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The effect of blade depth ratio on the performance of in-stream water wheels

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ABSTRACT

The power characteristics of an in-stream water wheel were measured experimentally to explore the influence of the blade depth ratio. The blade depth ratio has a significant effect on the performance of in-stream water wheels, but its influence has been overlooked throughout the literature. It was determined that the blade depth ratio has a greater impact on the power production than the number of blades at all tip-speed ratios. However, the variation between the maximum and minimum available power is greater at high blade depth ratios, so it is important to understand the relationship between the blade depth ratio and tip-speed ratio. Analysis of velocity triangles determined that at the inlet and outlet, the turbine blade contributes negatively to net torque. This effect is increased at higher blade depth ratios. It was also determined that the peak dry coefficient of power is linearly proportional to the blade submergence ratio, which is a measure of the total submerged blade area. This investigation progresses research in this area by highlighting an overlooked parameter and experimentally determining its influence on power characteristics.

Introduction

Affordable and Clean Energy is Goal 7 of the UN Sustainable Development Goals which aims to 'ensure access to affordable, reliable, sustainable and modern energy.' Hydropower is a major factor in achieving Goal 7 since it is a well-established means of generating renewable energy, with new large-scale systems (>100 MW) delivering up to 16,000MW per system, and total installations in 2021 of over 26 GW (Taylor, 2022). Hydropower systems designed for constant power can provide high power output and high capacity factor (up to 80% (Ottmar et al., 2011)) when compared with recent developments in solar photovoltaic and wind farm capacity (26% (Choudhary & Srivastava, 2019) and 35% (Boretti, 2019), respectively). However, large-scale hydropower systems require significant infrastructure, time to build, and are expensive (both in terms of capital and operational expenditure). They also require considerable water resources, in terms of head and flow rate, and can only be installed in very particular locations as a result. In resource-constrained communities, large-scale hydropower is often not a viable solution for energy generation. Alternatively, the water wheel is a hydropower option that can use low-head rivers and requires little additional infrastructure but is limited by low power production capabilities.

In-stream water wheels generate power exclusively from the kinetic energy in the fluid and have (nominally) zero head difference. The major downside of the in-stream water wheel is its poor efficiency, typically 20% to 40% depending on the flow regime (Cleynen, Kerikous, Hoerner, & Thévenin, 2018; Müller, Jenkins, & Batten, 2010; Zhao, Zheng, Yang, Zhang, & Tang, 2020), compared with overshot, breastshot, and undershot water wheels which operate at up to 85% efficiency, due to capturing additional potential energy (Müller & Kauppert, 2004). In-stream water wheels, however, are able to be deployed on any stream that has adequate flow velocity, and require little infrastructure compared to other water wheels. This makes them an attractive method of power generation in situations where no head is available but only a small amount of power is required, such as in resource-constrained communities where other methods of generating power are either too expensive, too polluting, or unsuited geographically. Notably, total power output per unit cost is likely to be more important than efficiency in the context of resource-constrained communities. There has recently been renewed interest in this water wheel technology due to the reasons mentioned above, although the literature is still sparse, with fewer than 25 papers investigating in-stream turbines up to 2018 (Quaranta, 2018). More research is required to modernise the technology and provide substantiated guidelines when determining suitable locations and designing power solutions for resource-constrained areas.

In-stream water wheels can further be broken down into subcategories based on the flow regime: deep flow wheels, where the blockage ratio of the fluid channel, defined as the ratio between the submerged turbine frontal area and the channel cross-sectional area, is <10% (Müller et al., 2010), subcritical shallow flow (where the blockage ratio is high and the Froude Number <1) and supercritical shallow flow (where the blockage ratio is high and the Froude

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Nomenclature

b	– turbine blade width (m)
BSR	– blade submergence ratio (–)
Cp_d	– dry coefficient of power, based on rotor area, Eq. (3) (–)
Cp_w	– wet coefficient of power, based on submerged area, Eq. (4) (–)
D	– blade diameter (m)
Fr	– Froude Number
ϵ	– measurement uncertainty (%)
h	– submerged blade depth (m)
h_n	– blade depth ratio, Eq. (1) (–)
λ	– tip-speed ratio (–)
n	– number of blades (–)
n_{min}	– minimum number of blades at which losses occur (–)
ω	– rotational velocity (s^{-1})
R	– blade radius (m)
ρ	– water density (kg m^{-3})
T	– torque (N m)
θ	– turbine angle (–)
U	– upstream flow velocity (m s^{-1})

Number >1) (Quaranta, 2018). The Froude Number in this context identifies whether the fluid flow is 'slow, tranquil, fluvial' (subcritical) or 'fast, shooting, torrential' (supercritical) (Chanson, 2004). Most riverine flows are subcritical and turbines have low blade capture areas compared to the river cross-sectional area. In general, turbines in shallow flow generate more power, as there is an induced head drop across the turbine due to blockage, but these turbines also usually require more infrastructure or have significant impacts on downstream ecology (Quaranta, 2018). In addition, results that are applicable for subcritical shallow flows may not be applicable for deep flows or supercritical flows (and vice versa). As a result of all this, the variables that influence power generation are varied and, in some cases, poorly understood.

The power production from in-stream water wheels is based on the flow of water around the turbine blades and the subsequent transfer of energy through dynamic pressure. Water wheel performance is dependent on several variables, including, but not limited to, the number of blades, the depth of the blade in the free stream, the blade capture area, and the blade shape. Many of these variables are complicated to change in situ, which contributes to overall system cost and feasibility. As a result, the present study aims to determine the influence of the blade depth on the power production of a deep flow in-stream water wheel. The blade depth can be controlled without major changes to the construction of an in-stream water wheel. This contrasts with, for example, the shape of the blade, which requires additional manufacturing.

The effect of the blade depth ratio (h_n), defined in Eq. (1) as the ratio between the submerged blade depth (h) and the total blade radius (R), has seen little research in the literature surrounding in-stream water wheels when compared to other parameters and design constraints.

$$h_n = \frac{h}{R} \tag{1}$$

Table 1 lists some key studies related to in-stream water wheels, with a particular focus on papers related to the blade depth ratio. The table lists the flow regimes, number of blades n, tip-speed ratios (defined as the ratio between the speed of the blade tip and the upstream fluid velocity, $\lambda = \frac{R\omega}{U}$), the blade depth ratio h_n, and dry coefficient of power range (defined in Eq. (3)).

A more complete summary of the research into in-stream water wheels (up to 2018) has been undertaken (Quaranta, 2018), however, the studies mentioned in Table 1 are highlighted to demonstrate the lack of consistency or justification in the choice of blade depth ratio. It is also unclear from the literature what impact the blade depth ratio has on the performance of in-stream water wheels. Of the studies listed, only four studies (Cleynen et al., 2021, 2018; Yah et al., 2016; Zhao et al., 2020) attempt to understand this relationship. Most of these studies are based on numerical work (CFD), with some experimental validation. CFD analysis of the effects of blade depth on power production (Yah et al., 2016) found that optimal power production occurred at $h_n = 0.4$, but these results relied on simplified flow conditions (laminar steady state flow) and have no experimental validation. This study in particular is described as having 'inconceivable flow fields' (Cleynen et al., 2018). Cleynen et al. (2018) attempted to characterise the performance of an in-stream water wheel using CFD. This study investigated several key parameters: the submerged depth (blade depth ratio), the free-stream velocity, the blade tip angle, and the number of blades. With respect to the blade depth ratio, the study determined that both the wet and dry coefficient of power increased as the blade depth ratio increased. However, a limitation of this study is that only low blade depth ratios were considered (0.17, 0.25 and 0.33), so it is unclear whether greater blade depth ratios continue to increase the power production or if there is a blade depth ratio corresponding to the peak power. Furthermore, the study only presented results in a narrow range of tip-speed ratios (0.5–0.65), which makes the conclusion difficult to generalise. The work from this study continued in a further study (Cleynen et al., 2021), in which the authors used genetic optimisation algorithms to influence the design characteristics of a 2D CFD simulation of an in-stream water wheel. The aim of this genetic optimisation was to determine the key power characteristics as well as arrive at an 'optimal' design. This work relied on experimental validation from the previous study (Cleynen et al., 2018). The genetic optimisation study determined that 'well-performing' wheels feature either large radius and low absolute blade depth, or small radius and high absolute blade depth. The study concludes by stating that for installations where cost is a key factor, operators should maximise power per unit area through a small-diameter, low-depth blade, with a relatively high blade depth ratio. The final study to explore the effect of the blade depth ratio investigated a wide ($b \gg R$) in-stream turbine in shallow flow (Zhao et al., 2020). This study determined that the optimal blade depth ratio is approximately 0.83, and occurs at a tip-speed ratio of approximately 0.2. However, due to the flow regime present in the study, it is likely that the dominant power mechanism is from the head loss across the turbine since the blockage ratio is over 0.5. This is discussed in the paper, stating that the water level is 'dramatically elevated' at the optimal blade depth ratio. Thus, it is difficult to draw conclusions about the effect of the blade depth ratio on the performance of in-stream water wheels from Zhao et al. (2020). The studies mentioned in this section highlight the importance of investigating power production at a wide range of blade depth ratios and tip-speed ratios in isolation of confounding factors such as high blockage ratios.

The remainder of the studies mentioned in Table 1 have a (usually implicit) blade depth ratio that influences the coefficient of power results without being adequately addressed as a design constraint. As an example, one study referred to the blade depth as the 'draft' and stated that this value was held constant across each experimental test, without an explanation for why its value was chosen or what influence it has on the power production (Batten et al., 2011). Another study (Al-Dabbagh, 2018) does not mention the blade depth ratio at all, and it has to be inferred from diagrams. This is not to say that these papers (or any papers discussed in this section) are inaccurate, rather that the effect of the blade depth ratio has not been comprehensively

Table 1
Studies of interest that investigated in-stream turbines with zero head, with a focus on the incongruence of blade depth ratio choice. Studies in bold deliberately mentioned or discussed the blade depth ratio effect.

Study	Flow Regime	n	λ	h_n	Cp_d
Müller et al. (2010)	Shallow flow[a] (exp.)	10	Not stated	0.2	0.4
Batten et al. (2011)	Shallow flow (exp.)	12	0.2–1.3[b]	0.4	0–0.8
Yah, Idris, and Oumer (2016)	Deep flow (CFD)	6	Not stated	0.2–0.8	0.01–0.03
Al-Dabbagh (2018)	Shallow flow (CFD)	12	0.2–0.9	0.33	0.01–0.14
Cleynen et al. (2018)	Deep flow (exp.) Deep flow (CFD)	10 6–12	0.28–0.65 0.5–0.65	0.33 0.17, 0.25, 0.33	Not stated 0.03–0.07
Nguyen, Jeong, and Yang (2018)	Shallow flow (CFD)	3–12	0.05–0.85	>1[c]	0–0.4
Zhao et al. (2020)	Shallow flow (exp.) Shallow flow (CFD)	8 8	Not stated 0.1–0.4	0.8 0.41–0.83	Not stated 0–0.18
Cleynen, Engel, Hoerner, and Thévenin (2021)	Deep flow (CFD)	Var.	Var.	0.1–1	Var.

[a] While physically in shallow flow, correction factors were introduced to account for blockage effects.
[b] Tip-speed ratio exceeds 1 due to a flow contraction.
[c] Blade depth ratio exceeds 1 due to high blockage ratio raising upstream water level.

investigated. This is best exemplified by the review paper (Quaranta, 2018), which defines the blade depth ratio as the ratio between h and D, where h is defined as the submerged depth of the blade, and D is the diameter of the turbine. The review paper states that $\frac{h}{D} = 0.2$ is a common ratio suggested in literature (Müller et al., 2010), however the setup described in the study (Müller et al., 2010) that the review paper references has a blade depth of 50 mm and a turbine diameter of 500 mm, giving $\frac{h}{D} = 0.1$. Instead, the ratio of 0.2 mentioned in this study refers to the ratio between submerged blade depth and channel depth (Müller et al., 2010). This confusion arises from a lack of shared terminology and, understandably, a misunderstanding of the impact of the blade depth ratio. A technical paper published in Germany in 1899 is cited as the design guideline for the optimal ratio (Müller, 1899). The technical paper refers to large turbines with reasonably high blockage ratios, such as those used for milling in ancient times, rather than smaller in-stream setups which are more common in recent research. In the review paper (Quaranta, 2018), the ratio $\frac{h}{D}$ is used to estimate the optimal number of blades. The authors of the review paper suggest an empirical relationship between the optimal number of turbine blades and the blade depth ratio as follows:

$$\frac{n_{min}}{n} = 7.76\frac{h}{D} - 0.31 \tag{2}$$

Here h and D are as defined above. The ratio $\frac{n_{min}}{n}$ is the ratio between the minimum number of blades before volumetric and leakage loss occur and the number of blades used, so this relationship is offered as a way to determine the optimal number of blades for a given blade depth ratio. The review paper explains that as n increases, the performance eventually decreases, implying that the turbines with fewer blades are preferable. However, there is a minimum number of blades, n_{min}, at which volumetric and leakage losses occur and subsequently decrease the performance. This explanation of $\frac{n_{min}}{n}$ refers to turbines in shallow supercritical flow (Quaranta, 2018), so it is uncertain how well this analysis actually relates to deep flow turbines where volumetric and leakage losses are present at any number of blades, since the blockage ratio is low and fluid is free to move around the turbine. Thus, using the recommended value of $\frac{h}{D} = 0.2$, and Eq. (2), the authors recommend a turbine with $\frac{n_{min}}{n} = 1.2$. This suggests that the number of blades required should be less than the minimum number of blades at which losses occur. This confusion around the blade depth ratio in the review paper (Quaranta, 2018), as well as incomplete research in the aforementioned studies

Cleynen et al. (2021, 2018), Yah et al. (2016) and Zhao et al. (2020) all highlight the fact that the blade depth ratio requires a more focused and deliberate analysis.

Another conclusion from Table 1 and the research review (Quaranta, 2018) is that the flow regime is another important design choice. As mentioned previously, the current study aims to investigate in-stream turbines in deep flow. This is for a number of reasons. The first is related to cost — shallow flow turbines are generally more expensive than deep flow turbines, since in almost all instances the shallow flow condition is achieved by constructing a channel or relying on a pre-existing channel to induce the high blockage ratio necessary. This flow regime can also be developed using a pontoon structure to develop a high blockage ratio and artificial head difference across the turbine (Batten et al., 2011), and this structure also has an associated cost. The second reason that the current study focuses on deep flow is to minimise the impact of other variables such as the blockage ratio and changes in the fluid velocity due to contractions in an attempt to isolate the effect of the blade depth ratio.

It is also worth noting that in the studies listed in Table 1, and in the broader literature, a variety of blade shapes is used. It has been determined that some curvature of the blade is beneficial for overshot, breastshot, and undershot water wheels (Quaranta & Revelli, 2018), but there is less analysis for in-stream water wheels. However, it has been identified that blades with curved tips (Cleynen et al., 2018) have a greater wet coefficient of power (defined in Eq. (4)), increasing from 0.39 to 0.42 at a tip-speed ratio of 0.65 when compared to a straight blade. It is, however, unclear whether this increase is meaningful in practice — in terms of cost per unit energy, the increased manufacturing complexity may make this change in blade structure of little importance.

The flow velocity available in streams and rivers is often overestimated (or not considered) in the literature. This overestimation phenomenon has been investigated (Kirke, 2019) and it was determined that studies should focus on flow velocities less than $1 ms^{-1}$, as anything higher is rare and generally only occurs during flood seasons. There are, however, several ways to accelerate water for an in-stream water wheel – by introducing a flow contraction, for example – but these often require significant infrastructure and cost when compared with a simple in-stream turbine. The focus of the present study is on low blockage ratio, deep flow turbines and consequently, the range of velocities is intentionally quite low.

To reiterate, the primary objective of this study is to investigate and quantify the effects of the blade depth ratio on the power production of in-stream water wheels in deep flow at a range of tip-speed ratios and blade numbers. While this important characteristic has been investigated to varying degrees in the literature, there are often limitations on the conclusions drawn, and in many cases, the importance of the blade depth ratio has been understated or ignored.

Methodology

Experimental setup

Experimental tests were performed in a recirculating flow water channel with a maximum operating flow area of 0.5 m by 0.5 m. The primary aim of the experiments was to improve the understanding of water wheels in the context of wide, deep river flows (i.e., the surface is largely unbounded), so to minimise boundary effects, the blockage ratio in the water tunnel had to be as low as practicable. A blockage ratio less than 0.15 is generally considered low, and a blockage ratio of 0.05 results in a theoretical change in the drag coefficient of less than 10% (Ramamurthy, Balachandar, & Vo, 1989). Unfortunately, there are no studies specific to in-stream water wheels quantifying the effect of blockage ratio on power production, so these results are used as a guideline. As detailed previously, deep flow turbines are categorised by a blockage ratio less than 10% (Müller et al., 2010), so the blockage ratio in the tunnel had to satisfy this criterion as well. Therefore, the water wheel used in the experiments had a radius $R = 0.125$ m and a width $b = 0.08$ m, giving a maximum blockage ratio of 0.05. The aspect ratio $\left(\frac{b}{R}\right)$ used was like that used in the HYLOW project (Müller, 2013). As discussed in Section "Introduction", curved blades may offer a slight increase in efficiency, but also come with increased manufacturing costs. This study aims to use the simplest geometry and blade shape available to both isolate the effect of the blade depth ratio and ensure the primary motivation – to enable access to low cost per unit energy turbines – is maintained.

The flow in the water channel was driven by a pump operated by a frequency-controlled 3-phase AC motor. The flow velocity near the surface was measured using dye injected at the centreline of the water channel, where its travel time between two points was then measured. While rudimentary, this flow measurement method is both minimally intrusive and accurate to an adequate level. These dye studies were also used to assess the uniformity of the velocity across the surface of the test section, and no velocity deficit regions were found. Most studies vastly overestimate the available flow velocities in rivers (Kirke, 2019), so a flow velocity of 0.33 m s^{-1} ($\pm$0.01 m s^{-1}) was considered a good representation of available river flows.

The water wheel was suspended over the water tunnel in such a way that the only object in the flow was the turbine itself. In the field, there will likely be some supporting structures in the flow, but for the purposes of this study, attempts were made to reduce interference as much as possible. The blade depth ratios tested were selected based on preliminary tests; blade depth ratios lower than 0.26 were too low for the turbine to rotate, and blade depth ratios higher than 0.56 introduced significant blade entry splash causing concerns about the potential for corrosion near the shaft and measurement mechanisms. This is largely consistent with other experimental studies (Cleynen et al., 2018). A high blade depth ratio can also cause maintenance issues in the field for the same reason, and if the turbine is fixed, sudden increases in the fluid level can endanger the electrical and drivetrain components of the turbine.

While the turbine was placed into stationary flow, it was found that the fluid surface dropped when the channel was in operation. Thus, the actual blade depth ratio was different to the blade depth ratio that the turbine was nominally placed at. Unless stated otherwise, the blade depth ratio discussed in the results is the *actual blade depth ratio* as measured when the turbine is moving. The actual blade depth ratio was determined using a flow visualisation method (Toole, Birzer, & Kelso, 2022). Particularly for low blade depth ratios, this can have a significant effect on the results, since the turbine is small.

Several different blade numbers were used, ranging from three to ten. However, results have not been presented for turbines with fewer than six blades, because at most blade depth ratios, these turbines do not rotate. The blade numbers investigated more closely ($n = 6$, 8, and 10) were chosen due to their relative ease to construct compared to turbines with a larger number of blades.

A custom torque measurement system was developed whereby a DC motor (Maxon RE-36) was attached to the water wheel shaft, and a load cell (TAL-221) was attached to the DC motor. This motor acted as the tension belt of a Prony brake (traditionally used to measure water wheel torque Batten et al., 2011), but, unlike a Prony brake, the counter-force it applied could be varied more finely using a DC power supply. The DC motor applies a torque to the turbine shaft, resisting its rotation, and that torque is measured by the load cell attached to a lever arm. The experimental apparatus is shown in Fig. 1. It should be noted that the DC motor used has an ironless core, specifically designed to have low inertia. As a result, the apparatus has minimal cogging torque and can operate at the very low torque values present in this study.

The rotational velocity of the turbine was measured using a Hall effect sensor and a small magnet attached to a turbine blade. In combination with flow velocity measurements, this allowed for dynamic measurement of the tip-speed ratio and dynamic torque of the system. The raw data was filtered and analysed using MATLAB. Details of the measurement uncertainty can be found in Section "Procedure and Errors".

Procedure and errors

The tests were performed with an inlet velocity of 0.33m s^{-1} and blade depth ratios from 0.26 to 0.56. For each case, the tip-speed ratio (λ) was varied over the operating range using the methods described in Section "Methodology". The tip-speed ratio is one of the primary variables affecting the coefficient of power, and it is used throughout water wheel research (e.g. Cleynen et al., 2018; Nguyen et al., 2018) as well as extensively in wind turbine research (Hansen, 2013). Each data point was gathered over approximately 120 s to minimise error and ensure that an adequate number of rotations were recorded (depending on the specific case, but always at least 10 rotations). A minimum of four repeats were conducted to reduce random error. In general, the uncertainty across measurements, ϵ, was less than 10%, and typically $\pm$2%–3%. The error was generally greater at lower blade depth ratios, i.e., when the absolute values for the measured force were lower. The errors present in the experiment can be separated into systematic and random errors, as follows.

Systematic errors refer to instrumentation errors and errors associated with experimental techniques. There were several sources of experimental technique error that caused some uncertainty, generally due to changing the experimental conditions. Sources of errors include small differences in the depth setting of the turbine, variations in the water depth in the tunnel leading to small velocity differences, and misalignment of the load cell and lever arm leading to minor differences in the measured torque. Based on experimental observations, the turbine placement was the largest source of systematic error - at low blade depth ratios, a misalignment of 3 mm led to errors of 10%. This effect was minimised by careful alignment of the turbine between tests.

In addition to these systematic errors, there were also several sources of random errors. These errors included load cell measurement errors due to electrical noise (approximately 0.05% of full scale according to its datasheet) and inaccuracies in the measurement of dye speed. The effects of random errors were reduced by experimental repetition, with the overall uncertainty ϵ below 10% as mentioned above.

In the experimental data represented graphically throughout this paper, the error bars represent $\pm$1 standard deviation from the mean value recorded over several repeated experiments. For the sake of clarity, only error bars for the vertical axis have been presented.

It should be noted that there is a 'generator and gearbox' efficiency loss associated with friction effects and the drivetrain. The measured value of approximately 1.9 mN m remained constant when different torques and angular velocities were applied.

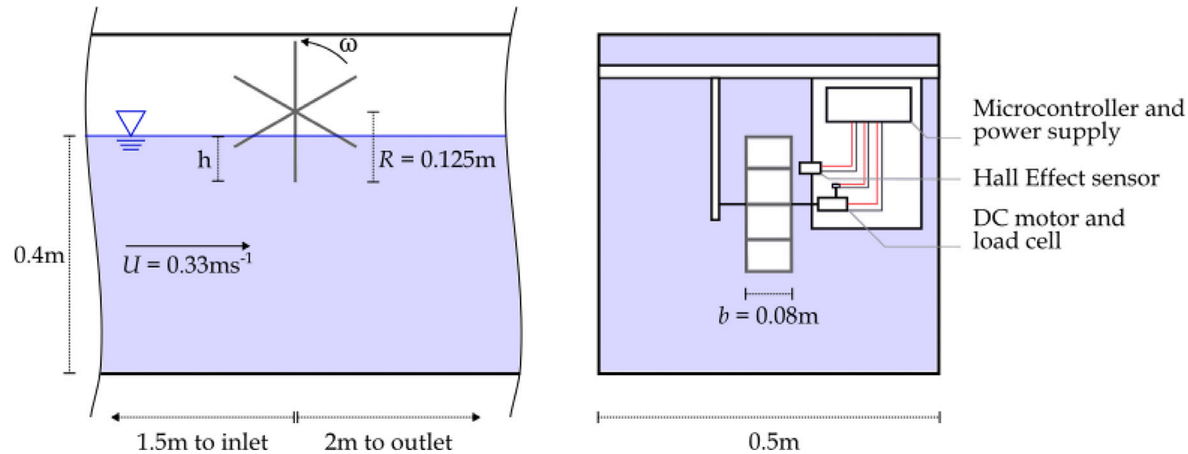

Fig. 1. Schematic of experimental apparatus, highlighting the turbine blades, water tunnel dimensions, and measurement equipment. Perpendicular to flow (left) and from above (right).

Power coefficient theory

This study explores the effect of the blade depth ratio on both the dry coefficient of power and the wet coefficient of power as defined in Cleynen et al. (2018).

The dry coefficient of power is defined as the power produced by the turbine divided by the theoretically available power in the fluid according to the drag equation, i.e.

$$Cp_d = \frac{\text{Measured Power}}{\frac{1}{2}\rho(2Rb)U^3} \tag{3}$$

where ρ is the fluid density (kg m^{-3}), R is the radius of the turbine (m), b is the width of the turbine (m), and U is the free stream velocity (m s^{-1}). This gives a non-dimensional form of the power produced based on the turbine frontal area and allows for comparisons of the overall system efficiency at different blade depth ratios and blade numbers. The dry coefficient of power allows for comparisons between different configurations for a given turbine radius, while the wet coefficient of power provides a clearer method of comparing the effect of blade depth ratio itself, since it is an area-normalised coefficient, rather than normalised based on the total size of the turbine.

In Cleynen et al. (2018), the wet coefficient of power is defined by the coefficient of power for each blade, averaged for each blade in the flow across the stroke angle of the blade. However, for the experimental setup used here it is not possible to separate the power coefficient by blade; only the sum of all torques on the shaft is measured. In this study, instead, the wet coefficient of power is defined by the nominal submerged depth of the blade, h, at an angle perpendicular to the flow. This provides an 'area-normalised' coefficient of power, i.e.

$$Cp_w = \frac{\text{Measured Power}}{\frac{1}{2}\rho(hb)U^3} = \frac{\text{Measured Power}}{\frac{1}{2}\rho(Rbh_n)U^3}, \tag{4}$$

where h is the submerged blade depth (m), h_n is the blade depth ratio (as defined in Eq. (1)) and the other variables are as defined above. This value allows for clearer comparison between different blade depth ratios since it normalises the power production on a submerged area basis.

Results

Dry coefficient of power

The dry coefficient of power is important when considering the overall power efficiency of the in-stream water wheel. It also allows for comparison between different turbine configurations. Table 2 shows the relative change in the peak dry coefficient of power for increasing blade depth ratio, averaged over the number of blades. The mean relative increase in the peak dry coefficient of power for different numbers of blades is 24% (6 to 8 blades) and 9% (8 to 10 blades). Using these values, it is evident that increasing the blade depth ratio produces a greater increase in power than increasing the number of blades over the viable range (as defined in Section "Methodology"). The increase is most important at low blade depth ratios (e.g., increasing from $h_n = 0.26$ to $h_n = 0.36$), but an increase in the blade depth ratio is still more impactful than increasing the number of blades. This is an important quantitative result for designing and siting an in-stream water wheel. While adding additional turbine blades can add significant cost (potentially eliminating any potential gains in power per unit cost), ensuring that the turbine is placed at an adequate depth can provide significant benefits for no additional cost, thus directly improving the power per unit cost.

Table 2
Relative Cp_d increase for increasing blade depth ratio h_n.

Blade depth ratio change	% Cp_d increase
0.26 to 0.36	64
0.36 to 0.46	39
0.46 to 0.56	33
0.26 to 0.56	203

Fig. 2 allows for an easier comparison of the effect of blade depth ratio for each blade number configuration. Fig. 2 can also be used to explore the effect of blade depth ratio on the variability of dry coefficients of power, or alternatively, the difference between the maximum and minimum dry coefficient of power for a given configuration. For each blade number and blade depth ratio, this range was calculated and can be seen in Table 3. The absolute values refer to the change in the absolute value of Cp_d for each blade depth ratio (averaged for each blade number). The variability percentage refers to the mean difference between the maximum and minimum dry coefficient of power relative to the minimum (for example, for a blade depth ratio of 0.56, the maximum dry coefficient of power is, on average, 360% of the minimum). This table indicates a potential trade-off between power generation and power variability. While a turbine with a greater blade depth ratio generates considerably more power, it also becomes more important to properly control the tip-speed ratio, as an improperly controlled tip-speed ratio will eliminate any potential efficiency gains. This is primarily caused by turbines at high tip-speed ratios having a low slip velocity, where the slip velocity is the difference between the blade tip velocity and the upstream velocity (i.e., as λ increases, $U - R\omega$ decreases).

To summarise, the most efficient turbine configuration, in terms of the dry coefficient of power, is a 10-blade turbine, with a blade depth

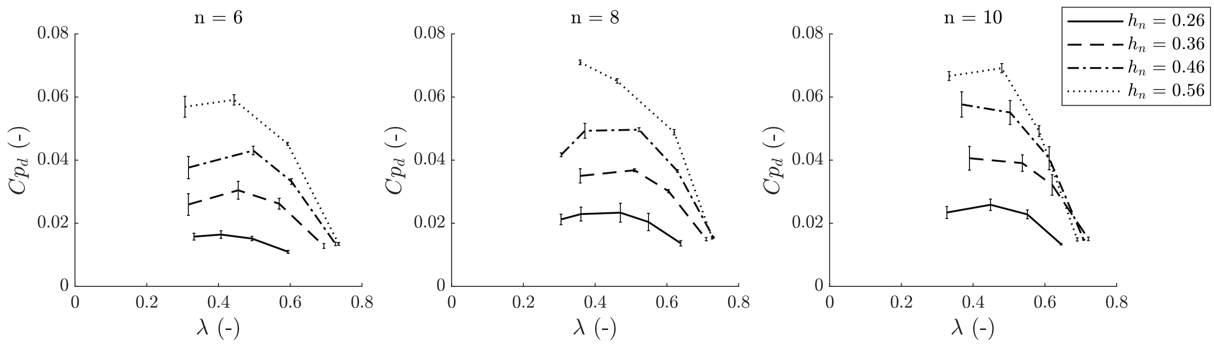

Fig. 2. Dry coefficient of power Cp_d versus tip-speed ratio λ with variable blade depth ratio h_n. $Cp_d - \lambda$ curves for (left: $n = 6$; middle: $n = 8$; right: $n = 10$) at constant flow velocity.

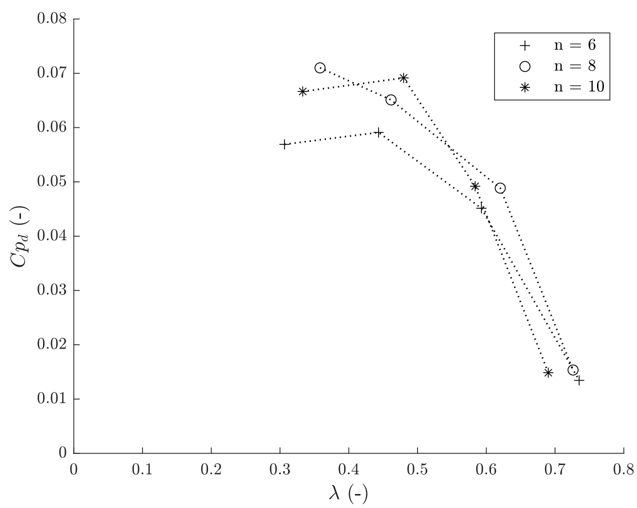

Fig. 3. Comparison of dry coefficient of power for $h_n = 0.56$ case for different numbers of blades.

Table 3
Table of absolute and relative differences in dry coefficient of power for different nominal blade depth ratios.

Blade depth ratio	Absolute Cp_d change	Variability of Cp_d
$h_n = 0.26$	0.01	75%
$h_n = 0.36$	0.021	150%
$h_n = 0.46$	0.035	240%
$h_n = 0.56$	0.05	360%

ratio of 0.56, operating at a tip-speed ratio near 0.5. However, there is evidence (see Fig. 3) that an 8-blade turbine would perform similarly, and a 6-blade turbine would only perform slightly worse under the same conditions, noting that the 6- and 8-blade turbines would operate at a lower material cost due to fewer blades. A key finding is that the blade depth ratio has a much greater impact on the dry coefficient of power than the number of blades. To better understand the fluid dynamics that govern the power production of in-stream water wheels, it is necessary to explore the wet coefficient of power.

Wet coefficient of power

Fig. 4 shows that the wet coefficient of power is dependent on both the number of blades and the blade depth ratio of the turbine. This is an expected result, as the previous section demonstrated that these variables significantly influence the power generated by the turbine. A clear trend is that, regardless of the blade depth ratio, increasing the number of blades increases the wet coefficient of power. Increasing from 6 to 8 blades, the mean increase in peak wet coefficient of power is 0.035 (in absolute terms), while increasing from 8 to 10 blades lifts Cp_w by 0.016. This is a mean relative increase of 24% and 9.1%, respectively. It should be noted that an 8-blade water wheel uses 33% more blade material than a 6-blade turbine, and a 10-blade turbine uses 25% more than an 8-blade turbine, so it is uncertain whether the increased efficiency is useful in terms of cost efficiency.

It is also evident that the wet coefficient of power typically improves as the blade depth ratio increases. This is clearest for the 6-blade turbine, but the effect is reduced for 8-blade turbines, and effectively negligible for 10-blade turbines. As a reminder, a 10-blade turbine's overall power output (Cp_d) does increase as the blade depth ratio increases, but the area-normalised efficiency remains relatively constant across blade depth ratios.

In addition to the quantitative results discussed above, there are some general trends observable from Fig. 4. Firstly, regardless of blade number, the optimal λ occurs between 0.4 and 0.5 for low blade depth ratios, and at a blade depth ratio of 0.56, the optimal λ is approximately 0.3. This can be explained by qualitative observations of the flow field, but quantitative studies of the flow fields are an object of further study. For an in-stream water wheel, the dynamic pressure of the flow that is harnessed to generate power is competing with a resistive force generated by the blade moving through and displacing the fluid behind the blade. Both forces are related to the blade depth ratio since it influences the frontal area of the blade, but the exact relationships cannot be discerned from these results. In addition to these competing forces, there are also significant power losses caused by the blades leaving and entering the flow. Generally, these power losses can be categorised using the following list, adapted from a description of power losses for breastshot water wheels (Quaranta & Revelli, 2018):

1. Inflow power losses. At high rotational velocities, more power is lost when the blades impact the water surface. At low rotational velocities, there is power loss from the blades moving through the flow without exploiting kinetic energy. As a result, there is a rotational velocity where both losses are minimised.
2. Outflow power losses. Near the outlet of the turbine, the blades experience drag while moving through the fluid without transferring kinetic energy. There are also outflow losses associated with water 'uplift' as the blade exits the flow.
3. Mechanical friction losses. Friction on the shaft is related to the wheel weight and constant across rotational velocities. There are also bearing friction losses.
4. Volumetric and buoyancy losses, which refer to losses due to flow moving around rather than 'through' the turbine, and the tendency for the blades to rise to the surface due to buoyancy. Both volumetric and buoyancy losses are not of particular relevance to in-stream water wheels in deep flow.

There are also minor losses due to aerodynamic drag as the blades travel through the air. Due to the low rotational velocity and the low density of air, these are minor. From the above power losses, it

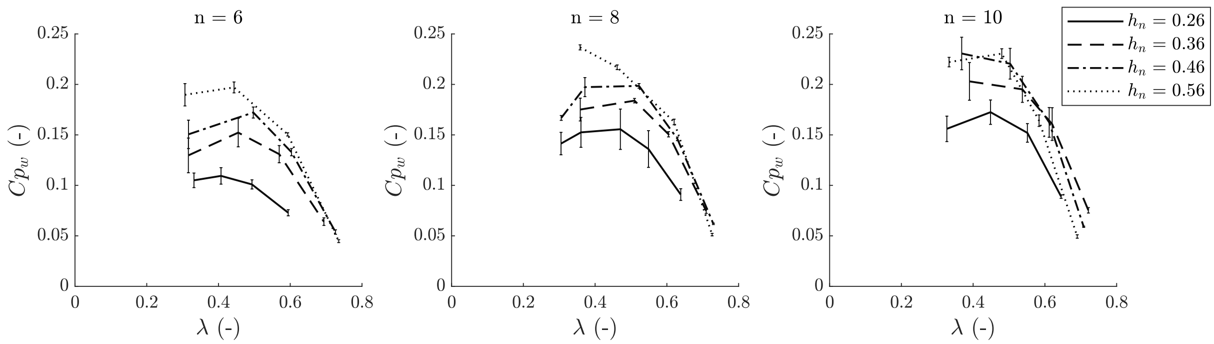

Fig. 4. Wet coefficient of power Cp_w versus tip-speed ratio λ with variable blade depth ratio h_n. $Cp_w - \lambda$ curves for (left: n = 6; middle: n = 8; right: n = 10) at constant flow velocity.

can be deduced that there is an optimal tip-speed ratio for a given turbine where the power losses are minimised. Most theoretical and numerical considerations place this optimal tip-speed ratio between 0.3 and 0.55, so the experimental results here agree with the literature (Cleynen et al., 2021; Denny, 2004). This finding is useful to validate the experimental results. Based on the power losses mentioned above and the results from the experiments, some general observations about the effect of blade depth ratio can be made.

1. Regarding inflow power losses, the effect of blade impact is greater at higher blade depth ratios. This is due to the blades hitting the fluid surface at a flatter angle, creating a larger impact force. At lower blade depth ratios, the blade hits the surface at an angle, reducing the impact force. However, this effect is balanced by a greater power production at higher blade depth ratios due to a greater submerged blade area.
2. Regarding outflow power losses, there appear to be greater water 'uplift' losses at higher blade depth ratios. This is due to the blades being closer to horizontal (or parallel to the flow surface) as they exit the flow, so the water flows off the blade more slowly. Further analysis is required to better understand this 'uplift.'

The angle of the blade as it enters the flow can also be explored by analysing the velocity at the inlet and outlet, as shown conceptually in Fig. 5. As mentioned in Section "Dry Coefficient of Power", the power generated by any blade is based on the velocity of the fluid with respect to the blade. This is part of the reason why high tip-speed ratio turbines are ineffective. The velocity of the water relative to the blade tip is defined as:

$$\underbrace{U_{wt}}_{\text{water relative to tip}} = \underbrace{U_{wg}}_{\text{water relative to ground}} - \underbrace{U_{tg}}_{\text{tip relative to ground}} \tag{5}$$

Near the inlet and outlet, the angle of the flow relative to the blade tip leads to negative torque being produced. The negative torque will persist until the flow relative to the tip is substantially aligned with the blade, the zero-torque angle, after which positive torque will be produced. The zero-torque angle is related solely to the tip-speed ratio and blade depth ratio. At low blade depth ratios, this effect is minimal, partially due to a small area in the flow, and partially due to the angle of entry. At higher tip-speed ratios, the zero-torque angle increases, and the phase of negative torque occurs over a larger angle of rotation, and subsequently has a greater impact. In addition to the risk to geartrain and electrical components, this analysis demonstrates that particularly high blade depth ratios can be detrimental. This also explains some of the losses associated with increasing the number of blades, since doing so increases the number of blades contributing negatively to the power generation at any given point of rotation.

Noting Eqs. (3) and (4), it is evident that

$$Cp_d = \frac{h_n}{2} Cp_w. \tag{6}$$

Thus even if the wet coefficient of power was constant for a given turbine configuration, the dry coefficient of power would increase linearly as the blade depth ratio increases linearly. However, from Fig. 4, it is evident that the wet coefficient of power is not constant for any turbine configuration and is governed by the blade depth ratio itself, as well as the tip-speed ratio. It is clear that the impact of the blade depth ratio and the tip-speed ratio on power production is not obvious, and that these variables depend on one another. For example, the results show that increasing the blade depth ratio typically increases Cp_w (and subsequently Cp_d), but not if the tip-speed ratio is inadequately controlled. Thus it is important to explore the relationship between the wet coefficient of power, the blade depth ratio, and the tip-speed ratio, more closely.

It is useful here to mention that the blade depth ratio is only a useful parameter, and does not correspond particularly well to any physical characteristic of the turbine when it is in operation — since the turbine is rotating, the actual depth of any blade is changing constantly, and each blade has a different blade depth ratio, notwithstanding any changes to the fluid surface due to interactions with the blades. To better explore the relationship between the blade depth ratio and power output, another variable is introduced – the blade submergence ratio BSR – to better account for the effects of the blade depth ratio.

Blade submergence ratio

The blade submergence ratio is an analytical estimation of the ratio between the total surface area of submerged blades in the flow and the total surface area of one blade, and can be used to better understand the results presented in this study. For turbines of 6, 8, and 10 blades, Fig. 6 represents the concept of the blade submergence ratio graphically. This ratio is conceptually simple but more complicated to determine. To simplify the formulation, it is assumed that the fluid surface is level and does not fluctuate as the blade rotates (in practice it does, but at low fluid velocities, not a significant amount). The blade submergence ratio also changes as the turbine rotates in the flow and blades enter and exit. Thus, rather than the instantaneous blade submergence ratio, instead the mean value is used.

For a turbine with n blades, after the turbine rotates by $\frac{2\pi}{n}$ radians, the turbine will return to the same position due to rotational symmetry around the turbine hub. This further simplifies the analysis — the mean blade submergence ratio across $0 < \theta < \frac{2\pi}{n}$ is equal to the mean blade submergence ratio across an entire turbine rotation. The next step is to define the submerged area of each blade as a function of θ. Using a six-blade turbine as an example, the submerged areas are calculated as follows. In this case, four blades interact with the flow.

$$A_1 = b\left(R - \frac{R - h_n}{cos\left(\frac{2\pi}{3} - \theta\right)}\right) \text{ for } \frac{2\pi}{3} - cos^{-1}\left(\frac{R - h_n}{R}\right) \le \theta \le \frac{2\pi}{6} \tag{7}$$

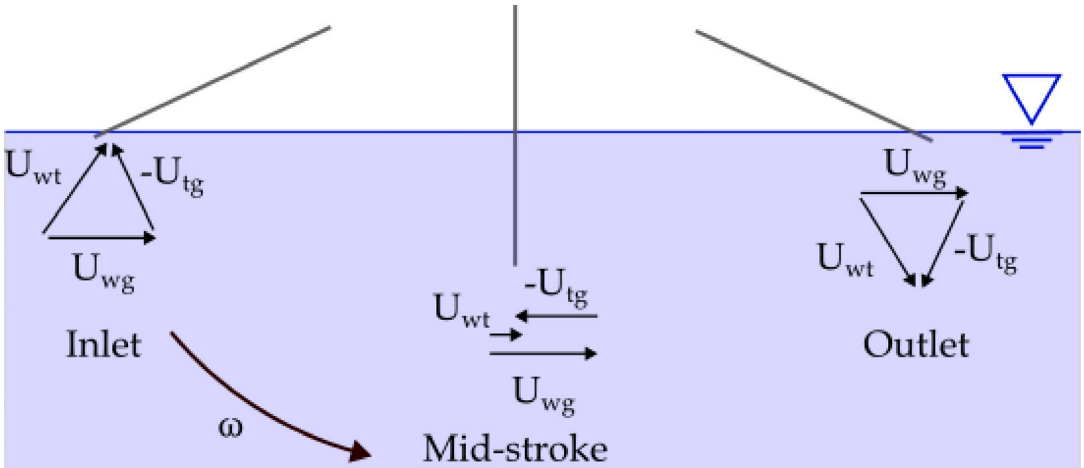

Fig. 5. Conceptual velocity triangles at blade inlet, mid-stroke, and outlet. Velocity triangles reflect the relationship identified in Eq. (5).

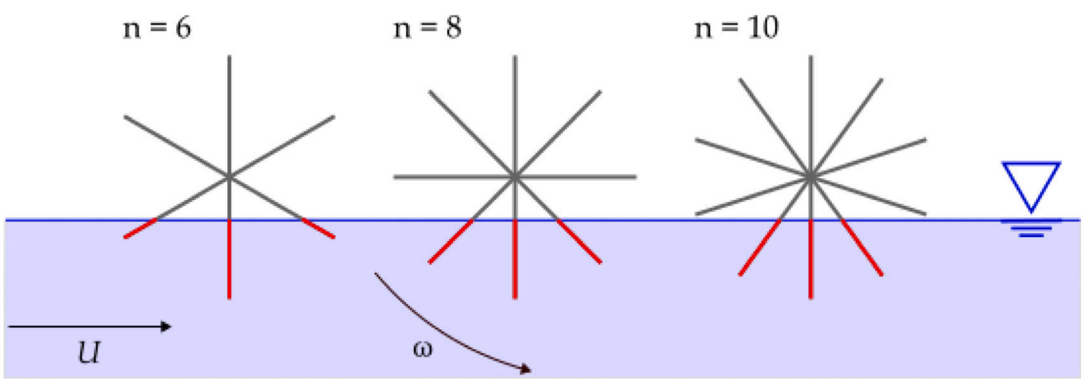

Fig. 6. Blade submergence ratio — the ratio between the red submerged area and the area of a single blade.

$$A_2 = b\left|R - \frac{R-h_n}{cos\left(\frac{2\pi}{6}-\theta\right)}\right| \text{ for } 0 \le \theta \le \frac{2\pi}{6} \quad (8)$$

$$A_3 = b\left(R - \frac{R-h_n}{cos\,(\theta)}\right) \text{ for } 0 \le \theta \le \frac{2\pi}{6} \quad (9)$$

$$A_4 = b\left|R - \frac{R-h_n}{cos\left(\frac{2\pi}{6}+\theta\right)}\right| \text{ for } 0 \le \theta \le cos^{-1}\left(\frac{R-h_n}{R}\right) - \frac{2\pi}{6} \quad (10)$$

Outside of the domains presented for θ, the areas are set to zero as they are no longer in the fluid flow. The limits of each domain are determined based on the geometry of the turbine. In addition to this, if at any θ the instantaneous blade submergence ratio is 0, the result is excluded (since if there is any point that the turbine has no blades in the flow, it will cease to rotate). The areas are then used to determine the instantaneous blade submergence ratio:

$$BSR(\theta) = \frac{A_1(\theta)+A_2(\theta)+A_3(\theta)+A_4(\theta)}{Rb} \quad (11)$$

and the mean blade submergence ratio:

$$\overline{BSR} = \frac{6}{2\pi}\int_0^{\frac{2\pi}{6}} BSR(\theta)d\theta \quad (12)$$

Due to the disparate domains of each area function, Eq. (12) is most easily determined computationally rather than analytically, by calculating the individual blade areas at discrete intervals of θ, then calculating the mean. This analysis was then repeated for turbines with 3, 4, 8 and 10 blades and the relationship between blade submergence ratio and blade depth ratio can be seen in Fig. 7. For the 3- and 4-blade turbines, this is a useful illustration of why these turbines are ineffective at almost all blade depth ratios. At low blade depth ratios both the 3- and 4-blade turbines have points where the total submerged area is 0, and even when at useful blade depth ratios (>0.5) their blade submergence ratio is low relative to the 6–10 blade turbines.

Before exploring the relationship between produced power and the blade submergence ratio, it is helpful to investigate the relationship between the blade depth ratio and the mean blade submergence ratio. As seen in Fig. 7, this relationship is exponential over the domain $0 < \frac{h_n}{R} \le 1$. It is evident that the relationship between the peak dry coefficient of power and the mean blade submergence ratio is linear across the middle blade depth ratio range between 0.26 and 0.56, with some variation due to experimental error. This can be seen in Fig. 9. It is uncertain whether this trend continues at higher blade depth ratios, but as discussed previously, it is often difficult for the blade depth ratio to be set greater than 0.56.

The relationship between the mean blade submergence ratio and the wet coefficient of power is shown in Fig. 8. As mentioned in Section "Wet Coefficient of Power", the blade depth ratio has an influence on the wet coefficient of power, but this is different for different blade numbers. However, when the peak wet coefficient of power is considered in terms of the mean blade submergence ratio, the number of blades makes little difference as shown in the figure. The wet coefficient of power increases as the mean blade submergence ratio increases regardless of the number of blades (other than minor variations). It is hypothesised, then, that the peak wet coefficient of power is primarily dependent on the blade submergence ratio. This is likely due to the blade submergence ratio accounting for the influence of all blades in

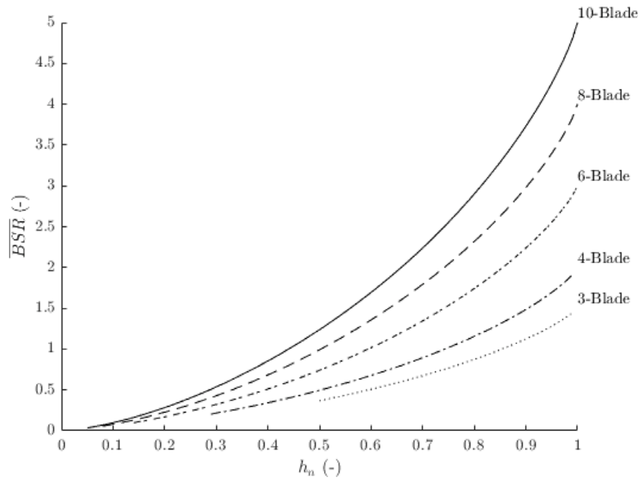

Fig. 7. Blade submergence ratio BSR vs. blade depth ratio for 3–10 blade turbines.

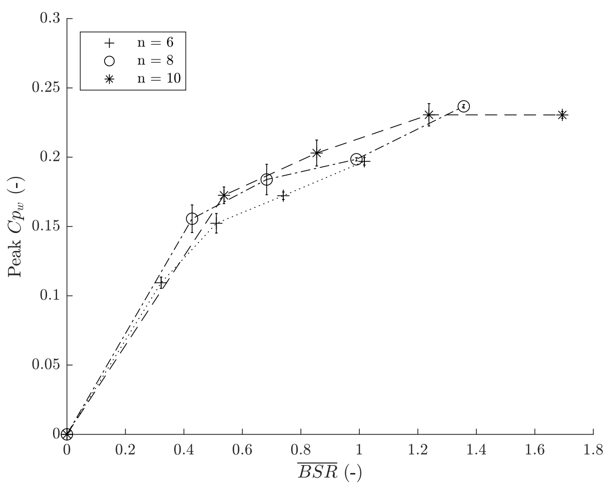

Fig. 8. Relationship between peak wet coefficient of power (Cp_w) and mean blade submergence ratio ($\overline{BSR}$) for different blade number (n) turbines.

the flow rather than just the maximum depth of a blade (i.e., the blade depth ratio). In addition, while the wet coefficient of power increases as the blade submergence ratio increases, this increases diminishes as the blade submergence ratio increases.

Fig. 9 leads to a simple relationship that can be used to assist in the deployment of in-stream water wheels: it is optimal to place a turbine as deep in the flow as practical when operating at feasible blade depth ratios between 0 and 0.56. Due to the drivetrains available to this type of turbine (e.g. geared DC generators), it is likely that blade depth ratios greater than 0.56 are impractical for small in-stream turbines, since water will cause detrimental wear on the drivetrain and electrical components.

Conclusion

This paper presents an experimental investigation of in-stream water wheels in deep flow. The main motivation behind the paper was to develop an understanding of a previously overlooked performance parameter, the blade depth ratio, so as to improve the ability to find suitable locations for installation. The experiments were conducted in a recirculating flow water tunnel. The major conclusions are as follows:

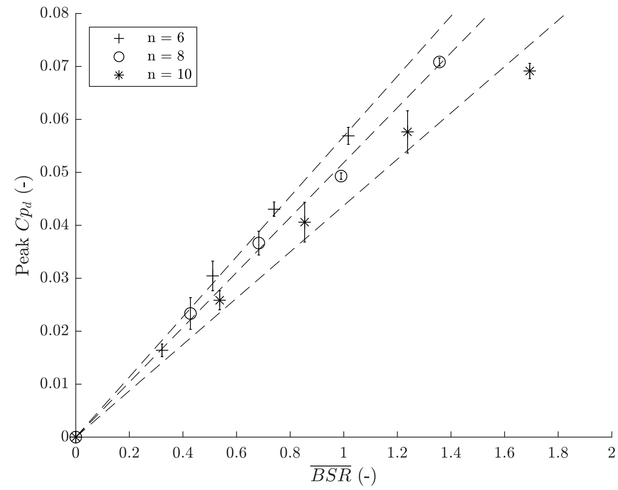

Fig. 9. Relationship between peak dry coefficient of power (Cp_d) and mean blade submergence ratio ($\overline{BSR}$) of different blade number (n) turbines.

- The blade depth ratio is a variable of unrecognised importance that has been found to have a significant effect on the performance of an in-stream water wheel. Throughout the literature the impact of this variable has been largely unexplored.
- The blade depth ratio has a greater impact on the power production of an in-stream turbine than the number of blades. Increasing the number of blades can increase the peak dry coefficient of power by up to 35% (from 6 to 10 blades), while increasing the blade depth ratio from 0.26 to 0.56 can increase the peak dry coefficient of power by over 200%.
- The importance of adequately controlling the tip-speed ratio increases as the blade depth ratio increases. The variation between the minimum and maximum available wet coefficient of power is an order of magnitude greater at high blade depth ratios.
- The peak wet coefficient of power increases as the blade submergence ratio increases, regardless of blade number, implying that the wet coefficient of power is primarily dependent on the blade submergence ratio.
- The peak dry coefficient of power is linearly proportional to the blade submergence ratio, a measure of the total submerged blade area. This relationship can be useful when designing or finding a location for an in-stream water wheel.
- It has been shown that a negative torque is produced at blade entry and exit, and its significance to net torque will depend on the blade depth ratio and tip-speed ratio.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data used to generate mean torque values is available at doi: 10.17632/pps4d75ntr.1. The MATLAB processing script can also be found in the repository.

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