Aspects of Topological Field Theories

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Contents

Abstract

Since their introduction in 1988, topological field theories have attracted a great deal of interest from both mathematicians and physicists. Mathematically they provide alternative formulations for certain topological invariants such as Donaldson invariants. Physically, topological field theories are important as they may be used to test different characteristics of their corresponding physical theory. In this thesis twisted $N = 2$ and $N = 4$ SYM theories in four and six dimensions are studied. We first provide the general background for topological field theories which can be obtained by twisting. A supersymmetric Yang-Mills theory is then constructed on a Calabi-Yau 3-fold by dimensional reduction. It is shown that this theory is a cohomological field theory and the corresponding path integral, in the weak coupling limit, localizes on the moduli space of Donaldson-Uhlenbeck-Yau equations. We also construct a partially twisted theory on a product six-manifold $X \times Y$. When Y is supersymmetrically embedded in a Calabi-Yau manifold M, it is argued how the moduli space on which the path integral localizes can be related to the mirror manifold of M. We also study the twisted $N = 4$ SYM theory on the product four-manifold $\Sigma \times S^2$. We derive the effective theory in the limit where S^2 shrinks. The correlators of the cohomology classes of the BRST operator are then computed in the mass deformed effective theory.