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Improved chiral properties of FLIC fermions

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The chiral properties of the fat-link irrelevant clover (FLIC) fermion action are examined. The improved chiral properties of fermion actions incorporating smoothed links are realized in the FLIC action where only the irrelevant operators of the fermion action are constructed with smoothed links. In particular, the histogram of the additive mass renormalization encountered in chiral-symmetry breaking Wilson-type fermion actions is seen to narrow upon introducing fat-links in the irrelevant operators. The exceptional configuration problem of quenched QCD is reduced, enabling access to the light quark mass regime of $m_\pi/m_\rho \sim 0.35$. In particular, quenched chiral non-analytic behavior is revealed in the light quark mass dependence of the Δ-baryon mass. FLIC fermions offer a promising approach to revealing the properties of full QCD at light quark masses.

The frontier of lattice QCD lies at the challenge of directly simulating the full theory, including the dynamical sea-quark loop contributions, at light quark masses. In principle, the problem is solved. The advent of fermion actions which provide exact lattice chiral symmetry, in particular the overlap action[1], allows one to directly simulate at the physical quark masses realized in QCD. However the computational cost of such fermion actions are prohibitively expensive. Simulations on physically large volumes with a cutoff sufficiently large to accommodate the physics of interest are not possible at present. Indeed significant breakthroughs are required to reduce the computational cost to a level where leading edge computing resources can have a significant impact.

In contrast the staggered fermion action is computationally cheap and is currently having a tremendous impact near the light quark mass regime [2]. Had nature presented us with four degenerate light-fermion flavors, the formalism would be ideal. In this formalism, the 16-fold degeneracy of fermion flavors encountered in naively discretising the continuum derivative is reduced to a 4-fold flavor degeneracy. In practice, the fermion determinant describing the contribution of sea-quark loops having their origin in the gluon field is reduced to a single flavor by taking the fourth-root. This operation renders a non-local lattice action, raising concerns about the continuum limit of the staggered theory [3].

On the other hand the Wilson fermion action defines a local lattice action for a single fermion flavor that is computationally efficient but at the expense of explicitly breaking global chiral symmetry through the introduction of the lattice Laplacian operator in an irrelevant, energy dimension-five operator. This operator solves the fermion doubling problem by giving the doublers a mass proportional to the inverse lattice spacing a, but at the expense of introducing large $\mathcal{O}(a)$ errors. A systematic approach [4] to achieving efficient $\mathcal{O}(a)$ improvement of the lattice fermion action leads to the Sheikholeslami-Wohlert fermion [5] action, or "clover action", so named because it is constructed by adding the clover term, $(i\, g\, a\, C_{\rm SW}\,/4)\, \bar{\psi}(x)\, \sigma_{\mu\nu} F_{\mu\nu}\, \psi(x)$ to the standard Wilson action [6]. The clover coefficient, $C_{\rm SW}$, can be tuned to remove $\mathcal{O}(a)$ artifacts to all orders in the gauge coupling constant g. Nonperturbative methods have been established [7, 8, 9] to accurately define improvement coefficients such as $C_{\rm SW}$ in the interacting theory. Traditionally, large renormalizations of the improvement coefficients have hindered the realization of lattice action improvement in practice.

While the nonperturbatively improved SW action correctly defines a lattice action for a single fermion flavor, scales well [10], and remains computationally efficient, the breaking of global chiral symmetry makes the approach to the chiral regime difficult. Global chiral symmetry breaking introduces an additive mass renormalization into the Dirac operator that can give rise to singularities in quark propagators at small quark masses. The problem is exacerbated through the use of large lattice volumes; large values of the strong coupling constant, g, providing large lattice spacings, a; or large values of the improvement coefficient, $C_{\rm SW}$ [7, 11, 12]. In practice, this prevents the use of coarse lattices (Wilson: $\beta < 5.7 \sim a > 0.18$ fm) [11, 12].

The singularities are caused by the shifting of the point where the renormalized mass is zero, away from the point where the bare mass parameter m is zero. The position of the singularity is both gauge configuration and action dependent [11]. Bare quark masses must be shifted by an amount dependent on the gauge action to restore chiral symmetry. That is, a "critical mass" is introduced and fine tuned such that the pion mass vanishes when the quark masses take the critical mass $m = m_{\rm cr}$. All other quark masses are measured relative to $m_{\rm cr}$.

The difficulty in approaching the chiral regime becomes apparent once one realizes the critical mass varies on a gauge-field configuration by configuration basis. For fixed input bare mass, the renormalized mass, relevant to the physical properties of the quark, is different on every configuration. Given that most observables encounter rapidly varying chiral nonanalytic behavior in the chiral regime [13], this small variation in the renormalized mass gives rise to widely varying hadron properties on a configuration by configuration basis which is ultimately realized as a large statistical error bar in hadronic observables. Thus, for the generation of gluon-field configurations incorporating the effects of dynamical fermion loops, the development of chirally improved fermion actions which minimize the breadth of the distribution of the critical mass obtained on a configuration by configuration basis (while maintaining only nearest neighbor interactions) is highly desirable.

In the generation of dynamical-fermion gauge configurations the fermion determinant acts to suppress the probability of creating configurations giving rise to approximate zeromodes of the Dirac operator, $D(m)$. In the quenched approximation, the fermion determinant is set to a constant and a proliferation of approximate zeromodes is encountered. At sufficiently light renormalized quark mass, it is possible to have the critical mass realized on a particular configuration near the bare mass, introducing a divergence in the quark propagator. Such "exceptional" configurations provide hadronic correlation functions which differ significantly in magnitude from the average, again introducing large statistical uncertainties and on some occasions spoiling the ensemble average result.

Fat-link actions [12, 14] have been identified as providing improved chiral properties while maintaining only nearest neighbor interactions. Fat links are obtained by averaging the gauge field links, $U_\mu(x) = \exp[ig \int_0^a A_\mu(x+\lambda\widehat{\mu})d\lambda]$, with their transverse neighbors in an iterative process of APE smearing [15] or HYP smearing [16]. The incorporation of smoothed links throughout the fermion action removes short distance interactions and thus suppresses renormalizations of the critical mass in the interacting theory. DeGrand *et al.* [12] noted that the appearance of spurious zero modes is due to local lattice dislocations with non-zero topology at the scale of the cutoff. By smoothing the gauge field at short distances it was possible to narrow the distribution of the critical mass by about a factor of three, thus reducing the problem of exceptional configurations.

However short distance effects are lost in the fat-link theory. FLIC fermions circumvent this problem by introducing fat-links only in the purely irrelevant lattice operators of the clover action [5], having energy dimension five or more. The untouched gauge fields generated via Monte Carlo methods form the relevant operators associated with the continuum action. FLIC fermions provide near continuum results at finite lattice spacing, while preserving short distance physics. The FLIC action provides an alternative form of nonperturbative $\mathcal{O}(a)$ improvement [9] which avoids the fine tuning problem of improvement coefficients.

The focus of this investigation is to determine the extent to which the improved chiral properties of fat-link fermion actions are realized with FLIC fermions. Previous work [8, 17] has shown that the FLIC fermion action has extremely impressive convergence rates for matrix inversion, which provides great promise for performing cost effective simulations at quark masses closer to the physical values. In the following, we will illustrate how the distribution of the critical mass $m_{\rm cr}$ is narrowed sufficiently to allow simulations at light quark masses providing $m_\pi/m_\rho \sim 0.35$ while exceptional configurations, encountered in the quenched approximation, are limited to $\sim 6\%$. Access to the light quark mass region is sufficient to reveal quenched chiral nonanalytic behavior in the quark mass dependence of the Δ-baryon mass.

Our simulations are based on gauge fields described by the $\mathcal{O}(a^2)$-mean-field improved Luscher-Weisz plaquette plus rectangle gauge action [18]. We begin with a study of the distribution of zero modes on 100 $12^3 \times 24$ lattices at $\beta = 4.60$, giving a lattice spacing of $a = 0.116(2)$ fm. The subsequent calculations of baryon masses are performed on a sample of 400 $20^3 \times 40$ lattices at $\beta = 4.53$, $a = 0.128(2)$ fm. The scale is set via r_0.

The mean-field improved FLIC action is [8, 9]

$$S^{\rm FL}_{\rm SW} = S^{\rm FL}_{\rm W} - \frac{i\, g\, C_{\rm SW}\, \kappa\, r}{2(u_0^{\rm FL})^4}\, \bar\psi(x)\, \sigma_{\mu\nu} F_{\mu\nu}\, \psi(x)\,, \qquad (1)$$

where $F_{\mu\nu}$ is an $\mathcal{O}(a^4)$-improved lattice definition [20] constructed using fat links, $u_0^{\rm FL}$ is the plaquette measure of the mean link calculated with fat links, and where the mean-field improved Fat-Link Irrelevant Wilson action is

$$\begin{aligned} S^{\rm FL}_{\rm W} = & \textstyle\sum_x \; \bar\psi(x)\psi(x) + \kappa \sum_{x,\mu} \bar\psi(x) \Big[\gamma_\mu \Big(\frac{U_\mu(x)}{u_0} \psi(x+\hat\mu) \\ & - \frac{U^\dagger_\mu(x-\hat\mu)}{u_0} \psi(x-\hat\mu) \Big) - r \Big(\frac{U^{\rm FL}_\mu(x)}{u_0^{\rm FL}} \psi(x+\hat\mu) \\ & + \frac{U^{\rm FL\dagger}_\mu(x-\hat\mu)}{u_0^{\rm FL}} \psi(x-\hat\mu) \Big) \Big] \,. \end{aligned} \qquad (2)$$

with $\kappa = 1/(2m+8r)$. We take the standard value $r = 1$. Our notation uses the Pauli representation of the Dirac γ-matrices [21], where the γ-matrices are hermitian and $\sigma_{\mu\nu} = [\gamma_\mu,\ \gamma_\nu]/(2i)$. Fat links are constructed by performing $n_{\rm APE}$ sweeps of APE smearing, where in each sweep the weights given to the original link and the six transverse staples are 0.3 and (0.7/6) respectively. The FLIC action is closely related to the mean-field improved clover (MFIC) fermion action in that the latter is described by Eqs. (1) and (2) with all fat-links replaced by untouched thin links and $F_{\mu\nu}$ defined by the 1×1-loop clover definition.

The critical mass for a particular configuration is determined by examining the spectral flow of the Hermitian FLIC Dirac operator, $\gamma_5 D(m)$, commencing with quark

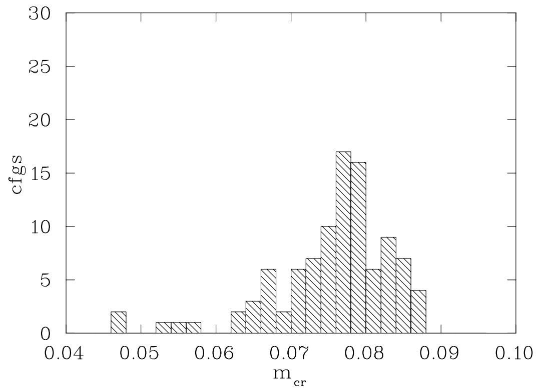

FIG. 1: Histogram of the critical mass obtained from the spectral flow of the mean-field improved clover (MFIC) fermion action on 100 configurations.

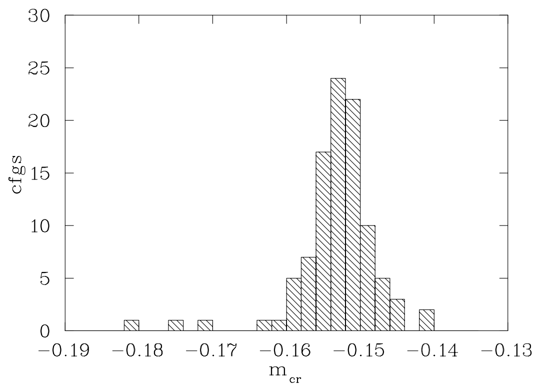

FIG. 2: Histogram of the critical mass obtained from the spectral flow of the fat-link irrelevant clover (FLIC) fermion action ($n_{\rm APE} = 4$) on 100 configurations.

masses in the physical mass regime and proceeding towards the negative quark mass regime relevant to overlap fermions. The physical regime is characterized by the magnitude of the lowest eigenvalue $\lambda_{\rm min}(m)$ decreasing as m decreases. The negative mass regime is characterized by the presence of eigenvalues crossing zero, due to nontrivial topology. The larger the topological object, the closer to $m_{\rm cr}$ is the induced zero crossing [22]. Hence, we can determine the regime transition point by calculating $\lambda_{\rm min}(m)$ and observing it decrease with m until it either crosses zero or turns away and begins to increase. The critical mass is then defined as the m at which the transition occurs.

Figures 1 and 2 illustrate histograms of the critical mass obtained from the MFIC fermion action and FLIC fermion action respectively. Each histogram is obtained from the same set of 100 configurations (cfgs). The FLIC fermion distribution of the critical mass is significantly narrower than that of the MFIC action, by a factor of two. Since the nonperturbatively improved estimate of $C_{\rm SW}$ typically exceeds that of the mean-field improved estimate, the distribution for the nonperturbatively improved clover (NPIC) action will be broader than that of Fig. 1. Thus FLIC fermions offer a chirally improved alternative to MFIC and NPIC fermion actions. FLIC

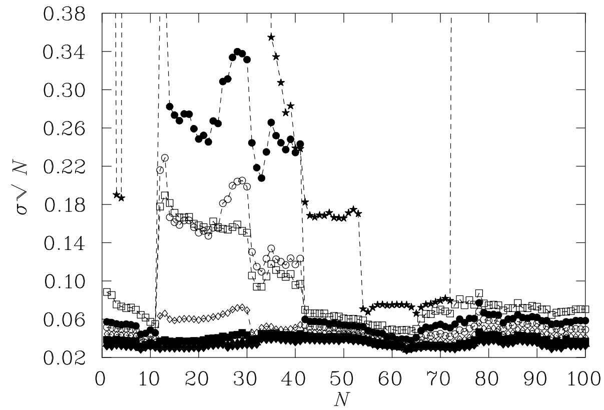

FIG. 3: The standard deviation in the error of the π mass calculated on 30 configurations plotted against the starting configuration number for the FLIC-fermion action on a $20^3 \times 40$ lattice with $a = 0.128(2)$ fm. See text for details.

fermions enjoy a significantly reduced exceptional configuration problem, and provide reduced fluctuations in hadronic observables as the chiral limit is approached in full QCD.

To directly examine the chiral properties of the FLIC fermion action, we determine the low-lying hadron mass spectrum. Hadron masses are extracted from the Euclidean-time dependence of the two-point correlation functions using standard techniques. Effective masses are calculated as a function of time and time-fitting intervals and are selected via standard covariance matrix estimates of the $\chi^2/N_{\rm DF}$. For quark masses lighter than the strange quark mass, effective mass splittings are calculated and fit using the same techniques. By examining mass splittings, excited-state contributions (less dependent on the quark mass) are suppressed and good $\chi^2/N_{\rm DF}$ are found one-to-two time slices earlier.

In searching for exceptional configurations, we follow the technique used by Della Morte *et al.* [23]. In the absence of exceptional configurations, the standard deviation of an observable is independent of the number of configurations considered in the average. Exceptional configurations reveal themselves by introducing a significant jump in the standard deviation as the configuration is introduced into the average.

Fig. 3 shows the standard deviation of the pion mass for eight quark masses on subsets of a total of 100 configurations. A moving average of 30 configurations is considered, with a cyclic property enforced from configuration 100 to configuration 1. Configuration 41 is revealed to be exceptional by noting that between $N = 12$ and $N = 41$ the error blows up for several quark masses and then drops again at $N = 42$ as configuration 41 leaves the moving average. The observation of a large change in the pion mass as config. 41 in included in the ensemble confirms the exceptional behavior. Exceptional behavior is also observed for the lightest quark mass for configurations 2, 13, 30, 34 and 53. Upon removal of these configurations, a near-constant behavior of the standard

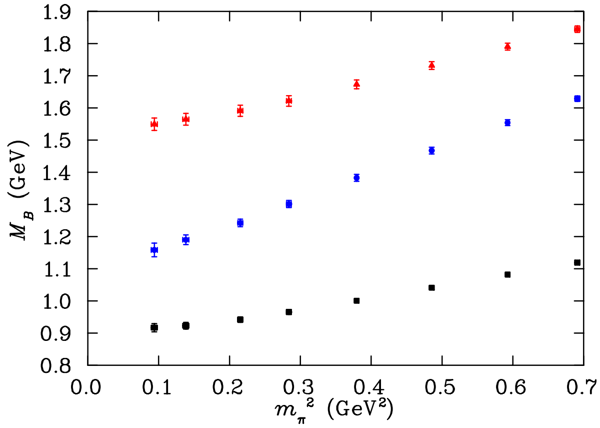

FIG. 4: ρ, N and Δ masses for the FLIC-fermion action ($n_{\rm APE} = 6$) on 400 $20^3 \times 40$ lattices with $a = 0.128(2)$ fm.

TABLE I: Values of κ and the corresponding π, ρ, N and Δ masses (in GeV) for the FLIC fermion action on a $20^3 \times 40$ lattice with $a = 0.128(2)$ fm.

κ	π	ρ	N	Δ
0.12780	0.831(2)	1.119(04)	1.629(08)	1.845(10)
0.12830	0.770(2)	1.082(05)	1.554(09)	1.791(11)
0.12885	0.697(2)	1.041(07)	1.468(11)	1.732(12)
0.12940	0.616(3)	1.001(07)	1.383(11)	1.673(14)
0.12990	0.533(3)	0.965(08)	1.301(11)	1.622(16)
0.13025	0.464(4)	0.941(09)	1.243(12)	1.592(17)
0.13060	0.372(6)	0.923(11)	1.190(15)	1.565(18)
0.13080	0.306(7)	0.917(13)	1.159(21)	1.550(19)

deviation is observed for all quark masses. Given the relatively coarse lattice spacing, large volume, and the lightest quark mass providing $m_\pi/m_\rho \sim 0.35$, an elimination rate of $\sim 6\%$ is a remarkable result.

Figure 4 shows the ρ, N and Δ masses as a function of m_π^2 for the FLIC-fermion action. An upward curvature in the Δ mass for decreasing quark mass is observed in the FLIC fermion results. This behavior, increasing the quenched $N - \Delta$ mass spitting, was predicted by Young *et al.* [24] using quenched chiral perturbation theory (QχPT) formulated with a finite-range regulator. This is the first time that clear quenched chiral nonanalytic behavior has been observed in a baryon mass. Simulations in quenched QCD with Wilson-type fermions on this lattice spacing for these light quark masses have been previously unattainable. Numerical results are summarized in Table I.

In summary, the Fat-Link Irrelevant Clover (FLIC) fermion action is an efficient lattice fermion operator with excellent scaling properties, providing near-continuum results at finite lattice spacing. The superior chiral properties of the FLIC fermion action enable access to the light quark-mass regime. With recent breakthroughs in dynamical fermion simulations of fat-link fermion actions [25] FLIC fermions offer a promising approach to revealing the properties of full QCD at light quark masses.

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