Network Routing And Design Problems With Piecewise Linear Costs

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Contents

Summary

In the thesis, I investigate network routing and design problems with a variety of cost functions. It is usual to treat this problem as a multicommodity flow problem on a directed network. In my thesis, however, I emphasise the undirected version of the problem and have found methods specifically for this situation.

Chapter 1 is a survey of network optimization problems. Three different formulations that are used for these problems are presented: the node–link model, the link–path model and the overflow model. The first two formulations are well known and are essentially directed network formulations, but can also be adapted to model undirected network problems. The overflow formulation is new and is particularly suitable for undirected networks, but can also be used for directed networks.

In Chapter 2 it is shown that the network design problem has a concave objective function, provided some reasonable conditions for the link operation and design costs hold. The proof of this result I believe to be new, and is a generalization of an earlier result.

Chapter 3 considers the problem of finding a minimum cost undirected communication network when the link costs are piecewise linear and concave. This problem is known to be NP–hard. Several techniques are applied to the problem and compared : a heuristic based on that proposed by Minoux, simulated annealing and also a lower bounding technique.

Routing problems in networks are similar to design problems, except that the link costs are often convex. In Chapters 4 and 5, I therefore investigate the case of an undirected network with piecewise linear and convex link costs. This problem is actually a linear program. I consider two different formulations; one based on the link–path model and the other based on the overflow model. In the first formulation there is a variable for each path, so the size of the linear program is not polynomial in the number of nodes. In the overflow formulation an overflow variable represents the rerouting of traffic on a link via another node, and therefore the number of overflow variables is cubic in the number of nodes. This shows that this problem can be solved in polynomial time.

In Chapter 4 the linear programs mentioned above, together with the dual problems, are described, and the complementary slackness conditions are discussed. Then I show how the column generation method can be developed for both models. In addition, a heuristic algorithm which generates integer solutions is presented. Computational results of these methods are compared.

In Chapter 5 I develop two primal–dual methods for the overflow model. In both methods dual feasibility and the complementary slackness conditions are maintained and the aim is to achieve primal feasibility. In the first method, this is accomplished by repeatedly solving the restricted primal problem using the simplex method and updating the dual variables. In the second method a labeling procedure for solving the restricted primal problem is used; the dual variables are updated by solving a system of equations and inequalities. This approach follows naturally from the definition of overflow variables which are indexed by 3 nodes. Instead of scanning nodes through the links, as is usual in other implementations, the links are fanned out by investigating the overflow triangles.