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MECHANICS OF SHEAR FAILURE IN FIBRE REINFORCED CONCRETE BEAMS

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ABSTRACT

In this paper, a new model, which can be solved either numerically or analytically, is presented for predicting the shear strength of fibre reinforced concrete beams. This approach is based on predicting the sliding capacity of an inclined crack through the application of fundamental partial-interaction and shear friction theories. A segmental approach is applied to predict this capacity because it has been shown to be able to produce simple analytical solutions while explicitly allowing for the influence of fibre reinforcement and tension stiffening. Once developed, the model is validated against a range of experimental tests and the accuracy is compared to both codified approaches and other approaches in the literature.

INTRODUCTION

Fibre reinforced concrete (FRC) beams have been shown experimentally to have superior shear capacity compared to conventional reinforced concrete beams (Lim et al. 1999; Kwak et al. 2002; Dinh et al. 2011; Aoude et al. 2012; Conforti et al. 2013; Amin & Foster 2016). This improvement has led to the suggestion that steel fibres could either reduce the quantity of transverse reinforcement, or completely replace it, particularly in ultra-high performance fibre reinforced concrete (UHPFRC) members (Casanova & Rossi 1997; Noghabai 2000; Singh & Jain 2014). Given the often catastrophic nature of shear failure, if this is to occur, it is essential that rational and reliable methods for predicting the shear capacity of FRC members are developed.

The observed increase in the shear capacity of FRC compared to RC members can be attributed to both the direct bridging of shear cracks (Choi et al. 2007), and also to an improvement in shear resistance of the uncracked FRC (Valle & Buyukozturk 1993; Sturm et al. 2018a). It is therefore necessary that models which predict the shear capacity of FRC members incorporate these behaviours.

In a recent review of the shear capacity of FRC members, Lansoght (2019) identified that the majority of approaches are empirical and are, therefore, difficult to extend to each new type of FRC developed. In addition to these empirical models, a number of mechanics based models have been developed. These can be categorised into two main types: (i) those that are based on the modified compression field theory (Minelli & Vecchio 2006; Baby et al. 2013; Lee et al. 2016b; Zhang et al. 2016a; Barros & Foster 2018), which was originally developed by Vecchio & Collins (1986) for conventional reinforced concrete, and (ii) those based on stresses that form along a critical diagonal shear crack (Voo et al. 2006; Choi et al. 2007; Lee et al. 2016a). Approaches based on modified compression field theory can be further subdivided into those that consider the full solution and those that apply the simplified approach. For the full solution, the beam is divided into a series of 2 dimensional elements while for simplified modified compression field theory a single element is considered. In both approaches, the shear capacity of an element is controlled by either the principal stresses on the element or is limited by the stresses that can be transferred across the shear crack due to aggregate interlock. The effect of the fibres is included into the approach either by modifying the constitutive relationships for the concrete (Minelli & Vecchio 2006; Baby et al. 2013; Lee et al. 2016b) or by adding an additional stress due to fibres in the element (Zhang et al. 2016a; Barros & Foster 2018). For Voo et al. (2006), the shear capacity is controlled by the intersection of the cracking and sliding load determined using an effective plastic compressive and tensile stress where, for FRC, the fibres alter the effective tensile stress.

For approaches that consider the development of stresses along the critical diagonal shear crack, Choi et al. (2007) define the shear capacity as being controlled by both: the uncracked concrete in the flexural compression region of the beam; and the stress carried by the fibres across the shear crack in the tension region. Alternatively in the work of Lee et al. (2016a), the shear capacity is controlled by the aggregate interlock in the flexural tension region and by the uncracked concrete in the flexural compression region. The effect of the fibres is allowed for by increasing the shear capacity of the flexural tension region.

In addition to models developed for research, numerous models are available in national codes of practice. These include the fib Model Code 2010 (fib 2013) which suggests two approaches which are either based on the expression in the Eurocode 2 (CEN 2004) or on a simplified modified compression field theory. The Australian concrete design standard AS3600:2018 (Standards Australia 2018) similarly suggests that the shear capacity of FRC members can be based on the application of a simplified version the modified compression field theory. French recommendations for UHPFRC (AFGC 2013) have a more simplified approach, in which a constant tensile stress due to the fibres is applied along the shear crack. As this crack is inclined, there is a vertical component of this stress that contributes to the shear capacity. The magnitude of this tensile stress is assumed to be equal to the average stress in the fibres at the ultimate limit state.

As highlighted in Lansoght's (2019) review, the existing empirical approaches have limited accuracy with the best performing empirical approach being that suggested by Kwak et al. (2002) which has a coefficient of variation of 28% and a mean of 1.01 when compared to a database of 488 experiments. While the best performing codified approach is that suggested by DAfSt B (2012) with a mean of 1.12 and a coefficient of variation of 27%. The accuracy of the mechanics-based approaches was not compared in Lansoght's (2019) review, however, it was highlighted that none of the approaches accounted for all the mechanisms that contribute to the shear capacity.

This paper will seek to address this limitation via extending the application of the mechanical model of Zhang et al. (2014a,b;2015;2016b) to FRC. This model has previously been applied to reinforced and prestressed concrete with steel or fibre reinforced polymer (FRP) reinforcement, and its accuracy has been demonstrated by comparison to more than 1100 experimental test results. In Zhang's approach, the shear capacity of a beam is based on the shear capacity of the critical diagonal crack, where the shear is primarily resisted by the flexural compression region. In this approach, the width of the shear crack is directly quantified through fundamental partial interaction theory. This is important because the direct application of partial interaction theory has made the approach able to predict the capacity of both steel and FRP reinforced concrete members without modification because the variation in bond and, therefore, tension stiffening between these two types of materials is explicitly considered. For application to FRC, this is also beneficial because it allows for the direct incorporation the fibre contribution through a stress-crack width relationship.

In the remainder of the paper, the extension of Zhang's approach to incorporate FRC is first explained qualitatively. Next, it is shown how the model can be implemented numerically and then analytically. The numerical and analytical models are then validated using a database of existing and new test results, and finally the accuracy of the approach is compared to other existing models. Importantly, having shown that an accurate analytical solution to predict shear capacity can be developed from fundamental mechanics, it is envisaged that further research can be conducted to further simplify the approach to produce more simplified design rules.

SHEAR FAILURE MODEL FOR FRC

Consider the simply supported beam subjected to a point load in Fig. 1(a). As the beam is loaded, discrete flexural-shear cracks form at the bottom face with a spacing of S_{cr} which is governed by tension stiffening and the tensile strength of the concrete (Balazs 1993; Lee et al. 2013; Sturm et al. 2018b). As the load is increased, these cracks propagate towards the neutral axis and are inclined as shown because they form perpendicular to the direction of the principal tensile stress. In reality these cracks are non-linear (Zarrinpour & Chao 2017). However, to simplify formulation and application of the approach, the non-linear shear crack has been approximated with a straight diagonal crack in Fig. 1(b). A similar assumption has been applied previously in a range of models to predict shear strengths; these include that by Zhang (1997), Hoang & Nielsen (1998) and Zhang et al. (2015,2016b) with which accurate predictions have been achieved. This assumption of a straight diagonal crack is also implicit in simplified modified compression field theory, as the crack forms perpendicular to the inclination angle of the stresses in the element (Bentz et al. 2006).

Sliding forces develop along the planes defined by each of these shear cracks in Fig. 1(a) to resist the applied shear force (Lucas et al. 2011; Zhang et al. 2015). When and where a sliding force exceeds the capacity of the compressed concrete above the shear crack to resist sliding, a crack penetrates the flexural compression region and the pre-sliding shear capacity is reached; this crack is referred to as the critical diagonal crack and sliding can now occur along the entirety of the shear crack. Once sliding commences, the shear force that can be resisted may or may not increase depending on the rate of increase in normal stress that develops along the sliding plane (σ_N) relative to the rate at which sliding (Δ) occurs. This can be seen in Fig. 2 where typical shear stress versus slip (τ_N/Δ) relationships are presented as a function of the applied normal stress (Chen et al. 2015). In Fig. 2(b), it can be seen that for a constant normal stress the shear resistance reduces as sliding occurs. However, the shear resistance can increase if the normal stress increases, for example if sliding causes the forces in the reinforcement to increase. In this paper, this post-sliding behaviour will be ignored and the shear capacity will be assumed to be equal to the pre-sliding shear capacity. This approach is taken because the pre-sliding capacity is either equal to the shear capacity or is a lower bound to it. Further, Zhang et al. (2015;2016b) showed in a broad validation, to over 1100 experimental test results on reinforced and prestressed concrete beams and columns with either steel or FRP reinforcement, that the pre-sliding capacity provided an accurate prediction of shear capacity. Further, as a result of ignoring post-sliding behaviour, dowel action can be ignored because as shown by Millard & Johnson (1984) in experiments specifically designed to investigate dowel action separately from aggregate interlock, some shear slip is required to generate significant forces due to dowel action.

Based on the assumption that the pre-sliding capacity is a reasonable approximation to the shear capacity, the shear capacity can be determined by quantifying the sliding force S along the shear crack in Fig. 1(b) as a function of the applied shear force V. Shear failure is then taken to occur when the capacity of the compressed concrete to resist the onset of sliding S_{cap} is reached.

Sliding force along critical diagonal shear crack

To determine the sliding force along the critical diagonal shear crack in Fig. 1(a), consider the free body in Fig. 1(b), where, as a simplification, the real crack geometry has been approximated with a straight line inclined at an angle β. The stress resultants acting on the free body include: the applied shear force V; bending moment M; the force in the longitudinal tension reinforcement F_{rt}; the force in the longitudinal compression reinforcement F_{rc}; the longitudinal force in the i^{th} stirrup $F_{st\text{-}i}$; the force in the fibres normal to the crack plane F_f; the compressive force in the concrete F_c; and the sliding force S.

From horizontal and vertical force equilibrium:

$$0 = F_{rt} + F_f \sin(\beta) - F_{rc} - F_c - S\cos(\beta) \quad (1)$$

$$V = F_f \cos(\beta) + \sum_i F_{st-i} + S\sin(\beta) \tag{2}$$

and from moment equilibrium:

$$M - V\frac{S_{cr}}{2} = Va' = F_{rt}d_{rt} + F_f d_f + \sum_i F_{st-i}\, d_{st-i} - F_{rc}d_{rc} - F_c d_c \tag{3}$$

where a' is the effective shear span, d_{rt} is the depth of the longitudinal tension reinforcement, d_f is the distance of the force in the fibres to the intersection of profile A-A with the top fibre, d_{st-i} is the horizontal distance between the i^{th} stirrup and the section A-A, d_{rc} is the depth of the compression reinforcement and d_c is the depth to the compressive force in the concrete.

The forces along the diagonal crack in Fig. 1(b) are a function of the deformations along the shear crack as the forces in the longitudinal tension reinforcement F_{rt}, in the transverse reinforcement F_{st-i} and in the fibres F_f are functions of the crack width. In contrast, the forces in the compressed concrete F_c and compression reinforcement F_{rc} are functions of the strain.

To determine these deformations, they are assumed to be the result of a linear rotation θ about a neutral axis depth d_{NA}. Consequently, the crack opening at a depth of y measured perpendicular to the crack is given by

$$w_p(y) = \frac{2\theta(y - d_{NA})}{\sin(\beta)} \tag{4}$$

which ignores the tensile strains in the concrete as the elastic deformation of the uncracked concrete away from the shear crack is negligible when compared to the crack opening. Resolving the crack width in Eq. 4, the horizontal component of the crack width is

$$w_x(x) = \frac{w_p(y)}{\sin(\beta)} = \frac{2\theta(y - d_{NA})}{\sin^2(\beta)} \tag{5}$$

and vertical components of is

$$w_y(y) = \frac{w_p(y)}{\cos(\beta)} = \frac{2\theta\left(x - \frac{d_{NA}}{\tan(\beta)}\right)}{\cos^2(\beta)} \tag{6}$$

where x is the horizontal distance measured from profile A-A in Fig. 1(b).

From Fig. 1 (b), the longitudinal strains in the compressed concrete at the location of the sliding plane, as shown in Fig. 1(d), is given by

$$\varepsilon_x(y) = \frac{\theta(d_{NA-y})}{\frac{S_{cr}}{2}\sin^2(\beta)+y\sin(\beta)\cos(\beta)} \tag{7}$$

and Eqs. (4-7) can be applied alongside the constitutive relations to solve Eqs. (1-3) for the sliding force *S* which can be compared with the sliding capacity of the compressed concrete S_{cap}. Hence it can be seen that the beneficial effects of fibres in the concrete can be allowed for directly by including the fibre concrete material properties for shear S_{cap} and for tension F_f. Significantly, the strain profile in Fig. 1(c) is seen to be non-linear. This is because in the segmental model, the strain in the compression region is taken as the deformation to cause the rotation θ divided by the length over which it is accommodated (the segment length) which varies along the height of the beam due to the inclined sliding plane (Zhang et al. 2014a). Further, in Fig. 1(c) the concrete strain has only been plotted in the compression region because below the neutral axis the concrete is cracked and the concrete strain is taken as zero at the crack face. While the strain in the concrete is taken as zero, the force in the reinforcement is not zero nor is the force in the fibres crossing the crack because these stresses are a function of the crack opening in Fig. 1(d). In the formulation of this approach, the forces in the reinforcement are taken to develop according to partial interaction theory, which describes the force in reinforcement crossing a crack as a function of the bond stresses developed along the segment length and the crack opening in Fig. 1(d).

Capacity to resist sliding S_{cap}

Shear friction theory has typically been applied to predict the stresses that can be transferred across a cracked sliding plane given the crack opening and the slip between the two surfaces (Walraven & Reinhardt 1981). However, it can equally be applied to determine the maximum shear stress that can be transferred for a given applied normal stress for an initially uncracked section (Mattock & Hawkins 1971, Haskett et al. 2011). Hence, shear friction theory can be applied to determine the magnitude of the sliding force that can be resisted along a potential sliding plane as a function of the magnitude of the compressive force normal to the sliding plane (Mohamed Ali et al. 2008; Lucas 2011). This is illustrated in Fig. 3(a), where the inclined shear plane is subjected to the sliding force S and the force in the compressive concrete F_c which is a function of the stresses in the concrete σ_c.

The magnitude of the normal stress σ_N can be found by considering the infinitesimal strip in Fig. 3(b) which has a cross-sectional area of dA in the vertical plane. The horizontal force applied to this strip is equal to $\sigma_c dA$, such that the component of this force normal to the sliding plane is $\sigma_c sin(\beta)dA$. Since the area of the sliding plane contained inside this infinitesimal strip is $dA/sin(\beta)$, dividing the normal component of the force by this area gives the normal stress σ_N on the sliding plane as equal to $\sigma_c sin^2(\beta)$.

Having determined the applied normal stress from the stress in the compressed concrete, the shear strength of the material $v(\sigma_N)$ can be determined. Integrating this shear strength gives the shear capacity of the initially uncracked plane as

$$Z_{cap} = \int^{A_c} \frac{v(\sigma_N)}{\sin(\beta)} dA \tag{8}$$

where A_c indicates that the integral is performed over the portions of the sliding plane which are in compression. When quantifying the capacity of the sliding plane, it is also important to consider that there is a component of σ_c parallel to the sliding plane which is equal to $\sigma_c cos(\beta)dA$ and acts to reduce the sliding capacity. Hence

$$S_{cap} = Z_{cap} - F_c \cos(\beta) \tag{9}$$

NUMERICAL IMPLEMENTATION

The above shear failure model can be applied numerically using the procedure in Fig. 4. In this approach, the shear angle β in Fig. 1(b) is varied, starting from the minimum value of β_{min} in Eq. 10, that corresponds to the critical diagonal shear crack that initiates at the support shown as A-B in Fig. 1(a).

$$\beta_{min} = \arctan\left(\frac{h}{a'}\right) \quad (10)$$

For each value of the shear angle β, the rotation θ is incrementally increased to give the relationship between the shear-force and rotation (S/θ), and hence from Eq. (1)

$$S = \frac{F_{rt} - F_{rc} - F_c}{\cos(\beta)} + F_f \tan(\beta) \quad (11)$$

For analysis, the rotation is incrementally increased until either shear failure occurs when $S = S_{cap}$, which then defines the shear capacity of that particular diagonal shear crack $V_{cap\text{-}\beta}$, or until flexural failure occurs. That is, the analysis is terminated when $V_{cap\text{-}\beta}$ exceeds the moment capacity M_{cap} of the beam.

For low values of β, failure occurs due to sliding, however, as the crack becomes more vertical, flexure will control failure and consequently the analysis is terminated as the flexural capacity is reached. Repeating the analysis for each crack inclination β yields the shear capacity V_{cap} which is given by the minimum value of V_{cap} obtained from all analyses in which β is varied ($V_{cap\text{-}\beta}$).

It may be worth noting that flexural cracks occur at discrete positions as in Fig. 1(a) such that the shear cracks occur at discrete positions and at discrete values of β. Hence this model which considers continuous values of β will give either the actual shear capacity or a lower bound to the shear capacity which explains some of the inherent scatter.

Applying the numerical solution in Fig. 4 requires the compressive stress-strain relationship for the concrete, the tensile-stress/crack-width for the fibres and the shear-strength/normal-stress relationship for the concrete all of which can be determined from simple experiments. It also requires the load-slip relationships for both the longitudinal tension reinforcement and the stirrups as well as the crack spacing which can be determined from established partial interaction theory (Visintin et al. 2013; Zhang et al. 2017b; Sturm et al. 2018b) and which rely on knowledge of the bond stress/slip relationship, which can also be determined from simple material tests.

This numerical implementation is also independent of the shape of the cross-section as the force in the concrete, the force in the fibres and the sliding capacity are integrated over the area of concrete in tension or compression. Hence, I-beams or T-beams can be accommodated without changing the underlying model.

Crack spacing and load-slip relationship of the reinforcement

In this section, the crack spacing and load-slip relationships of the reinforcement used in the validation are outlined. The primary assumption of partial interaction modelling is that after cracking, slip occurs between reinforcement and the surrounding concrete (Balazs 1993; Sturm et al. 2018b). The interface shear stress then becomes a function of this slip (Balazs 1993; Sturm et al. 2018b) which is given by the local bond stress/slip relationship. To analyse this situation a tension chord is extracted from the beam and by considering that the slip strain is equal to the difference in the reinforcement and concrete strains as well as equilibrium of the tension chord a governing equation can be developed relating the slip to the position along the tension chord, as (Balazs 1993; Sturm et al. 2018b)

$$\frac{d^2\delta}{dx^2} = \frac{\tau L_{per}}{\delta_1^{\alpha}}\left(\frac{1}{E_c A_{ct}} + \frac{1}{E_r A_{rt}}\right) \qquad (12)$$

where τ is the interface shear stress, L_{per} is the bonded perimeter of the reinforcement, A_{ct} is the area of concrete in the tension chord, A_{rt} is the area of tension reinforcement in the tension chord, E_c is the elastic modulus of the concrete, and E_r is the elastic modulus of the reinforcement. By imposing a local bond stress/slip relationship and boundary conditions, Eq. (12) can be solved for the variation of slip along the tension chord. From this variation of slip, the variation in interface shear stress along the tension chord can be determined. Hence by integrating the interface shear stresses, the stress in the concrete can be determined. The crack spacing is then determined by finding the location where the concrete stress is equal to the tensile strength. Previously this approach has been implemented numerically and a range of analytical solutions have been developed. Here the following approach of Sturm et al. (2018b) is applied because it has been developed for both conventional strength concrete with fibres and ultra-high performance fibre reinforced concrete

$$S_{cr} = \left[\frac{2^{\alpha}(1+\alpha)}{\lambda_2(1-\alpha)^{1+\alpha}}\right]^{\frac{1}{1+\alpha}} \left[\frac{f_{ct}-f_{pc}}{E_c}\left(\frac{E_c A_{ct}}{E_r A_{rt}}+1\right)\right]^{\frac{1-\alpha}{1+\alpha}} \tag{13}$$

in which

$$\lambda_2 = \frac{\tau_{max} L_{per}}{\delta_1^{\alpha}}\left(\frac{1}{E_c A_{ct}}+\frac{1}{E_r A_{rt}}\right) \tag{14}$$

and where, as shown in Fig. 5, τ_{max} is the maximum bond stress, α is the non-linearity of the bond stress-slip relationship, δ_1 is the slip when the maximum bond stress is achieved, f_{ct} is the tensile strength of the concrete and f_{pc} is the post-cracking strength. The validity of the expression was established in Sturm et al. (2018b) when it was used in conjunction with a load-slip relationship to predict the tension stiffening behaviour of 20 FRC specimens.

The load-slip relationship of the reinforcement can also be determined from the variation of slip along the tension chord yielding the load-slip relationship for the longitudinal tension reinforcement given by the bilinear load-slip relationship in Fig. 6(a) where the crack opening stiffness $\underline{K_{rt}}$ (Sturm et al. 2018b) is given by

$$K_{rt} = E_r A_{rt} \frac{\lambda_1}{\tanh\left(\frac{\lambda_1 S_{cr}}{2}\right)} \tag{15}$$

in which

$$\lambda_1 = \sqrt{k L_{per}\left(\frac{1}{E_r A_{rt}}+\frac{1}{E_c A_{ct}}\right)} \tag{16}$$

and where k is the effective linear bond stiffness taken as τ_{max}/δ_1.

The load-slip relationship of the stirrups is given by a bilinear relationship of the same form as that used for the longitudinal tension reinforcement such that

$$K_{st-i} = E_r A_{st-i} \frac{2\lambda_{1-st}}{\tanh(\lambda_{1-st} L_{st1})+\tanh(\lambda_{1-st} L_{st2})} \tag{17}$$

as derived in Appendix S1 in the supplementary material

$$\lambda_{1-st} = \sqrt{kL_{per-st}\left(\frac{1}{E_r A_{st-i}} + \frac{1}{E_c A_{ct-st}}\right)} \tag{18}$$

and where $A_{st\text{-}i}$ is the cross-sectional area of the i^{th} stirrup, L_{st1} is the embedded length above the shear crack, L_{st2} is the embedded below the crack, $L_{per\text{-}st}$ is the bonded perimeter of the stirrup and $A_{c\text{-}st}$ is the area of the tension chord surrounding the stirrup. These geometric properties are illustrated in Fig. 7.

ANALTYICAL IMPLEMENTATION

The shear failure model can also be implemented analytically, which for design may be more convenient to implement in a simple spreadsheet. As noted previously, the purpose of this paper is to develop a fundamental rational approach which captures the underlying mechanism, but it is envisaged that in future work further simplifications could be made. Here as initial approximations, the compression reinforcement will be neglected as too will be the action of the stirrups in the flexural compression region. These approximations are in line with those previously made by Placas & Regan (1971) and Tompos & Frosch (2002) respectively. The following analysis will be conducted assuming that the section is rectangular and the reinforcement is unyielded. However when this is not the case, some of the expressions in the following section can be replaced with the expressions in Appendix S2 in the supplementary material for when the section is either an I-beam or T-beam and with the expressions in Appendix S3 in the supplementary material when the reinforcement has yielded. Note that to determine whether the reinforcement is yielded or unyielded, it is recommended that the section is first analysed as unyielded and then this assumption is checked by determining the force in the reinforcement. Should this force exceed the yield force, then repeat the analysis assuming that that reinforcement has yielded. A worked example is provided in Appendix S4 in the supplementary material.

Idealised material and mechanical behaviours

Reinforcement

For the longitudinal tensile reinforcement, a bilinear load-slip relationship is assumed (Sturm et al. 2018b)

$$F_{rt} = K_{rt}\Delta_{rt} = \frac{K_{rt}\theta(d_{rt} - d_{NA})}{\sin^2(\beta)} \leq f_y A_{rt} \tag{19}$$

where: K_{rt} is the crack opening stiffness and an example of which is given in Appendix S1 in the supplementary material; Δ_{rt} is the slip of the reinforcement which is equal to $w_x(d_{rt})/2$; f_y is the yield stress; and A_{rt} is the cross-sectional area of the reinforcement.

For the transverse or vertical stirrups,

$$F_{st-i} = K_{st-i}\Delta_{st-i} = \frac{K_{st-i}\theta\left(d_{st-i} - \frac{d_{NA}}{\tan(\beta)}\right)}{\cos^2(\beta)} \leq f_{y-st} A_{st-i} \tag{20}$$

where: K_{st-i} is the crack opening stiffness, and an example of how to determine this is given in Appendix S1 in the supplementary material; Δ_{st-i} is the slip of the stirrup which is equal to $w_y(d_{st-i})/2$; f_{y-st} is the yield stress of the stirrup; and A_{st-i} is the cross-sectional area of the stirrup.

Fibres

As a simplification, the stress in the fibres is approximated by a constant stress f_f which is equal to the average tensile stress ranging from a crack width of 0 mm to the crack width at the bottom fibre w_D, as shown in Fig. 8. Since w_D is unknown before the analysis has been performed, it is proposed that f_f is imposed based on the expected crack width. A possible approach for estimating the expected crack width would be to determine this from a flexural analysis with same applied moment M. The crack width could then be estimated directly from a segmental analysis (Sturm et al. 2020) or alternatively from a sectional analysis by multiplying the bottom fibre strain by the crack spacing. This is permissible as the pre-sliding shear capacity is being predicted, hence, significant additional crack opening due to sliding has not yet occurred. This assumption can then be checked by determining the actual width of the shear crack and checking that the average fibre stress corresponding to this crack width is consistent with the value that was assumed. It is consistent if the difference is small and conservative. As a good rule of thumb, it is suggested that if the difference in stress is less than 10% and underestimated then the error introduced is small and conservative.

For a rectangular section, the force in the fibres is given by

$$F_f = \frac{f_f b(h-d_{NA})}{\sin(\beta)} \tag{21}$$

and the lever arm between the force in the fibres and the top fibre is given by

$$d_f = \frac{h+d_{NA}}{2\sin(\beta)} \tag{22}$$

For the case of a T-beam or I-beam Eqs. (21) and (22) are replaced by those in Appendix S2 in the supplementary material.

Concrete

Shear failure or sliding is assumed to occur before concrete crushing, hence, the concrete is approximated as linear elastic

$$\sigma_c = E_c \varepsilon_x \tag{23}$$

Above the neutral axis in Fig. 1(c), the strain profile in the concrete is non-linear, because even though the deformation varies linearly as shown, the longitudinal length of concrete over which it acts also varies. As a further simplification, this non-linear strain profile is approximated with the following linear strain profile

$$\varepsilon_x = \frac{\theta(d_{NA}-y)}{\frac{S_{cr}}{2}\sin^2(\beta)} \tag{24}$$

The reason for this simplification is that if the strain profile in Eq. (7) is used, then the integration of the stress to obtain the force in the concrete results in a functional form that prevents an analytical solution from being obtained for the neutral axis depth. Hence as a simplification, the non-linear strain profile is replaced by a linear strain profile where the strain at the neutral axis and at the top fibre are the same as for the actual non-linear strain distribution.

This simplification is shown to be acceptable because of the closeness of the numerical and analytical solutions in the validation.

Hence using the simplified stress-strain relationship, the following force in the concrete is obtained by integrating the stress in the concrete over the area of concrete in compression

$$F_c = \frac{1}{2} b d_{NA}^2 E_c \frac{\theta}{\frac{s_{cr}}{2}\sin^2(\beta)} \tag{25}$$

Using the simplified stress-strain relationship, the lever arm between the force in the compressed concrete and the top fibre is

$$d_c = \frac{d_{NA}}{3} \tag{26}$$

For the case of a T-beam or an I-beam, Eqs. (25) and (26) are replaced by the expressions in Appendix S2 in the supplementary material.

The shear strength of the concrete material along the potential sliding plane is assumed to be of the form (Regan & Yu 1973)

$$v = m\sigma_N + c \tag{27}$$

where *m* represents the frictional component of the shear capacity and *c* represents the cohesion.

Shear capacity

Substituting Eq. (27) into Eq. (8) and then substituting Eq. (8) into Eq. (9) gives the shear capacity of the sliding plane as

$$S_{cap} = \int^{A_c} \frac{m\sigma_c \sin^2(\beta) + c}{\sin(\beta)} dA - F_c \cos(\beta) = F_c[m\sin(\beta) - \cos(\beta)] + \frac{cA_c}{\sin(\beta)} \tag{28}$$

where A_c is the area of concrete in compression which is equal to bd_{NA} for a rectangular section. For the case of an I or T beam see Appendix S2 in the supplementary material.

If the sliding force S is equated with the sliding capacity S_{cap} in Eq. (28) and then substituted into Eqs. (1) and (2), the following is obtained which, as a reminder, ignores the contribution of the compression reinforcement.

$$0 = F_{rt} + F_f \sin(\beta) - F_c \sin(\beta)\,[m\cos(\beta) + \sin(\beta)] - \frac{cA_c}{\tan(\beta)} \quad (29)$$

$$V_{cap} = \sum_{i=1}^{N} F_{st-i} + F_f \cos(\beta) + F_c \sin(\beta)\,[m\sin(\beta) - \cos(\beta)] + cA_c \quad (30)$$

In Eq. (30), V has been replaced by the shear capacity V_{cap} as S=S_{cap} and where N refers to the number of stirrups crossing the shear crack below the neutral axis. As the neutral axis is not yet known at this stage of the analysis, as a simplification N can be approximated as the number of the stirrups crossing the shear crack at a depth between d_{rt} and $h/2$.

In order to determine the neutral axis depth, now consider the forces developed in the concrete in compression, the fibre reinforcement and the longitudinal tensile reinforcement as a function of the crack rotation θ. Substituting Eqs. (19), (21) and (25) into Eq. (29) and rearranging gives the following expression for the rotation

$$\theta = \frac{A_0 + A_1 d_{NA}}{B_0 + B_1 d_{NA} + B_2 d_{NA}^2} \quad (31)$$

where

$$A_0 = -f_f b h \quad (32a)$$

$$A_1 = \frac{cb}{\tan(\beta)} + f_f b \quad (32b)$$

$$B_0 = \frac{K_{rt} d_{rt}}{\sin^2(\beta)} \quad (32c)$$

$$B_1 = -\frac{K_{rt}}{\sin^2(\beta)} \quad (32d)$$

$$B_2 = -\frac{b}{2}\frac{E_c}{\frac{S_{cr}}{2}}\left[\frac{m}{\tan(\beta)} + 1\right] \quad (32e)$$

If the section is an I or T beam, Eqs. (32) are replaced by the expressions in Appendix S2 in the supplementary material. If the longitudinal tension reinforcement has yielded, the expressions in Eq. (32) are replaced by those in Appendix S3 in the supplementary material.

Now considering moment equilibrium, substituting Eq. (30) for V_{cap} into Eq. (3) gives

$$0 = F_{rt} d_{rt} + F_f\left[d_f - a'\cos(\beta)\right] + \sum_{i=1}^{N} F_{st-i}\,(d_{st-i} - a') - F_c\{d_c + a'\sin(\beta)\,[m\sin(\beta) - \cos(\beta)]\} - a' c A_c \quad (33)$$

Substituting Eqs. (19), (20), (21), (22), (25) and (26) into Eq. (33) and rearranging gives the following second equation for the rotation which can then be equated to the first to determine the neutral axis depth, d_{NA}

$$\theta = \frac{C_0 + C_1 d_{NA} + C_2 d_{NA}^2}{B_0 + B_1 d_{NA} + B_2 d_{NA}^2 + B_3 d_{NA}^3} \tag{34}$$

where

$$C_0 = -f_f b h \left[\frac{h}{2\sin^2(\beta)} - \frac{a'}{\tan(\beta)} \right] \tag{35a}$$

$$C_1 = a' c b - f_f b \frac{a'}{\tan(\beta)} \tag{35b}$$

$$C_2 = f_f \frac{b}{2\sin^2(\beta)} \tag{35c}$$

$$D_0 = \frac{K_{rt} d_{rt}^2}{\sin^2(\beta)} + \sum_{i=1}^{N} \frac{K_{st-i} d_{st-i} (d_{st-i} - a')}{\cos^2(\beta)} \tag{35d}$$

$$D_1 = -\frac{K_{rt} d_{rt}}{\sin^2(\beta)} - \sum_{i=1}^{N} \frac{K_{st-i} (d_{st-i} - a')}{\sin(\beta)\cos(\beta)} \tag{35e}$$

$$D_2 = -\frac{E_c}{\frac{S_{cr}}{2}} b \frac{a'}{2} \left[m - \frac{1}{\tan(\beta)} \right] \tag{35f}$$

$$D_3 = -\frac{E_c}{\frac{S_{cr}}{2}} \frac{b}{6\sin^2(\beta)} \tag{35g}$$

If the section is an I or T beam, Eqs. (35) are replaced by the expressions in Appendix S2 in the supplementary material. If the longitudinal tension reinforcement or stirrups has yielded, the expressions in Eq. (35) are replaced by those in Appendix S3 in the supplementary material. Equating Eqs. (24) and (27) and rearranging gives the following polynomial equation

$$0 = P_0 + P_1 d_{NA} + P_2 d_{NA}^2 + P_3 d_{NA}^3 + P_4 d_{NA}^4 \tag{36}$$

where

$$P_0 = A_0 D_0 - B_0 C_0 \tag{37a}$$

$$P_1 = A_0 D_1 + A_1 D_0 - B_0 C_1 - B_1 C_0 \tag{37b}$$

$$P_2 = A_0 D_2 + A_1 D_1 - B_0 C_2 - B_1 C_1 - B_2 C_0 \tag{37c}$$

$$P_3 = A_0 D_3 + A_1 D_2 - B_1 C_2 - B_2 C_1 \tag{37d}$$

$$P_4 = A_1 D_3 - B_2 C_2 \tag{37e}$$

and which can be solved for the neutral axis depth.

The neutral axis depth d_{NA} can now be determined noting that Eq. (37) has four solutions, two of which are complex, and of the real solutions only one will be positive which is the physical solution. This can then be substituted into Eq. (31) to give the rotation θ. The rotation and neutral axis depth can then be substituted into Eqs. (20), (21) and (25) to give the forces in the stirrups F_{st-i}, fibres F_f and compressed concrete F_c. These forces can then be substituted into Eq. (30) to give the shear capacity V_{cap}. The only unknown is the shear angle β.

Theoretically β can be found by minimising V_{cap} with respect to β, however, minimising this analytically does not lead to a simple closed-form solution. It is also impractical for an analytical solution to evaluate V_{cap} for a range of shear angles and then take the minimum value in the same way as is done for the numerical implementation. Instead, as a simplification, it will be assumed that the fibres do not significantly influence the shear angle β which is analogous to the assumption of Zhang et al. (2015) where stirrups were assumed to have no effect on the shear angle. The validity of this assumption is demonstrated by the accuracy of the validation. Hence, the shear capacity without stirrups or fibres from Zhang et al. (2016b) can be minimised to give the shear angle β. The shear capacity without stirrups or fibres is given by (Zhang et al. 2016b)

$$V_{cap-nf} = \frac{b d_{NA} c}{1-[m\sin(\beta)-\cos(\beta)]\left[\frac{a'\sin(\beta)-d_{rt}\cos(\beta)}{d_{rt}-d_c}\right]} \tag{38}$$

Minimising Eq. (38) with respect to β by differentiating and equating with zero yields

$$\frac{dV_{cap-nf}}{d\beta} = 0 =$$

$$b d_{NA} c \frac{[m\cos(\beta)+\sin(\beta)]\left[\frac{a'}{d_{rt}-d_c}\sin(\beta)-\frac{d_{rt}}{d_{rt}-d_c}\cos(\beta)\right]+[m\sin(\beta)-\cos(\beta)]\left[\frac{a'}{d_{rt}-d_c}\cos(\beta)+\frac{d_{rt}}{d_{rt}-d_c}\sin(\beta)\right]}{1-[m\sin(\beta)-\cos(\beta)]\left[\frac{a'}{d_{rt}-d_c}\sin(\beta)-\frac{d_{rt}}{d_{rt}-d_c}\cos(\beta)\right]^2} \tag{39}$$

Rearranging then gives the following expression for the shear angle

$$0 = \left(m\frac{a'}{d_{rt}} - 1\right) 2\sin(\beta)\cos(\beta) - \left(\frac{a'}{d_{rt}} + m\right)[\cos^2(\beta) - \sin^2(\beta)] \quad (40)$$

Next consider that

$$2\sin(\beta)\cos(\beta) = \frac{2\tan(\beta)}{1+\tan^2(\beta)} \quad (41)$$

and

$$\cos^2(\beta) - \sin^2(\beta) = \frac{1-\tan^2(\beta)}{1+\tan^2(\beta)} \quad (42)$$

Hence substituting Eqs. (41) and (42) into Eq. (40) gives the following quadratic equation in terms of *tan(β)*

$$0 = \left(m\frac{a'}{d_{rt}} - 1\right) 2\tan(\beta) - \left(\frac{a'}{d_{rt}} + m\right)[1 - \tan^2(\beta)] \quad (43)$$

and solving Eq. (43) gives

$$\beta = \arctan\left[\sqrt{1 + \left(\frac{m\frac{a'}{d_{rt}}-1}{m+\frac{a'}{d_{rt}}}\right)^2} - \frac{m\frac{a'}{d_{rt}}-1}{m+\frac{a'}{d_{rt}}}\right] \geq \beta_{min} \quad (44)$$

From Eq. (44), it is seen that the shear angle is a function of the ratio between the shear span and effective depth and the frictional component of the shear strength. The variation of the shear angle with these parameters is shown in Fig. 9. Note that the inequality comes from the fact that the shear angle cannot be less than β_{min} as defined earlier (Eq. 10) which is limited by the shear crack entering the support. From Fig. 9, it can be seen that as the shear span to depth ratio reduces β increases. An increase in the frictional component of the shear strength results in a decrease in shear angle.

The presented analytical solution has assumed a rectangular cross-section and unyielded reinforcement. However, the model can accommodate other cross-sections, for example, the expressions for I and T beams are given in Appendix S2 in the supplementary material while the expressions for yielded reinforcement are given in Appendix S3 in the supplementary material. To demonstrate the manner in which these different solutions fit together, a flow chart is given in Fig. 6 which outlines the procedure for determining the shear capacity using the analytical solutions. A worked example is also given in Appendix S4 in the supplementary material.

VALIDATION

The shear capacity models in this paper are compared with 29 experimental tests (Casanova & Rossi (1997), Noghabai (2000) and Amin & Foster (2016) from the literature, as well as an additional 2 tests performed by the authors with details in Appendix S5 in the supplementary material. The tests from the literature were chosen from the data base by Lansoght (2019) where direct tension tests were also available. The examples cover: concrete strengths from 34 to 125 MPa; fibre volumes from 0.29 to 1.28%; beam depths from 250 to 700 mm; and rectangular and I shaped sections.

Comparisons were also made to the codified approaches presented by fib Model Code 2010 (fib 2013), AS3600-2018 (Standards Australia 2018) and AFGC UHPFRC recommendations (AFGC 2013) as well with the approaches of Voo et al. (2006), Choi et al. (2007), Zhang et al. (2016a), Lee et al. (2017) and Foster and Barros (2018). The results are summarised in Fig. 11 which alongside the plot gives the means and coefficients of variation (COV). Note that the *n* in Fig. 11 refers to the number of tests the approach was applied to in the validation. The reason that Voo et al. (2006), Choi et al. (2007) and Zhang et al. (2016a) were compared to less than 31 tests is that they did not include a provision for the allowance of stirrups. In Fig.11(k) that is Foster & Barros, the number of tests for comparison was reduced as the model does not include the case where there is a mix of two different types of fibre. The fib Model Code #1 refers to the approach in the model code which is based on a modified Eurocode approach and fib Model Code #2 refers to an approach based on simplified modified compression field theory.

The results for the numerical approach developed in this paper are shown in Fig. 11(a); these specimens were with and without stirrups and had both normal and high strength FRC. It can be seen that the results are closely distributed about the ordinate 1 with a mean of 0.98 and COV of 0.19 demonstrating the accuracy of the proposed numerical implementation. When using the analytical formulation, the results in Fig. 1(b) have a similar mean to the numerical approach of 0.97, however, the COV has increased slightly to 0.24 due to the simplifications in this approach.

The codified predictions in Figs. 11(c) to (f) are conservative especially for the higher strength FRCs. The COVs are significantly higher than for the approaches developed in this paper of 0.19 and 0.24 with fib(2013)#1 the most accurate with a COV of 0.37 and AFGC (2013) the least with a COV of 0.44. The AFCG (2013) standard is also the least conservative with a mean of 1.33 while Standards Australia (2018) is the most conservative with a mean of 1.74.

Various approaches in the literature are also compared in Figs. 7(g-k). Zhang et al. (2016a), Lee et al. (2016a) and Foster & Barros (2018) approaches show similar patterns to the codified approaches of increasing conservativeness as the concrete strength increases. Voo et al. (2006) shows a different pattern where the approach is accurate for high strength FRC, however, it is unconservative for lower strength FRC. Choi et al. (2007) demonstrates similar accuracy for all concrete strengths. This is also reflected in the means, with Voo et al. (2006) being unconservative with a mean of 0.68 while Choi et al. (2007) is the closest to the experimental values with a mean of 1.01 and the other approaches are conservative with means between 1.67 and 1.84. Inspecting the COVs shows Voo et al. (2006) as being the most accurate with a COV of 0.23 while Foster & Barros (2017) is the least accurate with a COV of 0.59. For the other approaches, the COVs are in the range of the codified approaches. These are all greater than the COVs for the proposed approaches except for Voo et al. (2006) which has a similar COV to the analytical solution, however, Voo et al. (2006) tends to overestimates the shear capacity in most cases.

The following material properties were used in the numerical and analytical implementations for the approaches presented in this paper. The concrete stress-strain relationship in compression was obtained from Ou et al. (2011) for FRC with a strength less than 100 MPa or Sobuz et al. (2016) for FRC with a strength greater than 100 MPa. The tensile-stress/crack-width relationship was obtained from direct tension tests, although the equivalent material property obtained from inverse analysis of flexural tensile tests could also be employed. This was not, however, done here to avoid any increased scatter associated with obtaining the material properties. The material shear strength was obtained from Zhang et al. (2014b). The crack spacing, load-slip relationships and crack opening stiffness were determined in accordance with that presented in Appendix S1 in the supplementary material. Note that these approaches utilise an empirical bond-stress/slip relationship which was obtained from Harajli (2009) for compressive strengths less than 100 MPa and from Sturm & Visintin (2018) for compressive strengths exceeding 100 MPa.

EFFECT OF SIZE, FIBRE STRESS AND CRACK SPACING ON SHEAR CAPACITY

Effect of size on shear capacity

It is a well established phenomenon for both conventional reinforced (Bazant & Kim 1984; Bazant & Sun 1987) and fibre reinforced concrete beams (Shoaib et al. 2014; Minelli et al. 2014; Chao 2020) that the shear capacity does not scale linearly with the size of the beam. Hence, to demonstrate that the model in this paper generates a size effect, a series of analyses were performed using the analytical model. The results are shown in Fig. 12(a) where the shear capacity, that is normalised with respect to the size of the beam, is plotted against the effective depth. It can be seen that the normalised strength reduces with increasing depth, that is, there is a size effect and that this new model does not require an empirically derived factor to allow for the size effect but allows for it automatically through mechanics.

For the above analyses, the effective depth was varied from 100 mm to 1000 mm, the shear span-to-effective depth ratio ws 3, the beam width 250 mm, the cover of the longitudinal reinforcement 50 mm, the reinforcement ratio 0.01 and the concrete strength was 40 MPa. The fibre stress was assumed to be 50% of the tensile strength which was set to 3.5 MPa. The elastic modulus of the concrete was 36 GPa.

Effect of fibre stress on shear capacity

In this section the effect of adding fibres on the shear capacity is explored. Because the exact relationship between the volume of fibres and the stress in the fibres is strongly dependent on mix design, and is usually assessed experimentally, the effect of adding fibres will be simulated considering the simple case of the beam with an effective depth of 500 mm and all other parameters the same as those used to explore the size effect. By varying the fibre stress as a ratio of the fibre stress to the peak tensile strength the result shown in Fig. 12(b) is obtained, where a value of zero is indicative of a plain concrete beam. This demonstrates that the addition of fibres can result in significant improvements in shear capacity. To place these values in context an addition of 0.3% by volume of fibres resulted in a f_f/f_{ct} of 0.24 and 0.7% by volume of fibres resulted in a f_f/f_{ct} of 0.67 in Amin & Foster (2016) while 1% by volume of fibres resulted in a f_f/f_{ct} of 0.97 for the beams in Appendix S5 in the supplementary material as determined using the analytical solution presented in this paper. It can therefore be seen that the shear capacity increases in proportion to the stress in the fibres.

Sensitivity of the predicted shear capacity to the crack spacing

This model uses the crack spacing as a parameter in determining the shear capacity. As there is a significant random component to predicting crack spacings (Sturm et al. 2018c), the sensitivity analysis in Fig. 12(c) was performed to explore the effect of crack spacing on the predicted shear capacity. The results indicate that the model is insensitive to the assumed crack spacing with only minor variation in the shear capacity even when the crack spacing is varied from 25 mm to 200 mm. The reason for this insensitivity is that as the crack spacing is increased the rotation increases to maintain similar strains on the section. This can be seen by plotting the rotation versus crack spacing as well as top strain versus crack spacing, as shown in Fig. 12(d) and 12(e). In this analysis, the effective depth was taken as 500 mm and the other parameters were the same as those used to investigate the size effect.

CONCLUSION

An approach has been developed for quantifying the shear capacity of FRC beams. The approach is based on the mechanics of shear failure along a sliding plane and uses: the reinforcement partial-interaction bond-slip material property; the concrete partial-interaction shear-friction property; and the partial-interaction fibre properties across a crack or sliding plane. A unique component of this approach is that it quantifies the weakest plane of shear failure and, consequently, automatically allows for the effect of the shear-span/depth and beam size. Being mechanics based, it can cope with a wide variety of member shapes, such as rectangular or I sections, member sizes and FRC material properties and does not require calibration through member testing.

This novel partial-interaction mechanics based approach has been compared with thirty one member tests and shows very good correlation with the measured strengths and a low COV of 19%, which increases to 24% when simplifications are made to produce an analytical solution. The means of the proposed solutions are also 0.98 for the numerical and 0.97 for the analytical implementations. Thus, it has been found to be more accurate than code approaches where the COV was larger with a range of 37 to 44% while the means were conservative with a range of 1.33 to 1.74 and published prediction approaches where the COV ranged from 23% to

59%.Voo et al. (2006) was unconservative with a mean of 0.68 while the mean of Choi et al. (2007) was 1.01. The other published prediction approaches were conservative with means of 1.67 to 1.84.

As this new approach is mechanics-based, it only requires knowledge of the partial-interaction material properties of the FRC concrete for application and as such it does not require calibration by member testing. The procedure can be used to quantify the shear capacity of FRC RC sections and thus has the potential to be used to develop simplified rules for design for any type of FRC member.

DATA AVAILABILITY STATEMENT

All data, models, and code generated or used during the study appear in the submitted article.

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NOTATION

The following symbols are used in this paper:

A_0, A_1, B_0, B_1, B_2 = coefficients for Eq. (31);

A_c = area of concrete in compression;

A_{ct} = area of concrete in tension chord;

$A_{ct\text{-}st}$ = area of concrete in tension chord around the stirrup;

A_{rt} = area of the longitudinal tension reinforcement;

$A_{st\text{-}i}$ = area of i^{th} stirrup;

a = shear span;

a' = effective shear span;

b = width of section;

b_{f1} = width of top flange;

b_{f2} = width of bottom flange;

b_w = width of web;

C_0, C_1, C_2, D_0, D_1, D_2, D_3 = coefficients for Eq. (34);

c = cohesive component of shear capacity;

d_c = depth to compressive force in the concrete;

d_f = distance from the force in the fibres to the top fibre;

d_{NA} = depth to neutral axis;

d_{rc} = depth to the compression reinforcement;

d_{rt} = depth to the longitudinal tension reinforcement;

$d_{st\text{-}i}$ = horizontal distance between stirrup and profile A-A in Fig. 1(a);

E_c = elastic modulus of concrete;

E_r = elastic modulus of reinforcement;

F_c = compressive force in the concrete;

F_f = force in fibres bridging shear crack;

F_{rc} = force in the compression reinforcement;

F_{rt} = force in longitudinal tension reinforcement;

$F_{st\text{-}i}$ = force in the i^{th} stirrup;

f_{ct} = tensile strength;

f_f = average tensile stress in the fibres for a given crack opening displacement;

f_{pc} = post-cracking strength;

f_y = yield strength of longitudinal reinforcement;

f_{y-st} = yield strength of stirrups;

h = depth of section;

K_{rt} = stiffness of longitudinal tension reinforcement;

K_{st-i} = stiffness of the stirrups;

k = effective linear bond stiffness;

L_{per} = bonded perimeter;

L_{per-st} = bonded perimeter of the stirrup;

L_{st1}, L_{st2} = distance from crack face to intersection of stirrup and longitudinal reinforcement;

M = bending moment;

M_{cap} = moment capacity;

m = frictional component of material shear capacity;

N = number of stirrups crossing the shear crack below the neutral axis;

n = number of stirrups that have yielded crossing the shear crack below the neutral axis;

P_0, P_1, P_2, P_3, P_4 = coefficients for Eq. (30);

S = sliding force along shear crack;

S_{cap} = maximum sliding force;

S_{cr} = crack spacing;

s = stirrup spacing;

t_{f1} = thickness of top flange;

t_{f2} = thickness of bottom flange;

V = shear force;

V_{cap} = shear capacity;

$V_{cap-\beta}$ = shear capacity corresponding to shear angle β;

V_{cap-nf} = shear capacity without fibres;

V_{exp} = experimental shear capacity;

V_f = fibre volume;

v = material shear strength;

w_D = crack width at bottom fibre (measured perpendicular to the crack face);

w_p = crack opening perpendicular to the crack face;

w_x = horizontal crack opening;

w_y = vertical crack opening;

x= distance from profile A-A in Fig. 1(a);

y = depth with respect to the top fibre;

Z_{cap} = shear capacity of uncracked sliding plane;

α = non-linearity of bond-stress/slip relationship;

β = angle of critical diagonal shear crack to the horizontal;

β_{min} = minimum shear angle;

Δ_{rt} = slip of the longitudinal tension reinforcement;

$\Delta_{st\text{-}i}$ = average slip of ith stirrup;

Δ_{st1}, Δ_{st2} = slip of the stirrup from each crack face;

δ_1 = slip at maximum bond stress;

ε_x = longitudinal strain;

θ = rotation at critical diagonal shear crack;

λ_1 = bond parameter for load-slip relationship of the longitudinal reinforcement;

$\lambda_{1\text{-}st}$ = bond parameter for load-slip relationship of the stirrups;

λ_2 = bond parameter for crack spacing;

ρ = reinforcement ratio;

σ_c = stress in concrete;

σ_f = stress in fibres;

σ_N = normal stress;

τ_{max} = maximum bond stress;

τ_N = shear stress at sliding plane;

SUPPLEMENTARY MATERIAL

Appendixes S1, S2, S3, S4, S5 and S6 are available online in the ASCE Library (www.ascelibrary.org)

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