PROBLEM SOLVING

IN

HIGH-SCHOOL ALGEBRA:

A THEORY-BASED APPROACH

TO

CLASSROOM PRACTICE

by

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ABSTRACT

This study examines, develops and presents one possibility for a new view of classroom algebra. It is seen as a contribution to identifying and devising classroom approaches that reflect current knowledge about human learning and problem solving.

The study is based in a cognitive view of instruction. This has involved the construction of a framework from a synthesis of theoretical perspectives that are specifying and describing how knowledge and problem-solving skills in mathematics may be developed. The strength of this theoretical approach is seen in its generality, and in its usefulness in guiding, supporting and predicting effective classroom practice.

A book Talking Maths is presented as an interpretation of the instructional framework, and an attempt to apply it to classroom algebra. A trial implementation of the course is reported, and factors that are seen as affecting the testing of theoretically-based programs are elicited.