Fast-convergent iterative scheme for filtering velocity signals and finding Kolmogorov scales

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The present fundamental knowledge of fluid turbulence has been established primarily from hot- and cold-wire measurements. Unfortunately, however, these measurements necessarily suffer from contamination by noise since no certain method has previously been available to optimally filter noise from the measured signals. This limitation has impeded our progress of understanding turbulence profoundly. We address this limitation by presenting a simple, fast-convergent iterative scheme to digitally filter signals optimally and find Kolmogorov scales definitely. The great efficacy of the scheme is demonstrated by its application to the instantaneous velocity measured in a turbulent jet.

In his pioneering work [1] on turbulence, Kolmogorov derived, based on dimensional reasoning, the characteristic length of the finest-scale turbulent motions to be

$$\eta = (\nu^3/\epsilon)^{1/4},$$

(1)

which is hence called the “Kolmogorov length scale.” In Eq. (1), \(\nu\) is the kinematic viscosity and \(\epsilon\) is the average dissipation rate of the turbulence kinetic energy given by (e.g., Hinze [2])

$$\epsilon = \nu (\partial u_i/\partial x_j + \partial u_j/\partial x_i)\partial u_i/\partial x_j$$

(2)

with standard Cartesian tensor notation and summation on repeated indices, where \(i\) or \(j=1, 2, \) and 3 represent the streamwise, lateral, and spanwise directions, respectively.

The appropriate estimate of \(\eta\) is of significant importance for improving our understanding of the fine-scale turbulence. However, great difficulty occurs in directly measuring this characteristic scale. To obtain \(\eta\) requires all the 12 terms of gradient correlations in Eq. (2) to be measured. This task cannot be realized by presently available experimental techniques. Accurate measurements of even one component of \(\partial u_i/\partial x_j\), as is well known, requires a multisensor probe with extremely high spatial and temporal resolution to incorporate even the smallest scales of velocity fluctuations. Moreover, several cross-correlation terms in Eq. (2) simply cannot be measured directly now [3] or in the foreseeable future, let alone direct measurements of \(\epsilon\) and \(\eta\). In this context, it remains necessary to estimate \(\eta\) from hot-wire measurements of \(\epsilon\) using the isotropic relation

$$\epsilon = 15 \nu (\partial u_1/\partial x_1)^2$$

(3)

and also Taylor’s hypothesis

$$(\partial u_1/\partial x_1)^2 = U_1^2(\partial u_1/\partial t)^2,$$

(4)

where \(U_1\) is the local streamwise mean velocity. Substitution of Eq. (4) into Eq. (3) leads to

$$\epsilon = 15 \nu U_1^2(\partial u_1/\partial t)^2.$$  

(5)

In addition, the Kolmogorov frequency is defined as

$$f_K = U_1(2\pi \eta)^{-1}. $$

(6)

It is important to note that the nonfiltered or slightly filtered velocity signals \(u_{im}\) (the subscript \(m\) means “measured”) is inevitably contaminated by high-frequency electronic noise \(n\), i.e.,

$$u_{im} = u_i + n.$$

(7)

This contamination causes both the spatial and temporal gradient variances to be overestimated, i.e.,

$$\overline{(\partial u_{im}/\partial x_i)^2} = \overline{(\partial u_i/\partial x_i)^2} + \overline{(\partial n/\partial x_i)^2},$$

(8)

$$\overline{(\partial u_{im}/\partial t)^2} = \overline{(\partial u_i/\partial t)^2} + \overline{(\partial n/\partial t)^2}.$$ 

The extra terms \(\overline{(\partial n/\partial x_i)^2}\) and \(\overline{(\partial n/\partial t)^2}\) are the unwanted noise contributions. Therefore, to achieve accurate measurements of the velocity gradients, it is necessary that the raw velocity signals \(u_{im}\) be low-pass filtered at a specific cutoff frequency \(f_c\) to eliminate the effect of noise. The choice of \(f_c\) is critical. Too high and it will not remove noise contributions sufficiently, while too low and it will wipe out some content of the signal. The right choice for \(f_c\) is the Kolmogorov frequency \(f_K\), i.e., the characteristic frequency of the smallest structures. However, \(f_K\) not only is a function of the flow but also varies with spatial locations in the flow so that it cannot be determined \textit{a priori}. The method described in [4] to determine \(f_c\) in \textit{situ} is complex, requiring two mechanical analog filters, a differentiator, a real-time spectrum analyzer, visual inspection, and optimization at \textit{each} measurement location. This procedure is only realistic where the number of spatial locations or flow conditions is limited, and is prohibitive for experiments where these are large. In the absence of a simpler procedure, more arbitrary criteria are usually adopted, so that most previous measurements of \(\epsilon\) must be contaminated by noise to some extent. The importance of this issue is also evident from [5] from which it is deduced that even slightly over filtering \(u_{im}\) at \(f_c < f_K\) may cause substantial underestimate of the velocity gradients.

From the above discussion it is obvious that substantial benefit would arise from a procedure that could correctly obtain \(f_K\) without prior knowledge of \(\eta\), or obtain both \(\eta\) and \(f_K\) solely from a nonfiltered signal of \(u_{im}\). The present work aims to address this issue, i.e., to develop a simple and ef-
ffective scheme that can definitively obtain $\eta$ and $f_K$ from $u_{1m}(t)$, and thus a means by which to low-pass filter all the velocity components $u_{im}(t)$ at $f_c = f_K$, thereby minimizing the effect of noise on gradients $\partial u_i/\partial x_j$ and other derived quantities such as structure functions.

Let us first inspect the contributions of noise to the measured kinetic energy $\overline{u'^2}_1$ and dissipation $\epsilon$. This is illustrated using the one-dimensional spectral forms of $\overline{u'^2}_1$ and $\epsilon$ for isotropy, which are (see [6])

$$\overline{u'^2}_1 = \int_0^\infty E_1(k_1)dk_1$$

and

$$\epsilon = 15\nu \int_0^\infty k_1^3E_1(k_1)dk_1,$$

respectively, where $E_1$ is the one-dimensional spectrum function and $k_1$ is the wave number in the streamwise ($x_1$) direction. To demonstrate the influence of noise on $\overline{u'^2}_1$ and $\epsilon$, we consider the model spectrum function (e.g., [7,8])

$$E_1(k_1) = C_k\epsilon^{2/3}k_1^{-5/3}\exp[-(\alpha k_1\eta)^{3/5}]$$

where $C_k$ is a “constant” determined empirically by experiments and $\alpha = \frac{3}{2}C_k$. Note that Eq. (11) has been verified by a large body of experimental high-Re data (see [9] which references relevant experiments). Using Eq. (6) and $k_1 = 2\pi f/U_1$ (Taylor hypothesis), Eq. (11) may be rewritten as

$$E_1(f) = C_k(2\pi U_1)^{5/3}\epsilon^{5/3}\exp[-\alpha(f/f_K)^{3/4}]$$.

Suppose the noise-contaminated velocity spectrum to be $E_{1m} = E_1 + E_n$, where $E_n$ is the “noise” contribution. Then, from Eqs. (9) and (10) and $k_1 = 2\pi f/U_1$, we can obtain

$$\overline{u'^2}_{1m} = 2\pi U_1^2\int_0^\infty (E_1 + E_n)df$$

and

$$\epsilon_{m} = 120\pi^2\nu U_1^3\int_0^\infty \tilde{f}^3(E_1 + E_n)df,$$

Using Eq. (12) with $C_k = 1.7$ and then $\alpha = 2.55$ (e.g., [10]) we illustrate in Fig. 1 the spectral density distributions of $\overline{u'^2}_{1m}$ and $\epsilon_m$ with $E_n = 0$ and $E_n = 10^{-3}(f/f_K)^2$. This demonstrates that, when $f_c > f_K$, the contributions of the noise to $\overline{u'^2}_{1m}$ and to $\epsilon_m$ are very different. For example, if $f_c = 10f_K$, the ratios $(\overline{u'^2}_{1m} - \overline{u'^2}_1)/\overline{u'^2}_1$ and $(\epsilon_m - \epsilon)/\epsilon$ are approximately $3.3\%$ and $200\%$, respectively. This indicates that the high-frequency noise contamination, if not properly filtered out, has an extremely large impact on $\epsilon_m$ while its influence on $\overline{u'^2}_{1m}$ or more generally on $\overline{u'^2}_i$ is very much less.

Next we examine the effects of noise on the measured Kolmogorov scales $\eta_m$ and $f_{Km}$. Let us express the measured dissipation $\epsilon_m$ by

$$\epsilon_m = \epsilon + \epsilon_n = \epsilon(1 + \epsilon_n/\epsilon) = C\epsilon$$

with $C = (1 + \epsilon_n/\epsilon) > 1$, where $\epsilon_n$ denotes the noise contribution. Substituting Eq. (15) into Eq. (1) leads to

$$\eta_m = (u'^3/C\epsilon)^{1/4} = C^{-1/4}\eta.$$
The newly filtered velocity signal \( u_{1m}^{(1)} \) is thus generated. The above process may be repeated as many times as necessary to generate \( u_{1m}^{(2)}, u_{1m}^{(3)}, \ldots, u_{1m}^{(N)} \) and then \( \partial u_{1m}^{(2)} / \partial t, \partial u_{1m}^{(3)} / \partial t, \ldots, \partial u_{1m}^{(N)} / \partial t \), until \( \left( f^{(N)} - f^{(N)}_{Km} \right)/f^{(N)}_{c} < \delta \) or until \( f^{(N)}_{Km}, \eta^{(N)}_{m}, \) and \( \varepsilon^{(N)}_{m} \) have converged satisfactorily. From the converged data, we finally obtain \( \varepsilon = \varepsilon^{(N)}_{m}, f_{K} = f^{(N)}_{Km}, \) and \( \eta = \eta^{(N)}_{m} \).

Based on Eqs. (15)–(17), both \( f^{(i)}_{Km} \) and \( \eta^{(i)}_{m} \) should converge quickly to their respective "asymptotic" values. This is true as demonstrated in an example in Table I for the case of \( f^{(0)}_{c} = 10 f_{K} \), with reference to Figs. 1 and 2. Clearly, for this case, \( \varepsilon_{m} \rightarrow \varepsilon, \eta_{m} \rightarrow \eta, \) and \( f_{Km} \rightarrow f_{K} \) just in two iterations, with an accuracy of 99.8%.

To validate the present scheme for real measurements, the instantaneous streamwise velocity \( = U_{1m} + u_{1} \) is obtained using hot-wire anemometry along the centerline of a two-dimensional plane jet issuing from a rectangular \((w \times h = 340 \times 5.6 \, \text{mm}^{2})\) slot, with aspect ratio \( w/h = 60 \). Here only a brief description of the jet facility is given as details may be found in [11]. To ensure statistical two-dimensionality, two parallel plates \((2000 \times 1800 \, \text{mm}^{2})\) are attached to the short sides of the slot so that the jet mixes with ambient fluid only in the direction normal to the long sides, following, e.g., Gutmark and Wygnanski [12]. The jet exit velocity is \( U_{j} = 8 \, \text{m/s}, \) which corresponds to a Reynolds number \( Re = U_{j} h/\nu \) of approximately 3000.

Velocity measurements are performed over the region \( 20 \leq x_{1}/h \leq 160 \) using a single hot-wire (tungsten) probe.

### Table I. Use of the iterative scheme for \( f^{(0)}_{c} = 10 f_{K} \)

<table>
<thead>
<tr>
<th>Iterations</th>
<th>( f^{(i)}_{c} )</th>
<th>( \varepsilon^{(i)}_{m} )</th>
<th>( \eta^{(i)}_{m} )</th>
<th>( f^{(i)}_{Km} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10f_{K}</td>
<td>17.0%</td>
<td>0.5%</td>
<td>2.0f_{K}</td>
</tr>
<tr>
<td>1</td>
<td>2.0f_{K}</td>
<td>1.1%</td>
<td>0.98%</td>
<td>1.02f_{K}</td>
</tr>
<tr>
<td>2</td>
<td>1.02f_{K}</td>
<td>1.02%</td>
<td>0.998%</td>
<td>1.002f_{K}</td>
</tr>
</tbody>
</table>

The hot-wire sensor is 5 \( \mu m \) in diameter and approximately 0.8 mm in length, aligned in the spanwise \((x_{1})\) direction. Velocity signals obtained at all the measured locations are low-pass filtered with a high and identical cutoff frequency of \( f_{c} = 9.2 \, \text{kHz} \). Then they are digitized at \( f_{s} = 18.4 \, \text{kHz} \) via a 16-channel, 12-bit analog-to-digital converter on a personal computer. The sampling duration is approximately 22 s.

To validate the present scheme for real measurements, the instantaneous streamwise velocity \( = U_{1m} + u_{1} \) is obtained using hot-wire anemometry along the centerline of a two-dimensional plane jet issuing from a rectangular \((w \times h = 340 \times 5.6 \, \text{mm}^{2})\) slot, with aspect ratio \( w/h = 60 \). Here only a brief description of the jet facility is given as details may be found in [11]. To ensure statistical two-dimensionality, two parallel plates \((2000 \times 1800 \, \text{mm}^{2})\) are attached to the short sides of the slot so that the jet mixes with ambient fluid only in the direction normal to the long sides, following, e.g., Gutmark and Wygnanski [12]. The jet exit velocity is \( U_{j} = 8 \, \text{m/s}, \) which corresponds to a Reynolds number \( Re = U_{j} h/\nu \) of approximately 3000.

Velocity measurements are performed over the region \( 20 \leq x_{1}/h \leq 160 \) using a single hot-wire (tungsten) probe.

The above measurements yield the original velocity signals \( U^{(0)}_{1m}(t) = U^{(0)}_{1m} + u^{(0)}_{1m}(t) \), which are corrected for the hot-wire length of 0.8 mm using Wyngaard’s approach [13], and, consequently, the original time derivatives along the jet centerline at \( x_{1}/h = 20 \) are obtained as follows:

\[
\frac{\partial u^{(i)}_{1m}}{\partial t} = \Delta u^{(i)}_{1m} / \Delta t = f_{c} \left[ u_{1m}^{(i)}(t + t_{s}^{-1}) - u_{1m}^{(i)}(t) \right] \quad \text{with} \quad \Delta t = t_{s}^{-1}.
\]

This in the region, the original mean velocity \( U^{(0)}_{1m} \) (not presented) is found to follow closely the relation \( U_{c}/U_{1m} = (x_{1}/h)^{-1/2} \) (here \( U_{c} \) denotes the centreline mean velocity), which is required by self-preservation of the jet. This relation and also that for the half-velocity width, i.e., \( L_{1/2}/h \approx x_{1}/h \), are well satisfied by previous data (e.g., [4]) obtained in the far field.

For high-Re flows, it is usually considered that the dissipation of turbulent kinetic energy \( \varepsilon \) out of the smallest-scale structures is equal to the supply rate of the turbulence energy from the large-scale structures, which is of order \( U_{0}^{3}/L_{0} \), where \( U_{0} \) and \( L_{0} \) are the local characteristic velocity and length scales (see, e.g., [14]). Based on this argument, we obtain \( \varepsilon \sim U_{j}^{3}/L_{1/2} \) by taking \( U_{0} = U_{c} \) and \( L_{0} = L_{1/2} \), for a plane jet. It follows that self-preservation of the flow further requires

\[
\varepsilon(hU_{j}^{-3}) = C_{c}(x_{1}/h)^{-5/2}
\]

and
The present scheme also applies for the measurement of temperature (scalar) using cold-wire anemometer for estimates of the Batchelor, instead of the Kolmogorov, scale. The support of the Australian Research Council is gratefully acknowledged.