only the results for the 95% significance level as discussed in section 8.4. Table 8.20 summarises the dependency of the case IIIa critical values.



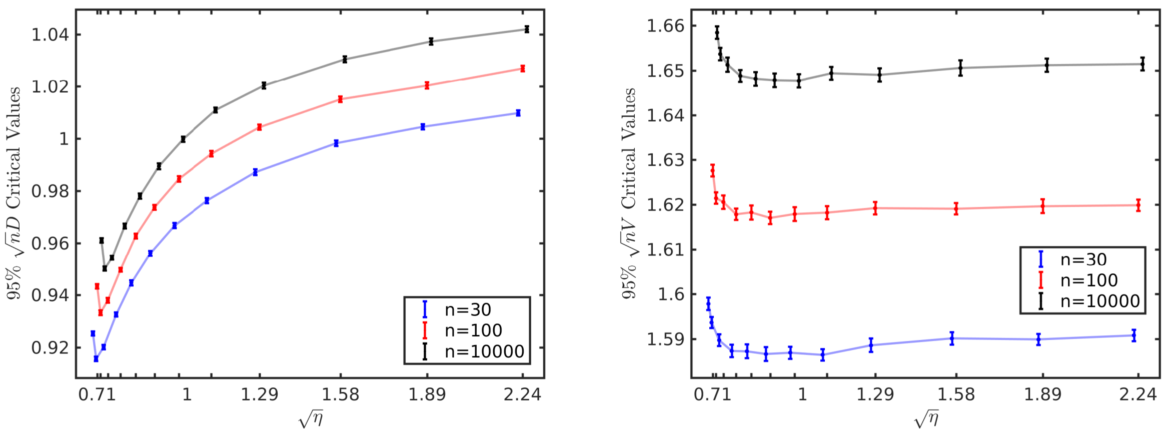
(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl.
(b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

Figure 8.14: Case IIIa Pareto Kolmogorov-Smirnov and Kuiper critical values
Critical values from different τ_l values are staggered on the n axis for clarity



(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl.   (b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

Figure 8.15: Case IIIa Pareto Cramér-von Mises and Anderson-Darling critical values
Critical values from different τ_l values are staggered on the n axis for clarity

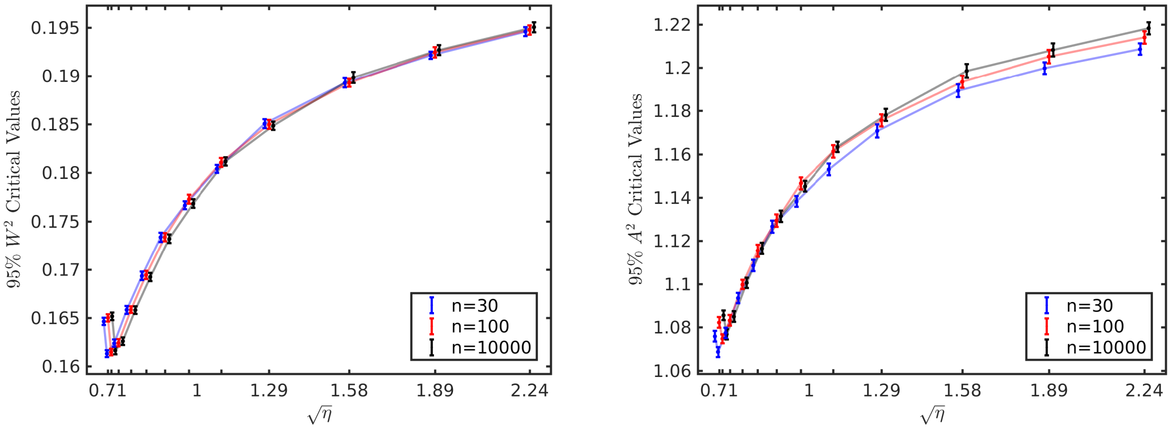


(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl.   (b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

Figure 8.16: Case IIIa Lognormal Kolmogorov-Smirnov and Kuiper critical values
Critical values from different n values are staggered on the $\sqrt{\eta}$ axis for clarity

Distribution	Test	n dependent	τ_L dependent
Weibull	KS & Kuiper	✓	✗
Loglogistic	KS & Kuiper	✓	✗
Lognormal	KS & Kuiper	✓	✓
Pareto	KS & Kuiper	✓	✗
Weibull	CvM & AD	-	-
Loglogistic	CvM & AD	-	-
Lognormal	CvM	✗	✓
Lognormal	AD	-	✓
Pareto	CvM & AD	-	-

Table 8.20: Case IIIa critical value dependences
A hyphen implies dependence is suggested but cannot be confirmed

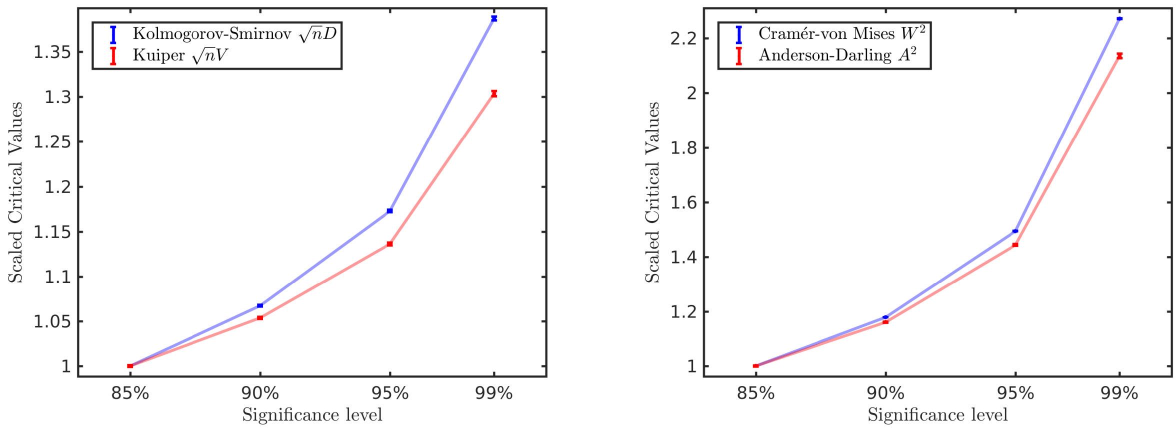


(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl. (b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

Figure 8.17: Case IIIa Lognormal Cramér-von Mises and Anderson-Darling critical values
Critical values from different n values are staggered on the $\sqrt{\eta}$ axis for clarity

8.6.2 Effect of Significance Level on Critical Values

Figure 8.18 displays the case IIIa Pareto critical values at $n = 10,000$ and τ_l=1, plotted against significance level. All critical values have been divided by their respective 85% critical values so that the change between different goodness-of-tests can be compared with increased clarity. As discussed in subsection 8.4.2, the critical values are necessarily monotone increasing and this behaviour is common to all sample sizes and truncation levels.



(a) Kolmogorov-Smirnov and Kuiper Critical Values (b) Cramér-von Mises and Anderson-Darling Critical Values

Figure 8.18: Case IIIa Pareto distribution τ_l=1, n=10,000 critical values
Scaled to the 85% significance level

8.6.3 Comparison with Kizilersü et al.

Tables 8.21 gives the Kolmogorov-Smirnov critical values for case IIIa at the 95% significance level. The results of the present work ($C = 100$ and $M = 10,000$) are displayed in bold below the results of Kizilersü et al. [10] ($C = 100$ and $M = 1,000$). There is strong agreement between the two studies, and the difference between the uncertainty margins is roughly $\frac{\sqrt{1000}}{1.984}$, as expected.

p	$\sqrt{\eta}$	30	50	100	200	500	1000
0	0	1.055(20) **1.0581(06)**	1.064(18) **1.0668(05)**	1.072(20) **1.0760(06)**	1.080(21) **1.0813(06)**	1.083(16) **1.0858(06)**	1.086(18) **1.0880(06)**
0.1	0.32	1.054(19) **1.0578(07)**	1.064(19) **1.0669(06)**	1.074(19) **1.0757(05)**	1.078(18) **1.0810(06)**	1.085(17) **1.0858(06)**	1.085(18) **1.0876(07)**
0.2	0.47	1.058(18) **1.0582(06)**	1.064(18) **1.0677(06)**	1.074(18) **1.0755(06)**	1.080(19) **1.0816(07)**	1.085(18) **1.0864(06)**	1.086(22) **1.0886(06)**
0.3	0.6	1.057(16) **1.0577(06)**	1.064(19) **1.0661(06)**	1.075(16) **1.0741(06)**	1.081(19) **1.0798(07)**	1.082(17) **1.0863(06)**	1.085(19) **1.0869(06)**
0.4	0.71	1.054(18) **1.0580(06)**	1.066(19) **1.0667(05)**	1.072(19) **1.0749(06)**	1.079(19) **1.0802(06)**	1.083(17) **1.0857(06)**	1.085(16) **1.0875(06)**
0.5	0.83	1.054(17) **1.0573(06)**	1.066(21) **1.0672(06)**	1.074(21) **1.0750(05)**	1.080(21) **1.0818(07)**	1.084(18) **1.0856(06)**	1.083(19) **1.0884(06)**
0.6	0.96	1.057(17) **1.0574(06)**	1.065(18) **1.0665(06)**	1.077(20) **1.0741(06)**	1.077(18) **1.0810(07)**	1.086(20) **1.0854(06)**	1.086(19) **1.0869(06)**
0.7	1.1	1.056(18) **1.0583(06)**	1.065(18) **1.0660(06)**	1.075(19) **1.0747(06)**	1.083(18) **1.0821(05)**	1.084(17) **1.0850(06)**	1.085(19) **1.0888(06)**
0.8	1.27	1.056(21) **1.0576(05)**	1.064(18) **1.0667(05)**	1.076(20) **1.0743(06)**	1.080(21) **1.0818(06)**	1.082(19) **1.0863(06)**	1.086(17) **1.0881(06)**
0.9	1.52	1.057(17) **1.0588(05)**	1.065(17) **1.0672(06)**	1.074(18) **1.0759(06)**	1.077(17) **1.0821(06)**	1.084(20) **1.0861(06)**	1.088(19) **1.0880(06)**

Table 8.21: Kolmogorov-Smirnov critical values of the case IIIa Weibull distrib. at 95% sig. lvl
Results from our work are **bold**, the others are from Kizilersü et al. [10]

8.6.4 Critical Values

Tables 8.22, 8.23 and 8.24 display the Kuiper, Cramér-von Mises and Anderson-Darling critical values for case IIIa Weibull distribution at the 95% significance level. These critical values are given for the loglogistic, lognormal and Pareto distributions in Tables 8.25-8.28, 8.29-8.32 and 8.33-8.36 respectively. The complete set of case IIIa critical values (found in Appendix A.3) is too extensive to include, hence, we have elected to show only the results for only one significance level. Again, the 95% significance level has been displayed because it was previously studied by Kizilersü et al. [10] and all the uncertainty margins are given at the 95% confidence level.

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5956(15)	1.6124(14)	1.6270(13)	1.6368(15)	1.6460(14)	1.6510(14)	1.6572(13)
0.0323	0.18	1.5967(15)	1.6116(13)	1.6277(15)	1.6374(15)	1.6464(13)	1.6517(12)	1.6597(13)
0.1	0.32	1.5962(15)	1.6132(15)	1.6270(15)	1.6369(15)	1.6481(14)	1.6501(13)	1.6589(14)
0.2	0.47	1.5968(13)	1.6117(16)	1.6266(13)	1.6393(14)	1.6470(13)	1.6516(13)	1.6573(16)
0.3	0.6	1.5967(16)	1.6115(15)	1.6257(13)	1.6358(15)	1.6467(13)	1.6507(14)	1.6573(15)
0.4	0.71	1.5978(17)	1.6122(13)	1.6271(14)	1.6374(15)	1.6462(14)	1.6510(17)	1.6590(16)
0.5	0.83	1.5959(15)	1.6129(12)	1.6264(14)	1.6381(16)	1.6458(15)	1.6507(16)	1.6594(13)
0.6	0.96	1.5957(15)	1.6122(16)	1.6255(15)	1.6371(16)	1.6459(15)	1.6497(14)	1.6570(14)
0.7	1.1	1.5971(13)	1.6122(13)	1.6272(15)	1.6399(12)	1.6459(13)	1.6511(15)	1.6583(16)
0.8	1.27	1.5960(12)	1.6123(15)	1.6262(14)	1.6375(13)	1.6462(14)	1.6506(13)	1.6584(14)
0.8605	1.4	1.5962(14)	1.6119(15)	1.6270(13)	1.6382(16)	1.6466(14)	1.6502(13)	1.6579(14)
0.9	1.52	1.5980(13)	1.6126(15)	1.6273(14)	1.6380(14)	1.6471(14)	1.6507(14)	1.6592(14)

Table 8.22: Kuiper critical values of the case IIIa Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.21948(58)	0.22098(55)	0.22173(59)	0.22173(63)	0.22118(56)	0.22150(56)	0.22115(62)
0.0323	0.18	0.21958(59)	0.22056(61)	0.22119(60)	0.22109(66)	0.22139(62)	0.22117(54)	0.22186(59)
0.1	0.32	0.21976(67)	0.22078(61)	0.22113(52)	0.22082(64)	0.22174(63)	0.22130(60)	0.22144(60)
0.2	0.47	0.21993(59)	0.22066(58)	0.22091(63)	0.22164(59)	0.22179(55)	0.22196(66)	0.22137(63)
0.3	0.6	0.21990(53)	0.22047(61)	0.22066(55)	0.22085(64)	0.22163(56)	0.22101(61)	0.22075(61)
0.4	0.71	0.21965(69)	0.22065(56)	0.22046(63)	0.22083(69)	0.22143(58)	0.22116(62)	0.22148(53)
0.5	0.83	0.21985(65)	0.22044(62)	0.22091(59)	0.22160(71)	0.22120(67)	0.22133(63)	0.22139(53)
0.6	0.96	0.21975(64)	0.22044(59)	0.22066(63)	0.22119(67)	0.22102(61)	0.22115(61)	0.22097(53)
0.7	1.1	0.21993(61)	0.22036(59)	0.22110(58)	0.22174(59)	0.22092(57)	0.22175(56)	0.22181(54)
0.8	1.27	0.21974(57)	0.22060(61)	0.22077(57)	0.22176(59)	0.22155(61)	0.22100(61)	0.22162(62)
0.8605	1.4	0.21983(61)	0.22063(61)	0.22093(62)	0.22116(64)	0.22129(60)	0.22076(56)	0.22077(68)
0.9	1.52	0.22018(62)	0.22034(58)	0.22130(55)	0.22185(61)	0.22155(57)	0.22136(58)	0.22184(50)

Table 8.23: Cramér-von Mises critical values of the case IIIa Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3043(31)	1.3107(31)	1.3179(34)	1.3187(35)	1.3190(32)	1.3169(30)	1.3201(33)
0.0323	0.18	1.3024(30)	1.3098(33)	1.3168(31)	1.3180(35)	1.3196(33)	1.3171(29)	1.3213(30)
0.1	0.32	1.3044(35)	1.3117(33)	1.3170(28)	1.3152(36)	1.3220(35)	1.3186(33)	1.3216(31)
0.2	0.47	1.3051(34)	1.3094(35)	1.3157(36)	1.3203(30)	1.3208(28)	1.3232(37)	1.3208(31)
0.3	0.6	1.3053(30)	1.3097(32)	1.3129(31)	1.3173(34)	1.3210(32)	1.3184(34)	1.3166(30)
0.4	0.71	1.3048(39)	1.3103(29)	1.3146(35)	1.3165(37)	1.3184(33)	1.3174(32)	1.3193(30)
0.5	0.83	1.3042(35)	1.3095(36)	1.3150(33)	1.3190(38)	1.3179(34)	1.3200(37)	1.3202(31)
0.6	0.96	1.3009(35)	1.3093(32)	1.3136(32)	1.3167(36)	1.3187(31)	1.3171(32)	1.3178(29)
0.7	1.1	1.3046(35)	1.3079(32)	1.3145(32)	1.3203(29)	1.3166(32)	1.3218(32)	1.3211(32)
0.8	1.27	1.3020(31)	1.3113(31)	1.3150(33)	1.3185(34)	1.3203(34)	1.3176(30)	1.3198(34)
0.8605	1.4	1.3045(32)	1.3098(32)	1.3145(31)	1.3177(34)	1.3183(32)	1.3165(30)	1.3191(36)
0.9	1.52	1.3061(32)	1.3082(32)	1.3179(31)	1.3199(32)	1.3201(35)	1.3184(33)	1.3235(33)

Table 8.24: Anderson-Darling critical values of the case IIIa Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8699(08)	0.8796(08)	0.8878(08)	0.8938(09)	0.8990(09)	0.9013(09)	0.9058(08)
0.0323	0.18	0.8696(07)	0.8790(07)	0.8885(07)	0.8946(08)	0.8996(09)	0.9019(09)	0.9063(07)
0.1	0.33	0.8700(08)	0.8798(08)	0.8883(09)	0.8941(08)	0.8989(08)	0.9016(09)	0.9059(08)
0.2	0.5	0.8697(08)	0.8800(08)	0.8881(08)	0.8945(08)	0.9003(07)	0.9017(07)	0.9055(08)
0.3	0.65	0.8692(08)	0.8792(07)	0.8890(09)	0.8944(08)	0.8986(09)	0.9017(09)	0.9053(09)
0.4	0.82	0.8700(08)	0.8791(07)	0.8880(08)	0.8945(08)	0.8993(08)	0.9022(08)	0.9056(08)
0.5	1	0.8702(07)	0.8793(07)	0.8886(08)	0.8944(07)	0.9003(08)	0.9024(09)	0.9062(08)
0.6	1.22	0.8699(08)	0.8788(08)	0.8886(08)	0.8948(09)	0.8988(08)	0.9018(09)	0.9059(08)
0.7	1.53	0.8701(08)	0.8793(08)	0.8884(08)	0.8947(08)	0.8992(08)	0.9022(09)	0.9060(08)
0.8	2	0.8693(07)	0.8798(08)	0.8886(09)	0.8944(09)	0.8991(10)	0.9014(09)	0.9065(07)
0.8605	2.48	0.8695(08)	0.8795(08)	0.8885(07)	0.8939(08)	0.8990(08)	0.9022(07)	0.9052(07)
0.9	3	0.8707(09)	0.8793(08)	0.8885(09)	0.8940(08)	0.8991(07)	0.9018(08)	0.9059(08)

Table 8.25: Kolmogorov-Smirnov critical values of the case IIIa loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5697(14)	1.5840(13)	1.5988(13)	1.6072(13)	1.6174(15)	1.6214(14)	1.6292(15)
0.0323	0.18	1.5692(14)	1.5840(13)	1.5983(13)	1.6094(16)	1.6174(15)	1.6213(15)	1.6307(15)
0.1	0.33	1.5707(14)	1.5850(15)	1.5976(15)	1.6082(15)	1.6168(14)	1.6222(16)	1.6301(13)
0.2	0.5	1.5694(16)	1.5846(13)	1.5977(12)	1.6084(15)	1.6194(14)	1.6210(13)	1.6292(15)
0.3	0.65	1.5689(15)	1.5841(13)	1.6004(14)	1.6097(15)	1.6176(14)	1.6222(16)	1.6289(13)
0.4	0.82	1.5698(14)	1.5843(13)	1.5980(15)	1.6085(15)	1.6165(13)	1.6222(15)	1.6295(13)
0.5	1	1.5693(14)	1.5846(14)	1.5989(15)	1.6089(13)	1.6187(14)	1.6231(14)	1.6304(17)
0.6	1.22	1.5699(14)	1.5828(14)	1.5995(15)	1.6094(14)	1.6160(16)	1.6230(16)	1.6298(14)
0.7	1.53	1.5697(13)	1.5839(16)	1.5988(14)	1.6091(14)	1.6166(15)	1.6223(14)	1.6301(14)
0.8	2	1.5695(14)	1.5848(14)	1.5988(17)	1.6087(15)	1.6177(17)	1.6215(15)	1.6290(15)
0.8605	2.48	1.5692(15)	1.5850(14)	1.5985(14)	1.6076(14)	1.6161(14)	1.6214(12)	1.6297(14)
0.9	3	1.5716(17)	1.5834(12)	1.5991(13)	1.6090(13)	1.6174(12)	1.6213(14)	1.6301(14)

Table 8.26: Kuiper critical values of the case IIIa loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.14813(30)	0.14804(35)	0.14792(33)	0.14772(32)	0.14789(38)	0.14753(35)	0.14780(38)
0.0323	0.18	0.14803(41)	0.14799(35)	0.14782(36)	0.14782(36)	0.14771(40)	0.14768(36)	0.14819(37)
0.1	0.33	0.14836(37)	0.14799(40)	0.14774(33)	0.14795(35)	0.14766(33)	0.14782(39)	0.14783(36)
0.2	0.5	0.14804(42)	0.14827(39)	0.14756(33)	0.14763(35)	0.14819(33)	0.14775(33)	0.14780(37)
0.3	0.65	0.14795(36)	0.14800(34)	0.14804(36)	0.14806(38)	0.14768(32)	0.14795(39)	0.14770(33)
0.4	0.82	0.14809(35)	0.14818(33)	0.14788(41)	0.14782(38)	0.14754(32)	0.14796(38)	0.14801(33)
0.5	1	0.14805(39)	0.14799(36)	0.14796(33)	0.14773(36)	0.14801(34)	0.14822(39)	0.14782(34)
0.6	1.22	0.14807(39)	0.14746(34)	0.14808(35)	0.14785(31)	0.14756(37)	0.14809(38)	0.14774(33)
0.7	1.53	0.14793(34)	0.14820(38)	0.14818(32)	0.14804(35)	0.14781(38)	0.14786(39)	0.14806(35)
0.8	2	0.14801(32)	0.14807(36)	0.14805(39)	0.14802(41)	0.14799(40)	0.14748(38)	0.14784(36)
0.8605	2.48	0.14805(36)	0.14808(36)	0.14777(38)	0.14758(34)	0.14786(35)	0.14761(31)	0.14783(34)
0.9	3	0.14836(42)	0.14778(34)	0.14799(38)	0.14786(37)	0.14782(31)	0.14771(35)	0.14815(38)

Table 8.27: Cramér-von Mises critical values of the case IIIa loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0379(24)	1.0408(22)	1.0449(22)	1.0431(24)	1.0444(25)	1.0431(23)	1.0449(26)
0.0323	0.18	1.0374(25)	1.0421(27)	1.0430(23)	1.0454(24)	1.0444(27)	1.0453(23)	1.0474(21)
0.1	0.33	1.0386(23)	1.0414(26)	1.0437(24)	1.0431(23)	1.0441(22)	1.0442(24)	1.0452(24)
0.2	0.5	1.0378(28)	1.0428(24)	1.0408(21)	1.0443(24)	1.0477(23)	1.0468(24)	1.0448(24)
0.3	0.65	1.0384(25)	1.0415(23)	1.0451(23)	1.0455(25)	1.0440(22)	1.0461(25)	1.0454(23)
0.4	0.82	1.0378(26)	1.0413(24)	1.0418(25)	1.0440(24)	1.0442(19)	1.0457(24)	1.0458(19)
0.5	1	1.0397(25)	1.0416(21)	1.0446(25)	1.0438(26)	1.0457(24)	1.0465(24)	1.0458(23)
0.6	1.22	1.0384(29)	1.0386(24)	1.0431(22)	1.0440(22)	1.0449(24)	1.0451(26)	1.0450(23)
0.7	1.53	1.0365(21)	1.0437(24)	1.0443(24)	1.0450(22)	1.0448(25)	1.0466(26)	1.0472(23)
0.8	2	1.0350(24)	1.0415(21)	1.0429(26)	1.0438(27)	1.0461(27)	1.0435(27)	1.0453(22)
0.8605	2.48	1.0379(23)	1.0402(25)	1.0417(25)	1.0420(22)	1.0445(24)	1.0445(21)	1.0441(25)
0.9	3	1.0387(27)	1.0389(26)	1.0437(24)	1.0429(26)	1.0445(21)	1.0443(22)	1.0460(25)

Table 8.28: Anderson-Darling critical values of the case IIIa loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.9253(07)	0.9346(08)	0.9434(09)	0.9506(09)	0.9545(10)	0.9576(09)	0.9611(09)
0.0323	0.72	0.9156(08)	0.9247(09)	0.9332(09)	0.9389(09)	0.9444(09)	0.9470(10)	0.9502(08)
0.1	0.75	0.9201(09)	0.9298(09)	0.9380(09)	0.9436(08)	0.9484(09)	0.9504(08)	0.9544(08)
0.2	0.79	0.9326(09)	0.9428(09)	0.9497(08)	0.9552(09)	0.9614(08)	0.9624(09)	0.9666(09)
0.3	0.85	0.9447(10)	0.9546(09)	0.9627(09)	0.9676(09)	0.9735(09)	0.9745(10)	0.9781(10)
0.4	0.91	0.9561(10)	0.9655(09)	0.9738(09)	0.9787(10)	0.9842(09)	0.9862(09)	0.9894(11)
0.5	1	0.9667(10)	0.9760(10)	0.9846(11)	0.9903(10)	0.9943(09)	0.9957(11)	0.9998(10)
0.6	1.12	0.9764(10)	0.9861(10)	0.9942(11)	0.9999(11)	1.0039(11)	1.0067(09)	1.0109(10)
0.7	1.29	0.9871(11)	0.9952(09)	1.0044(10)	1.0095(10)	1.0145(10)	1.0166(10)	1.0203(11)
0.8	1.58	0.9982(11)	1.0061(10)	1.0150(10)	1.0192(11)	1.0249(12)	1.0261(09)	1.0304(11)
0.8605	1.89	1.0045(10)	1.0141(10)	1.0203(12)	1.0264(10)	1.0309(11)	1.0336(09)	1.0373(12)
0.9	2.24	1.0097(10)	1.0185(11)	1.0269(11)	1.0311(11)	1.0357(10)	1.0388(10)	1.0419(12)

Table 8.29: Kolmogorov-Smirnov critical values of the case IIIa lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.5978(14)	1.6122(15)	1.6276(13)	1.6381(14)	1.6449(17)	1.6511(14)	1.6584(14)
0.0323	0.72	1.5937(12)	1.6093(16)	1.6215(13)	1.6335(13)	1.6418(14)	1.6467(17)	1.6536(14)
0.1	0.75	1.5897(13)	1.6055(15)	1.6206(15)	1.6301(15)	1.6381(14)	1.6435(14)	1.6513(16)
0.2	0.79	1.5873(14)	1.6045(15)	1.6179(12)	1.6284(16)	1.6384(13)	1.6407(16)	1.6488(13)
0.3	0.85	1.5873(15)	1.6035(15)	1.6183(16)	1.6285(14)	1.6374(13)	1.6409(15)	1.6481(15)
0.4	0.91	1.5867(16)	1.6031(14)	1.6171(14)	1.6275(15)	1.6373(15)	1.6419(15)	1.6478(15)
0.5	1	1.5870(13)	1.6032(15)	1.6179(15)	1.6280(14)	1.6372(15)	1.6406(15)	1.6477(15)
0.6	1.12	1.5865(12)	1.6031(13)	1.6182(14)	1.6289(13)	1.6365(14)	1.6414(14)	1.6494(14)
0.7	1.29	1.5886(15)	1.6030(14)	1.6192(14)	1.6283(14)	1.6383(14)	1.6421(13)	1.6490(14)
0.8	1.58	1.5901(14)	1.6045(14)	1.6191(13)	1.6281(14)	1.6394(15)	1.6417(13)	1.6505(17)
0.8605	1.89	1.5899(12)	1.6058(14)	1.6197(16)	1.6304(13)	1.6394(14)	1.6428(12)	1.6512(15)
0.9	2.24	1.5908(13)	1.6050(14)	1.6199(13)	1.6287(14)	1.6388(13)	1.6441(13)	1.6514(14)

Table 8.30: Kuiper critical values of the case IIIa lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.16464(37)	0.16492(38)	0.16501(38)	0.16534(36)	0.16485(38)	0.16506(42)	0.16515(40)
0.0323	0.72	0.16131(35)	0.16164(42)	0.16147(34)	0.16192(33)	0.16182(36)	0.16210(42)	0.16162(38)
0.1	0.75	0.16238(38)	0.16266(43)	0.16243(38)	0.16250(41)	0.16237(36)	0.16256(37)	0.16260(39)
0.2	0.79	0.16582(40)	0.16576(42)	0.16585(33)	0.16559(41)	0.16589(38)	0.16562(42)	0.16580(38)
0.3	0.85	0.16934(44)	0.16969(38)	0.16944(42)	0.16960(36)	0.16943(41)	0.16922(46)	0.16922(42)
0.4	0.91	0.17336(48)	0.17330(43)	0.17336(42)	0.17318(41)	0.17339(48)	0.17355(37)	0.17320(45)
0.5	1	0.17665(40)	0.17735(44)	0.17729(45)	0.17711(47)	0.17714(38)	0.17698(42)	0.17683(43)
0.6	1.12	0.18040(40)	0.18090(42)	0.18107(46)	0.18099(49)	0.18066(45)	0.18096(46)	0.18117(44)
0.7	1.29	0.18507(47)	0.18475(44)	0.18499(47)	0.18466(50)	0.18522(45)	0.18507(48)	0.18485(44)
0.8	1.58	0.18927(48)	0.18965(47)	0.18927(41)	0.18909(47)	0.18939(52)	0.18912(52)	0.18982(56)
0.8605	1.89	0.19212(38)	0.19256(46)	0.19244(54)	0.19286(47)	0.19267(52)	0.19246(49)	0.19267(51)
0.9	2.24	0.19457(46)	0.19484(52)	0.19475(48)	0.19465(43)	0.19473(47)	0.19490(53)	0.19502(52)

Table 8.31: Cramér-von Mises critical values of the case IIIa lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.0758(25)	1.0802(23)	1.0822(24)	1.0854(23)	1.0849(27)	1.0848(27)	1.0855(22)
0.0323	0.72	1.0686(23)	1.0733(26)	1.0748(21)	1.0776(23)	1.0764(26)	1.0790(24)	1.0768(24)
0.1	0.75	1.0774(24)	1.0814(25)	1.0832(25)	1.0827(25)	1.0842(23)	1.0850(25)	1.0850(24)
0.2	0.79	1.0935(24)	1.0966(26)	1.0998(21)	1.1015(27)	1.1010(25)	1.0990(25)	1.1006(24)
0.3	0.85	1.1085(26)	1.1130(22)	1.1155(28)	1.1171(23)	1.1157(28)	1.1160(27)	1.1165(26)
0.4	0.91	1.1266(27)	1.1287(27)	1.1294(29)	1.1303(27)	1.1328(29)	1.1340(23)	1.1314(26)
0.5	1	1.1383(25)	1.1442(28)	1.1466(29)	1.1477(28)	1.1475(23)	1.1464(27)	1.1453(25)
0.6	1.12	1.1531(27)	1.1600(29)	1.1613(29)	1.1616(28)	1.1617(29)	1.1638(30)	1.1634(24)
0.7	1.29	1.1707(30)	1.1735(28)	1.1756(27)	1.1772(32)	1.1795(29)	1.1788(27)	1.1781(28)
0.8	1.58	1.1893(30)	1.1931(28)	1.1934(28)	1.1936(27)	1.1954(30)	1.1943(26)	1.1984(32)
0.8605	1.89	1.1997(27)	1.2041(32)	1.2051(31)	1.2101(28)	1.2078(28)	1.2075(30)	1.2082(30)
0.9	2.24	1.2086(26)	1.2133(32)	1.2140(29)	1.2139(25)	1.2159(29)	1.2168(30)	1.2182(28)

Table 8.32: Anderson-Darling critical values of the case IIIa lognormal distrib. at 95% sig. lvl

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.0575(10)	1.0668(12)	1.0757(11)	1.0819(11)	1.0859(10)	1.0872(12)	1.0920(13)
10	3.16	1.0579(12)	1.0665(11)	1.0757(10)	1.0810(12)	1.0855(12)	1.0867(13)	1.0919(12)
100	10	1.0569(11)	1.0664(11)	1.0754(12)	1.0803(11)	1.0858(12)	1.0878(13)	1.0933(13)
250	15.81	1.0571(11)	1.0674(10)	1.0755(12)	1.0812(12)	1.0855(12)	1.0883(11)	1.0921(11)
500	22.36	1.0582(11)	1.0659(11)	1.0756(12)	1.0812(11)	1.0868(11)	1.0888(12)	1.0919(11)
1000	31.62	1.0579(10)	1.0669(10)	1.0752(12)	1.0803(11)	1.0862(12)	1.0876(12)	1.0910(12)

Table 8.33: Kolmogorov-Smirnov critical values of the case IIIa Pareto distrib. at 95% sig. lvl

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.5959(13)	1.6122(14)	1.6275(15)	1.6380(14)	1.6459(14)	1.6502(14)	1.6583(15)
10	3.16	1.5976(14)	1.6122(13)	1.6278(12)	1.6373(14)	1.6473(14)	1.6504(14)	1.6586(14)
100	10	1.5962(13)	1.6112(14)	1.6281(14)	1.6368(16)	1.6467(13)	1.6510(14)	1.6597(15)
250	15.81	1.5964(15)	1.6122(13)	1.6274(13)	1.6383(14)	1.6467(14)	1.6516(16)	1.6585(13)
500	22.36	1.5979(14)	1.6117(13)	1.6274(14)	1.6373(14)	1.6481(13)	1.6526(15)	1.6589(13)
1000	31.62	1.5975(13)	1.6125(14)	1.6277(15)	1.6376(14)	1.6468(15)	1.6504(15)	1.6580(15)

Table 8.34: Kuiper critical values of the case IIIa Pareto distrib. at 95% sig. lvl

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.22005(58)	0.22030(66)	0.22097(58)	0.22161(59)	0.22174(58)	0.22125(60)	0.22154(60)
10	3.16	0.21981(60)	0.22059(58)	0.22107(54)	0.22189(62)	0.22137(59)	0.22118(60)	0.22124(56)
100	10	0.21973(62)	0.22046(64)	0.22107(59)	0.22102(59)	0.22130(57)	0.22167(69)	0.22208(69)
250	15.81	0.21964(62)	0.22047(56)	0.22082(60)	0.22117(60)	0.22127(63)	0.22178(64)	0.22186(50)
500	22.36	0.22005(51)	0.22039(53)	0.22116(58)	0.22109(66)	0.22215(57)	0.22172(57)	0.22145(57)
1000	31.62	0.21986(54)	0.22052(59)	0.22096(65)	0.22076(58)	0.22135(63)	0.22110(67)	0.22120(61)

Table 8.35: Cramér-von Mises critical values of the case IIIa Pareto distrib. at 95% sig. lvl

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.3047(31)	1.3097(39)	1.3145(33)	1.3199(33)	1.3206(35)	1.3188(35)	1.3222(35)
10	3.16	1.3052(31)	1.3115(31)	1.3147(30)	1.3210(31)	1.3203(37)	1.3184(32)	1.3191(33)
100	10	1.3026(31)	1.3111(35)	1.3157(31)	1.3185(34)	1.3200(33)	1.3208(34)	1.3235(38)
250	15.81	1.3033(34)	1.3094(30)	1.3144(33)	1.3172(31)	1.3183(31)	1.3220(36)	1.3228(31)
500	22.36	1.3046(30)	1.3082(27)	1.3161(31)	1.3173(37)	1.3232(34)	1.3214(33)	1.3209(26)
1000	31.62	1.3045(31)	1.3135(34)	1.3163(34)	1.3168(36)	1.3185(34)	1.3177(33)	1.3189(28)

Table 8.36: Anderson-Darling critical values of the case IIIa Pareto distrib. at 95% sig. lvl

8.6.5 Comparison with the available literature

The case IIIa critical values from our work are compared to the literature in Appendix B.3. Tables B.10, B.11, B.12 and B.13 (cont. in Table B.14) display the comparison for the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests, respectively. As discussed in subsection 8.4.5, the critical values from this work are displayed in bold, below the critical values from other studies. Critical values from the literature that were determined from a formula of n, are preceded by an asterisk (*) and those determined from C code are preceded with a dagger (†). To the best of our knowledge, the work conducted by Kizilersü et al. [10] is the only one that has determined critical values corresponding to one of the case IIIa left-truncated distributions that we studied. Therefore, the comparison to literature in Appendix B.3 is restricted to complete (untruncated) distributions, i.e. $\tau_l = p = \eta = 0$.

The case IIIa Weibull and Pareto critical values have been displayed together because they can be related through a reparametrisation, hence, we expect them to yield identical critical values. Tables B.10, B.11, B.12 and B.13 (cont. in Table B.14) give slightly different critical values because they were calculated via independent Monte Carlo simulations. Closer inspection reveals that there is significant overlap of the confidence intervals in the vast majority of cases, thus, there is strong agreement between the Monte Carlo results.

Again, our comparison was hampered by the fact that most studies stated critical values without a corresponding confidence interval, even if the critical values were produced through Monte Carlo simulation[6]. The critical values we determined achieved broad agreement with the majority of values we consulted. However, there were some cases in which significant differences occured. In these situations previous studies had usually undertaken a Monte Carlo simulation with far less repetition than ours. Thus, it is more likely that the discrepancy is attributable to a large uncertainty margin in the literature. For example, Chandra et al. [9] estimated the case IIIa critical values of Kuiper's test for a sample of $n = 50$ observations drawn from the Weibull/exponential[7] distribution in their 1981 [9] paper. At the 95% significance level they give a value of 1.59, our work attained a result 1.6118 ± 0.0014, thus their result is outside of our uncertainty margin. Chandra et al. produced their results via a single-sample Monte Carlo simulation with $N = 10,000$ repetitions, in our work we used 100 times as many samples. Hence, there is likely to be more uncertainty in their estimate than ours. Table B.11 displays these results. This example is typical of the reason that our results differ from the literature.

[6] Kizilersü et al. [10] is a notable exception.

[7] The case IIIa Weibull and exponential distributions are equivalent and therefore have the same critical values.

8.7 Case IIIb

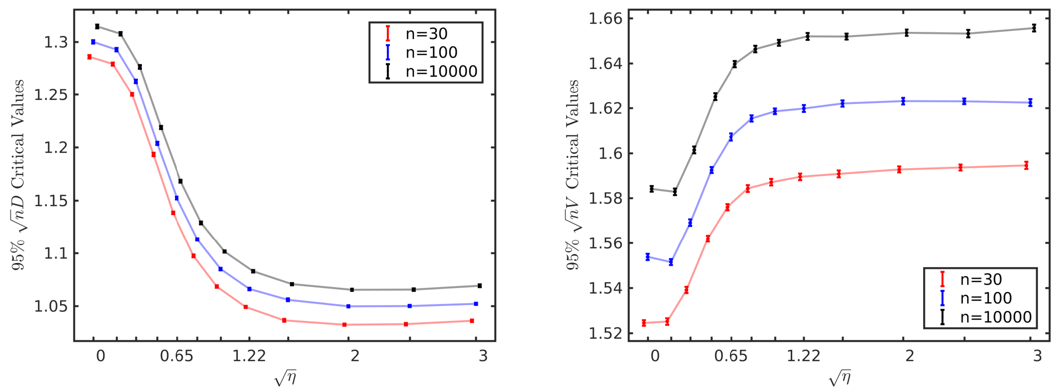
- Weibull
- Loglogistic
- Lognormal

8.7.1 Variable Dependence

In case IIIb, the shape parameter is determined from the data and the scale parameter is known *a priori*. As the Pareto distribution has just one parameter, knowing the scale parameter is the same as having a completely specified distribution, i.e. case I. Thus only the Weibull, loglogistic and lognormal distributions need to be considered. Figure 8.19 displays the Kolmogorov-Smirnov and Kuiper critical values at the 95% significance level for the case IIIb loglogistic distribution plotted against truncation level ($\sqrt{\eta}$) and Figure 8.21 displays the same critical values plotted against sample size (n). The corresponding plots for the Cramér-von Mises and Anderson-Darling tests are shown in Figures 8.20 and 8.22 respectively. Figure 8.21 shows that the Kolmogorov-Smirnov and Kuiper critical values have strong n dependence, however, we see in Figure 8.22 that the Cramér-von Mises and Anderson-Darling critical values are independent of sample size. Strong dependence on truncation level is depicted in Figures 8.19 and 8.20.

Only the loglogistic 95% critical values are depicted, however, these plots showcase the characteristics that are common to all case IIIb distributions and confidence levels, namely strong τ_l dependence, n dependence for the Kolmogorov-Smirnov and Kuiper tests and n independence for the Cramér-von Mises and Anderson-Darling tests. Table 8.37 summarises the dependency of the case IIIb critical values on n and τ_l.

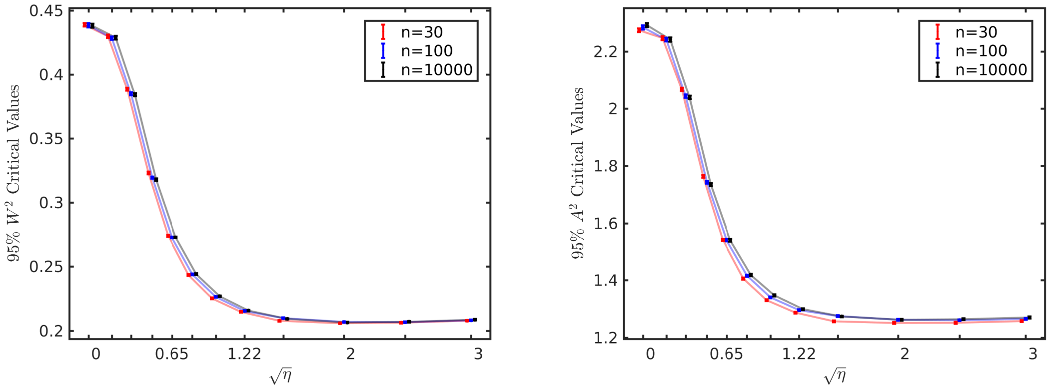
The critical values depicted in Figures 8.19 and 8.20 resulting from different sample size (n values) are staggered along the $\sqrt{\eta}$ axis; this does not imply that the critical were evaluated at distinct sample sizes. In Figures 8.21 and 8.22 the critical values resulting from different truncation levels (p values) are staggered along the n axis; this does not imply that the critical were evaluated at distinct truncation levels. There is no discernible relation between the confidence interval length and significance level, hence, we have elected to show only the results for the 95% significance level as discussed in section 8.4



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl.

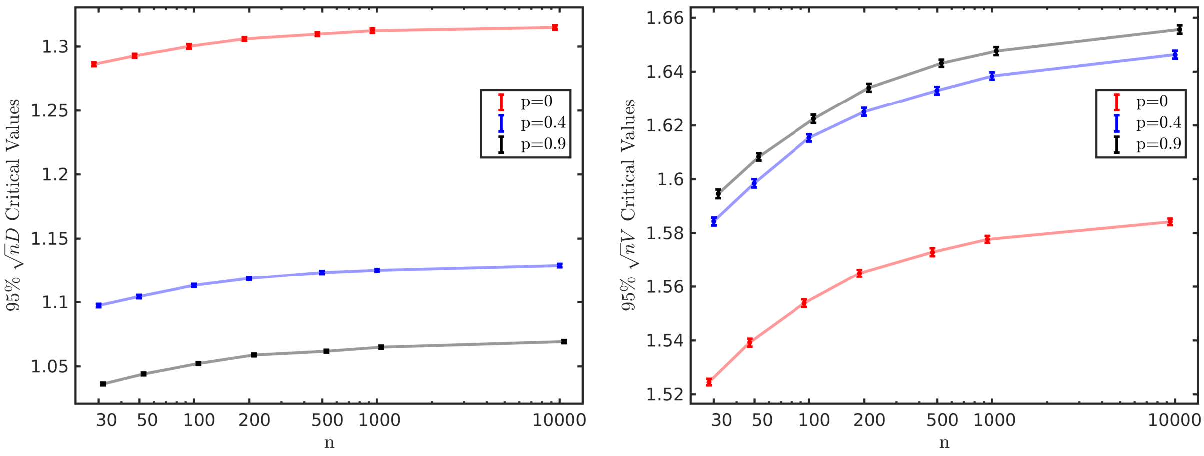
(b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

Figure 8.19: Case IIIb Loglogistic Kolmogorov-Smirnov and Kuiper critical values
Critical values from different n values are staggered on the $\sqrt{\eta}$ axis for clarity



(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl. (b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

Figure 8.20: Case IIIb Loglogistic Cramér-von Mises and Anderson-Darling critical values
Critical values from different n values are staggered on the $\sqrt{\eta}$ axis for clarity

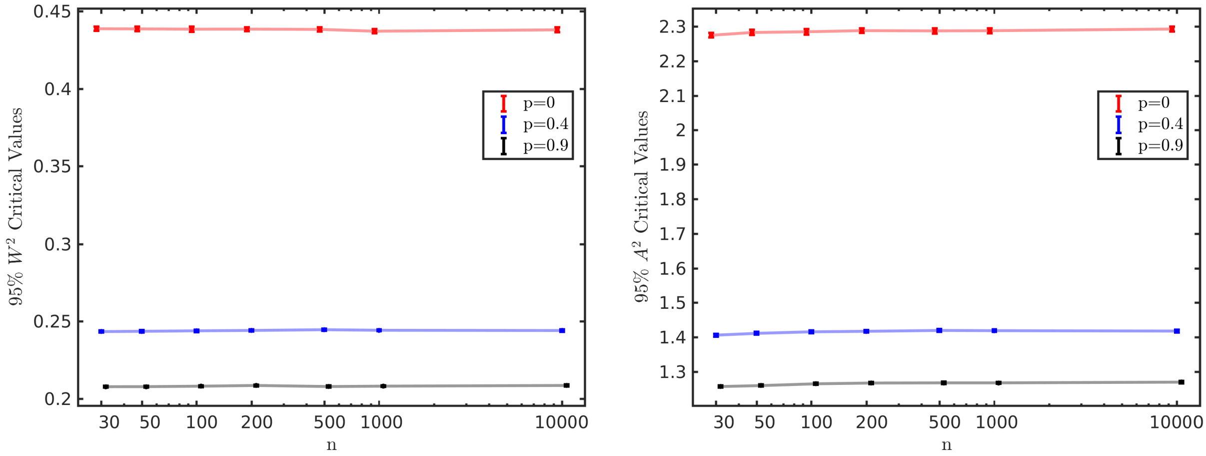


(a) Kolmogorov-Smirnov ($\sqrt{n}D$) crit. vals. at 95% sig. lvl. (b) Kuiper ($\sqrt{n}V$) crit. vals. at 95% sig. lvl.

Figure 8.21: Case IIIb Loglogistic Kolmogorov-Smirnov and Kuiper critical values
Critical values from different p values are staggered on the n axis for clarity

Distribution	Test	n dependent	τ_L dependent
Weibull	KS & Kuiper	✓	✓
Loglogistic	KS & Kuiper	✓	✓
Lognormal	KS & Kuiper	✓	✓
Weibull	CvM & AD	✗	✓
Loglogistic	CvM & AD	✗	✓
Lognormal	CvM & AD	✗	✓

Table 8.37: Case IIIb critical value dependences

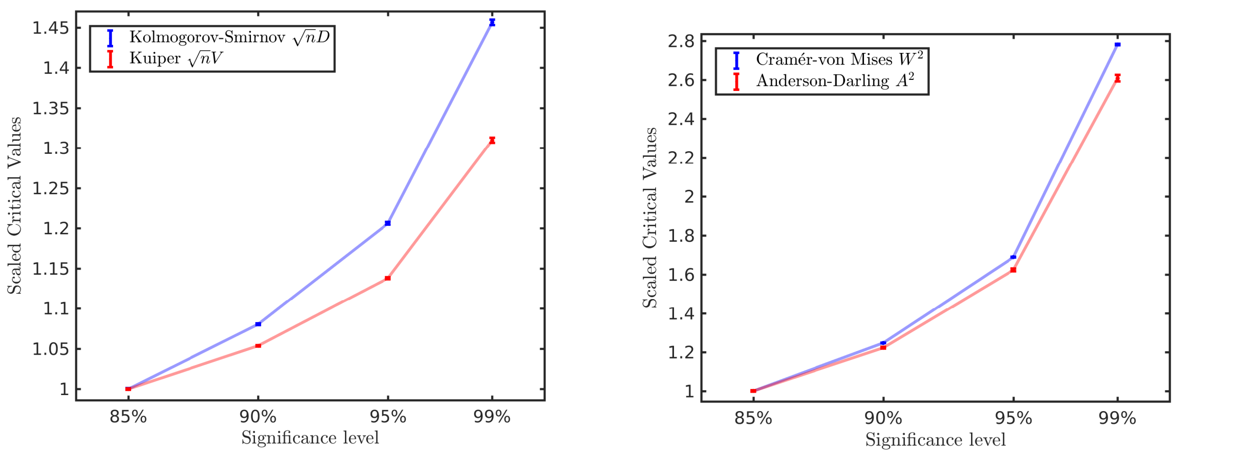


(a) Cramér-von Mises (W^2) crit. vals. at 95% sig. lvl. (b) Anderson-Darling (A^2) crit. vals. at 95% sig. lvl.

Figure 8.22: Case IIIb Loglogistic Cramér-von Mises and Anderson-Darling critical values
Critical values from different p values are staggered on the n axis for clarity

8.7.2 Effect of Significance Level on Critical Values

Figure 8.23 displays the complete (untruncated) case I critical values at $n = 10,000$ plotted against significance level. All critical values have been divided by their respective 85% critical values so that the change between different goodness-of-tests can be compared with increased clarity. **The critical values are necessarily monotone increasing with significance level** as shown in Figure 8.23. This behaviour is common to all sample sizes and truncation levels, even though only one example has been depicted. **Future studies should model how the critical values change with significance level.**



(a) Kolmogorov-Smirnov and Kuiper Critical Values (b) Cramér-von Mises and Anderson-Darling Critical Values

Figure 8.23: Case IIIb Loglogistic distribution p=0, n=10,000 critical values
Scaled to the 85% significance level

8.7.3 Comparison with Kizilersü et al.

Table 8.38 gives the Kolmogorov-Smirnov critical values for case IIIb at the 95% significance level. The results of the present work ($C = 100$ and $M = 10,000$) are displayed in bold below the results of Kizilersü et al. [10] ($C = 100$ and $M = 1,000$). There is strong agreement between the two studies, and the difference between the uncertainty margins is roughly $\frac{\sqrt{1000}}{1.984}$, as expected.

p	$\sqrt{\eta}$	30	50	100	200	500	1000
0	0	1.281(24) **1.2833(09)**	1.289(22) **1.2906(07)**	1.301(24) **1.2991(08)**	1.302(28) **1.3044(08)**	1.310(25) **1.3063(08)**	1.310(23) **1.3112(07)**
0.1	0.32	1.301(26) **1.3038(08)**	1.307(23) **1.3094(08)**	1.314(23) **1.3164(07)**	1.320(24) **1.3206(09)**	1.322(25) **1.3244(09)**	1.323(27) **1.3280(07)**
0.2	0.47	1.283(26) **1.2844(07)**	1.293(23) **1.2929(07)**	1.299(26) **1.2997(07)**	1.308(25) **1.3041(08)**	1.306(26) **1.3084(10)**	1.307(26) **1.3099(09)**
0.3	0.6	1.255(23) **1.2556(08)**	1.262(25) **1.2635(08)**	1.270(23) **1.2690(07)**	1.273(24) **1.2741(08)**	1.284(23) **1.2798(08)**	1.281(24) **1.2818(07)**
0.4	0.71	1.224(22) **1.2262(07)**	1.233(21) **1.2330(07)**	1.241(22) **1.2410(07)**	1.246(24) **1.2454(07)**	1.250(27) **1.2506(07)**	1.257(22) **1.2517(08)**
0.5	0.83	1.194(21) **1.1955(08)**	1.203(22) **1.2053(07)**	1.212(20) **1.2134(07)**	1.214(21) **1.2180(08)**	1.223(22) **1.2224(07)**	1.227(23) **1.2269(07)**
0.6	0.96	1.171(20) **1.1693(07)**	1.177(19) **1.1791(06)**	1.189(23) **1.1883(07)**	1.193(20) **1.1931(07)**	1.194(23) **1.1974(07)**	1.198(23) **1.2008(07)**
0.7	1.1	1.144(20) **1.1462(06)**	1.154(21) **1.1561(07)**	1.162(23) **1.1648(06)**	1.169(21) **1.1706(07)**	1.174(21) **1.1759(06)**	1.179(22) **1.1773(07)**
0.8	1.27	1.122(19) **1.1254(07)**	1.136(21) **1.1357(05)**	1.142(24) **1.1448(06)**	1.148(20) **1.1499(08)**	1.153(22) **1.1558(07)**	1.154(21) **1.1580(07)**
0.9	1.52	1.100(19) **1.1056(07)**	1.110(21) **1.1145(06)**	1.125(22) **1.1230(06)**	1.124(20) **1.1304(05)**	1.131(18) **1.1346(07)**	1.136(19) **1.1378(06)**

Table 8.38: Kolmogorov-Smirnov critical values of the case IIIb Weibull distrib. at 95% sig. lvl Results from our work are **bold**, the others are from Kizilersü et al. [10]

8.7.4 Critical Values

Tables 8.39, 8.40 and 8.41 display the Kuiper, Cramér-von Mises and Anderson-Darling critical values for case IIIb Weibull distribution at the 95% significance level. These critical values are given for the loglogistic and lognormal distributions in Tables 8.42-8.45 and 8.46-8.49 respectively. The complete set of case IIIb critical values (found in Appendix A.4) is too extensive to include, hence, we have elected to show only the results for only one significance level. Again, the 95% significance level has been displayed because it was previously studied by Kizilersü et al. [10] and all the uncertainty margins are given at the 95% confidence level.

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5541(14)	1.5703(13)	1.5879(16)	1.5978(14)	1.6058(14)	1.6117(15)	1.6206(14)
0.0323	0.18	1.5576(15)	1.5743(14)	1.5916(15)	1.6028(13)	1.6111(14)	1.6171(14)	1.6248(15)
0.1	0.32	1.5705(13)	1.5883(14)	1.6071(12)	1.6174(15)	1.6274(16)	1.6325(15)	1.6403(13)
0.2	0.47	1.5858(15)	1.6055(13)	1.6237(14)	1.6355(14)	1.6444(15)	1.6480(16)	1.6571(14)
0.3	0.6	1.5989(13)	1.6159(16)	1.6319(13)	1.6439(14)	1.6525(14)	1.6573(13)	1.6649(14)
0.4	0.71	1.6044(13)	1.6213(14)	1.6375(14)	1.6474(14)	1.6574(13)	1.6607(16)	1.6688(14)
0.5	0.83	1.6056(14)	1.6235(14)	1.6396(14)	1.6493(15)	1.6581(15)	1.6642(14)	1.6701(13)
0.6	0.96	1.6053(15)	1.6231(14)	1.6389(13)	1.6494(14)	1.6592(14)	1.6631(13)	1.6696(13)
0.7	1.1	1.6051(16)	1.6219(15)	1.6375(14)	1.6481(14)	1.6581(14)	1.6616(14)	1.6694(15)
0.8	1.27	1.6047(13)	1.6204(12)	1.6359(13)	1.6466(14)	1.6576(13)	1.6608(14)	1.6677(14)
0.8605	1.4	1.6046(13)	1.6190(14)	1.6356(16)	1.6452(15)	1.6550(14)	1.6586(15)	1.6660(13)
0.9	1.52	1.6033(16)	1.6188(14)	1.6336(12)	1.6456(12)	1.6546(16)	1.6587(14)	1.6672(15)

Table 8.39: Kuiper critical values of the case IIIb Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4289(14)	0.4306(13)	0.4310(14)	0.4314(13)	0.4300(16)	0.4320(13)	0.4320(15)
0.0323	0.18	0.4448(15)	0.4427(15)	0.4422(15)	0.4411(13)	0.4411(17)	0.4424(15)	0.4416(15)
0.1	0.32	0.4463(14)	0.4441(15)	0.4425(13)	0.4413(15)	0.4410(15)	0.4414(12)	0.4404(14)
0.2	0.47	0.4161(12)	0.4160(13)	0.4152(14)	0.4151(11)	0.4144(15)	0.4140(16)	0.4148(13)
0.3	0.6	0.3806(12)	0.3811(13)	0.3798(10)	0.3818(12)	0.3814(13)	0.3807(11)	0.3813(11)
0.4	0.71	0.3496(10)	0.3496(11)	0.3498(11)	0.3502(11)	0.3496(10)	0.3496(11)	0.3503(10)
0.5	0.83	0.3209(10)	0.3238(10)	0.3234(10)	0.3235(11)	0.3241(10)	0.3245(10)	0.3239(10)
0.6	0.96	0.2984(09)	0.3008(08)	0.3012(08)	0.3014(09)	0.3024(08)	0.3024(09)	0.3020(09)
0.7	1.1	0.2801(09)	0.2820(08)	0.2826(08)	0.2842(08)	0.2840(08)	0.2836(09)	0.2841(08)
0.8	1.27	0.2641(08)	0.2663(07)	0.2672(07)	0.2678(09)	0.2681(07)	0.2680(08)	0.2682(08)
0.8605	1.4	0.2561(06)	0.2565(07)	0.2587(07)	0.2592(07)	0.2593(06)	0.2584(08)	0.2592(07)
0.9	1.52	0.2498(08)	0.2512(07)	0.2515(06)	0.2532(07)	0.2530(07)	0.2537(07)	0.2536(07)

Table 8.40: Cramér-von Mises critical values of the case IIIb Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	2.2451(68)	2.2588(62)	2.2698(65)	2.2743(67)	2.2682(74)	2.2773(67)	2.2787(74)
0.0323	0.18	2.3289(78)	2.3241(73)	2.3246(71)	2.3179(68)	2.3194(80)	2.3231(74)	2.3184(82)
0.1	0.32	2.3429(63)	2.3229(76)	2.3116(63)	2.3049(73)	2.3015(73)	2.3044(60)	2.2960(71)
0.2	0.47	2.1873(64)	2.1743(65)	2.1676(66)	2.1658(62)	2.1603(69)	2.1587(79)	2.1638(66)
0.3	0.6	1.9988(53)	2.0013(66)	1.9951(55)	2.0048(58)	2.0051(61)	1.9985(57)	2.0005(52)
0.4	0.71	1.8513(49)	1.8514(54)	1.8539(53)	1.8575(55)	1.8567(53)	1.8588(55)	1.8636(51)
0.5	0.83	1.7186(50)	1.7346(46)	1.7405(48)	1.7405(53)	1.7442(45)	1.7463(51)	1.7450(53)
0.6	0.96	1.6203(44)	1.6376(39)	1.6453(46)	1.6432(44)	1.6497(41)	1.6506(43)	1.6487(44)
0.7	1.1	1.5432(45)	1.5581(38)	1.5642(41)	1.5709(41)	1.5728(44)	1.5724(39)	1.5758(39)
0.8	1.27	1.4772(42)	1.4909(33)	1.5009(39)	1.5031(45)	1.5079(40)	1.5080(38)	1.5071(41)
0.8605	1.4	1.4465(34)	1.4532(33)	1.4635(40)	1.4710(37)	1.4703(33)	1.4666(38)	1.4729(38)
0.9	1.52	1.4202(43)	1.4301(35)	1.4374(34)	1.4455(36)	1.4467(36)	1.4524(35)	1.4509(38)

Table 8.41: Anderson-Darling critical values of the case IIIb Weibull distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2860(16)	1.2925(17)	1.3000(17)	1.3059(14)	1.3095(15)	1.3122(17)	1.3147(17)
0.0323	0.18	1.2792(15)	1.2851(15)	1.2927(15)	1.2967(17)	1.3015(16)	1.3035(16)	1.3077(17)
0.1	0.33	1.2504(15)	1.2563(14)	1.2627(16)	1.2674(15)	1.2713(15)	1.2728(15)	1.2765(18)
0.2	0.5	1.1934(16)	1.1970(13)	1.2038(15)	1.2080(15)	1.2126(15)	1.2156(14)	1.2188(14)
0.3	0.65	1.1384(12)	1.1455(13)	1.1522(14)	1.1573(13)	1.1612(12)	1.1634(12)	1.1682(13)
0.4	0.82	1.0972(13)	1.1042(14)	1.1130(12)	1.1185(12)	1.1231(12)	1.1249(12)	1.1287(13)
0.5	1	1.0682(12)	1.0765(11)	1.0848(12)	1.0908(12)	1.0950(12)	1.0968(12)	1.1013(11)
0.6	1.22	1.0491(10)	1.0578(12)	1.0660(12)	1.0712(12)	1.0761(11)	1.0790(11)	1.0827(12)
0.7	1.53	1.0364(13)	1.0453(11)	1.0557(13)	1.0595(13)	1.0662(11)	1.0669(11)	1.0706(10)
0.8	2	1.0323(10)	1.0413(11)	1.0496(11)	1.0548(12)	1.0587(12)	1.0623(12)	1.0652(11)
0.8605	2.48	1.0328(11)	1.0408(10)	1.0499(10)	1.0548(12)	1.0594(13)	1.0622(11)	1.0654(12)
0.9	3	1.0360(11)	1.0438(11)	1.0520(11)	1.0586(11)	1.0615(11)	1.0647(12)	1.0690(13)

Table 8.42: Kolmogorov-Smirnov critical values of the case IIIb loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5245(12)	1.5391(14)	1.5538(14)	1.5650(13)	1.5728(15)	1.5776(13)	1.5841(12)
0.0323	0.18	1.5251(14)	1.5388(12)	1.5515(12)	1.5618(13)	1.5715(13)	1.5761(13)	1.5828(14)
0.1	0.33	1.5391(14)	1.5544(14)	1.5691(14)	1.5806(13)	1.5888(13)	1.5927(15)	1.6014(15)
0.2	0.5	1.5620(13)	1.5768(13)	1.5925(12)	1.6042(14)	1.6135(14)	1.6172(15)	1.6250(16)
0.3	0.65	1.5760(14)	1.5927(14)	1.6071(16)	1.6183(16)	1.6272(12)	1.6318(15)	1.6397(14)
0.4	0.82	1.5843(15)	1.5984(15)	1.6153(14)	1.6250(15)	1.6329(15)	1.6383(13)	1.6463(15)
0.5	1	1.5871(15)	1.6024(13)	1.6185(13)	1.6286(14)	1.6369(13)	1.6421(14)	1.6492(13)
0.6	1.22	1.5895(14)	1.6049(16)	1.6197(15)	1.6291(13)	1.6391(14)	1.6428(14)	1.6520(15)
0.7	1.53	1.5908(15)	1.6058(15)	1.6220(14)	1.6319(16)	1.6431(14)	1.6453(15)	1.6519(14)
0.8	2	1.5927(13)	1.6081(14)	1.6230(15)	1.6336(15)	1.6409(15)	1.6465(15)	1.6535(14)
0.8605	2.48	1.5936(14)	1.6085(13)	1.6229(13)	1.6333(14)	1.6429(15)	1.6464(16)	1.6532(16)
0.9	3	1.5945(16)	1.6082(14)	1.6224(15)	1.6340(15)	1.6431(13)	1.6476(15)	1.6556(15)

Table 8.43: Kuiper critical values of the case IIIb loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4387(14)	0.4386(15)	0.4384(16)	0.4385(12)	0.4383(14)	0.4371(14)	0.4380(16)
0.0323	0.18	0.4298(12)	0.4283(13)	0.4284(13)	0.4271(13)	0.4276(15)	0.4282(15)	0.4288(15)
0.1	0.33	0.3884(13)	0.3866(12)	0.3846(12)	0.3842(11)	0.3833(12)	0.3829(14)	0.3839(13)
0.2	0.5	0.3231(11)	0.3200(10)	0.3194(09)	0.3176(10)	0.3176(09)	0.3175(10)	0.3179(10)
0.3	0.65	0.2741(09)	0.2739(08)	0.2727(08)	0.2728(09)	0.2723(07)	0.2720(07)	0.2729(08)
0.4	0.82	0.2433(07)	0.2435(07)	0.2438(08)	0.2440(06)	0.2445(07)	0.2441(06)	0.2440(07)
0.5	1	0.2251(07)	0.2256(06)	0.2263(05)	0.2265(07)	0.2267(07)	0.2265(06)	0.2268(07)
0.6	1.22	0.2147(06)	0.2151(06)	0.2153(06)	0.2151(06)	0.2157(05)	0.2154(06)	0.2156(06)
0.7	1.53	0.2077(06)	0.2084(05)	0.2097(06)	0.2088(06)	0.2100(05)	0.2092(06)	0.2092(05)
0.8	2	0.2058(05)	0.2064(05)	0.2068(05)	0.2066(06)	0.2062(06)	0.2067(05)	0.2064(06)
0.8605	2.48	0.2062(06)	0.2062(05)	0.2067(05)	0.2066(06)	0.2070(06)	0.2067(05)	0.2069(06)
0.9	3	0.2078(05)	0.2078(06)	0.2081(06)	0.2086(06)	0.2079(06)	0.2081(05)	0.2086(06)

Table 8.44: Cramér-von Mises critical values of the case IIIb loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	2.2743(65)	2.2823(75)	2.2845(79)	2.2877(67)	2.2870(76)	2.2875(68)	2.2923(72)
0.0323	0.18	2.2473(70)	2.2424(70)	2.2417(63)	2.2372(63)	2.2410(73)	2.2429(73)	2.2424(75)
0.1	0.33	2.0675(65)	2.0559(60)	2.0434(59)	2.0391(58)	2.0332(64)	2.0294(65)	2.0387(64)
0.2	0.5	1.7633(50)	1.7507(46)	1.7425(46)	1.7379(48)	1.7354(48)	1.7345(50)	1.7347(53)
0.3	0.65	1.5408(44)	1.5428(39)	1.5399(45)	1.5377(45)	1.5371(41)	1.5371(36)	1.5389(44)
0.4	0.82	1.4060(39)	1.4112(40)	1.4156(39)	1.4171(36)	1.4196(40)	1.4189(34)	1.4178(41)
0.5	1	1.3303(36)	1.3356(30)	1.3402(31)	1.3456(34)	1.3469(37)	1.3434(34)	1.3473(36)
0.6	1.22	1.2871(32)	1.2904(30)	1.2956(35)	1.2945(31)	1.2992(31)	1.3006(33)	1.2991(33)
0.7	1.53	1.2566(35)	1.2656(33)	1.2751(34)	1.2701(31)	1.2776(29)	1.2729(31)	1.2734(29)
0.8	2	1.2509(31)	1.2561(33)	1.2621(31)	1.2606(32)	1.2610(34)	1.2638(29)	1.2619(30)
0.8605	2.48	1.2516(33)	1.2562(31)	1.2603(31)	1.2614(33)	1.2634(36)	1.2614(32)	1.2640(32)
0.9	3	1.2574(31)	1.2601(30)	1.2653(28)	1.2675(32)	1.2680(33)	1.2680(29)	1.2701(34)

Table 8.45: Anderson-Darling critical values of the case IIIb loglogistic distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.2967(15)	1.3022(16)	1.3105(15)	1.3147(17)	1.3203(16)	1.3205(16)	1.3249(16)
0.0323	0.72	1.2868(16)	1.2926(16)	1.2997(17)	1.3037(14)	1.3084(15)	1.3128(14)	1.3146(16)
0.1	0.75	1.2614(14)	1.2655(14)	1.2718(17)	1.2765(16)	1.2792(17)	1.2816(17)	1.2868(15)
0.2	0.79	1.2131(13)	1.2202(14)	1.2267(16)	1.2317(13)	1.2364(14)	1.2374(14)	1.2421(15)
0.3	0.85	1.1752(15)	1.1822(13)	1.1896(15)	1.1957(15)	1.2002(14)	1.2010(13)	1.2047(15)
0.4	0.91	1.1468(12)	1.1542(14)	1.1631(14)	1.1676(12)	1.1715(13)	1.1747(12)	1.1782(13)
0.5	1	1.1243(13)	1.1320(11)	1.1415(14)	1.1471(12)	1.1515(14)	1.1543(10)	1.1571(13)
0.6	1.12	1.1075(12)	1.1165(14)	1.1255(14)	1.1299(13)	1.1354(13)	1.1366(13)	1.1417(11)
0.7	1.29	1.0937(12)	1.1025(13)	1.1120(12)	1.1180(13)	1.1217(13)	1.1252(12)	1.1278(11)
0.8	1.58	1.0843(13)	1.0920(11)	1.1012(11)	1.1063(12)	1.1117(12)	1.1129(12)	1.1175(13)
0.8605	1.89	1.0781(13)	1.0869(13)	1.0957(12)	1.1003(11)	1.1047(12)	1.1059(13)	1.1110(12)
0.9	2.24	1.0735(12)	1.0826(12)	1.0917(11)	1.0974(13)	1.1009(12)	1.1041(11)	1.1072(14)

Table 8.46: Kolmogorov-Smirnov critical values of the case IIIb lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.5457(14)	1.5603(15)	1.5767(15)	1.5868(14)	1.5963(14)	1.5994(13)	1.6079(14)
0.0323	0.72	1.5437(14)	1.5574(13)	1.5738(15)	1.5837(14)	1.5927(13)	1.5987(14)	1.6056(13)
0.1	0.75	1.5626(13)	1.5789(14)	1.5958(16)	1.6069(14)	1.6144(13)	1.6197(15)	1.6277(14)
0.2	0.79	1.5830(15)	1.6001(14)	1.6160(14)	1.6266(14)	1.6365(14)	1.6406(13)	1.6492(13)
0.3	0.85	1.5935(14)	1.6096(14)	1.6244(13)	1.6348(14)	1.6446(14)	1.6482(14)	1.6560(15)
0.4	0.91	1.5968(13)	1.6121(15)	1.6289(13)	1.6397(13)	1.6481(14)	1.6521(14)	1.6597(13)
0.5	1	1.5986(15)	1.6131(13)	1.6295(17)	1.6403(12)	1.6505(15)	1.6539(15)	1.6612(15)
0.6	1.12	1.5988(14)	1.6151(15)	1.6319(14)	1.6404(14)	1.6494(14)	1.6530(14)	1.6627(14)
0.7	1.29	1.5994(13)	1.6141(14)	1.6289(15)	1.6409(14)	1.6491(14)	1.6543(14)	1.6612(14)
0.8	1.58	1.6004(15)	1.6144(14)	1.6303(15)	1.6395(14)	1.6497(14)	1.6537(13)	1.6615(16)
0.8605	1.89	1.6003(13)	1.6136(15)	1.6300(14)	1.6396(12)	1.6486(13)	1.6510(15)	1.6607(15)
0.9	2.24	1.5980(13)	1.6127(14)	1.6290(14)	1.6398(14)	1.6479(14)	1.6524(15)	1.6599(15)

Table 8.47: Kuiper critical values of the case IIIb lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.4438(13)	0.4428(14)	0.4423(14)	0.4422(16)	0.4421(15)	0.4402(14)	0.4419(13)
0.0323	0.72	0.4319(14)	0.4301(13)	0.4305(15)	0.4280(14)	0.4276(14)	0.4309(15)	0.4279(13)
0.1	0.75	0.3944(14)	0.3913(12)	0.3896(13)	0.3892(13)	0.3871(12)	0.3885(14)	0.3896(12)
0.2	0.79	0.3397(10)	0.3398(10)	0.3383(10)	0.3382(10)	0.3384(10)	0.3376(10)	0.3380(12)
0.3	0.85	0.3031(09)	0.3030(09)	0.3034(10)	0.3045(10)	0.3045(09)	0.3035(10)	0.3031(08)
0.4	0.91	0.2795(07)	0.2802(08)	0.2813(08)	0.2809(08)	0.2809(08)	0.2809(08)	0.2810(08)
0.5	1	0.2630(07)	0.2633(07)	0.2648(08)	0.2651(08)	0.2647(08)	0.2649(06)	0.2652(07)
0.6	1.12	0.2510(07)	0.2521(07)	0.2529(07)	0.2528(07)	0.2531(08)	0.2530(07)	0.2540(07)
0.7	1.29	0.2424(06)	0.2429(06)	0.2437(07)	0.2448(07)	0.2442(07)	0.2448(08)	0.2442(07)
0.8	1.58	0.2355(07)	0.2358(06)	0.2372(07)	0.2373(06)	0.2374(06)	0.2371(07)	0.2373(07)
0.8605	1.89	0.2319(07)	0.2326(06)	0.2334(07)	0.2334(06)	0.2330(06)	0.2330(07)	0.2334(06)
0.9	2.24	0.2290(06)	0.2302(06)	0.2309(06)	0.2311(07)	0.2309(07)	0.2313(06)	0.2315(06)

Table 8.48: Cramér-von Mises critical values of the case IIIb lognormal distrib. at 95% sig. lvl

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	2.3026(64)	2.3025(71)	2.3016(72)	2.3085(80)	2.3092(70)	2.3016(69)	2.3083(64)
0.0323	0.72	2.2640(62)	2.2523(67)	2.2502(72)	2.2379(64)	2.2364(68)	2.2549(73)	2.2390(68)
0.1	0.75	2.0913(72)	2.0662(58)	2.0592(65)	2.0523(61)	2.0437(57)	2.0483(67)	2.0527(58)
0.2	0.79	1.8223(48)	1.8218(54)	1.8165(49)	1.8137(52)	1.8170(49)	1.8103(49)	1.8127(59)
0.3	0.85	1.6524(50)	1.6552(41)	1.6571(50)	1.6614(50)	1.6638(45)	1.6616(47)	1.6593(42)
0.4	0.91	1.5472(37)	1.5549(43)	1.5610(40)	1.5624(42)	1.5638(40)	1.5637(42)	1.5634(42)
0.5	1	1.4780(37)	1.4821(36)	1.4940(41)	1.4967(39)	1.4943(46)	1.4943(37)	1.4963(36)
0.6	1.12	1.4290(41)	1.4370(38)	1.4459(38)	1.4450(40)	1.4480(43)	1.4481(38)	1.4496(43)
0.7	1.29	1.3886(33)	1.3991(36)	1.4045(37)	1.4105(40)	1.4094(38)	1.4125(38)	1.4111(35)
0.8	1.58	1.3647(39)	1.3706(34)	1.3792(36)	1.3818(38)	1.3830(30)	1.3826(35)	1.3819(35)
0.8605	1.89	1.3517(35)	1.3570(35)	1.3635(36)	1.3653(35)	1.3653(37)	1.3631(36)	1.3672(33)
0.9	2.24	1.3405(31)	1.3483(38)	1.3532(31)	1.3571(38)	1.3565(39)	1.3583(32)	1.3610(37)

Table 8.49: Anderson-Darling critical values of the case IIIb lognormal distrib. at 95% sig. lvl

8.7.5 Comparison with the available literature

The case IIIb critical values from our work are compared to the literature in Appendix B.4. Tables B.15, B.16, B.17 and B.18 display the comparison for the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests, respectively. As discussed in subsection 8.4.5, the critical values from this work are displayed in bold, below the critical values from other studies. Critical values from the literature that were determined from a formula of n, are preceded by an asterisk (*) and those determined from C code are preceded with a dagger (†). To the best of our knowledge, the work conducted by Kizilersü et al. [10] is the only one that has determined critical values corresponding to one of the case IIIb left-truncated distributions that we studied. Therefore, the comparison to literature in Appendix B.4 is restricted to complete (untruncated) distributions, i.e. $\tau_l = p = \eta = 0$.

Again, our comparison was hampered by the fact that most studies stated critical values without a corresponding confidence interval, even if the critical values were produced through Monte Carlo simulation[8]. The critical values we determined achieved broad agreement with the majority of values we consulted. However, there were some cases in which significant differences occured.In these situations previous studies had usually undertaken a Monte Carlo simulation with far less repetition than ours. Thus, it is more likely that the discrepancy is attributable to a large uncertainty margin in the literature. For example, in his 1974 work Stephens [12] gives a series of formulae for the determining the critical values as a function of n for all the goodness-of-fit tests we studied. These formulae were derived based on a theoretical asymptotic point and single-sample Monte Carlo simulations with $N = 10,000$ repetitions conducted at sample sizes $n = 10, 20, 50, 100$. Consider the Cramér-von Mises critical values at the 90% significance level resulting from a sample of size $n = 30$ drawn from the case IIIb normal/lognormal[9] distribution, we achieved a result of 0.3273 ± 0.0010 and Stephens' formula produced a value of 0.329. In our work, we took 100 times as many samples and conducted them at $n = 30$, rather than interpolating as Stephens did. Therefore, it is likely that the uncertainty margin on Stephens' result is far larger than ours and if this could be taken into account there may be well be agreement between the two values. Table B.17 displays these results. This example is typical of the reason and magnitude that our results differ from the literature.

[8] Kizilersü et al. [10] is a notable exception.

[9] The normal and lognormal distributions are equivalent and therefore have the same critical values.

Chapter 9

Modelling

The previous chapter displayed and discussed the critical values we calculated via Monte Carlo methods for every permutation-barring case II and IIIb for the Pareto distribution-of the cases, distributions, goodness-of-fit tests and significance levels listed below. For each of these permutations, the sample size, n, and the probability of truncation, p, took the values specified by Eq. (10.1). We determined the uncertainty margin at the 95% confidence level via Schafer's method [83] and managed to restrict the confidence interval lengths to 1% of the critical values in the worst case scenario. The complete set of critical values for cases I, II, IIIa and IIIb are displayed in Appendices A.1, A.2, A.3 and A.4 and compared to existing literature in Appendices B.1, B.2, B.3 and B.4 respectively.

Cases	Distributions:	Goodness-of-fit Tests	Significance Levels
• Case I: scale: known shape: known	• Weibull	• Kolmogorov-Smirnov	• 85%
• Case II: scale: acquired shape: acquired	• Loglogistic	• Kuiper	• 90%
• Case IIIa: scale: known shape: acquired	• Lognormal	• Cramér-von Mises	• 95%
• Case IIIb: scale: acquired shape: known	• Pareto	• Anderson-Darling	• 99%

$$
\begin{aligned}
&n \in \{30, 50, 100, 200, 500, 1000, 10000\} \\
&p \in \{0, 0.0323, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.8605, 0.9\}
\end{aligned} \tag{9.1}
$$

These critical values allow rigorous statistical tests to take place at the truncation levels (η or p) and sample sizes (n) for which the simulations were conducted. However, data sets from the real world seldom have the specific truncation levels and sample sizes required to employ the critical values we generated. For example, it is unlikely that someone measuring radiation with a Geiger counter will be able to control the number of particle detections (n) or the dead time of the detector τ_l ($\eta = \eta(\tau_l)$). Additionally, the time taken to compute the critical values is proportional to the sample size for large n, hence, in big data applications it is not feasible to calculate the critical values at each of the required sample sizes. Thus, in order for our results to be useful for practitioners and scientists from all disciplines, it is necessary to model the critical values across the truncation levels and sample sizes for which we have determined the critical values. In this chapter, we aim to provide the functional form of the critical values, such that one can access them without the need for excessive tables or further computation; this is a common method of displaying critical values [6, 12, 13, 13]. One can easily obtain critical values from the literature, however as the critical values have not previously been determined (to the best of our knowledge) for observations drawn from left-truncated distributions, our models will be a new and valuable addition to the literature.

9.1 Fitting Functions

Figures 8.1, 8.2, 8.3 and 8.4 from section 8.1 displayed strong agreement between critical values from different parameter sets, numerically verifying the parameter independence of our critical values. Figure 8.4 depicts parameter independence for the critical values of the Pareto distribution plotted against τ_l. In Figures 8.1, 8.2 and 8.3 the critical values are plotted against $\sqrt{\eta}$, not the left-truncation point, τ_l. Recall from Eq. (3.13), (4.13) and (5.16)-for the Weibull, loglogistic and lognormal distributions respectively-that τ_l is a function of η and the true parameter values, i.e. $\tau_l = \tau_l(\eta, \boldsymbol{\theta^0})$. Therefore, **the critical values of the Weibull, loglogistic and lognormal distributions are not parameter independent if presented as a function of $\boldsymbol{\tau_l}$.** The truncation probability (p) however, can be written as function of η alone; as specified by Eq. (3.14), (4.14) and (5.17) for the Weibull, loglogistic and lognormal distributions respectively. Thus, if the critical values are expressed as a function of p, they retain the parameter independence from the η space. That the critical values should be parameter dependent if specified in terms of τ_l is not initially obvious, however if one considers the situation in which a τ_l value is held fixed while the scale parameter (α) of the Weibull distribution is varied, it becomes clear that p will be adversely affected. This thought experiment elucidates the fact that specifying the critical values in terms of τ_l will introduce some parameter dependence. Following from Kizilersü et al. [10], in this work the left-truncation limit is specified by $\sqrt{\eta}$.

Kizilersü et al. [10] studied the Kolmogorov-Smirnov critical values of the Weibull distribution at the 95% significance level and proposed Eq. (9.7), (9.8) and (9.10), to model the individual η, n and combined dependence respectively. Our objective was to find similar functions that are applicable to all distributions, goodness-of-fit tests and significance levels studied in the present work. Initially, the functions employed by Kizilersü et al. were trialled, however, additional complexity was required to accurately describe the critical values for the distributions and goodness-of-fit tests which Kizilersü et al. did not analyse.

In this chapter, we will firstly discuss the metrics for analysing a fit (SSE, R^2, R^2_{adj}) and then individually address the three fitting scenarios (i) η dependence, (ii) n dependence and (iii) η and n dependence. The models employed by Kizilersü et al. [10] are analysed with the aforementioned metrics and more complex models are introduced and analysed where necessary. Finally the procedure for determining the optimal parameter values is discussed.

9.2 Define the error measurements

We evaluated the proposed models by analysing the sum of squared errors, SSE, coefficient of determination, R^2, and adjusted coefficient of determination, R^2_{adj}; analysis methods which are the subject of discussion of Rawlings et al. in chapters 1.4 and 7.5 of their book 'Applied Regression Analysis' [99]. In this section we define these metrics and briefly discuss their use.

Consider a set of n observations of a dependent variable, $\boldsymbol{y} = (y_1, y_2, ..., y_n)$, evaluated at values of the corresponding independent variable, $\boldsymbol{x} = (x_1, x_2, ..., x_n)$. These observations are modelled by a function, f_m, such that the model predicts the observation at x_i to be $f_m(x_i)$.

9.2.1 Sum of Squared Errors

In chapter 1.4 of 'Applied Regression Analysis' Rawlings et al. [99] introduced the sum of squared errors (SSE) as the sum of the squared differences between the predicted and observed values,

$$SSE = \sum_{i=1}^{n} \{y_i - f_m(x_i)\}^2 \ . \tag{9.2}$$

In general, lower SSE values are desirable as they result from more accurate models, however there are a few caveats to this. As the number of observations grows, the SSE also increases. This does not imply the model is becoming worse; one can only compare SSE values from data sets of the same size. Additionally, if the observations and predictions differ by a fixed percentage (error-percentage) larger magnitude observations will yield a larger SSE. Consider the case in which all of the predictions are 10% below the observed values, i.e. $f_m(x_i) = 0.9y_i \ \forall \ i \in (1, 2, ..., n)$. In this case the SSE is proportional to the mean-square of observed values,

$$\begin{aligned} SSE &= \sum_{i=1}^{n} \{y_i - 0.9y_i\}^2 = (0.1)^2 \sum_{i=1}^{n} y_i^2 \ , \\ SSE &\propto \sum_{i=1}^{n} y_i^2 \ . \end{aligned} \tag{9.3}$$

One must keep this in mind when comparing SSE values for distinct sets of observations, as the same SSE value implies varying levels of agreement in different contexts.

9.2.2 Coefficient of Determination

The coefficient of determination, R^2, is defined by Rawlings et al. (chapter 1.4) [99] as the proportion of the variation in the dependent variable that is predictable from the independent variable,

$$R^2 = 1 - \frac{Var(\epsilon)}{Var(y_t)}, \text{ where } \epsilon = y_i - f_m(x_i), \quad y_t = y_i - \bar{y} \text{ and } \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \ . \tag{9.4}$$

The coefficient of determination is restricted to $0 \le R^2 \le 1$; higher values result from more accurate models.
Note: "$Var(x)$" is the sample variance of a variable, "x"

$$Var(\epsilon) = \frac{1}{n}\sum_{i=1}^{n} \{y_i - f_m(x_i)\}^2 \ .$$

9.2.3 Adjusted Coefficient of Determination

The R^2 value necessarily increases as more parameters are added to a model, even if they have no relevance. As a result, employing R^2 as the only goodness-of-fit test can lead one to 'overfit' to a given data set. Theil [100] suggested replacing the variances in R^2 with their unbiased estimators,

$$Var(y_t) = \frac{1}{n-1}\sum_{i=1}^{n} \{y_i - \bar{y}\}^2 \quad \text{and} \quad Var(\epsilon) = \frac{1}{n-d-1}\sum_{i=1}^{n} \{y_i - f_m(x_i)\}^2 \ , \tag{9.5}$$

where d is the number of parameters in the model. The expression for R^2_{adj} can then be rearranged in terms of R^2;

$$R^2_{adj} = 1 - \left(1 - R^2\right) \frac{n-1}{n-d} \ . \tag{9.6}$$

If R^2 is held constant, the R^2_{adj} decreases as more parameters are added to the model (k increases), hence models are punished for using excessive parameters that do not contribute sufficiently to the goodness-of-fit. Therefore, using the R^2_{adj} reduces the probability of 'overfitting' to the data. Adjusted R^2 values are restricted to the range $R^2_{adj} \leq R^2 \leq 1$ and higher values suggest better agreement between the predictions and observations.

9.2.4 Metrics in High-uncertainty Conditions

The R^2 and R^2_{adj} are related to the proportion of variation in the dependent variable that can be explained by changes in the independent variable. Therefore, if the variation in the dependent variable is dominated by statistical uncertainty, extremely poor R^2 and R^2_{adj} values will result. This occurs when one attempts to model the critical values as a function of a variable (n or η) upon which they are not (or only weakly) dependent. In this situation, it is odd to model the critical values as a function of this variable(s) at all, and often using the mean for all values of the independent variable(s) will suffice. For example, we found that there was insufficient evidence in the results of the Monte Carlo procedure to declare n or η dependence for the case I Cramér-von Mises and Anderson-Darling critical values. In this situation there is only one parameter, thus, $R^2 \to R^2_{adj}$ and both values are very poor as the independent variable(s) are essentially irrelevant. If one still desires to use a more complex model for other reasons (e.g. continuity with other scenarios) then the SSE should be used as the sole goodness-of-fit metric.

9.3 η Dependence

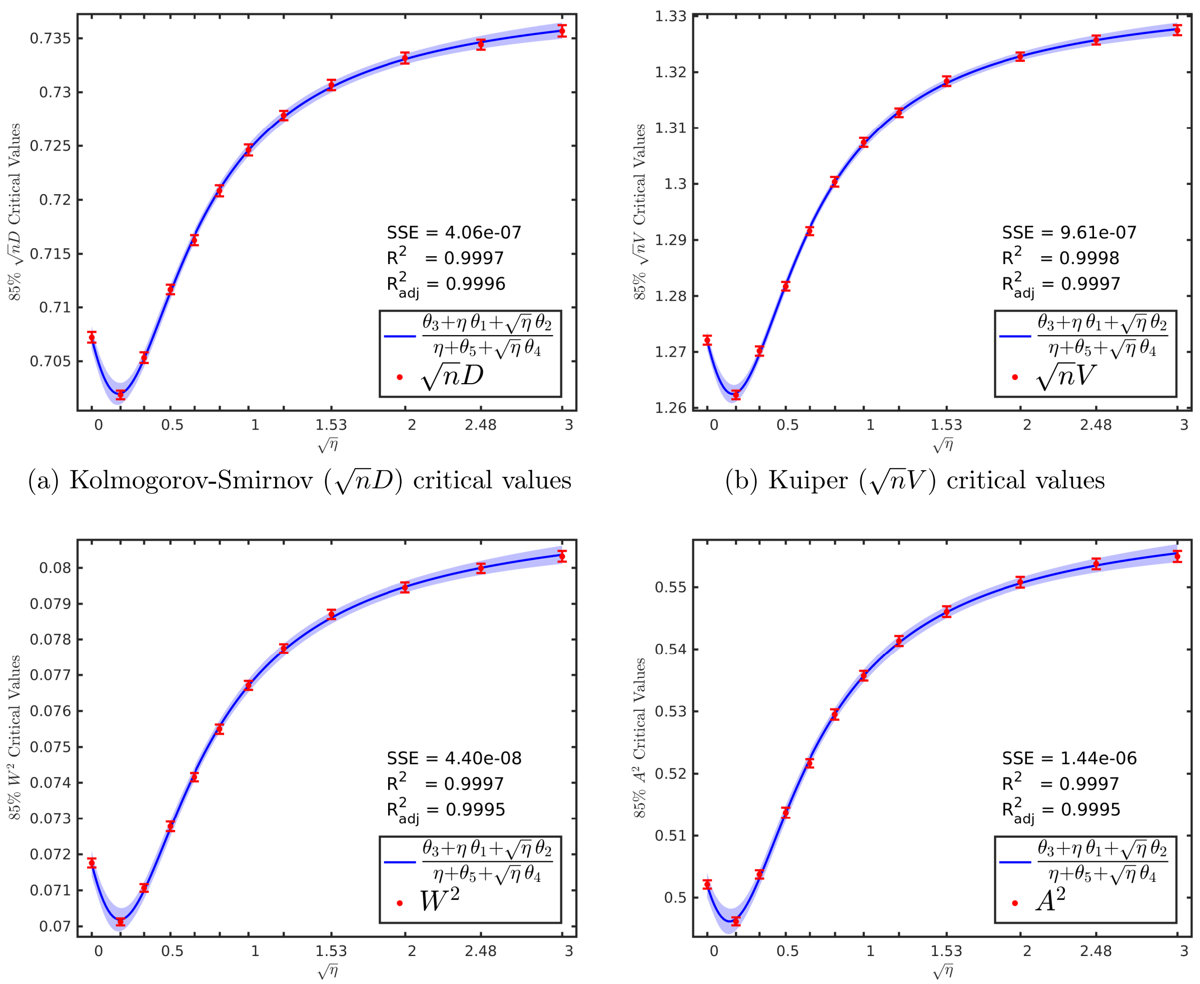
Kizilersü et al. [10] proposed a model defined by Eq. (9.7) to describe the η dependence of the Kolmogorov-Smirnov critical values of the Weibull distribution at the 95% significance level. We found that this model performs well for all of the distributions, goodness-of-fit tests and significance levels with η dependent critical values. Therefore, **we also use Eq. (9.7) to describe $\boldsymbol{\eta}$ dependent critical values**.

$$D^q_{cv}(\eta|n) = \frac{\theta_1(n)\eta + \theta_2(n)\sqrt{\eta} + \theta_3(n)}{\theta_4\sqrt{\eta} + \theta_5 + \eta} \tag{9.7}$$

Scenarios with η Dependent Critical Values

- Case II
- Case IIIa: Lognormal
- Case IIIb

Figure 9.1 depicts Eq. (9.7) fitted to the critical values of the loglogistic distribution with two unknown parameters (case II) and $n = 10,000$ observations at the 85% significance level; Figure 9.2 shows the corresponding plots for a sample size of $n = 30$ at the 99% significance level. There is excellent agreement between the model and observations, one can verify this via inspection and by looking at the extremely high R^2 and R^2_{adj} values. The shaded region represents the 95% simultaneous prediction interval of the function [101, 102]. The sample sizes and significance levels were selected as they correspond to the upper and lower limits of the signal to noise ratio, i.e. the ratio of the critical value range (over the relevant η interval) to the average confidence interval length. At the 85% significance level, a sample size of $n = 10,000$ maximises the signal to noise ratio, the small uncertainty margins in Figure 9.1 give a visual representation of this. A sample size of $n = 30$ and significance level of 99% produces the minimal signal to noise ratio; as a result the confidence interval lengths in Figure 9.2 appear substantially larger. The two combinations of significance level and sample size displayed give a good indication of the strong agreement between the critical values and Eq. (9.7) for all significance levels and sample sizes. Additionally, all the η dependent scenarios can be described by Eq. (9.7) with a similar level of agreement, we have not included more examples of this purely for the sake of brevity.



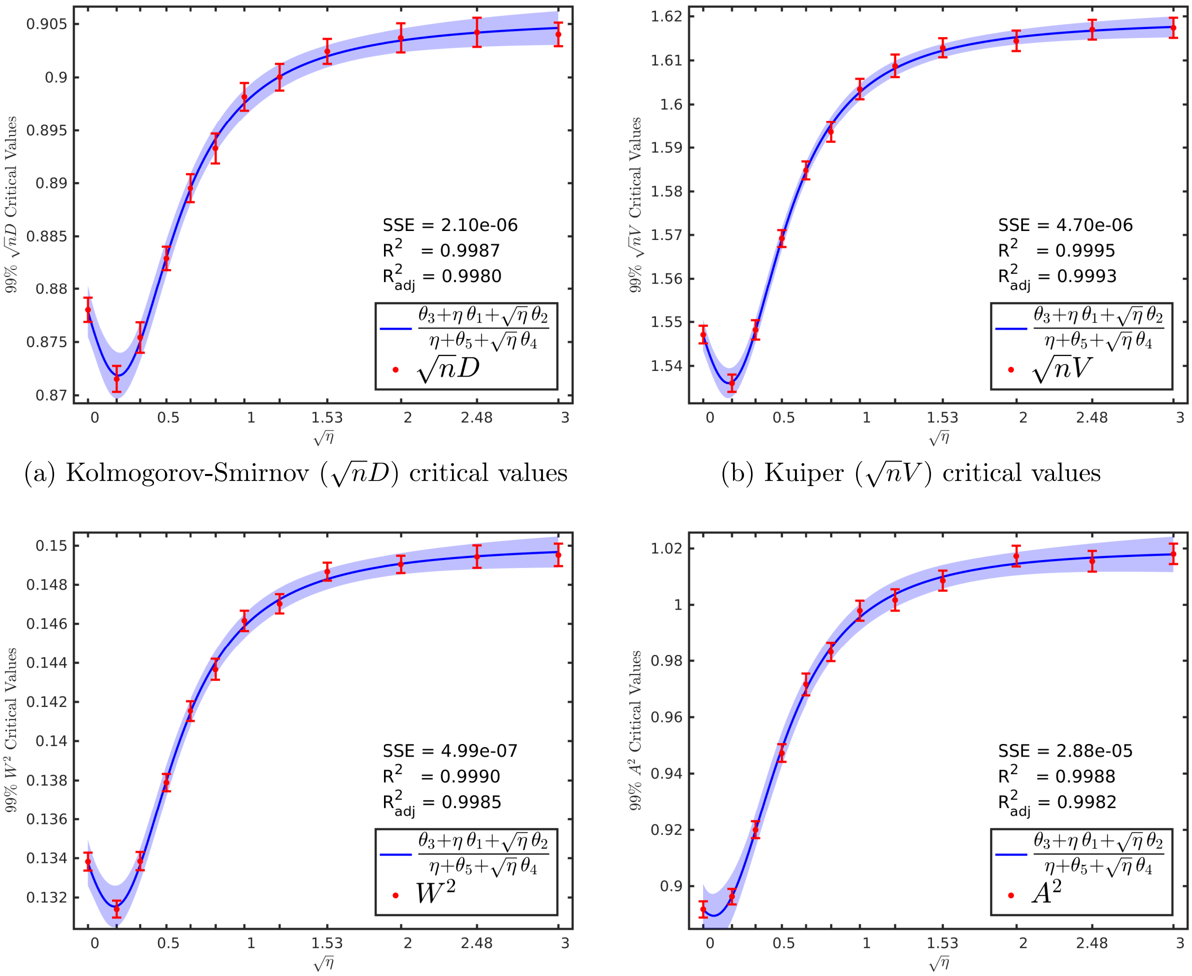
(a) Kolmogorov-Smirnov ($\sqrt{n}D$) critical values

(b) Kuiper ($\sqrt{n}V$) critical values

(c) Cramér-von Mises (W^2) critical values

(d) Anderson-Darling (A^2) critical values

Figure 9.1: Eq. (9.7) fitted to case II loglogistic distribution critical values at the 85% sig. lvl. for $n = 10,000$



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) critical values

(b) Kuiper ($\sqrt{n}V$) critical values

(c) Cramér-von Mises (W^2) critical values

(d) Anderson-Darling (A^2) critical values

Figure 9.2: Eq. (9.7) fitted to case II loglogistic distribution critical values at the 99% sig. lvl. for $n = 30$

In the scenarios not listed as having η dependence the critical values resulting from the Monte Carlo procedure did not have a discernible dependence upon η (for further discussion read Chapter 8). In these situations modelling the critical values as a function of η is illogical and thus was not completed. It is possible that future experiments could resolve η dependence and thus allow for reliable modelling to take place. Should this occur, we would be curious to see if Eq. (9.7) is a suitable model.

9.4 n Dependence

Kizilersü et al. [10] considered a linear fit-Eq. (9.8)-and a quadratic fit-Eq. (9.9)-to describe the n dependence of the Kolmogorov-Smirnov critical values of the Weibull distribution at the 95% significance level, and found that there was an insignificant difference in the R^2_{adj} values. However, the uncertainty margin of the parameter values was roughly an order magnitude higher for Eq. (9.9), hence they elected to employ Eq. (9.8) to describe the n dependence. It is unclear how fluctuations in one parameter affect the other parameter values, hence, in this study we analysed the uncertainty margin for the model as a whole, rather than for individual parameters. This is a point of difference between the two studies.

$$D_{cv}(n|\eta) = \theta_1(\eta) + \frac{\theta_2(\eta)}{\sqrt{n}} \tag{9.8}$$

$$D_{cv}(n|\eta) = \theta_1(\eta) + \frac{\theta_2(\eta)}{\sqrt{n}} + \frac{\theta_3(\eta)}{n} \tag{9.9}$$

n Dependent Scenarios

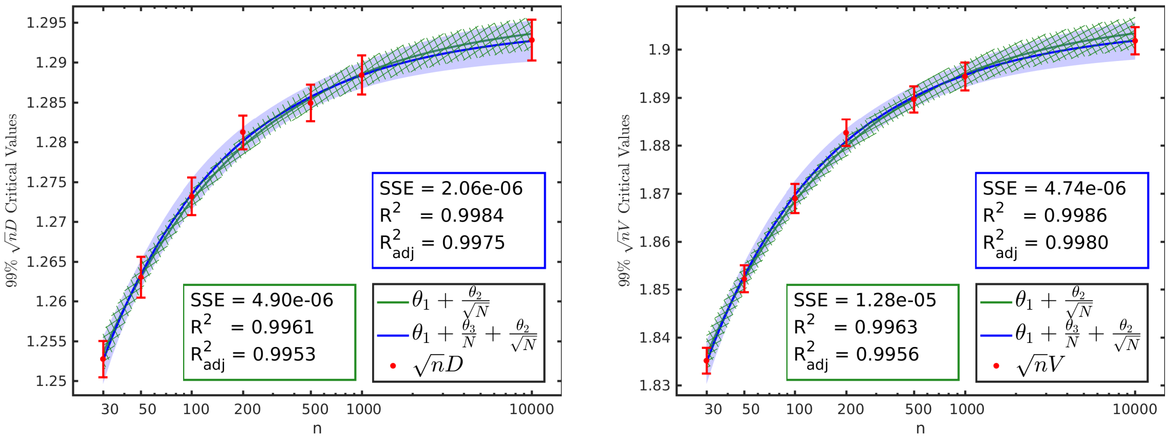
- Case I: Kolmogorov-Smirnov and Kuiper

- Case II

- Case IIIa: Kolmogorov-Smirnov and Kuiper

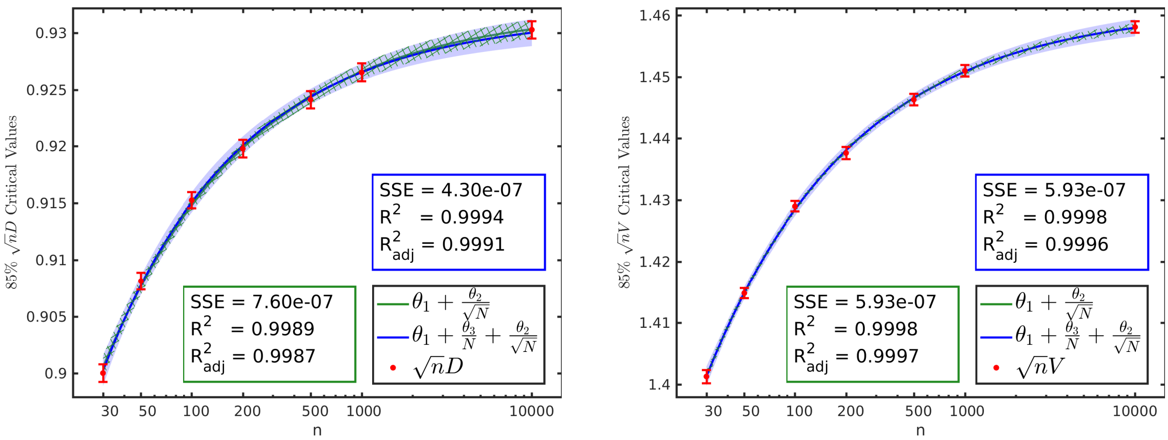
- Case IIIb: Kolmogorov-Smirnov and Kuiper

Figure 9.3 shows Eq. (9.8) and (9.9) fitted to the critical values of the Kolmogorov-Smirnov and Kuiper tests for observations drawn from the Weibull distribution with unknown scale parameter (case IIIa) at the 99% significance level with 90% truncation ($p = 0.9$, $\sqrt{\eta} = 1.52$). Figure 9.4 reproduces these plots for the complete ($p = 0$, $\sqrt{\eta} = 0$) distribution at the 85% significance level. Figures 9.5 and 9.6 show the equivalent plots for case IIIb in which the shape parameter is unknown. There is excellent agreement between both models and the observations, one can verify this via inspection and by looking at the extremely high R^2 and R^2_{adj} values. The green hashed region represents the 95% simultaneous prediction interval of the linear fit and the blue shaded region represents the corresponding interval for the quadratic model [101, 102]. Again, we have displayed the combinations of significance level and truncation level that correspond to the upper (85% and $\sqrt{\eta} = 0$ $\{p = 0\}$) and lower (99% and $\sqrt{\eta} = 1.52$ $\{p = 0.9\}$) limits of the signal to noise ratio. This is visible in the aforementioned figures as the uncertainty margins are noticeably smaller in Figures 9.4 and 9.6 than in Figures 9.3 and 9.5.



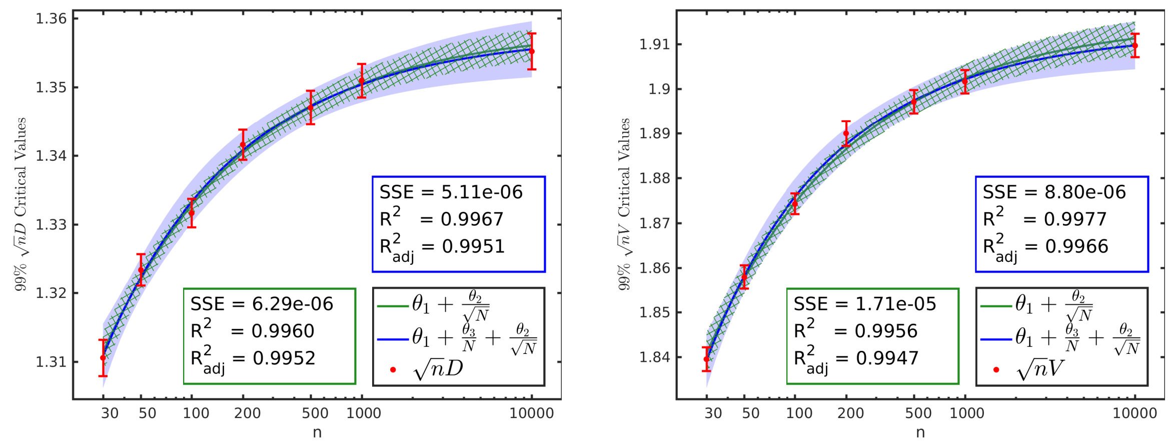
(a) Kolmogorov-Smirnov ($\sqrt{n}D$) critical values (b) Kuiper critical ($\sqrt{n}V$) values

Figure 9.3: Eq. (9.8) and Eq. (9.9) fitted to the critical values of the case IIIa Weibull distribution at the 99% sig. lvl. with $\sqrt{\eta}$=1.52 (p=0.9)



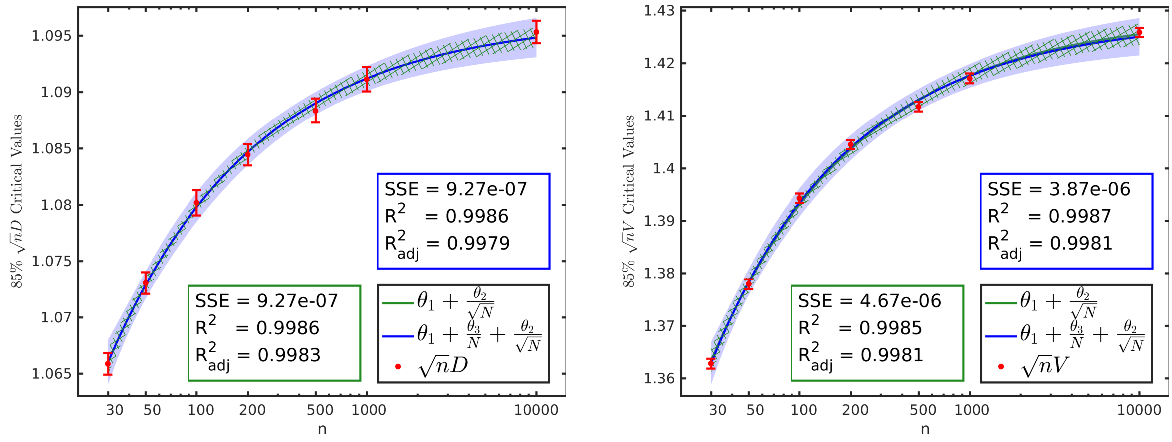
(a) Kolmogorov-Smirnov ($\sqrt{n}D$) critical values (b) Kuiper critical ($\sqrt{n}V$) values

Figure 9.4: Eq. (9.8) and Eq. (9.9) fitted to the critical values of the complete ($\sqrt{\eta}$=0, p=0) case IIIa Weibull distribution at the 85% sig. lvl.



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) critical values (b) Kuiper ($\sqrt{n}V$) critical values

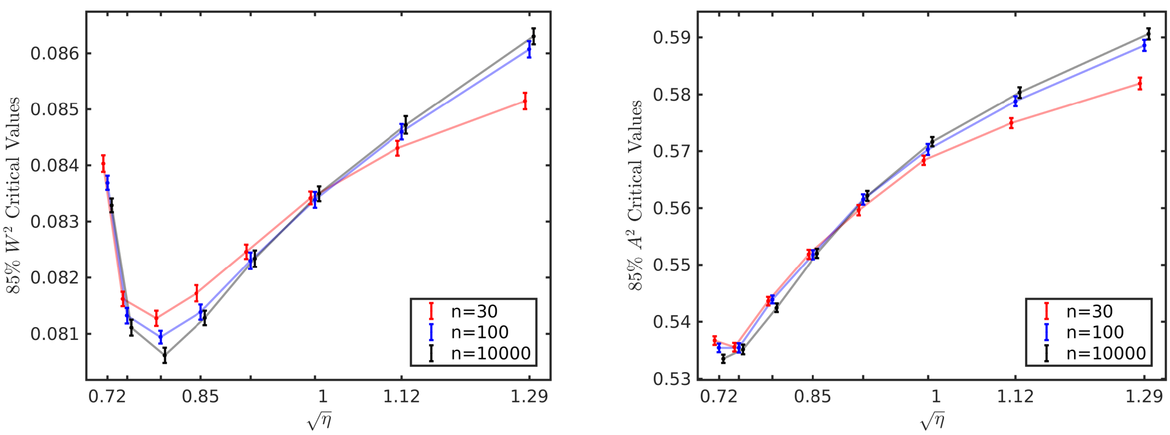
Figure 9.5: Eq. (9.8) and Eq. (9.9) fitted to the critical values of the case IIIb Weibull distribution at the 99% sig. lvl. with $\sqrt{\eta}$=1.52 (p=0.9)



(a) Kolmogorov-Smirnov ($\sqrt{n}D$) critical values (b) Kuiper ($\sqrt{n}V$) critical values

Figure 9.6: Eq. (9.8) and Eq. (9.9) fitted to critical values of the complete ($\sqrt{\eta}$=0, p=0) case IIIb Weibull distribution at the 85% sig. lvl.

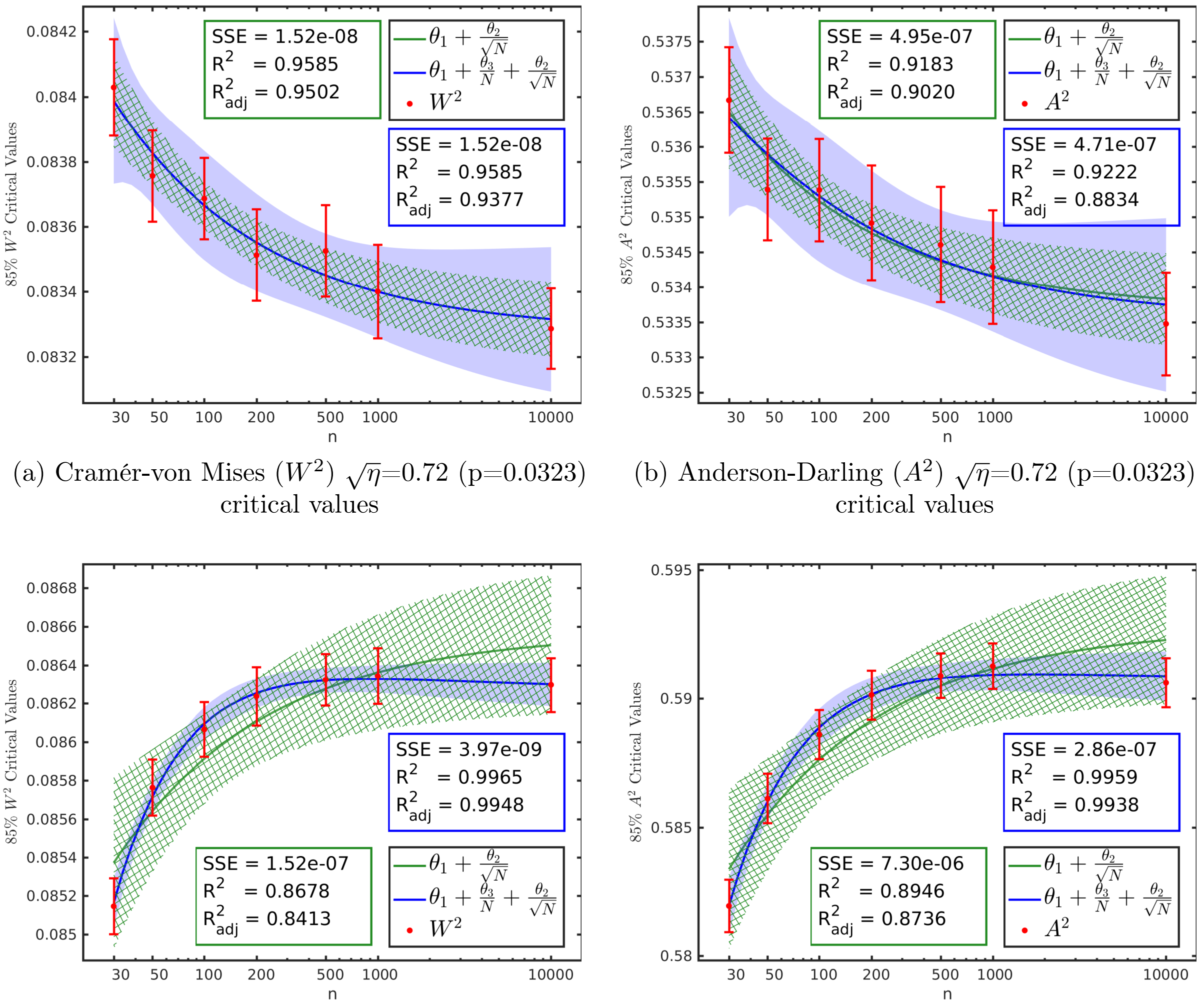
Figure 9.7 displays the Cramér-von Mises and Anderson-Darling critical values for observations drawn from the case II lognormal distribution at the 85% significance level. The $\sqrt{\eta}$ range is reduced to $\sqrt{\eta} \in [0.72, 1.29]$ ($p \in [0.0323, 0.7]$) from $\sqrt{\eta} \in [0.71, 2.24]$ ($p \in [0, 0.9]$) for the sake of clarity. One can see that at $\sqrt{\eta} = 0.72$ (p=0.0323) both the Cramér-von Mises and Anderson-Darling critical values decrease with respect to n, however as η increases this relationship reverses and the critical values increase with n. At intermediate truncation levels ($\sqrt{\eta} \approx 1, p \approx 0.5$ for the Cramér-von Mises test and $\sqrt{\eta} \approx 0.85, p \approx 0.3$ for the Anderson-Darling test) the critical values are n independent, which can be seen by the overlap of data points in Figure 9.7. This unusual behaviour also occurs for the loglogistic distribution, but not the Weibull distribution, and is not exhibited by the Kolmogorov-Smirnov or Kuiper critical values. Indeed, none of the other n dependent scenarios exhibit such a complex relationship. It is not clear to us why this should occur and further research into this area is encouraged. We note that this phenomena may be present in the Cramér-von Mises and Anderson-Darling critical values for cases IIIa and IIIb, however we cannot discern this from our data due to excessive noise. The 85% significance level has been displayed because it most clearly shows the odd relationship these critical values have with truncation level and sample size.



(a) Cramér-von Mises (W^2) critical values (b) Anderson-Darling (A^2) critical values

Figure 9.7: Critical values of the case II lognormal distribution at the 85% sig. lvl.

The linear and quadratic models are fitted to the Cramér-von Mises and Anderson-Darling critical values for data from the case II lognormal distribution at the 85% critical values with $\sqrt{\eta} = 0.72$ (p=0.0323) and $\sqrt{\eta} = 1.29$ (p=0.7) in Figure 9.8. At $\sqrt{\eta} = 0.72$ (Figures 9.8a and 9.8b) one can see the aforementioned decrease in the critical values and that both proposed models predict the observations very well, however, the quadratic model has a significantly larger prediction interval. Initially this difference in prediction interval size may lead one to favour the linear fit, however, perusal of Figures 9.8c and 9.8d ($\sqrt{\eta} = 1.29$) show how erroneous this would be. Above the intermediate truncation range the quadratic model does a significantly better job of predicting the Cramér-von Mises and Anderson-Darling critical values. The R^2_{adj} and prediction interval size from the quadratic fit outweigh the slight advantage of the linear model for the low truncation and Kolmogorov-Smirnov/Kuiper critical values. Therefore **we employ Eq. (9.9) as the function of choice for modelling n dependence** in this work. It is possible to apply each model to the situations in which they are most successful, however, we have elected to stick to one model for the sake of simplicity. We note that the range of distributions and goodness-of-fit tests studied in this work and not by Kizilersü et al. [10], provided the most support for the quadratic model over the linear one, hence, it is not surprising we selected a different model to describe the n dependence of the critical values. This discussion has been conducted with respect to the Cramér-von Mises and Anderson-Darling critical values for observations of the case II lognormal distribution at the 85% significance level, however, it is applicable to all significance levels and the loglogistic distribution. The Cramér-von Mises and Anderson-Darling critical values of the case II Weibull distribution increase with respect to n for all η values, hence, the previously discussed odd behaviour does not occur. The quadratic model becomes preferable for the case II Weibull distribution as the truncation level is increased.



(a) Cramér-von Mises (W^2) $\sqrt{\eta}$=0.72 (p=0.0323) critical values

(b) Anderson-Darling (A^2) $\sqrt{\eta}$=0.72 (p=0.0323) critical values

(c) Cramér-von Mises (W^2) $\sqrt{\eta}$=1.29 (p=0.7) critical values

(d) Anderson-Darling (A^2) $\sqrt{\eta}$=1.29 (p=0.7) critical values

Figure 9.8: Eq. (9.8) and Eq. (9.9) fitted to the critical values of the case II lognormal distribution at the 85% sig. lvl.

In the scenarios not listed at the start of this section as having n dependence, the critical values resulting from the Monte Carlo procedure did not have a discernible dependence upon n (for further discussion read Chapter 8). In these situations modelling the critical values as a function of n is illogical and thus was not completed. It is possible that future experiments could resolve n dependence and thus allow for reliable modelling to take place. Should this occur, we would be curious to see what functional form is required of a suitable model.

9.5 η and n Dependence

Kizilersü et al. [10] proposed a model defined by Eq. (9.10) to simultaneously describe both the n and η dependence of the Kolmogorov-Smirnov critical values for the case II Weibull distribution at the 95% significance level. We found that this model performs well for all the critical values of the case II Weibull distribution. The loglogistic and lognormal critical values, however, could not be adequately predicted by Eq. (9.10), as displayed in Table 9.1. In an attempt to find a suitable model, a number of additional terms were added to this function without significant improvement, however, we noticed that Eq. (9.7) and Eq. (9.9) successfully described the η and n dependence respectively. These functions were then combined with some mixing terms, yielding much better results. After some trial and error, Eq. (9.11) was produced and out-performed all other models in the goodness-of-fit metrics we considered. As a result, we suggest Eq. (9.11) for all case II critical values. The uncertainty margins of the two models were not considered in this case, as comparison of the SSE, R^2 and R^2_{adj} yielded a strong preference for Eq. (9.11).

$$D^q_{cv}(\eta, n) = \theta_1 + \frac{\theta_2}{\sqrt{n}} + \theta_3\sqrt{\eta} + \theta_4\sqrt{\frac{\eta}{n}} + \theta_5\eta + \theta_6\eta^{\frac{3}{2}} \tag{9.10}$$

$$D_{cv}(\eta, n) = \frac{\theta_1 + \theta_2\sqrt{\eta} + \theta_3\eta}{\theta_4\sqrt{\eta} + \theta_5 + \eta} + \theta_6\sqrt{\frac{\eta}{n}} + \theta_7\eta^{\frac{3}{2}} + \frac{\theta_8}{\sqrt{n}} + \frac{\theta_9}{n}\ . \tag{9.11}$$

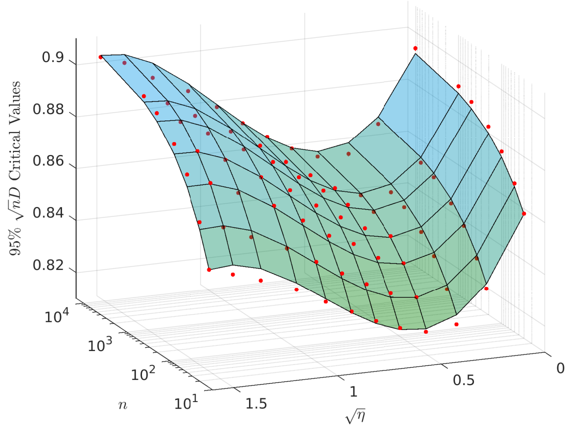
η and n Dependent Scenarios

- Case II
- Case IIIa: Lognormal Kolmogorov-Smirnov and Kuiper
- Case IIIb: Kolmogorov-Smirnov and Kuiper

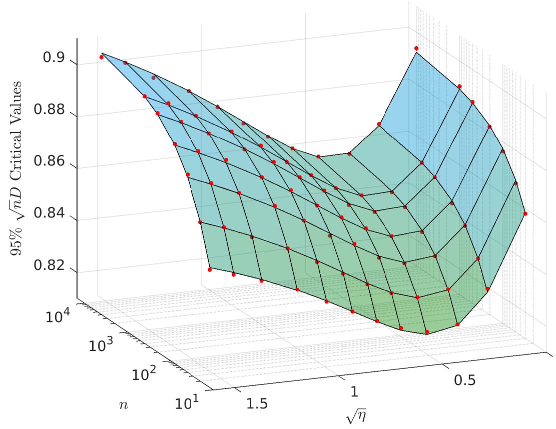
Figure 9.9 displays Eq. (9.10) and (9.11) fitted to the Kolmogorov-Smirnov critical values of the case II Weibull distribution at the 95% significance level. The red dots are the critical values determined from the Monte Carlo procedure and the surfaces are the values predicted by the respective models. We can see that there is very strong agreement between both models and the observations, and that Eq. (9.11) generally outperforms Eq. (9.10) (which is to be expected given the larger number of parameters). Table 9.1 shows that the SSE is roughly an order of magnitude smaller for Eq. (9.11), and that the R^2_{adj} is larger. We note that Eq. (9.10) also has a respectable R^2_{adj}.

Distribution	Case	GOF Test	Significance Level	Model	SSE	R^2	R^2_{adj}
Weibull	II	KS ($\sqrt{n}D$)	95%	Eq. (9.10)	0.000400	0.98968	0.98902
"	II	KS ($\sqrt{n}D$)	95%	Eq. (9.11)	0.000029	0.99924	0.99916
Lognormal	II	KS ($\sqrt{n}D$)	95%	Eq. (9.10)	0.013333	0.64706	0.62443
"	II	KS ($\sqrt{n}D$)	95%	Eq. (9.11)	0.000102	0.99731	0.99702
Loglogistic	II	KS ($\sqrt{n}D$)	95%	Eq. (9.10)	0.000923	0.97008	0.96817
"	II	KS ($\sqrt{n}D$)	95%	Eq. (9.11)	0.000028	0.99909	0.99900

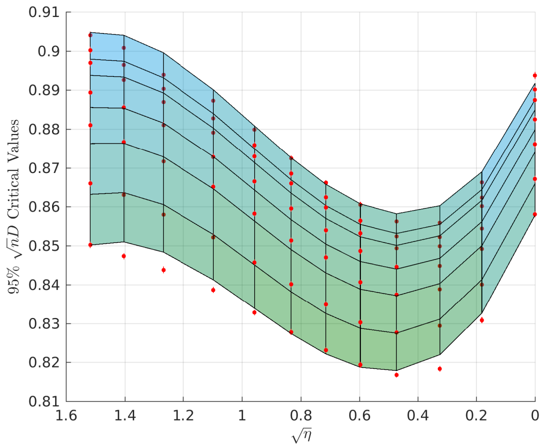
Table 9.1: Comparison of models for the case II 95% Kolmogorov-Smirnov critical values



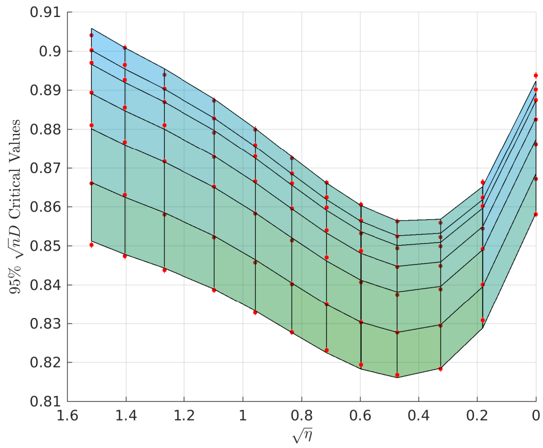
(a) Eq. (9.10) fitted to critical values (from an angle)



(b) Eq. (9.11) fitted to critical values (from an angle)



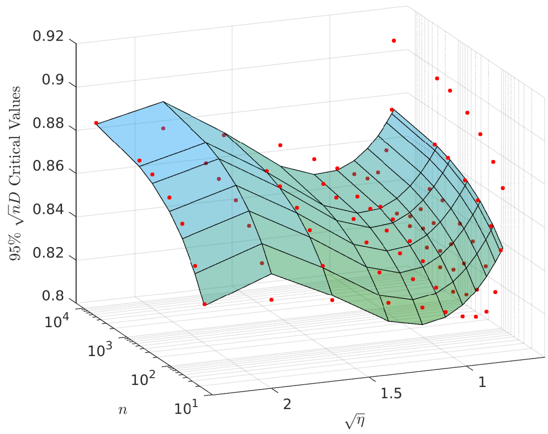
(c) Eq. (9.10) fitted to critical values (along n axis)



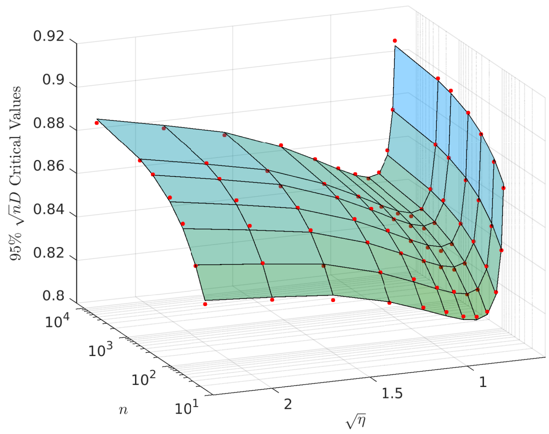
(d) Eq. (9.11) fitted to critical values (along n axis)

Figure 9.9: Eq. (9.10) and Eq. (9.11) fitted to the Kolmogorov-Smirnov critical values of the case II Weibull distribution at the 95% sig. lvl.

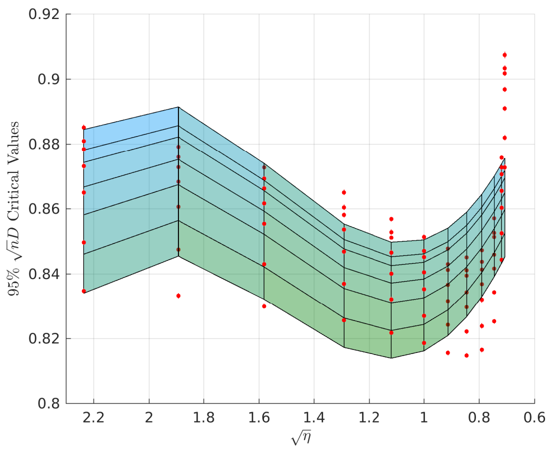
Figure 9.10 displays Eq. (9.10) and Eq. (9.11) fitted to the Kolmogorov-Smirnov critical values of the case II lognormal distribution at the 95% significance level. The red dots are the critical values determined from the Monte Carlo procedure and the surfaces are the values predicted by the respective models. For the case II lognormal distribution, Eq. (9.10) (Figures 9.10a and 9.10c) does a demonstrably worse job of predicting the observations than Eq. (9.11) (Figures 9.10b and 9.10d). The difference between Eq. (9.10) and the results of the Monte Carlo procedure are extremely large, therefore Eq. (9.10) is not a suitable model in this case. Table 9.1 shows that the SSE is more than two orders of magnitude larger for Eq. (9.10) and the difference between the R^2_{adj} values is also significant. Therefore, Eq. (9.11) is far better model for this scenario. This discussion was been conducted with regard to the Kolmogorov-Smirnov critical values of the case II lognormal distribution at the 95% significance level, however, it is indicative of all significance levels and goodness-of-fit tests for the case II lognormal and loglogistic distributions. Tables comparing the success of the two models (similar to Table 9.1) for more cases are presented in Appendix D.



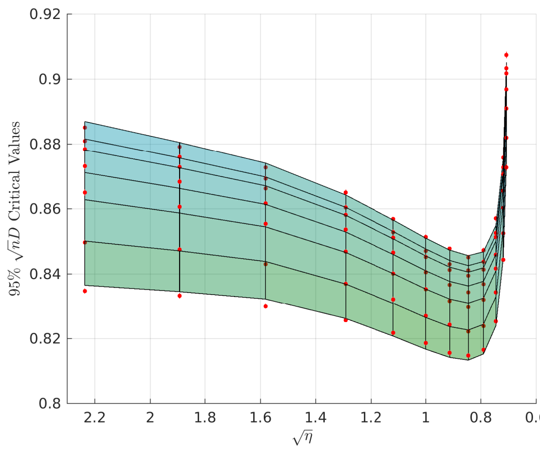
(a) Eq. (9.10) fitted to critical values (from an angle)



(b) Eq. (9.11) fitted to critical values (from an angle)



(c) Eq. (9.10) fitted to critical values (along n axis)



(d) Eq. (9.11) fitted to critical values (along n axis)

Figure 9.10: Eq. (9.10) and Eq. (9.11) fitted to the Kolmogorov-Smirnov critical values for the case II lognormal distribution at the 95% sig. lvl.

The Kolmogorov-Smirnov and Kuiper critical values for the case IIIa and case IIIb loglogistic and lognormal distributions behave in a very similar way to the Kolmogorov-Smirnov critical values of the case II Weibull distribution at the 95% significance level. Both Eq. (9.10) and Eq. (9.11) are good models for describing the n and η dependence, however Eq. (9.11) performs slightly better. **We conclude that Eq. (9.11) models the critical values as a function of n and η for all the distributions and goodness-of-fit tests studied in the present work successfully**.

9.6 Determining Optimal Models

For each of the scenarios discussed in this chapter, we tested several modelling functions and selected the most successful one. The success of a model was judged by its SSE, R^2 and R^2_{adj} values, with the most weight being placed on the R^2_{adj} as it punishes a model for having more parameters (reducing the probability of overfitting). The functions we tested were largely based on those discussed by Kizilersü et al. [10], making amendments when required.

9.6.1 Selecting Parameters

Throughout this work, we used MATLAB to conduct the Monte Carlo simulations and analysis, including selecting the parameters for a given model. The optimal parameters yield the smallest SSE, hence, running a multidimensional minimisation algorithm on the SSE allows one to determine these parameters numerically. We completed this procedure with the MATLAB functions *fit* [103] and *fitnlm* [104], both of which require one to specify the starting values[1]. The *fitnlm* function was passed randomly generated numbers as starting values; its result was then passed as a starting point to *fit*, which yielded a parameter estimate. The *fit* function also returns a 95% confidence interval for the parameters; if the results of the initial *fitnlm* run lay within this interval, the parameter estimate was considered valid. This procedure was undertaken to reduce the effect of the starting point on the parameter estimates and to ensure the stability of the solution.

For simple models (e.g. Eq. (9.8) and (9.9)) this procedure is sufficient, however, as the number of parameters increases (e.g. Eq. (9.7), (9.11) and (9.10)) the starting values play an increasingly large role, despite utilising multiple algorithms in combination. To combat this the SSE, R^2 and R^2_{adj} values were each compared to a heuristically determined threshold. The SSE thresholds were scaled for different goodness-of-fit tests (see 9.2.1); for example the Cramér-von Mises test statistics are far lower than the corresponding Kuiper ones, hence, a lower SSE threshold was required to convey the same error-percentage for the Cramér-von Mises critical values. Parameter estimates were only accepted if two of the three goodness-of-fit metrics were on the preferable side of the threshold (below for SSE and above for R^2 and R^2_{adj}). This process was repeated with different (randomly generated) starting points and the number of repetitions was proportional to the number of parameters in the model. Initially each of the parameters was drawn from a normal distribution of mean 0 and variance 16 ($\mathcal{N}(0,16)$), however as the repetition number grew the variance was also increased to extend the starting parameter space. If none of the estimates were accepted, and the best performing parameter set was repeated at least five times then that parameter set was selected. Otherwise the procedure was continued until the best performing parameter set had occurred five times, or 110 iterations of the procedure had been conducted. There were some additional nuances which have been omitted for the sake of brevity.

We are aware that this is a particularly complicated procedure for finding the optimal parameter values and recommend that **future studies should seek to simplify this procedure**. The results we obtained are sufficient, however, a more streamlined and systematic procedure might well be more versatile and transparent. We did not seek to simplify this procedure in the present work as the perceived effect on the results was minimal and we did not have the time required.

9.7 Summary

The modelling functions were selected such that they only accepted n and/or η as an argument(s) if the results of Monte Carlo simulation were clearly dependent upon them. For example, the case I, Cramér-von Mises and Anderson-Darling critical values have possible n dependence-as discussed in 8.4.1-however this was not resolvable from the Monte Carlo procedure, therefore they have been modelled with a function independent of n. Tables 8.2, 8.7, 8.20 and 8.37 summarise the functions we selected to model the critical values. The parameters values for each situation are given in Appendix C.

Distribution	Test	n dependent	τ_L dependent	Function
Weibull	KS & Kuiper	✓	✗	Eq. (9.9)
Loglogistic	KS & Kuiper	✓	✗	Eq. (9.9)
Lognormal	KS & Kuiper	✓	✗	Eq. (9.9)
Pareto	KS & Kuiper	✓	✗	Eq. (9.9)
Weibull	CvM & AD	-	✗	mean
Loglogistic	CvM & AD	-	✗	mean
Lognormal	CvM & AD	-	✗	mean
Pareto	CvM & AD	-	✗	mean

Table 8.2: Case I critical value dependence

Distribution	Test	n dependent	τ_L dependent	Function
Weibull	KS, Kuiper, CvM & AD	✓	✓	Eq. (9.11)
Loglogistic	KS, Kuiper, CvM & AD	✓	✓	Eq. (9.11)
Lognormal	KS, Kuiper, CvM & AD	✓	✓	Eq. (9.11)

Table 8.7: Case II critical value dependence

Distribution	Test	n dependent	τ_L dependent	Function
Weibull	KS & Kuiper	✓	✗	Eq. (9.9)
Loglogistic	KS & Kuiper	✓	✗	Eq. (9.9)
Lognormal	KS & Kuiper	✓	✓	Eq. (9.11)
Pareto	KS & Kuiper	✓	✗	Eq. (9.9)
Weibull	CvM & AD	-	-	mean
Loglogistic	CvM & AD	-	-	mean
Lognormal	CvM	✗	✓	Eq. (9.7)
Lognormal	AD	-	✓	Eq. (9.7)
Pareto	CvM & AD	-	-	mean

Table 8.20: Case IIIa critical value dependence

Distribution	Test	n dependent	τ_L dependent	Function
Weibull	KS & Kuiper	✓	✓	Eq. (9.11)
Loglogistic	KS & Kuiper	✓	✓	Eq. (9.11)
Lognormal	KS & Kuiper	✓	✓	Eq. (9.11)
Weibull	CvM & AD	✗	✓	Eq. (9.7)
Loglogistic	CvM & AD	✗	✓	Eq. (9.7)
Lognormal	CvM & AD	✗	✓	Eq. (9.7)

Table 8.37: Case IIIb critical value dependence

Chapter 10

Power testing

Fundamentally, hypothesis testing is conducted to determine whether the available data can justify rejecting a given hypothesis at a particular significance level. In order to do this effectively one must understand how the statistical tests at their disposal perform when said hypothesis is true and when it is false. Power testing is the procedure of measuring this performance, and allows one to select tests which have the greatest power to discriminate between different hypotheses which is of the utmost importance in hypothesis testing.

Consider the case in which an analyst wants to model the amount of lead in a public supply of drinking water, they test their set of measurements against a distribution A, which has been used to describe the lead levels in the past. The chosen goodness-of-fit test cannot reject the null hypothesis that the sample was drawn from A at the 95% significance level, hence, the analyst concludes that the distribution of lead levels are described by A. However, the analyst does not know how well the test can distinguish between samples drawn from distribution A and those drawn from distribution a distinct distribution, B. If power testing had been conducted, it would show that the test was passed by 90% of samples drawn from B. Therefore, if the lead levels were actually distributed by B, there is still a 90% chance that they would pass the test, i.e. this goodness-of-fit test lacks the power to accurately distinguish between distributions A and B. It may be the case that under distribution B, there is a significant probability that the lead levels exceed safety standards. Access to this information would surely cause the analyst to conduct further testing before declaring that distribution A described the observations sufficiently well. This example highlights the importance of power testing in real world applications.

Chapter 8 discusses the critical values we produced (via Monte Carlo simulations) for the cases, distributions, goodness-of-fit tests and significance levels listed below. It was not possible to assess case II or case IIIb for the Pareto distribution as it has no shape parameter, however, every other permutation of the listed options has been studied.

Cases		Distributions:	Goodness-of-fit Tests	Significance Levels
• Case I:	scale: known shape: known	• Weibull	• Kolmogorov-Smirnov	• 85%
• Case II:	scale: acquired shape: acquired	• Loglogistic	• Kuiper	• 90%
• Case IIIa:	scale: known shape: acquired	• Lognormal	• Cramér-von Mises	• 95%
• Case IIIb:	scale: acquired shape: known	• Pareto	• Anderson-Darling	• 99%

$$
\begin{aligned}
&n \in \{30, 50, 100, 200, 500, 1000, 10000\}\\
&p \in \{0, 0.0323, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.8605, 0.9\}
\end{aligned}
\tag{10.1}
$$

The significance level is specified by the probability of a type 1 error, α,

$$\alpha = \mathbb{P}\left(\text{reject } H_0 | H_0 \text{ is true}\right). \qquad (2.1 \text{ revisited})$$

For example, the 95% significance level is defined by having $\alpha = 0.05$. There is also the possibility that a false hypothesis is accepted, the probability of one of these type 2 errors occurring is denoted $1 - \beta$

$$1 - \beta = \mathbb{P}\left(\text{do not reject } H_0 | H_0 \text{ is false}\right), \qquad (2.2 \text{ revisited})$$

where β is the statistical power. Power testing allows us to determine the probability of type 1 and type 2 errors, that is, the probability that the null hypothesis is rejected when it is true, and the probability that the null hypothesis is not rejected when it is false. When we perform a power test, we draw observations from a distribution '2' and compare them to a distribution '1' under a goodness-of-fit test, at a particular significance level. If the correct critical values have been applied, the percentage of samples, P, which pass the test will be given by,

$$P_1 \approx 1 - a \qquad \text{if distribution } 1 = \text{ distribution } 2, \qquad (10.2)$$

$$P_2 = 1 - \beta \leq 1 - a \qquad \text{if distribution } 1 \neq \text{ distribution } 2. \qquad (10.3)$$

The power of the test comes from how well it can distinguish between distributions 1 and 2. For cases in which $P_2 << P_1$, the test can easily distinguish between the two distributions, however, if $P_2 \approx P_1$ the test does a poor job of telling them apart. If this occurs, the two distributions may often be confused under the given test.

An optimal test will have a low probability of both type 1 and type 2 errors, as this means there is a low chance of rejecting a true hypothesis or accepting a false one, i.e. the test has high discriminating power. In practice, decreasing the probability of a type 1 error typically increases the probability of a type 2, however, this is not always the case. As the sample size (n) increases, type 1 errors occur with the same frequency but type 2 errors become decreasingly common. Type 1 errors are unaffected by truncation, however, for the cases in which the parameters are estimated, a higher truncation level usually increases the probability of a type 2 error. Generally speaking, **large data sets with low truncation allow the statistical tests to perform the best**. In this chapter, we discuss the discriminating power of the goodness-of-fit tests under different circumstances.

Power testing is important as it gives the analysts a deeper understanding of the statistical tests which they use, however, in the present work it has an additional purpose. In Chapter 9 we modelled the critical values as functions of sample size (n) and truncation level (which was parametrised by η). η can be defined as a function the of the left-truncation limit, τ_l, and the parameters of the distribution, $\boldsymbol{\theta}$, i.e. $\eta = \eta(\tau_l, \boldsymbol{\theta})$. When developing the models, we employed the true parameter values, $\boldsymbol{\theta^0}$, hence, the resulting η values can more correctly be denoted η^0. In the real world however, the true parameters of the distribution are often unknown, thus, the estimated parameters, $\hat{\boldsymbol{\theta}}$, must be used to construct an estimate of η, which is denoted $\hat{\eta}$. It is not necessarily the case that the critical value models (which accept η^0) will accurately describe the critical values as a function of $\hat{\eta}$. We have implicitly made the assumption that $\eta^0 \approx \hat{\eta}$. The results of the power testing procedure show that the probability of type I errors is approximately α for all tests which employed the critical value models we determined in Chapter 9. Thus, **our assumption that $\boldsymbol{\eta^0 \approx \hat{\eta}}$ is valid** and we can employ our models as intended. All this confusion may reasonably lead one to ask why we didn't construct the models as functions of $\hat{\eta}$ in the first place, this question can most easily be answered by the following thought experiment. Consider a series of data sets drawn from a distribution with fixed (true) parameters, nearly every data set will produce a unique parameter estimate and thus a unique $\hat{\eta}$ value. As there will be only one test statistic for most $\hat{\eta}$ values, we cannot obtain percentile estimates for any fixed $\hat{\eta}$ value. Therefore, **it is not possible to determine critical values as a function of $\hat{\boldsymbol{\eta}}$**. Thus, the approximation $\hat{\eta} \approx \eta^0$ must be made in order to achieve any results at all.

We make extensive use of $\hat{\eta}$ in this chapter. As there is a 1-1 relationship between p and η, it is useful to define the truncation percentage which corresponds to $\hat{\eta}$, we denote this $\hat{p}$. Like $\hat{\eta}$, $\hat{p}$ is defined in terms of τ_l and the estimated parameter values $\hat{\boldsymbol{\theta}}$. The Monte Carlo procedure was undertaken for a range of p^0 values between 0 and 0.9, therefore the models derived in Chapter 9 are only valid for $\hat{p}$ values within that range. There is, however, no reason to believe that the $\hat{p}$ values will be restricted to this interval. The performance of the modelling functions is untested outside of the aforementioned range, hence it would be negligent to employ them for $\hat{p} > 0.9$. For this reason, the test statistics resulting from data sets with $\hat{p} > 0.9$ were not compared to a model of the critical values, only to the critical values themselves. In this way, we can be sure that the models are only employed when they are valid, however, the number of data sets trialled in the power testing procedure is also reduced. The analyst must invoke the same criteria upon real world data in order for the results of the power testing to be meaningful. **Future studies should include higher p^0 values** so that the models are valid over a larger range of $\hat{p}$ estimates.

When test statistics are compared to the critical values (rather than the model) $\hat{p}$ is still utilised. A cubic spline is fitted to the critical values resulting from the Monte Carlo procedure in the p direction for a given n. This spline is evaluated at $\hat{p}$ in order to determine the critical value that a test statistic should be compared to. This procedure was undertaken because in practice the analyst will not readily have access to p^0, therefore utilising it in the power testing belies the true behaviour of the system in the real world.

To conduct the power testing, we drew $10,000$ data sets from the range of distributions defined in Table 10.1 and compared them to the Weibull, loglogistic, lognormal and Pareto distributions by determining the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling test statistics. This procedure was completed for the sample sizes, $n = (30, 100, 1000)$, and truncation levels, $p^0 = (0, 0.1, 0.4, 0.6, 0.8)$. The proportion of test statistics which was lower than the relevant critical value was recorded as the pass rate. Previously we have shown that data sets which satisfy specific criteria and are drawn from the case II Weibull, loglogistic, and lognormal distributions can be shown to lack unique solutions to the MLE equations. The data sets which satisfied these criteria were disregarded without replacement because the parameter estimates could not be trusted. For this reason, there are several situations in which the pass rates were determined for less than $10,000$ samples. In the aforementioned tables, the number of samples is displayed below the relevant pass rates. Additionally, there may be a discrepancy between the number of test statistics (samples) which were compared to critical values and to the model. Where this occurs, the difference is because some of the $\hat{p}$ values exceed 0.9 and hence were not compared to the model. Figure 10.1 summarises the procedure for determining which samples to subject to which tests.

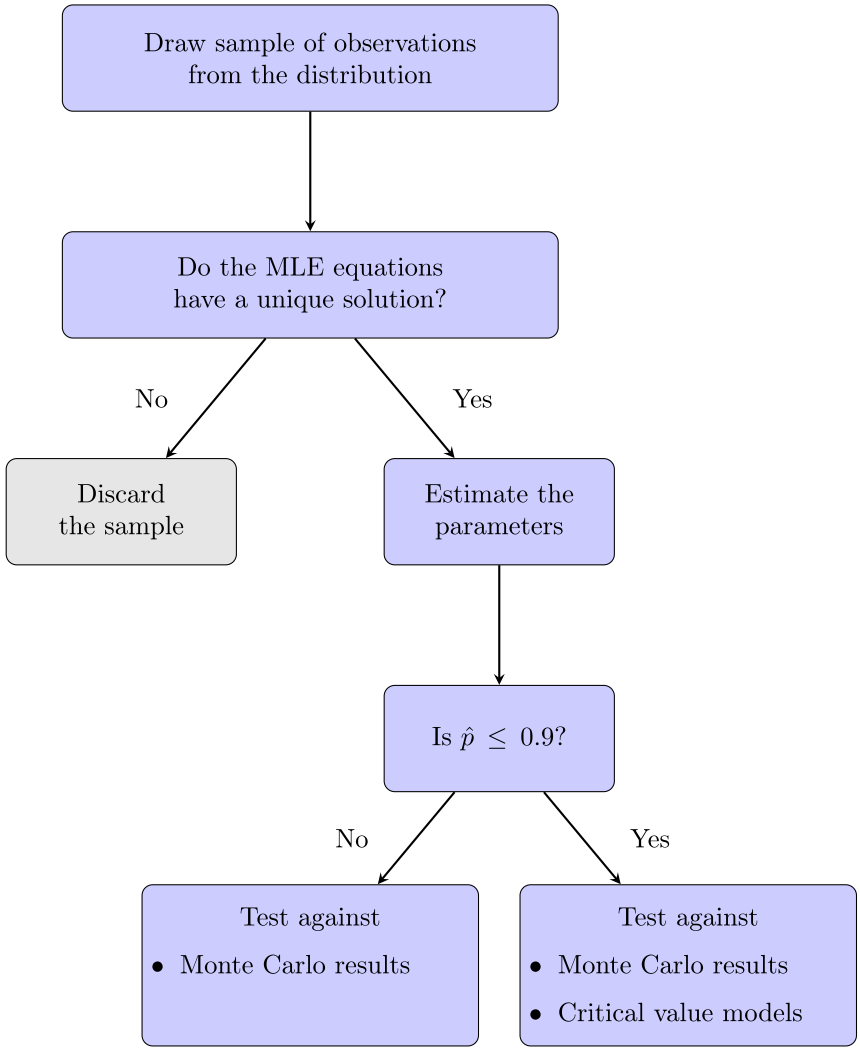


Figure 10.1: Flow chart for selecting the samples to test against the critical value models

This chapter features a section on each case (I, II, IIIa and IIIb) and discusses the performance of the relevant tests under the power testing procedure. The complete set of results for the power testing comprises several hundred tables, hence, we have elected to restrict our display to the Kolmogorov-Smirnov and Anderson-Darling tests at the 95% significance level and to observations tested against the Weibull distribution. The Weibull distribution was selected because it had been previously studied by Kizilersü et al. [10] and our work was an extension of theirs. **The remaining power testing results are displayed in Appendix E.** For the sake of brevity, only the 95% significance level is displayed anywhere in this thesis, however, there is a very limited difference between the results at differing significance levels. In each section we compare the anticipated and measured α values, and look into the discriminating power of each test by evaluating the prevalence of type 2 errors.

Distribution	Complete pdf (no truncation)	Scale Parameter	Shape Parameter	Support
Weibull	$f(x\|\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\alpha}\right)^{\beta}\right]$	$0 < \alpha < \infty$	$0 < \beta < \infty$	$0 < x < \infty$
Loglogistic	$f(x\|\phi,\rho) = \frac{\rho}{\phi^{\rho}} \frac{x^{\rho-1}}{\left(1+\left(\frac{x}{\phi}\right)^{\rho}\right)^2}$	$0 < \phi < \infty$	$0 < \rho < \infty$	$0 < x < \infty$
Lognormal	$f(x\|\mu,\sigma) = \frac{1}{\sigma x\sqrt{2\pi}} \exp\left[-\frac{(\log(x)-\mu)^2}{2\sigma^2}\right]$	$-\infty < \mu < \infty$	$0 < \sigma < \infty$	$0 < x < \infty$
Pareto	$f(x\|k) = \frac{k\tau_l^k}{x^{k+1}}$	$0 < k < \infty$	-	$\tau_l < x < \infty$
χ^2 (1 d.o.f.)	$f(x) = \frac{1}{\sqrt{2x}\Gamma\left(\frac{1}{2}\right)} \exp\left[-\frac{x}{2}\right]$	-	-	$0 < x < \infty$
χ^2 (3 d.o.f.)	$f(x) = \frac{1}{2^{\frac{3}{2}}\Gamma\left(\frac{3}{2}\right)}\sqrt{x} \exp\left[-\frac{x}{2}\right]$	-	-	$0 < x < \infty$
χ^2 (4 d.o.f.)	$f(x) = \frac{1}{4\Gamma(2)} x \exp\left[-\frac{x}{2}\right]$	-	-	$0 < x < \infty$
Loglaplace	$f(x\|\mu,b) = \frac{1}{2bx} \exp\left[-\frac{\|\log(x)-\mu\|}{b}\right]$	$-\infty < \mu < \infty$	$0 < b < \infty$	$0 < x < \infty$
Rayleigh	$f(x\|b) = \frac{x}{b^2} \exp\left[-\frac{x^2}{2b^2}\right]$	$0 < b < \infty$	-	$0 < x < \infty$
Gamma	$f(x\|k) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} \exp\left[-\frac{x}{\theta}\right]$	$0 < \theta < \infty$	$0 < k < \infty$	$0 < x < \infty$
Uniform	$f(x) = 1$	-	-	$0 < x < 1$
Mittag-Leffler	$f(x,\alpha,\beta) = \frac{d}{dx}\{1 - E_{\alpha,\beta}(-x^{\alpha})\}$, where $E_{\alpha,\beta}(x) = \sum_{i=0}^{\infty} \frac{x^i}{\Gamma(\alpha i+\beta)}$	$0 < \alpha < 1$	$0 < \beta < \infty$	$0 < x < \infty$

Table 10.1: Distributions from which the data for power testing was drawn

10.1 Case I

In case I, all parameters of the distribution are known *a priori*, therefore, the distribution is completely specified. Tables 10.2 and 10.3 display the pass rates which result from testing observations from various distributions against the Weibull distribution for the Kolmogorov-Smirnov and Anderson-Darling (respectively) tests at the 95% significance level. Table 10.4 displays the same information as Table 10.2 but for a different set of parameter values, the importance of this will be discussed later in this section. Table 10.1 specifies the distributions; the parameter values that we employed are displayed next to the distribution name in Tables 10.2, 10.3 and 10.4. The power testing tables for the other distributions and goodness-of-fit tests at the 95% significance level are given in Appendix E.1.

The aforementioned figures show that **the pass rates corresponding to observations drawn from the Weibull distribution were very close to the expected values for all sample sizes, truncation levels and goodness-of-fit tests.** Appendix E.1 shows that the loglogistic, lognormal and Pareto distributions also achieved this feat, thus, we can conclude that we have appropriately selected the critical values. Test statistics which were compared to the results of the Monte Carlo simulations attained almost identical pass rates to those which were compared to the models deduced in Chapter 9. This result is true of all distributions, goodness-of-fit tests, sample sizes, truncation levels and significance levels, thus our models have performed very well. However, as the parameters are known *a priori* in case I, $\hat{\eta} = \eta^0$, hence, we cannot claim that we have evidence to support the approximation that $\hat{\eta} \approx \eta^0$.

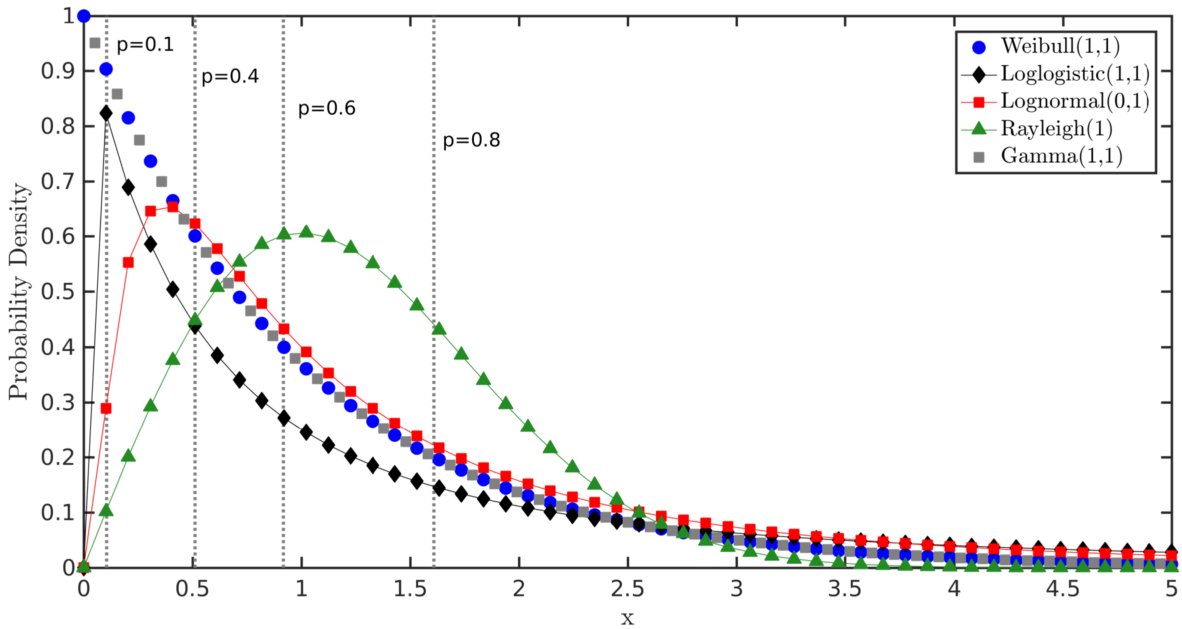
The probability of a type II error is greater for small sample sizes because less information about the probability distribution can be deduced from each sample. For example, Table 10.3 shows that the χ^2 distribution with 1 degree-of-freedom has Anderson-Darling pass rates that exceed 80% for $n = 30$ and $p = 0.1$, however, as the sample size increases to $n = 100$, the pass rate drops to approximately 50%. Further increasing the sample size to $n = 1000$ reduces the pass rates to zero. This effect is prominent for all of the goodness-of-fit tests and distributions which we studied. Additionally, this effect is also very clear in all the cases of parameters estimation we conducted. In sections 10.2, 10.3 and 10.4 we will not repeat this point, however, it will very clear from the relevant tables.

In case I, the statistical power of the test is entirely dependent upon the parameters selected. If the chosen parameters cause the compared distributions to approach one another, the test will struggle to distinguish them. Figure 10.2 displays five distributions with two sets of parameter values, one that makes it difficult for a goodness-of-fit test to tell them apart (Figure 10.2a), and one that makes it easier (Figure 10.2b). In Figure 10.2a there is an extended range over which the distributions have comparable pdf values, thus, observations drawn from them will be very similar, conversely, the distributions in Figure 10.2b all have drastically different pdf values for the vast majority of the displayed range. As there is no parameter estimation in case I, we cannot adjust the distribution the samples are tested against to better suit the observations. Therefore, the parameters of both the distribution from which the data was drawn, and tested against, play a huge role in deciding the pass rates. In Tables 10.2 and 10.3, and Appendix E.1, we have selected parameter values which make it relatively easy for the tests to distinguish between the distributions. This is why the pass rates are so low for observations drawn from distributions which the data was not tested against. At the end of this section, Table 10.4 displays the Kolmogorov-Smirnov pass rates at the 95% significance level with the parameters from Figure 10.2a. Unless explicitly stated otherwise, any discussion regarding the pass rates in this chapter is with respect to the parameter values which produce the most different pdfs, i.e. those in Tables 10.2 and 10.3.

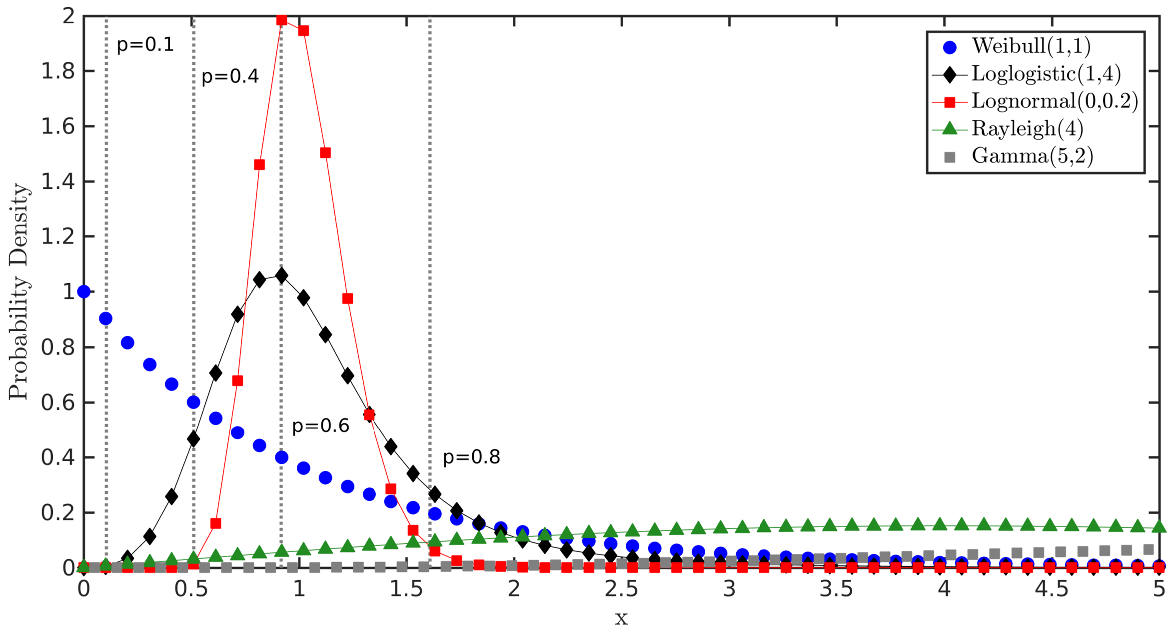
For small sample sizes, the loglogistic and Weibull distributions were not always distinguished. Observations drawn from the Weibull distribution were confused with the loglogistic distribution most at around $p = 0.1$, and pass rates were highest for data from the loglogistic distribution tested against the Weibull at roughly $p = 0.4$. This discrepancy is probably because a given p value corresponds to different τ_l values in distinct distributions, hence, it is likely the similarities in pdf occur within a particular x range. The most significant confusion with the Weibull distribution came from the χ^2 distribution with 1 degree-of-freedom for small sample size and moderate truncation, $p \in (0.1, 0.6)$. Pass rates from the Anderson-Darling test (which is most sensitive in the tails) were significantly lower (about 10 percentage points) than for all other tests (which performed equally well). Coupled with the decreased pass rates at high truncation, we conclude that the χ^2 and Weibull have appreciably different probability density in the tails.

For low truncation levels, and the smallest sample size, there were some problems distinguishing the loglogistic and lognormal distributions, this implies that the tails of the distributions differ, but that the pdfs have a similar shape for low x. Referring to Figure 10.2b we see that similar x values maximise both pdfs, which helps explain this phenomena in the pass rates. It is worth reiterating that this confusion only occurs for $n = 30$, hence, the difference in probability density at $x \approx 1$ is detectable with more samples. The Anderson-Darling test did the best job of distinguishing loglogistic observations against the lognormal distribution, which is not surprising, given it is most sensitive to the tails, which is a clear region of difference. Interestingly, Kuiper's test did a remarkable job when the data came from the lognormal distribution and was tested against the loglogistic. There was not a significant performance difference between the other tests in either case.

The tests confuse the loglogistic distribution with the Pareto distribution for small sample sizes and large truncation levels. This is not surprising, as both distributions have heavy tails, thus at large truncation, the pdfs will become similar in shape. Interestingly, the Kolmogorov-Smirnov test appears to confuse the two more frequently than the other goodness-of-fit tests. The lognormal distribution is also confused with the Pareto distribution, although, the pass rates are highest for $p \in (0.4, 0.6)$, which suggests that the shapes of the distributions are most similar in the moderate x range. The Anderson-Darling test is most sensitive in the tail regions, and also confuses the two distributions least frequently. This supports our assertion that the tails of the Pareto and lognormal distributions differ appreciably for the parameter values we selected.
In Table 10.4 we display the pass rates for the distributions tested against the Weibull distribution under the Kolmogorov-Smirnov test at the 95% significance level with the parameter values from Figure 10.2a. This table is analogous to Table 10.2, but with parameters that make it difficult to tell the distributions apart. We can see that there is now significantly higher pass rates for the loglogistic, lognormal, Pareto, Rayleigh, Gamma and Mittag-Leffler distributions. Only the Kolmogorov-Smirnov test is displayed for these parameters as it is sufficient to highlight the importance of the parameters upon the pass rates. The other goodness-of-fit tests show similar differences in pass rates between the two parameter sets. At the parameters specified, the Weibull, Gamma and Mittag-Leffler distributions all tend to the exponential distribution, hence, it is not surprising that the pass rates approach 95%. The Pareto, χ^2 (3 degrees of freedom), χ^2 (4 degrees of freedom), loglaplace and uniform distributions have near zero pass rates in Table 10.4. Hence, we can conclude that Kolmogorov-Smirnov test can easily discern the difference between these distributions and the Weibull distributions for all n and p values considered. The loglogistic, lognormal, χ^2 (1 degree of freedom) and Rayleigh distributions all have significant pass rates for particular values of n and p. Therefore, the Kolmogorov-Smirnov test can discriminate between those distributions more effectively for larger sample sizes. The pass rates change considerably as truncation is varied, with the greatest pass rates occurring between $p = 0.1$ and $p = 0.4$. This implies, that at those truncation levels there is the most similarity between the Weibull distribution and the loglogistic, lognormal, χ^2 (1 degree of freedom) and Rayleigh distributions.



(a) Parameters that reduce distinguishability



(b) Parameters that increase distinguishability

Figure 10.2: pdf of distributions with parameter values that alter their distinguishability

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.91 10000	94.91 10000	94.61 10000	94.63 10000	94.63 10000	94.61 10000	94.92 10000	94.91 10000	94.84 10000	94.88 10000
Weibull(1,1)	100	95.12 10000	95.12 10000	95.16 10000	95.13 10000	94.88 10000	94.89 10000	94.41 10000	94.42 10000	94.99 10000	94.98 10000
Weibull(1,1)	1000	95.11 10000	95.15 10000	95.29 10000	95.28 10000	94.82 10000	94.83 10000	94.94 10000	94.95 10000	95.11 10000	95.12 10000
Loglogistic(1,4)	30	0.14 10000	0.14 10000	4.45 10000	4.52 10000	43.48 10000	43.40 10000	7.10 10000	7.08 10000	18.60 10000	18.68 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.35 10000	0.35 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.75 10000	0.77 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	57.67 10000	57.67 10000	89.41 10000	89.47 10000	78.48 10000	78.39 10000	66.81 10000	66.78 10000	55.06 10000	55.18 10000
ChiSquared1	100	8.36 10000	8.36 10000	78.63 10000	78.52 10000	41.88 10000	41.91 10000	20.53 10000	20.55 10000	7.91 10000	7.90 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.18 10000	0.18 10000	0.77 10000	0.77 10000	1.71 10000	1.71 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.04 10000	0.04 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.08 10000	0.08 10000	1.83 10000	1.84 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table 10.2: Test against the case I Weibull dist. under Kolmogorov-Smirnov test at 95% sig. lvl.
Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV $\rightarrow$ critical values, F $\rightarrow$ model of critical values

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.14 10000	95.16 10000	94.88 10000	94.91 10000	94.75 10000	94.75 10000	94.99 10000	94.97 10000	94.83 10000	94.89 10000
Weibull(1,1)	100	95.09 10000	95.09 10000	95.12 10000	95.09 10000	94.92 10000	94.90 10000	94.67 10000	94.67 10000	94.83 10000	94.83 10000
Weibull(1,1)	1000	95.19 10000	95.18 10000	95.09 10000	95.06 10000	94.84 10000	94.84 10000	95.06 10000	95.06 10000	95.35 10000	95.36 10000
Loglogistic(1,4)	30	0.05 10000	0.05 10000	2.39 10000	2.39 10000	47.25 10000	47.28 10000	4.95 10000	4.92 10000	13.97 10000	14.03 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.16 10000	0.16 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.36 10000	0.36 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	30.45 10000	30.49 10000	80.41 10000	80.49 10000	64.03 10000	64.04 10000	50.39 10000	50.30 10000	37.76 10000	37.90 10000
ChiSquared1	100	0.81 10000	0.81 10000	52.24 10000	52.15 10000	21.32 10000	21.31 10000	7.81 10000	7.81 10000	2.10 10000	2.10 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.11 10000	0.11 10000	0.27 10000	0.27 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.08 10000	0.08 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table 10.3: Test against the case I Weibull dist. under Anderson-Darling test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted

CV $\rightarrow$ critical values, F $\rightarrow$ model of critical values

KS 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.91 10000	94.91 10000	94.61 10000	94.63 10000	94.63 10000	94.61 10000	94.92 10000	94.91 10000	94.84 10000	94.88 10000
Weibull(1,1)	100	94.97 10000	94.95 10000	95.49 10000	95.48 10000	95.20 10000	95.21 10000	95.01 10000	95.02 10000	95.07 10000	95.02 10000
Weibull(1,1)	1000	95.12 10000	95.16 10000	95.14 10000	95.13 10000	94.61 10000	94.62 10000	95.65 10000	95.65 10000	95.00 10000	95.05 10000
Loglogistic(1,1)	30	42.89 10000	42.90 10000	32.83 10000	32.86 10000	9.42 10000	9.37 10000	2.16 10000	2.16 10000	0.19 10000	0.19 10000
Loglogistic(1,1)	100	1.04 10000	1.04 10000	0.25 10000	0.24 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,1)	30	55.27 10000	55.27 10000	74.64 10000	74.68 10000	80.49 10000	80.45 10000	68.75 10000	68.72 10000	44.57 10000	44.65 10000
Lognormal(0,1)	100	3.50 10000	3.50 10000	34.69 10000	34.55 10000	44.28 10000	44.32 10000	21.26 10000	21.29 10000	2.89 10000	2.88 10000
Lognormal(0,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(1)	30	-	-	0.02 10000	0.02 10000	73.30 10000	73.20 10000	52.20 10000	52.19 10000	6.59 10000	6.62 10000
Pareto(1)	100	-	-	0.00 10000	0.00 10000	23.89 10000	23.93 10000	3.16 10000	3.16 10000	0.00 10000	0.00 10000
Pareto(1)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	57.64 10000	57.64 10000	89.84 10000	89.91 10000	78.98 10000	78.94 10000	67.31 10000	67.24 10000	55.88 10000	55.99 10000
ChiSquared1	100	8.01 10000	8.01 10000	79.79 10000	79.69 10000	42.27 10000	42.29 10000	20.20 10000	20.28 10000	7.93 10000	7.90 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.06 10000	0.06 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.27 10000	0.27 10000	0.73 10000	0.71 10000	1.57 10000	1.57 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.05 10000	0.05 10000	1.91 10000	1.91 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(1)	30	4.43 10000	4.43 10000	20.05 10000	20.09 10000	89.72 10000	89.64 10000	75.97 10000	75.94 10000	10.89 10000	10.97 10000
Rayleigh(1)	100	0.00 10000	0.00 10000	0.03 10000	0.03 10000	57.66 10000	57.70 10000	9.29 10000	9.31 10000	0.00 10000	0.00 10000
Rayleigh(1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(1,1)	30	95.07 10000	95.07 10000	95.28 10000	95.30 10000	94.80 10000	94.80 10000	94.84 10000	94.84 10000	94.65 10000	94.70 10000
Gamma(1,1)	100	94.89 10000	94.89 10000	94.99 10000	94.95 10000	94.94 10000	94.94 10000	94.75 10000	94.77 10000	95.38 10000	95.35 10000
Gamma(1,1)	1000	95.34 10000	95.39 10000	95.08 10000	95.08 10000	95.13 10000	95.16 10000	95.22 10000	95.22 10000	94.85 10000	94.86 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,1)	30	94.92 10000	94.92 10000	94.83 10000	94.86 10000	94.86 10000	94.85 10000	95.13 10000	95.12 10000	95.05 10000	95.07 10000
Mittag-Leffler(1,1)	100	95.16 10000	95.15 10000	95.43 10000	95.39 10000	95.27 10000	95.28 10000	94.78 10000	94.79 10000	94.91 10000	94.90 10000
Mittag-Leffler(1,1)	1000	95.24 10000	95.26 10000	95.35 10000	95.35 10000	94.84 10000	94.84 10000	94.74 10000	94.74 10000	95.04 10000	95.08 10000

Table 10.4: Test against the case I Weibull dist. under Kolmogorov-Smirnov test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV $\rightarrow$ critical values, F $\rightarrow$ model of critical values

10.2 Case II

In case II, both parameters of the distribution which we are testing against are determined from the data. Tables 10.5 and 10.6 display the pass rates which result from testing observations from various distributions against the Weibull distribution for the Kolmogorov-Smirnov and Anderson-Darling (respectively) tests at the 95% significance level. Table 10.1 specifies the distributions; the parameter values that we employed are displayed next to the distribution name in Tables 10.5 and 10.6. The power testing tables for the other distributions and goodness-of-fit tests at the 95% significance level are given in Appendix E.2.

We observe that **the pass rates corresponding to observations drawn from the Weibull distribution were very close to the expected values for all sample sizes, truncation levels and goodness-of-fit tests.** Appendix E.2 shows that the loglogistic, lognormal and Pareto distributions also achieved this feat, thus, we can conclude that we have appropriately selected the critical values. Test statistics which were compared to the results of the Monte Carlo simulations attained very similar pass rates to those which were compared to the models deduced in Chapter 9. This result is true of all distributions, goodness-of-fit tests, sample sizes, truncation levels and significance levels, with a handful of exceptions that do not call into question the validity of the models; one such exception is analysed later in this section. Thus, we can confirm that our models work sufficiently well to be used in place of the critical values. As both parameters were estimated from each sample, it was necessary to use $\hat{\eta}$ in the models to describe the truncation level. Thus, the success of the models also provides evidence that supports the necessarily invoked approximation that $\hat{\eta} \approx \eta^0$. This is an encouraging result, because the approximation is necessary for our modelling framework to be valid.

One of the examples in which the performance of the model appears to be strained, is when the lognormal data is tested against the loglogistic distribution (Figures E.21, E.22, E.23 and E.24). At high truncation levels ($p = 0.8$) the pass rates of test statistics compared to the models are significantly higher than those compared to the results of the Monte Carlo procedure; the discrepancy is most significant for the largest sample size, $n = 1,000$. The number of tested samples is displayed underneath the pass rates, through perusal of these values we notice that the number of samples is far less for the models, and that this discrepancy is largest when the difference in the pass rates is maximal. Samples are not tested against the critical value models when $\hat{p} > 0.9$ (i.e. when the answer to "Is $\hat{p} \leq 0.9$?" is no in Figure 10.1). Therefore, a low number of samples tested against the models, implies that a large proportion of samples yielded $\hat{p} > 0.9$. Therefore, the comparatively high model pass rates can be explained in two ways:

(i) Samples which correspond to $\hat{p} > 0.9$ are also more likely to pass the goodness-of-fit tests.

(ii) The critical value models work extremely well for every goodness-of-fit test, distribution, sample size, truncation level and significance level, except the ones that produce a high proportion of samples for which $\hat{p} > 0.9$.

Both of these options are possible, however, we believe (i) is overwhelmingly likely. As a result, we are confident that our models perform well enough to be used in-place of the results of the Monte Carlo simulations. It is also worth noting that in many of these situations, all of the samples yielded $\hat{p} > 0.9$, and thus no pass rate was recorded. When this occurred, a hyphen was entered into the relevant table.

Because both parameters of the distribution we are testing against are estimated from the data, the 'similarity' to the distribution from which the observations were drawn is optimised. Thus, case II is the most challenging scenario under which the goodness-of-fit tests attempt to distinguish different distributions. As a result, we expect high pass rates and in particular, far higher pass rates than in case I. Because the distributions we are testing against can adapt to the data, the pass rates are far less sensitive to the parameters of the distribution from which the data was taken, however, the pass rates are not completely insensitive to parameter change. Therefore, the pass rates quoted in Tables 10.5 and 10.6 are still specific to the parameter values stated in those tables.

Generally high pass rates are perhaps the most obvious feature of Tables 10.5 and 10.6 as compared to those from case I. If the Weibull shape parameter is set to $\beta = 2$, then the Weibull distribution exactly describes the Rayleigh distribution with $\alpha = \sqrt{2}b$. Thus, it is not surprising that when we are free to estimate the Weibull parameters from the data, observations from the Rayleigh distribution exhibit the same pass rates as the Weibull distribution itself. The Mittag-Leffler distribution is and extension of the exponential distribution. Given that at $\beta = 1$, the Weibull distribution is equivalent to the exponential, it is not surprising that the Mittag-Leffler distribution also produces pass rates comparable to the Weibull. Because these pass rates result from strong similarities between the pdfs, the pass rates are the same for all goodness-of-fit tests.

The Weibull, loglogistic, lognormal and Pareto distributions are indistinguishable for high truncation and small sample sizes (with the exception of lognormal data being tested against the loglogistic distribution). None of the goodness-of-fit tests we studied were able to differentiate any of these distributions from each other, however, as the sample size increased the Anderson-Darling test exhibited a better ability to distinguish the distributions. For larger n, data from the Weibull and lognormal distributions were less frequently confused with the loglogistic distribution, however, an increase in sample size did not have a significant affect on the pass rates for observations from the Pareto distribution. This feature was preserved under all goodness-of-fit tests which leads us to believe that the tail of the loglogistic distribution is more similar to the corresponding region in the Pareto than the Weibull or lognormal distributions.

Oddly, the goodness-of-fit tests could not tell when loglogistic observations were tested against the lognormal distribution, but could when lognormal data was tested against the loglogistic distribution. Kuiper's test (Figure E.6) did a noticeably worse job of distinguishing these distributions than the other three tests which all performed similarly (Figures E.5, E.7 and E.8). We presume that the specific loglogistic parameters cause the pdf to form a shape that the lognormal distribution cannot emulate. At intermediate truncation levels ($p \in [0.1, 0.6]$) the quadratic goodness-of-fit tests (Cramér-von Mises and Anderson-Darling) were better able to distinguish between the two distributions. The χ^2 distributions all had reasonably high pass rates for each of the Weibull, loglogistic and lognormal distributions, with the affect being exacerbated by high truncation. The Weibull distribution was confused with the χ^2 the most frequently. The lognormal distribution was often confused with the Mittag-Leffler and Gamma distributions by all four tests at high truncation levels and small sample sizes.

We reiterate that any comment regarding pass rates and thus distinguishability is specific to the parameter values that are listed in Tables 10.5 and 10.6. In conclusion, all four goodness-of-fit tests performed similarly, with the Anderson-Darling test having slightly higher statistical power.

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.76 10000	94.79 10000	95.20 10000	95.13 10000	94.74 10000	94.70 9906	95.22 9989	95.16 9281	94.21 9907	94.28 7618
Weibull(1,1)	100	95.51 10000	95.56 10000	95.37 10000	95.39 10000	94.73 10000	94.75 10000	94.37 10000	94.43 9903	94.89 9998	94.81 8177
Weibull(1,1)	1000	95.19 10000	95.10 10000	94.51 10000	94.53 10000	94.85 10000	94.83 10000	94.43 10000	94.40 10000	95.18 10000	95.18 9923
Loglogistic(1,4)	30	66.05 10000	66.06 10000	67.14 10000	66.24 10000	81.66 10000	81.69 9959	93.42 9552	94.44 6012	94.44 7289	94.88 2168
Loglogistic(1,4)	100	17.20 10000	17.30 10000	17.53 10000	16.84 10000	43.16 10000	43.20 9999	86.36 9934	90.28 4682	93.67 7177	92.19 256
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	16.69 10000	28.83 1516	90.81 8803	-
Lognormal(0,0.2)	30	76.04 10000	76.06 10000	77.44 10000	77.40 10000	80.95 10000	80.46 10000	93.79 9983	93.75 8998	94.68 8108	95.16 2810
Lognormal(0,0.2)	100	33.92 10000	34.10 10000	33.44 10000	33.60 10000	43.98 10000	43.74 10000	92.43 10000	92.46 9702	94.21 8811	94.77 669
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	66.25 10000	66.21 10000	93.99 9991	-
Pareto(5)	30	-	-	94.43 6539	94.94 1640	93.97 6567	94.62 1654	93.78 6655	94.50 1673	93.76 6603	94.63 1583
Pareto(5)	100	-	-	93.61 5963	92.67 150	94.30 5985	97.62 126	93.29 5920	94.96 119	94.08 6046	93.91 115
Pareto(5)	1000	-	-	93.87 5354	-	93.60 5357	-	93.94 5383	-	93.29 5337	-
ChiSquared1	30	92.32 10000	92.37 10000	95.02 10000	94.97 10000	94.42 10000	94.43 9857	95.15 9995	95.25 9384	94.75 9961	94.97 8384
ChiSquared1	100	86.38 10000	86.48 10000	94.43 10000	94.43 10000	94.61 10000	94.60 10000	94.94 10000	94.91 9948	94.70 10000	94.64 9289
ChiSquared1	1000	15.82 10000	15.76 10000	84.88 10000	84.87 10000	92.97 10000	92.97 10000	93.98 10000	93.96 10000	94.94 10000	94.93 10000
ChiSquared3	30	94.53 10000	94.55 10000	95.07 10000	94.80 10000	94.89 10000	94.83 10000	95.04 10000	94.97 9998	95.03 10000	95.06 9916
ChiSquared3	100	93.08 10000	93.17 10000	94.11 10000	93.88 10000	94.60 10000	94.61 10000	94.74 10000	94.76 10000	94.73 10000	94.74 10000
ChiSquared3	1000	74.19 10000	74.04 10000	81.46 10000	80.92 10000	89.84 10000	89.85 10000	92.72 10000	92.73 10000	93.76 10000	93.76 10000
ChiSquared4	30	93.63 10000	93.64 10000	94.03 10000	93.74 10000	94.61 10000	94.48 10000	94.06 10000	94.03 9999	95.16 10000	95.08 9997
ChiSquared4	100	90.16 10000	90.22 10000	91.27 10000	90.71 10000	92.71 10000	92.73 10000	93.51 10000	93.50 10000	94.47 10000	94.49 10000
ChiSquared4	1000	40.39 10000	40.21 10000	48.81 10000	47.36 10000	71.63 10000	71.66 10000	80.26 10000	80.27 10000	89.56 10000	89.58 10000
Loglaplace(1,1)	30	44.58 10000	44.61 10000	45.58 10000	45.02 10000	51.21 10000	51.21 9973	59.92 9993	61.40 9494	79.64 9536	83.28 5904
Loglaplace(1,1)	100	2.12 10000	2.13 10000	2.63 10000	2.54 10000	4.28 10000	4.29 9992	7.85 10000	8.05 9748	35.48 9899	43.39 4190
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9996	0.00 10000	0.00 627
Rayleigh(4)	30	95.28 10000	95.29 10000	95.25 10000	95.15 10000	95.36 10000	95.13 10000	95.08 10000	94.94 10000	94.54 10000	94.46 10000
Rayleigh(4)	100	94.75 10000	94.82 10000	94.93 10000	94.86 10000	95.13 10000	94.78 10000	95.23 10000	95.12 10000	94.82 10000	94.82 10000
Rayleigh(4)	1000	95.17 10000	95.12 10000	94.88 10000	94.70 10000	95.33 10000	94.97 10000	94.98 10000	94.93 10000	95.20 10000	95.23 10000
Gamma(5,2)	30	89.76 10000	89.76 10000	89.78 10000	89.73 10000	90.47 10000	90.16 10000	90.63 10000	90.10 10000	90.58 10000	90.14 10000
Gamma(5,2)	100	76.45 10000	76.56 10000	75.34 10000	75.38 10000	75.92 10000	75.42 10000	77.64 10000	76.94 10000	80.67 10000	80.03 10000
Gamma(5,2)	1000	0.53 10000	0.50 10000	0.65 10000	0.62 10000	0.70 10000	0.65 10000	1.11 10000	0.98 10000	2.70 10000	2.41 10000
Uniform	30	64.56 10000	64.57 10000	75.60 10000	75.01 10000	83.96 10000	83.89 10000	86.39 10000	86.33 9996	-	-
Uniform	100	11.88 10000	12.04 10000	27.14 10000	26.77 10000	46.08 10000	46.15 10000	56.13 10000	56.23 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	95.14 10000	95.14 10000	95.32 10000	95.16 10000	94.99 10000	94.95 10000	94.89 10000	94.85 10000	95.22 10000	95.17 9992
Mittag-Leffler(1,5)	100	94.83 10000	94.92 10000	95.25 10000	95.18 10000	95.18 10000	95.19 10000	94.96 10000	94.94 10000	95.32 10000	95.33 10000
Mittag-Leffler(1,5)	1000	95.28 10000	95.20 10000	95.02 10000	94.85 10000	95.00 10000	95.02 10000	95.01 10000	94.98 10000	94.97 10000	94.98 10000

Table 10.5: Test against the case II Weibull dist. under Kolmogorov-Smirnov test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV $\rightarrow$ critical values, F $\rightarrow$ model of critical values

AD 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.58 10000	94.67 10000	95.22 10000	95.22 10000	94.88 10000	94.80 9906	95.09 9989	95.05 9281	94.29 9907	94.18 7618
Weibull(1,1)	100	95.03 10000	95.08 10000	95.26 10000	95.26 10000	95.05 10000	95.04 10000	94.29 10000	94.31 9903	94.43 9998	94.40 8177
Weibull(1,1)	1000	94.87 10000	94.81 10000	94.96 10000	94.98 10000	94.98 10000	94.98 10000	94.74 10000	94.74 10000	95.33 10000	95.36 9923
Loglogistic(1,4)	30	53.99 10000	54.23 10000	55.19 10000	54.93 10000	78.58 10000	78.67 9959	93.81 9552	94.78 6012	94.70 7289	95.34 2168
Loglogistic(1,4)	100	6.28 10000	6.29 10000	6.36 10000	6.31 10000	32.49 10000	32.56 9999	85.14 9934	89.06 4682	94.20 7177	93.36 256
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	9.82 10000	18.07 1516	90.74 8803	-
Lognormal(0,0.2)	30	65.86 10000	66.11 10000	67.01 10000	67.11 10000	73.43 10000	73.23 10000	94.47 9983	94.38 8998	94.68 8108	95.05 2810
Lognormal(0,0.2)	100	12.30 10000	12.36 10000	12.10 10000	12.15 10000	22.26 10000	22.05 10000	92.10 10000	92.17 9702	94.77 8811	95.22 669
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	56.83 10000	56.81 10000	94.28 9991	-
Pareto(5)	30	-	-	94.02 6539	93.35 1640	94.43 6567	95.16 1654	94.24 6655	94.74 1673	94.18 6603	95.01 1583
Pareto(5)	100	-	-	94.15 5963	93.33 150	94.52 5985	92.06 126	94.07 5920	95.80 119	94.24 6046	97.39 115
Pareto(5)	1000	-	-	94.02 5354	-	93.52 5357	-	94.17 5383	-	93.91 5337	-
ChiSquared1	30	90.59 10000	90.71 10000	94.56 10000	94.51 10000	94.80 10000	94.78 9857	94.83 9995	94.88 9384	94.91 9961	95.00 8384
ChiSquared1	100	80.20 10000	80.27 10000	93.18 10000	93.19 10000	93.99 10000	93.98 10000	94.68 10000	94.71 9948	95.04 10000	94.98 9289
ChiSquared1	1000	2.63 10000	2.59 10000	77.43 10000	77.50 10000	92.20 10000	92.20 10000	93.56 10000	93.56 10000	94.88 10000	94.88 10000
ChiSquared3	30	94.24 10000	94.34 10000	94.78 10000	94.65 10000	94.81 10000	94.77 10000	94.98 10000	94.93 9998	95.14 10000	95.16 9916
ChiSquared3	100	92.06 10000	92.09 10000	93.14 10000	93.07 10000	94.46 10000	94.47 10000	94.49 10000	94.51 10000	94.86 10000	94.86 10000
ChiSquared3	1000	57.87 10000	57.73 10000	68.77 10000	68.21 10000	86.07 10000	86.13 10000	91.57 10000	91.59 10000	93.21 10000	93.21 10000
ChiSquared4	30	93.46 10000	93.53 10000	93.95 10000	93.86 10000	94.45 10000	94.42 10000	94.43 10000	94.42 9999	94.74 10000	94.72 9997
ChiSquared4	100	86.97 10000	87.04 10000	87.94 10000	87.67 10000	91.24 10000	91.25 10000	92.84 10000	92.87 10000	94.23 10000	94.26 10000
ChiSquared4	1000	15.80 10000	15.69 10000	22.43 10000	21.99 10000	51.53 10000	51.64 10000	70.66 10000	70.77 10000	85.64 10000	85.74 10000
Loglaplace(1,1)	30	35.41 10000	35.51 10000	36.71 10000	36.59 10000	44.54 10000	44.58 9973	56.53 9993	57.90 9494	79.44 9536	83.38 5904
Loglaplace(1,1)	100	0.83 10000	0.83 10000	1.03 10000	1.03 10000	1.93 10000	1.93 9992	4.53 10000	4.66 9748	30.32 9899	37.76 4190
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9996	0.00 10000	0.00 627
Rayleigh(4)	30	95.13 10000	95.17 10000	95.04 10000	95.02 10000	94.93 10000	94.85 10000	94.96 10000	94.92 10000	94.60 10000	94.59 10000
Rayleigh(4)	100	94.82 10000	94.85 10000	94.95 10000	94.93 10000	95.20 10000	95.07 10000	95.00 10000	94.90 10000	94.84 10000	94.84 10000
Rayleigh(4)	1000	95.18 10000	95.10 10000	95.09 10000	94.99 10000	94.93 10000	94.77 10000	94.46 10000	94.45 10000	95.10 10000	95.12 10000
Gamma(5,2)	30	86.79 10000	86.95 10000	86.96 10000	87.14 10000	88.14 10000	88.11 10000	88.25 10000	88.02 10000	89.01 10000	88.90 10000
Gamma(5,2)	100	62.83 10000	62.97 10000	61.24 10000	61.31 10000	62.37 10000	62.16 10000	64.49 10000	64.11 10000	68.79 10000	68.32 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.07 10000	0.06 10000
Uniform	30	44.64 10000	44.91 10000	57.80 10000	57.56 10000	71.91 10000	71.82 10000	77.98 10000	77.92 9996	-	-
Uniform	100	0.83 10000	0.83 10000	4.82 10000	4.77 10000	16.95 10000	17.01 10000	28.18 10000	28.35 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	95.19 10000	95.25 10000	95.63 10000	95.52 10000	95.20 10000	95.20 10000	94.95 10000	94.91 10000	94.94 10000	94.95 9992
Mittag-Leffler(1,5)	100	94.69 10000	94.71 10000	94.99 10000	94.91 10000	94.95 10000	94.98 10000	94.83 10000	94.86 10000	95.09 10000	95.11 10000
Mittag-Leffler(1,5)	1000	95.05 10000	94.99 10000	94.94 10000	94.85 10000	95.41 10000	95.45 10000	95.38 10000	95.39 10000	95.05 10000	95.10 10000

Table 10.6: Test against the case II Weibull dist. under Anderson-Darling test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV → critical values, F → model of critical values

10.3 Case IIIa

In case IIIa the shape parameter of the distribution which we are testing against is determined from the data. Tables 10.7 and 10.8 display the pass rates which result from testing observations from various distributions against the Weibull distribution for the Kolmogorov-Smirnov and Anderson-Darling (respectively) tests at the 95% significance level. Table 10.1 specifies the distributions; the parameter values that we employed are displayed next to the distribution names in Tables 10.7 and 10.8. The power testing tables for the other distributions and goodness-of-fit tests at the 95% significance level are given in Appendix E.3.

We observe that **the pass rates corresponding to observations drawn from the Weibull distribution were very close to the expected values for all sample sizes, truncation levels and goodness-of-fit tests.** Appendix E.3, shows that the loglogistic, lognormal and Pareto distributions also achieved this feat, thus, we can conclude that we have appropriately selected the critical values. Test statistics which were compared to the results of the Monte Carlo simulations attained very similar pass rates to those which were compared to the models deduced in Chapter 9. This result is true of all distributions, goodness-of-fit tests, sample sizes, truncation levels and significance levels, with a handful of exceptions that do not call into question the validity of the models (we will touch on this point again later in this section). Thus, we can confirm that our models work sufficiently well to be used in place of the critical values. As the scale parameter was estimated from each sample, it was necessary to use $\hat{\eta}$ in the models to describe the truncation level. Thus, the success of the models also provides evidence that supports the necessarily invoked approximation that $\hat{\eta} \approx \eta^0$. This is an encouraging result, because the approximation is necessary for our modelling framework to be valid.

When there is a discrepancy between the pass rates for test statistics compared to the results of the Monte Carlo simulations, and the critical value models, the difference occurs because $\hat{p} > 0.9$ for a significant proportion of the samples. Thus, only a subset of the samples is compared the critical value models, this is the subject of detailed discussion section 10.2, hence, we will refrain from repeating the discussion here. For case IIIa $\hat{p} > 0.9$ frequently occurs in the following situations:

- loglogistic, lognormal and Pareto distributions tested against the Weibull distribution (Figures E.29, E.30, E.31 and E.32)
- lognormal and Pareto distributions tested against the loglogistic distribution (Figures E.33, E.34, E.35 and E.36).

It is also worth noting, that in many of these situations, all of the samples yielded $\hat{p} > 0.9$, and thus no pass rate was recorded. When this occurred a hyphen was entered into the relevant table.

For the Pareto distribution, the concept of p is not well defined because it is intrinsically left-truncated, therefore, $\hat{p} < 0.9$ cannot be imposed. This is of no concern, because the critical values were found to be independent of τ_l, hence, we can apply our model for any τ_l value. There were several cases in which the τ_l was so high that there was insufficient probability mass above τ_l for observations to be drawn, in this case a hyphen was entered into tables presented in Appendix E.3. This effect is the result of the computer not being able to discriminate between extremely small numbers and zero, it is purely a shortcoming of the numerics.

Case IIIa has the greatest discriminatory power-of all the cases in which parameters are estimated. This is true for all distributions (for both drawing data from and testing against) and goodness-of-fit tests. Therefore, we conclude that the shape parameter makes the most difference to the shape of the pdf, which is perhaps not surprising given the name. In a similar fashion to the result from case II, Tables 10.7 and 10.8 show that the Mittag-Leffler distribution has pass rates which are comparable to that of the Weibull distribution. This is because at the shape parameter value, $\beta = 1$, the Weibull distribution is equivalent to the exponential distribution of which the Mittag-Leffler distribution is an extension. For all other distributions, pass rates are lower than for cases II and IIIb, even for small n values. As the sample size is increased the pass rates reduce even further. The pass rates for the Rayleigh and loglogistic distribution increase substantially with truncation. All goodness-of-fit tests perform equally well in distinguishing the loglogistic distribution from the Weibull, however, the quadratic tests (Cramér-von Mises and Anderson-Darling) seem to do a better job of determining when observations came from the Rayleigh distribution. There was a great deal of confusion between the χ^2 distributions and the Weibull, this increased with truncation. For low to moderate truncation levels, the Anderson-Darling test performed markedly better at distinguishing these two. As the Anderson-Darling test is most sensitive in the tail region, this leads us to believe that the χ^2 and the Weibull distributions have similar probability density in the tail region.

When observations are compared against the loglogistic distribution, there is a risk of confusing the Weibull and Gamma distributions at high truncation, although this can be mitigated with a sufficient sample size, $n \approx 1,000$. The Anderson-Darling test did a slightly better job of picking up the difference between these distributions, but the performance amongst the four goodness-of-fit tests was very similar. Again, the χ^2 distributions were a potential source of confusion for sufficiently high truncation. The Anderson-Darling test performed far better than the other tests (which all performed similarly) in determining when data came from the χ^2 distributions. It appears as though the loglogistic distribution also has similar probability density in the tail to the χ^2 distributions.

Observations that were compared with the lognormal distribution were very easily distinguished from it. Under the Anderson-Darling test the highest pass rate from another distribution was 37%. This occurred for observations from the Pareto distribution with a sample size of $n = 30$ and truncation of $p = 0.8$, this is a truly remarkable performance. The only other distributions from which pass rates were above 3% were the loglogistic and uniform distributions. Again, these only had appreciable pass rates for $n = 30$, and were insignificant for sample sizes as small as $n = 100$. Against the lognormal distribution, the Kolmogorov-Smirnov and Cramér-von Mises tests performed similarly, and Kuiper's test experienced the lowest distinguishability. However, it must be said that all tests performed very well, with the Anderson-Darling test performing the best.

Samples taken from the Pareto distribution and compared to the Weibull and loglogistic distributions often produced $\hat{p}$ estimates that exceeded 0.9, and thus were not compared to the critical values models. The samples that were only compared to the results of the Monte Carlo procedure produced pass rates of roughly 50% when compared to the loglogistic distribution and 80% when tested against the Weibull distribution (for $n = 30$) for the Kolmogorov-Smirnov, Cramér-von Mises and Anderson-Darling tests. Kuiper's test performed markedly worse, producing pass rates of roughly 85% against the loglogistic distribution and 95% against the Weibull. For observations tested against the Pareto distribution, no pass rates could be recorded for $\tau_l > 1$, hence we only have one truncation to evaluate. The lognormal, loglogistic, χ^2 (1 degree of freedom) and Weibull distributions had pass rates which may lead to some confusion, with the loglogistic being least distinguishable. All goodness-of-fit tests performed similarly well.

We reiterate that any comment regarding pass rates and thus distinguishability is specific to the distributions and parameter values that are listed in Tables 10.7 and 10.8.

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.12 10000	95.08 10000	94.80 10000	94.80 10000	94.84 10000	94.85 10000	95.31 10000	95.33 10000	94.66 10000	94.66 9617
Weibull(1,1)	100	95.12 10000	95.07 10000	95.32 10000	95.29 10000	95.04 10000	95.05 10000	94.91 10000	94.94 10000	94.80 10000	94.83 9995
Weibull(1,1)	1000	94.78 10000	94.79 10000	95.21 10000	95.27 10000	94.93 10000	94.94 10000	94.89 10000	94.93 10000	95.17 10000	95.17 10000
Loglogistic(1,4)	30	0.00 10000	0.00 10000	0.16 10000	0.16 10000	54.49 10000	54.58 10000	91.39 10000	91.10 8233	84.07 10000	65.32 1805
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	7.71 10000	7.74 10000	88.34 10000	88.20 9627	62.60 10000	28.12 729
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	45.48 10000	45.60 10000	0.25 10000	0.00 2
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	76.45 10000	-	95.12 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	36.28 10000	-	94.31 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	90.64 10000	-
Pareto(5)	30	-	-	85.54 10000	37.70 61	85.97 10000	33.90 59	86.29 10000	28.12 64	86.25 10000	40.51 79
Pareto(5)	100	-	-	67.84 10000	0.00 3	68.91 10000	-	68.06 10000	0.00 1	69.07 10000	-
Pareto(5)	1000	-	-	0.66 10000	-	0.47 10000	-	0.72 10000	-	0.62 10000	-
ChiSquared1	30	36.49 10000	36.46 10000	79.27 10000	79.28 10000	90.86 10000	90.86 10000	92.65 10000	92.65 10000	93.99 10000	94.00 9999
ChiSquared1	100	1.17 10000	1.16 10000	41.66 10000	41.60 10000	80.16 10000	80.25 10000	88.03 10000	88.05 10000	92.03 10000	92.03 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	5.41 10000	5.46 10000	31.29 10000	31.29 10000	65.64 10000	65.74 10000
ChiSquared3	30	75.30 10000	75.27 10000	82.42 10000	82.44 10000	88.68 10000	88.67 10000	91.84 10000	91.84 10000	93.92 10000	93.92 10000
ChiSquared3	100	33.30 10000	33.16 10000	47.38 10000	47.32 10000	73.96 10000	73.90 10000	84.18 10000	84.23 10000	90.30 10000	90.30 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.66 10000	0.66 10000	11.47 10000	11.52 10000	45.10 10000	45.09 10000
ChiSquared4	30	41.24 10000	41.19 10000	48.87 10000	48.90 10000	69.67 10000	69.67 10000	80.00 10000	79.99 10000	87.80 10000	87.83 10000
ChiSquared4	100	0.82 10000	0.81 10000	2.83 10000	2.81 10000	21.14 10000	21.12 10000	44.30 10000	44.31 10000	69.32 10000	69.36 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.26 10000	0.26 10000
Loglaplace(1,1)	30	44.18 10000	44.10 10000	46.80 10000	46.80 10000	51.61 10000	51.61 10000	50.10 10000	50.09 10000	42.12 10000	42.12 10000
Loglaplace(1,1)	100	3.20 10000	3.18 10000	5.23 10000	5.21 10000	9.77 10000	9.74 10000	10.07 10000	10.07 10000	4.36 10000	4.37 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	3.08 10000	3.07 10000	5.03 10000	5.03 10000	12.37 10000	12.37 10000	23.38 10000	23.33 10000	40.76 10000	40.79 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	1.31 10000	1.31 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.04 10000	0.04 10000	0.35 10000	0.35 10000	1.92 10000	1.91 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	27.47 10000	27.43 10000	27.56 10000	27.53 10000	27.67 10000	18.73 8060	26.34 10000	-	-	-
Uniform	100	0.04 10000	0.04 10000	0.03 10000	0.03 10000	0.03 10000	0.00 9483	0.07 10000	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-
Mittag-Leffler(1,5)	30	95.40 10000	95.39 10000	95.02 10000	95.04 10000	95.29 10000	95.29 10000	95.00 10000	94.99 10000	95.17 10000	95.16 10000
Mittag-Leffler(1,5)	100	95.06 10000	95.04 10000	95.13 10000	95.11 10000	95.09 10000	95.09 10000	95.35 10000	95.34 10000	95.23 10000	95.25 10000
Mittag-Leffler(1,5)	1000	95.05 10000	95.05 10000	95.21 10000	95.20 10000	94.98 10000	95.00 10000	95.31 10000	95.31 10000	94.77 10000	94.83 10000

Table 10.7: Test against the case IIIa Weibull dist. under Kolmogorov-Smirnov test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV $\rightarrow$ critical values, F $\rightarrow$ model of critical values

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.13 10000	95.13 10000	94.87 10000	94.87 10000	94.74 10000	94.72 10000	95.17 10000	95.21 10000	94.79 10000	94.78 9617
Weibull(1,1)	100	94.82 10000	94.79 10000	95.40 10000	95.39 10000	95.04 10000	95.05 10000	94.93 10000	94.97 10000	94.66 10000	94.70 9995
Weibull(1,1)	1000	94.97 10000	95.04 10000	95.02 10000	95.03 10000	94.65 10000	94.70 10000	94.74 10000	94.76 10000	95.22 10000	95.24 10000
Loglogistic(1,4)	30	0.00 10000	0.00 10000	0.05 10000	0.05 10000	50.19 10000	50.28 10000	90.90 10000	90.28 8233	80.75 10000	59.06 1805
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.69 10000	3.69 10000	86.72 10000	86.53 9627	56.85 10000	21.40 729
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	30.47 10000	30.67 10000	0.04 10000	0.00 2
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	76.38 10000	-	95.41 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	28.49 10000	-	94.51 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	90.16 10000	-
Pareto(5)	30	-	-	82.54 10000	37.70 61	83.03 10000	23.73 59	83.80 10000	28.12 64	83.46 10000	36.71 79
Pareto(5)	100	-	-	62.96 10000	0.00 3	62.56 10000	-	61.91 10000	0.00 1	62.83 10000	-
Pareto(5)	1000	-	-	0.19 10000	-	0.11 10000	-	0.26 10000	-	0.16 10000	-
ChiSquared1	30	15.61 10000	15.60 10000	71.76 10000	71.75 10000	88.40 10000	88.38 10000	91.14 10000	91.14 10000	93.31 10000	93.31 9999
ChiSquared1	100	0.08 10000	0.08 10000	30.08 10000	30.06 10000	75.05 10000	75.13 10000	85.35 10000	85.38 10000	90.91 10000	90.97 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	2.39 10000	2.42 10000	22.28 10000	22.31 10000	59.33 10000	59.41 10000
ChiSquared3	30	72.85 10000	72.80 10000	81.35 10000	81.37 10000	89.31 10000	89.31 10000	92.61 10000	92.56 10000	94.32 10000	94.30 10000
ChiSquared3	100	20.70 10000	20.60 10000	37.22 10000	37.17 10000	70.72 10000	70.67 10000	82.51 10000	82.59 10000	89.95 10000	89.95 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.18 10000	0.18 10000	6.02 10000	6.06 10000	36.87 10000	36.89 10000
ChiSquared4	30	32.81 10000	32.79 10000	41.45 10000	41.49 10000	67.80 10000	67.75 10000	79.42 10000	79.35 10000	88.29 10000	88.26 10000
ChiSquared4	100	0.07 10000	0.07 10000	0.74 10000	0.74 10000	12.89 10000	12.88 10000	35.96 10000	36.03 10000	65.79 10000	65.86 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000
Loglaplace(1,1)	30	38.24 10000	38.21 10000	42.11 10000	42.17 10000	49.72 10000	49.65 10000	49.93 10000	49.86 10000	40.52 10000	40.53 10000
Loglaplace(1,1)	100	0.59 10000	0.57 10000	1.79 10000	1.78 10000	5.72 10000	5.71 10000	8.15 10000	8.15 10000	3.75 10000	3.75 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.76 10000	0.75 10000	1.56 10000	1.56 10000	6.46 10000	6.43 10000	15.70 10000	15.62 10000	34.54 10000	34.47 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.32 10000	0.32 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.52 10000	0.52 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	15.11 10000	15.11 10000	15.31 10000	15.26 10000	15.65 10000	5.97 8060	15.16 10000	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9483	0.01 10000	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-
Mittag-Leffler(1,5)	30	95.17 10000	95.17 10000	95.02 10000	95.04 10000	94.89 10000	94.89 10000	94.83 10000	94.82 10000	95.06 10000	95.05 10000
Mittag-Leffler(1,5)	100	95.29 10000	95.28 10000	94.74 10000	94.72 10000	94.94 10000	94.93 10000	95.44 10000	95.41 10000	94.90 10000	94.91 10000
Mittag-Leffler(1,5)	1000	94.96 10000	94.99 10000	95.18 10000	95.21 10000	95.15 10000	95.18 10000	95.65 10000	95.63 10000	94.93 10000	94.95 10000

Table 10.8: Test against the case IIIa Weibull dist. under Anderson-Darling test at 95% sig. lvl.
Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV → critical values, F → model of critical values

10.4 Case IIIb

In case IIIb, the scale parameter of the distribution which we are testing against is determined from the data. Tables 10.9 and 10.10 display the pass rates which result from testing observations from various distributions against the Weibull distribution for the Kolmogorov-Smirnov and Anderson-Darling (respectively) tests at the 95% significance level. Table 10.1 specifies the distributions; the parameter values that we employed are displayed next to the distribution names in Tables 10.9 and 10.10. The power testing tables for the other distributions and goodness-of-fit tests at the 95% significance level are given in Appendix E.4.

We observe that **the pass rates corresponding to observations drawn from the Weibull distribution were very close to the expected values for all sample sizes, truncation levels and goodness-of-fit tests.** Appendix E.4 shows that the loglogistic, lognormal and Pareto distributions also achieved this feat, thus, we can conclude that we have appropriately selected the critical values. Test statistics which were compared to the results of the Monte Carlo simulations attained very similar pass rates to those which were compared to the models deduced in Chapter 9. This result is true of all distributions, goodness-of-fit tests, sample sizes, truncation levels and significance levels, with a handful of exceptions that do not call into question the validity of the models (as was discussed in section 10.2). Thus, we can confirm that our models work sufficiently well to be used in place of the critical values. As the shape parameter was estimated from each sample, it was necessary to use $\hat{\eta}$ in the models to describe the truncation level. Thus, the success of the models also provides evidence that supports the necessarily invoked approximation that $\hat{\eta} \approx \eta^0$. This is an encouraging result, because the approximation is necessary for our modelling framework to be valid.

For case IIIb, $\hat{p} > 0.9$ frequently occurs when the observations are drawn from the Pareto distribution and compared to Weibull. It is also worth noting that observations from the lognormal distribution all of the samples yielded $\hat{p} > 0.9$ when tested against the Weibull distribution. Thus no pass rate was recorded and a hyphen was entered into Tables E.45, E.46, E.47 and E.48. Hyphens were entered in the tables for data tested against the Pareto distribution when τ_l was too large, and thus, there was insufficient probability mass above τ_l for observations to be drawn from that region. This point is the subject of a more detailed discussion in section 10.3.

Tables 10.9 and 10.10 show that only observations from the Rayleigh, Gamma and uniform distributions consistently produced negligible pass rates. There was some chance of confusion for all other distributions that increased for small sample sizes. Loglogistic data were most likely to be misconstrued as Weibull in a moderate truncation range $p \approx 0.4$. For small sample sizes the χ^2 (1 degree of freedom) produced pass rates approaching 95% at truncation levels $p > 0.1$ and for all goodness-of-fit tests. Overall the tests performed very similarly except that Kuiper's test struggled to differentiate loglogistic observations from Weibull ones at high truncation levels. All of the tests found it very difficult to distinguish between the Weibull, lognormal and loglogistic distributions for high truncation.

Observations tested against the loglogistic distribution on the whole performed very similarly to those tested against Weibull. The most obvious exception is that for high truncation levels the Pareto observations were far more likely to be confused with the loglogistic than the Weibull distribution. This feature appears to be insensitive to goodness-of-fit test. For low truncation the quadratic tests were far better at distinguishing between χ^2 (1 degree of freedom) data and the loglogistic or lognormal distributions. Pareto observations were confused with the lognormal distribution slightly more than they were with the Weibull, but still far less than the loglogistic distribution. On the whole the tests performed very similarly with the Anderson-Darling having a slight discrimination advantage.

We reiterate that any comment regarding pass rates and thus distinguishability is specific to the distributions and parameter values that are listed in Tables 10.9 and 10.10.

KS 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.78 10000	94.79 10000	94.47 10000	94.40 10000	94.97 10000	94.97 10000	95.36 10000	95.38 10000	94.41 10000	94.49 10000
Weibull(1,1)	100	95.01 10000	95.01 10000	95.09 10000	95.06 10000	95.05 10000	95.05 10000	95.09 10000	95.06 10000	95.07 10000	95.07 10000
Weibull(1,1)	1000	95.12 10000	95.10 10000	95.31 10000	95.25 10000	94.90 10000	94.93 10000	94.94 10000	94.89 10000	95.23 10000	95.23 10000
Loglogistic(1,4)	30	33.33 10000	33.37 10000	37.65 10000	38.02 10000	91.12 10000	91.10 10000	79.13 10000	79.18 10000	74.63 10000	74.31 9811
Loglogistic(1,4)	100	0.48 10000	0.47 10000	0.57 10000	0.58 10000	73.07 10000	73.21 10000	42.17 10000	41.99 10000	34.25 10000	34.22 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	39.91 10000	39.95 10000	41.14 10000	41.28 10000	59.09 10000	59.01 10000	93.15 10000	93.16 10000	93.23 10000	-
Lognormal(0,0.2)	100	0.87 10000	0.85 10000	0.78 10000	0.77 10000	3.79 10000	3.85 10000	87.77 10000	87.71 10000	88.70 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	25.54 10000	25.25 10000	42.50 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	31.44 10000	31.46 10000	72.91 10000	68.21 7539
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.49 10000	0.48 10000	31.37 10000	28.74 9236
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	91.73 10000	91.75 10000	93.88 10000	93.78 10000	95.08 10000	95.05 10000	94.60 10000	94.62 10000	94.78 10000	94.86 10000
ChiSquared1	100	79.53 10000	79.44 10000	92.92 10000	92.83 10000	94.79 10000	94.77 10000	94.34 10000	94.28 10000	94.12 10000	94.12 10000
ChiSquared1	1000	0.64 10000	0.61 10000	73.65 10000	73.52 10000	92.58 10000	92.57 10000	89.80 10000	89.67 10000	88.75 10000	88.74 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.82 10000	0.82 10000	29.85 10000	29.87 10000	56.66 10000	56.73 10000	77.79 10000	78.05 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.20 10000	0.20 10000	8.33 10000	8.24 10000	39.65 10000	39.61 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.31 10000	3.34 10000	20.95 10000	21.04 10000	55.07 10000	55.31 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	6.23 10000	6.23 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.18 10000	0.18 10000	23.08 10000	23.13 10000	60.25 10000	60.33 10000	79.74 10000	79.83 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.17 10000	0.17 10000	13.89 10000	13.77 10000	47.18 10000	47.16 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.39 10000	0.40 10000	5.95 10000	6.02 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	1.95 10000	1.95 10000	24.18 10000	24.24 10000	43.38 10000	43.49 10000	63.38 10000	63.60 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.07 10000	0.07 10000	1.68 10000	1.66 10000	13.58 10000	13.57 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table 10.9: Test against the case IIIb Weibull dist. under Kolmogorov-Smirnov test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV $\rightarrow$ critical values, F $\rightarrow$ model of critical values

AD 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.94 10000	95.07 10000	95.08 10000	94.94 10000	94.96 10000	95.04 10000	95.21 10000	95.35 10000	94.59 10000	94.77 10000
Weibull(1,1)	100	95.15 10000	95.12 10000	94.94 10000	94.91 10000	94.88 10000	94.94 10000	95.03 10000	94.95 10000	94.70 10000	94.69 10000
Weibull(1,1)	1000	95.02 10000	94.96 10000	95.19 10000	95.14 10000	94.78 10000	94.80 10000	94.80 10000	94.71 10000	95.15 10000	95.03 10000
Loglogistic(1,4)	30	25.49 10000	25.77 10000	30.47 10000	31.11 10000	91.60 10000	91.65 10000	75.99 10000	76.42 10000	68.99 10000	69.08 9811
Loglogistic(1,4)	100	0.15 10000	0.15 10000	0.21 10000	0.23 10000	66.95 10000	67.24 10000	37.61 10000	37.47 10000	27.51 10000	27.48 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	27.00 10000	27.46 10000	27.91 10000	28.40 10000	51.90 10000	51.58 10000	92.70 10000	92.86 10000	91.51 10000	-
Lognormal(0,0.2)	100	0.22 10000	0.22 10000	0.13 10000	0.12 10000	1.35 10000	1.37 10000	86.41 10000	86.40 10000	87.15 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	20.37 10000	20.19 10000	34.57 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	24.39 10000	24.78 10000	67.20 10000	62.06 7539
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.18 10000	0.17 10000	24.86 10000	22.41 9236
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	89.23 10000	89.36 10000	93.77 10000	93.51 10000	95.39 10000	95.53 10000	94.97 10000	95.19 10000	94.83 10000	95.08 10000
ChiSquared1	100	62.52 10000	62.30 10000	91.90 10000	91.76 10000	94.85 10000	94.87 10000	94.51 10000	94.45 10000	94.79 10000	94.69 10000
ChiSquared1	1000	0.00 10000	0.00 10000	67.77 10000	67.72 10000	92.41 10000	92.36 10000	89.08 10000	88.95 10000	87.72 10000	87.50 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.33 10000	0.33 10000	24.80 10000	25.35 10000	53.09 10000	53.79 10000	77.02 10000	77.51 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	4.58 10000	4.54 10000	33.19 10000	33.10 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	1.83 10000	1.91 10000	15.09 10000	15.59 10000	51.07 10000	51.86 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	2.76 10000	2.75 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.14 10000	0.16 10000	21.07 10000	21.56 10000	59.69 10000	60.42 10000	80.55 10000	80.99 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	6.68 10000	6.61 10000	42.50 10000	42.39 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.10 10000	0.10 10000	3.17 10000	3.31 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.93 10000	0.94 10000	18.84 10000	19.45 10000	38.70 10000	39.46 10000	61.23 10000	61.97 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.65 10000	0.65 10000	8.23 10000	8.12 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table 10.10: Test against the case IIIb Weibull dist. under Anderson-Darling test at 95% sig. lvl. Sample number is displayed below pass rate, a hyphen implies the test was not conducted
CV → critical values, F → model of critical values

10.5 Summary

For case I, the Anderson-Darling test was superior to the other three tests for $n = 30$, however, for $n = 100$, all the tests performed very well. The statistical power of the tests takes the following order (1) Anderson-Darling, (2) Kolmogorov-Smirnov, (3) Cramér-von Mises, (4) Kuiper. In case II, the power of the tests are reduced by the fact that both parameters of the distribution we are testing against are estimated from the data. The power of all tests is weak, and decreases as the truncation level is increased. This because most of the distributions can take similar forms in the tail regions. Overall the statistical power ranking is as follows (1) Anderson-Darling, (2) Cramér-von Mises, (3) Kuiper, (4) Kolmogorov-Smirnov. In case IIIa the statistical powers are the greater than for any of the other cases in which parameters are estimated from the data. The power increases with sample size and is not heavily related to truncation level. The powers of the tests can be ranked as follows (1) Anderson-Darling, (2) Kolmogorov-Smirnov, (3) Cramér-von Mises, (4) Kuiper. Case IIIb, saw the Anderson-Darling test and Cramér-von Mises outperform the Kolmogorov-Smirnov and Kuiper tests. The ranking is (1) Anderson-Darling, (2) Cramér-von Mises, (3) Kolmogorov-Smirnov, (4) Kuiper. In conclusion we found that the Anderson-Darling test had the highest statistical power of all the goodness-of-fit tests for all cases of parameter estimation.

Chapter 11

Applications

In Chapter 8 we presented the critical values that were determined via Monte Carlo methods for the cases, distributions, goodness-of-fit tests and significance levels listed below. We did not assess case II or case IIIb for the Pareto distribution as it has no shape parameter, however, every other permutation of the listed options was studied. For each of these permutations, the sample size, n, and the probability of truncation, p took the values specified by Eq. (11.1). We then modelled the critical values as a function of sample size (n) and truncation level (η) in Chapter 9. Modelling is necessary because the results of the Monte Carlo procedure are only valid for particular n and η values, and analysts require a set of critical values that can be applied in the vast majority of cases they encounter. This requirement means that the additional flexibility afforded by a model, as opposed to a table, of critical values is required. To verify that these models performed sufficiently well, we conducted a rigorous array of power tests in Chapter 10. An additional benefit of the power testing procedure was that we were able to verify the ability of the goodness-of-fit tests (Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling) to distinguish between different distributions. In practice, drawing a distinction between competing models is one of the main reasons that goodness-of-fit testing is employed. In this chapter, we apply the critical value models to the analysis of the arrival time differences between executed orders for a range of stocks on the London Stock Exchange (LSE). We compare the pass rates to those determined from the power testing procedure in order to develop a better understanding of the underlying probability distribution of the data.

Cases		Distributions:	Goodness-of-fit Tests	Significance Levels
• Case I:	scale: known shape: known	• Weibull	• Kolmogorov-Smirnov	• 85%
• Case II:	scale: acquired shape: acquired	• Loglogistic	• Kuiper	• 90%
• Case IIIa:	scale: known shape: acquired	• Lognormal	• Cramér-von Mises	• 95%
• Case IIIb:	scale: acquired shape: known	• Pareto	• Anderson-Darling	• 99%

$$\begin{aligned} &n \in \{30, 50, 100, 200, 500, 1000, 10000\} \\ &p \in \{0, 0.0323, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.8605, 0.9\} \end{aligned} \tag{11.1}$$

11.1 Background

Stock markets allow people to exchange money for part-ownership of listed companies, i.e. buy stocks; equally, those who hold stocks are able to sell them. In order to buy or sell on the exchange one must first submit an order, of which there are two main types (i) limit orders (LO) and (ii) market orders (MO) [33].

(i) **Limit orders** are only executed at a specified price. If one wishes to sell stock at a desired price A, we call A the "ask price", if one intends to buy stock at a price B, we call B the "bid price". The lowest such A is called the "Best Ask" and equivalently, the highest such B is called the "Best Bid". For this type of order to be executed, one must wait until someone else is willing to exchange at the requested price.

(ii) **Market orders** are executed at the best available price. For example, if one submits an MO to buy stock, the transaction will occur at the "Best Ask" at the time the order is submitted. This type of order is executed almost immediately.

Modern stock exchanges manage the flow of orders and transactions electronically via the electronic order book (EOB). The exchange sends all EOB orders to data vendors, such as Reuters, Morning star, Bloomberg, etc. who distribute the orders to traders and financial institutions. Some of the key pieces of information the end users receive for each order are the stock ticker (e.g. BARC for Barclays the financial services company), type of order (bid or ask), limit price, the proposed volume of shares and the arrival time of the order on the EOB. Additionally, the limit order status is included ("addition" for newly arrived orders, "cancellation" for cancelled orders and "modification" for adjusted orders) in combination with a trade indicator stating whether the proposed transaction has taken place[1].

In this work, we focus our attention on the time separation between consecutive market orders. For any trade to occur, a market order must be placed so that a previously submitted limit order can be executed. The most recent price at which a transaction has occurred is commonly called the 'stock price'. Therefore, the frequency with which market orders are submitted is related to the rate of change in the stock price, and as a result, is of great value to financial institutions. Additionally, understanding the usual frequency of market orders allows us to deduce when the market is not behaving 'normally' and take preventative action. This may allow us to predict market drops, or simply let us know when our current financial models are no longer valid.

Left-truncation was applied to this data for a number of reasons. The EOB records the arrival time of orders with microsecond resolution in time, but releases the data with millisecond resolution. This gives rise to some strange phenomena. If we are observing time differences which are on the order of 1ms, the binning of the arrival times significantly affects the distribution of the inter-arrival times. As the truncation level is increased, this affect will become less significant because the ratio of bin width to inter-arrival times decreases. Also, there is a need to remove "zero inflated data", which arises when orders arrive less than a millisecond apart. In the data we have access to, these orders appear to have arrived simultaneously, hence, there is an inflated number of inter-arrival times recorded at zero time separation. Any non-zero τ_l value is sufficient to remove the "zero inflated data", however, failure to do so, has a noticeable affect on the testing procedure. Another reason for employing left-truncation is that there is some time taken for the orders to propagate from the location they were placed to the exchange, and then for the computers which are responsible for administering the EOB to process the order. By using a sufficiently high τ_l, we should be able to remove any potential affect this has on the results. Additionally, previous work by Kizilersü et al. [33] leads us to believe that prevalent high frequency trading may mean that the Weibull distribution does a poor job of describing the arrival of market orders with no, or very little truncation.

[1]The explanation in this paragraph is based heavily upon a discussion in [33]

11.2 Procedure

In this study we had access to all the market orders that were placed on the London Stock Exchange (LSE) between the 1^{st} of June 2010 until the 30^{th} of September 2011 on the following stocks:

- **ABF:** Associated British Foods
- **BARC:** Barclays
- **PUB:** Punch Tavern
- **RIO:** Rio Tinto
- **RR:** Rolls-Royce Holding
- **SSE:** Scottish and Southern Energy Company
- **VOD:** Vodafone Group
- **YELL:** Yell group

The arrival time of the i^{th} MO for a particular stock on a given day is denoted, t_i^{mo}, and is recorded in milliseconds. In our investigation, the 'observations' are the arrival time differences, which are denoted x_i and defined as,

$$x_i = t_i^{mo} - t_{i-1}^{mo} \ . \tag{11.2}$$

After converting our time stamped orders to a set of valid x_i values, we implemented truncation by removing all of the $x_i \leq \tau_l$. The left-truncation limit, τ_l, was varied amongst the values

$$\tau_l = \{0ms, 10ms, 30ms, 50ms, 80ms, 101ms, 200ms, 500ms, 1000ms, 5000ms\} \ . \tag{11.3}$$

The τ_l =0ms truncation level was employed to remove the "zero inflated data" points discussed earlier. In this application, a 'sample' was defined as a set of n consecutive x_i values taken after the truncation had been conducted. For a given sample, we required that all x_i must originate from the same trading day so that there were no irregularities associated with the large gap that occurs overnight and on weekends. The sample sizes we studied were,

$$n = \{30, 50, 100, 200, 500, 1000\} \ . \tag{11.4}$$

We then tried to find a distribution which could best describe these data sets. In this regard the Weibull, loglogistic, lognormal and Pareto distributions were considered. From each of the intervals, the scale and shape parameters for all of the distributions were estimated from the data. Afterwards, we employed the Kolmogorov-Smirnov, Kuiper, Cramér-von Mises and Anderson-Darling goodness-of-fit tests at the 95% significance level to assess the fits. In this procedure, we employed the models of the critical values determined in Chapter 9.

There were a number of samples for each combination of stock, sample size and truncation level. Every such sample either passed or failed each of the four goodness-of-fit tests we investigated in this thesis. The pass rate for a particular combination of variables and goodness-of-fit test was determined by finding the proportion of samples which passed the given test.

11.3 Results

Figures 11.1a and 11.2a display the percentage (pass rates) of samples (size $n = 30$) which passed the Kolmogorov-Smirnov and Anderson-Darling tests-respectively-at the 95% significance level when compared to the case II Weibull distribution. Figures 11.1b-11.2b, 11.1c-11.2c and 11.1d-11.2d display the equivalent plots for the loglogistic, lognormal and Pareto distributions respectively. From left to right the stocks are displayed in order of increasing liquidity, where the number of samples was treated as proxy for liquidity. Each plot features a dotted, red, horizontal line at the 95% pass rate. If the data follows the proposed distribution then we expect the pass rates to reach this level. For the sake of brevity, the pass rates of only two goodness-of-fit tests are shown. Of the four goodness-of-fit tests we studied, the Kolmogorov-Smirnov and Anderson-Darling are the most common supremum and quadratic class tests in the literature, hence, they were selected for display. Concision dictated that the pass rates for only one sample size could be displayed, and $n = 30$ was selected for this role. In Chapter 10 we learnt it is beneficial for the sample size to be large, as it allows the goodness-of-fit tests to more easily distinguish between distributions. However, the scale parameter of the Weibull distribution, α, is proportional to the inverse of the stock activity [33]. This activity varies quite considerably throughout the day, hence, if the sample is taken over a long enough period of time, the distribution may change substantially during the sample. Thus, the sample size must be small enough, such that the activity remains reasonably constant over the entire interval. The scale parameters of the other distributions are also likely to be dependent upon the stock activity. The sample size $n = 30$ is selected as the best compromise between having reliable parameter estimates (and statistical power) and having a constant scale parameter. Figures 11.3a and 11.3b display the pass rates of BARC and YELL against truncation level for the Weibull, loglogistic and lognormal distributions.

Five key conclusions that we can take from these figures are (i) the Weibull distribution has the highest pass rates for all truncation levels, (ii) the pass rates increase with truncation, (iii) the pass rates increase with liquidity, (iv) the pass rates for the Anderson-Darling test are generally lower than for the Kolmogorov-Smirnov test and (v) the Pareto distribution has abysmal pass rates. Possible explanations for these observations are:

(i) **Weibull distribution has the highest pass rates for all truncation levels:** The available data indicate that the left-truncated Weibull distribution describes the observations better than the left-truncated loglogistic, left-truncated lognormal and Pareto distributions.

(ii) **Pass rates increase with truncation:** As discussed previously, the EOB releases the data with millisecond time resolution, hence, the order arrival times are necessarily binned. For low truncation, this can have a significant impact on the distribution of our x_i. Increasing the left-truncation limit, τ_l, causes the bin widths to become small in comparison to the inter-arrival times of the MOs, which allows the data to be more accurately described by the continuous distributions we have employed throughout this thesis.

(iii) **Pass rates increase with liquidity:** For higher liquidity stocks, there are far more financial institutions and private investors involved in placing orders. The net behaviour of a large number of individuals is generally more consistent than that of a small group. Hence, the illiquid stocks have inter-arrival times that are more difficult to model.

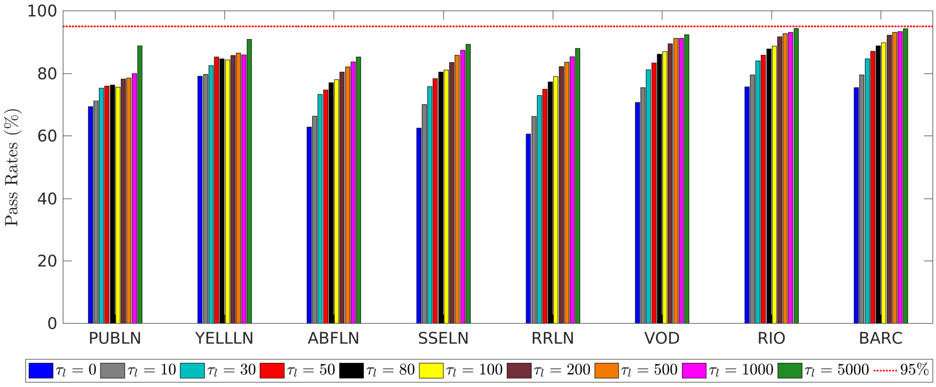
(iv) **Pass rates for Anderson-Darling are generally lower than for Kolmogorov-Smirnov:** The Anderson-Darling test is more sensitive to fluctuations in the tail regions of a distribution, hence, it may be the case that the models do not predict the low probability observations accurately. We do note though that situations with a higher Kolmogorov-Smirnov pass rate are less affected. Thus, the high truncation and liquidity cases seem to predict low probability events sufficiently well.

(v) **Pareto distribution has abysmal pass rates:** It appears the Pareto distribution does not describe the observations accurately for any combination of liquidity and truncation.

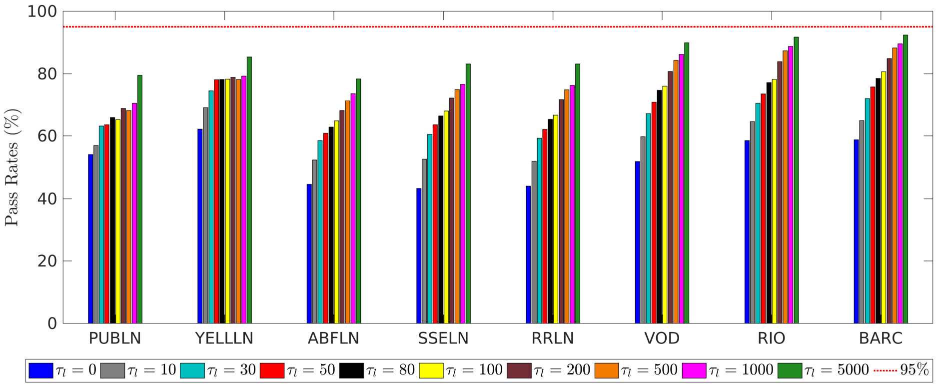
Figures 11.1a and 11.2a show that the most liquid stocks (BARC, RIO and VOD) have pass rates that approach 95% when compared to the Weibull distribution at high truncation levels ($\tau_l \geq 200ms$) under the Kolmogorov-Smirnov and Anderson-Darling tests respectively. For extremely high truncation ($\tau_l \approx 5000ms$) they attain pass rates which could reasonably result from data drawn from a Weibull distribution. The lower liquidity stocks have pass rates that are better than the other distributions we studied, however, they are far from the performance we would expect from observations taken from the Weibull distribution.

The pass rates of the loglogistic distribution are significantly less than the Weibull distribution. The Anderson-Darling pass rates are roughly half of the comparable Kolmogorov-Smirnov ones for lower truncation, as can be seen by comparing Figures 11.1b and 11.2b. There is a large increase in the pass rates for higher liquidity and truncation levels, this implies that majority of the discrepancy between the loglogistic distribution and the observations occurs for lower x_i values. The lognormal distribution has pass rates between the Weibull and loglogistic distributions. Figures 11.1c and 11.2c display that for the highest liquidity and truncation levels, the lognormal pass rates are only marginally worse than the corresponding Weibull ones. However, the lognormal pass rates are significantly more dependent upon truncation levels. Perhaps there is some phenomena occurring in a mid to low x_i range that induces a discrepancy between the observations and the lognormal distribution. As mentioned earlier and depicted in Figures 11.1d and 11.2d, the Pareto pass rates are extremely poor. Based on these pass rates, we can safely say that the Pareto distribution does not adequately describe the time separation of market orders. The comparison of these figures shows that the Weibull distribution clearly models the observations more accurately than any of the other distributions we studied. As a result, the remainder of this chapter will focus on the Weibull distribution.

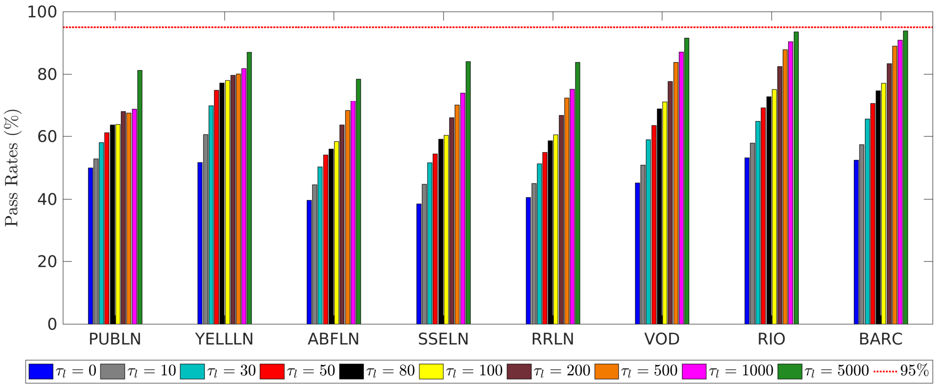
Throughout this chapter, we have elected use the standard deviation as the uncertainty margins because the inter-arrival times have been discretised. This has inevitably increased the uncertainty in all of our calculations. In lieu of a rigorous methodology to account for this, we have used a more conservative estimate of the uncertainty margins than in the earlier parts of this thesis.



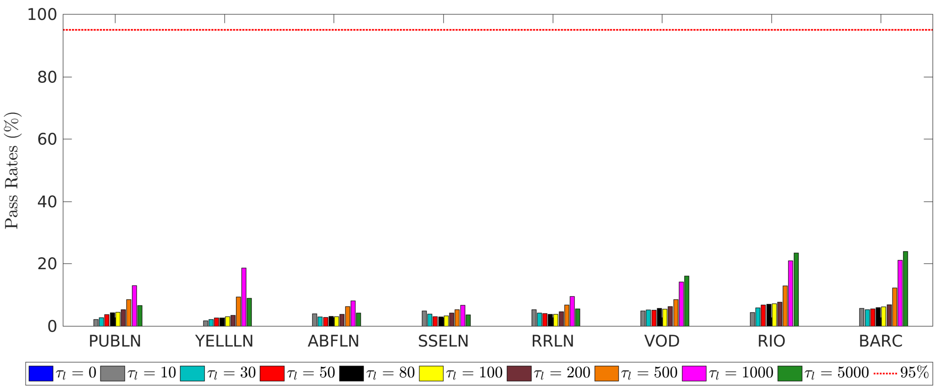
(a) Weibull



(b) Loglogistic

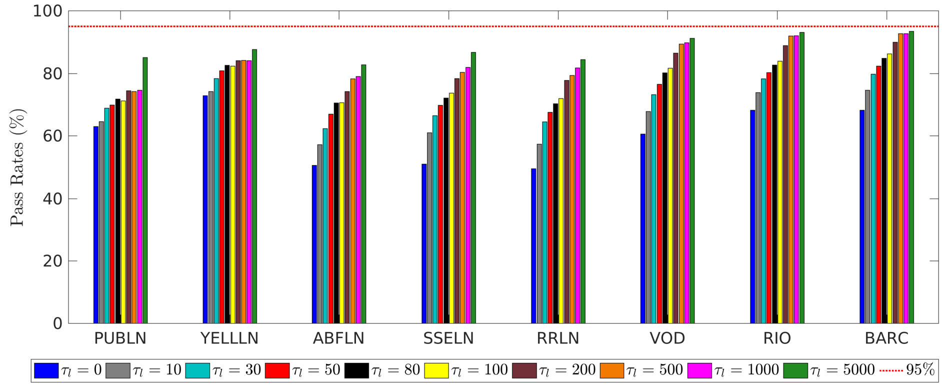


(c) Lognormal

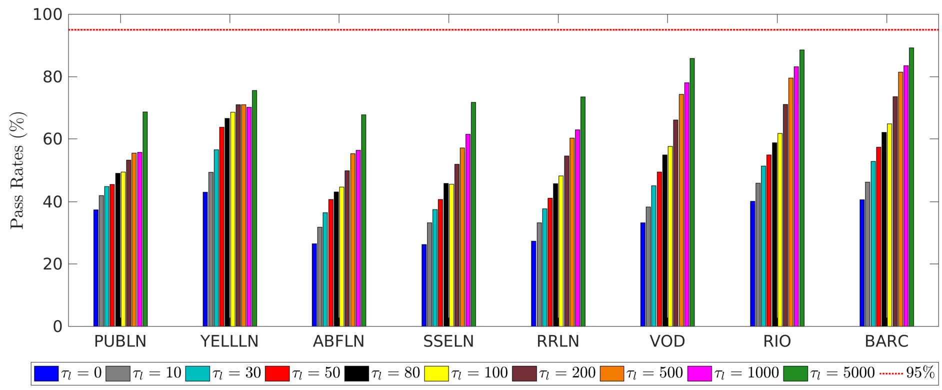


(d) Pareto (case IIIa)

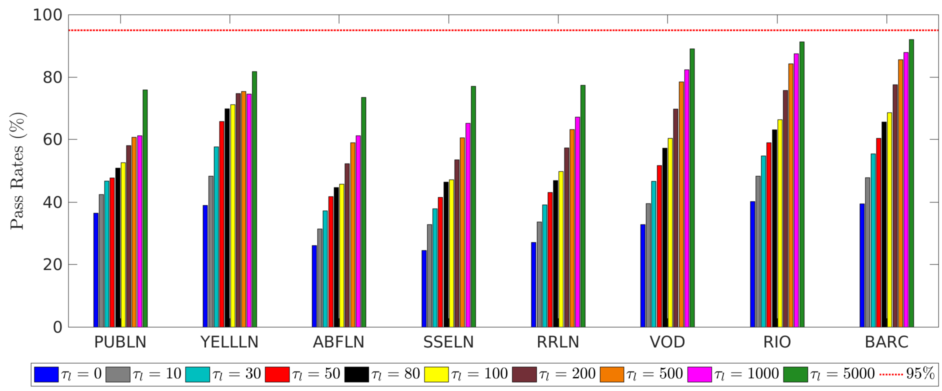
Figure 11.1: Kolmogorov-Smirnov pass-rates at the 95% sig. lvl. for case II samples ($n = 30$)



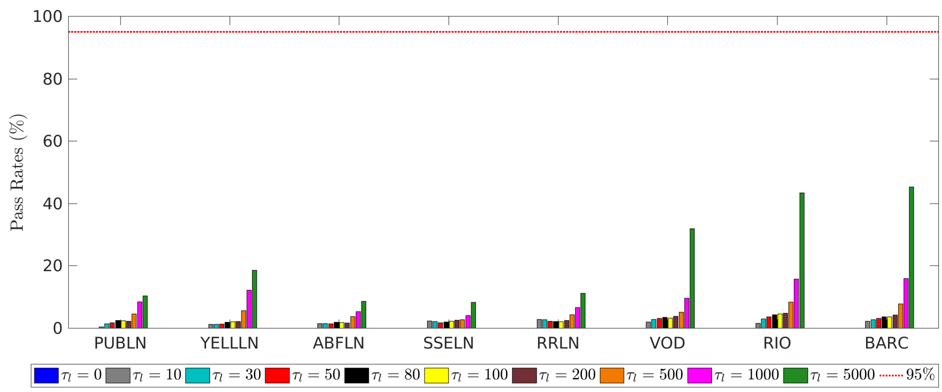
(a) Weibull



(b) Loglogistic

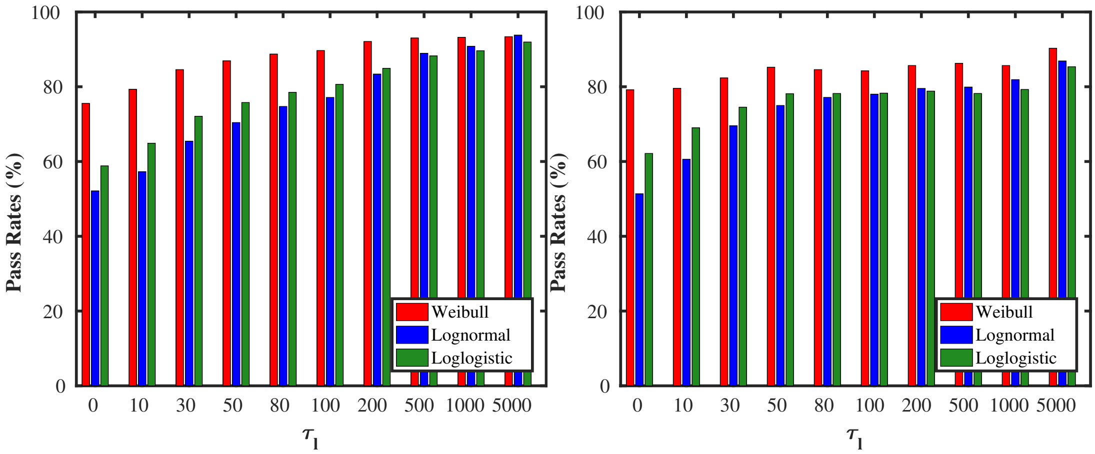


(c) Lognormal



(d) Pareto (case IIIa)

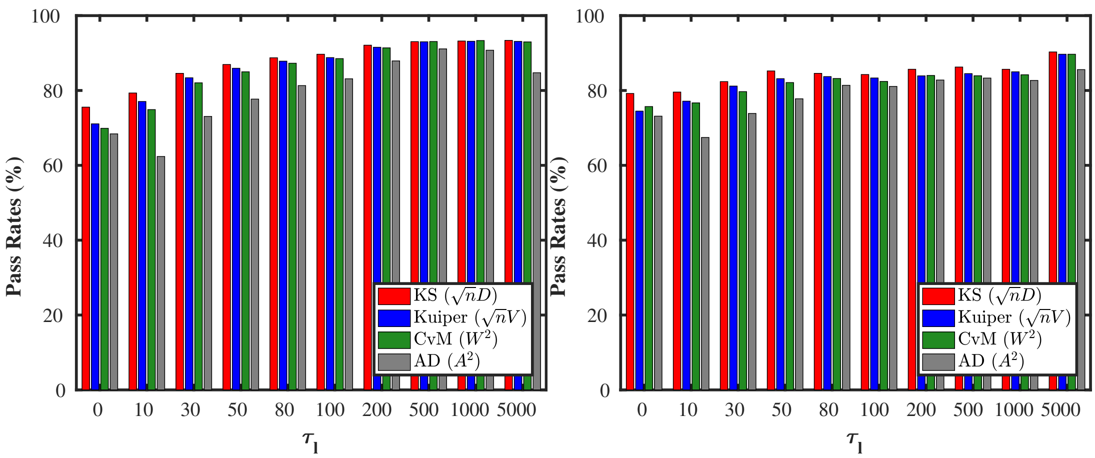
Figure 11.2: Anderson-Darling pass-rates at the 95% sig. lvl. for case II samples ($n = 30$)



(a) BARC (b) YELL

Figure 11.3: Kolmogorov-Smirnov pass rates for different distributions against trunc. lvl. for $n = 30$

Tables 11.1 , 11.2 and 11.3 display the pass rates of observations compared to the case II Weibull distribution for all sample sizes, truncation levels and goodness-of-fit tests at the 95% significance level for YELL (a low liquidity stock), RR (a medium liquidity stock) and BARC (a high liquidity stock) respectively. We have only presented tables for the Weibull distribution because Figures 11.1 and 11.2 showed that it best described the inter-arrival times. The column titled N gives the number of samples for each combination of the sample size and truncation level, $p(\%)$ displays the percentage of time differences that were removed by imposing the left-truncation limit τ_l, and $\hat{p}(\%)$ gives the estimated truncation percentage based upon the parameter estimates. Only n and τ_l combinations which had more than one hundred samples ($N > 100$) are displayed in these tables, so that the affect of statistical errors on the displayed pass rates was minimal. Figures 11.4a and 11.4b display the pass rates when the observations were tested against the Weibull distribution at each truncation level and goodness-of-fit test for BARC and YELL respectively.



(a) BARC (b) YELL

Figure 11.4: Pass rates of different goodness-of-fit tests against trunc. lvl. for Weibull, $n = 30$

For all three stocks, the pass rates increase significantly as sample size decreases, however, recall from Chapter 10 that a smaller sample size reduces a test's ability to distinguish between distributions. As discussed earlier, we have reached a compromise between having a large sample size so that the test has strong statistical power, and having a small sample size so that the scale parameter is constant across the whole interval. The result of this compromise is that we have goodness-of-fit tests with reasonably low statistical power. For both YELL and RR, $\hat{p}$ is restricted to less than roughly 30%, which is still a reasonably low truncation level, therefore, for those stocks we do not need to take into account the decrease in resolving power with increasing truncation. BARC, however, has several $\hat{p}$ values that exceed 50% for $\tau_l = 5,000$ms, therefore, the pass rates which occur for those situations should be given less weight. As it happens, at $\tau_l = 1,000$ms, BARC exhibits comparable pass rates (to those at $\tau_l = 5,000$ms) with $\hat{p}$ estimates capped at 32%, thus, the high pass rates of BARC are more than likely due to strong agreement between the Weibull distribution and the observations.

					Parameter Est.		Pass Rates (%)			
n	τ_l (ms)	N	p (%)	$\hat{p}$ (%)	scale	shape	KS	Kuiper	CvM	AD
30	0	2375	2(3)	0	2(3)e+4	5(1)e-1	79.3	74.5	75.7	73.1
50	0	1408	2(3)	0	2(2)e+4	5(1)e-1	69.2	63.8	65.1	60.6
100	0	681	2(3)	0	2(1)e+4	4(1)e-1	52.0	44.8	47.3	37.7
200	0	310	2(3)	0	2(1)e+4	43(9)e-2	30.3	23.9	26.8	14.5
500	0	101	2(3)	0	14(6)e+3	42(8)e-2	7.9	5.9	5.9	2.0
30	10	2212	8(4)	7(9)	2(3)e+4	5(1)e-1	79.3	77.4	76.4	74.2
50	10	1304	8(4)	7(8)	2(2)e+4	5(1)e-1	71.8	69.1	67.9	64.4
100	10	625	12(8)	6(7)	2(1)e+4	4(1)e-1	55.7	49.4	50.7	45.2
200	10	287	12(8)	6(6)	2(1)e+4	4(1)e-1	34.6	29.7	31.5	25.3
30	30	2087	12(5)	8(9)	2(3)e+4	5(1)e-1	82.5	81.3	79.7	78.4
50	30	1233	11(6)	7(8)	2(3)e+4	5(1)e-1	76.5	74.4	74.2	70.5
100	30	597	14(9)	7(7)	2(1)e+4	5(1)e-1	63.8	59.5	61.3	55.8
200	30	274	14(9)	7(5)	2(1)e+4	5(1)e-1	49.6	43.1	44.5	34.4
30	50	2028	14(7)	7.9(1)	3(3)e+4	5(1)e-1	84.9	83.2	82.0	80.9
50	50	1191	13(7)	8(8)	2(2)e+4	5(1)e-1	78.8	76.4	74.8	73.8
100	50	572	16.6(1)	7(6)	2(1)e+4	5(1)e-1	65.7	63.8	64.7	60.0
200	50	263	16.5(1)	7(5)	2(1)e+4	5(1)e-1	50.2	44.1	47.5	40.2
30	80	1974	15(8)	8.5(1)	3(3)e+4	6(1)e-1	84.5	83.9	83.2	82.7
50	80	1161	15(8)	8(9)	3(2)e+4	5(1)e-1	77.4	75.4	76.1	75.3
100	80	553	13(7)	8(7)	2(2)e+4	5(1)e-1	66.7	64.6	63.1	62.6
200	80	255	13(7)	8(5)	2(1)e+4	5(1)e-1	53.7	50.6	48.2	45.9
30	100	1948	15(8)	9.1(1)	3(3)e+4	6(1)e-1	84.2	83.5	82.4	82.4
50	100	1152	15(8)	9(9)	3(2)e+4	5(1)e-1	78.5	75.6	76.5	76.0
100	100	550	17.0(1)	9(8)	2(2)e+4	5(1)e-1	68.1	65.6	63.0	62.9
200	100	253	16.9(1)	9(6)	2(1)e+4	5(1)e-1	53.4	48.2	47.4	45.4
30	200	1874	18(9)	11.2(1)	3(3)e+4	6(1)e-1	85.7	84.0	83.8	84.1
50	200	1106	17(9)	11.5(1)	3(3)e+4	5(1)e-1	80.4	77.6	78.1	78.7
100	200	528	10(7)	11(9)	3(2)e+4	5(1)e-1	68.9	65.9	64.4	64.9
200	200	242	10(7)	12(7)	2(1)e+4	5(1)e-1	55.4	51.2	50.0	48.8
30	500	1756	20.5(1)	16.4(1)	3(3)e+4	6(2)e-1	86.8	85.0	84.5	84.3
50	500	1034	19.6(1)	16.5(1)	3(3)e+4	6(1)e-1	81.0	79.1	79.0	79.0
100	500	496	22.7(1)	16.8(1)	3(2)e+4	5(1)e-1	70.2	67.5	65.1	65.4
200	500	220	22.5(1)	17.0(1)	3(2)e+4	5(1)e-1	59.1	54.1	54.1	50.0
30	1000	1640	21.2(1)	22.6(1)	3(3)e+4	6(2)e-1	86.0	85.5	84.6	84.2
50	1000	965	20.2(1)	23.0(1)	3(3)e+4	5(1)e-1	82.2	79.1	78.8	78.2
100	1000	461	7.4(1)	23.3(1)	3(2)e+4	5(1)e-1	72.1	68.1	67.0	67.3
200	1000	203	7.2(1)	24.2(1)	2(1)e+4	5(1)e-1	56.7	52.2	51.7	48.2
30	5000	1136	28.5(2)	28.7(1)	5(4)e+4	7(2)e-1	90.8	90.2	90.0	87.6
50	5000	659	28.1(2)	29.2(1)	4(3)e+4	6(1)e-1	89.4	87.6	87.9	84.9
100	5000	308	16.1(1)	30.3(1)	4(2)e+4	6(1)e-1	85.1	85.1	83.5	77.6
200	5000	135	16.0(1)	31.8(1)	4(2)e+4	6(1)e-1	80.7	77.0	77.8	71.8

Table 11.1: Pass rates of YELL tested against the case II Weibull distribution

					Parameter Est.		Pass Rates (%)			
n	τ_l (ms)	N	p (%)	$\hat{p}$ (%)	scale	shape	KS	Kuiper	CvM	AD
30	0	5631	3(4)	0	2(7)e+4	5(1)e-1	60.6	54.1	52.9	49.7
50	0	3353	3(5)	0	2(5)e+4	4(1)e-1	45.4	39.0	37.3	33.5
100	0	1643	3(4)	0	2(2)e+4	4(1)e-1	24.5	19.4	18.4	15.4
200	0	784	3(5)	0	1(1)e+4	40(8)e-2	12.4	7.8	8.4	6.3
500	0	268	3(5)	0	11(7)e+3	38(7)e-2	1.9	0.7	1.5	1.5
30	10	5011	12(7)	8.7(1)	3(8)e+4	5(2)e-1	65.7	63.6	60.2	57.3
50	10	2988	10(7)	8.5(1)	2(6)e+4	5(1)e-1	52.1	48.4	44.5	42.4
100	10	1453	13(9)	8(9)	2(2)e+4	4(1)e-1	32.6	27.9	25.1	24.4
200	10	690	13(9)	8(9)	1(1)e+4	4(1)e-1	15.4	12.0	10.4	13.1
500	10	236	10(8)	9(8)	12(8)e+3	39(9)e-2	4.2	3.0	3.4	0.0
30	30	4690	16(9)	8.4(1)	3(8)e+4	5(2)e-1	72.9	71.1	69.1	64.4
50	30	2792	14(9)	8.1(1)	3(6)e+4	5(1)e-1	61.7	58.0	55.0	50.2
100	30	1360	16.2(1)	8(8)	2(3)e+4	5(1)e-1	42.2	38.6	35.3	32.9
200	30	639	15.8(1)	8(7)	2(1)e+4	5(1)e-1	25.8	20.7	20.3	17.1
500	30	213	14(9)	9(6)	15(9)e+3	4(1)e-1	7.0	5.6	5.6	2.6
30	50	4531	17.8(1)	8.3(1)	3(8)e+4	6(2)e-1	74.8	73.8	71.3	67.4
50	50	2696	15.6(1)	8.2(1)	3(6)e+4	5(1)e-1	65.7	62.9	60.6	55.9
100	50	1315	18.9(1)	8(8)	2(3)e+4	5(1)e-1	48.6	45.2	42.9	39.4
200	50	622	18.4(1)	8(7)	2(1)e+4	5(1)e-1	32.0	27.5	26.4	21.9
500	50	202	15.6(1)	9(6)	2(1)e+4	4(1)e-1	9.4	8.4	7.9	8.2
30	80	4394	19.4(1)	8.6(1)	3(9)e+4	6(2)e-1	77.1	75.8	74.0	70.2
50	80	2607	17.3(1)	8(9)	3(6)e+4	6(1)e-1	69.1	67.3	64.7	59.1
100	80	1270	15.2(1)	8(8)	2(3)e+4	5(1)e-1	54.3	50.5	47.4	42.8
200	80	599	14.6(1)	9(7)	2(1)e+4	5(1)e-1	36.1	32.7	31.1	26.5
500	80	194	17.2(1)	10(6)	2(1)e+4	5(1)e-1	11.3	11.3	10.8	10.2
30	100	4325	19.3(1)	8.7(1)	3(9)e+4	6(2)e-1	78.8	77.7	75.7	71.9
50	100	2568	17.1(1)	8(9)	3(6)e+4	6(1)e-1	70.9	68.2	67.3	61.8
100	100	1249	19.4(1)	9(8)	3(3)e+4	5(1)e-1	54.6	52.4	48.8	43.7
200	100	591	18.9(1)	9(7)	2(1)e+4	5(1)e-1	39.4	35.2	32.8	27.3
500	100	194	17.1(1)	10(6)	2(1)e+4	5(1)e-1	16.5	13.9	11.3	9.2
30	200	4093	21.8(1)	9.5(1)	4(9)e+4	7(2)e-1	82.0	81.0	79.6	77.8
50	200	2435	19.7(1)	9.3(1)	3(6)e+4	6(2)e-1	76.4	72.8	70.9	68.1
100	200	1183	13.2(1)	9(8)	3(3)e+4	6(1)e-1	61.9	58.8	56.9	52.8
200	200	558	12.7(1)	10(8)	2(1)e+4	6(1)e-1	46.8	44.8	42.8	38.4
500	200	183	19.7(1)	12(7)	2(1)e+4	5(1)e-1	25.1	21.9	21.3	16.3
30	500	3793	24.7(1)	12.0(1)	4(9)e+4	7(2)e-1	83.5	82.4	81.4	79.5
50	500	2248	22.9(1)	12.2(1)	4(7)e+4	7(2)e-1	77.6	75.8	75.1	72.8
100	500	1092	24.9(1)	12.6(1)	3(3)e+4	6(1)e-1	67.2	64.0	62.7	61.1
200	500	505	24.5(1)	14.0(1)	3(2)e+4	6(1)e-1	53.2	47.2	46.8	44.0
500	500	161	22.9(1)	16(9)	2(1)e+4	5(1)e-1	28.6	26.1	24.8	25.3
30	1000	3529	24.7(1)	14.8(1)	0(1)e+5	7(2)e-1	85.2	84.4	83.8	81.7
50	1000	2089	23.2(1)	15.1(1)	4(7)e+4	7(2)e-1	79.8	78.2	77.7	74.9
100	1000	1011	9.4(1)	16.0(1)	3(3)e+4	6(1)e-1	71.5	68.3	68.4	64.7
200	1000	469	9.0(1)	17.9(1)	3(2)e+4	6(1)e-1	58.3	54.9	56.0	52.2
500	1000	148	23.2(1)	21.3(1)	2(1)e+4	5(1)e-1	33.1	31.1	27.7	30.0
30	5000	2665	29.7(2)	24.4(2)	1(1)e+5	8(3)e-1	88.2	87.1	87.3	84.6
50	5000	1581	28.9(2)	25.3(1)	5(8)e+4	8(2)e-1	84.3	83.5	83.2	81.1
100	5000	757	18.2(1)	27.6(1)	4(3)e+4	7(2)e-1	79.5	77.1	77.1	72.6
200	5000	350	18.0(1)	31.6(1)	3(2)e+4	6(2)e-1	70.5	68.5	67.1	64.2

Table 11.2: Pass rates of RR tested against the case II Weibull distribution

					Parameter Est.		Pass Rates (%)			
n	τ_l (ms)	N	p (%)	$\hat{p}$ (%)	scale	shape	KS	Kuiper	CvM	AD
30	0	31699	3(4)	0	3(8)e+3	6(1)e-1	75.6	71.1	70.0	68.4
50	0	18993	3(5)	0	3(7)e+3	5(1)e-1	65.0	59.0	57.1	54.0
100	0	9457	3(5)	0	3(6)e+3	5(1)e-1	45.3	39.5	36.4	32.4
200	0	4691	3(5)	0	3(6)e+3	51(9)e-2	24.9	20.4	17.8	14.5
500	0	1833	3(5)	0	3(4)e+3	49(8)e-2	6.1	4.6	4.5	3.0
1000	0	884	3(4)	0	3(2)e+3	48(6)e-2	1.8	1.1	0.9	0.6
30	10	28341	12(6)	9.6(1)	4(8)e+3	6(2)e-1	79.4	77.3	74.9	74.7
50	10	17007	11(6)	9.4(1)	4(8)e+3	6(1)e-1	70.1	67.2	64.0	65.2
100	10	8476	14.2(1)	9(9)	4(7)e+3	5(1)e-1	52.8	47.9	44.8	49.2
200	10	4205	14.0(1)	9(8)	3(6)e+3	5(1)e-1	34.1	29.3	26.8	34.8
500	10	1645	11(6)	9(6)	3(5)e+3	5(1)e-1	12.0	9.4	8.0	16.0
1000	10	782	14.2(1)	9(5)	3(3)e+3	48(9)e-2	2.8	2.3	2.8	3.6
30	30	26125	17(8)	10.1(1)	5(8)e+3	6(2)e-1	84.6	83.5	82.0	79.8
50	30	15666	15(8)	10.0(1)	4(8)e+3	6(1)e-1	79.0	76.2	74.2	71.4
100	30	7798	18.4(1)	10(9)	4(7)e+3	6(1)e-1	65.8	61.8	59.3	56.5
200	30	3861	18.0(1)	10(7)	4(6)e+3	6(1)e-1	46.6	42.0	39.5	39.3
500	30	1498	15(8)	10(6)	4(5)e+3	5(1)e-1	21.0	16.6	15.6	21.1
1000	30	709	18.4(1)	10(5)	3(3)e+3	53(8)e-2	7.5	6.3	5.6	11.1
30	50	24944	19.5(1)	10.7(1)	5(9)e+3	7(2)e-1	87.0	86.1	85.1	82.4
50	50	14943	17.7(1)	10.6(1)	5(8)e+3	6(1)e-1	82.7	80.8	79.6	76.1
100	50	7436	21.4(1)	11(9)	5(7)e+3	6(1)e-1	72.6	69.1	67.1	63.0
200	50	3684	20.9(1)	11(8)	4(7)e+3	6(1)e-1	56.2	52.1	49.3	44.2
500	50	1432	17.5(1)	11(6)	4(5)e+3	6(1)e-1	28.4	24.1	22.8	21.3
1000	50	673	21.3(1)	11(5)	4(3)e+3	56(8)e-2	11.9	10.7	9.8	6.0
30	80	23820	21.9(1)	11.6(1)	5(9)e+3	7(2)e-1	88.8	88.0	87.3	84.9
50	80	14271	19.9(1)	11.5(1)	5(8)e+3	7(1)e-1	85.4	83.9	83.3	80.1
100	80	7103	17.5(1)	11.6(1)	5(8)e+3	6(1)e-1	78.1	75.8	74.2	70.1
200	80	3517	17.0(1)	12(8)	5(7)e+3	6(1)e-1	65.5	61.6	60.3	53.7
500	80	1360	19.7(1)	12(7)	4(5)e+3	6(1)e-1	39.9	35.5	34.2	28.2
1000	80	647	17.4(1)	12(6)	4(3)e+3	58(8)e-2	22.3	19.0	19.0	15.2
30	100	23246	21.5(1)	12.0(1)	5(9)e+3	7(2)e-1	89.7	88.9	88.6	86.3
50	100	13931	19.5(1)	12.1(1)	5(9)e+3	7(1)e-1	86.6	85.4	85.0	81.5
100	100	6921	19.4(1)	12(9)	5(8)e+3	7(1)e-1	80.5	78.5	77.3	72.8
200	100	3424	18.9(1)	12(8)	5(7)e+3	6(1)e-1	69.2	66.3	64.2	58.4
500	100	1329	19.3(1)	13(7)	5(5)e+3	6(1)e-1	46.8	43.1	42.5	38.1
1000	100	633	19.3(1)	13(6)	4(3)e+3	59(8)e-2	26.1	23.7	22.8	20.2
30	200	21317	25.2(1)	13.8(1)	6(9)e+3	8(2)e-1	92.1	91.7	91.4	90.0
50	200	12768	23.2(1)	13.9(1)	6(9)e+3	7(1)e-1	90.7	90.5	89.9	88.2
100	200	6342	14.4(1)	14.1(1)	6(8)e+3	7(1)e-1	88.2	87.1	86.4	83.9
200	200	3143	13.9(1)	14(9)	5(7)e+3	7(1)e-1	82.2	80.5	79.8	75.9
500	200	1212	22.9(1)	15(8)	5(5)e+3	7(1)e-1	69.9	67.6	66.4	62.0
1000	200	568	14.3(1)	15(7)	5(3)e+3	63(9)e-2	54.9	53.1	53.4	45.1
30	500	18578	28.8(1)	19.5(1)	1(1)e+4	8(2)e-1	93.2	93.2	93.2	92.8
50	500	11114	26.8(1)	20.0(1)	7(9)e+3	8(1)e-1	92.6	92.1	91.9	91.4
100	500	5528	21.6(1)	20.5(1)	6(9)e+3	7(1)e-1	90.6	89.9	89.7	88.7
200	500	2726	21.1(1)	21.0(1)	6(7)e+3	7(1)e-1	88.0	86.6	87.1	86.0
500	500	1048	26.6(1)	22(9)	6(5)e+3	7(1)e-1	80.3	79.2	78.9	75.6
1000	500	483	21.5(1)	23(8)	5(3)e+3	65(8)e-2	72.7	71.2	71.2	68.5
30	1000	15987	26.5(2)	27.0(1)	1(1)e+4	8(2)e-1	93.4	93.3	93.5	92.7
50	1000	9577	24.7(2)	28.0(1)	1(1)e+4	8(2)e-1	92.9	92.7	92.7	91.6
100	1000	4755	10.0(1)	28.9(1)	7(9)e+3	8(1)e-1	91.3	90.9	91.1	89.6
200	1000	2344	9.6(1)	29.7(1)	7(8)e+3	7(1)e-1	88.9	88.5	88.6	87.4
500	1000	895	24.6(2)	30.8(1)	6(6)e+3	7(1)e-1	84.4	83.6	83.0	80.9
1000	1000	409	9.9(1)	32.3(1)	5(3)e+3	65(9)e-2	76.7	77.2	74.5	68.9
30	5000	7664	27.5(2)	49.0(2)	1(1)e+4	9(4)e-1	94.6	94.2	94.1	93.6
50	5000	4605	26.8(2)	52.0(2)	1(1)e+4	9(3)e-1	94.0	93.8	93.8	93.2
100	5000	2271	18.9(1)	54.9(2)	1(1)e+4	8(2)e-1	93.2	92.2	93.0	92.1
200	5000	1104	18.7(1)	57.1(1)	1(1)e+4	7(1)e-1	92.3	91.1	91.7	90.8
500	5000	401	26.7(2)	60.8(1)	7(7)e+3	7(1)e-1	86.5	87.0	86.2	83.7
1000	5000	153	18.8(1)	62.2(1)	6(4)e+3	6(1)e-1	84.6	82.6	81.9	81.1

Table 11.3: Pass rates of BARC tested against the case II Weibull distribution

Figures 11.5 and 11.6 display the scale parameter (α) and shape parameter (β) estimates when the samples ($n = 30$) from all stocks were tested against the case II Weibull distribution. As discussed earlier, we have employed a more conservative estimate of the uncertainty margins than in the earlier parts of this thesis. On each plot there is a horizontal dotted blue line which represents the weighted mean and in Figure 11.6 there is a horizontal, dotted red line that represents the Euler-Mascheroni constant. Kizilersü et al. [33] found that the Euler-Mascheroni constant fell within the uncertainty margin for all $\hat{\beta}$ estimates at $n = 30$, with the larger uncertainty margins we employed in this section, we also found that this occured.

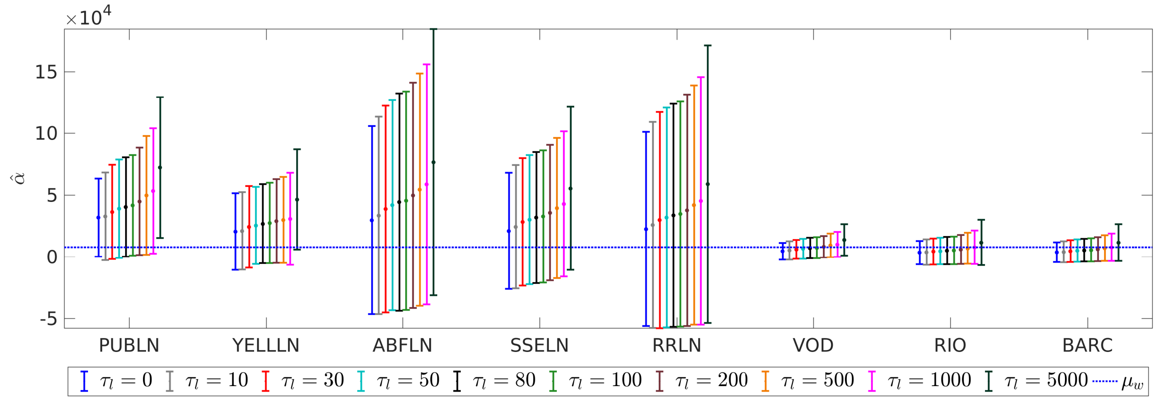


Figure 11.5: Case II Weibull distribution $\hat{\alpha}$ estimates with $n = 30$

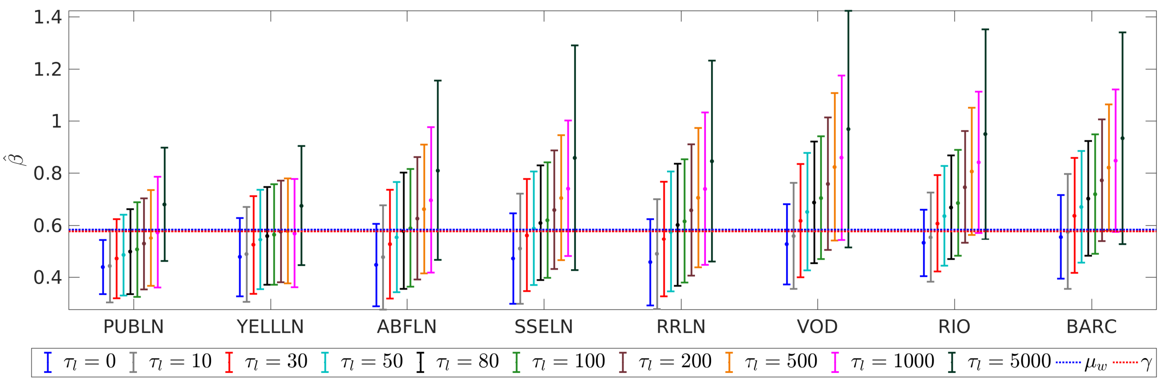


Figure 11.6: Case II Weibull distribution $\hat{\beta}$ estimates with $n = 30$

Chapter 12

Conclusion

Statistical analysis has been a key component of the progress made by those working in industry, the sciences, and clinical disciplines in recent history. Being able to accurately analyse data is becoming increasingly essential for people to thrive in the modern world. The rate of increase in data production is truly mind-boggling, by some accounts 90% of the data created in human history was produced in the last two years [105]. With the development of machine learning, companies are realising that they can harness the information they collect about their customers in order to offer them more targeted services, and thus, increase performance. Part of the reason for the financial success of the tech giants such as Facebook and Google, is that they have access to vast amounts of consumer data, offering them a huge advantage over smaller companies. However, the potential gains are not restricted to corporate entities. In 2017, a group from Stanford University developed a smart phone application that allows users to self-diagnose their skin lesions with comparable accuracy to a dermatologist [106]. There is huge scope for machine vision to automate the analysis of medical images, which has the potential to reduce the cost of medical treatment significantly. Each day, an Airbus A350 XWB collects 2.5 terabytes of data from 50,000 sensors, and Airbus is seeking to use as much of that as possible to make its aeroplanes even safer [107]. All of this investment and development surrounding the collection and use of data is based on the fact that we can analyse it accurately and derive some meaning from the results. Without the correct statistical infrastructure none of the aforementioned advances would be possible.

The use of hypothesis testing is ubiquitous in statistics, and by extension, data analysis. The hypothesis testing procedure typically begins by developing a null hypothesis against which we want to test our observations. In the context of this thesis, the null hypothesis specifies a model distribution from which we suspect the observations were drawn. Selecting an appropriate model is necessary if we want the result of the hypothesis test to answer a particular question. For instance, in Chapter 2 we gave the example of the ATLAS collaboration, which, prior to the discovery of the Higgs Boson in 2012 [43], was hoping to answer the question "Is there a particle which gives mass to fundamental particles?". They employed the null hypothesis that such a particle did not exist, therefore, the distribution of observations corresponding to this null hypothesis was what they expected to see if the particle was not extant.

The next step is to select a goodness-of-fit test which has the statistical power necessary to distinguish between observations drawn from the proposed distribution and another, distinct distribution. Without this feature, it is likely that we will not be able to reject the null hypothesis, even if the observations are drawn from a different distribution. The goodness-of-fit test produces a scalar valued *test statistic* which is compared to a *critical value* to determine whether to reject the null hypothesis. If the test statistic is greater than or equal to the critical value, we can reject the null hypothesis. Therefore, employing the correct critical values is necessary for the hypothesis test to yield correct results. When the parameters are estimated from the data, the relevant critical values are significantly lower. As an example, consider the critical values at the 95% significance level for a sample of size $n = 30$, drawn from complete Weibull distribution. The relevant critical value when no parameters (case I) are estimated from the data is 1.3237 ± 0.0017 and when both parameters (case II) are estimated from the data, the relevant critical value is 0.8581 ± 0.0007. Therefore, if one was to employ the case I critical values in case II, we would not be rejecting the null hypothesis in cases in which we should be. If we are modelling some health or safety related data, such as the amount of carbon monoxide in a mineshaft, this error could be fatal. Finally, we need to understand the probability that a sample from another distribution passes the test. For example, if samples from distribution 'A' pass the test for distribution 'B' at the nearly same rate as those from 'B', then we lack the power to distinguish between the two distributions.

When parameters are estimated from the data, the critical values are dependent upon the distribution which we are testing against. Therefore, the critical values need to be determined independently for every distribution which we want to test against. There are very few studies conducted that have determined the critical values of left-truncated distributions. Hence, there are many such distributions for which critical values are not available in the literature. At present, if an analyst wishes to employ one of these distributions in their null hypothesis, they must determine the critical values themselves. For an expert in the field, this procedure can take months, hence, it is not realistic to expect a novice who might wish to employ such a distribution to conduct the procedure. In practice this means one of two things occur (i) people don't employ left-truncated distributions or (ii) they employ the incorrect critical values. By avoiding left-truncated distributions, people are often shying away from a model which may describe their data more accurately, this can have drastic consequences. If the incorrect critical values are used, the results of the hypothesis testing procedure are not valid. Any decisions based on these results will not have the intended outcome, which can be ruinous if the decisions are of great importance. The main objective of this thesis is to find the previously undetermined critical values for a range of left-truncated distributions (with estimated parameters), and present them in such a way that they are readily available for use by the wider community. This will allow people model their data with a whole family of new functions, thus, enabling them to make more accurate predictions and answer more challenging questions.

In this work we determined the critical values via Monte Carlo simulations for the cases, distributions, goodness-of-fit tests and significance levels listed below. It was not possible to assess case II or case IIIb for the Pareto distribution as it has no shape parameter, however, every other permutation of the listed options has been studied. For each of these permutations, the number of observations in a data set, n, and the probability of truncation, p took the values specified by 12.1.

Cases	Distributions:	Goodness-of-fit Tests	Significance Levels
• Case I: scale: known, shape: known	• Weibull	• Kolmogorov-Smirnov	• 85%
• Case II: scale: acquired, shape: acquired	• Loglogistic	• Kuiper	• 90%
• Case IIIa: scale: known, shape: acquired	• Lognormal	• Cramér-von Mises	• 95%
• Case IIIb: scale: acquired, shape: known	• Pareto	• Anderson-Darling	• 99%

$$n \in \{30, 50, 100, 200, 500, 1000, 10000\}$$
$$p \in \{0, 0.0323, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.8605, 0.9\} \tag{12.1}$$

These critical values allow rigorous statistical tests to take place at the truncation levels (η or p) and sample sizes (n) for which the simulations were conducted. However, data sets from the real world seldom have the specific truncation levels and sample sizes required to employ the critical values we generated. Thus, in order for our results to be useful in the real world, it is necessary to model the critical values as a function of truncation level and sample size. We provided the functional form of the critical values, such that one can access them without the need for excessive tables. Armed with these models of the critical values, we conducted power testing for two main reasons (i) to verify that the models could describe the critical values sufficiently well and (ii) to develop an understanding of the discriminatory power of the goodness-of-fit tests. The second point allows us to understand how frequently we expect observations from other distributions to pass the given tests. Finally, we applied the statistical tests we developed to real world data acquired from the London Stock Exchange (LSE). We analysed the difference in arrival times of successive market orders for a range stocks with the complete set of distributions and goodness-of-fit tests at our disposal. The key findings from each of these procedures is given in the following sections.

12.1 Monte Carlo Results

For each combination of the aforementioned cases, distributions, sample sizes, truncation levels, goodness-of-fit tests and significance levels, we conducted independent Monte Carlo simulations. All such simulations were conducted using the multi-sample method (see section 7.6) with 100 repetitions (i.e. $C = 100$) of $10,000$ samples (i.e. $M = 10,000$) and employed Schafer's method (see section 7.3) of determining the uncertainty margin (at the 95% confidence level). Using this method of uncertainty determination, it was possible to restrict the confidence interval length to 1%. In the literature, critical values are often stated without reference to an uncertainty margin, however, there is some inherent uncertainty associated with result of any Monte Carlo simulation with a finite number of samples. Therefore, an exhaustive discussion was conducted in Chapter 7 in order to determine the most appropriate margins to use. In Chapter 9 we numerically verified that the critical values are parameter independent and discussed the dependence of the critical values upon sample size and truncation level. The key points which are specific to the cases are;

- **Case I, Completely specified distributions:**
  - The critical values are independent of the distribution we are testing against.
  - The critical values of the supremum class tests (Kolmogorov-Smirnov and Kuiper) are sensitive to sample size but not to truncation level.

- The critical values of the quadratic class tests (Cramér-von Mises and the Anderson-Darling) do not exhibit strong sensitivity to sample size and are independent of truncation level.

- **Case II, Both parameters are estimated from the data:**
  - The critical values of the supremum class tests (Kolmogorov-Smirnov and Kuiper) are very sensitive to both sample size and truncation level.
  - The critical values of the quadratic class tests (Cramér-von Mises and the Anderson-Darling) are sensitive to both sample size and truncation level, but vary more strongly with truncation level than sample size.

- **Case IIIa, The scale parameter is estimated from the data:**
  - Regarding the Weibull, loglogistic and Pareto distributions,
    * The critical values of the supremum class tests (Kolmogorov-Smirnov and Kuiper) are sensitive to sample size but not to truncation level.
    * The critical values of the quadratic class tests (Cramér-von Mises and the Anderson-Darling) do not exhibit strong sensitivity to truncation level or sample size.
  - Regarding the lognormal distribution,
    * The critical values of the supremum class tests (Kolmogorov-Smirnov and Kuiper) are strongly dependent upon both sample size and truncation level.
    * The critical values of the quadratic class tests (Cramér-von Mises and the Anderson-Darling) are sensitive to both sample size and truncation level, but vary more strongly with truncation level than sample size.

- **Case IIIb, The shape parameter is estimated from the data:**
  - The critical values of the supremum class tests (Kolmogorov-Smirnov and Kuiper) are not sensitive to both sample size and truncation level.
  - The critical values of the quadratic class tests (Cramér-von Mises and the Anderson-Darling) are not sensitive to sample size but are dependent upon truncation level.

Additionally, we found that the results of our Monte Carlo procedure generally agree with Kizilersü et al. [10] and the available literature on the untruncated distributions.

12.2 Modelling Results

Kizilersü et al. [10] studied the Kolmogorov-Smirnov critical values of the Weibull distribution at the 95% significance level and proposed equations to separately describe the three fitting scenarios (i) η dependence, (ii) n dependence and (iii) η and n dependence. In Chapter 9, we proposed equivalent models for all of the goodness-of-fit tests, distributions, and significance levels which we studied. In some cases, complicated functions with many parameters were required, thus an elaborate fitting procedure was necessary in order to find the optimal parameter values. The sum of squared errors (SSE), coefficient of determination (R^2) and adjusted coefficient of determination (R^2_{adj}) were used to evaluate the resulting fits. The most important findings for each of the three cases are:

(i) η **dependence**

- Kizilersü et al. [10] proposed,

$$D^q_{cv}(\eta|n) = \frac{\theta_1(n)\eta + \theta_2(n)\sqrt{\eta} + \theta_3(n)}{\theta_4\sqrt{\eta} + \theta_5 + \eta} \; , \qquad \text{(9.7 revisited)}$$

to describe the n dependence of the critical values studied in their work. Our analysis showed that it accurately describes the critical values of all the goodness-of-fit tests, distributions, and significance levels we studied.

(ii) n **dependence**

- Kizilersü et al. [10] proposed a fit linear in $\frac{1}{\sqrt{n}}$ to describe the critical values, however, we found that a function quadratic in this variable,

$$D_{cv}(n|\eta) = \theta_1(\eta) + \frac{\theta_2(\eta)}{\sqrt{n}} + \frac{\theta_3(\eta)}{n} \; , \qquad \text{(9.9 revisited)}$$

was required to sufficiently describe the Cramér-von Mises and Anderson-Darling critical values. We note that Kizilersü et al. [10] did not study the Cramér-von Mises or Anderson-Darling tests and that Eq. (9.9) performs equivalently to the linear model for the Kolmogorov-Smirnov and Kuiper critical values.

(iii) η **and** n **dependence**

- Kizilersü et al. [10] proposed a six parameter function to describe the η and n dependence of the critical values, however, we found that this did a poor job predicting the Kolmogorov-Smirnov and Cramér-von Mises critical values for observations drawn from the lognormal distribution. We propose,

$$D_{cv}(\eta, n) = \frac{\theta_1 + \theta_2\sqrt{\eta} + \theta_3\eta}{\theta_4\sqrt{\eta} + \theta_5 + \eta} + \theta_6\sqrt{\frac{\eta}{n}} + \theta_7\eta^{\frac{3}{2}} + \frac{\theta_8}{\sqrt{n}} + \frac{\theta_9}{n} \; , \qquad \text{(9.11 revisited)}$$

Eq. (9.11), which adequately describes the η and n dependence of all the critical values we investigated in this study.

12.3 Power Testing Results

In Chapter 10 we tested data sets (of a range of sample sizes and truncation levels) that were drawn from 12 distributions (defined in Table 10.1) against the Weibull, loglogistic, lognormal and Pareto distributions under the four goodness-of-fit tests employed in this work. We discussed the pass rates for observations drawn from these distributions and compared the performance of the critical value models to that of the Monte Carlo results.

- **Case I, Completely specified distributions:**
  - The pass rates are extremely dependent upon the values of the parameters selected.
  - The power of the goodness-of-tests to discriminate distinct distributions can be ranked as (1) Anderson-Darling, (2) Kolmogorov-Smirnov, (3) Cramér-von Mises, (4) Kuiper.
- **Case II, Both parameters are estimated from the data:**
  - This case had the worst discriminating power as the distribution we are testing against can be adapted to the form which 'best fits' the data.

  - The power of the goodness-of-tests to discriminate distinct distributions can be ranked as (1) Anderson-Darling, (2) Cramér-von Mises, (3) Kuiper, (4) Kolmogorov-Smirnov.

- **Case IIIa, The scale parameter is estimated from the data:**
  - Of the cases which estimated parameters from the data, this case has the highest discriminating power.
  - The power of the goodness-of-tests to discriminate distinct distributions can be ranked as (1) Anderson-Darling, (2) Kolmogorov-Smirnov, (3) Cramér-von Mises, (4) Kuiper.

- **Case IIIb, The shape parameter is estimated from the data:**
  - This case had discriminating power between that of cases IIIa and II.
  - The power of the goodness-of-tests to discriminate distinct distributions can be ranked as (1) Anderson-Darling, (2) Cramér-von Mises, (3) Kolmogorov-Smirnov, (4) Kuiper.

Across all cases it was observed that enlarging the sample size (n) increases the ability of the tests to distinguish distributions from each other. Generally speaking, the rate of confusion between distributions was elevated as the truncation level was increased. It was postulated that this is because the distributions have similar probability densities in the tail region. In some particular cases low or moderate truncation resulted in the most confusion, these cases are discussed in Chapter 10.

When observations were tested against the distributions they were drawn from, the pass rates were very close to the significance level. This was true whether the samples were tested against the critical value models or the results of the Monte Carlo simulations. This is the expected level of performance required of the true critical values, therefore, we concluded that the models accurately described the critical values. We found that all of the goodness-of-tests performed very similarly, however, the Anderson-Darling test was observed to have a marginally greater power to discriminate different distributions and thus is the best test we studied.

12.4 Application Results

The critical value models we devised in Chapter 9 were necessary because we wanted people without a background in statistics to be able to apply them to real world data. In Chapter 11 we used these models to analyse the arrival time differences between market orders (MO) for a range of stocks on the London Stock Exchange. The data we had access to was a set of lists of times at which market orders were placed on a particular day. We found the differences between these times and truncated the result at a range of left-truncation limits, τ_l. The remaining time differences were split it into groups of size n (for a range of n values) and tested against the Weibull, loglogistic, lognormal and Pareto distributions where the parameters were estimated from the data. We used the Kolmogorov-Smirnov and Anderson-Darling tests at the 95% significance level to conduct this testing.

We found that the data passed the goodness-of-fit tests most frequently when it was tested against the Weibull distribution for all sample sizes and truncation levels. Hence, we concluded that it described that time differences more accurately than the loglogistic, lognormal and Pareto distributions. Elevating the truncation level caused the pass rates to increase significantly for all the distributions and goodness-of-fit tests concerned in this study. This implies that the distributions do a better job of explaining the tail of the probability distribution than the region near zero. As the truncation point, τ_l, reached the highest level employed in this study, the Weibull pass rates approached 95% which would be expected for samples taken from the Weibull distribution.

Market orders are executed at the best offer available at the time, and hence are almost immediately followed by a transaction. The alternative to a market order, is a limit order (LO) which executes at a particular price. The distribution of time differences between limit orders is difficult to model near zero because a significant number of orders are placed with the sole intention of manipulating the stock price. These orders are cancelled shortly after they are submitted and are never intended to be executed. This behaviour requires truncation to be employed for an accurate model to be produced [33]. Market orders however, should be subject to far less of this effect because essentially all of the orders are executed. There may be still be a small amount of orders that are designed to manipulate the price, however it should be negligible compared to that experienced by limit orders.

Given that the Weibull distribution was selected as the best fit to the data, we evaluated the parameter estimates that were produced in the testing procedure. Regarding the shape parameter, β, it was found that there is significant overlap of the uncertainty margins for all of the stocks and truncation levels employed at the $n = 30$ sample size. This is in agreement with the findings of Kizilersü et al. [33]. It should be noted that we used larger error margins in this chapter to account for the fact that the data was discretized by the EOB. The scale parameter, α, is inversely related to liquidity, hence, its value varies throughout the trading day. It is necessary to keep the time spanned by any particular sample as short as possible so that the underlying distribution from which the observations are drawn does not change appreciably. This is at odds with the requirements of the goodness-of-fit tests which exhibit maximum discriminatory power for large sample sizes. Therefore, a compromise is necessary so that one can be reasonably sure that distribution is not altered and still have reasonable differentiability. We observed that pass rates increased for small sample sizes, but it is unclear whether this is attributable to the reduced statistical power of the tests, or because distribution is only stable for short periods of time.

12.5 Future Work

Throughout this thesis we have mentioned areas in which we believe that more research is required to further the development of the discipline. Below is a list of the most important improvements that could be made to our work.

- **Critical values as a function of significance level:** In section 2.7 we noted that it appeared as though the test statistics may be described by a skew normal distribution. If this is the case, we could avoid having to specify the critical values at a particular significance level. Additionally, in Chapter 8 we mentioned that future studies should seek to model the critical values as a function of significance level. Even if the test statistics do not follow a skew normal distribution, it is still possible to interpolate the critical values over the frequently used range (85% to 99%). This would increase the complexity of the critical value models, but allow them to be employed more generally.

- **Unbiased quantile estimates:** Section 7.5 points out that one can achieve unbiased estimates of the quantiles by selecting a number of samples N, such that at the significance levels of interest $1 - \alpha = p = \frac{K}{N-1}$, where K is an integer. By employing this strategy, one can simply increase the number of samples C in the multi-sample method and approach the true quantile value. This reduces the computational expense for large numbers of samples as discussed in section 7.6.

- **Minimum critical values at moderate truncation levels:** In section 8.5.1 we noted that the critical values are the smallest (i.e. the test is most difficult to pass) when a moderate truncation level ($\sqrt{\eta} = 0.32$) is employed. This is observed for all case II distributions, however, we are unsure why this should be the case. Hence, it would be interesting to understand the theoretical reason why this occurs.

- **Larger η range:** In specific circumstances the power testing was hampered by the fact that the parameter estimates consistently produced $\hat{p}$ values that exceeded 0.9, and thus could not be compared to the critical value models (see Chapter 10). Future studies could obtain critical values for a larger η range so that more samples could be tested against the critical value models.

- **More data:** As the number of samples increases, the uncertainty margin upon the critical value estimates decreases. This is particularly useful in situations such as the case I quadratic class tests (Cramér-von Mises and Anderson-Darling) where the critical values appeared to have some slight n dependence, however, the uncertainty margins were too large to make a firm statement. Collecting more data may be able to resolve some n dependence in this case (and η dependence in others) and thus, allow more accurate models of the critical values to be produced.

- **Simplifying the parameter selection for critical value models:** As mentioned in section 9.6.1, the procedure we employed to determine the optimal parameters of the critical value models was very complex. Streamlining this procedure may produce more transparent and versatile results. Additionally, if better parameter values result from the new procedure, it may be the case that the models can be simplified.

- **Computational procedure:** Throughout this work we used extensive parallelisation to accelerate the computation of the critical values, however, due to time constraints, we stopped short of conducting the Monte Carlo simulations on a Graphics Processing Unit (GPU). GPUs lend themselves to massively parallelisable problems such as Monte Carlo simulations, hence, future studies should definitely seek to harness this capability.

These improvements will allow analysts of the future to model data ever-more accurately and gain a deeper understanding of the phenomena they study. Improvements in analytic capability seldom go unnoticed and often contribute to growth and development of the world as we know it.

Appendix A

Tables of Critical Values

A.1 Case I: All Parameters are Known

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.1066(11)	1.1146(11)	1.1212(12)	1.1256(10)	1.1307(09)	1.1328(09)	1.1360(12)
0.0323	0.18	1.1063(10)	1.1133(11)	1.1212(09)	1.1250(11)	1.1298(12)	1.1318(10)	1.1368(11)
0.1	0.32	1.1061(09)	1.1140(12)	1.1217(10)	1.1256(09)	1.1302(10)	1.1330(11)	1.1369(09)
0.2	0.47	1.1067(10)	1.1140(12)	1.1201(11)	1.1260(10)	1.1306(11)	1.1331(12)	1.1362(10)
0.3	0.6	1.1071(10)	1.1143(09)	1.1212(11)	1.1252(09)	1.1298(11)	1.1330(10)	1.1364(09)
0.4	0.71	1.1073(10)	1.1134(11)	1.1213(10)	1.1261(11)	1.1305(12)	1.1331(11)	1.1359(08)
0.5	0.83	1.1077(11)	1.1142(12)	1.1217(11)	1.1268(10)	1.1300(10)	1.1336(10)	1.1361(10)
0.6	0.96	1.1075(10)	1.1140(10)	1.1218(11)	1.1255(11)	1.1309(10)	1.1320(10)	1.1362(11)
0.7	1.1	1.1063(10)	1.1148(10)	1.1210(11)	1.1267(09)	1.1292(09)	1.1318(10)	1.1369(11)
0.8	1.27	1.1060(10)	1.1135(10)	1.1212(10)	1.1262(11)	1.1305(10)	1.1318(11)	1.1362(09)
0.8605	1.4	1.1065(08)	1.1128(11)	1.1207(11)	1.1259(11)	1.1305(11)	1.1328(11)	1.1368(10)
0.9	1.52	1.1066(11)	1.1139(10)	1.1210(10)	1.1264(11)	1.1304(10)	1.1331(11)	1.1365(10)

Table A.1: Case I Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.1914(13)	1.2002(12)	1.2065(13)	1.2115(12)	1.2162(12)	1.2182(11)	1.2217(14)
0.0323	0.18	1.1913(12)	1.1991(13)	1.2071(10)	1.2100(14)	1.2149(14)	1.2179(10)	1.2222(13)
0.1	0.32	1.1905(11)	1.1990(13)	1.2074(14)	1.2113(10)	1.2163(11)	1.2188(12)	1.2229(11)
0.2	0.47	1.1921(11)	1.1991(13)	1.2062(12)	1.2117(12)	1.2167(13)	1.2191(14)	1.2216(11)
0.3	0.6	1.1930(13)	1.2003(12)	1.2064(12)	1.2117(12)	1.2155(13)	1.2188(11)	1.2228(11)
0.4	0.71	1.1920(11)	1.1987(13)	1.2070(12)	1.2121(13)	1.2162(14)	1.2185(12)	1.2219(11)
0.5	0.83	1.1925(12)	1.1996(13)	1.2073(13)	1.2118(11)	1.2154(11)	1.2201(11)	1.2226(12)
0.6	0.96	1.1916(12)	1.1997(11)	1.2073(12)	1.2109(12)	1.2166(12)	1.2177(12)	1.2216(13)
0.7	1.1	1.1915(13)	1.2001(13)	1.2068(14)	1.2125(12)	1.2139(11)	1.2179(11)	1.2231(12)
0.8	1.27	1.1907(12)	1.1988(12)	1.2068(13)	1.2118(12)	1.2160(12)	1.2177(13)	1.2219(11)
0.8605	1.4	1.1918(12)	1.1989(13)	1.2064(13)	1.2123(13)	1.2158(12)	1.2192(12)	1.2223(11)
0.9	1.52	1.1910(12)	1.1992(11)	1.2056(12)	1.2119(12)	1.2168(12)	1.2192(12)	1.2220(11)

Table A.2: Case I Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3237(17)	1.3332(14)	1.3405(16)	1.3446(15)	1.3500(16)	1.3520(13)	1.3564(16)
0.0323	0.18	1.3235(15)	1.3316(16)	1.3407(15)	1.3430(18)	1.3492(17)	1.3511(15)	1.3572(17)
0.1	0.32	1.3229(15)	1.3307(17)	1.3414(16)	1.3448(14)	1.3504(15)	1.3530(17)	1.3574(15)
0.2	0.47	1.3243(16)	1.3316(15)	1.3398(17)	1.3450(17)	1.3502(17)	1.3532(18)	1.3559(15)
0.3	0.6	1.3255(15)	1.3331(16)	1.3400(16)	1.3449(14)	1.3488(18)	1.3531(14)	1.3564(15)
0.4	0.71	1.3245(15)	1.3319(16)	1.3400(16)	1.3453(18)	1.3497(17)	1.3525(19)	1.3555(15)
0.5	0.83	1.3239(15)	1.3325(17)	1.3412(17)	1.3457(15)	1.3499(15)	1.3523(15)	1.3558(14)
0.6	0.96	1.3241(16)	1.3321(16)	1.3397(15)	1.3456(15)	1.3499(15)	1.3524(16)	1.3555(15)
0.7	1.1	1.3232(13)	1.3332(16)	1.3417(17)	1.3462(15)	1.3479(16)	1.3508(13)	1.3569(15)
0.8	1.27	1.3228(15)	1.3314(14)	1.3407(19)	1.3453(14)	1.3497(15)	1.3515(17)	1.3563(16)
0.8605	1.4	1.3239(16)	1.3328(16)	1.3402(17)	1.3460(16)	1.3517(15)	1.3529(18)	1.3555(15)
0.9	1.52	1.3228(16)	1.3332(16)	1.3390(17)	1.3448(15)	1.3511(15)	1.3528(17)	1.3554(14)

Table A.3: Case I Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5849(27)	1.5995(31)	1.6087(30)	1.6137(27)	1.6199(30)	1.6222(28)	1.6235(30)
0.0323	0.18	1.5868(27)	1.5990(31)	1.6068(27)	1.6118(30)	1.6201(32)	1.6206(29)	1.6273(30)
0.1	0.32	1.5886(27)	1.5945(31)	1.6030(33)	1.6116(27)	1.6186(30)	1.6193(26)	1.6248(30)
0.2	0.47	1.5874(29)	1.5969(30)	1.6078(27)	1.6120(29)	1.6190(29)	1.6220(34)	1.6262(27)
0.3	0.6	1.5889(30)	1.5970(31)	1.6054(32)	1.6139(26)	1.6184(38)	1.6220(31)	1.6237(32)
0.4	0.71	1.5861(26)	1.5980(30)	1.6077(28)	1.6141(29)	1.6183(29)	1.6187(31)	1.6267(29)
0.5	0.83	1.5880(30)	1.5977(31)	1.6080(37)	1.6117(29)	1.6180(30)	1.6211(28)	1.6245(26)
0.6	0.96	1.5891(31)	1.5972(30)	1.6057(33)	1.6141(27)	1.6185(31)	1.6203(29)	1.6238(31)
0.7	1.1	1.5861(29)	1.5990(30)	1.6073(30)	1.6173(32)	1.6134(33)	1.6188(28)	1.6257(32)
0.8	1.27	1.5835(29)	1.5975(28)	1.6090(29)	1.6111(27)	1.6201(25)	1.6200(31)	1.6261(32)
0.8605	1.4	1.5877(31)	1.5964(30)	1.6069(27)	1.6143(31)	1.6227(32)	1.6215(31)	1.6236(28)
0.9	1.52	1.5844(25)	1.5987(30)	1.6045(29)	1.6119(30)	1.6183(29)	1.6212(29)	1.6217(31)

Table A.4: Case I Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4752(10)	1.4896(10)	1.5033(10)	1.5137(10)	1.5215(10)	1.5263(09)	1.5334(09)
0.0323	0.18	1.4755(09)	1.4898(10)	1.5032(11)	1.5116(11)	1.5219(11)	1.5269(10)	1.5332(10)
0.1	0.32	1.4748(10)	1.4895(10)	1.5036(09)	1.5126(09)	1.5213(09)	1.5261(10)	1.5339(10)
0.2	0.47	1.4753(11)	1.4904(10)	1.5027(10)	1.5131(09)	1.5215(10)	1.5270(10)	1.5331(10)
0.3	0.6	1.4754(09)	1.4900(09)	1.5032(11)	1.5125(11)	1.5216(10)	1.5266(09)	1.5337(10)
0.4	0.71	1.4759(11)	1.4890(10)	1.5032(10)	1.5129(11)	1.5216(12)	1.5257(11)	1.5331(10)
0.5	0.83	1.4754(10)	1.4905(10)	1.5028(10)	1.5138(11)	1.5213(10)	1.5271(10)	1.5335(09)
0.6	0.96	1.4750(10)	1.4884(10)	1.5031(11)	1.5121(10)	1.5226(12)	1.5262(10)	1.5329(08)
0.7	1.1	1.4750(11)	1.4903(10)	1.5034(11)	1.5132(09)	1.5207(10)	1.5255(08)	1.5340(10)
0.8	1.27	1.4755(10)	1.4889(09)	1.5028(10)	1.5132(09)	1.5211(09)	1.5258(11)	1.5330(10)
0.8605	1.4	1.4755(11)	1.4894(09)	1.5035(11)	1.5132(10)	1.5222(10)	1.5258(10)	1.5338(09)
0.9	1.52	1.4753(09)	1.4898(10)	1.5039(10)	1.5140(10)	1.5219(11)	1.5264(10)	1.5336(09)

Table A.5: Case I Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5561(12)	1.5711(12)	1.5855(12)	1.5966(11)	1.6037(12)	1.6089(10)	1.6161(11)
0.0323	0.18	1.5561(11)	1.5713(12)	1.5868(11)	1.5939(13)	1.6039(12)	1.6102(12)	1.6162(11)
0.1	0.32	1.5560(11)	1.5703(12)	1.5860(11)	1.5952(10)	1.6043(11)	1.6085(12)	1.6164(11)
0.2	0.47	1.5562(12)	1.5715(13)	1.5852(11)	1.5953(10)	1.6039(13)	1.6094(12)	1.6157(11)
0.3	0.6	1.5571(11)	1.5715(10)	1.5852(13)	1.5952(11)	1.6042(12)	1.6092(11)	1.6163(13)
0.4	0.71	1.5574(13)	1.5705(11)	1.5854(10)	1.5953(12)	1.6040(13)	1.6082(13)	1.6156(11)
0.5	0.83	1.5567(12)	1.5720(13)	1.5852(12)	1.5958(12)	1.6040(11)	1.6099(12)	1.6163(11)
0.6	0.96	1.5560(12)	1.5697(12)	1.5849(12)	1.5951(11)	1.6047(12)	1.6093(11)	1.6158(10)
0.7	1.1	1.5558(12)	1.5716(12)	1.5851(12)	1.5954(12)	1.6028(12)	1.6078(11)	1.6166(12)
0.8	1.27	1.5566(11)	1.5706(11)	1.5849(11)	1.5957(11)	1.6038(11)	1.6089(11)	1.6156(11)
0.8605	1.4	1.5565(12)	1.5711(11)	1.5856(12)	1.5956(13)	1.6049(11)	1.6084(12)	1.6162(10)
0.9	1.52	1.5563(11)	1.5718(12)	1.5857(12)	1.5957(12)	1.6048(12)	1.6095(11)	1.6155(12)

Table A.6: Case I Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.6803(16)	1.6968(15)	1.7119(15)	1.7239(15)	1.7313(16)	1.7361(14)	1.7422(14)
0.0323	0.18	1.6805(14)	1.6972(15)	1.7131(15)	1.7207(15)	1.7324(16)	1.7373(15)	1.7442(14)
0.1	0.32	1.6817(14)	1.6960(16)	1.7122(15)	1.7230(15)	1.7313(15)	1.7357(15)	1.7443(14)
0.2	0.47	1.6807(17)	1.6968(16)	1.7122(14)	1.7225(15)	1.7315(15)	1.7362(16)	1.7445(16)
0.3	0.6	1.6817(13)	1.6976(13)	1.7115(15)	1.7230(11)	1.7305(15)	1.7370(13)	1.7436(16)
0.4	0.71	1.6810(15)	1.6963(16)	1.7110(14)	1.7232(14)	1.7313(16)	1.7366(17)	1.7429(13)
0.5	0.83	1.6815(16)	1.6986(15)	1.7125(17)	1.7224(16)	1.7319(16)	1.7379(14)	1.7437(12)
0.6	0.96	1.6804(15)	1.6943(14)	1.7109(14)	1.7217(14)	1.7327(15)	1.7359(13)	1.7438(14)
0.7	1.1	1.6803(14)	1.6968(15)	1.7119(17)	1.7230(15)	1.7303(15)	1.7354(14)	1.7445(16)
0.8	1.27	1.6804(14)	1.6967(14)	1.7111(16)	1.7234(14)	1.7310(15)	1.7357(15)	1.7430(17)
0.8605	1.4	1.6815(15)	1.6969(15)	1.7117(15)	1.7232(16)	1.7332(15)	1.7365(16)	1.7436(15)
0.9	1.52	1.6805(14)	1.6982(15)	1.7122(14)	1.7233(17)	1.7327(13)	1.7367(16)	1.7426(16)

Table A.7: Case I Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.9224(28)	1.9447(28)	1.9640(27)	1.9776(26)	1.9856(32)	1.9896(33)	1.9973(26)
0.0323	0.18	1.9255(24)	1.9458(28)	1.9632(24)	1.9730(29)	1.9865(29)	1.9887(29)	1.9967(28)
0.1	0.32	1.9267(29)	1.9421(29)	1.9615(30)	1.9743(33)	1.9845(31)	1.9860(26)	1.9987(29)
0.2	0.47	1.9270(30)	1.9445(32)	1.9623(24)	1.9750(30)	1.9839(28)	1.9877(25)	2.0002(26)
0.3	0.6	1.9271(24)	1.9463(28)	1.9631(29)	1.9732(28)	1.9825(31)	1.9898(29)	1.9978(31)
0.4	0.71	1.9257(30)	1.9453(26)	1.9617(26)	1.9760(25)	1.9830(29)	1.9908(29)	1.9954(28)
0.5	0.83	1.9299(27)	1.9456(28)	1.9643(34)	1.9760(30)	1.9853(27)	1.9899(31)	1.9962(26)
0.6	0.96	1.9232(29)	1.9445(28)	1.9614(28)	1.9746(28)	1.9844(28)	1.9888(31)	1.9956(32)
0.7	1.1	1.9264(29)	1.9450(26)	1.9644(28)	1.9768(26)	1.9829(27)	1.9909(25)	1.9971(28)
0.8	1.27	1.9246(28)	1.9438(30)	1.9628(28)	1.9772(27)	1.9828(28)	1.9848(29)	1.9963(28)
0.8605	1.4	1.9277(30)	1.9445(31)	1.9637(29)	1.9756(29)	1.9881(25)	1.9889(29)	1.9977(28)
0.9	1.52	1.9255(26)	1.9453(27)	1.9612(28)	1.9753(28)	1.9845(26)	1.9896(28)	1.9945(26)

Table A.8: Case I Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.2840(07)	0.2844(07)	0.2840(08)	0.2841(08)	0.2841(07)	0.2836(07)	0.2842(08)
0.0323	0.18	0.2839(06)	0.2835(07)	0.2843(07)	0.2832(07)	0.2838(07)	0.2839(07)	0.2844(08)
0.1	0.32	0.2834(07)	0.2840(08)	0.2847(08)	0.2839(07)	0.2840(08)	0.2844(07)	0.2841(06)
0.2	0.47	0.2838(07)	0.2838(08)	0.2839(08)	0.2840(07)	0.2843(07)	0.2846(08)	0.2840(07)
0.3	0.6	0.2843(07)	0.2848(07)	0.2845(07)	0.2834(07)	0.2841(07)	0.2845(08)	0.2842(07)
0.4	0.71	0.2844(06)	0.2841(08)	0.2839(07)	0.2838(08)	0.2839(07)	0.2841(08)	0.2840(06)
0.5	0.83	0.2843(07)	0.2842(08)	0.2841(08)	0.2842(07)	0.2838(07)	0.2844(07)	0.2839(07)
0.6	0.96	0.2841(07)	0.2838(06)	0.2842(07)	0.2837(07)	0.2844(08)	0.2834(07)	0.2839(07)
0.7	1.1	0.2839(07)	0.2843(07)	0.2835(08)	0.2847(07)	0.2835(07)	0.2834(07)	0.2844(07)
0.8	1.27	0.2837(07)	0.2837(08)	0.2837(08)	0.2839(07)	0.2843(07)	0.2839(07)	0.2840(07)
0.8605	1.4	0.2838(06)	0.2834(08)	0.2838(08)	0.2841(08)	0.2838(07)	0.2840(07)	0.2843(07)
0.9	1.52	0.2837(08)	0.2842(07)	0.2835(07)	0.2840(08)	0.2837(08)	0.2847(07)	0.2842(06)

Table A.9: Case I Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.3465(10)	0.3476(09)	0.3475(10)	0.3466(10)	0.3469(08)	0.3473(09)	0.3478(10)
0.0323	0.18	0.3463(09)	0.3461(10)	0.3471(09)	0.3456(09)	0.3463(10)	0.3465(09)	0.3479(10)
0.1	0.32	0.3457(09)	0.3470(10)	0.3475(11)	0.3465(08)	0.3472(08)	0.3475(09)	0.3478(08)
0.2	0.47	0.3465(09)	0.3465(10)	0.3469(10)	0.3468(10)	0.3472(09)	0.3477(10)	0.3474(09)
0.3	0.6	0.3471(10)	0.3472(08)	0.3474(10)	0.3468(10)	0.3469(10)	0.3477(10)	0.3475(08)
0.4	0.71	0.3469(08)	0.3469(10)	0.3472(09)	0.3468(09)	0.3472(10)	0.3480(09)	0.3475(09)
0.5	0.83	0.3470(10)	0.3472(11)	0.3476(11)	0.3475(09)	0.3467(09)	0.3476(09)	0.3468(09)
0.6	0.96	0.3467(10)	0.3472(08)	0.3473(09)	0.3467(09)	0.3476(09)	0.3469(09)	0.3469(10)
0.7	1.1	0.3467(08)	0.3471(09)	0.3470(10)	0.3477(09)	0.3463(10)	0.3464(09)	0.3474(10)
0.8	1.27	0.3461(09)	0.3466(09)	0.3473(10)	0.3473(10)	0.3476(09)	0.3467(11)	0.3474(10)
0.8605	1.4	0.3468(09)	0.3466(10)	0.3467(10)	0.3471(10)	0.3470(09)	0.3473(10)	0.3472(10)
0.9	1.52	0.3464(10)	0.3470(09)	0.3466(09)	0.3471(10)	0.3472(09)	0.3473(09)	0.3477(08)

Table A.10: Case I Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4589(15)	0.4612(14)	0.4610(15)	0.4601(13)	0.4611(13)	0.4612(14)	0.4610(15)
0.0323	0.18	0.4581(14)	0.4594(15)	0.4604(12)	0.4595(15)	0.4608(13)	0.4587(13)	0.4623(15)
0.1	0.32	0.4580(15)	0.4588(14)	0.4610(15)	0.4604(13)	0.4619(15)	0.4615(14)	0.4613(13)
0.2	0.47	0.4600(14)	0.4590(14)	0.4593(15)	0.4604(15)	0.4615(14)	0.4609(16)	0.4612(16)
0.3	0.6	0.4605(16)	0.4595(15)	0.4610(15)	0.4606(14)	0.4606(16)	0.4618(16)	0.4616(15)
0.4	0.71	0.4593(16)	0.4596(15)	0.4608(15)	0.4602(14)	0.4612(14)	0.4611(16)	0.4613(15)
0.5	0.83	0.4590(15)	0.4603(15)	0.4614(16)	0.4607(13)	0.4597(13)	0.4611(12)	0.4607(15)
0.6	0.96	0.4593(15)	0.4609(14)	0.4612(13)	0.4606(14)	0.4608(15)	0.4615(14)	0.4605(14)
0.7	1.1	0.4595(12)	0.4609(15)	0.4607(15)	0.4621(14)	0.4589(14)	0.4607(13)	0.4618(15)
0.8	1.27	0.4579(13)	0.4597(14)	0.4606(16)	0.4608(15)	0.4608(15)	0.4599(15)	0.4612(15)
0.8605	1.4	0.4593(14)	0.4593(14)	0.4600(14)	0.4617(16)	0.4618(14)	0.4617(15)	0.4602(15)
0.9	1.52	0.4585(15)	0.4604(15)	0.4595(14)	0.4614(15)	0.4616(15)	0.4615(15)	0.4609(14)

Table A.11: Case I Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7322(29)	0.7393(31)	0.7408(37)	0.7426(37)	0.7429(37)	0.7438(36)	0.7423(32)
0.0323	0.18	0.7339(34)	0.7358(36)	0.7381(32)	0.7393(38)	0.7407(34)	0.7384(37)	0.7452(40)
0.1	0.32	0.7360(32)	0.7371(34)	0.7364(34)	0.7416(35)	0.7439(36)	0.7432(36)	0.7405(36)
0.2	0.47	0.7344(30)	0.7346(35)	0.7407(33)	0.7400(38)	0.7437(31)	0.7436(40)	0.7420(32)
0.3	0.6	0.7355(35)	0.7377(39)	0.7381(33)	0.7424(32)	0.7415(37)	0.7431(35)	0.7424(34)
0.4	0.71	0.7305(30)	0.7400(32)	0.7400(37)	0.7397(33)	0.7409(38)	0.7405(40)	0.7454(36)
0.5	0.83	0.7336(33)	0.7363(40)	0.7404(39)	0.7394(33)	0.7397(39)	0.7431(36)	0.7411(38)
0.6	0.96	0.7359(39)	0.7365(34)	0.7379(36)	0.7413(29)	0.7436(37)	0.7410(41)	0.7407(35)
0.7	1.1	0.7330(34)	0.7378(35)	0.7389(34)	0.7442(35)	0.7367(35)	0.7415(29)	0.7445(36)
0.8	1.27	0.7317(38)	0.7379(34)	0.7429(34)	0.7383(28)	0.7404(32)	0.7380(38)	0.7444(42)
0.8605	1.4	0.7335(38)	0.7361(34)	0.7395(31)	0.7443(36)	0.7483(33)	0.7420(36)	0.7420(35)
0.9	1.52	0.7310(31)	0.7388(36)	0.7347(36)	0.7401(40)	0.7427(35)	0.7410(33)	0.7408(34)

Table A.12: Case I Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.6226(34)	1.6247(35)	1.6223(37)	1.6199(36)	1.6219(36)	1.6209(35)	1.6227(42)
0.0323	0.18	1.6217(32)	1.6187(34)	1.6241(33)	1.6172(35)	1.6203(35)	1.6188(35)	1.6234(40)
0.1	0.32	1.6215(34)	1.6222(37)	1.6243(41)	1.6212(33)	1.6211(36)	1.6232(36)	1.6216(31)
0.2	0.47	1.6231(36)	1.6214(38)	1.6195(37)	1.6221(36)	1.6218(33)	1.6240(38)	1.6226(34)
0.3	0.6	1.6238(38)	1.6247(32)	1.6230(35)	1.6185(35)	1.6214(38)	1.6237(36)	1.6221(34)
0.4	0.71	1.6250(32)	1.6220(42)	1.6231(38)	1.6206(36)	1.6195(36)	1.6233(35)	1.6220(33)
0.5	0.83	1.6238(38)	1.6228(36)	1.6235(43)	1.6230(34)	1.6193(38)	1.6225(34)	1.6199(35)
0.6	0.96	1.6231(34)	1.6231(32)	1.6229(35)	1.6189(36)	1.6231(36)	1.6174(33)	1.6211(33)
0.7	1.1	1.6216(34)	1.6245(37)	1.6198(36)	1.6244(32)	1.6184(36)	1.6171(32)	1.6210(38)
0.8	1.27	1.6199(36)	1.6200(35)	1.6203(38)	1.6221(36)	1.6221(32)	1.6199(36)	1.6208(36)
0.8605	1.4	1.6224(32)	1.6202(37)	1.6188(36)	1.6206(41)	1.6205(35)	1.6211(36)	1.6220(33)
0.9	1.52	1.6219(38)	1.6219(40)	1.6192(33)	1.6217(35)	1.6191(38)	1.6237(34)	1.6231(30)

Table A.13: Case I Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.9352(46)	1.9380(45)	1.9364(49)	1.9327(46)	1.9303(45)	1.9343(47)	1.9351(54)
0.0323	0.18	1.9350(45)	1.9330(47)	1.9338(43)	1.9257(48)	1.9297(50)	1.9294(44)	1.9345(51)
0.1	0.32	1.9345(46)	1.9362(48)	1.9349(49)	1.9325(42)	1.9329(46)	1.9350(42)	1.9335(43)
0.2	0.47	1.9370(43)	1.9342(49)	1.9327(48)	1.9325(45)	1.9320(48)	1.9330(50)	1.9340(46)
0.3	0.6	1.9376(50)	1.9375(44)	1.9365(52)	1.9330(49)	1.9312(50)	1.9328(45)	1.9338(41)
0.4	0.71	1.9377(42)	1.9346(48)	1.9350(48)	1.9326(46)	1.9326(48)	1.9368(46)	1.9348(44)
0.5	0.83	1.9401(51)	1.9385(50)	1.9348(50)	1.9350(43)	1.9294(48)	1.9348(40)	1.9318(43)
0.6	0.96	1.9381(45)	1.9359(42)	1.9348(45)	1.9311(49)	1.9314(45)	1.9297(43)	1.9320(44)
0.7	1.1	1.9357(41)	1.9374(51)	1.9310(48)	1.9366(44)	1.9293(46)	1.9301(44)	1.9330(51)
0.8	1.27	1.9322(43)	1.9336(45)	1.9342(49)	1.9338(47)	1.9341(48)	1.9299(49)	1.9318(50)
0.8605	1.4	1.9361(44)	1.9338(47)	1.9311(48)	1.9336(49)	1.9330(44)	1.9333(47)	1.9306(43)
0.9	1.52	1.9356(51)	1.9364(49)	1.9328(44)	1.9345(48)	1.9346(50)	1.9332(44)	1.9353(39)

Table A.14: Case I Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	2.4976(76)	2.5037(66)	2.4950(74)	2.4879(69)	2.4889(64)	2.4935(67)	2.4913(73)
0.0323	0.18	2.4924(72)	2.4966(78)	2.4931(61)	2.4858(68)	2.4912(69)	2.4830(68)	2.4940(76)
0.1	0.32	2.4948(74)	2.4936(68)	2.4973(77)	2.4894(64)	2.4942(67)	2.4948(64)	2.4930(63)
0.2	0.47	2.5011(74)	2.4931(69)	2.4885(69)	2.4931(72)	2.4918(71)	2.4914(75)	2.4927(72)
0.3	0.6	2.5071(72)	2.4984(74)	2.4947(75)	2.4923(68)	2.4916(76)	2.4924(79)	2.4927(70)
0.4	0.71	2.4980(69)	2.4991(67)	2.4947(75)	2.4903(66)	2.4926(75)	2.4932(75)	2.4908(78)
0.5	0.83	2.5002(72)	2.4961(72)	2.4963(80)	2.4908(71)	2.4839(67)	2.4920(61)	2.4879(71)
0.6	0.96	2.5021(74)	2.5002(68)	2.4935(67)	2.4929(63)	2.4899(77)	2.4912(75)	2.4879(71)
0.7	1.1	2.5009(62)	2.5015(79)	2.4914(71)	2.4990(70)	2.4832(65)	2.4900(62)	2.4911(74)
0.8	1.27	2.4926(66)	2.4963(68)	2.4964(81)	2.4906(68)	2.4918(71)	2.4851(75)	2.4910(72)
0.8605	1.4	2.5013(72)	2.4941(67)	2.4934(62)	2.4954(79)	2.4951(72)	2.4968(75)	2.4890(66)
0.9	1.52	2.4946(79)	2.4990(76)	2.4856(76)	2.4943(68)	2.4947(72)	2.4873(73)	2.4908(69)

Table A.15: Case I Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	3.8828(176)	3.8978(155)	3.8815(184)	3.8808(178)	3.8745(192)	3.8872(180)	3.8724(168)
0.0323	0.18	3.8894(194)	3.8729(188)	3.8720(157)	3.8636(174)	3.8633(163)	3.8551(180)	3.8765(183)
0.1	0.32	3.8895(169)	3.8858(180)	3.8710(181)	3.8871(164)	3.8888(166)	3.8813(170)	3.8632(181)
0.2	0.47	3.9013(167)	3.8749(189)	3.8779(157)	3.8797(194)	3.8892(155)	3.8714(189)	3.8737(163)
0.3	0.6	3.9000(180)	3.8878(179)	3.8668(165)	3.8770(172)	3.8649(186)	3.8789(169)	3.8655(179)
0.4	0.71	3.8807(180)	3.9028(163)	3.8744(183)	3.8723(155)	3.8714(170)	3.8680(196)	3.8854(183)
0.5	0.83	3.8923(169)	3.8779(167)	3.8708(201)	3.8653(160)	3.8634(187)	3.8788(161)	3.8652(172)
0.6	0.96	3.8943(177)	3.8819(183)	3.8683(161)	3.8739(145)	3.8792(193)	3.8751(198)	3.8689(176)
0.7	1.1	3.8875(173)	3.8865(194)	3.8766(168)	3.8920(169)	3.8568(178)	3.8611(158)	3.8852(169)
0.8	1.27	3.8746(192)	3.8868(175)	3.8823(172)	3.8676(156)	3.8719(174)	3.8522(178)	3.8890(180)
0.8605	1.4	3.8934(209)	3.8774(153)	3.8719(178)	3.8912(180)	3.9003(168)	3.8716(185)	3.8680(172)
0.9	1.52	3.8826(170)	3.8916(177)	3.8515(173)	3.8763(186)	3.8797(170)	3.8604(174)	3.8684(159)

Table A.16: Case I Anderson-Darling 99% critical values

A.2 Case II: Both Parameters are Unknown

A.2.1 Weibull Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7462(05)	0.7541(05)	0.7617(05)	0.7677(05)	0.7719(05)	0.7742(06)	0.7779(06)
0.0323	0.18	0.7248(05)	0.7328(06)	0.7407(05)	0.7460(05)	0.7506(04)	0.7531(05)	0.7568(06)
0.1	0.32	0.7167(06)	0.7255(05)	0.7335(05)	0.7387(05)	0.7441(05)	0.7464(05)	0.7503(06)
0.2	0.47	0.7158(04)	0.7249(05)	0.7336(05)	0.7393(05)	0.7443(06)	0.7466(05)	0.7508(05)
0.3	0.6	0.7182(05)	0.7269(05)	0.7361(05)	0.7425(05)	0.7480(06)	0.7501(05)	0.7538(05)
0.4	0.71	0.7208(05)	0.7304(05)	0.7404(05)	0.7466(05)	0.7522(05)	0.7552(05)	0.7584(05)
0.5	0.83	0.7245(05)	0.7348(05)	0.7444(05)	0.7509(05)	0.7567(05)	0.7593(05)	0.7636(05)
0.6	0.96	0.7279(05)	0.7382(05)	0.7488(05)	0.7554(05)	0.7613(05)	0.7642(05)	0.7686(06)
0.7	1.1	0.7321(05)	0.7430(05)	0.7533(05)	0.7607(05)	0.7663(06)	0.7692(06)	0.7735(05)
0.8	1.27	0.7356(06)	0.7478(05)	0.7587(05)	0.7661(05)	0.7721(05)	0.7748(05)	0.7789(05)
0.8605	1.4	0.7384(05)	0.7510(06)	0.7623(05)	0.7694(06)	0.7756(05)	0.7790(06)	0.7832(06)
0.9	1.52	0.7408(06)	0.7529(06)	0.7652(05)	0.7725(06)	0.7792(06)	0.7817(05)	0.7861(06)

Table A.17: Case II Weibull Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7899(06)	0.7983(06)	0.8065(06)	0.8127(06)	0.8168(06)	0.8191(07)	0.8231(07)
0.0323	0.18	0.7663(06)	0.7746(06)	0.7830(06)	0.7884(05)	0.7933(06)	0.7958(07)	0.7995(06)
0.1	0.32	0.7566(06)	0.7660(05)	0.7747(06)	0.7803(05)	0.7852(05)	0.7878(06)	0.7917(07)
0.2	0.47	0.7555(06)	0.7653(05)	0.7743(06)	0.7806(06)	0.7853(06)	0.7880(05)	0.7923(06)
0.3	0.6	0.7582(05)	0.7673(06)	0.7772(06)	0.7844(06)	0.7896(06)	0.7917(06)	0.7957(06)
0.4	0.71	0.7611(05)	0.7714(06)	0.7823(06)	0.7890(06)	0.7940(05)	0.7971(06)	0.8006(06)
0.5	0.83	0.7650(05)	0.7762(06)	0.7861(05)	0.7934(06)	0.7994(06)	0.8020(07)	0.8061(06)
0.6	0.96	0.7692(05)	0.7799(06)	0.7915(05)	0.7988(05)	0.8047(06)	0.8080(06)	0.8122(07)
0.7	1.1	0.7739(06)	0.7858(06)	0.7967(07)	0.8045(07)	0.8104(07)	0.8134(06)	0.8175(06)
0.8	1.27	0.7778(07)	0.7910(05)	0.8028(06)	0.8109(06)	0.8172(06)	0.8200(06)	0.8237(06)
0.8605	1.4	0.7812(06)	0.7948(06)	0.8068(06)	0.8145(07)	0.8212(06)	0.8250(07)	0.8289(07)
0.9	1.52	0.7835(07)	0.7972(08)	0.8105(06)	0.8180(06)	0.8250(07)	0.8279(06)	0.8322(07)

Table A.18: Case II Weibull Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8581(07)	0.8671(09)	0.8761(08)	0.8825(08)	0.8875(07)	0.8902(08)	0.8937(09)
0.0323	0.18	0.8308(08)	0.8400(08)	0.8492(08)	0.8544(07)	0.8601(07)	0.8624(09)	0.8663(08)
0.1	0.32	0.8183(07)	0.8294(07)	0.8388(08)	0.8448(07)	0.8499(07)	0.8523(08)	0.8559(09)
0.2	0.47	0.8167(07)	0.8276(07)	0.8374(07)	0.8446(07)	0.8494(07)	0.8524(07)	0.8563(07)
0.3	0.6	0.8193(08)	0.8302(07)	0.8406(07)	0.8487(08)	0.8532(08)	0.8564(07)	0.8605(07)
0.4	0.71	0.8231(07)	0.8349(07)	0.8470(08)	0.8540(08)	0.8598(08)	0.8624(08)	0.8662(07)
0.5	0.83	0.8278(08)	0.8401(08)	0.8513(07)	0.8596(08)	0.8660(07)	0.8686(09)	0.8725(08)
0.6	0.96	0.8328(07)	0.8457(08)	0.8582(07)	0.8665(07)	0.8730(08)	0.8759(08)	0.8799(09)
0.7	1.1	0.8387(08)	0.8522(09)	0.8651(09)	0.8728(09)	0.8791(08)	0.8827(08)	0.8873(08)
0.8	1.27	0.8438(09)	0.8580(08)	0.8717(07)	0.8810(08)	0.8870(08)	0.8904(08)	0.8939(09)
0.8605	1.4	0.8473(08)	0.8630(08)	0.8767(08)	0.8856(08)	0.8926(08)	0.8964(09)	0.9008(08)
0.9	1.52	0.8502(08)	0.8660(08)	0.8810(08)	0.8894(07)	0.8970(09)	0.9002(09)	0.9041(09)

Table A.19: Case II Weibull Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.9949(13)	1.0070(15)	1.0166(17)	1.0252(16)	1.0292(15)	1.0324(15)	1.0363(15)
0.0323	0.18	0.9603(16)	0.9721(16)	0.9823(13)	0.9888(14)	0.9944(15)	0.9967(17)	1.0006(15)
0.1	0.32	0.9424(14)	0.9555(14)	0.9673(16)	0.9735(15)	0.9792(14)	0.9812(14)	0.9857(16)
0.2	0.47	0.9387(14)	0.9515(14)	0.9635(14)	0.9718(14)	0.9767(17)	0.9808(14)	0.9845(14)
0.3	0.6	0.9404(13)	0.9547(12)	0.9682(13)	0.9779(15)	0.9819(15)	0.9862(15)	0.9913(15)
0.4	0.71	0.9464(12)	0.9616(13)	0.9762(14)	0.9845(16)	0.9928(15)	0.9931(14)	0.9983(15)
0.5	0.83	0.9531(16)	0.9686(15)	0.9829(14)	0.9935(16)	0.9997(15)	1.0028(14)	1.0064(14)
0.6	0.96	0.9595(13)	0.9769(15)	0.9947(15)	1.0035(14)	1.0102(15)	1.0130(17)	1.0176(14)
0.7	1.1	0.9697(14)	0.9874(15)	1.0019(17)	1.0119(18)	1.0179(15)	1.0232(18)	1.0270(14)
0.8	1.27	0.9755(14)	0.9956(15)	1.0117(15)	1.0226(17)	1.0301(15)	1.0339(14)	1.0383(15)
0.8605	1.4	0.9802(14)	1.0008(15)	1.0194(16)	1.0305(16)	1.0379(18)	1.0422(18)	1.0467(15)
0.9	1.52	0.9849(15)	1.0057(16)	1.0267(16)	1.0357(16)	1.0444(17)	1.0470(18)	1.0516(18)

Table A.20: Case II Weibull Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2790(08)	1.2944(08)	1.3098(08)	1.3201(08)	1.3287(08)	1.3333(08)	1.3406(10)
0.0323	0.18	1.2740(08)	1.2892(10)	1.3047(09)	1.3148(07)	1.3242(07)	1.3290(09)	1.3363(08)
0.1	0.32	1.2772(09)	1.2928(07)	1.3082(09)	1.3182(07)	1.3282(08)	1.3321(08)	1.3399(09)
0.2	0.47	1.2823(08)	1.2990(07)	1.3135(08)	1.3243(09)	1.3331(09)	1.3377(09)	1.3459(09)
0.3	0.6	1.2877(08)	1.3024(08)	1.3181(08)	1.3292(08)	1.3391(09)	1.3426(08)	1.3503(08)
0.4	0.71	1.2913(08)	1.3068(08)	1.3233(08)	1.3339(08)	1.3433(08)	1.3481(08)	1.3551(09)
0.5	0.83	1.2949(08)	1.3120(08)	1.3269(09)	1.3376(08)	1.3482(09)	1.3520(09)	1.3597(08)
0.6	0.96	1.2990(08)	1.3146(09)	1.3306(08)	1.3421(09)	1.3516(09)	1.3562(08)	1.3641(09)
0.7	1.1	1.3026(08)	1.3193(10)	1.3345(08)	1.3459(09)	1.3552(09)	1.3600(09)	1.3674(08)
0.8	1.27	1.3050(10)	1.3227(08)	1.3383(08)	1.3501(09)	1.3597(09)	1.3640(09)	1.3713(09)
0.8605	1.4	1.3073(07)	1.3245(09)	1.3416(08)	1.3521(08)	1.3623(10)	1.3674(09)	1.3750(10)
0.9	1.52	1.3092(09)	1.3259(09)	1.3442(09)	1.3544(08)	1.3652(09)	1.3693(08)	1.3767(10)

Table A.21: Case II Weibull Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3445(09)	1.3607(10)	1.3769(09)	1.3878(09)	1.3966(09)	1.4010(09)	1.4081(11)
0.0323	0.18	1.3396(09)	1.3556(11)	1.3721(10)	1.3822(09)	1.3917(08)	1.3965(10)	1.4041(10)
0.1	0.32	1.3432(10)	1.3605(09)	1.3759(10)	1.3861(09)	1.3964(09)	1.4008(10)	1.4084(10)
0.2	0.47	1.3494(09)	1.3666(08)	1.3819(09)	1.3934(10)	1.4019(09)	1.4067(10)	1.4150(09)
0.3	0.6	1.3555(09)	1.3705(09)	1.3880(10)	1.3989(10)	1.4086(11)	1.4126(10)	1.4199(09)
0.4	0.71	1.3594(10)	1.3759(10)	1.3936(09)	1.4040(11)	1.4132(09)	1.4187(09)	1.4253(10)
0.5	0.83	1.3629(09)	1.3815(10)	1.3973(10)	1.4085(10)	1.4185(10)	1.4227(10)	1.4307(10)
0.6	0.96	1.3679(09)	1.3845(10)	1.4019(09)	1.4133(11)	1.4226(10)	1.4275(09)	1.4353(11)
0.7	1.1	1.3720(10)	1.3897(11)	1.4059(11)	1.4170(10)	1.4269(10)	1.4319(10)	1.4392(09)
0.8	1.27	1.3748(11)	1.3930(09)	1.4103(09)	1.4223(10)	1.4320(10)	1.4358(11)	1.4437(10)
0.8605	1.4	1.3770(09)	1.3959(10)	1.4125(10)	1.4244(11)	1.4345(11)	1.4399(10)	1.4472(12)
0.9	1.52	1.3793(11)	1.3974(11)	1.4162(11)	1.4264(10)	1.4373(10)	1.4421(10)	1.4495(10)

Table A.22: Case II Weibull Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4452(12)	1.4634(13)	1.4800(12)	1.4925(14)	1.5013(12)	1.5051(11)	1.5139(12)
0.0323	0.18	1.4406(12)	1.4588(15)	1.4759(13)	1.4870(13)	1.4976(09)	1.5014(12)	1.5095(13)
0.1	0.32	1.4452(14)	1.4640(12)	1.4819(13)	1.4930(12)	1.5023(12)	1.5069(13)	1.5148(15)
0.2	0.47	1.4534(11)	1.4724(10)	1.4889(11)	1.5014(13)	1.5096(12)	1.5148(12)	1.5235(12)
0.3	0.6	1.4605(11)	1.4776(11)	1.4950(12)	1.5077(14)	1.5174(14)	1.5217(13)	1.5293(13)
0.4	0.71	1.4652(13)	1.4837(11)	1.5030(12)	1.5139(12)	1.5230(13)	1.5279(12)	1.5347(12)
0.5	0.83	1.4693(13)	1.4895(12)	1.5057(14)	1.5183(14)	1.5280(12)	1.5328(12)	1.5404(12)
0.6	0.96	1.4745(11)	1.4925(14)	1.5115(13)	1.5237(11)	1.5339(14)	1.5389(12)	1.5472(13)
0.7	1.1	1.4793(13)	1.4982(14)	1.5165(14)	1.5280(14)	1.5381(13)	1.5433(12)	1.5512(15)
0.8	1.27	1.4831(14)	1.5029(14)	1.5212(12)	1.5343(14)	1.5435(12)	1.5487(14)	1.5558(12)
0.8605	1.4	1.4854(13)	1.5058(14)	1.5236(13)	1.5363(13)	1.5472(13)	1.5523(14)	1.5603(14)
0.9	1.52	1.4873(14)	1.5075(13)	1.5281(13)	1.5393(13)	1.5503(13)	1.5552(16)	1.5624(15)

Table A.23: Case II Weibull Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.6468(24)	1.6691(24)	1.6844(23)	1.7006(25)	1.7097(19)	1.7153(23)	1.7238(23)
0.0323	0.18	1.6418(25)	1.6644(24)	1.6843(24)	1.6947(18)	1.7086(25)	1.7127(26)	1.7195(22)
0.1	0.32	1.6504(21)	1.6723(21)	1.6926(22)	1.7070(25)	1.7160(20)	1.7201(25)	1.7299(28)
0.2	0.47	1.6625(25)	1.6845(25)	1.7034(23)	1.7171(25)	1.7260(24)	1.7332(25)	1.7402(24)
0.3	0.6	1.6700(23)	1.6921(23)	1.7137(24)	1.7272(27)	1.7345(25)	1.7413(24)	1.7516(25)
0.4	0.71	1.6767(21)	1.6988(24)	1.7194(23)	1.7331(25)	1.7467(26)	1.7479(23)	1.7557(22)
0.5	0.83	1.6821(26)	1.7047(24)	1.7256(25)	1.7395(25)	1.7485(25)	1.7546(22)	1.7631(25)
0.6	0.96	1.6874(20)	1.7119(24)	1.7357(28)	1.7484(23)	1.7600(26)	1.7623(25)	1.7691(25)
0.7	1.1	1.6971(24)	1.7181(23)	1.7392(24)	1.7520(24)	1.7628(23)	1.7686(24)	1.7786(24)
0.8	1.27	1.6988(24)	1.7223(26)	1.7440(24)	1.7598(25)	1.7692(27)	1.7750(28)	1.7823(25)
0.8605	1.4	1.7004(26)	1.7267(26)	1.7492(20)	1.7656(26)	1.7733(25)	1.7812(27)	1.7869(23)
0.9	1.52	1.7048(25)	1.7294(23)	1.7531(23)	1.7669(25)	1.7766(25)	1.7817(26)	1.7905(28)

Table A.24: Case II Weibull Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.08874(14)	0.08884(14)	0.08896(13)	0.08897(15)	0.08907(14)	0.08907(14)	0.08913(17)
0.0323	0.18	0.08435(12)	0.08439(16)	0.08447(13)	0.08455(12)	0.08460(14)	0.08460(13)	0.08461(15)
0.1	0.32	0.08331(15)	0.08356(14)	0.08366(14)	0.08366(13)	0.08382(13)	0.08384(14)	0.08385(14)
0.2	0.47	0.08357(14)	0.08399(12)	0.08422(13)	0.08442(14)	0.08446(15)	0.08450(15)	0.08453(14)
0.3	0.6	0.08450(13)	0.08475(14)	0.08514(13)	0.08548(15)	0.08567(15)	0.08550(13)	0.08558(13)
0.4	0.71	0.08534(14)	0.08582(12)	0.08637(14)	0.08655(13)	0.08678(14)	0.08699(14)	0.08697(16)
0.5	0.83	0.08639(16)	0.08698(14)	0.08741(13)	0.08776(15)	0.08810(16)	0.08811(16)	0.08833(13)
0.6	0.96	0.08741(13)	0.08794(14)	0.08867(16)	0.08906(16)	0.08941(13)	0.08949(14)	0.08970(15)
0.7	1.1	0.08846(13)	0.08936(15)	0.08990(16)	0.09038(17)	0.09079(17)	0.09088(15)	0.09093(15)
0.8	1.27	0.08940(17)	0.09049(15)	0.09139(15)	0.09203(15)	0.09235(16)	0.09234(17)	0.09248(17)
0.8605	1.4	0.09015(15)	0.09145(16)	0.09233(15)	0.09296(16)	0.09334(15)	0.09361(15)	0.09372(18)
0.9	1.52	0.09079(16)	0.09194(16)	0.09323(17)	0.09368(16)	0.09433(17)	0.09437(17)	0.09454(18)

Table A.25: Case II Weibull Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.10119(19)	0.10150(20)	0.10166(16)	0.10177(19)	0.10187(18)	0.10182(18)	0.10183(21)
0.0323	0.18	0.09607(16)	0.09631(21)	0.09658(17)	0.09658(16)	0.09673(17)	0.09669(18)	0.09673(19)
0.1	0.32	0.09495(17)	0.09543(18)	0.09557(18)	0.09563(16)	0.09580(18)	0.09572(18)	0.09591(18)
0.2	0.47	0.09545(16)	0.09599(17)	0.09617(16)	0.09658(19)	0.09660(17)	0.09663(18)	0.09671(18)
0.3	0.6	0.09658(17)	0.09690(16)	0.09740(17)	0.09776(19)	0.09805(19)	0.09792(18)	0.09799(17)
0.4	0.71	0.09755(17)	0.09816(16)	0.09901(18)	0.09925(16)	0.09940(18)	0.09969(18)	0.09958(20)
0.5	0.83	0.09872(19)	0.09960(19)	0.10025(18)	0.10061(20)	0.10104(19)	0.10101(19)	0.10129(17)
0.6	0.96	0.10007(18)	0.10082(18)	0.10179(19)	0.10232(20)	0.10266(16)	0.10281(18)	0.10303(21)
0.7	1.1	0.10132(16)	0.10252(20)	0.10332(21)	0.10389(20)	0.10430(20)	0.10447(22)	0.10452(21)
0.8	1.27	0.10239(22)	0.10392(20)	0.10510(19)	0.10589(22)	0.10615(20)	0.10620(21)	0.10649(21)
0.8605	1.4	0.10335(18)	0.10505(18)	0.10609(20)	0.10691(20)	0.10756(20)	0.10786(19)	0.10790(24)
0.9	1.52	0.10419(22)	0.10566(22)	0.10724(21)	0.10780(20)	0.10852(21)	0.10886(21)	0.10897(22)

Table A.26: Case II Weibull Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.12253(25)	0.12285(26)	0.12326(25)	0.12354(29)	0.12372(26)	0.12379(27)	0.12368(28)
0.0323	0.18	0.11636(25)	0.11672(27)	0.11715(25)	0.11718(26)	0.11751(26)	0.11743(24)	0.11762(28)
0.1	0.32	0.11495(24)	0.11572(26)	0.11604(28)	0.11608(25)	0.11642(22)	0.11621(27)	0.11665(28)
0.2	0.47	0.11558(24)	0.11655(26)	0.11684(26)	0.11738(28)	0.11740(25)	0.11760(26)	0.11778(24)
0.3	0.6	0.11725(26)	0.11780(24)	0.11853(25)	0.11928(27)	0.11944(29)	0.11950(27)	0.11963(27)
0.4	0.71	0.11849(25)	0.11959(29)	0.12093(26)	0.12093(23)	0.12148(27)	0.12158(26)	0.12158(27)
0.5	0.83	0.12002(27)	0.12152(30)	0.12238(28)	0.12313(31)	0.12358(27)	0.12342(26)	0.12352(27)
0.6	0.96	0.12177(28)	0.12292(30)	0.12452(28)	0.12526(23)	0.12553(30)	0.12596(27)	0.12616(33)
0.7	1.1	0.12352(28)	0.12529(31)	0.12636(32)	0.12730(31)	0.12787(27)	0.12819(32)	0.12830(29)
0.8	1.27	0.12508(30)	0.12723(32)	0.12879(27)	0.12990(31)	0.13056(30)	0.13049(31)	0.13077(29)
0.8605	1.4	0.12629(30)	0.12851(30)	0.13005(33)	0.13136(30)	0.13211(30)	0.13246(30)	0.13263(35)
0.9	1.52	0.12728(34)	0.12957(34)	0.13187(32)	0.13265(28)	0.13337(32)	0.13384(31)	0.13388(35)

Table A.27: Case II Weibull Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.17185(55)	0.17360(63)	0.17375(60)	0.17461(63)	0.17455(57)	0.17478(61)	0.17525(70)
0.0323	0.18	0.16369(63)	0.16503(59)	0.16553(62)	0.16565(61)	0.16664(67)	0.16637(58)	0.16625(57)
0.1	0.32	0.16188(59)	0.16333(56)	0.16427(56)	0.16488(60)	0.16484(60)	0.16473(64)	0.16526(70)
0.2	0.47	0.16299(61)	0.16500(58)	0.16584(61)	0.16697(63)	0.16712(65)	0.16762(68)	0.16727(68)
0.3	0.6	0.16532(60)	0.16749(57)	0.16897(62)	0.17025(67)	0.17026(65)	0.17038(67)	0.17077(60)
0.4	0.71	0.16832(60)	0.17058(69)	0.17204(64)	0.17285(66)	0.17370(71)	0.17391(63)	0.17431(61)
0.5	0.83	0.17066(59)	0.17305(64)	0.17494(66)	0.17672(70)	0.17714(65)	0.17746(61)	0.17747(68)
0.6	0.96	0.17291(59)	0.17614(66)	0.17898(70)	0.18037(56)	0.18095(76)	0.18129(72)	0.18075(66)
0.7	1.1	0.17639(66)	0.17960(64)	0.18207(75)	0.18349(73)	0.18418(73)	0.18469(68)	0.18518(69)
0.8	1.27	0.17888(63)	0.18215(71)	0.18579(64)	0.18811(65)	0.18897(73)	0.18903(77)	0.18841(67)
0.8605	1.4	0.18035(66)	0.18457(68)	0.18863(72)	0.19074(73)	0.19107(69)	0.19246(74)	0.19181(62)
0.9	1.52	0.18158(66)	0.18668(67)	0.19063(75)	0.19210(64)	0.19378(77)	0.19379(75)	0.19431(78)

Table A.28: Case II Weibull Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.5583(09)	0.5599(09)	0.5615(07)	0.5623(09)	0.5632(08)	0.5632(07)	0.5640(09)
0.0323	0.18	0.5508(07)	0.5520(10)	0.5531(08)	0.5538(07)	0.5546(08)	0.5546(08)	0.5549(09)
0.1	0.32	0.5566(08)	0.5588(08)	0.5597(08)	0.5603(08)	0.5615(08)	0.5618(08)	0.5619(08)
0.2	0.47	0.5654(09)	0.5690(08)	0.5702(09)	0.5714(09)	0.5724(08)	0.5724(09)	0.5731(08)
0.3	0.6	0.5756(09)	0.5777(09)	0.5792(08)	0.5818(09)	0.5824(09)	0.5820(08)	0.5831(09)
0.4	0.71	0.5823(08)	0.5854(08)	0.5894(09)	0.5903(09)	0.5912(09)	0.5929(08)	0.5925(10)
0.5	0.83	0.5902(09)	0.5933(10)	0.5965(09)	0.5984(10)	0.6008(10)	0.6008(10)	0.6011(09)
0.6	0.96	0.5969(09)	0.6002(10)	0.6046(09)	0.6069(10)	0.6089(09)	0.6096(09)	0.6106(10)
0.7	1.1	0.6044(09)	0.6096(10)	0.6121(10)	0.6158(11)	0.6173(09)	0.6175(09)	0.6177(10)
0.8	1.27	0.6104(10)	0.6166(08)	0.6210(10)	0.6250(10)	0.6260(10)	0.6262(10)	0.6269(10)
0.8605	1.4	0.6146(10)	0.6224(10)	0.6271(10)	0.6302(08)	0.6321(10)	0.6341(09)	0.6337(11)
0.9	1.52	0.6184(10)	0.6251(12)	0.6317(11)	0.6344(10)	0.6378(10)	0.6381(09)	0.6385(12)

Table A.29: Case II Weibull Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.6282(11)	0.6303(11)	0.6324(09)	0.6334(11)	0.6343(11)	0.6347(10)	0.6345(11)
0.0323	0.18	0.6199(09)	0.6214(12)	0.6237(10)	0.6241(09)	0.6250(10)	0.6245(09)	0.6252(12)
0.1	0.32	0.6280(10)	0.6303(10)	0.6316(11)	0.6319(10)	0.6331(10)	0.6336(11)	0.6336(10)
0.2	0.47	0.6392(10)	0.6436(10)	0.6447(09)	0.6461(11)	0.6468(10)	0.6465(12)	0.6472(11)
0.3	0.6	0.6517(12)	0.6539(11)	0.6559(11)	0.6588(11)	0.6595(11)	0.6587(11)	0.6596(12)
0.4	0.71	0.6605(11)	0.6646(10)	0.6693(11)	0.6699(11)	0.6711(11)	0.6722(11)	0.6720(12)
0.5	0.83	0.6698(13)	0.6735(13)	0.6773(11)	0.6796(13)	0.6819(12)	0.6821(13)	0.6824(12)
0.6	0.96	0.6784(12)	0.6828(13)	0.6872(12)	0.6899(12)	0.6921(13)	0.6929(11)	0.6944(12)
0.7	1.1	0.6879(11)	0.6941(12)	0.6967(14)	0.7003(14)	0.7021(13)	0.7027(13)	0.7034(14)
0.8	1.27	0.6952(14)	0.7024(11)	0.7079(13)	0.7119(13)	0.7137(12)	0.7130(12)	0.7138(13)
0.8605	1.4	0.7006(13)	0.7095(13)	0.7140(13)	0.7184(12)	0.7206(14)	0.7225(13)	0.7226(16)
0.9	1.52	0.7058(14)	0.7132(16)	0.7208(14)	0.7232(13)	0.7266(13)	0.7280(13)	0.7286(14)

Table A.30: Case II Weibull Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7466(15)	0.7491(15)	0.7528(12)	0.7547(17)	0.7565(15)	0.7566(15)	0.7571(16)
0.0323	0.18	0.7386(15)	0.7410(17)	0.7443(15)	0.7446(14)	0.7459(16)	0.7454(14)	0.7457(16)
0.1	0.32	0.7493(16)	0.7534(15)	0.7551(17)	0.7558(16)	0.7570(14)	0.7565(16)	0.7587(17)
0.2	0.47	0.7666(16)	0.7721(14)	0.7720(15)	0.7753(17)	0.7752(16)	0.7754(16)	0.7766(13)
0.3	0.6	0.7846(16)	0.7867(16)	0.7893(15)	0.7928(16)	0.7936(17)	0.7917(16)	0.7935(15)
0.4	0.71	0.7966(16)	0.8013(16)	0.8068(18)	0.8069(16)	0.8083(16)	0.8095(17)	0.8092(18)
0.5	0.83	0.8103(19)	0.8148(18)	0.8180(17)	0.8203(18)	0.8233(19)	0.8227(18)	0.8228(18)
0.6	0.96	0.8212(19)	0.8260(18)	0.8323(19)	0.8349(16)	0.8362(18)	0.8376(18)	0.8399(19)
0.7	1.1	0.8346(18)	0.8426(22)	0.8452(19)	0.8478(19)	0.8512(18)	0.8514(22)	0.8524(19)
0.8	1.27	0.8446(20)	0.8521(15)	0.8581(18)	0.8645(21)	0.8667(19)	0.8652(19)	0.8665(19)
0.8605	1.4	0.8519(19)	0.8625(19)	0.8669(20)	0.8724(16)	0.8747(20)	0.8775(19)	0.8766(22)
0.9	1.52	0.8589(22)	0.8672(21)	0.8769(20)	0.8791(20)	0.8831(19)	0.8854(20)	0.8849(21)

Table A.31: Case II Weibull Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0196(31)	1.0320(35)	1.0359(33)	1.0395(37)	1.0402(33)	1.0417(34)	1.0420(40)
0.0323	0.18	1.0163(41)	1.0245(36)	1.0257(33)	1.0263(36)	1.0320(37)	1.0279(34)	1.0309(36)
0.1	0.32	1.0415(35)	1.0470(36)	1.0480(35)	1.0487(39)	1.0473(40)	1.0491(39)	1.0514(40)
0.2	0.47	1.0747(43)	1.0792(40)	1.0806(40)	1.0823(40)	1.0816(41)	1.0828(36)	1.0827(39)
0.3	0.6	1.1049(39)	1.1060(37)	1.1086(42)	1.1138(41)	1.1118(43)	1.1131(44)	1.1128(40)
0.4	0.71	1.1272(42)	1.1332(42)	1.1354(39)	1.1353(39)	1.1395(41)	1.1386(38)	1.1418(51)
0.5	0.83	1.1537(43)	1.1551(46)	1.1561(43)	1.1626(45)	1.1639(42)	1.1646(39)	1.1600(41)
0.6	0.96	1.1698(45)	1.1770(43)	1.1842(48)	1.1826(34)	1.1878(41)	1.1872(46)	1.1851(41)
0.7	1.1	1.1984(46)	1.2031(44)	1.2043(45)	1.2112(50)	1.2094(49)	1.2095(47)	1.2107(47)
0.8	1.27	1.2142(43)	1.2173(44)	1.2268(42)	1.2320(42)	1.2334(46)	1.2349(43)	1.2329(44)
0.8605	1.4	1.2265(55)	1.2358(42)	1.2440(45)	1.2485(48)	1.2477(47)	1.2584(49)	1.2485(42)
0.9	1.52	1.2347(48)	1.2464(50)	1.2597(51)	1.2620(49)	1.2664(45)	1.2659(44)	1.2658(51)

Table A.32: Case II Weibull Anderson-Darling 99% critical values

A.2.2 Loglogistic Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.6740(04)	0.6830(05)	0.6907(05)	0.6961(05)	0.7012(04)	0.7036(04)	0.7072(05)
0.0323	0.18	0.6687(05)	0.6775(04)	0.6854(04)	0.6913(05)	0.6961(04)	0.6982(05)	0.7019(04)
0.1	0.33	0.6719(04)	0.6806(05)	0.6891(04)	0.6941(05)	0.6993(04)	0.7022(04)	0.7053(05)
0.2	0.5	0.6774(04)	0.6864(04)	0.6943(04)	0.7000(05)	0.7047(05)	0.7075(05)	0.7117(05)
0.3	0.65	0.6818(04)	0.6909(05)	0.6997(05)	0.7056(05)	0.7104(04)	0.7130(04)	0.7162(05)
0.4	0.82	0.6846(04)	0.6948(05)	0.7040(06)	0.7098(05)	0.7145(05)	0.7171(05)	0.7208(05)
0.5	1	0.6875(05)	0.6974(05)	0.7078(04)	0.7139(05)	0.7184(05)	0.7207(05)	0.7246(05)
0.6	1.22	0.6889(05)	0.7000(04)	0.7098(05)	0.7163(04)	0.7218(05)	0.7239(06)	0.7278(04)
0.7	1.53	0.6904(05)	0.7011(05)	0.7119(05)	0.7189(05)	0.7240(05)	0.7269(05)	0.7306(05)
0.8	2	0.6916(05)	0.7025(05)	0.7139(04)	0.7200(05)	0.7268(05)	0.7294(05)	0.7331(05)
0.8605	2.48	0.6920(04)	0.7032(04)	0.7140(05)	0.7211(05)	0.7275(04)	0.7305(05)	0.7344(05)
0.9	3	0.6922(05)	0.7038(04)	0.7140(05)	0.7220(05)	0.7281(05)	0.7309(05)	0.7357(05)

Table A.33: Case II Loglogistic Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7101(05)	0.7199(06)	0.7280(05)	0.7334(06)	0.7388(05)	0.7412(05)	0.7450(06)
0.0323	0.18	0.7047(05)	0.7139(05)	0.7225(05)	0.7286(05)	0.7334(05)	0.7356(06)	0.7394(05)
0.1	0.33	0.7079(05)	0.7174(05)	0.7262(05)	0.7316(06)	0.7368(05)	0.7398(05)	0.7429(06)
0.2	0.5	0.7139(05)	0.7236(06)	0.7322(05)	0.7381(05)	0.7428(06)	0.7457(06)	0.7496(05)
0.3	0.65	0.7186(05)	0.7285(05)	0.7380(06)	0.7441(06)	0.7490(05)	0.7514(05)	0.7548(05)
0.4	0.82	0.7219(05)	0.7331(05)	0.7425(06)	0.7486(06)	0.7540(06)	0.7564(06)	0.7600(06)
0.5	1	0.7251(05)	0.7357(05)	0.7468(05)	0.7531(06)	0.7579(05)	0.7604(06)	0.7639(06)
0.6	1.22	0.7266(05)	0.7385(05)	0.7489(06)	0.7554(06)	0.7611(06)	0.7638(06)	0.7674(05)
0.7	1.53	0.7282(05)	0.7397(06)	0.7511(06)	0.7586(06)	0.7639(06)	0.7669(06)	0.7705(05)
0.8	2	0.7295(06)	0.7410(05)	0.7533(05)	0.7599(06)	0.7670(05)	0.7697(05)	0.7734(05)
0.8605	2.48	0.7300(05)	0.7417(05)	0.7532(05)	0.7609(06)	0.7676(06)	0.7707(06)	0.7747(05)
0.9	3	0.7299(05)	0.7424(05)	0.7539(06)	0.7622(06)	0.7684(06)	0.7714(05)	0.7763(06)

Table A.34: Case II Loglogistic Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7661(06)	0.7774(07)	0.7860(07)	0.7916(08)	0.7976(07)	0.8002(07)	0.8039(07)
0.0323	0.18	0.7602(06)	0.7711(07)	0.7799(07)	0.7871(07)	0.7915(06)	0.7942(08)	0.7981(06)
0.1	0.33	0.7640(07)	0.7746(07)	0.7840(07)	0.7898(07)	0.7952(07)	0.7982(07)	0.8017(07)
0.2	0.5	0.7705(06)	0.7809(06)	0.7912(07)	0.7972(06)	0.8017(07)	0.8048(08)	0.8095(08)
0.3	0.65	0.7761(07)	0.7869(07)	0.7975(08)	0.8032(07)	0.8090(06)	0.8117(07)	0.8148(07)
0.4	0.82	0.7795(07)	0.7919(07)	0.8024(08)	0.8094(08)	0.8144(07)	0.8171(08)	0.8211(07)
0.5	1	0.7832(07)	0.7952(08)	0.8071(07)	0.8138(07)	0.8192(07)	0.8210(07)	0.8253(07)
0.6	1.22	0.7843(06)	0.7981(07)	0.8098(07)	0.8164(07)	0.8224(07)	0.8248(07)	0.8292(07)
0.7	1.53	0.7865(07)	0.7991(07)	0.8124(08)	0.8205(07)	0.8258(07)	0.8287(08)	0.8326(07)
0.8	2	0.7880(07)	0.8015(07)	0.8146(08)	0.8215(07)	0.8292(07)	0.8319(07)	0.8352(08)
0.8605	2.48	0.7885(07)	0.8015(08)	0.8144(08)	0.8227(07)	0.8301(07)	0.8328(08)	0.8375(07)
0.9	3	0.7884(07)	0.8028(07)	0.8157(08)	0.8242(08)	0.8310(07)	0.8339(08)	0.8393(07)

Table A.35: Case II Loglogistic Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8780(11)	0.8918(14)	0.9015(15)	0.9093(13)	0.9160(14)	0.9180(13)	0.9227(14)
0.0323	0.18	0.8715(12)	0.8858(12)	0.8958(14)	0.9041(14)	0.9100(13)	0.9124(12)	0.9171(14)
0.1	0.33	0.8754(14)	0.8884(14)	0.9005(13)	0.9078(13)	0.9128(13)	0.9165(13)	0.9206(13)
0.2	0.5	0.8829(11)	0.8956(14)	0.9081(12)	0.9167(12)	0.9216(12)	0.9236(13)	0.9292(14)
0.3	0.65	0.8895(13)	0.9037(14)	0.9163(14)	0.9224(13)	0.9293(14)	0.9326(13)	0.9360(14)
0.4	0.82	0.8933(14)	0.9110(13)	0.9225(14)	0.9309(13)	0.9364(12)	0.9390(15)	0.9436(13)
0.5	1	0.8982(13)	0.9136(14)	0.9273(14)	0.9361(15)	0.9421(14)	0.9432(13)	0.9480(15)
0.6	1.22	0.9000(13)	0.9157(15)	0.9300(13)	0.9383(14)	0.9470(13)	0.9484(14)	0.9538(14)
0.7	1.53	0.9024(12)	0.9195(15)	0.9341(13)	0.9438(15)	0.9508(15)	0.9532(15)	0.9573(13)
0.8	2	0.9037(14)	0.9210(12)	0.9375(13)	0.9452(15)	0.9530(13)	0.9574(14)	0.9616(15)
0.8605	2.48	0.9042(14)	0.9205(14)	0.9381(16)	0.9477(15)	0.9560(13)	0.9591(16)	0.9641(11)
0.9	3	0.9040(11)	0.9238(12)	0.9375(13)	0.9480(15)	0.9562(15)	0.9598(12)	0.9666(15)

Table A.36: Case II Loglogistic Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2124(07)	1.2268(07)	1.2407(07)	1.2506(08)	1.2598(07)	1.2646(07)	1.2720(08)
0.0323	0.18	1.2030(07)	1.2182(08)	1.2312(07)	1.2424(08)	1.2507(07)	1.2551(08)	1.2623(08)
0.1	0.33	1.2106(07)	1.2246(08)	1.2396(07)	1.2488(08)	1.2581(07)	1.2630(07)	1.2701(08)
0.2	0.5	1.2207(07)	1.2364(08)	1.2502(08)	1.2605(07)	1.2691(08)	1.2745(07)	1.2818(07)
0.3	0.65	1.2297(07)	1.2451(08)	1.2603(08)	1.2713(08)	1.2800(07)	1.2848(07)	1.2916(07)
0.4	0.82	1.2358(07)	1.2529(09)	1.2691(08)	1.2791(07)	1.2882(09)	1.2928(08)	1.3003(09)
0.5	1	1.2410(08)	1.2579(09)	1.2753(07)	1.2865(08)	1.2954(07)	1.3000(08)	1.3074(08)
0.6	1.22	1.2431(08)	1.2623(07)	1.2795(08)	1.2917(07)	1.3014(08)	1.3054(08)	1.3127(08)
0.7	1.53	1.2461(08)	1.2653(08)	1.2837(08)	1.2967(08)	1.3058(09)	1.3112(08)	1.3184(08)
0.8	2	1.2482(08)	1.2678(09)	1.2872(08)	1.2985(07)	1.3108(08)	1.3158(08)	1.3227(08)
0.8605	2.48	1.2499(07)	1.2685(08)	1.2875(08)	1.3006(09)	1.3121(08)	1.3182(09)	1.3257(08)
0.9	3	1.2496(08)	1.2697(08)	1.2881(09)	1.3020(08)	1.3128(08)	1.3191(08)	1.3274(09)

Table A.37: Case II Loglogistic Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2721(07)	1.2873(09)	1.3020(08)	1.3121(09)	1.3215(09)	1.3260(08)	1.3335(09)
0.0323	0.18	1.2621(09)	1.2783(08)	1.2915(07)	1.3031(08)	1.3118(07)	1.3164(09)	1.3239(08)
0.1	0.33	1.2700(07)	1.2855(09)	1.3009(08)	1.3105(09)	1.3193(08)	1.3250(09)	1.3323(09)
0.2	0.5	1.2824(08)	1.2985(10)	1.3131(09)	1.3236(09)	1.3329(10)	1.3376(08)	1.3455(09)
0.3	0.65	1.2926(08)	1.3087(08)	1.3246(09)	1.3353(10)	1.3444(08)	1.3494(08)	1.3562(08)
0.4	0.82	1.2990(09)	1.3171(09)	1.3338(10)	1.3444(09)	1.3540(10)	1.3582(09)	1.3663(10)
0.5	1	1.3050(09)	1.3229(11)	1.3411(08)	1.3525(10)	1.3615(09)	1.3662(09)	1.3736(09)
0.6	1.22	1.3075(09)	1.3280(09)	1.3462(10)	1.3583(09)	1.3683(09)	1.3717(09)	1.3800(09)
0.7	1.53	1.3108(09)	1.3306(09)	1.3508(10)	1.3641(10)	1.3734(10)	1.3791(09)	1.3858(09)
0.8	2	1.3126(10)	1.3336(09)	1.3545(08)	1.3659(08)	1.3789(09)	1.3840(09)	1.3909(08)
0.8605	2.48	1.3147(09)	1.3349(09)	1.3547(10)	1.3680(11)	1.3803(09)	1.3858(10)	1.3940(09)
0.9	3	1.3147(10)	1.3361(09)	1.3556(09)	1.3703(10)	1.3813(09)	1.3870(09)	1.3963(09)

Table A.38: Case II Loglogistic Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3634(10)	1.3810(12)	1.3960(10)	1.4064(12)	1.4169(11)	1.4212(10)	1.4290(11)
0.0323	0.18	1.3542(11)	1.3709(11)	1.3852(10)	1.3976(12)	1.4060(10)	1.4109(11)	1.4184(10)
0.1	0.33	1.3632(10)	1.3794(12)	1.3960(12)	1.4059(12)	1.4154(11)	1.4217(11)	1.4281(12)
0.2	0.5	1.3779(11)	1.3945(12)	1.4112(11)	1.4220(11)	1.4308(13)	1.4366(11)	1.4444(12)
0.3	0.65	1.3898(11)	1.4071(12)	1.4242(13)	1.4348(13)	1.4447(10)	1.4500(13)	1.4567(12)
0.4	0.82	1.3972(12)	1.4176(13)	1.4348(13)	1.4459(13)	1.4561(13)	1.4606(13)	1.4685(12)
0.5	1	1.4039(12)	1.4245(15)	1.4436(09)	1.4555(12)	1.4653(12)	1.4684(14)	1.4770(12)
0.6	1.22	1.4076(11)	1.4292(13)	1.4493(12)	1.4622(12)	1.4727(12)	1.4764(14)	1.4847(12)
0.7	1.53	1.4112(10)	1.4335(13)	1.4540(13)	1.4685(13)	1.4786(14)	1.4842(14)	1.4908(12)
0.8	2	1.4135(14)	1.4360(11)	1.4590(12)	1.4710(12)	1.4845(12)	1.4904(13)	1.4972(13)
0.8605	2.48	1.4149(12)	1.4374(13)	1.4593(13)	1.4740(13)	1.4870(13)	1.4923(12)	1.5011(13)
0.9	3	1.4148(13)	1.4392(11)	1.4603(13)	1.4757(14)	1.4885(14)	1.4941(12)	1.5032(12)

Table A.39: Case II Loglogistic Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5471(20)	1.5660(22)	1.5836(21)	1.5964(23)	1.6071(25)	1.6095(22)	1.6208(24)
0.0323	0.18	1.5360(20)	1.5553(18)	1.5718(20)	1.5862(22)	1.5943(19)	1.6019(18)	1.6082(22)
0.1	0.33	1.5481(22)	1.5669(21)	1.5878(23)	1.5994(22)	1.6077(23)	1.6123(20)	1.6206(23)
0.2	0.5	1.5691(19)	1.5884(22)	1.6075(19)	1.6177(19)	1.6273(24)	1.6324(22)	1.6436(23)
0.3	0.65	1.5848(20)	1.6054(22)	1.6253(21)	1.6361(22)	1.6467(22)	1.6518(25)	1.6576(25)
0.4	0.82	1.5936(23)	1.6196(24)	1.6374(26)	1.6513(21)	1.6617(24)	1.6651(23)	1.6735(23)
0.5	1	1.6034(23)	1.6286(21)	1.6487(23)	1.6635(23)	1.6729(22)	1.6762(23)	1.6830(22)
0.6	1.22	1.6087(26)	1.6338(27)	1.6574(22)	1.6699(23)	1.6841(23)	1.6896(25)	1.6973(24)
0.7	1.53	1.6128(22)	1.6393(26)	1.6640(25)	1.6810(26)	1.6926(22)	1.6955(23)	1.7032(22)
0.8	2	1.6144(23)	1.6435(26)	1.6695(21)	1.6841(24)	1.6982(20)	1.7046(25)	1.7125(26)
0.8605	2.48	1.6170(22)	1.6443(22)	1.6705(26)	1.6879(26)	1.7017(24)	1.7090(26)	1.7170(21)
0.9	3	1.6174(23)	1.6484(22)	1.6701(25)	1.6895(28)	1.7031(23)	1.7113(22)	1.7234(25)

Table A.40: Case II Loglogistic Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.07182(11)	0.07178(11)	0.07176(12)	0.07166(12)	0.07170(10)	0.07176(12)	0.07176(12)
0.0323	0.18	0.07041(11)	0.07042(11)	0.07020(11)	0.07023(11)	0.07020(10)	0.07024(11)	0.07012(10)
0.1	0.33	0.07121(11)	0.07112(11)	0.07120(11)	0.07100(11)	0.07104(11)	0.07115(12)	0.07106(11)
0.2	0.5	0.07261(11)	0.07270(11)	0.07269(11)	0.07267(11)	0.07254(11)	0.07278(11)	0.07278(13)
0.3	0.65	0.07388(11)	0.07398(12)	0.07413(12)	0.07419(14)	0.07418(12)	0.07425(12)	0.07415(11)
0.4	0.82	0.07479(12)	0.07519(12)	0.07548(12)	0.07548(12)	0.07549(13)	0.07553(13)	0.07549(13)
0.5	1	0.07555(13)	0.07606(13)	0.07661(10)	0.07672(13)	0.07670(10)	0.07675(12)	0.07670(13)
0.6	1.22	0.07601(13)	0.07672(12)	0.07728(12)	0.07760(12)	0.07766(13)	0.07774(13)	0.07775(12)
0.7	1.53	0.07642(12)	0.07722(12)	0.07806(13)	0.07844(11)	0.07860(14)	0.07859(14)	0.07870(13)
0.8	2	0.07675(14)	0.07762(13)	0.07852(12)	0.07881(11)	0.07933(14)	0.07952(13)	0.07945(14)
0.8605	2.48	0.07694(13)	0.07772(12)	0.07862(13)	0.07915(13)	0.07968(12)	0.07994(14)	0.07998(13)
0.9	3	0.07697(14)	0.07798(12)	0.07869(14)	0.07936(13)	0.07985(13)	0.08005(14)	0.08031(15)

Table A.41: Case II Loglogistic Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.08126(13)	0.08125(15)	0.08118(14)	0.08123(16)	0.08129(14)	0.08137(14)	0.08133(14)
0.0323	0.18	0.07969(13)	0.07973(14)	0.07954(14)	0.07964(13)	0.07961(14)	0.07962(14)	0.07954(12)
0.1	0.33	0.08059(13)	0.08053(14)	0.08068(13)	0.08054(14)	0.08059(13)	0.08069(14)	0.08057(15)
0.2	0.5	0.08239(14)	0.08245(16)	0.08254(14)	0.08259(15)	0.08241(16)	0.08272(14)	0.08269(16)
0.3	0.65	0.08389(14)	0.08411(15)	0.08439(15)	0.08437(17)	0.08444(15)	0.08452(15)	0.08445(14)
0.4	0.82	0.08497(16)	0.08561(15)	0.08595(16)	0.08600(15)	0.08605(17)	0.08607(16)	0.08611(17)
0.5	1	0.08596(16)	0.08668(18)	0.08725(14)	0.08750(18)	0.08753(14)	0.08758(16)	0.08749(15)
0.6	1.22	0.08646(14)	0.08741(17)	0.08817(16)	0.08862(15)	0.08871(17)	0.08871(16)	0.08879(16)
0.7	1.53	0.08700(16)	0.08797(16)	0.08903(17)	0.08962(15)	0.08978(19)	0.08991(18)	0.08990(16)
0.8	2	0.08739(16)	0.08853(15)	0.08975(15)	0.09002(15)	0.09076(19)	0.09091(16)	0.09087(17)
0.8605	2.48	0.08760(16)	0.08863(17)	0.08978(17)	0.09047(19)	0.09106(17)	0.09143(17)	0.09143(17)
0.9	3	0.08759(15)	0.08896(16)	0.08989(17)	0.09074(17)	0.09136(17)	0.09160(16)	0.09188(18)

Table A.42: Case II Loglogistic Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.09709(20)	0.09732(23)	0.09748(18)	0.09762(23)	0.09749(20)	0.09771(20)	0.09777(18)
0.0323	0.18	0.09541(18)	0.09554(19)	0.09531(20)	0.09569(22)	0.09573(18)	0.09570(20)	0.09558(20)
0.1	0.33	0.09664(22)	0.09669(20)	0.09699(19)	0.09694(18)	0.09694(20)	0.09708(19)	0.09696(22)
0.2	0.5	0.09912(19)	0.09921(21)	0.09949(22)	0.09949(21)	0.09939(21)	0.09970(20)	0.09969(21)
0.3	0.65	0.10103(20)	0.10158(21)	0.10182(22)	0.10191(24)	0.10208(23)	0.10215(21)	0.10206(22)
0.4	0.82	0.10242(21)	0.10353(23)	0.10392(24)	0.10418(23)	0.10417(24)	0.10423(25)	0.10434(24)
0.5	1	0.10388(22)	0.10488(25)	0.10561(23)	0.10603(23)	0.10612(20)	0.10603(25)	0.10614(25)
0.6	1.22	0.10434(21)	0.10586(23)	0.10703(25)	0.10743(22)	0.10785(23)	0.10773(24)	0.10785(22)
0.7	1.53	0.10536(23)	0.10661(23)	0.10793(23)	0.10892(25)	0.10920(25)	0.10935(25)	0.10929(24)
0.8	2	0.10564(23)	0.10736(23)	0.10901(21)	0.10948(23)	0.11050(25)	0.11084(24)	0.11064(24)
0.8605	2.48	0.10588(22)	0.10755(26)	0.10914(23)	0.11001(26)	0.11106(25)	0.11145(29)	0.11157(24)
0.9	3	0.10587(23)	0.10789(20)	0.10935(27)	0.11054(27)	0.11131(26)	0.11174(23)	0.11206(25)

Table A.43: Case II Loglogistic Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.13382(46)	0.13424(46)	0.13483(44)	0.13539(50)	0.13541(47)	0.13553(52)	0.13562(50)
0.0323	0.18	0.13139(43)	0.13222(43)	0.13230(42)	0.13270(56)	0.13294(49)	0.13285(43)	0.13309(44)
0.1	0.33	0.13385(46)	0.13403(48)	0.13502(44)	0.13535(48)	0.13526(47)	0.13566(43)	0.13523(48)
0.2	0.5	0.13788(44)	0.13863(47)	0.13925(42)	0.13922(44)	0.13907(52)	0.13920(53)	0.14013(49)
0.3	0.65	0.14152(50)	0.14283(54)	0.14283(58)	0.14296(47)	0.14389(49)	0.14418(52)	0.14343(56)
0.4	0.82	0.14365(54)	0.14574(56)	0.14659(56)	0.14692(51)	0.14737(54)	0.14736(58)	0.14747(47)
0.5	1	0.14615(52)	0.14816(50)	0.14937(50)	0.15031(67)	0.15037(48)	0.14996(54)	0.14990(52)
0.6	1.22	0.14702(50)	0.14967(56)	0.15163(54)	0.15255(59)	0.15356(57)	0.15356(68)	0.15336(54)
0.7	1.53	0.14867(46)	0.15101(57)	0.15346(53)	0.15495(60)	0.15588(55)	0.15596(58)	0.15571(59)
0.8	2	0.14903(44)	0.15207(54)	0.15529(62)	0.15604(59)	0.15727(52)	0.15827(52)	0.15790(63)
0.8605	2.48	0.14943(57)	0.15223(48)	0.15487(60)	0.15713(59)	0.15845(61)	0.15934(51)	0.15974(58)
0.9	3	0.14951(58)	0.15321(49)	0.15550(52)	0.15723(57)	0.15899(60)	0.15995(48)	0.16118(59)

Table A.44: Case II Loglogistic Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4998(07)	0.5010(07)	0.5011(07)	0.5013(07)	0.5021(07)	0.5021(07)	0.5021(07)
0.0323	0.18	0.4973(07)	0.4974(07)	0.4964(07)	0.4968(08)	0.4969(06)	0.4976(07)	0.4962(06)
0.1	0.33	0.5031(07)	0.5030(07)	0.5040(06)	0.5032(07)	0.5035(07)	0.5038(07)	0.5037(07)
0.2	0.5	0.5120(07)	0.5125(07)	0.5133(07)	0.5131(06)	0.5126(07)	0.5142(07)	0.5137(08)
0.3	0.65	0.5190(07)	0.5197(08)	0.5209(08)	0.5213(08)	0.5218(07)	0.5220(07)	0.5216(07)
0.4	0.82	0.5233(07)	0.5266(07)	0.5286(08)	0.5289(07)	0.5289(08)	0.5292(08)	0.5294(08)
0.5	1	0.5283(07)	0.5314(09)	0.5340(07)	0.5353(08)	0.5356(07)	0.5357(08)	0.5357(08)
0.6	1.22	0.5309(08)	0.5352(07)	0.5384(07)	0.5406(08)	0.5413(09)	0.5406(08)	0.5413(08)
0.7	1.53	0.5332(07)	0.5379(09)	0.5419(08)	0.5442(08)	0.5457(09)	0.5461(09)	0.5460(08)
0.8	2	0.5349(08)	0.5401(08)	0.5453(08)	0.5462(08)	0.5496(08)	0.5516(08)	0.5508(08)
0.8605	2.48	0.5361(08)	0.5404(09)	0.5461(08)	0.5483(08)	0.5516(09)	0.5536(08)	0.5537(09)
0.9	3	0.5360(09)	0.5422(08)	0.5462(09)	0.5497(09)	0.5526(09)	0.5541(10)	0.5549(09)

Table A.45: Case II Loglogistic Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.5589(08)	0.5601(09)	0.5608(08)	0.5616(09)	0.5620(08)	0.5629(08)	0.5628(09)
0.0323	0.18	0.5568(08)	0.5570(09)	0.5560(08)	0.5570(09)	0.5572(08)	0.5574(09)	0.5561(09)
0.1	0.33	0.5643(09)	0.5641(09)	0.5657(09)	0.5647(09)	0.5648(09)	0.5657(09)	0.5651(09)
0.2	0.5	0.5756(08)	0.5759(10)	0.5768(09)	0.5763(09)	0.5757(09)	0.5776(09)	0.5769(10)
0.3	0.65	0.5839(09)	0.5848(10)	0.5865(10)	0.5868(10)	0.5875(09)	0.5873(10)	0.5874(09)
0.4	0.82	0.5897(09)	0.5933(10)	0.5955(10)	0.5956(09)	0.5957(09)	0.5960(10)	0.5971(11)
0.5	1	0.5960(09)	0.5987(10)	0.6025(09)	0.6043(10)	0.6040(09)	0.6043(11)	0.6039(10)
0.6	1.22	0.5985(09)	0.6040(11)	0.6076(11)	0.6096(09)	0.6112(11)	0.6106(10)	0.6104(10)
0.7	1.53	0.6022(09)	0.6070(10)	0.6122(10)	0.6155(09)	0.6165(11)	0.6174(11)	0.6169(10)
0.8	2	0.6038(11)	0.6102(11)	0.6165(11)	0.6173(10)	0.6220(12)	0.6234(11)	0.6224(10)
0.8605	2.48	0.6056(09)	0.6106(11)	0.6171(10)	0.6201(11)	0.6235(11)	0.6255(11)	0.6261(11)
0.9	3	0.6053(11)	0.6123(10)	0.6170(11)	0.6218(11)	0.6247(10)	0.6268(12)	0.6278(11)

Table A.46: Case II Loglogistic Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.6594(12)	0.6614(14)	0.6632(12)	0.6634(15)	0.6647(13)	0.6653(14)	0.6652(13)
0.0323	0.18	0.6580(12)	0.6589(14)	0.6572(12)	0.6598(13)	0.6592(12)	0.6595(13)	0.6589(13)
0.1	0.33	0.6695(13)	0.6689(14)	0.6707(13)	0.6699(12)	0.6699(13)	0.6705(13)	0.6698(13)
0.2	0.5	0.6857(14)	0.6845(14)	0.6851(13)	0.6855(13)	0.6849(14)	0.6865(14)	0.6869(13)
0.3	0.65	0.6972(12)	0.6987(13)	0.6995(14)	0.6995(15)	0.7006(13)	0.6996(14)	0.6996(15)
0.4	0.82	0.7055(14)	0.7091(15)	0.7108(16)	0.7113(13)	0.7110(15)	0.7114(15)	0.7127(15)
0.5	1	0.7133(14)	0.7166(15)	0.7203(14)	0.7224(15)	0.7217(13)	0.7224(15)	0.7222(16)
0.6	1.22	0.7161(14)	0.7235(16)	0.7280(15)	0.7296(14)	0.7319(14)	0.7312(14)	0.7317(15)
0.7	1.53	0.7219(14)	0.7270(15)	0.7340(15)	0.7376(15)	0.7392(15)	0.7400(15)	0.7388(14)
0.8	2	0.7231(16)	0.7316(16)	0.7394(15)	0.7403(15)	0.7460(16)	0.7479(15)	0.7468(15)
0.8605	2.48	0.7256(15)	0.7314(16)	0.7400(13)	0.7440(16)	0.7496(16)	0.7513(18)	0.7523(15)
0.9	3	0.7252(17)	0.7345(16)	0.7399(17)	0.7469(16)	0.7507(16)	0.7525(14)	0.7541(16)

Table A.47: Case II Loglogistic Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8917(29)	0.8956(30)	0.8981(28)	0.9035(30)	0.8998(31)	0.9050(28)	0.9027(32)
0.0323	0.18	0.8962(27)	0.8975(32)	0.8959(29)	0.8984(30)	0.8979(29)	0.8984(28)	0.8999(29)
0.1	0.33	0.9199(30)	0.9163(32)	0.9210(32)	0.9177(33)	0.9168(32)	0.9183(31)	0.9161(34)
0.2	0.5	0.9473(32)	0.9461(34)	0.9462(31)	0.9424(35)	0.9407(30)	0.9422(34)	0.9465(33)
0.3	0.65	0.9716(38)	0.9701(37)	0.9657(35)	0.9656(30)	0.9682(26)	0.9688(33)	0.9657(34)
0.4	0.82	0.9831(33)	0.9893(37)	0.9893(35)	0.9864(33)	0.9885(30)	0.9885(35)	0.9853(32)
0.5	1	0.9979(35)	1.0001(39)	1.0035(33)	1.0043(36)	1.0021(33)	1.0008(38)	1.0015(32)
0.6	1.22	1.0017(38)	1.0091(33)	1.0162(34)	1.0176(36)	1.0211(36)	1.0226(35)	1.0201(36)
0.7	1.53	1.0085(35)	1.0165(39)	1.0235(33)	1.0301(37)	1.0362(40)	1.0340(29)	1.0347(38)
0.8	2	1.0172(37)	1.0212(37)	1.0317(35)	1.0357(39)	1.0437(34)	1.0474(31)	1.0438(37)
0.8605	2.48	1.0154(37)	1.0241(38)	1.0343(40)	1.0452(40)	1.0485(39)	1.0544(35)	1.0524(37)
0.9	3	1.0180(36)	1.0319(34)	1.0350(34)	1.0436(40)	1.0523(36)	1.0540(32)	1.0605(32)

Table A.48: Case II Loglogistic Anderson-Darling 99% critical values

A.2.3 Lognormal Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.7569(05)	0.7646(05)	0.7718(05)	0.7772(05)	0.7822(05)	0.7840(06)	0.7878(06)
0.0323	0.72	0.7323(05)	0.7394(06)	0.7465(05)	0.7519(06)	0.7560(05)	0.7582(06)	0.7614(05)
0.1	0.75	0.7188(05)	0.7258(05)	0.7328(05)	0.7373(05)	0.7420(05)	0.7437(05)	0.7475(05)
0.2	0.79	0.7132(05)	0.7193(05)	0.7265(05)	0.7310(05)	0.7355(05)	0.7376(05)	0.7409(06)
0.3	0.85	0.7123(05)	0.7190(05)	0.7252(05)	0.7306(05)	0.7350(05)	0.7368(05)	0.7406(05)
0.4	0.91	0.7130(05)	0.7209(05)	0.7277(05)	0.7324(05)	0.7367(05)	0.7387(05)	0.7430(05)
0.5	1	0.7158(05)	0.7231(05)	0.7305(05)	0.7357(05)	0.7406(05)	0.7426(04)	0.7466(05)
0.6	1.12	0.7187(05)	0.7271(05)	0.7345(05)	0.7406(05)	0.7447(05)	0.7474(05)	0.7507(05)
0.7	1.29	0.7214(05)	0.7310(05)	0.7396(04)	0.7456(05)	0.7501(05)	0.7527(06)	0.7564(05)
0.8	1.58	0.7246(05)	0.7352(05)	0.7455(05)	0.7517(06)	0.7565(05)	0.7592(05)	0.7627(05)
0.8605	1.89	0.7270(06)	0.7384(05)	0.7503(06)	0.7567(06)	0.7611(05)	0.7638(04)	0.7675(06)
0.9	2.24	0.7282(05)	0.7402(06)	0.7527(05)	0.7601(06)	0.7649(05)	0.7675(05)	0.7717(06)

Table A.49: Case II Lognormal Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.8020(06)	0.8104(07)	0.8181(06)	0.8237(06)	0.8284(06)	0.8306(07)	0.8345(07)
0.0323	0.72	0.7759(06)	0.7836(07)	0.7908(06)	0.7961(06)	0.8009(07)	0.8029(06)	0.8058(05)
0.1	0.75	0.7606(06)	0.7681(05)	0.7751(06)	0.7797(06)	0.7847(06)	0.7863(06)	0.7901(06)
0.2	0.79	0.7536(05)	0.7602(06)	0.7676(06)	0.7723(06)	0.7768(06)	0.7791(07)	0.7824(06)
0.3	0.85	0.7525(06)	0.7595(06)	0.7662(05)	0.7714(05)	0.7756(06)	0.7777(06)	0.7816(05)
0.4	0.91	0.7534(06)	0.7613(06)	0.7683(06)	0.7732(06)	0.7776(06)	0.7796(06)	0.7837(07)
0.5	1	0.7563(06)	0.7638(06)	0.7714(06)	0.7766(05)	0.7814(06)	0.7835(05)	0.7875(06)
0.6	1.12	0.7593(05)	0.7682(06)	0.7757(06)	0.7820(07)	0.7864(06)	0.7885(06)	0.7924(06)
0.7	1.29	0.7619(06)	0.7726(05)	0.7817(06)	0.7880(06)	0.7924(06)	0.7952(06)	0.7989(06)
0.8	1.58	0.7657(06)	0.7773(06)	0.7882(06)	0.7951(06)	0.7996(07)	0.8022(07)	0.8060(06)
0.8605	1.89	0.7687(06)	0.7809(06)	0.7931(07)	0.8004(07)	0.8046(06)	0.8077(06)	0.8111(06)
0.9	2.24	0.7700(06)	0.7831(06)	0.7964(06)	0.8042(07)	0.8092(06)	0.8113(06)	0.8161(07)

Table A.50: Case II Lognormal Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.8728(10)	0.8820(08)	0.8909(08)	0.8968(08)	0.9017(08)	0.9033(09)	0.9074(10)
0.0323	0.72	0.8443(07)	0.8525(09)	0.8604(09)	0.8655(09)	0.8706(09)	0.8727(09)	0.8758(07)
0.1	0.75	0.8252(07)	0.8342(07)	0.8416(09)	0.8458(08)	0.8513(07)	0.8526(08)	0.8570(09)
0.2	0.79	0.8164(08)	0.8238(08)	0.8319(09)	0.8368(08)	0.8413(08)	0.8436(08)	0.8473(08)
0.3	0.85	0.8147(07)	0.8221(07)	0.8297(08)	0.8343(07)	0.8394(07)	0.8410(08)	0.8450(08)
0.4	0.91	0.8155(08)	0.8243(08)	0.8315(08)	0.8366(08)	0.8411(07)	0.8430(08)	0.8477(07)
0.5	1	0.8185(07)	0.8269(07)	0.8352(08)	0.8405(07)	0.8451(08)	0.8470(07)	0.8513(08)
0.6	1.12	0.8217(07)	0.8321(08)	0.8401(07)	0.8465(09)	0.8511(07)	0.8528(08)	0.8568(07)
0.7	1.29	0.8256(06)	0.8369(08)	0.8468(08)	0.8536(08)	0.8581(08)	0.8604(08)	0.8650(08)
0.8	1.58	0.8299(07)	0.8430(07)	0.8554(08)	0.8617(08)	0.8663(08)	0.8693(07)	0.8727(08)
0.8605	1.89	0.8332(08)	0.8474(08)	0.8607(08)	0.8684(08)	0.8729(09)	0.8761(08)	0.8791(07)
0.9	2.24	0.8347(07)	0.8496(08)	0.8650(10)	0.8732(10)	0.8784(07)	0.8809(08)	0.8851(09)

Table A.51: Case II Lognormal Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.0150(15)	1.0286(17)	1.0383(17)	1.0457(18)	1.0503(16)	1.0523(17)	1.0556(18)
0.0323	0.72	0.9824(14)	0.9928(19)	1.0019(14)	1.0062(16)	1.0122(17)	1.0150(19)	1.0189(16)
0.1	0.75	0.9570(15)	0.9674(13)	0.9768(15)	0.9810(15)	0.9862(16)	0.9894(17)	0.9937(16)
0.2	0.79	0.9433(15)	0.9527(16)	0.9609(15)	0.9676(14)	0.9714(15)	0.9736(16)	0.9780(17)
0.3	0.85	0.9389(12)	0.9483(13)	0.9563(15)	0.9629(14)	0.9661(15)	0.9676(14)	0.9732(14)
0.4	0.91	0.9404(15)	0.9508(14)	0.9578(14)	0.9640(14)	0.9697(13)	0.9699(13)	0.9744(12)
0.5	1	0.9425(12)	0.9533(15)	0.9633(15)	0.9684(15)	0.9728(14)	0.9750(14)	0.9795(14)
0.6	1.12	0.9492(13)	0.9607(15)	0.9686(14)	0.9760(15)	0.9816(14)	0.9831(13)	0.9855(15)
0.7	1.29	0.9539(13)	0.9660(15)	0.9784(14)	0.9857(15)	0.9912(17)	0.9920(14)	0.9974(16)
0.8	1.58	0.9590(14)	0.9758(13)	0.9912(15)	0.9975(15)	1.0020(13)	1.0052(18)	1.0093(14)
0.8605	1.89	0.9632(15)	0.9827(14)	0.9987(17)	1.0067(17)	1.0108(15)	1.0135(14)	1.0178(14)
0.9	2.24	0.9652(15)	0.9837(13)	1.0043(15)	1.0125(17)	1.0190(16)	1.0208(17)	1.0271(18)

Table A.52: Case II Lognormal Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.2731(07)	1.2880(07)	1.3020(07)	1.3121(07)	1.3215(08)	1.3253(09)	1.3326(08)
0.0323	0.72	1.2552(08)	1.2690(08)	1.2834(08)	1.2932(09)	1.3019(08)	1.3062(09)	1.3129(07)
0.1	0.75	1.2553(08)	1.2704(08)	1.2848(08)	1.2942(08)	1.3033(08)	1.3070(08)	1.3153(08)
0.2	0.79	1.2617(08)	1.2764(07)	1.2917(08)	1.3010(09)	1.3105(08)	1.3146(09)	1.3213(09)
0.3	0.85	1.2677(08)	1.2835(09)	1.2971(08)	1.3082(08)	1.3172(09)	1.3210(08)	1.3287(07)
0.4	0.91	1.2734(08)	1.2896(08)	1.3039(08)	1.3144(08)	1.3233(08)	1.3270(09)	1.3356(09)
0.5	1	1.2790(07)	1.2940(08)	1.3097(08)	1.3199(09)	1.3295(09)	1.3336(08)	1.3416(08)
0.6	1.12	1.2834(08)	1.2997(09)	1.3149(09)	1.3263(09)	1.3349(07)	1.3398(08)	1.3468(08)
0.7	1.29	1.2872(07)	1.3048(08)	1.3203(09)	1.3316(09)	1.3410(07)	1.3455(08)	1.3527(08)
0.8	1.58	1.2919(08)	1.3094(08)	1.3262(08)	1.3377(10)	1.3468(09)	1.3516(08)	1.3584(08)
0.8605	1.89	1.2940(09)	1.3119(09)	1.3306(10)	1.3423(09)	1.3504(09)	1.3551(08)	1.3623(08)
0.9	2.24	1.2951(08)	1.3140(09)	1.3323(09)	1.3441(10)	1.3538(09)	1.3582(09)	1.3658(09)

Table A.53: Case II Lognormal Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.3372(09)	1.3530(09)	1.3678(08)	1.3782(09)	1.3877(09)	1.3913(10)	1.3991(10)
0.0323	0.72	1.3186(09)	1.3333(09)	1.3482(09)	1.3580(09)	1.3668(09)	1.3719(10)	1.3777(09)
0.1	0.75	1.3189(08)	1.3353(10)	1.3502(10)	1.3598(09)	1.3695(09)	1.3731(09)	1.3810(10)
0.2	0.79	1.3269(08)	1.3420(09)	1.3582(10)	1.3679(10)	1.3772(09)	1.3815(10)	1.3885(10)
0.3	0.85	1.3336(10)	1.3498(10)	1.3648(09)	1.3759(09)	1.3849(10)	1.3886(09)	1.3967(09)
0.4	0.91	1.3398(10)	1.3574(10)	1.3724(11)	1.3831(10)	1.3914(10)	1.3954(10)	1.4046(10)
0.5	1	1.3462(09)	1.3621(09)	1.3791(09)	1.3891(09)	1.3982(10)	1.4025(08)	1.4109(09)
0.6	1.12	1.3509(09)	1.3686(10)	1.3840(10)	1.3958(11)	1.4049(09)	1.4095(09)	1.4164(10)
0.7	1.29	1.3550(10)	1.3736(09)	1.3900(10)	1.4016(10)	1.4111(09)	1.4159(10)	1.4232(11)
0.8	1.58	1.3603(10)	1.3788(10)	1.3967(09)	1.4089(11)	1.4179(11)	1.4221(10)	1.4290(10)
0.8605	1.89	1.3631(10)	1.3819(11)	1.4013(11)	1.4133(11)	1.4215(10)	1.4268(09)	1.4339(10)
0.9	2.24	1.3637(08)	1.3833(09)	1.4032(11)	1.4155(12)	1.4253(10)	1.4295(11)	1.4379(10)

Table A.54: Case II Lognormal Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.4361(11)	1.4530(11)	1.4695(10)	1.4810(13)	1.4901(10)	1.4942(12)	1.5013(14)
0.0323	0.72	1.4167(11)	1.4329(12)	1.4479(12)	1.4582(11)	1.4677(12)	1.4722(13)	1.4788(12)
0.1	0.75	1.4178(12)	1.4359(13)	1.4511(13)	1.4613(12)	1.4714(13)	1.4752(11)	1.4834(13)
0.2	0.79	1.4272(12)	1.4436(12)	1.4615(13)	1.4714(13)	1.4812(12)	1.4858(14)	1.4927(14)
0.3	0.85	1.4360(12)	1.4533(13)	1.4700(13)	1.4811(12)	1.4906(13)	1.4932(12)	1.5024(14)
0.4	0.91	1.4428(13)	1.4619(13)	1.4781(15)	1.4890(13)	1.4985(12)	1.5023(12)	1.5112(14)
0.5	1	1.4499(12)	1.4675(12)	1.4853(13)	1.4971(12)	1.5065(13)	1.5097(11)	1.5180(13)
0.6	1.12	1.4559(12)	1.4755(13)	1.4912(13)	1.5041(16)	1.5140(12)	1.5185(13)	1.5252(12)
0.7	1.29	1.4608(13)	1.4803(12)	1.4986(13)	1.5112(14)	1.5204(14)	1.5255(13)	1.5335(14)
0.8	1.58	1.4658(12)	1.4871(13)	1.5056(12)	1.5191(13)	1.5280(13)	1.5341(12)	1.5398(12)
0.8605	1.89	1.4687(13)	1.4911(13)	1.5110(14)	1.5239(15)	1.5319(13)	1.5376(14)	1.5455(12)
0.9	2.24	1.4705(14)	1.4921(12)	1.5138(14)	1.5267(14)	1.5375(14)	1.5415(14)	1.5505(13)

Table A.55: Case II Lognormal Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.6319(23)	1.6526(22)	1.6715(19)	1.6852(24)	1.6926(21)	1.6973(24)	1.7047(22)
0.0323	0.72	1.6114(20)	1.6294(24)	1.6470(21)	1.6586(23)	1.6670(25)	1.6727(25)	1.6818(20)
0.1	0.75	1.6156(23)	1.6356(21)	1.6540(25)	1.6638(22)	1.6744(25)	1.6783(22)	1.6888(25)
0.2	0.79	1.6293(25)	1.6480(23)	1.6681(24)	1.6803(19)	1.6896(27)	1.6939(24)	1.7034(23)
0.3	0.85	1.6394(24)	1.6626(26)	1.6797(28)	1.6935(25)	1.7017(23)	1.7053(25)	1.7145(29)
0.4	0.91	1.6490(23)	1.6737(23)	1.6900(21)	1.7041(22)	1.7146(22)	1.7172(24)	1.7276(24)
0.5	1	1.6581(21)	1.6784(24)	1.7008(26)	1.7137(25)	1.7229(27)	1.7282(21)	1.7368(27)
0.6	1.12	1.6689(24)	1.6896(24)	1.7075(24)	1.7209(26)	1.7319(24)	1.7376(26)	1.7431(24)
0.7	1.29	1.6715(24)	1.6943(25)	1.7161(25)	1.7323(26)	1.7419(26)	1.7473(27)	1.7564(26)
0.8	1.58	1.6779(22)	1.7039(24)	1.7277(25)	1.7395(26)	1.7500(24)	1.7575(26)	1.7643(26)
0.8605	1.89	1.6838(24)	1.7117(23)	1.7322(26)	1.7460(25)	1.7566(29)	1.7632(25)	1.7696(23)
0.9	2.24	1.6844(26)	1.7116(22)	1.7360(27)	1.7495(28)	1.7643(25)	1.7681(27)	1.7754(28)

Table A.56: Case II Lognormal Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.09044(15)	0.09046(14)	0.09032(13)	0.09051(16)	0.09057(15)	0.09042(16)	0.09045(17)
0.0323	0.72	0.08403(15)	0.08376(14)	0.08369(13)	0.08351(14)	0.08353(14)	0.08340(14)	0.08329(12)
0.1	0.75	0.08162(13)	0.08160(14)	0.08132(14)	0.08115(12)	0.08104(15)	0.08099(13)	0.08111(14)
0.2	0.79	0.08127(14)	0.08100(12)	0.08094(11)	0.08067(14)	0.08073(13)	0.08073(15)	0.08061(13)
0.3	0.85	0.08172(14)	0.08150(13)	0.08138(14)	0.08141(13)	0.08132(13)	0.08123(13)	0.08128(13)
0.4	0.91	0.08246(13)	0.08245(14)	0.08230(15)	0.08219(13)	0.08214(14)	0.08212(12)	0.08234(15)
0.5	1	0.08342(11)	0.08341(15)	0.08338(14)	0.08335(14)	0.08339(14)	0.08340(14)	0.08349(13)
0.6	1.12	0.08430(13)	0.08462(15)	0.08460(14)	0.08469(15)	0.08476(12)	0.08471(14)	0.08472(16)
0.7	1.29	0.08515(15)	0.08576(15)	0.08607(14)	0.08624(15)	0.08632(13)	0.08634(15)	0.08630(14)
0.8	1.58	0.08612(14)	0.08698(14)	0.08777(16)	0.08800(16)	0.08809(15)	0.08807(15)	0.08800(15)
0.8605	1.89	0.08679(16)	0.08784(14)	0.08899(16)	0.08927(16)	0.08933(16)	0.08927(13)	0.08931(16)
0.9	2.24	0.08710(13)	0.08837(16)	0.08968(16)	0.09026(18)	0.09041(17)	0.09032(16)	0.09042(17)

Table A.57: Case II Lognormal Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.10326(17)	0.10329(19)	0.10333(18)	0.10356(20)	0.10370(17)	0.10354(19)	0.10358(23)
0.0323	0.72	0.09585(18)	0.09563(18)	0.09566(16)	0.09540(18)	0.09546(17)	0.09533(18)	0.09522(18)
0.1	0.75	0.09305(17)	0.09313(18)	0.09279(18)	0.09258(17)	0.09261(16)	0.09253(17)	0.09268(18)
0.2	0.79	0.09267(16)	0.09241(15)	0.09250(16)	0.09230(16)	0.09218(16)	0.09221(19)	0.09211(16)
0.3	0.85	0.09321(17)	0.09303(19)	0.09288(17)	0.09305(16)	0.09292(17)	0.09280(16)	0.09283(15)
0.4	0.91	0.09406(17)	0.09422(18)	0.09415(18)	0.09393(17)	0.09399(17)	0.09397(16)	0.09413(19)
0.5	1	0.09533(17)	0.09529(17)	0.09537(20)	0.09540(16)	0.09542(18)	0.09541(15)	0.09555(18)
0.6	1.12	0.09630(16)	0.09691(20)	0.09683(18)	0.09710(19)	0.09710(15)	0.09702(18)	0.09700(20)
0.7	1.29	0.09739(20)	0.09820(18)	0.09858(20)	0.09881(20)	0.09896(16)	0.09900(19)	0.09903(19)
0.8	1.58	0.09862(19)	0.09976(18)	0.10063(18)	0.10100(20)	0.10108(20)	0.10119(20)	0.10096(18)
0.8605	1.89	0.09931(21)	0.10078(19)	0.10218(21)	0.10256(20)	0.10252(20)	0.10265(18)	0.10267(19)
0.9	2.24	0.09978(17)	0.10133(20)	0.10303(21)	0.10371(23)	0.10396(21)	0.10382(20)	0.10401(20)

Table A.58: Case II Lognormal Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.12507(29)	0.12561(27)	0.12581(26)	0.12609(29)	0.12617(26)	0.12594(28)	0.12587(32)
0.0323	0.72	0.11613(26)	0.11614(26)	0.11602(25)	0.11596(25)	0.11594(24)	0.11579(24)	0.11582(26)
0.1	0.75	0.11274(24)	0.11287(28)	0.11242(27)	0.11245(25)	0.11252(24)	0.11244(24)	0.11247(28)
0.2	0.79	0.11222(22)	0.11196(24)	0.11212(26)	0.11205(22)	0.11199(24)	0.11195(25)	0.11198(25)
0.3	0.85	0.11290(24)	0.11286(24)	0.11291(27)	0.11301(26)	0.11294(27)	0.11276(26)	0.11281(23)
0.4	0.91	0.11406(27)	0.11439(28)	0.11451(28)	0.11434(24)	0.11427(23)	0.11413(24)	0.11446(26)
0.5	1	0.11562(23)	0.11584(23)	0.11614(28)	0.11618(23)	0.11631(27)	0.11628(27)	0.11629(24)
0.6	1.12	0.11722(26)	0.11795(28)	0.11811(25)	0.11837(30)	0.11858(25)	0.11832(26)	0.11822(27)
0.7	1.29	0.11862(29)	0.11960(27)	0.12047(26)	0.12078(33)	0.12066(25)	0.12091(29)	0.12094(27)
0.8	1.58	0.12012(29)	0.12187(24)	0.12290(27)	0.12359(30)	0.12374(28)	0.12394(30)	0.12352(28)
0.8605	1.89	0.12104(31)	0.12327(27)	0.12510(29)	0.12548(30)	0.12571(29)	0.12582(31)	0.12594(26)
0.9	2.24	0.12163(31)	0.12394(29)	0.12626(30)	0.12719(32)	0.12757(30)	0.12742(31)	0.12756(28)

Table A.59: Case II Lognormal Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.17581(62)	0.17766(64)	0.17826(61)	0.17882(64)	0.17843(56)	0.17854(67)	0.17840(62)
0.0323	0.72	0.16372(55)	0.16370(62)	0.16406(62)	0.16437(56)	0.16378(61)	0.16451(67)	0.16449(60)
0.1	0.75	0.15858(60)	0.15913(61)	0.15892(61)	0.15905(58)	0.15929(55)	0.15899(57)	0.15945(57)
0.2	0.79	0.15834(55)	0.15812(56)	0.15920(57)	0.15877(59)	0.15863(58)	0.15844(61)	0.15899(58)
0.3	0.85	0.15933(60)	0.16017(57)	0.15981(57)	0.16016(50)	0.16022(64)	0.16012(58)	0.16015(56)
0.4	0.91	0.16165(64)	0.16291(60)	0.16285(56)	0.16272(52)	0.16310(55)	0.16224(59)	0.16344(61)
0.5	1	0.16395(62)	0.16496(53)	0.16562(63)	0.16602(64)	0.16606(59)	0.16593(65)	0.16639(63)
0.6	1.12	0.16704(63)	0.16838(61)	0.16867(57)	0.16880(71)	0.16970(56)	0.16930(62)	0.16882(60)
0.7	1.29	0.16847(64)	0.17112(72)	0.17240(68)	0.17305(64)	0.17319(67)	0.17343(63)	0.17372(72)
0.8	1.58	0.17111(61)	0.17414(56)	0.17726(65)	0.17770(67)	0.17791(61)	0.17840(68)	0.17761(65)
0.8605	1.89	0.17341(72)	0.17751(62)	0.17991(70)	0.18108(67)	0.18096(74)	0.18149(69)	0.18102(67)
0.9	2.24	0.17343(65)	0.17869(71)	0.18220(74)	0.18368(68)	0.18431(77)	0.18409(74)	0.18398(73)

Table A.60: Case II Lognormal Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.5564(08)	0.5577(07)	0.5583(07)	0.5594(09)	0.5599(08)	0.5599(09)	0.5598(09)
0.0323	0.72	0.5367(08)	0.5354(07)	0.5354(07)	0.5349(08)	0.5346(08)	0.5343(08)	0.5335(07)
0.1	0.75	0.5355(07)	0.5366(08)	0.5354(08)	0.5345(07)	0.5348(08)	0.5343(08)	0.5351(08)
0.2	0.79	0.5436(07)	0.5425(07)	0.5438(07)	0.5430(08)	0.5428(09)	0.5431(09)	0.5424(08)
0.3	0.85	0.5518(08)	0.5516(08)	0.5518(08)	0.5528(08)	0.5528(09)	0.5523(07)	0.5520(08)
0.4	0.91	0.5596(09)	0.5613(08)	0.5615(09)	0.5610(08)	0.5609(09)	0.5609(08)	0.5621(09)
0.5	1	0.5684(08)	0.5691(09)	0.5703(10)	0.5704(09)	0.5707(09)	0.5708(09)	0.5716(08)
0.6	1.12	0.5749(09)	0.5783(10)	0.5788(09)	0.5800(09)	0.5811(09)	0.5801(09)	0.5803(09)
0.7	1.29	0.5819(10)	0.5861(10)	0.5886(10)	0.5901(10)	0.5909(09)	0.5913(09)	0.5906(10)
0.8	1.58	0.5889(09)	0.5949(09)	0.5987(10)	0.6010(10)	0.6013(10)	0.6017(10)	0.6010(09)
0.8605	1.89	0.5936(11)	0.6003(10)	0.6071(10)	0.6085(11)	0.6090(10)	0.6092(09)	0.6092(10)
0.9	2.24	0.5963(10)	0.6038(10)	0.6111(10)	0.6144(11)	0.6151(10)	0.6150(10)	0.6158(10)

Table A.61: Case II Lognormal Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.6261(10)	0.6274(10)	0.6290(10)	0.6305(11)	0.6306(10)	0.6308(10)	0.6304(12)
0.0323	0.72	0.6033(10)	0.6022(10)	0.6024(09)	0.6014(10)	0.6015(10)	0.6007(10)	0.6002(10)
0.1	0.75	0.6030(09)	0.6038(11)	0.6024(11)	0.6016(09)	0.6019(09)	0.6018(10)	0.6019(10)
0.2	0.79	0.6127(09)	0.6117(09)	0.6129(09)	0.6116(10)	0.6119(10)	0.6119(10)	0.6117(10)
0.3	0.85	0.6227(10)	0.6231(11)	0.6232(11)	0.6238(10)	0.6236(11)	0.6226(09)	0.6229(09)
0.4	0.91	0.6332(11)	0.6346(11)	0.6352(11)	0.6345(11)	0.6335(09)	0.6340(10)	0.6343(11)
0.5	1	0.6437(10)	0.6443(10)	0.6451(12)	0.6459(11)	0.6457(11)	0.6460(11)	0.6464(11)
0.6	1.12	0.6527(11)	0.6567(13)	0.6567(11)	0.6576(12)	0.6579(09)	0.6571(13)	0.6569(12)
0.7	1.29	0.6612(12)	0.6660(12)	0.6687(12)	0.6691(13)	0.6698(11)	0.6704(12)	0.6700(11)
0.8	1.58	0.6699(13)	0.6770(11)	0.6808(13)	0.6832(12)	0.6830(13)	0.6836(12)	0.6824(11)
0.8605	1.89	0.6755(14)	0.6831(12)	0.6910(13)	0.6923(13)	0.6923(12)	0.6933(11)	0.6929(11)
0.9	2.24	0.6784(13)	0.6874(13)	0.6958(13)	0.7000(14)	0.7005(13)	0.7000(13)	0.7005(13)

Table A.62: Case II Lognormal Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.7437(15)	0.7468(15)	0.7493(14)	0.7511(16)	0.7518(15)	0.7510(16)	0.7510(17)
0.0323	0.72	0.7172(14)	0.7165(15)	0.7161(15)	0.7159(14)	0.7154(14)	0.7149(13)	0.7150(13)
0.1	0.75	0.7185(13)	0.7190(16)	0.7165(15)	0.7162(14)	0.7160(14)	0.7159(13)	0.7174(15)
0.2	0.79	0.7330(15)	0.7306(15)	0.7318(16)	0.7300(12)	0.7307(16)	0.7299(15)	0.7298(15)
0.3	0.85	0.7471(15)	0.7462(16)	0.7458(15)	0.7465(18)	0.7458(17)	0.7442(16)	0.7449(15)
0.4	0.91	0.7607(17)	0.7622(17)	0.7622(16)	0.7603(16)	0.7598(15)	0.7601(15)	0.7609(18)
0.5	1	0.7767(15)	0.7758(17)	0.7751(17)	0.7752(16)	0.7768(17)	0.7757(16)	0.7767(16)
0.6	1.12	0.7883(19)	0.7921(16)	0.7905(17)	0.7914(20)	0.7923(15)	0.7902(18)	0.7902(17)
0.7	1.29	0.7994(16)	0.8053(18)	0.8071(19)	0.8081(20)	0.8081(17)	0.8085(18)	0.8080(16)
0.8	1.58	0.8125(19)	0.8201(16)	0.8247(17)	0.8271(18)	0.8268(20)	0.8271(17)	0.8242(17)
0.8605	1.89	0.8190(20)	0.8299(19)	0.8364(19)	0.8386(19)	0.8390(19)	0.8394(18)	0.8388(16)
0.9	2.24	0.8237(19)	0.8336(17)	0.8440(20)	0.8489(18)	0.8493(19)	0.8483(18)	0.8483(17)

Table A.63: Case II Lognormal Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.0142(32)	1.0255(33)	1.0295(32)	1.0324(30)	1.0338(30)	1.0338(36)	1.0325(38)
0.0323	0.72	0.9829(32)	0.9823(38)	0.9844(33)	0.9844(33)	0.9832(35)	0.9867(36)	0.9858(36)
0.1	0.75	0.9908(32)	0.9906(34)	0.9858(35)	0.9885(38)	0.9857(31)	0.9851(30)	0.9873(32)
0.2	0.79	1.0204(36)	1.0111(34)	1.0157(37)	1.0100(35)	1.0105(36)	1.0087(35)	1.0110(36)
0.3	0.85	1.0456(37)	1.0423(42)	1.0370(35)	1.0412(36)	1.0360(38)	1.0326(36)	1.0332(36)
0.4	0.91	1.0736(42)	1.0765(42)	1.0665(40)	1.0632(34)	1.0623(37)	1.0580(40)	1.0624(37)
0.5	1	1.0991(37)	1.0965(35)	1.0936(42)	1.0907(39)	1.0886(38)	1.0876(40)	1.0888(35)
0.6	1.12	1.1234(44)	1.1245(45)	1.1161(42)	1.1125(48)	1.1179(44)	1.1125(43)	1.1125(36)
0.7	1.29	1.1421(47)	1.1443(47)	1.1432(39)	1.1430(43)	1.1401(47)	1.1404(44)	1.1458(47)
0.8	1.58	1.1645(45)	1.1736(40)	1.1765(45)	1.1738(43)	1.1731(42)	1.1751(40)	1.1675(43)
0.8605	1.89	1.1770(46)	1.1942(45)	1.1946(41)	1.1944(47)	1.1940(44)	1.1913(39)	1.1904(35)
0.9	2.24	1.1841(45)	1.2013(53)	1.2066(49)	1.2102(43)	1.2097(45)	1.2079(49)	1.2077(44)

Table A.64: Case II Lognormal Anderson-Darling 99% critical values

A.3 Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

A.3.1 Weibull Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.9000(08)	0.9081(07)	0.9152(07)	0.9198(08)	0.9241(08)	0.9265(08)	0.9303(08)
0.0323	0.18	0.8994(08)	0.9072(08)	0.9154(07)	0.9198(08)	0.9241(09)	0.9269(07)	0.9310(07)
0.1	0.32	0.9006(08)	0.9076(08)	0.9152(07)	0.9199(07)	0.9245(07)	0.9265(09)	0.9304(07)
0.2	0.47	0.9003(07)	0.9077(08)	0.9150(08)	0.9203(07)	0.9251(08)	0.9273(08)	0.9306(08)
0.3	0.6	0.9002(08)	0.9077(08)	0.9146(07)	0.9195(08)	0.9243(07)	0.9269(07)	0.9296(08)
0.4	0.71	0.9002(08)	0.9077(07)	0.9145(06)	0.9201(08)	0.9249(07)	0.9269(08)	0.9304(07)
0.5	0.83	0.9003(07)	0.9085(07)	0.9150(08)	0.9201(08)	0.9246(07)	0.9264(08)	0.9306(08)
0.6	0.96	0.9001(08)	0.9073(08)	0.9147(09)	0.9199(08)	0.9246(08)	0.9268(08)	0.9297(08)
0.7	1.1	0.9006(08)	0.9074(07)	0.9150(06)	0.9209(06)	0.9242(08)	0.9268(08)	0.9303(07)
0.8	1.27	0.9000(07)	0.9082(08)	0.9146(07)	0.9200(07)	0.9248(07)	0.9269(07)	0.9304(08)
0.8605	1.4	0.9003(07)	0.9075(08)	0.9152(08)	0.9204(07)	0.9244(09)	0.9260(08)	0.9300(08)
0.9	1.52	0.9002(07)	0.9080(07)	0.9155(07)	0.9200(07)	0.9248(08)	0.9267(08)	0.9302(08)

Table A.65: Case IIIa Weibull Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.9612(09)	0.9701(09)	0.9779(08)	0.9827(09)	0.9867(09)	0.9894(09)	0.9926(08)
0.0323	0.18	0.9613(10)	0.9691(08)	0.9779(08)	0.9821(10)	0.9868(10)	0.9895(08)	0.9938(08)
0.1	0.32	0.9615(11)	0.9695(10)	0.9774(08)	0.9823(09)	0.9873(08)	0.9891(11)	0.9934(09)
0.2	0.47	0.9619(08)	0.9700(10)	0.9773(09)	0.9827(10)	0.9878(08)	0.9901(09)	0.9927(09)
0.3	0.6	0.9621(09)	0.9694(09)	0.9766(08)	0.9817(09)	0.9871(09)	0.9890(09)	0.9921(10)
0.4	0.71	0.9620(09)	0.9699(08)	0.9770(08)	0.9823(09)	0.9874(08)	0.9893(09)	0.9936(09)
0.5	0.83	0.9614(09)	0.9703(09)	0.9772(09)	0.9826(10)	0.9869(09)	0.9891(10)	0.9931(08)
0.6	0.96	0.9611(09)	0.9694(09)	0.9768(08)	0.9823(10)	0.9869(09)	0.9891(10)	0.9923(09)
0.7	1.1	0.9619(09)	0.9694(09)	0.9772(08)	0.9837(08)	0.9867(09)	0.9896(09)	0.9935(09)
0.8	1.27	0.9617(08)	0.9698(09)	0.9767(09)	0.9831(08)	0.9873(08)	0.9897(09)	0.9930(09)
0.8605	1.4	0.9615(09)	0.9695(09)	0.9777(09)	0.9829(09)	0.9867(10)	0.9883(09)	0.9924(10)
0.9	1.52	0.9622(08)	0.9701(08)	0.9776(08)	0.9831(09)	0.9879(10)	0.9895(09)	0.9929(09)

Table A.66: Case IIIa Weibull Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0581(12)	1.0668(11)	1.0760(12)	1.0813(12)	1.0858(13)	1.0880(12)	1.0909(11)
0.0323	0.18	1.0577(12)	1.0667(11)	1.0756(13)	1.0807(13)	1.0849(13)	1.0883(10)	1.0928(10)
0.1	0.32	1.0578(14)	1.0669(12)	1.0757(11)	1.0810(12)	1.0858(11)	1.0876(13)	1.0922(12)
0.2	0.47	1.0582(12)	1.0677(12)	1.0755(13)	1.0816(13)	1.0864(11)	1.0886(12)	1.0908(12)
0.3	0.6	1.0577(12)	1.0661(11)	1.0741(11)	1.0798(14)	1.0863(12)	1.0869(12)	1.0908(13)
0.4	0.71	1.0580(12)	1.0667(10)	1.0749(12)	1.0802(12)	1.0857(12)	1.0875(13)	1.0919(12)
0.5	0.83	1.0573(11)	1.0672(12)	1.0750(10)	1.0818(13)	1.0856(11)	1.0884(13)	1.0910(11)
0.6	0.96	1.0574(12)	1.0665(12)	1.0741(11)	1.0810(14)	1.0854(12)	1.0869(13)	1.0904(11)
0.7	1.1	1.0583(12)	1.0660(11)	1.0747(13)	1.0821(10)	1.0850(11)	1.0888(12)	1.0910(13)
0.8	1.27	1.0576(11)	1.0667(11)	1.0743(13)	1.0818(12)	1.0863(11)	1.0881(12)	1.0917(11)
0.8605	1.4	1.0574(10)	1.0666(11)	1.0756(12)	1.0804(13)	1.0860(12)	1.0872(12)	1.0911(12)
0.9	1.52	1.0588(10)	1.0672(12)	1.0759(12)	1.0821(12)	1.0861(11)	1.0880(11)	1.0920(11)

Table A.67: Case IIIa Weibull Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2519(22)	1.2628(22)	1.2724(20)	1.2807(24)	1.2852(21)	1.2857(23)	1.2914(24)
0.0323	0.18	1.2508(22)	1.2616(22)	1.2728(22)	1.2791(24)	1.2851(26)	1.2883(20)	1.2926(25)
0.1	0.32	1.2513(20)	1.2655(22)	1.2739(21)	1.2796(24)	1.2859(20)	1.2871(20)	1.2918(23)
0.2	0.47	1.2497(25)	1.2629(23)	1.2727(23)	1.2812(26)	1.2861(22)	1.2895(23)	1.2899(20)
0.3	0.6	1.2505(21)	1.2624(21)	1.2734(24)	1.2770(20)	1.2871(24)	1.2874(23)	1.2906(24)
0.4	0.71	1.2510(21)	1.2615(20)	1.2730(23)	1.2811(22)	1.2839(23)	1.2886(24)	1.2936(23)
0.5	0.83	1.2500(22)	1.2621(22)	1.2723(22)	1.2808(22)	1.2851(21)	1.2880(21)	1.2928(21)
0.6	0.96	1.2509(20)	1.2640(25)	1.2713(23)	1.2789(23)	1.2839(20)	1.2849(22)	1.2909(23)
0.7	1.1	1.2521(23)	1.2613(22)	1.2721(21)	1.2816(23)	1.2840(23)	1.2899(23)	1.2914(22)
0.8	1.27	1.2496(21)	1.2638(20)	1.2726(21)	1.2801(21)	1.2839(21)	1.2866(21)	1.2918(21)
0.8605	1.4	1.2512(21)	1.2627(22)	1.2724(24)	1.2806(26)	1.2856(22)	1.2864(23)	1.2929(25)
0.9	1.52	1.2527(23)	1.2630(26)	1.2732(24)	1.2812(21)	1.2849(23)	1.2884(24)	1.2928(25)

Table A.68: Case IIIa Weibull Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4013(11)	1.4149(08)	1.4290(08)	1.4376(10)	1.4464(09)	1.4510(09)	1.4581(09)
0.0323	0.18	1.4007(10)	1.4148(09)	1.4287(08)	1.4383(10)	1.4472(10)	1.4512(09)	1.4593(09)
0.1	0.32	1.4008(10)	1.4155(10)	1.4286(09)	1.4379(09)	1.4476(10)	1.4514(10)	1.4582(09)
0.2	0.47	1.4016(09)	1.4146(11)	1.4286(09)	1.4378(08)	1.4480(10)	1.4518(09)	1.4587(09)
0.3	0.6	1.4011(09)	1.4146(09)	1.4280(10)	1.4375(10)	1.4476(09)	1.4512(10)	1.4575(11)
0.4	0.71	1.4014(10)	1.4152(09)	1.4285(08)	1.4381(11)	1.4472(09)	1.4514(09)	1.4586(10)
0.5	0.83	1.4013(09)	1.4155(09)	1.4284(09)	1.4390(10)	1.4472(09)	1.4513(10)	1.4594(09)
0.6	0.96	1.4014(11)	1.4150(10)	1.4278(10)	1.4376(11)	1.4467(09)	1.4508(09)	1.4574(10)
0.7	1.1	1.4018(09)	1.4144(10)	1.4292(09)	1.4394(08)	1.4467(09)	1.4514(10)	1.4583(11)
0.8	1.27	1.4010(09)	1.4148(09)	1.4284(10)	1.4388(09)	1.4471(08)	1.4514(09)	1.4580(09)
0.8605	1.4	1.4015(08)	1.4145(09)	1.4293(10)	1.4385(08)	1.4471(10)	1.4508(10)	1.4584(11)
0.9	1.52	1.4011(09)	1.4150(09)	1.4293(10)	1.4391(10)	1.4477(10)	1.4509(10)	1.4586(09)

Table A.69: Case IIIa Weibull Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4773(12)	1.4921(10)	1.5067(10)	1.5164(13)	1.5246(11)	1.5291(11)	1.5357(11)
0.0323	0.18	1.4773(11)	1.4920(10)	1.5060(10)	1.5166(12)	1.5251(11)	1.5295(08)	1.5375(10)
0.1	0.32	1.4776(12)	1.4926(11)	1.5062(10)	1.5156(10)	1.5261(11)	1.5300(11)	1.5366(11)
0.2	0.47	1.4784(11)	1.4923(12)	1.5057(11)	1.5165(10)	1.5264(11)	1.5304(11)	1.5364(10)
0.3	0.6	1.4777(10)	1.4915(11)	1.5054(11)	1.5154(10)	1.5256(10)	1.5296(12)	1.5359(14)
0.4	0.71	1.4782(12)	1.4916(10)	1.5062(11)	1.5163(12)	1.5252(10)	1.5298(12)	1.5366(11)
0.5	0.83	1.4776(11)	1.4931(11)	1.5068(10)	1.5175(12)	1.5248(11)	1.5293(12)	1.5372(10)
0.6	0.96	1.4774(12)	1.4927(11)	1.5051(11)	1.5157(12)	1.5247(10)	1.5284(10)	1.5358(11)
0.7	1.1	1.4781(10)	1.4919(11)	1.5067(11)	1.5176(09)	1.5247(11)	1.5295(12)	1.5365(13)
0.8	1.27	1.4773(10)	1.4921(12)	1.5061(10)	1.5169(11)	1.5255(10)	1.5296(10)	1.5358(12)
0.8605	1.4	1.4780(10)	1.4917(11)	1.5066(11)	1.5165(11)	1.5249(12)	1.5288(11)	1.5367(11)
0.9	1.52	1.4787(10)	1.4921(12)	1.5072(11)	1.5172(12)	1.5257(12)	1.5287(11)	1.5368(12)

Table A.70: Case IIIa Weibull Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5956(15)	1.6124(14)	1.6270(13)	1.6368(15)	1.6460(14)	1.6510(14)	1.6572(13)
0.0323	0.18	1.5967(15)	1.6116(13)	1.6277(15)	1.6374(15)	1.6464(13)	1.6517(12)	1.6597(13)
0.1	0.32	1.5962(15)	1.6132(15)	1.6270(15)	1.6369(15)	1.6481(14)	1.6501(13)	1.6589(14)
0.2	0.47	1.5968(13)	1.6117(16)	1.6266(13)	1.6393(14)	1.6470(13)	1.6516(13)	1.6573(16)
0.3	0.6	1.5967(16)	1.6115(15)	1.6257(13)	1.6358(15)	1.6467(13)	1.6507(14)	1.6573(15)
0.4	0.71	1.5978(17)	1.6122(13)	1.6271(14)	1.6374(15)	1.6462(14)	1.6510(17)	1.6590(16)
0.5	0.83	1.5959(15)	1.6129(12)	1.6264(14)	1.6381(16)	1.6458(15)	1.6507(16)	1.6594(13)
0.6	0.96	1.5957(15)	1.6122(16)	1.6255(15)	1.6371(16)	1.6459(15)	1.6497(14)	1.6570(14)
0.7	1.1	1.5971(13)	1.6122(13)	1.6272(15)	1.6399(12)	1.6459(13)	1.6511(15)	1.6583(16)
0.8	1.27	1.5960(12)	1.6123(15)	1.6262(14)	1.6375(13)	1.6462(14)	1.6506(13)	1.6584(14)
0.8605	1.4	1.5962(14)	1.6119(15)	1.6270(13)	1.6382(16)	1.6466(14)	1.6502(13)	1.6579(14)
0.9	1.52	1.5980(13)	1.6126(15)	1.6273(14)	1.6380(14)	1.6471(14)	1.6507(14)	1.6592(14)

Table A.71: Case IIIa Weibull Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.8344(28)	1.8524(25)	1.8684(26)	1.8800(28)	1.8897(28)	1.8918(27)	1.9016(25)
0.0323	0.18	1.8338(27)	1.8535(30)	1.8716(29)	1.8807(26)	1.8896(30)	1.8926(27)	1.9014(29)
0.1	0.32	1.8336(27)	1.8532(29)	1.8684(27)	1.8804(30)	1.8905(29)	1.8948(25)	1.9023(27)
0.2	0.47	1.8340(31)	1.8530(29)	1.8688(28)	1.8819(28)	1.8927(26)	1.8957(29)	1.8998(25)
0.3	0.6	1.8329(25)	1.8519(25)	1.8673(28)	1.8783(28)	1.8906(27)	1.8922(28)	1.9013(27)
0.4	0.71	1.8356(27)	1.8525(24)	1.8656(28)	1.8817(26)	1.8887(28)	1.8943(25)	1.9039(34)
0.5	0.83	1.8335(29)	1.8510(27)	1.8691(24)	1.8826(26)	1.8893(25)	1.8961(31)	1.9034(25)
0.6	0.96	1.8325(28)	1.8516(30)	1.8682(25)	1.8807(28)	1.8868(28)	1.8945(28)	1.9046(25)
0.7	1.1	1.8341(28)	1.8512(25)	1.8691(26)	1.8823(25)	1.8895(23)	1.8962(27)	1.9022(26)
0.8	1.27	1.8323(26)	1.8533(28)	1.8674(28)	1.8800(28)	1.8901(25)	1.8918(25)	1.9032(24)
0.8605	1.4	1.8343(27)	1.8530(24)	1.8690(27)	1.8798(30)	1.8898(28)	1.8936(28)	1.9025(29)
0.9	1.52	1.8351(27)	1.8522(29)	1.8690(31)	1.8827(28)	1.8896(27)	1.8944(29)	1.9018(28)

Table A.72: Case IIIa Weibull Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.14778(28)	0.14810(27)	0.14815(30)	0.14791(31)	0.14789(30)	0.14774(28)	0.14793(33)
0.0323	0.18	0.14749(29)	0.14776(32)	0.14793(26)	0.14781(35)	0.14804(31)	0.14774(28)	0.14814(30)
0.1	0.32	0.14755(34)	0.14783(33)	0.14808(29)	0.14764(29)	0.14803(31)	0.14797(32)	0.14793(32)
0.2	0.47	0.14768(31)	0.14794(33)	0.14787(32)	0.14795(30)	0.14816(28)	0.14814(32)	0.14803(30)
0.3	0.6	0.14770(30)	0.14791(29)	0.14754(30)	0.14773(31)	0.14803(29)	0.14801(30)	0.14767(30)
0.4	0.71	0.14784(30)	0.14779(28)	0.14800(27)	0.14776(33)	0.14794(31)	0.14793(33)	0.14808(28)
0.5	0.83	0.14775(29)	0.14781(31)	0.14798(31)	0.14814(35)	0.14781(26)	0.14782(33)	0.14809(30)
0.6	0.96	0.14767(35)	0.14782(31)	0.14786(29)	0.14791(31)	0.14778(30)	0.14792(34)	0.14760(29)
0.7	1.1	0.14789(32)	0.14767(30)	0.14802(30)	0.14820(27)	0.14756(29)	0.14802(32)	0.14797(30)
0.8	1.27	0.14774(29)	0.14786(32)	0.14778(28)	0.14809(29)	0.14806(29)	0.14795(28)	0.14794(30)
0.8605	1.4	0.14793(29)	0.14776(30)	0.14806(34)	0.14800(28)	0.14801(31)	0.14763(29)	0.14797(32)
0.9	1.52	0.14788(29)	0.14773(29)	0.14803(27)	0.14792(30)	0.14811(32)	0.14792(26)	0.14784(30)

Table A.73: Case IIIa Weibull Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.17367(36)	0.17452(39)	0.17463(39)	0.17442(43)	0.17422(37)	0.17427(40)	0.17432(38)
0.0323	0.18	0.17360(39)	0.17390(37)	0.17445(37)	0.17422(47)	0.17451(42)	0.17412(35)	0.17455(37)
0.1	0.32	0.17379(43)	0.17403(39)	0.17446(36)	0.17405(38)	0.17450(37)	0.17429(39)	0.17444(38)
0.2	0.47	0.17387(36)	0.17419(40)	0.17421(40)	0.17446(38)	0.17470(37)	0.17470(40)	0.17450(41)
0.3	0.6	0.17389(37)	0.17415(39)	0.17388(36)	0.17405(41)	0.17457(36)	0.17411(41)	0.17418(39)
0.4	0.71	0.17392(42)	0.17399(34)	0.17427(37)	0.17423(43)	0.17424(40)	0.17424(43)	0.17444(37)
0.5	0.83	0.17382(39)	0.17401(38)	0.17430(39)	0.17442(44)	0.17428(34)	0.17427(43)	0.17456(40)
0.6	0.96	0.17361(42)	0.17393(45)	0.17410(40)	0.17437(43)	0.17432(36)	0.17446(41)	0.17395(38)
0.7	1.1	0.17404(40)	0.17397(36)	0.17435(38)	0.17461(35)	0.17416(36)	0.17458(41)	0.17444(39)
0.8	1.27	0.17370(36)	0.17406(33)	0.17398(39)	0.17467(37)	0.17454(40)	0.17436(37)	0.17448(39)
0.8605	1.4	0.17401(40)	0.17396(43)	0.17449(43)	0.17445(37)	0.17441(40)	0.17418(39)	0.17428(40)
0.9	1.52	0.17392(34)	0.17403(37)	0.17458(35)	0.17440(38)	0.17464(38)	0.17445(37)	0.17430(37)

Table A.74: Case IIIa Weibull Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.21948(58)	0.22098(55)	0.22173(59)	0.22173(63)	0.22118(56)	0.22150(56)	0.22115(62)
0.0323	0.18	0.21958(59)	0.22056(61)	0.22119(60)	0.22109(66)	0.22139(62)	0.22117(54)	0.22186(59)
0.1	0.32	0.21976(67)	0.22078(61)	0.22113(52)	0.22082(64)	0.22174(63)	0.22130(60)	0.22144(60)
0.2	0.47	0.21993(59)	0.22066(58)	0.22091(63)	0.22164(59)	0.22179(55)	0.22196(66)	0.22137(63)
0.3	0.6	0.21990(53)	0.22047(61)	0.22066(55)	0.22085(64)	0.22163(56)	0.22101(61)	0.22075(61)
0.4	0.71	0.21965(69)	0.22065(56)	0.22046(63)	0.22083(69)	0.22143(58)	0.22116(62)	0.22148(53)
0.5	0.83	0.21985(65)	0.22044(62)	0.22091(59)	0.22160(71)	0.22120(67)	0.22133(63)	0.22139(53)
0.6	0.96	0.21975(64)	0.22044(59)	0.22066(63)	0.22119(67)	0.22102(61)	0.22115(61)	0.22097(53)
0.7	1.1	0.21993(61)	0.22036(59)	0.22110(58)	0.22174(59)	0.22092(57)	0.22175(56)	0.22181(54)
0.8	1.27	0.21974(57)	0.22060(61)	0.22077(57)	0.22176(59)	0.22155(61)	0.22100(61)	0.22162(62)
0.8605	1.4	0.21983(61)	0.22063(61)	0.22093(62)	0.22116(64)	0.22129(60)	0.22076(56)	0.22077(68)
0.9	1.52	0.22018(62)	0.22034(58)	0.22130(55)	0.22185(61)	0.22155(57)	0.22136(58)	0.22184(50)

Table A.75: Case IIIa Weibull Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.33176(124)	0.33325(154)	0.33545(137)	0.33676(152)	0.33753(143)	0.33571(147)	0.33765(153)
0.0323	0.18	0.33137(135)	0.33423(154)	0.33613(147)	0.33626(154)	0.33729(141)	0.33708(138)	0.33693(130)
0.1	0.32	0.33101(139)	0.33420(139)	0.33557(145)	0.33729(139)	0.33643(141)	0.33735(139)	0.33730(140)
0.2	0.47	0.33193(155)	0.33462(154)	0.33558(133)	0.33770(133)	0.33708(143)	0.33822(156)	0.33560(124)
0.3	0.6	0.33210(132)	0.33321(150)	0.33522(133)	0.33519(140)	0.33775(157)	0.33666(147)	0.33683(153)
0.4	0.71	0.33102(127)	0.33346(135)	0.33522(167)	0.33738(145)	0.33565(139)	0.33601(147)	0.33827(158)
0.5	0.83	0.33099(135)	0.33386(135)	0.33507(140)	0.33758(161)	0.33629(138)	0.33699(153)	0.33837(145)
0.6	0.96	0.33142(150)	0.33341(135)	0.33498(132)	0.33552(139)	0.33659(145)	0.33597(144)	0.33703(155)
0.7	1.1	0.33264(129)	0.33398(139)	0.33472(140)	0.33691(150)	0.33624(158)	0.33855(146)	0.33748(131)
0.8	1.27	0.33065(143)	0.33386(141)	0.33381(142)	0.33650(154)	0.33638(160)	0.33586(145)	0.33757(139)
0.8605	1.4	0.33208(145)	0.33349(139)	0.33594(129)	0.33733(159)	0.33629(156)	0.33574(144)	0.33717(142)
0.9	1.52	0.33351(141)	0.33460(147)	0.33616(156)	0.33777(147)	0.33606(151)	0.33719(130)	0.33929(138)

Table A.76: Case IIIa Weibull Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.9057(16)	0.9110(16)	0.9134(18)	0.9135(18)	0.9140(17)	0.9136(16)	0.9147(17)
0.0323	0.18	0.9041(15)	0.9086(18)	0.9123(17)	0.9130(19)	0.9151(19)	0.9136(16)	0.9153(16)
0.1	0.32	0.9052(20)	0.9084(17)	0.9130(16)	0.9112(16)	0.9149(18)	0.9146(16)	0.9153(17)
0.2	0.47	0.9053(17)	0.9097(18)	0.9120(18)	0.9134(17)	0.9155(14)	0.9155(16)	0.9152(18)
0.3	0.6	0.9053(18)	0.9098(16)	0.9104(16)	0.9124(17)	0.9142(16)	0.9146(18)	0.9132(16)
0.4	0.71	0.9057(16)	0.9085(15)	0.9127(16)	0.9133(19)	0.9140(17)	0.9141(17)	0.9154(16)
0.5	0.83	0.9049(17)	0.9089(17)	0.9124(17)	0.9136(18)	0.9136(14)	0.9130(17)	0.9150(16)
0.6	0.96	0.9045(19)	0.9081(18)	0.9117(16)	0.9130(17)	0.9130(17)	0.9144(17)	0.9129(17)
0.7	1.1	0.9061(16)	0.9078(17)	0.9121(17)	0.9148(14)	0.9128(17)	0.9149(16)	0.9149(16)
0.8	1.27	0.9045(16)	0.9096(16)	0.9115(17)	0.9142(15)	0.9147(16)	0.9140(15)	0.9141(17)
0.8605	1.4	0.9059(16)	0.9094(16)	0.9129(16)	0.9143(16)	0.9144(18)	0.9138(16)	0.9148(16)
0.9	1.52	0.9053(16)	0.9091(17)	0.9128(15)	0.9134(18)	0.9146(17)	0.9142(15)	0.9146(14)

Table A.77: Case IIIa Weibull Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0489(18)	1.0555(19)	1.0591(24)	1.0596(24)	1.0604(20)	1.0593(21)	1.0610(21)
0.0323	0.18	1.0485(22)	1.0520(22)	1.0589(19)	1.0592(26)	1.0609(24)	1.0590(19)	1.0620(19)
0.1	0.32	1.0493(25)	1.0534(21)	1.0601(19)	1.0576(20)	1.0617(23)	1.0603(21)	1.0616(21)
0.2	0.47	1.0496(20)	1.0540(23)	1.0570(21)	1.0604(19)	1.0624(21)	1.0619(23)	1.0615(22)
0.3	0.6	1.0501(21)	1.0546(21)	1.0556(20)	1.0570(22)	1.0613(20)	1.0599(22)	1.0587(22)
0.4	0.71	1.0497(21)	1.0534(17)	1.0580(21)	1.0586(26)	1.0597(22)	1.0597(23)	1.0615(21)
0.5	0.83	1.0489(22)	1.0537(21)	1.0576(22)	1.0606(24)	1.0600(20)	1.0597(23)	1.0620(20)
0.6	0.96	1.0487(22)	1.0529(23)	1.0570(20)	1.0596(25)	1.0597(21)	1.0604(24)	1.0585(21)
0.7	1.1	1.0501(23)	1.0531(17)	1.0573(22)	1.0611(21)	1.0593(19)	1.0613(21)	1.0614(22)
0.8	1.27	1.0488(22)	1.0542(20)	1.0574(23)	1.0604(18)	1.0614(21)	1.0600(20)	1.0612(21)
0.8605	1.4	1.0499(21)	1.0537(22)	1.0584(21)	1.0611(21)	1.0614(21)	1.0579(19)	1.0608(22)
0.9	1.52	1.0501(21)	1.0533(21)	1.0590(20)	1.0596(21)	1.0611(21)	1.0609(20)	1.0615(22)

Table A.78: Case IIIa Weibull Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3043(31)	1.3107(31)	1.3179(34)	1.3187(35)	1.3190(32)	1.3169(30)	1.3201(33)
0.0323	0.18	1.3024(30)	1.3098(33)	1.3168(31)	1.3180(35)	1.3196(33)	1.3171(29)	1.3213(30)
0.1	0.32	1.3044(35)	1.3117(33)	1.3170(28)	1.3152(36)	1.3220(35)	1.3186(33)	1.3216(31)
0.2	0.47	1.3051(34)	1.3094(35)	1.3157(36)	1.3203(30)	1.3208(28)	1.3232(37)	1.3208(31)
0.3	0.6	1.3053(30)	1.3097(32)	1.3129(31)	1.3173(34)	1.3210(32)	1.3184(34)	1.3166(30)
0.4	0.71	1.3048(39)	1.3103(29)	1.3146(35)	1.3165(37)	1.3184(33)	1.3174(32)	1.3193(30)
0.5	0.83	1.3042(35)	1.3095(36)	1.3150(33)	1.3190(38)	1.3179(34)	1.3200(37)	1.3202(31)
0.6	0.96	1.3009(35)	1.3093(32)	1.3136(32)	1.3167(36)	1.3187(31)	1.3171(32)	1.3178(29)
0.7	1.1	1.3046(35)	1.3079(32)	1.3145(32)	1.3203(29)	1.3166(32)	1.3218(32)	1.3211(32)
0.8	1.27	1.3020(31)	1.3113(31)	1.3150(33)	1.3185(34)	1.3203(34)	1.3176(30)	1.3198(34)
0.8605	1.4	1.3045(32)	1.3098(32)	1.3145(31)	1.3177(34)	1.3183(32)	1.3165(30)	1.3191(36)
0.9	1.52	1.3061(32)	1.3082(32)	1.3179(31)	1.3199(32)	1.3201(35)	1.3184(33)	1.3235(33)

Table A.79: Case IIIa Weibull Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.9392(73)	1.9426(83)	1.9504(84)	1.9563(79)	1.9557(81)	1.9496(82)	1.9525(80)
0.0323	0.18	1.9344(90)	1.9435(80)	1.9524(73)	1.9548(72)	1.9584(74)	1.9549(73)	1.9549(74)
0.1	0.32	1.9353(82)	1.9452(84)	1.9570(73)	1.9584(79)	1.9543(80)	1.9605(75)	1.9585(77)
0.2	0.47	1.9441(85)	1.9444(86)	1.9446(70)	1.9586(76)	1.9574(82)	1.9575(93)	1.9519(76)
0.3	0.6	1.9418(79)	1.9434(75)	1.9469(77)	1.9488(81)	1.9574(83)	1.9519(77)	1.9523(75)
0.4	0.71	1.9405(72)	1.9447(82)	1.9475(97)	1.9547(82)	1.9469(76)	1.9463(80)	1.9577(85)
0.5	0.83	1.9355(83)	1.9416(80)	1.9502(78)	1.9553(90)	1.9493(79)	1.9573(85)	1.9610(81)
0.6	0.96	1.9334(91)	1.9443(79)	1.9461(82)	1.9513(82)	1.9502(79)	1.9492(83)	1.9603(82)
0.7	1.1	1.9370(83)	1.9415(81)	1.9512(88)	1.9601(80)	1.9534(82)	1.9579(76)	1.9607(81)
0.8	1.27	1.9332(78)	1.9436(80)	1.9441(80)	1.9508(84)	1.9573(92)	1.9534(86)	1.9594(81)
0.8605	1.4	1.9389(87)	1.9430(70)	1.9506(81)	1.9586(89)	1.9530(83)	1.9480(77)	1.9513(76)
0.9	1.52	1.9424(81)	1.9450(91)	1.9504(82)	1.9585(79)	1.9488(82)	1.9523(76)	1.9669(75)

Table A.80: Case IIIa Weibull Anderson-Darling 99% critical values

A.3.2 Loglogistic Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7587(05)	0.7665(05)	0.7745(05)	0.7794(05)	0.7844(05)	0.7867(05)	0.7906(06)
0.0323	0.18	0.7587(05)	0.7669(05)	0.7745(05)	0.7801(06)	0.7843(06)	0.7872(06)	0.7912(06)
0.1	0.33	0.7588(06)	0.7668(05)	0.7746(05)	0.7803(05)	0.7842(05)	0.7869(06)	0.7907(05)
0.2	0.5	0.7588(05)	0.7670(05)	0.7741(05)	0.7803(05)	0.7850(05)	0.7869(05)	0.7903(06)
0.3	0.65	0.7585(04)	0.7668(06)	0.7753(05)	0.7801(05)	0.7843(06)	0.7872(06)	0.7907(05)
0.4	0.82	0.7589(05)	0.7668(05)	0.7747(05)	0.7798(05)	0.7842(05)	0.7870(05)	0.7907(05)
0.5	1	0.7588(05)	0.7667(05)	0.7748(06)	0.7800(05)	0.7848(05)	0.7872(06)	0.7905(06)
0.6	1.22	0.7585(05)	0.7664(05)	0.7748(05)	0.7804(06)	0.7845(06)	0.7866(06)	0.7912(05)
0.7	1.53	0.7587(05)	0.7672(06)	0.7749(05)	0.7803(05)	0.7847(06)	0.7872(06)	0.7907(05)
0.8	2	0.7584(05)	0.7672(06)	0.7744(06)	0.7800(05)	0.7843(06)	0.7866(06)	0.7903(05)
0.8605	2.48	0.7590(05)	0.7665(06)	0.7750(05)	0.7798(06)	0.7839(06)	0.7870(05)	0.7909(05)
0.9	3	0.7588(06)	0.7667(06)	0.7748(05)	0.7801(05)	0.7845(05)	0.7866(06)	0.7907(05)

Table A.81: Case IIIa Loglogistic Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8022(06)	0.8108(06)	0.8188(06)	0.8244(06)	0.8291(07)	0.8315(06)	0.8356(07)
0.0323	0.18	0.8022(07)	0.8110(06)	0.8191(06)	0.8247(07)	0.8297(07)	0.8322(06)	0.8363(07)
0.1	0.33	0.8026(06)	0.8112(06)	0.8191(06)	0.8251(06)	0.8292(06)	0.8317(06)	0.8360(06)
0.2	0.5	0.8025(06)	0.8112(06)	0.8187(06)	0.8247(06)	0.8301(06)	0.8318(06)	0.8354(07)
0.3	0.65	0.8017(05)	0.8111(06)	0.8197(07)	0.8249(07)	0.8295(06)	0.8320(07)	0.8357(06)
0.4	0.82	0.8027(06)	0.8109(06)	0.8191(06)	0.8244(06)	0.8292(06)	0.8319(07)	0.8358(06)
0.5	1	0.8025(06)	0.8110(06)	0.8195(06)	0.8250(06)	0.8300(06)	0.8324(07)	0.8358(07)
0.6	1.22	0.8021(06)	0.8106(06)	0.8197(06)	0.8251(06)	0.8291(07)	0.8319(07)	0.8362(06)
0.7	1.53	0.8020(06)	0.8114(07)	0.8195(06)	0.8251(07)	0.8297(06)	0.8323(07)	0.8359(06)
0.8	2	0.8018(06)	0.8114(06)	0.8191(06)	0.8247(06)	0.8293(08)	0.8314(06)	0.8358(06)
0.8605	2.48	0.8026(06)	0.8109(07)	0.8197(06)	0.8247(07)	0.8290(06)	0.8319(06)	0.8357(06)
0.9	3	0.8027(07)	0.8112(06)	0.8195(06)	0.8246(06)	0.8294(06)	0.8317(07)	0.8359(06)

Table A.82: Case IIIa Loglogistic Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8699(08)	0.8796(08)	0.8878(08)	0.8938(09)	0.8990(09)	0.9013(09)	0.9058(08)
0.0323	0.18	0.8696(07)	0.8790(07)	0.8885(07)	0.8946(08)	0.8996(09)	0.9019(09)	0.9063(07)
0.1	0.33	0.8700(08)	0.8798(08)	0.8883(09)	0.8941(08)	0.8989(08)	0.9016(09)	0.9059(08)
0.2	0.5	0.8697(08)	0.8800(08)	0.8881(08)	0.8945(08)	0.9003(07)	0.9017(07)	0.9055(08)
0.3	0.65	0.8692(08)	0.8792(07)	0.8890(09)	0.8944(08)	0.8986(09)	0.9017(09)	0.9053(09)
0.4	0.82	0.8700(08)	0.8791(07)	0.8880(08)	0.8945(08)	0.8993(08)	0.9022(08)	0.9056(08)
0.5	1	0.8702(07)	0.8793(07)	0.8886(08)	0.8944(07)	0.9003(08)	0.9024(09)	0.9062(08)
0.6	1.22	0.8699(08)	0.8788(08)	0.8886(08)	0.8948(09)	0.8988(08)	0.9018(09)	0.9059(08)
0.7	1.53	0.8701(08)	0.8793(08)	0.8884(08)	0.8947(08)	0.8992(08)	0.9022(09)	0.9060(08)
0.8	2	0.8693(07)	0.8798(08)	0.8886(09)	0.8944(09)	0.8991(10)	0.9014(09)	0.9065(07)
0.8605	2.48	0.8695(08)	0.8795(08)	0.8885(07)	0.8939(08)	0.8990(08)	0.9022(07)	0.9052(07)
0.9	3	0.8707(09)	0.8793(08)	0.8885(09)	0.8940(08)	0.8991(07)	0.9018(08)	0.9059(08)

Table A.83: Case IIIa Loglogistic Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0039(17)	1.0173(16)	1.0276(14)	1.0336(16)	1.0407(16)	1.0428(17)	1.0450(15)
0.0323	0.18	1.0038(15)	1.0158(16)	1.0287(15)	1.0341(17)	1.0387(16)	1.0425(17)	1.0461(16)
0.1	0.33	1.0036(15)	1.0163(18)	1.0273(15)	1.0329(17)	1.0393(15)	1.0430(15)	1.0465(16)
0.2	0.5	1.0033(14)	1.0161(17)	1.0263(13)	1.0324(16)	1.0391(14)	1.0410(16)	1.0464(16)
0.3	0.65	1.0019(14)	1.0171(17)	1.0280(16)	1.0336(16)	1.0392(16)	1.0418(16)	1.0461(15)
0.4	0.82	1.0028(16)	1.0147(15)	1.0262(17)	1.0328(14)	1.0395(15)	1.0420(15)	1.0467(16)
0.5	1	1.0038(15)	1.0173(15)	1.0264(16)	1.0344(16)	1.0404(16)	1.0423(17)	1.0459(15)
0.6	1.22	1.0039(14)	1.0162(14)	1.0261(15)	1.0343(16)	1.0393(17)	1.0436(18)	1.0457(15)
0.7	1.53	1.0045(13)	1.0165(16)	1.0279(16)	1.0344(14)	1.0400(15)	1.0433(15)	1.0466(17)
0.8	2	1.0033(15)	1.0156(14)	1.0278(16)	1.0335(17)	1.0398(17)	1.0418(19)	1.0455(14)
0.8605	2.48	1.0028(15)	1.0177(13)	1.0272(15)	1.0337(14)	1.0376(14)	1.0415(17)	1.0460(15)
0.9	3	1.0051(14)	1.0160(18)	1.0263(16)	1.0333(17)	1.0388(14)	1.0428(17)	1.0466(17)

Table A.84: Case IIIa Loglogistic Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3732(09)	1.3866(08)	1.3996(09)	1.4084(09)	1.4179(10)	1.4219(09)	1.4293(10)
0.0323	0.18	1.3732(11)	1.3861(10)	1.3998(09)	1.4093(10)	1.4168(10)	1.4224(09)	1.4304(10)
0.1	0.33	1.3736(10)	1.3870(09)	1.4000(09)	1.4098(10)	1.4177(09)	1.4224(09)	1.4298(10)
0.2	0.5	1.3741(10)	1.3869(09)	1.3985(09)	1.4097(09)	1.4189(09)	1.4219(09)	1.4290(09)
0.3	0.65	1.3733(08)	1.3870(09)	1.4009(08)	1.4096(10)	1.4175(10)	1.4230(10)	1.4298(09)
0.4	0.82	1.3734(08)	1.3865(09)	1.3996(08)	1.4087(09)	1.4176(09)	1.4230(09)	1.4294(09)
0.5	1	1.3736(08)	1.3870(09)	1.4002(10)	1.4098(08)	1.4184(09)	1.4228(09)	1.4299(10)
0.6	1.22	1.3731(09)	1.3860(09)	1.4002(08)	1.4101(08)	1.4177(09)	1.4223(10)	1.4297(09)
0.7	1.53	1.3736(08)	1.3874(11)	1.4006(09)	1.4092(09)	1.4185(09)	1.4232(10)	1.4300(10)
0.8	2	1.3728(08)	1.3872(09)	1.4004(09)	1.4092(09)	1.4179(10)	1.4217(10)	1.4291(09)
0.8605	2.48	1.3741(09)	1.3864(10)	1.4006(10)	1.4094(12)	1.4172(09)	1.4224(09)	1.4296(09)
0.9	3	1.3736(11)	1.3870(10)	1.4002(09)	1.4102(09)	1.4178(08)	1.4217(10)	1.4294(09)

Table A.85: Case IIIa Loglogistic Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4498(10)	1.4633(10)	1.4775(11)	1.4862(10)	1.4951(12)	1.4996(11)	1.5074(11)
0.0323	0.18	1.4496(11)	1.4635(10)	1.4775(11)	1.4871(11)	1.4950(11)	1.5000(11)	1.5086(11)
0.1	0.33	1.4504(11)	1.4641(10)	1.4778(11)	1.4872(11)	1.4955(10)	1.4999(12)	1.5079(12)
0.2	0.5	1.4508(12)	1.4643(10)	1.4762(09)	1.4876(11)	1.4971(10)	1.5001(11)	1.5064(11)
0.3	0.65	1.4497(09)	1.4634(10)	1.4785(11)	1.4876(12)	1.4957(11)	1.5003(12)	1.5074(09)
0.4	0.82	1.4503(09)	1.4640(10)	1.4773(11)	1.4865(10)	1.4955(10)	1.5009(11)	1.5076(10)
0.5	1	1.4504(10)	1.4640(10)	1.4781(12)	1.4871(09)	1.4965(11)	1.5007(11)	1.5076(11)
0.6	1.22	1.4500(11)	1.4629(11)	1.4785(10)	1.4873(11)	1.4951(12)	1.5003(12)	1.5079(11)
0.7	1.53	1.4500(11)	1.4644(12)	1.4783(11)	1.4877(11)	1.4961(12)	1.5013(12)	1.5082(10)
0.8	2	1.4496(09)	1.4646(11)	1.4776(11)	1.4872(10)	1.4953(12)	1.4994(13)	1.5070(10)
0.8605	2.48	1.4503(11)	1.4642(12)	1.4777(11)	1.4870(12)	1.4949(11)	1.5002(09)	1.5080(10)
0.9	3	1.4509(13)	1.4633(11)	1.4781(10)	1.4876(10)	1.4954(10)	1.4997(12)	1.5082(09)

Table A.86: Case IIIa Loglogistic Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5697(14)	1.5840(13)	1.5988(13)	1.6072(13)	1.6174(15)	1.6214(14)	1.6292(15)
0.0323	0.18	1.5692(14)	1.5840(13)	1.5983(13)	1.6094(16)	1.6174(15)	1.6213(15)	1.6307(15)
0.1	0.33	1.5707(14)	1.5850(15)	1.5976(15)	1.6082(15)	1.6168(14)	1.6222(16)	1.6301(13)
0.2	0.5	1.5694(16)	1.5846(13)	1.5977(12)	1.6084(15)	1.6194(14)	1.6210(13)	1.6292(15)
0.3	0.65	1.5689(15)	1.5841(13)	1.6004(14)	1.6097(15)	1.6176(14)	1.6222(16)	1.6289(13)
0.4	0.82	1.5698(14)	1.5843(13)	1.5980(15)	1.6085(15)	1.6165(13)	1.6222(15)	1.6295(13)
0.5	1	1.5693(14)	1.5846(14)	1.5989(15)	1.6089(13)	1.6187(14)	1.6231(14)	1.6304(17)
0.6	1.22	1.5699(14)	1.5828(14)	1.5995(15)	1.6094(14)	1.6160(16)	1.6230(16)	1.6298(14)
0.7	1.53	1.5697(13)	1.5839(16)	1.5988(14)	1.6091(14)	1.6166(15)	1.6223(14)	1.6301(14)
0.8	2	1.5695(14)	1.5848(14)	1.5988(17)	1.6087(15)	1.6177(17)	1.6215(15)	1.6290(15)
0.8605	2.48	1.5692(15)	1.5850(14)	1.5985(14)	1.6076(14)	1.6161(14)	1.6214(12)	1.6297(14)
0.9	3	1.5716(17)	1.5834(12)	1.5991(13)	1.6090(13)	1.6174(12)	1.6213(14)	1.6301(14)

Table A.87: Case IIIa Loglogistic Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.8091(29)	1.8283(31)	1.8427(26)	1.8543(27)	1.8638(27)	1.8669(26)	1.8732(26)
0.0323	0.18	1.8101(27)	1.8274(27)	1.8441(26)	1.8539(30)	1.8625(29)	1.8669(31)	1.8758(29)
0.1	0.33	1.8102(30)	1.8256(28)	1.8418(30)	1.8527(29)	1.8620(28)	1.8703(29)	1.8766(27)
0.2	0.5	1.8094(26)	1.8275(29)	1.8419(27)	1.8511(26)	1.8641(26)	1.8663(27)	1.8738(29)
0.3	0.65	1.8060(29)	1.8284(26)	1.8446(29)	1.8525(27)	1.8619(27)	1.8703(29)	1.8747(26)
0.4	0.82	1.8098(27)	1.8264(26)	1.8401(28)	1.8516(25)	1.8656(25)	1.8675(27)	1.8771(27)
0.5	1	1.8094(25)	1.8292(26)	1.8459(26)	1.8534(26)	1.8654(27)	1.8670(30)	1.8766(28)
0.6	1.22	1.8099(26)	1.8261(30)	1.8428(30)	1.8531(29)	1.8617(29)	1.8694(31)	1.8774(27)
0.7	1.53	1.8100(26)	1.8273(31)	1.8424(27)	1.8530(25)	1.8643(30)	1.8669(27)	1.8749(26)
0.8	2	1.8077(30)	1.8271(27)	1.8428(29)	1.8542(31)	1.8643(29)	1.8696(30)	1.8754(26)
0.8605	2.48	1.8081(28)	1.8290(27)	1.8443(30)	1.8546(27)	1.8605(26)	1.8667(27)	1.8751(29)
0.9	3	1.8125(27)	1.8266(27)	1.8426(30)	1.8510(25)	1.8618(28)	1.8683(26)	1.8773(29)

Table A.88: Case IIIa Loglogistic Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.10342(19)	0.10312(18)	0.10297(18)	0.10279(18)	0.10285(20)	0.10281(17)	0.10292(21)
0.0323	0.18	0.10337(19)	0.10299(18)	0.10305(18)	0.10297(19)	0.10279(18)	0.10292(19)	0.10303(18)
0.1	0.33	0.10350(19)	0.10329(18)	0.10301(17)	0.10304(19)	0.10280(17)	0.10298(17)	0.10297(18)
0.2	0.5	0.10354(19)	0.10330(18)	0.10283(16)	0.10299(19)	0.10303(16)	0.10281(18)	0.10280(18)
0.3	0.65	0.10337(16)	0.10313(19)	0.10320(18)	0.10296(18)	0.10279(19)	0.10307(19)	0.10293(17)
0.4	0.82	0.10356(17)	0.10320(16)	0.10298(20)	0.10288(17)	0.10287(16)	0.10292(18)	0.10284(18)
0.5	1	0.10353(18)	0.10330(17)	0.10307(19)	0.10297(19)	0.10285(20)	0.10294(19)	0.10295(20)
0.6	1.22	0.10336(18)	0.10311(19)	0.10312(17)	0.10300(16)	0.10287(19)	0.10292(21)	0.10285(17)
0.7	1.53	0.10351(19)	0.10347(19)	0.10307(19)	0.10309(16)	0.10288(19)	0.10305(22)	0.10289(17)
0.8	2	0.10345(15)	0.10333(18)	0.10300(17)	0.10296(18)	0.10289(22)	0.10276(19)	0.10277(19)
0.8605	2.48	0.10355(17)	0.10316(19)	0.10319(19)	0.10290(21)	0.10279(18)	0.10284(18)	0.10288(18)
0.9	3	0.10351(22)	0.10319(19)	0.10306(17)	0.10307(18)	0.10291(17)	0.10286(20)	0.10289(18)

Table A.89: Case IIIa Loglogistic Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.11959(22)	0.11935(22)	0.11929(25)	0.11905(22)	0.11914(27)	0.11900(25)	0.11929(26)
0.0323	0.18	0.11953(25)	0.11934(24)	0.11930(23)	0.11929(25)	0.11913(25)	0.11924(26)	0.11953(23)
0.1	0.33	0.11969(23)	0.11955(26)	0.11929(23)	0.11924(24)	0.11911(21)	0.11924(25)	0.11928(24)
0.2	0.5	0.11979(25)	0.11956(22)	0.11902(21)	0.11921(24)	0.11942(23)	0.11903(23)	0.11908(25)
0.3	0.65	0.11966(23)	0.11951(23)	0.11961(24)	0.11931(25)	0.11915(22)	0.11934(25)	0.11911(22)
0.4	0.82	0.11970(22)	0.11955(24)	0.11927(25)	0.11917(22)	0.11901(21)	0.11926(21)	0.11918(22)
0.5	1	0.11974(25)	0.11960(25)	0.11954(25)	0.11929(26)	0.11925(25)	0.11933(24)	0.11929(25)
0.6	1.22	0.11957(24)	0.11933(27)	0.11947(23)	0.11938(23)	0.11923(25)	0.11923(27)	0.11906(22)
0.7	1.53	0.11970(25)	0.11965(22)	0.11946(26)	0.11929(23)	0.11915(25)	0.11945(27)	0.11925(21)
0.8	2	0.11969(22)	0.11958(25)	0.11931(23)	0.11928(24)	0.11918(26)	0.11891(24)	0.11907(23)
0.8605	2.48	0.11979(24)	0.11948(25)	0.11943(25)	0.11921(24)	0.11916(22)	0.11908(23)	0.11922(23)
0.9	3	0.11990(29)	0.11946(25)	0.11934(23)	0.11926(24)	0.11927(21)	0.11920(25)	0.11932(25)

Table A.90: Case IIIa Loglogistic Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.14813(30)	0.14804(35)	0.14792(33)	0.14772(32)	0.14789(38)	0.14753(35)	0.14780(38)
0.0323	0.18	0.14803(41)	0.14799(35)	0.14782(36)	0.14782(36)	0.14771(40)	0.14768(36)	0.14819(37)
0.1	0.33	0.14836(37)	0.14799(40)	0.14774(33)	0.14795(35)	0.14766(33)	0.14782(39)	0.14783(36)
0.2	0.5	0.14804(42)	0.14827(39)	0.14756(33)	0.14763(35)	0.14819(33)	0.14775(33)	0.14780(37)
0.3	0.65	0.14795(36)	0.14800(34)	0.14804(36)	0.14806(38)	0.14768(32)	0.14795(39)	0.14770(33)
0.4	0.82	0.14809(35)	0.14818(33)	0.14788(41)	0.14782(38)	0.14754(32)	0.14796(38)	0.14801(33)
0.5	1	0.14805(39)	0.14799(36)	0.14796(33)	0.14773(36)	0.14801(34)	0.14822(39)	0.14782(34)
0.6	1.22	0.14807(39)	0.14746(34)	0.14808(35)	0.14785(31)	0.14756(37)	0.14809(38)	0.14774(33)
0.7	1.53	0.14793(34)	0.14820(38)	0.14818(32)	0.14804(35)	0.14781(38)	0.14786(39)	0.14806(35)
0.8	2	0.14801(32)	0.14807(36)	0.14805(39)	0.14802(41)	0.14799(40)	0.14748(38)	0.14784(36)
0.8605	2.48	0.14805(36)	0.14808(36)	0.14777(38)	0.14758(34)	0.14786(35)	0.14761(31)	0.14783(34)
0.9	3	0.14836(42)	0.14778(34)	0.14799(38)	0.14786(37)	0.14782(31)	0.14771(35)	0.14815(38)

Table A.91: Case IIIa Loglogistic Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.21625(95)	0.21744(86)	0.21721(81)	0.21739(84)	0.21726(87)	0.21679(89)	0.21647(84)
0.0323	0.18	0.21661(90)	0.21673(77)	0.21793(82)	0.21777(87)	0.21749(89)	0.21786(93)	0.21754(87)
0.1	0.33	0.21699(76)	0.21672(98)	0.21744(83)	0.21692(85)	0.21707(84)	0.21831(97)	0.21764(92)
0.2	0.5	0.21698(90)	0.21733(80)	0.21700(86)	0.21662(90)	0.21787(85)	0.21803(90)	0.21721(95)
0.3	0.65	0.21590(91)	0.21766(82)	0.21755(91)	0.21742(97)	0.21695(89)	0.21781(87)	0.21776(85)
0.4	0.82	0.21631(85)	0.21676(87)	0.21626(79)	0.21675(86)	0.21816(88)	0.21779(86)	0.21805(76)
0.5	1	0.21708(92)	0.21782(88)	0.21850(92)	0.21726(95)	0.21753(86)	0.21742(92)	0.21799(77)
0.6	1.22	0.21625(96)	0.21672(95)	0.21717(79)	0.21681(93)	0.21699(91)	0.21733(85)	0.21731(75)
0.7	1.53	0.21671(80)	0.21673(84)	0.21697(81)	0.21779(85)	0.21754(93)	0.21795(82)	0.21768(91)
0.8	2	0.21613(83)	0.21695(86)	0.21705(84)	0.21692(95)	0.21789(90)	0.21747(83)	0.21748(80)
0.8605	2.48	0.21668(90)	0.21728(85)	0.21732(86)	0.21715(90)	0.21722(79)	0.21733(79)	0.21748(89)
0.9	3	0.21752(85)	0.21699(83)	0.21733(98)	0.21700(75)	0.21725(94)	0.21729(84)	0.21837(86)

Table A.92: Case IIIa Loglogistic Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7439(13)	0.7463(12)	0.7475(12)	0.7467(13)	0.7481(14)	0.7475(13)	0.7495(13)
0.0323	0.18	0.7433(13)	0.7450(12)	0.7468(12)	0.7487(13)	0.7476(14)	0.7488(13)	0.7497(12)
0.1	0.33	0.7437(13)	0.7458(12)	0.7471(12)	0.7475(13)	0.7475(12)	0.7483(12)	0.7490(13)
0.2	0.5	0.7442(14)	0.7461(12)	0.7455(10)	0.7481(13)	0.7489(11)	0.7481(13)	0.7473(12)
0.3	0.65	0.7435(11)	0.7455(10)	0.7480(13)	0.7478(12)	0.7475(12)	0.7489(13)	0.7479(11)
0.4	0.82	0.7449(12)	0.7452(13)	0.7467(12)	0.7475(11)	0.7476(11)	0.7484(11)	0.7484(12)
0.5	1	0.7439(12)	0.7465(12)	0.7478(12)	0.7482(13)	0.7477(14)	0.7489(12)	0.7488(13)
0.6	1.22	0.7435(14)	0.7448(13)	0.7480(11)	0.7478(11)	0.7480(12)	0.7479(14)	0.7483(11)
0.7	1.53	0.7433(12)	0.7463(12)	0.7478(13)	0.7486(12)	0.7486(14)	0.7498(14)	0.7491(12)
0.8	2	0.7436(11)	0.7459(12)	0.7466(12)	0.7479(13)	0.7482(13)	0.7478(13)	0.7487(13)
0.8605	2.48	0.7440(10)	0.7444(12)	0.7473(14)	0.7477(13)	0.7476(11)	0.7473(12)	0.7487(13)
0.9	3	0.7438(13)	0.7449(13)	0.7470(12)	0.7480(13)	0.7489(12)	0.7480(14)	0.7482(12)

Table A.93: Case IIIa Loglogistic Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.8508(14)	0.8533(14)	0.8564(16)	0.8550(16)	0.8564(18)	0.8550(17)	0.8571(18)
0.0323	0.18	0.8505(18)	0.8528(16)	0.8542(15)	0.8567(16)	0.8556(18)	0.8574(16)	0.8578(15)
0.1	0.33	0.8510(17)	0.8530(16)	0.8543(17)	0.8561(16)	0.8556(15)	0.8569(16)	0.8573(16)
0.2	0.5	0.8514(16)	0.8540(16)	0.8528(14)	0.8558(17)	0.8575(16)	0.8561(15)	0.8556(16)
0.3	0.65	0.8506(17)	0.8531(15)	0.8555(16)	0.8565(15)	0.8554(14)	0.8564(16)	0.8551(15)
0.4	0.82	0.8510(16)	0.8533(16)	0.8547(18)	0.8556(16)	0.8558(14)	0.8567(14)	0.8562(13)
0.5	1	0.8508(16)	0.8537(16)	0.8557(17)	0.8560(17)	0.8558(16)	0.8570(17)	0.8569(17)
0.6	1.22	0.8509(18)	0.8520(16)	0.8565(15)	0.8559(15)	0.8559(17)	0.8567(19)	0.8559(15)
0.7	1.53	0.8493(15)	0.8540(15)	0.8558(17)	0.8570(16)	0.8563(17)	0.8582(18)	0.8578(14)
0.8	2	0.8497(15)	0.8532(14)	0.8549(16)	0.8564(16)	0.8569(17)	0.8551(17)	0.8562(16)
0.8605	2.48	0.8512(13)	0.8522(15)	0.8542(17)	0.8550(16)	0.8559(15)	0.8558(16)	0.8564(16)
0.9	3	0.8512(17)	0.8517(16)	0.8543(14)	0.8559(16)	0.8565(14)	0.8558(16)	0.8566(17)

Table A.94: Case IIIa Loglogistic Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0379(24)	1.0408(22)	1.0449(22)	1.0431(24)	1.0444(25)	1.0431(23)	1.0449(26)
0.0323	0.18	1.0374(25)	1.0421(27)	1.0430(23)	1.0454(24)	1.0444(27)	1.0453(23)	1.0474(21)
0.1	0.33	1.0386(23)	1.0414(26)	1.0437(24)	1.0431(23)	1.0441(22)	1.0442(24)	1.0452(24)
0.2	0.5	1.0378(28)	1.0428(24)	1.0408(21)	1.0443(24)	1.0477(23)	1.0468(24)	1.0448(24)
0.3	0.65	1.0384(25)	1.0415(23)	1.0451(23)	1.0455(25)	1.0440(22)	1.0461(25)	1.0454(23)
0.4	0.82	1.0378(26)	1.0413(24)	1.0418(25)	1.0440(24)	1.0442(19)	1.0457(24)	1.0458(19)
0.5	1	1.0397(25)	1.0416(21)	1.0446(25)	1.0438(26)	1.0457(24)	1.0465(24)	1.0458(23)
0.6	1.22	1.0384(29)	1.0386(24)	1.0431(22)	1.0440(22)	1.0449(24)	1.0451(26)	1.0450(23)
0.7	1.53	1.0365(21)	1.0437(24)	1.0443(24)	1.0450(22)	1.0448(25)	1.0466(26)	1.0472(23)
0.8	2	1.0350(24)	1.0415(21)	1.0429(26)	1.0438(27)	1.0461(27)	1.0435(27)	1.0453(22)
0.8605	2.48	1.0379(23)	1.0402(25)	1.0417(25)	1.0420(22)	1.0445(24)	1.0445(21)	1.0441(25)
0.9	3	1.0387(27)	1.0389(26)	1.0437(24)	1.0429(26)	1.0445(21)	1.0443(22)	1.0460(25)

Table A.95: Case IIIa Loglogistic Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4951(63)	1.5006(58)	1.5005(55)	1.5019(58)	1.5043(59)	1.4973(61)	1.4996(58)
0.0323	0.18	1.4929(54)	1.4978(53)	1.5020(54)	1.5061(60)	1.5056(68)	1.5061(61)	1.5004(55)
0.1	0.33	1.5013(60)	1.4982(58)	1.5014(47)	1.5005(56)	1.4998(60)	1.5055(60)	1.5031(55)
0.2	0.5	1.4990(67)	1.5010(50)	1.5006(54)	1.5025(52)	1.5073(61)	1.5040(58)	1.5012(59)
0.3	0.65	1.4954(61)	1.5008(56)	1.5033(62)	1.5015(62)	1.4997(54)	1.5052(53)	1.5037(51)
0.4	0.82	1.4912(60)	1.4996(54)	1.4985(50)	1.4960(57)	1.5054(60)	1.5051(56)	1.5079(49)
0.5	1	1.4961(61)	1.5030(59)	1.5051(54)	1.5003(59)	1.5072(60)	1.4993(61)	1.5076(56)
0.6	1.22	1.4898(56)	1.4983(64)	1.5007(54)	1.5024(57)	1.5006(63)	1.4993(56)	1.5016(56)
0.7	1.53	1.4920(58)	1.4976(62)	1.5018(59)	1.5036(56)	1.5023(62)	1.5012(47)	1.5078(62)
0.8	2	1.4889(57)	1.4967(57)	1.4985(56)	1.4988(64)	1.5038(61)	1.5051(56)	1.5007(52)
0.8605	2.48	1.4943(61)	1.4987(57)	1.4989(60)	1.5014(55)	1.5004(58)	1.5005(57)	1.5022(55)
0.9	3	1.4995(58)	1.4955(55)	1.5043(60)	1.4969(60)	1.5013(58)	1.5000(55)	1.5126(58)

Table A.96: Case IIIa Loglogistic Anderson-Darling 99% critical values

A.3.3 Lognormal Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.8054(06)	0.8137(06)	0.8212(06)	0.8273(06)	0.8308(06)	0.8338(06)	0.8375(05)
0.0323	0.72	0.7973(06)	0.8051(06)	0.8128(06)	0.8181(07)	0.8222(06)	0.8250(06)	0.8287(06)
0.1	0.75	0.8001(06)	0.8079(06)	0.8152(06)	0.8200(06)	0.8246(06)	0.8272(06)	0.8311(06)
0.2	0.79	0.8077(05)	0.8155(06)	0.8231(06)	0.8283(06)	0.8330(05)	0.8349(06)	0.8387(05)
0.3	0.85	0.8163(06)	0.8243(07)	0.8310(06)	0.8364(06)	0.8414(06)	0.8430(06)	0.8468(07)
0.4	0.91	0.8243(07)	0.8316(06)	0.8390(06)	0.8443(07)	0.8487(07)	0.8513(06)	0.8539(06)
0.5	1	0.8314(06)	0.8394(07)	0.8466(07)	0.8522(07)	0.8561(06)	0.8577(07)	0.8617(06)
0.6	1.12	0.8388(07)	0.8462(06)	0.8537(07)	0.8591(07)	0.8629(08)	0.8653(06)	0.8692(06)
0.7	1.29	0.8460(07)	0.8530(07)	0.8613(07)	0.8658(07)	0.8708(06)	0.8727(07)	0.8762(07)
0.8	1.58	0.8543(06)	0.8613(07)	0.8687(07)	0.8733(07)	0.8784(07)	0.8796(06)	0.8842(07)
0.8605	1.89	0.8588(07)	0.8667(07)	0.8729(07)	0.8786(07)	0.8826(08)	0.8844(07)	0.8885(07)
0.9	2.24	0.8628(06)	0.8705(07)	0.8770(07)	0.8819(06)	0.8865(06)	0.8893(07)	0.8930(07)

Table A.97: Case IIIa Lognormal Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.8527(06)	0.8615(07)	0.8692(07)	0.8753(07)	0.8794(08)	0.8826(06)	0.8859(06)
0.0323	0.72	0.8442(06)	0.8522(07)	0.8600(07)	0.8656(08)	0.8703(07)	0.8726(08)	0.8766(06)
0.1	0.75	0.8473(07)	0.8554(07)	0.8632(07)	0.8684(07)	0.8733(07)	0.8755(07)	0.8792(06)
0.2	0.79	0.8565(07)	0.8653(08)	0.8727(06)	0.8774(07)	0.8833(07)	0.8849(06)	0.8887(07)
0.3	0.85	0.8664(08)	0.8750(07)	0.8827(07)	0.8873(07)	0.8928(07)	0.8942(07)	0.8980(08)
0.4	0.91	0.8759(08)	0.8839(07)	0.8915(07)	0.8968(08)	0.9013(08)	0.9040(07)	0.9070(07)
0.5	1	0.8840(07)	0.8924(08)	0.9003(09)	0.9055(08)	0.9097(08)	0.9117(08)	0.9156(07)
0.6	1.12	0.8926(08)	0.9000(08)	0.9085(07)	0.9137(08)	0.9177(09)	0.9206(07)	0.9246(08)
0.7	1.29	0.9005(08)	0.9086(07)	0.9170(08)	0.9215(08)	0.9268(07)	0.9284(09)	0.9324(08)
0.8	1.58	0.9102(08)	0.9174(08)	0.9254(08)	0.9295(08)	0.9356(09)	0.9366(07)	0.9413(09)
0.8605	1.89	0.9160(08)	0.9239(09)	0.9305(08)	0.9362(08)	0.9402(10)	0.9426(08)	0.9462(08)
0.9	2.24	0.9202(08)	0.9279(09)	0.9351(08)	0.9399(08)	0.9445(08)	0.9472(08)	0.9511(09)

Table A.98: Case IIIa Lognormal Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.9253(07)	0.9346(08)	0.9434(09)	0.9506(09)	0.9545(10)	0.9576(09)	0.9611(09)
0.0323	0.72	0.9156(08)	0.9247(09)	0.9332(09)	0.9389(09)	0.9444(09)	0.9470(10)	0.9502(08)
0.1	0.75	0.9201(09)	0.9298(09)	0.9380(09)	0.9436(08)	0.9484(09)	0.9504(08)	0.9544(08)
0.2	0.79	0.9326(09)	0.9428(09)	0.9497(08)	0.9552(09)	0.9614(08)	0.9624(09)	0.9666(09)
0.3	0.85	0.9447(10)	0.9546(09)	0.9627(09)	0.9676(09)	0.9735(09)	0.9745(10)	0.9781(10)
0.4	0.91	0.9561(10)	0.9655(09)	0.9738(09)	0.9787(10)	0.9842(09)	0.9862(09)	0.9894(11)
0.5	1	0.9667(10)	0.9760(10)	0.9846(11)	0.9903(10)	0.9943(09)	0.9957(11)	0.9998(10)
0.6	1.12	0.9764(10)	0.9861(10)	0.9942(11)	0.9999(11)	1.0039(11)	1.0067(09)	1.0109(10)
0.7	1.29	0.9871(11)	0.9952(09)	1.0044(10)	1.0095(10)	1.0145(10)	1.0166(10)	1.0203(11)
0.8	1.58	0.9982(11)	1.0061(10)	1.0150(10)	1.0192(11)	1.0249(12)	1.0261(09)	1.0304(11)
0.8605	1.89	1.0045(10)	1.0141(10)	1.0203(12)	1.0264(10)	1.0309(11)	1.0336(09)	1.0373(12)
0.9	2.24	1.0097(10)	1.0185(11)	1.0269(11)	1.0311(11)	1.0357(10)	1.0388(10)	1.0419(12)

Table A.99: Case IIIa Lognormal Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.0678(17)	1.0807(16)	1.0922(18)	1.0986(17)	1.1046(18)	1.1071(16)	1.1112(16)
0.0323	0.72	1.0572(14)	1.0691(17)	1.0768(16)	1.0869(15)	1.0934(14)	1.0961(17)	1.0969(15)
0.1	0.75	1.0661(17)	1.0774(17)	1.0878(17)	1.0953(17)	1.0993(17)	1.1030(14)	1.1049(16)
0.2	0.79	1.0846(17)	1.0974(17)	1.1072(16)	1.1129(21)	1.1206(17)	1.1193(19)	1.1250(20)
0.3	0.85	1.1034(19)	1.1156(16)	1.1256(18)	1.1308(20)	1.1369(18)	1.1380(18)	1.1418(19)
0.4	0.91	1.1192(18)	1.1302(17)	1.1395(20)	1.1469(19)	1.1526(18)	1.1551(20)	1.1586(20)
0.5	1	1.1333(20)	1.1457(21)	1.1572(19)	1.1604(19)	1.1679(21)	1.1683(18)	1.1729(19)
0.6	1.12	1.1470(20)	1.1600(19)	1.1696(24)	1.1750(21)	1.1808(18)	1.1837(19)	1.1852(21)
0.7	1.29	1.1621(21)	1.1716(20)	1.1822(19)	1.1872(18)	1.1960(20)	1.1969(20)	1.1996(21)
0.8	1.58	1.1760(20)	1.1874(19)	1.1960(19)	1.1998(19)	1.2084(21)	1.2081(22)	1.2135(20)
0.8605	1.89	1.1833(21)	1.1948(19)	1.2037(23)	1.2106(19)	1.2170(22)	1.2170(16)	1.2231(23)
0.9	2.24	1.1910(21)	1.2007(20)	1.2100(21)	1.2161(20)	1.2204(20)	1.2255(20)	1.2286(22)

Table A.100: Case IIIa Lognormal Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.4009(10)	1.4145(10)	1.4279(10)	1.4378(10)	1.4448(09)	1.4506(10)	1.4573(09)
0.0323	0.72	1.3968(09)	1.4102(10)	1.4228(09)	1.4335(10)	1.4408(10)	1.4465(11)	1.4529(08)
0.1	0.75	1.3943(09)	1.4074(10)	1.4210(09)	1.4299(09)	1.4384(09)	1.4427(09)	1.4510(09)
0.2	0.79	1.3915(10)	1.4055(10)	1.4187(09)	1.4291(09)	1.4380(10)	1.4411(10)	1.4488(09)
0.3	0.85	1.3915(09)	1.4064(10)	1.4190(10)	1.4282(10)	1.4380(09)	1.4414(09)	1.4488(11)
0.4	0.91	1.3918(10)	1.4053(09)	1.4186(10)	1.4284(09)	1.4375(11)	1.4412(10)	1.4480(10)
0.5	1	1.3915(09)	1.4056(09)	1.4194(11)	1.4296(09)	1.4373(09)	1.4409(10)	1.4479(10)
0.6	1.12	1.3917(10)	1.4059(08)	1.4191(10)	1.4295(09)	1.4370(10)	1.4420(09)	1.4492(09)
0.7	1.29	1.3923(09)	1.4051(09)	1.4202(09)	1.4295(10)	1.4379(10)	1.4425(09)	1.4494(09)
0.8	1.58	1.3946(09)	1.4070(10)	1.4211(10)	1.4298(10)	1.4392(10)	1.4430(08)	1.4509(11)
0.8605	1.89	1.3946(09)	1.4084(10)	1.4208(11)	1.4319(09)	1.4390(10)	1.4436(10)	1.4513(09)
0.9	2.24	1.3949(09)	1.4088(09)	1.4219(09)	1.4309(08)	1.4395(09)	1.4455(07)	1.4516(10)

Table A.101: Case IIIa Lognormal Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.4777(10)	1.4922(11)	1.5060(12)	1.5165(12)	1.5235(12)	1.5289(12)	1.5354(10)
0.0323	0.72	1.4744(10)	1.4876(12)	1.5016(10)	1.5111(11)	1.5192(12)	1.5248(13)	1.5314(12)
0.1	0.75	1.4710(10)	1.4849(12)	1.4987(11)	1.5082(12)	1.5173(11)	1.5212(10)	1.5292(11)
0.2	0.79	1.4686(11)	1.4836(11)	1.4962(09)	1.5068(11)	1.5165(10)	1.5194(12)	1.5272(10)
0.3	0.85	1.4683(11)	1.4837(11)	1.4964(11)	1.5073(12)	1.5160(10)	1.5192(11)	1.5266(12)
0.4	0.91	1.4676(11)	1.4821(11)	1.4966(10)	1.5066(12)	1.5155(12)	1.5194(11)	1.5260(11)
0.5	1	1.4679(10)	1.4832(11)	1.4971(12)	1.5079(10)	1.5149(10)	1.5191(11)	1.5258(12)
0.6	1.12	1.4683(11)	1.4832(10)	1.4971(12)	1.5074(11)	1.5151(11)	1.5200(11)	1.5276(10)
0.7	1.29	1.4693(11)	1.4829(10)	1.4984(10)	1.5073(11)	1.5163(11)	1.5204(10)	1.5279(10)
0.8	1.58	1.4712(10)	1.4843(11)	1.4986(11)	1.5076(10)	1.5173(11)	1.5205(10)	1.5293(13)
0.8605	1.89	1.4712(11)	1.4859(11)	1.4987(12)	1.5096(11)	1.5174(12)	1.5219(11)	1.5289(12)
0.9	2.24	1.4718(10)	1.4857(11)	1.4996(10)	1.5082(10)	1.5175(10)	1.5236(09)	1.5299(12)

Table A.102: Case IIIa Lognormal Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.5978(14)	1.6122(15)	1.6276(13)	1.6381(14)	1.6449(17)	1.6511(14)	1.6584(14)
0.0323	0.72	1.5937(12)	1.6093(16)	1.6215(13)	1.6335(13)	1.6418(14)	1.6467(17)	1.6536(14)
0.1	0.75	1.5897(13)	1.6055(15)	1.6206(15)	1.6301(15)	1.6381(14)	1.6435(14)	1.6513(16)
0.2	0.79	1.5873(14)	1.6045(15)	1.6179(12)	1.6284(16)	1.6384(13)	1.6407(16)	1.6488(13)
0.3	0.85	1.5873(15)	1.6035(15)	1.6183(16)	1.6285(14)	1.6374(13)	1.6409(15)	1.6481(15)
0.4	0.91	1.5867(16)	1.6031(14)	1.6171(14)	1.6275(15)	1.6373(15)	1.6419(15)	1.6478(15)
0.5	1	1.5870(13)	1.6032(15)	1.6179(15)	1.6280(14)	1.6372(15)	1.6406(15)	1.6477(15)
0.6	1.12	1.5865(12)	1.6031(13)	1.6182(14)	1.6289(13)	1.6365(14)	1.6414(14)	1.6494(14)
0.7	1.29	1.5886(15)	1.6030(14)	1.6192(14)	1.6283(14)	1.6383(14)	1.6421(13)	1.6490(14)
0.8	1.58	1.5901(14)	1.6045(14)	1.6191(13)	1.6281(14)	1.6394(15)	1.6417(13)	1.6505(17)
0.8605	1.89	1.5899(12)	1.6058(14)	1.6197(16)	1.6304(13)	1.6394(14)	1.6428(12)	1.6512(15)
0.9	2.24	1.5908(13)	1.6050(14)	1.6199(13)	1.6287(14)	1.6388(13)	1.6441(13)	1.6514(14)

Table A.103: Case IIIa Lognormal Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.8355(29)	1.8524(29)	1.8711(26)	1.8796(26)	1.8891(28)	1.8965(29)	1.9029(28)
0.0323	0.72	1.8322(25)	1.8502(28)	1.8637(27)	1.8769(27)	1.8887(29)	1.8911(28)	1.8980(24)
0.1	0.75	1.8271(28)	1.8460(27)	1.8643(25)	1.8754(26)	1.8835(29)	1.8893(26)	1.8941(28)
0.2	0.79	1.8249(25)	1.8454(30)	1.8622(26)	1.8719(30)	1.8838(25)	1.8846(29)	1.8948(29)
0.3	0.85	1.8251(23)	1.8460(28)	1.8645(29)	1.8717(25)	1.8814(24)	1.8858(27)	1.8920(30)
0.4	0.91	1.8267(26)	1.8448(25)	1.8617(27)	1.8726(27)	1.8806(26)	1.8874(29)	1.8931(27)
0.5	1	1.8247(27)	1.8459(26)	1.8605(28)	1.8709(28)	1.8827(28)	1.8849(29)	1.8927(29)
0.6	1.12	1.8249(27)	1.8440(25)	1.8621(28)	1.8708(30)	1.8802(24)	1.8880(27)	1.8920(29)
0.7	1.29	1.8272(29)	1.8429(27)	1.8611(25)	1.8722(30)	1.8841(28)	1.8876(24)	1.8943(28)
0.8	1.58	1.8282(27)	1.8471(27)	1.8615(25)	1.8708(27)	1.8830(26)	1.8868(28)	1.8939(24)
0.8605	1.89	1.8287(23)	1.8459(26)	1.8619(28)	1.8742(27)	1.8835(30)	1.8870(27)	1.8965(28)
0.9	2.24	1.8286(25)	1.8459(30)	1.8610(28)	1.8733(24)	1.8823(27)	1.8904(29)	1.8953(28)

Table A.104: Case IIIa Lognormal Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.11660(22)	0.11645(22)	0.11646(23)	0.11642(21)	0.11608(23)	0.11630(22)	0.11637(22)
0.0323	0.72	0.11409(20)	0.11388(23)	0.11357(21)	0.11384(22)	0.11345(22)	0.11372(22)	0.11349(20)
0.1	0.75	0.11447(20)	0.11417(23)	0.11401(19)	0.11373(22)	0.11385(19)	0.11374(21)	0.11385(20)
0.2	0.79	0.11596(22)	0.11593(21)	0.11566(22)	0.11563(21)	0.11576(18)	0.11557(22)	0.11570(20)
0.3	0.85	0.11822(21)	0.11814(23)	0.11797(24)	0.11782(22)	0.11790(21)	0.11774(19)	0.11770(22)
0.4	0.91	0.12049(23)	0.12017(24)	0.11987(24)	0.12003(24)	0.11998(24)	0.11994(21)	0.11976(23)
0.5	1	0.12239(25)	0.12250(24)	0.12212(26)	0.12238(23)	0.12212(23)	0.12206(26)	0.12183(23)
0.6	1.12	0.12464(24)	0.12449(21)	0.12438(27)	0.12454(20)	0.12423(24)	0.12437(24)	0.12436(23)
0.7	1.29	0.12692(25)	0.12686(22)	0.12697(25)	0.12661(27)	0.12664(22)	0.12668(24)	0.12660(24)
0.8	1.58	0.12968(24)	0.12960(27)	0.12944(25)	0.12919(24)	0.12938(27)	0.12898(25)	0.12936(26)
0.8605	1.89	0.13129(22)	0.13153(24)	0.13106(29)	0.13119(25)	0.13109(27)	0.13092(24)	0.13103(26)
0.9	2.24	0.13278(26)	0.13274(24)	0.13254(24)	0.13220(23)	0.13232(23)	0.13250(23)	0.13230(24)

Table A.105: Case IIIa Lognormal Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.13439(26)	0.13434(29)	0.13433(27)	0.13445(27)	0.13415(29)	0.13429(28)	0.13424(26)
0.0323	0.72	0.13148(23)	0.13151(29)	0.13120(24)	0.13147(25)	0.13114(25)	0.13130(31)	0.13110(26)
0.1	0.75	0.13208(25)	0.13185(30)	0.13181(26)	0.13169(28)	0.13153(25)	0.13159(26)	0.13165(27)
0.2	0.79	0.13417(27)	0.13414(29)	0.13390(25)	0.13391(27)	0.13403(26)	0.13389(29)	0.13389(26)
0.3	0.85	0.13698(27)	0.13706(27)	0.13675(27)	0.13669(25)	0.13676(28)	0.13657(27)	0.13654(30)
0.4	0.91	0.13963(30)	0.13953(29)	0.13930(28)	0.13938(30)	0.13935(27)	0.13932(27)	0.13918(28)
0.5	1	0.14227(31)	0.14232(31)	0.14222(33)	0.14233(28)	0.14213(29)	0.14202(33)	0.14182(28)
0.6	1.12	0.14503(28)	0.14490(28)	0.14496(32)	0.14500(30)	0.14477(31)	0.14487(30)	0.14510(29)
0.7	1.29	0.14797(32)	0.14779(30)	0.14793(30)	0.14756(32)	0.14780(31)	0.14780(32)	0.14782(33)
0.8	1.58	0.15147(29)	0.15124(35)	0.15127(30)	0.15084(33)	0.15122(36)	0.15067(33)	0.15122(35)
0.8605	1.89	0.15333(30)	0.15365(32)	0.15308(34)	0.15370(31)	0.15317(33)	0.15313(30)	0.15341(34)
0.9	2.24	0.15526(34)	0.15508(33)	0.15514(29)	0.15497(31)	0.15478(31)	0.15506(32)	0.15514(32)

Table A.106: Case IIIa Lognormal Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.16464(37)	0.16492(38)	0.16501(38)	0.16534(36)	0.16485(38)	0.16506(42)	0.16515(40)
0.0323	0.72	0.16131(35)	0.16164(42)	0.16147(34)	0.16192(33)	0.16182(36)	0.16210(42)	0.16162(38)
0.1	0.75	0.16238(38)	0.16266(43)	0.16243(38)	0.16250(41)	0.16237(36)	0.16256(37)	0.16260(39)
0.2	0.79	0.16582(40)	0.16576(42)	0.16585(33)	0.16559(41)	0.16589(38)	0.16562(42)	0.16580(38)
0.3	0.85	0.16934(44)	0.16969(38)	0.16944(42)	0.16960(36)	0.16943(41)	0.16922(46)	0.16922(42)
0.4	0.91	0.17336(48)	0.17330(43)	0.17336(42)	0.17318(41)	0.17339(48)	0.17355(37)	0.17320(45)
0.5	1	0.17665(40)	0.17735(44)	0.17729(45)	0.17711(47)	0.17714(38)	0.17698(42)	0.17683(43)
0.6	1.12	0.18040(40)	0.18090(42)	0.18107(46)	0.18099(49)	0.18066(45)	0.18096(46)	0.18117(44)
0.7	1.29	0.18507(47)	0.18475(44)	0.18499(47)	0.18466(50)	0.18522(45)	0.18507(48)	0.18485(44)
0.8	1.58	0.18927(48)	0.18965(47)	0.18927(41)	0.18909(47)	0.18939(52)	0.18912(52)	0.18982(56)
0.8605	1.89	0.19212(38)	0.19256(46)	0.19244(54)	0.19286(47)	0.19267(52)	0.19246(49)	0.19267(51)
0.9	2.24	0.19457(46)	0.19484(52)	0.19475(48)	0.19465(43)	0.19473(47)	0.19490(53)	0.19502(52)

Table A.107: Case IIIa Lognormal Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.23522(89)	0.23595(99)	0.23691(88)	0.23689(88)	0.23675(98)	0.23770(92)	0.23726(73)
0.0323	0.72	0.23140(78)	0.23278(94)	0.23252(91)	0.23309(88)	0.23392(79)	0.23361(92)	0.23322(83)
0.1	0.75	0.23350(85)	0.23499(85)	0.23552(84)	0.23612(91)	0.23538(80)	0.23649(81)	0.23557(86)
0.2	0.79	0.24021(91)	0.24126(99)	0.24210(87)	0.24237(100)	0.24300(88)	0.24131(110)	0.24252(84)
0.3	0.85	0.24670(97)	0.24847(87)	0.24940(98)	0.24902(98)	0.24877(97)	0.24812(103)	0.24810(108)
0.4	0.91	0.25368(97)	0.25471(106)	0.25484(103)	0.25510(104)	0.25522(99)	0.25648(113)	0.25454(120)
0.5	1	0.26083(106)	0.26153(106)	0.26207(101)	0.26095(110)	0.26272(109)	0.26251(108)	0.26130(104)
0.6	1.12	0.26632(103)	0.26747(103)	0.26843(129)	0.26891(117)	0.26828(98)	0.26917(117)	0.26881(107)
0.7	1.29	0.27360(121)	0.27368(99)	0.27540(112)	0.27576(109)	0.27639(104)	0.27627(115)	0.27609(121)
0.8	1.58	0.28114(97)	0.28276(116)	0.28276(115)	0.28196(111)	0.28347(115)	0.28345(113)	0.28343(108)
0.8605	1.89	0.28536(111)	0.28714(130)	0.28745(118)	0.28832(113)	0.28867(122)	0.28809(107)	0.28868(120)
0.9	2.24	0.29005(122)	0.29060(127)	0.29151(110)	0.29206(115)	0.29114(114)	0.29289(121)	0.29298(128)

Table A.108: Case IIIa Lognormal Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.7749(13)	0.7767(14)	0.7792(14)	0.7796(13)	0.7789(14)	0.7807(13)	0.7812(14)
0.0323	0.72	0.7681(14)	0.7704(14)	0.7707(13)	0.7739(13)	0.7714(14)	0.7739(14)	0.7729(13)
0.1	0.75	0.7729(12)	0.7738(13)	0.7753(12)	0.7750(11)	0.7760(13)	0.7754(14)	0.7767(13)
0.2	0.79	0.7796(14)	0.7819(14)	0.7836(13)	0.7843(13)	0.7859(12)	0.7846(15)	0.7855(13)
0.3	0.85	0.7894(13)	0.7922(15)	0.7928(14)	0.7933(14)	0.7950(13)	0.7946(13)	0.7935(14)
0.4	0.91	0.7984(15)	0.8002(14)	0.8013(14)	0.8029(15)	0.8020(15)	0.8031(12)	0.8021(13)
0.5	1	0.8060(14)	0.8093(15)	0.8104(15)	0.8121(14)	0.8119(14)	0.8119(15)	0.8109(13)
0.6	1.12	0.8155(14)	0.8167(14)	0.8190(15)	0.8208(13)	0.8195(15)	0.8210(14)	0.8218(13)
0.7	1.29	0.8239(15)	0.8253(13)	0.8285(14)	0.8286(15)	0.8295(14)	0.8301(14)	0.8296(16)
0.8	1.58	0.8352(15)	0.8368(15)	0.8391(14)	0.8389(14)	0.8409(16)	0.8393(14)	0.8407(15)
0.8605	1.89	0.8411(13)	0.8449(15)	0.8444(18)	0.8473(15)	0.8467(14)	0.8465(15)	0.8479(15)
0.9	2.24	0.8476(15)	0.8488(15)	0.8501(14)	0.8508(14)	0.8517(15)	0.8524(14)	0.8521(13)

Table A.109: Case IIIa Lognormal Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.8853(16)	0.8881(17)	0.8906(17)	0.8917(16)	0.8914(19)	0.8920(17)	0.8929(17)
0.0323	0.72	0.8772(16)	0.8807(18)	0.8816(16)	0.8842(17)	0.8834(17)	0.8846(18)	0.8834(17)
0.1	0.75	0.8840(17)	0.8858(16)	0.8882(15)	0.8873(18)	0.8889(15)	0.8886(18)	0.8894(17)
0.2	0.79	0.8937(17)	0.8969(17)	0.8983(15)	0.8992(17)	0.8998(18)	0.8994(17)	0.8998(16)
0.3	0.85	0.9065(18)	0.9093(15)	0.9098(18)	0.9110(17)	0.9110(16)	0.9110(18)	0.9119(19)
0.4	0.91	0.9175(18)	0.9199(17)	0.9211(18)	0.9211(19)	0.9219(18)	0.9230(17)	0.9222(17)
0.5	1	0.9263(19)	0.9302(18)	0.9323(20)	0.9344(17)	0.9336(17)	0.9337(18)	0.9322(17)
0.6	1.12	0.9374(17)	0.9404(18)	0.9428(19)	0.9453(18)	0.9439(19)	0.9445(18)	0.9456(19)
0.7	1.29	0.9495(17)	0.9512(17)	0.9542(18)	0.9543(19)	0.9561(19)	0.9567(19)	0.9569(20)
0.8	1.58	0.9634(18)	0.9657(19)	0.9681(20)	0.9675(19)	0.9691(19)	0.9676(19)	0.9696(21)
0.8605	1.89	0.9700(17)	0.9754(19)	0.9745(20)	0.9793(17)	0.9773(20)	0.9773(19)	0.9781(19)
0.9	2.24	0.9782(20)	0.9800(20)	0.9826(17)	0.9823(17)	0.9840(18)	0.9839(20)	0.9840(18)

Table A.110: Case IIIa Lognormal Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.0758(25)	1.0802(23)	1.0822(24)	1.0854(23)	1.0849(27)	1.0848(27)	1.0855(22)
0.0323	0.72	1.0686(23)	1.0733(26)	1.0748(21)	1.0776(23)	1.0764(26)	1.0790(24)	1.0768(24)
0.1	0.75	1.0774(24)	1.0814(25)	1.0832(25)	1.0827(25)	1.0842(23)	1.0850(25)	1.0850(24)
0.2	0.79	1.0935(24)	1.0966(26)	1.0998(21)	1.1015(27)	1.1010(25)	1.0990(25)	1.1006(24)
0.3	0.85	1.1085(26)	1.1130(22)	1.1155(28)	1.1171(23)	1.1157(28)	1.1160(27)	1.1165(26)
0.4	0.91	1.1266(27)	1.1287(27)	1.1294(29)	1.1303(27)	1.1328(29)	1.1340(23)	1.1314(26)
0.5	1	1.1383(25)	1.1442(28)	1.1466(29)	1.1477(28)	1.1475(23)	1.1464(27)	1.1453(25)
0.6	1.12	1.1531(27)	1.1600(29)	1.1613(29)	1.1616(28)	1.1617(29)	1.1638(30)	1.1634(24)
0.7	1.29	1.1707(30)	1.1735(28)	1.1756(27)	1.1772(32)	1.1795(29)	1.1788(27)	1.1781(28)
0.8	1.58	1.1893(30)	1.1931(28)	1.1934(28)	1.1936(27)	1.1954(30)	1.1943(26)	1.1984(32)
0.8605	1.89	1.1997(27)	1.2041(32)	1.2051(31)	1.2101(28)	1.2078(28)	1.2075(30)	1.2082(30)
0.9	2.24	1.2086(26)	1.2133(32)	1.2140(29)	1.2139(25)	1.2159(29)	1.2168(30)	1.2182(28)

Table A.111: Case IIIa Lognormal Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.5372(62)	1.5433(53)	1.5430(60)	1.5442(53)	1.5469(61)	1.5485(61)	1.5471(55)
0.0323	0.72	1.5358(59)	1.5418(65)	1.5365(62)	1.5397(61)	1.5421(52)	1.5417(55)	1.5363(54)
0.1	0.75	1.5519(59)	1.5533(61)	1.5569(64)	1.5568(61)	1.5564(58)	1.5606(56)	1.5582(57)
0.2	0.79	1.5826(60)	1.5826(63)	1.5902(56)	1.5907(58)	1.5905(60)	1.5819(58)	1.5878(61)
0.3	0.85	1.6129(66)	1.6192(64)	1.6179(59)	1.6144(62)	1.6185(68)	1.6116(63)	1.6107(65)
0.4	0.91	1.6399(62)	1.6433(76)	1.6368(65)	1.6359(69)	1.6430(68)	1.6437(62)	1.6392(66)
0.5	1	1.6688(68)	1.6684(74)	1.6699(60)	1.6664(57)	1.6732(69)	1.6688(73)	1.6606(66)
0.6	1.12	1.6917(70)	1.6906(74)	1.6950(72)	1.6993(70)	1.6908(63)	1.6970(76)	1.6915(67)
0.7	1.29	1.7201(75)	1.7181(63)	1.7227(65)	1.7232(68)	1.7275(68)	1.7252(66)	1.7230(73)
0.8	1.58	1.7437(64)	1.7529(76)	1.7513(67)	1.7447(64)	1.7483(79)	1.7501(74)	1.7495(63)
0.8605	1.89	1.7650(76)	1.7661(85)	1.7619(71)	1.7721(72)	1.7699(72)	1.7727(55)	1.7727(71)
0.9	2.24	1.7828(69)	1.7862(68)	1.7837(67)	1.7865(71)	1.7801(72)	1.7907(75)	1.7901(80)

Table A.112: Case IIIa Lognormal Anderson-Darling 99% critical values

A.3.4 Pareto Distribution

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.9002(07)	0.9075(07)	0.9151(07)	0.9202(07)	0.9241(08)	0.9268(08)	0.9311(08)
10	3.16	0.9007(07)	0.9078(08)	0.9153(06)	0.9201(07)	0.9243(08)	0.9268(07)	0.9301(07)
100	10	0.9001(07)	0.9078(08)	0.9148(07)	0.9196(07)	0.9250(07)	0.9266(08)	0.9311(08)
250	15.81	0.8999(08)	0.9077(07)	0.9147(08)	0.9203(09)	0.9241(07)	0.9266(08)	0.9299(07)
500	22.36	0.9006(08)	0.9079(07)	0.9150(08)	0.9202(07)	0.9252(07)	0.9272(07)	0.9303(08)
1000	31.62	0.9005(07)	0.9077(08)	0.9149(08)	0.9205(07)	0.9244(09)	0.9270(08)	0.9301(07)

Table A.113: Case IIIa Pareto Kolmogorov-Smirnov 85% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.9614(08)	0.9693(09)	0.9776(08)	0.9827(08)	0.9867(09)	0.9892(09)	0.9936(10)
10	3.16	0.9622(09)	0.9698(09)	0.9777(08)	0.9827(10)	0.9867(09)	0.9888(08)	0.9933(09)
100	10	0.9616(09)	0.9693(09)	0.9772(10)	0.9818(08)	0.9878(08)	0.9889(09)	0.9940(10)
250	15.81	0.9613(09)	0.9699(09)	0.9770(09)	0.9830(10)	0.9868(09)	0.9896(09)	0.9925(08)
500	22.36	0.9621(09)	0.9702(08)	0.9773(08)	0.9826(08)	0.9882(10)	0.9897(08)	0.9937(09)
1000	31.62	0.9618(08)	0.9697(08)	0.9775(09)	0.9825(09)	0.9871(10)	0.9901(08)	0.9927(09)

Table A.114: Case IIIa Pareto Kolmogorov-Smirnov 90% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.0575(10)	1.0668(12)	1.0757(11)	1.0819(11)	1.0859(10)	1.0872(12)	1.0920(13)
10	3.16	1.0579(12)	1.0665(11)	1.0757(10)	1.0810(12)	1.0855(12)	1.0867(13)	1.0919(12)
100	10	1.0569(11)	1.0664(11)	1.0754(12)	1.0803(11)	1.0858(12)	1.0878(13)	1.0933(13)
250	15.81	1.0571(11)	1.0674(10)	1.0755(12)	1.0812(12)	1.0855(12)	1.0883(11)	1.0921(11)
500	22.36	1.0582(11)	1.0659(11)	1.0756(12)	1.0812(11)	1.0868(11)	1.0888(12)	1.0919(11)
1000	31.62	1.0579(10)	1.0669(10)	1.0752(12)	1.0803(11)	1.0862(12)	1.0876(12)	1.0910(12)

Table A.115: Case IIIa Pareto Kolmogorov-Smirnov 95% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.2504(21)	1.2625(21)	1.2733(19)	1.2800(20)	1.2852(21)	1.2873(25)	1.2914(22)
10	3.16	1.2510(18)	1.2625(20)	1.2744(19)	1.2802(23)	1.2836(20)	1.2861(25)	1.2917(23)
100	10	1.2504(22)	1.2624(22)	1.2722(21)	1.2793(23)	1.2865(21)	1.2869(21)	1.2926(23)
250	15.81	1.2500(23)	1.2635(22)	1.2726(23)	1.2793(25)	1.2873(24)	1.2872(22)	1.2918(22)
500	22.36	1.2521(20)	1.2625(21)	1.2745(22)	1.2790(23)	1.2865(21)	1.2885(26)	1.2928(23)
1000	31.62	1.2510(23)	1.2623(22)	1.2724(22)	1.2794(21)	1.2858(25)	1.2872(25)	1.2935(24)

Table A.116: Case IIIa Pareto Kolmogorov-Smirnov 99% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.4011(08)	1.4148(09)	1.4288(09)	1.4382(09)	1.4470(09)	1.4510(10)	1.4593(10)
10	3.16	1.4018(10)	1.4157(09)	1.4293(09)	1.4387(10)	1.4474(10)	1.4513(09)	1.4587(09)
100	10	1.4009(09)	1.4145(10)	1.4286(09)	1.4376(10)	1.4474(10)	1.4515(09)	1.4592(10)
250	15.81	1.4014(09)	1.4154(09)	1.4283(09)	1.4386(10)	1.4464(10)	1.4516(09)	1.4587(09)
500	22.36	1.4024(09)	1.4158(08)	1.4283(09)	1.4386(10)	1.4477(10)	1.4517(09)	1.4586(10)
1000	31.62	1.4015(09)	1.4152(10)	1.4292(10)	1.4380(09)	1.4469(10)	1.4515(10)	1.4580(09)

Table A.117: Case IIIa Pareto Kuiper 85% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.4774(10)	1.4925(11)	1.5064(11)	1.5164(11)	1.5249(11)	1.5295(11)	1.5376(11)
10	3.16	1.4786(10)	1.4926(11)	1.5069(10)	1.5166(11)	1.5257(10)	1.5291(10)	1.5363(11)
100	10	1.4774(10)	1.4918(11)	1.5069(10)	1.5156(12)	1.5251(11)	1.5292(11)	1.5374(11)
250	15.81	1.4777(11)	1.4930(10)	1.5063(11)	1.5164(11)	1.5248(11)	1.5301(11)	1.5369(10)
500	22.36	1.4790(11)	1.4927(10)	1.5059(10)	1.5164(11)	1.5260(10)	1.5302(11)	1.5372(10)
1000	31.62	1.4784(10)	1.4934(11)	1.5067(11)	1.5164(11)	1.5254(12)	1.5293(11)	1.5359(12)

Table A.118: Case IIIa Pareto Kuiper 90% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.5959(13)	1.6122(14)	1.6275(15)	1.6380(14)	1.6459(14)	1.6502(14)	1.6583(15)
10	3.16	1.5976(14)	1.6122(13)	1.6278(12)	1.6373(14)	1.6473(14)	1.6504(14)	1.6586(14)
100	10	1.5962(13)	1.6112(14)	1.6281(14)	1.6368(16)	1.6467(13)	1.6510(14)	1.6597(15)
250	15.81	1.5964(15)	1.6122(13)	1.6274(13)	1.6383(14)	1.6467(14)	1.6516(16)	1.6585(13)
500	22.36	1.5979(14)	1.6117(13)	1.6274(14)	1.6373(14)	1.6481(13)	1.6526(15)	1.6589(13)
1000	31.62	1.5975(13)	1.6125(14)	1.6277(15)	1.6376(14)	1.6468(15)	1.6504(15)	1.6580(15)

Table A.119: Case IIIa Pareto Kuiper 95% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.8340(27)	1.8522(27)	1.8708(27)	1.8794(26)	1.8927(25)	1.8942(28)	1.9023(29)
10	3.16	1.8344(25)	1.8527(25)	1.8699(24)	1.8834(30)	1.8889(26)	1.8936(27)	1.9007(23)
100	10	1.8317(25)	1.8515(27)	1.8709(24)	1.8800(30)	1.8922(27)	1.8948(28)	1.9039(29)
250	15.81	1.8334(29)	1.8517(26)	1.8684(26)	1.8824(25)	1.8898(30)	1.8933(27)	1.9039(25)
500	22.36	1.8330(29)	1.8511(22)	1.8694(26)	1.8806(29)	1.8926(28)	1.8954(28)	1.9040(26)
1000	31.62	1.8320(24)	1.8519(29)	1.8691(28)	1.8802(26)	1.8904(29)	1.8946(31)	1.9026(28)

Table A.120: Case IIIa Pareto Kuiper 99% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.14778(32)	0.14792(31)	0.14793(27)	0.14806(28)	0.14793(31)	0.14785(31)	0.14820(29)
10	3.16	0.14789(33)	0.14791(31)	0.14798(28)	0.14804(32)	0.14791(29)	0.14762(30)	0.14787(30)
100	10	0.14763(31)	0.14770(33)	0.14774(31)	0.14772(30)	0.14814(27)	0.14779(31)	0.14831(31)
250	15.81	0.14772(32)	0.14773(29)	0.14778(32)	0.14781(31)	0.14780(31)	0.14789(30)	0.14782(29)
500	22.36	0.14796(28)	0.14791(26)	0.14799(27)	0.14814(30)	0.14806(29)	0.14803(28)	0.14799(31)
1000	31.62	0.14782(30)	0.14809(30)	0.14780(30)	0.14798(26)	0.14806(33)	0.14792(28)	0.14773(31)

Table A.121: Case IIIa Pareto Cramér-von Mises 85% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.17384(38)	0.17409(43)	0.17427(33)	0.17447(35)	0.17438(40)	0.17437(42)	0.17465(40)
10	3.16	0.17413(39)	0.17429(44)	0.17428(38)	0.17475(39)	0.17433(41)	0.17402(38)	0.17426(43)
100	10	0.17356(38)	0.17391(43)	0.17424(39)	0.17410(41)	0.17461(34)	0.17414(43)	0.17475(43)
250	15.81	0.17391(43)	0.17393(36)	0.17419(41)	0.17430(40)	0.17411(39)	0.17448(44)	0.17456(38)
500	22.36	0.17413(36)	0.17407(32)	0.17431(32)	0.17430(39)	0.17453(37)	0.17443(40)	0.17438(40)
1000	31.62	0.17384(41)	0.17443(40)	0.17426(40)	0.17427(36)	0.17437(41)	0.17429(39)	0.17419(36)

Table A.122: Case IIIa Pareto Cramér-von Mises 90% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.22005(58)	0.22030(66)	0.22097(58)	0.22161(59)	0.22174(58)	0.22125(60)	0.22154(60)
10	3.16	0.21981(60)	0.22059(58)	0.22107(54)	0.22189(62)	0.22137(59)	0.22118(60)	0.22124(56)
100	10	0.21973(62)	0.22046(64)	0.22107(59)	0.22102(59)	0.22130(57)	0.22167(69)	0.22208(69)
250	15.81	0.21964(62)	0.22047(56)	0.22082(60)	0.22117(60)	0.22127(63)	0.22178(64)	0.22186(50)
500	22.36	0.22005(51)	0.22039(53)	0.22116(58)	0.22109(66)	0.22215(57)	0.22172(57)	0.22145(57)
1000	31.62	0.21986(54)	0.22052(59)	0.22096(65)	0.22076(58)	0.22135(63)	0.22110(67)	0.22120(61)

Table A.123: Case IIIa Pareto Cramér-von Mises 95% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.33165(140)	0.33318(132)	0.33557(136)	0.33722(140)	0.33675(152)	0.33680(146)	0.33670(147)
10	3.16	0.33170(135)	0.33385(125)	0.33578(131)	0.33660(140)	0.33689(143)	0.33593(142)	0.33677(131)
100	10	0.33114(131)	0.33407(149)	0.33587(152)	0.33589(147)	0.33686(141)	0.33729(138)	0.33759(151)
250	15.81	0.33048(146)	0.33445(150)	0.33504(167)	0.33705(153)	0.33754(152)	0.33723(144)	0.33727(134)
500	22.36	0.33242(146)	0.33271(143)	0.33568(145)	0.33497(161)	0.33761(149)	0.33689(152)	0.33788(141)
1000	31.62	0.33135(131)	0.33361(140)	0.33582(146)	0.33665(147)	0.33654(151)	0.33677(154)	0.33805(144)

Table A.124: Case IIIa Pareto Cramér-von Mises 99% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	0.9054(17)	0.9088(17)	0.9124(16)	0.9138(18)	0.9139(18)	0.9143(17)	0.9155(16)
10	3.16	0.9066(16)	0.9095(17)	0.9125(16)	0.9147(16)	0.9138(16)	0.9124(16)	0.9135(17)
100	10	0.9049(16)	0.9079(19)	0.9122(17)	0.9122(18)	0.9151(16)	0.9129(17)	0.9163(17)
250	15.81	0.9048(18)	0.9088(16)	0.9113(17)	0.9131(17)	0.9127(16)	0.9150(17)	0.9147(16)
500	22.36	0.9060(15)	0.9091(13)	0.9121(15)	0.9146(17)	0.9150(16)	0.9141(17)	0.9155(17)
1000	31.62	0.9051(16)	0.9120(18)	0.9111(17)	0.9140(15)	0.9150(19)	0.9145(17)	0.9132(17)

Table A.125: Case IIIa Pareto Anderson-Darling 85% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.0492(21)	1.0536(23)	1.0583(19)	1.0606(22)	1.0610(23)	1.0603(23)	1.0623(21)
10	3.16	1.0512(22)	1.0551(21)	1.0571(21)	1.0619(23)	1.0592(23)	1.0593(23)	1.0596(21)
100	10	1.0480(21)	1.0542(25)	1.0565(21)	1.0582(21)	1.0607(21)	1.0596(24)	1.0632(24)
250	15.81	1.0497(23)	1.0528(20)	1.0574(24)	1.0592(20)	1.0597(21)	1.0610(23)	1.0611(21)
500	22.36	1.0495(21)	1.0523(18)	1.0579(21)	1.0601(23)	1.0613(21)	1.0602(21)	1.0609(21)
1000	31.62	1.0493(23)	1.0563(22)	1.0567(22)	1.0599(23)	1.0602(23)	1.0601(23)	1.0609(22)

Table A.126: Case IIIa Pareto Anderson-Darling 90% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.3047(31)	1.3097(39)	1.3145(33)	1.3199(33)	1.3206(35)	1.3188(35)	1.3222(35)
10	3.16	1.3052(31)	1.3115(31)	1.3147(30)	1.3210(31)	1.3203(37)	1.3184(32)	1.3191(33)
100	10	1.3026(31)	1.3111(35)	1.3157(31)	1.3185(34)	1.3200(33)	1.3208(34)	1.3235(38)
250	15.81	1.3033(34)	1.3094(30)	1.3144(33)	1.3172(31)	1.3183(31)	1.3220(36)	1.3228(31)
500	22.36	1.3046(30)	1.3082(27)	1.3161(31)	1.3173(37)	1.3232(34)	1.3214(33)	1.3209(26)
1000	31.62	1.3045(31)	1.3135(34)	1.3163(34)	1.3168(36)	1.3185(34)	1.3177(33)	1.3189(28)

Table A.127: Case IIIa Pareto Anderson-Darling 95% critical values

τ_L	$\sqrt{\tau_L}$	30	50	100	200	500	1000	10000
1	1	1.9411(84)	1.9413(80)	1.9486(70)	1.9570(76)	1.9550(78)	1.9500(82)	1.9552(76)
10	3.16	1.9385(83)	1.9444(74)	1.9505(69)	1.9510(79)	1.9560(83)	1.9480(83)	1.9576(70)
100	10	1.9366(81)	1.9438(76)	1.9570(88)	1.9516(76)	1.9554(82)	1.9571(72)	1.9596(86)
250	15.81	1.9335(74)	1.9457(86)	1.9511(91)	1.9536(79)	1.9603(80)	1.9527(84)	1.9529(76)
500	22.36	1.9420(78)	1.9328(78)	1.9541(82)	1.9462(85)	1.9572(80)	1.9529(79)	1.9611(70)
1000	31.62	1.9317(77)	1.9368(80)	1.9484(84)	1.9543(83)	1.9504(78)	1.9563(88)	1.9604(72)

Table A.128: Case IIIa Pareto Anderson-Darling 99% critical values

A.4 Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

A.4.1 Weibull Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0659(09)	1.0730(10)	1.0802(11)	1.0844(09)	1.0884(11)	1.0911(11)	1.0953(10)
0.0323	0.18	1.0772(10)	1.0826(12)	1.0882(12)	1.0936(10)	1.0969(11)	1.0999(10)	1.1034(11)
0.1	0.32	1.0832(10)	1.0883(11)	1.0947(10)	1.0988(11)	1.1020(12)	1.1048(09)	1.1068(11)
0.2	0.47	1.0693(09)	1.0757(10)	1.0811(10)	1.0864(10)	1.0906(12)	1.0925(10)	1.0965(11)
0.3	0.6	1.0476(10)	1.0539(10)	1.0604(09)	1.0657(09)	1.0692(11)	1.0711(10)	1.0748(09)
0.4	0.71	1.0254(09)	1.0319(09)	1.0391(10)	1.0431(09)	1.0477(10)	1.0494(11)	1.0538(09)
0.5	0.83	1.0035(10)	1.0121(08)	1.0183(10)	1.0227(10)	1.0275(10)	1.0300(09)	1.0335(08)
0.6	0.96	0.9842(08)	0.9924(08)	0.9997(09)	1.0044(09)	1.0090(08)	1.0113(09)	1.0149(08)
0.7	1.1	0.9683(09)	0.9754(08)	0.9827(08)	0.9881(09)	0.9926(09)	0.9942(09)	0.9986(09)
0.8	1.27	0.9514(08)	0.9596(08)	0.9677(07)	0.9728(09)	0.9771(08)	0.9803(07)	0.9829(08)
0.8605	1.4	0.9431(07)	0.9504(09)	0.9581(09)	0.9632(08)	0.9679(07)	0.9696(07)	0.9734(08)
0.9	1.52	0.9372(09)	0.9453(08)	0.9514(09)	0.9580(07)	0.9620(08)	0.9645(08)	0.9680(08)

Table A.129: Case IIIb Weibull Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.1503(12)	1.1582(12)	1.1665(13)	1.1701(12)	1.1733(14)	1.1765(12)	1.1810(11)
0.0323	0.18	1.1633(13)	1.1683(13)	1.1747(13)	1.1801(13)	1.1838(13)	1.1863(11)	1.1897(11)
0.1	0.32	1.1698(12)	1.1744(14)	1.1815(11)	1.1850(12)	1.1886(13)	1.1917(11)	1.1937(12)
0.2	0.47	1.1536(12)	1.1602(12)	1.1666(13)	1.1718(12)	1.1757(14)	1.1769(12)	1.1820(13)
0.3	0.6	1.1296(12)	1.1360(12)	1.1423(10)	1.1476(11)	1.1521(13)	1.1532(12)	1.1573(11)
0.4	0.71	1.1036(10)	1.1106(10)	1.1173(11)	1.1220(11)	1.1270(11)	1.1283(12)	1.1331(11)
0.5	0.83	1.0783(11)	1.0871(10)	1.0942(11)	1.0984(11)	1.1036(10)	1.1063(11)	1.1095(10)
0.6	0.96	1.0561(09)	1.0656(09)	1.0730(10)	1.0779(10)	1.0827(09)	1.0854(11)	1.0881(10)
0.7	1.1	1.0378(11)	1.0456(10)	1.0537(11)	1.0593(10)	1.0637(10)	1.0651(10)	1.0700(10)
0.8	1.27	1.0195(09)	1.0282(09)	1.0365(09)	1.0413(11)	1.0465(09)	1.0492(10)	1.0521(09)
0.8605	1.4	1.0096(09)	1.0177(10)	1.0256(10)	1.0313(09)	1.0357(09)	1.0372(10)	1.0414(10)
0.9	1.52	1.0027(10)	1.0111(09)	1.0184(10)	1.0248(08)	1.0291(09)	1.0321(09)	1.0357(09)

Table A.130: Case IIIb Weibull Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2833(17)	1.2906(15)	1.2991(15)	1.3044(15)	1.3063(16)	1.3112(14)	1.3150(17)
0.0323	0.18	1.2956(19)	1.3016(16)	1.3089(18)	1.3151(18)	1.3182(18)	1.3220(14)	1.3261(17)
0.1	0.32	1.3038(15)	1.3094(17)	1.3164(13)	1.3206(18)	1.3244(17)	1.3280(15)	1.3297(16)
0.2	0.47	1.2844(14)	1.2929(15)	1.2997(14)	1.3041(16)	1.3084(19)	1.3099(18)	1.3144(16)
0.3	0.6	1.2556(15)	1.2635(16)	1.2690(14)	1.2741(16)	1.2798(15)	1.2818(15)	1.2847(15)
0.4	0.71	1.2262(14)	1.2330(14)	1.2410(14)	1.2454(13)	1.2506(14)	1.2517(16)	1.2573(14)
0.5	0.83	1.1955(15)	1.2053(13)	1.2134(14)	1.2180(16)	1.2224(14)	1.2269(15)	1.2292(13)
0.6	0.96	1.1693(14)	1.1791(13)	1.1883(13)	1.1931(15)	1.1974(14)	1.2008(13)	1.2039(13)
0.7	1.1	1.1462(12)	1.1561(13)	1.1648(13)	1.1706(14)	1.1759(13)	1.1773(14)	1.1817(14)
0.8	1.27	1.1254(13)	1.1357(11)	1.1448(12)	1.1499(16)	1.1558(13)	1.1580(13)	1.1607(14)
0.8605	1.4	1.1141(12)	1.1227(13)	1.1326(14)	1.1377(13)	1.1425(11)	1.1439(13)	1.1484(12)
0.9	1.52	1.1056(13)	1.1145(11)	1.1230(12)	1.1304(11)	1.1346(13)	1.1378(12)	1.1420(11)

Table A.131: Case IIIb Weibull Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5414(27)	1.5555(25)	1.5664(31)	1.5728(28)	1.5763(32)	1.5793(29)	1.5832(29)
0.0323	0.18	1.5585(34)	1.5695(28)	1.5790(31)	1.5835(28)	1.5871(32)	1.5920(30)	1.5972(30)
0.1	0.32	1.5678(29)	1.5755(32)	1.5871(31)	1.5902(31)	1.5949(36)	1.5997(27)	1.6032(33)
0.2	0.47	1.5474(28)	1.5574(29)	1.5653(31)	1.5700(29)	1.5737(27)	1.5742(31)	1.5813(31)
0.3	0.6	1.5069(27)	1.5182(29)	1.5246(25)	1.5344(29)	1.5401(28)	1.5391(33)	1.5441(30)
0.4	0.71	1.4676(28)	1.4762(28)	1.4866(29)	1.4932(28)	1.4986(28)	1.4988(31)	1.5053(27)
0.5	0.83	1.4280(25)	1.4401(23)	1.4519(23)	1.4585(26)	1.4618(26)	1.4634(26)	1.4707(26)
0.6	0.96	1.3955(25)	1.4080(29)	1.4185(27)	1.4243(26)	1.4298(25)	1.4325(25)	1.4359(28)
0.7	1.1	1.3647(24)	1.3768(26)	1.3856(25)	1.3952(28)	1.4004(26)	1.4040(25)	1.4096(26)
0.8	1.27	1.3365(25)	1.3490(22)	1.3625(27)	1.3680(23)	1.3746(22)	1.3760(23)	1.3817(27)
0.8605	1.4	1.3229(19)	1.3342(24)	1.3470(24)	1.3513(26)	1.3594(24)	1.3589(23)	1.3660(24)
0.9	1.52	1.3105(26)	1.3234(23)	1.3316(21)	1.3416(22)	1.3470(25)	1.3509(24)	1.3552(26)

Table A.132: Case IIIb Weibull Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3628(09)	1.3779(09)	1.3943(09)	1.4045(09)	1.4117(09)	1.4171(09)	1.4258(09)
0.0323	0.18	1.3660(09)	1.3820(09)	1.3972(10)	1.4091(09)	1.4178(09)	1.4227(08)	1.4301(09)
0.1	0.32	1.3812(09)	1.3965(09)	1.4141(08)	1.4239(09)	1.4333(09)	1.4379(08)	1.4450(10)
0.2	0.47	1.3968(09)	1.4130(10)	1.4288(09)	1.4396(09)	1.4491(11)	1.4532(10)	1.4611(09)
0.3	0.6	1.4064(09)	1.4220(11)	1.4369(09)	1.4471(09)	1.4553(10)	1.4604(09)	1.4675(10)
0.4	0.71	1.4107(09)	1.4247(09)	1.4404(10)	1.4497(09)	1.4595(10)	1.4630(10)	1.4710(09)
0.5	0.83	1.4116(10)	1.4270(08)	1.4408(10)	1.4500(11)	1.4599(09)	1.4653(10)	1.4712(08)
0.6	0.96	1.4115(09)	1.4254(09)	1.4408(09)	1.4502(10)	1.4595(09)	1.4628(10)	1.4704(09)
0.7	1.1	1.4110(10)	1.4250(10)	1.4390(10)	1.4494(11)	1.4582(09)	1.4621(09)	1.4692(10)
0.8	1.27	1.4091(10)	1.4234(09)	1.4374(10)	1.4470(10)	1.4568(09)	1.4616(10)	1.4676(10)
0.8605	1.4	1.4084(10)	1.4221(08)	1.4364(10)	1.4459(08)	1.4550(09)	1.4587(10)	1.4664(10)
0.9	1.52	1.4078(09)	1.4221(09)	1.4349(10)	1.4458(08)	1.4541(08)	1.4587(09)	1.4666(09)

Table A.133: Case IIIb Weibull Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4377(11)	1.4530(10)	1.4703(11)	1.4800(11)	1.4876(12)	1.4932(11)	1.5018(11)
0.0323	0.18	1.4412(11)	1.4577(11)	1.4731(12)	1.4846(11)	1.4939(10)	1.4985(10)	1.5060(12)
0.1	0.32	1.4550(11)	1.4719(10)	1.4894(10)	1.4995(11)	1.5091(11)	1.5140(11)	1.5210(10)
0.2	0.47	1.4706(12)	1.4888(11)	1.5052(10)	1.5164(10)	1.5258(13)	1.5303(12)	1.5379(10)
0.3	0.6	1.4821(11)	1.4983(14)	1.5140(11)	1.5242(10)	1.5324(11)	1.5378(10)	1.5450(11)
0.4	0.71	1.4871(10)	1.5017(11)	1.5180(11)	1.5280(12)	1.5372(11)	1.5407(12)	1.5489(10)
0.5	0.83	1.4878(11)	1.5041(10)	1.5189(12)	1.5279(12)	1.5375(10)	1.5435(12)	1.5494(09)
0.6	0.96	1.4875(11)	1.5028(11)	1.5185(11)	1.5282(13)	1.5381(12)	1.5419(10)	1.5485(11)
0.7	1.1	1.4872(12)	1.5024(11)	1.5172(11)	1.5272(11)	1.5367(11)	1.5398(10)	1.5481(11)
0.8	1.27	1.4859(11)	1.5009(10)	1.5153(10)	1.5257(12)	1.5351(10)	1.5397(11)	1.5464(11)
0.8605	1.4	1.4857(10)	1.4993(10)	1.5144(11)	1.5246(10)	1.5330(10)	1.5370(11)	1.5446(10)
0.9	1.52	1.4847(13)	1.4986(11)	1.5127(10)	1.5241(10)	1.5324(11)	1.5374(11)	1.5450(10)

Table A.134: Case IIIb Weibull Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5541(14)	1.5703(13)	1.5879(16)	1.5978(14)	1.6058(14)	1.6117(15)	1.6206(14)
0.0323	0.18	1.5576(15)	1.5743(14)	1.5916(15)	1.6028(13)	1.6111(14)	1.6171(14)	1.6248(15)
0.1	0.32	1.5705(13)	1.5883(14)	1.6071(12)	1.6174(15)	1.6274(16)	1.6325(15)	1.6403(13)
0.2	0.47	1.5858(15)	1.6055(13)	1.6237(14)	1.6355(14)	1.6444(15)	1.6480(16)	1.6571(14)
0.3	0.6	1.5989(13)	1.6159(16)	1.6319(13)	1.6439(14)	1.6525(14)	1.6573(13)	1.6649(14)
0.4	0.71	1.6044(13)	1.6213(14)	1.6375(14)	1.6474(14)	1.6574(13)	1.6607(16)	1.6688(14)
0.5	0.83	1.6056(14)	1.6235(14)	1.6396(14)	1.6493(15)	1.6581(15)	1.6642(14)	1.6701(13)
0.6	0.96	1.6053(15)	1.6231(14)	1.6389(13)	1.6494(14)	1.6592(14)	1.6631(13)	1.6696(13)
0.7	1.1	1.6051(16)	1.6219(15)	1.6375(14)	1.6481(14)	1.6581(14)	1.6616(14)	1.6694(15)
0.8	1.27	1.6047(13)	1.6204(12)	1.6359(13)	1.6466(14)	1.6576(13)	1.6608(14)	1.6677(14)
0.8605	1.4	1.6046(13)	1.6190(14)	1.6356(16)	1.6452(15)	1.6550(14)	1.6586(15)	1.6660(13)
0.9	1.52	1.6033(16)	1.6188(14)	1.6336(12)	1.6456(12)	1.6546(16)	1.6587(14)	1.6672(15)

Table A.135: Case IIIb Weibull Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.7868(27)	1.8085(27)	1.8232(28)	1.8357(26)	1.8471(29)	1.8498(28)	1.8593(27)
0.0323	0.18	1.7931(30)	1.8095(28)	1.8271(29)	1.8407(26)	1.8503(28)	1.8564(27)	1.8646(26)
0.1	0.32	1.8032(26)	1.8235(29)	1.8442(27)	1.8560(25)	1.8653(27)	1.8691(26)	1.8770(26)
0.2	0.47	1.8145(23)	1.8391(23)	1.8616(24)	1.8731(26)	1.8815(26)	1.8835(25)	1.8958(26)
0.3	0.6	1.8284(24)	1.8541(26)	1.8703(22)	1.8828(29)	1.8931(25)	1.8956(29)	1.9037(27)
0.4	0.71	1.8379(24)	1.8567(26)	1.8739(27)	1.8872(26)	1.8992(28)	1.9007(28)	1.9105(27)
0.5	0.83	1.8399(29)	1.8571(26)	1.8787(26)	1.8889(29)	1.9009(26)	1.9040(30)	1.9135(24)
0.6	0.96	1.8411(27)	1.8622(26)	1.8782(31)	1.8894(28)	1.9010(28)	1.9061(25)	1.9120(26)
0.7	1.1	1.8406(26)	1.8592(30)	1.8766(27)	1.8899(27)	1.9004(26)	1.9048(28)	1.9138(26)
0.8	1.27	1.8413(25)	1.8575(27)	1.8782(22)	1.8902(26)	1.9020(23)	1.9024(28)	1.9119(27)
0.8605	1.4	1.8399(25)	1.8587(31)	1.8767(29)	1.8871(27)	1.8982(28)	1.8994(31)	1.9094(24)
0.9	1.52	1.8395(26)	1.8579(27)	1.8743(23)	1.8899(28)	1.8971(26)	1.9016(26)	1.9097(26)

Table A.136: Case IIIb Weibull Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.2602(06)	0.2595(07)	0.2590(07)	0.2585(07)	0.2578(08)	0.2586(07)	0.2586(06)
0.0323	0.18	0.2688(08)	0.2665(08)	0.2655(08)	0.2652(07)	0.2645(08)	0.2645(07)	0.2644(08)
0.1	0.32	0.2687(07)	0.2667(07)	0.2659(07)	0.2653(07)	0.2644(08)	0.2648(07)	0.2639(06)
0.2	0.47	0.2532(06)	0.2526(07)	0.2517(07)	0.2517(06)	0.2510(07)	0.2510(06)	0.2513(07)
0.3	0.6	0.2348(07)	0.2344(07)	0.2336(06)	0.2344(06)	0.2340(06)	0.2336(06)	0.2337(06)
0.4	0.71	0.2185(05)	0.2176(06)	0.2178(06)	0.2177(05)	0.2178(06)	0.2174(05)	0.2179(06)
0.5	0.83	0.2035(05)	0.2043(05)	0.2040(05)	0.2037(05)	0.2039(04)	0.2043(05)	0.2040(05)
0.6	0.96	0.1915(04)	0.1922(04)	0.1925(04)	0.1922(04)	0.1923(05)	0.1924(04)	0.1922(04)
0.7	1.1	0.1819(04)	0.1821(04)	0.1826(04)	0.1827(05)	0.1827(04)	0.1824(04)	0.1829(05)
0.8	1.27	0.1729(04)	0.1734(04)	0.1737(03)	0.1740(04)	0.1743(04)	0.1745(03)	0.1740(04)
0.8605	1.4	0.1682(03)	0.1683(04)	0.1688(04)	0.1690(03)	0.1693(03)	0.1692(04)	0.1690(03)
0.9	1.52	0.1649(03)	0.1658(04)	0.1658(03)	0.1662(03)	0.1658(03)	0.1662(04)	0.1662(03)

Table A.137: Case IIIb Weibull Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.3205(09)	0.3205(09)	0.3206(09)	0.3203(08)	0.3194(11)	0.3204(09)	0.3203(09)
0.0323	0.18	0.3324(09)	0.3298(10)	0.3291(11)	0.3283(09)	0.3279(10)	0.3283(09)	0.3283(10)
0.1	0.32	0.3322(09)	0.3303(10)	0.3292(08)	0.3283(10)	0.3274(10)	0.3278(08)	0.3268(09)
0.2	0.47	0.3117(09)	0.3112(09)	0.3101(10)	0.3104(09)	0.3096(09)	0.3094(09)	0.3096(09)
0.3	0.6	0.2873(08)	0.2869(08)	0.2861(07)	0.2868(08)	0.2870(08)	0.2861(07)	0.2865(07)
0.4	0.71	0.2656(07)	0.2648(08)	0.2649(08)	0.2649(06)	0.2651(07)	0.2645(08)	0.2650(07)
0.5	0.83	0.2456(06)	0.2468(06)	0.2465(06)	0.2465(07)	0.2464(06)	0.2472(06)	0.2467(07)
0.6	0.96	0.2299(05)	0.2309(06)	0.2313(05)	0.2314(06)	0.2315(05)	0.2317(06)	0.2315(06)
0.7	1.1	0.2171(06)	0.2180(05)	0.2184(05)	0.2187(06)	0.2189(06)	0.2182(05)	0.2192(06)
0.8	1.27	0.2059(05)	0.2068(05)	0.2071(05)	0.2073(05)	0.2081(05)	0.2080(05)	0.2076(05)
0.8605	1.4	0.1998(04)	0.2001(05)	0.2011(05)	0.2011(05)	0.2017(04)	0.2011(05)	0.2013(04)
0.9	1.52	0.1956(05)	0.1964(05)	0.1966(04)	0.1977(05)	0.1973(04)	0.1979(05)	0.1977(05)

Table A.138: Case IIIb Weibull Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4289(14)	0.4306(13)	0.4310(14)	0.4314(13)	0.4300(16)	0.4320(13)	0.4320(15)
0.0323	0.18	0.4448(15)	0.4427(15)	0.4422(15)	0.4411(13)	0.4411(17)	0.4424(15)	0.4416(15)
0.1	0.32	0.4463(14)	0.4441(15)	0.4425(13)	0.4413(15)	0.4410(15)	0.4414(12)	0.4404(14)
0.2	0.47	0.4161(12)	0.4160(13)	0.4152(14)	0.4151(11)	0.4144(15)	0.4140(16)	0.4148(13)
0.3	0.6	0.3806(12)	0.3811(13)	0.3798(10)	0.3818(12)	0.3814(13)	0.3807(11)	0.3813(11)
0.4	0.71	0.3496(10)	0.3496(11)	0.3498(11)	0.3502(11)	0.3496(10)	0.3496(11)	0.3503(10)
0.5	0.83	0.3209(10)	0.3238(10)	0.3234(10)	0.3235(11)	0.3241(10)	0.3245(10)	0.3239(10)
0.6	0.96	0.2984(09)	0.3008(08)	0.3012(08)	0.3014(09)	0.3024(08)	0.3024(09)	0.3020(09)
0.7	1.1	0.2801(09)	0.2820(08)	0.2826(08)	0.2842(08)	0.2840(08)	0.2836(09)	0.2841(08)
0.8	1.27	0.2641(08)	0.2663(07)	0.2672(07)	0.2678(09)	0.2681(07)	0.2680(08)	0.2682(08)
0.8605	1.4	0.2561(06)	0.2565(07)	0.2587(07)	0.2592(07)	0.2593(06)	0.2584(08)	0.2592(07)
0.9	1.52	0.2498(08)	0.2512(07)	0.2515(06)	0.2532(07)	0.2530(07)	0.2537(07)	0.2536(07)

Table A.139: Case IIIb Weibull Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.6886(31)	0.6964(31)	0.6988(32)	0.7025(33)	0.7042(35)	0.7058(33)	0.7049(33)
0.0323	0.18	0.7122(38)	0.7177(29)	0.7195(37)	0.7197(33)	0.7178(36)	0.7206(36)	0.7216(35)
0.1	0.32	0.7228(33)	0.7195(34)	0.7206(34)	0.7210(38)	0.7195(37)	0.7225(36)	0.7182(33)
0.2	0.47	0.6720(31)	0.6745(34)	0.6736(29)	0.6737(33)	0.6726(31)	0.6733(35)	0.6746(31)
0.3	0.6	0.6097(25)	0.6112(28)	0.6101(27)	0.6146(27)	0.6174(25)	0.6136(28)	0.6136(27)
0.4	0.71	0.5527(25)	0.5544(26)	0.5596(26)	0.5591(26)	0.5619(27)	0.5610(26)	0.5624(28)
0.5	0.83	0.5038(23)	0.5097(22)	0.5138(22)	0.5152(23)	0.5157(25)	0.5144(23)	0.5161(24)
0.6	0.96	0.4650(23)	0.4703(19)	0.4742(22)	0.4761(23)	0.4765(21)	0.4770(21)	0.4773(26)
0.7	1.1	0.4331(18)	0.4385(20)	0.4421(18)	0.4434(21)	0.4453(21)	0.4456(20)	0.4480(21)
0.8	1.27	0.4061(18)	0.4095(18)	0.4150(18)	0.4173(20)	0.4165(17)	0.4172(17)	0.4193(18)
0.8605	1.4	0.3924(16)	0.3953(20)	0.3993(19)	0.4008(20)	0.4010(19)	0.4002(18)	0.4033(18)
0.9	1.52	0.3822(20)	0.3861(16)	0.3883(15)	0.3918(17)	0.3910(19)	0.3921(18)	0.3922(17)

Table A.140: Case IIIb Weibull Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4188(31)	1.4196(36)	1.4202(38)	1.4170(34)	1.4171(37)	1.4208(34)	1.4205(31)
0.0323	0.18	1.4678(36)	1.4569(38)	1.4503(40)	1.4496(31)	1.4459(40)	1.4469(36)	1.4446(38)
0.1	0.32	1.4677(34)	1.4547(38)	1.4487(32)	1.4436(33)	1.4401(41)	1.4407(32)	1.4348(32)
0.2	0.47	1.3833(30)	1.3793(32)	1.3729(33)	1.3722(31)	1.3689(34)	1.3695(30)	1.3713(32)
0.3	0.6	1.2904(33)	1.2889(31)	1.2853(28)	1.2889(27)	1.2894(31)	1.2868(28)	1.2879(31)
0.4	0.71	1.2114(27)	1.2099(27)	1.2119(30)	1.2128(24)	1.2150(30)	1.2131(27)	1.2142(26)
0.5	0.83	1.1434(24)	1.1491(24)	1.1501(23)	1.1503(24)	1.1522(23)	1.1551(24)	1.1551(23)
0.6	0.96	1.0888(21)	1.0952(22)	1.0991(21)	1.1009(21)	1.1018(23)	1.1021(22)	1.1020(21)
0.7	1.1	1.0470(22)	1.0514(20)	1.0569(19)	1.0596(22)	1.0606(22)	1.0580(19)	1.0611(26)
0.8	1.27	1.0081(20)	1.0145(17)	1.0187(18)	1.0222(22)	1.0242(19)	1.0246(17)	1.0234(20)
0.8605	1.4	0.9894(16)	0.9922(21)	0.9986(22)	1.0007(17)	1.0033(16)	1.0021(20)	1.0022(18)
0.9	1.52	0.9747(18)	0.9822(19)	0.9842(19)	0.9895(18)	0.9885(18)	0.9911(20)	0.9906(19)

Table A.141: Case IIIb Weibull Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.7147(45)	1.7212(44)	1.7225(46)	1.7223(41)	1.7203(52)	1.7264(44)	1.7247(44)
0.0323	0.18	1.7810(49)	1.7682(50)	1.7657(50)	1.7605(43)	1.7580(56)	1.7603(45)	1.7589(54)
0.1	0.32	1.7801(43)	1.7670(50)	1.7566(41)	1.7500(50)	1.7457(51)	1.7484(39)	1.7418(44)
0.2	0.47	1.6710(40)	1.6638(43)	1.6571(48)	1.6565(37)	1.6526(44)	1.6512(43)	1.6539(44)
0.3	0.6	1.5437(41)	1.5432(39)	1.5399(37)	1.5443(40)	1.5454(39)	1.5410(38)	1.5425(35)
0.4	0.71	1.4411(34)	1.4404(35)	1.4409(39)	1.4442(33)	1.4454(37)	1.4426(38)	1.4448(34)
0.5	0.83	1.3494(31)	1.3578(31)	1.3617(33)	1.3601(34)	1.3632(30)	1.3672(35)	1.3642(31)
0.6	0.96	1.2791(26)	1.2903(28)	1.2943(25)	1.2968(27)	1.2987(26)	1.2974(29)	1.2976(29)
0.7	1.1	1.2258(31)	1.2335(27)	1.2389(27)	1.2428(28)	1.2441(28)	1.2425(28)	1.2451(31)
0.8	1.27	1.1772(24)	1.1867(22)	1.1921(24)	1.1958(29)	1.1989(25)	1.1995(25)	1.1969(25)
0.8605	1.4	1.1535(23)	1.1581(27)	1.1663(27)	1.1684(22)	1.1713(23)	1.1693(25)	1.1718(23)
0.9	1.52	1.1358(26)	1.1441(23)	1.1474(24)	1.1533(24)	1.1535(23)	1.1571(24)	1.1559(25)

Table A.142: Case IIIb Weibull Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	2.2451(68)	2.2588(62)	2.2698(65)	2.2743(67)	2.2682(74)	2.2773(67)	2.2787(74)
0.0323	0.18	2.3289(78)	2.3241(73)	2.3246(71)	2.3179(68)	2.3194(80)	2.3231(74)	2.3184(82)
0.1	0.32	2.3429(63)	2.3229(76)	2.3116(63)	2.3049(73)	2.3015(73)	2.3044(60)	2.2960(71)
0.2	0.47	2.1873(64)	2.1743(65)	2.1676(66)	2.1658(62)	2.1603(69)	2.1587(79)	2.1638(66)
0.3	0.6	1.9988(53)	2.0013(66)	1.9951(55)	2.0048(58)	2.0051(61)	1.9985(57)	2.0005(52)
0.4	0.71	1.8513(49)	1.8514(54)	1.8539(53)	1.8575(55)	1.8567(53)	1.8588(55)	1.8636(51)
0.5	0.83	1.7186(50)	1.7346(46)	1.7405(48)	1.7405(53)	1.7442(45)	1.7463(51)	1.7450(53)
0.6	0.96	1.6203(44)	1.6376(39)	1.6453(46)	1.6432(44)	1.6497(41)	1.6506(43)	1.6487(44)
0.7	1.1	1.5432(45)	1.5581(38)	1.5642(41)	1.5709(41)	1.5728(44)	1.5724(39)	1.5758(39)
0.8	1.27	1.4772(42)	1.4909(33)	1.5009(39)	1.5031(45)	1.5079(40)	1.5080(38)	1.5071(41)
0.8605	1.4	1.4465(34)	1.4532(33)	1.4635(40)	1.4710(37)	1.4703(33)	1.4666(38)	1.4729(38)
0.9	1.52	1.4202(43)	1.4301(35)	1.4374(34)	1.4455(36)	1.4467(36)	1.4524(35)	1.4509(38)

Table A.143: Case IIIb Weibull Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	3.5182(154)	3.5737(149)	3.5966(167)	3.6217(163)	3.6248(177)	3.6432(173)	3.6391(175)
0.0323	0.18	3.6284(182)	3.6783(144)	3.6939(170)	3.6886(168)	3.6854(183)	3.7036(177)	3.7014(175)
0.1	0.32	3.7164(165)	3.6772(172)	3.6915(174)	3.6781(183)	3.6624(170)	3.6757(174)	3.6588(151)
0.2	0.47	3.4655(147)	3.4496(166)	3.4296(147)	3.4229(158)	3.4182(147)	3.4152(170)	3.4209(144)
0.3	0.6	3.1428(123)	3.1336(144)	3.1174(127)	3.1398(124)	3.1498(119)	3.1329(140)	3.1345(128)
0.4	0.71	2.8567(133)	2.8623(134)	2.8922(132)	2.8849(134)	2.8875(138)	2.8914(138)	2.8956(137)
0.5	0.83	2.6368(118)	2.6622(120)	2.6747(112)	2.6864(111)	2.6904(121)	2.6805(103)	2.6908(129)
0.6	0.96	2.4633(114)	2.4931(110)	2.5053(122)	2.5176(108)	2.5195(103)	2.5199(100)	2.5182(120)
0.7	1.1	2.3311(108)	2.3537(101)	2.3726(92)	2.3831(104)	2.3870(105)	2.3884(101)	2.3967(105)
0.8	1.27	2.2233(93)	2.2437(97)	2.2664(78)	2.2705(100)	2.2684(89)	2.2736(97)	2.2807(95)
0.8605	1.4	2.1742(85)	2.1824(101)	2.2023(93)	2.2051(101)	2.2087(98)	2.2008(96)	2.2200(89)
0.9	1.52	2.1277(103)	2.1476(84)	2.1491(88)	2.1669(78)	2.1685(88)	2.1728(94)	2.1709(95)

Table A.144: Case IIIb Weibull Anderson-Darling 99% critical values

A.4.2 Loglogistic Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.0645(10)	1.0702(11)	1.0758(10)	1.0805(10)	1.0847(09)	1.0868(11)	1.0900(10)
0.0323	0.18	1.0578(10)	1.0630(10)	1.0693(11)	1.0732(11)	1.0781(10)	1.0794(10)	1.0836(10)
0.1	0.33	1.0360(12)	1.0420(10)	1.0476(09)	1.0522(10)	1.0558(10)	1.0577(10)	1.0625(11)
0.2	0.5	0.9947(09)	0.9995(09)	1.0055(09)	1.0099(08)	1.0138(09)	1.0159(09)	1.0192(09)
0.3	0.65	0.9555(08)	0.9618(09)	0.9677(09)	0.9725(08)	0.9764(09)	0.9782(09)	0.9823(08)
0.4	0.82	0.9256(08)	0.9324(08)	0.9397(08)	0.9445(08)	0.9486(08)	0.9509(08)	0.9548(09)
0.5	1	0.9054(08)	0.9121(08)	0.9201(07)	0.9249(07)	0.9292(09)	0.9312(07)	0.9352(07)
0.6	1.22	0.8914(08)	0.8990(08)	0.9059(08)	0.9113(08)	0.9156(08)	0.9179(07)	0.9221(08)
0.7	1.53	0.8830(08)	0.8901(07)	0.8986(07)	0.9031(07)	0.9083(07)	0.9102(08)	0.9134(07)
0.8	2	0.8807(07)	0.8871(07)	0.8951(07)	0.8997(07)	0.9041(07)	0.9073(07)	0.9104(07)
0.8605	2.48	0.8809(07)	0.8883(07)	0.8953(08)	0.9003(07)	0.9050(07)	0.9064(08)	0.9110(07)
0.9	3	0.8833(07)	0.8896(07)	0.8975(07)	0.9031(08)	0.9068(07)	0.9090(08)	0.9134(08)

Table A.145: Case IIIb Loglogistic Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.1517(12)	1.1581(13)	1.1636(13)	1.1690(10)	1.1726(11)	1.1751(12)	1.1776(12)
0.0323	0.18	1.1444(11)	1.1502(12)	1.1570(11)	1.1604(12)	1.1650(12)	1.1666(12)	1.1717(14)
0.1	0.33	1.1197(12)	1.1252(12)	1.1315(13)	1.1362(11)	1.1394(12)	1.1419(12)	1.1466(13)
0.2	0.5	1.0720(11)	1.0769(11)	1.0828(10)	1.0873(10)	1.0907(11)	1.0935(11)	1.0970(11)
0.3	0.65	1.0266(09)	1.0338(11)	1.0398(11)	1.0441(09)	1.0487(10)	1.0497(09)	1.0548(09)
0.4	0.82	0.9921(11)	0.9993(09)	1.0077(08)	1.0123(09)	1.0162(10)	1.0191(09)	1.0222(11)
0.5	1	0.9688(10)	0.9764(09)	0.9849(09)	0.9892(10)	0.9935(10)	0.9953(08)	0.9997(09)
0.6	1.22	0.9530(08)	0.9613(08)	0.9683(10)	0.9732(09)	0.9783(09)	0.9802(09)	0.9848(09)
0.7	1.53	0.9431(10)	0.9499(08)	0.9596(09)	0.9638(09)	0.9700(09)	0.9709(09)	0.9745(07)
0.8	2	0.9395(08)	0.9471(08)	0.9554(09)	0.9603(08)	0.9642(09)	0.9676(08)	0.9706(09)
0.8605	2.48	0.9405(08)	0.9477(09)	0.9547(09)	0.9605(09)	0.9650(10)	0.9672(09)	0.9705(09)
0.9	3	0.9427(08)	0.9492(09)	0.9573(08)	0.9634(09)	0.9669(08)	0.9693(09)	0.9738(09)

Table A.146: Case IIIb Loglogistic Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.2860(16)	1.2925(17)	1.3000(17)	1.3059(14)	1.3095(15)	1.3122(17)	1.3147(17)
0.0323	0.18	1.2792(15)	1.2851(15)	1.2927(15)	1.2967(17)	1.3015(16)	1.3035(16)	1.3077(17)
0.1	0.33	1.2504(15)	1.2563(14)	1.2627(16)	1.2674(15)	1.2713(15)	1.2728(15)	1.2765(18)
0.2	0.5	1.1934(16)	1.1970(13)	1.2038(15)	1.2080(15)	1.2126(15)	1.2156(14)	1.2188(14)
0.3	0.65	1.1384(12)	1.1455(13)	1.1522(14)	1.1573(13)	1.1612(12)	1.1634(12)	1.1682(13)
0.4	0.82	1.0972(13)	1.1042(14)	1.1130(12)	1.1185(12)	1.1231(12)	1.1249(12)	1.1287(13)
0.5	1	1.0682(12)	1.0765(11)	1.0848(12)	1.0908(12)	1.0950(12)	1.0968(12)	1.1013(11)
0.6	1.22	1.0491(10)	1.0578(12)	1.0660(12)	1.0712(12)	1.0761(11)	1.0790(11)	1.0827(12)
0.7	1.53	1.0364(13)	1.0453(11)	1.0557(13)	1.0595(13)	1.0662(11)	1.0669(11)	1.0706(10)
0.8	2	1.0323(10)	1.0413(11)	1.0496(11)	1.0548(12)	1.0587(12)	1.0623(12)	1.0652(11)
0.8605	2.48	1.0328(11)	1.0408(10)	1.0499(10)	1.0548(12)	1.0594(13)	1.0622(11)	1.0654(12)
0.9	3	1.0360(11)	1.0438(11)	1.0520(11)	1.0586(11)	1.0615(11)	1.0647(12)	1.0690(13)

Table A.147: Case IIIb Loglogistic Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5536(30)	1.5602(33)	1.5721(29)	1.5791(31)	1.5827(33)	1.5855(31)	1.5875(32)
0.0323	0.18	1.5452(31)	1.5563(30)	1.5606(24)	1.5680(28)	1.5759(32)	1.5755(31)	1.5799(32)
0.1	0.33	1.5108(28)	1.5196(29)	1.5258(29)	1.5297(30)	1.5360(30)	1.5340(28)	1.5422(31)
0.2	0.5	1.4347(28)	1.4394(26)	1.4474(30)	1.4512(29)	1.4579(26)	1.4595(26)	1.4635(26)
0.3	0.65	1.3604(22)	1.3718(23)	1.3787(23)	1.3844(27)	1.3901(23)	1.3907(24)	1.3967(26)
0.4	0.82	1.3080(26)	1.3159(26)	1.3285(21)	1.3331(23)	1.3376(30)	1.3415(24)	1.3446(23)
0.5	1	1.2683(23)	1.2812(24)	1.2888(21)	1.2950(24)	1.3012(24)	1.3034(24)	1.3072(22)
0.6	1.22	1.2424(21)	1.2544(24)	1.2629(22)	1.2700(20)	1.2766(24)	1.2787(21)	1.2810(25)
0.7	1.53	1.2253(23)	1.2373(20)	1.2505(21)	1.2545(22)	1.2634(20)	1.2613(24)	1.2675(24)
0.8	2	1.2199(20)	1.2326(23)	1.2400(22)	1.2464(22)	1.2505(22)	1.2548(24)	1.2574(21)
0.8605	2.48	1.2208(21)	1.2305(21)	1.2410(20)	1.2475(24)	1.2520(23)	1.2550(22)	1.2584(21)
0.9	3	1.2239(21)	1.2343(21)	1.2403(18)	1.2503(22)	1.2534(21)	1.2602(25)	1.2619(26)

Table A.148: Case IIIb Loglogistic Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.3356(09)	1.3492(10)	1.3627(08)	1.3729(09)	1.3815(09)	1.3861(09)	1.3922(09)
0.0323	0.18	1.3347(09)	1.3476(09)	1.3615(08)	1.3705(09)	1.3799(09)	1.3839(09)	1.3913(10)
0.1	0.33	1.3498(10)	1.3649(09)	1.3781(09)	1.3888(08)	1.3975(08)	1.4015(08)	1.4096(10)
0.2	0.5	1.3723(09)	1.3862(08)	1.4005(10)	1.4101(08)	1.4191(10)	1.4242(10)	1.4316(08)
0.3	0.65	1.3840(08)	1.3991(10)	1.4117(09)	1.4225(08)	1.4309(09)	1.4349(08)	1.4428(09)
0.4	0.82	1.3904(09)	1.4033(09)	1.4184(09)	1.4276(09)	1.4361(09)	1.4405(09)	1.4478(10)
0.5	1	1.3931(09)	1.4062(10)	1.4211(10)	1.4308(08)	1.4388(10)	1.4428(09)	1.4506(09)
0.6	1.22	1.3948(09)	1.4086(09)	1.4224(09)	1.4316(09)	1.4404(09)	1.4444(09)	1.4527(09)
0.7	1.53	1.3954(10)	1.4086(09)	1.4234(10)	1.4330(10)	1.4418(09)	1.4461(10)	1.4527(09)
0.8	2	1.3974(08)	1.4108(08)	1.4252(10)	1.4341(10)	1.4420(10)	1.4470(10)	1.4541(10)
0.8605	2.48	1.3976(09)	1.4111(11)	1.4242(09)	1.4345(09)	1.4430(10)	1.4470(10)	1.4546(10)
0.9	3	1.3986(10)	1.4111(08)	1.4253(10)	1.4351(10)	1.4436(10)	1.4474(11)	1.4563(10)

Table A.149: Case IIIb Loglogistic Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4090(10)	1.4236(11)	1.4380(10)	1.4480(09)	1.4560(10)	1.4608(11)	1.4669(09)
0.0323	0.18	1.4086(11)	1.4216(09)	1.4358(09)	1.4449(10)	1.4545(11)	1.4588(09)	1.4659(12)
0.1	0.33	1.4237(11)	1.4383(11)	1.4530(10)	1.4637(10)	1.4714(10)	1.4765(10)	1.4842(12)
0.2	0.5	1.4464(10)	1.4611(10)	1.4759(10)	1.4856(10)	1.4947(11)	1.4995(12)	1.5074(10)
0.3	0.65	1.4593(10)	1.4747(11)	1.4882(12)	1.4988(10)	1.5075(10)	1.5113(10)	1.5192(10)
0.4	0.82	1.4664(11)	1.4804(10)	1.4953(10)	1.5049(11)	1.5131(11)	1.5183(10)	1.5253(12)
0.5	1	1.4695(10)	1.4834(11)	1.4982(10)	1.5082(11)	1.5161(10)	1.5207(11)	1.5285(11)
0.6	1.22	1.4715(11)	1.4856(11)	1.4996(12)	1.5088(12)	1.5178(11)	1.5222(11)	1.5304(11)
0.7	1.53	1.4718(12)	1.4856(11)	1.5015(10)	1.5108(12)	1.5207(10)	1.5237(11)	1.5308(10)
0.8	2	1.4741(09)	1.4879(11)	1.5030(12)	1.5117(11)	1.5200(11)	1.5252(12)	1.5320(12)
0.8605	2.48	1.4744(10)	1.4884(12)	1.5020(10)	1.5124(12)	1.5212(12)	1.5247(11)	1.5320(13)
0.9	3	1.4753(11)	1.4880(10)	1.5026(11)	1.5134(11)	1.5213(12)	1.5250(12)	1.5339(12)

Table A.150: Case IIIb Loglogistic Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.5245(12)	1.5391(14)	1.5538(14)	1.5650(13)	1.5728(15)	1.5776(13)	1.5841(12)
0.0323	0.18	1.5251(14)	1.5388(12)	1.5515(12)	1.5618(13)	1.5715(13)	1.5761(13)	1.5828(14)
0.1	0.33	1.5391(14)	1.5544(14)	1.5691(14)	1.5806(13)	1.5888(13)	1.5927(15)	1.6014(15)
0.2	0.5	1.5620(13)	1.5768(13)	1.5925(12)	1.6042(14)	1.6135(14)	1.6172(15)	1.6250(16)
0.3	0.65	1.5760(14)	1.5927(14)	1.6071(16)	1.6183(16)	1.6272(12)	1.6318(15)	1.6397(14)
0.4	0.82	1.5843(15)	1.5984(15)	1.6153(14)	1.6250(15)	1.6329(15)	1.6383(13)	1.6463(15)
0.5	1	1.5871(15)	1.6024(13)	1.6185(13)	1.6286(14)	1.6369(13)	1.6421(14)	1.6492(13)
0.6	1.22	1.5895(14)	1.6049(16)	1.6197(15)	1.6291(13)	1.6391(14)	1.6428(14)	1.6520(15)
0.7	1.53	1.5908(15)	1.6058(15)	1.6220(14)	1.6319(16)	1.6431(14)	1.6453(15)	1.6519(14)
0.8	2	1.5927(13)	1.6081(14)	1.6230(15)	1.6336(15)	1.6409(15)	1.6465(15)	1.6535(14)
0.8605	2.48	1.5936(14)	1.6085(13)	1.6229(13)	1.6333(14)	1.6429(15)	1.6464(16)	1.6532(16)
0.9	3	1.5945(16)	1.6082(14)	1.6224(15)	1.6340(15)	1.6431(13)	1.6476(15)	1.6556(15)

Table A.151: Case IIIb Loglogistic Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.7593(28)	1.7751(27)	1.7922(24)	1.8018(29)	1.8108(28)	1.8162(26)	1.8234(31)
0.0323	0.18	1.7618(29)	1.7779(26)	1.7897(25)	1.8005(25)	1.8116(28)	1.8150(28)	1.8235(31)
0.1	0.33	1.7728(29)	1.7901(25)	1.8073(27)	1.8188(27)	1.8270(27)	1.8288(25)	1.8403(29)
0.2	0.5	1.7965(28)	1.8117(28)	1.8275(26)	1.8419(25)	1.8529(27)	1.8548(28)	1.8647(26)
0.3	0.65	1.8081(23)	1.8313(24)	1.8475(28)	1.8594(24)	1.8681(27)	1.8726(24)	1.8794(28)
0.4	0.82	1.8197(28)	1.8369(28)	1.8553(23)	1.8660(26)	1.8751(32)	1.8811(29)	1.8870(28)
0.5	1	1.8227(29)	1.8439(27)	1.8585(26)	1.8669(27)	1.8771(28)	1.8836(28)	1.8908(27)
0.6	1.22	1.8255(29)	1.8440(30)	1.8598(30)	1.8747(29)	1.8819(26)	1.8860(27)	1.8942(31)
0.7	1.53	1.8287(28)	1.8474(27)	1.8672(30)	1.8742(26)	1.8867(26)	1.8877(28)	1.8964(28)
0.8	2	1.8311(21)	1.8483(27)	1.8647(31)	1.8771(30)	1.8823(27)	1.8884(31)	1.8980(26)
0.8605	2.48	1.8307(27)	1.8468(26)	1.8690(29)	1.8786(27)	1.8862(31)	1.8882(29)	1.8968(29)
0.9	3	1.8319(27)	1.8510(26)	1.8646(27)	1.8771(31)	1.8852(26)	1.8902(29)	1.9015(29)

Table A.152: Case IIIb Loglogistic Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.2626(07)	0.2612(08)	0.2604(07)	0.2601(06)	0.2596(06)	0.2593(06)	0.2593(07)
0.0323	0.18	0.2562(07)	0.2547(07)	0.2542(06)	0.2540(07)	0.2538(07)	0.2534(07)	0.2538(06)
0.1	0.33	0.2331(07)	0.2325(06)	0.2309(06)	0.2308(06)	0.2302(07)	0.2299(06)	0.2306(07)
0.2	0.5	0.1994(05)	0.1976(05)	0.1969(05)	0.1962(04)	0.1959(05)	0.1963(05)	0.1959(05)
0.3	0.65	0.1744(04)	0.1739(04)	0.1731(04)	0.1729(04)	0.1728(04)	0.1726(04)	0.1728(04)
0.4	0.82	0.1584(04)	0.1583(04)	0.1584(03)	0.1584(03)	0.1585(04)	0.1584(03)	0.1582(04)
0.5	1	0.1494(03)	0.1491(03)	0.1493(03)	0.1493(03)	0.1492(04)	0.1491(03)	0.1494(03)
0.6	1.22	0.1435(03)	0.1438(03)	0.1437(03)	0.1437(03)	0.1436(03)	0.1435(03)	0.1438(03)
0.7	1.53	0.1402(03)	0.1403(03)	0.1406(03)	0.1405(03)	0.1407(03)	0.1404(03)	0.1405(03)
0.8	2	0.1394(02)	0.1394(03)	0.1395(03)	0.1390(03)	0.1392(03)	0.1395(03)	0.1391(03)
0.8605	2.48	0.1398(03)	0.1396(03)	0.1395(03)	0.1394(03)	0.1394(03)	0.1394(02)	0.1395(03)
0.9	3	0.1405(03)	0.1402(03)	0.1405(03)	0.1405(03)	0.1403(03)	0.1403(03)	0.1406(03)

Table A.153: Case IIIb Loglogistic Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.3259(10)	0.3248(10)	0.3240(10)	0.3239(09)	0.3231(09)	0.3232(09)	0.3235(10)
0.0323	0.18	0.3182(09)	0.3170(09)	0.3173(08)	0.3162(09)	0.3158(09)	0.3155(08)	0.3160(09)
0.1	0.33	0.2887(09)	0.2873(08)	0.2861(09)	0.2859(07)	0.2850(09)	0.2850(08)	0.2855(09)
0.2	0.5	0.2437(06)	0.2412(06)	0.2403(06)	0.2395(05)	0.2393(06)	0.2395(06)	0.2397(06)
0.3	0.65	0.2100(05)	0.2097(05)	0.2090(05)	0.2085(05)	0.2084(05)	0.2081(05)	0.2084(05)
0.4	0.82	0.1890(05)	0.1892(05)	0.1892(04)	0.1890(05)	0.1894(04)	0.1890(04)	0.1887(05)
0.5	1	0.1767(05)	0.1767(04)	0.1768(04)	0.1770(04)	0.1768(05)	0.1767(04)	0.1771(04)
0.6	1.22	0.1691(03)	0.1693(04)	0.1695(04)	0.1694(04)	0.1694(04)	0.1695(04)	0.1696(04)
0.7	1.53	0.1648(04)	0.1647(03)	0.1654(04)	0.1651(04)	0.1656(03)	0.1651(04)	0.1653(04)
0.8	2	0.1634(03)	0.1636(04)	0.1639(04)	0.1633(04)	0.1634(04)	0.1637(04)	0.1634(04)
0.8605	2.48	0.1638(04)	0.1637(04)	0.1637(04)	0.1636(04)	0.1638(04)	0.1636(04)	0.1639(04)
0.9	3	0.1647(03)	0.1646(04)	0.1649(04)	0.1651(03)	0.1649(04)	0.1647(04)	0.1651(04)

Table A.154: Case IIIb Loglogistic Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.4387(14)	0.4386(15)	0.4384(16)	0.4385(12)	0.4383(14)	0.4371(14)	0.4380(16)
0.0323	0.18	0.4298(12)	0.4283(13)	0.4284(13)	0.4271(13)	0.4276(15)	0.4282(15)	0.4288(15)
0.1	0.33	0.3884(13)	0.3866(12)	0.3846(12)	0.3842(11)	0.3833(12)	0.3829(14)	0.3839(13)
0.2	0.5	0.3231(11)	0.3200(10)	0.3194(09)	0.3176(10)	0.3176(09)	0.3175(10)	0.3179(10)
0.3	0.65	0.2741(09)	0.2739(08)	0.2727(08)	0.2728(09)	0.2723(07)	0.2720(07)	0.2729(08)
0.4	0.82	0.2433(07)	0.2435(07)	0.2438(08)	0.2440(06)	0.2445(07)	0.2441(06)	0.2440(07)
0.5	1	0.2251(07)	0.2256(06)	0.2263(05)	0.2265(07)	0.2267(07)	0.2265(06)	0.2268(07)
0.6	1.22	0.2147(06)	0.2151(06)	0.2153(06)	0.2151(06)	0.2157(05)	0.2154(06)	0.2156(06)
0.7	1.53	0.2077(06)	0.2084(05)	0.2097(06)	0.2088(06)	0.2100(05)	0.2092(06)	0.2092(05)
0.8	2	0.2058(05)	0.2064(05)	0.2068(05)	0.2066(06)	0.2062(06)	0.2067(05)	0.2064(06)
0.8605	2.48	0.2062(06)	0.2062(05)	0.2067(05)	0.2066(06)	0.2070(06)	0.2067(05)	0.2069(06)
0.9	3	0.2078(05)	0.2078(06)	0.2081(06)	0.2086(06)	0.2079(06)	0.2081(05)	0.2086(06)

Table A.155: Case IIIb Loglogistic Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	0.7082(32)	0.7105(34)	0.7166(37)	0.7214(35)	0.7197(39)	0.7175(39)	0.7213(36)
0.0323	0.18	0.6962(31)	0.7001(31)	0.6997(33)	0.7028(34)	0.7066(40)	0.7025(33)	0.7044(38)
0.1	0.33	0.6323(29)	0.6316(32)	0.6291(33)	0.6254(31)	0.6276(26)	0.6246(29)	0.6301(33)
0.2	0.5	0.5170(25)	0.5131(24)	0.5133(24)	0.5106(25)	0.5113(27)	0.5106(25)	0.5115(25)
0.3	0.65	0.4287(19)	0.4310(21)	0.4302(18)	0.4309(20)	0.4305(20)	0.4306(21)	0.4303(19)
0.4	0.82	0.3760(19)	0.3779(17)	0.3796(16)	0.3804(17)	0.3810(20)	0.3808(18)	0.3814(17)
0.5	1	0.3442(16)	0.3458(17)	0.3470(15)	0.3479(16)	0.3476(16)	0.3481(16)	0.3494(15)
0.6	1.22	0.3245(13)	0.3260(14)	0.3278(15)	0.3270(15)	0.3292(15)	0.3285(14)	0.3287(16)
0.7	1.53	0.3129(13)	0.3145(14)	0.3180(14)	0.3172(13)	0.3192(13)	0.3170(14)	0.3185(15)
0.8	2	0.3088(12)	0.3110(14)	0.3118(13)	0.3111(14)	0.3114(15)	0.3116(15)	0.3117(14)
0.8605	2.48	0.3092(13)	0.3093(13)	0.3117(13)	0.3127(12)	0.3121(15)	0.3123(13)	0.3120(14)
0.9	3	0.3119(11)	0.3125(14)	0.3131(12)	0.3142(14)	0.3134(12)	0.3153(13)	0.3150(14)

Table A.156: Case IIIb Loglogistic Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.4194(36)	1.4145(39)	1.4132(37)	1.4122(33)	1.4116(33)	1.4115(34)	1.4119(34)
0.0323	0.18	1.3945(35)	1.3876(33)	1.3866(29)	1.3859(38)	1.3843(34)	1.3821(34)	1.3860(33)
0.1	0.33	1.2922(34)	1.2886(32)	1.2800(30)	1.2798(29)	1.2749(34)	1.2753(30)	1.2775(29)
0.2	0.5	1.1374(23)	1.1301(24)	1.1267(26)	1.1245(21)	1.1233(24)	1.1248(26)	1.1238(27)
0.3	0.65	1.0236(21)	1.0241(19)	1.0227(21)	1.0225(18)	1.0219(20)	1.0220(21)	1.0234(21)
0.4	0.82	0.9534(19)	0.9575(19)	0.9592(18)	0.9592(18)	0.9613(20)	0.9607(19)	0.9610(20)
0.5	1	0.9147(20)	0.9165(17)	0.9205(16)	0.9223(16)	0.9218(19)	0.9215(16)	0.9227(18)
0.6	1.22	0.8892(15)	0.8937(17)	0.8967(18)	0.8963(17)	0.8977(18)	0.8980(16)	0.8991(18)
0.7	1.53	0.8760(17)	0.8798(16)	0.8846(17)	0.8846(16)	0.8851(14)	0.8845(17)	0.8857(15)
0.8	2	0.8726(15)	0.8766(18)	0.8793(16)	0.8782(15)	0.8795(17)	0.8816(16)	0.8790(16)
0.8605	2.48	0.8740(15)	0.8759(16)	0.8782(15)	0.8786(16)	0.8806(18)	0.8793(15)	0.8814(18)
0.9	3	0.8751(15)	0.8777(15)	0.8817(16)	0.8836(16)	0.8826(15)	0.8828(19)	0.8855(17)

Table A.157: Case IIIb Loglogistic Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	1.7282(44)	1.7264(52)	1.7245(48)	1.7256(45)	1.7246(43)	1.7240(47)	1.7257(49)
0.0323	0.18	1.6999(43)	1.6935(44)	1.6945(40)	1.6898(46)	1.6877(41)	1.6880(44)	1.6912(45)
0.1	0.33	1.5689(45)	1.5626(42)	1.5522(40)	1.5504(39)	1.5456(45)	1.5458(45)	1.5487(41)
0.2	0.5	1.3612(30)	1.3501(31)	1.3457(32)	1.3419(28)	1.3404(33)	1.3425(30)	1.3434(34)
0.3	0.65	1.2090(30)	1.2097(25)	1.2077(27)	1.2069(28)	1.2075(27)	1.2054(27)	1.2079(26)
0.4	0.82	1.1152(25)	1.1208(27)	1.1231(24)	1.1236(24)	1.1259(26)	1.1253(22)	1.1240(27)
0.5	1	1.0639(23)	1.0672(21)	1.0722(20)	1.0735(22)	1.0736(26)	1.0728(22)	1.0759(25)
0.6	1.22	1.0321(22)	1.0383(22)	1.0402(24)	1.0402(25)	1.0420(24)	1.0432(20)	1.0437(22)
0.7	1.53	1.0136(21)	1.0178(20)	1.0242(22)	1.0238(22)	1.0272(20)	1.0253(22)	1.0264(19)
0.8	2	1.0089(21)	1.0141(21)	1.0181(22)	1.0163(20)	1.0171(22)	1.0203(21)	1.0173(20)
0.8605	2.48	1.0100(20)	1.0146(21)	1.0164(19)	1.0159(22)	1.0188(23)	1.0183(20)	1.0195(22)
0.9	3	1.0127(21)	1.0161(21)	1.0207(19)	1.0221(20)	1.0215(19)	1.0218(21)	1.0240(23)

Table A.158: Case IIIb Loglogistic Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	2.2743(65)	2.2823(75)	2.2845(79)	2.2877(67)	2.2870(76)	2.2875(68)	2.2923(72)
0.0323	0.18	2.2473(70)	2.2424(70)	2.2417(63)	2.2372(63)	2.2410(73)	2.2429(73)	2.2424(75)
0.1	0.33	2.0675(65)	2.0559(60)	2.0434(59)	2.0391(58)	2.0332(64)	2.0294(65)	2.0387(64)
0.2	0.5	1.7633(50)	1.7507(46)	1.7425(46)	1.7379(48)	1.7354(48)	1.7345(50)	1.7347(53)
0.3	0.65	1.5408(44)	1.5428(39)	1.5399(45)	1.5377(45)	1.5371(41)	1.5371(36)	1.5389(44)
0.4	0.82	1.4060(39)	1.4112(40)	1.4156(39)	1.4171(36)	1.4196(40)	1.4189(34)	1.4178(41)
0.5	1	1.3303(36)	1.3356(30)	1.3402(31)	1.3456(34)	1.3469(37)	1.3434(34)	1.3473(36)
0.6	1.22	1.2871(32)	1.2904(30)	1.2956(35)	1.2945(31)	1.2992(31)	1.3006(33)	1.2991(33)
0.7	1.53	1.2566(35)	1.2656(33)	1.2751(34)	1.2701(31)	1.2776(29)	1.2729(31)	1.2734(29)
0.8	2	1.2509(31)	1.2561(33)	1.2621(31)	1.2606(32)	1.2610(34)	1.2638(29)	1.2619(30)
0.8605	2.48	1.2516(33)	1.2562(31)	1.2603(31)	1.2614(33)	1.2634(36)	1.2614(32)	1.2640(32)
0.9	3	1.2574(31)	1.2601(30)	1.2653(28)	1.2675(32)	1.2680(33)	1.2680(29)	1.2701(34)

Table A.159: Case IIIb Loglogistic Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0	3.5698(162)	3.6038(165)	3.6516(181)	3.6774(163)	3.6827(177)	3.6623(190)	3.6815(169)
0.0323	0.18	3.5494(147)	3.5749(154)	3.5865(163)	3.5894(172)	3.6096(196)	3.5958(165)	3.6056(171)
0.1	0.33	3.3289(146)	3.2959(162)	3.2612(154)	3.2417(145)	3.2477(124)	3.2327(148)	3.2590(156)
0.2	0.5	2.7853(121)	2.7442(116)	2.7293(120)	2.7163(118)	2.7168(125)	2.7078(114)	2.7187(128)
0.3	0.65	2.3622(97)	2.3688(108)	2.3625(98)	2.3587(107)	2.3646(104)	2.3578(104)	2.3599(103)
0.4	0.82	2.1301(98)	2.1388(92)	2.1416(78)	2.1461(93)	2.1447(101)	2.1491(92)	2.1520(85)
0.5	1	1.9972(85)	2.0003(83)	2.0040(82)	2.0070(84)	2.0096(85)	2.0077(92)	2.0141(83)
0.6	1.22	1.9162(84)	1.9194(84)	1.9279(86)	1.9206(78)	1.9297(80)	1.9282(72)	1.9295(82)
0.7	1.53	1.8700(79)	1.8695(80)	1.8923(77)	1.8876(75)	1.8877(68)	1.8800(68)	1.8878(80)
0.8	2	1.8528(66)	1.8564(78)	1.8603(77)	1.8590(72)	1.8598(82)	1.8609(87)	1.8612(78)
0.8605	2.48	1.8504(74)	1.8488(73)	1.8637(71)	1.8629(76)	1.8624(86)	1.8613(72)	1.8583(73)
0.9	3	1.8625(72)	1.8656(75)	1.8643(71)	1.8689(76)	1.8721(74)	1.8721(69)	1.8747(83)

Table A.160: Case IIIb Loglogistic Anderson-Darling 99% critical values

A.4.3 Lognormal Distribution

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.0760(09)	1.0803(11)	1.0862(10)	1.0912(10)	1.0954(11)	1.0969(09)	1.1008(11)
0.0323	0.72	1.0661(11)	1.0717(11)	1.0785(12)	1.0815(10)	1.0859(11)	1.0893(10)	1.0914(11)
0.1	0.75	1.0474(10)	1.0522(11)	1.0578(11)	1.0620(09)	1.0663(10)	1.0682(11)	1.0725(10)
0.2	0.79	1.0144(09)	1.0192(10)	1.0257(09)	1.0296(09)	1.0338(08)	1.0364(09)	1.0394(09)
0.3	0.85	0.9855(08)	0.9921(09)	0.9987(09)	1.0038(09)	1.0086(08)	1.0097(09)	1.0140(09)
0.4	0.91	0.9652(08)	0.9719(09)	0.9792(08)	0.9841(08)	0.9881(09)	0.9904(08)	0.9940(08)
0.5	1	0.9491(09)	0.9560(08)	0.9637(09)	0.9692(09)	0.9730(09)	0.9755(08)	0.9789(08)
0.6	1.12	0.9371(09)	0.9448(08)	0.9529(09)	0.9570(08)	0.9609(09)	0.9635(08)	0.9677(08)
0.7	1.29	0.9272(08)	0.9348(08)	0.9424(08)	0.9477(09)	0.9520(08)	0.9545(08)	0.9577(07)
0.8	1.58	0.9196(09)	0.9271(07)	0.9351(08)	0.9394(08)	0.9444(07)	0.9460(08)	0.9500(08)
0.8605	1.89	0.9155(09)	0.9224(09)	0.9303(08)	0.9352(08)	0.9389(08)	0.9409(09)	0.9451(07)
0.9	2.24	0.9121(08)	0.9194(07)	0.9277(07)	0.9316(08)	0.9364(08)	0.9390(08)	0.9421(08)

Table A.161: Case IIIb Lognormal Kolmogorov-Smirnov 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.1622(11)	1.1676(13)	1.1738(11)	1.1792(13)	1.1830(12)	1.1840(12)	1.1884(13)
0.0323	0.72	1.1527(12)	1.1579(13)	1.1650(13)	1.1686(12)	1.1730(12)	1.1766(12)	1.1782(12)
0.1	0.75	1.1312(13)	1.1354(12)	1.1412(13)	1.1458(12)	1.1493(12)	1.1514(12)	1.1566(11)
0.2	0.79	1.0918(10)	1.0978(11)	1.1043(11)	1.1081(10)	1.1127(11)	1.1144(10)	1.1178(12)
0.3	0.85	1.0593(10)	1.0665(11)	1.0735(11)	1.0781(10)	1.0831(09)	1.0845(10)	1.0886(11)
0.4	0.91	1.0361(09)	1.0427(10)	1.0505(10)	1.0553(10)	1.0596(11)	1.0619(09)	1.0660(10)
0.5	1	1.0175(10)	1.0248(09)	1.0328(10)	1.0385(10)	1.0424(10)	1.0451(10)	1.0482(10)
0.6	1.12	1.0034(10)	1.0120(11)	1.0205(09)	1.0241(10)	1.0286(11)	1.0312(10)	1.0354(09)
0.7	1.29	0.9925(09)	1.0001(09)	1.0081(10)	1.0139(09)	1.0180(10)	1.0205(09)	1.0241(09)
0.8	1.58	0.9837(11)	0.9913(09)	0.9999(09)	1.0043(09)	1.0095(09)	1.0106(09)	1.0150(10)
0.8605	1.89	0.9784(10)	0.9865(10)	0.9945(08)	0.9996(09)	1.0034(09)	1.0053(10)	1.0096(09)
0.9	2.24	0.9752(09)	0.9831(08)	0.9914(08)	0.9958(10)	1.0005(09)	1.0034(10)	1.0065(09)

Table A.162: Case IIIb Lognormal Kolmogorov-Smirnov 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.2967(15)	1.3022(16)	1.3105(15)	1.3147(17)	1.3203(16)	1.3205(16)	1.3249(16)
0.0323	0.72	1.2868(16)	1.2926(16)	1.2997(17)	1.3037(14)	1.3084(15)	1.3128(14)	1.3146(16)
0.1	0.75	1.2614(14)	1.2655(14)	1.2718(17)	1.2765(16)	1.2792(17)	1.2816(17)	1.2868(15)
0.2	0.79	1.2131(13)	1.2202(14)	1.2267(16)	1.2317(13)	1.2364(14)	1.2374(14)	1.2421(15)
0.3	0.85	1.1752(15)	1.1822(13)	1.1896(15)	1.1957(15)	1.2002(14)	1.2010(13)	1.2047(15)
0.4	0.91	1.1468(12)	1.1542(14)	1.1631(14)	1.1676(12)	1.1715(13)	1.1747(12)	1.1782(13)
0.5	1	1.1243(13)	1.1320(11)	1.1415(14)	1.1471(12)	1.1515(14)	1.1543(10)	1.1571(13)
0.6	1.12	1.1075(12)	1.1165(14)	1.1255(14)	1.1299(13)	1.1354(13)	1.1366(13)	1.1417(11)
0.7	1.29	1.0937(12)	1.1025(13)	1.1120(12)	1.1180(13)	1.1217(13)	1.1252(12)	1.1278(11)
0.8	1.58	1.0843(13)	1.0920(11)	1.1012(11)	1.1063(12)	1.1117(12)	1.1129(12)	1.1175(13)
0.8605	1.89	1.0781(13)	1.0869(13)	1.0957(12)	1.1003(11)	1.1047(12)	1.1059(13)	1.1110(12)
0.9	2.24	1.0735(12)	1.0826(12)	1.0917(11)	1.0974(13)	1.1009(12)	1.1041(11)	1.1072(14)

Table A.163: Case IIIb Lognormal Kolmogorov-Smirnov 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.5620(31)	1.5688(30)	1.5822(34)	1.5867(31)	1.5928(34)	1.5929(29)	1.5961(31)
0.0323	0.72	1.5533(28)	1.5610(31)	1.5690(30)	1.5746(29)	1.5792(28)	1.5822(30)	1.5852(28)
0.1	0.75	1.5208(30)	1.5269(24)	1.5334(33)	1.5373(30)	1.5391(30)	1.5432(27)	1.5478(30)
0.2	0.79	1.4555(29)	1.4638(27)	1.4722(23)	1.4778(25)	1.4849(27)	1.4862(28)	1.4906(29)
0.3	0.85	1.4034(24)	1.4141(23)	1.4229(26)	1.4299(27)	1.4343(28)	1.4345(20)	1.4406(26)
0.4	0.91	1.3653(23)	1.3754(24)	1.3869(26)	1.3950(27)	1.3984(24)	1.4009(23)	1.4043(22)
0.5	1	1.3381(22)	1.3472(20)	1.3601(25)	1.3673(24)	1.3709(25)	1.3745(19)	1.3770(23)
0.6	1.12	1.3165(24)	1.3276(25)	1.3371(23)	1.3450(26)	1.3480(25)	1.3528(26)	1.3567(24)
0.7	1.29	1.2971(22)	1.3098(21)	1.3203(23)	1.3287(23)	1.3333(27)	1.3346(21)	1.3393(23)
0.8	1.58	1.2848(23)	1.2948(22)	1.3068(22)	1.3111(26)	1.3198(21)	1.3201(22)	1.3250(20)
0.8605	1.89	1.2766(26)	1.2883(22)	1.2996(23)	1.3039(22)	1.3103(22)	1.3106(21)	1.3187(22)
0.9	2.24	1.2731(22)	1.2823(23)	1.2931(22)	1.3000(19)	1.3035(23)	1.3078(25)	1.3107(24)

Table A.164: Case IIIb Lognormal Kolmogorov-Smirnov 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.3574(09)	1.3712(09)	1.3860(10)	1.3957(09)	1.4046(10)	1.4085(08)	1.4162(09)
0.0323	0.72	1.3541(08)	1.3684(09)	1.3837(10)	1.3936(09)	1.4026(08)	1.4079(09)	1.4139(09)
0.1	0.75	1.3740(08)	1.3884(10)	1.4038(10)	1.4140(10)	1.4228(10)	1.4277(10)	1.4351(10)
0.2	0.79	1.3919(09)	1.4070(10)	1.4213(10)	1.4310(08)	1.4403(08)	1.4447(08)	1.4521(10)
0.3	0.85	1.3997(09)	1.4140(10)	1.4284(09)	1.4381(10)	1.4480(10)	1.4513(09)	1.4586(09)
0.4	0.91	1.4031(09)	1.4168(09)	1.4314(08)	1.4410(09)	1.4497(09)	1.4546(10)	1.4615(09)
0.5	1	1.4040(09)	1.4172(09)	1.4320(09)	1.4426(09)	1.4505(10)	1.4550(09)	1.4620(11)
0.6	1.12	1.4042(10)	1.4186(10)	1.4326(09)	1.4415(10)	1.4509(10)	1.4546(09)	1.4628(09)
0.7	1.29	1.4041(11)	1.4168(10)	1.4318(10)	1.4417(09)	1.4501(09)	1.4553(09)	1.4611(09)
0.8	1.58	1.4040(10)	1.4178(10)	1.4317(09)	1.4413(10)	1.4501(09)	1.4538(09)	1.4618(10)
0.8605	1.89	1.4038(10)	1.4167(11)	1.4310(09)	1.4418(10)	1.4488(10)	1.4525(11)	1.4610(10)
0.9	2.24	1.4031(09)	1.4164(09)	1.4316(09)	1.4399(09)	1.4488(09)	1.4530(09)	1.4607(11)

Table A.165: Case IIIb Lognormal Kuiper 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.4309(10)	1.4456(11)	1.4607(11)	1.4703(11)	1.4796(10)	1.4833(10)	1.4911(10)
0.0323	0.72	1.4281(10)	1.4420(09)	1.4577(11)	1.4685(12)	1.4775(10)	1.4823(10)	1.4888(11)
0.1	0.75	1.4479(10)	1.4633(11)	1.4789(12)	1.4895(10)	1.4977(10)	1.5025(11)	1.5106(11)
0.2	0.79	1.4672(10)	1.4830(12)	1.4973(11)	1.5079(10)	1.5172(10)	1.5217(09)	1.5294(12)
0.3	0.85	1.4758(11)	1.4906(12)	1.5054(12)	1.5151(13)	1.5251(11)	1.5284(10)	1.5357(10)
0.4	0.91	1.4795(10)	1.4935(11)	1.5084(09)	1.5187(10)	1.5274(11)	1.5319(11)	1.5389(11)
0.5	1	1.4803(12)	1.4940(10)	1.5101(11)	1.5205(09)	1.5286(12)	1.5337(10)	1.5399(12)
0.6	1.12	1.4801(12)	1.4958(11)	1.5110(11)	1.5187(13)	1.5290(12)	1.5324(11)	1.5409(11)
0.7	1.29	1.4803(11)	1.4936(11)	1.5092(12)	1.5193(11)	1.5285(12)	1.5331(10)	1.5398(10)
0.8	1.58	1.4815(12)	1.4948(11)	1.5096(10)	1.5185(11)	1.5287(09)	1.5321(10)	1.5402(12)
0.8605	1.89	1.4806(12)	1.4939(12)	1.5093(10)	1.5191(09)	1.5271(11)	1.5307(11)	1.5392(12)
0.9	2.24	1.4797(10)	1.4936(11)	1.5093(11)	1.5176(11)	1.5268(11)	1.5316(10)	1.5385(12)

Table A.166: Case IIIb Lognormal Kuiper 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.5457(14)	1.5603(15)	1.5767(15)	1.5868(14)	1.5963(14)	1.5994(13)	1.6079(14)
0.0323	0.72	1.5437(14)	1.5574(13)	1.5738(15)	1.5837(14)	1.5927(13)	1.5987(14)	1.6056(13)
0.1	0.75	1.5626(13)	1.5789(14)	1.5958(16)	1.6069(14)	1.6144(13)	1.6197(15)	1.6277(14)
0.2	0.79	1.5830(15)	1.6001(14)	1.6160(14)	1.6266(14)	1.6365(14)	1.6406(13)	1.6492(13)
0.3	0.85	1.5935(14)	1.6096(14)	1.6244(13)	1.6348(14)	1.6446(14)	1.6482(14)	1.6560(15)
0.4	0.91	1.5968(13)	1.6121(15)	1.6289(13)	1.6397(13)	1.6481(14)	1.6521(14)	1.6597(13)
0.5	1	1.5986(15)	1.6131(13)	1.6295(17)	1.6403(12)	1.6505(15)	1.6539(15)	1.6612(15)
0.6	1.12	1.5988(14)	1.6151(15)	1.6319(14)	1.6404(14)	1.6494(14)	1.6530(14)	1.6627(14)
0.7	1.29	1.5994(13)	1.6141(14)	1.6289(15)	1.6409(14)	1.6491(14)	1.6543(14)	1.6612(14)
0.8	1.58	1.6004(15)	1.6144(14)	1.6303(15)	1.6395(14)	1.6497(14)	1.6537(13)	1.6615(16)
0.8605	1.89	1.6003(13)	1.6136(15)	1.6300(14)	1.6396(12)	1.6486(13)	1.6510(15)	1.6607(15)
0.9	2.24	1.5980(13)	1.6127(14)	1.6290(14)	1.6398(14)	1.6479(14)	1.6524(15)	1.6599(15)

Table A.167: Case IIIb Lognormal Kuiper 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.7760(27)	1.7922(27)	1.8109(29)	1.8209(26)	1.8307(26)	1.8330(25)	1.8440(25)
0.0323	0.72	1.7777(28)	1.7914(29)	1.8080(23)	1.8195(24)	1.8290(28)	1.8332(27)	1.8423(28)
0.1	0.75	1.7945(29)	1.8125(24)	1.8297(27)	1.8418(28)	1.8502(22)	1.8559(28)	1.8641(31)
0.2	0.79	1.8149(26)	1.8346(26)	1.8539(24)	1.8650(25)	1.8759(27)	1.8811(25)	1.8879(30)
0.3	0.85	1.8273(24)	1.8463(27)	1.8632(26)	1.8773(29)	1.8854(27)	1.8883(29)	1.8970(26)
0.4	0.91	1.8320(25)	1.8484(30)	1.8680(27)	1.8814(27)	1.8897(24)	1.8914(26)	1.9013(25)
0.5	1	1.8339(27)	1.8526(24)	1.8692(28)	1.8819(27)	1.8899(27)	1.8966(28)	1.9020(29)
0.6	1.12	1.8358(26)	1.8547(28)	1.8717(28)	1.8831(29)	1.8936(28)	1.8974(26)	1.9057(30)
0.7	1.29	1.8367(25)	1.8545(24)	1.8728(29)	1.8843(29)	1.8925(27)	1.8965(26)	1.9057(28)
0.8	1.58	1.8374(28)	1.8538(27)	1.8718(24)	1.8820(29)	1.8932(31)	1.8974(33)	1.9043(27)
0.8605	1.89	1.8361(27)	1.8518(24)	1.8709(30)	1.8806(27)	1.8906(26)	1.8940(26)	1.9054(27)
0.9	2.24	1.8337(24)	1.8517(30)	1.8685(29)	1.8827(20)	1.8908(27)	1.8959(30)	1.9038(29)

Table A.168: Case IIIb Lognormal Kuiper 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.2669(06)	0.2647(07)	0.2638(06)	0.2635(07)	0.2635(07)	0.2628(07)	0.2632(08)
0.0323	0.72	0.2589(07)	0.2572(07)	0.2569(07)	0.2559(07)	0.2555(07)	0.2563(06)	0.2554(06)
0.1	0.75	0.2386(06)	0.2369(06)	0.2358(06)	0.2351(06)	0.2350(06)	0.2348(07)	0.2350(06)
0.2	0.79	0.2112(05)	0.2099(05)	0.2097(05)	0.2091(05)	0.2091(05)	0.2088(05)	0.2086(05)
0.3	0.85	0.1923(04)	0.1919(05)	0.1918(04)	0.1916(05)	0.1916(04)	0.1914(04)	0.1916(05)
0.4	0.91	0.1800(04)	0.1797(05)	0.1801(04)	0.1799(04)	0.1796(04)	0.1799(04)	0.1799(04)
0.5	1	0.1716(04)	0.1714(04)	0.1716(04)	0.1714(04)	0.1712(05)	0.1714(04)	0.1717(04)
0.6	1.12	0.1651(04)	0.1653(04)	0.1657(04)	0.1652(03)	0.1651(04)	0.1655(04)	0.1656(04)
0.7	1.29	0.1603(04)	0.1603(04)	0.1607(03)	0.1608(04)	0.1604(03)	0.1608(03)	0.1606(03)
0.8	1.58	0.1564(04)	0.1567(03)	0.1571(03)	0.1569(03)	0.1568(03)	0.1569(04)	0.1570(03)
0.8605	1.89	0.1545(04)	0.1546(03)	0.1547(03)	0.1548(03)	0.1545(03)	0.1547(04)	0.1549(03)
0.9	2.24	0.1531(03)	0.1532(03)	0.1536(03)	0.1534(03)	0.1534(03)	0.1534(03)	0.1534(03)

Table A.169: Case IIIb Lognormal Cramér-von Mises 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.3307(09)	0.3283(09)	0.3276(09)	0.3275(10)	0.3272(09)	0.3262(09)	0.3273(11)
0.0323	0.72	0.3210(10)	0.3198(10)	0.3187(10)	0.3173(09)	0.3174(10)	0.3189(09)	0.3170(10)
0.1	0.75	0.2940(08)	0.2920(08)	0.2905(09)	0.2900(08)	0.2895(09)	0.2892(09)	0.2904(08)
0.2	0.79	0.2573(06)	0.2563(07)	0.2555(07)	0.2553(07)	0.2551(06)	0.2547(06)	0.2546(07)
0.3	0.85	0.2321(06)	0.2316(06)	0.2315(06)	0.2317(06)	0.2317(05)	0.2314(06)	0.2314(06)
0.4	0.91	0.2160(05)	0.2155(05)	0.2162(05)	0.2159(05)	0.2159(05)	0.2160(05)	0.2161(05)
0.5	1	0.2045(05)	0.2044(05)	0.2049(05)	0.2051(05)	0.2047(05)	0.2048(05)	0.2051(05)
0.6	1.12	0.1961(05)	0.1964(05)	0.1972(05)	0.1969(05)	0.1968(05)	0.1967(05)	0.1972(05)
0.7	1.29	0.1899(05)	0.1899(05)	0.1904(04)	0.1910(05)	0.1903(04)	0.1909(04)	0.1908(04)
0.8	1.58	0.1848(05)	0.1853(04)	0.1859(04)	0.1856(04)	0.1858(04)	0.1858(05)	0.1857(04)
0.8605	1.89	0.1826(05)	0.1827(04)	0.1829(04)	0.1829(04)	0.1828(04)	0.1827(04)	0.1830(04)
0.9	2.24	0.1803(04)	0.1810(04)	0.1814(04)	0.1814(04)	0.1811(04)	0.1814(04)	0.1815(04)

Table A.170: Case IIIb Lognormal Cramér-von Mises 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.4438(13)	0.4428(14)	0.4423(14)	0.4422(16)	0.4421(15)	0.4402(14)	0.4419(13)
0.0323	0.72	0.4319(14)	0.4301(13)	0.4305(15)	0.4280(14)	0.4276(14)	0.4309(15)	0.4279(13)
0.1	0.75	0.3944(14)	0.3913(12)	0.3896(13)	0.3892(13)	0.3871(12)	0.3885(14)	0.3896(12)
0.2	0.79	0.3397(10)	0.3398(10)	0.3383(10)	0.3382(10)	0.3384(10)	0.3376(10)	0.3380(12)
0.3	0.85	0.3031(09)	0.3030(09)	0.3034(10)	0.3045(10)	0.3045(09)	0.3035(10)	0.3031(08)
0.4	0.91	0.2795(07)	0.2802(08)	0.2813(08)	0.2809(08)	0.2809(08)	0.2809(08)	0.2810(08)
0.5	1	0.2630(07)	0.2633(07)	0.2648(08)	0.2651(08)	0.2647(08)	0.2649(06)	0.2652(07)
0.6	1.12	0.2510(07)	0.2521(07)	0.2529(07)	0.2528(07)	0.2531(08)	0.2530(07)	0.2540(07)
0.7	1.29	0.2424(06)	0.2429(06)	0.2437(07)	0.2448(07)	0.2442(07)	0.2448(08)	0.2442(07)
0.8	1.58	0.2355(07)	0.2358(06)	0.2372(07)	0.2373(06)	0.2374(06)	0.2371(07)	0.2373(07)
0.8605	1.89	0.2319(07)	0.2326(06)	0.2334(07)	0.2334(06)	0.2330(06)	0.2330(07)	0.2334(06)
0.9	2.24	0.2290(06)	0.2302(06)	0.2309(06)	0.2311(07)	0.2309(07)	0.2313(06)	0.2315(06)

Table A.171: Case IIIb Lognormal Cramér-von Mises 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	0.7146(34)	0.7186(34)	0.7217(35)	0.7213(33)	0.7256(39)	0.7244(36)	0.7209(31)
0.0323	0.72	0.6993(34)	0.7018(34)	0.7010(32)	0.7004(36)	0.6987(30)	0.7043(30)	0.7028(34)
0.1	0.75	0.6394(32)	0.6361(30)	0.6324(32)	0.6326(34)	0.6301(36)	0.6306(32)	0.6317(32)
0.2	0.79	0.5419(23)	0.5411(24)	0.5414(25)	0.5415(27)	0.5448(25)	0.5446(25)	0.5436(29)
0.3	0.85	0.4768(23)	0.4797(22)	0.4802(21)	0.4823(22)	0.4823(26)	0.4807(21)	0.4818(23)
0.4	0.91	0.4351(19)	0.4376(22)	0.4404(22)	0.4412(21)	0.4420(22)	0.4426(20)	0.4421(21)
0.5	1	0.4046(19)	0.4075(16)	0.4112(18)	0.4129(18)	0.4127(20)	0.4136(17)	0.4135(19)
0.6	1.12	0.3861(15)	0.3883(18)	0.3912(18)	0.3921(18)	0.3922(16)	0.3933(18)	0.3938(19)
0.7	1.29	0.3695(17)	0.3722(17)	0.3741(18)	0.3768(18)	0.3772(18)	0.3786(18)	0.3766(16)
0.8	1.58	0.3577(15)	0.3608(15)	0.3625(14)	0.3639(17)	0.3652(16)	0.3652(16)	0.3648(15)
0.8605	1.89	0.3517(16)	0.3542(15)	0.3571(17)	0.3571(14)	0.3570(14)	0.3564(16)	0.3586(14)
0.9	2.24	0.3477(14)	0.3502(17)	0.3509(13)	0.3526(14)	0.3523(15)	0.3550(15)	0.3530(17)

Table A.172: Case IIIb Lognormal Cramér-von Mises 99% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.4434(32)	1.4353(36)	1.4332(31)	1.4301(39)	1.4320(36)	1.4270(33)	1.4294(37)
0.0323	0.72	1.4106(32)	1.4026(37)	1.3986(35)	1.3928(31)	1.3910(35)	1.3964(34)	1.3910(32)
0.1	0.75	1.3160(31)	1.3066(31)	1.3017(31)	1.2979(31)	1.2955(31)	1.2926(31)	1.2957(33)
0.2	0.79	1.1848(24)	1.1793(26)	1.1782(25)	1.1776(24)	1.1763(23)	1.1761(27)	1.1753(26)
0.3	0.85	1.0959(21)	1.0970(24)	1.0976(21)	1.0986(25)	1.0994(24)	1.0995(22)	1.0999(24)
0.4	0.91	1.0403(22)	1.0428(21)	1.0467(20)	1.0473(21)	1.0470(20)	1.0484(19)	1.0486(22)
0.5	1	1.0037(20)	1.0066(20)	1.0102(21)	1.0115(21)	1.0110(22)	1.0113(21)	1.0139(18)
0.6	1.12	0.9764(20)	0.9812(22)	0.9861(21)	0.9847(19)	0.9860(20)	0.9863(19)	0.9892(19)
0.7	1.29	0.9557(20)	0.9600(20)	0.9643(17)	0.9659(21)	0.9642(17)	0.9670(16)	0.9667(19)
0.8	1.58	0.9399(20)	0.9451(16)	0.9495(18)	0.9504(18)	0.9502(16)	0.9514(19)	0.9519(16)
0.8605	1.89	0.9330(20)	0.9370(18)	0.9396(16)	0.9414(18)	0.9401(19)	0.9418(18)	0.9422(18)
0.9	2.24	0.9267(17)	0.9307(16)	0.9355(16)	0.9353(20)	0.9356(18)	0.9365(18)	0.9368(18)

Table A.173: Case IIIb Lognormal Anderson-Darling 85% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	1.7534(41)	1.7468(48)	1.7441(44)	1.7432(49)	1.7443(44)	1.7386(43)	1.7440(52)
0.0323	0.72	1.7151(49)	1.7072(47)	1.7030(50)	1.6960(44)	1.6934(47)	1.7033(43)	1.6952(49)
0.1	0.75	1.5911(40)	1.5775(43)	1.5699(39)	1.5641(39)	1.5613(43)	1.5607(43)	1.5676(41)
0.2	0.79	1.4124(32)	1.4081(33)	1.4059(34)	1.4052(34)	1.4045(30)	1.4036(33)	1.4033(34)
0.3	0.85	1.2943(30)	1.2969(30)	1.2984(29)	1.3010(33)	1.3008(31)	1.3004(31)	1.3016(31)
0.4	0.91	1.2231(25)	1.2268(27)	1.2314(28)	1.2309(27)	1.2328(27)	1.2330(25)	1.2325(28)
0.5	1	1.1742(27)	1.1768(26)	1.1837(27)	1.1847(26)	1.1837(29)	1.1854(24)	1.1877(26)
0.6	1.12	1.1388(26)	1.1444(25)	1.1508(27)	1.1499(25)	1.1521(27)	1.1511(24)	1.1541(29)
0.7	1.29	1.1130(24)	1.1183(28)	1.1229(22)	1.1262(27)	1.1243(25)	1.1270(23)	1.1263(24)
0.8	1.58	1.0927(25)	1.0982(18)	1.1054(24)	1.1055(23)	1.1067(21)	1.1063(26)	1.1066(22)
0.8605	1.89	1.0844(24)	1.0877(24)	1.0928(19)	1.0943(22)	1.0930(26)	1.0945(22)	1.0958(22)
0.9	2.24	1.0765(23)	1.0819(24)	1.0873(22)	1.0883(26)	1.0875(23)	1.0884(23)	1.0896(24)

Table A.174: Case IIIb Lognormal Anderson-Darling 90% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	2.3026(64)	2.3025(71)	2.3016(72)	2.3085(80)	2.3092(70)	2.3016(69)	2.3083(64)
0.0323	0.72	2.2640(62)	2.2523(67)	2.2502(72)	2.2379(64)	2.2364(68)	2.2549(73)	2.2390(68)
0.1	0.75	2.0913(72)	2.0662(58)	2.0592(65)	2.0523(61)	2.0437(57)	2.0483(67)	2.0527(58)
0.2	0.79	1.8223(48)	1.8218(54)	1.8165(49)	1.8137(52)	1.8170(49)	1.8103(49)	1.8127(59)
0.3	0.85	1.6524(50)	1.6552(41)	1.6571(50)	1.6614(50)	1.6638(45)	1.6616(47)	1.6593(42)
0.4	0.91	1.5472(37)	1.5549(43)	1.5610(40)	1.5624(42)	1.5638(40)	1.5637(42)	1.5634(42)
0.5	1	1.4780(37)	1.4821(36)	1.4940(41)	1.4967(39)	1.4943(46)	1.4943(37)	1.4963(36)
0.6	1.12	1.4290(41)	1.4370(38)	1.4459(38)	1.4450(40)	1.4480(43)	1.4481(38)	1.4496(43)
0.7	1.29	1.3886(33)	1.3991(36)	1.4045(37)	1.4105(40)	1.4094(38)	1.4125(38)	1.4111(35)
0.8	1.58	1.3647(39)	1.3706(34)	1.3792(36)	1.3818(38)	1.3830(30)	1.3826(35)	1.3819(35)
0.8605	1.89	1.3517(35)	1.3570(35)	1.3635(36)	1.3653(35)	1.3653(37)	1.3631(36)	1.3672(33)
0.9	2.24	1.3405(31)	1.3483(38)	1.3532(31)	1.3571(38)	1.3565(39)	1.3583(32)	1.3610(37)

Table A.175: Case IIIb Lognormal Anderson-Darling 95% critical values

p	$\sqrt{\eta}$	30	50	100	200	500	1000	10000
0	0.71	3.6174(164)	3.6423(163)	3.6820(158)	3.6797(161)	3.7005(194)	3.6953(164)	3.6862(157)
0.0323	0.72	3.5943(171)	3.5993(144)	3.5862(169)	3.5845(174)	3.5709(144)	3.6022(152)	3.5894(155)
0.1	0.75	3.3391(164)	3.2967(162)	3.2683(158)	3.2515(151)	3.2355(164)	3.2379(153)	3.2421(150)
0.2	0.79	2.8616(120)	2.8362(123)	2.8379(124)	2.8302(137)	2.8388(128)	2.8389(114)	2.8383(138)
0.3	0.85	2.5432(124)	2.5551(110)	2.5489(117)	2.5626(111)	2.5541(123)	2.5476(111)	2.5562(114)
0.4	0.91	2.3569(106)	2.3628(113)	2.3805(103)	2.3871(101)	2.3842(108)	2.3796(105)	2.3827(105)
0.5	1	2.2273(99)	2.2379(82)	2.2535(104)	2.2544(98)	2.2581(97)	2.2617(93)	2.2643(97)
0.6	1.12	2.1479(89)	2.1600(95)	2.1776(98)	2.1744(100)	2.1716(88)	2.1780(102)	2.1833(99)
0.7	1.29	2.0800(86)	2.0916(86)	2.1005(97)	2.1118(84)	2.1111(98)	2.1160(91)	2.1117(95)
0.8	1.58	2.0328(88)	2.0498(90)	2.0554(87)	2.0578(88)	2.0640(94)	2.0695(88)	2.0583(76)
0.8605	1.89	2.0097(93)	2.0221(80)	2.0310(97)	2.0329(90)	2.0340(80)	2.0309(85)	2.0408(77)
0.9	2.24	1.9951(83)	2.0086(97)	2.0103(71)	2.0182(75)	2.0131(76)	2.0250(84)	2.0143(87)

Table A.176: Case IIIb Lognormal Anderson-Darling 99% critical values

Appendix B

Comparison of the Critical Values with Literature

Several of the comparison papers give formulas for the critical values as a function of n. Where this is the case, the values are preceded by an asterisk (*). Additionally, some papers provide C code from which the critical values can be quickly computed; values that were determined via this method are preceded with a dagger (†).

B.1 Case I: All Parameters are Known

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Smirnov '48 [54]	All	30	1.14	1.23	1.36	1.63
	Weibull	"	**1.1066(10)**	**1.1914(14)**	**1.3237(18)**	**1.5849(26)**
Massey '51 [55]	All	30	1.10	1.20	1.31	1.59
	Weibull	"	**1.1066(10)**	**1.1914(14)**	**1.3237(18)**	**1.5849(26)**
Birnbaum '52 [98]	All	30	-	-	1.3238	1.5873
	Weibull	"			**1.3237(18)**	**1.5849(26)**
"	"	50	-	-	1.3322	1.5981
	Weibull	"			**1.3332(14)**	**1.5995(30)**
"	"	100	-	-	1.3400	-
	Weibull	"			**1.3405(16)**	
"	"	∞	-	-	1.3581	1.6276
	Weibull	10000			**1.3564(16)**	**1.6235(30)**
Miller '56 [56]	All	30	-	1.19163	1.3238	1.58768
	Weibull	"		**1.1914(14)**	**1.3237(18)**	**1.5849(26)**
"	"	50	-	1.19918	1.33226	1.59834
	Weibull	"		**1.2002(12)**	**1.3332(14)**	**1.5995(30)**
"	"	100	-	1.2067	1.3403	1.6081
	Weibull	"		**1.2065(14)**	**1.3405(16)**	**1.6087(30)**
"	"	∞	1.1380	1.2239	1.3581	1.6276
	Weibull	10000	**1.1360(12)**	**1.2217(14)**	**1.3564(16)**	**1.6235(30)**
Stephens '74 [12]	All	30	*1.110	*1.193	*1.324	*1.587
	Weibull	"	**1.1066(10)**	**1.1914(14)**	**1.3237(18)**	**1.5849(26)**
"	"	50	*1.117	*1.201	*1.332	*1.597
	Weibull	"	**1.1146(10)**	**1.2002(12)**	**1.3332(14)**	**1.5995(30)**
"	"	100	*1.123	*1.208	*1.340	*1.607
	Weibull	"	**1.1212(12)**	**1.2065(14)**	**1.3405(16)**	**1.6087(30)**
"	"	200	*1.128	*1.213	*1.346	*1.613
	Weibull	"	**1.1256(10)**	**1.2115(12)**	**1.3446(16)**	**1.6137(28)**
"	"	500	*1.132	*1.217	*1.350	*1.619
	Weibull	"	**1.1307(10)**	**1.2162(12)**	**1.3500(16)**	**1.6199(30)**
"	"	1000	*1.134	*1.219	*1.353	*1.622
	Weibull	"	**1.1328(10)**	**1.2182(12)**	**1.3520(14)**	**1.6222(28)**
"	"	10000	*1.137	*1.223	*1.356	*1.626
	Weibull	"	**1.1360(12)**	**1.2217(14)**	**1.3564(16)**	**1.6235(30)**
"	"	∞	*1.138	*1.224	*1.358	*1.628
	Weibull	10000	**1.1360(12)**	**1.2217(14)**	**1.3564(16)**	**1.6235(30)**
Tsang et al. '03 [57]	All	30	†1.1068	†1.1916	†1.3239	†1.5877
	Weibull	"	**1.1066(10)**	**1.1914(14)**	**1.3237(18)**	**1.5849(26)**
"	"	50	†1.1139	†1.1992	†1.3322	†1.5983
	Weibull	"	**1.1146(10)**	**1.2002(12)**	**1.3332(14)**	**1.5995(30)**
"	"	100	†1.1210	†1.2066	†1.3403	†1.6081
	Weibull	"	**1.1212(12)**	**1.2065(14)**	**1.3405(16)**	**1.6087(30)**
"	"	200	†1.1260	†1.2117	†1.3457	†1.6144
	Weibull	"	**1.1256(10)**	**1.2115(12)**	**1.3446(16)**	**1.6137(28)**
"	"	500	†1.1304	†1.2163	†1.3504	†1.6196
	Weibull	"	**1.1307(10)**	**1.2162(12)**	**1.3500(16)**	**1.6199(30)**
"	"	1000	†1.1327	†1.2185	†1.3527	†1.6221
	Weibull	"	**1.1328(10)**	**1.2182(12)**	**1.3520(14)**	**1.6222(28)**
"	"	10000	†1.1363	†1.2222	†1.3564	†1.6259
	Weibull	"	**1.1360(12)**	**1.2217(14)**	**1.3564(16)**	**1.6235(30)**

Table B.1: Case I Kolmogorov-Smirnov critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Kuiper '60 [47]	All	30	-	1.5503	1.6760	1.9153
	Weibull	"		**1.5561(12)**	**1.6803(16)**	**1.9224(28)**
"	"	100	-	1.5839	1.7110	1.9637
	Weibull	"		**1.5855(12)**	**1.7119(16)**	**1.9640(26)**
"	"	∞	-	1.6196	1.7473	2.0010
	Weibull	10000	**1.5334(10)**	**1.6161(12)**	**1.7422(14)**	**1.9973(26)**
Stephens '65 [108]	All	30	1.476	1.562	1.684	1.930
	Weibull	"	**1.4752(10)**	**1.5561(12)**	**1.6803(16)**	**1.9224(28)**
"	"	50	1.490	1.576	1.701	1.949
	Weibull	"	**1.4896(10)**	**1.5711(12)**	**1.6968(14)**	**1.9447(28)**
"	"	100	1.505	1.590	1.716	1.967
	Weibull	"	**1.5033(10)**	**1.5855(12)**	**1.7119(16)**	**1.9640(26)**
"	"	∞	1.537	1.620	1.747	2.0010
	Weibull	10000	**1.5334(10)**	**1.6161(12)**	**1.7422(14)**	**1.9973(26)**
Stephens '74 [12]	All	30	*1.483	*1.563	*1.686	*1.931
	Weibull	"	**1.4752(10)**	**1.5561(12)**	**1.6803(16)**	**1.9224(28)**
"	All	50	*1.497	*1.578	*1.702	*1.949
	Weibull	"	**1.4896(10)**	**1.5711(12)**	**1.6968(14)**	**1.9447(28)**
"	All	100	*1.510	*1.592	*1.716	*1.966
	Weibull	"	**1.5033(10)**	**1.5855(12)**	**1.7119(16)**	**1.9640(26)**
"	All	200	*1.519	*1.601	*1.726	*1.977
	Weibull	"	**1.5137(10)**	**1.5966(12)**	**1.7239(16)**	**1.9776(26)**
"	All	500	*1.526	*1.608	*1.734	*1.986
	Weibull	"	**1.5215(10)**	**1.6037(12)**	**1.7313(16)**	**1.9856(32)**
"	All	1000	*1.529	*1.612	*1.738	*1.991
	Weibull	"	**1.5263(10)**	**1.6089(10)**	**1.7361(14)**	**1.9896(34)**
"	All	10000	*1.535	*1.617	*1.744	*1.998
	Weibull	"	**1.5334(10)**	**1.6161(12)**	**1.7422(14)**	**1.9973(26)**
"	All	∞	*1.537	*1.620	*1.747	*2.001
	Weibull	10000	**1.5334(10)**	**1.6161(12)**	**1.7422(14)**	**1.9973(26)**

Table B.2: Case I Kuiper critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Anderson '52 [60]	All	∞	0.28406	0.34730	0.46136	0.74346
	Weibull	1000	**0.2842(08)**	**0.3478(10)**	**0.4610(16)**	**0.7423(32)**
Darling '69 [109]	All	50	-	-	0.45996	0.73784
	Weibull	"			**0.4612(14)**	**0.7393(30)**
"	"	200	-	-	0.46101	0.74205
	Weibull	"			**0.4601(14)**	**0.7426(38)**
"	"	1000	-	-	0.46129	0.74318
	Weibull	"			**0.4612(14)**	**0.7438(36)**
"	"	∞	-	-	0.46136	0.74346
	Weibull	10000	**0.2842(08)**	**0.3478(10)**	**0.4610(16)**	**0.7423(32)**
Stephens '74 [12]	All	30	*0.288	*0.348	*0.459	*0.732
	Weibull	"	**0.2840(06)**	**0.3465(10)**	**0.4589(14)**	**0.7322(28)**
"	All	50	*0.286	*0.348	*0.460	*0.736
	Weibull	"	**0.2844(08)**	**0.3476(10)**	**0.4612(14)**	**0.7393(30)**
"	All	100	*0.285	*0.348	*0.460	*0.740
	Weibull	"	**0.2840(08)**	**0.3475(10)**	**0.4610(14)**	**0.7408(38)**
"	All	200	*0.285	*0.347	*0.461	*0.741
	Weibull	"	**0.2841(08)**	**0.3466(10)**	**0.4601(14)**	**0.7426(38)**
"	All	500	*0.284	*0.347	*0.461	*0.742
	Weibull	"	**0.2841(08)**	**0.3469(08)**	**0.4611(14)**	**0.7429(38)**
"	All	1000	*0.284	*0.347	*0.461	*0.743
	Weibull	"	**0.2836(06)**	**0.3473(10)**	**0.4612(14)**	**0.7438(36)**
"	All	10000	*0.284	*0.347	*0.461	*0.743
	Weibull	"	**0.2842(08)**	**0.3478(10)**	**0.4610(16)**	**0.7423(32)**
"	All	∞	*0.284	*0.347	*0.461	*0.743
	Weibull	10000	**0.2842(08)**	**0.3478(10)**	**0.4610(16)**	**0.7423(32)**
Knott'74 [110]	All	50	0.28398	0.34686	0.45996	0.73784
	Weibull	"	**0.2844(08)**	**0.3476(10)**	**0.4612(14)**	**0.7393(30)**
"	"	200	0.28404	0.34719	0.46101	0.74205
	Weibull	"	**0.2841(08)**	**0.3466(10)**	**0.4601(14)**	**0.7426(38)**
"	"	1000	0.28406	0.34728	0.46129	0.74318
	Weibull	"	**0.2836(06)**	**0.3473(10)**	**0.4612(14)**	**0.7438(36)**
"	"	∞	0.28406	0.34730	0.46136	0.74346
	Weibull	10000	**0.2842(08)**	**0.3478(10)**	**0.4610(16)**	**0.7423(32)**
Csorgo '96 [111]	All	50	0.28396	0.34682	0.45986	0.73728
	Weibull	"	**0.2844(08)**	**0.3476(10)**	**0.4612(14)**	**0.7393(30)**
"	"	200	0.28402	0.34715	0.46091	0.74149
	Weibull	"	**0.2841(08)**	**0.3466(10)**	**0.4601(14)**	**0.7426(38)**
"	"	1000	0.28403	0.34724	0.46119	0.74262
	Weibull	"	**0.2836(06)**	**0.3473(10)**	**0.4612(14)**	**0.7438(36)**
"	"	∞	0.28406	0.34730	0.46136	0.74346
	Weibull	10000	**0.2842(08)**	**0.3478(10)**	**0.4610(16)**	**0.7423(32)**

Table B.3: Case I Cramér-von Mises critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Anderson '54 [45]	All	∞	-	1.933	2.492	3.857
	Weibull	10000		**1.9351(54)**	**2.4913(73)**	**3.872 (17)**
Stephens '74 [12]	$n > 5$	∞	1.610	1.933	2.492	3.857
	Weibull	10000	**1.6227(42)**	**1.9351(54)**	**2.4913(73)**	**3.872 (17)**
Marsaglia '04 [93]	All	30	†1.6224	†1.9361	†2.4992	†3.8920
	Weibull	"	**1.6226(34)**	**1.9352(46)**	**2.4976(75)**	**3.882 (18)**
"	All	50	†1.6219	†1.9348	†2.4964	†3.8865
	Weibull	"	**1.6247(36)**	**1.9380(46)**	**2.5037(65)**	**3.897 (15)**
"	All	100	†1.6216	†1.9339	†2.4944	†3.8823
	Weibull	"	**1.6223(38)**	**1.9364(48)**	**2.4950(73)**	**3.881 (18)**
"	All	200	†1.6214	†1.9334	†2.4934	†3.8802
	Weibull	"	**1.6199(36)**	**1.9327(46)**	**2.4879(69)**	**3.880 (18)**
"	All	500	†1.6213	†1.9331	†2.4928	†3.8790
	Weibull	"	**1.6219(36)**	**1.9303(46)**	**2.4889(63)**	**3.874 (19)**
"	All	1,000	†1.6213	†1.9331	†2.4926	†3.8785
	Weibull	"	**1.6209(36)**	**1.9343(48)**	**2.4935(67)**	**3.887 (18)**
"	All	10,000	†1.6212	†1.9330	†2.4924	†3.8782
	Weibull	"	**1.6227(42)**	**1.9351(54)**	**2.4913(73)**	**3.872 (17)**
"	All	∞	†1.6212	†1.9330	†2.4924	†3.8781
	Weibull	10000	**1.6227(42)**	**1.9351(54)**	**2.4913(73)**	**3.872 (17)**

Table B.4: Case I Anderson-Darling critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

B.2 Case II: Both Parameters are Unknown

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Littell et al. '70 [61]	Weibull	30	0.745	0.789	0.854	0.980
	Weibull	"	**0.7462(06)**	**0.7899(06)**	**0.8581(08)**	**0.9949(14)**
Stephens '74 [12]	Normal	30	*0.755	*0.798	*0.872	*1.008
	Lognormal	"	**0.7572(06)**	**0.8028(06)**	**0.8744(08)**	**1.0163(16)**
"	Normal	50	*0.763	*0.806	*0.881	*1.019
	Lognormal	"	**0.7643(06)**	**0.8102(06)**	**0.8825(08)**	**1.0284(16)**
"	Normal	100	*0.769	*0.813	*0.888	*1.027
	Lognormal	"	**0.7718(06)**	**0.8182(06)**	**0.8903(08)**	**1.0368(18)**
"	Normal	200	*0.772	*0.816	*0.892	*1.031
	Lognormal	"	**0.7768(06)**	**0.8232(06)**	**0.8967(10)**	**1.0438(18)**
"	Normal	500	*0.774	*0.818	*0.894	*1.034
	Lognormal	"	**0.7820(06)**	**0.8287(06)**	**0.9018(10)**	**1.0484(16)**
"	Normal	1000	*0.775	*0.819	*0.895	*1.034
	Lognormal	"	**0.7842(06)**	**0.8308(06)**	**0.9037(10)**	**1.0522(16)**
"	Normal	10000	*0.775	*0.819	*0.895	*1.035
	Lognormal	"	**0.7875(06)**	**0.8343(06)**	**0.9071(08)**	**1.0559(16)**
"	Normal	∞	*0.775	*0.819	*0.895	*1.035
	Lognormal	10000	**0.7875(06)**	**0.8343(06)**	**0.9071(08)**	**1.0559(16)**
Stephens '79 [8]	Logistic	50	-	0.708	0.770	0.873
	Loglogistic	"		**0.7199(06)**	**0.7774(08)**	**0.8918(14)**
"	Logistic	∞	-	0.715	0.780	0.886
	Loglogistic	10000		**0.7450(06)**	**0.8039(06)**	**0.9227(14)**
Chandra et al. '81 [9]	Extreme value	50	-	0.790	0.856	0.988
	Weibull	"		**0.7983(06)**	**0.8671(08)**	**1.0070(16)**
"	Extreme value	∞	-	0.803	0.874	1.007
	Weibull	10000		**0.8231(08)**	**0.8937(10)**	**1.0363(16)**
Parsons et al. '82 [7]	Weibull	30	0.745	0.789	0.854	0.991
	Weibull	"	**0.7462(06)**	**0.7899(06)**	**0.8581(08)**	**0.9949(14)**
"	Weibull	$n > 30$	0.749	0.794	0.865	1.003
	Weibull	10000	**0.7779(06)**	**0.8231(08)**	**0.8937(10)**	**1.0363(16)**
D'Agostino et al. '86 [112]	Extreme value	50	-	0.790	0.856	0.988
	Weibull	"		**0.7983(06)**	**0.8671(08)**	**1.0070(16)**
"	Extreme value	∞	-	0.803	0.874	1.007
	Weibull	10000		**0.8231(08)**	**0.8937(10)**	**1.0363(16)**
Evans et al. '89 [6]	Weibull	30	0.7460	0.7904	0.8599	0.9936
	Weibull	"	**0.7462(06)**	**0.7899(06)**	**0.8581(08)**	**0.9949(14)**
"	Weibull	50	0.7559	0.7997	0.8697	1.0097
	Weibull	"	**0.7541(06)**	**0.7983(06)**	**0.8671(08)**	**1.0070(16)**
"	Weibull	100	0.7570	0.8010	0.8740	1.0170
	Weibull	"	**0.7617(06)**	**0.8065(06)**	**0.8761(08)**	**1.0166(16)**
"	Weibull	200	0.7665	0.8118	0.8796	1.0225
	Weibull	"	**0.7678(04)**	**0.8127(06)**	**0.8825(08)**	**1.0252(16)**
"	Weibull	500	-	*0.8176	*0.8883	*1.0329
	Weibull	"		**0.8168(06)**	**0.8875(08)**	**1.0292(16)**
"	Weibull	1000	-	*0.8202	*0.8912	*1.0366
	Weibull	"		**0.8191(06)**	**0.8902(08)**	**1.0324(16)**
"	Weibull	10000	-	*0.8245	*0.8960	*1.0427
	Weibull	"		**0.8231(08)**	**0.8937(10)**	**1.0363(16)**
"	Weibull	∞	-	*0.8265	*0.8982	*1.0455
	Weibull	10000		**0.8231(08)**	**0.8937(10)**	**1.0363(16)**

Table B.5: Case II Kolmogorov-Smirnov critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Normal	30	*1.274	*1.337	*1.437	*1.633
	Lognormal	"	**1.2737(08)**	**1.3379(08)**	**1.4367(12)**	**1.6320(24)**
"	Normal	50	*1.290	*1.354	*1.455	*1.654
	Lognormal	"	**1.2869(08)**	**1.3521(10)**	**1.4539(10)**	**1.6551(22)**
"	Normal	100	*1.303	*1.368	*1.470	*1.671
	Lognormal	"	**1.3016(08)**	**1.3675(08)**	**1.4698(12)**	**1.6704(24)**
"	Normal	200	*1.310	*1.375	*1.478	*1.680
	Lognormal	"	**1.3119(08)**	**1.3780(10)**	**1.4802(12)**	**1.6828(26)**
"	Normal	500	*1.315	*1.381	*1.483	*1.686
	Lognormal	"	**1.3214(08)**	**1.3879(10)**	**1.4902(12)**	**1.6925(22)**
"	Normal	1000	*1.317	*1.383	*1.485	*1.689
	Lognormal	"	**1.3251(08)**	**1.3913(08)**	**1.4938(10)**	**1.6978(24)**
"	Normal	10000	*1.319	*1.385	*1.488	*1.692
	Lognormal	"	**1.3330(08)**	**1.3989(10)**	**1.5010(14)**	**1.7056(24)**
"	Normal	∞	*1.320	*1.386	*1.489	*1.693
	Lognormal	10000	**1.3330(08)**	**1.3989(10)**	**1.5010(14)**	**1.7056(24)**
Stephens '79 [8]	Logistic	50	-	1.277	1.364	1.542
	Loglogistic	"		**1.2873(08)**	**1.3810(12)**	**1.5660(22)**
"	Logistic	∞	-	1.289	1.376	1.560
	Loglogistic	10000		**1.3335(10)**	**1.4290(12)**	**1.6208(24)**
Chandra et al. '81 [9]	Extreme value	50	-	1.344	1.453	1.639
	Weibull	10000		**1.3607(10)**	**1.4634(12)**	**1.6691(24)**
"	Extreme value	∞	-	1.372	1.477	1.671
	Weibull	10000		**1.4081(10)**	**1.5139(12)**	**1.7238(24)**

Table B.6: Case II Kuiper critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Littell et al. '70 [61]	Weibull	30	0.088	0.101	0.123	0.172
	Weibull	"	**0.08874(14)**	**0.10119(18)**	**0.12253(26)**	**0.17185(56)**
Stephens '74 [12]	Normal	30	*0.090	*0.102	*0.124	*0.175
	Lognormal	"	**0.09055(16)**	**0.10348(18)**	**0.12535(28)**	**0.17670(60)**
"	Normal	50	*0.090	*0.103	*0.125	*0.176
	Lognormal	"	**0.09024(14)**	**0.10333(18)**	**0.12571(28)**	**0.17784(65)**
"	Normal	100	*0.091	*0.103	*0.125	*0.177
	Lognormal	"	**0.09039(16)**	**0.10339(18)**	**0.12568(26)**	**0.17808(67)**
"	Normal	200	*0.091	*0.104	*0.126	*0.178
	Lognormal	"	**0.09044(14)**	**0.10351(18)**	**0.12580(28)**	**0.17853(65)**
"	Normal	500	*0.091	*0.104	*0.126	*0.178
	Lognormal	"	**0.09052(16)**	**0.10352(22)**	**0.12596(30)**	**0.17797(62)**
"	Normal	1000	*0.091	*0.104	*0.126	*0.178
	Lognormal	"	**0.09049(16)**	**0.10369(20)**	**0.12601(30)**	**0.17804(71)**
"	Normal	10000	*0.091	*0.104	*0.126	*0.178
	Lognormal	"	**0.09047(16)**	**0.10351(18)**	**0.12593(30)**	**0.17796(62)**
"	Normal	∞	*0.091	*0.104	*0.126	*0.178
	Lognormal	10000	**0.09047(16)**	**0.10351(18)**	**0.12593(30)**	**0.17796(62)**
Stephens '77 [13]	Extreme value	30	-	*0.098	*0.120	*0.169
	Weibull	"		**0.10119(18)**	**0.12253(26)**	**0.17185(56)**
"	Extreme value	50	-	*0.099	*0.121	*0.170
	Weibull	"		**0.10150(20)**	**0.12285(26)**	**0.17360(63)**
"	Extreme value	100	-	*0.100	*0.122	*0.172
	Weibull	"		**0.10166(16)**	**0.12326(26)**	**0.17375(60)**
"	Extreme value	200	-	*0.101	*0.122	*0.173
	Weibull	"		**0.10177(18)**	**0.12354(30)**	**0.17461(63)**
"	Extreme value	500	-	*0.101	*0.123	*0.173
	Weibull	"		**0.10187(18)**	**0.12372(26)**	**0.17455(58)**
"	Extreme value	1000	-	*0.101	*0.123	*0.174
	Weibull	"		**0.10182(18)**	**0.12379(28)**	**0.17478(62)**
"	Extreme value	10000	-	*0.102	*0.124	*0.174
	Weibull	"		**0.10183(20)**	**0.12368(28)**	**0.17525(69)**
"	Extreme value	∞	-	*0.102	*0.124	*0.175
	Weibull	10000		**0.10183(20)**	**0.12368(28)**	**0.17525(69)**
Stephens '79 [8]	Logistic	$n \geq 50$	-	*0.081	*0.098	*0.136
	Loglogistic	10000		**0.08141(12)**	**0.09764(20)**	**0.13537(50)**

Table B.7: Case II Cramér-von Mises critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Littell et al. '70 [61]	Weibull	30	0.557	0.626	0.741	1.007
	Weibull	"	**0.5583(08)**	**0.6282(10)**	**0.7466(16)**	**1.0196(30)**
Stephens '74 [12]	Normal	30	*0.521	*0.593	*0.712	*0.988
	Lognormal	"	**0.5577(08)**	**0.6270(10)**	**0.7438(16)**	**1.0179(34)**
"	Normal	50	*0.538	*0.613	*0.736	*1.021
	Lognormal	"	**0.5575(08)**	**0.6277(10)**	**0.7472(14)**	**1.0269(36)**
"	Normal	100	*0.555	*0.632	*0.759	*1.053
	Lognormal	"	**0.5587(08)**	**0.6291(10)**	**0.7483(16)**	**1.0301(36)**
"	Normal	200	*0.565	*0.644	*0.772	*1.071
	Lognormal	"	**0.5595(08)**	**0.6300(08)**	**0.7500(16)**	**1.0321(34)**
"	Normal	500	*0.571	*0.651	*0.781	*1.083
	Lognormal	"	**0.5599(10)**	**0.6303(12)**	**0.7507(16)**	**1.0303(34)**
"	Normal	1000	*0.574	*0.653	*0.784	*1.088
	Lognormal	"	**0.5600(08)**	**0.6312(10)**	**0.7512(14)**	**1.0316(32)**
"	Normal	10000	*0.576	*0.656	*0.787	*1.092
	Lognormal	"	**0.5598(08)**	**0.6305(10)**	**0.7516(16)**	**1.0312(36)**
"	Normal	∞	*0.576	*0.656	*0.787	*1.092
	Lognormal	10000	**0.5598(08)**	**0.6305(10)**	**0.7516(16)**	**1.0312(36)**
Stephens '77 [13]	Extreme value	30	-	*0.615	*0.730	*1.001
	Weibull	"		**0.6282(10)**	**0.7466(16)**	**1.0196(30)**
"	Extreme value	50	-	*0.619	*0.736	*1.009
	Weibull	"		**0.6303(10)**	**0.7491(14)**	**1.0320(36)**
"	Extreme value	100	-	*0.625	*0.742	*1.018
	Weibull	"		**0.6324(08)**	**0.7528(12)**	**1.0359(34)**
"	Extreme value	200	-	*0.628	*0.746	*1.024
	Weibull	"		**0.6334(12)**	**0.7547(18)**	**1.0395(36)**
"	Extreme value	500	-	*0.631	*0.750	*1.029
	Weibull	"		**0.6343(10)**	**0.7564(16)**	**1.0402(34)**
"	Extreme value	1000	-	*0.633	*0.752	*1.031
	Weibull	"		**0.6347(10)**	**0.7566(16)**	**1.0417(34)**
"	Extreme value	10000	-	*0.636	*0.755	*1.036
	Weibull	"		**0.6345(12)**	**0.7571(16)**	**1.0420(40)**
"	Extreme value	∞	-	*0.637	*0.757	*1.038
	Weibull	10000		**0.6345(12)**	**0.7571(16)**	**1.0420(40)**
Stephens '79 [8]	Logistic	30	-	*0.547	*0.641	*0.878
	Loglogistic	"		**0.5595(08)**	**0.6609(12)**	**0.8933(30)**
"	Logistic	50	-	*0.553	*0.648	*0.889
	Loglogistic	"		**0.5610(08)**	**0.6631(12)**	**0.9003(28)**
"	Logistic	100	-	*0.558	*0.654	*0.898
	Loglogistic	"		**0.5617(08)**	**0.6637(14)**	**0.9012(30)**
"	Logistic	200	-	*0.561	*0.657	*0.902
	Loglogistic	"		**0.5623(08)**	**0.6649(12)**	**0.9012(30)**
"	Logistic	500	-	*0.562	*0.659	*0.904
	Loglogistic	"		**0.5624(10)**	**0.6650(14)**	**0.9037(30)**
"	Logistic	1000	-	*0.563	*0.659	*0.905
	Loglogistic	"		**0.5621(10)**	**0.6651(14)**	**0.9037(28)**
"	Logistic	10000	-	*0.563	*0.660	*0.906
	Loglogistic	"		**0.5632(10)**	**0.6654(12)**	**0.9040(32)**
"	Logistic	∞	-	*0.563	*0.660	*0.906
	Loglogistic	10000		**0.5632(10)**	**0.6654(12)**	**0.9040(32)**

Table B.8: Case II Anderson-Darling critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Evans et al. '89 [6]	Weibull	30	0.5602	0.6302	0.7489	1.0235
	Weibull	"	**0.5583(08)**	**0.6282(10)**	**0.7466(16)**	**1.0196(30)**
"	Weibull	50	0.5618	0.6336	0.7559	1.0405
	Weibull	"	**0.5599(08)**	**0.6303(10)**	**0.7491(14)**	**1.0320(36)**
"	Weibull	100	0.5573	0.6269	0.7467	1.0309
	Weibull	"	**0.5615(08)**	**0.6324(08)**	**0.7528(12)**	**1.0359(34)**
"	Weibull	200	0.5587	0.6309	0.7550	1.0411
	Weibull	"	**0.5623(10)**	**0.6334(12)**	**0.7547(18)**	**1.0395(36)**

Table B.9: Cont. Case II Anderson-Darling critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

B.3 Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Lilliefors '69 [113]	Exponential	30	0.898	0.953	1.052	1.238
	Weibull	"	**0.9003(08)**	**0.9615(08)**	**1.0581(12)**	**1.2517(22)**
	Pareto	"	**0.9002(08)**	**0.9614(08)**	**1.0575(10)**	**1.2504(22)**
"	Exponential	∞	0.91	0.96	1.06	1.25
	Weibull	10000	**0.9307(08)**	**0.9934(08)**	**1.0921(12)**	**1.2934(22)**
	Pareto	10000	**0.9311(08)**	**0.9936(10)**	**1.0920(12)**	**1.2914(22)**
Stephens '74 [12]	Exponential	30	*0.907	*0.967	*1.065	*1.266
	Weibull	"	**0.9003(08)**	**0.9615(08)**	**1.0581(12)**	**1.2517(22)**
	Pareto	"	**0.9002(08)**	**0.9614(08)**	**1.0575(10)**	**1.2504(22)**
"	Exponential	50	*0.913	*0.974	*1.073	*1.278
	Weibull	"	**0.9075(08)**	**0.9694(08)**	**1.0668(12)**	**1.2638(22)**
	Pareto	"	**0.9075(08)**	**0.9693(10)**	**1.0668(12)**	**1.2625(20)**
"	Exponential	100	*0.918	*0.980	*1.081	*1.289
	Weibull	"	**0.9147(08)**	**0.9773(08)**	**1.0751(12)**	**1.2741(22)**
	Pareto	"	**0.9151(08)**	**0.9777(08)**	**1.0757(10)**	**1.2733(20)**
"	Exponential	200	*0.921	*0.984	*1.086	*1.295
	Weibull	"	**0.9198(08)**	**0.9824(10)**	**1.0807(12)**	**1.2797(24)**
	Pareto	"	**0.9202(08)**	**0.9827(08)**	**1.0819(12)**	**1.2800(20)**
"	Exponential	500	*0.923	*0.987	*1.089	*1.301
	Weibull	"	**0.9245(08)**	**0.9871(08)**	**1.0855(12)**	**1.2857(22)**
	Pareto	"	**0.9241(08)**	**0.9867(08)**	**1.0859(10)**	**1.2852(20)**
"	Exponential	1000	*0.924	*0.988	*1.091	*1.303
	Weibull	"	**0.9268(08)**	**0.9893(08)**	**1.0877(12)**	**1.2879(22)**
	Pareto	"	**0.9268(08)**	**0.9892(10)**	**1.0872(12)**	**1.2873(24)**
"	Exponential	10000	*0.926	*0.989	*1.093	*1.307
	Weibull	"	**0.9307(08)**	**0.9934(08)**	**1.0921(12)**	**1.2934(22)**
	Pareto	"	**0.9311(08)**	**0.9936(10)**	**1.0920(12)**	**1.2914(22)**
"	Exponential	∞	*0.926	*0.990	*1.094	*1.308
	Weibull	10000	**0.9307(08)**	**0.9934(08)**	**1.0921(12)**	**1.2934(22)**
	Pareto	10000	**0.9311(08)**	**0.9936(10)**	**1.0920(12)**	**1.2914(22)**
Durbin '75 [114]	Exponential	30	-	0.9617	1.0580	1.2519
	Weibull	"		**0.9615(08)**	**1.0581(12)**	**1.2517(22)**
	Pareto	"		**0.9614(08)**	**1.0575(10)**	**1.2504(22)**
"	Exponential	50	-	0.9696	1.0668	1.2634
	Weibull	"		**0.9694(08)**	**1.0668(12)**	**1.2638(22)**
	Pareto	"		**0.9693(10)**	**1.0668(12)**	**1.2625(20)**
"	Exponential	100	-	0.9773	1.0753	1.2743
	Weibull	"		**0.9773(08)**	**1.0751(12)**	**1.2741(22)**
	Pareto	"		**0.9777(08)**	**1.0757(10)**	**1.2733(20)**
Stephens '79 [8]	Logistic	50	-	0.808	0.874	1.011
	Loglogistic	"		**0.8108(06)**	**0.8793(08)**	**1.0161(18)**
"	Logistic	∞	-	0.816	0.883	1.025
	Loglogistic	10000		**0.8362(06)**	**0.9063(08)**	**1.0470(14)**
Chandra et al. '81 [9]	Extreme value	50	-	0.970	1.067	1.263
	Weibull	"		**0.9694(08)**	**1.0668(12)**	**1.2638(22)**
	Pareto	"		**0.9693(10)**	**1.0668(12)**	**1.2625(20)**
"	Extreme value	∞	-	0.995	1.094	1.298
	Weibull	10000		**0.9934(08)**	**1.0921(12)**	**1.2934(22)**
	Pareto	10000		**0.9936(10)**	**1.0920(12)**	**1.2914(22)**

Table B.10: Case IIIa Kolmogorov-Smirnov critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Exponential	30	*1.406	*1.483	*1.605	*1.846
	Weibull	"	**1.4014(10)**	**1.4780(10)**	**1.5970(14)**	**1.8341(28)**
	Pareto	"	**1.4011(08)**	**1.4774(10)**	**1.5959(14)**	**1.8340(28)**
"	Exponential	50	*1.416	*1.495	*1.618	*1.863
	Weibull	"	**1.4149(10)**	**1.4920(12)**	**1.6118(14)**	**1.8523(26)**
	Pareto	"	**1.4148(10)**	**1.4925(10)**	**1.6122(14)**	**1.8522(26)**
"	Exponential	100	*1.426	*1.506	*1.631	*1.879
	Weibull	"	**1.4285(10)**	**1.5062(12)**	**1.6273(14)**	**1.8699(28)**
	Pareto	"	**1.4288(10)**	**1.5064(12)**	**1.6275(14)**	**1.8708(28)**
"	Exponential	200	*1.433	*1.513	*1.639	*1.889
	Weibull	"	**1.4379(10)**	**1.5156(10)**	**1.6366(14)**	**1.8804(28)**
	Pareto	"	**1.4382(10)**	**1.5164(12)**	**1.6380(14)**	**1.8794(26)**
"	Exponential	500	*1.438	*1.519	*1.645	*1.897
	Weibull	"	**1.4471(10)**	**1.5253(12)**	**1.6469(14)**	**1.8909(28)**
	Pareto	"	**1.4470(10)**	**1.5249(10)**	**1.6459(14)**	**1.8927(26)**
"	Exponential	1000	*1.440	*1.521	*1.648	*1.901
	Weibull	"	**1.4512(10)**	**1.5292(12)**	**1.6509(14)**	**1.8955(28)**
	Pareto	"	**1.4510(10)**	**1.5295(12)**	**1.6502(14)**	**1.8942(28)**
"	Exponential	10000	*1.443	*1.525	*1.653	*1.907
	Weibull	"	**1.4591(10)**	**1.5373(12)**	**1.6588(14)**	**1.9040(28)**
	Pareto	"	**1.4593(10)**	**1.5376(12)**	**1.6583(16)**	**1.9023(28)**
"	Exponential	∞	*1.445	*1.527	*1.655	*1.910
	Weibull	10000	**1.4591(10)**	**1.5373(12)**	**1.6588(14)**	**1.9040(28)**
	Pareto	10000	**1.4593(10)**	**1.5376(12)**	**1.6583(16)**	**1.9023(28)**
Stephens '79 [8]	Logistic	50	-	1.447	1.564	1.815
	Loglogistic	"		**1.4636(12)**	**1.5841(14)**	**1.8269(28)**
"	Logistic	∞	-	1.454	1.574	1.832
	Loglogistic	10000		**1.5078(12)**	**1.6305(14)**	**1.8772(28)**
Chandra et al. '81 [9]	Extreme value	50	-	1.48	1.59	1.84
	Weibull	"		**1.4920(12)**	**1.6118(14)**	**1.8523(26)**
	Pareto	"		**1.4925(10)**	**1.6122(14)**	**1.8522(26)**
"	Extreme value	∞	-	1.53	1.65	1.91
	Weibull	10000		**1.5373(12)**	**1.6588(14)**	**1.9040(28)**
	Pareto	10000		**1.5376(12)**	**1.6583(16)**	**1.9023(28)**

Table B.11: Case IIIa Kuiper critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Exponential	30	*0.148	*0.176	*0.223	*0.335
	Weibull	"	**0.14774(30)**	**0.17376(38)**	**0.21997(60)**	**0.3319 (15)**
	Pareto	"	**0.14778(32)**	**0.17384(38)**	**0.22005(58)**	**0.3316 (14)**
"	Exponential	50	*0.149	*0.176	*0.223	*0.336
	Weibull	"	**0.14786(30)**	**0.17407(40)**	**0.22040(60)**	**0.3349 (14)**
	Pareto	"	**0.14792(32)**	**0.17409(44)**	**0.22030(65)**	**0.3331 (13)**
"	Exponential	100	*0.149	*0.177	*0.224	*0.336
	Weibull	"	**0.14781(32)**	**0.17424(40)**	**0.22100(62)**	**0.3361 (14)**
	Pareto	"	**0.14793(28)**	**0.17427(34)**	**0.22097(58)**	**0.3355 (14)**
"	Exponential	200	*0.149	*0.177	*0.224	*0.337
	Weibull	"	**0.14781(32)**	**0.17425(40)**	**0.22105(62)**	**0.3363 (15)**
	Pareto	"	**0.14806(28)**	**0.17447(34)**	**0.22161(60)**	**0.3372 (14)**
"	Exponential	500	*0.149	*0.177	*0.224	*0.337
	Weibull	"	**0.14797(30)**	**0.17442(40)**	**0.22159(60)**	**0.3370 (15)**
	Pareto	"	**0.14793(30)**	**0.17438(40)**	**0.22174(58)**	**0.3367 (15)**
"	Exponential	1000	*0.149	*0.177	*0.224	*0.337
	Weibull	"	**0.14800(30)**	**0.17445(40)**	**0.22140(62)**	**0.3369 (14)**
	Pareto	"	**0.14785(32)**	**0.17437(42)**	**0.22125(60)**	**0.3368 (15)**
"	Exponential	10000	*0.149	*0.177	*0.224	*0.337
	Weibull	"	**0.14810(30)**	**0.17466(40)**	**0.22172(60)**	**0.3377 (15)**
	Pareto	"	**0.14820(28)**	**0.17465(40)**	**0.22154(60)**	**0.3367 (15)**
"	Exponential	∞	*0.149	*0.177	*0.224	*0.337
	Weibull	10000	**0.14810(30)**	**0.17466(40)**	**0.22172(60)**	**0.3377 (15)**
	Pareto	10000	**0.14820(28)**	**0.17465(40)**	**0.22154(60)**	**0.3367 (15)**
Stephens '74 [12]	Normal	∞	-	0.135	0.165	0.237
	Lognormal	10000		**0.13424(26)**	**0.16515(40)**	**0.23726(73)**
Stephens '77 [13]	Extreme value	30	-	*0.174	*0.221	*0.336
	Weibull	"		**0.17376(38)**	**0.21997(60)**	**0.3319 (15)**
	Pareto	"		**0.17384(38)**	**0.22005(58)**	**0.3316 (14)**
"	Extreme value	50	-	*0.174	*0.221	*0.337
	Weibull	"		**0.17407(40)**	**0.22040(60)**	**0.3349 (14)**
	Pareto	"		**0.17409(44)**	**0.22030(65)**	**0.3331 (13)**
"	Extreme value	100	-	*0.175	*0.222	*0.337
	Weibull	"		**0.17424(40)**	**0.22100(62)**	**0.3361 (14)**
	Pareto	"		**0.17427(34)**	**0.22097(58)**	**0.3355 (14)**
"	Extreme value	200	-	*0.175	*0.222	*0.338
	Weibull	"		**0.17425(40)**	**0.22105(62)**	**0.3363 (15)**
	Pareto	"		**0.17447(34)**	**0.22161(60)**	**0.3372 (14)**
"	Extreme value	500	-	*0.175	*0.222	*0.338
	Weibull	"		**0.17442(40)**	**0.22159(60)**	**0.3370 (15)**
	Pareto	"		**0.17438(40)**	**0.22174(58)**	**0.3367 (15)**
"	Extreme value	1000	-	*0.175	*0.222	*0.338
	Weibull	"		**0.17445(40)**	**0.22140(62)**	**0.3369 (14)**
	Pareto	"		**0.17437(42)**	**0.22125(60)**	**0.3368 (15)**
"	Extreme value	10000	-	*0.175	*0.222	*0.338
	Weibull	"		**0.17466(40)**	**0.22172(60)**	**0.3377 (15)**
	Pareto	"		**0.17465(40)**	**0.22154(60)**	**0.3367 (15)**
"	Extreme value	∞	-	*0.175	*0.222	*0.338
	Weibull	10000		**0.17466(40)**	**0.22172(60)**	**0.3377 (15)**
	Pareto	10000		**0.17465(40)**	**0.22154(60)**	**0.3367 (15)**
Stephens '79 [8]	Logistic	$n \geq 100$	-	*0.119	*0.148	*0.218
	Loglogistic	10000		**0.11923(24)**	**0.14797(36)**	**0.21790(89)**

Table B.12: Case IIIa Cramér-von Mises critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Exponential	30	*0.904	*1.057	*1.315	*1.919
	Weibull	"	**0.9047(16)**	**1.0483(22)**	**1.3038(32)**	**1.9370(81)**
	Pareto	"	**0.9054(16)**	**1.0492(22)**	**1.3047(32)**	**1.9411(83)**
"	Exponential	50	*0.911	*1.065	*1.325	*1.934
	Weibull	"	**0.9090(16)**	**1.0541(22)**	**1.3100(32)**	**1.9485(79)**
	Pareto	"	**0.9088(18)**	**1.0536(24)**	**1.3097(38)**	**1.9413(79)**
"	Exponential	100	*0.917	*1.072	*1.333	*1.945
	Weibull	"	**0.9115(16)**	**1.0575(22)**	**1.3160(32)**	**1.9522(77)**
	Pareto	"	**0.9124(16)**	**1.0583(20)**	**1.3145(34)**	**1.9486(69)**
"	Exponential	200	*0.919	*1.075	*1.337	*1.951
	Weibull	"	**0.9127(16)**	**1.0587(22)**	**1.3164(34)**	**1.9522(83)**
	Pareto	"	**0.9138(18)**	**1.0606(22)**	**1.3199(34)**	**1.9570(75)**
"	Exponential	500	*0.921	*1.077	*1.339	*1.955
	Weibull	"	**0.9138(16)**	**1.0604(22)**	**1.3206(32)**	**1.9567(79)**
	Pareto	"	**0.9139(18)**	**1.0610(24)**	**1.3206(36)**	**1.9550(77)**
"	Exponential	1000	*0.921	*1.077	*1.340	*1.956
	Weibull	"	**0.9147(18)**	**1.0610(22)**	**1.3196(32)**	**1.9534(79)**
	Pareto	"	**0.9143(18)**	**1.0603(22)**	**1.3188(34)**	**1.9500(83)**
"	Exponential	10000	*0.922	*1.078	*1.341	*1.957
	Weibull	"	**0.9157(16)**	**1.0620(22)**	**1.3223(34)**	**1.9595(79)**
	Pareto	"	**0.9155(16)**	**1.0623(20)**	**1.3222(36)**	**1.9552(75)**
"	Exponential	∞	*0.922	*1.078	*1.341	*1.957
	Weibull	10000	**0.9157(16)**	**1.0620(22)**	**1.3223(34)**	**1.9595(79)**
	Pareto	10000	**0.9155(16)**	**1.0623(20)**	**1.3222(36)**	**1.9552(75)**
Stephens '74 [12]	Normal	∞	-	0.908	1.105	1.573
	Lognormal	10000		**0.8929(18)**	**1.0855(22)**	**1.5471(56)**
Stephens '77 [13]	Extreme value	30	-	*1.051	*1.308	*1.940
	Weibull	"		**1.0483(22)**	**1.3038(32)**	**1.9370(81)**
	Pareto	"		**1.0492(22)**	**1.3047(32)**	**1.9411(83)**
"	Extreme value	50	-	*1.056	*1.313	*1.947
	Weibull	"		**1.0541(22)**	**1.3100(32)**	**1.9485(79)**
	Pareto	"		**1.0536(24)**	**1.3097(38)**	**1.9413(79)**
"	Extreme value	100	-	*1.059	*1.317	*1.953
	Weibull	"		**1.0575(22)**	**1.3160(32)**	**1.9522(77)**
	Pareto	"		**1.0583(20)**	**1.3145(34)**	**1.9486(69)**
"	Extreme value	200	-	*1.060	*1.319	*1.956
	Weibull	"		**1.0587(22)**	**1.3164(34)**	**1.9522(83)**
	Pareto	"		**1.0606(22)**	**1.3199(34)**	**1.9570(75)**
"	Extreme value	500	-	*1.061	*1.320	*1.958
	Weibull	"		**1.0604(22)**	**1.3206(32)**	**1.9567(79)**
	Pareto	"		**1.0610(24)**	**1.3206(36)**	**1.9550(77)**
"	Extreme value	1000	-	*1.062	*1.321	*1.958
	Weibull	"		**1.0610(22)**	**1.3196(32)**	**1.9534(79)**
	Pareto	"		**1.0603(22)**	**1.3188(34)**	**1.9500(83)**
"	Extreme value	10000	-	*1.062	*1.321	*1.959
	Weibull	"		**1.0620(22)**	**1.3223(34)**	**1.9595(79)**
	Pareto	"		**1.0623(20)**	**1.3222(36)**	**1.9552(75)**
"	Extreme value	∞	-	*1.062	*1.321	*1.959
	Weibull	10000		**1.0620(22)**	**1.3223(34)**	**1.9595(79)**
	Pareto	10000		**1.0623(20)**	**1.3222(36)**	**1.9552(75)**

Table B.13: Case IIIa Anderson-Darling critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '79 [8]	Logistic	30	-	*0.852	*1.041	*1.500
	Loglogistic	"		**0.8511(16)**	**1.0393(24)**	**1.4996(60)**
"	Logistic	50	-	*0.854	*1.043	*1.502
	Loglogistic	"		**0.8532(16)**	**1.0419(24)**	**1.5019(62)**
"	Logistic	100	-	*0.856	*1.045	*1.503
	Loglogistic	"		**0.8548(16)**	**1.0436(24)**	**1.5024(58)**
"	Logistic	200	-	*0.856	*1.045	*1.504
	Loglogistic	"		**0.8552(16)**	**1.0440(24)**	**1.5016(58)**
"	Logistic	500	-	*0.857	*1.046	*1.505
	Loglogistic	"		**0.8565(16)**	**1.0459(24)**	**1.5067(58)**
"	Logistic	1000	-	*0.857	*1.046	*1.505
	Loglogistic	"		**0.8565(16)**	**1.0445(24)**	**1.5058(58)**
"	Logistic	10000	-	*0.857	*1.046	*1.505
	Loglogistic	"		**0.8566(16)**	**1.0462(24)**	**1.5066(62)**
"	Logistic	∞	-	*0.857	*1.046	*1.505
	Loglogistic	10000		**0.8566(16)**	**1.0462(24)**	**1.5066(62)**

Table B.14: Cont. Case IIIa Anderson-Darling critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

B.4 Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Normal Lognormal	50 "	1.080 **1.0803(10)**	1.170 **1.1676(12)**	1.310 **1.3022(16)**	1.595 **1.5688(30)**
"	Normal Lognormal	100 "	1.100 **1.0862(10)**	1.180 **1.1738(12)**	1.320 **1.3105(16)**	1.610 **1.5822(34)**
"	Normal Lognormal	∞ 10000	1.120 **1.1008(12)**	1.190 **1.1884(14)**	1.333 **1.3249(16)**	1.625 **1.5961(32)**
Stephens '79 [8]	Logistic Loglogistic	50 "	-	1.179 **1.1577(12)**	1.305 **1.2932(16)**	1.559 **1.5638(30)**
"	Logistic Loglogistic	∞ 10000	-	1.187 **1.1783(12)**	1.313 **1.3151(16)**	1.568 **1.5903(30)**
D'Agostino et al. '86 [112]	Extreme value Weibull	50 "	-	1.16 **1.1583(12)**	1.29 **1.2909(16)**	1.53 **1.5553(30)**
"	Extreme value Weibull	∞ 10000	-	1.16 **1.1801(12)**	1.29 **1.3146(16)**	1.53 **1.5835(30)**

Table B.15: Case IIIb Kolmogorov-Smirnov critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Normal Lognormal	50 "	1.380 **1.3712(08)**	1.450 **1.4456(10)**	1.570 **1.5603(16)**	1.810 **1.7922(26)**
"	Normal Lognormal	100 "	1.390 **1.3860(10)**	1.470 **1.4607(10)**	1.590 **1.5767(16)**	1.825 **1.8109(28)**
"	Normal Lognormal	∞ 10000	1.410 **1.4162(10)**	1.490 **1.4911(10)**	1.612 **1.6079(14)**	1.845 **1.8440(26)**
Stephens '79 [8]	Logistic Loglogistic	50 "	-	1.417 **1.4237(10)**	1.525 **1.5399(14)**	1.741 **1.7776(28)**
"	Logistic Loglogistic	∞ 10000	-	1.429 **1.4679(10)**	1.535 **1.5848(14)**	1.748 **1.8241(28)**
Chandra et al. '81 [9]	Extreme value Weibull	50 "	-	1.45 **1.4534(10)**	1.56 **1.5710(14)**	1.79 **1.8079(26)**
"	Extreme value Weibull	∞ 10000	-	1.46 **1.5003(10)**	1.58 **1.6188(14)**	1.81 **1.8593(28)**

Table B.16: Case IIIb Kuiper critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Normal Lognormal	$n \geq 5$ 10000	-	0.329 **0.3273(10)**	0.443 **0.4419(14)**	0.723 **0.7209(32)**
Stephens '77 [13]	Extreme value Weibull	All 10000	-	*0.320 **0.3198(10)**	*0.431 **0.4308(14)**	*0.705 **0.7052(36)**
Stephens '79 [8]	Logistic Loglogistic	30 "	-	*0.327 **0.3262(10)**	*0.438 **0.4390(14)**	*0.711 **0.7099(32)**
"	Logistic Loglogistic	50 "	-	*0.326 **0.3251(10)**	*0.438 **0.4382(16)**	*0.715 **0.7141(34)**
"	Logistic Loglogistic	100 "	-	*0.324 **0.3243(10)**	*0.438 **0.4384(14)**	*0.718 **0.7164(36)**
"	Logistic Loglogistic	200 "	-	*0.324 **0.3236(10)**	*0.438 **0.4381(14)**	*0.720 **0.7194(34)**
"	Logistic Loglogistic	500 "	-	*0.323 **0.3235(10)**	*0.438 **0.4384(14)**	*0.720 **0.7215(36)**
"	Logistic Loglogistic	1000 "	-	*0.323 **0.3234(10)**	*0.438 **0.4381(14)**	*0.721 **0.7200(36)**
"	Logistic Loglogistic	10000 "	-	*0.323 **0.3232(10)**	*0.438 **0.4381(14)**	*0.721 **0.7214(36)**
"	Logistic Loglogistic	∞ 10000	-	*0.323 **0.3232(10)**	*0.438 **0.4381(14)**	*0.721 **0.7214(36)**

Table B.17: Case IIIb Cramér-von Mises critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Author	Distribution	n	$D85$	$D90$	$D95$	$D99$
Stephens '74 [12]	Normal Lognormal	$n \geq 5$ 10000	-	1.760 **1.7440(52)**	2.323 **2.3083(63)**	3.690 **3.686 (16)**
Stephens '77 [13]	Extreme value Weibull	All 10000	-	*1.725 **1.7228(46)**	*2.277 **2.2765(69)**	*3.640 **3.638 (17)**
Stephens '79 [8]	Logistic Loglogistic	30 "	-	*1.729 **1.7285(46)**	*2.263 **2.2771(69)**	*3.580 **3.580 (17)**
"	Logistic Loglogistic	50 "	-	*1.728 **1.7265(48)**	*2.274 **2.2794(69)**	*3.622 **3.620 (16)**
"	Logistic Loglogistic	100 "	-	*1.726 **1.7256(50)**	*2.282 **2.2859(71)**	*3.654 **3.648 (18)**
"	Logistic Loglogistic	200 "	-	*1.726 **1.7241(46)**	*2.286 **2.2876(73)**	*3.669 **3.669 (17)**
"	Logistic Loglogistic	500 "	-	*1.725 **1.7237(48)**	*2.288 **2.2901(71)**	*3.679 **3.683 (18)**
"	Logistic Loglogistic	1000 "	-	*1.725 **1.7245(48)**	*2.289 **2.2877(73)**	*3.682 **3.679 (17)**
"	Logistic Loglogistic	10000 "	-	*1.725 **1.7232(48)**	*2.290 **2.2899(73)**	*3.685 **3.686 (18)**
"	Logistic Loglogistic	∞ 10000	-	*1.725 **1.7232(48)**	*2.290 **2.2899(73)**	*3.685 **3.686 (18)**

Table B.18: Case IIIb Anderson-Darling critical values from literature
Critical values preceded by a (*) were calculated from a formula of n
Critical values preceded by a (†) were calculated from C code
Results of the present work are **bold**

Appendix C

Fitted Parameter Values

C.1 Variable η

C.1.1 Case II: Both Parameters are Unknown

C.1.1.1 Weibull Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.77139	-0.03584	0.24677	0.01635	0.33070
50	0.79049	-0.01398	0.29081	0.05681	0.38565
100	0.80935	0.05098	0.32540	0.15320	0.42717
200	0.81644	0.03904	0.31121	0.13540	0.40533
500	0.82750	0.09336	0.33723	0.21202	0.43685
1000	0.82555	0.00387	0.32370	0.08750	0.41811
10000	0.83059	0.00181	0.32981	0.08539	0.42397

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.81680	-0.08535	0.26169	-0.04257	0.33128
50	0.84136	-0.02006	0.32619	0.05864	0.40861
100	0.86170	0.04013	0.36151	0.14455	0.44821
200	0.86872	0.04721	0.34002	0.15015	0.41839
500	0.87547	-0.00693	0.35703	0.08217	0.43709
1000	0.87785	-0.03445	0.35970	0.04675	0.43914
10000	0.88521	0.01561	0.36904	0.11247	0.44834

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.88825	-0.17041	0.29103	-0.13049	0.33916
50	0.91575	-0.10866	0.36498	-0.03825	0.42091
100	0.93525	-0.10992	0.39247	-0.03233	0.44794
200	0.94999	-0.01220	0.39426	0.08704	0.44677
500	0.95274	-0.11952	0.39309	-0.04115	0.44289
1000	0.96143	-0.06420	0.40757	0.02845	0.45784
10000	0.96318	-0.09304	0.40359	-0.00738	0.45157

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.03334	-0.34710	0.36251	-0.27512	0.36439
50	1.06201	-0.36837	0.41389	-0.28118	0.41104
100	1.09674	-0.29608	0.49469	-0.18589	0.48658
200	1.10983	-0.22420	0.46859	-0.11070	0.45704
500	1.11604	-0.29166	0.48589	-0.17821	0.47203
1000	1.12298	-0.26299	0.49042	-0.14590	0.47499
10000	1.13157	-0.21954	0.50532	-0.09843	0.48757

(d) 99% Sig. Lvl.

Table C.1: Params. of Eq. 9.7 fitted to the case II **Weibull** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.33008	0.24873	0.39238	0.20974	0.30678
50	1.35270	0.44369	0.40827	0.35961	0.31542
100	1.38713	1.10399	0.43692	0.86479	0.33358
200	1.38558	0.56985	0.42560	0.45028	0.32239
500	1.40269	0.84946	0.45520	0.65859	0.34259
1000	1.40032	0.48059	0.53231	0.37854	0.39926
10000	1.41275	0.74765	0.49852	0.57626	0.37187

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.40340	0.32124	0.40684	0.25459	0.30259
50	1.43125	0.65065	0.43782	0.49604	0.32178
100	1.44945	0.59673	0.47186	0.45187	0.34269
200	1.45928	0.54224	0.42502	0.40926	0.30626
500	1.47745	0.80307	0.48632	0.59472	0.34823
1000	1.47937	0.66296	0.52978	0.49201	0.37816
10000	1.48802	0.72114	0.55723	0.53034	0.39574

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.51373	0.32895	0.40856	0.24263	0.28270
50	1.54409	0.67476	0.42761	0.47816	0.29220
100	1.57300	0.99532	0.45452	0.69089	0.30711
200	1.58611	1.05798	0.35984	0.72853	0.24109
500	1.59136	0.70039	0.57436	0.48451	0.38257
1000	1.59972	0.83921	0.53606	0.57564	0.35616
10000	1.60833	0.90021	0.50844	0.61359	0.33585

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.73949	0.54492	0.34819	0.34612	0.21144
50	1.77661	0.89413	0.37949	0.55303	0.22737
100	1.79506	0.67521	0.60752	0.41305	0.36067
200	1.85298	2.14285	0.14389	1.28280	0.08461
500	1.80176	-0.04242	0.79590	-0.01341	0.46552
1000	1.84125	1.11176	0.52097	0.66555	0.30372
10000	1.85987	1.61170	0.30461	0.95438	0.17671

(d) 99% Sig. Lvl.

Table C.2: Params. of Eq. 9.7 fitted to the case II **Weibull** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.09908	0.00030	0.03172	0.13730	0.35751
50	0.10378	0.01657	0.03882	0.36335	0.43696
100	0.10896	0.03764	0.04468	0.64572	0.50227
200	0.10863	0.02710	0.04328	0.51353	0.48641
500	0.11012	0.02973	0.04444	0.55238	0.49891
1000	0.10993	0.02595	0.04441	0.50650	0.49857
10000	0.10935	0.02102	0.04269	0.44277	0.47890

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.11523	0.01165	0.03768	0.26713	0.37241
50	0.12008	0.01851	0.04565	0.36772	0.44982
100	0.12234	0.01401	0.04781	0.33018	0.47028
200	0.12461	0.02217	0.04839	0.42300	0.47549
500	0.12678	0.02514	0.05196	0.46634	0.51007
1000	0.12843	0.03064	0.05368	0.53273	0.52720
10000	0.12725	0.02149	0.05301	0.43002	0.52060

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.14133	0.00475	0.04836	0.19562	0.39468
50	0.14821	0.01893	0.05856	0.34674	0.47678
100	0.15229	0.01994	0.06151	0.36960	0.49898
200	0.15481	0.02545	0.06043	0.42275	0.48911
500	0.15460	0.01034	0.06306	0.29411	0.50970
1000	0.15753	0.02434	0.06389	0.42585	0.51606
10000	0.15703	0.01687	0.06816	0.35968	0.55109

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.19798	-0.04035	0.06828	-0.10060	0.39736
50	0.21315	0.00700	0.08278	0.23326	0.47683
100	0.22243	0.00509	0.09802	0.24719	0.56419
200	0.22799	0.02836	0.09248	0.39426	0.52972
500	0.22484	-0.01211	0.10028	0.14555	0.57442
1000	0.22874	0.00876	0.09746	0.27931	0.55759
10000	0.23654	0.05633	0.09890	0.58601	0.56428

(d) 99% Sig. Lvl.

Table C.3: Params. of Eq. 9.7 fitted to the case II **Weibull** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.66411	0.11292	0.21573	0.27044	0.38641
50	0.70425	0.38736	0.21731	0.78132	0.38812
100	0.71975	0.42132	0.22513	0.84963	0.40096
200	0.71654	0.33224	0.23092	0.68779	0.41065
500	0.72879	0.39706	0.23566	0.80856	0.41843
1000	0.72943	0.38704	0.23802	0.79066	0.42261
10000	0.72819	0.38214	0.22756	0.78123	0.40348

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.76874	0.19264	0.24065	0.38138	0.38311
50	0.80628	0.40235	0.24097	0.72977	0.38236
100	0.81740	0.38070	0.26443	0.69991	0.41811
200	0.81582	0.31039	0.26386	0.58777	0.41655
500	0.82307	0.32225	0.27599	0.60884	0.43510
1000	0.83517	0.39460	0.26861	0.73037	0.42319
10000	0.82535	0.30845	0.28328	0.58799	0.44645

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.93374	0.13977	0.29989	0.26175	0.40167
50	0.97318	0.32755	0.31581	0.52268	0.42159
100	0.98105	0.25835	0.35296	0.43541	0.46882
200	1.00505	0.38594	0.32419	0.61432	0.42956
500	1.00150	0.29876	0.34779	0.49868	0.45976
1000	1.02324	0.41209	0.34648	0.65899	0.45792
10000	1.01568	0.38570	0.33862	0.61799	0.44732

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.34418	0.08980	0.47201	0.15198	0.46303
50	1.39419	0.25817	0.48199	0.33476	0.46707
100	1.48764	0.65784	0.46186	0.74797	0.44582
200	1.48690	0.61461	0.42855	0.70974	0.41227
500	1.42664	0.20864	0.54595	0.30121	0.52470
1000	1.51439	0.67690	0.45441	0.77804	0.43619
10000	1.49106	0.60780	0.48639	0.70004	0.46676

(d) 99% Sig. Lvl.

Table C.4: Params. of Eq. 9.7 fitted to the case II **Weibull** Anderson-Darling critical values

C.1.1.2 Loglogistic Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.69375	-0.19175	0.11465	-0.27277	0.17011
50	0.70535	-0.19743	0.13095	-0.27577	0.19175
100	0.71584	-0.24518	0.17027	-0.34057	0.24660
200	0.72352	-0.23082	0.18110	-0.31593	0.26018
500	0.73126	-0.16456	0.18301	-0.21753	0.26104
1000	0.73525	-0.08355	0.15608	-0.10128	0.22186
10000	0.74008	-0.05362	0.15201	-0.05775	0.21496

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.73146	-0.22737	0.13267	-0.30821	0.18684
50	0.74390	-0.22568	0.14134	-0.29996	0.19637
100	0.75551	-0.25629	0.17954	-0.33720	0.24667
200	0.76395	-0.22373	0.19373	-0.28901	0.26416
500	0.77162	-0.17721	0.19085	-0.22221	0.25833
1000	0.77557	-0.12102	0.17997	-0.14539	0.24284
10000	0.78095	-0.06380	0.16904	-0.06684	0.22691

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.79017	-0.23442	0.13930	-0.29376	0.18186
50	0.80399	-0.27226	0.16998	-0.33612	0.21869
100	0.81759	-0.25201	0.18516	-0.30498	0.23560
200	0.82551	-0.28212	0.23710	-0.34013	0.29952
500	0.83463	-0.19434	0.21006	-0.22508	0.26340
1000	0.83838	-0.13364	0.19838	-0.14841	0.24794
10000	0.84428	-0.06668	0.18687	-0.06370	0.23247

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.90574	-0.32596	0.18684	-0.35871	0.21283
50	0.92332	-0.38507	0.23127	-0.41711	0.25930
100	0.94061	-0.28879	0.24519	-0.30450	0.27201
200	0.95081	-0.28182	0.27305	-0.29313	0.30029
500	0.95922	-0.31625	0.28705	-0.32627	0.31336
1000	0.96537	-0.16568	0.27484	-0.16118	0.29940
10000	0.97270	-0.07914	0.25980	-0.06546	0.28155

(d) 99% Sig. Lvl.

Table C.5: Params. of Eq. 9.7 fitted to the case II **Loglogistic** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.25258	-0.32316	0.20198	-0.25430	0.16661
50	1.27217	-0.37259	0.25437	-0.29005	0.20736
100	1.29184	-0.36458	0.27872	-0.27878	0.22469
200	1.30499	-0.39130	0.32470	-0.29695	0.25966
500	1.31843	-0.29185	0.31371	-0.21447	0.24905
1000	1.32721	-0.13021	0.26778	-0.08493	0.21176
10000	1.33568	-0.09321	0.25978	-0.05490	0.20423

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.31730	-0.36998	0.21888	-0.27787	0.17206
50	1.33855	-0.39570	0.27296	-0.29300	0.21205
100	1.35967	-0.38380	0.29251	-0.27873	0.22472
200	1.37350	-0.39685	0.33886	-0.28550	0.25828
500	1.38759	-0.29544	0.32322	-0.20530	0.24461
1000	1.39522	-0.18039	0.30500	-0.11751	0.23003
10000	1.40484	-0.11157	0.28976	-0.06475	0.21729

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.41724	-0.44091	0.26830	-0.30997	0.19679
50	1.44092	-0.48141	0.31205	-0.33312	0.22598
100	1.46482	-0.40203	0.31863	-0.27098	0.22828
200	1.47925	-0.43563	0.38920	-0.29210	0.27677
500	1.49549	-0.30894	0.34725	-0.19825	0.24509
1000	1.50439	-0.13000	0.31396	-0.07183	0.22093
10000	1.51324	-0.11828	0.30664	-0.06249	0.21459

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.61942	-0.52160	0.30852	-0.32196	0.19943
50	1.64921	-0.53242	0.35734	-0.32257	0.22817
100	1.67661	-0.43104	0.37113	-0.25394	0.23443
200	1.69447	-0.47820	0.46148	-0.28000	0.28912
500	1.71068	-0.42593	0.43142	-0.24321	0.26850
1000	1.72036	-0.30481	0.48672	-0.16848	0.30239
10000	1.73852	0.01662	0.33366	0.03413	0.20586

(d) 99% Sig. Lvl.

Table C.6: Params. of Eq. 9.7 fitted to the case II **Loglogistic** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.07738	-0.02378	0.01391	-0.29947	0.19379
50	0.07833	-0.02653	0.01715	-0.33203	0.23892
100	0.07926	-0.02753	0.01912	-0.34090	0.26664
200	0.07994	-0.02616	0.02068	-0.31776	0.28868
500	0.08088	-0.01975	0.02098	-0.22222	0.29267
1000	0.08147	-0.01134	0.01885	-0.10331	0.26278
10000	0.08188	-0.00640	0.01727	-0.03144	0.24066

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.08805	-0.02867	0.01657	-0.31935	0.20397
50	0.08926	-0.03248	0.02052	-0.36004	0.25255
100	0.09052	-0.03159	0.02290	-0.34426	0.28223
200	0.09135	-0.03143	0.02501	-0.33708	0.30798
500	0.09246	-0.02478	0.02509	-0.24892	0.30880
1000	0.09325	-0.01411	0.02206	-0.11535	0.27114
10000	0.09358	-0.01062	0.02087	-0.07040	0.25659

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.10635	-0.03652	0.02210	-0.34106	0.22766
50	0.10825	-0.03931	0.02538	-0.36109	0.26085
100	0.11036	-0.03294	0.02618	-0.28774	0.26881
200	0.11132	-0.03670	0.03110	-0.32134	0.31870
500	0.11265	-0.03194	0.03406	-0.26861	0.34943
1000	0.11401	-0.01605	0.02899	-0.10083	0.29682
10000	0.11441	-0.01197	0.02621	-0.05639	0.26815

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.15010	-0.05161	0.03182	-0.34454	0.23789
50	0.15328	-0.05566	0.03748	-0.36651	0.27911
100	0.15636	-0.05614	0.04601	-0.36296	0.34166
200	0.15829	-0.05924	0.05193	-0.37833	0.38404
500	0.16048	-0.05145	0.05113	-0.31487	0.37778
1000	0.16323	-0.02728	0.04587	-0.13094	0.33857
10000	0.16591	0.00082	0.03840	0.08222	0.28309

(d) 99% Sig. Lvl.

Table C.7: Params. of Eq. 9.7 fitted to the case II **Loglogistic** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.53741	-0.14095	0.12426	-0.26229	0.24866
50	0.54305	-0.16456	0.14135	-0.30376	0.28218
100	0.54954	-0.13259	0.13942	-0.23630	0.27834
200	0.55200	-0.15007	0.15466	-0.26861	0.30862
500	0.55752	-0.10951	0.15391	-0.18294	0.30664
1000	0.56229	-0.03195	0.13935	-0.02741	0.27755
10000	0.56368	-0.00860	0.11816	0.02147	0.23538

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.60703	-0.14896	0.14328	-0.24613	0.25638
50	0.61349	-0.17566	0.16702	-0.28835	0.29821
100	0.62082	-0.15376	0.16553	-0.24513	0.29532
200	0.62405	-0.17902	0.19034	-0.28657	0.33906
500	0.62947	-0.15080	0.20059	-0.23300	0.35704
1000	0.63583	-0.05268	0.16614	-0.05524	0.29524
10000	0.63842	-0.00421	0.13123	0.03397	0.23320

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.72682	-0.17385	0.16524	-0.24250	0.25062
50	0.73451	-0.21927	0.20502	-0.30523	0.30997
100	0.74391	-0.20576	0.21116	-0.27872	0.31863
200	0.74959	-0.19286	0.24114	-0.25803	0.36352
500	0.75872	-0.13597	0.22882	-0.16562	0.34434
1000	0.76368	-0.07916	0.21614	-0.07835	0.32497
10000	0.76753	-0.01503	0.17582	0.02081	0.26433

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.01890	-0.19143	0.22745	-0.19542	0.25506
50	1.03073	-0.21741	0.24716	-0.21514	0.27583
100	1.03686	-0.26068	0.33184	-0.26539	0.36984
200	1.04976	-0.27370	0.35761	-0.26661	0.39602
500	1.05801	-0.24171	0.39828	-0.23509	0.44278
1000	1.07085	-0.13723	0.32525	-0.10747	0.35954
10000	1.08072	0.00828	0.30010	0.05219	0.33246

(d) 99% Sig. Lvl.

Table C.8: Params. of Eq. 9.7 fitted to the case II **Loglogistic** Anderson-Darling critical values

C.1.1.3 Lognormal Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.73738	-0.70349	0.13449	-0.92988	0.16510
50	0.75300	-0.68321	0.11346	-0.87312	0.12636
100	0.77405	-0.54932	0.01137	-0.64809	-0.02881
200	0.78214	-0.56971	0.02184	-0.66466	-0.01705
500	0.78811	-0.54206	-0.00042	-0.61979	-0.04853
1000	0.78989	-0.58555	0.02908	-0.67765	-0.00810
10000	0.79537	-0.54379	-0.00297	-0.61431	-0.05268

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.78038	-0.73531	0.13660	-0.91596	0.15622
50	0.79800	-0.71168	0.11256	-0.85382	0.11402
100	0.82034	-0.57515	0.00815	-0.63486	-0.03687
200	0.83015	-0.58597	0.01114	-0.63471	-0.03678
500	0.83533	-0.58209	0.00638	-0.62435	-0.04348
1000	0.83749	-0.60547	0.02137	-0.65274	-0.02401
10000	0.84431	-0.53973	-0.02819	-0.55955	-0.08956

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.84694	-0.81223	0.15926	-0.93086	0.16784
50	0.86785	-0.77122	0.12203	-0.84611	0.11033
100	0.89555	-0.58557	-0.01854	-0.57437	-0.07681
200	0.90571	-0.59996	-0.01356	-0.57827	-0.07430
500	0.91240	-0.56989	-0.03698	-0.53501	-0.10368
1000	0.91603	-0.58207	-0.02991	-0.54474	-0.09651
10000	0.91995	-0.55506	-0.05139	-0.51113	-0.12083

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.98051	-1.01017	0.23550	-1.00108	0.21917
50	1.01076	-0.88026	0.13283	-0.81762	0.09349
100	1.04858	-0.70640	-0.00119	-0.57593	-0.07002
200	1.06494	-0.55334	-0.11696	-0.39681	-0.19622
500	1.06711	-0.65299	-0.04799	-0.50154	-0.12266
1000	1.06862	-0.71051	-0.00697	-0.55920	-0.08088
10000	1.08393	-0.54589	-0.12646	-0.36841	-0.21153

(d) 99% Sig. Lvl.

Table C.9: Params. of Eq. 9.7 fitted to the case II **Lognormal** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.30202	-1.60470	0.48516	-1.22384	0.36655
50	1.32221	-1.57973	0.45714	-1.18388	0.33807
100	1.34332	-1.52513	0.40798	-1.12061	0.29332
200	1.35563	-1.54223	0.41404	-1.12243	0.29469
500	1.36451	-1.55295	0.41700	-1.12310	0.29503
1000	1.36948	-1.55993	0.41968	-1.12383	0.29571
10000	1.37614	-1.57547	0.42703	-1.13026	0.30003

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.37144	-1.69833	0.51671	-1.22946	0.37050
50	1.39231	-1.68688	0.49798	-1.20087	0.35012
100	1.41465	-1.63093	0.44706	-1.13837	0.30579
200	1.42779	-1.63910	0.44633	-1.13259	0.30173
500	1.43764	-1.62551	0.43152	-1.11449	0.28873
1000	1.44213	-1.64595	0.44429	-1.12544	0.29688
10000	1.44931	-1.65428	0.44588	-1.12589	0.29668

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.47833	-1.85259	0.57248	-1.24429	0.38100
50	1.50284	-1.82273	0.53892	-1.20127	0.35044
100	1.52649	-1.76558	0.48599	-1.14129	0.30754
200	1.53995	-1.79966	0.50366	-1.15322	0.31618
500	1.54978	-1.79270	0.49361	-1.14090	0.30732
1000	1.55590	-1.78718	0.48697	-1.13186	0.30114
10000	1.56219	-1.82247	0.50855	-1.15114	0.31463

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.69484	-2.13352	0.66287	-1.24907	0.38425
50	1.72433	-2.11519	0.63455	-1.21427	0.35925
100	1.75052	-2.08872	0.60246	-1.17799	0.33342
200	1.76309	-2.14469	0.63596	-1.20239	0.35080
500	1.78428	-1.98032	0.50819	-1.08803	0.26938
1000	1.78508	-2.10225	0.59501	-1.16019	0.32100
10000	1.78964	-2.13610	0.61663	-1.17800	0.33356

(d) 99% Sig. Lvl.

Table C.10: Params. of Eq. 9.7 fitted to the case II **Lognormal** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.08924	-0.09870	0.02550	-1.06263	0.25507
50	0.09160	-0.09366	0.02084	-0.95432	0.17944
100	0.09480	-0.08390	0.01251	-0.77102	0.05161
200	0.09591	-0.08214	0.01071	-0.72682	0.02038
500	0.09603	-0.08336	0.01151	-0.73923	0.02912
1000	0.09580	-0.08450	0.01241	-0.75758	0.04188
10000	0.09603	-0.08199	0.01050	-0.72315	0.01735

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.10238	-0.11317	0.02922	-1.05892	0.25256
50	0.10526	-0.10880	0.02480	-0.96302	0.18574
100	0.10931	-0.09619	0.01405	-0.75754	0.04229
200	0.11060	-0.09437	0.01210	-0.71590	0.01263
500	0.11075	-0.09590	0.01311	-0.72932	0.02220
1000	0.11061	-0.09744	0.01426	-0.74799	0.03527
10000	0.11102	-0.09382	0.01148	-0.70277	0.00324

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.12508	-0.13942	0.03654	-1.06514	0.25717
50	0.12933	-0.13342	0.03033	-0.95260	0.17874
100	0.13420	-0.12159	0.01966	-0.78007	0.05784
200	0.13607	-0.11836	0.01649	-0.72532	0.01945
500	0.13698	-0.11595	0.01436	-0.69115	-0.00447
1000	0.13673	-0.11927	0.01683	-0.72316	0.01814
10000	0.13733	-0.11402	0.01284	-0.66911	-0.01990

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.17893	-0.20539	0.05651	-1.09629	0.27939
50	0.18763	-0.19154	0.04251	-0.92633	0.16000
100	0.19545	-0.17779	0.02914	-0.76418	0.04677
200	0.19848	-0.17485	0.02566	-0.71567	0.01308
500	0.19875	-0.17495	0.02549	-0.71540	0.01238
1000	0.19839	-0.18463	0.03268	-0.77605	0.05615
10000	0.19807	-0.17694	0.02734	-0.73445	0.02639

(d) 99% Sig. Lvl.

Table C.11: Params. of Eq. 9.7 fitted to the case II **Lognormal** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.60898	-0.72910	0.21212	-1.16037	0.32242
50	0.62141	-0.71140	0.19332	-1.09356	0.27505
100	0.63498	-0.67991	0.16423	-0.99831	0.20762
200	0.63936	-0.68147	0.16316	-0.98856	0.20075
500	0.63979	-0.68594	0.16608	-0.99561	0.20569
1000	0.64043	-0.68376	0.16419	-0.98967	0.20144
10000	0.64080	-0.67994	0.16114	-0.98205	0.19575

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.69456	-0.83614	0.24520	-1.16393	0.32500
50	0.70886	-0.82037	0.22687	-1.10320	0.28200
100	0.72459	-0.78695	0.19529	-1.01017	0.21609
200	0.73042	-0.78119	0.18822	-0.98600	0.19886
500	0.73166	-0.77665	0.18440	-0.97552	0.19147
1000	0.73207	-0.77780	0.18494	-0.97626	0.19188
10000	0.73212	-0.77285	0.18127	-0.96832	0.18606

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.84529	-1.02451	0.30326	-1.16769	0.32765
50	0.86339	-1.00992	0.28398	-1.11004	0.28699
100	0.88238	-0.96797	0.24456	-1.01350	0.21837
200	0.89088	-0.95668	0.23238	-0.97997	0.19474
500	0.89049	-0.96161	0.23599	-0.98753	0.19999
1000	0.89145	-0.95719	0.23244	-0.97902	0.19404
10000	0.89014	-0.95043	0.22815	-0.97324	0.18974

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.22185	-1.50726	0.45703	-1.18113	0.33731
50	1.25240	-1.48638	0.42686	-1.11386	0.28960
100	1.26974	-1.43615	0.38266	-1.03751	0.23559
200	1.27916	-1.39993	0.35210	-0.98767	0.20011
500	1.27651	-1.41593	0.36491	-1.00623	0.21339
1000	1.27644	-1.41836	0.36763	-1.00711	0.21496
10000	1.27124	-1.41879	0.37006	-1.01668	0.22128

(d) 99% Sig. Lvl.

Table C.12: Params. of Eq. 9.7 fitted to the case II **Lognormal** Anderson-Darling critical values

C.1.2 Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

C.1.2.1 Lognormal Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.87808	-0.98335	0.25783	-1.08718	0.27066
50	0.88600	-0.98259	0.25309	-1.07565	0.26219
100	0.89126	-1.01259	0.27218	-1.10620	0.28439
200	0.89588	-1.02376	0.27769	-1.11375	0.28962
500	0.90102	-1.01704	0.27006	-1.09864	0.27856
1000	0.90363	-1.01145	0.26499	-1.08827	0.27144
10000	0.90824	-1.00737	0.25978	-1.07698	0.26343

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.93779	-1.05483	0.27865	-1.08962	0.27244
50	0.94523	-1.06057	0.27860	-1.08737	0.27037
100	0.95108	-1.09002	0.29690	-1.11463	0.29010
200	0.95583	-1.09784	0.30022	-1.11756	0.29235
500	0.96110	-1.09177	0.29300	-1.10403	0.28244
1000	0.96338	-1.09455	0.29391	-1.10416	0.28261
10000	0.96744	-1.10081	0.29660	-1.10598	0.28422

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.02999	-1.18197	0.32239	-1.11110	0.28741
50	1.03874	-1.18420	0.31906	-1.10363	0.28153
100	1.04528	-1.21228	0.33625	-1.12609	0.29805
200	1.04959	-1.22404	0.34247	-1.13367	0.30345
500	1.05458	-1.22141	0.33800	-1.12530	0.29741
1000	1.05785	-1.22182	0.33709	-1.12114	0.29491
10000	1.06138	-1.22597	0.33801	-1.12149	0.29490

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.21559	-1.44055	0.41238	-1.14768	0.31298
50	1.22469	-1.46307	0.42371	-1.15959	0.32135
100	1.23310	-1.47923	0.43024	-1.16581	0.32511
200	1.24014	-1.46798	0.41954	-1.14819	0.31334
500	1.24431	-1.50180	0.44212	-1.17470	0.33275
1000	1.24738	-1.49468	0.43608	-1.16464	0.32612
10000	1.25329	-1.48506	0.42472	-1.14920	0.31374

(d) 99% Sig. Lvl.

Table C.13: Params. of Eq. 9.7 fitted to the case IIIa **Lognormal** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.39952	-0.64475	-0.20537	-0.45298	-0.15223
50	1.38161	-14.29752	9.22018	-10.20469	6.57519
100	1.42530	-0.32473	-0.45738	-0.22105	-0.32573
200	1.43277	-1.37512	0.26906	-0.95757	0.18620
500	1.45467	9.00864	-6.96062	6.28864	-4.85206
1000	1.45469	1.48642	-1.71088	1.04840	-1.19478
10000	1.45734	0.03708	-0.70628	0.03665	-0.49256

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.47604	-1.04864	0.04731	-0.70433	0.02770
50	1.58388	47.76770	-33.71775	32.33974	-22.79728
100	1.50065	-1.68344	0.45216	-1.12019	0.30014
200	1.50971	-1.40360	0.24708	-0.92826	0.16261
500	1.52599	3.43263	-3.09817	2.27762	-2.05009
1000	1.72324	78.31776	-54.74801	51.80018	-36.15203
10000	1.53292	-1.12358	0.06213	-0.72855	0.03737

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.59535	-0.91476	-0.10856	-0.56677	-0.07277
50	1.61175	1.03056	-1.43362	0.65335	-0.89937
100	1.62362	0.45394	-1.10034	0.28680	-0.68286
200	1.63022	-1.59040	0.32352	-0.97443	0.19760
500	1.64149	-0.98227	-0.09746	-0.59533	-0.06158
1000	1.65908	6.35698	-5.16412	3.89481	-3.15685
10000	1.65571	-0.74065	-0.26104	-0.44111	-0.16209

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.83097	-1.92062	0.46536	-1.04675	0.25256
50	1.85750	9.87948	-7.62875	5.36946	-4.14248
100	1.85939	-0.02390	-0.90663	-0.01536	-0.48584
200	1.91726	25.46153	-17.74347	13.65846	-9.50931
500	1.88388	-2.47608	0.82808	-1.31376	0.43911
1000	1.88472	-6.18456	3.39224	-3.28046	1.79931
10000	1.90599	3.20687	-3.12012	1.70708	-1.65429

(d) 99% Sig. Lvl.

Table C.14: Params. of Eq. 9.7 fitted to the case IIIa **Lognormal** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.13885	-0.14557	0.03387	-0.96045	0.18229
50	0.13906	-0.14533	0.03358	-0.95509	0.17829
100	0.13863	-0.14649	0.03458	-0.96937	0.18817
200	0.13764	-0.15163	0.03885	-1.02592	0.22934
500	0.13849	-0.14565	0.03403	-0.96415	0.18419
1000	0.13820	-0.14790	0.03589	-0.98642	0.20104
10000	0.13863	-0.14518	0.03366	-0.95758	0.17980

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.16287	-0.17222	0.04074	-0.96383	0.18449
50	0.16282	-0.17228	0.04082	-0.96459	0.18512
100	0.16246	-0.17408	0.04226	-0.98108	0.19671
200	0.16251	-0.17385	0.04213	-0.97865	0.19544
500	0.16212	-0.17464	0.04281	-0.98896	0.20218
1000	0.16233	-0.17364	0.04206	-0.97890	0.19545
10000	0.16297	-0.17262	0.04098	-0.96403	0.18468

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.20499	-0.21931	0.05299	-0.96892	0.18762
50	0.20520	-0.22062	0.05389	-0.97545	0.19267
100	0.20461	-0.22296	0.05582	-0.99449	0.20602
200	0.20497	-0.22163	0.05482	-0.98300	0.19863
500	0.20495	-0.22231	0.05533	-0.98741	0.20192
1000	0.20472	-0.22340	0.05637	-0.99582	0.20876
10000	0.20580	-0.22004	0.05321	-0.96583	0.18611

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.30540	-0.34456	0.09163	-1.02848	0.23019
50	0.30720	-0.34181	0.08878	-1.00870	0.21616
100	0.30771	-0.34313	0.08913	-1.01198	0.21710
200	0.30968	-0.33725	0.08401	-0.97599	0.19173
500	0.30748	-0.34759	0.09304	-1.03259	0.23434
1000	0.30847	-0.34851	0.09314	-1.02960	0.23201
10000	0.31134	-0.33893	0.08459	-0.97176	0.18969

(d) 99% Sig. Lvl.

Table C.15: Params. of Eq. 9.7 fitted to the case IIIa **Lognormal** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.87137	-0.89013	0.19463	-0.96630	0.18443
50	0.87271	-0.90044	0.20147	-0.97826	0.19317
100	0.87224	-0.92325	0.21761	-1.00902	0.21463
200	0.87145	-0.95198	0.24045	-1.04755	0.24460
500	0.87544	-0.90785	0.20445	-0.98424	0.19624
1000	0.87272	-0.94516	0.23434	-1.03667	0.23607
10000	0.87646	-0.91093	0.20716	-0.98604	0.19885

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.00795	-1.03348	0.22684	-0.96566	0.18286
50	1.01179	-1.03177	0.22387	-0.95897	0.17831
100	1.01033	-1.06659	0.24977	-1.00095	0.20861
200	1.00970	-1.09472	0.27162	-1.03352	0.23349
500	1.01273	-1.06654	0.24899	-0.99752	0.20669
1000	1.01041	-1.08786	0.26629	-1.02506	0.22736
10000	1.01325	-1.06833	0.24979	-0.99872	0.20731

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.24930	-1.31188	0.30295	-0.98641	0.19747
50	1.25090	-1.34579	0.32788	-1.01698	0.22070
100	1.25108	-1.34573	0.32695	-1.01760	0.22040
200	1.25261	-1.35260	0.33208	-1.02219	0.22459
500	1.25329	-1.35564	0.33339	-1.02410	0.22549
1000	1.25194	-1.36830	0.34564	-1.03770	0.23752
10000	1.25943	-1.32330	0.30709	-0.98632	0.19844

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.83681	-2.07054	0.54821	-1.06840	0.25712
50	1.84310	-2.05143	0.53269	-1.05115	0.24565
100	1.83995	-2.05427	0.53133	-1.05563	0.24561
200	1.85211	-2.00456	0.49031	-1.01269	0.21539
500	1.83623	-2.08077	0.55548	-1.07675	0.26284
1000	1.85290	-2.03547	0.51655	-1.03183	0.23200
10000	1.86669	-2.47497	0.60366	-1.24327	0.24049

(d) 99% Sig. Lvl.

Table C.16: Params. of Eq. 9.7 fitted to the case IIIa **Lognormal** Anderson-Darling critical values

C.1.3 Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

C.1.3.1 Weibull Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.92983	-0.83961	0.67466	-0.85178	0.63362
50	0.93943	-0.87682	0.71465	-0.87861	0.66674
100	0.94546	-0.87831	0.73691	-0.87383	0.68311
200	0.95363	-0.89357	0.72103	-0.88320	0.66564
500	0.95751	-0.90438	0.74125	-0.88980	0.68189
1000	0.95811	-0.87897	0.73448	-0.86446	0.67389
10000	0.96534	-0.93619	0.77510	-0.91127	0.70840

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.99636	-0.91703	0.73003	-0.86412	0.63529
50	1.00726	-0.96813	0.78814	-0.89956	0.68132
100	1.01390	-0.95988	0.80365	-0.88532	0.68997
200	1.02192	-0.97775	0.79066	-0.89684	0.67653
500	1.02469	-0.97736	0.79935	-0.89494	0.68205
1000	1.02518	-0.95121	0.79997	-0.86994	0.68069
10000	1.03629	-1.03923	0.83842	-0.93818	0.71074

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.10465	-1.08023	0.85548	-0.90962	0.66748
50	1.11144	-1.08151	0.87953	-0.90573	0.68253
100	1.11757	-1.05782	0.89474	-0.88075	0.68982
200	1.12638	-1.07223	0.89232	-0.88604	0.68495
500	1.12906	-1.06741	0.88036	-0.88298	0.67482
1000	1.13005	-1.05527	0.90376	-0.86943	0.69008
10000	1.14000	-1.11797	0.92808	-0.91196	0.70647

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.30872	-1.28463	1.01627	-0.90839	0.66030
50	1.32648	-1.34726	1.08345	-0.93617	0.69759
100	1.32716	-1.26288	1.08118	-0.87727	0.69137
200	1.34818	-1.41285	1.15540	-0.96427	0.73578
500	1.35574	-1.41839	1.13719	-0.96542	0.72266
1000	1.34274	-1.25718	1.08442	-0.86445	0.68762
10000	1.35030	-1.29161	1.09348	-0.88408	0.69156

(d) 99% Sig. Lvl.

Table C.17: Params. of Eq. 9.7 fitted to the case IIIb **Weibull** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.39594	-0.83343	0.33718	-0.60997	0.24746
50	1.41002	-0.84046	0.33215	-0.60853	0.24106
100	1.42432	-0.84260	0.31903	-0.60325	0.22889
200	1.43467	-0.82475	0.31961	-0.58630	0.22758
500	1.44179	-0.83000	0.35105	-0.58870	0.24870
1000	1.44643	-0.84562	0.34894	-0.59711	0.24627
10000	1.45554	-0.83592	0.31050	-0.58504	0.21778

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.47175	-0.92423	0.38791	-0.64115	0.26981
50	1.48539	-0.92566	0.38777	-0.63645	0.26689
100	1.50105	-0.93145	0.36242	-0.63250	0.24657
200	1.51271	-0.86630	0.33742	-0.58402	0.22800
500	1.51904	-0.89356	0.39434	-0.60165	0.26511
1000	1.52455	-0.89489	0.36892	-0.59923	0.24709
10000	1.53348	-0.89002	0.33925	-0.59127	0.22590

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.58993	-1.02270	0.44629	-0.65616	0.28714
50	1.60379	-1.06430	0.45925	-0.67704	0.29248
100	1.62064	-1.01462	0.43273	-0.63857	0.27258
200	1.63286	-0.94487	0.38632	-0.59017	0.24178
500	1.64252	-0.90693	0.39319	-0.56426	0.24489
1000	1.64421	-1.00452	0.45689	-0.62399	0.28352
10000	1.65573	-0.90442	0.35675	-0.55643	0.22014

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.81879	-1.46368	0.83162	-0.82103	0.46526
50	1.84652	-1.12440	0.44726	-0.61859	0.24731
100	1.86535	-0.93732	0.40177	-0.51220	0.22039
200	1.87883	-0.90543	0.40497	-0.49128	0.22059
500	1.88372	-1.20606	0.54025	-0.65200	0.29252
1000	1.87935	-1.41235	0.78959	-0.76727	0.42686
10000	1.89457	-1.21914	0.57569	-0.65568	0.30963

(d) 99% Sig. Lvl.

Table C.18: Params. of Eq. 9.7 fitted to the case IIIb **Weibull** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.16133	-0.13951	0.12858	-0.69486	0.49510
50	0.16427	-0.15159	0.13617	-0.73549	0.52602
100	0.16351	-0.14774	0.13657	-0.72064	0.52866
200	0.16631	-0.15821	0.13847	-0.75905	0.53691
500	0.16490	-0.15385	0.13929	-0.74476	0.54149
1000	0.16558	-0.15499	0.14005	-0.74463	0.54303
10000	0.16712	-0.16375	0.14552	-0.77470	0.56401

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.19242	-0.17295	0.15625	-0.70811	0.48838
50	0.19490	-0.18275	0.16514	-0.73204	0.51665
100	0.19546	-0.18271	0.16752	-0.72731	0.52395
200	0.19993	-0.19946	0.17323	-0.77478	0.54233
500	0.19886	-0.19535	0.17268	-0.76460	0.54194
1000	0.19851	-0.19289	0.17318	-0.75267	0.54181
10000	0.19920	-0.19741	0.17702	-0.76516	0.55387

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.24820	-0.23505	0.20921	-0.72637	0.48909
50	0.25059	-0.24570	0.22501	-0.74131	0.52430
100	0.25239	-0.24714	0.22725	-0.73848	0.52893
200	0.25978	-0.27332	0.23930	-0.79032	0.55648
500	0.25428	-0.25291	0.23145	-0.75104	0.53983
1000	0.25441	-0.25252	0.23448	-0.74335	0.54429
10000	0.25922	-0.27096	0.24186	-0.78164	0.56148

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.39174	-0.41395	0.34274	-0.79102	0.50003
50	0.39359	-0.41634	0.36468	-0.77787	0.52561
100	0.38683	-0.38548	0.36435	-0.73186	0.52312
200	0.39971	-0.42426	0.38238	-0.77415	0.54625
500	0.40555	-0.45643	0.40916	-0.81173	0.58322
1000	0.39749	-0.41689	0.38691	-0.75881	0.55035
10000	0.39753	-0.41504	0.39131	-0.75505	0.55685

(d) 99% Sig. Lvl.

Table C.19: Params. of Eq. 9.7 fitted to the case IIIb **Weibull** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.95884	-0.73718	0.60641	-0.65634	0.42812
50	0.97443	-0.79590	0.66186	-0.68744	0.46713
100	0.97552	-0.78235	0.68189	-0.67252	0.48119
200	0.98465	-0.81236	0.68984	-0.69244	0.48763
500	0.98593	-0.82485	0.71691	-0.69834	0.50674
1000	0.99086	-0.84020	0.72648	-0.70432	0.51220
10000	0.99766	-0.88815	0.76416	-0.73463	0.53883

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.12612	-0.90933	0.72191	-0.67601	0.42174
50	1.14036	-0.95348	0.78909	-0.68927	0.45954
100	1.13907	-0.92226	0.80422	-0.66548	0.46782
200	1.16214	-1.01888	0.85209	-0.71476	0.49567
500	1.15846	-1.00303	0.86227	-0.70541	0.50206
1000	1.16007	-0.99335	0.86167	-0.69481	0.50001
10000	1.16923	-1.04553	0.89063	-0.72296	0.51718

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.43718	-1.27913	0.96333	-0.72470	0.43038
50	1.43355	-1.24486	1.03439	-0.69711	0.45920
100	1.44004	-1.23062	1.07043	-0.67932	0.47275
200	1.48781	-1.41575	1.15607	-0.74894	0.50958
500	1.45951	-1.29425	1.11784	-0.70153	0.49380
1000	1.46013	-1.28208	1.12614	-0.69054	0.49549
10000	1.48812	-1.41044	1.19440	-0.74017	0.52523

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	2.23908	-2.31411	1.58537	-0.82537	0.45353
50	2.22245	-2.20126	1.70473	-0.76940	0.47866
100	2.16491	-1.92431	1.66141	-0.68889	0.46344
200	2.22620	-2.15194	1.83005	-0.73402	0.50695
500	2.24938	-2.26565	1.92722	-0.75854	0.53315
1000	2.22124	-2.10832	1.85714	-0.71250	0.51116
10000	2.23650	-2.15708	1.89631	-0.72360	0.52234

(d) 99% Sig. Lvl.

Table C.20: Params. of Eq. 9.7 fitted to the case IIIb **Weibull** Anderson-Darling critical values

C.1.3.2 Loglogistic Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.90273	-0.60858	0.51287	-0.59033	0.48210
50	0.90826	-0.59228	0.50534	-0.57188	0.47249
100	0.91461	-0.57748	0.49334	-0.55563	0.45881
200	0.92047	-0.58922	0.50633	-0.56267	0.46887
500	0.92334	-0.56862	0.48903	-0.54262	0.45107
1000	0.92564	-0.57072	0.49377	-0.54298	0.45464
10000	0.92951	-0.57754	0.48951	-0.54901	0.44931

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.96452	-0.65067	0.54813	-0.58611	0.47626
50	0.97016	-0.62912	0.54657	-0.56293	0.47224
100	0.97535	-0.59943	0.52011	-0.53682	0.44719
200	0.98342	-0.63000	0.54664	-0.55744	0.46791
500	0.98514	-0.59826	0.52380	-0.53030	0.44695
1000	0.98875	-0.61567	0.53562	-0.54316	0.45613
10000	0.99103	-0.60985	0.51798	-0.54023	0.44005

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.06117	-0.73000	0.60924	-0.59335	0.47406
50	1.06684	-0.69364	0.58904	-0.56185	0.45603
100	1.07378	-0.67048	0.57563	-0.54065	0.44305
200	1.08123	-0.68975	0.60394	-0.54945	0.46273
500	1.08234	-0.65784	0.57671	-0.52629	0.44067
1000	1.08741	-0.68304	0.59633	-0.54240	0.45473
10000	1.09032	-0.68288	0.59396	-0.54226	0.45194

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.25669	-0.88344	0.73068	-0.59992	0.47079
50	1.26044	-0.81055	0.66780	-0.55456	0.42827
100	1.26970	-0.79747	0.70507	-0.53603	0.44899
200	1.27942	-0.81414	0.71989	-0.54088	0.45616
500	1.27736	-0.75534	0.66010	-0.50899	0.41735
1000	1.28877	-0.82535	0.73551	-0.54380	0.46403
10000	1.28847	-0.81328	0.69958	-0.54188	0.44094

(d) 99% Sig. Lvl.

Table C.21: Params. of Eq. 9.7 fitted to the case IIIb **Loglogistic** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.39541	-0.47894	0.22229	-0.34920	0.16639
50	1.40890	-0.46141	0.20279	-0.33268	0.15027
100	1.42222	-0.50028	0.22348	-0.35807	0.16397
200	1.43261	-0.47392	0.20022	-0.33584	0.14582
500	1.44085	-0.47567	0.20663	-0.33537	0.14955
1000	1.44514	-0.47811	0.19934	-0.33576	0.14379
10000	1.45320	-0.44594	0.19539	-0.31149	0.14033

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.47185	-0.51606	0.25446	-0.35713	0.18055
50	1.48526	-0.53516	0.23705	-0.36650	0.16649
100	1.49997	-0.53776	0.23954	-0.36461	0.16656
200	1.51070	-0.49675	0.21328	-0.33349	0.14728
500	1.51861	-0.52212	0.23487	-0.34949	0.16128
1000	1.52248	-0.53095	0.23228	-0.35447	0.15899
10000	1.53036	-0.50251	0.22503	-0.33379	0.15338

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.59082	-0.55465	0.30208	-0.35521	0.19809
50	1.60521	-0.57640	0.29441	-0.36578	0.19124
100	1.61942	-0.62553	0.28757	-0.39340	0.18506
200	1.63157	-0.54752	0.24203	-0.34036	0.15464
500	1.64036	-0.55613	0.26472	-0.34458	0.16828
1000	1.64388	-0.59670	0.27471	-0.36914	0.17409
10000	1.65134	-0.57513	0.26439	-0.35428	0.16687

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.82857	-0.62655	0.42274	-0.34914	0.24022
50	1.84402	-0.75481	0.45089	-0.41827	0.25391
100	1.86361	-0.69029	0.37034	-0.37653	0.20663
200	1.87603	-0.57525	0.30976	-0.31077	0.17190
500	1.88191	-0.67397	0.35709	-0.36458	0.19716
1000	1.88416	-0.83167	0.38684	-0.44926	0.21291
10000	1.89744	-0.56824	0.32295	-0.30367	0.17709

(d) 99% Sig. Lvl.

Table C.22: Params. of Eq. 9.7 fitted to the case IIIb **Loglogistic** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.14969	-0.09902	0.08062	-0.42613	0.30732
50	0.14880	-0.09483	0.07735	-0.41491	0.29653
100	0.14821	-0.09053	0.07442	-0.39993	0.28608
200	0.14781	-0.08904	0.07327	-0.39543	0.28195
500	0.14742	-0.08697	0.07188	-0.38913	0.27708
1000	0.14785	-0.08946	0.07353	-0.39778	0.28383
10000	0.14757	-0.08799	0.07167	-0.39559	0.27661

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.17679	-0.12290	0.09976	-0.43483	0.30658
50	0.17552	-0.11570	0.09529	-0.41464	0.29379
100	0.17467	-0.10977	0.09016	-0.40086	0.27855
200	0.17481	-0.11031	0.09119	-0.39955	0.28184
500	0.17428	-0.10761	0.08953	-0.39329	0.27738
1000	0.17461	-0.10940	0.09076	-0.39733	0.28115
10000	0.17486	-0.11021	0.09081	-0.40033	0.28106

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.22471	-0.16491	0.13338	-0.44493	0.30451
50	0.22359	-0.15626	0.12939	-0.42227	0.29546
100	0.22287	-0.14930	0.12485	-0.40655	0.28515
200	0.22313	-0.14985	0.12654	-0.40366	0.28897
500	0.22154	-0.14210	0.12177	-0.38887	0.27817
1000	0.22126	-0.14159	0.11946	-0.39240	0.27353
10000	0.22194	-0.14426	0.12133	-0.39695	0.27725

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.34129	-0.27059	0.21686	-0.46750	0.30705
50	0.33806	-0.24952	0.20493	-0.43786	0.28899
100	0.34008	-0.24655	0.20915	-0.41929	0.29249
200	0.33951	-0.23836	0.20817	-0.39779	0.28885
500	0.33544	-0.22242	0.19332	-0.38900	0.26890
1000	0.33832	-0.23353	0.20227	-0.39875	0.28215
10000	0.33842	-0.23326	0.20141	-0.39877	0.27969

(d) 99% Sig. Lvl.

Table C.23: Params. of Eq. 9.7 fitted to the case IIIb **Loglogistic** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.91603	-0.54454	0.41572	-0.43066	0.29324
50	0.91516	-0.51789	0.40057	-0.41270	0.28358
100	0.91421	-0.48326	0.38064	-0.38700	0.26959
200	0.91426	-0.48056	0.37735	-0.38556	0.26742
500	0.91316	-0.46480	0.37276	-0.37262	0.26422
1000	0.91527	-0.47690	0.38315	-0.37935	0.27166
10000	0.91509	-0.47300	0.37299	-0.37965	0.26434

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.06619	-0.66643	0.50204	-0.44138	0.29095
50	1.06514	-0.62539	0.48211	-0.41534	0.27967
100	1.06267	-0.58197	0.45361	-0.39057	0.26327
200	1.06313	-0.57848	0.45917	-0.38484	0.26637
500	1.06214	-0.56109	0.45316	-0.37352	0.26300
1000	1.06375	-0.56968	0.45443	-0.37925	0.26385
10000	1.06420	-0.57087	0.45319	-0.38067	0.26285

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.33104	-0.87938	0.65030	-0.45610	0.28647
50	1.33054	-0.82985	0.63906	-0.42574	0.28048
100	1.32892	-0.77665	0.60985	-0.39964	0.26731
200	1.32971	-0.77083	0.61771	-0.39207	0.27035
500	1.32362	-0.72414	0.58594	-0.37420	0.25646
1000	1.32288	-0.72120	0.58420	-0.37234	0.25558
10000	1.32853	-0.74644	0.59948	-0.38150	0.26181

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.99982	-1.51311	1.05849	-0.51937	0.29794
50	1.98107	-1.35677	1.00814	-0.46177	0.28049
100	1.98153	-1.25547	0.99287	-0.41477	0.27245
200	1.98262	-1.22590	1.00808	-0.39398	0.27451
500	1.97439	-1.18172	0.96656	-0.38658	0.26275
1000	1.96900	-1.16422	0.95361	-0.38436	0.26060
10000	1.97509	-1.18942	0.96976	-0.39004	0.26381

(d) 99% Sig. Lvl.

Table C.24: Params. of Eq. 9.7 fitted to the case IIIb **Loglogistic** Anderson-Darling critical values

C.1.3.3 Lognormal Distribution

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.90388	-1.15472	0.37843	-1.29211	0.42654
50	0.91008	-1.16424	0.37891	-1.29558	0.42603
100	0.91785	-1.17544	0.38149	-1.29737	0.42590
200	0.92200	-1.16654	0.37517	-1.28222	0.41702
500	0.92693	-1.17359	0.37869	-1.28219	0.41788
1000	0.92843	-1.18658	0.38340	-1.29537	0.42379
10000	0.93236	-1.18632	0.38293	-1.28918	0.42091

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.96473	-1.24228	0.40750	-1.30464	0.43237
50	0.97267	-1.24440	0.40605	-1.29616	0.42719
100	0.98022	-1.25399	0.40700	-1.29708	0.42604
200	0.98489	-1.25247	0.40458	-1.28949	0.42148
500	0.98959	-1.25523	0.40550	-1.28571	0.41988
1000	0.99050	-1.26965	0.41001	-1.30105	0.42631
10000	0.99515	-1.27115	0.41140	-1.29534	0.42447

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.06111	-1.37108	0.45046	-1.31079	0.43576
50	1.07016	-1.37756	0.45166	-1.30533	0.43273
100	1.07830	-1.37904	0.44756	-1.29816	0.42678
200	1.08393	-1.38347	0.44949	-1.29491	0.42567
500	1.08798	-1.37557	0.44352	-1.28309	0.41852
1000	1.08809	-1.39551	0.44965	-1.30416	0.42742
10000	1.09374	-1.40130	0.45456	-1.30064	0.42766

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.25578	-1.64008	0.54250	-1.32647	0.44480
50	1.26554	-1.64658	0.54162	-1.32229	0.44143
100	1.27647	-1.64123	0.53493	-1.30686	0.43208
200	1.27960	-1.63291	0.52530	-1.30005	0.42587
500	1.28790	-1.63401	0.52838	-1.28940	0.42236
1000	1.28790	-1.65077	0.53319	-1.30489	0.42890
10000	1.29485	-1.66517	0.54250	-1.30697	0.43198

(d) 99% Sig. Lvl.

Table C.25: Params. of Eq. 9.7 fitted to the case IIIb **Lognormal** Kolmogorov-Smirnov critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.40450	-2.00592	0.71794	-1.42806	0.51112
50	1.41750	-2.02987	0.72855	-1.43203	0.51403
100	1.43213	-2.04715	0.73342	-1.42942	0.51214
200	1.44174	-2.06023	0.73786	-1.42898	0.51182
500	1.44976	-2.07837	0.74700	-1.43375	0.51542
1000	1.45375	-2.08047	0.74685	-1.43134	0.51396
10000	1.46179	-2.09062	0.74920	-1.43019	0.51257

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.48116	-2.11402	0.75639	-1.42712	0.51062
50	1.49471	-2.13905	0.76710	-1.43103	0.51321
100	1.51032	-2.15577	0.77113	-1.42723	0.51052
200	1.51913	-2.17176	0.77839	-1.42965	0.51247
500	1.52812	-2.18931	0.78651	-1.43280	0.51483
1000	1.53212	-2.19196	0.78659	-1.43083	0.51357
10000	1.54007	-2.20012	0.78761	-1.42854	0.51142

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.60016	-2.27889	0.81425	-1.42399	0.50879
50	1.61435	-2.30724	0.82671	-1.42914	0.51211
100	1.63066	-2.32598	0.83164	-1.42626	0.50995
200	1.64080	-2.34057	0.83671	-1.42633	0.50988
500	1.64945	-2.36301	0.84863	-1.43261	0.51455
1000	1.65303	-2.36083	0.84593	-1.42830	0.51189
10000	1.66150	-2.37372	0.85009	-1.42863	0.51166

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.83557	-2.59093	0.92311	-1.41187	0.50332
50	1.85376	-2.63499	0.94033	-1.42134	0.50727
100	1.87177	-2.66759	0.95339	-1.42497	0.50927
200	1.88255	-2.68913	0.96468	-1.42861	0.51262
500	1.89220	-2.70388	0.96949	-1.42897	0.51243
1000	1.89676	-2.69589	0.96113	-1.42120	0.50670
10000	1.90567	-2.70879	0.96546	-1.42118	0.50649

(d) 99% Sig. Lvl.

Table C.26: Params. of Eq. 9.7 fitted to the case IIIb **Lognormal** Kuiper critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.14873	-0.19150	0.06250	-1.33257	0.44772
50	0.14877	-0.19179	0.06245	-1.33568	0.44906
100	0.14875	-0.19162	0.06209	-1.33760	0.44948
200	0.14880	-0.19124	0.06194	-1.33328	0.44701
500	0.14886	-0.19091	0.06192	-1.32861	0.44460
1000	0.14849	-0.19142	0.06187	-1.34030	0.45062
10000	0.14870	-0.19090	0.06166	-1.33325	0.44664

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.17487	-0.22533	0.07357	-1.33840	0.45144
50	0.17518	-0.22634	0.07387	-1.34245	0.45355
100	0.17510	-0.22519	0.07287	-1.34013	0.45126
200	0.17534	-0.22462	0.07265	-1.33342	0.44742
500	0.17503	-0.22416	0.07242	-1.33382	0.44753
1000	0.17457	-0.22470	0.07237	-1.34506	0.45347
10000	0.17544	-0.22516	0.07288	-1.33597	0.44889

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.22146	-0.28645	0.09391	-1.34750	0.45754
50	0.22213	-0.28574	0.09329	-1.34037	0.45299
100	0.22186	-0.28454	0.09181	-1.34359	0.45354
200	0.22263	-0.28430	0.09188	-1.33466	0.44863
500	0.22219	-0.28213	0.09073	-1.32848	0.44464
1000	0.22143	-0.28415	0.09129	-1.34778	0.45548
10000	0.22290	-0.28560	0.09245	-1.33885	0.45112

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.33448	-0.43368	0.14274	-1.35623	0.46365
50	0.33575	-0.43192	0.14051	-1.35070	0.45923
100	0.33631	-0.42844	0.13767	-1.34287	0.45353
200	0.33726	-0.42901	0.13750	-1.34208	0.45284
500	0.33871	-0.42662	0.13694	-1.32375	0.44241
1000	0.33785	-0.42920	0.13703	-1.34227	0.45253
10000	0.33745	-0.42930	0.13721	-1.34436	0.45390

(d) 99% Sig. Lvl.

Table C.27: Params. of Eq. 9.7 fitted to the case IIIb **Lognormal** Cramér-von Mises critical values

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	0.91012	-1.20089	0.40134	-1.34767	0.45797
50	0.91318	-1.20170	0.39920	-1.34609	0.45605
100	0.91653	-1.20262	0.39766	-1.34335	0.45376
200	0.91753	-1.19824	0.39502	-1.33670	0.44973
500	0.91676	-1.19031	0.39053	-1.32939	0.44507
1000	0.91553	-1.19976	0.39428	-1.34510	0.45371
10000	0.91746	-1.19630	0.39259	-1.33688	0.44910

(a) 85% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.05541	-1.39855	0.46917	-1.35572	0.46318
50	1.05824	-1.39467	0.46425	-1.35069	0.45920
100	1.06228	-1.39588	0.46190	-1.34890	0.45726
200	1.06394	-1.38991	0.45838	-1.34032	0.45207
500	1.06282	-1.38230	0.45396	-1.33529	0.44874
1000	1.06015	-1.39105	0.45728	-1.35104	0.45747
10000	1.06506	-1.39268	0.45917	-1.34217	0.45303

(b) 90% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.31102	-1.74578	0.58799	-1.36609	0.46990
50	1.31569	-1.73503	0.57865	-1.35496	0.46229
100	1.31851	-1.73283	0.57331	-1.35427	0.46077
200	1.32354	-1.72684	0.56950	-1.34259	0.45378
500	1.32263	-1.71538	0.56303	-1.33533	0.44915
1000	1.31781	-1.72829	0.56800	-1.35516	0.46034
10000	1.32625	-1.73444	0.57292	-1.34527	0.45541

(c) 95% Sig. Lvl.

	$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}$				
n	θ_1	θ_2	θ_3	θ_4	θ_5
30	1.94582	-2.60784	0.88576	-1.37801	0.47845
50	1.95146	-2.58756	0.86643	-1.36816	0.47086
100	1.95453	-2.55744	0.84586	-1.35302	0.46073
200	1.95369	-2.54534	0.83619	-1.35034	0.45842
500	1.96074	-2.52938	0.82957	-1.33266	0.44811
1000	1.95872	-2.55570	0.83910	-1.35247	0.45940
10000	1.95328	-2.53979	0.83143	-1.34937	0.45744

(d) 99% Sig. Lvl.

Table C.28: Params. of Eq. 9.7 fitted to the case IIIb **Lognormal** Anderson-Darling critical values

C.2 Variable n

C.2.1 Case I: All Parameters are Known

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.13761	-0.15745	-0.06112
0.18	1.13803	-0.18096	0.04475
0.32	1.13823	-0.16777	-0.03758
0.47	1.13832	-0.17808	0.02850
0.60	1.13811	-0.17843	0.05445
0.71	1.13799	-0.16853	-0.00632
0.83	1.13797	-0.16057	-0.03311
0.96	1.13770	-0.16343	-0.01218
1.10	1.13756	-0.15892	-0.05416
1.27	1.13743	-0.15405	-0.09849
1.40	1.13875	-0.18803	0.05621
1.52	1.13825	-0.16874	-0.02554

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.22312	-0.15219	-0.10868
0.18	1.22339	-0.17452	0.00345
0.32	1.22414	-0.16468	-0.10022
0.47	1.22400	-0.17256	-0.01617
0.60	1.22444	-0.18866	0.09806
0.71	1.22360	-0.16088	-0.07518
0.83	1.22453	-0.17832	0.01684
0.96	1.22290	-0.14936	-0.11640
1.10	1.22358	-0.16712	-0.03119
1.27	1.22311	-0.15122	-0.14326
1.40	1.22424	-0.17519	-0.01690
1.52	1.22424	-0.17315	-0.04645

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.35757	-0.16300	-0.10830
0.18	1.35819	-0.19784	0.05531
0.32	1.35891	-0.18087	-0.09441
0.47	1.35827	-0.18330	-0.01974
0.60	1.35826	-0.19167	0.07572
0.71	1.35744	-0.16678	-0.07982
0.83	1.35695	-0.14009	-0.22447
0.96	1.35737	-0.16238	-0.11076
1.10	1.35644	-0.13260	-0.25316
1.27	1.35726	-0.15486	-0.18422
1.40	1.35750	-0.14358	-0.22501
1.52	1.35750	-0.15700	-0.16803

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.62486	-0.09004	-0.69105
0.18	1.62859	-0.21006	-0.07971
0.32	1.62816	-0.26645	0.25053
0.47	1.62835	-0.20784	-0.09025
0.60	1.62691	-0.19218	-0.10351
0.71	1.62657	-0.16177	-0.31357
0.83	1.62625	-0.17381	-0.19257
0.96	1.62625	-0.18111	-0.13714
1.10	1.62441	-0.11878	-0.47708
1.27	1.62646	-0.14453	-0.47441
1.40	1.62679	-0.15048	-0.37986
1.52	1.62417	-0.12931	-0.46646

(d) 99% Sig. Lvl.

Table C.29: Params. of Eq. 9.9 fitted to the case I Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.53657	-0.32436	-0.06141
0.18	1.53696	-0.34481	0.05251
0.32	1.53684	-0.33599	-0.01136
0.47	1.53665	-0.33052	-0.02128
0.60	1.53707	-0.34304	0.03864
0.71	1.53649	-0.33705	0.02458
0.83	1.53684	-0.32970	-0.02693
0.96	1.53679	-0.33887	-0.00515
1.10	1.53639	-0.32750	-0.03150
1.27	1.53636	-0.33437	0.00115
1.40	1.53688	-0.33334	-0.01611
1.52	1.53659	-0.31625	-0.10536

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.61909	-0.32031	-0.13583
0.18	1.61963	-0.33532	-0.05941
0.32	1.61950	-0.33449	-0.07480
0.47	1.61922	-0.33042	-0.07407
0.60	1.61981	-0.34737	0.02588
0.71	1.61908	-0.33914	-0.00187
0.83	1.61981	-0.33722	-0.04013
0.96	1.61988	-0.34821	-0.01934
1.10	1.61893	-0.33298	-0.05604
1.27	1.61917	-0.33486	-0.04854
1.40	1.61940	-0.33052	-0.07618
1.52	1.61898	-0.31352	-0.15989

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.74542	-0.29240	-0.35721
0.18	1.74772	-0.34471	-0.11616
0.32	1.74747	-0.35139	-0.05493
0.47	1.74753	-0.34451	-0.11174
0.60	1.74710	-0.34359	-0.07277
0.71	1.74677	-0.33571	-0.14039
0.83	1.74735	-0.33114	-0.14792
0.96	1.74801	-0.37109	-0.01057
1.10	1.74689	-0.33767	-0.13851
1.27	1.74620	-0.32186	-0.21067
1.40	1.74744	-0.33586	-0.14515
1.52	1.74598	-0.29384	-0.35042

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.99964	-0.26689	-0.85557
0.18	2.00007	-0.32615	-0.44180
0.32	2.00167	-0.40029	-0.07267
0.47	2.00263	-0.40135	-0.06627
0.60	2.00090	-0.36728	-0.18377
0.71	1.99946	-0.31664	-0.48023
0.83	2.00023	-0.32674	-0.34283
0.96	1.99901	-0.30755	-0.58396
1.10	2.00019	-0.31159	-0.51430
1.27	1.99737	-0.27821	-0.66764
1.40	2.00158	-0.34204	-0.36554
1.52	1.99870	-0.30517	-0.52868

(d) 99% Sig. Lvl.

Table C.30: Params. of Eq. 9.9 fitted to the case I Kuiper critical values

C.2.2 Case II: Both Parameters are Known

C.2.2.1 Weibull Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.77963	-0.16760	-0.08741
0.18	0.75849	-0.17185	-0.06919
0.32	0.75205	-0.17706	-0.08747
0.47	0.75243	-0.17532	-0.13763
0.60	0.75605	-0.18452	-0.13167
0.71	0.76067	-0.18362	-0.19494
0.83	0.76559	-0.19476	-0.16544
0.96	0.77077	-0.20464	-0.16888
1.10	0.77566	-0.19997	-0.21284
1.27	0.78089	-0.18544	-0.34480
1.40	0.78529	-0.19868	-0.31683
1.52	0.78837	-0.19889	-0.34424

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.82471	-0.16793	-0.12597
0.18	0.80135	-0.17573	-0.09079
0.32	0.79331	-0.17056	-0.16390
0.47	0.79393	-0.17944	-0.16924
0.60	0.79792	-0.18629	-0.18123
0.71	0.80265	-0.17632	-0.28806
0.83	0.80830	-0.19552	-0.22944
0.96	0.81457	-0.20981	-0.21896
1.10	0.81973	-0.19798	-0.29337
1.27	0.82597	-0.18362	-0.44323
1.40	0.83129	-0.20772	-0.36273
1.52	0.83437	-0.19479	-0.46357

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.89579	-0.18008	-0.14905
0.18	0.86810	-0.17676	-0.15092
0.32	0.85739	-0.15141	-0.33921
0.47	0.85814	-0.17849	-0.26514
0.60	0.86255	-0.18884	-0.26675
0.71	0.86814	-0.17163	-0.41891
0.83	0.87493	-0.19575	-0.34794
0.96	0.88234	-0.19792	-0.41121
1.10	0.88942	-0.20651	-0.39307
1.27	0.89636	-0.18604	-0.56905
1.40	0.90324	-0.21302	-0.51013
1.52	0.90650	-0.19231	-0.64210

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.03806	-0.17028	-0.36218
0.18	1.00226	-0.16297	-0.36364
0.32	0.98693	-0.14912	-0.51634
0.47	0.98660	-0.18852	-0.40783
0.60	0.99298	-0.19641	-0.50461
0.71	1.00029	-0.17340	-0.67760
0.83	1.00902	-0.18630	-0.67097
0.96	1.01927	-0.16082	-0.92487
1.10	1.02939	-0.20462	-0.66785
1.27	1.04062	-0.19578	-0.87933
1.40	1.04903	-0.19789	-0.98726
1.52	1.05340	-0.15864	-1.20153

(d) 99% Sig. Lvl.

Table C.31: Params. of Eq. 9.9 fitted to the case II **Weibull** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.34375	-0.32460	-0.16457
0.18	1.33983	-0.34491	-0.08559
0.32	1.34320	-0.33958	-0.11968
0.47	1.34892	-0.33990	-0.12720
0.60	1.35411	-0.34966	-0.08696
0.71	1.35876	-0.33967	-0.16982
0.83	1.36317	-0.34223	-0.16826
0.96	1.36787	-0.36287	-0.08333
1.10	1.37094	-0.34414	-0.16323
1.27	1.37478	-0.33180	-0.27406
1.40	1.37857	-0.35395	-0.19886
1.52	1.38035	-0.33966	-0.28321

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.41136	-0.31839	-0.26565
0.18	1.40745	-0.34180	-0.16467
0.32	1.41176	-0.34127	-0.17976
0.47	1.41793	-0.33814	-0.19750
0.60	1.42366	-0.33998	-0.19886
0.71	1.42887	-0.33008	-0.28340
0.83	1.43382	-0.33409	-0.28812
0.96	1.43885	-0.35083	-0.21211
1.10	1.44272	-0.34295	-0.24062
1.27	1.44703	-0.32817	-0.37238
1.40	1.45096	-0.35318	-0.27892
1.52	1.45310	-0.34272	-0.33976

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.51677	-0.33013	-0.33836
0.18	1.51282	-0.33869	-0.30953
0.32	1.51768	-0.31890	-0.42529
0.47	1.52631	-0.34018	-0.31515
0.60	1.53316	-0.35056	-0.26973
0.71	1.53781	-0.30082	-0.54048
0.83	1.54368	-0.33192	-0.40509
0.96	1.55092	-0.36808	-0.28460
1.10	1.55466	-0.35024	-0.34127
1.27	1.55935	-0.32781	-0.49708
1.40	1.56410	-0.36599	-0.35124
1.52	1.56593	-0.32874	-0.56491

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.72747	-0.37315	-0.36244
0.18	1.72344	-0.33437	-0.61046
0.32	1.73260	-0.33510	-0.63346
0.47	1.74374	-0.34548	-0.53960
0.60	1.75374	-0.35152	-0.58295
0.71	1.75974	-0.32001	-0.75335
0.83	1.76619	-0.34276	-0.64489
0.96	1.77201	-0.24702	-1.20135
1.10	1.78205	-0.40020	-0.35605
1.27	1.78612	-0.33491	-0.79581
1.40	1.79029	-0.29354	-1.09361
1.52	1.79318	-0.31357	-0.93798

(d) 99% Sig. Lvl.

Table C.32: Params. of Eq. 9.9 fitted to the case II **Weibull** Kuiper critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.08914	-0.00196	-0.00129
0.18	0.08464	-0.00137	-0.00156
0.32	0.08387	-0.00133	-0.00872
0.47	0.08452	0.00032	-0.02989
0.60	0.08566	-0.00202	-0.02579
0.71	0.08707	-0.00484	-0.02586
0.83	0.08842	-0.00810	-0.01635
0.96	0.08984	-0.00987	-0.02017
1.10	0.09106	-0.00620	-0.04405
1.27	0.09255	-0.00294	-0.07954
1.40	0.09384	-0.00751	-0.06896
1.52	0.09469	-0.00771	-0.07641

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.10182	0.00110	-0.02503
0.18	0.09673	0.00018	-0.02092
0.32	0.09585	-0.00073	-0.02144
0.47	0.09672	-0.00117	-0.03147
0.60	0.09811	-0.00383	-0.02724
0.71	0.09968	-0.00233	-0.05283
0.83	0.10134	-0.00663	-0.04179
0.96	0.10317	-0.01018	-0.03903
1.10	0.10464	-0.00548	-0.06932
1.27	0.10648	-0.00229	-0.11060
1.40	0.10812	-0.01031	-0.08594
1.52	0.10920	-0.01216	-0.08510

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.12381	-0.00237	-0.02719
0.18	0.11764	-0.00408	-0.01613
0.32	0.11652	-0.00210	-0.03305
0.47	0.11776	-0.00344	-0.04451
0.60	0.11980	-0.00777	-0.03672
0.71	0.12161	0.00038	-0.09685
0.83	0.12357	0.00096	-0.11218
0.96	0.12633	-0.01146	-0.07697
1.10	0.12850	-0.01084	-0.08941
1.27	0.13087	-0.00411	-0.15203
1.40	0.13292	-0.01474	-0.11866
1.52	0.13405	-0.00652	-0.16923

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.17509	-0.00369	-0.07083
0.18	0.16638	0.00064	-0.08240
0.32	0.16501	0.00381	-0.11359
0.47	0.16743	0.00124	-0.13921
0.60	0.17067	0.00124	-0.16758
0.71	0.17442	-0.01213	-0.11429
0.83	0.17775	-0.00726	-0.17665
0.96	0.18092	0.01284	-0.31567
1.10	0.18526	-0.01202	-0.19965
1.27	0.18893	0.00666	-0.35041
1.40	0.19216	0.00143	-0.36835
1.52	0.19432	-0.00071	-0.37769

(d) 99% Sig. Lvl.

Table C.33: Params. of Eq. 9.9 fitted to the case II **Weibull** Cramér-von Mises critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.56413	-0.02082	-0.06046
0.18	0.55515	-0.01509	-0.04893
0.32	0.56215	-0.01521	-0.07877
0.47	0.57295	-0.00611	-0.18621
0.60	0.58338	-0.02631	-0.09200
0.71	0.59297	-0.01883	-0.22199
0.83	0.60198	-0.03724	-0.15629
0.96	0.61147	-0.05596	-0.13908
1.10	0.61834	-0.02156	-0.30077
1.27	0.62712	-0.00870	-0.45875
1.40	0.63430	-0.02187	-0.46541
1.52	0.63934	-0.03158	-0.46369

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.63489	-0.00894	-0.15638
0.18	0.62513	-0.00355	-0.14106
0.32	0.63381	-0.01289	-0.09970
0.47	0.64683	0.00984	-0.27332
0.60	0.65993	-0.01719	-0.16054
0.71	0.67212	-0.00031	-0.35117
0.83	0.68322	-0.03484	-0.22077
0.96	0.69516	-0.06415	-0.15289
1.10	0.70395	-0.03385	-0.28930
1.27	0.71398	0.00401	-0.59091
1.40	0.72343	-0.03923	-0.46271
1.52	0.72947	-0.04765	-0.45325

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.75780	-0.03422	-0.15834
0.18	0.74573	0.00629	-0.25447
0.32	0.75800	-0.01341	-0.17332
0.47	0.77625	-0.00546	-0.24157
0.60	0.79371	-0.01765	-0.18872
0.71	0.80919	0.00728	-0.42087
0.83	0.82364	-0.02371	-0.27479
0.96	0.84012	-0.06205	-0.23405
1.10	0.85277	-0.03408	-0.34102
1.27	0.86718	-0.01611	-0.60691
1.40	0.87779	-0.03905	-0.55164
1.52	0.88617	-0.04467	-0.58397

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.04106	0.03978	-0.83462
0.18	1.03031	-0.00304	-0.37504
0.32	1.04987	-0.00172	-0.21294
0.47	1.08233	0.01729	-0.31409
0.60	1.11366	-0.02219	-0.16428
0.71	1.14148	-0.05548	-0.09978
0.83	1.16318	-0.01837	-0.21755
0.96	1.18559	0.04570	-0.73023
1.10	1.21082	-0.01058	-0.32080
1.27	1.23516	-0.03212	-0.50186
1.40	1.25177	0.01398	-0.85573
1.52	1.26593	0.04057	-1.17953

(d) 99% Sig. Lvl.

Table C.34: Params. of Eq. 9.9 fitted to the case II **Weibull** Anderson-Darling critical values

C.2.2.2 Loglogistic Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.70897	-0.17091	-0.10777
0.18	0.70359	-0.16507	-0.14103
0.33	0.70723	-0.17144	-0.12026
0.50	0.71330	-0.18181	-0.07639
0.65	0.71810	-0.16397	-0.19437
0.82	0.72232	-0.15774	-0.26540
1.00	0.72610	-0.15418	-0.31833
1.22	0.72949	-0.16363	-0.32068
1.53	0.73262	-0.17726	-0.30103
2.00	0.73544	-0.19155	-0.27117
2.48	0.73681	-0.20361	-0.23395
3.00	0.73795	-0.21608	-0.18792

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.74667	-0.17060	-0.15627
0.18	0.74114	-0.16639	-0.18265
0.33	0.74480	-0.16884	-0.17978
0.50	0.75125	-0.17418	-0.16411
0.65	0.75657	-0.15839	-0.27516
0.82	0.76165	-0.15925	-0.31750
1.00	0.76555	-0.15159	-0.38978
1.22	0.76918	-0.16557	-0.36961
1.53	0.77258	-0.17591	-0.37591
2.00	0.77584	-0.19602	-0.32489
2.48	0.77734	-0.21412	-0.25425
3.00	0.77856	-0.21615	-0.27759

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.80557	-0.17048	-0.24184
0.18	0.79969	-0.16521	-0.27726
0.33	0.80354	-0.17202	-0.24177
0.50	0.81085	-0.17856	-0.23022
0.65	0.81673	-0.16100	-0.34152
0.82	0.82262	-0.15919	-0.41810
1.00	0.82669	-0.14991	-0.48946
1.22	0.83062	-0.16017	-0.50665
1.53	0.83459	-0.17118	-0.51809
2.00	0.83770	-0.18429	-0.48911
2.48	0.84010	-0.22146	-0.34281
3.00	0.84138	-0.21562	-0.40750

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.92455	-0.18670	-0.36342
0.18	0.91878	-0.18002	-0.42131
0.33	0.92230	-0.17675	-0.43836
0.50	0.93063	-0.17876	-0.45986
0.65	0.93787	-0.16806	-0.52725
0.82	0.94456	-0.13997	-0.75481
1.00	0.94938	-0.14150	-0.76786
1.22	0.95601	-0.20307	-0.57090
1.53	0.95943	-0.17020	-0.78352
2.00	0.96384	-0.20378	-0.68922
2.48	0.96685	-0.22597	-0.65868
3.00	0.96860	-0.24894	-0.55464

(d) 99% Sig. Lvl.

Table C.35: Params. of Eq. 9.9 fitted to the case II **Loglogistic** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.27554	-0.35156	0.03475
0.18	1.26558	-0.32864	-0.07375
0.33	1.27361	-0.34337	-0.01127
0.50	1.28503	-0.34165	-0.04953
0.65	1.29513	-0.33010	-0.15923
0.82	1.30325	-0.31735	-0.28120
1.00	1.31062	-0.32184	-0.33278
1.22	1.31600	-0.31414	-0.46713
1.53	1.32195	-0.33472	-0.44821
2.00	1.32704	-0.36328	-0.38123
2.48	1.33052	-0.41066	-0.17867
3.00	1.33180	-0.40996	-0.21929

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.33694	-0.34389	-0.05804
0.18	1.32732	-0.34237	-0.07157
0.33	1.33561	-0.34472	-0.07554
0.50	1.34873	-0.34713	-0.08062
0.65	1.35969	-0.33039	-0.20453
0.82	1.36916	-0.32375	-0.32483
1.00	1.37672	-0.31708	-0.42141
1.22	1.38282	-0.30814	-0.56998
1.53	1.38966	-0.33230	-0.56006
2.00	1.39524	-0.36274	-0.49868
2.48	1.39863	-0.41079	-0.27233
3.00	1.40041	-0.41084	-0.32173

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.43236	-0.34567	-0.16364
0.18	1.42184	-0.34205	-0.15234
0.33	1.43192	-0.34913	-0.14997
0.50	1.44773	-0.35382	-0.15463
0.65	1.46038	-0.33665	-0.27566
0.82	1.47141	-0.32115	-0.45705
1.00	1.47954	-0.29799	-0.63745
1.22	1.48782	-0.31850	-0.66688
1.53	1.49478	-0.33162	-0.69853
2.00	1.50185	-0.37531	-0.60427
2.48	1.50578	-0.41772	-0.44775
3.00	1.50772	-0.41798	-0.49431

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.62364	-0.37493	-0.23948
0.18	1.61242	-0.36635	-0.28484
0.33	1.62324	-0.30635	-0.58747
0.50	1.64579	-0.37494	-0.23200
0.65	1.66112	-0.29566	-0.67664
0.82	1.67580	-0.28299	-0.89466
1.00	1.68571	-0.24797	-1.11181
1.22	1.70163	-0.37675	-0.72354
1.53	1.70695	-0.30007	-1.20231
2.00	1.71670	-0.37462	-1.01531
2.48	1.72248	-0.43659	-0.78826
3.00	1.72804	-0.51698	-0.45637

(d) 99% Sig. Lvl.

Table C.36: Params. of Eq. 9.9 fitted to the case II **Loglogistic** Kuiper critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.07176	-0.00133	0.00948
0.18	0.07015	0.00099	0.00317
0.33	0.07108	-0.00025	0.00519
0.50	0.07275	-0.00147	0.00488
0.65	0.07418	0.00092	-0.01471
0.82	0.07543	0.00420	-0.04193
1.00	0.07665	0.00505	-0.06137
1.22	0.07774	0.00173	-0.06170
1.53	0.07870	0.00059	-0.07296
2.00	0.07963	-0.00598	-0.05473
2.48	0.08025	-0.01323	-0.02856
3.00	0.08050	-0.01451	-0.02640

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.08140	-0.00310	0.01313
0.18	0.07956	0.00082	-0.00023
0.33	0.08060	0.00019	-0.00196
0.50	0.08269	-0.00221	0.00342
0.65	0.08446	0.00097	-0.02274
0.82	0.08602	0.00421	-0.05316
1.00	0.08746	0.00546	-0.07541
1.22	0.08876	0.00281	-0.08500
1.53	0.08998	-0.00039	-0.08957
2.00	0.09106	-0.00664	-0.07518
2.48	0.09175	-0.01482	-0.04548
3.00	0.09211	-0.01638	-0.04566

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.09775	-0.00197	-0.00841
0.18	0.09569	-0.00145	-0.00065
0.33	0.09699	0.00072	-0.01558
0.50	0.09970	-0.00311	-0.00052
0.65	0.10208	0.00151	-0.03898
0.82	0.10418	0.00585	-0.08252
1.00	0.10606	0.00512	-0.09380
1.22	0.10779	0.00500	-0.13093
1.53	0.10949	-0.00341	-0.10927
2.00	0.11097	-0.00928	-0.11056
2.48	0.11198	-0.02127	-0.06812
3.00	0.11235	-0.02017	-0.08372

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.13573	-0.00525	-0.03067
0.18	0.13307	-0.00297	-0.03160
0.33	0.13543	0.00079	-0.05715
0.50	0.13977	-0.00655	-0.01681
0.65	0.14377	-0.00107	-0.05832
0.82	0.14731	0.00776	-0.14803
1.00	0.14972	0.02073	-0.22222
1.22	0.15355	0.00305	-0.21403
1.53	0.15613	-0.00425	-0.20745
2.00	0.15831	-0.01176	-0.21693
2.48	0.16051	-0.04390	-0.09578
3.00	0.16165	-0.05491	-0.05702

(d) 99% Sig. Lvl.

Table C.37: Params. of Eq. 9.9 fitted to the case II **Loglogistic** Cramér-von Mises critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.50223	-0.00589	-0.03764
0.18	0.49669	0.00159	0.00970
0.33	0.50367	-0.00005	-0.01852
0.50	0.51377	-0.00632	-0.01675
0.65	0.52190	-0.00256	-0.07604
0.82	0.52891	0.02152	-0.27994
1.00	0.53561	0.01274	-0.29084
1.22	0.54111	0.01408	-0.38724
1.53	0.54652	-0.00897	-0.35511
2.00	0.55199	-0.04502	-0.26812
2.48	0.55528	-0.08154	-0.13909
3.00	0.55606	-0.07322	-0.19850

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.56313	-0.02014	-0.01519
0.18	0.55667	0.00290	-0.01526
0.33	0.56526	-0.00021	-0.03033
0.50	0.57703	-0.00677	-0.00339
0.65	0.58755	-0.00270	-0.09930
0.82	0.59634	0.01165	-0.25411
1.00	0.60391	0.02104	-0.36532
1.22	0.61045	0.02393	-0.49198
1.53	0.61762	-0.00877	-0.42711
2.00	0.62371	-0.03906	-0.38809
2.48	0.62770	-0.09125	-0.17542
3.00	0.62925	-0.09407	-0.20168

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.66552	-0.01493	-0.10190
0.18	0.65936	-0.00576	-0.00888
0.33	0.66995	0.00621	-0.05695
0.50	0.68731	-0.04392	0.18737
0.65	0.69950	0.01592	-0.15626
0.82	0.71183	0.00948	-0.23139
1.00	0.72205	0.02064	-0.38480
1.22	0.73142	0.02334	-0.58155
1.53	0.73989	-0.00247	-0.55157
2.00	0.74833	-0.04753	-0.49787
2.48	0.75479	-0.13379	-0.16359
3.00	0.75590	-0.11509	-0.28912

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.90354	-0.01274	-0.29414
0.18	0.89999	-0.04459	0.14143
0.33	0.91628	0.03411	-0.10610
0.50	0.94446	-0.04138	0.33714
0.65	0.96764	-0.03510	0.31562
0.82	0.98464	0.10261	-0.58253
1.00	1.00005	0.08080	-0.51771
1.22	1.02130	0.01167	-0.66238
1.53	1.03695	-0.06585	-0.51562
2.00	1.04897	-0.16275	-0.10653
2.48	1.05683	-0.16906	-0.35441
3.00	1.06295	-0.27500	0.20044

(d) 99% Sig. Lvl.

Table C.38: Params. of Eq. 9.9 fitted to the case II **Loglogistic** Anderson-Darling critical values

C.2.2.3 Lognormal Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.78961	-0.17235	-0.03681
0.72	0.76317	-0.15869	-0.05910
0.75	0.74903	-0.16132	-0.02278
0.79	0.74270	-0.16611	0.02089
0.85	0.74232	-0.17105	0.03619
0.91	0.74424	-0.16287	-0.03727
1.00	0.74840	-0.17946	0.00559
1.12	0.75250	-0.16613	-0.10240
1.29	0.75802	-0.16213	-0.20858
1.58	0.76435	-0.15696	-0.33526
1.89	0.76885	-0.14223	-0.48436
2.24	0.77327	-0.16070	-0.48010

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.83616	-0.17093	-0.08671
0.72	0.80768	-0.15548	-0.10363
0.75	0.79168	-0.16170	-0.04329
0.79	0.78422	-0.16719	-0.00651
0.85	0.78322	-0.17021	0.01096
0.91	0.78503	-0.16232	-0.05346
1.00	0.78918	-0.17585	-0.02444
1.12	0.79403	-0.16558	-0.13235
1.29	0.80033	-0.15544	-0.29685
1.58	0.80745	-0.14954	-0.43727
1.89	0.81263	-0.14402	-0.53680
2.24	0.81741	-0.15492	-0.58285

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.90891	-0.15681	-0.22832
0.72	0.87764	-0.15862	-0.13356
0.75	0.85817	-0.15530	-0.12835
0.79	0.84893	-0.16579	-0.06944
0.85	0.84663	-0.16775	-0.03825
0.91	0.84887	-0.16642	-0.07959
1.00	0.85272	-0.16440	-0.12486
1.12	0.85820	-0.14950	-0.27085
1.29	0.86630	-0.16035	-0.34002
1.58	0.87393	-0.13052	-0.60825
1.89	0.88056	-0.12754	-0.72853
2.24	0.88650	-0.14023	-0.79921

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.05675	-0.11850	-0.59688
0.72	1.02049	-0.17399	-0.18006
0.75	0.99492	-0.17083	-0.19147
0.79	0.97929	-0.16059	-0.20002
0.85	0.97387	-0.15376	-0.20224
0.91	0.97569	-0.14658	-0.24795
1.00	0.98032	-0.14117	-0.35748
1.12	0.98757	-0.13727	-0.39940
1.29	0.99859	-0.15413	-0.50388
1.58	1.00961	-0.10030	-0.96581
1.89	1.01775	-0.07574	-1.21644
2.24	1.02781	-0.14053	-1.12458

(d) 99% Sig. Lvl.

Table C.39: Params. of Eq. 9.9 fitted to the case II **Lognormal** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.33593	-0.32890	-0.08255
0.72	1.31629	-0.32158	-0.07579
0.75	1.31811	-0.32891	-0.07535
0.79	1.32477	-0.31970	-0.14362
0.85	1.33193	-0.33115	-0.10696
0.91	1.33838	-0.33107	-0.12610
1.00	1.34499	-0.34803	-0.07571
1.12	1.35017	-0.32809	-0.20721
1.29	1.35606	-0.32776	-0.26532
1.58	1.36185	-0.32049	-0.34764
1.89	1.36523	-0.29810	-0.51515
2.24	1.36907	-0.32306	-0.45413

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.40228	-0.32956	-0.14412
0.72	1.38134	-0.31678	-0.15090
0.75	1.38384	-0.31894	-0.19233
0.79	1.39188	-0.32309	-0.18539
0.85	1.39978	-0.32882	-0.18237
0.91	1.40683	-0.32193	-0.23444
1.00	1.41383	-0.33506	-0.19597
1.12	1.41984	-0.32219	-0.30079
1.29	1.42656	-0.32899	-0.33937
1.58	1.43230	-0.30273	-0.51020
1.89	1.43688	-0.30423	-0.55718
2.24	1.44103	-0.32910	-0.52391

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.50468	-0.31689	-0.32791
0.72	1.48226	-0.32165	-0.20321
0.75	1.48622	-0.32614	-0.25316
0.79	1.49614	-0.32299	-0.30627
0.85	1.50505	-0.32017	-0.31701
0.91	1.51373	-0.32196	-0.35211
1.00	1.52106	-0.31706	-0.40637
1.12	1.52885	-0.32402	-0.41017
1.29	1.53673	-0.33767	-0.42981
1.58	1.54347	-0.30591	-0.65737
1.89	1.54791	-0.29613	-0.75045
2.24	1.55365	-0.33811	-0.65019

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.70732	-0.28066	-0.73170
0.72	1.68467	-0.35872	-0.22946
0.75	1.69098	-0.34766	-0.34230
0.79	1.70629	-0.34744	-0.41518
0.85	1.71638	-0.28487	-0.73307
0.91	1.72973	-0.32101	-0.63720
1.00	1.73995	-0.33927	-0.61097
1.12	1.74764	-0.34269	-0.48912
1.29	1.76012	-0.37046	-0.63853
1.58	1.76730	-0.31147	-0.97058
1.89	1.77267	-0.29740	-1.01779
2.24	1.77947	-0.33228	-1.03618

(d) 99% Sig. Lvl.

Table C.40: Params. of Eq. 9.9 fitted to the case II **Lognormal** Kuiper critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.09049	-0.00081	0.00265
0.72	0.08329	0.00344	0.00236
0.75	0.08098	0.00233	0.00842
0.79	0.08064	0.00090	0.01375
0.85	0.08126	0.00029	0.01157
0.91	0.08224	-0.00145	0.01618
1.00	0.08349	-0.00239	0.01151
1.12	0.08470	0.00154	-0.01918
1.29	0.08627	0.00345	-0.05205
1.58	0.08794	0.00754	-0.09698
1.89	0.08919	0.00918	-0.12394
2.24	0.09036	0.00616	-0.13418

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.10366	-0.00217	-0.00126
0.72	0.09521	0.00414	-0.00416
0.75	0.09257	0.00088	0.01259
0.79	0.09208	0.00333	-0.00172
0.85	0.09283	0.00077	0.00626
0.91	0.09402	0.00034	0.00154
1.00	0.09555	-0.00308	0.01018
1.12	0.09694	0.00492	-0.04457
1.29	0.09900	0.00174	-0.05684
1.58	0.10094	0.00887	-0.11907
1.89	0.10250	0.01099	-0.15681
2.24	0.10395	0.00622	-0.16214

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.12584	0.00721	-0.06294
0.72	0.11575	0.00361	-0.00813
0.75	0.11245	-0.00015	0.01174
0.79	0.11198	0.00016	0.00490
0.85	0.11277	0.00315	-0.01448
0.91	0.11423	0.00410	-0.02540
1.00	0.11631	0.00015	-0.02205
1.12	0.11819	0.00789	-0.07201
1.29	0.12084	0.00464	-0.09216
1.58	0.12356	0.01158	-0.16712
1.89	0.12571	0.01251	-0.20795
2.24	0.12749	0.01046	-0.23671

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.17808	0.02120	-0.18133
0.72	0.16451	-0.00623	0.00945
0.75	0.15932	-0.00264	-0.00415
0.79	0.15872	0.00247	-0.02784
0.85	0.16008	0.00403	-0.04063
0.91	0.16289	0.00398	-0.05119
1.00	0.16621	0.00104	-0.07237
1.12	0.16895	0.01082	-0.11523
1.29	0.17343	0.00856	-0.19098
1.58	0.17748	0.03019	-0.36078
1.89	0.18070	0.03147	-0.38888
2.24	0.18359	0.03676	-0.50601

(d) 99% Sig. Lvl.

Table C.41: Params. of Eq. 9.9 fitted to the case II **Lognormal** Cramér-von Mises critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.56011	-0.00770	-0.07012
0.72	0.53356	0.01951	-0.02093
0.75	0.53448	0.00834	0.00002
0.79	0.54248	0.01123	-0.03713
0.85	0.55229	0.00339	-0.03974
0.91	0.56136	0.00359	-0.06077
1.00	0.57156	-0.01508	-0.01268
1.12	0.58019	0.01630	-0.24050
1.29	0.59074	0.01716	-0.35813
1.58	0.60084	0.03894	-0.57218
1.89	0.60853	0.05128	-0.73658
2.24	0.61548	0.02665	-0.73138

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.63076	0.00040	-0.15078
0.72	0.60009	0.02430	-0.04342
0.75	0.60151	0.00907	0.00665
0.79	0.61172	0.00302	0.00738
0.85	0.62262	0.01904	-0.10468
0.91	0.63347	0.02693	-0.14892
1.00	0.64646	-0.01155	-0.02010
1.12	0.65639	0.04268	-0.33297
1.29	0.66962	0.02890	-0.40556
1.58	0.68192	0.06345	-0.70566
1.89	0.69216	0.06039	-0.83654
2.24	0.70001	0.05181	-0.94546

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.75099	0.02021	-0.33534
0.72	0.71476	0.01307	0.00078
0.75	0.71678	-0.01608	0.15865
0.79	0.72989	0.00395	0.05928
0.85	0.74452	0.01955	-0.03529
0.91	0.75977	0.02442	-0.08851
1.00	0.77711	-0.03618	0.18423
1.12	0.78960	0.05055	-0.30176
1.29	0.80730	0.04979	-0.49822
1.58	0.82387	0.10670	-0.93292
1.89	0.83765	0.08909	-1.04095
2.24	0.84773	0.07996	-1.18478

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.03185	0.08138	-0.96057
0.72	0.98640	-0.03688	0.09022
0.75	0.98649	-0.01108	0.20817
0.79	1.01069	-0.02920	0.41329
0.85	1.03210	0.07975	-0.04466
0.91	1.05858	0.07952	0.08730
1.00	1.08719	0.04281	0.13561
1.12	1.11217	0.03843	0.16674
1.29	1.14316	-0.01803	0.09986
1.58	1.16640	0.22365	-1.26948
1.89	1.18624	0.25129	-1.59383
2.24	1.20469	0.19279	-1.64444

(d) 99% Sig. Lvl.

Table C.42: Params. of Eq. 9.9 fitted to the case II **Lognormal** Anderson-Darling critical values

C.2.3 Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

C.2.3.1 Weibull Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.93161	-0.15760	-0.07763
0.18	0.93234	-0.17066	-0.05026
0.32	0.93194	-0.16708	-0.02759
0.47	0.93253	-0.16956	-0.04004
0.60	0.93162	-0.16544	-0.03520
0.71	0.93234	-0.17301	-0.01621
0.83	0.93187	-0.16053	-0.06039
0.96	0.93162	-0.16073	-0.07056
1.10	0.93195	-0.16259	-0.05566
1.27	0.93212	-0.16558	-0.05194
1.40	0.93136	-0.15237	-0.10102
1.52	0.93171	-0.15371	-0.10063

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.99386	-0.14106	-0.20293
0.18	0.99520	-0.17552	-0.05462
0.32	0.99492	-0.16995	-0.07009
0.47	0.99479	-0.15993	-0.11215
0.60	0.99418	-0.17047	-0.03161
0.71	0.99530	-0.18189	0.00176
0.83	0.99440	-0.15842	-0.11514
0.96	0.99407	-0.15751	-0.12707
1.10	0.99506	-0.16924	-0.07242
1.27	0.99487	-0.16667	-0.08248
1.40	0.99355	-0.14217	-0.18500
1.52	0.99459	-0.15441	-0.12947

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.09249	-0.13904	-0.27779
0.18	1.09410	-0.18111	-0.09457
0.32	1.09344	-0.16266	-0.17708
0.47	1.09276	-0.13941	-0.27474
0.60	1.09289	-0.17325	-0.11146
0.71	1.09360	-0.17957	-0.08214
0.83	1.09277	-0.14224	-0.28420
0.96	1.09211	-0.15023	-0.22095
1.10	1.09328	-0.16353	-0.16627
1.27	1.09371	-0.16797	-0.16710
1.40	1.09256	-0.14870	-0.24154
1.52	1.09349	-0.15588	-0.19198

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.29264	-0.16220	-0.33800
0.18	1.29469	-0.20164	-0.21374
0.32	1.29255	-0.13690	-0.46607
0.47	1.29244	-0.12150	-0.62313
0.60	1.29290	-0.16445	-0.37005
0.71	1.29519	-0.20333	-0.21777
0.83	1.29443	-0.18248	-0.33144
0.96	1.29165	-0.16275	-0.31200
1.10	1.29437	-0.18994	-0.24950
1.27	1.29239	-0.14834	-0.45029
1.40	1.29404	-0.18333	-0.27910
1.52	1.29459	-0.18851	-0.22816

(d) 99% Sig. Lvl.

Table C.43: Params. of Eq. 9.9 fitted to the case IIIa **Weibull** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.46129	-0.32835	0.00019
0.18	1.46236	-0.33923	0.01120
0.32	1.46161	-0.32375	-0.04404
0.47	1.46255	-0.34735	0.07032
0.60	1.46143	-0.33169	0.00508
0.71	1.46200	-0.33659	0.02891
0.83	1.46249	-0.34039	0.03451
0.96	1.46101	-0.33092	0.02638
1.10	1.46154	-0.32024	-0.05005
1.27	1.46142	-0.31938	-0.06769
1.40	1.46152	-0.32379	-0.03273
1.52	1.46156	-0.31315	-0.10091

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.53876	-0.30342	-0.18300
0.18	1.54073	-0.34405	-0.01606
0.32	1.54030	-0.33426	-0.04471
0.47	1.54066	-0.34026	-0.00948
0.60	1.54004	-0.33462	-0.00350
0.71	1.54034	-0.34134	-0.00139
0.83	1.53984	-0.31468	-0.13557
0.96	1.53899	-0.32541	-0.05790
1.10	1.53961	-0.31513	-0.12814
1.27	1.53935	-0.30941	-0.17058
1.40	1.53975	-0.32825	-0.06018
1.52	1.53971	-0.31725	-0.10180

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.66047	-0.30967	-0.24178
0.18	1.66299	-0.35641	-0.03702
0.32	1.66182	-0.32999	-0.14912
0.47	1.66122	-0.31149	-0.24215
0.60	1.66140	-0.35137	-0.01855
0.71	1.66239	-0.35626	0.01455
0.83	1.66205	-0.33933	-0.11241
0.96	1.66025	-0.31913	-0.18281
1.10	1.66131	-0.31354	-0.21613
1.27	1.66160	-0.33689	-0.11468
1.40	1.66093	-0.31356	-0.22581
1.52	1.66241	-0.34727	-0.03085

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.90409	-0.32316	-0.31665
0.18	1.90286	-0.26053	-0.63464
0.32	1.90567	-0.33088	-0.33795
0.47	1.90416	-0.27481	-0.61233
0.60	1.90447	-0.33794	-0.28521
0.71	1.90760	-0.40678	0.07385
0.83	1.90679	-0.34080	-0.34151
0.96	1.90679	-0.37290	-0.16687
1.10	1.90604	-0.32932	-0.36733
1.27	1.90511	-0.33479	-0.32268
1.40	1.90526	-0.33464	-0.28426
1.52	1.90499	-0.31441	-0.38459

(d) 99% Sig. Lvl.

Table C.44: Params. of Eq. 9.9 fitted to the case IIIa **Weibull** Kuiper critical values

C.2.3.2 Loglogistic Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.79236	-0.17741	-0.03703
0.18	0.79283	-0.17937	-0.03785
0.33	0.79224	-0.16827	-0.08116
0.50	0.79221	-0.16567	-0.09556
0.65	0.79218	-0.15934	-0.13703
0.82	0.79234	-0.17302	-0.05385
1.00	0.79233	-0.16503	-0.10503
1.22	0.79255	-0.17054	-0.08809
1.53	0.79233	-0.16190	-0.12090
2.00	0.79177	-0.15759	-0.13391
2.48	0.79242	-0.17434	-0.04891
3.00	0.79217	-0.16505	-0.09857

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.83726	-0.17621	-0.08309
0.18	0.83802	-0.18138	-0.07939
0.33	0.83748	-0.17282	-0.09862
0.50	0.83733	-0.17052	-0.11026
0.65	0.83708	-0.15446	-0.21377
0.82	0.83761	-0.18186	-0.05128
1.00	0.83773	-0.16947	-0.13146
1.22	0.83752	-0.16912	-0.13631
1.53	0.83746	-0.16120	-0.17702
2.00	0.83708	-0.16345	-0.15536
2.48	0.83725	-0.16844	-0.11636
3.00	0.83743	-0.17224	-0.09509

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	0.90742	-0.18359	-0.11401
0.18	0.90796	-0.17858	-0.17343
0.33	0.90744	-0.17699	-0.14900
0.50	0.90732	-0.16368	-0.22871
0.65	0.90666	-0.15233	-0.28744
0.82	0.90769	-0.17921	-0.15414
1.00	0.90826	-0.18200	-0.14995
1.22	0.90752	-0.17403	-0.18079
1.53	0.90788	-0.17987	-0.15046
2.00	0.90758	-0.17120	-0.20202
2.48	0.90707	-0.16430	-0.22416
3.00	0.90774	-0.18630	-0.09185

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.04705	-0.13857	-0.53153
0.18	1.04737	-0.14943	-0.49256
0.33	1.04841	-0.18139	-0.34287
0.50	1.04780	-0.18653	-0.30068
0.65	1.04680	-0.13177	-0.60703
0.82	1.04875	-0.20039	-0.28083
1.00	1.04778	-0.15950	-0.43826
1.22	1.04825	-0.17653	-0.36458
1.53	1.04857	-0.16994	-0.39323
2.00	1.04711	-0.14949	-0.49846
2.48	1.04658	-0.14398	-0.50099
3.00	1.04887	-0.20731	-0.17892

(d) 99% Sig. Lvl.

Table C.45: Params. of Eq. 9.9 fitted to the case IIIa **Loglogistic** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.43265	-0.33997	0.08491
0.18	1.43343	-0.35451	0.13812
0.33	1.43290	-0.33269	0.04554
0.50	1.43286	-0.34173	0.10524
0.65	1.43281	-0.32355	-0.00941
0.82	1.43314	-0.34732	0.11394
1.00	1.43322	-0.33355	0.03990
1.22	1.43279	-0.32834	0.00262
1.53	1.43332	-0.33400	0.04441
2.00	1.43186	-0.30968	-0.06791
2.48	1.43278	-0.33647	0.08065
3.00	1.43233	-0.31951	-0.00985

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.51045	-0.33996	0.04620
0.18	1.51139	-0.34990	0.06883
0.33	1.51089	-0.33774	0.03926
0.50	1.51040	-0.32814	0.00457
0.65	1.51040	-0.31527	-0.09753
0.82	1.51120	-0.34921	0.09104
1.00	1.51102	-0.32987	-0.01328
1.22	1.51091	-0.33464	0.00139
1.53	1.51142	-0.32992	-0.03065
2.00	1.50965	-0.31028	-0.09305
2.48	1.51092	-0.34151	0.05923
3.00	1.51114	-0.34468	0.07588

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.63242	-0.34348	0.00376
0.18	1.63348	-0.35057	-0.00424
0.33	1.63365	-0.37323	0.16687
0.50	1.63268	-0.33456	-0.06316
0.65	1.63165	-0.28948	-0.30058
0.82	1.63288	-0.35000	0.02891
1.00	1.63390	-0.34385	-0.05206
1.22	1.63311	-0.34311	-0.02988
1.53	1.63322	-0.34749	-0.00104
2.00	1.63201	-0.31885	-0.12358
2.48	1.63234	-0.34192	-0.00259
3.00	1.63338	-0.36060	0.10948

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.87622	-0.27341	-0.50834
0.18	1.87809	-0.31995	-0.27904
0.33	1.88123	-0.40641	0.09500
0.50	1.87730	-0.32854	-0.22556
0.65	1.87738	-0.27702	-0.59146
0.82	1.88155	-0.41979	0.15637
1.00	1.87837	-0.29193	-0.44960
1.22	1.88099	-0.39378	0.03059
1.53	1.87835	-0.33453	-0.21344
2.00	1.87906	-0.31611	-0.40243
2.48	1.87621	-0.27024	-0.53463
3.00	1.88114	-0.42836	0.29418

(d) 99% Sig. Lvl.

Table C.46: Params. of Eq. 9.9 fitted to the case IIIa **Loglogistic** Kuiper critical values

C.2.3.3 Lognormal Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.83904	-0.16660	-0.09465
0.72	0.83029	-0.17036	-0.05613
0.75	0.83269	-0.17533	-0.01348
0.79	0.84038	-0.16572	-0.07320
0.85	0.84849	-0.16530	-0.05819
0.91	0.85591	-0.15894	-0.08236
1.00	0.86295	-0.14716	-0.14004
1.12	0.87063	-0.16342	-0.06131
1.29	0.87795	-0.16276	-0.07246
1.58	0.88569	-0.17266	0.00526
1.89	0.88987	-0.16021	-0.04843
2.24	0.89470	-0.17982	0.03489

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.88771	-0.17149	-0.10812
0.72	0.87835	-0.17811	-0.04914
0.75	0.88092	-0.16991	-0.07812
0.79	0.89042	-0.16774	-0.09243
0.85	0.89953	-0.15591	-0.13720
0.91	0.90884	-0.16259	-0.09888
1.00	0.91686	-0.14992	-0.16335
1.12	0.92621	-0.17673	-0.04368
1.29	0.93393	-0.16094	-0.12251
1.58	0.94291	-0.18103	0.01031
1.89	0.94776	-0.16237	-0.06117
2.24	0.95275	-0.17895	0.00968

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	0.96298	-0.17257	-0.18984
0.72	0.95226	-0.17458	-0.14619
0.75	0.95578	-0.15599	-0.21148
0.79	0.96811	-0.16208	-0.16741
0.85	0.97949	-0.13978	-0.27375
0.91	0.99110	-0.15229	-0.21393
1.00	1.00084	-0.13385	-0.29078
1.12	1.01214	-0.16161	-0.18088
1.29	1.02203	-0.16542	-0.14623
1.58	1.03189	-0.16356	-0.11655
1.89	1.03890	-0.17039	-0.08631
2.24	1.04340	-0.15493	-0.15672

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.11280	-0.16342	-0.45334
0.72	1.10058	-0.17990	-0.32406
0.75	1.10693	-0.14028	-0.46500
0.79	1.12595	-0.14003	-0.46510
0.85	1.14296	-0.12676	-0.48747
0.91	1.16083	-0.18156	-0.25573
1.00	1.17370	-0.12733	-0.50632
1.12	1.18698	-0.11796	-0.54641
1.29	1.20255	-0.18185	-0.23037
1.58	1.21483	-0.17048	-0.21531
1.89	1.22429	-0.17487	-0.26483
2.24	1.23076	-0.19953	-0.09813

(d) 99% Sig. Lvl.

Table C.47: Params. of Eq. 9.9 fitted to the case IIIa **Lognormal** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.46042	-0.32552	0.00039
0.72	1.45643	-0.33600	0.05212
0.75	1.45389	-0.34350	0.09970
0.79	1.45199	-0.32378	-0.04144
0.85	1.45191	-0.32313	-0.03269
0.91	1.45150	-0.32458	-0.01495
1.00	1.45074	-0.29420	-0.16758
1.12	1.45223	-0.32595	-0.02742
1.29	1.45278	-0.32820	-0.02446
1.58	1.45429	-0.34979	0.12350
1.89	1.45434	-0.33270	0.03452
2.24	1.45520	-0.34069	0.06462

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.53848	-0.31077	-0.12073
0.72	1.53490	-0.33886	0.03851
0.75	1.53233	-0.34010	0.02520
0.79	1.53052	-0.33259	-0.02890
0.85	1.52963	-0.31077	-0.13074
0.91	1.52935	-0.31056	-0.15615
1.00	1.52852	-0.28362	-0.26590
1.12	1.53055	-0.32693	-0.07246
1.29	1.53108	-0.32851	-0.05891
1.58	1.53243	-0.35341	0.10006
1.89	1.53210	-0.32101	-0.06422
2.24	1.53362	-0.35424	0.09224

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.66145	-0.33476	-0.07630
0.72	1.65725	-0.34248	-0.02262
0.75	1.65398	-0.32624	-0.13249
0.79	1.65157	-0.30607	-0.23830
0.85	1.65102	-0.29886	-0.27331
0.91	1.65153	-0.32054	-0.18519
1.00	1.65068	-0.29947	-0.26927
1.12	1.65192	-0.31429	-0.23510
1.29	1.65253	-0.32412	-0.15051
1.58	1.65385	-0.35384	0.02924
1.89	1.65405	-0.32876	-0.11587
2.24	1.65512	-0.36653	0.08187

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.90642	-0.34756	-0.22268
0.72	1.90244	-0.34681	-0.20556
0.75	1.89728	-0.26540	-0.65994
0.79	1.89678	-0.29989	-0.49732
0.85	1.89374	-0.23023	-0.78455
0.91	1.89642	-0.30917	-0.39739
1.00	1.89539	-0.29257	-0.49472
1.12	1.89556	-0.29417	-0.50321
1.29	1.89886	-0.35705	-0.21334
1.58	1.89734	-0.33375	-0.22731
1.89	1.89948	-0.35373	-0.18491
2.24	1.89999	-0.37051	-0.11105

(d) 99% Sig. Lvl.

Table C.48: Params. of Eq. 9.9 fitted to the case IIIa **Lognormal** Kuiper critical values

C.2.3.4 Pareto Distribution: Kolmogorov-Smirnov test and Kuiper's test

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	0.93256	-0.17777	0.00546
3.16	0.93173	-0.16068	-0.05230
10.00	0.93275	-0.18329	0.02934
15.81	0.93150	-0.15631	-0.09118
22.36	0.93230	-0.16659	-0.04011
31.62	0.93191	-0.16141	-0.06185

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	0.99494	-0.16978	-0.07498
3.16	0.99456	-0.16568	-0.06213
10.00	0.99558	-0.18834	0.01406
15.81	0.99413	-0.15069	-0.15883
22.36	0.99551	-0.17546	-0.03965
31.62	0.99470	-0.16320	-0.09409

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	1.09308	-0.14820	-0.25588
3.16	1.09293	-0.15820	-0.18598
10.00	1.09450	-0.18736	-0.09494
15.81	1.09336	-0.15484	-0.23002
22.36	1.09416	-0.17230	-0.14826
31.62	1.09280	-0.15593	-0.19508

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	1.29274	-0.14955	-0.45152
3.16	1.29202	-0.14116	-0.45700
10.00	1.29429	-0.19000	-0.27123
15.81	1.29349	-0.15940	-0.42258
22.36	1.29478	-0.18862	-0.25308
31.62	1.29507	-0.20946	-0.16957

(d) 99% Sig. Lvl.

Table C.49: Params. of Eq. 9.9 fitted to the case IIIa **Pareto** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	1.46233	-0.34239	0.04189
3.16	1.46175	-0.32261	-0.02778
10.00	1.46265	-0.35100	0.07195
15.81	1.46200	-0.33811	0.03881
22.36	1.46230	-0.34062	0.06794
31.62	1.46136	-0.32204	-0.03016

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	1.54051	-0.33832	-0.03233
3.16	1.53949	-0.31611	-0.10041
10.00	1.54041	-0.33996	-0.02556
15.81	1.54017	-0.33069	-0.05409
22.36	1.54114	-0.35447	0.07551
31.62	1.53907	-0.30740	-0.13176

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	1.66088	-0.31222	-0.23582
3.16	1.66175	-0.33300	-0.10443
10.00	1.66268	-0.35116	-0.07129
15.81	1.66181	-0.32357	-0.19231
22.36	1.66337	-0.36172	0.00655
31.62	1.66114	-0.32053	-0.15615

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
1.00	1.90554	-0.31425	-0.42815
3.16	1.90298	-0.26209	-0.63057
10.00	1.90659	-0.32140	-0.48104
15.81	1.90623	-0.34121	-0.31395
22.36	1.90762	-0.35251	-0.31349
31.62	1.90551	-0.32024	-0.44370

(d) 99% Sig. Lvl.

Table C.50: Params. of Eq. 9.9 fitted to the case IIIa **Pareto** Kuiper critical values

C.2.4 Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

C.2.4.1 Weibull Distribution:

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.09658	-0.17042	0.01770
0.18	1.10531	-0.18397	0.16408
0.32	1.10855	-0.13949	-0.00432
0.47	1.09813	-0.17569	0.10147
0.60	1.07628	-0.15559	-0.01049
0.71	1.05518	-0.16877	0.03266
0.83	1.03498	-0.16156	-0.04800
0.96	1.01638	-0.15889	-0.09073
1.10	1.00014	-0.17269	-0.01032
1.27	0.98479	-0.15866	-0.13401
1.40	0.97508	-0.16378	-0.06463
1.52	0.96987	-0.17300	-0.03005

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.18173	-0.15484	-0.08605
0.18	1.19177	-0.18295	0.13976
0.32	1.19565	-0.15168	0.04674
0.47	1.18315	-0.17027	0.04941
0.60	1.15898	-0.16710	0.03284
0.71	1.13458	-0.17943	0.05810
0.83	1.11104	-0.16110	-0.08979
0.96	1.08971	-0.14554	-0.20335
1.10	1.07128	-0.16783	-0.08753
1.27	1.05398	-0.16153	-0.14900
1.40	1.04282	-0.15725	-0.13837
1.52	1.03761	-0.18289	-0.04364

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.31624	-0.17584	-0.02447
0.18	1.32805	-0.20522	0.14655
0.32	1.33182	-0.15833	0.02153
0.47	1.31524	-0.14487	-0.12217
0.60	1.28680	-0.17242	0.01424
0.71	1.25855	-0.18683	0.05336
0.83	1.23095	-0.16153	-0.17033
0.96	1.20526	-0.14852	-0.26192
1.10	1.18313	-0.15979	-0.23105
1.27	1.16255	-0.14788	-0.30409
1.40	1.14965	-0.15274	-0.23544
1.52	1.14400	-0.19831	-0.06955

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.58364	-0.10989	-0.65479
0.18	1.59788	-0.18722	-0.13574
0.32	1.60494	-0.19107	-0.07180
0.47	1.58105	-0.14345	-0.20631
0.60	1.54590	-0.16001	-0.29314
0.71	1.50664	-0.18919	-0.14353
0.83	1.47055	-0.15106	-0.44478
0.96	1.43736	-0.14595	-0.45098
1.10	1.41185	-0.24316	-0.07472
1.27	1.38285	-0.16911	-0.46767
1.40	1.36698	-0.18192	-0.32624
1.52	1.35782	-0.23062	-0.14647

(d) 99% Sig. Lvl.

Table C.51: Params. of Eq. 9.9 fitted to the case IIIb **Weibull** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.42837	-0.33706	-0.12049
0.18	1.43368	-0.34570	-0.13789
0.32	1.44849	-0.33030	-0.21876
0.47	1.46440	-0.33845	-0.17405
0.60	1.47063	-0.32311	-0.15298
0.71	1.47426	-0.33944	-0.05222
0.83	1.47498	-0.33639	-0.05172
0.96	1.47358	-0.32089	-0.11144
1.10	1.47262	-0.32743	-0.05712
1.27	1.47149	-0.33658	-0.02961
1.40	1.46959	-0.33177	-0.01945
1.52	1.46987	-0.34989	0.05963

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.50446	-0.33979	-0.14238
0.18	1.50961	-0.34572	-0.15820
0.32	1.52433	-0.32533	-0.30139
0.47	1.54100	-0.32334	-0.33832
0.60	1.54804	-0.32333	-0.20263
0.71	1.55202	-0.33147	-0.14028
0.83	1.55295	-0.32932	-0.14240
0.96	1.55202	-0.31578	-0.21185
1.10	1.55113	-0.32895	-0.11492
1.27	1.55007	-0.33735	-0.07833
1.40	1.54786	-0.32973	-0.06717
1.52	1.54872	-0.36221	0.05984

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.62329	-0.35110	-0.15028
0.18	1.62810	-0.34869	-0.20593
0.32	1.64360	-0.34189	-0.32491
0.47	1.65947	-0.30846	-0.51700
0.60	1.66825	-0.33608	-0.23983
0.71	1.67172	-0.31842	-0.27224
0.83	1.67326	-0.30432	-0.35710
0.96	1.67283	-0.29911	-0.38598
1.10	1.67266	-0.32818	-0.22685
1.27	1.67163	-0.33698	-0.16792
1.40	1.66944	-0.32801	-0.15934
1.52	1.67067	-0.36175	-0.04002

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.86212	-0.34129	-0.36581
0.18	1.86915	-0.41374	-0.02732
0.32	1.87968	-0.28966	-0.71731
0.47	1.89588	-0.25612	-1.02578
0.60	1.90551	-0.24941	-0.91777
0.71	1.91413	-0.36405	-0.29768
0.83	1.91702	-0.36655	-0.32208
0.96	1.91586	-0.32528	-0.44766
1.10	1.91767	-0.38473	-0.20701
1.27	1.91545	-0.33457	-0.42302
1.40	1.91173	-0.31053	-0.45140
1.52	1.91306	-0.33320	-0.39039

(d) 99% Sig. Lvl.

Table C.52: Params. of Eq. 9.9 fitted to the case IIIb **Weibull** Kuiper critical values

C.2.4.2 Loglogistic Distribution:

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.09182	-0.16662	0.09296
0.18	1.08526	-0.17670	0.14239
0.33	1.06373	-0.17559	0.13700
0.50	1.02098	-0.16732	0.12435
0.65	0.98372	-0.16540	0.06352
0.82	0.95637	-0.16867	-0.00001
1.00	0.93675	-0.16715	-0.02873
1.22	0.92369	-0.17753	0.00864
1.53	0.91528	-0.16112	-0.09272
2.00	0.91238	-0.18002	0.03025
2.48	0.91236	-0.16928	-0.01374
3.00	0.91496	-0.17751	0.01502

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.17946	-0.15301	0.00559
0.18	1.17281	-0.17853	0.12945
0.33	1.14787	-0.18066	0.14527
0.50	1.09874	-0.17654	0.16292
0.65	1.05581	-0.16495	0.03732
0.82	1.02377	-0.15484	-0.10600
1.00	1.00082	-0.15395	-0.11844
1.22	0.98628	-0.18001	-0.00401
1.53	0.97651	-0.16397	-0.11834
2.00	0.97232	-0.16362	-0.09085
2.48	0.97243	-0.17053	-0.02642
3.00	0.97546	-0.18401	0.01682

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.31671	-0.15677	-0.07177
0.18	1.30937	-0.18039	0.07965
0.33	1.27796	-0.15338	0.00960
0.50	1.22129	-0.20692	0.28649
0.65	1.16948	-0.17829	0.04838
0.82	1.13051	-0.16776	-0.09018
1.00	1.10277	-0.16652	-0.12619
1.22	1.08442	-0.17652	-0.09041
1.53	1.07216	-0.14390	-0.29249
2.00	1.06679	-0.15568	-0.17987
2.48	1.06721	-0.16411	-0.13705
3.00	1.07057	-0.17929	-0.05800

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.58995	-0.15047	-0.30209
0.18	1.58215	-0.18778	-0.06299
0.33	1.54223	-0.16632	-0.01103
0.50	1.46605	-0.21532	0.22878
0.65	1.39742	-0.16605	-0.17890
0.82	1.34651	-0.17198	-0.23145
1.00	1.30870	-0.16120	-0.31040
1.22	1.28345	-0.15988	-0.35444
1.53	1.26838	-0.13780	-0.54462
2.00	1.25887	-0.14893	-0.33291
2.48	1.26027	-0.16601	-0.28260
3.00	1.26512	-0.23032	0.03171

(d) 99% Sig. Lvl.

Table C.53: Params. of Eq. 9.9 fitted to the case IIIb **Loglogistic** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.39587	-0.32367	-0.04023
0.18	1.39472	-0.34156	0.06737
0.33	1.41268	-0.33938	-0.01904
0.50	1.43508	-0.35394	0.05578
0.65	1.44583	-0.33230	-0.02621
0.82	1.45113	-0.33271	-0.00642
1.00	1.45363	-0.32548	-0.03962
1.22	1.45559	-0.33965	0.04268
1.53	1.45629	-0.32580	-0.05143
2.00	1.45710	-0.32243	-0.02620
2.48	1.45790	-0.33716	0.03841
3.00	1.45938	-0.35786	0.13487

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.47013	-0.30249	-0.18007
0.18	1.46957	-0.34845	0.07743
0.33	1.48732	-0.33827	-0.05537
0.50	1.51065	-0.35062	-0.00488
0.65	1.52219	-0.32627	-0.09437
0.82	1.52854	-0.33034	-0.05802
1.00	1.53149	-0.33078	-0.04930
1.22	1.53343	-0.34695	0.04849
1.53	1.53425	-0.31701	-0.14712
2.00	1.53513	-0.32211	-0.06833
2.48	1.53528	-0.32355	-0.05726
3.00	1.53700	-0.35156	0.06824

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.58758	-0.31830	-0.15574
0.18	1.58686	-0.35990	0.12150
0.33	1.60441	-0.34158	-0.08749
0.50	1.62870	-0.34794	-0.10584
0.65	1.64287	-0.33964	-0.13754
0.82	1.64934	-0.34234	-0.08260
1.00	1.65232	-0.32396	-0.18600
1.22	1.65486	-0.35580	-0.00339
1.53	1.65571	-0.31162	-0.25393
2.00	1.65653	-0.31977	-0.16139
2.48	1.65673	-0.32042	-0.14243
3.00	1.65959	-0.37893	0.11921

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.00	1.82664	-0.33315	-0.19988
0.18	1.82719	-0.37551	0.11278
0.33	1.84192	-0.31699	-0.33623
0.50	1.86905	-0.40387	0.01948
0.65	1.88155	-0.25554	-0.78260
0.82	1.89072	-0.32104	-0.38257
1.00	1.89327	-0.32043	-0.32432
1.22	1.89724	-0.31941	-0.40537
1.53	1.89858	-0.27366	-0.59794
2.00	1.89997	-0.33097	-0.24748
2.48	1.89853	-0.24262	-0.73401
3.00	1.90367	-0.38178	-0.03141

(d) 99% Sig. Lvl.

Table C.54: Params. of Eq. 9.9 fitted to the case IIIb **Loglogistic** Kuiper critical values

C.2.4.3 Lognormal Distribution:

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.10277	-0.18257	0.18964
0.72	1.09328	-0.15893	0.05357
0.75	1.07419	-0.18818	0.22776
0.79	1.04130	-0.17277	0.13580
0.85	1.01562	-0.16975	0.02619
0.91	0.99563	-0.16365	-0.01923
1.00	0.98065	-0.16351	-0.05616
1.12	0.96881	-0.15915	-0.07542
1.29	0.95945	-0.16163	-0.08494
1.58	0.95145	-0.15942	-0.08139
1.89	0.94633	-0.15660	-0.06927
2.24	0.94382	-0.16073	-0.07205

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.18991	-0.16600	0.07316
0.72	1.18051	-0.16564	0.06549
0.75	1.15814	-0.20200	0.29826
0.79	1.11940	-0.15658	0.03089
0.85	1.09015	-0.16596	-0.01529
0.91	1.06759	-0.17597	0.01685
1.00	1.05008	-0.16316	-0.09032
1.12	1.03641	-0.15435	-0.13810
1.29	1.02589	-0.16977	-0.07525
1.58	1.01632	-0.15738	-0.11988
1.89	1.01073	-0.15031	-0.14584
2.24	1.00820	-0.16233	-0.10096

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.32663	-0.16637	0.00110
0.72	1.31714	-0.19013	0.12928
0.75	1.28805	-0.19178	0.24963
0.79	1.24342	-0.16665	0.00811
0.85	1.20647	-0.15269	-0.10963
0.91	1.17976	-0.16469	-0.08953
1.00	1.15919	-0.16239	-0.16861
1.12	1.14282	-0.16408	-0.15604
1.29	1.12966	-0.15392	-0.24235
1.58	1.11913	-0.17247	-0.10725
1.89	1.11175	-0.14702	-0.20072
2.24	1.10865	-0.14597	-0.25688

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.59806	-0.13265	-0.39030
0.72	1.58727	-0.17531	-0.06410
0.75	1.54856	-0.17203	0.11933
0.79	1.49299	-0.20745	0.00903
0.85	1.44132	-0.15027	-0.30983
0.91	1.40605	-0.14500	-0.44736
1.00	1.37916	-0.15389	-0.41372
1.12	1.35839	-0.18803	-0.22204
1.29	1.34053	-0.14484	-0.51248
1.58	1.32710	-0.19023	-0.23522
1.89	1.31890	-0.18229	-0.25672
2.24	1.31281	-0.17503	-0.24342

(d) 99% Sig. Lvl.

Table C.55: Params. of Eq. 9.9 fitted to the case IIIb **Lognormal** Kolmogorov-Smirnov critical values

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.41941	-0.33269	-0.04223
0.72	1.41770	-0.33200	-0.09751
0.75	1.43859	-0.34630	-0.04405
0.79	1.45540	-0.33597	-0.05890
0.85	1.46201	-0.32813	-0.07491
0.91	1.46502	-0.33838	-0.00657
1.00	1.46541	-0.32831	-0.05432
1.12	1.46570	-0.33443	-0.00658
1.29	1.46501	-0.33108	-0.02566
1.58	1.46482	-0.33212	-0.00333
1.89	1.46372	-0.32494	-0.02348
2.24	1.46368	-0.32788	-0.02296

(a) 85% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.49418	-0.32722	-0.10728
0.72	1.49279	-0.34317	-0.07456
0.75	1.51363	-0.33829	-0.11806
0.79	1.53275	-0.34527	-0.07052
0.85	1.53904	-0.32401	-0.12649
0.91	1.54240	-0.33346	-0.06669
1.00	1.54358	-0.32212	-0.14879
1.12	1.54346	-0.32338	-0.11783
1.29	1.54368	-0.34461	-0.02547
1.58	1.54363	-0.35115	0.05732
1.89	1.54200	-0.32675	-0.05953
2.24	1.54178	-0.32681	-0.07396

(b) 90% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.61097	-0.33393	-0.13759
0.72	1.60960	-0.36749	0.02753
0.75	1.63055	-0.32690	-0.25003
0.79	1.65229	-0.34779	-0.16864
0.85	1.65920	-0.32996	-0.16129
0.91	1.66277	-0.31574	-0.25928
1.00	1.66509	-0.34261	-0.13153
1.12	1.66465	-0.32104	-0.20824
1.29	1.66506	-0.34951	-0.06194
1.58	1.66505	-0.35347	-0.00777
1.89	1.66334	-0.33675	-0.05546
2.24	1.66307	-0.32502	-0.18196

(c) 95% Sig. Lvl.

	$\theta_1 + \frac{\theta_2}{\sqrt{n}} + \frac{\theta_3}{n}$		
$\sqrt{\eta}$	θ_1	θ_2	θ_3
0.71	1.84629	-0.34501	-0.22213
0.72	1.84613	-0.39232	0.08459
0.75	1.86730	-0.35047	-0.26132
0.79	1.89160	-0.33401	-0.47506
0.85	1.89979	-0.30598	-0.50438
0.91	1.90398	-0.31416	-0.46365
1.00	1.90578	-0.31702	-0.42224
1.12	1.90895	-0.33830	-0.33657
1.29	1.90808	-0.31822	-0.40185
1.58	1.90834	-0.34604	-0.24263
1.89	1.90790	-0.37870	-0.08257
2.24	1.90739	-0.34918	-0.30389

(d) 99% Sig. Lvl.

Table C.56: Params. of Eq. 9.9 fitted to the case IIIb **Lognormal** Kuiper critical values

C.3 Variable n and η

C.3.1 Case II: Both Parameters are Unknown

C.3.1.1 Weibull Distribution

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
AD (A^2)	85%	4.510359	-3.564436	4.924154	-4.849546	8.768972	-0.065053	-0.580058	0.027504	-0.222700
AD (A^2)	90%	0.835806	0.388793	0.280850	0.715421	0.444129	-0.073358	0.000137	0.036531	-0.279331
AD (A^2)	95%	0.168850	-1.964362	-0.491428	-2.815311	-0.651929	-0.075478	-0.082915	0.035847	-0.335809
AD (A^2)	99%	1.410189	0.259362	0.465820	0.340757	0.449211	-0.057322	0.002584	0.045893	-0.483867
CvM (W^2)	85%	0.109632	0.023230	0.044260	0.466958	0.498831	-0.014662	0.000087	0.006982	-0.033215
CvM (W^2)	90%	0.127391	0.024092	0.052828	0.452420	0.521055	-0.017971	0.000078	0.009542	-0.050892
CvM (W^2)	95%	0.156103	0.017849	0.064506	0.356750	0.523998	-0.023468	0.000161	0.012809	-0.081076
CvM (W^2)	99%	0.228137	0.007781	0.098701	0.264666	0.568637	-0.040872	0.000236	0.031238	-0.206357
KS ($\sqrt{n}D$)	85%	0.826498	-0.023327	0.310762	0.045077	0.399174	-0.057915	0.000842	-0.141656	-0.188446
KS ($\sqrt{n}D$)	90%	0.876073	-0.060845	0.333801	0.004655	0.405420	-0.065723	0.001072	-0.135842	-0.252540
KS ($\sqrt{n}D$)	95%	0.957076	-0.137771	0.386774	-0.067525	0.432877	-0.078910	0.001053	-0.124969	-0.371957
KS ($\sqrt{n}D$)	99%	1.110961	-0.351589	0.452020	-0.250242	0.436825	-0.111828	0.002012	-0.091441	-0.680331
Kuiper ($\sqrt{n}V$)	85%	1.417917	0.762998	0.487140	0.586929	0.362557	-0.024407	-0.000011	-0.323695	-0.160397
Kuiper ($\sqrt{n}V$)	90%	1.222232	-3.912594	-0.672342	-2.812797	-0.476430	-0.025926	-0.028795	-0.318229	-0.251816
Kuiper ($\sqrt{n}V$)	95%	1.613312	0.938768	0.495958	0.637706	0.327130	-0.027902	-0.000004	-0.315893	-0.386894
Kuiper ($\sqrt{n}V$)	99%	1.490270	-5.076977	-0.173325	-2.972597	-0.100460	-0.036070	-0.037509	-0.304481	-0.709327

Table C.57: Params. of Eq. 9.11 fitted to the case II **Weibull** critical values

C.3.1.2 Loglogistic Distribution

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
AD (A^2)	85%	0.564400	0.002555	0.130651	0.040622	0.261206	-0.043237	0.000053	0.034976	-0.171506
AD (A^2)	90%	0.639005	-0.000470	0.154046	0.036052	0.275048	-0.049662	0.000048	0.039840	-0.205598
AD (A^2)	95%	0.766913	-0.018942	0.192745	0.009432	0.291362	-0.064149	0.000092	0.049369	-0.236376
AD (A^2)	99%	1.071415	-0.058887	0.293288	-0.030088	0.327225	-0.094328	0.000218	0.059767	-0.178732
CvM (W^2)	85%	0.081952	-0.005853	0.018257	-0.025957	0.255668	-0.008254	0.000011	0.007500	-0.028306
CvM (W^2)	90%	0.093586	-0.010338	0.022046	-0.069920	0.272420	-0.010031	0.000018	0.009048	-0.039819
CvM (W^2)	95%	0.114334	-0.011777	0.027883	-0.059158	0.287073	-0.013574	0.000026	0.011984	-0.061922
CvM (W^2)	99%	0.163556	-0.021036	0.043098	-0.088752	0.320036	-0.022446	0.000051	0.017475	-0.113000
KS ($\sqrt{n}D$)	85%	0.742051	-0.049184	0.156498	-0.051658	0.220976	-0.023814	0.000033	-0.149204	-0.211524
KS ($\sqrt{n}D$)	90%	0.782091	-0.081832	0.174212	-0.091745	0.233591	-0.026620	0.000055	-0.145974	-0.272292
KS ($\sqrt{n}D$)	95%	0.844303	-0.120691	0.198706	-0.131807	0.246983	-0.029753	0.000084	-0.141732	-0.375360
KS ($\sqrt{n}D$)	99%	0.971053	-0.172919	0.267239	-0.168693	0.289515	-0.038204	0.000119	-0.141621	-0.582486
Kuiper ($\sqrt{n}V$)	85%	1.339777	-0.075488	0.268398	-0.041307	0.210737	-0.043011	0.000057	-0.298086	-0.213964
Kuiper ($\sqrt{n}V$)	90%	1.408110	-0.117277	0.290962	-0.069255	0.217921	-0.046676	0.000081	-0.294467	-0.288279
Kuiper ($\sqrt{n}V$)	95%	1.515856	-0.144586	0.323296	-0.081577	0.226075	-0.054909	0.000110	-0.287839	-0.408540
Kuiper ($\sqrt{n}V$)	99%	1.733360	-0.209820	0.395813	-0.108242	0.244416	-0.074105	0.000202	-0.269770	-0.684393

Table C.58: Params. of Eq. 9.11 fitted to the case II **Loglogistic** critical values

C.3.1.3 Lognormal Distribution

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
AD (A^2)	85%	0.636423	-0.692563	0.172496	-1.015972	0.220181	-0.079942	0.000559	0.106664	-0.240010
AD (A^2)	90%	0.726351	-0.792909	0.198616	-1.013623	0.218526	-0.089238	0.000609	0.129453	-0.306665
AD (A^2)	95%	0.884383	-0.974399	0.248192	-1.016172	0.220390	-0.101165	0.000641	0.150424	-0.334570
AD (A^2)	99%	1.259187	-1.445622	0.394791	-1.060089	0.251722	-0.077273	0.000764	0.163651	-0.359316
CvM (W^2)	85%	0.097444	-0.078268	0.007256	-0.647315	-0.035230	-0.015552	0.000014	0.020451	-0.029990
CvM (W^2)	90%	0.112588	-0.090346	0.008335	-0.636398	-0.042765	-0.018581	0.000014	0.024509	-0.043002
CvM (W^2)	95%	0.139115	-0.112090	0.010648	-0.624496	-0.050863	-0.024402	0.000016	0.033585	-0.074360
CvM (W^2)	99%	0.199784	-0.176270	0.026024	-0.709886	0.009153	-0.036914	0.000100	0.054375	-0.160828
KS ($\sqrt{n}D$)	85%	0.811432	-0.362977	-0.135523	-0.337307	-0.244312	-0.059293	-0.000174	-0.094951	-0.142001
KS ($\sqrt{n}D$)	90%	0.465562	-3.211798	1.992867	-4.704248	2.772373	-0.065761	-0.019837	-0.085258	-0.191100
KS ($\sqrt{n}D$)	95%	0.477230	-3.458735	2.150679	-4.739351	2.789537	-0.075951	-0.022293	-0.066011	-0.295769
KS ($\sqrt{n}D$)	99%	0.469855	-3.921768	2.452511	-4.813194	2.822903	-0.098699	-0.028552	-0.025622	-0.515521
Kuiper ($\sqrt{n}V$)	85%	1.378413	-1.577075	0.427281	-1.129565	0.299717	-0.045730	0.000326	-0.272830	-0.206296
Kuiper ($\sqrt{n}V$)	90%	1.450501	-1.675020	0.460448	-1.140059	0.307057	-0.049926	0.000367	-0.264206	-0.293081
Kuiper ($\sqrt{n}V$)	95%	1.562506	-1.833446	0.516397	-1.158689	0.320120	-0.059577	0.000517	-0.252017	-0.422000
Kuiper ($\sqrt{n}V$)	99%	1.788175	-2.158446	0.633210	-1.192685	0.343968	-0.075229	0.000801	-0.241067	-0.654345

Table C.59: Params. of Eq. 9.11 fitted to the case II **Lognormal** critical values

C.3.2 Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

C.3.2.1 Lognormal Distribution: Kolmogorov-Smirnov test and Kuiper's test

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
KS ($\sqrt{n}D$)	85%	0.906008	-1.040559	0.284506	-1.121283	0.294922	0.006334	0.000127	-0.172995	-0.055008
KS ($\sqrt{n}D$)	90%	0.965368	-1.121697	0.312328	-1.133761	0.303778	0.007832	0.000161	-0.177006	-0.077884
KS ($\sqrt{n}D$)	95%	1.058842	-1.246585	0.353919	-1.147449	0.313315	0.010486	0.000173	-0.171014	-0.181671
KS ($\sqrt{n}D$)	99%	1.249626	-1.508586	0.443773	-1.175757	0.333053	0.012560	0.000155	-0.173149	-0.359337
Kuiper ($\sqrt{n}V$)	85%	1.451448	-2.359332	0.938812	-1.625582	0.646884	0.002875	0.000320	-0.332318	0.005524
Kuiper ($\sqrt{n}V$)	90%	1.529495	-2.665650	1.113403	-1.742928	0.728051	0.003262	0.000305	-0.329710	-0.053498
Kuiper ($\sqrt{n}V$)	95%	1.651170	-5.241999	2.817551	-3.176013	1.707534	0.001568	0.000918	-0.327788	-0.133871
Kuiper ($\sqrt{n}V$)	99%	1.895975	-3.589430	1.575400	-1.893291	0.831032	0.001885	0.000212	-0.318912	-0.375164

Table C.60: Params. of Eq. 9.11 fitted to the case IIIa **Lognormal** critical values

C.3.3 Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

C.3.3.1 Weibull Distribution: Kolmogorov-Smirnov test and Kuiper's test

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
KS ($\sqrt{n}D$)	85%	1.000984	-1.035807	0.642843	-0.989286	0.587475	-0.021104	-0.006042	-0.148737	-0.006386
KS ($\sqrt{n}D$)	90%	1.074672	-1.132263	0.700156	-1.004099	0.593574	-0.026060	-0.006967	-0.144860	-0.039240
KS ($\sqrt{n}D$)	95%	1.187654	-1.257147	0.788650	-1.004042	0.600534	-0.030698	-0.008122	-0.143727	-0.098614
KS ($\sqrt{n}D$)	99%	1.429449	-1.581894	0.955658	-1.046610	0.604238	-0.035188	-0.012746	-0.147754	-0.284682
Kuiper ($\sqrt{n}V$)	85%	1.461056	-0.814121	0.313309	-0.568060	0.219194	0.024633	-0.000882	-0.353977	-0.088846
Kuiper ($\sqrt{n}V$)	90%	1.537912	-0.895506	0.356679	-0.593578	0.236936	0.022700	-0.000709	-0.350413	-0.153168
Kuiper ($\sqrt{n}V$)	95%	1.658107	-0.989293	0.415788	-0.608137	0.256050	0.020719	-0.000501	-0.346421	-0.253956
Kuiper ($\sqrt{n}V$)	99%	1.899288	-1.193850	0.565164	-0.640334	0.303366	0.012176	-0.000193	-0.340265	-0.466101

Table C.61: Params. of Eq. 9.11 fitted to the case IIIb **Weibull** critical values

C.3.3.2 Loglogistic Distribution: Kolmogorov-Smirnov test and Kuiper's test

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
KS ($\sqrt{n}D$)	85%	0.933672	-0.608559	0.509338	-0.575037	0.466808	-0.011209	0.000007	-0.158258	0.039910
KS ($\sqrt{n}D$)	90%	0.996362	-0.647179	0.546130	-0.568071	0.463199	-0.012168	-0.000002	-0.154803	0.002777
KS ($\sqrt{n}D$)	95%	1.094229	-0.709880	0.601798	-0.562273	0.457419	-0.012835	0.000017	-0.154449	-0.051819
KS ($\sqrt{n}D$)	99%	1.294185	-0.846781	0.715975	-0.561124	0.450766	-0.013882	-0.000017	-0.155963	-0.195909
Kuiper ($\sqrt{n}V$)	85%	1.452325	-0.576353	0.238088	-0.404393	0.170462	0.002670	0.000126	-0.339048	0.010835
Kuiper ($\sqrt{n}V$)	90%	1.529991	-0.621196	0.264773	-0.413792	0.179928	0.002561	0.000119	-0.335296	-0.043377
Kuiper ($\sqrt{n}V$)	95%	1.650930	-0.692724	0.312798	-0.427909	0.196879	0.000830	0.000146	-0.339298	-0.089634
Kuiper ($\sqrt{n}V$)	99%	1.893068	-0.828236	0.426484	-0.446323	0.233413	-0.001845	0.000184	-0.320805	-0.325797

Table C.62: Params. of Eq. 9.11 fitted to the case IIIb **Loglogistic** critical values

C.3.3.3 Lognormal Distribution: Kolmogorov-Smirnov test and Kuiper's test

		$\frac{\theta_1\eta+\theta_2\sqrt{\eta}+\theta_3}{\theta_4\sqrt{\eta}+\theta_5+\eta}+\theta_6\sqrt{\frac{\eta}{n}}+\theta_7\eta^{\frac{3}{2}}+\frac{\theta_8}{\sqrt{n}}+\frac{\theta_9}{n}$								
Test	Sig. Lvl.	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7	θ_8	θ_9
KS ($\sqrt{n}D$)	85%	0.936598	-1.197512	0.391302	-1.292555	0.425616	-0.017587	-0.000032	-0.146125	0.014541
KS ($\sqrt{n}D$)	90%	0.998261	-1.279395	0.417463	-1.297680	0.427621	-0.020212	0.000056	-0.142476	-0.016749
KS ($\sqrt{n}D$)	95%	1.096053	-1.406612	0.458690	-1.301479	0.429401	-0.021864	0.000122	-0.139632	-0.078577
KS ($\sqrt{n}D$)	99%	1.297171	-1.675691	0.548834	-1.311828	0.435439	-0.026065	0.000056	-0.138024	-0.247234
Kuiper ($\sqrt{n}V$)	85%	1.467209	-2.090545	0.746353	-1.424665	0.508602	0.009718	-0.000371	-0.343892	-0.038374
Kuiper ($\sqrt{n}V$)	90%	1.545546	-2.200672	0.785201	-1.423657	0.507924	0.009033	-0.000371	-0.344272	-0.077654
Kuiper ($\sqrt{n}V$)	95%	1.667305	-2.370222	0.844439	-1.421280	0.506294	0.008377	-0.000400	-0.347179	-0.133015
Kuiper ($\sqrt{n}V$)	99%	1.911481	-2.702904	0.958359	-1.413554	0.501081	-0.002907	-0.000389	-0.337432	-0.302643

Table C.63: Params. of Eq. 9.11 fitted to the case IIIb **Lognormal** critical values

Appendix D

Quality of Fit Comparison

In this appendix we compare how well Eq. (9.10) and (9.11) fit each the cases which have both n and η dependence.

D.1 Variable n and η

D.1.1 Case II: Both Parameters are Unknown

D.1.1.1 Weibull Distribution

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0004003	0.98968	0.98902
"	"	Eq. (9.11)	0.0000295	0.99924	0.99916
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0002509	0.98991	0.98927
"	"	Eq. (9.11)	0.0000114	0.99954	0.99949
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0003058	0.98966	0.98900
"	"	Eq. (9.11)	0.0000175	0.99941	0.99934
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0007682	0.98835	0.98760
"	"	Eq. (9.11)	0.0000931	0.99859	0.99844
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0003599	0.99506	0.99474
"	"	Eq. (9.11)	0.0000254	0.99965	0.99961
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0001778	0.99667	0.99646
"	"	Eq. (9.11)	0.0000130	0.99976	0.99973
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0002329	0.99613	0.99588
"	"	Eq. (9.11)	0.0000149	0.99975	0.99973
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0008549	0.99204	0.99152
"	"	Eq. (9.11)	0.0001386	0.99871	0.99857
Cramér-von Mises (W^2)	95%	Eq. (9.10)	0.0000388	0.98422	0.98320
"	"	Eq. (9.11)	0.0000037	0.99849	0.99833
Cramér-von Mises (W^2)	85%	Eq. (9.10)	0.0000169	0.98124	0.98003
"	"	Eq. (9.11)	0.0000011	0.99883	0.99871
Cramér-von Mises (W^2)	90%	Eq. (9.10)	0.0000234	0.98268	0.98157
"	"	Eq. (9.11)	0.0000016	0.99884	0.99871
Cramér-von Mises (W^2)	99%	Eq. (9.10)	0.0001068	0.98485	0.98388
"	"	Eq. (9.11)	0.0000164	0.99768	0.99743
Anderson-Darling (A^2)	95%	Eq. (9.10)	0.0012866	0.99266	0.99219
"	"	Eq. (9.11)	0.0001341	0.99924	0.99915
Anderson-Darling (A^2)	85%	Eq. (9.10)	0.0005257	0.99134	0.99078
"	"	Eq. (9.11)	0.0000559	0.99908	0.99898
Anderson-Darling (A^2)	90%	Eq. (9.10)	0.0007673	0.99181	0.99128
"	"	Eq. (9.11)	0.0000548	0.99941	0.99935
Anderson-Darling (A^2)	99%	Eq. (9.10)	0.0036807	0.99308	0.99264
"	"	Eq. (9.11)	0.0006539	0.99877	0.99864

Table D.1: Comparison of Eq. (9.10) and (9.11) fitted to the case II **Weibull** critical values

D.1.1.2 Loglogistic Distribution

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0009227	0.97008	0.96817
"	"	Eq. (9.11)	0.0000280	0.99909	0.99900
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0006012	0.97300	0.97127
"	"	Eq. (9.11)	0.0000164	0.99926	0.99918
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0007184	0.97181	0.97000
"	"	Eq. (9.11)	0.0000179	0.99930	0.99922
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0013148	0.97050	0.96861
"	"	Eq. (9.11)	0.0000552	0.99876	0.99863
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0032510	0.97100	0.96914
"	"	Eq. (9.11)	0.0000827	0.99926	0.99918
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0020239	0.97366	0.97198
"	"	Eq. (9.11)	0.0000553	0.99928	0.99920
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0024885	0.97205	0.97026
"	"	Eq. (9.11)	0.0000674	0.99924	0.99916
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0054034	0.96910	0.96712
"	"	Eq. (9.11)	0.0002085	0.99881	0.99868
Cramér-von Mises (W^2)	95%	Eq. (9.10)	0.0001133	0.94975	0.94652
"	"	Eq. (9.11)	0.0000046	0.99797	0.99775
Cramér-von Mises (W^2)	85%	Eq. (9.10)	0.0000485	0.94136	0.93760
"	"	Eq. (9.11)	0.0000018	0.99788	0.99766
Cramér-von Mises (W^2)	90%	Eq. (9.10)	0.0000696	0.94408	0.94049
"	"	Eq. (9.11)	0.0000026	0.99791	0.99769
Cramér-von Mises (W^2)	99%	Eq. (9.10)	0.0002808	0.95690	0.95414
"	"	Eq. (9.11)	0.0000153	0.99765	0.99740
Anderson-Darling (A^2)	95%	Eq. (9.10)	0.0024698	0.96906	0.96707
"	"	Eq. (9.11)	0.0001649	0.99793	0.99771
Anderson-Darling (A^2)	85%	Eq. (9.10)	0.0010672	0.96276	0.96037
"	"	Eq. (9.11)	0.0000559	0.99805	0.99784
Anderson-Darling (A^2)	90%	Eq. (9.10)	0.0014923	0.96568	0.96348
"	"	Eq. (9.11)	0.0000882	0.99797	0.99776
Anderson-Darling (A^2)	99%	Eq. (9.10)	0.0060191	0.97426	0.97261
"	"	Eq. (9.11)	0.0008784	0.99624	0.99584

Table D.2: Comparison of Eq. (9.10) and (9.11) fitted to the case II **Loglogistic** critical values

D.1.1.3 Lognormal Distribution

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0133330	0.64706	0.62443
"	"	Eq. (9.11)	0.0000979	0.99741	0.99713
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0081924	0.66898	0.64776
"	"	Eq. (9.11)	0.0000500	0.99798	0.99776
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0099703	0.65824	0.63633
"	"	Eq. (9.11)	0.0000655	0.99775	0.99751
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0221370	0.64217	0.61924
"	"	Eq. (9.11)	0.0002546	0.99588	0.99544
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0027899	0.96711	0.96501
"	"	Eq. (9.11)	0.0000516	0.99939	0.99933
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0021441	0.96407	0.96177
"	"	Eq. (9.11)	0.0000290	0.99951	0.99946
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0023738	0.96519	0.96296
"	"	Eq. (9.11)	0.0000356	0.99948	0.99942
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0037560	0.97119	0.96935
"	"	Eq. (9.11)	0.0001701	0.99870	0.99856
Cramér-von Mises (W^2)	95%	Eq. (9.10)	0.0009092	0.58938	0.56305
"	"	Eq. (9.11)	0.0000072	0.99673	0.99639
Cramér-von Mises (W^2)	85%	Eq. (9.10)	0.0004302	0.52943	0.49926
"	"	Eq. (9.11)	0.0000031	0.99665	0.99629
Cramér-von Mises (W^2)	90%	Eq. (9.10)	0.0005857	0.55201	0.52329
"	"	Eq. (9.11)	0.0000045	0.99657	0.99620
Cramér-von Mises (W^2)	99%	Eq. (9.10)	0.0019123	0.65986	0.63806
"	"	Eq. (9.11)	0.0000289	0.99486	0.99431
Anderson-Darling (A^2)	95%	Eq. (9.10)	0.0078198	0.94993	0.94672
"	"	Eq. (9.11)	0.0002795	0.99821	0.99802
Anderson-Darling (A^2)	85%	Eq. (9.10)	0.0038279	0.93072	0.92628
"	"	Eq. (9.11)	0.0000877	0.99841	0.99824
Anderson-Darling (A^2)	90%	Eq. (9.10)	0.0051808	0.93843	0.93448
"	"	Eq. (9.11)	0.0001256	0.99851	0.99835
Anderson-Darling (A^2)	99%	Eq. (9.10)	0.0160840	0.96629	0.96413
"	"	Eq. (9.11)	0.0015716	0.99671	0.99635

Table D.3: Comparison of Eq. (9.10) and (9.11) fitted to the case II **Lognormal** critical values

D.2 Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

D.2.0.1 Lognormal Distribution: Kolmogorov-Smirnov test and Kuiper's test

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0010319	0.98928	0.98860
"	"	Eq. (9.11)	0.0000212	0.99978	0.99976
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0006136	0.98770	0.98692
"	"	Eq. (9.11)	0.0000095	0.99981	0.99979
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0007539	0.98844	0.98770
"	"	Eq. (9.11)	0.0000108	0.99983	0.99982
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0018991	0.98987	0.98923
"	"	Eq. (9.11)	0.0000947	0.99950	0.99944
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0003107	0.99116	0.99059
"	"	Eq. (9.11)	0.0000436	0.99876	0.99863
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0002599	0.99147	0.99093
"	"	Eq. (9.11)	0.0000331	0.99891	0.99880
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0002770	0.99136	0.99080
"	"	Eq. (9.11)	0.0000376	0.99883	0.99870
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0004262	0.99006	0.98942
"	"	Eq. (9.11)	0.0001259	0.99706	0.99675

Table D.4: Comparison of Eq. (9.10) and (9.11) fitted to the case IIIa **Lognormal** critical values

D.3 Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

D.3.0.1 Weibull Distribution: Kolmogorov-Smirnov test and Kuiper's test

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0020242	0.99523	0.99492
"	"	Eq. (9.11)	0.0002080	0.99951	0.99946
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0011113	0.99521	0.99490
"	"	Eq. (9.11)	0.0001043	0.99955	0.99950
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0014412	0.99522	0.99491
"	"	Eq. (9.11)	0.0001288	0.99957	0.99953
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0037294	0.99488	0.99455
"	"	Eq. (9.11)	0.0003822	0.99947	0.99942
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0013447	0.98019	0.97892
"	"	Eq. (9.11)	0.0000632	0.99907	0.99897
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0011612	0.97957	0.97826
"	"	Eq. (9.11)	0.0000346	0.99939	0.99933
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0012581	0.97948	0.97816
"	"	Eq. (9.11)	0.0000423	0.99931	0.99924
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0014718	0.98184	0.98068
"	"	Eq. (9.11)	0.0001956	0.99759	0.99733

Table D.5: Comparison of Eq. (9.10) and (9.11) fitted to the case IIIb **Weibull** critical values

D.3.0.2 Loglogistic Distribution: Kolmogorov-Smirnov test and Kuiper's test

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0236950	0.96928	0.96731
"	"	Eq. (9.11)	0.0001363	0.99982	0.99980
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0120190	0.97008	0.96816
"	"	Eq. (9.11)	0.0000780	0.99981	0.99979
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0160140	0.97000	0.96808
"	"	Eq. (9.11)	0.0001021	0.99981	0.99979
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0432790	0.96798	0.96593
"	"	Eq. (9.11)	0.0003708	0.99973	0.99970
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0029348	0.96871	0.96670
"	"	Eq. (9.11)	0.0000629	0.99933	0.99926
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0024405	0.96849	0.96647
"	"	Eq. (9.11)	0.0000251	0.99968	0.99964
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0026813	0.96800	0.96595
"	"	Eq. (9.11)	0.0000312	0.99963	0.99959
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0031152	0.97102	0.96917
"	"	Eq. (9.11)	0.0002577	0.99760	0.99735

Table D.6: Comparison of Eq. (9.10) and (9.11) fitted to the case IIIb **Loglogistic** critical values

D.3.0.3 Lognormal Distribution: Kolmogorov-Smirnov test and Kuiper's test

GOF Test	Sig. Lvl.	Model	SSE	R^2	R^2_{adj}
Kolmogorov-Smirnov ($\sqrt{n}D$)	95%	Eq. (9.10)	0.0141430	0.97277	0.97102
"	"	Eq. (9.11)	0.0000832	0.99984	0.99982
Kolmogorov-Smirnov ($\sqrt{n}D$)	85%	Eq. (9.10)	0.0074909	0.97278	0.97104
"	"	Eq. (9.11)	0.0000477	0.99983	0.99981
Kolmogorov-Smirnov ($\sqrt{n}D$)	90%	Eq. (9.10)	0.0099557	0.97253	0.97077
"	"	Eq. (9.11)	0.0000555	0.99985	0.99983
Kolmogorov-Smirnov ($\sqrt{n}D$)	99%	Eq. (9.10)	0.0246390	0.97263	0.97088
"	"	Eq. (9.11)	0.0002263	0.99975	0.99972
Kuiper ($\sqrt{n}V$)	95%	Eq. (9.10)	0.0063957	0.91202	0.90638
"	"	Eq. (9.11)	0.0000599	0.99918	0.99909
Kuiper ($\sqrt{n}V$)	85%	Eq. (9.10)	0.0052036	0.91112	0.90542
"	"	Eq. (9.11)	0.0000416	0.99929	0.99921
Kuiper ($\sqrt{n}V$)	90%	Eq. (9.10)	0.0057170	0.91066	0.90494
"	"	Eq. (9.11)	0.0000452	0.99929	0.99922
Kuiper ($\sqrt{n}V$)	99%	Eq. (9.10)	0.0069706	0.92276	0.91781
"	"	Eq. (9.11)	0.0001197	0.99867	0.99853

Table D.7: Comparison of Eq. (9.10) and (9.11) fitted to the case IIIb **Lognormal** critical values

Appendix E

Power Testing Tables

E.1 Power testing v.s. Case I: All Parameters are Known

E.1.1 Weibull Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.91 10000	94.91 10000	94.61 10000	94.63 10000	94.63 10000	94.61 10000	94.92 10000	94.91 10000	94.84 10000	94.88 10000
Weibull(1,1)	100	95.12 10000	95.12 10000	95.16 10000	95.13 10000	94.88 10000	94.89 10000	94.41 10000	94.42 10000	94.99 10000	94.98 10000
Weibull(1,1)	1000	95.11 10000	95.15 10000	95.29 10000	95.28 10000	94.82 10000	94.83 10000	94.94 10000	94.95 10000	95.11 10000	95.12 10000
Loglogistic(1,4)	30	0.14 10000	0.14 10000	4.45 10000	4.52 10000	43.48 10000	43.40 10000	7.10 10000	7.08 10000	18.60 10000	18.68 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.35 10000	0.35 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.75 10000	0.77 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	57.67 10000	57.67 10000	89.41 10000	89.47 10000	78.48 10000	78.39 10000	66.81 10000	66.78 10000	55.06 10000	55.18 10000
ChiSquared1	100	8.36 10000	8.36 10000	78.63 10000	78.52 10000	41.88 10000	41.91 10000	20.53 10000	20.55 10000	7.91 10000	7.90 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.18 10000	0.18 10000	0.77 10000	0.77 10000	1.71 10000	1.71 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.04 10000	0.04 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.08 10000	0.08 10000	1.83 10000	1.84 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.1: Testing v.s. Case I Weibull under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.03 10000	95.03 10000	94.93 10000	94.91 10000	94.93 10000	94.92 10000	95.24 10000	95.25 10000	94.65 10000	94.66 10000
Weibull(1,1)	100	95.15 10000	95.15 10000	95.71 10000	95.70 10000	94.95 10000	94.98 10000	94.52 10000	94.56 10000	94.90 10000	94.91 10000
Weibull(1,1)	1000	94.88 10000	94.88 10000	94.96 10000	95.02 10000	94.95 10000	94.95 10000	94.94 10000	94.95 10000	95.37 10000	95.39 10000
Loglogistic(1,4)	30	0.01 10000	0.01 10000	0.39 10000	0.38 10000	28.65 10000	28.64 10000	19.85 10000	19.90 10000	37.63 10000	37.65 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.09 10000	0.09 10000	0.01 10000	0.01 10000	0.38 10000	0.39 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	4.87 10000	4.87 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	48.87 10000	48.91 10000	83.34 10000	83.22 10000	83.85 10000	83.85 10000	78.63 10000	78.64 10000	71.72 10000	71.75 10000
ChiSquared1	100	3.17 10000	3.17 10000	51.99 10000	51.99 10000	53.71 10000	53.80 10000	37.32 10000	37.41 10000	20.97 10000	21.13 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.17 10000	0.17 10000	0.42 10000	0.42 10000	1.94 10000	1.94 10000	4.33 10000	4.34 10000	7.87 10000	7.88 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.08 10000	0.08 10000	0.33 10000	0.33 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	0.94 10000	0.94 10000	9.49 10000	9.50 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.02 10000	0.02 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.02 10000	0.02 10000	0.03 10000	0.03 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.2: Testing v.s. Case I Weibull (1,1) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.19 10000	95.19 10000	94.79 10000	94.80 10000	94.54 10000	94.53 10000	94.88 10000	94.87 10000	94.80 10000	94.83 10000
Weibull(1,1)	100	95.14 10000	95.12 10000	95.07 10000	95.07 10000	94.92 10000	94.92 10000	94.69 10000	94.69 10000	95.07 10000	95.07 10000
Weibull(1,1)	1000	95.14 10000	95.14 10000	94.90 10000	94.88 10000	94.84 10000	94.84 10000	95.20 10000	95.19 10000	95.20 10000	95.23 10000
Loglogistic(1,4)	30	0.05 10000	0.05 10000	3.06 10000	3.12 10000	45.50 10000	45.41 10000	4.34 10000	4.33 10000	13.93 10000	14.00 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.23 10000	0.23 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.29 10000	0.29 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	56.60 10000	56.60 10000	90.13 10000	90.14 10000	75.86 10000	75.79 10000	62.57 10000	62.54 10000	49.59 10000	49.74 10000
ChiSquared1	100	6.92 10000	6.90 10000	80.26 10000	80.23 10000	38.29 10000	38.29 10000	16.61 10000	16.59 10000	5.57 10000	5.57 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.06 10000	0.06 10000	0.34 10000	0.34 10000	0.96 10000	0.96 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.02 10000	0.02 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.74 10000	0.75 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.3: Testing v.s. Case I Weibull (1,1) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.14 10000	95.16 10000	94.88 10000	94.91 10000	94.75 10000	94.75 10000	94.99 10000	94.97 10000	94.83 10000	94.89 10000
Weibull(1,1)	100	95.09 10000	95.09 10000	95.12 10000	95.09 10000	94.92 10000	94.90 10000	94.67 10000	94.67 10000	94.83 10000	94.83 10000
Weibull(1,1)	1000	95.19 10000	95.18 10000	95.09 10000	95.06 10000	94.84 10000	94.84 10000	95.06 10000	95.06 10000	95.35 10000	95.36 10000
Loglogistic(1,4)	30	0.05 10000	0.05 10000	2.39 10000	2.39 10000	47.25 10000	47.28 10000	4.95 10000	4.92 10000	13.97 10000	14.03 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.16 10000	0.16 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.36 10000	0.36 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	30.45 10000	30.49 10000	80.41 10000	80.49 10000	64.03 10000	64.04 10000	50.39 10000	50.30 10000	37.76 10000	37.90 10000
ChiSquared1	100	0.81 10000	0.81 10000	52.24 10000	52.15 10000	21.32 10000	21.31 10000	7.81 10000	7.81 10000	2.10 10000	2.10 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.11 10000	0.11 10000	0.27 10000	0.27 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.08 10000	0.08 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.4: Testing v.s. Case I Weibull (1,1) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.1.2 Loglogistic Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	3.99 10000	3.99 10000	36.07 10000	36.07 10000	11.72 10000	11.70 10000	9.55 10000	9.52 10000	13.22 10000	13.19 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.69 10000	0.69 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	94.99 10000	95.03 10000	94.76 10000	94.76 10000	95.21 10000	95.21 10000	94.87 10000	94.83 10000	94.83 10000	94.79 10000
Loglogistic(1,4)	100	95.19 10000	95.18 10000	95.07 10000	95.10 10000	94.77 10000	94.83 10000	95.04 10000	95.04 10000	94.96 10000	95.01 10000
Loglogistic(1,4)	1000	95.09 10000	95.13 10000	95.07 10000	95.07 10000	95.21 10000	95.22 10000	94.58 10000	94.58 10000	94.75 10000	94.76 10000
Lognormal(0,0.2)	30	54.90 10000	55.01 10000	47.27 10000	47.27 10000	1.73 10000	1.73 10000	0.03 10000	0.03 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	3.93 10000	3.93 10000	29.07 10000	28.94 10000	65.02 10000	64.93 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.42 10000	0.42 10000	14.62 10000	14.72 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.06 10000	0.07 10000	7.31 10000	7.31 10000	0.99 10000	0.99 10000	0.37 10000	0.36 10000	0.63 10000	0.63 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.5: Testing v.s. Case I Loglogistic (1,4) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.66 10000	0.67 10000	35.21 10000	35.14 10000	24.51 10000	24.56 10000	22.73 10000	22.73 10000	29.14 10000	29.18 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.62 10000	0.62 10000	0.04 10000	0.04 10000	0.07 10000	0.07 10000	0.14 10000	0.14 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	94.99 10000	95.00 10000	94.83 10000	94.83 10000	94.89 10000	94.92 10000	95.09 10000	95.09 10000	95.07 10000	95.07 10000
Loglogistic(1,4)	100	95.17 10000	95.17 10000	95.23 10000	95.27 10000	94.79 10000	94.81 10000	94.85 10000	94.87 10000	94.98 10000	94.98 10000
Loglogistic(1,4)	1000	95.14 10000	95.14 10000	95.16 10000	95.17 10000	94.93 10000	94.83 10000	94.83 10000	94.83 10000	94.81 10000	94.81 10000
Lognormal(0,0.2)	30	1.18 10000	1.18 10000	9.97 10000	9.94 10000	9.24 10000	9.27 10000	0.93 10000	0.95 10000	0.01 10000	0.01 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	14.57 10000	14.59 10000	52.40 10000	52.40 10000	79.52 10000	79.54 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.01 10000	0.01 10000	2.95 10000	2.98 10000	34.26 10000	34.26 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	8.69 10000	8.68 10000	3.66 10000	3.70 10000	2.16 10000	2.17 10000	3.30 10000	3.30 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.6: Testing v.s. Case I Loglogistic (1,4) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	3.79 10000	3.85 10000	36.55 10000	36.57 10000	9.16 10000	9.11 10000	6.92 10000	6.91 10000	9.37 10000	9.35 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.77 10000	0.77 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	95.03 10000	95.10 10000	94.81 10000	94.82 10000	95.20 10000	95.19 10000	95.01 10000	94.98 10000	94.85 10000	94.85 10000
Loglogistic(1,4)	100	95.04 10000	95.04 10000	94.92 10000	94.92 10000	94.96 10000	94.98 10000	95.15 10000	95.15 10000	95.05 10000	95.07 10000
Loglogistic(1,4)	1000	95.08 10000	95.08 10000	95.19 10000	95.18 10000	95.16 10000	95.16 10000	94.88 10000	94.85 10000	94.77 10000	94.77 10000
Lognormal(0,0.2)	30	40.44 10000	40.81 10000	47.57 10000	47.76 10000	0.77 10000	0.75 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	2.04 10000	2.04 10000	22.64 10000	22.55 10000	59.16 10000	59.13 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.13 10000	0.13 10000	9.40 10000	9.46 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.05 10000	0.05 10000	7.60 10000	7.64 10000	0.73 10000	0.72 10000	0.26 10000	0.26 10000	0.33 10000	0.33 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.7: Testing v.s. Case I Loglogistic (1,4) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.04 10000	0.04 10000	19.60 10000	19.71 10000	5.56 10000	5.54 10000	4.50 10000	4.46 10000	7.32 10000	7.28 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.05 10000	0.05 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	94.98 10000	95.01 10000	94.80 10000	94.82 10000	95.17 10000	95.14 10000	95.10 10000	95.08 10000	94.87 10000	94.84 10000
Loglogistic(1,4)	100	95.03 10000	95.01 10000	94.79 10000	94.79 10000	94.97 10000	94.98 10000	95.12 10000	95.12 10000	95.17 10000	95.20 10000
Loglogistic(1,4)	1000	95.08 10000	95.08 10000	95.01 10000	95.01 10000	95.15 10000	95.15 10000	94.93 10000	94.91 10000	94.93 10000	94.93 10000
Lognormal(0,0.2)	30	26.64 10000	26.88 10000	41.11 10000	41.43 10000	0.72 10000	0.72 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Pareto(5)	30	-	-	0.00 10000	0.00 10000	1.90 10000	1.89 10000	22.72 10000	22.69 10000	59.76 10000	59.67 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.10 10000	0.10 10000	8.96 10000	8.97 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	1.70 10000	1.71 10000	0.16 10000	0.16 10000	0.07 10000	0.07 10000	0.05 10000	0.05 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.8: Testing v.s. Case I Loglogistic (1,4) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.1.3 Lognormal Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.01 10000	0.01 10000	0.03 10000	0.03 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	37.42 10000	37.43 10000	22.32 10000	22.32 10000	5.29 10000	5.29 10000	2.20 10000	2.20 10000	0.50 10000	0.50 10000
Loglogistic(1,4)	100	0.19 10000	0.20 10000	0.06 10000	0.06 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	94.87 10000	94.88 10000	95.05 10000	95.05 10000	95.21 10000	95.19 10000	95.13 10000	95.13 10000	95.11 10000	95.08 10000
Lognormal(0,0.2)	100	94.84 10000	94.89 10000	94.57 10000	94.57 10000	95.04 10000	95.07 10000	94.55 10000	94.57 10000	94.81 10000	94.81 10000
Lognormal(0,0.2)	1000	94.83 10000	94.83 10000	95.07 10000	95.10 10000	94.58 10000	94.62 10000	95.20 10000	95.21 10000	94.94 10000	94.98 10000
Pareto(5)	30	-	-	4.96 10000	4.96 10000	86.02 10000	85.97 10000	75.88 10000	75.88 10000	35.90 10000	35.86 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	66.94 10000	67.02 10000	31.12 10000	31.17 10000	0.81 10000	0.82 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.9: Testing v.s. Case I Lognormal (0,0.2) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.08 10000	0.08 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	6.08 10000	6.08 10000	28.42 10000	28.39 10000	13.95 10000	13.93 10000	7.54 10000	7.55 10000	3.10 10000	3.10 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.12 10000	0.12 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	94.88 10000	94.88 10000	95.31 10000	95.29 10000	95.21 10000	95.21 10000	95.28 10000	95.29 10000	94.85 10000	94.85 10000
Lognormal(0,0.2)	100	94.85 10000	94.87 10000	94.89 10000	94.89 10000	94.84 10000	94.84 10000	94.58 10000	94.51 10000	94.65 10000	94.66 10000
Lognormal(0,0.2)	1000	94.86 10000	94.93 10000	94.95 10000	94.97 10000	94.90 10000	94.91 10000	95.21 10000	95.21 10000	94.97 10000	94.99 10000
Pareto(5)	30	-	-	8.22 10000	8.18 10000	74.15 10000	74.14 10000	75.63 10000	75.64 10000	52.85 10000	52.85 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	29.23 10000	29.29 10000	31.98 10000	31.90 10000	4.06 10000	4.09 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.10: Testing v.s. Case I Lognormal (0,0.2) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.06 10000	0.06 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	32.51 10000	32.61 10000	22.38 10000	22.35 10000	4.26 10000	4.26 10000	1.36 10000	1.36 10000	0.35 10000	0.35 10000
Loglogistic(1,4)	100	0.07 10000	0.07 10000	0.07 10000	0.07 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	94.87 10000	94.89 10000	94.96 10000	94.96 10000	95.20 10000	95.20 10000	95.20 10000	95.20 10000	94.98 10000	94.98 10000
Lognormal(0,0.2)	100	94.91 10000	94.95 10000	94.55 10000	94.56 10000	94.99 10000	95.00 10000	94.55 10000	94.56 10000	94.81 10000	94.81 10000
Lognormal(0,0.2)	1000	94.85 10000	94.83 10000	94.99 10000	95.01 10000	94.91 10000	94.91 10000	95.15 10000	95.17 10000	94.84 10000	94.87 10000
Pareto(5)	30	-	-	3.80 10000	3.80 10000	86.54 10000	86.54 10000	75.21 10000	75.19 10000	31.99 10000	31.98 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	66.83 10000	66.96 10000	34.42 10000	34.45 10000	0.76 10000	0.75 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.11: Testing v.s. Case I Lognormal (0,0.2) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	1.13 10000	1.13 10000	4.23 10000	4.22 10000	0.58 10000	0.57 10000	0.18 10000	0.18 10000	0.02 10000	0.02 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	94.88 10000	94.90 10000	94.93 10000	94.89 10000	95.28 10000	95.27 10000	95.16 10000	95.15 10000	95.11 10000	95.11 10000
Lognormal(0,0.2)	100	95.09 10000	95.15 10000	94.52 10000	94.52 10000	95.04 10000	95.08 10000	94.58 10000	94.59 10000	94.95 10000	94.95 10000
Lognormal(0,0.2)	1000	94.78 10000	94.77 10000	95.03 10000	95.03 10000	94.93 10000	94.97 10000	95.15 10000	95.19 10000	94.83 10000	94.84 10000
Pareto(5)	30	-	-	2.03 10000	2.03 10000	63.79 10000	63.72 10000	44.02 10000	43.97 10000	11.54 10000	11.53 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	21.64 10000	21.71 10000	5.64 10000	5.64 10000	0.04 10000	0.04 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.12: Testing v.s. Case I Lognormal (0,0.2) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.1.4 Pareto Distribution

KS 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	0.00 10000	0.00 10000	-	-	-	-
ChiSquared1	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared1	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	30	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	30	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	1000	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	30	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	100	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	1000	0.00 10000	0.00 10000	-	-	-	-
Loglogistic(1,4)	30	15.60 10000	15.63 10000	-	-	-	-
Loglogistic(1,4)	100	0.03 10000	0.02 10000	-	-	-	-
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	-	-	-	-
Lognormal(0,0.2)	30	93.37 10000	93.39 10000	-	-	-	-
Lognormal(0,0.2)	100	68.48 10000	68.41 10000	-	-	-	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	-	-	-	-
Pareto(5)	30	95.38 10000	95.41 10000	95.09 10000	95.07 10000	95.25 10000	95.25 10000
Pareto(5)	100	95.01 10000	95.01 10000	95.16 10000	95.14 10000	94.70 10000	94.70 10000
Pareto(5)	1000	95.28 10000	95.24 10000	95.05 10000	95.06 10000	95.00 10000	94.97 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	100	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	1000	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	30	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	100	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	1000	0.00 10000	0.00 10000	-	-	-	-

Table E.13: Testing v.s. Case I Pareto (5) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared1	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared1	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	30	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	30	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Loglogistic(1,4)	30	35.28 (10000)	35.28 (10000)	-	-	-	-
Loglogistic(1,4)	100	0.26 (10000)	0.26 (10000)	-	-	-	-
Loglogistic(1,4)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Lognormal(0,0.2)	30	80.05 (10000)	80.06 (10000)	-	-	-	-
Lognormal(0,0.2)	100	37.76 (10000)	37.63 (10000)	-	-	-	-
Lognormal(0,0.2)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Pareto(5)	30	94.98 (10000)	94.98 (10000)	94.93 (10000)	94.88 (10000)	94.95 (10000)	94.93 (10000)
Pareto(5)	100	94.71 (10000)	94.70 (10000)	95.08 (10000)	95.07 (10000)	94.76 (10000)	94.77 (10000)
Pareto(5)	1000	95.16 (10000)	95.17 (10000)	94.81 (10000)	94.81 (10000)	95.07 (10000)	95.07 (10000)
Rayleigh(4)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Rayleigh(4)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Rayleigh(4)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Weibull(1,1)	30	0.14 (10000)	0.14 (10000)	-	-	-	-
Weibull(1,1)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Weibull(1,1)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-

Table E.14: Testing v.s. Case I Pareto (5) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	0.00 10000	0.00 10000	-	-	-	-
ChiSquared1	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared1	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	30	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	30	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	1000	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	30	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	100	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	1000	0.00 10000	0.00 10000	-	-	-	-
Loglogistic(1,4)	30	11.44 10000	11.45 10000	-	-	-	-
Loglogistic(1,4)	100	0.01 10000	0.01 10000	-	-	-	-
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	-	-	-	-
Lognormal(0,0.2)	30	94.10 10000	94.11 10000	-	-	-	-
Lognormal(0,0.2)	100	74.86 10000	74.86 10000	-	-	-	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	-	-	-	-
Pareto(5)	30	95.37 10000	95.39 10000	95.19 10000	95.14 10000	95.21 10000	95.22 10000
Pareto(5)	100	94.70 10000	94.70 10000	95.01 10000	95.00 10000	94.59 10000	94.58 10000
Pareto(5)	1000	95.17 10000	95.13 10000	95.21 10000	95.18 10000	94.95 10000	94.94 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	100	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	1000	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	30	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	100	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	1000	0.00 10000	0.00 10000	-	-	-	-

Table E.15: Testing v.s. Case I Pareto (5) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared1	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared1	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	30	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	30	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Loglogistic(1,4)	30	9.13 (10000)	9.15 (10000)	-	-	-	-
Loglogistic(1,4)	100	0.01 (10000)	0.01 (10000)	-	-	-	-
Loglogistic(1,4)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Lognormal(0,0.2)	30	93.93 (10000)	93.96 (10000)	-	-	-	-
Lognormal(0,0.2)	100	56.60 (10000)	56.62 (10000)	-	-	-	-
Lognormal(0,0.2)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Pareto(5)	30	95.26 (10000)	95.28 (10000)	95.15 (10000)	95.08 (10000)	95.06 (10000)	95.06 (10000)
Pareto(5)	100	94.76 (10000)	94.77 (10000)	95.09 (10000)	95.07 (10000)	94.57 (10000)	94.57 (10000)
Pareto(5)	1000	95.02 (10000)	94.95 (10000)	95.18 (10000)	95.14 (10000)	94.80 (10000)	94.80 (10000)
Rayleigh(4)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Rayleigh(4)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Rayleigh(4)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Weibull(1,1)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Weibull(1,1)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Weibull(1,1)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-

Table E.16: Testing v.s. Case I Pareto (5) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.2 Power testing v.s. Case II: Both Parameters are Known

E.2.1 Weibull Distribution

KS 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.76 10000	94.79 10000	95.20 10000	95.13 10000	94.74 10000	94.70 9906	95.22 9989	95.16 9281	94.21 9907	94.28 7618
Weibull(1,1)	100	95.51 10000	95.56 10000	95.37 10000	95.39 10000	94.73 10000	94.75 10000	94.37 10000	94.43 9903	94.89 9998	94.81 8177
Weibull(1,1)	1000	95.19 10000	95.10 10000	94.51 10000	94.53 10000	94.85 10000	94.83 10000	94.43 10000	94.40 10000	95.18 10000	95.18 9923
Loglogistic(1,4)	30	66.05 10000	66.06 10000	67.14 10000	66.24 10000	81.66 10000	81.69 9959	93.42 9552	94.44 6012	94.44 7289	94.88 2168
Loglogistic(1,4)	100	17.20 10000	17.30 10000	17.53 10000	16.84 10000	43.16 10000	43.20 9999	86.36 9934	90.28 4682	93.67 7177	92.19 256
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	16.69 10000	28.83 1516	90.81 8803	-
Lognormal(0,0.2)	30	76.04 10000	76.06 10000	77.44 10000	77.40 10000	80.95 10000	80.46 10000	93.79 9983	93.75 8998	94.68 8108	95.16 2810
Lognormal(0,0.2)	100	33.92 10000	34.10 10000	33.44 10000	33.60 10000	43.98 10000	43.74 10000	92.43 10000	92.46 9702	94.21 8811	94.77 669
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	66.25 10000	66.21 10000	93.99 9991	-
Pareto(5)	30	-	-	94.43 6539	94.94 1640	93.97 6567	94.62 1654	93.78 6655	94.50 1673	93.76 6603	94.63 1583
Pareto(5)	100	-	-	93.61 5963	92.67 150	94.30 5985	97.62 126	93.29 5920	94.96 119	94.08 6046	93.91 115
Pareto(5)	1000	-	-	93.87 5354	-	93.60 5357	-	93.94 5383	-	93.29 5337	-
ChiSquared1	30	92.32 10000	92.37 10000	95.02 10000	94.97 10000	94.42 10000	94.43 9857	95.15 9995	95.25 9384	94.75 9961	94.97 8384
ChiSquared1	100	86.38 10000	86.48 10000	94.43 10000	94.43 10000	94.61 10000	94.60 10000	94.94 10000	94.91 9948	94.70 10000	94.64 9289
ChiSquared1	1000	15.82 10000	15.76 10000	84.88 10000	84.87 10000	92.97 10000	92.97 10000	93.98 10000	93.96 10000	94.94 10000	94.93 10000
ChiSquared3	30	94.53 10000	94.55 10000	95.07 10000	94.80 10000	94.89 10000	94.83 10000	95.04 10000	94.97 9998	95.03 10000	95.06 9916
ChiSquared3	100	93.08 10000	93.17 10000	94.11 10000	93.88 10000	94.60 10000	94.61 10000	94.74 10000	94.76 10000	94.73 10000	94.74 10000
ChiSquared3	1000	74.19 10000	74.04 10000	81.46 10000	80.92 10000	89.84 10000	89.85 10000	92.72 10000	92.73 10000	93.76 10000	93.76 10000
ChiSquared4	30	93.63 10000	93.64 10000	94.03 10000	93.74 10000	94.61 10000	94.48 10000	94.06 10000	94.03 9999	95.16 10000	95.08 9997
ChiSquared4	100	90.16 10000	90.22 10000	91.27 10000	90.71 10000	92.71 10000	92.73 10000	93.51 10000	93.50 10000	94.47 10000	94.49 10000
ChiSquared4	1000	40.39 10000	40.21 10000	48.81 10000	47.36 10000	71.63 10000	71.66 10000	80.26 10000	80.27 10000	89.56 10000	89.58 10000
Loglaplace(1,1)	30	44.58 10000	44.61 10000	45.58 10000	45.02 10000	51.21 10000	51.21 9973	59.92 9993	61.40 9494	79.64 9536	83.28 5904
Loglaplace(1,1)	100	2.12 10000	2.13 10000	2.63 10000	2.54 10000	4.28 10000	4.29 9992	7.85 10000	8.05 9748	35.48 9899	43.39 4190
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9996	0.00 10000	0.00 627
Rayleigh(4)	30	95.28 10000	95.29 10000	95.25 10000	95.15 10000	95.36 10000	95.13 10000	95.08 10000	94.94 10000	94.54 10000	94.46 10000
Rayleigh(4)	100	94.75 10000	94.82 10000	94.93 10000	94.86 10000	95.13 10000	94.78 10000	95.23 10000	95.12 10000	94.82 10000	94.82 10000
Rayleigh(4)	1000	95.17 10000	95.12 10000	94.88 10000	94.70 10000	95.33 10000	94.97 10000	94.98 10000	94.93 10000	95.20 10000	95.23 10000
Gamma(5,2)	30	89.76 10000	89.76 10000	89.78 10000	89.73 10000	90.47 10000	90.16 10000	90.63 10000	90.10 10000	90.58 10000	90.14 10000
Gamma(5,2)	100	76.45 10000	76.56 10000	75.34 10000	75.38 10000	75.92 10000	75.42 10000	77.64 10000	76.94 10000	80.67 10000	80.03 10000
Gamma(5,2)	1000	0.53 10000	0.50 10000	0.65 10000	0.62 10000	0.70 10000	0.65 10000	1.11 10000	0.98 10000	2.70 10000	2.41 10000
Uniform	30	64.56 10000	64.57 10000	75.60 10000	75.01 10000	83.96 10000	83.89 10000	86.39 10000	86.33 9996	-	-
Uniform	100	11.88 10000	12.04 10000	27.14 10000	26.77 10000	46.08 10000	46.15 10000	56.13 10000	56.23 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	95.14 10000	95.14 10000	95.32 10000	95.16 10000	94.99 10000	94.95 10000	94.89 10000	94.85 10000	95.22 10000	95.17 9992
Mittag-Leffler(1,5)	100	94.83 10000	94.92 10000	95.25 10000	95.18 10000	95.18 10000	95.19 10000	94.96 10000	94.94 10000	95.32 10000	95.33 10000
Mittag-Leffler(1,5)	1000	95.28 10000	95.20 10000	95.02 10000	94.85 10000	95.00 10000	95.02 10000	95.01 10000	94.98 10000	94.97 10000	94.98 10000

Table E.17: Testing v.s. Case II Weibull (1,1) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.87 10000	94.90 10000	95.03 10000	95.04 10000	94.68 10000	94.68 9906	95.08 9989	95.12 9281	94.27 9907	94.25 7618
Weibull(1,1)	100	94.96 10000	94.98 10000	95.32 10000	95.31 10000	94.89 10000	94.88 10000	94.48 10000	94.53 9903	94.48 9998	94.33 8177
Weibull(1,1)	1000	94.82 10000	94.83 10000	94.65 10000	94.67 10000	94.93 10000	94.93 10000	94.43 10000	94.43 10000	95.24 10000	95.23 9923
Loglogistic(1,4)	30	61.28 10000	61.30 10000	61.43 10000	61.05 10000	79.96 10000	80.02 9959	93.10 9552	94.00 6012	94.61 7289	94.70 2168
Loglogistic(1,4)	100	12.71 10000	12.73 10000	12.16 10000	12.04 10000	40.21 10000	40.22 9999	86.12 9934	89.49 4682	94.04 7177	91.02 256
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	16.87 10000	28.56 1516	92.33 8803	-
Lognormal(0,0.2)	30	73.69 10000	73.71 10000	74.92 10000	74.90 10000	78.22 10000	78.15 10000	93.70 9983	93.62 8998	94.98 8108	95.02 2810
Lognormal(0,0.2)	100	26.11 10000	26.19 10000	25.51 10000	25.54 10000	36.27 10000	36.14 10000	92.08 10000	92.10 9702	94.62 8811	94.92 669
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	65.84 10000	65.85 10000	94.98 9991	-
Pareto(5)	30	-	-	94.71 6539	94.94 1640	94.46 6567	94.62 1654	94.37 6655	94.74 1673	94.37 6603	94.69 1583
Pareto(5)	100	-	-	94.31 5963	94.00 150	94.74 5985	96.83 126	94.19 5920	96.64 119	94.41 6046	93.91 115
Pareto(5)	1000	-	-	94.47 5354	-	94.68 5357	-	94.91 5383	-	94.44 5337	-
ChiSquared1	30	93.92 10000	93.96 10000	94.82 10000	94.82 10000	94.51 10000	94.53 9857	94.79 9995	94.78 9384	95.25 9961	95.30 8384
ChiSquared1	100	88.62 10000	88.68 10000	93.95 10000	93.96 10000	94.68 10000	94.66 10000	95.03 10000	95.07 9948	95.24 10000	95.22 9289
ChiSquared1	1000	16.18 10000	16.28 10000	82.41 10000	82.44 10000	92.92 10000	92.92 10000	93.82 10000	93.82 10000	94.85 10000	94.85 10000
ChiSquared3	30	94.04 10000	94.04 10000	94.83 10000	94.77 10000	94.39 10000	94.37 10000	94.51 10000	94.47 9998	94.62 10000	94.62 9916
ChiSquared3	100	93.18 10000	93.23 10000	93.82 10000	93.61 10000	94.47 10000	94.47 10000	94.74 10000	94.74 10000	94.83 10000	94.81 10000
ChiSquared3	1000	74.08 10000	74.16 10000	78.45 10000	78.20 10000	88.43 10000	88.48 10000	92.45 10000	92.45 10000	93.74 10000	93.74 10000
ChiSquared4	30	93.58 10000	93.59 10000	93.87 10000	93.71 10000	94.24 10000	94.24 10000	94.17 10000	94.18 9999	94.93 10000	94.93 9997
ChiSquared4	100	89.81 10000	89.86 10000	90.56 10000	90.42 10000	92.07 10000	92.08 10000	93.25 10000	93.25 10000	94.32 10000	94.33 10000
ChiSquared4	1000	37.31 10000	37.39 10000	41.44 10000	41.04 10000	65.21 10000	65.30 10000	77.18 10000	77.21 10000	88.35 10000	88.35 10000
Loglaplace(1,1)	30	40.26 10000	40.27 10000	40.45 10000	40.26 10000	46.70 10000	46.80 9973	56.64 9993	58.08 9494	78.08 9536	81.89 5904
Loglaplace(1,1)	100	1.34 10000	1.34 10000	1.72 10000	1.71 10000	2.93 10000	2.95 9992	6.30 10000	6.44 9748	35.55 9899	42.82 4190
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9996	0.00 10000	0.00 627
Rayleigh(4)	30	95.30 10000	95.34 10000	95.12 10000	95.06 10000	95.06 10000	94.91 10000	94.77 10000	94.71 10000	94.16 10000	94.18 10000
Rayleigh(4)	100	94.74 10000	94.76 10000	95.44 10000	95.40 10000	94.91 10000	94.75 10000	94.97 10000	94.93 10000	95.03 10000	95.05 10000
Rayleigh(4)	1000	94.88 10000	94.91 10000	94.92 10000	94.87 10000	94.95 10000	94.84 10000	94.78 10000	94.77 10000	95.18 10000	95.22 10000
Gamma(5,2)	30	89.23 10000	89.25 10000	89.20 10000	89.18 10000	89.95 10000	89.78 10000	89.76 10000	89.55 10000	89.72 10000	89.56 10000
Gamma(5,2)	100	74.52 10000	74.58 10000	73.92 10000	73.94 10000	74.00 10000	73.72 10000	74.85 10000	74.67 10000	77.52 10000	77.24 10000
Gamma(5,2)	1000	0.21 10000	0.21 10000	0.18 10000	0.18 10000	0.15 10000	0.15 10000	0.19 10000	0.19 10000	0.95 10000	0.92 10000
Uniform	30	66.66 10000	66.71 10000	73.83 10000	73.63 10000	81.29 10000	81.30 10000	84.11 10000	84.11 9996	-	-
Uniform	100	8.22 10000	8.25 10000	21.51 10000	21.32 10000	38.70 10000	38.74 10000	48.98 10000	49.01 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	95.10 10000	95.10 10000	95.27 10000	95.21 10000	95.17 10000	95.19 10000	94.65 10000	94.65 10000	94.98 10000	94.96 9992
Mittag-Leffler(1,5)	100	94.89 10000	94.91 10000	94.80 10000	94.72 10000	94.98 10000	94.99 10000	94.77 10000	94.80 10000	95.07 10000	95.04 10000
Mittag-Leffler(1,5)	1000	94.98 10000	95.00 10000	94.84 10000	94.75 10000	94.94 10000	94.95 10000	95.39 10000	95.40 10000	94.72 10000	94.72 10000

Table E.18: Testing v.s. Case II Weibull (1,1) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		p = 0		p = 0.1		p = 0.4		p = 0.6		p = 0.8	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.81 10000	94.86 10000	95.21 10000	95.14 10000	94.70 10000	94.62 9906	95.23 9989	95.26 9281	94.33 9907	94.32 7618
Weibull(1,1)	100	95.00 10000	95.04 10000	95.23 10000	95.23 10000	94.96 10000	94.94 10000	94.29 10000	94.30 9903	94.50 9998	94.23 8177
Weibull(1,1)	1000	94.90 10000	94.85 10000	94.57 10000	94.63 10000	94.91 10000	94.90 10000	94.70 10000	94.70 10000	95.38 10000	95.38 9923
Loglogistic(1,4)	30	56.69 10000	56.79 10000	57.54 10000	56.91 10000	77.05 10000	77.11 9959	92.60 9552	93.80 6012	94.38 7289	95.16 2168
Loglogistic(1,4)	100	8.71 10000	8.74 10000	8.67 10000	8.37 10000	33.81 10000	33.82 9999	83.80 9934	88.21 4682	93.67 7177	92.19 256
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	10.38 10000	19.06 1516	90.12 8803	-
Lognormal(0,0.2)	30	68.77 10000	68.92 10000	69.91 10000	69.92 10000	74.98 10000	74.59 10000	93.50 9983	93.43 8998	94.65 8108	94.95 2810
Lognormal(0,0.2)	100	18.71 10000	18.79 10000	18.18 10000	18.25 10000	29.48 10000	29.19 10000	91.05 10000	91.09 9702	94.52 8811	95.96 669
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	58.15 10000	58.12 10000	94.13 9991	-
Pareto(5)	30	-	-	94.33 6539	94.63 1640	94.12 6567	94.80 1654	94.02 6655	94.80 1673	94.18 6603	94.57 1583
Pareto(5)	100	-	-	93.95 5963	92.00 150	93.92 5985	96.03 126	93.73 5920	95.80 119	93.91 6046	97.39 115
Pareto(5)	1000	-	-	93.93 5354	-	93.21 5357	-	93.98 5383	-	93.48 5337	-
ChiSquared1	30	91.92 10000	91.96 10000	94.73 10000	94.69 10000	94.69 10000	94.65 9857	94.75 9995	94.73 9384	95.19 9961	95.28 8384
ChiSquared1	100	83.80 10000	83.92 10000	94.01 10000	94.10 10000	94.38 10000	94.33 10000	94.78 10000	94.76 9948	95.18 10000	95.14 9289
ChiSquared1	1000	6.41 10000	6.37 10000	80.46 10000	80.50 10000	92.39 10000	92.38 10000	93.87 10000	93.87 10000	94.75 10000	94.75 10000
ChiSquared3	30	94.03 10000	94.06 10000	94.56 10000	94.30 10000	94.46 10000	94.39 10000	94.95 10000	94.91 9998	94.77 10000	94.70 9916
ChiSquared3	100	92.50 10000	92.57 10000	93.23 10000	93.01 10000	94.35 10000	94.37 10000	94.36 10000	94.41 10000	94.52 10000	94.52 10000
ChiSquared3	1000	65.72 10000	65.52 10000	74.20 10000	73.51 10000	87.23 10000	87.35 10000	91.60 10000	91.62 10000	93.05 10000	93.05 10000
ChiSquared4	30	93.04 10000	93.15 10000	93.83 10000	93.58 10000	94.08 10000	93.99 10000	94.01 10000	93.96 9999	94.70 10000	94.67 9997
ChiSquared4	100	87.96 10000	87.98 10000	89.30 10000	88.77 10000	91.52 10000	91.53 10000	92.67 10000	92.68 10000	93.96 10000	94.00 10000
ChiSquared4	1000	24.90 10000	24.74 10000	32.47 10000	31.35 10000	59.68 10000	59.68 10000	74.05 10000	74.28 10000	86.53 10000	86.55 10000
Loglaplace(1,1)	30	35.77 10000	35.84 10000	36.31 10000	35.83 10000	41.61 10000	41.57 9973	51.48 9993	52.91 9494	75.48 9536	79.69 5904
Loglaplace(1,1)	100	0.85 10000	0.90 10000	1.11 10000	1.10 10000	1.76 10000	1.77 9992	3.86 10000	3.96 9748	28.68 9899	36.16 4190
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9996	0.00 10000	0.00 627
Rayleigh(4)	30	95.41 10000	95.43 10000	94.99 10000	94.84 10000	95.16 10000	94.81 10000	94.90 10000	94.79 10000	94.59 10000	94.50 10000
Rayleigh(4)	100	94.94 10000	94.99 10000	95.09 10000	95.05 10000	95.27 10000	95.03 10000	94.90 10000	94.87 10000	94.88 10000	94.89 10000
Rayleigh(4)	1000	95.11 10000	95.06 10000	95.06 10000	94.81 10000	95.10 10000	94.82 10000	94.62 10000	94.58 10000	95.25 10000	95.27 10000
Gamma(5,2)	30	87.59 10000	87.66 10000	87.76 10000	87.76 10000	88.26 10000	87.98 10000	88.45 10000	87.96 10000	88.77 10000	88.32 10000
Gamma(5,2)	100	68.72 10000	68.89 10000	67.04 10000	67.09 10000	67.71 10000	67.19 10000	69.69 10000	68.92 10000	73.50 10000	72.77 10000
Gamma(5,2)	1000	0.02 10000	0.02 10000	0.04 10000	0.04 10000	0.03 10000	0.03 10000	0.04 10000	0.04 10000	0.43 10000	0.35 10000
Uniform	30	54.90 10000	54.95 10000	67.53 10000	67.12 10000	78.61 10000	78.52 10000	82.18 10000	82.12 9996	-	-
Uniform	100	3.59 10000	3.67 10000	12.27 10000	12.05 10000	28.49 10000	28.60 10000	39.37 10000	39.52 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	95.18 10000	95.29 10000	95.43 10000	95.21 10000	95.15 10000	95.08 10000	95.03 10000	94.96 10000	95.21 10000	95.18 9992
Mittag-Leffler(1,5)	100	94.73 10000	94.76 10000	95.14 10000	95.01 10000	95.07 10000	95.10 10000	94.98 10000	95.03 10000	95.18 10000	95.19 10000
Mittag-Leffler(1,5)	1000	95.01 10000	94.91 10000	95.03 10000	94.93 10000	95.20 10000	95.27 10000	95.34 10000	95.37 10000	94.73 10000	94.73 10000

Table E.19: Testing v.s. Case II Weibull (1,1) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.58 10000	94.67 10000	95.22 10000	95.22 10000	94.88 10000	94.80 9906	95.09 9989	95.05 9281	94.29 9907	94.18 7618
Weibull(1,1)	100	95.03 10000	95.08 10000	95.26 10000	95.26 10000	95.05 10000	95.04 10000	94.29 10000	94.31 9903	94.43 9998	94.40 8177
Weibull(1,1)	1000	94.87 10000	94.81 10000	94.96 10000	94.98 10000	94.98 10000	94.98 10000	94.74 10000	94.74 10000	95.33 10000	95.36 9923
Loglogistic(1,4)	30	53.99 10000	54.23 10000	55.19 10000	54.93 10000	78.58 10000	78.67 9959	93.81 9552	94.78 6012	94.70 7289	95.34 2168
Loglogistic(1,4)	100	6.28 10000	6.29 10000	6.36 10000	6.31 10000	32.49 10000	32.56 9999	85.14 9934	89.06 4682	94.20 7177	93.36 256
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	9.82 10000	18.07 1516	90.74 8803	-
Lognormal(0,0.2)	30	65.86 10000	66.11 10000	67.01 10000	67.11 10000	73.43 10000	73.23 10000	94.47 9983	94.38 8998	94.68 8108	95.05 2810
Lognormal(0,0.2)	100	12.30 10000	12.36 10000	12.10 10000	12.15 10000	22.26 10000	22.05 10000	92.10 10000	92.17 9702	94.77 8811	95.22 669
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	56.83 10000	56.81 10000	94.28 9991	-
Pareto(5)	30	-	-	94.02 6539	93.35 1640	94.43 6567	95.16 1654	94.24 6655	94.74 1673	94.18 6603	95.01 1583
Pareto(5)	100	-	-	94.15 5963	93.33 150	94.52 5985	92.06 126	94.07 5920	95.80 119	94.24 6046	97.39 115
Pareto(5)	1000	-	-	94.02 5354	-	93.52 5357	-	94.17 5383	-	93.91 5337	-
ChiSquared1	30	90.59 10000	90.71 10000	94.56 10000	94.51 10000	94.80 10000	94.78 9857	94.83 9995	94.88 9384	94.91 9961	95.00 8384
ChiSquared1	100	80.20 10000	80.27 10000	93.18 10000	93.19 10000	93.99 10000	93.98 10000	94.68 10000	94.71 9948	95.04 10000	94.98 9289
ChiSquared1	1000	2.63 10000	2.59 10000	77.43 10000	77.50 10000	92.20 10000	92.20 10000	93.56 10000	93.56 10000	94.88 10000	94.88 10000
ChiSquared3	30	94.24 10000	94.34 10000	94.78 10000	94.65 10000	94.81 10000	94.77 10000	94.98 10000	94.93 9998	95.14 10000	95.16 9916
ChiSquared3	100	92.06 10000	92.09 10000	93.14 10000	93.07 10000	94.46 10000	94.47 10000	94.49 10000	94.51 10000	94.86 10000	94.86 10000
ChiSquared3	1000	57.87 10000	57.73 10000	68.77 10000	68.21 10000	86.07 10000	86.13 10000	91.57 10000	91.59 10000	93.21 10000	93.21 10000
ChiSquared4	30	93.46 10000	93.53 10000	93.95 10000	93.86 10000	94.45 10000	94.42 10000	94.43 10000	94.42 9999	94.74 10000	94.72 9997
ChiSquared4	100	86.97 10000	87.04 10000	87.94 10000	87.67 10000	91.24 10000	91.25 10000	92.84 10000	92.87 10000	94.23 10000	94.26 10000
ChiSquared4	1000	15.80 10000	15.69 10000	22.43 10000	21.99 10000	51.53 10000	51.64 10000	70.66 10000	70.77 10000	85.64 10000	85.74 10000
Loglaplace(1,1)	30	35.41 10000	35.51 10000	36.71 10000	36.59 10000	44.54 10000	44.58 9973	56.53 9993	57.90 9494	79.44 9536	83.38 5904
Loglaplace(1,1)	100	0.83 10000	0.83 10000	1.03 10000	1.03 10000	1.93 10000	1.93 9992	4.53 10000	4.66 9748	30.32 9899	37.76 4190
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9996	0.00 10000	0.00 627
Rayleigh(4)	30	95.13 10000	95.17 10000	95.04 10000	95.02 10000	94.93 10000	94.85 10000	94.96 10000	94.92 10000	94.60 10000	94.59 10000
Rayleigh(4)	100	94.82 10000	94.85 10000	94.95 10000	94.93 10000	95.20 10000	95.07 10000	95.00 10000	94.90 10000	94.84 10000	94.84 10000
Rayleigh(4)	1000	95.18 10000	95.10 10000	95.09 10000	94.99 10000	94.93 10000	94.77 10000	94.46 10000	94.45 10000	95.10 10000	95.12 10000
Gamma(5,2)	30	86.79 10000	86.95 10000	86.96 10000	87.14 10000	88.14 10000	88.11 10000	88.25 10000	88.02 10000	89.01 10000	88.90 10000
Gamma(5,2)	100	62.83 10000	62.97 10000	61.24 10000	61.31 10000	62.37 10000	62.16 10000	64.49 10000	64.11 10000	68.79 10000	68.32 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.07 10000	0.06 10000
Uniform	30	44.64 10000	44.91 10000	57.80 10000	57.56 10000	71.91 10000	71.82 10000	77.98 10000	77.92 9996	-	-
Uniform	100	0.83 10000	0.83 10000	4.82 10000	4.77 10000	16.95 10000	17.01 10000	28.18 10000	28.35 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	95.19 10000	95.25 10000	95.63 10000	95.52 10000	95.20 10000	95.20 10000	94.95 10000	94.91 10000	94.94 10000	94.95 9992
Mittag-Leffler(1,5)	100	94.69 10000	94.71 10000	94.99 10000	94.91 10000	94.95 10000	94.98 10000	94.83 10000	94.86 10000	95.09 10000	95.11 10000
Mittag-Leffler(1,5)	1000	95.05 10000	94.99 10000	94.94 10000	94.85 10000	95.41 10000	95.45 10000	95.38 10000	95.39 10000	95.05 10000	95.10 10000

Table E.20: Testing v.s. Case II Weibull (1,1) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.2.2 Loglogistic Distribution

KS 95 Pass Rate		p = 0		p = 0.1		p = 0.4		p = 0.6		p = 0.8	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	86.32 10000	86.33 10000	93.04 9988	92.99 9982	93.41 9937	93.38 9902	93.52 9890	93.49 9825	93.73 9800	93.72 9683
Weibull(1,1)	100	60.91 10000	61.15 10000	84.86 10000	84.86 10000	87.46 10000	87.47 9999	88.69 10000	88.67 10000	90.08 9998	90.05 9990
Weibull(1,1)	1000	0.00 10000	0.00 10000	4.90 10000	4.90 10000	14.17 10000	14.25 10000	21.41 10000	21.45 10000	31.78 10000	31.77 10000
Loglogistic(1,4)	30	95.08 10000	95.08 10000	94.92 10000	94.84 10000	94.76 9580	94.73 9388	94.37 8477	94.40 8053	95.25 7175	95.16 6565
Loglogistic(1,4)	100	95.22 10000	95.27 10000	94.89 10000	94.89 10000	95.16 9985	95.12 9955	94.96 9449	94.96 9076	94.70 7761	94.64 6701
Loglogistic(1,4)	1000	95.13 10000	95.10 10000	94.89 10000	94.83 10000	95.33 10000	95.33 10000	94.87 10000	94.86 10000	95.17 9832	95.19 8665
Lognormal(0,0.2)	30	94.50 10000	94.52 10000	94.97 10000	94.85 10000	94.83 9969	94.77 9945	94.50 9327	94.89 8921	79.39 7970	94.70 3132
Lognormal(0,0.2)	100	91.49 10000	91.53 10000	93.14 10000	93.09 10000	92.82 10000	92.79 10000	93.22 9942	94.02 9690	67.58 8946	94.46 2546
Lognormal(0,0.2)	1000	39.26 10000	39.15 10000	58.84 10000	58.26 10000	56.59 10000	56.68 10000	72.94 10000	73.93 9869	10.39 10000	87.53 834
Pareto(5)	30	-	-	94.86 5835	94.94 5215	95.47 5872	95.48 5243	94.49 5826	94.80 5149	94.52 5857	94.80 5150
Pareto(5)	100	-	-	94.25 5478	94.44 4230	94.91 5477	95.00 4262	95.04 5447	95.15 4165	95.22 5527	95.35 4258
Pareto(5)	1000	-	-	95.73 5181	95.44 1798	94.50 5145	94.48 1810	94.75 5051	94.91 1788	95.10 5061	95.32 1753
ChiSquared1	30	75.55 10000	75.58 10000	92.19 9985	92.18 9969	93.00 9963	92.97 9928	93.26 9919	93.27 9867	93.33 9878	93.33 9805
ChiSquared1	100	24.09 10000	24.26 10000	82.66 10000	82.66 10000	86.18 10000	86.16 10000	87.41 10000	87.43 9999	88.01 9999	88.01 9997
ChiSquared1	1000	0.00 10000	0.00 10000	2.41 10000	2.44 10000	7.68 10000	7.72 10000	12.17 10000	12.19 10000	19.41 10000	19.43 10000
ChiSquared3	30	89.55 10000	89.55 10000	91.46 10000	91.35 10000	91.36 10000	91.27 10000	92.30 10000	92.24 10000	92.30 9999	92.26 9997
ChiSquared3	100	74.22 10000	74.33 10000	78.51 10000	78.70 10000	80.30 10000	80.42 10000	82.50 10000	82.52 10000	84.27 10000	84.29 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.41 10000	0.41 10000	0.89 10000	0.88 10000	1.77 10000	1.74 10000	3.59 10000	3.59 10000
ChiSquared4	30	91.36 10000	91.39 10000	91.30 10000	91.14 10000	91.81 10000	91.74 10000	92.20 10000	92.15 10000	92.41 10000	92.36 10000
ChiSquared4	100	80.03 10000	80.11 10000	81.11 10000	81.03 10000	82.15 10000	82.32 10000	82.45 10000	82.50 10000	83.04 10000	83.15 10000
ChiSquared4	1000	0.04 10000	0.04 10000	0.54 10000	0.50 10000	1.00 10000	1.00 10000	1.07 10000	1.07 10000	2.23 10000	2.23 10000
Loglaplace(1,1)	30	86.66 10000	86.67 10000	88.56 10000	88.47 10000	87.29 10000	87.23 9999	87.12 9994	86.98 9988	87.81 9943	87.84 9910
Loglaplace(1,1)	100	68.41 10000	68.49 10000	72.72 10000	72.85 10000	71.20 10000	71.28 10000	69.40 10000	69.45 10000	70.59 10000	70.59 10000
Loglaplace(1,1)	1000	0.04 10000	0.04 10000	0.42 10000	0.42 10000	0.22 10000	0.22 10000	0.19 10000	0.19 10000	0.23 10000	0.23 10000
Rayleigh(4)	30	86.83 10000	86.84 10000	88.45 10000	88.24 10000	89.46 10000	89.27 10000	89.65 10000	89.41 10000	89.47 10000	89.35 10000
Rayleigh(4)	100	60.85 10000	60.97 10000	66.52 10000	66.19 10000	69.77 10000	69.35 10000	70.61 10000	70.20 10000	71.68 10000	71.65 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000
Gamma(5,2)	30	93.43 10000	93.44 10000	93.30 10000	93.30 10000	93.32 10000	93.28 10000	93.20 10000	93.15 10000	93.51 10000	93.39 10000
Gamma(5,2)	100	87.83 10000	87.86 10000	87.25 10000	87.28 10000	87.45 10000	87.46 10000	88.05 10000	87.96 10000	88.02 10000	87.94 10000
Gamma(5,2)	1000	10.36 10000	10.28 10000	10.18 10000	10.09 10000	10.47 10000	10.34 10000	10.69 10000	10.43 10000	10.10 10000	9.83 10000
Uniform	30	23.75 10000	23.76 10000	72.94 9999	72.84 9999	78.95 9999	78.83 9997	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	12.32 10000	12.39 10000	24.86 10000	24.92 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	87.10 10000	87.13 10000	90.31 10000	90.20 10000	91.27 10000	91.17 10000	91.04 10000	90.89 10000	91.84 10000	91.74 10000
Mittag-Leffler(1,5)	100	61.60 10000	61.70 10000	73.72 10000	73.80 10000	76.88 10000	76.97 10000	77.58 10000	77.71 10000	79.37 10000	79.39 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.14 10000	0.14 10000	0.22 10000	0.22 10000	0.56 10000	0.56 10000

Table E.21: Testing v.s. Case II Loglogistic (1,4) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	85.04 10000	85.10 10000	92.85 9988	92.74 9982	93.11 9937	93.08 9902	93.39 9890	93.35 9825	93.84 9800	93.91 9683
Weibull(1,1)	100	53.50 10000	53.68 10000	83.47 10000	83.52 10000	86.40 10000	86.44 9999	87.53 10000	87.56 10000	89.43 9998	89.43 9990
Weibull(1,1)	1000	0.00 10000	0.00 10000	2.51 10000	2.51 10000	9.68 10000	9.68 10000	16.03 10000	16.05 10000	25.59 10000	25.61 10000
Loglogistic(1,4)	30	95.17 10000	95.19 10000	94.75 10000	94.70 10000	94.63 9580	94.59 9388	94.53 8477	94.44 8053	95.09 7175	94.94 6565
Loglogistic(1,4)	100	95.10 10000	95.12 10000	95.30 10000	95.32 10000	94.99 9985	94.98 9955	95.07 9449	95.05 9076	94.58 7761	94.63 6701
Loglogistic(1,4)	1000	95.04 10000	95.01 10000	94.87 10000	94.85 10000	94.98 10000	94.99 10000	94.76 10000	94.78 10000	95.33 9832	95.35 8665
Lognormal(0,0.2)	30	94.09 10000	94.12 10000	95.07 10000	94.95 10000	94.67 9969	94.61 9945	94.61 9327	94.83 8921	89.64 7970	94.89 3132
Lognormal(0,0.2)	100	90.37 10000	90.44 10000	92.52 10000	92.42 10000	92.19 10000	92.20 10000	93.08 9942	93.44 9690	84.05 8946	94.30 2546
Lognormal(0,0.2)	1000	31.02 10000	30.96 10000	52.01 10000	51.22 10000	51.49 10000	51.56 10000	69.73 10000	70.77 9869	21.92 10000	86.21 834
Pareto(5)	30	-	-	94.50 5835	94.61 5215	95.38 5872	95.56 5243	94.58 5826	94.68 5149	94.54 5857	94.78 5150
Pareto(5)	100	-	-	94.51 5478	94.66 4230	94.72 5477	94.89 4262	94.99 5447	95.10 4165	95.11 5527	95.37 4258
Pareto(5)	1000	-	-	94.71 5181	94.83 1798	94.79 5145	94.70 1810	94.63 5051	94.52 1788	94.82 5061	95.38 1753
ChiSquared1	30	71.75 10000	71.81 10000	91.73 9985	91.64 9969	92.86 9963	92.84 9928	92.93 9919	92.94 9867	92.94 9878	92.95 9805
ChiSquared1	100	18.31 10000	18.40 10000	80.72 10000	80.77 10000	84.82 10000	84.83 10000	86.30 10000	86.32 9999	87.23 9999	87.25 9997
ChiSquared1	1000	0.00 10000	0.00 10000	1.17 10000	1.17 10000	4.61 10000	4.61 10000	8.05 10000	8.05 10000	14.10 10000	14.10 10000
ChiSquared3	30	88.74 10000	88.78 10000	91.15 10000	91.06 10000	90.87 10000	90.74 10000	92.00 10000	91.92 10000	92.14 9999	92.06 9997
ChiSquared3	100	69.46 10000	69.65 10000	75.86 10000	76.05 10000	78.09 10000	78.17 10000	80.48 10000	80.51 10000	82.81 10000	82.85 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.13 10000	0.13 10000	0.36 10000	0.36 10000	0.71 10000	0.71 10000	1.90 10000	1.90 10000
ChiSquared4	30	91.42 10000	91.43 10000	91.47 10000	91.23 10000	91.76 10000	91.60 10000	92.14 10000	92.04 10000	91.86 10000	91.76 10000
ChiSquared4	100	77.31 10000	77.41 10000	78.31 10000	78.10 10000	79.39 10000	79.53 10000	79.90 10000	80.06 10000	81.14 10000	81.23 10000
ChiSquared4	1000	0.03 10000	0.03 10000	0.15 10000	0.14 10000	0.23 10000	0.24 10000	0.34 10000	0.34 10000	0.88 10000	0.87 10000
Loglaplace(1,1)	30	85.74 10000	85.79 10000	87.66 10000	87.51 10000	86.36 10000	86.19 9999	86.47 9994	86.39 9988	86.88 9943	86.86 9910
Loglaplace(1,1)	100	64.68 10000	64.84 10000	69.92 10000	70.12 10000	67.41 10000	67.48 10000	65.60 10000	65.65 10000	66.75 10000	66.77 10000
Loglaplace(1,1)	1000	0.03 10000	0.03 10000	0.15 10000	0.16 10000	0.12 10000	0.12 10000	0.06 10000	0.06 10000	0.10 10000	0.10 10000
Rayleigh(4)	30	85.40 10000	85.49 10000	87.68 10000	87.29 10000	88.55 10000	88.32 10000	88.75 10000	88.46 10000	88.53 10000	88.33 10000
Rayleigh(4)	100	53.47 10000	53.59 10000	59.73 10000	59.20 10000	63.48 10000	62.97 10000	65.67 10000	65.36 10000	67.72 10000	67.70 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	93.37 10000	93.39 10000	93.39 10000	93.37 10000	93.04 10000	93.01 10000	92.69 10000	92.64 10000	93.26 10000	93.14 10000
Gamma(5,2)	100	86.28 10000	86.38 10000	86.09 10000	86.12 10000	85.94 10000	85.90 10000	86.81 10000	86.73 10000	86.66 10000	86.57 10000
Gamma(5,2)	1000	5.29 10000	5.26 10000	4.95 10000	4.86 10000	5.29 10000	5.16 10000	5.19 10000	5.04 10000	4.88 10000	4.80 10000
Uniform	30	20.32 10000	20.39 10000	68.04 9999	67.79 9999	75.94 9999	75.71 9997	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	6.92 10000	6.98 10000	15.97 10000	16.05 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	85.56 10000	85.59 10000	90.26 10000	90.16 10000	90.75 10000	90.68 10000	90.57 10000	90.44 10000	91.79 10000	91.68 10000
Mittag-Leffler(1,5)	100	54.48 10000	54.65 10000	70.14 10000	70.36 10000	73.72 10000	73.98 10000	74.61 10000	74.72 10000	77.26 10000	77.33 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.05 10000	0.05 10000	0.16 10000	0.16 10000

Table E.22: Testing v.s. Case II Loglogistic (1,4) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	81.21 10000	81.28 10000	92.24 9988	92.16 9982	92.73 9937	92.68 9902	92.99 9890	92.96 9825	93.80 9800	93.81 9683
Weibull(1,1)	100	44.48 10000	44.73 10000	80.03 10000	80.08 10000	84.57 10000	84.59 9999	86.12 10000	86.14 10000	88.22 9998	88.25 9990
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.66 10000	0.66 10000	4.34 10000	4.36 10000	8.31 10000	8.32 10000	16.17 10000	16.24 10000
Loglogistic(1,4)	30	95.31 10000	95.37 10000	94.71 10000	94.59 10000	94.45 9580	94.40 9388	94.63 8477	94.59 8053	95.26 7175	95.28 6565
Loglogistic(1,4)	100	94.95 10000	94.98 10000	95.12 10000	95.14 10000	94.97 9985	94.96 9955	94.68 9449	94.61 9076	94.51 7761	94.58 6701
Loglogistic(1,4)	1000	95.14 10000	95.03 10000	94.87 10000	94.87 10000	95.02 10000	95.03 10000	94.79 10000	94.79 10000	95.31 9832	95.37 8665
Lognormal(0,0.2)	30	94.36 10000	94.42 10000	94.75 10000	94.60 10000	94.62 9969	94.55 9945	94.48 9327	94.89 8921	79.02 7970	94.99 3132
Lognormal(0,0.2)	100	90.06 10000	90.16 10000	92.19 10000	92.10 10000	91.76 10000	91.78 10000	92.21 9942	93.09 9690	66.06 8946	93.79 2546
Lognormal(0,0.2)	1000	21.84 10000	21.71 10000	43.08 10000	42.30 10000	41.13 10000	41.22 10000	64.15 10000	65.09 9869	8.57 10000	83.69 834
Pareto(5)	30	-	-	94.48 5835	94.59 5215	94.99 5872	95.25 5243	94.85 5826	95.07 5149	94.57 5857	94.87 5150
Pareto(5)	100	-	-	94.49 5478	94.54 4230	94.80 5477	95.05 4262	94.93 5447	94.91 4165	95.44 5527	95.63 4258
Pareto(5)	1000	-	-	95.00 5181	94.88 1798	95.20 5145	94.81 1810	95.03 5051	94.85 1788	94.74 5061	95.21 1753
ChiSquared1	30	66.08 10000	66.24 10000	91.38 9985	91.26 9969	92.45 9963	92.43 9928	92.40 9919	92.38 9867	92.99 9878	92.96 9805
ChiSquared1	100	13.75 10000	13.80 10000	77.12 10000	77.14 10000	82.22 10000	82.23 10000	83.90 10000	83.92 9999	85.82 9999	85.83 9997
ChiSquared1	1000	0.00 10000	0.00 10000	0.28 10000	0.28 10000	1.54 10000	1.54 10000	3.16 10000	3.16 10000	7.12 10000	7.14 10000
ChiSquared3	30	86.26 10000	86.34 10000	89.55 10000	89.39 10000	89.86 10000	89.64 10000	91.32 10000	91.17 10000	91.36 9999	91.30 9997
ChiSquared3	100	61.10 10000	61.27 10000	69.31 10000	69.66 10000	72.64 10000	72.75 10000	75.87 10000	75.91 10000	79.51 10000	79.54 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.07 10000	0.07 10000	0.06 10000	0.06 10000	0.42 10000	0.42 10000
ChiSquared4	30	89.58 10000	89.68 10000	89.78 10000	89.56 10000	91.05 10000	90.88 10000	91.15 10000	91.03 10000	91.25 10000	91.12 10000
ChiSquared4	100	69.99 10000	70.22 10000	71.58 10000	71.46 10000	73.23 10000	73.50 10000	74.67 10000	74.82 10000	76.85 10000	76.96 10000
ChiSquared4	1000	0.01 10000	0.01 10000	0.01 10000	0.01 10000	0.02 10000	0.02 10000	0.05 10000	0.05 10000	0.11 10000	0.11 10000
Loglaplace(1,1)	30	85.16 10000	85.23 10000	86.92 10000	86.72 10000	85.49 10000	85.33 9999	85.16 9994	84.99 9988	85.50 9943	85.40 9910
Loglaplace(1,1)	100	60.60 10000	60.75 10000	66.01 10000	66.23 10000	62.77 10000	62.85 10000	60.74 10000	60.79 10000	62.38 10000	62.44 10000
Loglaplace(1,1)	1000	0.01 10000	0.01 10000	0.03 10000	0.03 10000	0.01 10000	0.01 10000	0.01 10000	0.01 10000	0.05 10000	0.05 10000
Rayleigh(4)	30	82.07 10000	82.17 10000	84.32 10000	84.05 10000	85.67 10000	85.37 10000	85.83 10000	85.44 10000	85.84 10000	85.62 10000
Rayleigh(4)	100	43.71 10000	43.84 10000	49.47 10000	49.13 10000	52.37 10000	51.95 10000	55.31 10000	55.07 10000	57.15 10000	57.18 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	92.78 10000	92.83 10000	92.74 10000	92.73 10000	92.36 10000	92.33 10000	92.50 10000	92.44 10000	92.56 10000	92.46 10000
Gamma(5,2)	100	83.46 10000	83.57 10000	82.25 10000	82.31 10000	82.39 10000	82.33 10000	83.32 10000	83.18 10000	83.19 10000	83.05 10000
Gamma(5,2)	1000	1.72 10000	1.71 10000	1.32 10000	1.29 10000	1.81 10000	1.81 10000	1.73 10000	1.65 10000	1.65 10000	1.57 10000
Uniform	30	22.61 10000	22.68 10000	60.51 9999	60.08 9999	69.02 9999	68.71 9997	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	5.05 10000	5.12 10000	11.07 10000	11.10 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	81.91 10000	81.96 10000	88.15 10000	87.96 10000	89.35 10000	89.22 10000	89.39 10000	89.21 10000	90.22 10000	90.03 10000
Mittag-Leffler(1,5)	100	44.42 10000	44.60 10000	61.45 10000	61.77 10000	66.60 10000	66.79 10000	68.32 10000	68.38 10000	72.01 10000	72.05 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000

Table E.23: Testing v.s. Case II Loglogistic (1,4) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	73.37 10000	73.66 10000	90.59 9988	90.45 9982	91.81 9937	91.80 9902	92.14 9890	92.10 9825	92.96 9800	92.98 9683
Weibull(1,1)	100	21.43 10000	21.54 10000	75.46 10000	75.49 10000	82.06 10000	82.09 9999	84.00 10000	83.98 10000	86.89 9998	86.88 9990
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.11 10000	0.11 10000	1.55 10000	1.58 10000	3.83 10000	3.88 10000	10.06 10000	10.12 10000
Loglogistic(1,4)	30	95.28 10000	95.42 10000	94.76 10000	94.66 10000	94.67 9580	94.59 9388	94.54 8477	94.57 8053	95.26 7175	95.25 6565
Loglogistic(1,4)	100	95.13 10000	95.15 10000	95.08 10000	95.08 10000	95.09 9985	95.07 9955	94.99 9449	94.93 9076	94.34 7761	94.52 6701
Loglogistic(1,4)	1000	95.24 10000	95.15 10000	95.03 10000	95.03 10000	94.77 10000	94.85 10000	94.72 10000	94.69 10000	95.32 9832	95.31 8665
Lognormal(0,0.2)	30	95.46 10000	95.58 10000	95.47 10000	95.46 10000	94.20 9969	94.13 9945	93.97 9327	94.26 8921	82.41 7970	94.80 3132
Lognormal(0,0.2)	100	91.03 10000	91.04 10000	92.71 10000	92.59 10000	90.85 10000	90.85 10000	91.66 9942	92.35 9690	70.58 8946	94.30 2546
Lognormal(0,0.2)	1000	15.11 10000	14.93 10000	33.63 10000	32.96 10000	32.58 10000	32.67 10000	60.19 10000	61.06 9869	8.85 10000	82.25 834
Pareto(5)	30	-	-	94.79 5835	94.94 5215	94.91 5872	95.08 5243	94.59 5826	94.99 5149	94.72 5857	94.89 5150
Pareto(5)	100	-	-	94.74 5478	94.78 4230	94.61 5477	94.74 4262	95.26 5447	95.29 4165	95.42 5527	95.58 4258
Pareto(5)	1000	-	-	94.92 5181	94.94 1798	95.32 5145	95.03 1810	95.07 5051	94.69 1788	94.63 5061	95.09 1753
ChiSquared1	30	51.43 10000	51.73 10000	89.49 9985	89.43 9969	90.89 9963	90.86 9928	91.81 9919	91.76 9867	91.50 9878	91.42 9805
ChiSquared1	100	2.49 10000	2.50 10000	72.23 10000	72.24 10000	78.56 10000	78.59 10000	80.92 10000	80.97 9999	83.39 9999	83.41 9997
ChiSquared1	1000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.25 10000	0.25 10000	1.06 10000	1.09 10000	3.28 10000	3.32 10000
ChiSquared3	30	81.91 10000	82.17 10000	86.63 10000	86.55 10000	87.79 10000	87.66 10000	89.53 10000	89.44 10000	89.52 9999	89.41 9997
ChiSquared3	100	39.88 10000	39.99 10000	58.04 10000	58.28 10000	63.82 10000	63.94 10000	68.77 10000	68.81 10000	74.23 10000	74.22 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000
ChiSquared4	30	86.59 10000	86.74 10000	87.06 10000	86.99 10000	88.57 10000	88.52 10000	88.89 10000	88.78 10000	89.28 10000	89.23 10000
ChiSquared4	100	53.02 10000	53.15 10000	58.80 10000	58.76 10000	62.49 10000	62.73 10000	65.57 10000	65.92 10000	69.04 10000	69.14 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	84.05 10000	84.22 10000	86.49 10000	86.43 10000	86.34 10000	86.20 9999	85.95 9994	85.86 9988	87.16 9943	87.09 9910
Loglaplace(1,1)	100	62.53 10000	62.64 10000	67.80 10000	68.03 10000	64.51 10000	64.56 10000	62.69 10000	62.72 10000	63.93 10000	63.95 10000
Loglaplace(1,1)	1000	0.02 10000	0.02 10000	0.08 10000	0.08 10000	0.01 10000	0.01 10000	0.03 10000	0.03 10000	0.04 10000	0.04 10000
Rayleigh(4)	30	73.64 10000	74.01 10000	77.12 10000	77.12 10000	79.34 10000	79.27 10000	80.69 10000	80.59 10000	80.99 10000	80.86 10000
Rayleigh(4)	100	21.00 10000	21.10 10000	26.36 10000	26.14 10000	30.91 10000	30.69 10000	35.09 10000	34.96 10000	38.70 10000	38.69 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	92.61 10000	92.69 10000	92.19 10000	92.33 10000	91.96 10000	92.05 10000	91.92 10000	91.99 10000	92.00 10000	92.03 10000
Gamma(5,2)	100	77.68 10000	77.74 10000	76.63 10000	76.65 10000	76.50 10000	76.49 10000	77.50 10000	77.45 10000	77.08 10000	76.95 10000
Gamma(5,2)	1000	0.03 10000	0.03 10000	0.02 10000	0.02 10000	0.05 10000	0.05 10000	0.03 10000	0.03 10000	0.01 10000	0.01 10000
Uniform	30	10.11 10000	10.24 10000	49.74 9999	49.60 9999	60.83 9999	60.64 9997	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.92 10000	0.95 10000	3.37 10000	3.41 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	73.54 10000	73.98 10000	83.79 10000	83.73 10000	86.51 10000	86.37 10000	86.07 10000	85.93 10000	87.93 10000	87.83 10000
Mittag-Leffler(1,5)	100	20.52 10000	20.57 10000	47.67 10000	47.85 10000	54.15 10000	54.38 10000	57.65 10000	57.84 10000	62.98 10000	63.03 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.24: Testing v.s. Case II Loglogistic (1,4) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.2.3 Lognormal Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	70.91 10000	71.22 10000	94.48 9997	94.41 9874	95.00 9990	95.00 9716	95.09 9983	95.19 9629	95.11 9975	95.27 9479
Weibull(1,1)	100	26.45 10000	26.55 10000	93.68 10000	93.73 9999	93.62 10000	93.67 9998	93.88 10000	93.93 9993	94.27 10000	94.32 9976
Weibull(1,1)	1000	0.00 10000	0.00 10000	71.61 10000	71.84 10000	78.09 10000	78.31 10000	80.40 10000	80.51 10000	83.19 10000	83.31 10000
Loglogistic(1,4)	30	91.17 10000	91.37 10000	93.55 9912	93.80 9336	94.24 9464	94.91 7577	94.81 9009	95.39 6490	93.92 8489	94.91 5361
Loglogistic(1,4)	100	85.54 10000	85.58 10000	88.67 9997	88.94 9886	91.79 9903	92.74 7946	92.46 9644	93.11 6066	93.56 9098	94.05 4120
Loglogistic(1,4)	1000	17.46 10000	17.36 10000	32.39 10000	32.61 10000	53.96 10000	54.62 9730	65.81 10000	69.69 6111	78.30 9993	85.62 1057
Lognormal(0,0.2)	30	95.16 10000	95.22 10000	95.34 10000	95.32 10000	94.98 9964	95.01 9452	95.01 9806	95.22 8405	94.56 9383	95.12 6973
Lognormal(0,0.2)	100	94.82 10000	94.83 10000	95.18 10000	95.19 10000	95.56 10000	95.58 9961	95.14 9996	95.14 9341	94.72 9929	94.81 7269
Lognormal(0,0.2)	1000	94.79 10000	94.75 10000	94.88 10000	94.85 10000	94.82 10000	94.87 10000	94.95 10000	94.95 10000	95.12 10000	95.22 9403
Pareto(5)	30	-	-	93.71 6535	95.06 3037	94.02 6509	95.31 3047	94.57 6540	95.49 3084	93.95 6534	95.29 3080
Pareto(5)	100	-	-	93.25 5956	93.66 899	93.62 5992	94.35 973	93.66 5932	94.73 930	93.64 5939	94.12 885
Pareto(5)	1000	-	-	90.06 5263	-	89.28 5344	-	89.44 5237	-	90.26 5390	-
ChiSquared1	30	53.77 10000	54.22 10000	94.83 10000	94.73 9909	94.82 9994	94.77 9818	94.83 9993	94.89 9745	94.90 9982	94.93 9673
ChiSquared1	100	5.83 10000	5.84 10000	92.15 10000	92.23 9999	93.44 10000	93.49 10000	93.26 10000	93.32 9996	93.94 10000	93.97 9993
ChiSquared1	1000	0.00 10000	0.00 10000	58.94 10000	59.19 10000	66.64 10000	66.86 10000	69.71 10000	70.03 10000	74.01 10000	74.29 10000
ChiSquared3	30	79.91 10000	80.16 10000	93.18 10000	93.18 10000	93.85 10000	93.80 10000	94.39 10000	94.31 9998	94.44 10000	94.41 9997
ChiSquared3	100	44.66 10000	44.74 10000	86.89 10000	86.92 10000	88.30 10000	88.29 10000	90.18 10000	90.16 10000	90.32 10000	90.32 10000
ChiSquared3	1000	0.00 10000	0.00 10000	16.63 10000	16.51 10000	26.55 10000	26.43 10000	33.48 10000	33.33 10000	40.48 10000	40.38 10000
ChiSquared4	30	83.95 10000	84.13 10000	91.93 10000	91.95 10000	93.06 10000	93.05 10000	92.70 10000	92.67 10000	93.25 10000	93.23 10000
ChiSquared4	100	57.03 10000	57.10 10000	83.84 10000	83.87 10000	85.71 10000	85.80 10000	86.81 10000	86.83 10000	88.06 10000	88.08 10000
ChiSquared4	1000	0.00 10000	0.00 10000	6.20 10000	6.10 10000	12.52 10000	12.46 10000	17.25 10000	17.22 10000	23.37 10000	23.31 10000
Loglaplace(1,1)	30	72.15 10000	72.46 10000	76.80 10000	76.76 9977	77.05 9987	77.40 9894	77.60 9975	77.91 9826	78.32 9942	79.05 9616
Loglaplace(1,1)	100	30.46 10000	30.56 10000	34.56 10000	34.55 10000	34.85 10000	34.85 9992	34.34 10000	34.47 9981	35.12 9999	35.34 9954
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	70.94 10000	71.14 10000	82.84 10000	82.90 10000	84.23 10000	84.35 10000	84.86 10000	84.98 10000	86.36 10000	86.46 10000
Rayleigh(4)	100	26.36 10000	26.45 10000	49.21 10000	49.30 10000	53.71 10000	53.66 10000	58.54 10000	58.49 10000	61.74 10000	61.65 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000
Gamma(5,2)	30	90.57 10000	90.69 10000	90.31 10000	90.45 10000	91.17 10000	91.35 10000	91.81 10000	92.04 10000	91.22 10000	91.30 10000
Gamma(5,2)	100	80.52 10000	80.61 10000	80.38 10000	80.41 10000	81.46 10000	81.59 10000	80.14 10000	80.19 10000	81.08 10000	81.18 10000
Gamma(5,2)	1000	4.37 10000	4.34 10000	4.28 10000	4.25 10000	4.76 10000	4.75 10000	5.25 10000	5.21 10000	5.18 10000	5.14 10000
Uniform	30	21.54 10000	21.90 10000	77.51 10000	77.44 9999	80.09 10000	79.96 9999	-	-	-	-
Uniform	100	0.01 10000	0.01 10000	30.72 10000	30.72 10000	37.87 10000	37.87 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	72.25 10000	72.54 10000	92.07 10000	92.06 10000	92.58 10000	92.55 10000	92.51 10000	92.49 10000	93.26 10000	93.17 10000
Mittag-Leffler(1,5)	100	27.08 10000	27.21 10000	81.73 10000	81.86 10000	84.54 10000	84.59 10000	84.74 10000	84.72 10000	86.19 10000	86.19 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	3.09 10000	3.09 10000	6.60 10000	6.56 10000	9.54 10000	9.43 10000	12.56 10000	12.52 10000

Table E.25: Testing v.s. Case II Lognormal (0,0.2) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	73.37 10000	73.37 10000	94.40 9997	94.39 9874	94.74 9990	94.87 9716	94.36 9983	94.59 9629	94.92 9975	95.07 9479
Weibull(1,1)	100	27.98 10000	28.03 10000	92.98 10000	92.97 9999	93.47 10000	93.47 9998	93.50 10000	93.51 9993	93.94 10000	93.97 9976
Weibull(1,1)	1000	0.00 10000	0.00 10000	66.94 10000	67.04 10000	74.39 10000	74.45 10000	77.41 10000	77.49 10000	80.81 10000	80.93 10000
Loglogistic(1,4)	30	90.97 10000	90.96 10000	93.07 9912	93.25 9336	94.01 9464	94.46 7577	94.51 9009	94.76 6490	94.09 8489	94.61 5361
Loglogistic(1,4)	100	82.18 10000	82.27 10000	87.20 9997	87.41 9886	91.09 9903	92.00 7946	92.33 9644	92.88 6066	93.28 9098	93.69 4120
Loglogistic(1,4)	1000	8.51 10000	8.50 10000	26.62 10000	26.69 10000	50.90 10000	51.42 9730	65.66 10000	69.20 6111	79.17 9993	85.05 1057
Lognormal(0,0.2)	30	95.14 10000	95.12 10000	95.19 10000	95.14 10000	94.68 9964	94.70 9452	94.72 9806	94.86 8405	94.62 9383	94.87 6973
Lognormal(0,0.2)	100	94.59 10000	94.62 10000	94.86 10000	94.87 10000	95.54 10000	95.55 9961	95.09 9996	95.03 9341	94.51 9929	94.51 7269
Lognormal(0,0.2)	1000	94.85 10000	94.85 10000	94.99 10000	95.00 10000	94.70 10000	94.72 10000	95.38 10000	95.38 10000	95.22 10000	95.17 9403
Pareto(5)	30	-	-	93.94 6535	94.34 3037	94.25 6509	94.62 3047	94.74 6540	95.04 3084	94.31 6534	94.87 3080
Pareto(5)	100	-	-	93.91 5956	93.44 899	94.33 5992	94.24 973	94.47 5932	94.84 930	94.58 5939	94.92 885
Pareto(5)	1000	-	-	93.63 5263	-	93.32 5344	-	93.01 5237	-	93.49 5390	-
ChiSquared1	30	54.65 10000	54.65 10000	94.46 10000	94.51 9909	94.48 9994	94.47 9818	94.47 9993	94.52 9745	94.27 9982	94.29 9673
ChiSquared1	100	4.87 10000	4.90 10000	91.88 10000	91.87 9999	92.86 10000	92.86 9999	92.87 10000	92.86 9996	93.44 10000	93.45 9993
ChiSquared1	1000	0.00 10000	0.00 10000	51.65 10000	51.73 10000	60.91 10000	61.03 10000	64.30 10000	64.44 10000	70.16 10000	70.20 10000
ChiSquared3	30	82.18 10000	82.18 10000	93.51 10000	93.42 10000	93.75 10000	93.70 10000	94.48 10000	94.41 9998	94.62 10000	94.58 9997
ChiSquared3	100	49.26 10000	49.33 10000	87.49 10000	87.48 10000	88.35 10000	88.35 10000	89.70 10000	89.71 10000	90.48 10000	90.50 10000
ChiSquared3	1000	0.00 10000	0.00 10000	13.40 10000	13.40 10000	21.63 10000	21.64 10000	27.87 10000	27.91 10000	33.74 10000	33.80 10000
ChiSquared4	30	86.07 10000	86.07 10000	93.11 10000	92.79 10000	93.93 10000	93.71 10000	93.06 10000	92.83 10000	93.89 10000	93.68 10000
ChiSquared4	100	62.64 10000	62.69 10000	85.42 10000	85.20 10000	87.51 10000	87.44 10000	88.28 10000	88.22 10000	88.84 10000	88.83 10000
ChiSquared4	1000	0.00 10000	0.00 10000	5.72 10000	5.67 10000	10.84 10000	10.78 10000	15.47 10000	15.48 10000	20.05 10000	20.06 10000
Loglaplace(1,1)	30	69.27 10000	69.26 10000	74.01 10000	73.95 9977	73.83 9987	74.13 9894	74.59 9975	75.02 9826	75.06 9942	75.88 9616
Loglaplace(1,1)	100	21.54 10000	21.63 10000	29.65 10000	29.69 10000	29.84 10000	29.88 9992	29.26 10000	29.35 9981	29.70 9999	29.82 9954
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	73.75 10000	73.73 10000	86.44 10000	86.20 10000	87.30 10000	87.04 10000	87.80 10000	87.43 10000	88.95 10000	88.59 10000
Rayleigh(4)	100	28.29 10000	28.35 10000	55.56 10000	55.26 10000	60.86 10000	60.43 10000	65.29 10000	64.71 10000	67.51 10000	66.86 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.01 10000
Gamma(5,2)	30	91.54 10000	91.53 10000	92.06 10000	92.03 10000	92.29 10000	92.28 10000	92.76 10000	92.75 10000	92.40 10000	92.38 10000
Gamma(5,2)	100	84.82 10000	84.86 10000	84.38 10000	84.46 10000	85.37 10000	85.40 10000	85.00 10000	85.04 10000	85.66 10000	85.70 10000
Gamma(5,2)	1000	6.27 10000	6.27 10000	6.51 10000	6.49 10000	6.72 10000	6.71 10000	6.98 10000	6.97 10000	7.47 10000	7.45 10000
Uniform	30	12.09 10000	12.09 10000	76.83 10000	76.63 9999	78.79 10000	78.64 9999	-	-	-	-
Uniform	100	0.01 10000	0.01 10000	24.62 10000	24.65 10000	31.33 10000	31.37 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	74.49 10000	74.48 10000	92.37 10000	92.25 10000	92.90 10000	92.80 10000	92.90 10000	92.79 10000	93.32 10000	93.22 10000
Mittag-Leffler(1,5)	100	28.95 10000	29.07 10000	82.50 10000	82.48 10000	84.51 10000	84.50 10000	84.89 10000	84.91 10000	86.23 10000	86.29 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	2.08 10000	2.08 10000	4.51 10000	4.51 10000	6.51 10000	6.51 10000	9.17 10000	9.19 10000

Table E.26: Testing v.s. Case II Lognormal (0,0.2) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	63.86 10000	64.12 10000	94.51 9997	94.46 9874	94.94 9990	95.00 9716	94.96 9983	95.16 9629	94.96 9975	95.06 9479
Weibull(1,1)	100	14.85 10000	14.93 10000	92.98 10000	93.01 9999	93.41 10000	93.45 9998	93.52 10000	93.59 9993	93.78 10000	93.84 9976
Weibull(1,1)	1000	0.00 10000	0.00 10000	61.31 10000	61.43 10000	70.53 10000	70.70 10000	73.96 10000	74.11 10000	77.61 10000	77.75 10000
Loglogistic(1,4)	30	89.57 10000	89.75 10000	92.65 9912	92.87 9336	93.85 9464	94.44 7577	94.38 9009	94.85 6490	94.23 8489	95.06 5361
Loglogistic(1,4)	100	80.45 10000	80.51 10000	85.67 9997	85.90 9886	90.59 9903	91.78 7946	91.64 9644	92.60 6066	92.88 9098	93.83 4120
Loglogistic(1,4)	1000	6.16 10000	6.12 10000	19.99 10000	20.18 10000	42.75 10000	43.41 9730	58.25 10000	63.12 6111	73.67 9993	81.36 1057
Lognormal(0,0.2)	30	95.17 10000	95.22 10000	95.32 10000	95.25 10000	95.24 9964	95.26 9452	94.82 9806	95.04 8405	94.70 9383	95.02 6973
Lognormal(0,0.2)	100	94.71 10000	94.75 10000	94.70 10000	94.74 10000	95.45 10000	95.47 9961	95.11 9996	95.10 9341	94.76 9929	94.81 7269
Lognormal(0,0.2)	1000	94.69 10000	94.64 10000	94.77 10000	94.73 10000	94.54 10000	94.55 10000	95.14 10000	95.14 10000	95.26 10000	95.18 9403
Pareto(5)	30	-	-	93.68 6535	94.20 3037	94.07 6509	95.24 3047	94.39 6540	95.30 3084	93.91 6534	95.10 3080
Pareto(5)	100	-	-	93.65 5956	94.88 899	93.69 5992	94.55 973	93.41 5932	94.73 930	93.77 5939	95.25 885
Pareto(5)	1000	-	-	90.50 5263	-	89.52 5344	-	90.20 5237	-	90.45 5390	-
ChiSquared1	30	43.17 10000	43.48 10000	94.54 10000	94.54 9909	94.57 9994	94.51 9818	94.62 9993	94.67 9745	94.48 9982	94.43 9673
ChiSquared1	100	1.50 10000	1.50 10000	91.39 10000	91.46 9999	92.31 10000	92.39 9999	92.41 10000	92.44 9996	93.20 10000	93.24 9993
ChiSquared1	1000	0.00 10000	0.00 10000	43.79 10000	43.87 10000	54.57 10000	54.71 10000	58.89 10000	59.05 10000	64.87 10000	65.01 10000
ChiSquared3	30	75.71 10000	75.96 10000	93.01 10000	92.95 10000	93.05 10000	92.95 10000	94.02 10000	93.86 9998	94.21 10000	94.11 9997
ChiSquared3	100	32.05 10000	32.18 10000	84.04 10000	84.07 10000	86.19 10000	86.26 10000	88.04 10000	88.08 10000	88.96 10000	89.03 10000
ChiSquared3	1000	0.00 10000	0.00 10000	6.16 10000	6.11 10000	12.53 10000	12.49 10000	18.16 10000	18.10 10000	24.17 10000	24.08 10000
ChiSquared4	30	80.96 10000	81.12 10000	91.48 10000	91.35 10000	92.63 10000	92.57 10000	92.01 10000	91.85 10000	93.04 10000	92.93 10000
ChiSquared4	100	46.45 10000	46.59 10000	80.06 10000	79.94 10000	83.18 10000	83.21 10000	84.74 10000	84.79 10000	85.81 10000	85.82 10000
ChiSquared4	1000	0.00 10000	0.00 10000	1.54 10000	1.52 10000	3.92 10000	3.91 10000	6.65 10000	6.67 10000	10.71 10000	10.64 10000
Loglaplace(1,1)	30	66.28 10000	66.60 10000	71.61 10000	71.56 9977	71.19 9987	71.38 9894	71.75 9975	72.06 9826	72.27 9942	73.08 9616
Loglaplace(1,1)	100	18.65 10000	18.73 10000	24.53 10000	24.57 10000	23.81 10000	23.85 9992	23.70 10000	23.83 9981	23.92 9999	24.12 9954
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	64.75 10000	65.05 10000	79.61 10000	79.61 10000	81.72 10000	81.63 10000	83.01 10000	82.95 10000	84.19 10000	84.14 10000
Rayleigh(4)	100	15.13 10000	15.16 10000	36.64 10000	36.65 10000	43.55 10000	43.40 10000	47.86 10000	47.70 10000	52.07 10000	51.72 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	89.09 10000	89.28 10000	89.48 10000	89.57 10000	89.91 10000	90.09 10000	90.98 10000	91.11 10000	90.44 10000	90.53 10000
Gamma(5,2)	100	76.16 10000	76.27 10000	76.34 10000	76.40 10000	77.15 10000	77.23 10000	76.24 10000	76.37 10000	77.76 10000	77.83 10000
Gamma(5,2)	1000	1.04 10000	1.04 10000	1.04 10000	1.01 10000	1.07 10000	1.05 10000	1.11 10000	1.10 10000	1.09 10000	1.09 10000
Uniform	30	10.20 10000	10.37 10000	71.49 10000	71.23 9999	73.71 10000	73.35 9999	-	-	-	-
Uniform	100	0.01 10000	0.01 10000	15.66 10000	15.71 10000	20.70 10000	20.77 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	65.59 10000	65.85 10000	91.02 10000	90.87 10000	91.82 10000	91.74 10000	92.35 10000	92.24 10000	92.72 10000	92.61 10000
Mittag-Leffler(1,5)	100	15.62 10000	15.74 10000	77.06 10000	77.05 10000	79.66 10000	79.70 10000	80.85 10000	80.92 10000	83.15 10000	83.18 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.44 10000	0.44 10000	1.38 10000	1.38 10000	2.38 10000	2.35 10000	4.00 10000	3.97 10000

Table E.27: Testing v.s. Case II Lognormal (0,0.2) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	60.44 (10000)	60.93 (10000)	93.89 (9997)	93.85 (9874)	94.37 (9990)	94.39 (9716)	94.22 (9983)	94.42 (9629)	94.69 (9975)	94.64 (9479)
Weibull(1,1)	100	10.45 (10000)	10.54 (10000)	91.92 (10000)	91.93 (9999)	92.54 (10000)	92.59 (9998)	92.97 (10000)	92.99 (9993)	93.10 (10000)	93.09 (9976)
Weibull(1,1)	1000	0.00 (10000)	0.00 (10000)	56.33 (10000)	56.40 (10000)	66.37 (10000)	66.45 (10000)	71.00 (10000)	71.09 (10000)	75.48 (10000)	75.54 (10000)
Loglogistic(1,4)	30	88.36 (10000)	88.53 (10000)	93.17 (9912)	93.27 (9336)	94.09 (9464)	94.52 (7577)	95.02 (9009)	95.29 (6490)	94.50 (8489)	95.21 (5361)
Loglogistic(1,4)	100	77.79 (10000)	77.88 (10000)	86.03 (9997)	86.29 (9886)	91.20 (9903)	92.00 (7946)	92.31 (9644)	92.99 (6066)	93.43 (9098)	94.15 (4120)
Loglogistic(1,4)	1000	3.84 (10000)	3.77 (10000)	17.79 (10000)	17.81 (10000)	41.62 (10000)	42.29 (9730)	57.62 (10000)	62.07 (6111)	73.63 (9993)	82.12 (1057)
Lognormal(0,0.2)	30	94.89 (10000)	95.15 (10000)	95.43 (10000)	95.35 (10000)	94.94 (9964)	94.94 (9452)	94.66 (9806)	94.71 (8405)	94.58 (9383)	95.02 (6973)
Lognormal(0,0.2)	100	94.70 (10000)	94.74 (10000)	94.74 (10000)	94.83 (10000)	95.23 (10000)	95.25 (9961)	94.72 (9996)	94.62 (9341)	94.79 (9929)	94.72 (7269)
Lognormal(0,0.2)	1000	94.70 (10000)	94.59 (10000)	95.07 (10000)	95.08 (10000)	94.90 (10000)	94.94 (10000)	95.02 (10000)	95.05 (10000)	95.08 (10000)	95.04 (9403)
Pareto(5)	30	-	-	94.03 (6535)	94.53 (3037)	93.58 (6509)	94.22 (3047)	94.11 (6540)	94.71 (3084)	94.02 (6534)	94.58 (3080)
Pareto(5)	100	-	-	93.82 (5956)	94.88 (899)	93.94 (5992)	94.14 (973)	93.83 (5932)	94.41 (930)	94.22 (5939)	95.59 (885)
Pareto(5)	1000	-	-	91.01 (5263)	-	90.76 (5344)	-	90.80 (5237)	-	90.89 (5390)	-
ChiSquared1	30	38.54 (10000)	38.92 (10000)	93.72 (10000)	93.73 (9909)	93.90 (9994)	93.90 (9818)	94.00 (9993)	94.04 (9745)	93.40 (9982)	93.50 (9673)
ChiSquared1	100	0.77 (10000)	0.77 (10000)	89.99 (10000)	90.02 (9999)	91.27 (10000)	91.32 (9999)	91.51 (10000)	91.50 (9996)	92.41 (10000)	92.43 (9993)
ChiSquared1	1000	0.00 (10000)	0.00 (10000)	36.56 (10000)	36.69 (10000)	48.93 (10000)	49.03 (10000)	53.62 (10000)	53.73 (10000)	60.08 (10000)	60.17 (10000)
ChiSquared3	30	72.99 (10000)	73.34 (10000)	92.42 (10000)	92.32 (10000)	92.39 (10000)	92.25 (10000)	93.10 (10000)	93.00 (9998)	92.94 (10000)	92.85 (9997)
ChiSquared3	100	25.62 (10000)	25.73 (10000)	81.47 (10000)	81.47 (10000)	84.24 (10000)	84.30 (10000)	85.77 (10000)	85.85 (10000)	87.35 (10000)	87.40 (10000)
ChiSquared3	1000	0.00 (10000)	0.00 (10000)	2.71 (10000)	2.70 (10000)	7.29 (10000)	7.25 (10000)	11.92 (10000)	11.88 (10000)	16.78 (10000)	16.81 (10000)
ChiSquared4	30	78.67 (10000)	78.99 (10000)	90.45 (10000)	90.35 (10000)	91.66 (10000)	91.55 (10000)	91.34 (10000)	91.27 (10000)	92.26 (10000)	92.15 (10000)
ChiSquared4	100	40.24 (10000)	40.36 (10000)	77.09 (10000)	76.91 (10000)	80.01 (10000)	79.91 (10000)	81.97 (10000)	81.97 (10000)	83.46 (10000)	83.45 (10000)
ChiSquared4	1000	0.00 (10000)	0.00 (10000)	0.57 (10000)	0.54 (10000)	1.46 (10000)	1.44 (10000)	3.26 (10000)	3.26 (10000)	5.91 (10000)	5.89 (10000)
Loglaplace(1,1)	30	65.26 (10000)	65.71 (10000)	72.29 (10000)	72.22 (9977)	72.69 (9987)	72.93 (9894)	73.98 (9975)	74.30 (9826)	74.83 (9942)	75.43 (9616)
Loglaplace(1,1)	100	18.10 (10000)	18.18 (10000)	25.01 (10000)	25.07 (10000)	24.98 (10000)	25.02 (9992)	24.88 (10000)	25.00 (9981)	25.31 (9999)	25.52 (9954)
Loglaplace(1,1)	1000	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)
Rayleigh(4)	30	60.94 (10000)	61.38 (10000)	76.45 (10000)	76.61 (10000)	79.22 (10000)	79.25 (10000)	80.39 (10000)	80.27 (10000)	81.75 (10000)	81.72 (10000)
Rayleigh(4)	100	10.88 (10000)	10.92 (10000)	29.24 (10000)	29.15 (10000)	36.13 (10000)	35.76 (10000)	40.76 (10000)	40.42 (10000)	44.51 (10000)	43.99 (10000)
Rayleigh(4)	1000	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)
Gamma(5,2)	30	88.11 (10000)	88.31 (10000)	88.46 (10000)	88.68 (10000)	89.40 (10000)	89.75 (10000)	90.19 (10000)	90.51 (10000)	89.50 (10000)	89.73 (10000)
Gamma(5,2)	100	72.90 (10000)	73.01 (10000)	73.20 (10000)	73.31 (10000)	73.81 (10000)	73.89 (10000)	72.88 (10000)	72.98 (10000)	74.37 (10000)	74.53 (10000)
Gamma(5,2)	1000	0.41 (10000)	0.41 (10000)	0.30 (10000)	0.30 (10000)	0.40 (10000)	0.39 (10000)	0.41 (10000)	0.41 (10000)	0.46 (10000)	0.46 (10000)
Uniform	30	6.69 (10000)	6.90 (10000)	64.84 (10000)	64.56 (9999)	67.62 (10000)	67.46 (9999)	-	-	-	-
Uniform	100	0.00 (10000)	0.00 (10000)	6.72 (10000)	6.76 (10000)	10.54 (10000)	10.58 (10000)	-	-	-	-
Uniform	1000	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	30	61.45 (10000)	61.89 (10000)	89.78 (10000)	89.69 (10000)	90.36 (10000)	90.17 (10000)	91.24 (10000)	91.15 (10000)	91.44 (10000)	91.33 (10000)
Mittag-Leffler(1,5)	100	11.23 (10000)	11.27 (10000)	72.40 (10000)	72.35 (10000)	76.54 (10000)	76.56 (10000)	77.54 (10000)	77.60 (10000)	79.88 (10000)	79.90 (10000)
Mittag-Leffler(1,5)	1000	0.00 (10000)	0.00 (10000)	0.10 (10000)	0.10 (10000)	0.40 (10000)	0.40 (10000)	0.79 (10000)	0.78 (10000)	1.79 (10000)	1.78 (10000)

Table E.28: Testing v.s. Case II Lognormal (0,0.2) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.3 Power testing v.s. Case IIIa: Scale Parameter is Unknown, Shape Parameter is Known

E.3.1 Weibull Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.12 10000	95.08 10000	94.80 10000	94.80 10000	94.84 10000	94.85 10000	95.31 10000	95.33 10000	94.66 10000	94.66 9617
Weibull(1,1)	100	95.12 10000	95.07 10000	95.32 10000	95.29 10000	95.04 10000	95.05 10000	94.91 10000	94.94 10000	94.80 10000	94.83 9995
Weibull(1,1)	1000	94.78 10000	94.79 10000	95.21 10000	95.27 10000	94.93 10000	94.94 10000	94.89 10000	94.93 10000	95.17 10000	95.17 10000
Loglogistic(1,4)	30	0.00 10000	0.00 10000	0.16 10000	0.16 10000	54.49 10000	54.58 10000	91.39 10000	91.10 8233	84.07 10000	65.32 1805
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	7.71 10000	7.74 10000	88.34 10000	88.20 9627	62.60 10000	28.12 729
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	45.48 10000	45.60 10000	0.25 10000	0.00 2
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	76.45 10000	-	95.12 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	36.28 10000	-	94.31 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	90.64 10000	-
Pareto(5)	30	-	-	85.54 10000	37.70 61	85.97 10000	33.90 59	86.29 10000	28.12 64	86.25 10000	40.51 79
Pareto(5)	100	-	-	67.84 10000	0.00 3	68.91 10000	-	68.06 10000	0.00 1	69.07 10000	-
Pareto(5)	1000	-	-	0.66 10000	-	0.47 10000	-	0.72 10000	-	0.62 10000	-
ChiSquared1	30	36.49 10000	36.46 10000	79.27 10000	79.28 10000	90.86 10000	90.86 10000	92.65 10000	92.65 10000	93.99 10000	94.00 9999
ChiSquared1	100	1.17 10000	1.16 10000	41.66 10000	41.60 10000	80.16 10000	80.25 10000	88.03 10000	88.05 10000	92.03 10000	92.03 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	5.41 10000	5.46 10000	31.29 10000	31.29 10000	65.64 10000	65.74 10000
ChiSquared3	30	75.30 10000	75.27 10000	82.42 10000	82.44 10000	88.68 10000	88.67 10000	91.84 10000	91.84 10000	93.92 10000	93.92 10000
ChiSquared3	100	33.30 10000	33.16 10000	47.38 10000	47.32 10000	73.96 10000	73.90 10000	84.18 10000	84.23 10000	90.30 10000	90.30 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.66 10000	0.66 10000	11.47 10000	11.52 10000	45.10 10000	45.09 10000
ChiSquared4	30	41.24 10000	41.19 10000	48.87 10000	48.90 10000	69.67 10000	69.67 10000	80.00 10000	79.99 10000	87.80 10000	87.83 10000
ChiSquared4	100	0.82 10000	0.81 10000	2.83 10000	2.81 10000	21.14 10000	21.12 10000	44.30 10000	44.31 10000	69.32 10000	69.36 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.26 10000	0.26 10000
Loglaplace(1,1)	30	44.18 10000	44.10 10000	46.80 10000	46.80 10000	51.61 10000	51.61 10000	50.10 10000	50.09 10000	42.12 10000	42.12 10000
Loglaplace(1,1)	100	3.20 10000	3.18 10000	5.23 10000	5.21 10000	9.77 10000	9.74 10000	10.07 10000	10.07 10000	4.36 10000	4.37 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	3.08 10000	3.07 10000	5.03 10000	5.03 10000	12.37 10000	12.37 10000	23.38 10000	23.33 10000	40.76 10000	40.79 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	1.31 10000	1.31 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.04 10000	0.04 10000	0.35 10000	0.35 10000	1.92 10000	1.91 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	27.47 10000	27.43 10000	27.56 10000	27.53 10000	27.67 10000	18.73 8060	26.34 10000	-	-	-
Uniform	100	0.04 10000	0.04 10000	0.03 10000	0.03 10000	0.03 10000	0.00 9483	0.07 10000	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-
Mittag-Leffler(1,5)	30	95.40 10000	95.39 10000	95.02 10000	95.04 10000	95.29 10000	95.29 10000	95.00 10000	94.99 10000	95.17 10000	95.16 10000
Mittag-Leffler(1,5)	100	95.06 10000	95.04 10000	95.13 10000	95.11 10000	95.09 10000	95.09 10000	95.35 10000	95.34 10000	95.23 10000	95.25 10000
Mittag-Leffler(1,5)	1000	95.05 10000	95.05 10000	95.21 10000	95.20 10000	94.98 10000	95.00 10000	95.31 10000	95.31 10000	94.77 10000	94.83 10000

Table E.29: Testing v.s. Case IIIa Weibull (1,1) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.09 10000	95.11 10000	94.77 10000	94.82 10000	94.92 10000	94.95 10000	95.27 10000	95.31 10000	94.69 10000	94.70 9617
Weibull(1,1)	100	95.06 10000	95.07 10000	95.59 10000	95.59 10000	94.94 10000	94.95 10000	94.62 10000	94.64 10000	94.82 10000	94.83 9995
Weibull(1,1)	1000	94.98 10000	94.98 10000	94.86 10000	94.89 10000	94.84 10000	94.84 10000	94.57 10000	94.60 10000	95.25 10000	95.28 10000
Loglogistic(1,4)	30	0.00 10000	0.00 10000	0.16 10000	0.16 10000	54.06 10000	54.16 10000	92.85 10000	92.58 8233	89.53 10000	77.45 1805
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	6.07 10000	6.11 10000	88.69 10000	88.53 9627	76.12 10000	47.87 729
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	40.58 10000	40.69 10000	0.96 10000	0.00 2
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	78.86 10000	-	95.18 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	42.00 10000	-	94.67 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	92.29 10000	-
Pareto(5)	30	-	-	90.45 10000	54.10 61	90.44 10000	54.24 59	90.35 10000	56.25 64	90.65 10000	56.96 79
Pareto(5)	100	-	-	79.50 10000	0.00 3	80.38 10000	-	79.36 10000	0.00 1	80.41 10000	-
Pareto(5)	1000	-	-	2.12 10000	-	1.90 10000	-	2.09 10000	-	1.99 10000	-
ChiSquared1	30	44.67 10000	44.80 10000	83.77 10000	83.80 10000	92.38 10000	92.36 10000	93.75 10000	93.76 10000	94.71 10000	94.75 9999
ChiSquared1	100	2.44 10000	2.44 10000	51.49 10000	51.49 10000	84.74 10000	84.79 10000	90.72 10000	90.72 10000	93.44 10000	93.46 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	9.93 10000	9.94 10000	41.91 10000	41.91 10000	73.94 10000	74.06 10000
ChiSquared3	30	77.52 10000	77.63 10000	83.88 10000	83.91 10000	89.79 10000	89.79 10000	92.47 10000	92.50 10000	93.85 10000	93.86 10000
ChiSquared3	100	39.33 10000	39.34 10000	52.92 10000	52.90 10000	78.24 10000	78.28 10000	86.23 10000	86.32 10000	91.33 10000	91.35 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	1.24 10000	1.24 10000	17.11 10000	17.13 10000	53.53 10000	53.60 10000
ChiSquared4	30	45.73 10000	45.85 10000	52.80 10000	52.85 10000	72.81 10000	72.84 10000	82.00 10000	82.01 10000	88.90 10000	88.89 10000
ChiSquared4	100	1.54 10000	1.54 10000	4.28 10000	4.26 10000	25.75 10000	25.78 10000	49.70 10000	49.86 10000	73.81 10000	73.86 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.40 10000	0.40 10000
Loglaplace(1,1)	30	35.16 10000	35.34 10000	39.81 10000	39.84 10000	48.95 10000	49.01 10000	52.89 10000	52.89 10000	50.59 10000	50.57 10000
Loglaplace(1,1)	100	0.53 10000	0.54 10000	1.35 10000	1.34 10000	5.60 10000	5.60 10000	8.89 10000	8.95 10000	6.93 10000	6.94 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	3.04 10000	3.09 10000	4.85 10000	4.86 10000	12.73 10000	12.74 10000	23.93 10000	23.96 10000	42.71 10000	42.66 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.07 10000	0.07 10000	1.27 10000	1.27 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.01 10000	0.01 10000	0.04 10000	0.04 10000	0.09 10000	0.09 10000	0.39 10000	0.39 10000	2.44 10000	2.46 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	12.95 10000	13.02 10000	13.28 10000	13.28 10000	13.46 10000	3.87 8060	12.66 10000	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9483	0.01 10000	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-
Mittag-Leffler(1,5)	30	95.19 10000	95.21 10000	95.18 10000	95.18 10000	95.06 10000	95.07 10000	94.66 10000	94.67 10000	95.36 10000	95.37 10000
Mittag-Leffler(1,5)	100	95.17 10000	95.18 10000	95.10 10000	95.09 10000	95.03 10000	95.03 10000	95.31 10000	95.32 10000	95.32 10000	95.34 10000
Mittag-Leffler(1,5)	1000	95.08 10000	95.08 10000	95.13 10000	95.12 10000	94.81 10000	94.82 10000	95.27 10000	95.27 10000	94.84 10000	94.85 10000

Table E.30: Testing v.s. Case IIIa Weibull (1,1) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.26 10000	95.29 10000	94.78 10000	94.79 10000	94.73 10000	94.75 10000	95.30 10000	95.29 10000	94.71 10000	94.70 9617
Weibull(1,1)	100	94.98 10000	94.96 10000	95.44 10000	95.44 10000	94.98 10000	95.01 10000	95.03 10000	95.05 10000	94.85 10000	94.85 9995
Weibull(1,1)	1000	94.99 10000	94.99 10000	94.93 10000	94.94 10000	95.02 10000	95.02 10000	94.99 10000	95.01 10000	95.25 10000	95.30 10000
Loglogistic(1,4)	30	0.00 10000	0.00 10000	0.04 10000	0.04 10000	47.33 10000	47.35 10000	90.58 10000	90.17 8233	81.85 10000	60.33 1805
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.91 10000	3.91 10000	86.70 10000	86.60 9627	56.62 10000	21.95 729
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	35.82 10000	36.06 10000	0.04 10000	0.00 2
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	72.41 10000	-	95.19 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	26.54 10000	-	94.49 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	89.14 10000	-
Pareto(5)	30	-	-	83.64 10000	37.70 61	84.34 10000	27.12 59	84.78 10000	29.69 64	84.69 10000	34.18 79
Pareto(5)	100	-	-	62.99 10000	0.00 3	63.15 10000	-	62.44 10000	0.00 1	63.65 10000	-
Pareto(5)	1000	-	-	0.15 10000	-	0.12 10000	-	0.22 10000	-	0.19 10000	-
ChiSquared1	30	30.31 10000	30.36 10000	76.26 10000	76.30 10000	89.96 10000	89.96 10000	92.16 10000	92.16 10000	93.90 10000	93.90 9999
ChiSquared1	100	0.52 10000	0.51 10000	34.82 10000	34.80 10000	76.62 10000	76.67 10000	86.66 10000	86.68 10000	91.33 10000	91.36 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	2.61 10000	2.64 10000	22.42 10000	22.43 10000	59.29 10000	59.33 10000
ChiSquared3	30	70.62 10000	70.73 10000	78.92 10000	78.97 10000	87.63 10000	87.63 10000	91.47 10000	91.47 10000	93.51 10000	93.54 10000
ChiSquared3	100	24.49 10000	24.31 10000	38.97 10000	38.90 10000	68.74 10000	68.75 10000	81.02 10000	81.10 10000	89.10 10000	89.12 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.23 10000	0.23 10000	6.24 10000	6.28 10000	36.00 10000	36.04 10000
ChiSquared4	30	32.51 10000	32.62 10000	39.99 10000	40.08 10000	64.56 10000	64.57 10000	76.53 10000	76.53 10000	86.07 10000	86.09 10000
ChiSquared4	100	0.13 10000	0.13 10000	1.21 10000	1.21 10000	13.32 10000	13.32 10000	34.91 10000	34.99 10000	63.67 10000	63.81 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000
Loglaplace(1,1)	30	36.60 10000	36.66 10000	40.37 10000	40.45 10000	47.80 10000	47.80 10000	48.08 10000	48.08 10000	38.96 10000	38.98 10000
Loglaplace(1,1)	100	0.81 10000	0.81 10000	1.88 10000	1.88 10000	5.75 10000	5.74 10000	7.59 10000	7.60 10000	3.34 10000	3.34 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.72 10000	0.72 10000	1.49 10000	1.50 10000	5.47 10000	5.47 10000	13.48 10000	13.48 10000	30.36 10000	30.41 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.24 10000	0.25 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.06 10000	0.06 10000	0.42 10000	0.42 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	13.38 10000	13.45 10000	13.69 10000	13.68 10000	13.85 10000	4.90 8060	13.49 10000	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9483	0.01 10000	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-
Mittag-Leffler(1,5)	30	95.14 10000	95.16 10000	94.88 10000	94.90 10000	95.23 10000	95.25 10000	94.73 10000	94.73 10000	95.22 10000	95.21 10000
Mittag-Leffler(1,5)	100	95.26 10000	95.21 10000	94.75 10000	94.71 10000	95.02 10000	95.02 10000	95.41 10000	95.42 10000	95.04 10000	95.04 10000
Mittag-Leffler(1,5)	1000	95.09 10000	95.09 10000	95.04 10000	95.05 10000	95.11 10000	95.12 10000	95.47 10000	95.47 10000	94.99 10000	95.03 10000

Table E.31: Testing v.s. Case IIIa Weibull (1,1) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	95.13 10000	95.13 10000	94.87 10000	94.87 10000	94.74 10000	94.72 10000	95.17 10000	95.21 10000	94.79 10000	94.78 9617
Weibull(1,1)	100	94.82 10000	94.79 10000	95.40 10000	95.39 10000	95.04 10000	95.05 10000	94.93 10000	94.97 10000	94.66 10000	94.70 9995
Weibull(1,1)	1000	94.97 10000	95.04 10000	95.02 10000	95.03 10000	94.65 10000	94.70 10000	94.74 10000	94.76 10000	95.22 10000	95.24 10000
Loglogistic(1,4)	30	0.00 10000	0.00 10000	0.05 10000	0.05 10000	50.19 10000	50.28 10000	90.90 10000	90.28 8233	80.75 10000	59.06 1805
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.69 10000	3.69 10000	86.72 10000	86.53 9627	56.85 10000	21.40 729
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	30.47 10000	30.67 10000	0.04 10000	0.00 2
Lognormal(0,0.2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	76.38 10000	-	95.41 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	28.49 10000	-	94.51 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	90.16 10000	-
Pareto(5)	30	-	-	82.54 10000	37.70 61	83.03 10000	23.73 59	83.80 10000	28.12 64	83.46 10000	36.71 79
Pareto(5)	100	-	-	62.96 10000	0.00 3	62.56 10000	-	61.91 10000	0.00 1	62.83 10000	-
Pareto(5)	1000	-	-	0.19 10000	-	0.11 10000	-	0.26 10000	-	0.16 10000	-
ChiSquared1	30	15.61 10000	15.60 10000	71.76 10000	71.75 10000	88.40 10000	88.38 10000	91.14 10000	91.14 10000	93.31 10000	93.31 9999
ChiSquared1	100	0.08 10000	0.08 10000	30.08 10000	30.06 10000	75.05 10000	75.13 10000	85.35 10000	85.38 10000	90.91 10000	90.97 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	2.39 10000	2.42 10000	22.28 10000	22.31 10000	59.33 10000	59.41 10000
ChiSquared3	30	72.85 10000	72.80 10000	81.35 10000	81.37 10000	89.31 10000	89.31 10000	92.61 10000	92.56 10000	94.32 10000	94.30 10000
ChiSquared3	100	20.70 10000	20.60 10000	37.22 10000	37.17 10000	70.72 10000	70.67 10000	82.51 10000	82.59 10000	89.95 10000	89.95 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.18 10000	0.18 10000	6.02 10000	6.06 10000	36.87 10000	36.89 10000
ChiSquared4	30	32.81 10000	32.79 10000	41.45 10000	41.49 10000	67.80 10000	67.75 10000	79.42 10000	79.35 10000	88.29 10000	88.26 10000
ChiSquared4	100	0.07 10000	0.07 10000	0.74 10000	0.74 10000	12.89 10000	12.88 10000	35.96 10000	36.03 10000	65.79 10000	65.86 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000
Loglaplace(1,1)	30	38.24 10000	38.21 10000	42.11 10000	42.17 10000	49.72 10000	49.65 10000	49.93 10000	49.86 10000	40.52 10000	40.53 10000
Loglaplace(1,1)	100	0.59 10000	0.57 10000	1.79 10000	1.78 10000	5.72 10000	5.71 10000	8.15 10000	8.15 10000	3.75 10000	3.75 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.76 10000	0.75 10000	1.56 10000	1.56 10000	6.46 10000	6.43 10000	15.70 10000	15.62 10000	34.54 10000	34.47 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.32 10000	0.32 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.52 10000	0.52 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	15.11 10000	15.11 10000	15.31 10000	15.26 10000	15.65 10000	5.97 8060	15.16 10000	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 9483	0.01 10000	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-
Mittag-Leffler(1,5)	30	95.17 10000	95.17 10000	95.02 10000	95.04 10000	94.89 10000	94.89 10000	94.83 10000	94.82 10000	95.06 10000	95.05 10000
Mittag-Leffler(1,5)	100	95.29 10000	95.28 10000	94.74 10000	94.72 10000	94.94 10000	94.93 10000	95.44 10000	95.41 10000	94.90 10000	94.91 10000
Mittag-Leffler(1,5)	1000	94.96 10000	94.99 10000	95.18 10000	95.21 10000	95.15 10000	95.18 10000	95.65 10000	95.63 10000	94.93 10000	94.95 10000

Table E.32: Testing v.s. Case IIIa Weibull (1,1) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.3.2 Loglogistic Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.62 10000	0.62 10000	48.26 10000	48.27 10000	74.46 10000	74.45 10000	83.75 10000	83.74 10000	91.12 10000	91.14 9996
Weibull(1,1)	100	0.00 10000	0.00 10000	2.43 10000	2.43 10000	28.82 10000	28.86 10000	50.73 10000	50.78 10000	77.07 10000	77.08 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	1.13 10000	1.13 10000
Loglogistic(1,4)	30	95.04 10000	95.04 10000	94.47 10000	94.46 10000	94.87 10000	94.89 9952	94.21 10000	94.67 9113	90.53 10000	95.04 6752
Loglogistic(1,4)	100	95.23 10000	95.26 10000	94.78 10000	94.79 10000	94.86 10000	94.86 10000	95.34 10000	95.38 9906	93.66 10000	95.16 7538
Loglogistic(1,4)	1000	94.77 10000	94.81 10000	94.61 10000	94.63 10000	94.85 10000	94.83 10000	94.59 10000	94.60 10000	95.09 10000	95.11 9850
Lognormal(0,0.2)	30	1.03 10000	1.03 10000	9.41 10000	9.40 10000	38.39 10000	39.16 2906	0.26 10000	-	0.00 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.79 10000	0.66 1055	0.00 10000	-	0.00 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	0.00 10000	-	0.00 10000	-
Pareto(5)	30	-	-	59.42 10000	92.65 1279	50.04 10000	93.02 960	51.35 10000	93.62 1035	47.87 10000	93.33 869
Pareto(5)	100	-	-	22.71 10000	91.75 97	17.98 10000	88.37 43	17.40 10000	90.48 42	16.90 10000	84.85 33
Pareto(5)	1000	-	-	0.00 10000	-	0.00 10000	-	0.00 10000	-	0.00 10000	-
ChiSquared1	30	0.00 10000	0.00 10000	18.09 10000	18.10 10000	41.46 10000	41.49 10000	53.87 10000	53.86 10000	69.18 10000	69.17 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.05 10000	0.05 10000	1.23 10000	1.23 10000	5.20 10000	5.20 10000	19.23 10000	19.25 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	5.61 10000	5.61 10000	17.08 10000	17.08 10000	31.03 10000	31.04 10000	40.97 10000	41.01 10000	54.06 10000	54.07 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.36 10000	0.36 10000	1.12 10000	1.12 10000	5.07 10000	5.07 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	19.88 10000	19.86 10000	26.52 10000	26.50 10000	35.98 10000	35.98 10000	44.07 10000	44.08 10000	52.91 10000	52.93 10000
ChiSquared4	100	0.02 10000	0.02 10000	0.13 10000	0.13 10000	0.67 10000	0.67 10000	1.51 10000	1.51 10000	4.75 10000	4.75 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	23.75 10000	23.74 10000	34.73 10000	34.71 10000	40.56 10000	40.57 10000	42.96 10000	42.98 10000	44.54 10000	44.57 10000
Loglaplace(1,1)	100	0.06 10000	0.06 10000	0.43 10000	0.43 10000	1.19 10000	1.20 10000	1.85 10000	1.85 10000	2.10 10000	2.10 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	61.36 10000	61.33 10000	65.09 10000	65.09 10000	69.13 10000	69.13 10000	72.43 10000	72.42 10000	77.66 10000	77.68 10000
Rayleigh(4)	100	9.66 10000	9.68 10000	12.86 10000	12.95 10000	18.59 10000	18.74 10000	24.10 10000	24.17 10000	34.32 10000	34.42 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	92.25 10000	92.25 10000	92.05 10000	92.04 10000	92.18 10000	92.18 10000	91.73 10000	91.72 10000	91.79 10000	91.77 10000
Gamma(5,2)	100	81.73 10000	81.79 10000	81.42 10000	81.50 10000	81.92 10000	82.01 10000	81.52 10000	81.59 10000	81.34 10000	81.47 10000
Gamma(5,2)	1000	3.15 10000	3.18 10000	3.34 10000	3.37 10000	3.72 10000	3.75 10000	3.28 10000	3.30 10000	3.35 10000	3.40 10000
Uniform	30	11.90 10000	11.89 10000	4.63 10000	4.10 9938	0.00 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.51 10000	0.51 10000	3.16 10000	3.16 10000	5.84 10000	5.84 10000	8.60 10000	8.60 10000	14.03 10000	14.03 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.33: Testing v.s. Case IIIa Loglogistic (1,4) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.50 10000	0.50 10000	45.08 10000	45.08 10000	72.79 10000	72.76 10000	82.55 10000	82.55 10000	91.11 10000	91.12 9996
Weibull(1,1)	100	0.00 10000	0.00 10000	1.74 10000	1.74 10000	25.48 10000	25.53 10000	47.22 10000	47.38 10000	75.45 10000	75.43 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.76 10000	0.76 10000
Loglogistic(1,4)	30	95.15 10000	95.15 10000	94.54 10000	94.51 10000	94.68 10000	94.68 9952	94.79 10000	94.72 9113	94.59 10000	95.13 6752
Loglogistic(1,4)	100	95.32 10000	95.32 10000	94.84 10000	94.89 10000	95.08 10000	95.10 10000	95.23 10000	95.24 9906	95.20 10000	95.30 7538
Loglogistic(1,4)	1000	94.67 10000	94.69 10000	94.70 10000	94.70 10000	95.02 10000	95.02 10000	94.61 10000	94.57 10000	94.84 10000	94.84 9850
Lognormal(0,0.2)	30	0.60 10000	0.60 10000	7.03 10000	7.00 10000	51.22 10000	36.79 2906	19.98 10000	-	0.05 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	4.77 10000	0.38 1055	0.00 10000	-	0.00 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	0.00 10000	-	0.00 10000	-
Pareto(5)	30	-	-	88.19 10000	91.87 1279	84.78 10000	92.81 960	85.69 10000	93.04 1035	84.12 10000	91.94 869
Pareto(5)	100	-	-	69.59 10000	90.72 97	65.62 10000	86.05 43	65.34 10000	92.86 42	64.33 10000	87.88 33
Pareto(5)	1000	-	-	0.05 10000	-	0.00 10000	-	0.02 10000	-	0.02 10000	-
ChiSquared1	30	0.00 10000	0.00 10000	16.07 10000	16.10 10000	38.68 10000	38.72 10000	51.53 10000	51.53 10000	66.96 10000	66.90 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.86 10000	0.86 10000	3.89 10000	3.93 10000	16.30 10000	16.39 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	4.71 10000	4.71 10000	14.84 10000	14.84 10000	28.30 10000	28.33 10000	37.90 10000	37.99 10000	51.46 10000	51.47 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.15 10000	0.15 10000	0.67 10000	0.67 10000	3.61 10000	3.61 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	17.64 10000	17.64 10000	23.83 10000	23.83 10000	32.91 10000	32.94 10000	40.85 10000	40.89 10000	50.02 10000	50.11 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.10 10000	0.10 10000	0.45 10000	0.45 10000	1.12 10000	1.12 10000	3.72 10000	3.74 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	21.46 10000	21.46 10000	32.64 10000	32.65 10000	38.82 10000	38.86 10000	41.64 10000	41.66 10000	43.12 10000	43.17 10000
Loglaplace(1,1)	100	0.04 10000	0.04 10000	0.32 10000	0.32 10000	1.25 10000	1.25 10000	1.83 10000	1.83 10000	2.00 10000	2.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	59.21 10000	59.23 10000	63.12 10000	63.12 10000	67.35 10000	67.36 10000	70.66 10000	70.67 10000	76.55 10000	76.59 10000
Rayleigh(4)	100	7.94 10000	7.94 10000	11.05 10000	11.05 10000	15.98 10000	15.98 10000	20.90 10000	20.90 10000	30.90 10000	30.90 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	92.03 10000	92.03 10000	91.85 10000	91.86 10000	92.29 10000	92.29 10000	91.75 10000	91.75 10000	91.82 10000	91.82 10000
Gamma(5,2)	100	80.74 10000	80.74 10000	80.33 10000	80.33 10000	80.59 10000	80.60 10000	80.35 10000	80.36 10000	80.46 10000	80.46 10000
Gamma(5,2)	1000	1.99 10000	2.00 10000	2.25 10000	2.25 10000	2.33 10000	2.33 10000	2.21 10000	2.25 10000	2.29 10000	2.30 10000
Uniform	30	10.39 10000	10.39 10000	9.01 10000	8.52 9938	0.00 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.36 10000	0.36 10000	2.59 10000	2.59 10000	4.64 10000	4.64 10000	7.12 10000	7.14 10000	12.05 10000	12.06 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.34: Testing v.s. Case IIIa Loglogistic (1,4) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.29 10000	0.29 10000	40.70 10000	40.71 10000	70.43 10000	70.36 10000	80.96 10000	80.93 10000	90.48 10000	90.48 9996
Weibull(1,1)	100	0.00 10000	0.00 10000	1.01 10000	1.01 10000	20.24 10000	20.33 10000	42.14 10000	42.24 10000	72.39 10000	72.42 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.42 10000	0.42 10000
Loglogistic(1,4)	30	94.87 10000	94.87 10000	94.63 10000	94.63 10000	94.90 10000	94.92 9952	94.45 10000	94.92 9113	91.12 10000	95.23 6752
Loglogistic(1,4)	100	95.18 10000	95.18 10000	95.02 10000	95.04 10000	95.00 10000	94.99 10000	95.13 10000	95.15 9906	93.99 10000	95.20 7538
Loglogistic(1,4)	1000	94.64 10000	94.68 10000	94.88 10000	94.89 10000	94.92 10000	94.89 10000	94.55 10000	94.53 10000	94.75 10000	94.83 9850
Lognormal(0,0.2)	30	0.21 10000	0.21 10000	3.88 10000	3.86 10000	32.99 10000	29.73 2906	0.16 10000	-	0.00 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.57 10000	0.00 1055	0.00 10000	-	0.00 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	0.00 10000	-	0.00 10000	-
Pareto(5)	30	-	-	57.81 10000	91.79 1279	48.44 10000	92.92 960	49.29 10000	92.27 1035	45.83 10000	91.94 869
Pareto(5)	100	-	-	19.76 10000	87.63 97	15.07 10000	88.37 43	14.96 10000	95.24 42	14.10 10000	84.85 33
Pareto(5)	1000	-	-	0.00 10000	-	0.00 10000	-	0.00 10000	-	0.00 10000	-
ChiSquared1	30	0.00 10000	0.00 10000	12.46 10000	12.46 10000	33.56 10000	33.56 10000	47.34 10000	47.26 10000	63.63 10000	63.59 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.38 10000	0.38 10000	2.42 10000	2.42 10000	12.47 10000	12.51 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	3.36 10000	3.36 10000	11.77 10000	11.77 10000	23.95 10000	23.95 10000	33.21 10000	33.21 10000	47.12 10000	47.14 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.07 10000	0.07 10000	0.30 10000	0.30 10000	2.23 10000	2.24 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	14.72 10000	14.72 10000	19.64 10000	19.64 10000	28.42 10000	28.44 10000	36.18 10000	36.19 10000	45.81 10000	45.85 10000
ChiSquared4	100	0.02 10000	0.02 10000	0.06 10000	0.06 10000	0.20 10000	0.20 10000	0.61 10000	0.61 10000	2.37 10000	2.37 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	18.72 10000	18.72 10000	29.11 10000	29.11 10000	35.45 10000	35.45 10000	37.90 10000	37.92 10000	39.78 10000	39.79 10000
Loglaplace(1,1)	100	0.04 10000	0.04 10000	0.19 10000	0.19 10000	0.79 10000	0.79 10000	1.39 10000	1.39 10000	1.44 10000	1.44 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	56.15 10000	56.15 10000	59.82 10000	59.82 10000	64.82 10000	64.82 10000	68.37 10000	68.37 10000	74.51 10000	74.51 10000
Rayleigh(4)	100	6.10 10000	6.10 10000	8.72 10000	8.72 10000	12.80 10000	12.80 10000	17.36 10000	17.37 10000	26.91 10000	26.94 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	91.84 10000	91.84 10000	91.85 10000	91.84 10000	92.02 10000	92.02 10000	91.47 10000	91.47 10000	91.64 10000	91.64 10000
Gamma(5,2)	100	79.06 10000	79.06 10000	78.52 10000	78.52 10000	78.81 10000	78.82 10000	78.49 10000	78.50 10000	78.52 10000	78.52 10000
Gamma(5,2)	1000	0.96 10000	0.96 10000	1.11 10000	1.11 10000	0.98 10000	1.00 10000	1.09 10000	1.09 10000	1.16 10000	1.17 10000
Uniform	30	8.67 10000	8.67 10000	15.23 10000	14.79 9938	0.00 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.24 10000	0.24 10000	1.60 10000	1.60 10000	3.30 10000	3.30 10000	5.07 10000	5.07 10000	9.42 10000	9.42 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.35: Testing v.s. Case IIIa Loglogistic (1,4) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	26.31 10000	26.33 10000	60.15 10000	60.14 10000	74.73 10000	74.72 10000	87.92 10000	87.93 9996
Weibull(1,1)	100	0.00 10000	0.00 10000	0.31 10000	0.31 10000	14.62 10000	14.62 10000	35.69 10000	35.74 10000	68.87 10000	68.90 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.35 10000	0.35 10000
Loglogistic(1,4)	30	95.13 10000	95.13 10000	94.75 10000	94.74 10000	95.04 10000	95.02 9952	94.55 10000	94.91 9113	91.77 10000	95.02 6752
Loglogistic(1,4)	100	95.07 10000	95.03 10000	94.96 10000	94.96 10000	95.15 10000	95.14 10000	95.28 10000	95.30 9906	94.22 10000	95.17 7538
Loglogistic(1,4)	1000	94.69 10000	94.73 10000	94.96 10000	94.97 10000	95.02 10000	94.99 10000	94.47 10000	94.47 10000	94.87 10000	94.89 9850
Lognormal(0,0.2)	30	0.13 10000	0.13 10000	3.43 10000	3.43 10000	34.78 10000	30.63 2906	0.20 10000	-	0.00 10000	-
Lognormal(0,0.2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.10 10000	0.00 1055	0.00 10000	-	0.00 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	0.00 10000	-	0.00 10000	-
Pareto(5)	30	-	-	63.25 10000	93.43 1279	53.07 10000	93.65 960	54.65 10000	94.30 1035	51.53 10000	93.79 869
Pareto(5)	100	-	-	21.91 10000	88.66 97	16.88 10000	83.72 43	16.80 10000	92.86 42	15.88 10000	87.88 33
Pareto(5)	1000	-	-	0.00 10000	-	0.00 10000	-	0.00 10000	-	0.00 10000	-
ChiSquared1	30	0.00 10000	0.00 10000	4.69 10000	4.69 10000	20.02 10000	20.06 10000	33.01 10000	33.01 10000	51.42 10000	51.39 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.06 10000	0.06 10000	0.81 10000	0.81 10000	7.17 10000	7.17 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.38 10000	0.38 10000	4.17 10000	4.17 10000	12.69 10000	12.70 10000	19.80 10000	19.80 10000	33.94 10000	33.95 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.03 10000	0.03 10000	1.00 10000	1.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	4.55 10000	4.55 10000	8.20 10000	8.20 10000	15.48 10000	15.49 10000	22.35 10000	22.37 10000	32.69 10000	32.71 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.05 10000	0.04 10000	0.24 10000	0.24 10000	0.99 10000	0.99 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	2.63 10000	2.63 10000	7.23 10000	7.23 10000	11.16 10000	11.16 10000	13.31 10000	13.31 10000	15.58 10000	15.58 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.09 10000	0.09 10000	0.10 10000	0.10 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	36.01 10000	36.01 10000	41.96 10000	41.98 10000	50.12 10000	50.13 10000	56.13 10000	56.14 10000	65.05 10000	65.07 10000
Rayleigh(4)	100	1.66 10000	1.65 10000	3.12 10000	3.11 10000	5.94 10000	5.92 10000	9.78 10000	9.77 10000	17.24 10000	17.19 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	89.73 10000	89.73 10000	89.65 10000	89.65 10000	90.10 10000	90.10 10000	89.29 10000	89.30 10000	89.89 10000	89.89 10000
Gamma(5,2)	100	75.34 10000	75.29 10000	74.70 10000	74.65 10000	75.17 10000	75.07 10000	74.33 10000	74.27 10000	74.17 10000	74.13 10000
Gamma(5,2)	1000	0.23 10000	0.23 10000	0.14 10000	0.14 10000	0.10 10000	0.10 10000	0.16 10000	0.16 10000	0.22 10000	0.22 10000
Uniform	30	1.35 10000	1.35 10000	13.20 10000	12.75 9938	0.00 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.19 10000	0.19 10000	0.54 10000	0.54 10000	1.38 10000	1.38 10000	2.92 10000	2.93 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.36: Testing v.s. Case IIIa Loglogistic (1,4) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.3.3 Lognormal Distribution

KS 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.24 10000	0.24 10000	1.37 10000	1.38 10000	2.11 10000	2.11 10000	3.67 10000	3.65 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	7.50 10000	7.53 10000	21.26 10000	21.18 10000	24.15 10000	24.14 10000	25.43 10000	25.45 10000	26.89 10000	26.93 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.07 10000	0.07 10000	0.26 10000	0.26 10000	0.39 10000	0.39 10000	0.44 10000	0.43 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	95.15 10000	95.15 10000	95.38 10000	95.42 10000	95.08 10000	95.08 9999	94.92 10000	94.97 9941	95.01 10000	95.15 8373
Lognormal(0,0.2)	100	94.65 10000	94.67 10000	94.71 10000	94.71 10000	95.05 10000	95.04 10000	94.80 10000	94.80 10000	94.93 10000	94.89 9664
Lognormal(0,0.2)	1000	95.12 10000	95.09 10000	95.31 10000	95.33 10000	94.60 10000	94.60 10000	95.46 10000	95.48 10000	94.88 10000	94.92 10000
Pareto(5)	30	-	-	49.47 10000	49.47 9997	48.55 10000	48.55 9999	49.48 10000	49.47 9999	48.37 10000	48.37 9999
Pareto(5)	100	-	-	7.41 10000	7.40 10000	7.67 10000	7.67 10000	7.32 10000	7.32 10000	7.16 10000	7.14 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.05 10000	0.05 10000	0.17 10000	0.17 10000	0.22 10000	0.22 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.02 10000	0.02 10000	0.03 10000	0.03 10000	0.03 10000	0.03 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.01 10000	0.01 10000	0.02 10000	0.02 10000	0.03 10000	0.03 10000	0.03 10000	0.03 10000	0.10 10000	0.10 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.03 10000	0.03 10000	0.04 10000	0.04 10000	0.04 10000	0.04 10000	0.14 10000	0.14 10000	0.06 10000	0.06 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.07 10000	0.08 10000	0.27 10000	0.27 10000	0.35 10000	0.35 10000	0.29 10000	0.30 10000	0.45 10000	0.45 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	2.81 10000	2.81 10000	2.98 10000	3.00 10000	2.74 10000	2.76 10000	2.81 10000	2.85 10000	2.64 10000	2.64 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	49.76 10000	49.69 9980	24.02 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.57 10000	0.55 10000	0.03 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.37: Testing v.s. Case IIIa Lognormal (0,0.2) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.23 10000	0.23 10000	0.98 10000	0.98 10000	1.64 10000	1.64 10000	3.37 10000	3.36 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	4.80 10000	4.78 10000	23.40 10000	23.34 10000	30.72 10000	30.71 10000	32.96 10000	32.98 10000	35.87 10000	35.92 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.10 10000	0.10 10000	0.38 10000	0.38 10000	0.62 10000	0.62 10000	1.00 10000	1.00 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	95.11 10000	95.09 10000	95.21 10000	95.23 10000	94.92 10000	94.94 9999	94.77 10000	94.79 9941	94.99 10000	95.02 8373
Lognormal(0,0.2)	100	94.79 10000	94.74 10000	94.73 10000	94.71 10000	95.11 10000	95.12 10000	94.76 10000	94.76 10000	94.93 10000	94.92 9664
Lognormal(0,0.2)	1000	95.06 10000	95.05 10000	95.18 10000	95.18 10000	94.72 10000	94.72 10000	95.26 10000	95.26 10000	94.78 10000	94.79 10000
Pareto(5)	30	-	-	60.71 10000	60.79 9997	60.68 10000	60.72 9999	61.24 10000	61.30 9999	60.56 10000	60.61 9999
Pareto(5)	100	-	-	12.82 10000	12.83 10000	13.25 10000	13.25 10000	13.11 10000	13.10 10000	12.37 10000	12.38 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.04 10000	0.04 10000	0.13 10000	0.13 10000	0.13 10000	0.13 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.01 10000	0.01 10000	0.01 10000	0.01 10000	0.01 10000	0.01 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.03 10000	0.03 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.03 10000	0.03 10000	0.09 10000	0.09 10000	0.02 10000	0.02 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.03 10000	0.03 10000	0.14 10000	0.14 10000	0.20 10000	0.20 10000	0.19 10000	0.19 10000	0.19 10000	0.19 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	1.52 10000	1.51 10000	1.53 10000	1.53 10000	1.45 10000	1.44 10000	1.46 10000	1.46 10000	1.49 10000	1.49 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	27.88 10000	27.70 9980	12.22 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.38: Testing v.s. Case IIIa Lognormal (0,0.2) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.09 10000	0.09 10000	0.52 10000	0.52 10000	0.98 10000	0.98 10000	1.85 10000	1.85 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	3.38 10000	3.39 10000	14.88 10000	14.87 10000	18.11 10000	18.13 10000	19.69 10000	19.73 10000	22.14 10000	22.15 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.06 10000	0.06 10000	0.08 10000	0.08 10000	0.16 10000	0.16 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	94.97 10000	94.99 10000	95.39 10000	95.42 10000	94.88 10000	94.86 9999	94.93 10000	94.99 9941	95.14 10000	95.20 8373
Lognormal(0,0.2)	100	95.12 10000	95.12 10000	95.06 10000	95.08 10000	94.96 10000	94.95 10000	94.78 10000	94.77 10000	94.79 10000	94.78 9664
Lognormal(0,0.2)	1000	94.58 10000	94.58 10000	95.03 10000	95.02 10000	94.79 10000	94.76 10000	95.20 10000	95.20 10000	94.64 10000	94.68 10000
Pareto(5)	30	-	-	43.60 10000	43.63 9997	43.54 10000	43.52 9999	44.04 10000	44.04 9999	43.04 10000	43.01 9999
Pareto(5)	100	-	-	4.16 10000	4.15 10000	4.45 10000	4.44 10000	4.19 10000	4.19 10000	4.08 10000	4.08 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.06 10000	0.06 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.02 10000	0.02 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.02 10000	0.02 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.02 10000	0.02 10000	0.04 10000	0.04 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.02 10000	0.02 10000	0.06 10000	0.06 10000	0.10 10000	0.10 10000	0.06 10000	0.06 10000	0.06 10000	0.06 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	1.09 10000	1.09 10000	0.94 10000	0.94 10000	0.96 10000	0.98 10000	0.94 10000	0.96 10000	0.85 10000	0.86 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	31.33 10000	31.23 9980	10.91 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.06 10000	0.06 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.39: Testing v.s. Case IIIa Lognormal (0,0.2) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.04 10000	0.04 10000	0.11 10000	0.11 10000	0.31 10000	0.31 10000
Weibull(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	0.17 10000	0.17 10000	5.70 10000	5.75 10000	9.86 10000	9.93 10000	11.84 10000	11.92 10000	14.50 10000	14.60 10000
Loglogistic(1,4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.07 10000	0.07 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	94.85 10000	95.01 10000	95.25 10000	95.39 10000	94.82 10000	94.94 9999	95.03 10000	95.17 9941	94.88 10000	95.14 8373
Lognormal(0,0.2)	100	94.82 10000	94.82 10000	94.83 10000	94.83 10000	95.34 10000	95.35 10000	94.66 10000	94.66 10000	95.01 10000	94.94 9664
Lognormal(0,0.2)	1000	94.95 10000	94.93 10000	94.85 10000	94.83 10000	94.74 10000	94.65 10000	95.22 10000	95.15 10000	94.66 10000	94.65 10000
Pareto(5)	30	-	-	36.60 10000	36.76 9997	36.09 10000	36.36 9999	37.10 10000	37.27 9999	36.88 10000	37.01 9999
Pareto(5)	100	-	-	3.11 10000	3.10 10000	3.24 10000	3.23 10000	3.05 10000	3.04 10000	3.20 10000	3.20 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000
ChiSquared1	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.04 10000	0.04 10000	0.01 10000	0.01 10000	0.02 10000	0.02 10000	0.02 10000	0.02 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	33.87 10000	34.16 9980	12.86 10000	-	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.00 10000	-	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.40: Testing v.s. Case IIIa Lognormal (0,0.2) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.3.4 Pareto Distribution

KS 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	61.66 (10000)	61.62 (10000)	-	-	-	-
ChiSquared1	100	9.47 (10000)	9.46 (10000)	-	-	-	-
ChiSquared1	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	30	13.12 (10000)	13.08 (10000)	-	-	-	-
ChiSquared3	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared3	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	30	2.16 (10000)	2.14 (10000)	-	-	-	-
ChiSquared4	100	0.00 (10000)	0.00 (10000)	-	-	-	-
ChiSquared4	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	30	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Gamma(5,2)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Loglogistic(1,4)	30	85.57 (10000)	85.55 (10000)	-	-	-	-
Loglogistic(1,4)	100	59.14 (10000)	59.07 (10000)	-	-	-	-
Loglogistic(1,4)	1000	0.01 (10000)	0.01 (10000)	-	-	-	-
Lognormal(0,0.2)	30	74.82 (10000)	74.77 (10000)	-	-	-	-
Lognormal(0,0.2)	100	31.80 (10000)	31.71 (10000)	-	-	-	-
Lognormal(0,0.2)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	30	7.98 (10000)	7.96 (10000)	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Pareto(5)	30	95.04 (10000)	95.02 (10000)	95.01 (10000)	95.02 (10000)	94.89 (10000)	94.88 (10000)
Pareto(5)	100	94.33 (10000)	94.30 (10000)	95.23 (10000)	95.22 (10000)	94.93 (10000)	94.93 (10000)
Pareto(5)	1000	94.94 (10000)	94.99 (10000)	95.13 (10000)	95.14 (10000)	94.85 (10000)	94.86 (10000)
Rayleigh(4)	30	0.02 (10000)	0.02 (10000)	-	-	-	-
Rayleigh(4)	100	0.00 (10000)	0.00 (10000)	-	-	-	-
Rayleigh(4)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-
Weibull(1,1)	30	64.89 (10000)	64.85 (10000)	-	-	-	-
Weibull(1,1)	100	13.05 (10000)	12.97 (10000)	-	-	-	-
Weibull(1,1)	1000	0.00 (10000)	0.00 (10000)	-	-	-	-

Table E.41: Testing v.s. Case IIIb Pareto (5) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	62.14 10000	62.02 10000	-	-	-	-
ChiSquared1	100	8.38 10000	8.35 10000	-	-	-	-
ChiSquared1	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	30	9.72 10000	9.67 10000	-	-	-	-
ChiSquared3	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	30	1.11 10000	1.09 10000	-	-	-	-
ChiSquared4	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	1000	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	30	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	100	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	1000	0.00 10000	0.00 10000	-	-	-	-
Loglogistic(1,4)	30	86.73 10000	86.66 10000	-	-	-	-
Loglogistic(1,4)	100	63.85 10000	63.82 10000	-	-	-	-
Loglogistic(1,4)	1000	0.01 10000	0.01 10000	-	-	-	-
Lognormal(0,0.2)	30	77.52 10000	77.48 10000	-	-	-	-
Lognormal(0,0.2)	100	35.30 10000	35.26 10000	-	-	-	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	4.79 10000	4.78 10000	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	-	-	-	-
Pareto(5)	30	95.20 10000	95.18 10000	94.83 10000	94.78 10000	94.84 10000	94.74 10000
Pareto(5)	100	94.55 10000	94.54 10000	95.21 10000	95.14 10000	94.75 10000	94.75 10000
Pareto(5)	1000	94.93 10000	94.93 10000	94.94 10000	94.94 10000	94.90 10000	94.93 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	100	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	1000	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	30	66.44 10000	66.37 10000	-	-	-	-
Weibull(1,1)	100	12.67 10000	12.65 10000	-	-	-	-
Weibull(1,1)	1000	0.00 10000	0.00 10000	-	-	-	-

Table E.42: Testing v.s. Case IIIb Pareto (5) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	52.02 10000	52.01 10000	-	-	-	-
ChiSquared1	100	3.57 10000	3.57 10000	-	-	-	-
ChiSquared1	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	30	5.60 10000	5.60 10000	-	-	-	-
ChiSquared3	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	30	0.48 10000	0.48 10000	-	-	-	-
ChiSquared4	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	1000	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	30	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	100	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	1000	0.00 10000	0.00 10000	-	-	-	-
Loglogistic(1,4)	30	83.11 10000	83.11 10000	-	-	-	-
Loglogistic(1,4)	100	51.75 10000	51.75 10000	-	-	-	-
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	-	-	-	-
Lognormal(0,0.2)	30	70.28 10000	70.27 10000	-	-	-	-
Lognormal(0,0.2)	100	20.46 10000	20.46 10000	-	-	-	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	2.57 10000	2.57 10000	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	-	-	-	-
Pareto(5)	30	95.04 10000	95.04 10000	95.21 10000	95.21 10000	94.88 10000	94.91 10000
Pareto(5)	100	94.39 10000	94.39 10000	95.26 10000	95.26 10000	95.02 10000	95.02 10000
Pareto(5)	1000	95.31 10000	95.34 10000	95.16 10000	95.13 10000	95.18 10000	95.20 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	100	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	1000	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	30	57.08 10000	57.07 10000	-	-	-	-
Weibull(1,1)	100	5.33 10000	5.33 10000	-	-	-	-
Weibull(1,1)	1000	0.00 10000	0.00 10000	-	-	-	-

Table E.43: Testing v.s. Case IIIb Pareto (5) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$\tau_L = 1$		$\tau_L = 100$		$\tau_L = 1000$	
Generation Distribution	n	CV	F	CV	F	CV	F
ChiSquared1	30	57.09 10000	57.08 10000	-	-	-	-
ChiSquared1	100	3.85 10000	3.85 10000	-	-	-	-
ChiSquared1	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	30	6.92 10000	6.91 10000	-	-	-	-
ChiSquared3	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared3	1000	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	30	0.60 10000	0.60 10000	-	-	-	-
ChiSquared4	100	0.00 10000	0.00 10000	-	-	-	-
ChiSquared4	1000	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	30	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	100	0.00 10000	0.00 10000	-	-	-	-
Gamma(5,2)	1000	0.00 10000	0.00 10000	-	-	-	-
Loglogistic(1,4)	30	85.73 10000	85.72 10000	-	-	-	-
Loglogistic(1,4)	100	54.45 10000	54.51 10000	-	-	-	-
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	-	-	-	-
Lognormal(0,0.2)	30	74.26 10000	74.25 10000	-	-	-	-
Lognormal(0,0.2)	100	22.72 10000	22.77 10000	-	-	-	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	3.38 10000	3.38 10000	-	-	-	-
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	-	-	-	-
Pareto(5)	30	95.01 10000	95.01 10000	95.30 10000	95.32 10000	94.81 10000	94.81 10000
Pareto(5)	100	94.29 10000	94.30 10000	95.08 10000	95.07 10000	94.79 10000	94.76 10000
Pareto(5)	1000	95.06 10000	95.10 10000	94.92 10000	94.92 10000	94.83 10000	94.88 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	100	0.00 10000	0.00 10000	-	-	-	-
Rayleigh(4)	1000	0.00 10000	0.00 10000	-	-	-	-
Weibull(1,1)	30	62.02 10000	62.00 10000	-	-	-	-
Weibull(1,1)	100	6.00 10000	6.05 10000	-	-	-	-
Weibull(1,1)	1000	0.00 10000	0.00 10000	-	-	-	-

Table E.44: Testing v.s. Case IIIb Pareto (5) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.4 Power testing v.s. Case IIIb: Scale Parameter is Known, Shape Parameter is Unknown

E.4.1 Weibull Distribution

KS 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.78 10000	94.79 10000	94.47 10000	94.40 10000	94.97 10000	94.97 10000	95.36 10000	95.38 10000	94.41 10000	94.49 10000
Weibull(1,1)	100	95.01 10000	95.01 10000	95.09 10000	95.06 10000	95.05 10000	95.05 10000	95.09 10000	95.06 10000	95.07 10000	95.07 10000
Weibull(1,1)	1000	95.12 10000	95.10 10000	95.31 10000	95.25 10000	94.90 10000	94.93 10000	94.94 10000	94.89 10000	95.23 10000	95.23 10000
Loglogistic(1,4)	30	33.33 10000	33.37 10000	37.65 10000	38.02 10000	91.12 10000	91.10 10000	79.13 10000	79.18 10000	74.63 10000	74.31 9811
Loglogistic(1,4)	100	0.48 10000	0.47 10000	0.57 10000	0.58 10000	73.07 10000	73.21 10000	42.17 10000	41.99 10000	34.25 10000	34.22 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	39.91 10000	39.95 10000	41.14 10000	41.28 10000	59.09 10000	59.01 10000	93.15 10000	93.16 10000	93.23 10000	-
Lognormal(0,0.2)	100	0.87 10000	0.85 10000	0.78 10000	0.77 10000	3.79 10000	3.85 10000	87.77 10000	87.71 10000	88.70 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	25.54 10000	25.25 10000	42.50 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	31.44 10000	31.46 10000	72.91 10000	68.21 7539
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.49 10000	0.48 10000	31.37 10000	28.74 9236
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	91.73 10000	91.75 10000	93.88 10000	93.78 10000	95.08 10000	95.05 10000	94.60 10000	94.62 10000	94.78 10000	94.86 10000
ChiSquared1	100	79.53 10000	79.44 10000	92.92 10000	92.83 10000	94.79 10000	94.77 10000	94.34 10000	94.28 10000	94.12 10000	94.12 10000
ChiSquared1	1000	0.64 10000	0.61 10000	73.65 10000	73.52 10000	92.58 10000	92.57 10000	89.80 10000	89.67 10000	88.75 10000	88.74 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.82 10000	0.82 10000	29.85 10000	29.87 10000	56.66 10000	56.73 10000	77.79 10000	78.05 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.20 10000	0.20 10000	8.33 10000	8.24 10000	39.65 10000	39.61 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.31 10000	3.34 10000	20.95 10000	21.04 10000	55.07 10000	55.31 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	6.23 10000	6.23 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.18 10000	0.18 10000	23.08 10000	23.13 10000	60.25 10000	60.33 10000	79.74 10000	79.83 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.17 10000	0.17 10000	13.89 10000	13.77 10000	47.18 10000	47.16 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.39 10000	0.40 10000	5.95 10000	6.02 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	1.95 10000	1.95 10000	24.18 10000	24.24 10000	43.38 10000	43.49 10000	63.38 10000	63.60 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.07 10000	0.07 10000	1.68 10000	1.66 10000	13.58 10000	13.57 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.45: Testing v.s. Case IIIb Weibull (1,1) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.83 10000	94.77 10000	94.69 10000	94.76 10000	94.79 10000	94.75 10000	95.12 10000	95.14 10000	94.58 10000	94.58 10000
Weibull(1,1)	100	95.10 10000	95.08 10000	94.91 10000	94.89 10000	95.00 10000	94.99 10000	94.59 10000	94.58 10000	95.04 10000	95.07 10000
Weibull(1,1)	1000	94.98 10000	95.00 10000	95.25 10000	95.18 10000	94.60 10000	94.65 10000	94.67 10000	94.68 10000	95.31 10000	95.31 10000
Loglogistic(1,4)	30	48.17 10000	48.08 10000	49.97 10000	49.82 10000	84.18 10000	84.27 10000	84.26 10000	84.26 10000	82.66 10000	82.40 9811
Loglogistic(1,4)	100	1.91 10000	1.91 10000	2.34 10000	2.31 10000	47.51 10000	47.54 10000	55.12 10000	55.08 10000	49.26 10000	49.33 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	58.73 10000	58.54 10000	60.95 10000	60.70 10000	69.31 10000	69.37 10000	93.58 10000	93.58 10000	94.64 10000	-
Lognormal(0,0.2)	100	4.37 10000	4.36 10000	4.63 10000	4.59 10000	12.54 10000	12.51 10000	89.79 10000	89.74 10000	92.39 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	36.28 10000	36.17 10000	56.74 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	42.27 10000	42.28 10000	81.13 10000	77.58 7539
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	1.47 10000	1.46 10000	46.62 10000	44.43 9236
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	94.15 10000	94.08 10000	94.37 10000	94.52 10000	95.01 10000	94.99 10000	94.37 10000	94.38 10000	95.00 10000	94.99 10000
ChiSquared1	100	91.22 10000	91.12 10000	93.54 10000	93.51 10000	94.76 10000	94.73 10000	94.94 10000	94.93 10000	94.80 10000	94.82 10000
ChiSquared1	1000	7.32 10000	7.37 10000	77.14 10000	77.13 10000	92.68 10000	92.67 10000	92.06 10000	92.06 10000	91.65 10000	91.67 10000
ChiSquared3	30	0.04 10000	0.04 10000	2.15 10000	2.19 10000	38.36 10000	38.35 10000	63.77 10000	63.78 10000	81.94 10000	81.93 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.64 10000	0.64 10000	12.90 10000	12.89 10000	49.20 10000	49.27 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	5.68 10000	5.68 10000	27.10 10000	27.13 10000	61.40 10000	61.40 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.06 10000	0.06 10000	9.16 10000	9.19 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.29 10000	0.29 10000	20.28 10000	20.27 10000	51.43 10000	51.47 10000	77.77 10000	77.77 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	3.69 10000	3.68 10000	35.07 10000	35.19 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.52 10000	0.53 10000	6.97 10000	6.97 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	3.35 10000	3.35 10000	30.60 10000	30.59 10000	50.29 10000	50.32 10000	69.53 10000	69.54 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.18 10000	0.18 10000	3.09 10000	3.08 10000	19.00 10000	19.04 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.46: Testing v.s. Case IIIb Weibull (1,1) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.96 10000	95.00 10000	95.03 10000	94.89 10000	95.01 10000	95.08 10000	95.25 10000	95.39 10000	94.57 10000	94.69 10000
Weibull(1,1)	100	94.97 10000	94.95 10000	95.05 10000	95.03 10000	95.02 10000	95.05 10000	95.14 10000	95.09 10000	94.72 10000	94.64 10000
Weibull(1,1)	1000	94.98 10000	94.95 10000	94.99 10000	94.91 10000	94.94 10000	94.98 10000	94.95 10000	94.85 10000	95.15 10000	95.04 10000
Loglogistic(1,4)	30	29.69 10000	29.76 10000	34.37 10000	34.71 10000	91.51 10000	91.53 10000	76.50 10000	76.68 10000	70.48 10000	70.38 9811
Loglogistic(1,4)	100	0.32 10000	0.32 10000	0.41 10000	0.41 10000	73.66 10000	73.84 10000	36.31 10000	36.29 10000	27.22 10000	27.13 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	33.59 10000	33.70 10000	34.17 10000	34.37 10000	55.07 10000	54.87 10000	92.82 10000	93.04 10000	92.30 10000	-
Lognormal(0,0.2)	100	0.59 10000	0.59 10000	0.55 10000	0.55 10000	2.95 10000	3.01 10000	86.18 10000	86.18 10000	87.14 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	19.76 10000	19.64 10000	32.27 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	26.13 10000	26.47 10000	68.74 10000	63.28 7539
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.20 10000	0.20 10000	24.89 10000	22.44 9236
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	90.27 10000	90.33 10000	94.02 10000	93.89 10000	95.22 10000	95.33 10000	94.68 10000	94.79 10000	94.66 10000	94.82 10000
ChiSquared1	100	71.45 10000	71.28 10000	92.77 10000	92.63 10000	95.02 10000	95.06 10000	94.18 10000	94.17 10000	94.17 10000	94.13 10000
ChiSquared1	1000	0.08 10000	0.07 10000	72.93 10000	72.67 10000	92.11 10000	92.12 10000	88.44 10000	88.31 10000	86.98 10000	86.75 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.23 10000	0.23 10000	22.17 10000	22.49 10000	49.01 10000	49.48 10000	74.31 10000	74.61 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	4.23 10000	4.20 10000	30.54 10000	30.37 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	1.51 10000	1.57 10000	13.03 10000	13.31 10000	47.02 10000	47.26 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000	2.37 10000	2.34 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.13 10000	0.13 10000	20.19 10000	20.72 10000	59.04 10000	59.48 10000	79.28 10000	79.55 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	8.39 10000	8.35 10000	43.27 10000	43.11 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.05 10000	0.05 10000	2.47 10000	2.51 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.88 10000	0.89 10000	16.55 10000	16.85 10000	34.95 10000	35.29 10000	56.78 10000	57.15 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.53 10000	0.53 10000	7.12 10000	7.07 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.47: Testing v.s. Case IIIb Weibull (1,1) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	94.94 10000	95.07 10000	95.08 10000	94.94 10000	94.96 10000	95.04 10000	95.21 10000	95.35 10000	94.59 10000	94.77 10000
Weibull(1,1)	100	95.15 10000	95.12 10000	94.94 10000	94.91 10000	94.88 10000	94.94 10000	95.03 10000	94.95 10000	94.70 10000	94.69 10000
Weibull(1,1)	1000	95.02 10000	94.96 10000	95.19 10000	95.14 10000	94.78 10000	94.80 10000	94.80 10000	94.71 10000	95.15 10000	95.03 10000
Loglogistic(1,4)	30	25.49 10000	25.77 10000	30.47 10000	31.11 10000	91.60 10000	91.65 10000	75.99 10000	76.42 10000	68.99 10000	69.08 9811
Loglogistic(1,4)	100	0.15 10000	0.15 10000	0.21 10000	0.23 10000	66.95 10000	67.24 10000	37.61 10000	37.47 10000	27.51 10000	27.48 10000
Loglogistic(1,4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Lognormal(0,0.2)	30	27.00 10000	27.46 10000	27.91 10000	28.40 10000	51.90 10000	51.58 10000	92.70 10000	92.86 10000	91.51 10000	-
Lognormal(0,0.2)	100	0.22 10000	0.22 10000	0.13 10000	0.12 10000	1.35 10000	1.37 10000	86.41 10000	86.40 10000	87.15 10000	-
Lognormal(0,0.2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	20.37 10000	20.19 10000	34.57 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	24.39 10000	24.78 10000	67.20 10000	62.06 7539
Pareto(5)	100	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.18 10000	0.17 10000	24.86 10000	22.41 9236
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	89.23 10000	89.36 10000	93.77 10000	93.51 10000	95.39 10000	95.53 10000	94.97 10000	95.19 10000	94.83 10000	95.08 10000
ChiSquared1	100	62.52 10000	62.30 10000	91.90 10000	91.76 10000	94.85 10000	94.87 10000	94.51 10000	94.45 10000	94.79 10000	94.69 10000
ChiSquared1	1000	0.00 10000	0.00 10000	67.77 10000	67.72 10000	92.41 10000	92.36 10000	89.08 10000	88.95 10000	87.72 10000	87.50 10000
ChiSquared3	30	0.00 10000	0.00 10000	0.33 10000	0.33 10000	24.80 10000	25.35 10000	53.09 10000	53.79 10000	77.02 10000	77.51 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	4.58 10000	4.54 10000	33.19 10000	33.10 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	1.83 10000	1.91 10000	15.09 10000	15.59 10000	51.07 10000	51.86 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	2.76 10000	2.75 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.00 10000	0.00 10000	0.14 10000	0.16 10000	21.07 10000	21.56 10000	59.69 10000	60.42 10000	80.55 10000	80.99 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	6.68 10000	6.61 10000	42.50 10000	42.39 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.10 10000	0.10 10000	3.17 10000	3.31 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-
Mittag-Leffler(1,5)	30	0.00 10000	0.00 10000	0.93 10000	0.94 10000	18.84 10000	19.45 10000	38.70 10000	39.46 10000	61.23 10000	61.97 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.65 10000	0.65 10000	8.23 10000	8.12 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.48: Testing v.s. Case IIIb Weibull (1,1) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.4.2 Loglogistic Distribution

KS 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	58.57 10000	58.52 10000	93.57 10000	93.49 10000	89.73 10000	89.82 10000	88.08 10000	88.11 10000	86.70 10000	86.84 10000
Weibull(1,1)	100	6.39 10000	6.38 10000	89.55 10000	89.57 10000	73.94 10000	73.94 10000	68.09 10000	68.01 10000	64.66 10000	64.57 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	17.72 10000	17.73 10000	0.17 10000	0.17 10000	0.02 10000	0.02 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	95.16 10000	95.16 10000	94.61 10000	94.53 10000	95.19 10000	95.24 10000	94.52 10000	94.56 10000	94.90 10000	94.95 9964
Loglogistic(1,4)	100	95.06 10000	95.05 10000	95.26 10000	95.23 10000	94.73 10000	94.73 10000	94.91 10000	94.87 10000	95.19 10000	95.19 10000
Loglogistic(1,4)	1000	95.30 10000	95.25 10000	95.19 10000	95.18 10000	94.86 10000	94.87 10000	94.87 10000	94.79 10000	94.94 10000	94.94 10000
Lognormal(0,0.2)	30	94.32 10000	94.31 10000	94.92 10000	94.97 10000	94.31 10000	94.31 10000	92.11 10000	92.20 10000	92.02 10000	-
Lognormal(0,0.2)	100	93.77 10000	93.76 10000	94.33 10000	94.44 10000	92.25 10000	92.33 10000	85.20 10000	85.11 10000	85.95 10000	-
Lognormal(0,0.2)	1000	82.14 10000	81.99 10000	86.33 10000	86.42 10000	51.47 10000	51.69 10000	10.40 10000	10.38 10000	21.02 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	59.79 10000	59.83 10000	91.08 10000	91.17 10000	94.73 10000	94.81 8234
Pareto(5)	100	-	-	0.00 10000	0.00 10000	11.50 10000	11.55 10000	81.29 10000	81.19 10000	93.42 10000	93.34 9611
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	7.12 10000	7.09 10000	80.71 10000	80.72 10000
ChiSquared1	30	24.64 10000	24.60 10000	92.14 10000	92.12 10000	87.60 10000	87.63 10000	86.53 10000	86.56 10000	85.03 10000	85.16 10000
ChiSquared1	100	0.01 10000	0.01 10000	83.15 10000	83.27 10000	67.74 10000	67.74 10000	61.98 10000	61.92 10000	58.43 10000	58.24 10000
ChiSquared1	1000	0.00 10000	0.00 10000	1.65 10000	1.66 10000	0.01 10000	0.01 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000
ChiSquared3	30	2.29 10000	2.29 10000	22.24 10000	22.31 10000	35.96 10000	36.11 10000	42.92 10000	42.99 10000	50.31 10000	50.46 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.07 10000	0.07 10000	0.51 10000	0.51 10000	1.62 10000	1.61 10000	3.36 10000	3.36 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	2.40 10000	2.41 10000	9.65 10000	9.76 10000	15.75 10000	15.84 10000	25.66 10000	25.81 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.08 10000	0.07 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.04 10000	0.04 10000	11.08 10000	11.19 10000	30.66 10000	30.82 10000	43.94 10000	44.05 10000	66.23 10000	66.42 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.31 10000	0.31 10000	2.41 10000	2.41 10000	18.31 10000	18.20 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.07 10000	0.07 10000	0.27 10000	0.27 10000	1.15 10000	1.16 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	1.05 10000	1.05 10000	16.62 10000	16.75 10000	24.86 10000	25.09 10000	30.14 10000	30.29 10000	35.73 10000	35.92 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.05 10000	0.05 10000	0.19 10000	0.19 10000	0.65 10000	0.64 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.49: Testing v.s. Case IIIb Loglogistic (1,4) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	74.72 10000	74.67 10000	92.09 10000	92.14 10000	89.71 10000	89.72 10000	88.29 10000	88.37 10000	87.81 10000	87.83 10000
Weibull(1,1)	100	19.93 10000	19.95 10000	84.37 10000	84.38 10000	75.21 10000	75.10 10000	70.72 10000	70.75 10000	67.90 10000	67.79 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	2.91 10000	2.91 10000	0.02 10000	0.02 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	94.85 10000	94.81 10000	94.89 10000	94.96 10000	94.78 10000	94.79 10000	94.67 10000	94.72 10000	95.10 10000	95.09 9964
Loglogistic(1,4)	100	95.14 10000	95.16 10000	95.32 10000	95.35 10000	94.83 10000	94.81 10000	94.71 10000	94.72 10000	95.45 10000	95.44 10000
Loglogistic(1,4)	1000	95.20 10000	95.22 10000	95.33 10000	95.35 10000	94.86 10000	94.85 10000	94.83 10000	94.87 10000	94.85 10000	94.82 10000
Lognormal(0,0.2)	30	93.95 10000	93.89 10000	94.62 10000	94.50 10000	93.93 10000	93.95 10000	92.68 10000	92.71 10000	92.96 10000	-
Lognormal(0,0.2)	100	91.90 10000	91.92 10000	93.32 10000	93.20 10000	91.76 10000	91.74 10000	87.23 10000	87.20 10000	89.73 10000	-
Lognormal(0,0.2)	1000	47.32 10000	47.37 10000	66.17 10000	65.65 10000	44.70 10000	44.52 10000	11.72 10000	11.71 10000	31.01 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	67.33 10000	67.38 10000	92.45 10000	92.46 10000	95.15 10000	95.18 8234
Pareto(5)	100	-	-	0.00 10000	0.00 10000	18.26 10000	18.23 10000	85.12 10000	85.12 10000	93.92 10000	93.92 9611
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	11.01 10000	11.06 10000	84.35 10000	84.35 10000
ChiSquared1	30	45.99 10000	45.89 10000	90.62 10000	90.66 10000	88.00 10000	88.03 10000	87.42 10000	87.54 10000	85.91 10000	85.97 10000
ChiSquared1	100	0.32 10000	0.32 10000	78.47 10000	78.47 10000	67.95 10000	67.90 10000	62.89 10000	62.94 10000	60.04 10000	60.02 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.11 10000	0.11 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	2.68 10000	2.63 10000	19.75 10000	19.75 10000	33.12 10000	33.25 10000	40.75 10000	40.87 10000	48.67 10000	48.78 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.19 10000	0.19 10000	0.79 10000	0.79 10000	2.03 10000	2.03 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	1.57 10000	1.57 10000	7.28 10000	7.28 10000	12.81 10000	12.83 10000	21.70 10000	21.76 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.02 10000	0.02 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.14 10000	0.14 10000	13.92 10000	13.93 10000	33.94 10000	34.01 10000	47.68 10000	47.76 10000	67.99 10000	68.07 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.54 10000	0.54 10000	3.46 10000	3.46 10000	20.42 10000	20.44 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.09 10000	0.09 10000	0.31 10000	0.31 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	1.21 10000	1.18 10000	12.99 10000	12.96 10000	20.71 10000	20.80 10000	25.96 10000	26.06 10000	31.73 10000	31.85 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.01 10000	0.00 10000	0.05 10000	0.05 10000	0.26 10000	0.26 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.50: Testing v.s. Case IIIb Loglogistic (1,4) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	48.74 10000	48.63 10000	93.26 10000	93.07 10000	87.93 10000	88.09 10000	86.24 10000	86.23 10000	84.51 10000	84.64 10000
Weibull(1,1)	100	2.52 10000	2.51 10000	88.81 10000	88.83 10000	67.74 10000	67.75 10000	60.08 10000	59.90 10000	55.95 10000	55.58 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	12.42 10000	12.71 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	95.09 10000	95.08 10000	94.82 10000	94.62 10000	95.23 10000	95.26 10000	94.79 10000	94.76 10000	94.94 10000	94.97 9964
Loglogistic(1,4)	100	95.10 10000	95.06 10000	95.04 10000	95.02 10000	94.87 10000	94.91 10000	94.63 10000	94.60 10000	95.41 10000	95.40 10000
Loglogistic(1,4)	1000	95.07 10000	95.10 10000	95.03 10000	95.04 10000	94.95 10000	94.99 10000	94.95 10000	94.90 10000	94.82 10000	94.79 10000
Lognormal(0,0.2)	30	94.30 10000	94.26 10000	94.96 10000	95.10 10000	94.37 10000	94.38 10000	91.45 10000	91.49 10000	91.46 10000	-
Lognormal(0,0.2)	100	94.46 10000	94.43 10000	94.52 10000	94.75 10000	91.39 10000	91.47 10000	82.33 10000	82.09 10000	84.02 10000	-
Lognormal(0,0.2)	1000	89.24 10000	89.24 10000	90.34 10000	90.63 10000	42.99 10000	43.44 10000	3.44 10000	3.37 10000	11.20 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	54.89 10000	54.90 10000	90.05 10000	90.08 10000	94.54 10000	94.56 8234
Pareto(5)	100	-	-	0.00 10000	0.00 10000	7.45 10000	7.50 10000	78.30 10000	78.09 10000	93.26 10000	93.23 9611
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.67 10000	3.63 10000	77.57 10000	77.64 10000
ChiSquared1	30	17.92 10000	17.85 10000	91.60 10000	91.51 10000	85.52 10000	85.68 10000	84.02 10000	84.03 10000	82.75 10000	82.79 10000
ChiSquared1	100	0.00 10000	0.00 10000	80.47 10000	80.61 10000	59.18 10000	59.17 10000	52.60 10000	52.36 10000	47.59 10000	47.17 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.60 10000	0.64 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	1.44 10000	1.42 10000	13.59 10000	13.65 10000	24.74 10000	24.91 10000	31.20 10000	31.25 10000	39.17 10000	39.18 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.10 10000	0.10 10000	0.20 10000	0.20 10000	0.66 10000	0.65 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.75 10000	0.75 10000	4.09 10000	4.13 10000	7.75 10000	7.83 10000	14.39 10000	14.39 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.02 10000	0.02 10000	8.35 10000	8.45 10000	26.16 10000	26.33 10000	39.63 10000	39.71 10000	63.88 10000	63.83 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.15 10000	0.15 10000	1.59 10000	1.58 10000	15.11 10000	15.04 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.01 10000	0.01 10000	0.16 10000	0.16 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.65 10000	0.65 10000	8.62 10000	8.69 10000	14.65 10000	14.76 10000	18.27 10000	18.33 10000	24.00 10000	23.96 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.06 10000	0.05 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.51: Testing v.s. Case IIIb Loglogistic (1,4) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p=0$		$p=0.1$		$p=0.4$		$p=0.6$		$p=0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	39.55 10000	39.71 10000	93.73 10000	93.44 10000	90.25 10000	90.50 10000	88.23 10000	88.36 10000	87.21 10000	87.57 10000
Weibull(1,1)	100	0.60 10000	0.60 10000	88.85 10000	88.88 10000	70.19 10000	70.19 10000	62.32 10000	62.22 10000	58.98 10000	58.59 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	4.58 10000	4.68 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglogistic(1,4)	30	94.86 10000	94.94 10000	94.79 10000	94.65 10000	95.29 10000	95.39 10000	94.83 10000	94.85 10000	94.97 10000	95.10 9964
Loglogistic(1,4)	100	94.92 10000	94.91 10000	94.95 10000	94.89 10000	95.13 10000	95.16 10000	94.73 10000	94.69 10000	95.33 10000	95.30 10000
Loglogistic(1,4)	1000	95.09 10000	95.04 10000	95.18 10000	95.19 10000	95.13 10000	95.12 10000	95.15 10000	94.99 10000	95.06 10000	95.01 10000
Lognormal(0,0.2)	30	94.48 10000	94.57 10000	94.81 10000	94.98 10000	94.90 10000	94.93 10000	93.08 10000	93.20 10000	92.75 10000	-
Lognormal(0,0.2)	100	94.45 10000	94.43 10000	94.50 10000	94.55 10000	92.20 10000	92.24 10000	84.29 10000	84.07 10000	86.60 10000	-
Lognormal(0,0.2)	1000	85.08 10000	84.98 10000	87.43 10000	87.70 10000	41.38 10000	41.75 10000	3.38 10000	3.34 10000	12.92 10000	-
Pareto(5)	30	-	-	0.00 10000	0.00 10000	51.20 10000	51.31 10000	88.72 10000	88.94 10000	94.30 10000	94.33 8234
Pareto(5)	100	-	-	0.00 10000	0.00 10000	6.61 10000	6.67 10000	77.01 10000	76.75 10000	92.72 10000	92.72 9611
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	3.66 10000	3.59 10000	77.81 10000	77.88 10000
ChiSquared1	30	10.35 10000	10.43 10000	92.47 10000	92.49 10000	88.01 10000	88.17 10000	86.68 10000	86.84 10000	85.28 10000	85.61 10000
ChiSquared1	100	0.00 10000	0.00 10000	81.48 10000	81.65 10000	61.66 10000	61.66 10000	54.73 10000	54.55 10000	50.30 10000	49.82 10000
ChiSquared1	1000	0.00 10000	0.00 10000	0.11 10000	0.11 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared3	30	1.55 10000	1.56 10000	16.28 10000	16.50 10000	28.90 10000	29.26 10000	35.69 10000	35.95 10000	44.03 10000	44.50 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.08 10000	0.08 10000	0.20 10000	0.20 10000	0.67 10000	0.64 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	0.98 10000	0.99 10000	5.24 10000	5.38 10000	9.73 10000	9.87 10000	17.13 10000	17.31 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.03 10000	0.03 10000	10.47 10000	10.63 10000	30.18 10000	30.50 10000	44.74 10000	45.10 10000	67.44 10000	67.65 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.20 10000	0.20 10000	1.88 10000	1.85 10000	15.90 10000	15.70 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.04 10000	0.04 10000	0.24 10000	0.24 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.69 10000	0.70 10000	10.45 10000	10.71 10000	17.57 10000	17.86 10000	21.73 10000	22.00 10000	27.87 10000	28.11 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.03 10000	0.03 10000	0.08 10000	0.08 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.52: Testing v.s. Case IIIb Loglogistic (1,4) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

E.4.3 Lognormal Distribution

KS 95 Pass Rate		p = 0		p = 0.1		p = 0.4		p = 0.6		p = 0.8	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	50.56 10000	50.42 10000	93.77 10000	93.78 10000	93.38 10000	93.41 10000	92.85 10000	92.94 10000	93.20 10000	93.30 10000
Weibull(1,1)	100	1.84 10000	1.83 10000	90.59 10000	90.55 10000	88.88 10000	88.86 10000	88.09 10000	88.09 10000	88.30 10000	88.26 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	41.47 10000	41.43 10000	30.51 10000	30.36 10000	27.40 10000	27.27 10000	25.10 10000	25.11 10000
Loglogistic(1,4)	30	95.13 10000	95.10 10000	92.64 10000	92.64 10000	90.57 10000	90.62 10000	89.22 10000	89.30 10000	88.89 10000	89.07 10000
Loglogistic(1,4)	100	93.77 10000	93.74 10000	87.20 10000	87.22 10000	80.21 10000	80.16 10000	76.92 10000	76.89 10000	74.52 10000	74.50 10000
Loglogistic(1,4)	1000	61.79 10000	61.91 10000	21.60 10000	21.67 10000	6.16 10000	6.09 10000	3.61 10000	3.60 10000	2.57 10000	2.55 10000
Lognormal(0,0.2)	30	94.95 10000	94.90 10000	95.07 10000	95.00 10000	94.99 10000	95.00 10000	94.82 10000	94.88 10000	95.03 10000	95.06 9994
Lognormal(0,0.2)	100	94.94 10000	94.94 10000	94.47 10000	94.47 10000	94.98 10000	94.96 10000	94.65 10000	94.64 10000	94.87 10000	94.84 10000
Lognormal(0,0.2)	1000	94.85 10000	94.86 10000	95.06 10000	95.11 10000	94.56 10000	94.55 10000	95.03 10000	95.03 10000	94.82 10000	94.83 10000
Pareto(5)	30	-	-	1.81 10000	1.78 10000	54.86 10000	54.95 10000	69.51 10000	69.66 10000	77.81 10000	77.96 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	8.40 10000	8.38 10000	24.49 10000	24.45 10000	43.90 10000	43.85 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	15.21 10000	15.13 10000	93.22 10000	93.24 10000	92.23 10000	92.33 10000	92.19 10000	92.26 10000	92.02 10000	92.07 10000
ChiSquared1	100	0.00 10000	0.00 10000	87.63 10000	87.62 10000	85.58 10000	85.55 10000	84.67 10000	84.65 10000	83.97 10000	83.92 10000
ChiSquared1	1000	0.00 10000	0.00 10000	16.78 10000	16.71 10000	11.12 10000	11.02 10000	9.65 10000	9.57 10000	7.85 10000	7.85 10000
ChiSquared3	30	3.12 10000	3.10 10000	39.37 10000	39.40 10000	47.13 10000	47.37 10000	51.72 10000	51.95 10000	56.82 10000	57.13 10000
ChiSquared3	100	0.00 10000	0.00 10000	1.04 10000	1.03 10000	2.87 10000	2.86 10000	4.02 10000	4.01 10000	6.73 10000	6.71 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.01 10000	0.01 10000	9.20 10000	9.26 10000	15.66 10000	15.78 10000	19.71 10000	19.82 10000	25.36 10000	25.56 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000	0.02 10000	0.02 10000	0.13 10000	0.13 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.03 10000	0.03 10000	34.76 10000	34.90 10000	49.55 10000	49.75 10000	57.46 10000	57.75 10000	67.55 10000	67.81 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	1.11 10000	1.11 10000	5.43 10000	5.43 10000	10.58 10000	10.56 10000	21.72 10000	21.69 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.10 10000	0.10 10000	0.17 10000	0.17 10000	0.55 10000	0.55 10000	0.94 10000	0.95 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	1.33 10000	1.33 10000	28.88 10000	28.95 10000	34.38 10000	34.56 10000	37.99 10000	38.23 10000	40.91 10000	41.28 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.23 10000	0.23 10000	0.53 10000	0.53 10000	0.93 10000	0.93 10000	1.18 10000	1.17 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.53: Testing v.s. Case IIIb Lognormal (0,0.2) under Kolmogorov-Smirnov at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Kuiper 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	65.09 10000	64.98 10000	94.07 10000	94.07 10000	93.87 10000	93.80 10000	93.43 10000	93.41 10000	93.43 10000	93.43 10000
Weibull(1,1)	100	7.72 10000	7.67 10000	91.64 10000	91.57 10000	90.86 10000	90.80 10000	90.50 10000	90.45 10000	90.58 10000	90.43 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	47.61 10000	47.61 10000	39.48 10000	39.41 10000	35.53 10000	35.53 10000	33.64 10000	33.78 10000
Loglogistic(1,4)	30	92.56 10000	92.50 10000	93.12 10000	93.16 10000	92.57 10000	92.55 10000	91.90 10000	91.89 10000	91.98 10000	92.00 10000
Loglogistic(1,4)	100	86.16 10000	86.12 10000	88.67 10000	88.68 10000	86.61 10000	86.59 10000	84.66 10000	84.62 10000	83.92 10000	83.89 10000
Loglogistic(1,4)	1000	12.82 10000	12.87 10000	26.31 10000	26.42 10000	12.50 10000	12.49 10000	8.37 10000	8.39 10000	6.33 10000	6.33 10000
Lognormal(0,0.2)	30	94.87 10000	94.86 10000	95.19 10000	95.23 10000	94.67 10000	94.63 10000	94.87 10000	94.87 10000	94.87 10000	94.84 9994
Lognormal(0,0.2)	100	94.61 10000	94.60 10000	94.86 10000	94.85 10000	95.05 10000	94.97 10000	94.76 10000	94.71 10000	94.86 10000	94.86 10000
Lognormal(0,0.2)	1000	94.81 10000	94.85 10000	94.77 10000	94.76 10000	94.54 10000	94.52 10000	95.45 10000	95.48 10000	94.94 10000	94.95 10000
Pareto(5)	30	-	-	5.48 10000	5.53 10000	65.44 10000	65.41 10000	79.07 10000	79.06 10000	85.08 10000	85.08 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	15.36 10000	15.29 10000	38.36 10000	38.23 10000	57.72 10000	57.88 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	29.64 10000	29.53 10000	93.30 10000	93.28 10000	92.85 10000	92.85 10000	92.89 10000	92.89 10000	92.97 10000	92.98 10000
ChiSquared1	100	0.02 10000	0.02 10000	89.34 10000	89.24 10000	88.50 10000	88.48 10000	87.57 10000	87.54 10000	87.26 10000	87.19 10000
ChiSquared1	1000	0.00 10000	0.00 10000	20.42 10000	20.39 10000	14.87 10000	14.80 10000	13.17 10000	13.16 10000	11.73 10000	11.76 10000
ChiSquared3	30	4.90 10000	4.86 10000	43.50 10000	43.47 10000	51.04 10000	50.98 10000	55.85 10000	55.80 10000	60.78 10000	60.77 10000
ChiSquared3	100	0.00 10000	0.00 10000	1.29 10000	1.29 10000	3.36 10000	3.35 10000	5.45 10000	5.45 10000	8.13 10000	8.08 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.01 10000	0.01 10000	10.39 10000	10.37 10000	17.66 10000	17.63 10000	21.43 10000	21.39 10000	27.60 10000	27.58 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.01 10000	0.01 10000	0.00 10000	0.00 10000	0.12 10000	0.12 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.29 10000	0.29 10000	34.88 10000	34.81 10000	48.13 10000	48.08 10000	55.00 10000	54.95 10000	64.16 10000	64.15 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.76 10000	0.72 10000	3.54 10000	3.54 10000	6.83 10000	6.82 10000	14.12 10000	14.05 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.09 10000	0.09 10000	0.32 10000	0.32 10000	0.60 10000	0.60 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	1.83 10000	1.81 10000	30.67 10000	30.63 10000	36.92 10000	36.84 10000	40.70 10000	40.62 10000	44.22 10000	44.20 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.30 10000	0.30 10000	0.59 10000	0.59 10000	1.03 10000	1.02 10000	1.41 10000	1.40 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.54: Testing v.s. Case IIIb Lognormal (0,0.2) under Kuiper at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

CvM 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	46.78 10000	46.55 10000	93.59 10000	93.67 10000	93.06 10000	93.09 10000	92.88 10000	92.98 10000	92.49 10000	92.68 10000
Weibull(1,1)	100	1.20 10000	1.20 10000	89.25 10000	89.20 10000	87.20 10000	87.18 10000	86.31 10000	86.30 10000	86.68 10000	86.67 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	32.55 10000	32.50 10000	20.71 10000	20.58 10000	17.42 10000	17.40 10000	15.60 10000	15.59 10000
Loglogistic(1,4)	30	95.59 10000	95.52 10000	91.89 10000	91.89 10000	89.49 10000	89.64 10000	88.17 10000	88.32 10000	87.39 10000	87.54 10000
Loglogistic(1,4)	100	94.84 10000	94.84 10000	85.84 10000	85.86 10000	77.10 10000	77.03 10000	72.86 10000	72.85 10000	70.10 10000	70.12 10000
Loglogistic(1,4)	1000	68.55 10000	68.99 10000	16.94 10000	16.95 10000	3.07 10000	3.06 10000	1.64 10000	1.63 10000	1.03 10000	1.01 10000
Lognormal(0,0.2)	30	94.86 10000	94.77 10000	94.93 10000	94.81 10000	95.00 10000	95.04 10000	94.99 10000	95.12 10000	94.86 10000	94.98 9994
Lognormal(0,0.2)	100	95.05 10000	95.05 10000	94.46 10000	94.47 10000	95.13 10000	95.08 10000	94.55 10000	94.55 10000	95.09 10000	95.07 10000
Lognormal(0,0.2)	1000	94.66 10000	94.74 10000	95.15 10000	95.17 10000	94.97 10000	94.96 10000	95.13 10000	95.13 10000	94.57 10000	94.54 10000
Pareto(5)	30	-	-	1.08 10000	1.02 10000	48.90 10000	49.06 10000	65.44 10000	65.66 10000	74.87 10000	75.16 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	5.15 10000	5.13 10000	18.51 10000	18.50 10000	36.09 10000	36.14 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	14.35 10000	14.24 10000	92.30 10000	92.38 10000	91.78 10000	91.91 10000	91.57 10000	91.73 10000	91.66 10000	91.81 10000
ChiSquared1	100	0.01 10000	0.01 10000	86.25 10000	86.07 10000	83.00 10000	82.95 10000	82.16 10000	82.12 10000	80.64 10000	80.63 10000
ChiSquared1	1000	0.00 10000	0.00 10000	9.87 10000	9.82 10000	5.11 10000	5.10 10000	4.05 10000	4.03 10000	2.95 10000	2.93 10000
ChiSquared3	30	2.08 10000	2.08 10000	29.34 10000	29.53 10000	37.47 10000	37.73 10000	42.51 10000	42.86 10000	47.91 10000	48.41 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.21 10000	0.19 10000	0.88 10000	0.88 10000	1.47 10000	1.47 10000	2.44 10000	2.44 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	4.18 10000	4.24 10000	8.65 10000	8.80 10000	11.72 10000	11.87 10000	16.26 10000	16.56 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.02 10000	0.02 10000	31.47 10000	31.67 10000	47.27 10000	47.60 10000	54.81 10000	55.19 10000	66.28 10000	66.65 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.54 10000	0.54 10000	3.41 10000	3.41 10000	7.69 10000	7.65 10000	17.26 10000	17.26 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.09 10000	0.09 10000	0.18 10000	0.18 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.85 10000	0.84 10000	19.12 10000	19.31 10000	24.48 10000	24.65 10000	27.88 10000	28.24 10000	31.37 10000	31.70 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.05 10000	0.05 10000	0.10 10000	0.10 10000	0.16 10000	0.16 10000	0.31 10000	0.31 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.55: Testing v.s. Case IIIb Lognormal (0,0.2) under Cramér-von Mises at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

AD 95 Pass Rate		$p = 0$		$p = 0.1$		$p = 0.4$		$p = 0.6$		$p = 0.8$	
Generation Distribution	n	CV	F	CV	F	CV	F	CV	F	CV	F
Weibull(1,1)	30	36.68 10000	36.77 10000	94.28 10000	94.46 10000	93.72 10000	93.92 10000	93.39 10000	93.53 10000	93.66 10000	93.93 10000
Weibull(1,1)	100	0.26 10000	0.26 10000	90.35 10000	90.34 10000	88.57 10000	88.55 10000	87.97 10000	87.87 10000	88.15 10000	88.03 10000
Weibull(1,1)	1000	0.00 10000	0.00 10000	34.18 10000	34.01 10000	21.74 10000	21.62 10000	18.43 10000	18.30 10000	16.85 10000	16.70 10000
Loglogistic(1,4)	30	95.37 10000	95.39 10000	92.01 10000	92.04 10000	89.39 10000	89.53 10000	87.90 10000	88.08 10000	86.88 10000	87.15 10000
Loglogistic(1,4)	100	94.32 10000	94.32 10000	85.96 10000	85.97 10000	77.39 10000	77.32 10000	73.52 10000	73.43 10000	70.27 10000	70.27 10000
Loglogistic(1,4)	1000	54.38 10000	54.54 10000	16.38 10000	16.36 10000	3.22 10000	3.19 10000	1.78 10000	1.74 10000	1.10 10000	1.10 10000
Lognormal(0,0.2)	30	94.93 10000	94.94 10000	95.24 10000	95.02 10000	94.84 10000	94.99 10000	94.98 10000	95.17 10000	94.86 10000	95.12 9994
Lognormal(0,0.2)	100	94.98 10000	95.03 10000	94.44 10000	94.44 10000	95.10 10000	95.05 10000	94.68 10000	94.63 10000	95.19 10000	95.16 10000
Lognormal(0,0.2)	1000	94.73 10000	94.75 10000	95.15 10000	95.23 10000	94.82 10000	94.76 10000	95.19 10000	95.10 10000	94.67 10000	94.60 10000
Pareto(5)	30	-	-	0.90 10000	0.87 10000	46.53 10000	46.86 10000	63.07 10000	63.50 10000	73.30 10000	73.80 10000
Pareto(5)	100	-	-	0.00 10000	0.00 10000	4.66 10000	4.65 10000	18.23 10000	18.18 10000	35.53 10000	35.60 10000
Pareto(5)	1000	-	-	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared1	30	8.27 10000	8.29 10000	93.24 10000	93.33 10000	92.84 10000	93.00 10000	92.63 10000	92.86 10000	92.68 10000	93.00 10000
ChiSquared1	100	0.00 10000	0.00 10000	87.58 10000	87.48 10000	85.07 10000	85.02 10000	83.94 10000	83.88 10000	83.05 10000	82.99 10000
ChiSquared1	1000	0.00 10000	0.00 10000	9.93 10000	9.84 10000	5.13 10000	5.13 10000	4.26 10000	4.19 10000	3.23 10000	3.20 10000
ChiSquared3	30	2.14 10000	2.15 10000	33.23 10000	33.69 10000	41.66 10000	42.24 10000	47.11 10000	47.72 10000	52.71 10000	53.33 10000
ChiSquared3	100	0.00 10000	0.00 10000	0.21 10000	0.21 10000	1.10 10000	1.08 10000	1.81 10000	1.81 10000	2.92 10000	2.91 10000
ChiSquared3	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
ChiSquared4	30	0.00 10000	0.00 10000	5.37 10000	5.53 10000	10.53 10000	10.94 10000	13.85 10000	14.32 10000	19.31 10000	19.66 10000
ChiSquared4	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.02 10000	0.02 10000
ChiSquared4	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Loglaplace(1,1)	30	0.02 10000	0.02 10000	34.21 10000	34.60 10000	49.84 10000	50.48 10000	57.41 10000	58.10 10000	67.97 10000	68.65 10000
Loglaplace(1,1)	100	0.00 10000	0.00 10000	0.53 10000	0.53 10000	3.27 10000	3.22 10000	7.19 10000	7.16 10000	16.02 10000	15.90 10000
Loglaplace(1,1)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.04 10000	0.04 10000	0.13 10000	0.13 10000	0.23 10000	0.24 10000
Rayleigh(4)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Rayleigh(4)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Gamma(5,2)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000
Uniform	30	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	100	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Uniform	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	-	-	-	-
Mittag-Leffler(1,5)	30	0.88 10000	0.88 10000	22.27 10000	22.73 10000	28.06 10000	28.59 10000	32.09 10000	32.58 10000	35.82 10000	36.29 10000
Mittag-Leffler(1,5)	100	0.00 10000	0.00 10000	0.05 10000	0.05 10000	0.08 10000	0.08 10000	0.23 10000	0.23 10000	0.41 10000	0.41 10000
Mittag-Leffler(1,5)	1000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000	0.00 10000

Table E.56: Testing v.s. Case IIIb Lognormal (0,0.2) under Anderson-Darling at 95% sig. lvl.
The number of samples is displayed under the pass rate
A dash implies there were zero samples

Appendix F

Glossary

- $\log(a)$ denotes the natural logarithm of a real, scalar value a
- x denotes a single observation of the independent variable
- $\boldsymbol{X}$ is a sample of observations of the independent variable. $\boldsymbol{X}$ and $\boldsymbol{x}$ are used interchangeably throughout this thesis
- x_i is the i^{th} measurement of the independent variable in a sample of observations, i.e. $\boldsymbol{X} = (x_1, x_2, ..., x_i, ...x_n)$ where $\boldsymbol{X}$ is a sample of n observations
- n is the number of data points in a sample
- m is the number of parameters required to completely specify a probability distribution
- N is the number of test statistics in a single-sample run
- M is the number of test statistics in a multi-sample run
- C is the number of runs in a multi-sample simulation
- θ denotes a general parameter used to specify a probability distribution
- $\boldsymbol{\theta}$ denotes the complete set of parameters required to specify a probability distribution
- θ_i is the i^{th} parameter in the set of parameters required to specify a distribution, i.e. $\boldsymbol{\theta} = (\theta_1, \theta_2, ..., \theta_i, ...\theta_n)$ where $\boldsymbol{\theta}$ is a set of m parameters
- $\boldsymbol{\theta^0}$ denotes a set of parameters that are known in advance
- $\hat{\boldsymbol{\theta}}$ denotes a set of parameters that have been estimated from a sample of observations, $\boldsymbol{x}$
- $f(t|\boldsymbol{\theta})$ denotes the value of the probability density function defined by the parameter set $\boldsymbol{\theta}$ evaluated at a point t
- pdf is an abbreviation of probability density function
- $F(t|\boldsymbol{\theta})$ denotes the value of the cumulative distribution function defined by the parameter set $\boldsymbol{\theta}$ evaluated at a point t
- cdf is an abbreviation of cumulative distribution function
- $F_n(t)$ denotes the value of the empirical distribution function evaluated at a point t. See section 2.2 for more information.
- edf is an abbreviation of empirical distribution function
- $L(\boldsymbol{\theta}|\boldsymbol{x})$ is the likelihood function for a sample of observations, $\boldsymbol{x}$, of a probability distribution which is specified by a set of parameters, $\boldsymbol{\theta}$
- $l(\boldsymbol{\theta}|\boldsymbol{x})$ is the loglikelihood function for a sample of observations, $\boldsymbol{x}$, of a probability distribution which is specified by a set of parameters, $\boldsymbol{\theta}$
- α is used to denote both
  - the significance level
  - the scale parameter of the Weibull distribution

  The definition applied in any particular situation should be clear from the context.
- D denotes the Kolmogorov-Smirnov test statistic

- V denotes Kuiper's test statistic
- W^2 denotes the Cramér von-Mises test statistic
- A^2 denotes the Anderson-Darling test statistic
- u denotes a random number drawn from a uniform distribution uniform distribution on $(0, 1)$
- $\boldsymbol{u}$ denotes a set of random numbers drawn from a uniform distribution on $(0, 1)$
- u_i is the i^{th} measurement of the independent variable in a sample of observations, i.e. $\boldsymbol{u} = (u_1, u_2, ..., u_i, ...u_n)$ where $\boldsymbol{u}$ is a set of n random numbers.

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