DETAILED COMPUTER MODELING OF A POWER STATION GENERATOR

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Minor amendments to the Thesis

The following amendments should be read in conjunction with this thesis:

[A] Points of clarification

A1. It is assumed that all machine parameters are expressed in per unit system in which synchronous speed is unity. Thus inductance and its associated inductive reactance have the same value. It follows also that the torque of electromagnetic origin has the same value as the internal electrical power developed under study-stay conditions.

A2. The data for the multi-mass generator model used on page 38 was taken from the paper by Bowler, Ewart and Concordia (Reference 11).

A3. Comparing the graphs on pages 84 and 93, it is evident that the increase in transformer resistance that was introduced in the series-capacitor case (page 93) has drastically damped some of the higher-frequency oscillations in torque response of the shaft. It is also notable that the level of high-frequency oscillations excited by a single-phase fault for a capacitor of 0.21 per unit (page 107) is greater than for any other type of fault with the this capacitor value, probably due to a higher 100 Hz component in the electrical phenomena.

[B] Errata

B1. page 37: replace " $\vec{I}_q R_a$ " with $\vec{I} R_a$ in Equation (4.2-6);

B2. page 37: replace $|\vec{V}| \sin \delta$ with $|\vec{V}| \cos \delta$ in equation (4.2-7);

B3. page 82: replace $\Theta_1 = \delta$ with $\Theta_3 = \delta$ in Equation (5.4-7);

B4. page 84: the labels T_1 and T_2 on the two graphs should be interchanged &

B5. page 114: in the coefficient of y_0, remove the term $(x-x_0)$.

CONTENTS

ABSTRACT

In this thesis the main aim is to isolate the features of the mathematical model of the power station generator that are critical for the accurate simulation of specific operational conditions. The analysis is based on a real-time three-phase model of the synchronous generator, augmented by mathematical implementations of automatic voltage control and a generator transformer (in a Dy5 connection).

Three-phase model variables are retained in order to obtain the model response to unbalanced as well as balanced system disturbances and to enable machine designers to track the transient operational conditions associated with specific machine regions.

The studies start with a base model which makes a number of common simplifying assumptions. A range of balanced and unbalanced close-up faults is applied to establish base-case system responses. Successive enhancements such as variable speed contribution to induced e.m.f's, magnetic saturation of flux paths and multiple-mass mechanical shaft representation are then added to test their effect over the complete disturbance range.

The results indicate the features of the machine model which are critical in providing an accurate response for specific purposes, and those which can be confidently ignored in the interests of computing efficiency.

THE STATEMENT REGARDING ORIGINALITY

This work contains no material which has been accepted for the award of any other degree or diploma in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text.

I give consent to this copy of my thesis, when deposited in the University Library, being available for loan and photocopying.

Vlatka Zivotic-Kukolj

Dedicated to Dr A. M. Parker for his brilliant supervision and to my husband Radan and daughters Dahlia and Nica for their moral support in accomplishment of this work.

1. INTRODUCTION

In the area of electrical engineering very often we use mathematical modeling of synchronous generators with the aim of predicting their impact on the electrical power system. The level of detail of such modeling depends on the particular purpose of the study, and whether steady state or dynamic information is required. Almost always it is the impact of the generators on power system performance which is the primary concern of the study, and the form of the generator model is normally optimized to produce acceptable electro-mechanical behavior in a computationally efficient form. In the case of dynamic modeling we usually study the effect on the system stability using well-established synchronous machine model.

In this thesis the emphasis of the model is rather different. We will study the impact of a variety of system disturbances on the internal behavior of the synchronous machine and investigate the level of detail needed in the machine model to reproduce accurately the physical phenomena. Thus, as well as the long-term electromechanical behavior of the generator, represented by variations in rotor angle δ, which is the normal region of interest in power system stability analysis, we will also be seeking the response of variables such as electromagnetic torque and field current, with frequencies of interest up to tens of Hertz.

In this thesis the main aim is to use analytical methods of examining the performance of the synchronous machine in response to a variety of feasible network disturbances. In practical power systems the great majority of disturbances give rise to unbalanced three-phase conditions, and the form of the model to be developed will be directed towards these conditions. The thesis will initially describe well-known three-phase and d/q representations of a synchronous generator dynamic model, and then propose an integrated generator/transformer model which will be subjected to a number of disturbances, and to which a number of model enhancements will be applied to test their effect on performance. In this way, a user of the models will be provided with guidelines which will assist in balancing the contradictory demands of model accuracy and computational efficiency.

2. MODELING OF THE SYNCHRONOUS MACHINE

The essential features of a three phase synchronous machine are given by figure (2.0-1):

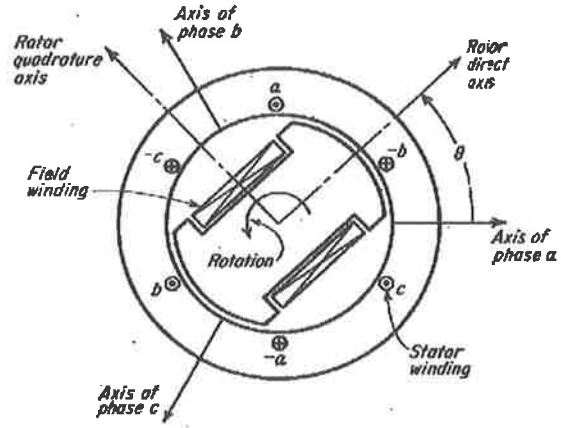


Figure (2.0-1)

The stator contains three distributed windings a, b and c, one in each phase. They are presented by the corresponding labeled concentrated coils where the magnetic axes of the phase winding coincide with the coil axes. The dc field winding fd is situated on the rotor. The rotor's damper bars are not shown in this diagram.

It is noted that this is a salient pole machine. A salient pole rotor has two axes of symmetry, the polar or DIRECT axis d and the interpolar or QUADRATURE axis q. It is obvious that the magnetic flux paths have different permeances in the two axes. The d- and q- axes revolve with the rotor, while the magnetic axes of the three stator phases remain fixed. θ is the angle from the axis of phase a to the d-axis at the instant time t. The corresponding angle from the phase b axis to the d-axis is θ-120 °, while the corresponding angle from the phase c axis to the d-axis is θ+120°. As the rotor turns θ varies with time. With a constant rotor angular velocity ω, θ is given by expression (2.0-1):

$$\theta = \omega * t \qquad (2.0\text{-}1)$$

All the circuits a, b, c and fd have their own resistances and their own self and mutual reactances with respect to every other circuit. It is important to emphasize that the self and mutual reactances associated with the stator circuit are functions of rotor position, varying periodically as the rotor revolves. Voltage–current relations are given by expression (2.0-2):

$$v = r * i + \frac{d\Psi}{dt} = r * i + p\psi \qquad (2.0\text{-}2)$$

where:

- v ... is the applied voltage,
- i ... is the winding current,
- r ... is the winding resistance,
- Ψ ... is the flux linkages with winding &
- p ... is the differential operator $\frac{d}{dt}$;

If this equation is applied to each winding from the figure (2.0-1) we can write:

$$\Psi_a = x_{aa}\, i_a + x_{ab}\, i_b + x_{ac}\, i_c + x_{afd}\, i_{fd} \qquad (2.0\text{-}2a)$$

$$\Psi_b = x_{ba}\, i_a + x_{bb}\, i_b + x_{bc}\, i_c + x_{bfd}\, i_{fd} \qquad (2.0\text{-}2b)$$

$$\Psi_c = x_{ca}\, i_a + x_{cb}\, i_b + x_{cc}\, i_c + x_{cfd}\, i_{fd} \qquad (2.0\text{-}2c)$$

$$\Psi_{fd} = x_{fda}\, i_a + x_{fdb}\, i_b + x_{fdc}\, i_c + x_{fd}\, i_{fd} \qquad (2.0\text{-}2d)$$

or in matrix form:

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \\ \Psi_{fd} \end{bmatrix} = \begin{bmatrix} x_{aa} & x_{ab} & x_{ac} & x_{afd} \\ x_{ba} & x_{bb} & x_{bc} & x_{bfd} \\ x_{ca} & x_{cb} & x_{cc} & x_{cfd} \\ x_{fda} & x_{fdb} & x_{fdc} & x_{fd} \end{bmatrix} \times \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_{fd} \end{bmatrix} \qquad (2.0\text{-}2e)$$

In these equation all reactances except x_{fd} are functions of θ and thus time-varying. It is desirable to examine the nature of these reactances. The nature of the reactance terms in equations (2.0-2a) to (2.0-2e) can be determined from inspection of the machine geometry in Figure (2.0-1). As the stator has a cylindrical bore, the self-reactance of the field winding fd will not depend on rotor position.

The stator-to-rotor mutual inductances will vary periodically with θ. Between phase a and the field winding the mutual-reactance will reach its own maximum x_{afd} at θ=0°, zero at θ=90°, a negative maximum - x_{afd} at θ=180°, zero again at θ=270°. Accordingly with space *mmf* and flux distribution assumed co-sinusoidal:

$$x_{afd}=x_{afd}\cos\theta \qquad (2.0\text{-}3)$$

with similar expressions for phases b and c except that θ is replaced by θ-120° and θ+120°. We will ignore higher order terms in θ in this analysis, effectively assuming a co-sinusoidal distribution of fluxes in the air gap. So we can write:

$$x_{bfd}=x_{bfd}\cos(\theta-120°) \qquad (2.0\text{-}3a)$$

$$x_{cfd}=x_{cfd}\cos(\theta+120°) \qquad (2.0\text{-}3b)$$

The self-inductance of any stator phase will always be positive, but there will be a second harmonic variation because of the different air-gap geometries in the d- and q-axes as the rotor body rotates relative to the stator winding axis (see Ref. No.1). So we can write:

$$x_{aa}=x_{aa0}+x_{aa2}\cos 2\theta \qquad (2.0\text{-}4)$$

$$x_{bb}=x_{aa0}+x_{aa2}\cos(2\theta+120°) \qquad (2.0\text{-}4a)$$

$$x_{cc}=x_{aa0}+x_{aa2}\cos(2\theta-120°) \qquad (2.0\text{-}4b)$$

The mutual-reactance between stator phases will also exhibit a second-harmonic variation with θ because of the rotor shape but the dc component x_{ab0} will be negative because the angle between the magnetic cores of those phases is greater then 90° (see Ref. No.1). So we can write:

$$x_{ab}=x_{ba}=x_{ab0}+x_{aa2}\cos(2\theta-120°) \qquad (2.0\text{-}5)$$

$$x_{bc}=x_{cb}=x_{ab0}+x_{aa2}\cos 2\theta \qquad (2.0\text{-}5a)$$

$$x_{ac}=x_{ca}=x_{ab0}+x_{aa2}\cos(2\theta+120°) \qquad (2.0\text{-}5b)$$

If we substitute all values from (2.0-3) to (2.0-5c) into the matrix from (2.0-2e), it is obvious that the flux linkages with the windings are changing in time as functions of the angle θ.

2.1 Transformation to Direct- and Quadrature-axis Variables

The general idea underlying the transformation is to resolve the mmf's provided by the three stator phases into two axes (d,q) rotating with the rotor. This well known procedure is justified in a number of texts (See: Ref. No. 1,2) and the essential features are summarised below. Now i_d, i_q and i_0 are given by:

$$i_d = \tfrac{2}{3}\left[i_a \cos\Theta + i_b \cos(\Theta-120°) + i_c \cos(\Theta+120°) \right] \quad (2.1\text{-}1a)$$

$$i_q = \tfrac{2}{3}\left[-i_a \sin\Theta - i_b \sin(\Theta-120°) - i_c \sin(\Theta+120°) \right] \quad (2.1\text{-}1b)$$

$$i_0 = \tfrac{1}{3}\left[i_a + i_b + i_c \right] \quad (2.1\text{-}1c)$$

or in matrix form:

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \tfrac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta-120°) & \cos(\theta+120°) \\ -\sin\theta & -\sin(\theta-120°) & -\sin(\theta+120°) \\ \tfrac{1}{2} & \tfrac{1}{2} & \tfrac{1}{2} \end{bmatrix} \times \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad (2.1\text{-}2)$$

In these equations the zero sequence component i_0 is included to allow for any residual unbalance in the original phase currents. If we wish to calculate the values of i_a, i_b and i_c then the following matrix will gave us the answer on that:

$$\begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 1 \\ \cos(\theta-120°) & -\sin(\theta-120°) & 1 \\ \cos(\theta+120°) & -\sin(\theta+120°) & 1 \end{bmatrix} \times \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} \quad (2.1\text{-}2a)$$

Voltage-current expressions in d- and q- axes are:

$$v_{fd} = r_{fd}\, i_{fd} + p\Psi_{fd} \quad (2.1\text{-}3)$$

$$v_d = r_a\, i_d + p\Psi_d + \omega\Psi_q \quad (2.1\text{-}3a)$$

$$v_q = r_a\, i_q + p\Psi_q - \omega\Psi_d \quad (2.1\text{-}3b)$$

$$v_0 = r_a\, i_0 + p\Psi_0 \quad (2.1\text{-}3c)$$

When the machine speed ω is constant the differational equations from (2.1-3) to (2.1-3c) are linear with constant coefficients. This model is very widely used in dynamic studies of power systems. It has major advantage of time-invariant reactances, and so allows the use of longer time-steps in solution and hence more efficient use of computer time.

However, the model variables representing stator quantities are no longer direct analogues of physical variables, and unbalanced operation (representing the vast majority of power system fault conditions) is not easyily accommodated.

Thus in this work the full three-phase stator representation is retained, with the physical response of the machine individual windings directly traceable from the computer results.

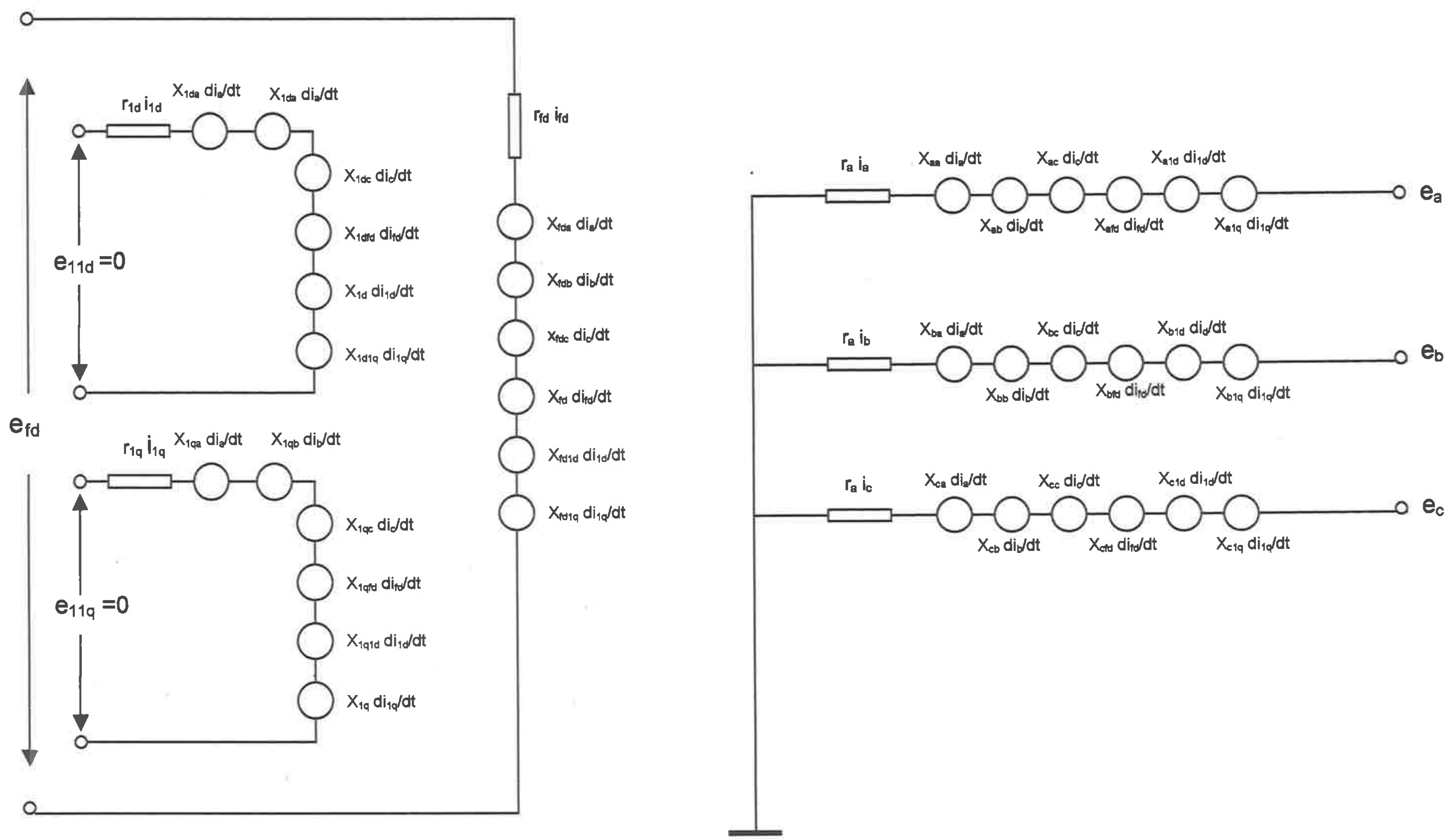
2.2 Three phase model of the synchronous generator

The form of representation of the synchronous generator chosen for this work **three-phase model** that would be the best described as a real-time model of 6^{th} order. In this case synchronous generator is represented through 6 coupled electro-magnetic circuits and those are:

1- winding of the phase a on the stator,
2- winding of the phase b on the stator,
3- winding of the phase c on the stator,
4- field winding on the rotor,
5- damper winding of the direct-axis amortisseur &
6- damper winding of the quadrature-axis amortisseur
(5 & 6 are situated on the rotor);

This assumes that the effects of damping of rotor oscillations can be represented by two equivalent concentrated rotor windings, one on the direct and one on the quadrature axis, denoted by $_{1d}$ and $_{1q}$ subscripts in the analysis which follows.

By the figure (2.2-1) it has been given electro-magnetic description of those relations:



r1d i1d
X1da dia/dt
X1db dib/dt
X1dc dic/dt
X1dfd difd/dt
X1d di1d/dt
X1d1q di1q/dt
e11d =0
efd
r1q i1q
X1qa dia/dt
X1qb dib/dt
X1qc dic/dt
X1qfd difd/dt
X1q1d di1d/dt
X1q di1q/dt
e11q =0
rfd ifd
Xfda dia/dt
Xfdb dib/dt
Xfdc dic/dt
Xfd difd/dt
Xfd1d di1d/dt
Xfd1q di1q/dt
ra ia
Xaa dia/dt
Xab dib/dt
Xac dic/dt
Xafd difd/dt
Xa1d di1d/dt
Xa1q di1q/dt
ea
ra ib
Xba dia/dt
Xbb dib/dt
Xbc dic/dt
Xbfd difd/dt
Xb1d di1d/dt
Xb1q di1q/dt
eb
ra ic
Xca dia/dt
Xcb dib/dt
Xcc dic/dt
Xcfd difd/dt
Xc1d di1d/dt
Xc1q di1q/dt
ec

Figure 2.2-1

The matrix form of the electro-magnetic flux equations has been represented by expression (2.2-1):

$$[\Psi_g] = [x_g] \times [i_g] \qquad (2.2\text{-}1)$$

where: $[\Psi_g]$ ... is vector of the electro-magnetic fluxes of the generator;

$[x_g]$ ... is the matrix of the generator's reactances;

$[i_g]$ ... is vector of the generator's currents;

or it could be written:

$$[\Psi_{abc}] = -[x_{abc}] \times [i_{abc}] + [x_{abcr}] \times [i_r] \qquad (2.2\text{-}1a)$$

$$[\Psi_r] = -\tfrac{2}{3}\,[x_{abcr}]^t \times [i_{abc}] + \tfrac{2}{3}\,[x_r] \times [i_r] \qquad (2.2\text{-}1b)$$

where: $[\Psi_{abc}]$ ... is vector of the flux linkages with the stator;

$[\Psi_r]$ ... is vector of the flux linkages with the rotor;

$[x_{abc}]$ ... is the matrix of the reactance for the stator;

$[x_{abcr}]$ ... is the matrix of the mutual-reactance between rotor and stator;

$[x_r]$ ... is the matrix of the reactance for the rotor;

$[i_{abc}]$ ... is vector of the stator's currents;

$[i_r]$ ... is vector of the rotor's currents;

Vector $[\Psi_{abc}]$ is described by equation (2.2-2):

$$[\Psi_{abc}] = \begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \end{bmatrix} \qquad (2.2\text{-}2)$$

where: Ψ_a ... is the total flux linkage of the phase a;

Ψ_b ... is the total flux linkage of the phase b;

Ψ_c ... is the total flux linkage of the phase c;

Vector $[\Psi_r]$ is described by equation (2.2-3):

$$[\Psi_r] = \begin{bmatrix} \Psi_{fd} \\ \Psi_{1d} \\ \Psi_{1q} \end{bmatrix} \tag{2.2-3}$$

where: Ψ_{fd} ... is the total flux linkage of the field winding;

Ψ_{1d} ... is the total flux linkage of the direct-axis amortisseur;

Ψ_{1q} ... is the total flux linkage of the quadrature-axis amortisseur;

Vector $[i_{abc}]$ is described by equation (2.2-4):

$$[i_{abc}] = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \tag{2.2-4}$$

where: i_a ... is the current of the phase a;

i_b ... is the current of the phase b;

i_c ... is the current of the phase c;

Vector $[i_r]$ is described by equation (2.2-5):

$$[i_r] = \begin{bmatrix} i_{fd} \\ i_{1d} \\ i_{1q} \end{bmatrix} \tag{2.2-5}$$

where: i_{fd} ... is the current of the field current;

i_{1d} ... is the current of the direct-axis amortisseur;

i_{1q} ... is the current of the quadrature-axis amortisseur;

Matrix $[x_{abc}]$ is described by equation (2.2-6):

$$[x_{abc}] = \begin{bmatrix} x_{aa} & x_{ab} & x_{ac} \\ x_{ba} & x_{bb} & x_{bc} \\ x_{ca} & x_{cb} & x_{cc} \end{bmatrix} \qquad (2.2\text{-}6)$$

where: x_{aa} , x_{bb} & x_{cc} ... are self-reactances of the each phase (on the stator);

$x_{ab} = x_{ba}$, $x_{ac} = x_{ca}$ & $x_{bc} = x_{cb}$ ... are mutual-reactances between any two phases (on the stator);

Expressions for x_{aa} , x_{bb} & x_{cc} are given by equations from (2.2-6a) to (2.2-6c):

$$x_{aa} = x_{aa0} + x_{aa2}\cos(2\theta) \qquad (2.2\text{-}6a)$$

$$x_{bb} = x_{aa0} + x_{aa2}\cos(2\theta + \frac{2}{3}\pi) \qquad (2.2\text{-}6b)$$

$$x_{cc} = x_{aa0} + x_{aa2}\cos(2\theta - \frac{2}{3}\pi) \qquad (2.2\text{-}6c)$$

where: x_{aa0} ... is a constant component of the self-reactance of a winding;

x_{aa2} ... is an amplitude of the second harmonic component of the reactances;

Expressions for $x_{ab} = x_{ba}$, $x_{ac} = x_{ca}$ & $x_{bc} = x_{cb}$ are given by equations from (2.2-6d) to (2.2-6f):

$$x_{ab} = x_{ba} = x_{ab0} + x_{aa2}\cos(2\theta - \frac{2}{3}\pi) \qquad (2.2\text{-}6d)$$

$$x_{ac} = x_{ca} = x_{ab0} + x_{aa2}\cos(2\theta + \frac{2}{3}\pi) \qquad (2.2\text{-}6e)$$

$$x_{bc} = x_{cb} = x_{ab0} + x_{aa2}\cos(2\theta) \qquad (2.2\text{-}6f)$$

where: x_{ab0} ... is a constant component of the mutual-reactance between two windings (and is negative);

x_{aa2} ... is an amplitude of the second harmonic component of the reactances;

Matrix $[x_{abcr}]$ is described by equation (2.2-7):

$$[x_{abcr}] = \begin{bmatrix} x_{afd} & x_{a1d} & x_{a1q} \\ x_{bfd} & x_{b1d} & x_{b1q} \\ x_{cfd} & x_{c1d} & x_{c1q} \end{bmatrix} \quad (2.2\text{-}7)$$

where: x_{afd} , x_{bfd} & x_{cfd} ... are mutual-reactances between each of the phases and the field (exited winding);

x_{a1d} , x_{b1d} & x_{c1d} ... are mutual-reactances between each of the phases and the direct-axis amortisseur winding;

x_{a1q} , x_{b1q} & x_{c1q} ... are mutual-reactances between each of the phases and the quadrature-axis amortisseur winding;

Expressions for x_{afd}, x_{bfd}, x_{cfd}, x_{a1d} , x_{b1d} , x_{c1d} x_{a1q} , x_{b1q} & x_{c1q} are given by equations from (2.2-7a) to (2.2-7i):

$$x_{afd} = x_{ad} \cos(2\theta) \quad (2.2\text{-}7a)$$

$$x_{bfd} = x_{ad} \cos(2\theta - \frac{2}{3}\pi) \quad (2.2\text{-}7b)$$

$$x_{cfd} = x_{ad} \cos(2\theta + \frac{2}{3}\pi) \quad (2.2\text{-}7c)$$

$$x_{a1d} = x_{ad} \cos(2\theta) \quad (2.2\text{-}7d)$$

$$x_{b1d} = x_{ad} \cos(2\theta - \frac{2}{3}\pi) \quad (2.2\text{-}7e)$$

$$x_{c1d} = x_{ad} \cos(2\theta + \frac{2}{3}\pi) \quad (2.2\text{-}7f)$$

$$x_{a1q} = - x_{aq} \cos(2\theta) \quad (2.2\text{-}7g)$$

$$x_{b1q} = - x_{aq} \cos(2\theta - \frac{2}{3}\pi) \quad (2.2\text{-}7h)$$

$$x_{c1q} = - x_{aq} \cos(2\theta + \frac{2}{3}\pi) \qquad (2.2\text{-}7i)$$

where: x_{ad} & x_{aq} ... are amplitudes of the mutual-reactances between winding;

$[x_{abcr}]^t$ ... is the transpose matrix of the matrix $[x_{abcr}]$;

Matrix $[x_r]$ is described by equation (2.2-8):

$$[x_r] = \begin{bmatrix} x_{fd} & x_{fd1d} & x_{fd1q} \\ x_{1dfd} & x_{1d} & x_{1d1q} \\ x_{1qfd} & x_{1q1d} & x_{1q} \end{bmatrix} \qquad (2.2\text{-}8)$$

where: x_{fd} ... is the reactance of the field (excited) winding;

x_{1d} ... is the reactance of the direct-axis amortisseur winding;

x_{1q} ... is the reactance of the quadrature-axis amortisseur winding;

Expressions for $x_{fd1d} = x_{1dfd}$, $x_{fd1q} = x_{1qfd}$ & $x_{1d1q} = x_{1q1d}$ ... are given by equations from(2.2-8a) to (2.2-8g):

$$x_{fd} = x_{fdfd} \qquad (2.2\text{-}8a)$$

$$x_{11d} = x_{1d1d} \qquad (2.2\text{-}8b)$$

$$x_{11q} = x_{1q1q} \qquad (2.2\text{-}8c)$$

$$x_{11d} = x_{1dfd} = x_{fd1d} = x_{ad} \qquad (2.2\text{-}8d)$$

In the equation (2.2-8d) it was assumed that the mutual-reactance between all pairs of coils on the direct axis had the same magnitude.

$$x_{fd1q} = x_{1qfd} = 0 \qquad (2.2\text{-}8e)$$

$$x_{1d1q} = x_{1q1d} = 0 \qquad (2.2\text{-}8f)$$

where: x_{11d} ... is the self-reactance of the direct-axis amortisseur winding;

x_{11q} ... is the self-reactance of the quadrature-axis amortisseur winding;

It is assumed that all quantities are expressed in per unit with the exception of angle θ that has been given in radians. It is necessary to emphasise the list of constants and their calculations needed for the mathematical modeling of the synchronous generator although some of them will be changed to real time functions in some of the later modifications. At this stage they all represent what is called the BASE MODEL of the synchronous machine. List of constants contains:

r_{fd} ... is working resistance of the field (excited) winding;
r_{11d} ... is working resistance of the direct-axis amortisseur winding;
r_{11q} ... is working resistance of the quadrature-axis amortisseur winding;
x_{aa0} ... is constant component of the self-reactance of a winding;
x_{ab0} ... is constant component of the mutual-reactance between two windings;
x_{aa2} ... is an amplitude of the second harmonic component of the reactances;
x_{fd} ... is an amplitude of the reactance of the field (excited) winding;
x_{11d} ... is an amplitude of the reactance of the direct-axis amortisseur winding;
x_{11q} ... is an amplitude of the reactance of the quadrature-axis amortisseur winding;
x_{ad} & x_{aq} ... are constants of reactances between windings;
ω ... the angular velocity of the rotor;

For the calculation of the constants the following list of generator entry data is required:

x_d ... is the direct axis synchronous reactance;
x_{dc} ... is the direct axis transient reactance;
x_{dcc} ... is the direct axis subtransient reactance;
x_q ... is the quadrature axis synchronous reactance;
x_{qcc} ... is the quadrature axis subtransient reactance;
x_0 ... is the zero seqence reactance;
x_l ... is the leakage reactance of the stator;
r_a ... is the working resistance of the phase winding;
t_{d0c} ... is the transient time constant of the direct-axis with open armature terminal;
t_{d0cc} ... is the subtransient time constant of the direct-axis with open armature terminal;
t_{q0cc} ... is the subtransient time constant of the quadrature-axis with open armature terminal;
f ... is the system frequency in hertz (Hz);

Calculations of constants is finally given by equations from (2.2-9) to (2.2-24):

$$\pi = 4 * \text{arctg}(1) = 3.141592 \qquad (2.2\text{-}9)$$

$$\omega = 2 \pi f \qquad (2.2\text{-}10)$$

$$x_{ad} = x_d - x_l \qquad (2.2\text{-}11)$$

$$x_{aq} = x_q - x_l \qquad (2.2\text{-}12)$$

$$x_{fdl} = x_{ad} * \frac{x_{dc} - x_l}{x_d - x_{dc}} \tag{2.2-13}$$

$$x_{fd} = x_{ad} - x_{fdl} \tag{2.2-14}$$

$$x_{1dl} = \frac{(x_{dc} - x_l)*(x_{dcc} - x_l)}{(x_{dc} - x_{dcc})} \tag{2.2-15}$$

$$x_{11d} = x_{ad} + x_{1dl} \tag{2.2-16}$$

$$x_{1ql} = \frac{(x_q - x_l)*(x_{qcc} - x_l)}{(x_q - x_{qcc})} \tag{2.2-17}$$

$$x_{11q} = x_{aq} + x_{1ql} \tag{2.2-18}$$

$$r_{fd} = \frac{x_{fd}}{\omega * t_{d0c}} \tag{2.2-19}$$

$$r_{11d} = \frac{(x_{dc} - x_l)*(x_{dcc} - x_l)}{\omega * t_{d0cc}} \tag{2.2-20}$$

$$r_{11q} = \frac{x_{11q}}{\omega * t_{q0cc}} \tag{2.2-21}$$

$$x_{aa2} = 2/3 * (x_d - x_q) \tag{2.2-22}$$

$$x_{ab0} = -((1/6 *(x_d + x_q)) - 1/3 * x_0) \tag{2.2-23}$$

$$x_{aa0} = x_0 - 2 * x_{ab0} \tag{2.2-24}$$

The equation that describes instantaneous rotor position θ given in radians is expressed as:

$$\theta = \omega * t + \varphi \tag{2.2-25}$$

where: θ ... is measured between the axis of the stator phase a (rad);
ω ... is the angular velocity of the rotor (rad/s);

t ... is time (s);
φ ... is the offset of position at t=0s (rad);

The final form of the electromagnetic flux linkages, given generically in equation (2.2-1), can now be evaluated as:

$$\begin{bmatrix} \Psi_a \\ \Psi_b \\ \Psi_c \\ \Psi_{fd} \\ \Psi_{1d} \\ \Psi_{1q} \end{bmatrix} = \begin{bmatrix} -(x_{aa0} + x_{aa2}\cos(2\Theta)) & -(x_{ab0} + x_{aa2}\cos(2\Theta - 2\pi/3)) & -(x_{ab0} + x_{aa2}\cos(2\Theta + 2\pi/3)) \\ -(x_{ab0} + x_{aa2}\cos(2\Theta - 2\pi/3)) & -(x_{aa0} + x_{aa2}\cos(2\Theta + 2\pi/3)) & -(x_{ab0} + x_{aa2}\cos(2\Theta)) \\ -(x_{ab0} + x_{aa2}\cos(2\Theta + 2\pi/3)) & -(x_{ab0} + x_{aa2}\cos(2\Theta)) & -(x_{aa0} + x_{aa2}\cos(2\Theta - 2\pi/3)) \\ -(2/3)*(x_{ad}\cos(\Theta)) & -(2/3)*(x_{ad}\cos(\Theta - 2\pi/3)) & -(2/3)*(x_{ad}\cos(\Theta + 2\pi/3)) \\ -(2/3)*(x_{ad}\cos(\Theta)) & -(2/3)*(x_{ad}\cos(\Theta - 2\pi/3)) & -(2/3)*(x_{ad}\cos(\Theta + 2\pi/3)) \\ (2/3)*(x_{aq}\sin(\Theta)) & (2/3)*(x_{aq}\sin(\Theta - 2\pi/3)) & -(2/3)*(x_{ad}\cos(\Theta + 2\pi/3)) \end{bmatrix}$$

$$\begin{matrix} (2/3)*(x_{ad}\cos(\Theta)) & (2/3)*(x_{ad}\cos(\Theta)) & -(2/3)*(x_{aq}\sin(\Theta)) \\ (2/3)*(x_{ad}\cos(\Theta - 2\pi/3)) & (2/3)*(x_{ad}\cos(\Theta - 2\pi/3)) & -(2/3)*(x_{aq}\sin(\Theta - 2\pi/3)) \\ (2/3)*(x_{ad}\cos(\Theta - 2\pi/3)) & (2/3)*(x_{ad}\cos(\Theta - 2\pi/3)) & -(2/3)*(x_{aq}\sin(\Theta + 2\pi/3)) \\ (2/3)*x_{fd} & (2/3)*x_{ad} & 0 \\ (2/3)*x_{ad} & (2/3)*x_{11d} & 0 \\ 0 & 0 & (2/3)*x_{11q} \end{matrix} \Bigg] * \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_{fd} \\ i_{1d} \\ i_{1q} \end{bmatrix} \quad (2.2\text{-}26)$$

This version of the generator model is basically that of Subramaniam and Malik (ref. 3) and Rafian and Laughton (ref. 4) with scaling factors (such as 2/3) for the per unit system here included explicitly rather than implied by the adapting the input data from standard generator parameters.

In addition to the existing generator model it is desirable to introduce equation of relative change of the angular velocity with respect to synchronous speed:

$$\omega' = d\delta/dt = \omega - \omega_s \qquad (2.2\text{-}27)$$

where: ω' ... is relative change of the angular velocity;

$d\delta/dt$ ... is derivative of rotor angle with respect to time;

ω ... is instantaneous angular velocity of the rotor;

ω_s ... is synchronous speed [rad/s];

and equation of motion is given by:

$$d\omega'/dt = (T_m - T_e)/2h \qquad (2.2\text{-}28)$$

where: $d\omega'/dt$ ... is derivative of relative change of angular frequency with time, or simply rotor acceleration;

h ... is the inertia constant;

T_m ... is the mechanical torque on the shaft [per unit];

$$T_m = P_m/(1+\omega') \qquad (2.2\text{-}29)$$

T_e ...is a calculated value of the electrical torque of the synchronous generator [per unit];

$$T_e = f(i_a, i_b, i_c, \psi_I, \psi_{II}, \psi_{III}) \qquad (2.2\text{-}30)$$

P_m ... is mechanical power on the shaft of the synchronous generator delivered by the turbine;

For the definition of the flux linkages ψ_I, ψ_{II}, ψ_{III} see chapter 3.1;

3. TESTING

3.1 Test system

The test system is not confined to the generator model alone. To analyse transients in a practical system context it will be necessary to extend the existing generator model in such a way that it will include block generator-transformer and an automatic voltage regulator (AVR). The first step in this is to mathematically describe generator-transformer block. The system is shown in Figure (3.1-1) where:

e_a ...is the terminal voltage of the phase a of the generator stator;

e_b ...is the terminal voltage of the phase b of the generator stator;

e_c ...is the terminal voltage of the phase c of the generator stator;

i_a ...is a current of the phase a;

i_b ...is a current of the phase b;

i_c ...is a current of the phase c;

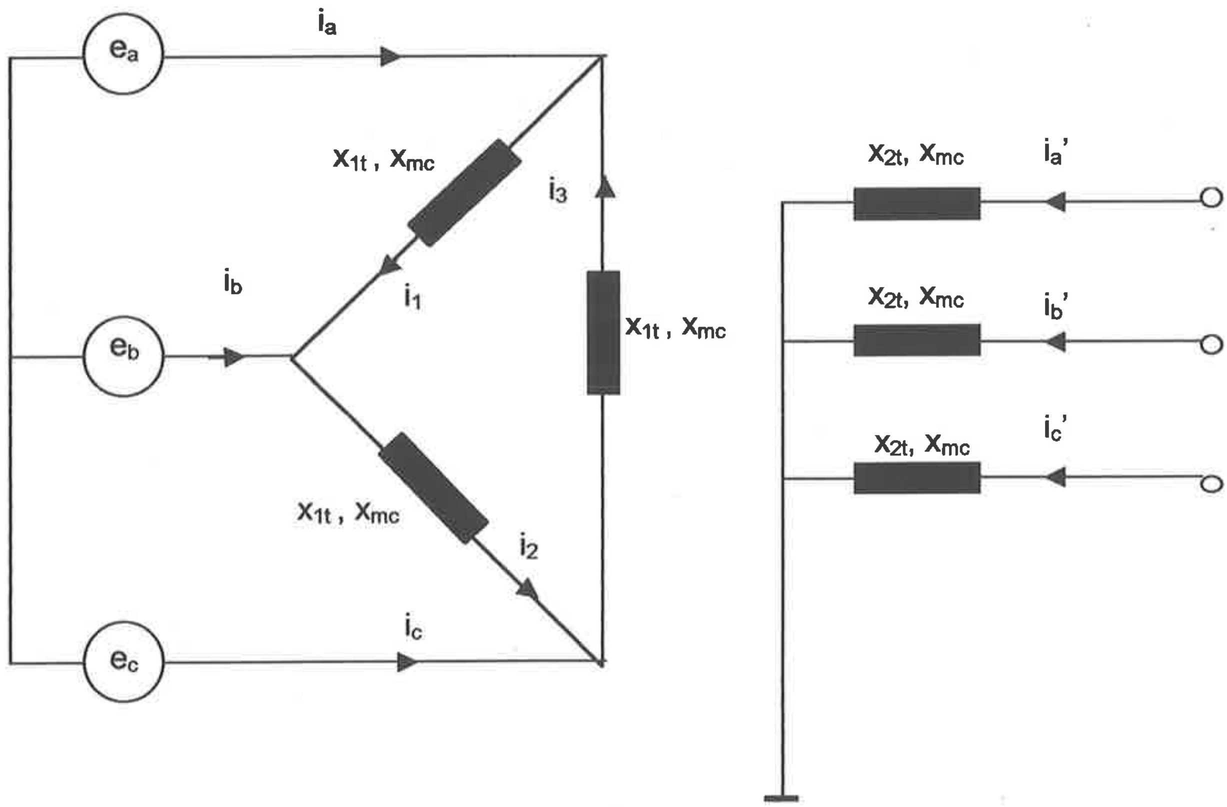


Figure 3.1-1

i_1 ...is a current of the primary winding of the transformer connected between generator phases a and b;

i_2 ...is a current of the primary winding of the transformer connected between generator phases b and c;

i_3 ...is a current of the primary winding of the transformer connected between generator phases c and a;

i_a' ...is a current of the phase a of the secondary side of the transformer;

i_b' ...is a current of the phase b of the secondary side of the transformer;

i_c' ...is a current of the phase c of the secondary side of the transformer;

x_{t1} ... is a self-reactance of the primary side of the transformer and is the same for all three phases;

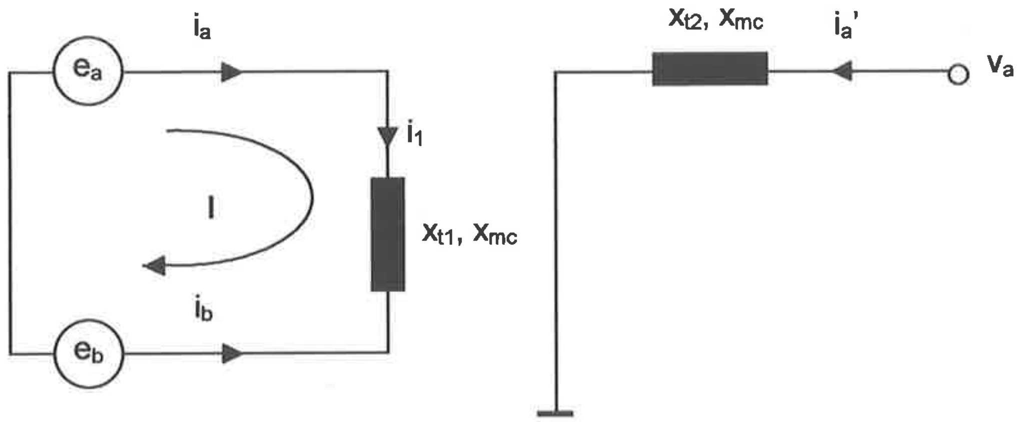
x_{t2} ... is a self-reactance of the secondary side of the transformer and is the same for all three phases;

x_{mc} ...is a mutual-reactance between primary and secondary windings and it is the same for all three phases;

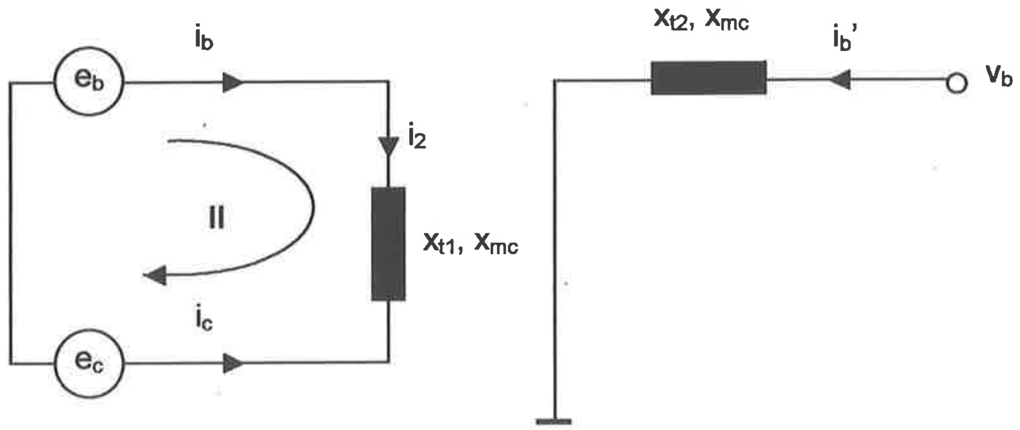
It is important to emphasise that the transformer consists of three single phase transformers that are on the primary side connected in Δ connection, while their secondaries are connected in star forming Dy5 transformer. This type of connection is widely used because if any of single phase transformers fail in it's function it is cheaper to replace it then in a case of three-phase transformer with common magnetic core and in a same time this is the simplest case of a three phase transformer because mutual reactances between phases are treated as zero.

From the Figure (3.1-2) it is possible to analyze three separate loops:

Loop I:



Loop II:



Loop III:

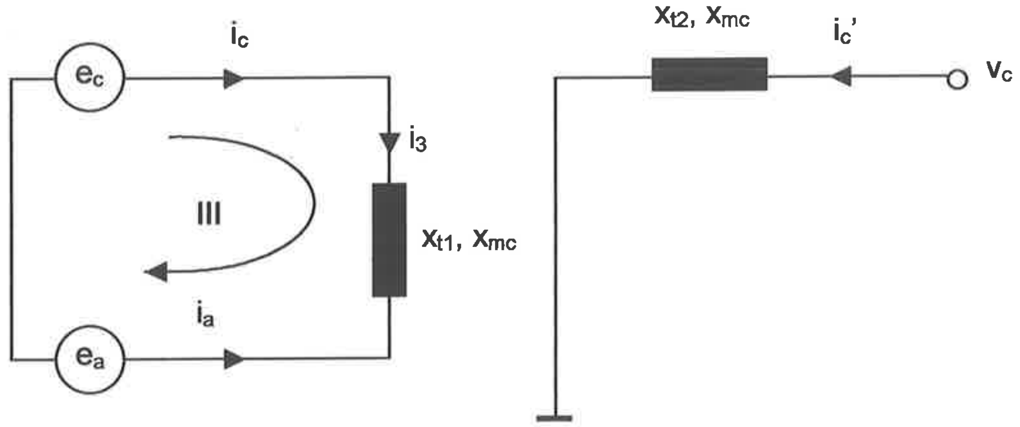


Figure 3.1-2

If we return to Figure 3.1-1 we could write according to Kirchoff's current law that:

$$i_a + i_3 - i_1 = 0 \quad (3.1\text{-}1)$$

$$i_b + i_1 - i_2 = 0 \quad (3.1\text{-}2)$$

$$i_c + i_2 - i_3 = 0 \quad (3.1\text{-}3)$$

From this expressions follow:

$$i_a = i_1 - i_3 \quad (3.1\text{-}1a)$$

$$i_b = i_2 - i_1 \quad (3.1\text{-}2b)$$

$$i_c = i_3 - i_2 \quad (3.1\text{-}3c)$$

To extend the existing generator model by adding the three-phase transformer to its stator the expression (2.2-1) needs to be expressed as:

$$\begin{bmatrix} \Psi_g \\ \Psi_t \end{bmatrix} = \begin{bmatrix} x_g & x_{g-t} \\ x_{t-g} & x_t \end{bmatrix} * \begin{bmatrix} i_g \\ i_t \end{bmatrix} \quad (3.1\text{-}4)$$

where: $[\Psi_g]$ ... is vector of the electromagnetic flux linkages of the generator;

$[\Psi_t]$ ... is vector of the electromagnetic flux linkages of the transformer;

$[x_g]$ ... is matrix of the generator reactances;

$[x_t]$ ... is matrix of the transformer reactances;

$[x_{g\text{-}t}]$ ... is matrix of the mutual reactances between generator and transformer (observe from the generator side) ;

$[x_{t\text{-}g}]$ ... is matrix of the mutual reactances between transformer and generator (observe from the transformer side) ;

$[\Psi_g]$, $[x_g]$ and $[i_g]$ are fully defined in Chapter (2.2). If the equations from (3.1-1a) to (3.1-2c) are substituted in (2.2-26) then the following set of equations describes the electro-magnetic fluxes of the generator:

$$\psi_a = x_{11}\, i_a + x_{12}\, i_b + x_{13}\, i_c + x_{14}\, i_{fd} + x_{15}\, i_{1d} + x_{16}\, i_{1q} =$$

$$= (x_{11} - x_{12})i_1 + (x_{12} - x_{13})i_2 + (x_{13} - x_{11})i_3 + x_{14} i_{fd} + x_{15}\, i_{1d} + x_{16}\, i_{1q} \quad (3.1\text{-}5)$$

$$\psi_b = x_{21}\, i_a + x_{22}\, i_b + x_{23}\, i_c + x_{24}\, i_{fd} + x_{25}\, i_{1d} + x_{26}\, i_{1q} =$$

$$= (x_{21} - x_{22})i_1 + (x_{22} - x_{23})i_2 + (x_{23} - x_{21})i_3 + x_{24}i_{fd} + x_{25}\, i_{1d} + x_{26}\, i_{1q} \qquad (3.1\text{-}6)$$

$$\psi_c = x_{31}\, i_a + x_{32}\, i_b + x_{33}\, i_c + x_{34}\, i_{fd} + x_{35}\, i_{1d} + x_{36}\, i_{1q} =$$

$$= (x_{31} - x_{32})i_1 + (x_{32} - x_{33})i_2 + (x_{33} - x_{31})i_3 + x_{34}i_{fd} + x_{35}\, i_{1d} + x_{36}\, i_{1q} \qquad (3.1\text{-}7)$$

$$\psi_{fd} = x_{41}\, i_a + x_{42}\, i_b + x_{43}\, i_c + x_{44}\, i_{fd} + x_{45}\, i_{1d} + x_{46}\, i_{4q} =$$

$$= (x_{41} - x_{42})i_1 + (x_{42} - x_{43})i_2 + (x_{43} - x_{41})i_3 + x_{44}i_{fd} + x_{45}\, i_{1d} + x_{46}\, i_{1q} \qquad (3.1\text{-}8)$$

$$\psi_{1d} = x_{51}\, i_a + x_{52}\, i_b + x_{53}\, i_c + x_{54}\, i_{fd} + x_{55}\, i_{1d} + x_{56}\, i_{1q} =$$

$$= (x_{51} - x_{52})i_1 + (x_{52} - x_{53})i_2 + (x_{53} - x_{51})i_3 + x_{54}i_{fd} + x_{55}\, i_{1d} + x_{56}\, i_{1q} \qquad (3.1\text{-}9)$$

$$\psi_{1q} = x_{61}\, i_a + x_{62}\, i_b + x_{63}\, i_c + x_{64}\, i_{fd} + x_{65}\, i_{1d} + x_{66}\, i_{1q} =$$

$$= (x_{61} - x_{62})i_1 + (x_{62} - x_{63})i_2 + (x_{63} - x_{61})i_3 + x_{64}i_{fd} + x_{65}\, i_{1d} + x_{66}\, i_{1q} \qquad (3.1\text{-}10)$$

It is obvious that the current i_1, i_2 and i_3 are nothing else than transformer's currents of the primary side. Now to describe the transformer model it is necessary to observe each loop of the Figure (3.1-2). For transformer primary side we can write the following set of equations:

$$\Psi_1 = -3\, x_{t1}\, i_1 + \sqrt{3}\, x_{mc}\, i_a' \qquad (3.1\text{-}11)$$

$$\Psi_2 = -3\, x_{t1}\, i_2 + \sqrt{3}\, x_{mc}\, i_b' \qquad (3.1\text{-}12)$$

$$\Psi_3 = -3\, x_{t1}\, i_3 + \sqrt{3}\, x_{mc}\, i_c' \qquad (3.1\text{-}13)$$

(where the factors 3 and $\sqrt{3}$ arise from the Δ connection of the primary winding and the per unit system used in the study);

For transformer secondary side we can write the following set of equations:

$$\Psi_1' = x_{t2}\, i_a' - \sqrt{3}\, x_{mc}\, i_1 \qquad (3.1\text{-}14)$$
$$\Psi_2' = x_{t2}\, i_b' - \sqrt{3}\, x_{mc}\, i_2 \qquad (3.1\text{-}15)$$
$$\Psi_3' = x_{t2}\, i_c' - \sqrt{3}\, x_{mc}\, i_3 \qquad (3.1\text{-}16)$$

Where: Ψ_1', Ψ_2' & Ψ_3' ... are the electro-magnetic flux linkages of the secondary side of the transformer;

i_a', i_b' & i_c' ... are currents of the secondary side of the transformer;

If loop I in Figure (3.1-2) is observed it could be written:

$$\Psi_a - \Psi_b - \Psi_1 = \Psi_I \quad (3.1\text{-}17)$$

Same follows for loop II & loop III:

$$\Psi_b - \Psi_c - \Psi_2 = \Psi_{II} \quad (3.1\text{-}18)$$

$$\Psi_c - \Psi_a - \Psi_3 = \Psi_{III} \quad (3.1\text{-}19)$$

The flux linkages Ψ_I, Ψ_{II} & Ψ_{III} defined by equations (3.1-17), (3.1-18) and (3.1-19) together with secondary flux linkages from expressions (3.1-14) to (3.1-16) will be the principle variables defining the generator-transformer assembly.

3.2 Automatic Voltage Regulator (AVR)

A basic automatic voltage regulator (AVR) model is included in the test system to provide realistic variations in field current in response to specific system faults. Variations in field current are major contributors to electromagnetic torque variations which impact on the electromechanical performance of the generator. Figure (3.2-1) shows the general arrangement of the AVR modeled:

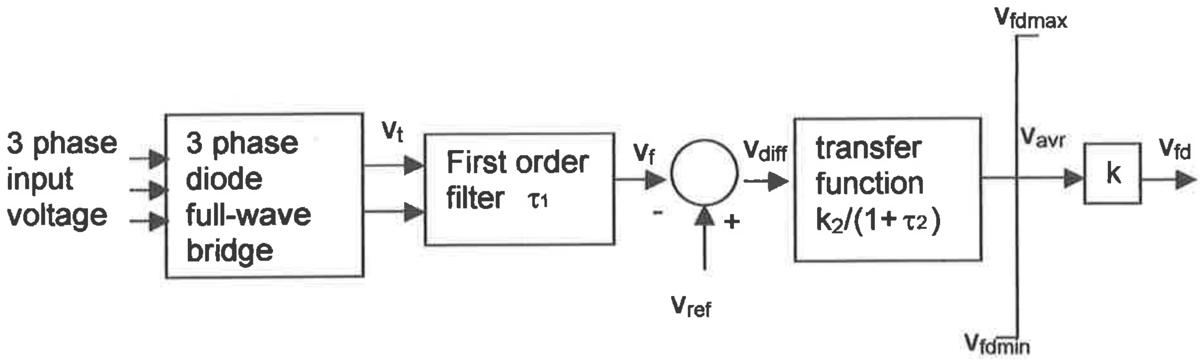


Figure 3.2-1

The generator's 3-phase terminal voltage is applied to a 3-phase full-wave bridge rectifier after which the signal enters a first order filter to reduce the phase ripple. After the filter the signal enters a summing junction where it is compared to a reference voltage v_{ref}. The difference between these two voltages is then applied to a first order transfer function. The output from this stage is then limited by $+v_{fdmax}$ and $-v_{fdmin}$, before being scaled to the correct per-unit value for application to the generator's field winding. The generator terminal phase voltages are obtained from:

$$v_a = \frac{d\Psi_a}{dt} - i_a r_a \qquad (3.2\text{-}1)$$

$$v_b = \frac{d\Psi_b}{dt} - i_b r_a \qquad (3.2\text{-}2)$$

$$v_c = \frac{d\Psi_c}{dt} - i_c r_a \qquad (3.2\text{-}1)$$

where: v_a ... is stator terminal voltage of the phase a;

v_b ... is stator terminal voltage of the phase b;

v_c ... is stator terminal voltage of the phase c;

i_a ... is the current of the phase a;

i_b ... is the current of the phase b;

i_c ... is the current of the phase c;

$\frac{d\Psi_a}{dt}$ ... is induced emf of the phase a;

$\frac{d\Psi_b}{dt}$ ... is induced emf of the phase b;

$\frac{d\Psi_c}{dt}$ ... is induced emf of the phase c;

$i_a r_a$ ...is a voltage drop across the working resistance of the phase a;

$i_b r_a$ ...is a voltage drop across the working resistance of the phase b;

$i_c r_a$ ...is a voltage drop across the working resistance of the phase c;

These simple equations contain the basic principle of the work of any synchronous machine where direct application of Faraday's Law is obvious. The action of the rectifier could be represented by equation (3.2-4):

$$v_t = \frac{[\max(v_a, v_b, v_c) - \min(v_a, v_b, v_c)] * \pi}{3\sqrt{3}\sqrt{2}} \qquad (3.3\text{-}4)$$

where: $\max(v_a, v_b, v_c)$ ...is the function that chooses the maximum instantaneous value between three stator terminal voltages;

$\min(v_a, v_b, v_c)$ ...is the function that chooses the minimum instantaneous value between three stator terminal voltages;

After that the equation for the voltage of the filter is given by expression (3.2-5):

$$\frac{dv_f}{dt} = \frac{1}{\tau_1}(v_t - v_f) \tag{3.2-5}$$

where: v_t ... is the voltage after the rectifier;

v_f ... is the voltage after the filter;

τ_1 ... is the time constant of the filter;

Now:

$$v_{diff} = v_{ref} - v_f \tag{3.2-6}$$

$$\frac{dv_{AVR}}{dt} = \frac{1}{\tau_2}(k_2 v_{diff} - v_{AVR}) \tag{3.2-7}$$

where: v_{diff} ... is the difference between v_{ref} and v_f;

v_{ref} ... is the voltage set by the operator in the power plant for this particular generator;

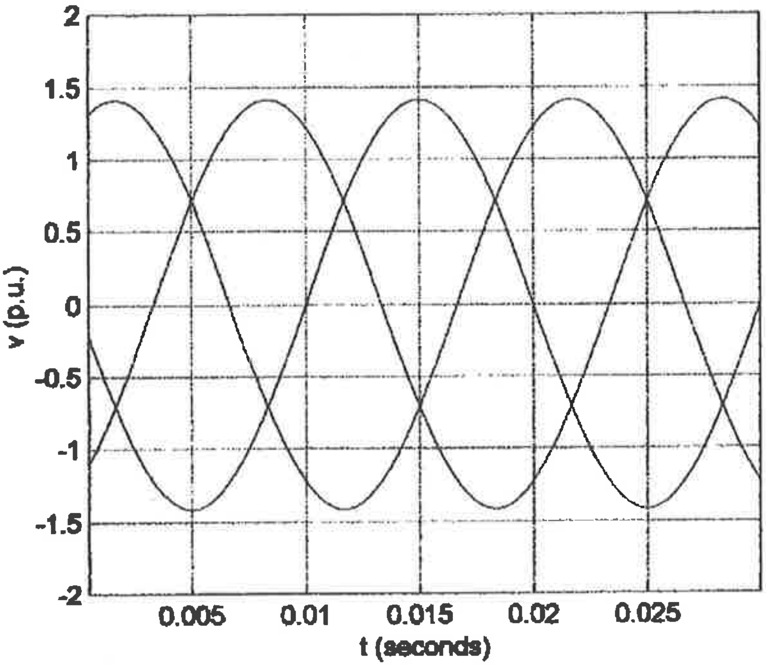
$\frac{dv_{AVR}}{dt}$ ... is derivative of the voltage after AVR in time;

τ_2 ... is the time constant of the transfer function;

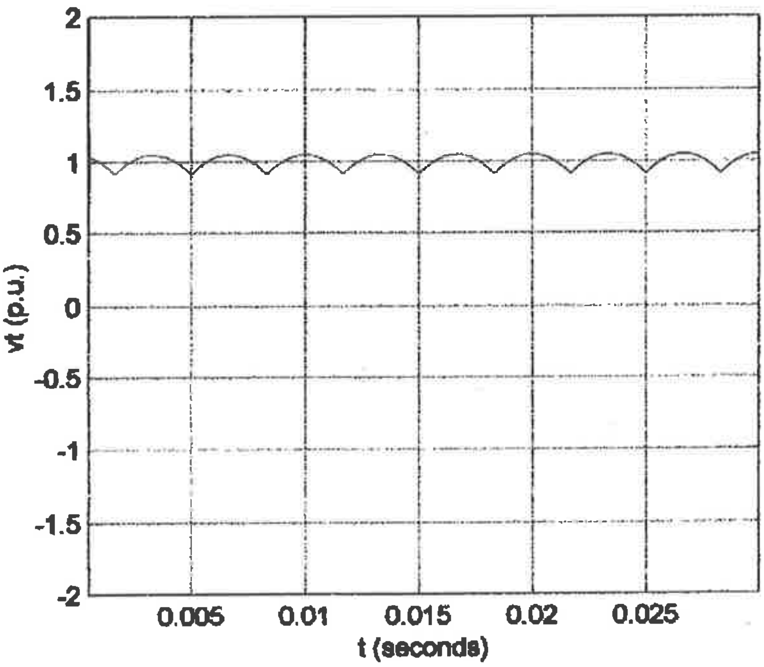
k_2 ... is the gain of the transfer function;

v_{AVR} ... is the voltage after the transfer function block;

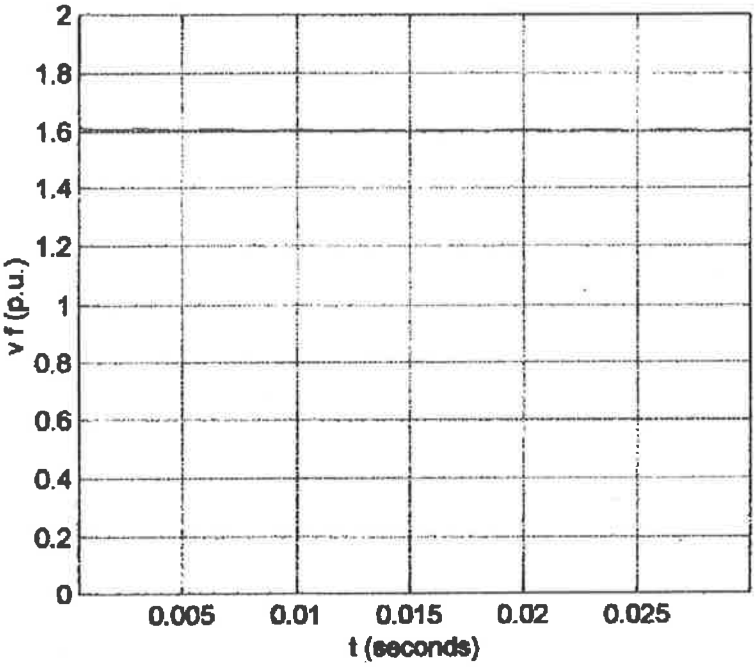
The scaling block k in Figure (3.2-1) is to convert the AVR (per unit) output voltage, which is normally expressed in terms of excitation for rated stator voltage on open circuit, into the appropriate (per unit) value of v_{fd} for the generator model. The scale factor is $\frac{r_{fd}}{X_{ad}}$. As an example of this model in action, the following graphs show the model variables for a steady-state condition.



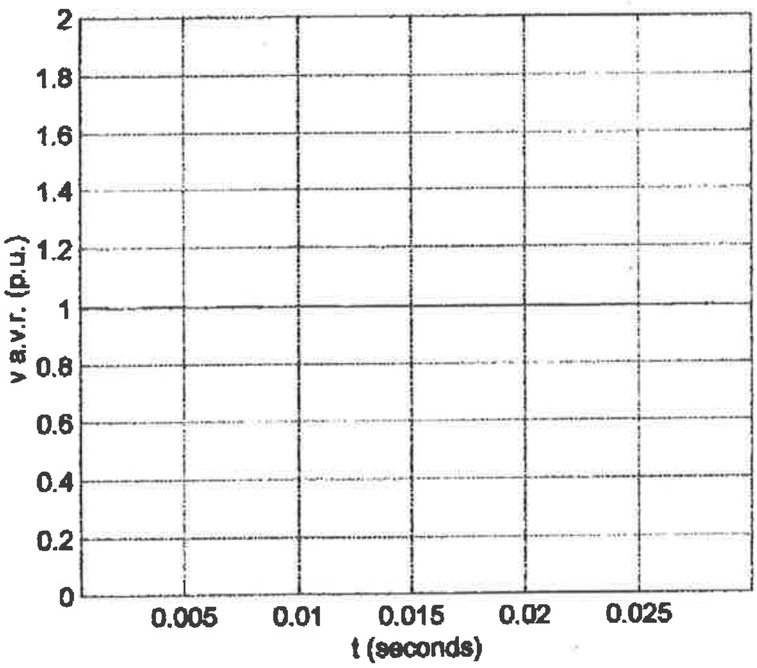
3-phase input voltage



Voltage v_t after 3-phase diode full-wave bridge rectifier



Voltage vf after C-R Filter



Voltage v_{avr} after automatic voltage regulator

Since the focus in this work is on short-term transient behavior within the generator and not the long-term electro-mechanical behavior of the power system, no dynamic model of the speed governing system is provided and the mechanical power delivered by the turbine will be assumed constant over the relatively short simulation periods.

3.3 Types of faults

For the given test system in the chapters (3.1) and (3.2) the following set of faults will be considered:

a) 1-phase short circuit;
b) 2-phase short circuit;
c) 2-phase short circuit with ground;
d) 3-phase short circuit with ground;

All will be applied directly to the secondary terminal of the generator transformer, thus applying severe disturbances to the model. A full description for each of these set of faults is given in Figure (3.3-1):

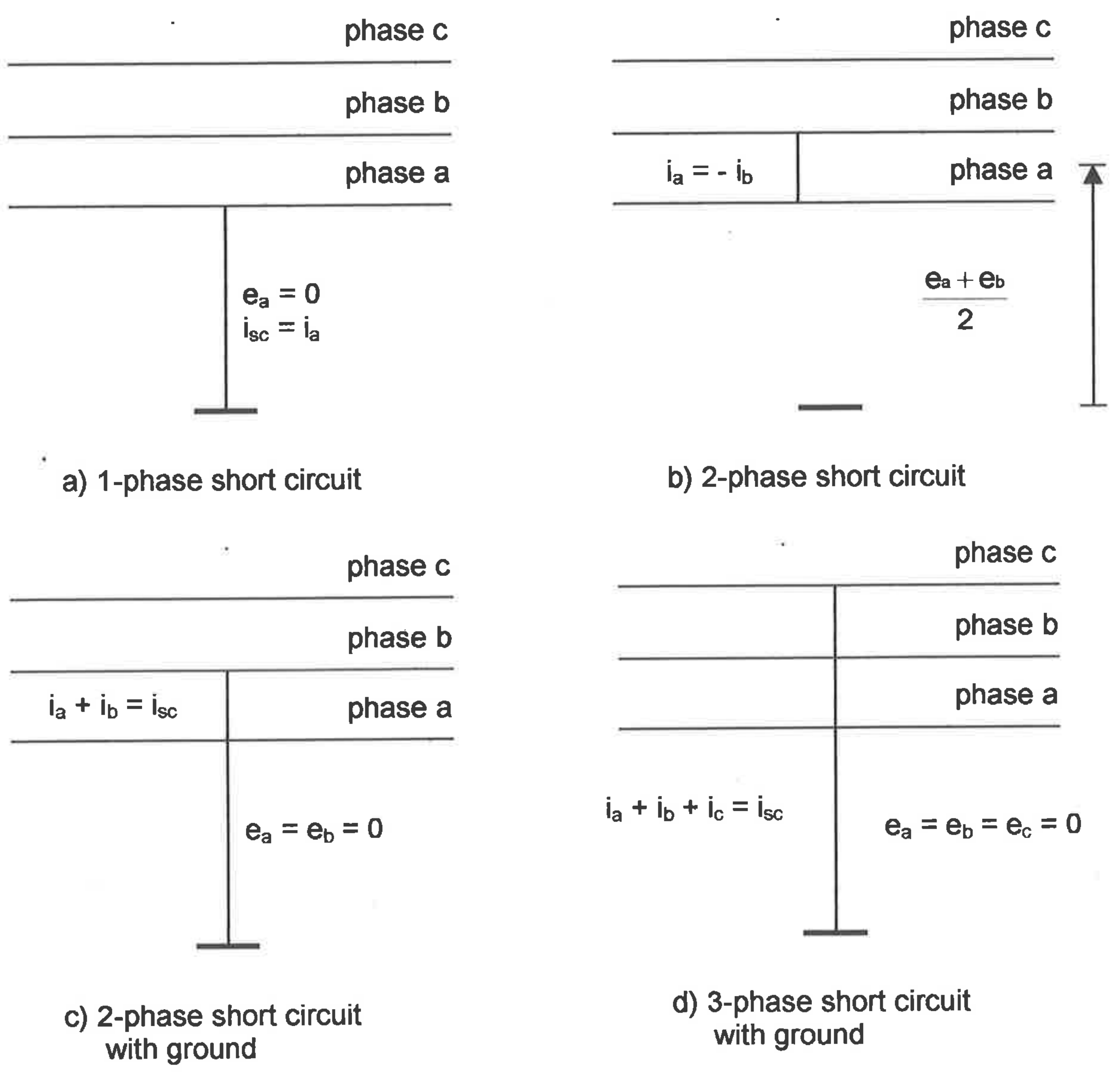


Figure (3.3-1)

4. MATHEMATICAL MODELING OF THE SYSTEM

4.1 First-order differential equations

The test system proposed in Chapter 3 can be described by set of simultaneous first-order differential equations. This approach leads naturally to state-space analysis for which an extensive body of literature exists (see for example refs 5 and 6). The general mathematical form of such a system is given by equation (4.1-1):

$$\frac{d[X]}{dt} = [A] * [X] + [B] * [U] \qquad (4.1\text{-}1)$$

where: $[X]$ ... is a vector of N state variables (for a N^{th} order system);

$[U]$ ... is a vector of M external inputs to the system;

$[A]$ ... is the N-square system matrix;

$[B]$ ... is the N×M input matrix;

In order to get a time response of this system we use step-by-step numerical integration of this set of equations where for the j^{th} time step it is given expression (4.1-2):

$$\frac{d[X]_j}{dt} = [A]_{j-1} * [X]_{j-1} + [B]_j * [U]_j \qquad (4.1\text{-}2)$$

The right hand side of the equations are known, the values of the state variables $[X]$ at the last time-step are known, the present values of the external inputs $[U]$ are known (because the study is done for a specific set of external events) and the values of the non-linear matrixes [A] and [B] evaluated at the last-known state and input values are also known.

We thus evaluate the right-hand sides and obtain estimates of $\frac{d[X]}{dt}$, the rate of change of the state variables. These rates of change, together with the previous state values are then fed into standard NUMERICAL INTEGRATION ROUTINE (in this case RUNGE-KUTTA – see ref. 7) resulting in estimates of the new state vector $[X]_j$. The whole solution advances by one time step and expression (4.1-2) is re-computed. This process is repeated, feeding in the sequence of inputs [U] at appropriate times until the simulation period is completed. A flow chart of the fourth-order Runge Kutta algorithm used is shown in Figure (4.1-1):

START

↓

INPUT DATA:

$[\psi]_{t=0sec}$	... is a vector of the electro-magnetic fluxes for the initial moment (t=0sec);
$[X]_{t=0sec}$	... is a matrix of the reactances for the initial moment (t=0sec);
$[e]_{t=0sec}$	... is a matrix of the voltages for the initial moment (t=0sec);
$[r]$	... is a matrix of the resistances (that is CONSTANT);
m	... is a total time in that the whole system has been observed;
Δt	... is the time increment in seconds;
h	... is a constant of inertia;
τ_1	... is the time constant of the AVR filter;
τ_2	... is the time constant of the AVR transfer function;
k	... is a gain of the AVR transfer function;
v_{ref}	... is the reference voltage set by the operator in the power plant;

↓

n = 0

(A) →

↓

$$[\,i\,]_{t=n} = [\,X\,]^{-1}_{t=n} * [\,\psi\,]_{t=n}$$

$[\,e\,]_{t=n}$ ...is given for the first step and then calculated for the particular moment t;

$$\frac{d[\psi]_{t=n}}{dt} = \omega\,[\,e\,]_{t=n} + [\,r\,] * [\,i\,]_{t=n}$$

t_e, t_m & v_t ... are calculated for the particular moment;

$$\left(\frac{d\omega}{dt}\right)_{t=n} = \frac{[(t_m)_{t=n} - (t_e)_{t=n}]}{2*h} \qquad \left(\frac{dv_f}{dt}\right)_{t=n} = \frac{[v_t - (v_f)_{t=n}]}{\tau_1}$$

$$\left(\frac{d\delta}{dt}\right)_{t=n} = 2\pi f * \omega_{t=n} \qquad v_{diff} = v_{ref} - (v_f)_{t=n}$$

$$\left(\frac{dv_{AVR}}{dt}\right)_{t=n} = \frac{[k_2 * v_{diff} - (v_{AVR})_{t=n}]}{\tau_2}$$

↓

(B)

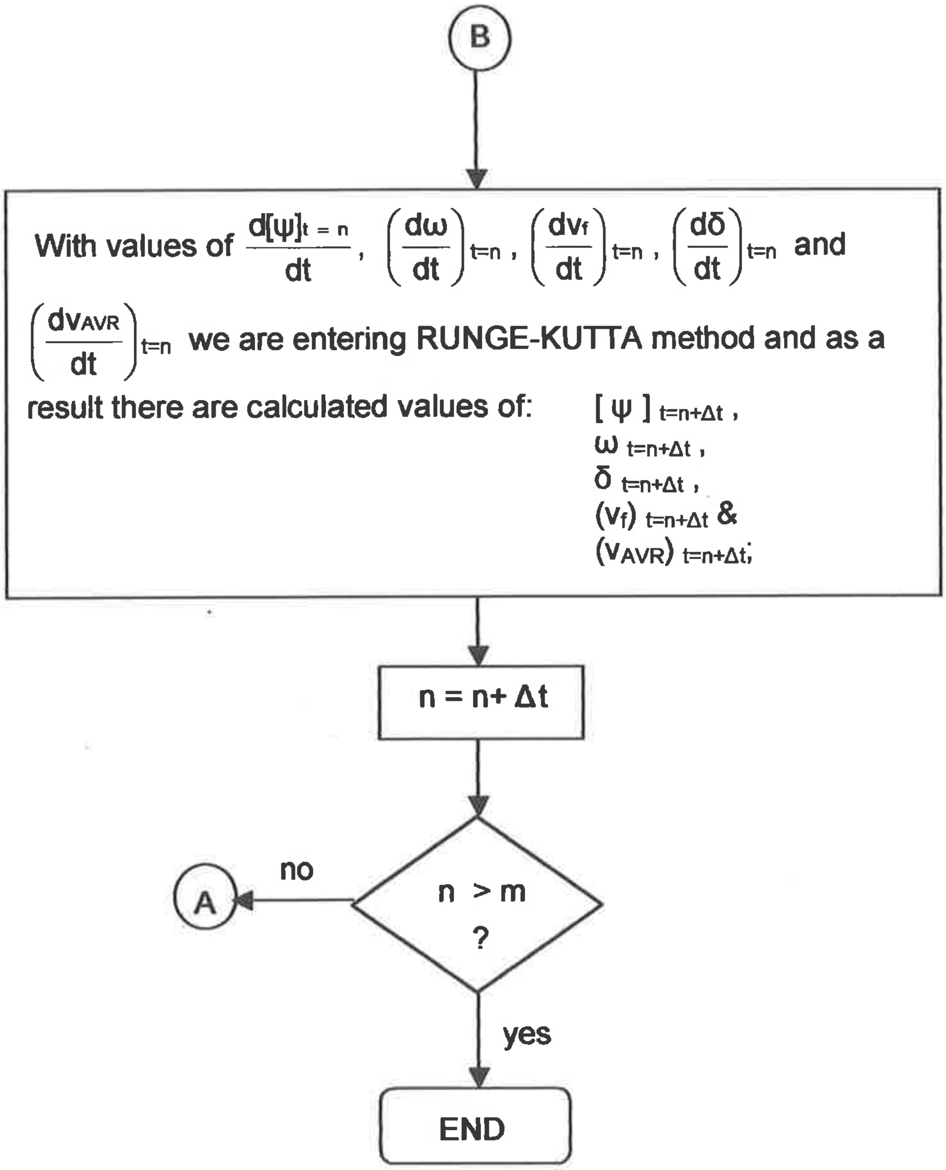


Figure 4.1-1

The equations in chapters 2 and 3 give rise to the following vectors for the system model. State vector [X] is given by:

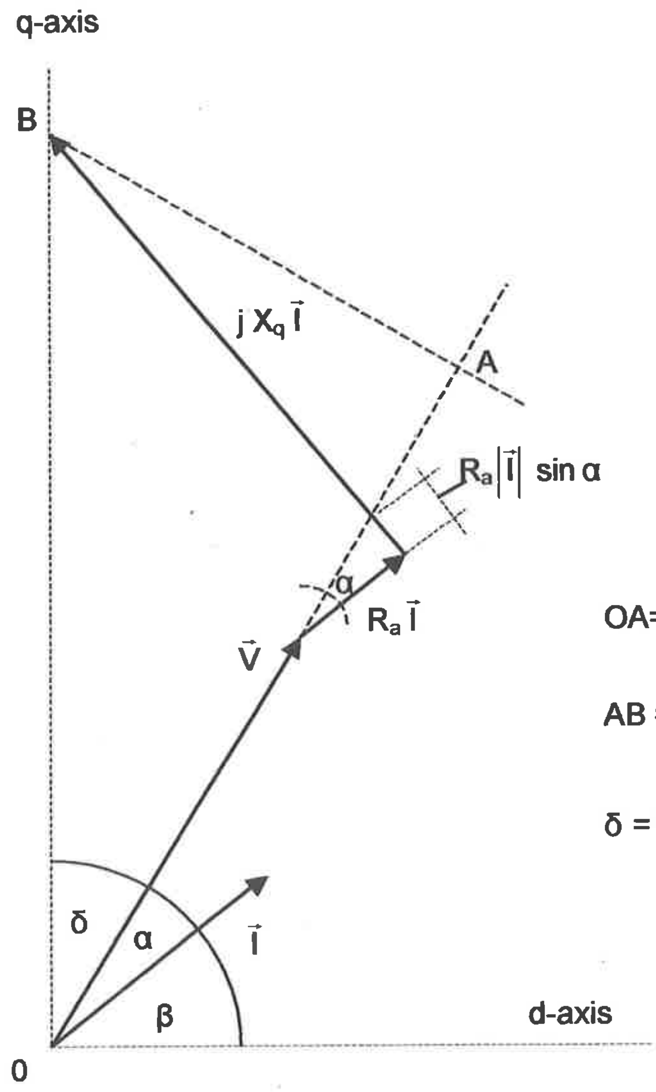
$[X]^t = [\Psi_I, \Psi_{II}, \Psi_{III}, \Psi_{fd}, \Psi_{1d}, \Psi_{1q}, \Psi_1', \Psi_2', \Psi_3', \acute{\omega}, \delta, v_f, v_{AVR}]$;

while input vector [U] is given by :

$[U]^t = [e_a, e_b, e_c, v_{ref}, P_{mech}]$;

4.2 Set up the initial conditions for the synchronous generator

A portion of the phasor diagram for a salient-pole synchronous machine acting as a generator is given in Figure (4.2-1) (ref. 8):



$$OA = |\vec{V}| + |\vec{I}| R_a \cos\alpha + |\vec{I}| X_q \sin\alpha$$

$$AB = (-|\vec{I}| R_a \sin\alpha + |\vec{I}| X_q) \cos\alpha$$

$$\delta = \tan^{-1}\left(\frac{AB}{OA}\right)$$

Figure (4.2-1)

where: $\vec{V}$ ... is the vector of the terminal voltage of the synchronous generator;

$\vec{I}$ ... is the vector of the stator current of the synchronous generator;

α ... is the angle between $\vec{V}$ and $\vec{I}$ in radians;

R_a ... is the working resistance in each of the 3-phases of the synchronous generator;

X_q ... is the quadrature-axis synchronous reactance;

From the Figure (4.2-1) follows the expressions (4.2-1), (4.2-2) and (4.2-3):

$$OA = |\vec{V}| + |\vec{I}| R_a \cos\alpha + |\vec{I}| X_q \sin\delta \quad (4.2\text{-}1)$$

$$AB = (-|\vec{I}| R_a \sin\alpha + |\vec{I}| X_q)\cos\alpha \quad (4.2\text{-}2)$$

$$\delta = \tan^{-1}\left(\frac{AB}{OA}\right) = = \tan^{-1}\left(\frac{(-|\vec{I}| R_a \sin\alpha + |\vec{I}| X_q)\cos\alpha}{|\vec{V}| + |\vec{I}| R_a \cos\alpha + |\vec{I}| X_q \sin\alpha}\right) \quad (4.2\text{-}3)$$

This procedure establishes the rotor angle δ. From Figure (4.2-1) also follows:

$$\delta + \alpha + \beta = \frac{\pi}{2} \quad (4.2\text{-}4)$$

where: δ ... is the rotor angle (angle between $\vec{V}$ and q-axis) in radians;

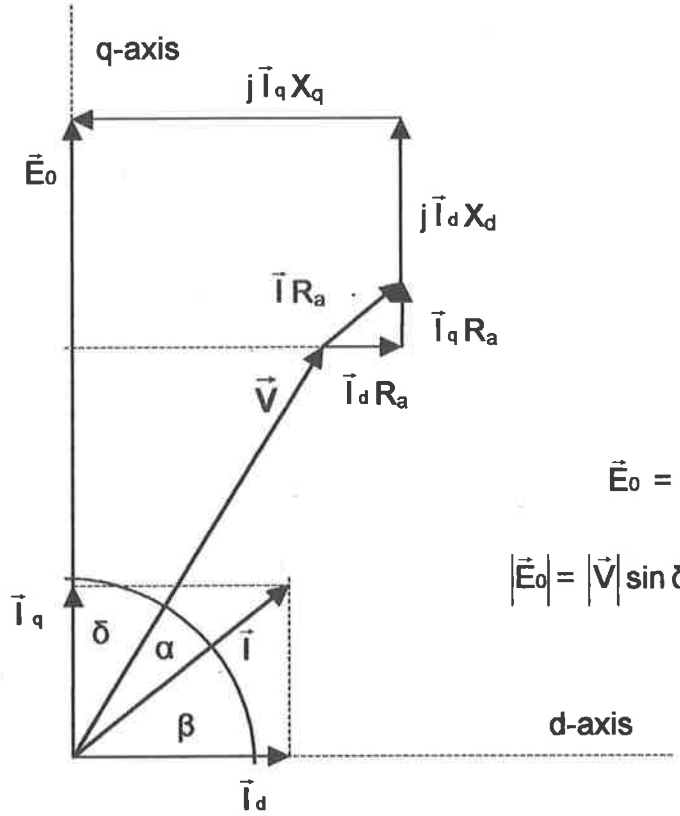
α ... is the phase angle between $\vec{V}$ and $\vec{I}$ in radians;

β ... is the angle between $\vec{I}$ and d-axis in radians;

As α is known for a given load and δ is calculated, the value of β is calculated from (4.2-4) and given by expression (4.2-5):

$$\beta = \frac{\pi}{2} - \delta - \alpha \quad (4.2\text{-}5)$$

From Figure (4.2-2) now is easy to determine $\vec{E}_0$.



$$\vec{E}_0 = \vec{V} + \vec{I}_q R_a + j\vec{I}_d X_d + j\vec{I}_q X_q$$

$$|\vec{E}_0| = |\vec{V}| \sin\delta + |\vec{I}| (R_a \sin\beta + X_d \cos\beta)$$

Figure (4.2-2)

As it could be obvious the following set of equations could be written:

$$\vec{E}_0 = \vec{V} + \vec{I}_q R_a + j\vec{I}_d X_d + j\vec{I}_q X_q \tag{4.2-6}$$

$$E_0 = |\vec{E}_0| = |\vec{V}| \sin\delta + |\vec{I}| (R_a \sin\beta + X_d \cos\beta) \tag{4.2-7}$$

where: E_0 ... is the emf induced in the stator on the open circuit test; $E_0 = f(I_{fd})$

By knowing E_0 from (4.2-7) and X_{ad} from (2.2-11) it is simple to determine i_{fd0}:

$$i_{fd0} = \frac{3}{2} * \frac{E_0}{X_{ad}} \tag{4.2-8}$$

To determine voltage supplied on the rotor winding e_{fd0} we use Ohm's law:

$$e_{fd0} = r_{fd} * i_{fd0} \qquad (4.2\text{-}9)$$

By knowing P, Q and V_n of the generator it is very easy to determine:

$$i_{phase} = \frac{S}{\sqrt{3}V_n} \qquad (4.2\text{-}10)$$

$$S = (P^2 + Q^2)^{½} \qquad (4.2\text{-}10a)$$

where: i_{phase} ... is the current in each stator phase;

P ... is the real power;

Q ... is the reactive power;

S ... is the apparent power;

V_n ... is a terminal voltage (for Δ connection);

And the voltage in each of the transformer phases is:

$$e_{phase} = \sqrt{3}\ V_n \qquad (4.2\text{-}10b)$$

By using the equivalent circuit in Figure (4.2-3) of the transformer per phase it is easy to calculate the initial conditions on the secondary side of the transformer:

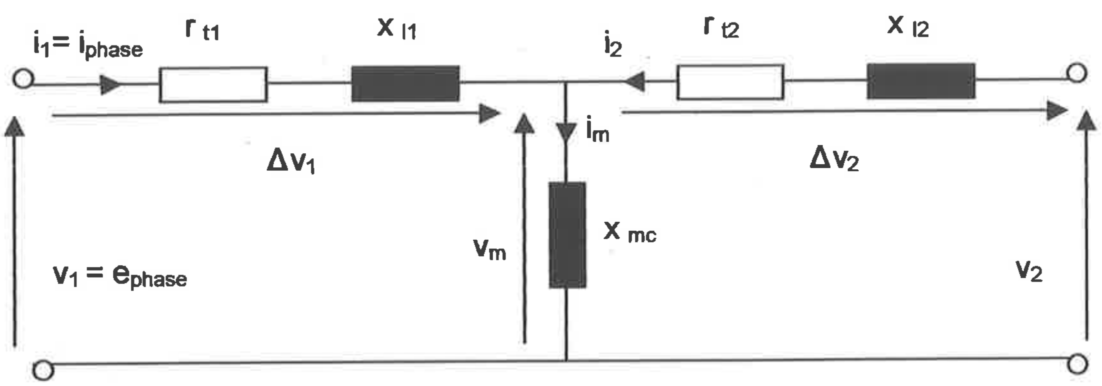


Figure (4.2-3)

$$i_1 = i_{phase} = I_{phase} \angle (\mathrm{atan}((\frac{-Q}{P}) + \frac{\pi}{6})) \quad (4.2\text{-}11)$$

$$v_1 = v_{phase} = V_{phase} \angle (\frac{\pi}{6}) \quad (4.2\text{-}12)$$

In each case, the $\frac{\pi}{6}$ phase shift is caused by delta connection on the primary side of transformer.

$$\Delta v_1 = i_1 z_{1t} \quad (4.2\text{-}13)$$

$$x_{l1} = x_{t1} - x_{mc} \quad (4.2\text{-}14)$$

$$z_{1t} = (r_{1t}^2 + x_{l1}^2)^{½} \angle (\mathrm{atan}(\frac{x_{l1}}{r_{t1}})) \quad (4.2\text{-}15)$$

$$v_m = v_1 - \Delta v_1 \quad (4.2\text{-}16)$$

$$i_m = \frac{v_m}{3\, x_{mc}} \qquad \text{(for } \Delta \text{ connection)} \quad (4.2\text{-}17)$$

$$i_2 = i_m - i_1 \quad (4.2\text{-}18)$$

$$\Delta v_2 = i_2 z_{2t} \quad (4.2\text{-}19)$$

$$z_{2t} = (r_{2t}^2 + x_{l2}^2)^{½} \angle (\mathrm{atan}(\frac{x_{l2}}{r_{t2}})) \quad (4.2\text{-}20)$$

$$v_2 = v_m - \Delta v_2 \quad (4.2\text{-}16)$$

where:

Δv_1 ... is the voltage drop across the z_{1t} on the primary side of the transformer;

z_{1t} ... is the impedance of the primary side of the transformer;

v_m ...is the voltage drop caused by magnetizing current i_m;

i_m ... is the magnetizing current;

Δv_2 ... is the voltage drop across the z_{2t} on the secondary side of the transformer;

z_{2t} ... is the impedance of the secondary side of the transformer;

x_{l1} … is the leakage reactance of the primary side of the transformer;

x_{l2} … is the leakage reactance of the secondary side of the transformer;

x_m … is mutual-reactance between the primary and secondary windings;

5. TEST OF GENERATOR MODELS

The sequence of faults described in Section (3.3) will be applied to the test system at the location shown in Figure (5.1). For the first test runs a basic form of the generator model will be used, in which a number of simplifying assumptions are made:

- the stator induced emf's assume a constant rotational speed,
- there is no saturation of the generator's magnetic circuits &
- the generator/turbine rotors act as a single inertial mass.

This will be referred to as the 'base model' whose performance will be compared in later runs to that of models with specific enhancement. The data for the base model are typical for a salient-pole machine in the range 200-300 MW, and are given in Section (8) [Appendix].

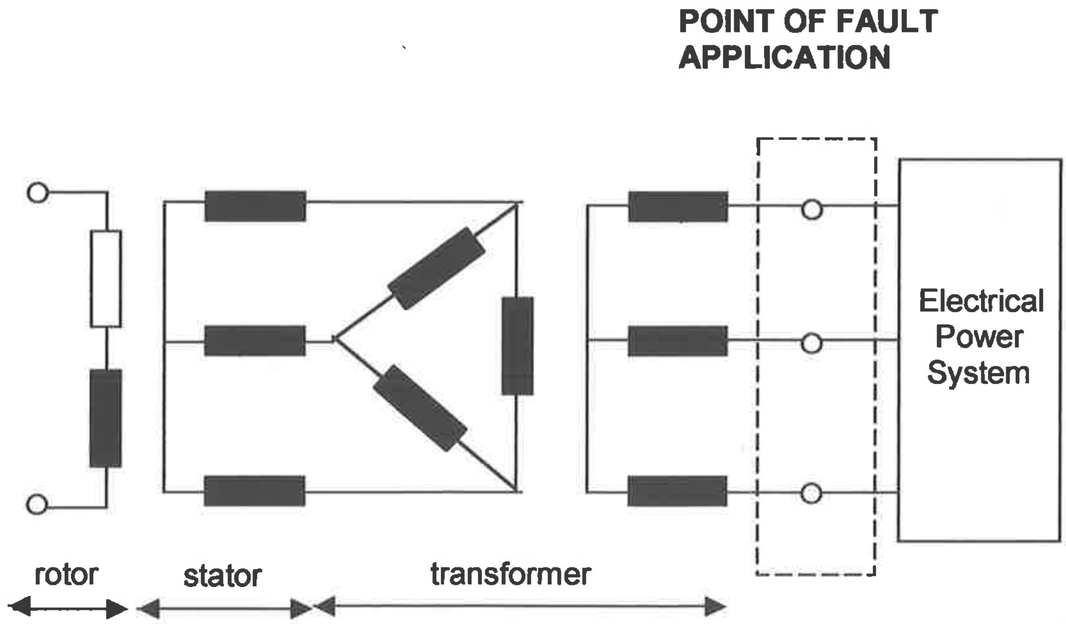
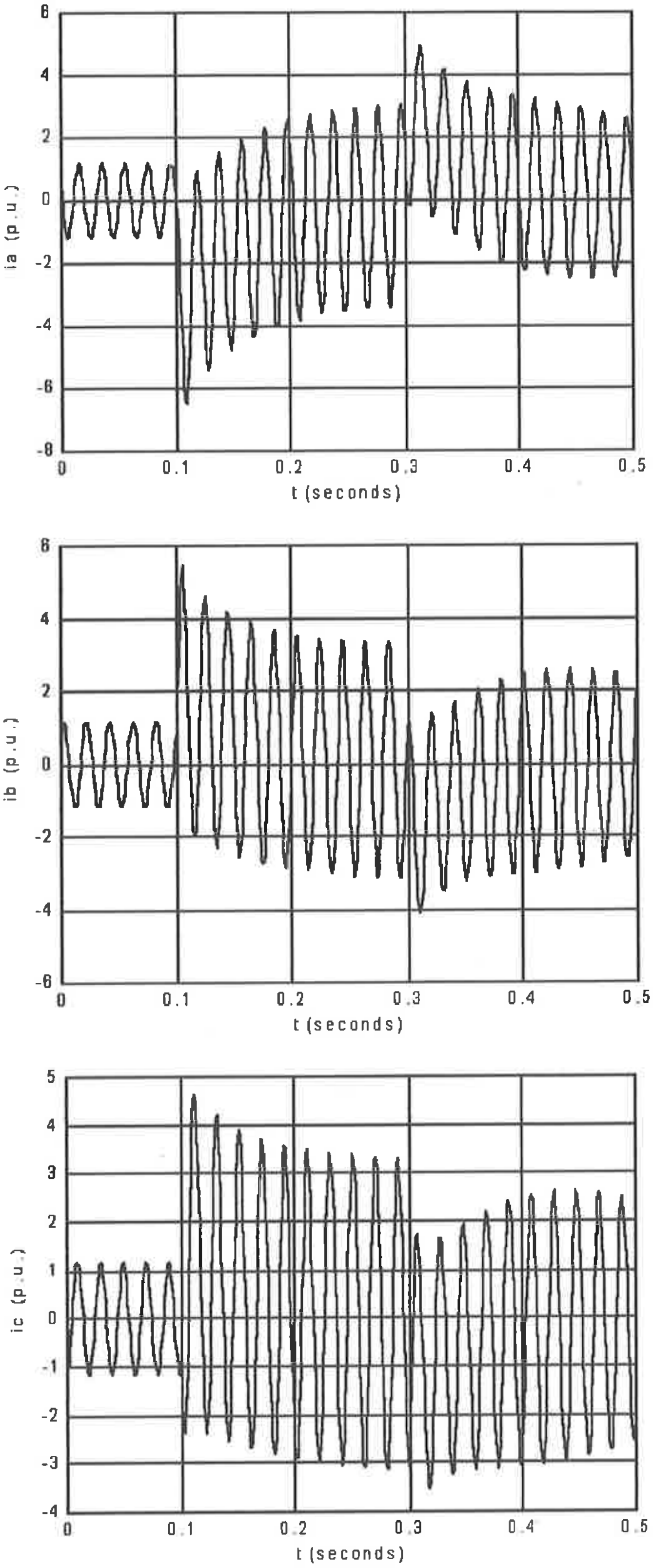


Figure (5.1)

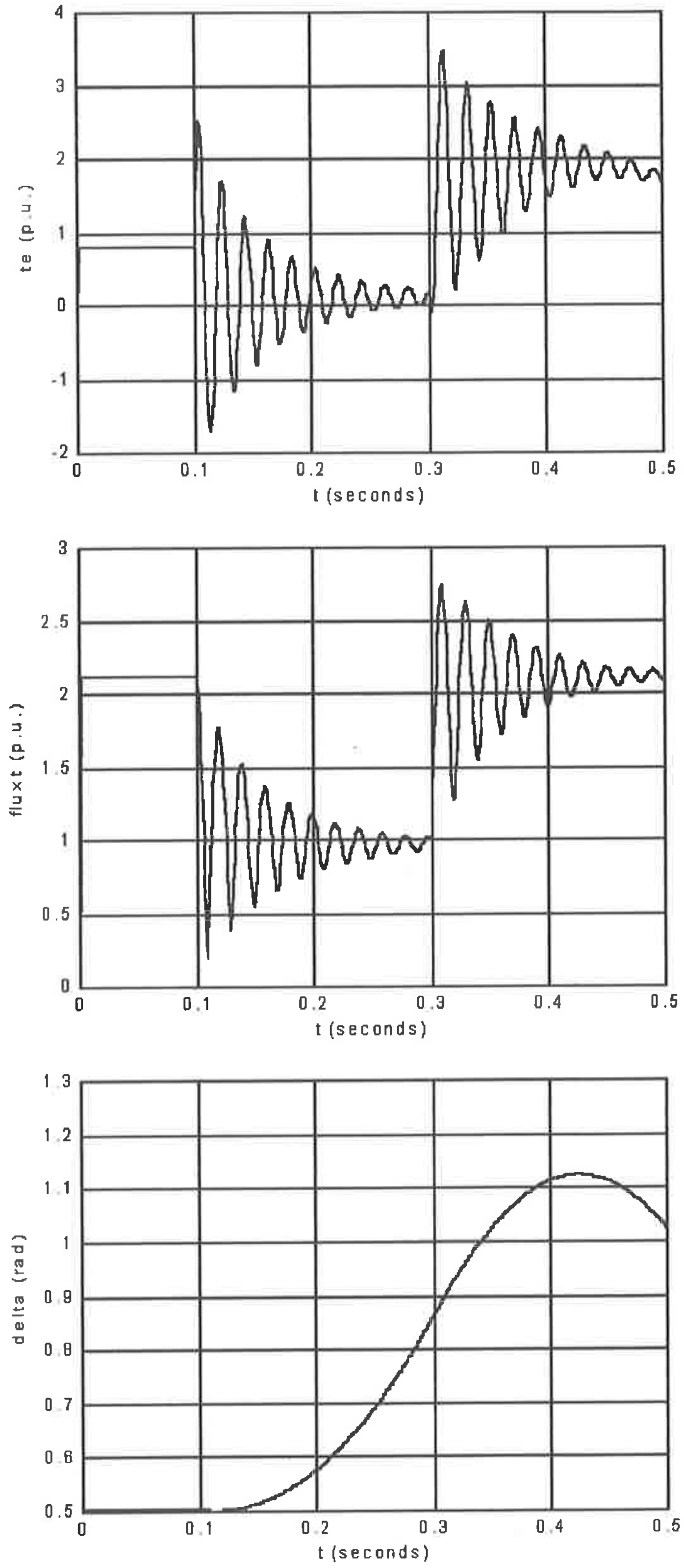
5.1 Short circuit test performed on the base model

The time period for which the behavior of the synchronous generator has been observed is 0.5 seconds. Short circuit tests have been applied between 0.1 and 0.3 seconds and then each of them has been removed. The analysis gave us the following set of graphical results in which the variables plotted are:

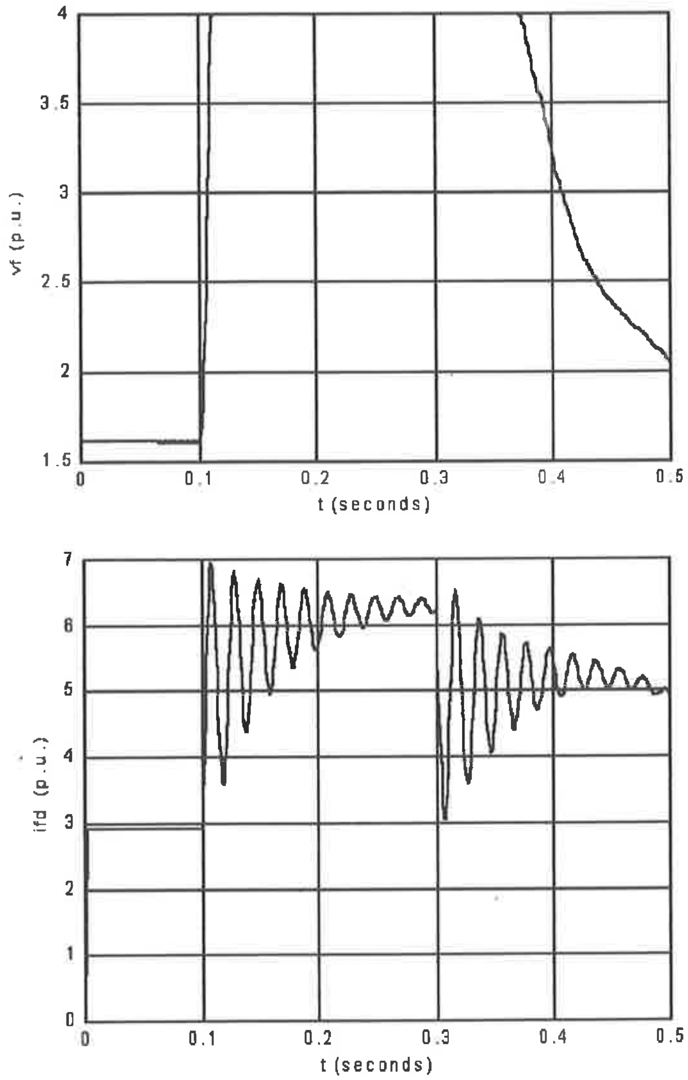
- 3-phase stator currents (i_a, i_b ,i_c),
- electromagnetic torque (t_e),
- air gap flux magnitude ($flux_t$),
- rotor angle (delta),
- field voltage (v_f) &
- field current (i_f).



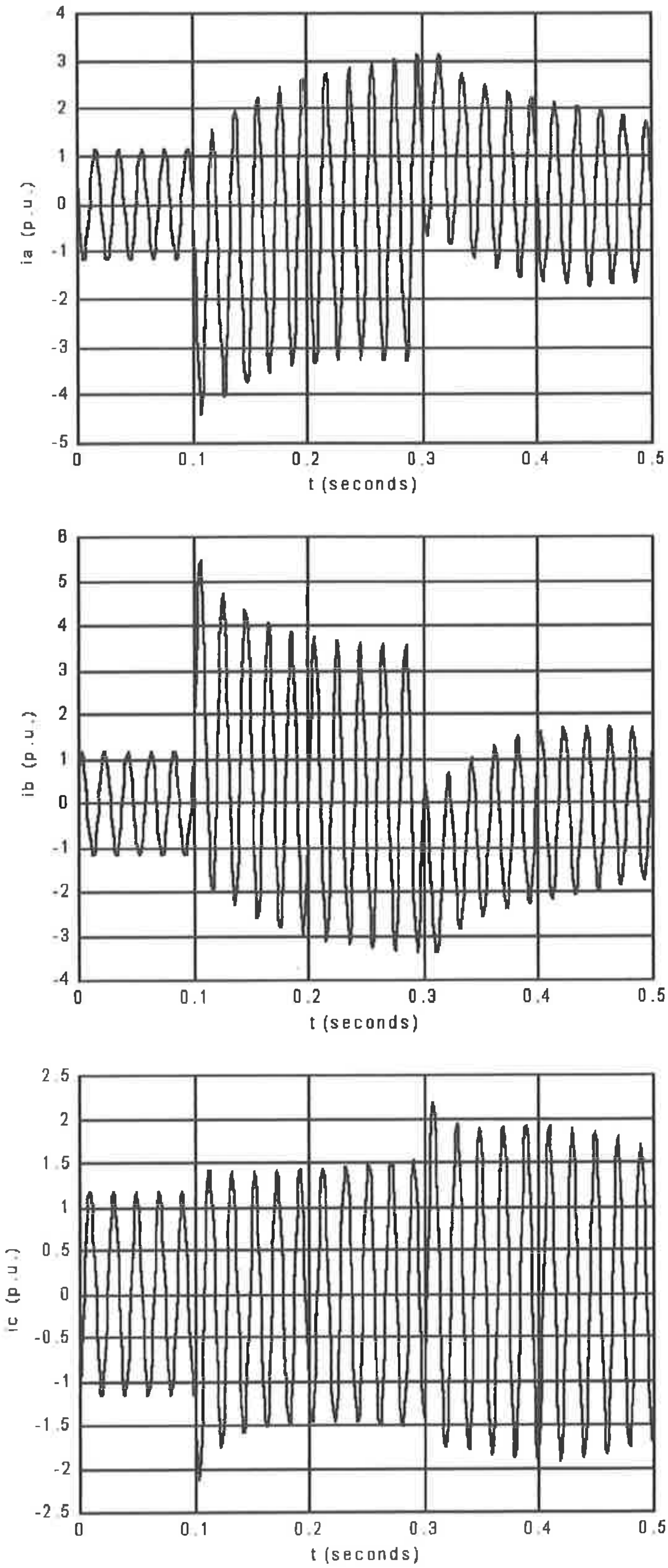
3-phase short circuit



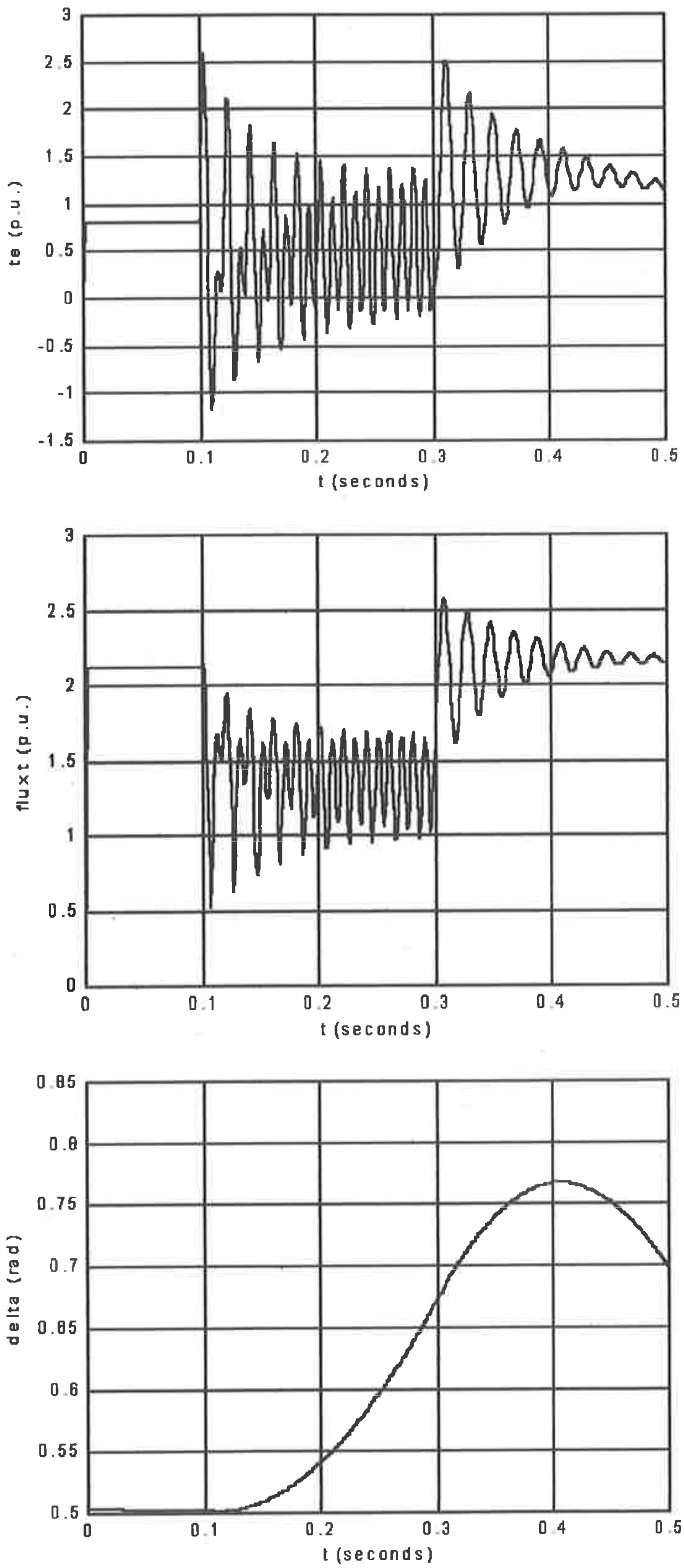
3-phase short circuit



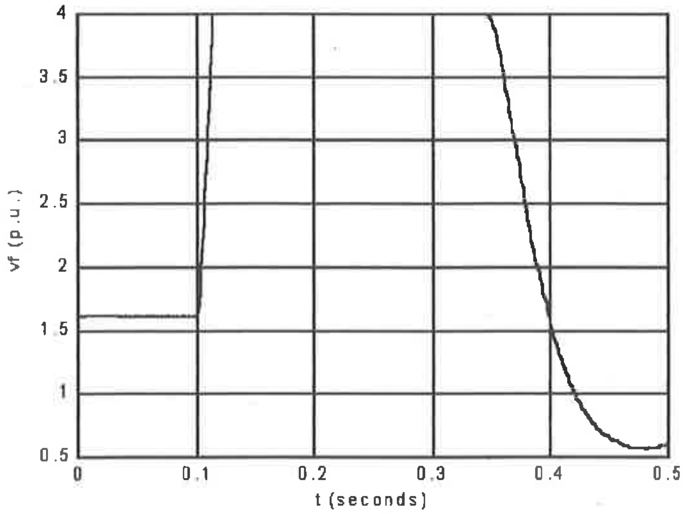
3-phase short circuit

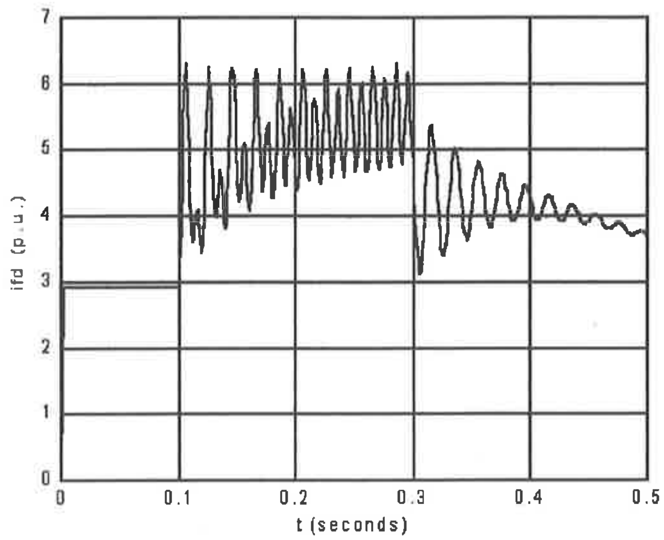


2-phase short circuit with ground

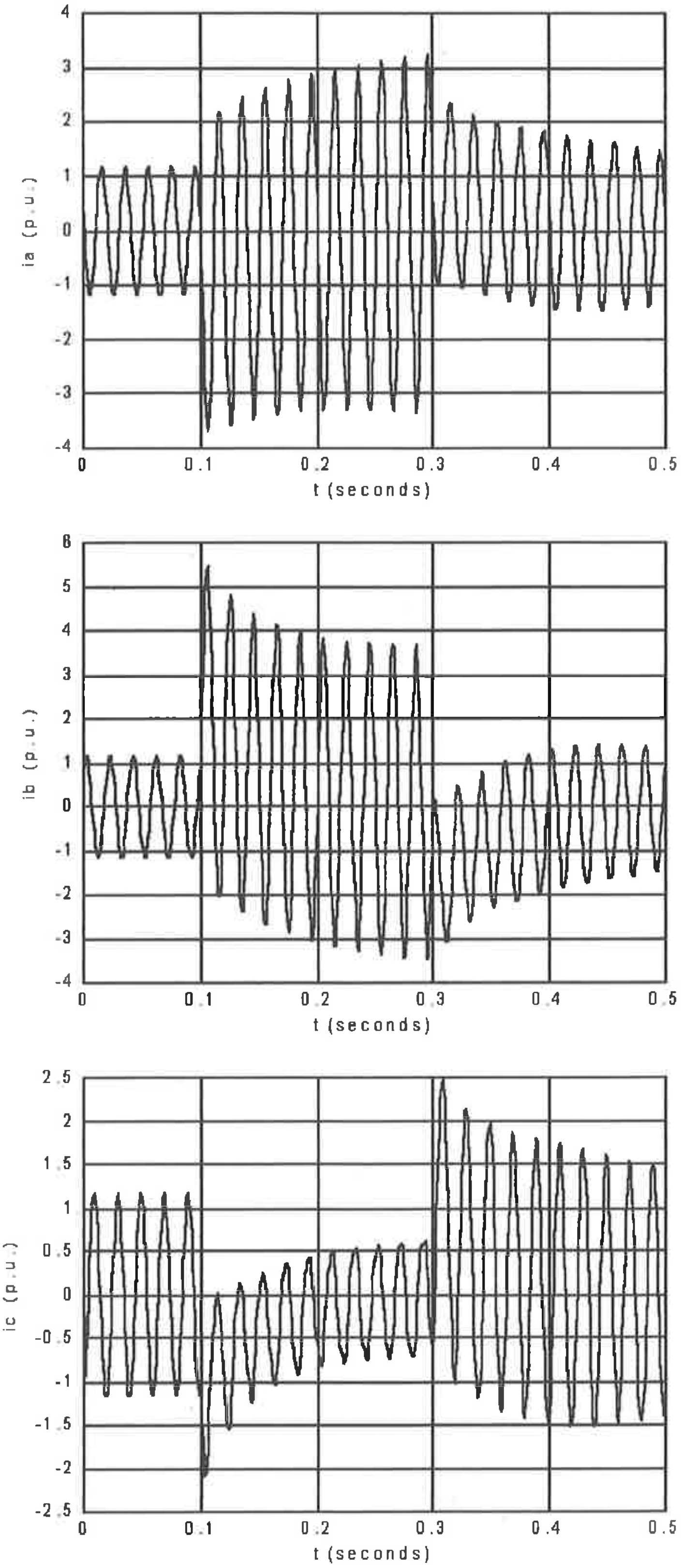


2-phase short circuit with ground

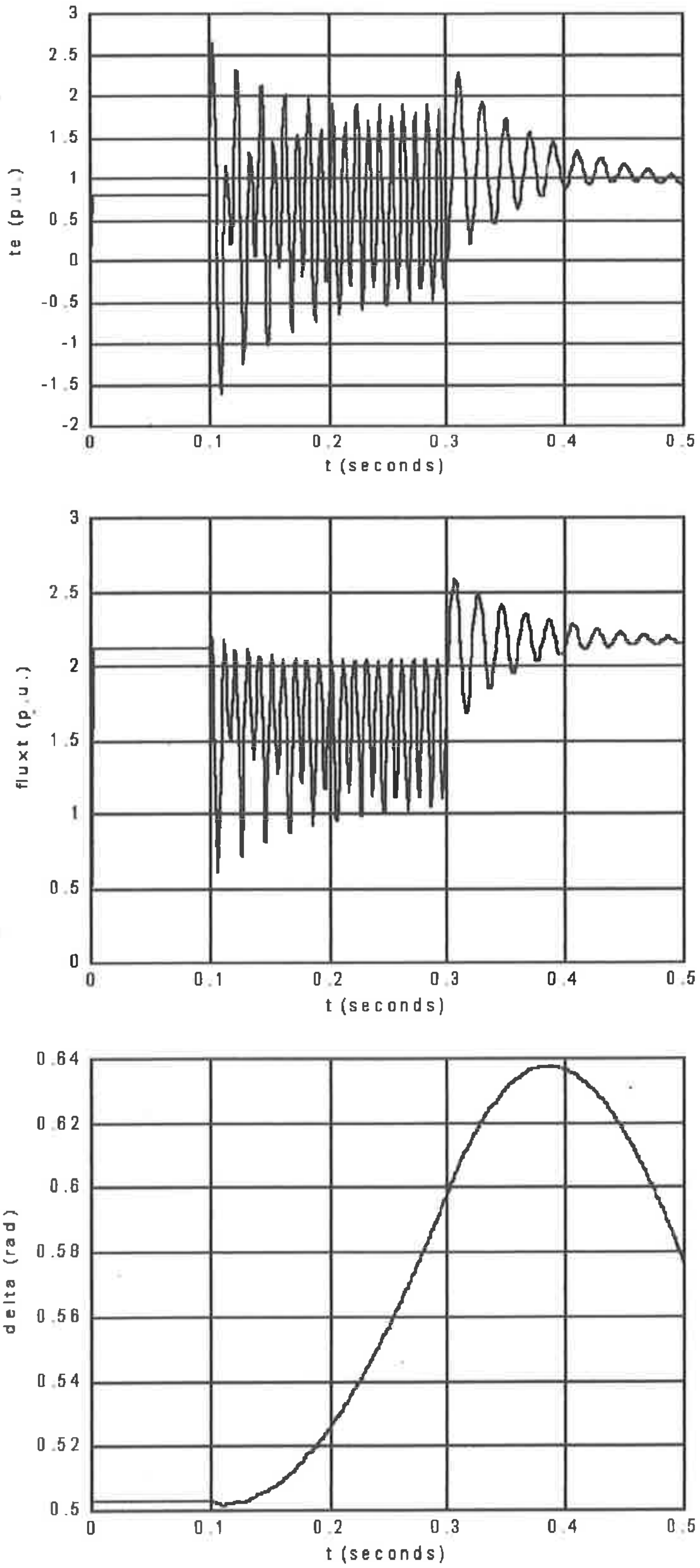




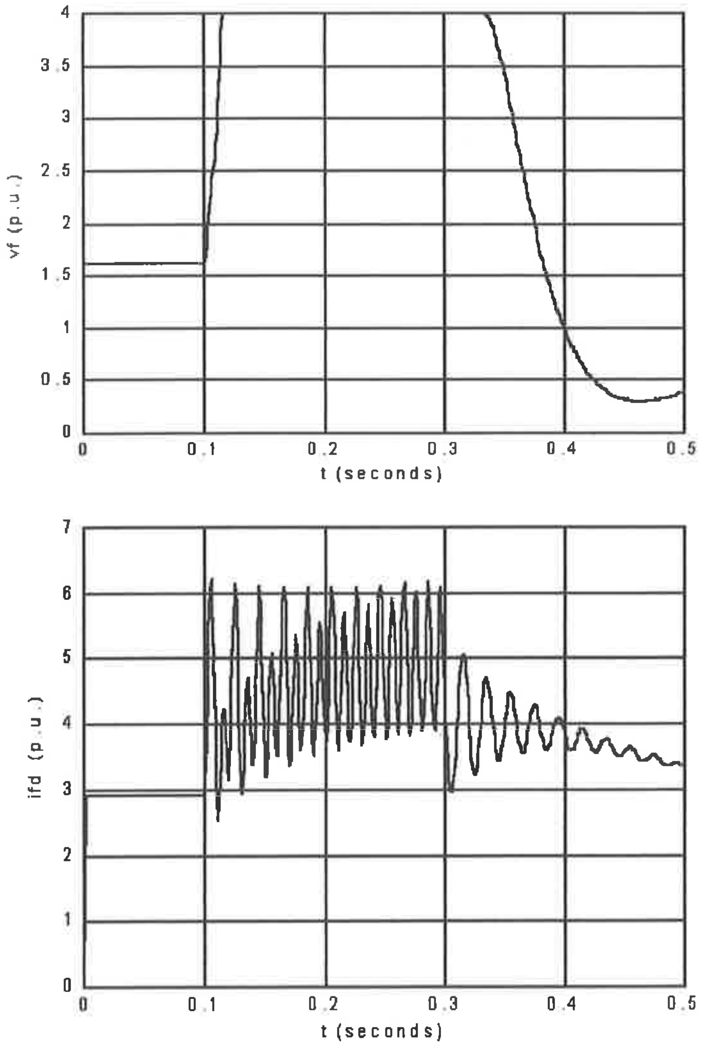
2-phase short circuit with ground



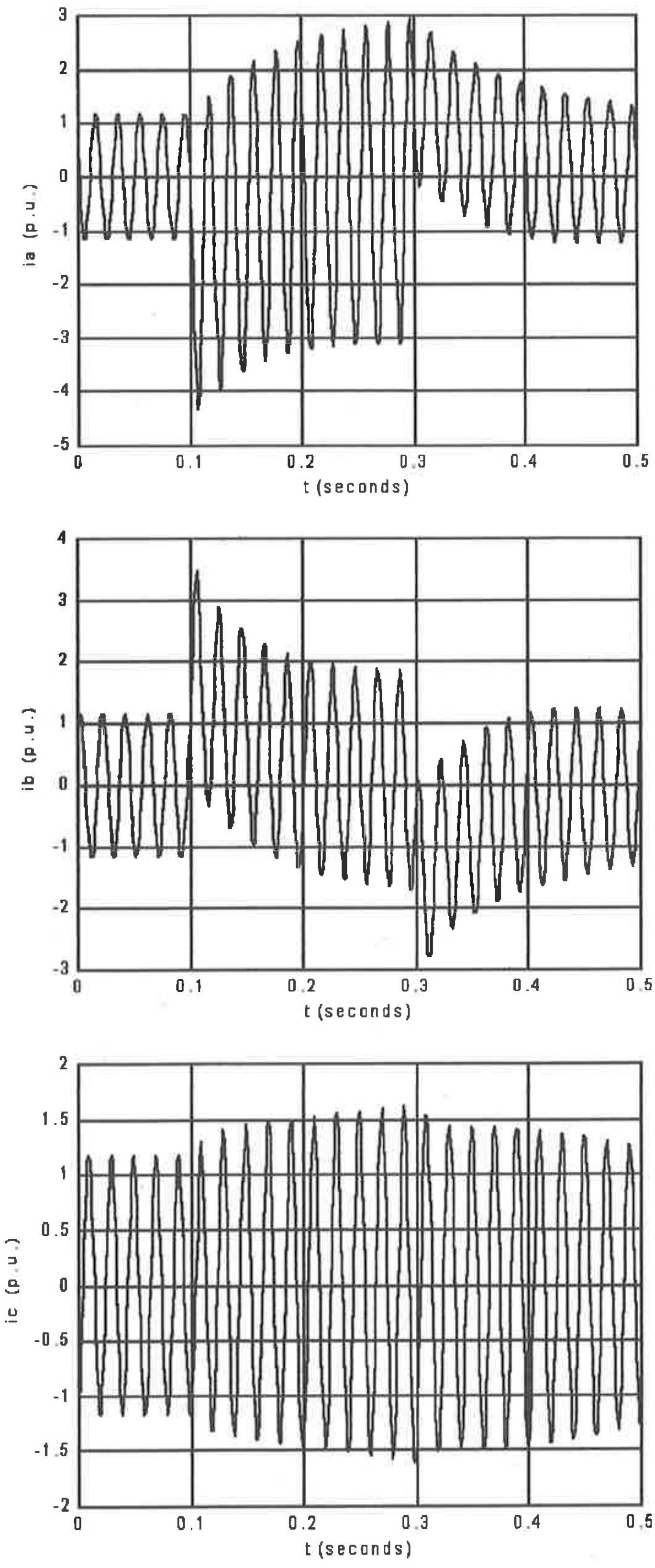
2-phase short circuit



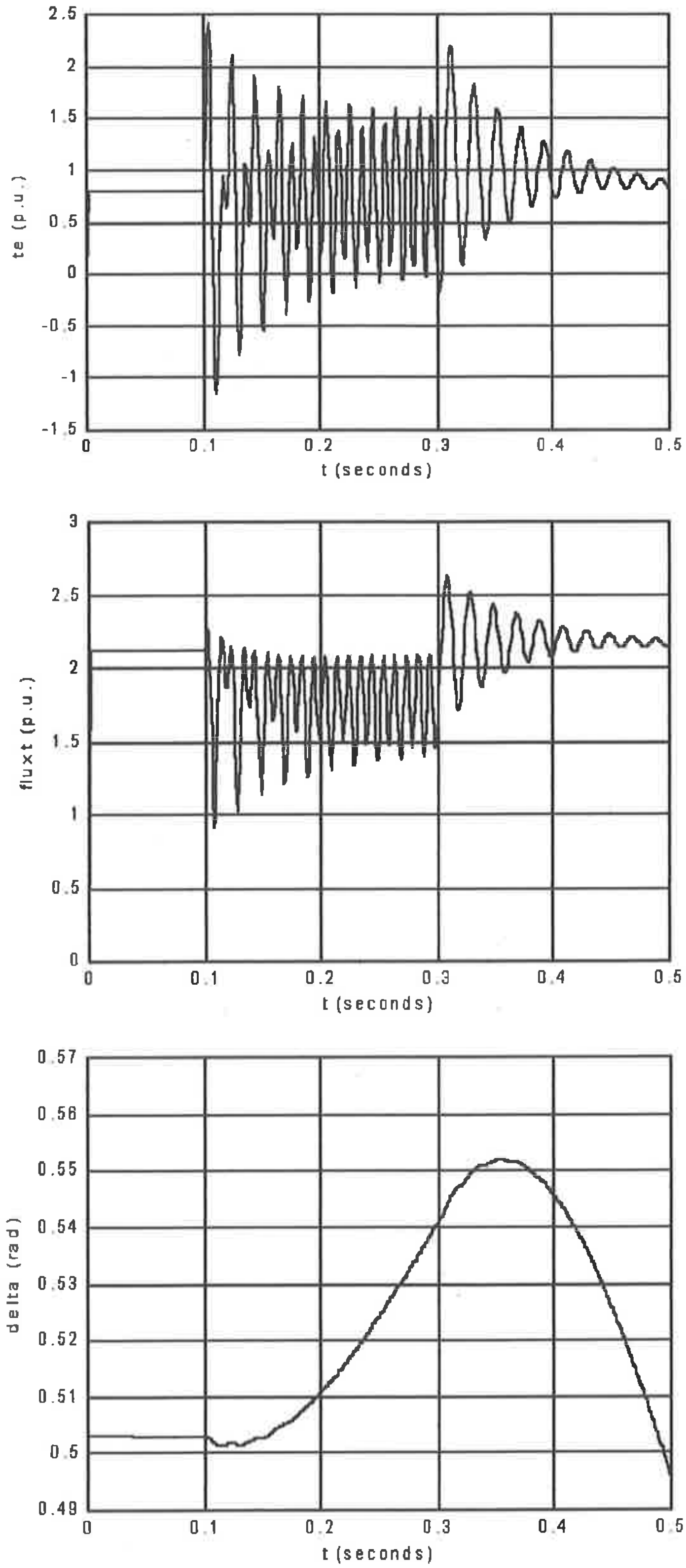
2-phase short circuit



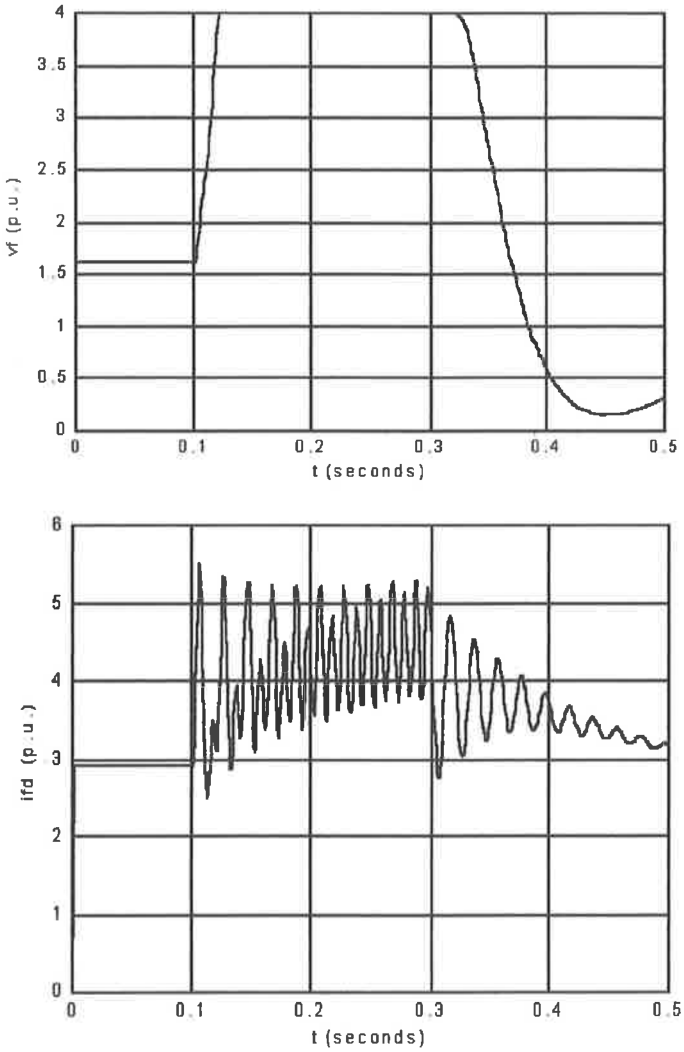
2-phase short circuit



1-phase short circuit



1-phase short circuit



1-phase short circuit

As the nature of the 3-phase short circuit is such that is known as a 'balanced fault' the presence of 50Hz oscillations on the graph of t_e (electrical torque) is what was expected. The same phenomena reflects on graphs $i_{fd} = f_1(t)$ (field current as a function of time) and $\Psi = f_2(t)$ (total air-gap flux as a function of time), while in all other cases the presence of 100 Hz oscillations during the short-circuit for t_e , i_{fd} and Ψ was caused by negative sequence components of the fault currents in these unbalanced cases. These components convert to double frequency terms for rotor-based phenomena. These results clearly show the amount of detail given by this form of the generator model, allowing direct comparison with site measurements.

5.2 Variable speed model

In the base model, it was assumed that the rotor speed was constant when computing induced emf's. In this section, we investigate the change in response due to an accurate calculation of the rotor speed at all times. To implement this change it is necessary to implement the following equation at the very beginning of the Runge-Kutta loop:

$$\omega = 2\,\pi\, f\,(\,1+\acute{\omega})\qquad (5.2\text{-}1)$$

where: ω ... is angular frequency [rad/s];

f ... is the supply frequency [1/sec or Hz];

$\acute{\omega}=\dfrac{\Delta\delta}{\Delta t}$ ... is deviation in rotor speed;

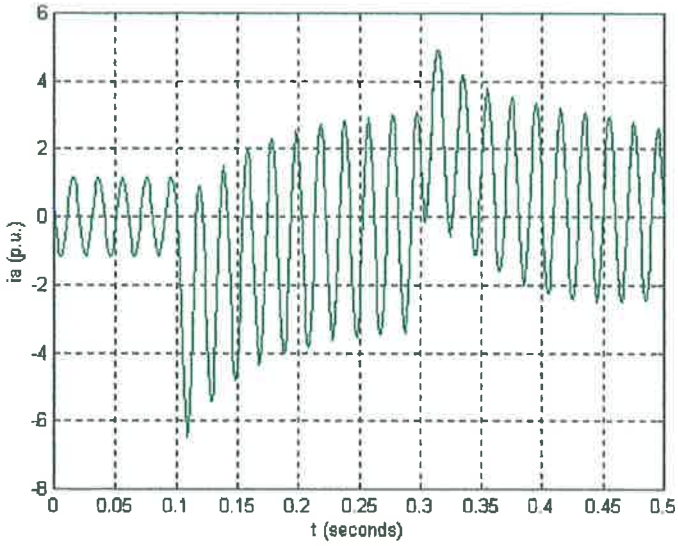
The rotor acceleration is already described in chapter (2.2) with expression (2.2-28). It is also important to mention that before we start to calculate matrix of reactances for each step of Runge-Kutta the following equation (5.2-2) describes the instantaneous value of the angle Θ:

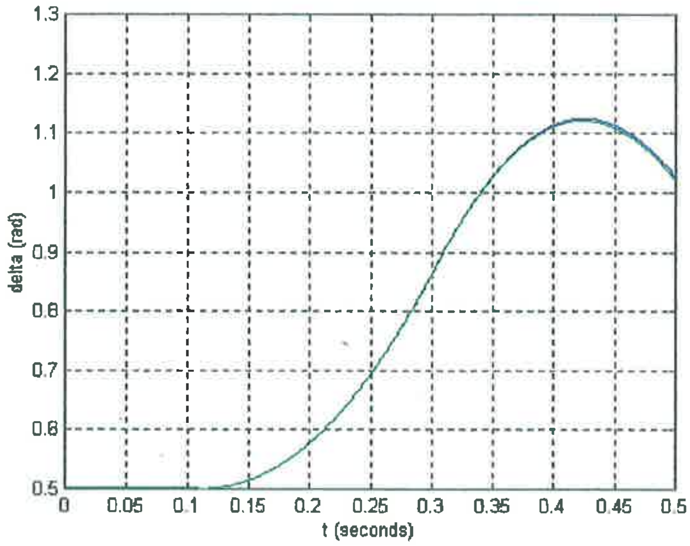
$$\Theta = 2\,\pi\, f * t + \delta \qquad (5.2\text{-}2)$$

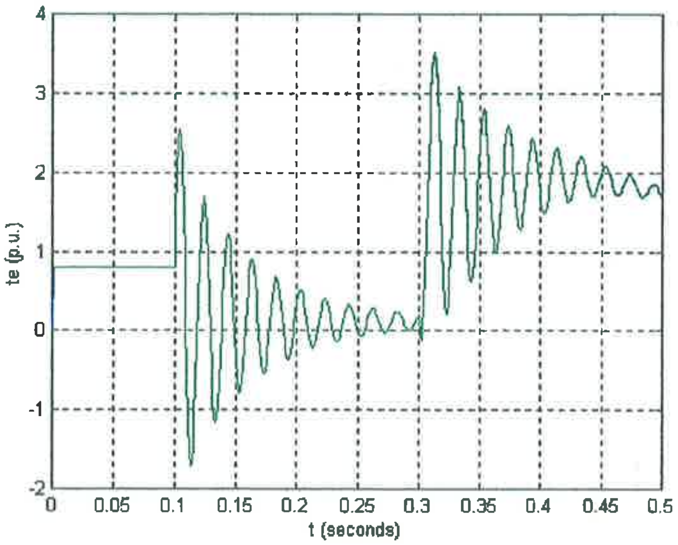
If we compare this equation with the similar one (2.2-25) it is obvious that the angular frequency of the rotor is no longer a constant. Instead of that it becomes the function of the time. The relative change of the angle δ in time is also been considered:

$$\frac{d\delta}{dt} = \acute{\omega} \qquad (5.2\text{-}3)$$

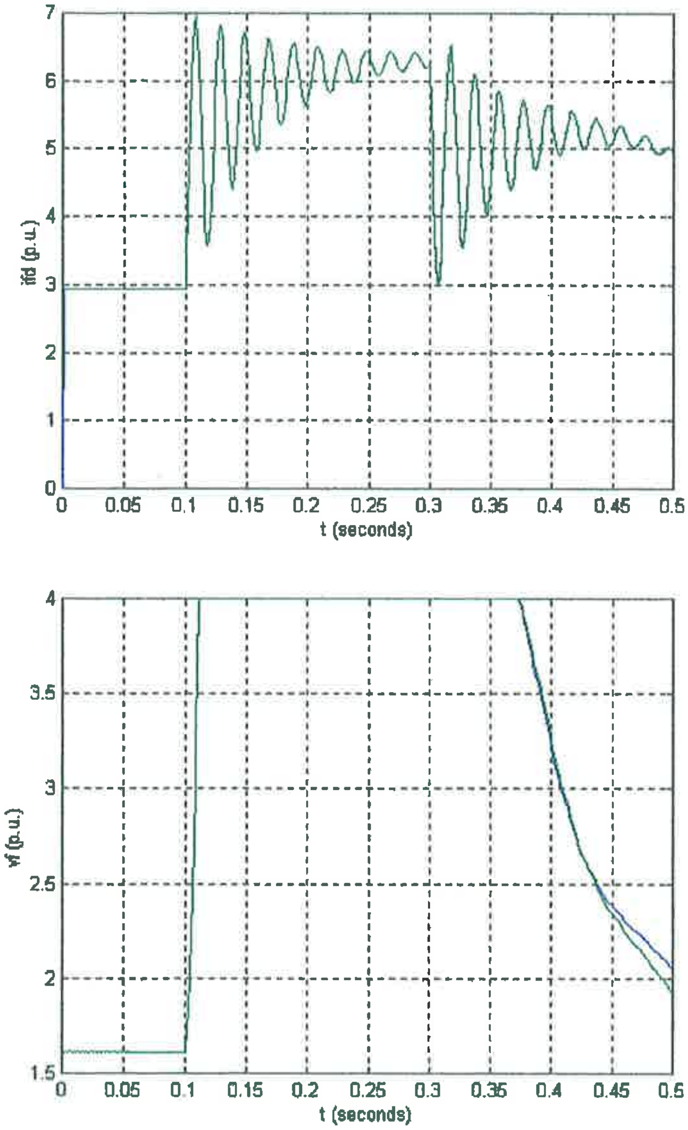
The time period in that the behavior of synchronous generator has been observed is again 0.5 sec as well as the applied short circuit test were observed again between 0.1 and 0.3 seconds. The analysis gave us the following set of graphical results that had been compared to base model (graphs for the base model have been represented with dark blue, while graphs for the variable speed model have been represented with green):



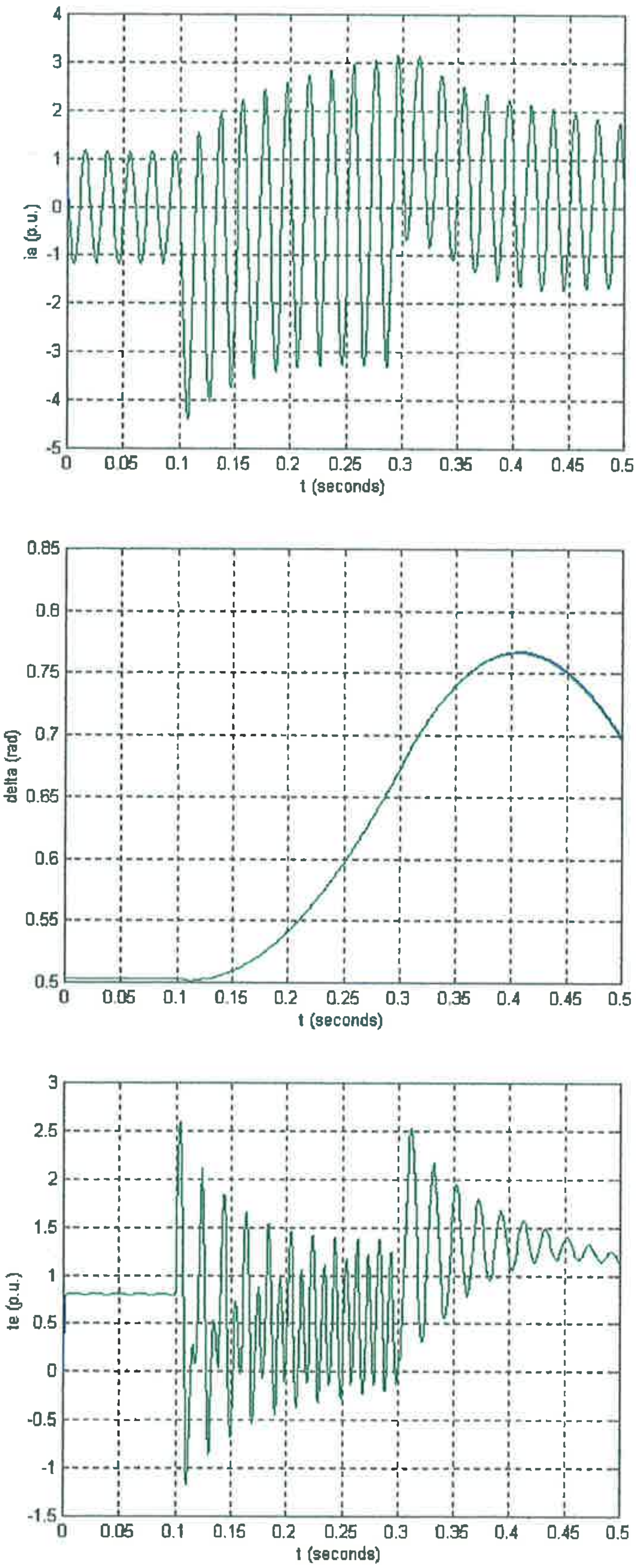




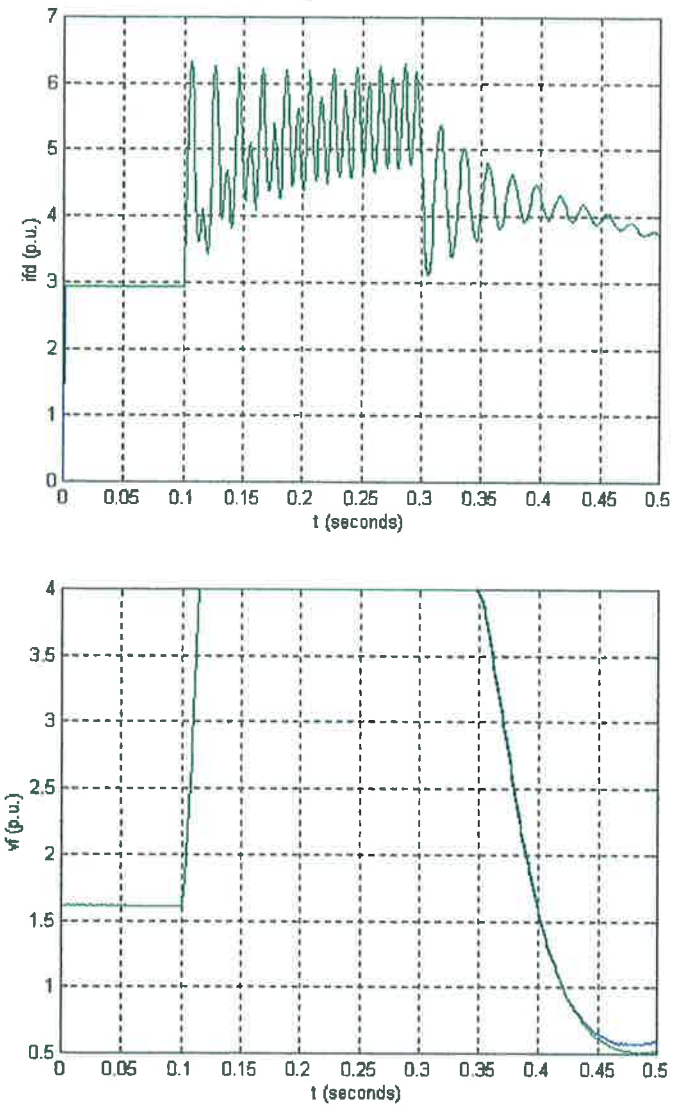
3-phase short circuit



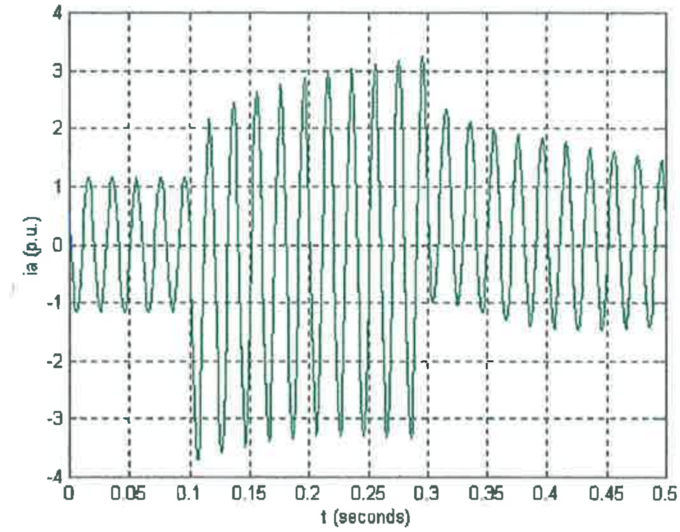
3-phase short circuit

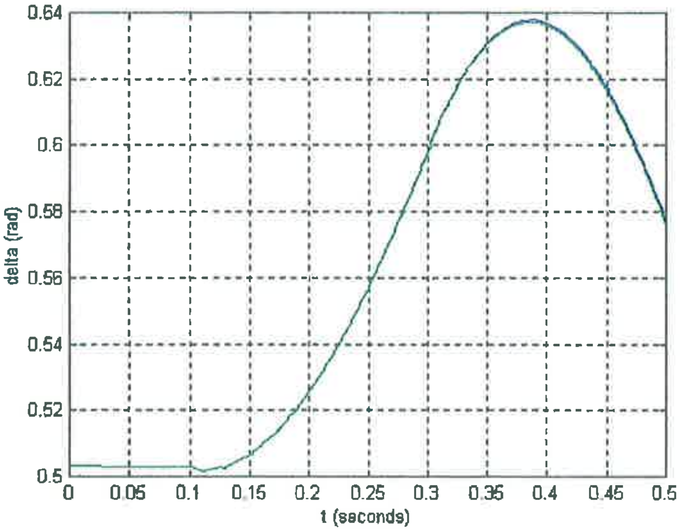


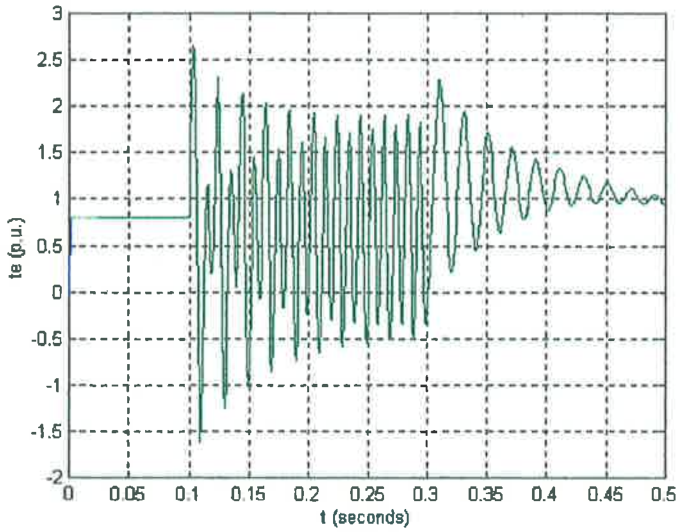
2-phase short circuit with ground



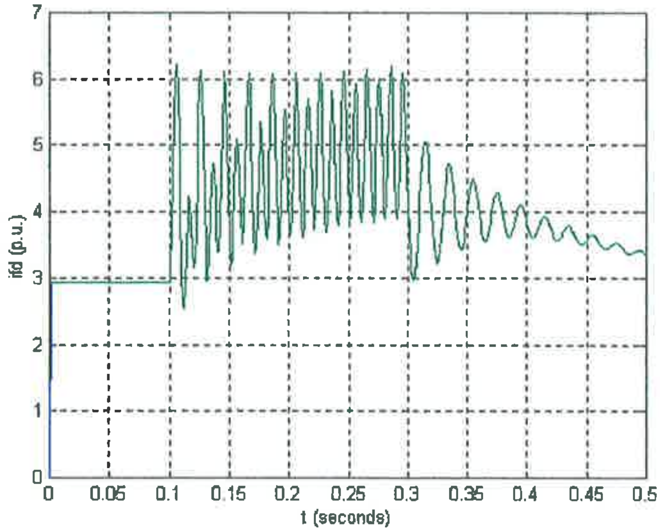
2-phase short circuit with ground

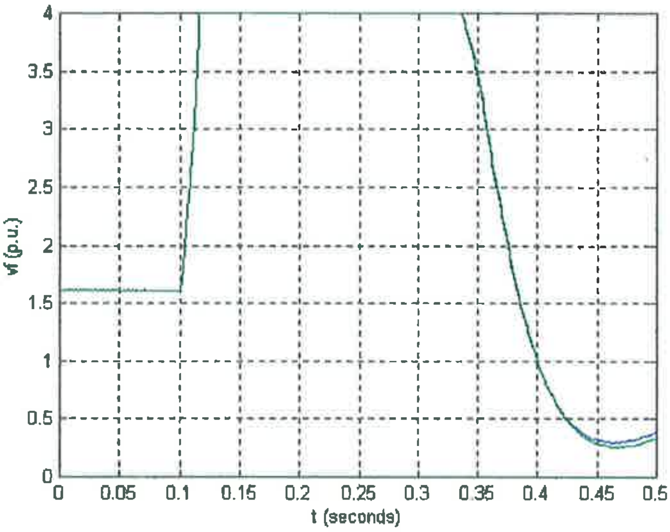




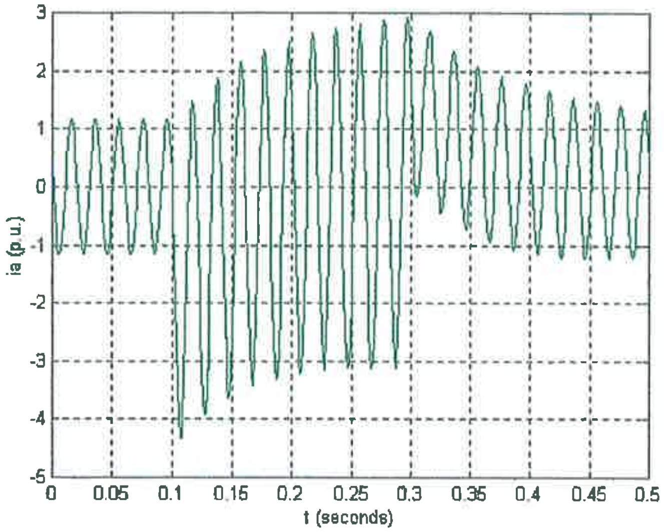


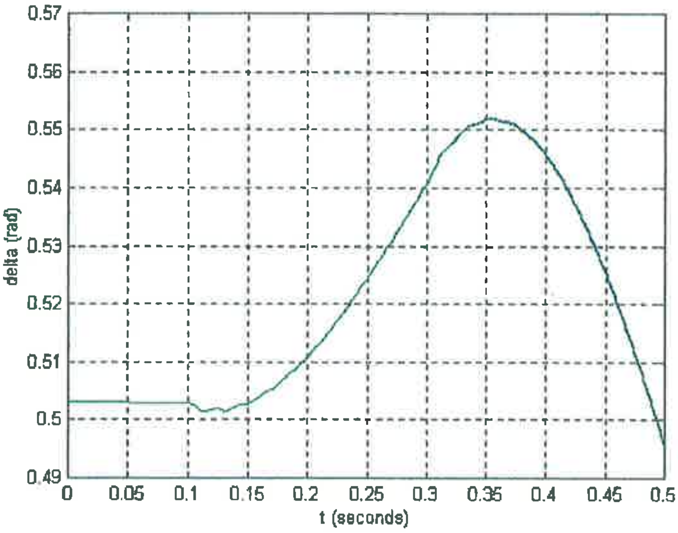
2-phase short circuit

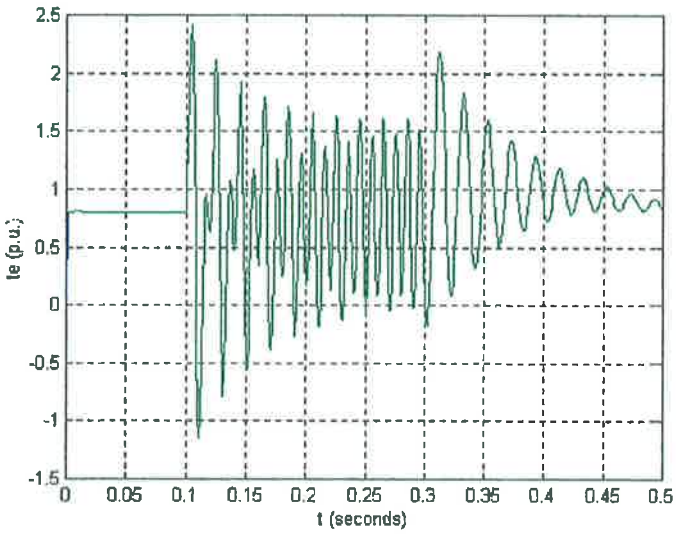




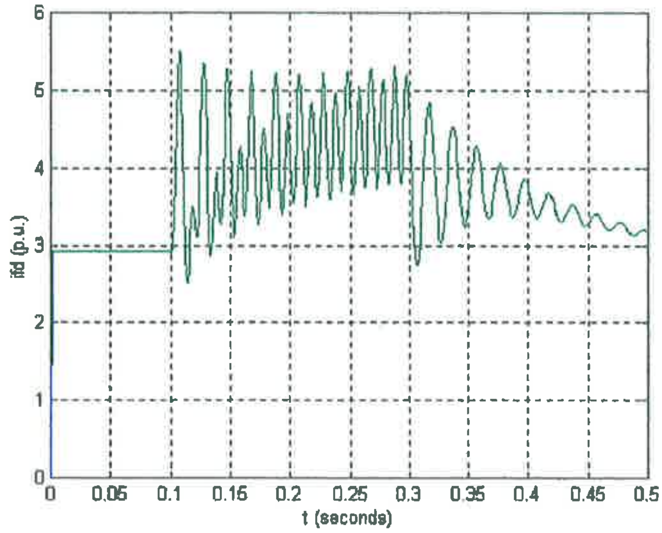
2-phase short circuit

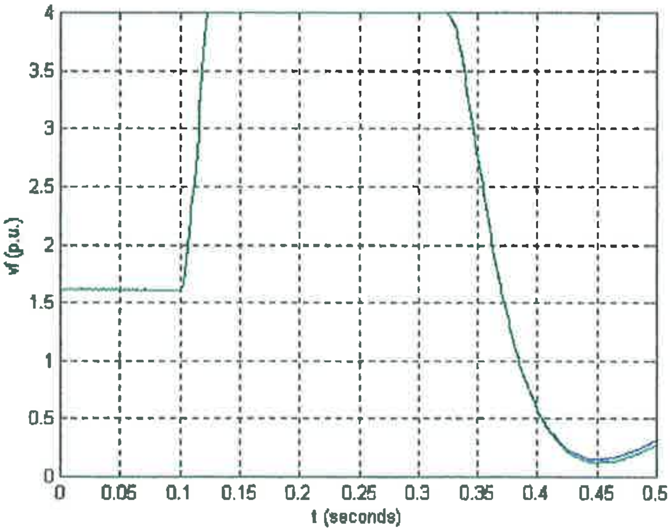






1-phase short circuit





1-phase short circuit

From the given graphs it is obvious that the difference between the results driven by base model and with speed variation model is practically indistinguishable, with insignificant differences in graphs for v_f and δ (for each separate case of specific kind of short circuit). The slight differences were observable only after removing the faults.

5.3 Saturation model

There are two basic cases that will be considered in this chapter:

1- saturation of the synchronous generator in d-axis only &
2- saturation of the synchronous generator in both d- and q-axes;

All previous results were obtained assuming the machine operated in a linear part of the B-H characteristic. Now our consideration will be directed to observe the synchronous machine in saturation, using similar methods to those of Harley et al (ref. 9).

5.3.1 Saturation of the synchronous generator in d-axis only

The starting point for this procedure is the calculation of the total electro-magnetic flux linkage of the machine for the initial operating condition (this absolute value of the total electro-magnetic flux linkage was calculated and located relative to the position of the phase a of the stator of the synchronous generator). By assuming the B-H characteristic (represented by the OPEN CIRCUIT CHARACTERISTIC of the synchronous generator) and by knowing the absolute value of the total electro-magnetic flux linkage, we use a LAGRANGE INTERPOLATION POLYNOMIAL of the 6^{-th} level to determine the coefficient of saturation k0 for the starting point (see Appendix for details). In order to give a valid comparison with the previous (unsaturated) results, it was decided to set the initial value of $x_{ad(saturated)}$ in this case to that used in those studies. Hence a new "unsaturated" value must be defined. Unsaturated value for the $x_{ad(unsaturated)}$ now will be given by equation (5.3-1):

$$x_{ad(unsaturated)} = x_{ad}/k_0 \qquad (5.3\text{-}1)$$

where: x_{ad} ... is the dataset value used previously;

On entering the RUNGE-KUTTA method for each step of the calculation it will be necessary to calculate total electro-magnetic flux linkage before and after that particular step, as well as the value of $x_{ad(saturated)}$ that was given by expression (5.3-2):

$$x_{ad(saturated)} = x_{ad(unsaturated)} * k \qquad (5.3\text{-}2)$$

The value of coefficient of saturation k is assigned from the previously known value of the total electro-magnetic flux for that particular step. One of the major changes in this model is calculation of the matrix of reactances for the given system where x_{ad}, x_{fdl}, x_{fd}, x_{1d}, x_{aa2}, x_{ab0} and x_{aa0} are no more constants. By recalculating these values we are implementing the saturation of the synchronous machine in the d-axis. Expression for x_d is given by (5.3-3):

$$x_d = x_{ad(saturated)} + x_l \qquad (5.3\text{-}3)$$

(we assume here that the leakage paths never saturate); By knowing the electro-magnetic flux for each of the stator windings before and after that particular step we are able to determine the change of each of these electro-magnetic flux linkages in time $d\psi_a/dt$, $d\psi_b/dt$ & $d\psi_c/dt$. We have already estimated instantaneous stator voltages v_a, v_b and v_c and flux linkages Ψ_a, Ψ_b and Ψ_c (see section 3.2). Comparison and discussion of given results will follow in section 5.3.3.

5.3.2 Saturation of the synchronous generator in both d- and q-axes

The case for the direct axis is now extended to the quadrature axis. As well as the change of the $x_{ad(unsaturated)}$, $x_{ad(saturated)}$ and x_d now follows the change of $x_{aq(unsaturated)}$, $x_{aq(saturated)}$ and x_q as well. Equations that describe that change are given by expressions (5.3-4) to (5.3-6):

$$x_{aq(unsaturated)} = x_{aq}/k_0 \tag{5.3-4}$$

$$x_{aq(saturated)} = x_{aq(unsaturated)} * k \tag{5.3-5}$$

$$x_q = x_{aq(saturated)} + x_l \tag{5.3-6}$$

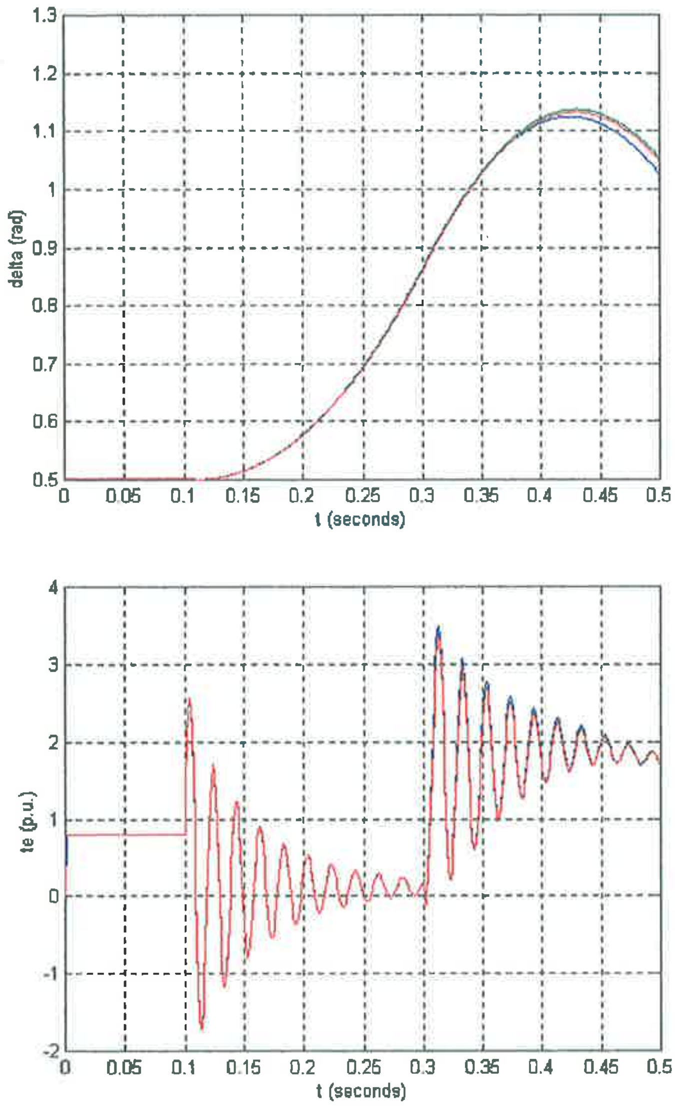
The assumption here is that the same saturation factor applies on both axes. This almost certainly exaggerates the degree of saturation on the q-axis, but will provide a useful comparative result. List of variable that will recalculate before calculation of the matrix of reluctances now contains x_{ad}, x_{aq}, x_{fdl}, x_{fd}, x_{1dl}, x_{11d}, x_{1ql}, x_{11q}, x_{aa2}, x_{ab0} and x_{aa0}. The same principle from section (3.3-1) applies for the recalculation of voltages v_a, v_b and v_c.

5.3.3 Comparison of the results between base model, saturation in d-axis & saturation in d- and q-axes

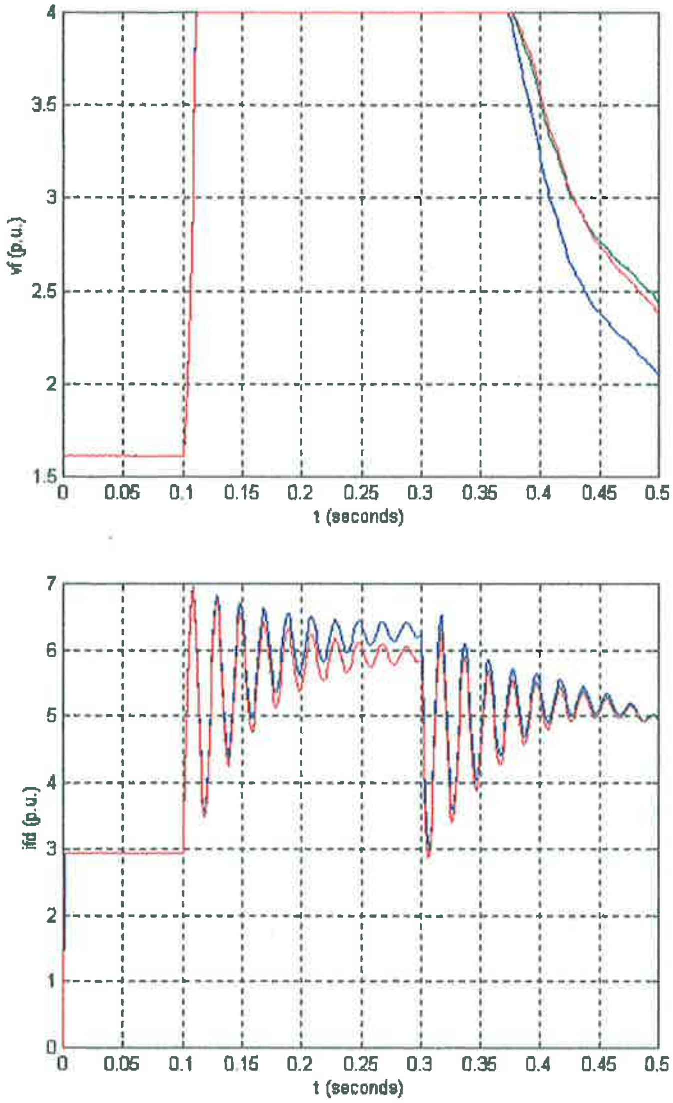
In all four cases there is a noticeable change in the field current i_{fd} immediately after applying the fault. This does not appear to have affected the torque t_e significantly, and so has little effect on the rotor angle δ. The stator currents are not affected at all, while certain differences of the v_f are noticeable after removing the faults.

It is clear that the approximation made in assuming equal saturation factors on each axis is not critical in generator response. From this point on all results presented by the following set of graphs will be just ones that show differences between base model, saturation in d-axis only and saturation in both d- and q- axes.

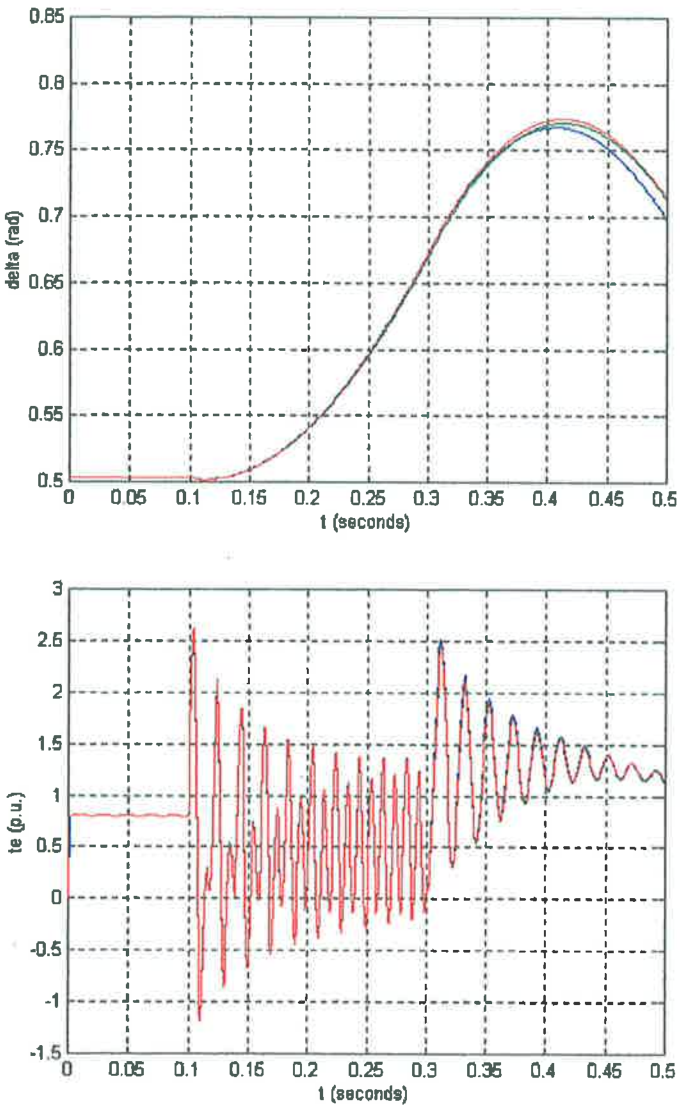
Color chart:
- blue represents base model;
- green represents saturation in d-axis only;
- red represents saturation in both d- and q- axes;



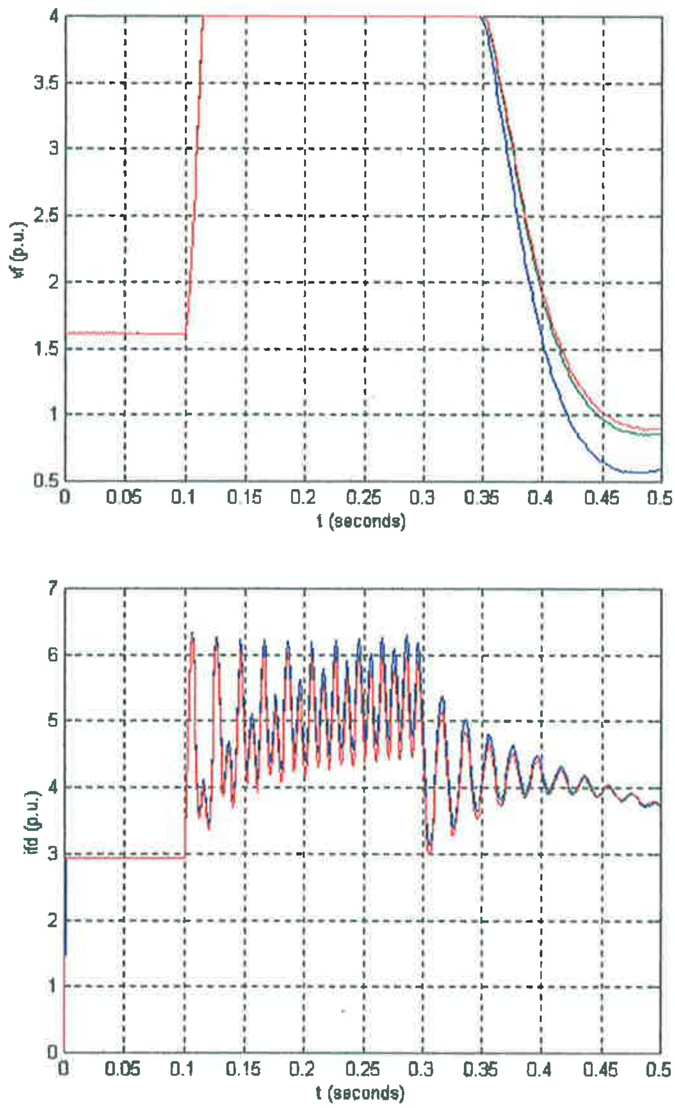
3-phase short circuit



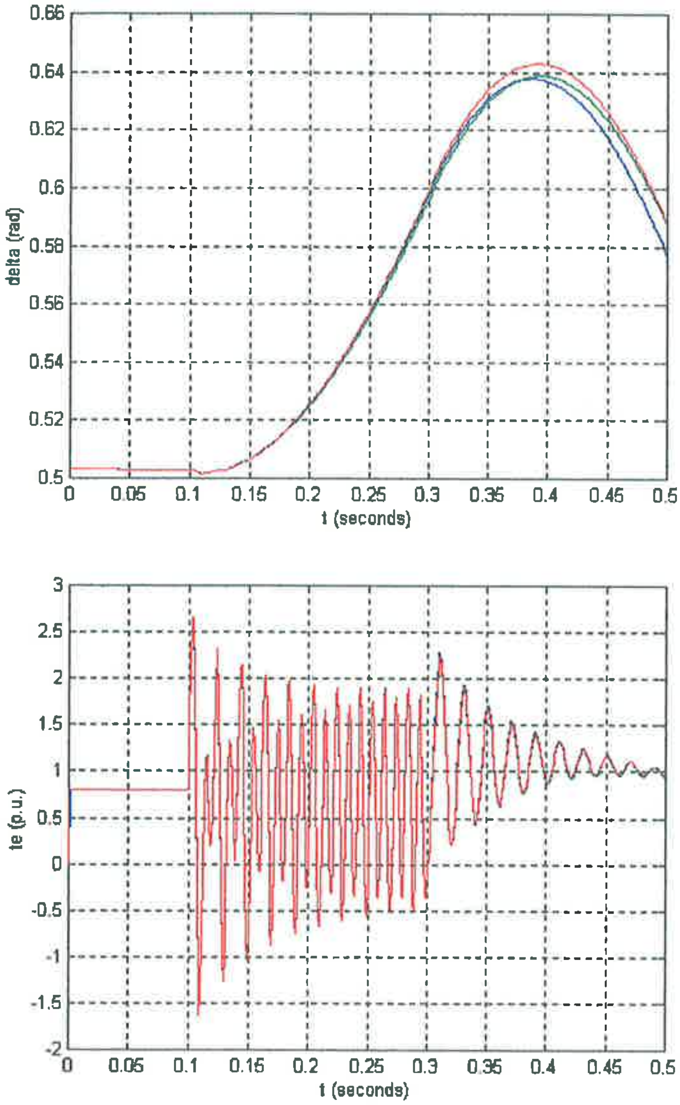
3-phase short circuit



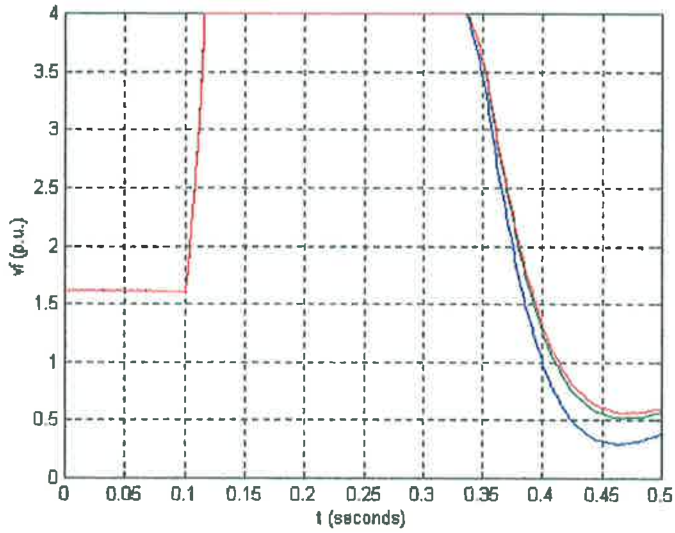
2-phase short circuit with ground

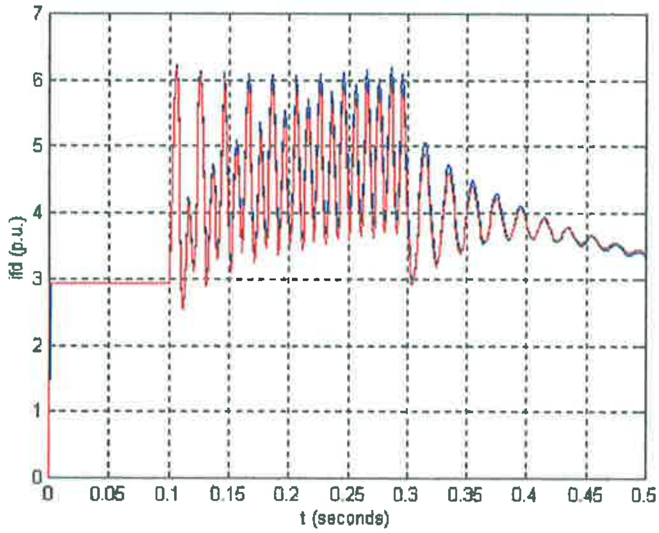


2-phase short circuit with ground

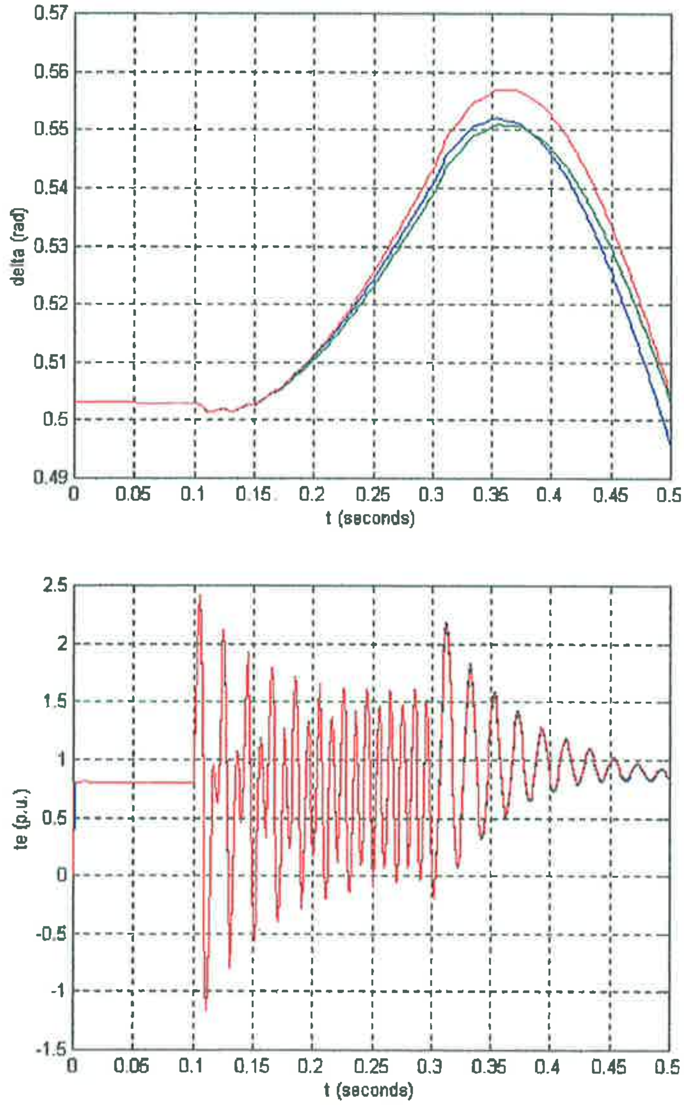


2-phase short circuit

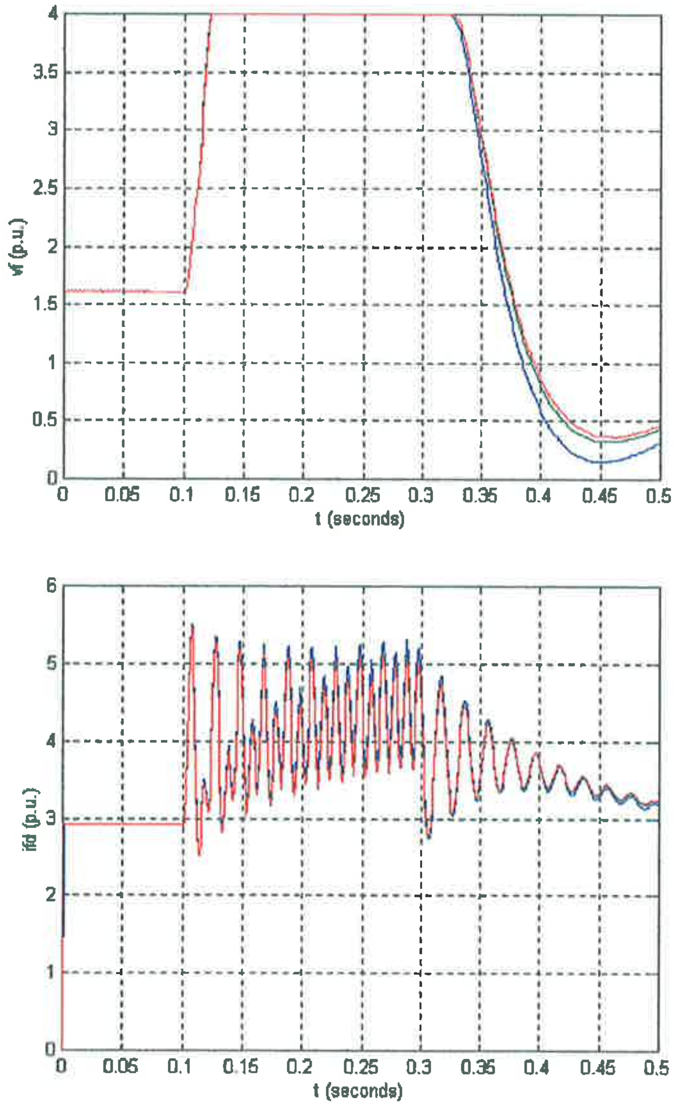




2-phase short circuit



1-phase short circuit



1-phase short circuit

5.4 Multiple rotor masses

There are two basic cases that will be considered in this chapter:

1- multiple rotor masses on the shaft of the synchronous generator &
2- multiple rotor masses on the shaft of the synchronous generator with capacitor C connected in series with the generator-transformer block, located between the observed generator-transformer system and electrical power system to form a SERIES RESONANT CIRCUIT to excite sub-synchronous resonant frequencies;

5.4.1 Multiple rotor masses on the shaft of the synchronous generator

The variable speed model described in section (5.2) will now be modified by treating the mechanical system as three lumped inertias connected by a common shaft. To illustrate such a modification we use Figure (5.4-1):

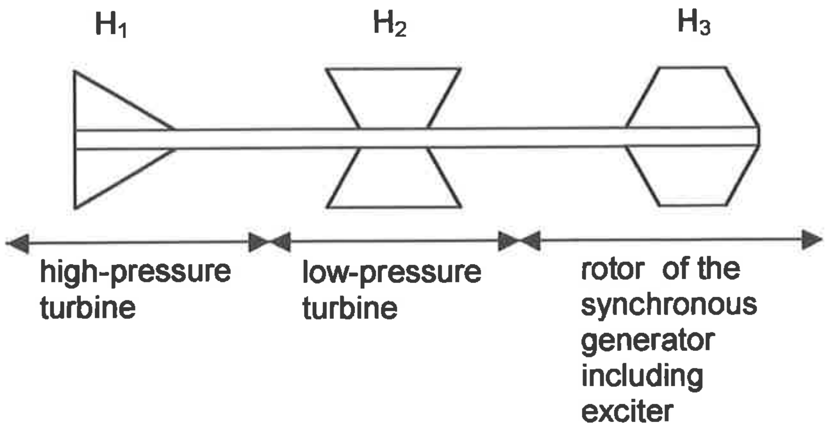


Figure (5.4-1)

where: H_1 ... is inertia of the high-pressure turbine;

H_2 ... is inertia of the low-pressure turbine;

H_3 ... is inertia of the rotor of the synchronous generator including the mass of the exciter;

This figure could be simplified to Figure (5.4-2) in which each of the given masses are represented by a disks on the same shaft and where the angular displacement between disks gives the measure of torsional forces that could in extreme conditions damage the shaft of the synchronous generator.

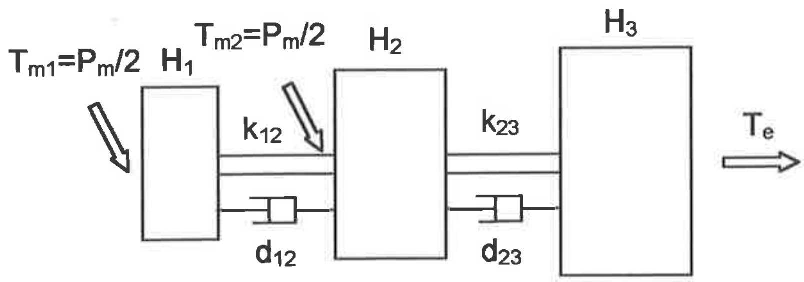


Figure (5.4-2)

where: H_1 ... is inertia constant of the disk 1;

H_2 ... is inertia constant of the disk 2;

H_3 ... is inertia constant of the disk 3;

k_{12} ... is the coefficient of the torsional stiffness between low-pressure and high-pressure turbine;

k_{23} ... is the coefficient of the torsional stiffness between high-pressure turbine and rotor exciter mass;

d_{12} ... is the damping coefficient between low-pressure and high-pressure turbine;

d_{23} ... is the damping coefficient between low-pressure and rotor-exciter mass;

We will assume that the total torque T_m delivered by the steam entering the turbine is constant, and is distributed equally between the high-pressure and low-pressure turbines. The model analysis of the shaft assembly could be represented by the equation (5.4-1) (ref. 10):

$$\frac{2}{\omega_0}[\,H\,]\, d^2[\,\Theta\,]/dt^2 + [\,D\,]\, d[\,\Theta\,]/dt + [\,K\,]\,[\,\Theta\,] = [\,T\,] \qquad (5.4\text{-}1)$$

where: $[\,H\,]$ ... is the diagonal matrix whose elements are the inertia constants of the disks [p.u.];

ω_0 ... is the synchronous speed [rad/sec];

$d^2[\,\Theta\,]/dt^2$ ... is the vector of angular acceleration for each of the shaft masses [rad/sec^2];

$[\,D\,]$ ... is the damping matrix;

$d[\,\Theta\,]/dt$ ... is the vector of angular speed for each of shaft masses;

$[\,K\,]$ ... is the matrix of torsional stiffness;

$[\,\Theta\,]$ ... is the vector of angles for each of the shaft masses;

$[\,T\,]$ ... is vector of externally applied torques on the disks;

$$[H]=\begin{bmatrix}H_1 & 0 & 0\\ 0 & H_2 & 0\\ 0 & 0 & H_3\end{bmatrix};\quad d^2[\Theta]/dt^2 = d^2/dt^2\begin{bmatrix}\Theta_1\\ \Theta_2\\ \Theta_3\end{bmatrix};\quad d[\Theta]/dt = d/dt\begin{bmatrix}\Theta_1\\ \Theta_2\\ \Theta_3\end{bmatrix};\quad [\Theta]=\begin{bmatrix}\Theta_1\\ \Theta_2\\ \Theta_3\end{bmatrix};$$

$$[D]=\begin{bmatrix}d_{12} & -d_{21} & 0\\ -d_{21} & d_{12}+d_{23} & -d_{23}\\ 0 & -d_{23} & d_{32}\end{bmatrix}$$ where: $d_{12}=d_{21}$ & $d_{23}=d_{32}$;

$$[K]=\begin{bmatrix}k_{12} & -k_{21} & 0\\ -k_{21} & k_{12}+k_{23} & -k_{23}\\ 0 & -k_{23} & k_{32}\end{bmatrix}$$ where: $k_{12}=k_{21}$ & $k_{23}=k_{32}$;

$$[T]=\begin{bmatrix}T_{m1}\\ T_{m2}\\ -T_e\end{bmatrix} = \begin{bmatrix}T_m/2\\ T_m/2\\ -T_e\end{bmatrix};$$

From the equation (5.4-1) it is easy to develop the following set of equations (5.4-1a) to (5.4-1c):

$$\frac{2}{\omega_0}H_1\, d^2\Theta_1/dt^2 + d_{12}\, d\Theta_1/dt - d_{21}\, d\Theta_2/dt + k_{12}\,\Theta_1 - k_{12}\,\Theta_2 = \frac{T_m}{2} \qquad (5.4\text{-}1a)$$

$$\frac{2}{\omega_0}H_2\, d^2\Theta_2/dt^2 - d_{12}\, d\Theta_1/dt + (d_{12}+d_{23})\, d\Theta_2/dt - d_{23}\, d\Theta_3/dt - k_{12}\,\Theta_1 +$$

$$+ (k_{12}+k_{23})\Theta_2 - k_{23}\,\Theta_2 = \frac{T_m}{2} \qquad (5.4\text{-}1b)$$

$$\frac{2}{\omega_0}H_3\, d^2\Theta_3/dt^2 - d_{23}\, d\Theta_2/dt + d_{23}\, d\Theta_3/dt - k_{23}\,\Theta_2 + k_{23}\,\Theta_3 = -T_e \qquad (5.4\text{-}1c)$$

From the equations (5.4-1a), (5.4-1b) and (5.4-1c) we could express values for $d^2\Theta_1/dt^2$, $d^2\Theta_2/dt^2$ and $d^2\Theta_3/dt^2$:

$$d^2\Theta_1/dt^2 = \frac{\omega_0}{2H_1}\left[-d_{12}\, d\Theta_1/dt + d_{21}\, d\Theta_2/dt - k_{12}\,\Theta_1 + k_{12}\,\Theta_2 + \frac{T_m}{2}\right] \qquad (5.4\text{-}2)$$

$$d^2\Theta_2/dt^2 = \frac{\omega_0}{2H_2} [\, d_{12}\, d\Theta_1/dt - (d_{12}+d_{23})\, d\Theta_2/dt + d_{23}\, d\Theta_3/dt + k_{12}\,\Theta_1 -$$

$$- (k_{12}+k_{23})\Theta_2 + k_{23}\,\Theta_2 + \frac{T_m}{2}\,] \quad (5.4\text{-}3)$$

$$d^2\Theta_3/dt^2 = \frac{\omega_0}{2H_3} [\, d_{23}\, d\Theta_2/dt - d_{23}\, d\Theta_3/dt + k_{23}\,\Theta_2 - k_{23}\,\Theta_3 - T_e\,] \quad (5.4\text{-}4)$$

From the variable speed model we know:

$\Theta_3 = \delta$

$d\Theta_3/dt$ ... is rotational speed of the generator ω;

$d^2\Theta_3/dt^2$ ... is rotor acceleration or $\frac{d\omega}{dt}$;

Our system now consists of four new additional stator variables: $\Theta_1, d\Theta_1/dt$, Θ_2 & $d\Theta_2/dt$;
From the initial step we can determine Θ_1, Θ_2, Θ_3, $d^2\Theta_1/dt^2$, $d^2\Theta_2/dt^2$ & $d^2\Theta_3/dt^2$:

$$\Theta_1 = \frac{p_m}{2k_{12}} + \Theta_2 = \frac{p_m}{2k_{12}} + \frac{p_m}{k_{23}} + \delta \quad (5.4\text{-}5)$$

$$\Theta_2 = \frac{p_m}{k_{23}} + \Theta_3 = \frac{p_m}{k_{23}} + \delta \quad (5.4\text{-}6)$$

$$\Theta_1 = \delta \quad (5.4\text{-}7)$$

By using system from the equation (5.4-1) we could easily calculate values of $d\Theta_1/dt$, $d\Theta_2/dt$ and $d\Theta_3/dt$ for the first step of the RUNGE-KUTTA method. For the next following step we can calculate:

$d^2\Theta_3/dt^2$ ... from the equation (5.4-4);

$d\Theta_3/dt = 2\pi f\, d^2\Theta_3/dt^2$

$d^2\Theta_2/dt^2$ ... from the equation (5.4-3);

$d\Theta_2/dt = 2\pi f\, d^2\Theta_2/dt^2$

$d^2\Theta_1/dt^2$ ... from the equation (5.4-2);

$d\Theta_1/dt = 2\pi f\, d^2\Theta_1/dt^2$

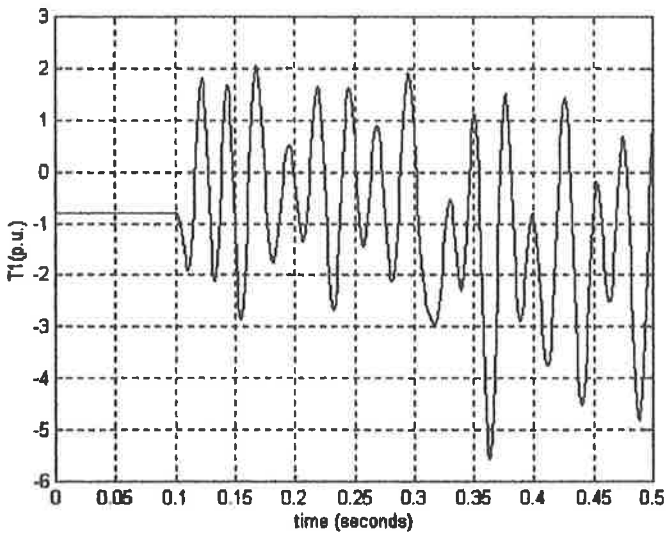
to determine values of Θ_1, Θ_2 & Θ_3.

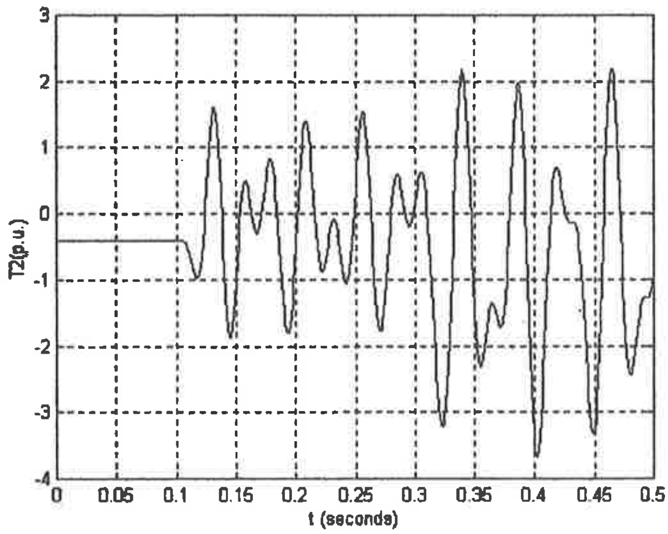
We take as our reference mass the rotor-exciter and to measure the torsional effect between the masses on the shaft we simply need to determine angles Θ_{12} & Θ_{23} by expressions (5.4-8) and (5.4-9):

$$\Theta_{12} = \Theta_2 - \Theta_1 \quad (5.4\text{-}8)$$

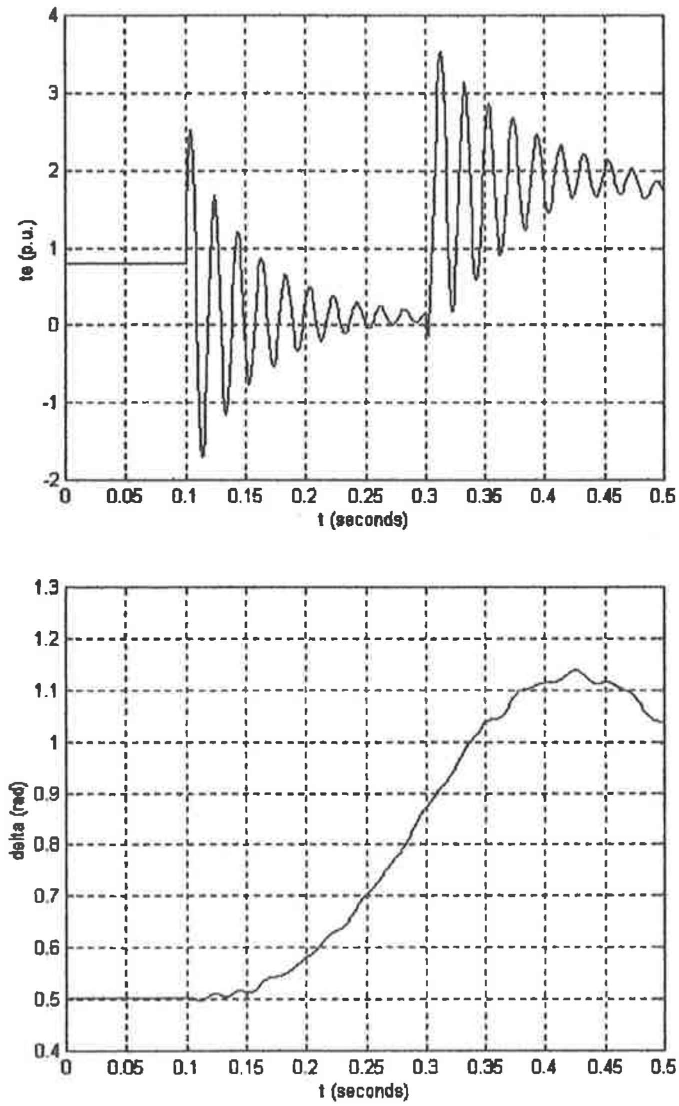
$$\Theta_{23} = \Theta_3 - \Theta_2 \quad (5.4\text{-}9)$$

The values of the inertias of the three masses were taken from typical generators of rating approximately 300 MW. The undamped torsional frequencies with these inertias are approximately 22 and 13.5 Hz. When the test was run on the model itself without any short-circuit condition on the secondary of the transformer, angles Θ_{12} and Θ_{23} were constant. This actually confirmed the viability of the model itself. After that the set of short circuit tests were performed between 0.1 and 0.3 seconds. The effects of these disturbances will be expressed by shaft torques T_1 (between disk 1and disk 2) and T_2 (between disk 2 and disk 3) that are directly proportional to angles Θ_{12} and Θ_{23}. The following set of graphs show the results:

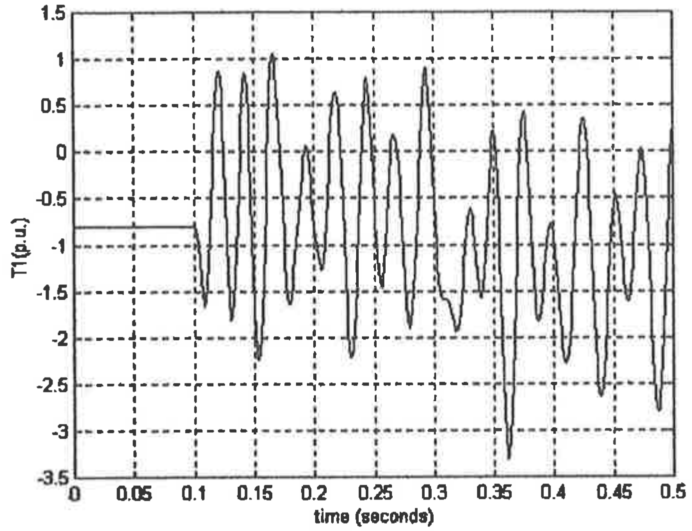


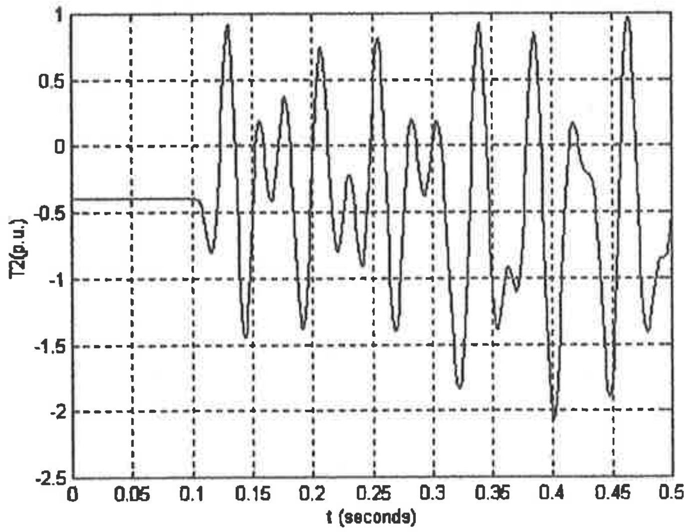


3-phase short circuit

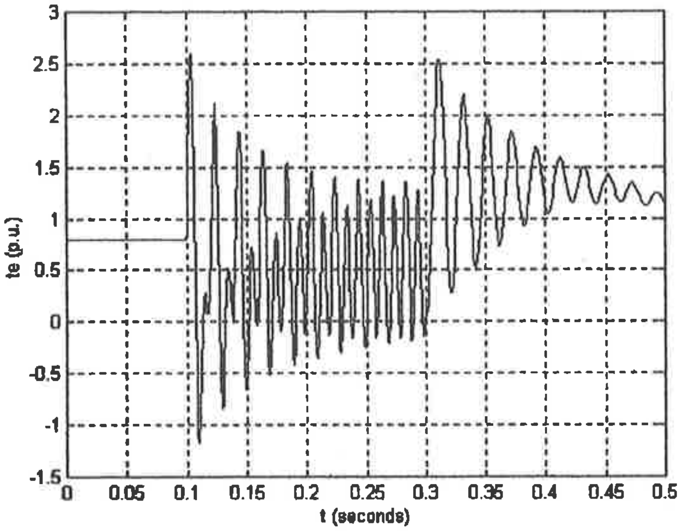


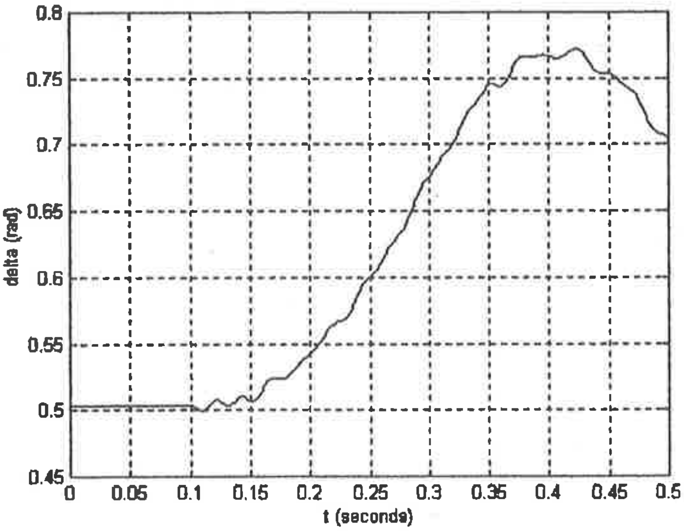
3-phase short circuit



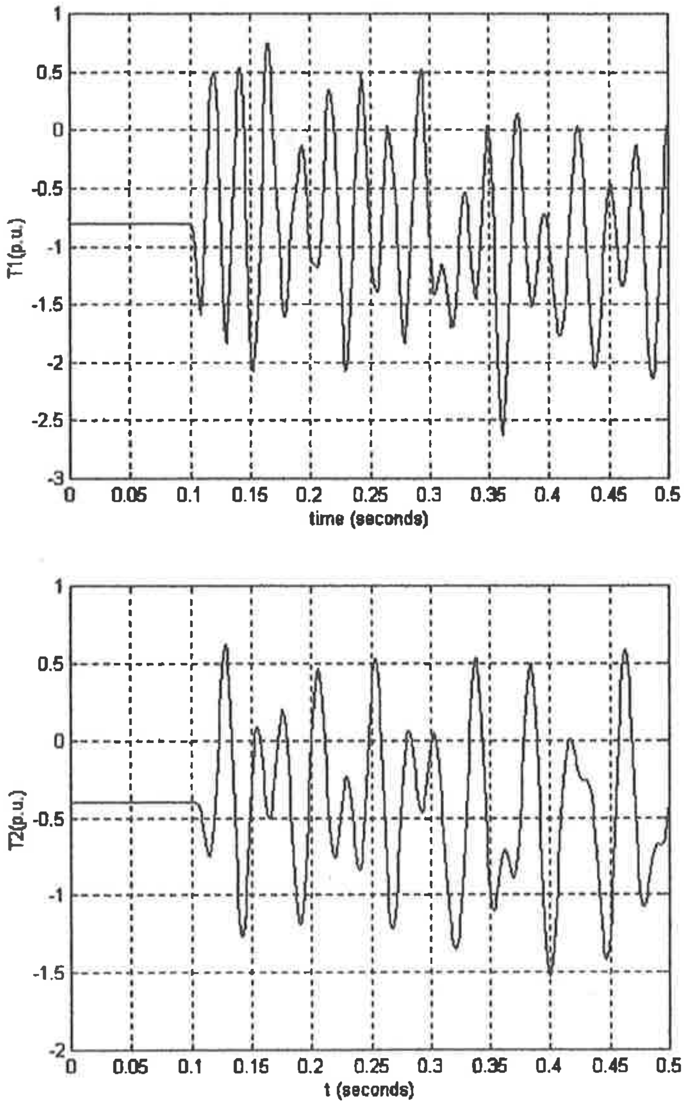


2-phase short circuit with ground

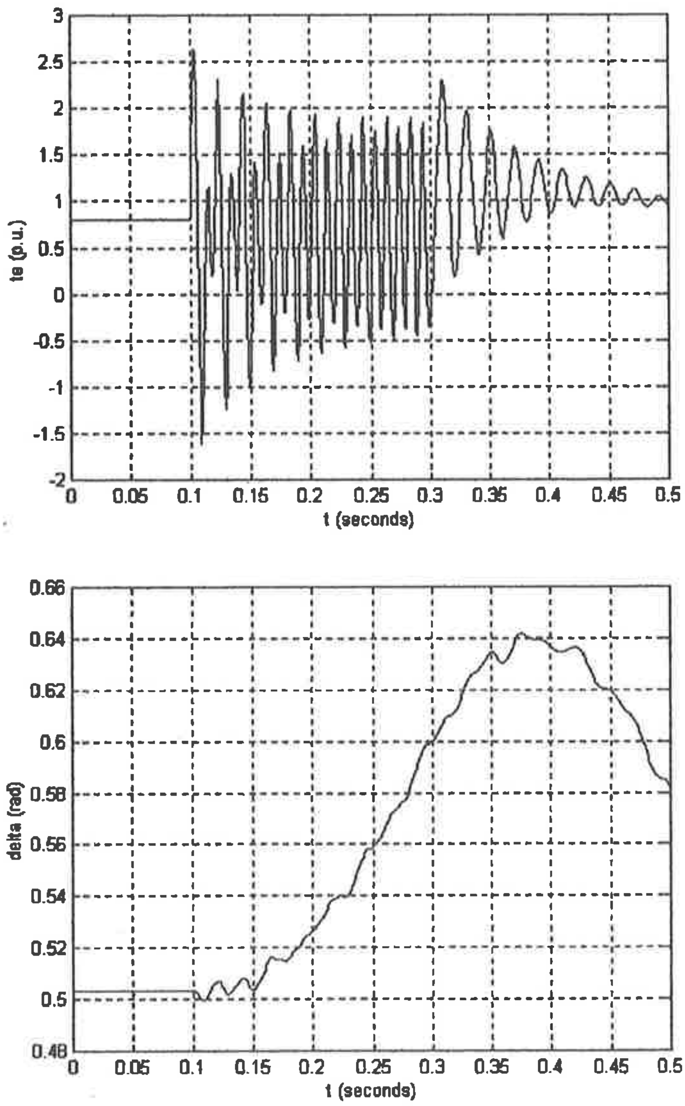




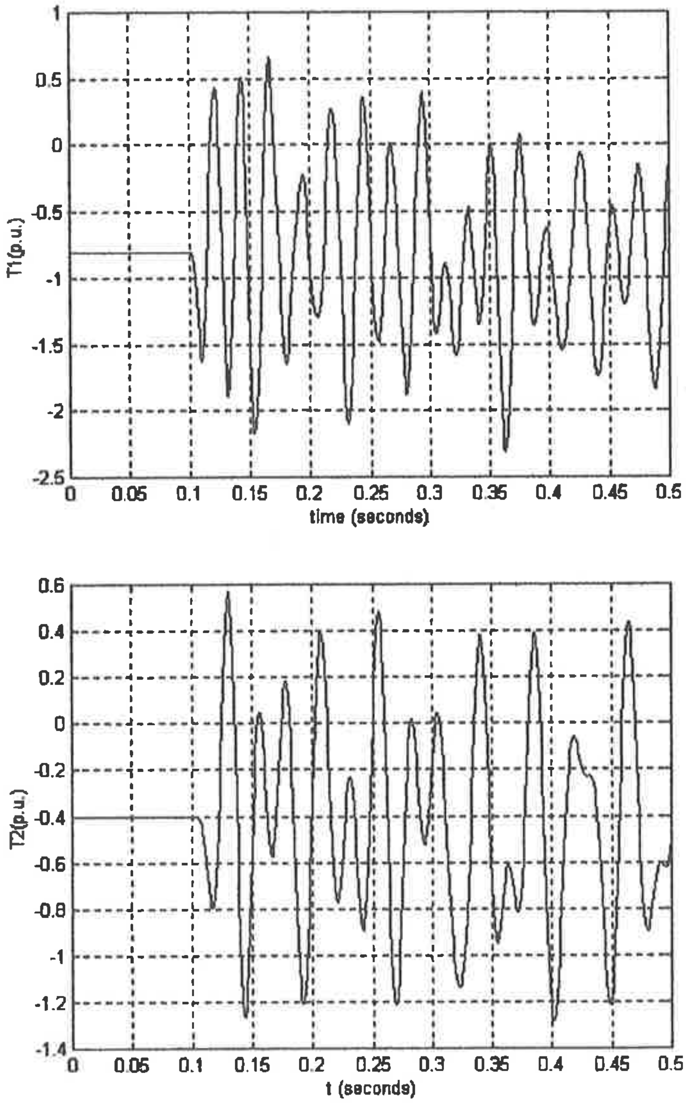
2-phase short circuit with ground



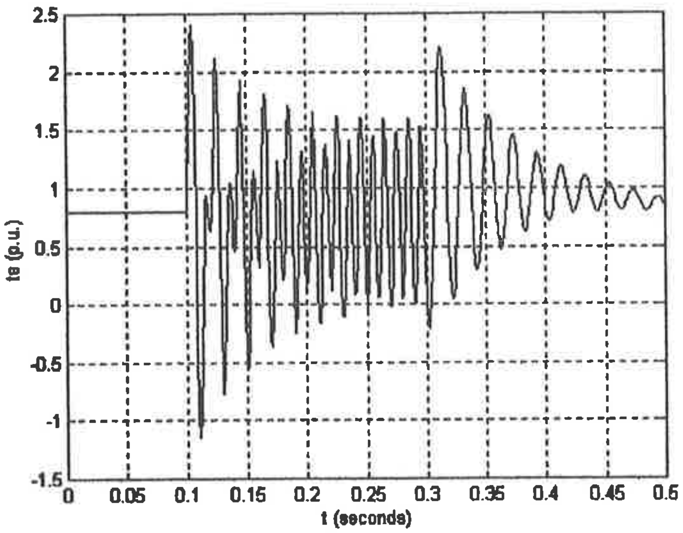
2-phase short circuit

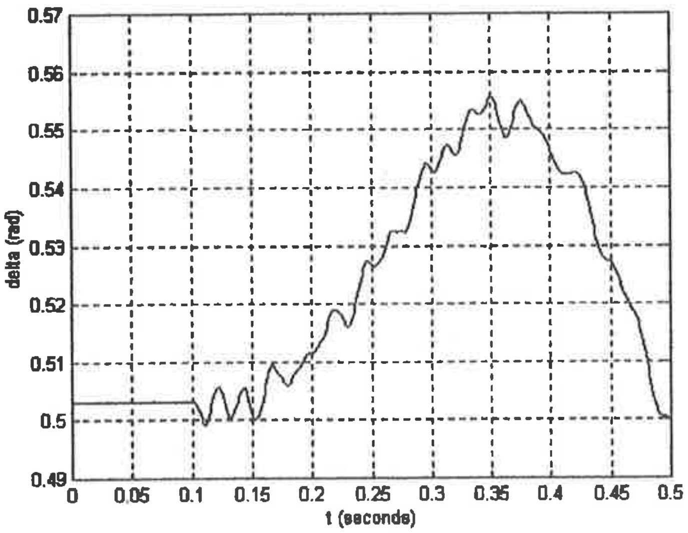


2-phase short circuit



1-phase short circuit





1-phase short circuit

From all fault cases, the part of the shaft between rotor and the low-pressure turbine shows a clear oscillation frequency of approximately 22 Hz, while the other section of the shaft shows the interaction of the different modes of oscillation.

5.4.2 Multiple rotor masses on the shaft of the synchronous generator with series system capacitance

On the existing model described in section (5.4.1) we will implement capacitor C between the generator-transformer block on one side and electrical power system on the other. The Figure (5.4-4) best describes that condition.

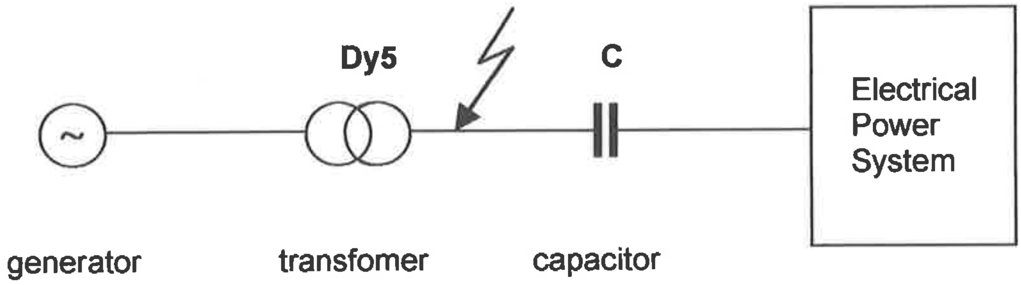


Figure (5.4-4)

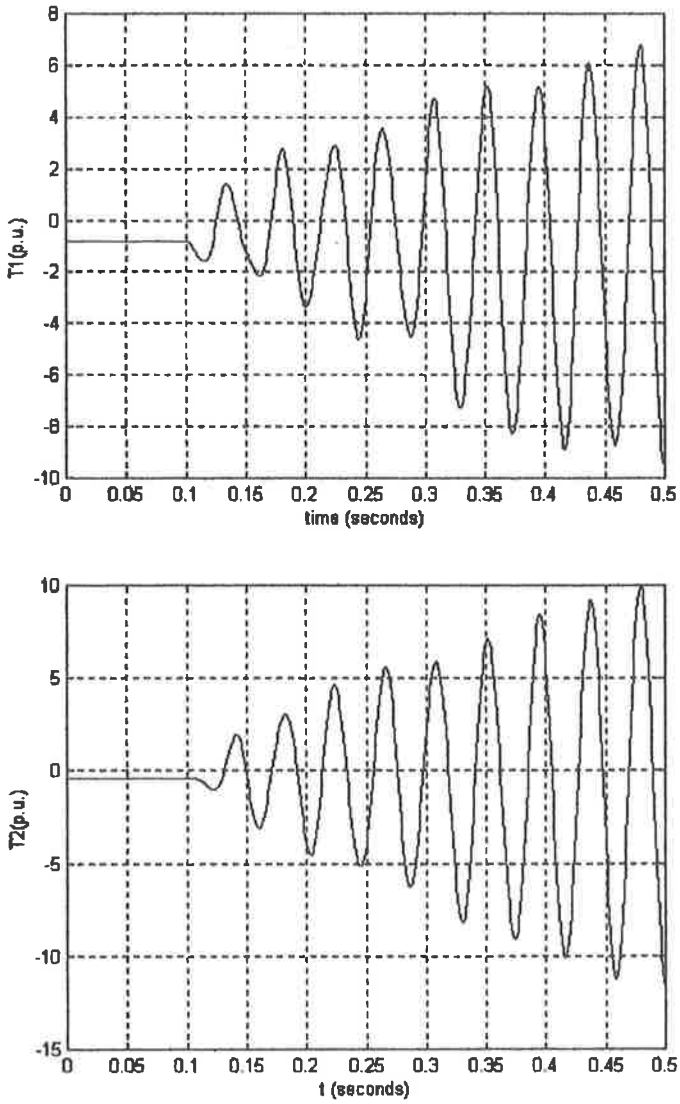
In systems involving series capacitance there is always a risk of resonance with the inductive reactances of lines, transformers and generators. This will normally give rise to oscillations on system and machine quantities which are damped mainly by system resistance. In order to give a realistic amount of damping the transformer secondary resistance is increased from 0.01 to 0.04 p.u. for these tests only.

In our case two values of x_c have been chosen and that were 0.21p.u. and 0.035 p.u.. These values were chosen so as to produce resonant frequencies close to the torsional frequencies of shaft oscillation. In the case when the test was run on the model itself without any short-circuit condition on the secondary side of the transformer angles Θ_{12} and Θ_{23} were constant. This confirmed the viability of the model itself. After that the set of short circuit test were performed between 0.1 and 0.3 seconds between the secondary side of transformer and capacitor C.

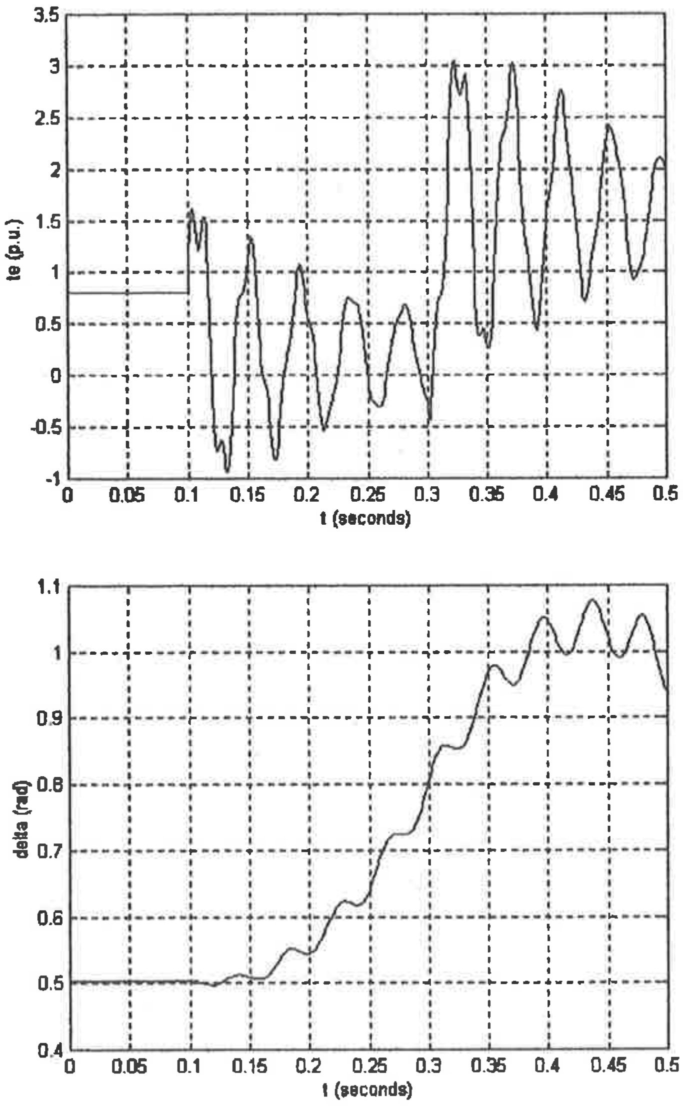
If the electrical resonances in the power system occur at frequencies less than the fundamental system frequency (so-called "sub-synchronous resonance") then special problems may occur. The current components at the resonant frequency give rise to mmf's rotating in the air gaps of generators at less than synchronous speed. Closed current paths on the generator rotor (such as damper windings) act as induction generators at this frequency and the energy thus injected into the system may exceed that removed by resistive damping. Under these circumstances the resonant component will be sustained or even increased.

If these components coincide with one of the natural torsional modes of the multi-mass shaft system, then the potential exists for excessive torsional oscillations of the shaft masses and consequent fatigue damage to the shaft (ref. 11).

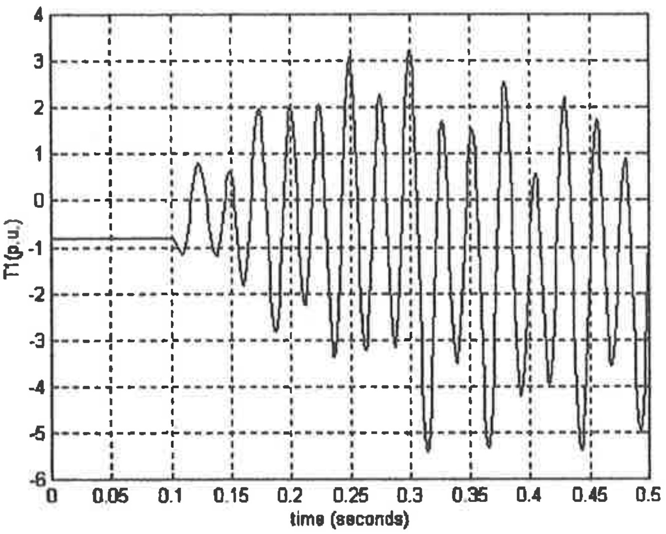
Between the x_{c1} = 0.21 p.u. and x_{c1} = 0.035 p.u. the worst possible case is the balanced fault (three phase short-circuit) with value of x_{c1} = 0.21 p.u.. The torques T_1 and T_2 (that are proportional to Θ_{12} and Θ_{23}) are oscillating in phase with increasing amplitude such that the shaft of synchronous generator could be easily damaged, which is very obvious from graph $\delta = \delta(t)$. If we compare these results with these given by x_{c1} = 0.035 p.u., the higher frequency resonance excites a mode such that the torques T_1 and T_2 are anti-phase which means that T_1 and T_2 will oscillate in opposite directions. With this mode, δ is not so seriously affected, and the oscillations do not increase in amplitude:

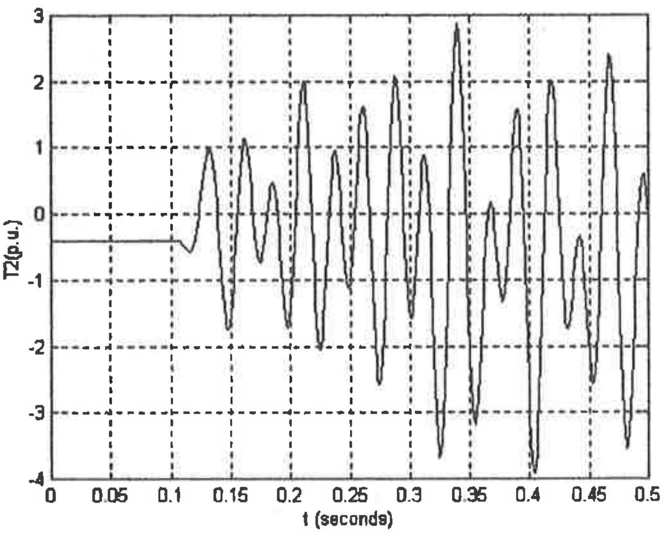


3-phase short circuit (x_c=0.21)

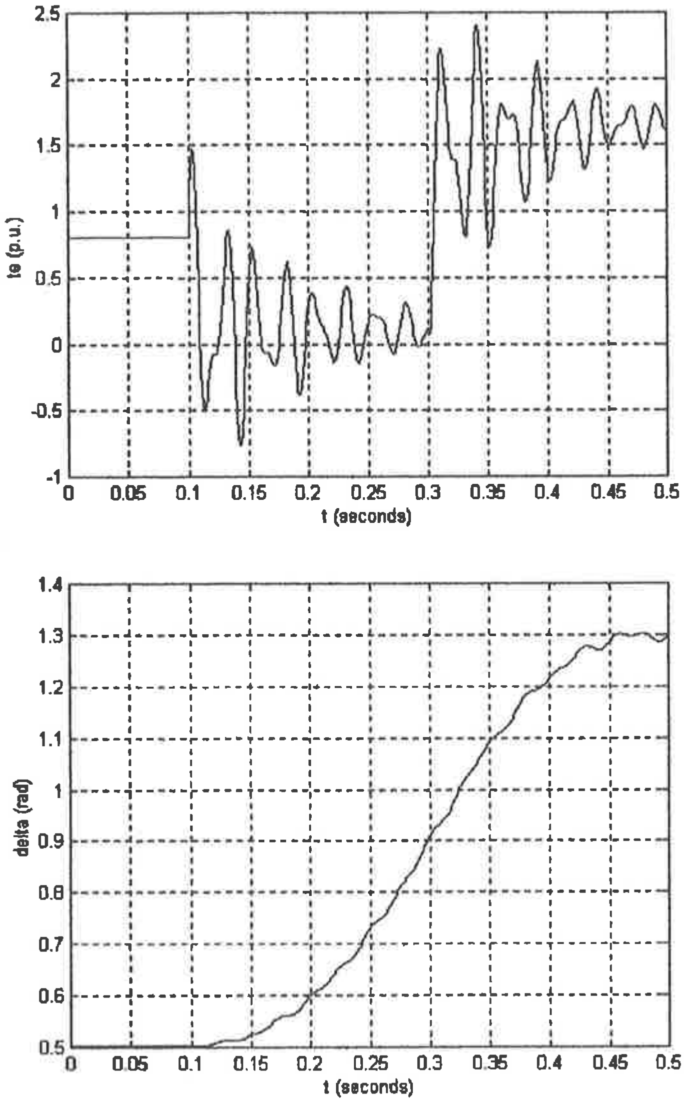


3-phase short circuit (x_c=0.21)

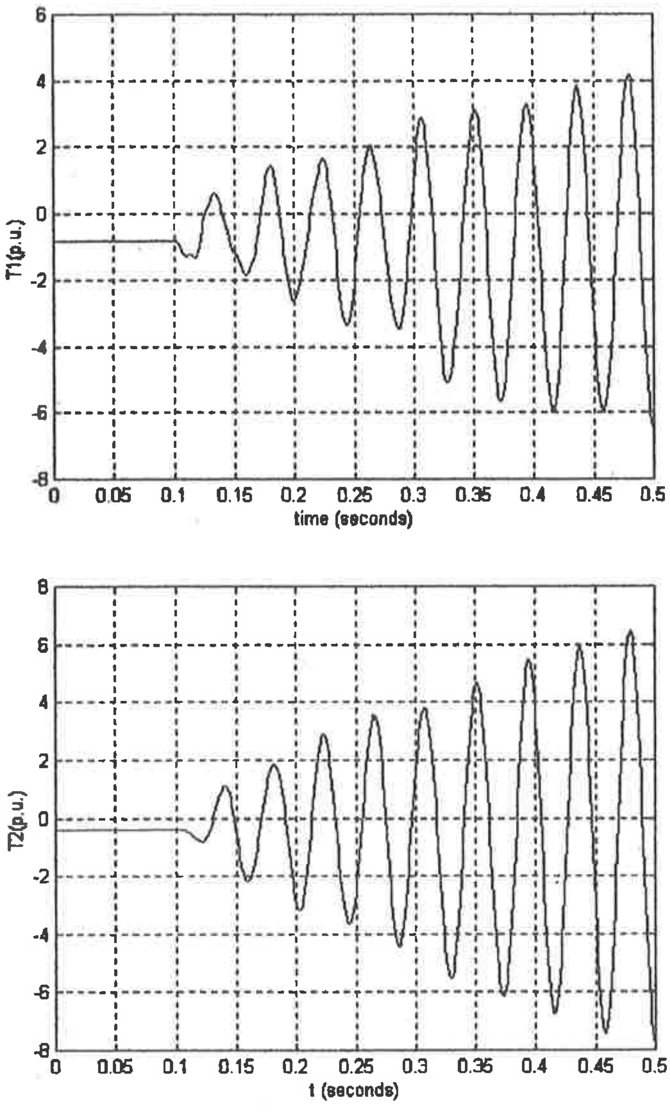




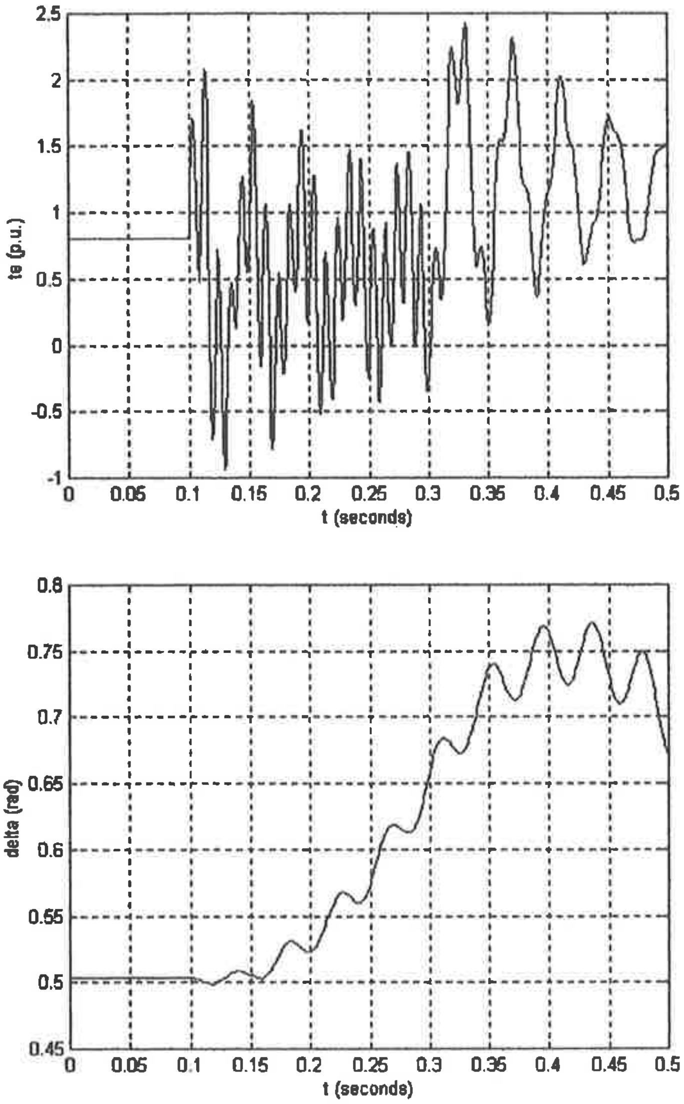
3-phase short circuit (x_c=0.035)



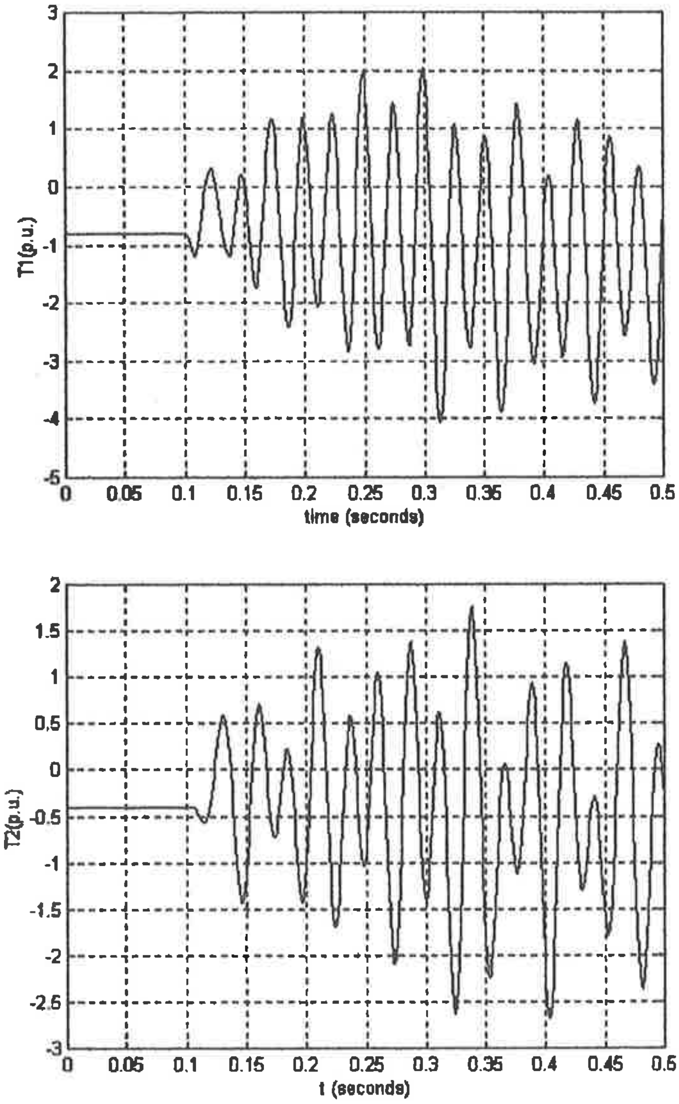
3-phase short circuit (x_c=0.035)



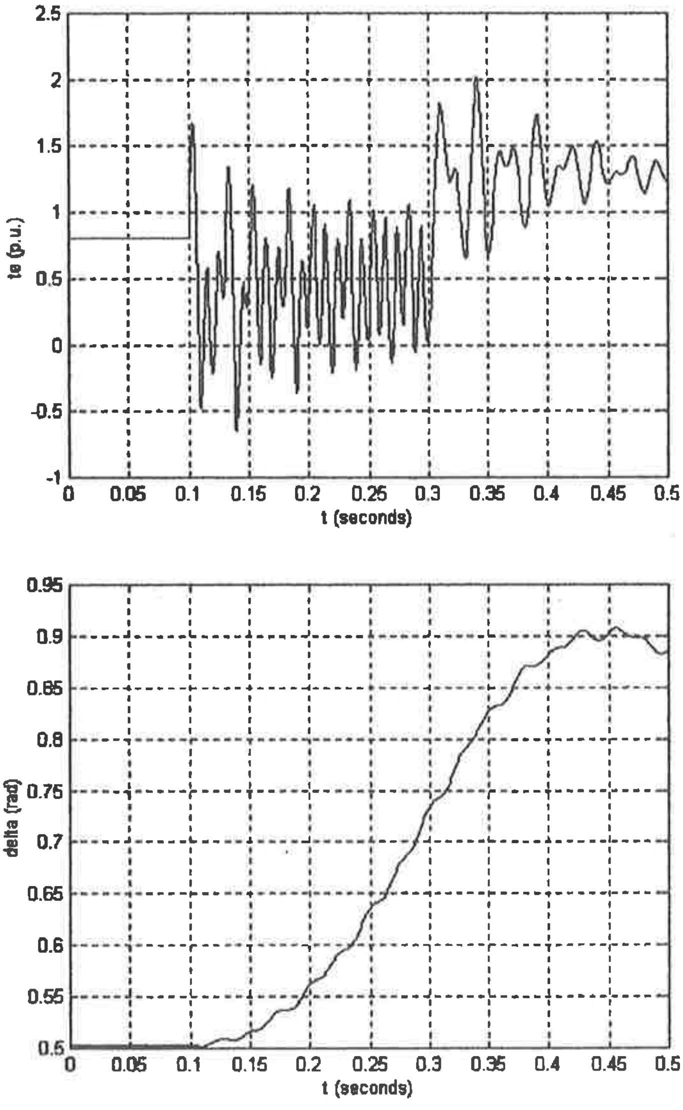
2-phase short circuit with ground (x_c=0.21)



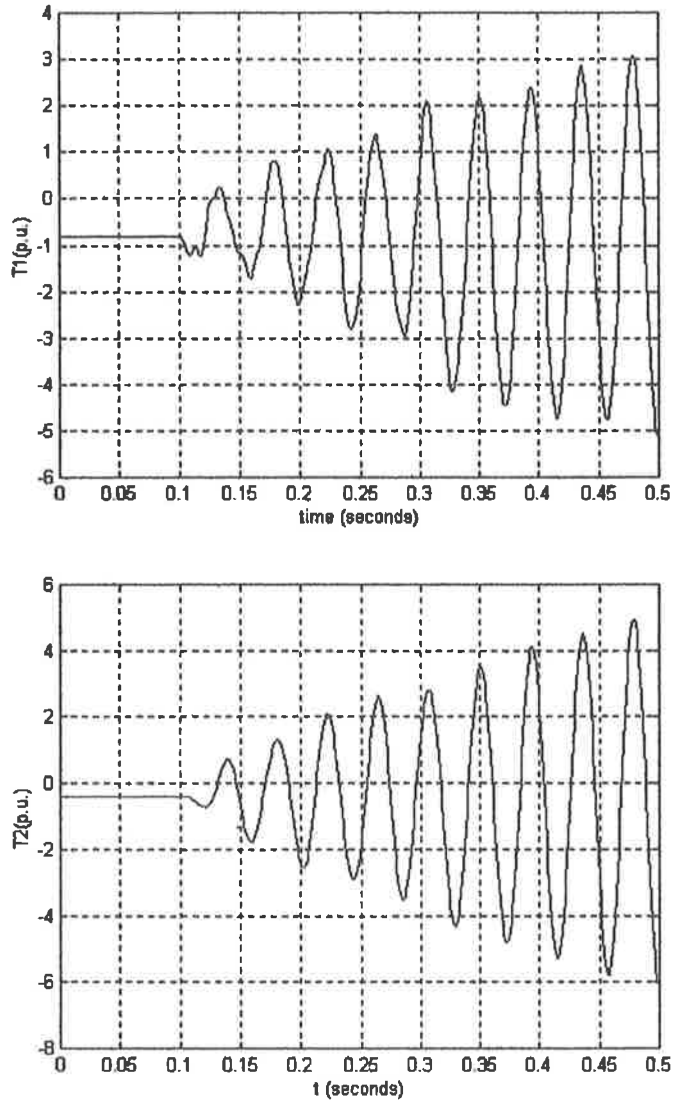
2-phase short circuit with ground (x_c=0.21)



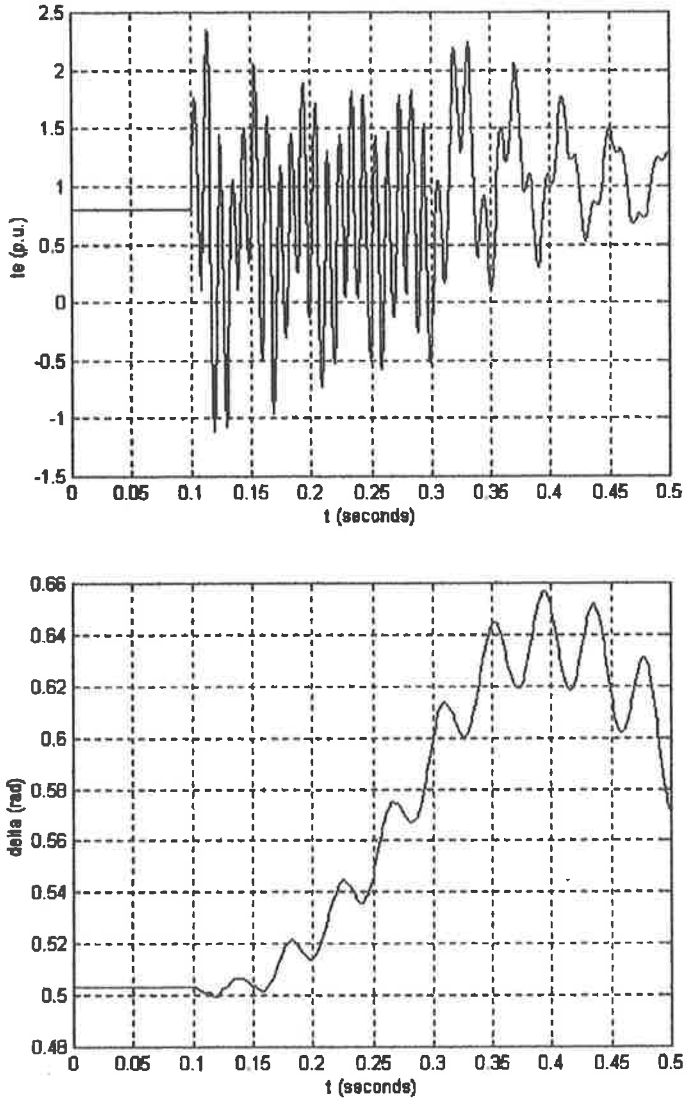
2-phase short circuit with ground (x_c=0.035)



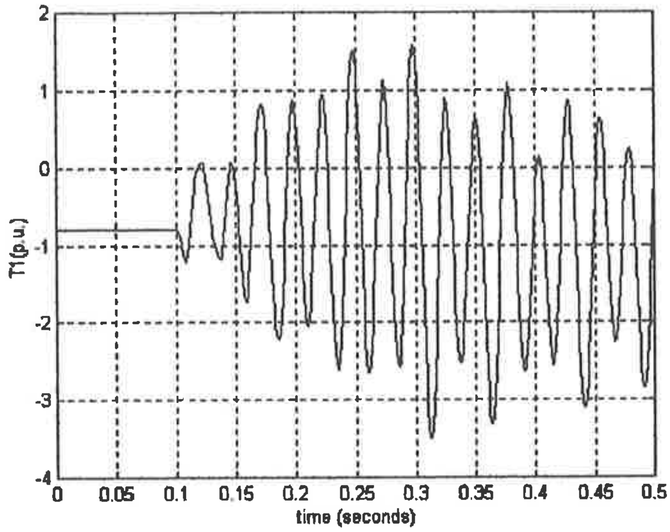
2-phase short circuit with ground (x_c=0.035)

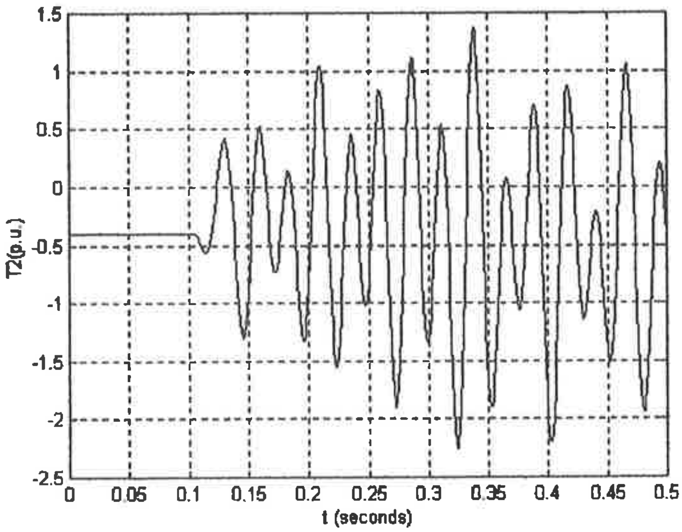


2-phase short circuit (x_c=0.21)

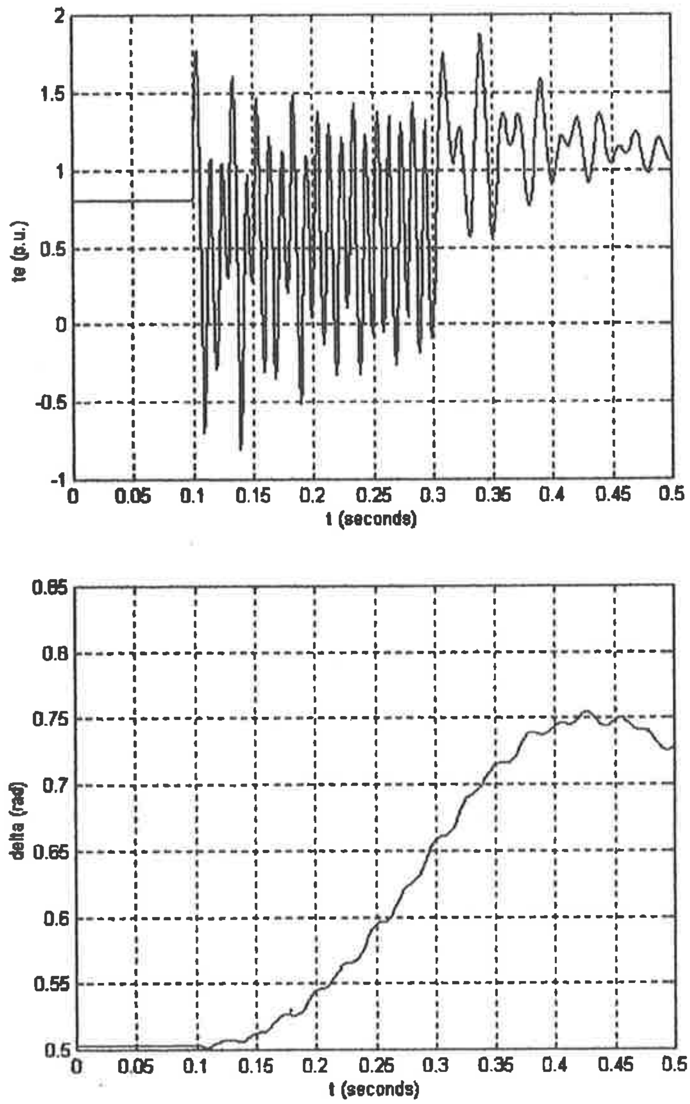


2-phase short circuit (x_c=0.21)

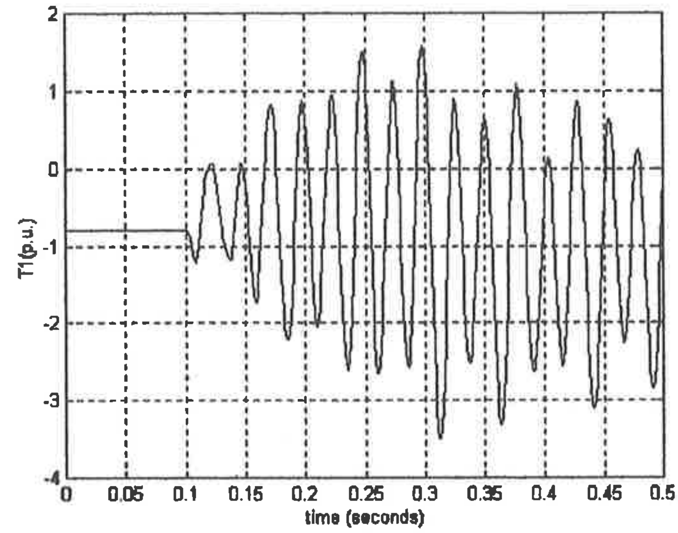


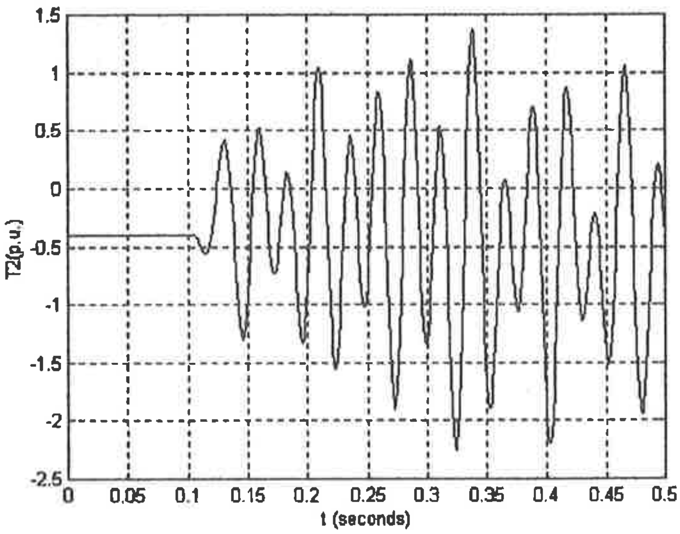


2-phase short circuit (x_c=0.035)

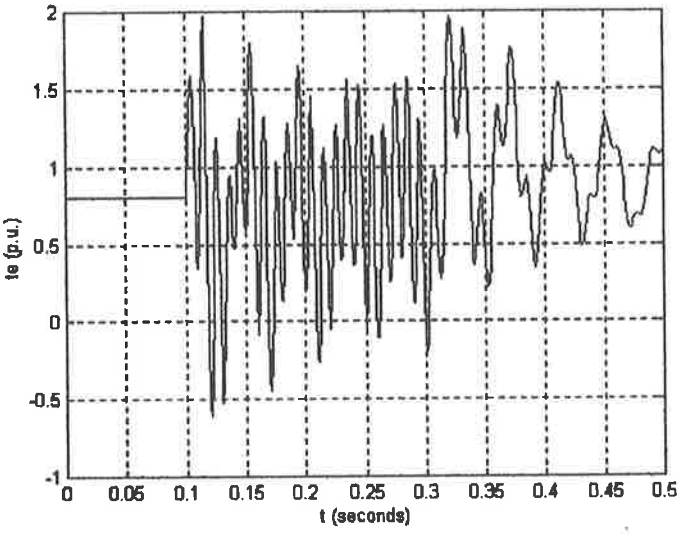


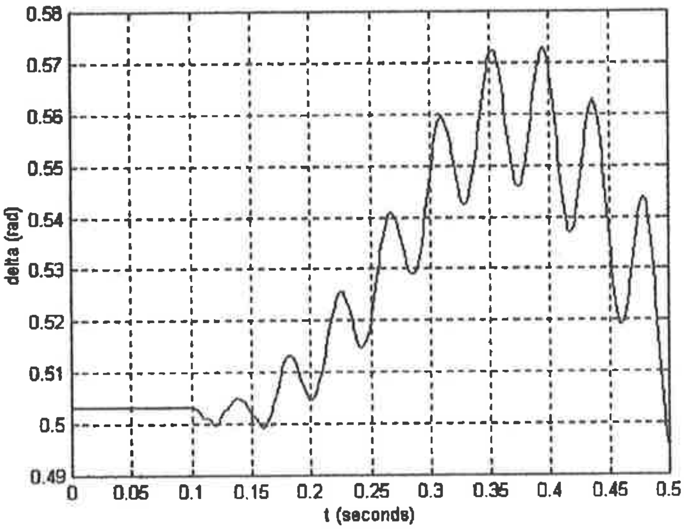
2-phase short circuit (x_c=0.035)



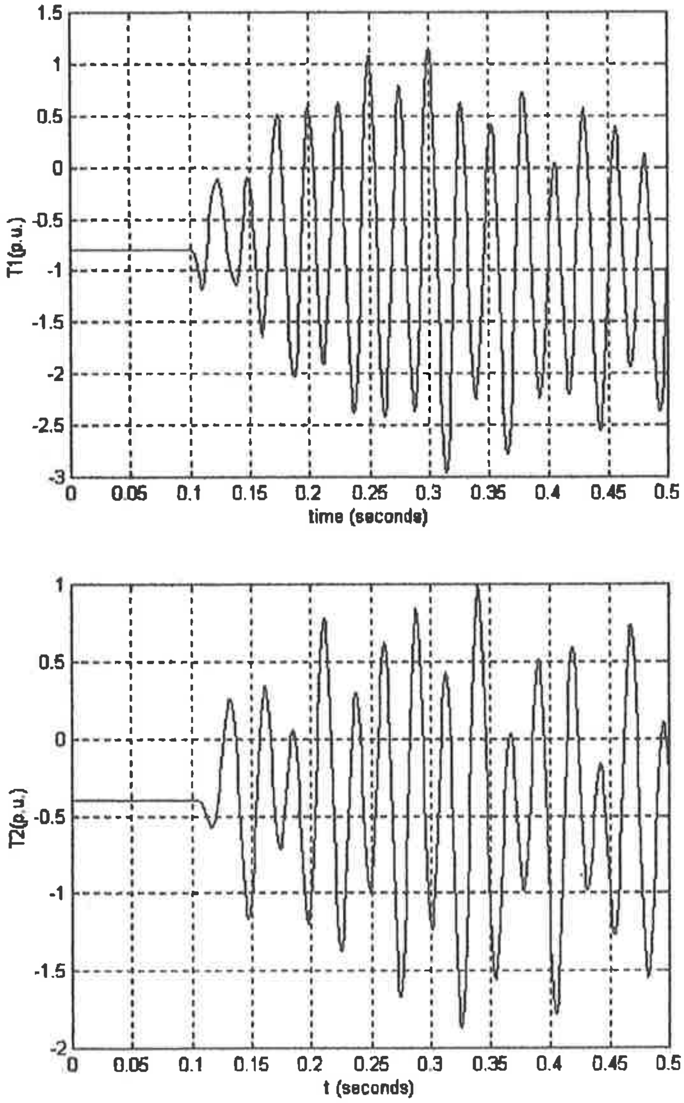


1-phase short circuit (x_c=0.21)

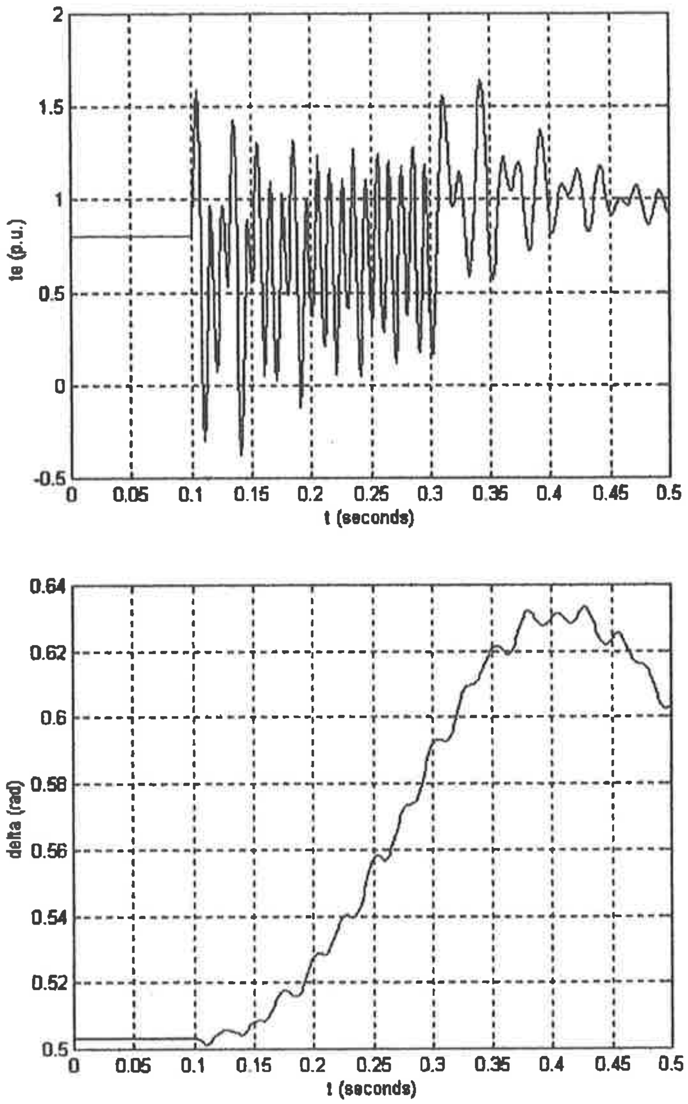




1-phase short circuit (x_c=0.21)



1-phase short circuit (x_c=0.035)



1-phase short circuit (x_c=0.035)

If we compare these two cases it is obvious that x_c = 0.21 p.u. gives us a worse condition on the shaft then x_c = 0.035 p.u.. After examining all these cases, it is evident that the value x_c = 0.21 p.u., giving rise to a resonant frequency of approximately 25 Hz, is critical in the torsional response of the shaft system, and that the generator model is capable of tracking this response for both balanced and unbalanced faults.

6. CONCLUSION

It is important to re-iterate that the focus of these studies has been the short-term transient response of selected generator models over a relatively short time-scale immediately following major system disturbances. In this context the results described in the previous section lead to the conclusion that the enhancement associated with the variable speed model and saturation of the machine have not shown any major impact on the response of the synchronous generator over the time-scale of the study. Thus the extra computing effort to represent these phenomena accurately is not justified in this context.

Multiple-mass representation of the mechanical system, however, has a significant effect on the mechanical behavior following a major disturbance. The effects are propagated from oscillations of the rotor body which give rise to electrical oscillations on most system variables.

Adding multiple rotor masses on the shaft of the synchronous generator is critical in representing the response of the generator to systems which involve series system capacitance. The worst possible case is 3-phase or what is known as a balanced fault. The torques on the shaft are oscillating in phase with increasing amplitude such that the shaft of the synchronous generator could be easily damaged. Resonant frequency for the generator in this case is around 22 Hz. This frequency is critical as well in all other unbalanced faults and it shows critical torsional response of the shaft system.

The most important fact is that this generator model is capable of detecting such responses and give designers of the electrical power network valuable information about undesirable conditions in the network such as series system capacitor between electrical power system on one and block generator-transformer on the other side.

The overall model responses have shown that this 3-phase representation gives a designer or machine operator information in a form which can be easily compared with design or operational data, and its use can be justified if detailed transient information is required.

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8. APPENDIX

8.1 Lagrange's Interpolation Formula

This method assumes that a non-linear function is described by a series of pairs of co-ordinates (in our case the open-circuit characteristic of the generator) in a look-up table.

From the table n+1 values given as

x	$x_0, x_1, x_2, \ldots, x_n$
f(x)	$y_0, y_1, y_2, \ldots, y_n$

And not necessarily equally spaced, the substitute (approximate) function for a given value is:

$$\bar{y}(x) = \frac{(x-x_0)(x-x_1)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1 + \ldots$$

$$\ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_0-x_{n-1})} y_n$$

If the number of given values is small ($n \leq 6$) this formula is very powerful and convenient approximation model.

8.2 Data for the studies

Generator data:

x_d =1.313
x_{dc} =0.26
x_{dcc} =0.2
x_q =0.8
x_{qcc} =0.24
x_0 =0.1
x_l =0.143
r_a =0.00254
t_{d0c} =6.42
t_{d0cc} =0.04
t_{q0cc} =0.07
h =5.0

AVR data:

τ_1 =0.02
τ_2 =0.02
k =50.0
$v_{fd\ max}$ =4.0
$v_{fd\ min}$ =-2.0

Transformer data:

r_{t1} =0.01
r_{t2} =0.01
x_{t1} =20.1
x_{mc} =20.0
x_{t2} =20.1

Multi-mass model data:

k_{12} =200.0
k_{23} =250.0
h_1 =1.75
h_2 =2.0
h_3 =1.25
d_{12} =0.0
d_{23} =0.0

System conditions:

v =1.0
p =0.8
q =0.2
f =50 Hz

Computational data:

Integral step length: step=0.0001
Total number of steps: nstep=5002