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Study on the sensitive factors of structural nonlinear damage based on the innovation series

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Abstract: Constructing a damage-sensitive factor (DSF) is one of the key steps in structural damage detection. In this paper, innovation series extracted from the auto-regressive conditional heteroscedasticity (ARCH) model are proposed to construct a DSF, which is defined as the standard deviation of innovation (SDI). A three-story shear building structure is used to demonstrate and verify the performance of the proposed method, and the results are compared with the standard deviation of the residuals (SDR) based on an auto-regressive (AR) model. In the proposed method, the AR model is established using the acceleration responses obtained from the reference and test states. The residual series are then extracted for fitting the SDR. Subsequently, the ARCH model is constructed based on the residual series from the AR model, and a new DSF of SDI is defined. This study focuses on analyzing the accuracy of fitting AR model and ARCH model to vibration response data via the normal probability distribution, and identifying the characteristics of the residual and innovation series. The mean squared error (MSE) is used as the loss function to calculate the loss on residual and innovation series from the AR model and ARCH model, respectively. The results demonstrate that the SDR can be used for nonlinear damage detection. However, the proposed SDI can provide more accurate nonlinear damage identification and is robust to varying environmental condition and small damages. Thus, the innovation series developed based on ARCH model are promising for expressing and constructing nonlinear DSFs.

Keywords: nonlinear damage identification; nonlinear damage-sensitive factor; innovation series;

auto-regressive conditional heteroscedasticity

1. INTRODUCTION

Civil engineering structures are continuously subjected to loading, fatigue, material aging and various external environmental conditions, which can cause damage accumulation. The accumulation of damage can lead to structural failure, which poses a serious threat to the social economy and public[1]. Structural damage identification of engineering structures during the service period can improve the structural performance in terms of reliability and safety for effectively prevention of accidents due to structural failure. In recent years, vibration based structural damage detection approaches, such as Bayesian statistical theory and time series analysis, have been attracted increasing attention by many researchers. For the damage identification method based on Bayesian statistical theory, the posterior probability distribution of the parameters to be identified for the structure is calculated by combining information of the test data with other prior information of the structure that has been determined. By comparing with the original theoretical finite element model (FEM), the distribution of structural damage probability is then obtained[2]. A lot of research results were acquired by applying Bayesian statistical theory to structural damage identification. For example, Papadimitriou and Papadioti[3] presented a method of component mode synthesis techniques based on reference FEM. Through this method, the model updating and damage identification of a highway bridges were studied theoretically and computationally. Au and Zhang[4] proposed a fundamental theory of the Bayesian two-stage problem, and the theory revealed a fundamental principle that ensures no double-counting of prior information in the two-stage identification process. Huang et al[5] presented a more rigorous formulation along with a corresponding efficient and scale-invariant hierarchical sparse Bayesian learning (SBL) algorithm. The algorithm was then applied to synthetic data and real vibration response obtained from a steel-frame structure so that the modeling error was considered in the study. The results showed that the algorithm has a good performance on structural damage identification. Zhang et al[6] proposed a Bayesian model updating method incorporating modal identification information in multiple setups, and the synthetic and experimental data were used to illustrate the proposed method. Ding et al[7] proposed a new heuristic algorithm, named improved Jaya (I-Jaya) algorithm for structural damage identification with the modified objective function based on sparse regularization and Bayesian inference, and numerical studies on a truss structure and experimental validations of a reinforced concrete bridge model were performed to verify the developed approach.

On the other hand, damage detection methods developed based on time series analysis have attracted significant research interests. The procedure of time series analysis method for structural damage identification are as follows: (1) collecting vibration response data from the in-service structures; (2) constructing suitable time series models; and (3) extracting structural characteristic information to determine the damage-sensitive factor (DSF) for structural damage identification analysis.

Linear time series models have been widely used in this research field, such as auto-regressive (AR), moving average (MA), auto-regressive exogenous (ARX), and auto-regressive moving average (ARMA) model. These models have been widely used in structural damage identification. In linear time series models, damage characteristic factors are typically determined based on residual errors or model coefficients that are obtained via modeling. Various studies on determining DSF by residual errors from linear time series models were conducted. Zhu and Yu[8] presented a comprehensive index of high-order statistical moments that combined skewness and kurtosis based on the residual error of the AR model. It was applied to fuzzy clustering analysis for structural damage identification. Hu et al[9] took the estimated residual series standard deviation of the ARMA model to diagnose the damage conditions. The experimental results showed that the method can detect damages with satisfactory precision, and it can be extended to identify the damage magnitude of nonlinear system. Liu et al[10] proposed an improved substructure-based damage detection approach to locate and quantify damages in a shear building structure. They used an ARMAX model residual-based technique to correct the damage indicator. The method was verified using simulation and measured data from a shaking table test of a five-story frame structure. The results showed that the improved method can provide much better performance in damage detection than the existing methods in the literature, especially in real applications.

Ostermann et al[11] proposed a new distance measure that relies on one-step prediction errors and sampling strategies in the detection of structural changes using ARMA processes. The method and its practicability were validated using measured data from a steel truss railway bridge and an aluminum shear frame construction. The results show that the distance measure increases with damage extent. Monavari et al[12] proposed a novel method to estimate the order of AR model. The method is based on the statistical hypotheses of two-sample *f*-test conducted on the AR model residuals. A FEM of a three-story reinforced concrete (RC) frame was used to validate the method. The results show that the method has a promising sensitivity in detecting small level of structural deterioration prior to damage, even in the presence of noise. There were different studies involved fitting the DSF by varying coefficients of linear models and then used the identified coefficients for damage detection. Bao et al[13] used AR model coefficients to form a damage feature vector, which was used as a damage indicator (DI), based on the Mahalanobis distance between ARMA models. The DI was employed for detecting and locating the damage. The proposed method was validated using the dynamic responses of a submarine pipeline model subjected to ambient wave forces. The results demonstrated that the proposed method is sensitive to damage but insensitive to noise, which is reliable and efficient in detecting and locating damages in subsea pipeline system. Roy et al[14] employed ARX model coefficients, Kolmogorov–Smirnov (KS) test statistical distance, and the model residual error as the DSF for damage detection. The efficiency of the proposed methods was evaluated by numerical studies using a shear building and a steel moment-resisting frame. The results showed that the proposed method can reasonably locate the damage in the structures. Wang et al[15] proposed a technique for structural damage detection based on the AR model coefficients, and validated using a cantilever Euler beam and Phase I of the IASC-ASCE benchmark structure. The numerical results showed that the control charts by utilizing the sensitivity-enhanced AR model coefficients are more sensitive to damage and able to detect small level of structural damage in the presence of measurement noise. Liao et al[16] used the function of AR model coefficients as the DSF and it was validated using experimental data from a shaking table test. The results showed that the change of mean value of DSF is statistically significant, and the angular velocity-based DSFs are sensitive to damage.

For most types of structural linear damage, fitting the damage characteristic factors with the residual errors and the model parameters after the establishment of the linear time series model is an effective method. The linear damage assumes that the structure is linearly elastic before and after the damage occurring. When changes in the geometrical conditions and material properties of the structure directly lead to changes in the modal properties, the response of the structure can be described by the linear motion equation. However, in practice, the responses of civil engineering structures are affected by the structural configuration, material, construction technology, external environment during service life and other factors, and the collected structural vibration responses often behaves nonlinear, such as the fatigue cracks that open and close upon dynamic loading[17]. If only the traditional linear time series model is used, nonlinear damage information of the structure is lost. Therefore, to more accurately reveal the state of structural damage, the introduction of nonlinear structural damage identification techniques has attracted significant attention in this field[18, 19] as it has practical engineering application value. Nonlinear time series models, which include the general expression for nonlinear auto-regressive exogenous (GNARX) model, the auto-regressive conditional heteroscedasticity (ARCH) model, and the generalized auto-regressive conditional heteroscedasticity (GARCH) model, have been widely applied in damage detection.

Kai et al[20] proposed a structural identification method using GNARX model. The feasibility and efficient were verified by numerical simulation data. The method was applied to damage detection of a steel plate, and the results showed that the effect of damage detection using GNARX model is better than those using AR, ARX and GNAR models. This indicated the superiority of GNARX model in damage detection. Chen and Yu[21] fitted the linear index of standard deviation residuals using the ARMA model and the nonlinear index of standard deviation residuals using the ARMA/GARCH model. These two indicators were used to identify nonlinear damage of a three-story shear building structure at Los Alamos laboratory. The results demonstrated the superior performance of the nonlinear index in nonlinear damage identification. Cheng et al[17] proposed a second-order variance damage characteristic index (SOVI) as the damage index using the AR/ARCH model. It was used to identify the nonlinear damage source of the three-story shear building at the Los Alamos laboratory. Xin et al[22] proposed a method for bridge deformation prediction. The Kalman-ARIMA model was obtained by first combining Kalman filter with ARIMA model, and then it was combined with the GARCH model to form the Kalman-ARIMA-GARCH model. Global Navigation Satellite System (GNSS) deformation monitoring system has been used to measured data for damage identification. It showed that the Kalman-ARIMA-GARCH model is better than Kalman-ARIMA model in damage detection. The method can lay a foundation for the early warning of bridge health monitoring system based on sensor data. The application of the ARCH and GARCH models for structural damage identification has attracted attention in the field of civil engineering. However, studies on damage identification based on nonlinear time series models were limited. In particular, significant attention is required for the extraction of effective nonlinear damage features from the established nonlinear time series model.

In this paper, an AR model is constructed from the measured vibration responses of a three-story shear building from Los Alamos laboratory. Then, the standard deviation of the residuals (SDR), which is obtained by fitting the AR models with residual errors, is defined as DSF. The ARCH models are established based on the residual errors and the parameter estimated from AR models and the standard deviation of innovation (SDI) obtained by fitting ARCH models with the innovation series, is defined as a new DSF for nonlinear damage identification. The damage detection performance of the proposed method is then compared using the AR and ARCH models. The results demonstrate that the proposed SDI has substantial advantages in nonlinear damage indentification. To study the nonlinear characteristics of the damage index SDI, the methods of normal probability distribution and fitting analysis are applied to analyze the residual series from the AR model and the innovation series from the ARCH model. Furthermore, a loss function is defined to determine whether the nonlinear damage characteristic information of the structure exists in the innovation series. It is believed that DSF obtained by fitting the innovation series is an effective approach for nonlinear damage detection.

The paper is organized as follow. Section 2 describes the AR/ARCH models and construction of the damage index. Then, the identification process using AR/ARCH model is presented in Section 3. Section 4 shows the results of residual errors in the AR model and innovations in the ARCH model. Finally, Section 5 provides conclusions and prospects of the study.

2. TIMES SERIES MODELS AND CONSTRUCTION OF DAMAGE INDEX

2.1. Time series model

Time series analysis originated from the AR model was proposed by Yule[23]. This model was combined with the MA model and was proposed by Walker[24] to form the ARMA model. The models that form the basis of time series analysis, such as AR, MA, and ARMA models, have been widely used in the literature. These three models are univariate, homoscedastic and linear. They have been applied to stationary sequence. As the theory of time series analysis developed, researchers found that these assumptions were invalid under certain conditions. For example, irregular features were discovered during the modeling of the Canadian bobcat by Moran[25], e.g., the residuals of sample points with value larger than the mean value are significantly smaller than that with value smaller than the mean value. Therefore, heteroscedastic, multivariate and nonlinear time series have attracted increasing attention.

For studying the heteroscedasticity, Engle[26] proposed the ARCH model. It is assumed that, for the ARCH model, the mean value of the noise is equal to zero at the same moment, the variance follows a normal distribution of the time-varying quantity, i.e. conditional heteroscedasticity, and the time-varying variance is a linear combination of the squared values of the past finite sequence, i.e. autoregressive. The ARCH model has developed substantially in recent decades and widely used to prove the regularity description in financial theory, forecast and make decisions in financial markets. The ARCH model has been recognized as the an important development of financial econometrics in recent years[27]. With the in-depth study of the theory and application of time series analysis, the time series analysis methods have been continuously improved, especially for parameter estimation, model recognition and order determination. These methods have become increasingly intelligent, thereby, driving the grow of their applications[28]. The ARCH model has been applied in nuclear engineering, environmental engineering, medical engineering, marine engineering, metallurgical engineering, mechanical engineering, civil engineering and other fields of engineering technology[29, 30].

2.1.1. AR model

The AR model has been widely used in different applications, including analysis of structural vibration data, due to its simple and accurate representation of the characteristics of the data. The general p order of AR(p) model is defined as:

$$y_t = \sum_{i=1}^{p} \varphi_i y_{t-i} + \varepsilon_t \tag{1}$$

where y_t represents the state at time t; y_{t-i} are observations in the past; p is the order of the AR model; φ_i is the coefficient of autoregression; ε_t is the residual error, which obeys the distribution of white noise $\varepsilon_t \sim N(0,\sigma^2)$ with zero expected value and constant variance σ^2.

2.1.2 ARCH model

A time series with the available information varying with time has different conditional variances. The general q order of ARCH(q) model can describe the conditional variance changing with time, and can be expressed as:

$$a_t = \sigma_t z_t \tag{2}$$

$$\sigma_t^2 = k + \sum_{j=1}^{q} A_j a_{t-j}^2 \tag{3}$$

where σ_t is the conditional standard deviation; a_t is the innovation series; z_t is the number of random draws from the normal distribution with $z_t \sim N(0,1)$; σ_t^2 is the conditional variance; q the order of the AR model; k and A_j are the model parameters.

In this study, vibration responses of a three-story shear building with nonlinear damage conditions were used to evaluate the performance of the ARCH model. If only the traditional linear time series models are used to fit the measured vibration responses, the nonlinear information may lose during the construction of the model and information extraction. As a result, misjudgment is prone to occur in the process of damage identification. Therefore, the ARCH model has been developed for nonlinear damage identification of the structures.

2.2. Construction of DSF

2.2.1. SDR based on the AR model

The AR models are established based on the experimentally measured vibration responses from structures at the reference (healthy) state and at the test state, by which the residual error ε_t can be obtained. The SDR of the AR model is defined as the ratio of the standard deviation at the test state to that at the reference state, which is expressed as:

$$\text{SDR}=\frac{std(\varepsilon_t^{Test})}{std(\varepsilon_t^{Ref})} \tag{4}$$

where ε_t^{Ref} and ε_t^{Test} are the residual errors at the reference (healthy) state and test state, respectively. If SDR=1, it means the structure is at the healthy state, whereas if SDR>1, the structure is considered damaged, and the larger value of SDR means the damage is more substantial.

2.2.2. SDI based on the ARCH model

In the area of finance, the value of a_t predicted by the ARCH model is named as “disturbance” or "innovation" of the return on the asset at time t. In this paper, a_t sequence, which is obtained by the fitted ARCH model, is named as “innovation”. The independence of a_t can be represented by its delayed simple quadratic function due to the irrelevance and independent of sequence a_t. Through the ARCH model, the square of the past innovation can be determined. If the value of $\{a_{t-j}^2\}_{j=1}^q$ is too large, the conditional variance σ_t^2 of a_t is also large, thereby, resulting in a tendency toward larger value. This situation is called “volatility clustering”, in the observed innovation data. The ARCH models are established by using the residual errors of the AR models after the parameter estimation for obtaining innovation series a_t. The SDI of the ARCH model is defined as the ratio of the standard deviation at the test state to that at the reference state, which is expressed as follows:

$$\text{SDI}=\frac{std(a_t^{Test})}{std(a_t^{Ref})} \tag{5}$$

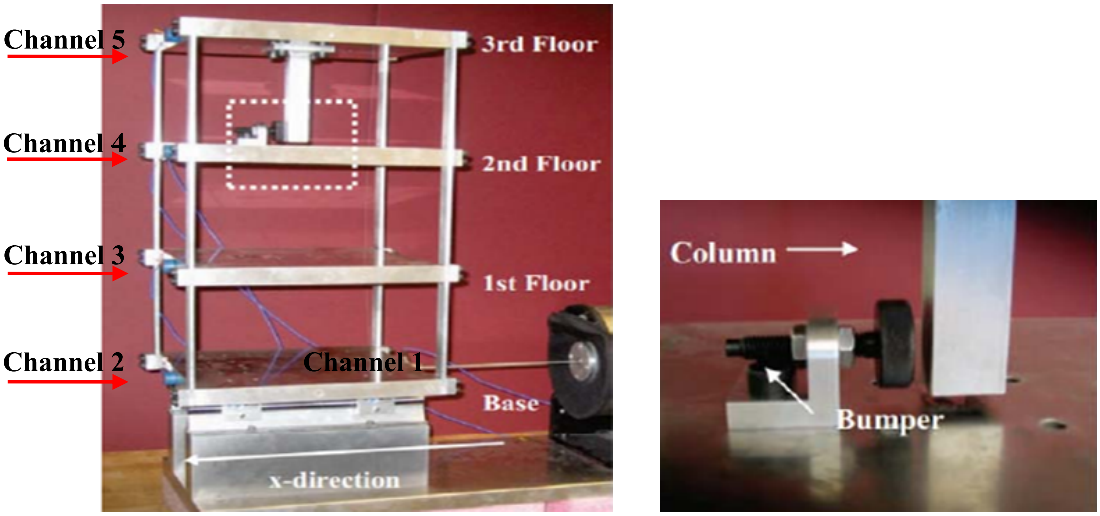
where a_t^{Ref} and a_t^{Test} are the innovation series at the reference state and test state, respectively. Similar to SDR in Section 2.2.1, the structure at healthy state is defined as reference state. If SDI=1, the structure is at the healthy state. If SDI>1, the structure is considered damaged and the larger value of SDI means the larger severity of damage.

3. AR/ARCH MODEL IDENTIFICATION PROCESS

In this paper, the experimentally measured vibration responses of a three-story shear building[31] were used to evaluate the performance of the proposed method. The three-story shear building is shown in Figure 1(a). The structure comprises aluminum columns and plates that are connected by bolts, and the structure is mounted on a one-way track to ensure that it slides along the *x*-axis only. Figure 1(a) shows a cantilever column and a bumper is fixed at the bottom middle of the third floor slab and on the top of the second floor slabs, respectively. During the excitation, the cantilever beam touches the bumper and this simulates the nonlinear damage behavior of the structure (Figure 1(b)). The distance between the bumper and the cantilever column can be adjusted to vary the level of nonlinearity. The size of the structure is shown in Figure 2 and the material of the structure is aluminum. The dimensions of the column are $17.7\times2.5\times0.6\text{cm}^3$, aluminum plate is $30.5\times30.5\times2.5\text{cm}^3$, and suspension column is $15.0\times2.5\times2.5\text{cm}^3$.

A force sensor (Sensitivity: 2.2mV / N) for measuring the excitation source is connected between the exciter rod and the structure, and 4 acceleration sensors (Sensitivity: 1000mV / g) are installed on the center line of each floor in the other side of the excitation source. The data acquisition system measures the signals of 5 channels, by which Channel 1 acquires the excitation source, and Channel 2-5 acquire the acceleration responses at the base level, first floor, second floor and third floor. The bandwidth of the excitation signal is set as 20Hz – 150Hz. The magnitude of the excitation is 2.6VRMS. The sampling frequency is 320Hz, the sampling time is 25.6s, and the number of samples is 8192.

The experimental conditions can be divided into four main groups, State#1, State#2–State#9, State#10–State#14, and State#15–State#17. State#1 is the reference state. State#2–State#9 are used to simulate the influence of the operating conditions and environmental changes in practical situation, in which a mass is added or stiffnesses of columns are changed. State#10–State#14 consider different severities of the nonlinear damage by changing the gap between the bumper and the cantilever column. State#15–State#17 considers different sizes of the gaps between the bumper and the cantilever column to simulate nonlinear damage. The effect of operating environment is simulated by changing the mass. These cases are summarized in Table 1.



(a) Three-story shear building structure and shaker (b)Adjustable bumper and suspended column

Figure 1. Three-story shear building structure[31]

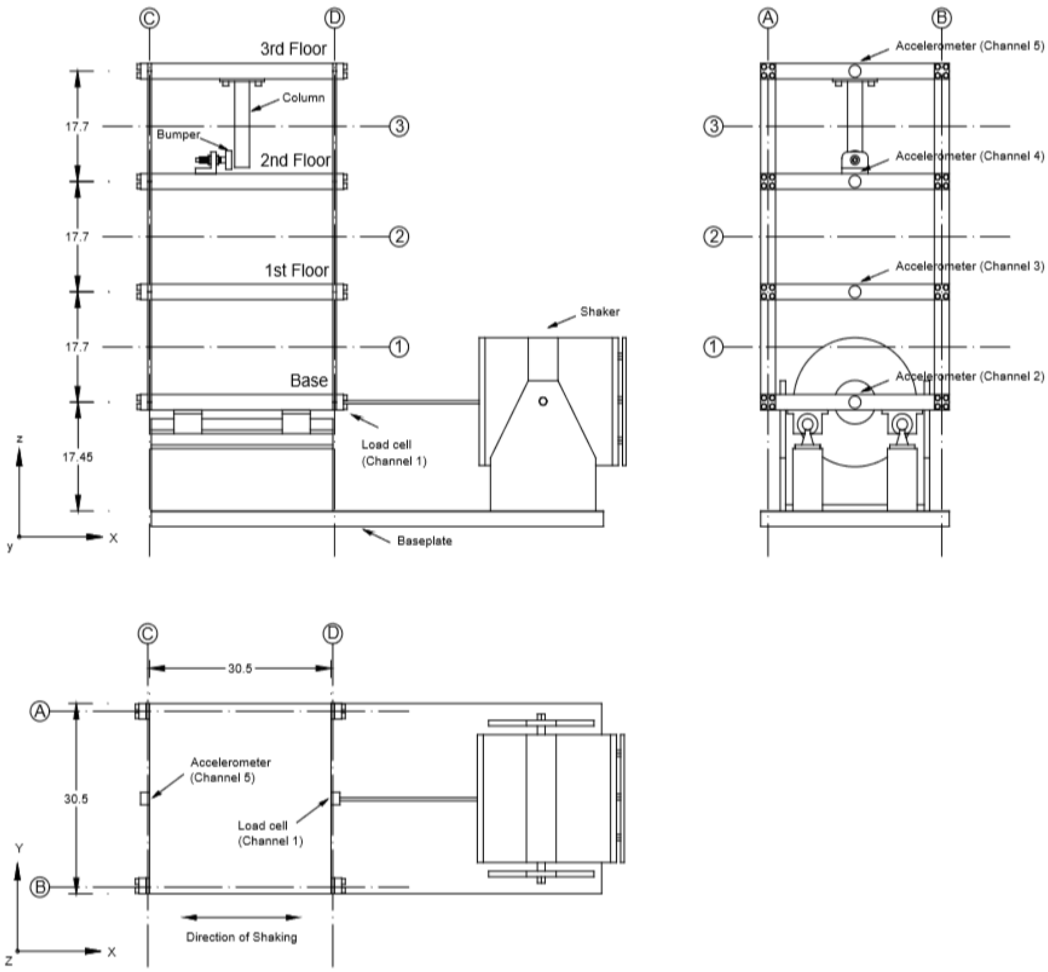


Figure 2. Basic dimensions of the three-story shear building structure[31]

Table 1. State condition settings for the structural experiment

Group	State#	Cases	Condition	Configuration				
				Perturbation			Damage	
				Content	Magnitude	Location	Content	Magnitude
Reference	1	1-9	Health	/	/	/	/	/
Operating conditions	2	10-19	Health	Mass	1.2 kg	Base	/	/
	3	20-29	Health	Mass	1.2 kg	1st floor	/	/
	4	30-39	Health	Stiffness	-87.50%	Column 1BD	/	/
	5	40-49	Health	Stiffness	-87.50%	Column 1AD and 1BD	/	/
	6	50-59	Health	Stiffness	-87.50%	Column 2BD	/	/
	7	60-69	Health	Stiffness	-87.50%	Column 2AD and 2BD	/	/
	8	70-79	Health	Stiffness	-87.50%	Column 3BD	/	/
	9	80-89	Health	Stiffness	-87.50%	Column 3AD and 3BD	/	/
Damage conditions	10	90-99	Damage	/	/	/	Gap	0.20 mm
	11	100-109	Damage	/	/	/	Gap	0.15 mm
	12	110-119	Damage	/	/	/	Gap	0.13 mm
	13	120-129	Damage	/	/	/	Gap	0.10 mm
	14	130-139	Damage	/	/	/	Gap	0.05 mm
Operating+ Damage	15	140-149	Damage	mass	1.2 kg	Base	Gap	0.20 mm
	16	150-159	Damage	mass	1.2 kg	1st floor	Gap	0.20 mm
	17	160-169	Damage	mass	1.2 kg	1st floor	Gap	0.10 mm

3.1. The Procedure of Constructing the AR/ARCH Models

This section will introduce the procedure and results of the parameter estimation for constructing the linear AR model and nonlinear ARCH model. The explanation of the procedures for nonlinear damage identification based on AR model and ARCH is shown in Figure 3.

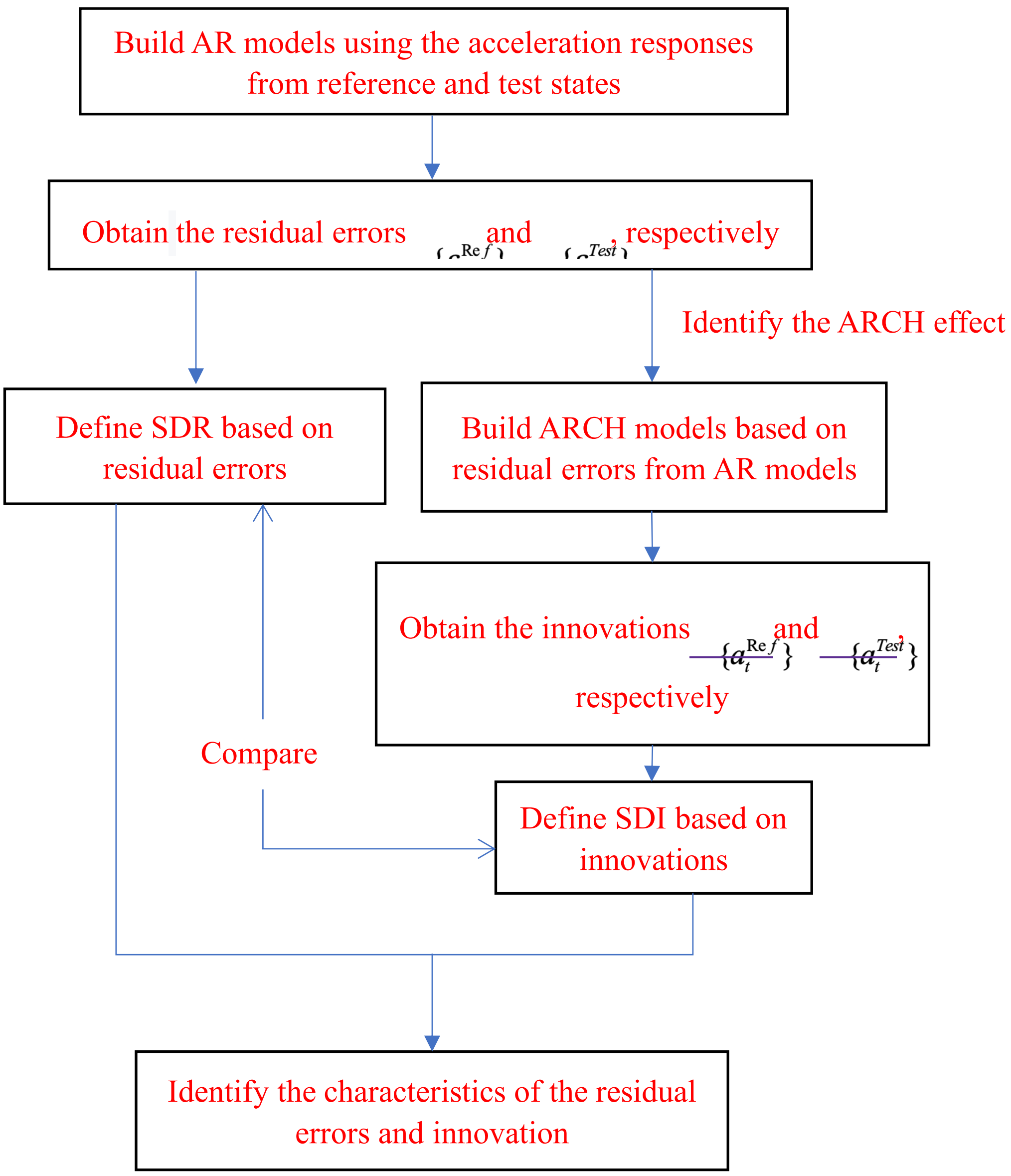


Figure 3. Damage identification flow chart of nonlinear ARCH model

3.1.1. Constructing the AR model

(1) Recognition and ordering of the linear AR model

First, stationarity tests were performed on the vibration responses collected at the reference state (health) and test state. According to the identification principle of the Box-Jenkins model[32], the autocorrelation function (ACF) and the partial autocorrelation function (PACF) were used to identify the characteristics of the vibration response data at the reference state, which were used to select a suitable linear time series model and determine the order of the model[33].

The experimentally measured vibration responses were divided into the reference state (State#1) and test states (State#2 – State#17) and each condition was tested 10 times, in which each time has 8192 data samples. The stationarity tests were carried out at the reference state and test states. Figures 4 and 5 show time history diagrams of the vibration responses at the reference state (State #1) and test state (State #12), respectively. According to Figures 4 and 5, the vibration responses of State#1 and State#12 satisfy the requirement of stationarity. ACF and PACF were calculated based on the vibration responses at the reference state (State #1), then the test states (State#2 – State#17) can be modeled based on the order selected by the reference state. The plots of the ACF and PACF are shown in Figure 6.

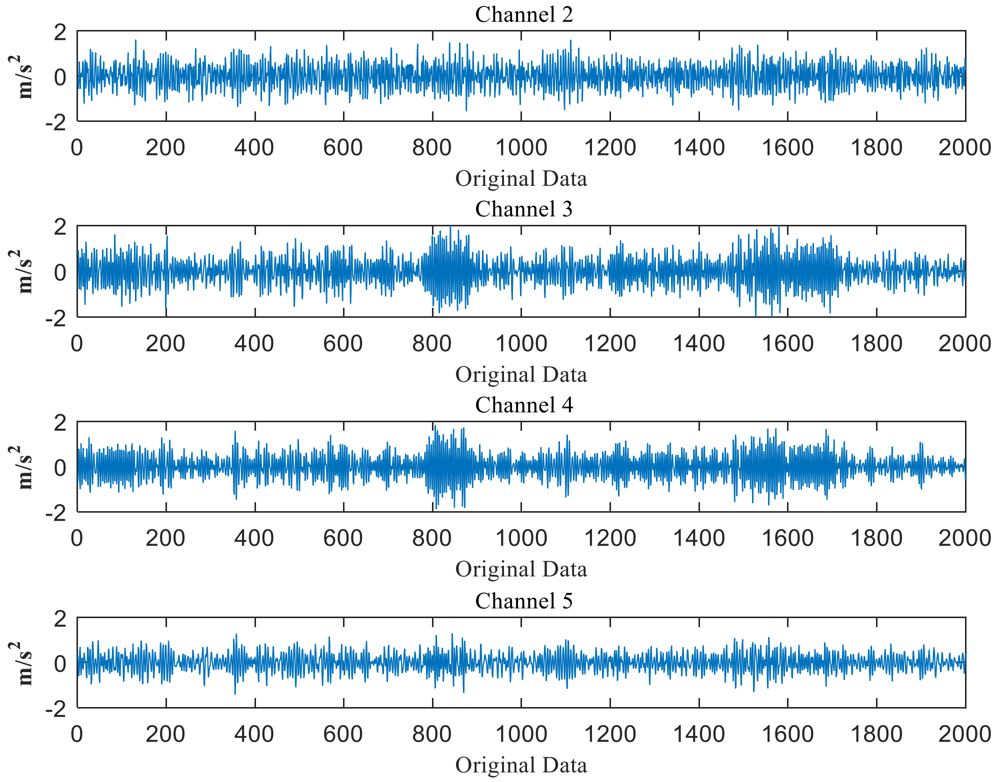


Figure 4. Acceleration response time series of State#1 in Channels 2 – 5

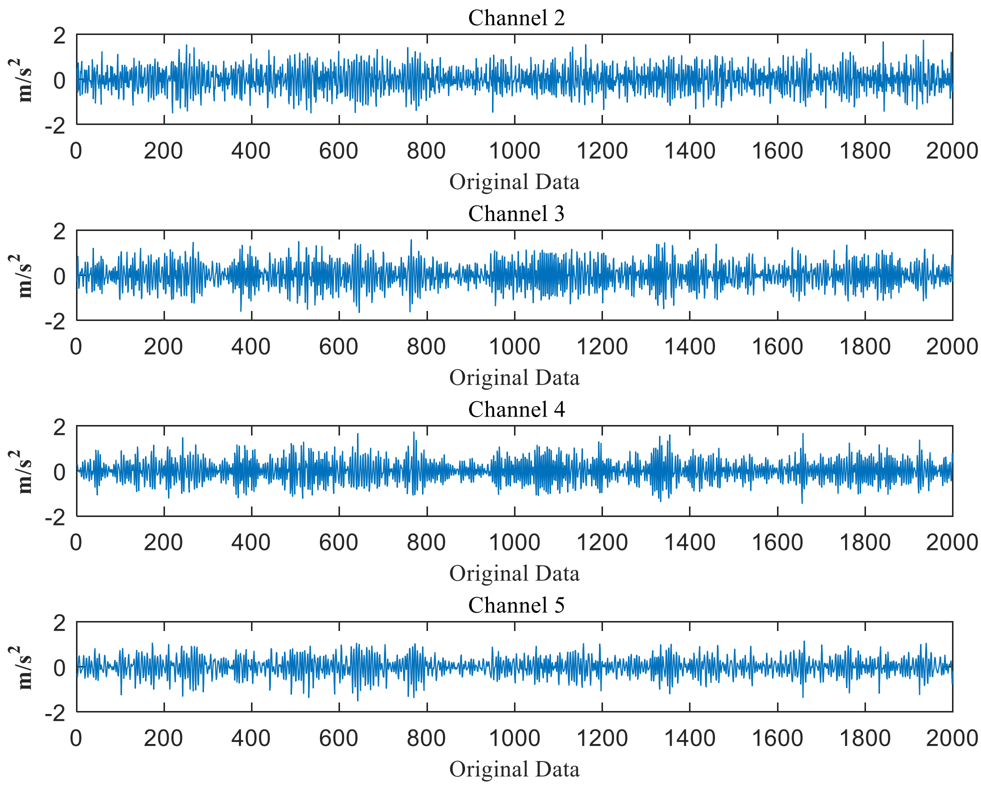


Figure 5. Acceleration response time series of State#12 in Channels 2 – 5

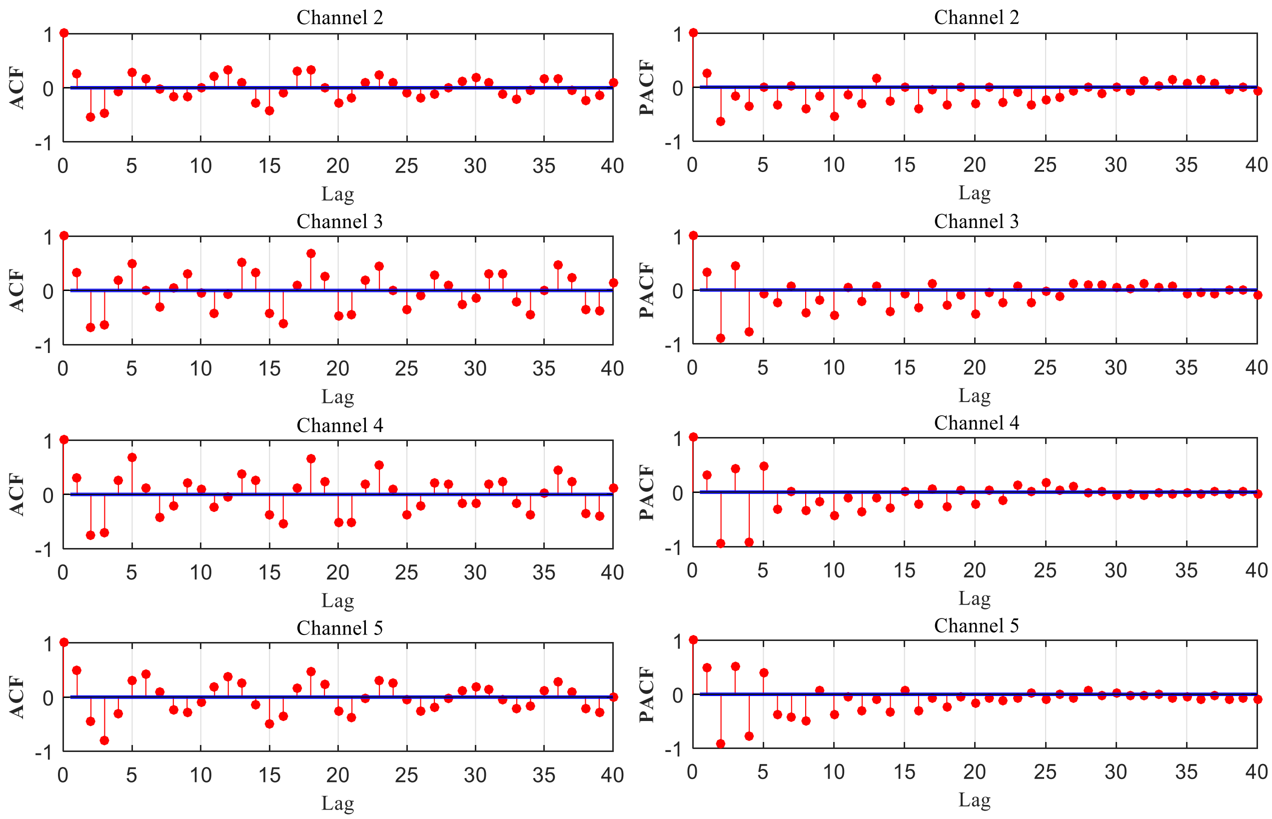


Figure 6. Graphs of ACF and PACF for State#1 in Channels 2 – 5

According to Figure 6, the ACF values of Channels 2–5 exhibit a tailing phenomenon and the PACF values of Channels 2–5 exhibit a tail truncation phenomenon after lag 25. Table 2 lists the statistical characteristics of ACF and PACF for the linear AR, MA and ARMA models. Considering Figure 6 and Table 2, the AR model is selected to model the experimentally measured vibration responses and the modeling order p is 25.

Table 2. Statistical characteristics of ACF and PACF for the AR, MA and ARMA models[33]

Model	AR(p)	MA(q)	ARMA(p, q) p>0,q>0
ACF	Trailing	q-order truncation	Trailing
PACF	p-order truncation	Trailing	Trailing

(2) Parameter estimation and validation of the linear AR model

After establishing the AR model, parameter estimation of the reference state was conducted, and the suitability of each order was evaluated based on the values of the identified parameters. ACF testing was then conducted to evaluate the residual error at the reference state that was obtained after the parameter estimation. The distribution of the ACF values was used to evaluate the accuracy of the fitted AR model. Finally, the model fitting rates were calculated and compared between the reference state and test state. Using the reference state as the standard, the difference in the fitting rates between the reference state and test state was used to quantify the damage.

(a) Model parameter estimation

Autoregressive parameters of the AR model were estimated via the least-square method using the Yule-Walker equation. The AR model lag operator is expressed as $A(q)y(t)=e(t)$. The polynomial expressions of the AR(25) model for State#1 in Channels 2–5 are:

Channel 2:

$$\begin{aligned}A(q) =&1+0.09406q^{-1}+1.94q^{-2}+0.5024q^{-3}+3.01q^{-4}+0.8741q^{-5}+4.165q^{-6}+1.389q^{-7}+5.131q^{-8}+1.866q^{-9}\\&+5.611q^{-10}+2.096q^{-11}+5.461q^{-12}+2.147q^{-13}+4.964q^{-14}+2.146q^{-15}+4.086q^{-16}+1.832q^{-17}\\&+2.911q^{-18}+1.366q^{-19}+1.821q^{-20}+0.9271q^{-21}+0.955q^{-22}+0.5296q^{-23}+0.3427q^{-24}+0.2272q^{-25}\end{aligned} \tag{6}$$

Channel 3:

$$\begin{aligned}A(q) =&1-1.205q^{-1}+2.979q^{-2}-2.411q^{-3}+4.598q^{-4}-3.299q^{-5}+6.194q^{-6}-4.286q^{-7}+7.442q^{-8}-4.874q^{-9}\\&+8.048q^{-10}-5.014q^{-11}+7.796q^{-12}-4.657q^{-13}+6.881q^{-14}-3.845q^{-15}+5.418q^{-16}-2.832q^{-17}\\&+3.728q^{-18}-1.787q^{-19}+2.201q^{-20}-0.9281q^{-21}+0.9856q^{-22}-0.3136q^{-23}+0.2223q^{-24}+0.0131q^{-25}\end{aligned} \tag{7}$$

Channel 4:

$$\begin{aligned}A(q) =&1-1.797q^{-1}+3.858q^{-2}-4.099q^{-3}+5.439q^{-4}-4.708q^{-5}+6.014q^{-6}-5.37q^{-7}+6.834q^{-8}-5.907q^{-9}\\&+7.022q^{-10}-5.935q^{-11}+6.601q^{-12}-5.677q^{-13}+5.834q^{-14}-5.051q^{-15}+4.725q^{-16}-3.997q^{-17}\\&+3.391q^{-18}-2.798q^{-19}+2.127q^{-20}-1.723q^{-21}+1.073q^{-22}-0.7954q^{-23}+0.3072q^{-24}-0.1736q^{-25}\end{aligned} \tag{8}$$

Channel 5:

$$\begin{aligned}A(q) =&1-1.815q^{-1}+3.344q^{-2}-2.89q^{-3}+3.504q^{-4}-2.448q^{-5}+3.997q^{-6}-3.156q^{-7}+4.537q^{-8}-2.716q^{-9}\\&+3.723q^{-10}-2.028q^{-11}+3.144q^{-12}-1.557q^{-13}+2.218q^{-14}-0.6339q^{-15}+1.024q^{-16}+0.07332q^{-17}\\&+0.3095q^{-18}+0.3274q^{-19}-0.0833q^{-20}+0.4791q^{-21}-0.3567q^{-22}+0.4297q^{-23}-0.1967q^{-24}+0.09511q^{-25}\end{aligned} \tag{9}$$

Equations (6)–(9) express the parameter estimation results based on the AR(25) model in State#1, the parameters before $\{q^{-n}\}$ are not equal to or close to zero, and hence, the selected order is not too high.

(b) Model validation

The applicability of a time series model refers to the degree to which the model describes the dynamics of the system. A suitable time series model can present the dynamics of the system and the residual errors of the model obey the white noise distribution. In this paper, the AR(25) models were established and the residual errors were determined from parameter estimation. The ACFs of the residual errors in State#1 were calculated, which are plotted in Figure 7. The ACF values from Channels 2–5 tend to zero within order 40 (Lag=40) and fall within the 95% confidence interval. Hence, the residual errors obey the white noise distribution. The AR(25) model is suitable for fitting the structural vibration responses and can extract relevant information for describing the structural system.

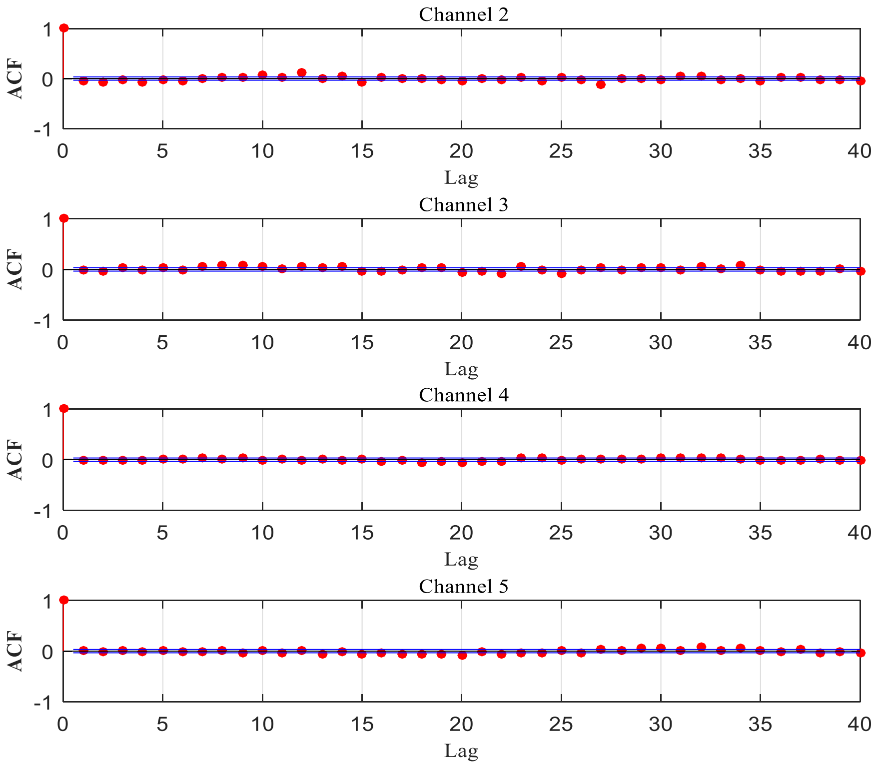


Figure 7. ACFs of the residuals of State#1 in Channels 2 – 5

To further evaluate the fitting degree of the AR(25) model to the structural vibration responses, the model fitting rates of the reference state and test state were calculated. The prediction of the AR(25) model for State#1 and State#12 are shown in Figures 8 and 9, respectively.

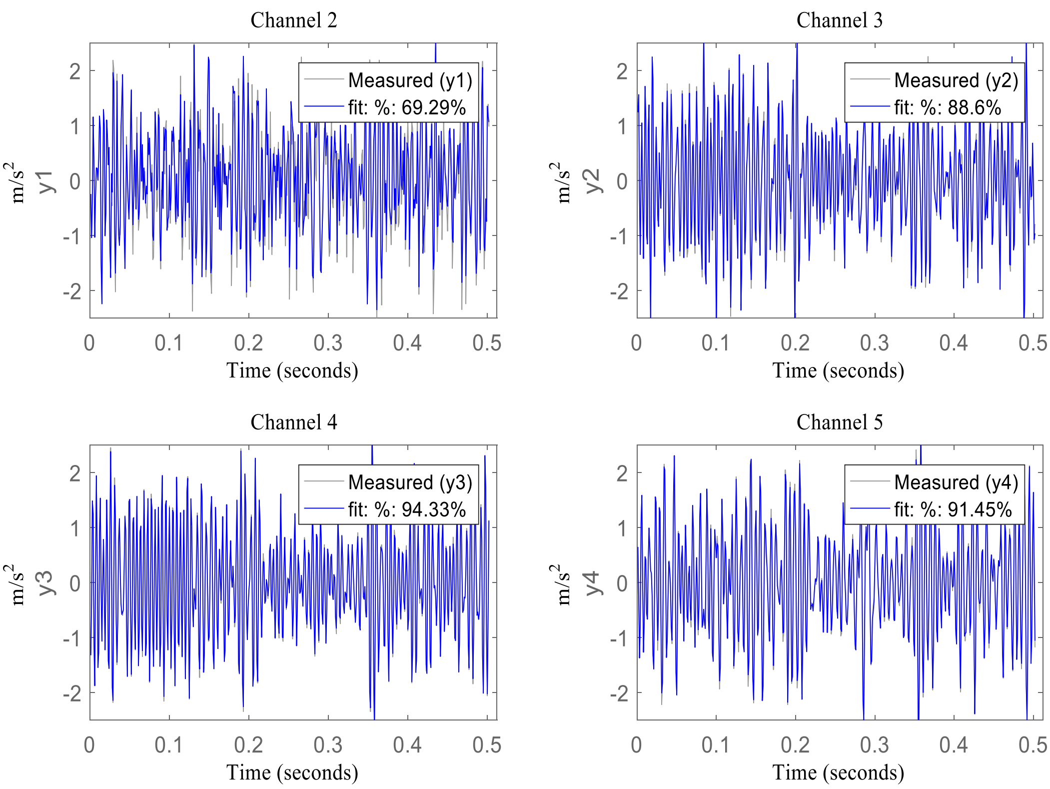


Figure 8. Prediction of the AR(25) model for State#1 in Channels 2 – 5

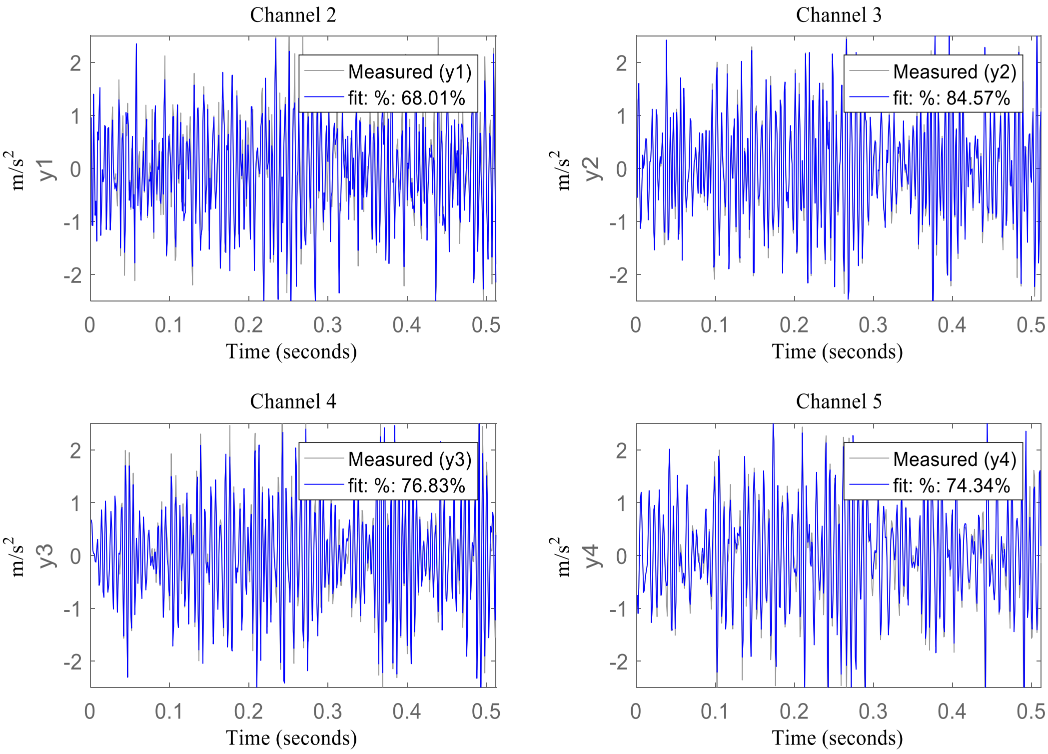


Figure 9. Prediction of the AR(25) model for State#12 in Channels 2 – 5

As shown in Figures 8 and 9, the prediction fitting rates of State#1 at Channels 2–5 are 69.29%, 88.6%, 94.33%, and 91.45%; the prediction fitting rates of State#12 at Channels 2–5 are 68.01%, 84.57%, 76.83%, and 74.34%. Comparing with the reference state, the fitting rates of the test state decrease significantly. Hence, the test State#12 is substantially different from the reference state. If the reference state is treated as the standard (healthy) state, it can be judged that damage occur in State#12.

3.1.2. Constructing the nonlinear ARCH model

(1) Recognition and order of the nonlinear ARCH model

The ARCH models were constructed based on the residual errors acquired from the AR models parameter estimation. First, the ARCH effect (conditional heteroscedasticity) of the residual errors from the AR models in the reference and test states has been evaluated. The Akaike information criterion (AIC)[34] method was then used to determine the orders of the ARCH models at the reference state and test state based on the residual errors and suitable order of the ARCH models was chosen. Finally, the ARCH models were established at the reference and test states based on the residual errors from the AR models. Subsequently, the innovation series of the reference state and the test states are obtained after the parameter estimation of ARCH models.

The Ljung-Box test and the Lagrange multiplier (LM) are both effective methods commonly used to identify the ARCH effect. In this paper, LM was selected to calculate the statistics of orders 5, 10, 15 and 20 of the residual errors from the AR(25) models in State#1 and State#12. If H=1 and p-value<a (the default value is a=0.05), the time series data are conditionally heteroscedastic; otherwise, they are not conditionally heteroscedastic. Tables 3 and 4 present the results of the ARCH effect tests with the residual errors of State#1 and State#12, respectively.

Table 3. LM test results of residuals series of State#1 in Channels 2 – 5

Order	Channel 2		Channel 3		Channel 4		Channel 5	
	H	*p*-value	*H*	*p*-value	*H*	*p*-value	*H*	*p*-value
5	0	0.3407	1	0.0379	1	0	1	0.0033
10	0	0.2672	1	0	1	0	1	0
15	1	0.0138	1	0	1	0	1	0
20	1	0.0067	1	0	1	0	1	0

Table 4. LM test results of residuals series of State#12 in Channels 2 – 5

Order	Channel 2		Channel 3		Channel 4		Channel 5	
	H	*p*-value	*H*	*p*-value	*H*	*p*-value	*H*	*p*-value
5	1	0	1	0	1	0	1	0
10	1	0	1	0	1	0	1	0
15	1	0	1	0	1	0	1	0
20	1	0	1	0	1	0	1	0

In Tables 3 and 4, H=1 and p-value<a (the default value is a=0.05) are in all cases except Channel 2 of State#1, in which H=0. Hence, the residual errors of the AR(25) models at the reference and test states are conditionally heteroscedastic and the ARCH model must be further established. Subsequently, the AIC was used to determine the orders of the ARCH model. Figures 9 and 10 present the AIC values for various orders of the ARCH models by residual series for State#1 and State#12, respectively, in Channels 2–5. According to the results in Figures 10 and 11, when q=10, the change of AIC values at State#1 and State#12 is minor and ARCH (10) model was finally selected.

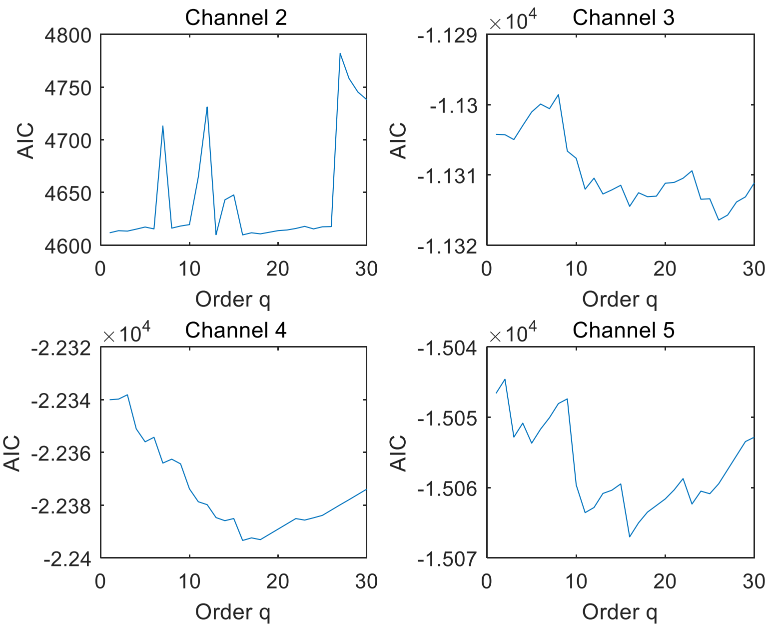


Figure 10. AIC values of the ARCH models for State#1 in Channels 2 – 5

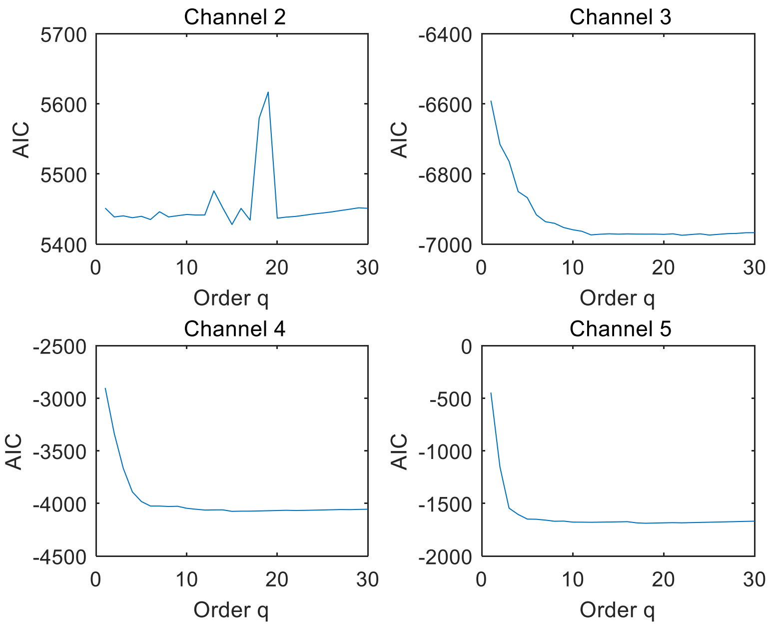


Figure 11. AIC values of the ARCH models for State#12 in Channels 2 – 5

(2) Parameter estimation and applicability testing of the ARCH model

In this section, parameter estimation was conducted at the reference and test states of the ARCH model and the performances of various orders for the ARCH model were evaluated based on the values of the identified parameters. The innovations of the reference and test states were then identified after estimating the parameters of the ARCH models. The ACF tests were conducted on the standardized innovation series at the reference and test states. The distribution of the ACF values could be used to determine the fitting degrees of the ARCH models. Finally, the standardized innovations of the reference and test states were inspected via the ARCH effect test. The heteroscedasticity results were compared between the AR model and the ARCH model to determine the difference in the heteroscedasticity of the data before and after the establishment of the ARCH model.

(a) Parameter estimation

The ARCH models were estimated using the maximum likelihood method, which has been commonly used method for parameter estimation in ARCH models. Tables 5 and 6 present the results of the parameter estimation for the ARCH(10) model at State#1 and State#12. According to Tables 5 and 6, the model parameters of State#1 and State#12 are close to 0, and hence, a higher-order ARCH model is no longer needed.

Table 5. Parameter estimation results of ARCH(10) for State#1 in Channels 2–5

ARCH(10)				
Parameters	Channel 2	Channel 3	Channel 4	Channel 5
c	-0.0042	-0.0066	0	0.0021
k	0.0931	0.0128	0.003	0.0082
A_1	0.0127	0.0153	0.0307	0
A_2	0	0.0161	0.0074	0
A_3	0.0171	0.0172	0.0034	0.0335
A_4	0.0061	0	0.0481	0.0037
A_5	0	0.0019	0.0233	0.026
A_6	0.0247	0.0085	0.0014	0
A_7	0.0036	0.0161	0.0374	0.0074
A_8	0.0206	0	0.0079	0
A_9	0	0.0372	0.0185	0.0105
A_{10}	0.0094	0.0191	0.04	0.0427

Table 6. Parameter estimation results of ARCH(10) for State#12 in Channels 2–5

ARCH（10）				
Parameters	Channel 2	Channel 3	Channel 4	Channel 5
c	0.0042	-0.0032	-0.001	-0.0151
k	0.1001	0.0141	0.0097	0.014
A_1	0	0.0262	0.4418	0.3862
A_2	0.04	0.0568	0.1058	0.1019
A_3	0.0085	0.0392	0.1309	0.2176
A_4	0.0249	0.0838	0.0671	0.0192
A_5	0	0.0329	0.0537	0.0468
A_6	0.0301	0.0693	0.0301	0.0066
A_7	0.0058	0.0474	0.0044	0.0084
A_8	0	0.026	0.0151	0.0287
A_9	0.0056	0.039	0	0
A_{10}	0.0056	0.0309	0.0263	0.0198

(b) Model applicability testing

To evaluate the accuracy of the fitted ARCH model, according to Section 3.1.1, ACF tests were conducted to determine the squares of the standardized innovations of the ARCH models at the reference and test states. Figures 12 and 13 present the ACF maps of the standardized innovations of the ARCH(10) models for State#1 and State#12.

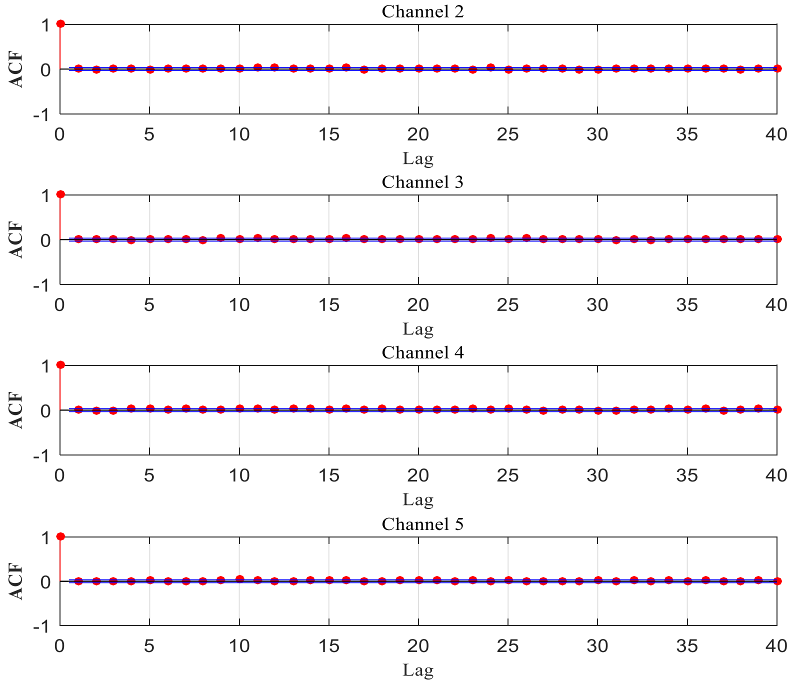


Figure 12. ACF maps of the standardized innovation of State#1 in Channels 2 – 5

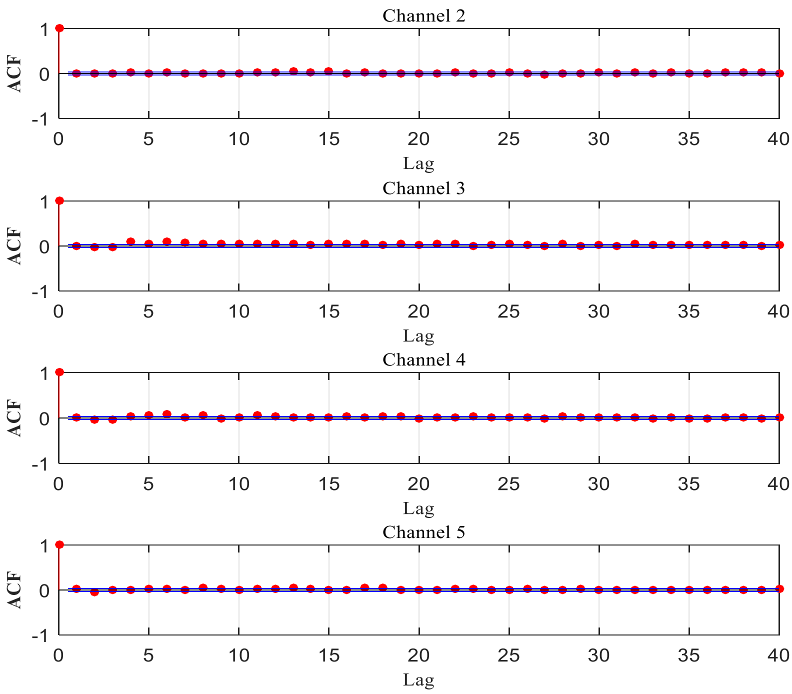


Figure 13. ACF maps of the standardized innovation of State#12 in Channels 2 – 5

According to Figures 12 and 13, the ACF values of Channels 2–5 for State#1 and State#12 tend to zero within order 40 (Lag=40) and fall within the 95% confidence interval. Hence, the squares of the standardized innovations obey the white noise distribution and the ARCH(10) model exhibits satisfactory fitting performance. To further evaluate the applicability of the ARCH model, tests on the standardized innovations of the ARCH model were conducted. Tables 7 and 8 present the results of the tests on the standardized innovations of the ARCH(10) model for State#1 and State#12, respectively. The results show that the establishment of ARCH(10) model can eliminate part of heteroscedasticity influence and provide a good fitting effect.

Table 7. LM test results of the standardized innovations of State#1 in Channels 2 – 5

Order	Channel 2		Channel 3		Channel 4		Channel 5	
	H	*p*-value	*H*	*p*-value	*H*	*p*-value	*H*	*p*-value
5	0	0.901	0	0.7945	0	0.993	0	0.971
10	0	0.9979	0	0.962	0	0.9998	0	0.9995
15	0	0.02432	0	0.3319	0	0.2204	0	0.8198
20	0	0.1075	0	0.1709	1	0.013	0	0.3087

Table 8. LM test results of the standardized innovations of State#12 in Channels 2 – 5

Order	Channel 2		Channel 3		Channel 4		Channel 5	
	H	*p*-value	*H*	*p*-value	*H*	*p*-value	*H*	*p*-value
5	0	0.9889	0	0.9999	1	0.0016	1	0
10	0	0.9999	0	0.9912	1	0	1	0
15	1	0.0176	0	0.4938	1	0.0025	1	0
20	0	0.0761	0	0.1295	1	0	1	0

3.2. Comparison of linear and nonlinear indices in terms of recognition performance

3.2.1. Nonlinear damage identification using SDR that is based on the AR model

Regarding the first experimental data (State#1) at the reference state, the remaining nine instances of experimental data of State#1 and State#10 of the other states from State#2 to State#17 constitute a total of 169 test states for all test states. Damage identification analysis was conducted for the 169 test states in Channels 2–5. Figure 12 presents the damage identification result of SDR that was obtained using the AR(25) model.

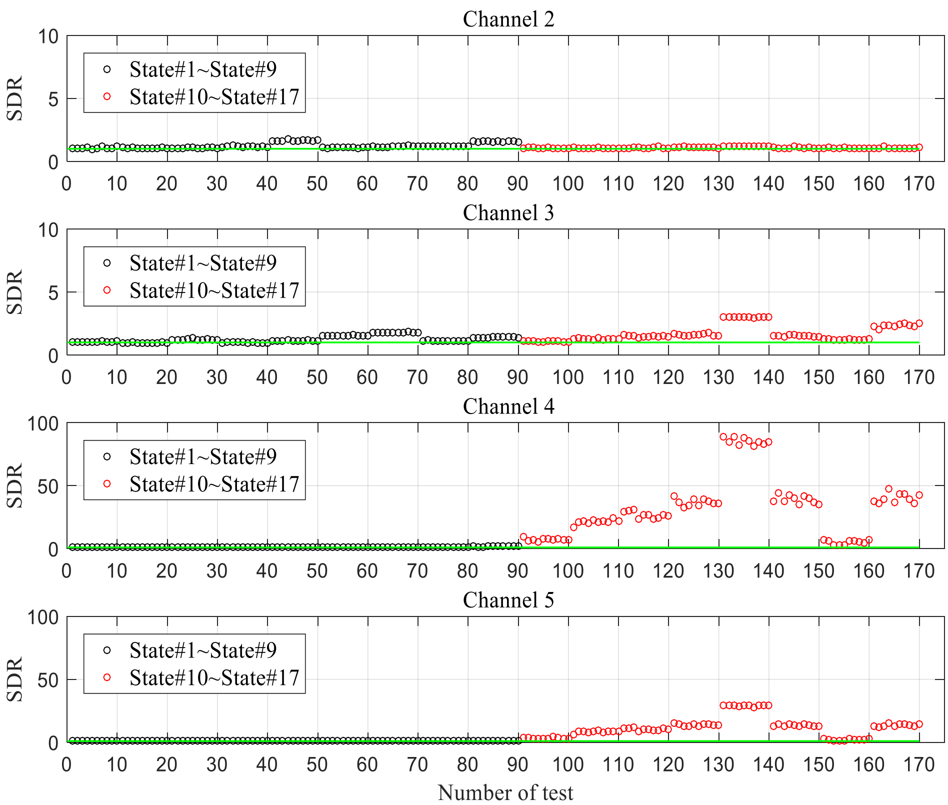


Figure 14. SDR of the AR(25) model for State#1 – 17 at Channels 2 – 5

The results Figure 14 show that the structural damage occurs between Channels 4 and 5. With further detailed analysis, the following conclusions can be drawn:

i. The SDR values of State#10–State#17 in Channels 2–4 increase gradually, where the SDR values in Channels 4 and 5 are much higher than that in Channels 2 and 3. Hence, the source of the damage is located near Channels 4 and 5. The SDR values of State#10–State#17 in Channel 4 exceed that in Channel 5. Therefore, the source of the damage is located closer to Channel 4 than Channel 5. The damage identification results suggest that the channels located closer to the source of the damage have the larger value of SDR.

ii. The SDR values of State#10–State#14 in Channels 4 and 5 range from small to large and conform to the specified degree of nonlinear damage in the experiment

iii. The SDR values of State#10, State#16, State#13 and State#17 in Channels 4 and 5 are similar. Hence, adding mass on the first floor (see Figure1(a)) to simulate the change in environmental factors has little effect on the nonlinear damage. The SDR values of State#15 and State#16 at Channels 4 and 5 have substantial difference. Therefore, adding mass on the base (see Figure1(a)) to simulate a change in environmental factors has a substantial impact on the damage.

iv. Under the small damage condition, such as State#10 and State#16, the SDR values at Channels 4 and 5 are approximately equal to 1. The SDR is inaccurate in judging the small damage in both states. Therefore, SDR is not sensitive to small damage.

v. The SDR values of State#5 and State#9 in Channel 2 and State#6, State#7 and State#9 in Channel 3 are greater than 1. The damage misjudgment occurs, thereby, the linear SDR is substantially affected by environmental factors.

3.2.2. Nonlinear damage identification using SDI based on AR /ARCH models

The ARCH(10) models were established based on the residual errors of the AR(25) model. Using the experimental data of State#1 as the reference state data, the remaining nine instances of State#1 and State#10 instances in each of State#2–State#17 are treated as the test states and comprise a total of 169 test data sets. Damage identification analysis was conducted for Channels 2–5 and the identification effect diagram is shown in Figure 15.

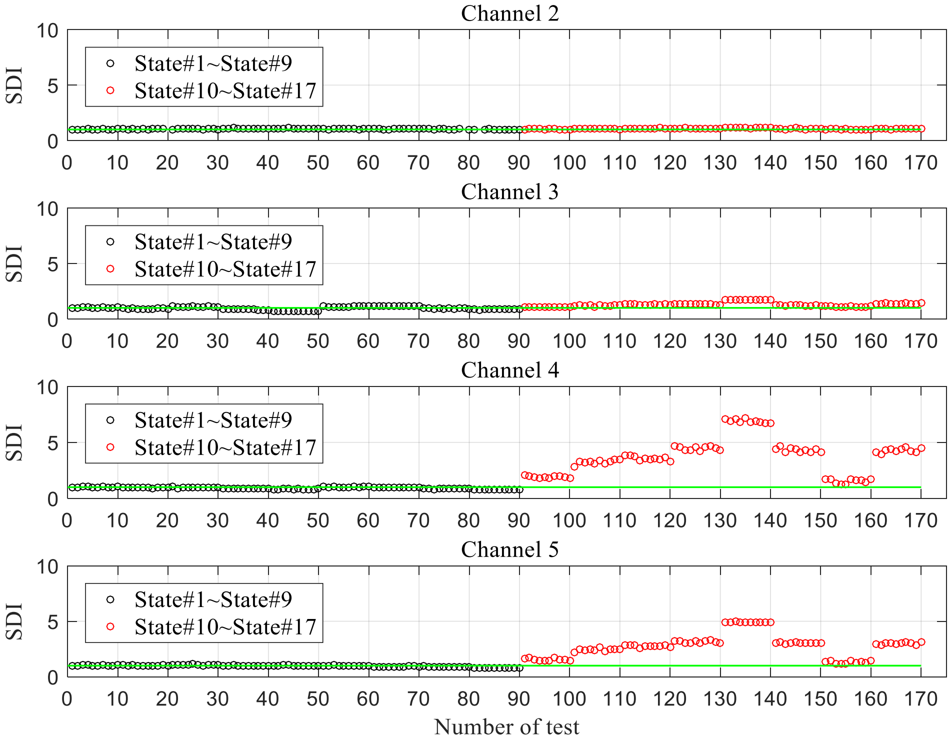


Figure 15. SDI of AR/ARCH models for State#1 to State#17 in Channels 2 – 5

Figure 15 presents the damage identification results that are based on SDI of the ARCH (10) model, from which several conclusions are drawn:

i. The SDI values of State#1–State#17 in Channels 2–3 are all close to 1; hence, Channels 2–3 are far away from the source of the structural damage and are less affected by the damage.

ii. The SDI values of State#1–State#9 in Channels 4–5 are all close to 1; hence, State#1–State#9 are health states. However, SDI values of State#10–State#17 all exceed 1; hence, State#10–State#17 are damage states. Moreover, Channels 4–5 are close to the source of the damage; therefore, they are substantially affected by the damage.

iii. The SDI values of State#10–State#14 in Channels 4–5 vary from small to large and conform to the specified degree of nonlinear damage in the experiment.

iv. State#10 and State#16 were set as small damage conditions in the test. The SDI values of the two cases are slightly larger than 1 in Channels 4–5 and exhibit performance volatility, thereby suggesting that damage occurs. Thus, the nonlinear SDI is sensitive to small nonlinear damage.

v. During the experiment, the effects of environmental factors for State#1–State#9 are simulated by increasing the mass and by reducing the stiffness of the structure. According to the damage recognition results, the SDI values of State#1–State#9 in Channels 25 are all close to 1; hence, the method is less affected by environmental factors.

3.2.3. Comparison of damage results between linear and nonlinear indices

The following conclusions are drawn by comparing the results of linear and nonlinear indicators shown in

Figures 14 and 15, respectively.

i. When using the linear indicator of the AR model for nonlinear damage identification, the SDR values of State#1–State#9 in Channels 2–3 fluctuate slightly and misjudgment occurs in various states; hence, the linear indicator SDR, which is based on the AR model, is sensitive to environmental factors and will be substantially affected by environmental factors. When the nonlinear indicator SDI obtained based on the ARCH model is adopted for structural nonlinear damage identification, the SDI values of State#1–State#9 in Channels 2–3 are all close to 1 and tend to be stable, thereby suggesting that SDI is not sensitive to and is less affected by environmental factors.

ii. When the indicator SDR obtained based on the AR model is adopted for damage identification, Channel 3 is located far from the source of damage while the SDR values of State#10–State#17 in Channel 3 fluctuate substantially. This suggests that the indicator SDR may misjudge in the process of nonlinear damage identification. When the nonlinear indicator SDI obtained based on the ARCH model is used for damage identification, the SDI values of State#10–State#17 in Channel 3 are close to 1 and tend to be stable and to yield satisfactory recognition performance; hence, SDI has substantial advantages in nonlinear damage identification.

iii. Both SDR obtained based on the linear AR model and SDI obtained based on the nonlinear ARCH model can identify the nonlinear damage for State#10–State#14 in Channels 4–5, in which the damages varies from small to large. When identifying the small damages in State#10 and State#16, SDI substantially outperforms SDR.

iv. Compared with SDI, SDR is more accurate in locating the damage near Channel 4. However, SDI can locate the damage near Channels 4–5. Therefore, the SDR obtained based on the linear AR model provides more information regarding the damage location.

v. Compared with SDR obtained based on the AR model, SDI is more accurate in identifying nonlinear damage. Moreover, SDI is insensitive to environmental factors and robust to small damage.

3.2.4. Discussion the damage results of different nonlinear indices and methods

In this section, the proposed method is verified by comparing different nonlinear damage identification methods with different nonlinear damage indices and different linear models with damage characteristic indices of Markov distance. All of the algorithms are implemented and compared using the acceleration responses obtained from the same three-story shear building structure used in this paper.

Compared to the residual condition of standard deviation index based on nonlinear GARCH model proposed by Chen and Yu[21], the SDI proposed in this paper has no misjudgment for damage identification in Channel 2 – 3, which are located far from the nonlinear damage. In addition, for small damage condition, such as State #16, SDI has better performance in damage detection. Comparing to the SOVI of the ARCH model presented by Cheng et al[17], the advantages of using the proposed SDI indicator are (1) the damage warning threshold is clear. When SDI > 1, it indicates that the structure is damaged; (2) For the damage identification in conditions State #10 – State #14, the SDI index indicates taht the damage size grow from small to large, which is in line with the experimental setting for the nonlinear damage degree in State #10 – State #14; (3) The SDI index is sensitive to small damage.

Li[35] proposed the structural damage identification method based on Symplectic Geometric Spectrum Analysis (SGSA) and frequency response function. Fitting a function of structural stiffness and excitation frequency as the damage factor of D value, the D values are decomposed by SGSA to analyze the first component. The method has been verified by the undamaged conditions and conditions affected by environment using the three-story shear building structure. The results show that the status of the two working conditions of the structure can be initially determined. The identification of all operating conditions, as well as damage location and damage degree discrimination still need to further study.

The improved Cross-Model Cross-Modal (CMCM) method proposed by Zhan[36] takes the right singular vector corresponding to the least singular value of the core matrix as the damage indication vector (DIV). The abnormal elements are used to locate the damage. The algorithm is suitable for estimating the damage degree of the mass and stiffness of each element, where nonlinear damage conditions have not been discussed.

4. ANALYSIS OF THE NONLINEAR INNOVATION INDEX

To investigate the information and feature of the parameters such as residuals and innovations from different time series models, the methods of normal probability distribution and fitting analysis were applied to analyze linear SDR and nonlinear SDI in this section. Normal distribution and fitting histogram are commonly used visual graphic methods in statistics, which play an important role in data analysis. The normal distribution corresponds to the probability distribution of the continuous random variable and the histogram obtained via fitting analysis is referred as the quality distribution diagram, which can be used to observe the distribution of the data[37, 38]. To investigate the differences in the fitting degree and in the representation degree of nonlinear information in vibration response data between the linear AR model and the nonlinear ARCH model, the loss function algorithm in statistics was applied in this paper. The loss function method[39] can measure the quality of the model prediction and the inconsistency between the predicted value and the actual value of the model. The loss function is a non-negative real-valued function and the smaller the value of the loss function means the more robust the model.

Both the residual errors with respect to the AR model and the innovations with respect to the ARCH model can represent the characteristics of the structure. The damage identification factors, SDR and SDI, which are fitted based on residual errors and innovations, can reflect the information of vibration responses in the experiments and can be used for nonlinear damage identification analysis.

4.1. Analysis of the residual errors in the AR model

To investigate the characteristics of the residual errors for the AR(25) models, the normal probability distribution and the normal fitting histogram were calculated for the reference condition (State#1) and the damage condition (State#12), respectively, and the results are presented in Figures 16–19. The ordinate of the normal fitting histograms is the frequency of the data.

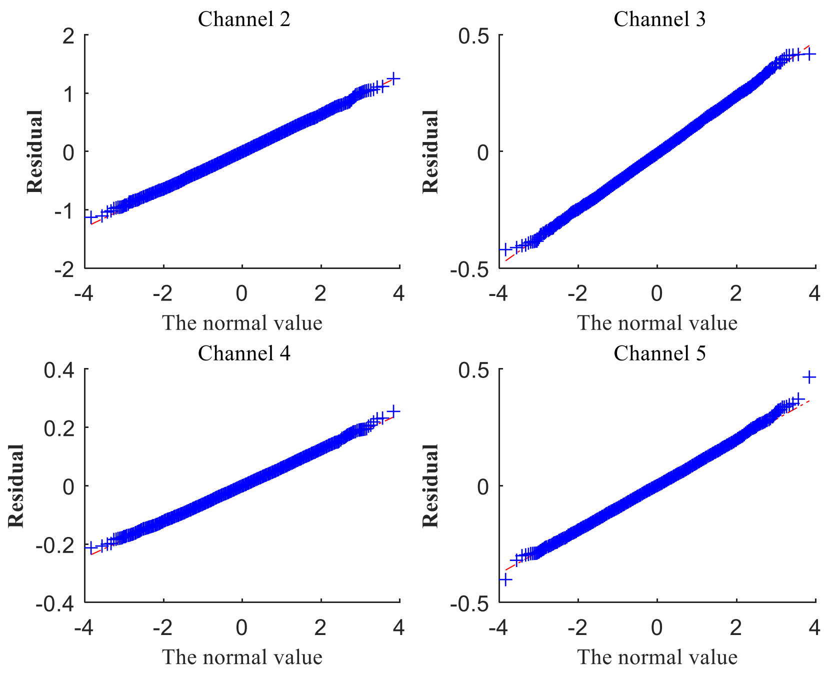


Figure 16. Normal probability distributions of residuals for State#1 in Channels 2–5

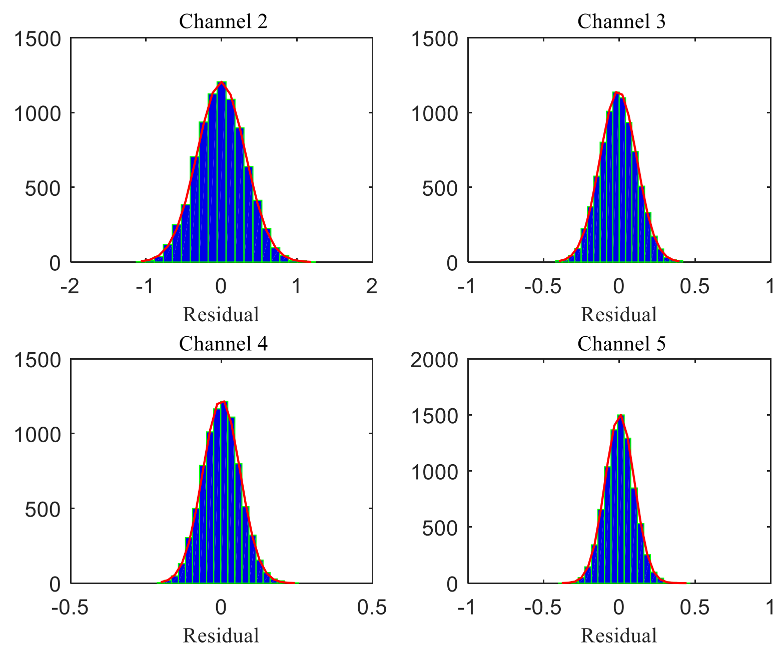


Figure 17. Normal fitting histograms of the residuals for State#1 in Channels 2–5

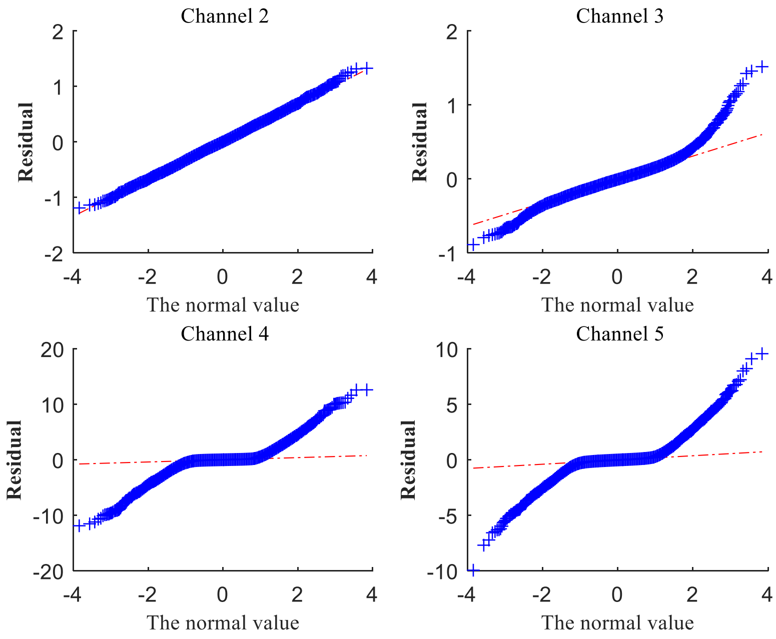


Figure 18. Normal probability distributions of residuals for State#12 in Channels 2–5

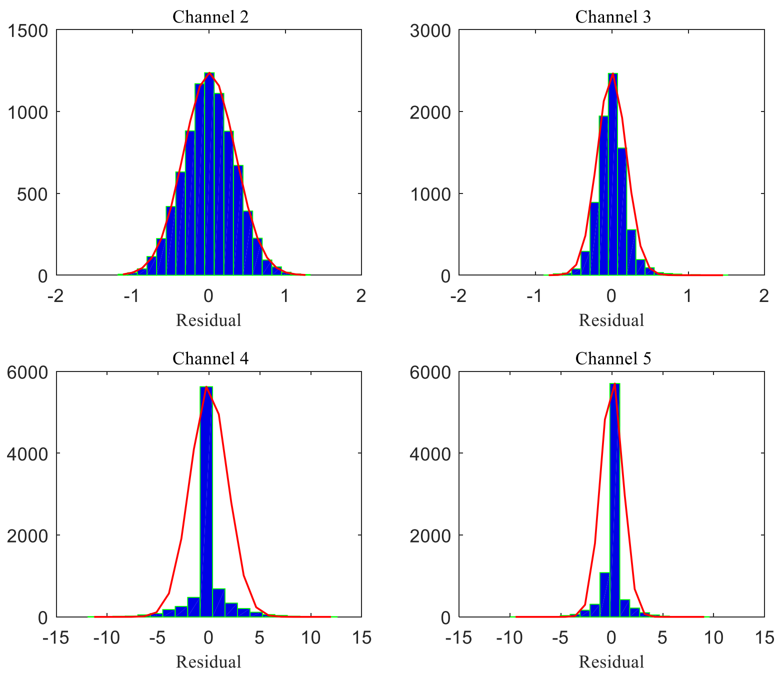


Figure 19. Normal fitting histograms of the residuals for State#12 in Channels 2–5

Figure 16 shows that the residual (State#1) follows a normal distribution, and hence, the distribution function is a straight line. According to Figure 18, the normal probability distributions in Channels 4–5 deviate from the standard line position. Compared with Figure 17, the normal fitting histogram at Channels 4–5 exhibits a clear spike and thick tail phenomenon in Figure 19. Hence, the residual errors obtained from the AR models are conditionally heteroscedastic. The AR model can filter the interference from the linear part of the structural vibration response data so that it can be used for locating the structural damage. But this loses substantial amount of nonlinear damage characteristic information. The residual errors lose the nonlinear damage information, and the linear indicator is easily affected by the environment and insensitive to small damage. This is the main shortcoming of using the linear model for analyzing the nonlinear damage.

4.2. Analysis of innovations in the ARCH model

To investigate the characteristics of the innovations for the ARCH(10) models, a normal probability distribution and a normal fitting histogram were calculated for State#12 and the results are shown in Figures 16 and 17.

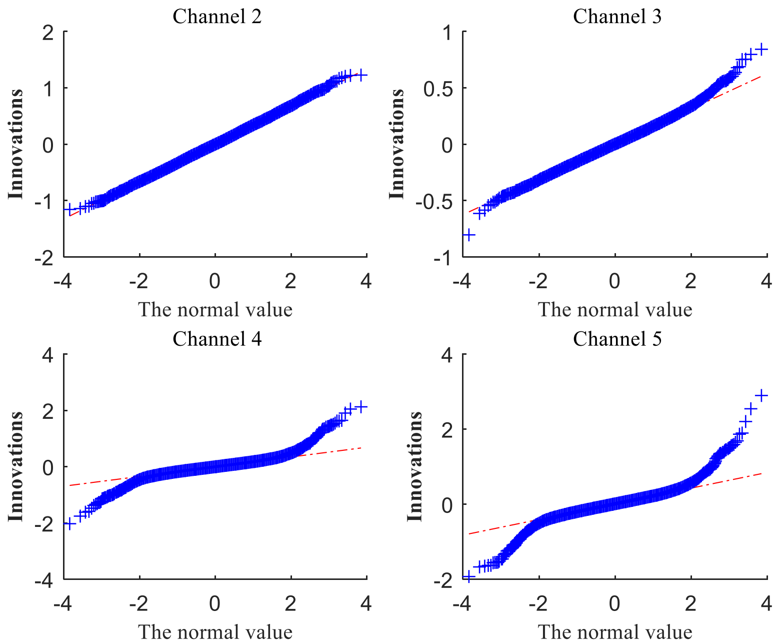


Figure 20. Normal probability distributions of innovations for State#12 in Channels 2–5

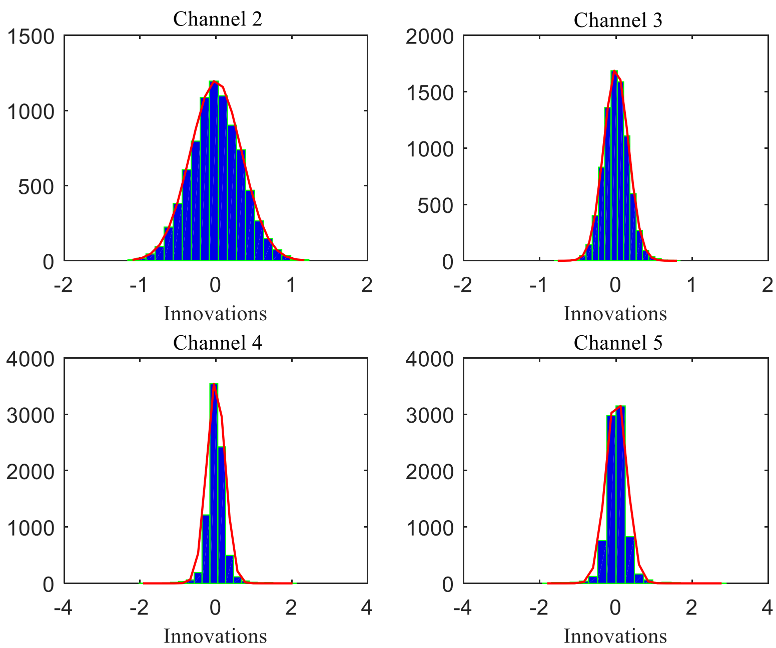


Figure 21. Normal fitting histograms of innovations for State#12 in Channels 2–5

Comparing with Figure 16 in Channels 4–5, the normal distribution curve of the innovations for the ARCH model in Figure 20 is closer to the normal distribution curve than that of the residual errors from the AR model in Figure 18. The normal fit histogram of the innovations from the ARCH model in Figure 21 has substantial changes in the peak and a thick tail phenomenon comparing with Figure 17. Hence, the ARCH model offers significant improvement in the characterization of the structural nonlinear damage. The innovations from the ARCH model can eliminate conditional heteroskedastic influence in the residual errors. Thus, nonlinear indicators that are fitted by the innovations can extract the structural nonlinear features accurately for superior nonlinear damage identification.

4.3 Advantage of the innovation index in characterizing nonlinear damage

In this study a loss function was utilized to further evaluate the differences in the fitting degree and the representation degree of nonlinear information for the vibration responses that are based on the linear AR model versus the nonlinear ARCH model. MSE was selected as the loss function for analyzing the two quantities separately. MSE can be defined as:

$$\text{MSE}=\frac{1}{m}\sum_{i=1}^{m}(f(x)-y_i)^2 \quad (10)$$

where $f(x)$ is the true value, y_i is the predicted value, and m is the total number of elements in the series.

Table 9. MSEs of residuals and innovations of State#1 in Channels 2–5

MSE	Channel 2	Channel 3	Channel 4	Channel 5
AR(25)	0.1028	0.0148	0.0038	0.0093
ARCH(10)	0.1027	0.0147	0.0038	0.0093

Table 10. MSEs of residuals and innovations of State#12 in Channels 2–5

MSE	Channel 2	Channel 3	Channel 4	Channel 5
AR(25)	0.1163	0.0365	3.3716	1.1582
ARCH(10)	0.1138	0.0264	0.0572	0.0752

The residual errors of the AR(25) model and the innovations of the ARCH(10) model were calculated to determine the MSEs for State#1 and State#12, respectively, and the results are listed in Tables 9 and10.

By analyzing the results in Table 9, the findings are summarized as below.

i. The MSE values of AR(25) model residuals and ARCH(10) model innovations for reference condition (State#1) are both small (MSEs<<1), which indicates that AR model and ARCH model have better fitting effect on the structural vibration response data.

ii. Regardless of AR model or ARCH model, the MSE values in Channel 2 are larger than that in the other channels because Channel 2 is closer to the base and the collected data is greatly affected by vibration.

The analysis of the results in Table 10 leads to the following findings.

i. Compared with Table 9, the MSE values of AR(25) model residuals and ARCH(10) model innovations in Table 10 significantly increased, indicating that State#12 has been damaged.

ii. Overall, comparing the ARCH(10) model with the AR(25) model, the MSE values of the ARCH model are much smaller than those of the AR model. It can be judged that the ARCH model has a significant advantage in fitting data with conditional heteroscedasticity and the innovations obtained by ARCH(10) model, which well retains the nonlinear damage information of the structure.

iii. On the other hand, the residuals obtained from linear AR model retain more linear information of the structure, but at the same time, the nonlinear information of the structure is lost. For the AR(25) model, the maximum of the MSE value in Channels 4–5 is 92 ($3.3716/0.0365 \approx 92$) times larger than the minimum of the MSE value in Channels 2–3. It is estimated that the damage is near Channels 4–5. Moreover, the MSE value in Channel 4 is much larger than that in Channel 5, this suggests that the damage is closer to Channel 4. It is predicted that the information of structural damage location is retained in linear data.

Thus, the establishment of the ARCH model can filter out the interference of nonlinear contents in the structural vibration responses. The obtained innovations can provide better characterization of the structural features than those of the residual errors in nonlinear damage detection. The nonlinear indicators, which are fitted by innovation series, can be used for robust identification of nonlinear damage.

5. CONCLUSIONS AND PROSPECTS

The experimentally measured vibration responses of the three-story shear building structure have been used to evaluate the performance of two proposed damage detection approaches. First, the AR model has been established to determine the SDR. Then, the ARCH model has been constructed based on the residual errors of the AR model. The SDI is determined from the ARCH model. The damage characteristic indicators of the two models have been analyzed and compared. Finally, the differences between linear and nonlinear damage indicators have been analyzed and evaluated using the normal probability distribution, histogram and loss function MSE. The advantages of innovations in characterizing the nonlinear damage have been discussed in detail. The following conclusions are drawn:

i. SDR, which is determined based on the AR model, can accurately locate damage in nonlinear damage identification and determine the damage condition of the structure. However, due to the loss of nonlinear damage features, misjudgments may occur during the identification process. Moreover, linear indicator, SDR, is substantially affected by environmental factors and is not sensitive to small damage.

ii. The innovations that obtained from ARCH model can eliminate the influence of heteroscedasticity and more accurately in representing the nonlinear damage information characteristics of the structures. Therefore, using the innovations to fit various damage characteristic indexes has advantages in expressing nonlinear damage characteristics of structures.

iii. Nonlinear SDI is more sensitive to small damage than linear SDR.

iv. Compared with nonlinear SDI, linear SDR has more comprehensive information for damage location.

v. The overall performance of the identification, which is realized by fitting nonlinear SDI using the innovations, is superior to that realized by fitting linear SDR using the residual errors. SDI can accurately locate the nonlinear damage and discriminate the damage degree, less affected by environmental factors and sensitive to small damage. The proposed SDI has practical application value in structural health monitoring.

In the future, other similar nonlinear models can be looked for modeling or the current model could be improved for extracting the innovations and constructing nonlinear damage identification factors. In a real engineering structure, nonlinear characteristics such as irregularity, chaotic and other nonlinear characteristics may be mixed in the collected time series data. It can be effectively combined with artificial neural network, fuzzy clustering, entropy principle and other methods for realizing new methodologies application in practical engineering structure.

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References

[1] H.V.D. Auweraer, International Research Projects on Structural Damage Detection. Key Engineering Materials. **429**(2001) 97-112.

[2] K. Law, H. Sohn, Bayesian probabilistic damage detection of a reinforced-concrete bridge column. **29**(2015) 1131-1152.

[3] C. Papadimitriou, D.C. Papadioti, Component mode synthesis techniques for finite element model updating. Computers & Structures. **126**(2013) 15-28.

[4] S.K. Au, F.L. Zhang, Fundamental two-stage formulation for Bayesian system identification, Part I: General theory. Mechanical Systems and Signal Processing. **66-67**(2016) 31-42.

[5] Y. Huang, J.L. Beck, H. Li, Hierarchical sparse Bayesian learning for structural damage detection:

Theory, computation and application. Structural Safety. **64**(2017) 37-53.

[6] F.L. Zhang, Y.C. Ni, H.F. Lam, Bayesian structural model updating using ambient vibration data collected by multiple setups. Structural Control & Health Monitoring. **24**(2017) 1-18.

[7] Z. Ding, J. Li, H. Hao, Structural damage identification using improved Jaya algorithm based on sparse regularization and Bayesian inference. Mechanical Systems and Signal Processing. **132**(2019) 211-231.

[8] J.H. Zhu, L. Yu, Damage detection based on time series analysis and higher statistical moments. Journal of Southeast University. **42**(2012) 137-143.

[9] M.H. Hu, S.T. Tu, F.Z. Xuan. Statistical Moments of ARMA(n,m) Model Residuals for Damage Detection. In Pressure Vessel Technology: Preparing for the Future. Shanghai, China(2015)

[10] L. Mei, A. Mita, J. Zhou, An improved substructural damage detection approach of shear structure based on ARMAX model residual. Structural Control & Health Monitoring. **23**(2016) 218-236.

[11] A. Ostermann, G. Spielberger, A. Tributsch, Detecting structural changes with ARMA processes. Mathematical Modelling of Systems. **22**(2018) 524-538.

[12] B. Monavari, T.H.T. Chan, A. Nguyen, D.P. Thambiratnam, Structural Deterioration Detection Using Enhanced Autoregressive Residuals. International Journal of Structural Stability & Dynamics. **18**(2018) 1850160:1-19.

[13] C.X. Bao, H. Hao, Z.X. Li, Integrated ARMA model method for damage detection of subsea pipeline system. Engineering Structures. **48**(2013) 176-192.

[14] K. Roy, B. Bhattacharya, S. Ray-Chaudhuri, ARX model-based damage sensitive features for structural damage localization using output-only measurements. Journal of Sound & Vibration. **349**(2015) 99-122.

[15] Z. Wang, F. Liu, L.Q. Yu, S.H. Chen, Structural Damage Detection Using Sensitivity-Enhanced Autoregressive Coefficients. International Journal of Structural Stability and Dynamics. **16**(2015) 1550001:1-16.

[16] Y.Z. Liao, A.S. Kiremidjian, R. Rajagopal, C.H. Loh. Angular velocity-based structural damage detection. In Conference on Sensors and Smart Structures Technologies for Civil, Mechanical, and Aerospace Systems.Las Vegas(2016)

[17] J.J. Cheng, H.Y. Guo, Y.S. Wang, Structural Nonlinear Damage Detection Method Using AR/ARCH Model. International Journal of Structural Stability and Dynamics. **17**(2017) 1750083.

[18] J.N. Yang, S. Lin, On-line damage identification of nonlinear structures. Proceedings of SPIE - The International Society for Optical Engineering. **5765**(2005) 01-10.

[19] H. Ebrahimian, R. Astroza, J.P. Conte, R.a.D. Callafon, Nonlinear finite element model updating for damage identification of civil structures using batch Bayesian estimation. Mechanical Systems & Signal Processing. **84**(2017) 194-222.

[20] H. Kai, J.X. Ma, Y.H. Zhang, F.Y. Xu. Nonlinear time series model identification and its application in structural damage detection. In Information Technology, Networking, Electronic & Automation Control Conference, IEEE.Chongqing, China(2016)

[21] L.J. Chen, L. Yu, Structural Nonlinear Damage Identification Algorithm Based on Time Series ARMA/GARCH Model. Advances In Structural Engineering. **16**(2013) 1597-1609.

[22] J.Z. Xin, J.T. Zhou, S.X. Yang, X.Q. Li, W. Yu, Bridge Structure Deformation Prediction Based on GNSS Data Using Kalman-ARIMA-GARCH Model. Sensors. **18**(2018) 298:01-17.

[23] G.U. Yule, On Reading a Scale. Journal of the Royal Statistical Society. **90**(1927) 570-587.

[24] G.T.Walker, G. T. WALKER ON SEASONAL FORESHADOWING. Mon.wea.rev. **59**(1931) 202.

[25] P.A.P.Moran, The statistical analysis of the Canadian Lynx cycle. Australian Journal of Zoology(1953).

[26] R.F. Engle, Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica. **50**(1982) 987-1007.

[27] S. Degiannakis, Volatility forecasting: evidence from a fractional integrated asymmetric power ARCH skewed-t model. Applied Financial Economics. **14**(2004) 1333-1342.

[28] F.C. Lin, Y.X. Chen, K. Rui. Degradation analysis method based on regression time series model under equal and unequal variances. In Prognostics & System Health Management Conference.Beijing, China(2011)

[29] R. Lund, Time Series Analysis and Its Applications: With R Examples: Publications of the American Statistical Association. Vol. 102. 479(2006).

[30] Y.D. Zhou, H.Q. Wang, W.Y. Lv, Analysis and application of time series: Higher Education Press. 308(2015).

[31] E. Figueiredo, G. Park, J. Figueiras, C. Farrar, K. Worden, Structural Health Monitoring Algorithm Comparisons Using Standard Data Sets. Algorithms(2009).

[32] K.W. Hipel, A.I. Mcleod, W.C. Lennox, Advances in Box-Jenkins modeling: 1. Model construction. Water Resources Research. **13**(1977) 567-575.

[33] X. Zhou. Research on damage identification of beam bridge based on autoregressive moving average model.master's degree thesis, Southwest Jiaotong University(2008).

[34] H. Akaike, A new look at the statistical model identification. Automatic Control IEEE Transactions on. **19**(1974) 215-222.

[35] X.H. Li. Structural Damage Identification Based on Symplectic Geometric Spectrum Analysis.master's degree thesis, Dalian University of Technology(2016).

[36] C. Zhan, D.S. Li, L. Ren, H.N. Li, Local damage identification approach based on improved cross-model cross-mode method. JOURNAL OF VIBRATION AND SHOCK. **34**(2015) 127-133.

[37] D.J. Zhang, Analysis and application of characteristic parameters of normal probability frequency distribution map of apparent formation water resistivity. Fault-Block Oil & Gas Field. **16**(2009) 123-125.

[38] J.T. Shiau, F.C. Wu, A Histogram Matching Approach for assessment of flow regime alteration: application to environmental flow optimization. River Research & Applications. **24**(2010) 914-928.

[39] Y.X. Liu. The BP neural network classification method under Linex loss function and the application to face recognition. In International Conference on Computer & Automation Engineering.Singapore(2010)