Electron Transport
in Photon and Electron Beam Modelling

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Abstract

To address the deficiencies of currently available dose calculation algorithms for radiotherapy planning, two rigorous dose calculation methods have been devised.

The first method incorporates Fermi-Eyges multiple scattering theory into the primary dose calculation of the superposition method for external X-ray beam radiotherapy. The inclusion of scattering theory into the superposition technique accounts for the density distribution between the primary photon interaction and energy deposition sites, whereas conventional superposition methods only consider the average density between these two points. This method gives depth dose curves which show better agreement with Monte Carlo calculations in a lung phantom than a standard superposition method, especially at high energies and small field sizes where lateral electronic disequilibrium exists. For a $5\times5$ cm$^2$ 18 MV beam incident on the lung phantom, a reduction in the maximum error between the superposition and Monte Carlo depth dose curves from 5% to 2.5% is obtained when scattering theory is used in the primary dose calculation.

The second method developed is the Super-Monte Carlo (SMC) method. SMC calculates dose by a superposition of pre-generated Monte Carlo electron track kernels. For X-ray beams, the primary dose is calculated by transporting pre-generated (in water) Monte Carlo electron tracks from each primary photon interaction site. The length of each electron step is scaled by the inverse of the density of the medium at the beginning of the step. Because the density scaling of the electron tracks is performed for each
individual transport step, the limitations of the macroscopic scaling of kernels (in the
superposition algorithm) are overcome. The scatter dose is calculated by superposition.
In both a lung-slab phantom and a two lung-block phantom, SMC dose distributions
are more consistent with 'standard' Monte Carlo generated dose distributions than are
superposition dose distributions.

SMC can also be applied to electron beam dose calculation. Pre-generated electron tracks
are transported through media of varying density and atomic number. The perturbation
of the electron fluence due to each material encountered by the electrons is explicitly
accounted for by considering the effect of variations in stopping power, scattering power
and radiation yield. For each step of every electron track, these parameters affect the
step length, the step direction and the energy deposited in that step respectively. Dose
distributions in a variety of phantoms show good agreement with Monte Carlo results.

SMC is an accurate, 3-dimensional unified photon/electron dose calculation algorithm.