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CONVEX SETS WITH LATTICE POINT CONSTRAINTS

by

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Contents

Abstract

Every convex set in the plane gives rise to certain geometric functionals such as the area, perimeter, diameter, width, inradius and circumradius. When the convex set is constrained by lattice points (points having integer coordinates), certain inequalities occur amongst these functionals. In this thesis, we are primarily concerned with obtaining new inequalities for a planar, convex set containing exactly 0, 1 or 2 lattice points in its interior.

This thesis consists of two parts. The first part comprising Chapters 3, 4 and 5 deals with problems concerning single geometric functionals. We obtain results concerning the maximal area, circumradius and width respectively.

The second part of the thesis comprising Chapters 6 to 12 deals with a larger class of problems concerning relationships between pairs of the above-mentioned functionals for lattice constrained sets. In a number of the problems concerning 1 or 2 interior lattice points, the solution is readily obtained by reducing the problem to one concerning a set with interior containing no point of the rectangular lattice.

Chapters 1 and 2 contain basic ideas and results which are used throughout the thesis. In the concluding chapter, we comment on the scope for future research in the area. It will be seen that there remain many new and interesting problems.