Matrix-Analytic Methods in Applied Probability

Leslie William Bright, B.Sc. (Hons.)

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Contents

Abstract

This thesis considers several questions in the field of matrix-analytic methods. These methods have been used extensively in applied probability to analyse a wide variety of problems. However, there still appears to be scope for further development both in the range of models that can be analysed, and in the type of analysis which can be employed.

The level dependent quasi-birth-and-death process (LDQBD) is an example of a matrix-analytic model. The LDQBD is a generalisation of the level independent quasi-birth-and-death process (QBD). QBDs are used frequently as a modelling tool when applying matrix-analytic methods but the LDQBD is used rather infrequently. In this thesis we develop theoretical results for LDQBDs as well as a number of algorithms that can be used to analyse LDQBDs. We model a number of systems using LDQBDs and provide numerical results.

The level independent M/G/1-type process and the level independent GI/M/1-type process are two other matrix-analytic models. Both of these models can be thought of as extensions of the QBD. Ramaswami developed a duality relationship for these two models. We give a new interpretation of Ramaswami's duality result and use this interpretation to develop an alternative duality relationship. We also present a duality result for level dependent M/G/1-type and GI/M/1-type processes which are generalisations of the level independent processes.

The thesis also considers quasi-stationary distributions for QBDs. We present an expression for the quasi-stationary distribution as well as algorithms for its numerical computation.