Singularity Structure of Scalar Field Cosmologies

Scott Foster B.Sc. (Hons)

Thesis submitted for the degree of
Doctor of Philosophy
at the
University of Adelaide
(Department of Physics and Mathematical Physics)
March 1996

Contents

Abstract

The classical dynamical structure of cosmological models in which the matter content of the universe consists of a scalar field with arbitrary non-negative potential is analyzed in detail. Emphasis is placed on the global features of solutions including the existence of an initial space-time singularity and particle horizons.

A natural class of potentials is defined which includes as an elementary subset all non-negative polynomial potentials. For the special case where the space-time metric is of Bianchi type I it is shown that almost all solutions of these models possess initial space-time singularities and particle horizons. Close to the singularity the dynamics is essentially independent of the form of the potential and the space of solutions may be identified with that of the exactly integrable model for which potential is identically zero. Furthermore, it is shown that there exist at least two solutions for which no particle horizons exist. These special solutions are isotropic and are exponential attractors in solution space. They are associated with the existence of inflationary behavior in the system, which they completely characterize.

These results, which are consistent with earlier work done for particular potentials, demonstrate the robustness of scalar field cosmological models and help to clarify the significance of the large number of horizon free and singularity free exact solutions.

Implications to more general space-times are considered making use of a form of long wavelength approximation to the Einstein Field Equations and an attempt is made to construct a general asymptotic solution. This is shown to be valid for general spatially homogeneous space-times subject to a generic condition on the spatial curvature.

In addition, an interesting class of exponential potentials which do not fall into the above mentioned class are investigated and their behavior compared with that above. These solutions possess an oscillatory behavior near an initial space-time singularity which seems to be associated with the inability of the scalar field to negotiate the walls of extremely steep potentials.