Closed Set Logic in Categories

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Abstract

In this work we investigate two related aspects of a dualisation program for the usual intuitionist logic in categories. The dualisation program has as its end the presentation of closed set, or paraconsistent, logic in place of the usual open set, or intuitionist, logic found in association with toposes. We address ourselves particularly to Brouwerian algebras in categories as the duals of the usual Heyting algebras. The first aspect of the program is that of external or ex-categorial dualisation of logic structures by interpretation of order. This appears in the work as an examination of the notion of a complement classifier. We also use ex-categorial dualisation as a tool to prompt the development of a categorial proof and model theory adequate to the task of modelling theories generated by inconsistency tolerant logics. We make an initial attempt to develop dual logic structures by considering quotient object classifiers in place of subobject classifiers. Ex-categorial dualisation of structure was always meant to act as an indication of the existence of categorial entities that directly satisfy dual descriptions, so the bulk of the work is concerned with the second aspect of the dualisation program: the discovery of logic objects within categories that exhibit paraconsistent algebras in their own right. Our investigation focuses on sheaves for their algebraic properties in relation to base space topologies. We define the notion of a sheaf over the closed sets of a topological space. We find essentially two things. First, logic objects in contravariant sheaf categories contain component Brouwerian algebras but are not generally themselves Brouwerian algebras within their categories. A corollary is that subobject lattices in Grothendieck toposes are Brouwerian algebras (but not naturally so). Second, paraconsistent logic objects do exist. We describe one such within a category of covariant sheaves. As a corollary we find that the original ex-categorial dualisation idea represented by the notion of a complement classifier has an instantiation in categories. Our paraconsistent logic object proves to be the object of a genuine complement classifier.