



Spin Dependent Deep Inelastic Scattering

Andreas Wolfgang Schreiber

Thesis submitted for the degree of

Doctor of Philosophy

at the University of Adelaide

(Department of Physics and Mathematical Physics)

August 1990

Contents

1	Introduction	1
2	Deep Inelastic Scattering	6
2.1	Introduction	6
2.2	$W_{\mu\nu}$ and the Structure Functions	8
2.3	The Björken Limit	12
2.4	The Operator Product Expansion	13
2.5	The Matrix Elements $\mathcal{A}_{(i,n)}$	18
2.6	Properties of $H_i(x)$	21
2.7	Conclusion	26
3	A Model for the Quark Distribution Functions	28
3.1	Introduction	28
3.2	The Quark Distributions	30
3.3	The Peierls-Yoccoz Procedure	34
3.4	The Peierls-Yoccoz quark distributions	37
3.5	Properties of the distribution functions $q_i^{(2)}(x)$	42
3.6	$q_i^{(4)}(x)$ and $\bar{q}_i^{(4)}(x)$	49
3.7	Normalization	50
3.8	Improvements of the model	55
3.8.1	One gluon exchange	55
3.8.2	Pions	63

3.9	Conclusion	69
4	QCD evolution of the spin structure functions	70
4.1	Introduction	70
4.2	The renormalization group	73
4.3	The reconstruction of the distributions	78
4.4	The First Moment Problem	83
4.5	Numerical Results	87
4.6	Conclusion	100
5	The spin structure function in the cloudy bag model	101
5.1	Introduction	101
5.2	The Cloudy Bag Model	104
5.3	g_1^p and g_1^n in the cloudy bag model	107
5.4	Centre of mass corrections	114
5.5	Conclusion	115
6	Numerical Results - Comparison with Data	117
6.1	Introduction	117
6.2	The anomaly and the first moment of g_1^p	120
6.2.1	The box diagram	122
6.2.2	Phenomenology of the gluon contribution	126
6.3	Bare Bag results	128
6.4	Bare bag with one gluon exchange	132
6.5	Pions	142
6.6	Predictions	147
6.7	Conclusion	148
7	Conclusion & Outlook	154

Abstract

The central theme of this thesis is the connection between low energy models of hadron structure and deep inelastic scattering, with particular emphasis on the spin dependence of the models. We concentrate on quark wavefunctions derived from the MIT bag model and investigate effects on the structure functions due to refinements such as one gluon exchange and pionic corrections. A comparison with experimental data is made possible by the inversion of the moment equations arising from the renormalization group for spin dependent perturbative QCD.

Statement

This thesis contains no material which has been accepted for the award of any other degree or diploma in any University and, to the best of my knowledge and belief, the thesis contains no material previously published or written by another person, except where due reference is made in the text of the thesis. I consent to the thesis being made available for photocopying and loan if applicable, providing due acknowledgement is given.

Acknowledgements

I am deeply indebted to Tony Thomas for taking me on as a student, for the continuing expert guidance that I received from him throughout my time in Adelaide and for being instrumental in creating a working environment to which it was a pleasure to belong.

An important part of this environment was the large number of visitors to the theory group over the years, two of which require special mention: Michael Weyrauch, whose critical approach always kept me on my toes and without whom this thesis would look quite different and Tim Londergan whose unlimited enthusiasm for the subject I hope I have caught something of.

I have of course also benefited from many discussions with other members of the theory group, in particular Steven Bass, Doug Driscoll, Jenny Henderson, Alex Kalloniatis, Andy Rawlinson, Marco Rossi and Tony Signal.

The preparation of this thesis was greatly aided by Sue Higgins, who diligently typed large parts of it, and Jenny Henderson who pointed out many obscurities and mistakes in an earlier version.

Finally, I would like to acknowledge financial support from the University of Adelaide through a scholarship and from my parents.

Chapter 1

Introduction

Ever since Gell-Mann, Zweig and Ne'eman [Gel64,GN64,Zwe64b,Zwe64a] introduced the quarks in order to explain the regularities observed in the hadron spectrum [Gel61,Nee61] efforts have been made to identify experimental consequences of their existence - at first it was not even clear whether or not they were just a useful notational device. In the late sixties however Björken, Paschos and Feynman [Bjo66,Bjo69,BP69,Fey69,Fey70] proposed that the proton consists of pointlike constituents, evidence for which should be found by looking for so-called scaling behaviour of the cross sections of inelastic scattering of high energy electrons by hadron targets. The first experiments of this kind were being performed at SLAC at around the same time, and indeed confirmed the above scaling hypothesis [Pan68,B*69a,B*69b].

Nevertheless, there were some problems with the original quark model. In particular, the classification required for the low lying baryons was the $\underline{56}$ -plet of spin-flavour $SU(6)$, which is symmetric. Together with the expectation that the lowest energy spatial wavefunction is symmetric this leads to a totally symmetric wavefunction for a three fermion state, something that seemed to violate the Pauli principle.

Han and Nambu [HN65] proposed that if one associated an extra $SU(3)$ degree of freedom (colour) with the quarks, with the hadrons belonging to its totally antisymmetric representation, then this problem could be resolved. Later on this idea tied

in neatly [FG72,FGL73] with the non-abelian $SU(3)$ gauge field theory (QCD) that was being developed in the early seventies to describe the interactions between the quarks [FP67,t H71,GW73,Pol73].

A great deal of progress was made from around 1973 to 1980 in investigating the consequences of perturbative QCD [MP78,Bur80,Alt82], in particular the renormalization group [Pet79] was used to predict scaling violations in deep inelastic lepton scattering which were later confirmed experimentally at Fermilab [W*75,C*75].

In comparison to perturbative QCD relatively little progress has been made with the long range, or non-perturbative, part of the theory. For example, both the confinement mechanism and the generation of the nuclear forces are still very poorly understood. The train of thought that is perhaps most promising is that of lattice QCD [Wil74,Cre83,Neg89] but it still seems to be much too early for any definite predictions to come out of this work.

Because of this a large number of models of the (postulated) confinement mechanism of QCD have come into existence. The earliest of these models rely on a non-relativistic hamiltonian formulation, the quark confinement being achieved by a suitable potential, for example of a harmonic oscillator type [FH68,FH69]. Inclusion of refinements such as one gluon exchange then enables one to include hyperfine splitting and so one can develop a quantitative understanding of the hadron spectrum [DGG75,IK78,IK79b,IK79a,Won86].

Any sensible calculation of structure functions, in particular spin structure functions, needs to be formulated in a relativistic framework [KW71]. It was soon realized that the classification of hadrons into multiplets of $SU(6)$, well established from the spectroscopy, could definitely not be the same classification needed for the description of moving hadrons (see, for example, [Bel64,Bel74,BH74]), especially for protons in the infinite momentum frame in which deep inelastic scattering was (and is) often formulated. Complicated schemes were developed to relate the two classifications [Mel74,Clo74,CK77,LOPR74,LOPR75,LOPR77,BKS*70,BNP72] so that one could begin to try to connect the low

energy models to deep inelastic scattering.

Concurrently with the above, efforts were being made at introducing a physically motivated basis for the confining potential. Attention has mainly been given to the so-called soliton [Lee81] and bag models [CJJ*74,CJTT74,BCD*75,HK78]. The latter introduce the confining potential by solving the free Dirac equation with appropriate boundary conditions, these having their origin in a phenomenological vacuum pressure balancing the pressure exerted by the quarks on the inside of the cavity. Refinements of these models that incorporate pions, necessary because of PCAC which is broken by the bare bag model, were introduced somewhat later [IM75,CT75,BR79,TTM81,TMT82,Tho84].

The study of deep inelastic structure functions within these models is the topic of this thesis. We shall concentrate on the spin dependence of the MIT and CBM bag models. Although important in its own right, the investigation of the spin structure of theories in the past has led to much more than a mere filling in of details - the most prominent example probably being the discovery of parity violation in the decay of spin-polarized ^{60}Co , which had a fundamental impact on the theory of weak interactions. In fact, shortly after the work presented here was commenced, an experiment by the European Muon Collaboration [EMC88] led to a thorough, and continuing, re-examination of our understanding of the proton spin (the so-called 'proton spin crisis').

The content of this thesis is organized as follows:

In Chapter 2 we develop the deep inelastic scattering formalism. The basic ideas on which this chapter is based can of course be found in the abundant literature on the subject, although the presentation here emphasizes the spin dependence which is done more rarely. Furthermore, the chapter serves to establish notation and a framework for what is to follow. As we shall see, the cross sections of interest can be expressed in terms of products of matrix elements calculated within models and coefficient functions given by perturbative QCD.

The matrix elements are evaluated for modified versions of the MIT bag model in Chapter 3. Care is taken to work only with momentum eigenstates which are extracted from the MIT bag wavefunction by the Peierls-Yoccoz projection. A feature of the work presented in this chapter is that vanishing support is ensured for the structure functions above $x = 1$. This is done by extracting a momentum conserving delta function before any approximations are made. The extraction of this delta function has the added benefit that it enables us to investigate the consequences of one gluon exchange within the bag, an important ingredient in any description of the spin structure of baryons, especially if one wants to make predictions for the d/u quark ratio or, for that matter, the asymmetry $2xg_1/F_2$. Finally, we make a start at including the effects of pions on the structure functions.

In Chapter 4 we concentrate on the calculation of the variation in Q^2 of the coefficient functions, again specifically paying attention to the spin dependent case. We do this by extending the method of Gonzalez-Arroyo et al. [GLY79b, GLY79a, GL80, GLY80] who performed a similar calculation for the spin independent structure functions some years ago. Descriptions of the material in this chapter may be found in [STL90a, STL90b] and the formalism was put to use by Thomas et al. in [TMSG89].

The particular case of the first moment of one of the spin structure functions is especially interesting as it can be directly linked to low energy quantities such as the nucleon's axial coupling constant and should therefore be directly calculable in bag models. In Chapter 5 we calculate these quantities in the cloudy bag model and will in fact find that some of the strength of the proton structure function is shifted to that of the neutron (the calculation includes centre of mass motion corrections). A summary of this work may be found in [ST88a] and [ST89a].

In Chapter 6 we return again to the predictions of the model of Chapter 3 for the distributions, not just their first moments, and compare them to data using the evolution formalism of Chapter 4. We also include in this chapter a discussion of the likely effect

of the $U(1)$ anomaly on the spin structure functions. As we shall see, it is unlikely that it provides much help with solving the 'spin crisis'. The agreement between theory and experiment is quite good, at least above $x = 0.1$, for quantities such as $g_1^p(x)$, the valence distribution $u(x)+d(x)$ and the d/u ratio. We also make predictions for as yet unmeasured quantities such as $g_1^n(x)$. Some of the work in this chapter will be included in [SST].

With Chapter 7 we conclude this thesis, summarizing what has been achieved and what still needs to be done. This is followed by an appendix needed for Chapter 4 and, finally, we include some reprints and preprints that arose out of the work presented here.

Chapter 2

Deep Inelastic Scattering

2.1 Introduction

Perhaps the cleanest probe of hadron structure is that of deep inelastic lepton scattering, where a pointlike lepton interacts via a virtual spin one particle with a hadron target. In the case of the weak interaction the exchanged particle is one of the intermediate vector bosons. In this thesis we will be primarily concerned with the electromagnetic interaction (Figure 2.1), although all of the work is immediately generalizable to the weak force.

In this chapter we shall provide an introduction to the formalism needed to describe these sort of interactions, and as such it will act as a starting point for the rest of the work presented in this thesis. Many of the ideas expressed here may be found in review articles and books on the subject [Alt82,Bur80,Clo79,Hey74,Jaf83,Jaf85,Jaf90,LP83,Mut87,Ynd83] although it is common to concentrate on the spin-independent part of the interaction. We shall in fact do the opposite and concentrate on that part of the interaction that is dependent on both the spins of the hadron and lepton. Furthermore, we shall develop the formalism without resorting to ideas that are true only for the free parton model and are invalid in an interacting field theory like QCD.

In Section 2.2 we introduce the structure functions relevant to the hadron vertex of Figure 2.1 and examine their behaviour in the Bjørken limit in Section 2.3. In Section

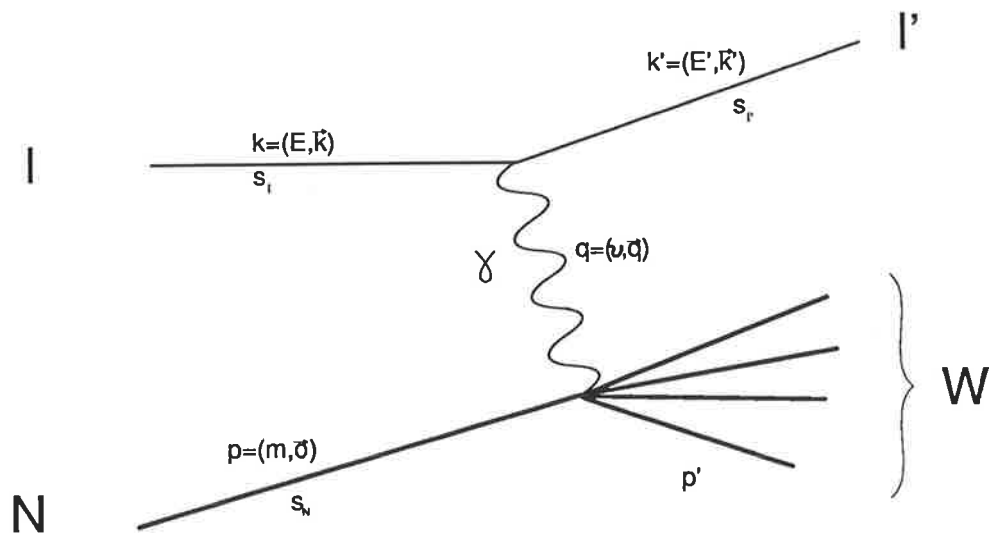
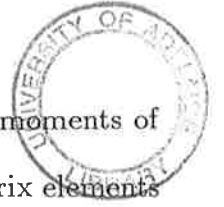


Figure 2.1: The dominant contribution to spin dependent deep inelastic scattering



2.4 we discuss the operator product expansion, which yields results for the moments of the structure functions in terms of moments of matrix elements. These matrix elements will be determined and related to the parton model in Sections 2.5 and 2.6. In Section 2.7 we conclude.

2.2 $W_{\mu\nu}$ and the Structure Functions

In general we may write the differential cross section corresponding to Figure 2.1 as

$$d\sigma \sim \sum_{s'_l, W} |\bar{u}(k', s'_l) \gamma^\mu u(k, s_l) \frac{g_{\mu\nu}}{q^2} \langle W | J^\nu(0) | p, s_N \rangle|^2, \quad (2.1)$$

where the sum is over final lepton spins and all final hadronic states [Hey74]. $u(k, s_l)$ and $|p, s_N\rangle$ refer to the incoming lepton and nucleon spinors respectively, while $u(k', s'_l)$ refers to the outgoing lepton and $\langle W |$ is any one of the possible final hadronic states. $J^\nu(0)$ is the renormalized current operator acting at the origin, which is arbitrary because of the translational invariance of the matrix element. As indicated in Figure 2.1, we shall usually work in the target rest frame so that $p = (M_N, \mathbf{0})$ and $q_0 = \nu$. We have neglected higher order electromagnetic contributions and so the cross-section factorises into a leptonic and hadronic part

$$d\sigma \sim \frac{1}{q^4} L^{\mu\nu} W_{\mu\nu}, \quad (2.2)$$

where

$$\begin{aligned} L^{\mu\nu} &= \sum_{s'_l} \bar{u}_\alpha(k, s_l) (\gamma^\mu)_{\alpha\beta} u_\beta(k', s'_l) \bar{u}_a(k', s'_l) (\gamma^\nu)_{ab} u_b(k, s_l) \\ &= \frac{1}{2} \text{Tr} \{ (\not{k} + m_l) (1 + \gamma_5 \not{s}_l) \gamma^\mu (\not{k}' + m_l) \gamma^\nu \} \\ &= 2 \{ k^\mu k'^\nu + k^\nu k'^\mu + (m_l^2 - k \cdot k') g^{\mu\nu} + i m_l \epsilon^{\mu\nu\rho\sigma} s_\rho q_\sigma \} \end{aligned} \quad (2.3)$$

Here $\epsilon^{\mu\nu\rho\sigma}$ is antisymmetric in its indices, with

$$\epsilon^{0123} = +1 \quad , \quad (2.4)$$

and

$$u(k, s_l) \bar{u}(k, s_l) = (\not{k} + m_l) \left(\frac{1 + \gamma_5 \not{s}_l}{2} \right) \quad (2.5)$$

has been used. Note that the part of the lepton tensor that depends on spin is antisymmetric in the indices μ and ν .

The hadron tensor $W_{\mu\nu}$ may depend on $p, q, g_{\mu\nu}, \epsilon_{\mu\nu\lambda\delta}$ and, because of the analogue to equation 2.5 for the hadronic case, at most linearly on s_N . This results in 12 possible independent terms, reduced to 6 after taking into account gauge invariance, which requires $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$. PT invariance means that the symmetric part of $W_{\mu\nu}$ must be independent of s_N reducing $W_{\mu\nu}$ to consist of four types of terms, which we shall write as follows:

$$\begin{aligned} W_{\mu\nu} = & \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) W_1(p \cdot q, q^2) \\ & + \frac{1}{M_N^2} \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu \right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu \right) W_2(p \cdot q, q^2) \\ & + i \epsilon_{\mu\nu\lambda\delta} q^\lambda \left\{ s_N^\delta \left[M_N G_1(p \cdot q, q^2) + \frac{p \cdot q}{M_N} G_2(p \cdot q, q^2) \right] - \frac{s \cdot q}{M_N} p^\delta G_2(p \cdot q, q^2) \right\} . \end{aligned} \quad (2.6)$$

The functions W_1, W_2, G_1, G_2 are independent of s_N and contain all the information about the structure of the target.

In order to obtain more information on these structure functions we need to go back to expression 2.1 which defines $W_{\mu\nu}$ to be

$$W_{\mu\nu} = \frac{1}{4\pi} \sum_W (2\pi)^4 \delta^4(p + q - p') \langle p, s_N | J_\mu(0) | W \rangle \cdot \langle W | J_\nu(0) | p, s_N \rangle \quad , \quad (2.7)$$

where we have extracted a δ -function in order to conform with the convention established in the literature. Diagrammatically, equation 2.7 refers to the process shown in Figure 2.2.

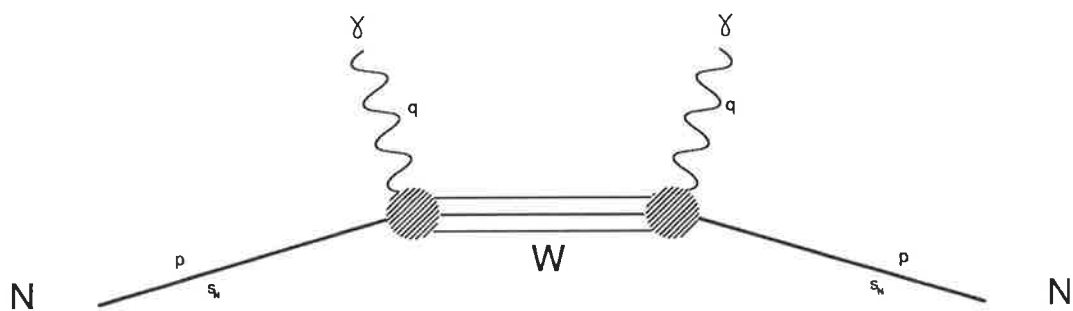


Figure 2.2: $W_{\mu\nu}$

Using

$$(2\pi)^4 \delta^4(p + q - p') = \int d^4 y e^{i(p+q-p') \cdot y} \quad (2.8)$$

and

$$J_\mu(y) = e^{ip \cdot y} J_\mu(0) e^{-ip \cdot y} \quad (2.9)$$

we obtain

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4 y e^{iq \cdot y} \langle p, s | J_\mu(y) J_\nu(0) | p, s \rangle \quad (2.10)$$

where, here and from now on, we drop the subscript on s .

In order to obtain the last expression we have summed over all possible intermediate states. One needs to be careful at this stage to exclude those intermediate states in 2.10 that are "disconnected". These correspond to diagrams where the photon does not interact with the target and could therefore take place in the vacuum, i.e. no scattering occurs.

It is clear from Figure 2.2 that $W_{\mu\nu}$ should be related to the forward (virtual) compton scattering amplitude $T_{\mu\nu}$

$$T_{\mu\nu} = \frac{i}{2} \int d^4 y e^{iq \cdot y} \langle p, s | T[J_\mu(y) J_\nu(0)] | p, s \rangle \quad (2.11)$$

In fact

$$W_{\mu\nu} = \frac{1}{\pi} \text{Abs } T_{\mu\nu} \quad (2.12)$$

where Abs indicates the absorptive (i.e. imaginary) part of $T_{\mu\nu}$

$$\text{Abs } T_{\mu\nu} = \frac{1}{2i} \{T_{\mu\nu}(q_0 + i\epsilon) - T_{\mu\nu}(q_0 - i\epsilon)\} \quad (2.13)$$

Equation 2.12 is, of course, nothing more than the optical theorem. For reasons that will become apparent later, we shall often work with equation 2.11 instead of equation 2.10.

One more property of equation 2.10 that will be needed later is its behaviour under change of the sign of q . By changing the integration variable to $-y$, interchanging the (bosonic) currents and using the translational invariance of the matrix element we find

$$W_{\mu\nu}(-q) = \pm W_{\mu\nu}(q) \quad , \quad (2.14)$$

where the $+(-)$ sign should be chosen if $W_{\mu\nu}$ is symmetric (antisymmetric) in $\mu\nu$.

2.3 The Bjørken Limit

Let us return briefly to the lepton vertex of Figure 2.1. Ignoring the lepton masses we find that $Q^2 = -q^2 \sim 2k_0 k'_0(1 - \cos\theta) \geq 0$, where $\cos\theta$ is the angle between $\vec{k}$ and $\vec{k}'$. Furthermore, because the nucleons are the lightest baryons, we have at the hadron vertex

$$(p + q)^2 \geq M_N^2 \quad , \quad (2.15)$$

i.e. $2p \cdot q (= 2M_N\nu$ in the nucleon rest frame) $\geq Q^2$. Hence the kinematics at the lepton vertex dictates that the photon probe is spacelike, while that of the hadron vertex demands the energy transfer (in the nucleon rest frame) to be positive, if Q^2 is positive. Therefore the Bjørken variable x , defined as

$$x = \frac{Q^2}{2p \cdot q} \quad , \quad (2.16)$$

must lie between 0 and 1.

In order to evaluate equation 2.10 it is customary to subtract from it the term

$$\frac{1}{4\pi} \int d^4y \, e^{iq \cdot y} \langle p, s | J_\nu(0) J_\mu(y) | p, s \rangle \quad . \quad (2.17)$$

"Undoing" the integration by inserting a complete set of intermediate states this term becomes proportional to $\delta(q + p' - p)$, i.e. it requires

$$p'^2 = -\frac{Q^2}{x}(1 + x) + M_N^2 < M_N^2 \quad (2.18)$$

and hence cannot contribute because the nucleon is the lightest baryon. Hence $W_{\mu\nu}$ becomes

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4y e^{iq \cdot y} \langle p, s | [J_\mu(y), J_\nu(0)] | p, s \rangle \quad (2.19)$$

Causality requires that the commutator appearing in 2.19 vanishes for $y^2 < 0$.

The Bjørken limit is defined as the kinematical region $Q^2 \rightarrow \infty$, $\nu \rightarrow \infty$, with x fixed. In order to ascertain what this means for the process of equation 2.11 we shall use the light cone coordinates $a^\pm = \frac{1}{\sqrt{2}}(a^0 \pm a^3)$ and shall take the photon to move along the negative z direction, i.e.

$$\begin{aligned} q \cdot y &= q^+ y^- + q^- y^+ - \vec{q}_\perp \cdot \vec{y}_\perp \\ &\sim -\frac{M_N x}{\sqrt{2}} y^- + \sqrt{2} \nu y^+ \end{aligned} \quad (2.20)$$

The main contribution to 2.11 will be in the region where $e^{iq \cdot y}$ fluctuates least rapidly [Lig58], i.e. in the Bjørken limit we require $y^+ \sim O(\frac{1}{\nu})$ and $y^- \sim O(\frac{1}{x})$. The Bjørken limit therefore probes the region $y^+ y^- \rightarrow 0$. However, given that the commutator is only non zero for timelike y^2 we see that in fact the Bjørken limit probes only that region where $y_\perp^2 \sim 0$. Hence $W_{\mu\nu}$ (and $T_{\mu\nu}$) are sensitive to the behaviour of the commutator when $y^2 \sim 0$. Note that this does not imply sensitivity to only short distances: y^- may in fact be quite large. Because in model field theories the commutator is in fact singular at $y^2 \sim 0$, deep inelastic scattering is often said to probe the strength of its lightcone singularity.

2.4 The Operator Product Expansion

In order to evaluate 2.11 in the Bjørken limit it is convenient to expand the product of currents in such a way that all singularities occurring at $y^2 = 0$ are gathered into coefficient

functions of operators regular at $y^2 = 0$. That this can be done relies on the short distance operator product expansion introduced by Wilson [Wil64,Wil69]:

$$T[j(y)j(0)] \xrightarrow{y \rightarrow 0} \sum_i \mathcal{C}^{(i)}(y^2) \mathcal{O}^{(i)}(0) \quad . \quad (2.21)$$

Here $\mathcal{C}^{(i)}$ is the singular coefficient function corresponding to the operator $\mathcal{O}^{(i)}(0)$ and we have suppressed Lorentz indices for simplicity. Note that only a finite number of terms appears in 2.21. This expansion was used by Zimmermann [Zim71,Zim73] as well as Brandt and Preparata [Bra69,BP71] to obtain a similar expansion, this time however with an infinite number of terms, valid on the lightcone rather than at short distances:

$$T[j_\mu(y)j_\nu(0)] \xrightarrow{y^2 \rightarrow 0} \sum_i \sum_{n=0}^{\infty} \mathcal{C}^{(i,n+1)}(y^2) y_{\mu_1} \dots y_{\mu_n} \mathcal{O}_{(i,n+1)}^{\mu,\nu,\mu_1,\dots,\mu_n}(0) \quad . \quad (2.22)$$

Although this expansion has not been proven generally in non-perturbative QCD it has been shown to hold for low orders of $\frac{1}{Q^2}$ [NSVZ80,SVZ79].

Inserting this expansion into 2.11 we may write the antisymmetric piece of $T_{\mu\nu}$ as

$$T_{\mu\nu}^A = i\epsilon_{\mu\nu\lambda\delta} \frac{2q^\lambda}{-q^2} \sum_{i,n} \langle p, s | \mathcal{O}_{(i,n+1)}^{\delta,\mu_1,\dots,\mu_n}(0) | p, s \rangle \left(\frac{2q_{\mu_1}}{-q^2} \right) \dots \left(\frac{2q_{\mu_n}}{-q^2} \right) \tilde{\mathcal{C}}^{(i,n+1)}(q^2) \quad , \quad (2.23)$$

where we have defined

$$\left(\frac{2q_{\mu_1}}{-q^2} \right) \dots \left(\frac{2q_{\mu_n}}{-q^2} \right) \tilde{\mathcal{C}}^{(i,n+1)}(q^2) = \int d^4 y e^{iq \cdot y} y_{\mu_1} \dots y_{\mu_n} \mathcal{C}^{(i,n+1)}(y^2) \quad (2.24)$$

and we have used the general form of the Lorentz structure of $T_{\mu\nu}^A$ (it is the same as that for $W_{\mu\nu}^A$ in equation 2.6) to suitably define the operators $\mathcal{O}_{(i,n+1)}^{\delta,\mu_1,\dots,\mu_n}(0)$. The sum over n can only include even values of n [HM72] because of the requirement that $T_{\mu\nu}^A$ must be antisymmetric with respect to a change of sign of q (see equation 2.14).

Note furthermore that the matrix element can only depend linearly on s (see the argument in Section 2.2). We can therefore write down its general form as

$$\langle p, s | \mathcal{O}_{(i,n+1)}^{\delta, \mu_1, \dots, \mu_n}(0) | p, s \rangle \equiv (s \, p \, p \, p \dots p)^{\delta, \mu_1, \dots, \mu_n} \mathcal{A}_{(i,n+1)} + \text{terms containing } g^{\mu_i, \mu_j} \quad . \quad (2.25)$$

When contracted with $q_{\mu_1} \dots q_{\mu_n}$ the terms that contain g^{μ_i, μ_j} (the so-called trace terms) are of a lower power in q^2 than the term that does not contain a g^{μ_i, μ_j} - we shall therefore ignore them from now on. Note that the matrix elements $\mathcal{A}_{(i,n+1)}$ are independent of s .

$(s \, p \, p \, p \dots p)^{\delta, \mu_1, \dots, \mu_n}$ denotes all possible permutations of the indices. We note that only terms symmetric in $\mu_1, \dots, \mu_n$ can contribute to equation 2.23. There are only two possible terms that have this property [HM72, Hei73, AR75, AG76, WW77, KMSU79, Jaf90]: one completely symmetric in $\delta, \mu_1, \dots, \mu_n$ and one antisymmetric in δ and one of the μ 's. We may therefore write the general contribution as [Jaf90]:

$$\begin{aligned} \langle p, s | \mathcal{O}_{(i,n+1)}^{\delta, \mu_1, \dots, \mu_n}(0) | p, s \rangle = & \\ & \frac{1}{2} \{ s^\delta p^{\mu_1} p^{\mu_2} \dots p^{\mu_n} + s^{\mu_1} p^\delta p^{\mu_2} \dots p^{\mu_n} + \dots + s^{\mu_n} p^{\mu_1} p^{\mu_2} \dots p^\delta \} \frac{\mathcal{A}_{(i,n+1)}^1}{n+1} \\ & + \frac{1}{2} \{ (s^\delta p^{\mu_1} - p^\delta s^{\mu_1}) p^{\mu_2} \dots p^{\mu_n} + (s^\delta p^{\mu_2} - p^\delta s^{\mu_2}) p^{\mu_1} p^{\mu_3} \dots p^{\mu_n} + \dots \\ & + (s^\delta p^{\mu_n} - p^\delta s^{\mu_n}) p^{\mu_1} \dots p^{\mu_{n-1}} \} \frac{\mathcal{A}_{(i,n+1)}^2}{n+1} \quad . \quad (2.26) \end{aligned}$$

The factor of 1/2 is included for later convenience. Note that if $n = 0$ there is in fact only one possible term - the one proportional to $\mathcal{A}_{(i,n+1)}^1$.

Inserting equation 2.26 into equation 2.23 we obtain

$$\begin{aligned} T_{\mu\nu}^A = \frac{i}{2} \epsilon_{\mu\nu\lambda\delta} q^\lambda \sum_i & \left\{ \sum_{n=0,2,\dots}^\infty \left[\frac{s^\delta}{M_{N\nu}} + n \frac{s \cdot q \, p^\delta}{M_{N\nu}^2} \right] \frac{1}{x^{n+1}} \frac{\mathcal{A}_{(i,n+1)}^1}{n+1} \tilde{\mathcal{C}}_1^{(i,n+1)}(q^2) \right. \\ & \left. + \sum_{n=2,4,\dots}^\infty n \left[\frac{s^\delta}{M_{N\nu}} - \frac{s \cdot q \, p^\delta}{M_{N\nu}^2} \right] \frac{1}{x^{n+1}} \frac{\mathcal{A}_{(i,n+1)}^2}{n+1} \tilde{\mathcal{C}}_2^{(i,n+1)}(q^2) \right\} \quad (2.27) \end{aligned}$$

Comparison with equation 2.6 gives

$$2M_N^2 \nu G_1^{(T)} = \sum_{n=0,2,\dots}^\infty \frac{1}{x^{n+1}} \mathcal{A}_{(i,n+1)}^1 \tilde{\mathcal{C}}_1^{(i,n+1)}(q^2)$$

$$2M_N \nu^2 G_2^{(T)} = \sum_{n=2,4,\dots}^{\infty} \frac{n}{n+1} \frac{1}{x^{n+1}} \left(\mathcal{A}_{(i,n+1)}^2 \tilde{\mathcal{C}}_2^{(i,n+1)}(q^2) - \mathcal{A}_{(i,n+1)}^1 \tilde{\mathcal{C}}_1^{(i,n+1)}(q^2) \right) \quad (2.28)$$

The superscripts 'T' remind us that we are really dealing with $T_{\mu\nu}^A$ rather than $W_{\mu\nu}^A$. The right hand sides of equation 2.28 contain power series expansions in x and might not converge. In fact, the region $0 \leq x \leq 1$ corresponds to $(p+q)^2 \geq M_N^2$ (i.e. as in equation 2.15) and so the forward Compton scattering may proceed via a physical intermediate state, i.e. $T_{\mu\nu}$ has a cut in this interval. There is also a cut from $-1 \leq x \leq 0$ because the process $p \rightarrow \gamma^* X$ is physical here. We see therefore that the summation in equation 2.28 cannot be carried out in the region of physical interest, i.e. $x \leq 1$.

This turns out to be the reason why we have worked with $T_{\mu\nu}$ rather than $W_{\mu\nu}$. The two cuts correspond to cuts in the complex ν plane from $-\infty$ to $-\frac{Q^2}{2M_N}$ and from $\frac{Q^2}{2M_N}$ to ∞ . Assuming that there are no more cuts let us perform a contour integration around the cuts

$$\begin{aligned} & \frac{1}{2\pi i} \oint x^{m+1} T_{\mu\nu}(\nu, Q^2) d\nu \\ &= \frac{1}{\pi} \left(\frac{Q^2}{2M_N} \right)^{m+1} \left\{ \int_{\frac{Q^2}{2M_N}}^{\infty} \frac{d\nu}{\nu^{m+1}} \text{Abs} T_{\mu\nu}(\nu, Q^2) + \int_{-\infty}^{-\frac{Q^2}{2M_N}} \frac{d\nu}{\nu^{m+1}} \text{Abs} T_{\mu\nu}(\nu, Q^2) \right\} \\ &= \frac{Q^2}{2M_N} \int_{-1}^1 dx x^{m-1} W_{\mu\nu}(x, Q^2) \quad , \end{aligned} \quad (2.29)$$

where we have used the definitions in equations 2.12 and 2.13. (We have assumed that $T_{\mu\nu}(\nu, Q^2)$ falls off faster than $|\nu|^{-1}$ as $\nu \rightarrow \infty$. If it doesn't, one should use subtracted dispersion relations instead, yielding the same result.) Using

$$\frac{1}{2\pi i} \oint dx x^{m-n-2} = \delta_{m,n+1} \quad (2.30)$$

equation 2.28 then gives

$$\int_{-1}^1 x^{n-1} g_1(x, Q^2) dx = \frac{1}{2} \sum_i \mathcal{A}_{(i,n)}^1 \tilde{\mathcal{C}}_1^{(i,n)}(q^2) \quad n = 1, 3, 5, \dots$$

$$\int_{-1}^1 x^{n-1} g_2(x, Q^2) dx = \frac{1}{2} \sum_i \frac{n-1}{n} \left(\mathcal{A}_{(i,n)}^2 \tilde{\mathcal{C}}_2^{(i,n)}(q^2) - \mathcal{A}_{(i,n)}^1 \tilde{\mathcal{C}}_1^{(i,n)}(q^2) \right) \quad n = 3, 5, 7, \dots \quad (2.31)$$

where we have defined the scaling structure functions g_1 and g_2 as

$$\begin{aligned} g_1(x, Q^2) &= M_N^2 \nu G_1(p, q, Q^2) \\ g_2(x, Q^2) &= M_N \nu^2 G_2(p, q, Q^2) \quad . \end{aligned} \quad (2.32)$$

Finally, using equations 2.14 and 2.32 we establish that both g_1 and g_2 must be even, so that we can restrict the integrals in equation 2.31 to the physical region, i.e.

$$\begin{aligned} \int_0^1 x^{n-1} g_1(x, Q^2) dx &= \sum_i \mathcal{A}_{(i,n)}^1 \tilde{\mathcal{C}}_1^{(i,n)}(q^2) \quad n = 1, 3, 5, \dots \\ \int_0^1 x^{n-1} g_2(x, Q^2) dx &= \sum_i \frac{n-1}{n} \left(\mathcal{A}_{(i,n)}^2 \tilde{\mathcal{C}}_2^{(i,n)}(q^2) - \mathcal{A}_{(i,n)}^1 \tilde{\mathcal{C}}_1^{(i,n)}(q^2) \right) \quad n = 3, 5, 7, \dots \quad . \end{aligned} \quad (2.33)$$

An interesting observation is that if it weren't for the $\mathcal{A}_{(i,n)}^2$'s appearing in the second expression, g_2 and g_1 would be related. In fact, by using the convolution theorem for Mellin transforms ¹(see, for example, [Gro74]; also [Obe74]) and noting that the inverse Mellin transform of $-\frac{n-1}{n}$ is given by $1 - \frac{1}{2}\delta(1-x)$, we find that

$$g_2'(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2) \quad (2.35)$$

where g_2' is that piece of g_2 originating from the $\mathcal{A}_{(i,n)}^1$'s.

The use of the operator product expansion has therefore enabled us to write the moments of the structure functions g_1 and g_2 as a product of Q^2 independent matrix

¹If f_n, g_n, h_n are the n^{th} moments of $f(x), g(x), h(x)$ and $f_n = g_n h_n$, then

$$f(x) = \int_x^1 \frac{dy}{y} g\left(\frac{x}{y}\right) h(y) \quad (2.34)$$

elements times Q^2 dependent coefficient functions. The variation of these coefficient functions with Q^2 that is to be expected in QCD will be examined in Chapter 4. In the rest of this chapter we shall concentrate on model independent observations one can make about the matrix elements themselves, to be followed in the next chapter with their derivation within a model.

2.5 The Matrix Elements $\mathcal{A}_{(i,n)}$

In order to evaluate the matrix elements $\mathcal{A}_{(i,n+1)}^{1,2}$, as defined by equation 2.26, we need to know the operators $\mathcal{O}_{(i,n+1)}^{\delta,\mu_1,\dots,\mu_n}$. In model field theories there are no problems constructing these directly (see, for example, reference [BP71]). Rather than do this we shall construct them (more precisely, the parts that will give rise to the dominant terms in equation 2.33 in the limit of large Q^2) by appealing to our knowledge of their general form. They must be made up of gauge invariant objects, such as

$$\mathcal{D}^{\mu_i}(\text{dimension} = 1), \Psi(d = \frac{3}{2}), \gamma^{\mu_i}(d = 0), \mathcal{F}^{\alpha\beta}(d = 2) \quad , \quad (2.36)$$

where we have indicated the relevant mass dimensions.

It is not difficult to determine via a dimensional argument what sort of operators dominate in the Björken limit. In order to do this let us look at equation 2.33. The Q^2 dependence of $W_{\mu\nu}$ is contained exclusively in the coefficients $\mathcal{C}_{1,2}^{(i,n+1)}(Q^2)$. Their mass dimension can only be due to Q^2 . (In fact $\mathcal{C}_{1,2}^{(i,n+1)}(Q^2)$ also depends on the renormalization scale μ , or equivalently Λ_{QCD} . It can however be shown that this dependence is only logarithmic in μ , introducing corrections of the form $\log \frac{\mu^2}{Q^2}$. We shall discuss these further in Chapter 4. Note also that we have assumed all quark masses to be zero; Jaffe [Jaf85] has pointed out, using Weinberg's theorem [Wei60], that the conclusions are in fact unaffected by this.) The leading contribution to $W_{\mu\nu}$ is therefore given by those terms for which $\mathcal{C}_{1,2}^{(i,n+1)}$ has the largest mass dimension, i.e. those terms for which $\mathcal{A}_{(i,n+1)}^{1,2}$ has the smallest mass dimension.

From equation 2.26 we write the essential part (the mass dimension of the wavefunction is irrelevant here) of the mass dimension of $\mathcal{A}_{(i,n+1)}^{1,2}$ as

$$\mathcal{T}_{(i)} = \mathcal{D}_{(i)} - n \quad , \quad (2.37)$$

where $\mathcal{D}_{(i)}$ is the mass dimension of the operator $\mathcal{O}_{(i,n+1)}^{\delta,\mu_1,\dots,\mu_n}$. $\mathcal{T}_{(i)}$ is referred to as its twist.

The operators of lowest twist that contribute to a connected matrix element of the type in equation 2.25 have twist 2. They include nine quark operators and one gluon operator, and are given by [Ynd83, page 78]

$$\begin{aligned} \mathcal{O}_{(i,n+1)}^{\delta,\mu_1,\dots,\mu_n} &= \mathcal{S} \left\{ (i)^n \bar{\Psi} \gamma^\delta \gamma_5 \mathcal{D}^{\mu_1} \dots \mathcal{D}^{\mu_n} \lambda_i \Psi \right\} \\ \text{and } \mathcal{O}_{(i,n+1)}^{\delta,\mu_1,\dots,\mu_n} &= \mathcal{S} \left\{ (i)^{n-1} \mathcal{F}^{\alpha\delta} \mathcal{D}^{\mu_1} \dots \mathcal{D}^{\mu_{n-1}} \mathcal{F}_\alpha^{\mu_n} \right\} \end{aligned} \quad (2.38)$$

where $\mathcal{S}$ implies symmetrization over the indices (including the δ and a division by $(n+1)!$) and we have explicitly shown the SU(3) flavour matrices λ_i distinguishing the operators of equal twist. (λ_0 is included and defined to be the identity). From now on we shall concentrate on the quark operators only.

Note that the operators in equation 2.38 are all completely symmetric and therefore correspond to $\mathcal{A}_{(i,n+1)}^1$ only. There are no twist two operators contributing to $\mathcal{A}_{(i,n+1)}^2$. In fact, the lowest twist contributing to the latter is three and operators of this type are listed, for example in [AR75,JJ90] while the QCD evolution thereof is discussed in [CHJR83]. We shall not discuss these twist three operators further, so that we effectively limit ourselves to looking at g_1 only.

In order to investigate the form of $\mathcal{A}_{(i,n+1)}^1$ further let us choose the $\mathcal{A}^+ = 0$ gauge. We obtain

$$\begin{aligned} \mathcal{A}_{(i,n+1)}^1 &= \frac{2}{s^+(p^+)^n} \langle p, s | \bar{\Psi}(0) \gamma^+ \gamma_5 \lambda_i (i\partial^+)^n \Psi(0) | p, s \rangle \\ &= \frac{2}{(p^+)^n} \langle p, s | \Psi^\dagger(0) \gamma^- \gamma^+ \gamma_5 \lambda_i (i\partial^+)^n \Psi(0) | p, s \rangle \quad , \end{aligned} \quad (2.39)$$

where we have defined

$$\gamma^\pm = \frac{1}{\sqrt{2}}(\gamma^0 \pm \gamma^3) \quad (2.40)$$

and we have assumed the hadron to be at rest with its spin in the positive z direction so that $s^+ = \frac{1}{\sqrt{2}}$.

Consider now the function (the discussion here essentially paraphrases that of Jaffe [Jaf83], with the spin dependence spelled out)

$$H_i(x) = \frac{p^+}{2\pi} \int_{-\infty}^{\infty} d\xi^- e^{-ixp^+\xi^-} \langle p, s | T [\Psi^\dagger(\xi^-) \lambda_i \Gamma \Psi(0)] | p, s \rangle \quad (2.41)$$

Here Γ is the matrix $2\gamma^- \gamma^+ \gamma_5$. We may relate this to $\mathcal{A}_{(i,n+1)}^1$ in the following way :

$$\begin{aligned} \int_{-\infty}^{\infty} H_i(x) x^{n-1} dx &= \frac{p^+}{2\pi} \int_{-\infty}^{\infty} d\xi^- dx \left\{ \left(\frac{1}{-ip^+} \frac{\partial}{\partial \xi^-} \right)^{n-1} e^{-ixp^+\xi^-} \right\} \\ &\quad \langle p, s | T [\Psi^\dagger(\xi^-) \lambda_i \Gamma \Psi(0)] | p, s \rangle \quad (2.42) \end{aligned}$$

By assuming that the matrix element and its derivatives vanish at infinity, we may transfer the derivative to the matrix element by integrating by parts, i.e.

$$\begin{aligned} \int_{-\infty}^{\infty} H_i(x) x^{n-1} dx &= \frac{p^+}{2\pi} \int_{-\infty}^{\infty} \frac{d\xi^- dx}{(-ip^+)^{n-1}} e^{-ixp^+\xi^-} (-1)^{n-1} \\ &\quad \cdot \langle p, s | T \left[\Psi^\dagger(\xi^-) \left(\frac{\partial}{\partial \xi^-} \right)^{n-1} \lambda_i \Gamma \Psi(0) \right] | p, s \rangle \\ &= \frac{1}{(p^+)^{n-1}} \langle p, s | \Psi^\dagger(0) (-i\partial^+)^{n-1} \lambda_i \Gamma \Psi(0) | p, s \rangle \quad (2.43) \end{aligned}$$

Comparing with equation 2.39 we obtain

$$\mathcal{A}_{(i,n)}^1 = \int_{-\infty}^{\infty} H_i(x) x^{n-1} dx \quad i = 1, 3, 5, \dots, \quad (2.44)$$

so that g_1 in equation 2.33 can be written as

$$\int_0^1 x^{n-1} g_1(x, Q^2) dx = \sum_i c_i \left\{ \int_{-\infty}^{\infty} H_i(x) x^{n-1} dx \right\} \tilde{\mathcal{C}}_1^{(i,n)}(q^2) \quad n = 1, 3, 5, \dots \quad (2.45)$$

where we have explicitly shown the flavour dependence of the scattering process by including the coefficients c_i - because $W_{\mu\nu}$ corresponds to an electromagnetic process the flavour dependence is proportional to the square of the charge of the quarks. Explicitly

$$\frac{1}{2}Q_f^2 = \frac{1}{9}\lambda_0 + \frac{1}{12}\lambda_3 + \frac{1}{36}\lambda_8 \quad , \quad (2.46)$$

i.e. we define

$$\begin{aligned} c_0 &= \frac{1}{9} \\ c_3 &= \frac{1}{12} \\ c_8 &= \frac{1}{36} \end{aligned} \quad (2.47)$$

together with the relevant SU(3) flavour matrices

$$\lambda_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ and } \lambda_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \quad (2.48)$$

Equation 2.45 is still somewhat cumbersome. To simplify it further we need to look closer at the matrix elements $H_i(x)$.

2.6 Properties of $H_i(x)$

In order to unravel the meaning of the functions $H_i(x)$ it is instructive to write the time ordered product appearing in the matrix element in such a way that the dependence on the matrix Γ is projected into the fields, i.e. let us define

$$\Psi^\dagger(\xi^-)\Gamma\Psi(0) \equiv \sum_{\Gamma} \Psi_{\Gamma}^{\dagger i}(\xi^-)\Psi_{\Gamma}^i(0) \quad . \quad (2.49)$$

We will discuss the explicit form of the Ψ_{Γ} 's in the next chapter. The superscript i indicates the three possible flavour combinations (we shall suppress this index on Ψ_{Γ} from

now on). It can be shown [Jaf83] that the T ordering of the (interacting) field operators is not necessary and that one indeed has a choice in the ordering:

$$\begin{aligned}\langle p, s | T \left[\Psi_{\Gamma}^{\dagger}(\xi^{-}) \Psi_{\Gamma}(0) \right] | p, s \rangle &= \langle p, s | \Psi_{\Gamma}^{\dagger}(\xi^{-}) \Psi_{\Gamma}(0) | p, s \rangle \\ &= -\langle p, s | \Psi_{\Gamma}(0) \Psi_{\Gamma}^{\dagger}(\xi^{-}) | p, s \rangle .\end{aligned}\quad (2.50)$$

Inserting a complete set of intermediate states, and using the translational invariance of the matrix element, we obtain from equation 2.41 for the two different orderings

$$\begin{aligned}H_i(x) &= p^+ \sum_{n, \Gamma} |\langle n | \Psi_{\Gamma}(0) | p, s \rangle|^2 \delta(p^+(1-x) - p_n^+) \\ &= -p^+ \sum_{n, \Gamma} |\langle n | \Psi_{\Gamma}^{\dagger}(0) | p, s \rangle|^2 \delta(p^+(1+x) - p_n^+) .\end{aligned}\quad (2.51)$$

Although either equation could be used to calculate $H_i(x)$ it turns out that particular choices are convenient for particular regions of x .

Consider the hadronic part of the forward scattering matrix element corresponding to the first choice for equation 2.51. Four connected diagrams are allowed by the δ -function (Figure 2.3): The intermediate state must have a positive plus component of momentum which places the following restrictions on x : Figure 2.3a: $x \leq 1$; Figure 2.3b and Figure 2.3c: $x \leq 0$; Figure 2.3d: $x \leq -1$.

The corresponding diagrams for the second option in equation 2.51 are identical, with x being replaced by $-x$. (See Figure 2.4) i.e. Figure 2.4a requires $x \geq -1$, Figure 2.4b and 2.4c: $x \geq 0$ and Figure 2.4d: $x \geq 1$.

One can see that $H_i(0 \leq x \leq 1)$ may be calculated from diagram 2.3(a) or 2.4(b)-(c). Similarly, $H_i(-1 \leq x \leq 0)$ is obtained from diagram 2.4(a) or 2.3(b)-(c). We shall choose to calculate $H_i(x)$ in terms of the diagrams 2.3 (a) and 2.4(a) because of their appealing interpretation: 2.3(a) corresponds to the photon colliding with a non-interacting parton with (plus component of) momentum $x p^+$, leaving a (coloured) intermediate state with momentum $p_n^+ = p^+ - x p^+$. The parton then re-emits the photon with momentum $x p^+$,

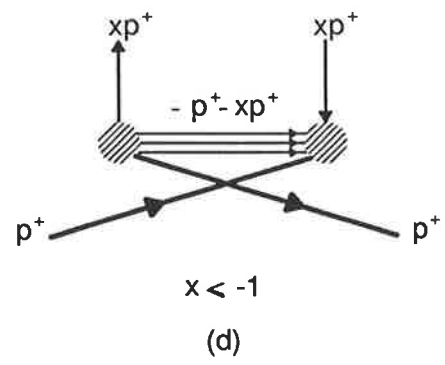
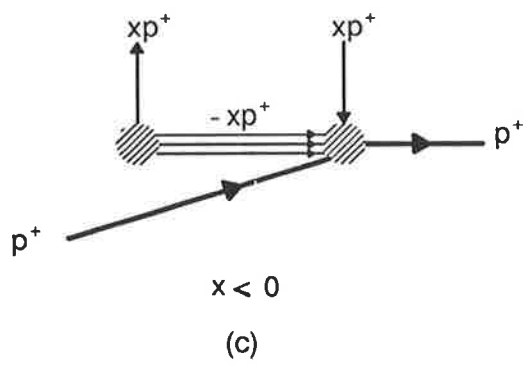
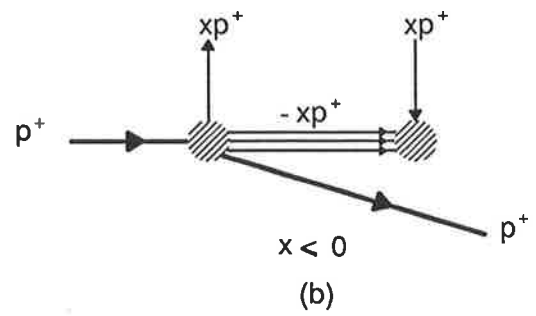
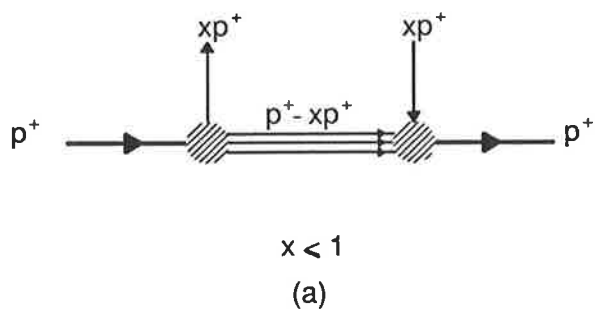


Figure 2.3: Possible diagrams for the first option in equation 2.51

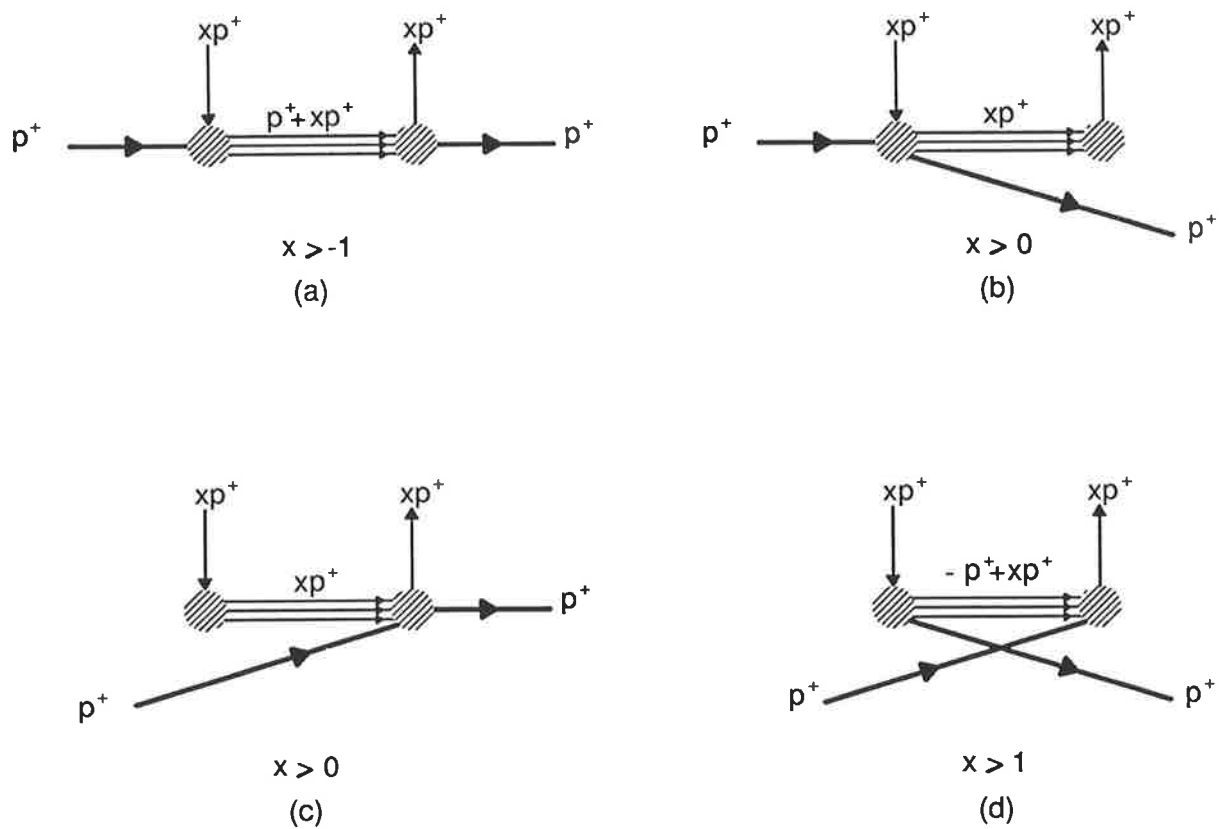


Figure 2.4: Possible diagrams for the second option in equation 2.51

resulting in its scattering in the forward direction. This interpretation is just that of Feynman's parton model [Fey72], and is of course in accord with the usual interpretation of the Björken variable x as the fraction of the hadron's (plus component of) momentum carried by the struck quark. An analogous interpretation holds for figure 2.4(a); with the quark replaced by an antiquark.

There is one more important observation to make from Figures 2.3 and 2.4: There are no diagrams possible for Figure 2.3 with $x > 1$. Because we are free to choose either of the two interpretations this means that the contributions of Figure 2.4(b)-(d) must sum to 0 in this region; i.e. $H_i(x)=0$ for $x > 1$. A similar argument for Figure 2.4 gives $H_i(x)=0$ for $x < -1$. We are therefore free to change the limits in equation 2.44 so that the integration runs from -1 to 1.

The integration on the left hand side of equation 2.45 however runs only from 0 to 1 while now the right hand side is defined by an integral from -1 to 1. In order to obtain a probabilistic parton interpretation we must write the latter as an integral from 0 to 1. We may do this by the use of equation 2.51:

$$\mathcal{A}_{(i,n)}^1 = \int_{-1}^1 x^{n-1} H_i(x) = \int_0^1 x^{n-1} \{H_i(x) + H_i(-x)\} dx \quad n = 1, 3, \dots \quad (2.52)$$

Let us therefore define the quark distribution associated with the flavour SU(3) matrix λ_i as

$$\begin{aligned} q_i(x) &= (H_i(x) + H_i(-x)) \\ &= p^+ \sum_{\Gamma, n} \left\{ |\langle n | \Psi_{\Gamma}(0) | p, s \rangle|^2 - |\langle n | \Psi_{\Gamma}^{\dagger}(0) | p, s \rangle|^2 \right\} \delta(p^+(1-x) - p_n^+) \end{aligned} \quad (2.53)$$

We obtain

$$\mathcal{A}_{(i,n)}^1 = \int_0^1 x^{n-1} q_i(x) dx \quad , \quad (2.54)$$

i.e. equation 2.45 becomes

$$\int_0^1 x^{n-1} g_1(x, Q^2) dx = \sum_i c_i \left\{ \int_0^1 q_i(x) x^{n-1} dx \right\} \tilde{\mathcal{C}}_1^{(i,n)}(q^2) \quad n = 1, 3, 5, \dots \quad (2.55)$$

As a final step we may replace the last equation by one running over all n by analytic continuation in n .

This is the result that we have been aiming to derive. It is easy to derive the (pre-QCD) quark-parton model (QPM) from this. As we will see in Chapter 4 the coefficients $\tilde{\mathcal{C}}_1^{(i,n)}(Q^2)$ can be found at any order in QCD by solving the relevant renormalisation group equation at that order. If there are no interactions, as in the QPM, the anomalous dimension for the operator $\mathcal{O}_{(i,n)}^{\delta, \mu_1, \dots, \mu_n}$ is zero - i.e. there is no such thing as renormalisation in a non-interacting theory. Hence the coefficients just become constants and, in fact, with the definitions of the $\tilde{\mathcal{C}}_1^{(i,n)}(Q^2)$'s and the c_i 's adopted here, can be shown to be unity [Mut87, page 291]. We therefore obtain the result that the n^{th} moment of the hadron tensor is proportional to the n^{th} moment of a sum of $q_i(x)$'s. If all the moments of two distributions are equal then the distributions are themselves identical, hence

$$g_1(x) \longrightarrow \sum_i c_i q_i(x) \quad . \quad (2.56)$$

$q_i(x)$ is then interpreted to be the probability of observing a quark of type i with momentum fraction x . If interactions are present a simple interpretation like this is not possible. Nevertheless, we see from equation 2.55 that the deviations from the free parton model enter in a well defined way through the coefficient functions $\tilde{\mathcal{C}}_1^{(i,n)}(q^2)$ - if we develop a model for $q_i(x)$ we may still derive from 2.55 the structure function g_1 at arbitrary (high) Q^2 .

2.7 Conclusion

The central result of this chapter is that we have shown that, in an interacting field theory, deep inelastic lepton scattering on a bound state target can be expressed in terms

of the moments of a hadron tensor $W_{\mu\nu}$. These are given by a sum of products of a Q^2 dependent coefficient function and a Q^2 independent matrix element. Furthermore, the matrix element can be expressed in terms of the moments of a distribution $q_i(x)$ that measures the probability of finding a quark of type i with momentum fraction x minus the probability of finding an antiquark with momentum fraction x . In the limit that all strong interactions are ignored one finds that the hadron tensor can be written in terms of the distributions themselves. These distributions will be calculated within a model in the next chapter, to be followed by a discussion of the Q^2 dependence predicted by QCD in Chapter 4.

Chapter 3

A Model for the Quark Distribution Functions

3.1 Introduction

We have seen in the last chapter that in order to describe and predict the results of deep inelastic scattering we require two ingredients: We need to know the Q^2 dependent coefficients $\mathcal{C}_1^{(i,n)}(Q^2)$ and we require the matrix elements $\mathcal{A}_{(i,n)}^1$ (we shall from now on suppress the subscript 1 on both of these, i.e. we shall consider only g_1) which we found we could write as moments of quark and antiquark distribution functions.

The $\mathcal{C}^{(i,n)}(Q^2)$ can be calculated in perturbative QCD. Their Q^2 dependence arises out of radiative corrections as well as gluon and quark loop corrections to the electromagnetic vertex. Their calculation will be left for the next chapter.

The matrix element $\mathcal{A}_{(i,n)}$, or equivalently the quark distribution $q_i(x)$, should in principle also be calculable in QCD. We immediately recognize however that these distribution functions must be connected in some way to the process that keeps the quarks localized inside a hadron. This process is by definition something concerned with the long range nature of the quark-gluon interaction, which in QCD at least is not a perturbative problem. At the present time non-perturbative QCD is not able to describe the confine-

ment mechanism that is necessary to explain the lack of observation of free quarks, let alone is it able to make predictions for quark distributions inside a hadron.

Because of this we shall try, in this chapter, to evaluate the quark distribution functions within a model of the non-perturbative aspects of QCD. The model is largely a matter of choice - we shall use primarily a modified version of the MIT bag model because of its simplicity and intuitive appeal. In order to see how this is to be done, let us look again at the definition of $H_i(x)$ in equation 2.41

$$H_i(x) = \frac{p^+}{2\pi} \int_{-\infty}^{\infty} d\xi^- e^{-ixp^+\xi^-} \langle p, s | T [\Psi^\dagger(\xi^-) \lambda_i \Gamma \Psi(0)] | p, s \rangle . \quad (3.1)$$

The wavefunction $|p, s\rangle$ contains the information about the quark distribution of a hadron of four momentum p and spin s . If we are to construct this from bag wavefunctions we are immediately faced with a problem. The individual quark wavefunctions of the MIT bag are those of a massless fermion confined in a stationary, infinitely deep well. The simplest hadron wavefunction would be a simple product of three quarks. Structure functions have indeed been calculated for this wavefunction, both for the spin independent case [Jaf75] as well as for the spin dependent case [Hug77, Bar79]. These calculations are however deficient in the sense that the wavefunction carries an unphysical dependence on the position of the bag, and indeed the wavefunction is not a momentum eigenstate (Remember that the manipulations of Chapter 2 required this because it was assumed that $e^{i\hat{p}\cdot x}|p, s\rangle = e^{ip\cdot x}|p, s\rangle$). We shall attempt to overcome this problem in Section 3.3 by the use of the Peierls-Yoccoz projection ¹.

The other problem that arises in the calculation of $\mathcal{A}_{(i,n)}$ is the choice of the quark field operators Ψ . According to the formalism of Chapter 2 they should be operators renormalized at some scale μ . Let us for a minute set all higher twist contributions, as well as the trace terms that we have neglected in the high Q^2 limit, to zero. In the limit $Q^2 = \mu^2$ the structure functions would then be given purely in terms of the matrix

¹This method of obtaining a momentum eigenstate in order to calculate structure functions is also being used by Benesh and Miller [BM87, BM88a, BM88b].

elements $\mathcal{A}_{(i,n)}$ calculated within the model, i.e. *the distribution functions $q_i(x)$ calculated from the model describe the structure of the target seen at the scale μ^2 modulo higher twist effects etc.* If we decide that at this scale the renormalized wavefunction $|p, s\rangle$ looks like three MIT bag quarks it follows that a sensible Ansatz is to take the operators Ψ to also be those of the bag model.

We are now ready to proceed with the detailed development of the model. We shall derive the form of the spin dependent quark distribution functions $q_i(x)$ in Section 3.2. In Section 3.3 we review the Peierls-Yoccoz wavefunctions which we shall use to obtain numerical results in Sections 3.4 to 3.6. In 3.7 we shall discuss their normalizations and in Section 3.8 we look at how these distribution functions might be modified in more realistic models. In particular we shall include phenomenologically the effects of one gluon exchange and pionic corrections.

3.2 The Quark Distributions

We saw in the previous chapter that the quark distribution corresponds to a combination of two terms, labeled $H_i(x)$ and $H_i(-x)$:

$$H_i(x) = p^+ \sum_n \delta(p^+[1-x] - p_n^+) |\langle n | \Psi(0) | \mathbf{p}, s \rangle|^2 \quad (3.2)$$

and

$$H_i(-x) = -p^+ \sum_n \delta(p^+[1-x] - p_n^+) |\langle n | \Psi^\dagger(0) | \mathbf{p}, s \rangle|^2 \quad (3.3)$$

An advantage of calculating the quark distributions using this form for H_i [ST88b,ST89b] rather than the original expression (equation 2.41) is that by explicitly extracting the delta function one guarantees vanishing support above $x = 1$ (note that both p^+ and p_n^+ are positive), something that was not the case in the calculations of Jaffe and Hughes [Jaf75, Hug77] or in those by Benesh and Miller [BM87,BM88a,BM88b]. In a fundamental field

theoretic description (i.e. where one can derive the initial and intermediate state masses) it should of course not matter whether one uses equation 2.41 or the expressions above..

In order to be able to calculate the H_i 's within the bag model we need to decide on the action of the quark field operator $\Psi(\mathbf{r}, t)$. We shall define it in the following manner:

$$\hat{\Psi}(\mathbf{r}, t) \equiv \sum_{\alpha} \hat{\Psi}_{\alpha}(\mathbf{r}, t) = \sum_{\alpha} b_{\alpha} \Psi_{\alpha}(\mathbf{r}, t) \quad (3.4)$$

where b_{α} creates (for negative energies) or destroys (for positive energies) a particle with quantum numbers α and $\Psi_{\alpha}(\mathbf{r}, t)$ is the corresponding spinor. We have marked the field operators by a " ^ " to distinguish them from the spinors. Let us write the wavefunction of a particle at position $\mathbf{r}'$ with quantum numbers β as $|\mathbf{r}'_{\beta}\rangle$. For positive energies we shall define

$$\hat{\Psi}_{\alpha}(\mathbf{r}, t)|\mathbf{r}'_{\beta}\rangle = \delta_{\alpha,\beta}\delta(\mathbf{r} - \mathbf{r}')|0\rangle \quad , \quad (3.5)$$

while for negative energies

$$\hat{\Psi}_{\alpha}(\mathbf{r}, t)|0\rangle = |\mathbf{r}_{\alpha}\rangle \quad . \quad (3.6)$$

The state $|0\rangle$ is not to be identified with free vacuum, but the bag vacuum. The operator $\hat{\Psi}_{\alpha}$ (negative energy) inserts a particle into this bag vacuum which satisfies the boundary conditions of the bag. This is not a disconnected process - it could not occur in free space - and should therefore be included in a calculation of quark distribution functions. We emphasize also that it is a twist two contribution, as it corresponds to operators of the type 2.38.

We see then that the quark distributions consist of a combination of four terms, two for $H_i(x)$

$$H_i(x) \equiv q_i^{(2)}(x) + q_i^{(4)}(x)$$

$$= p^+ \sum_{n,\alpha} \delta(p^+[1-x] - p_n^+) \left\{ |\langle n | \hat{\Psi}_{\Gamma,\alpha(+)}(\mathbf{0}) | \mathbf{p}, s \rangle|^2 + |\langle n | \hat{\Psi}_{\Gamma,\alpha(-)}(\mathbf{0}) | \mathbf{p}, s \rangle|^2 \right\} \quad (3.7)$$

and two for $H_i(-x)$

$$\begin{aligned} H_i(-x) &\equiv \bar{q}_i^{(2)}(x) + \bar{q}_i^{(4)}(x) \\ &= -p^+ \sum_{n,\alpha} \delta(p^+[1-x] - p_n^+) \left\{ |\langle n | \hat{\Psi}_{\Gamma,\alpha(-)}^\dagger(\mathbf{0}) | \mathbf{p}, s \rangle|^2 + |\langle n | \hat{\Psi}_{\Gamma,\alpha(+)}^\dagger(\mathbf{p}) | \mathbf{p}, s \rangle|^2 \right\} . \end{aligned} \quad (3.8)$$

Here $q_i^{(2)}(x)$ corresponds to a quark being destroyed by $\hat{\Psi}_{\Gamma,\alpha(+)}(\mathbf{0})$,
 $q_i^{(4)}(x)$ corresponds to an antiquark being created by $\hat{\Psi}_{\Gamma,\alpha(-)}(\mathbf{0})$,
 $\bar{q}_i^{(4)}(x)$ corresponds to a quark being created by $\hat{\Psi}_{\Gamma,\alpha(+)}^\dagger(\mathbf{0})$,
and $\bar{q}_i^{(2)}(x)$ corresponds to an antiquark being destroyed by $\hat{\Psi}_{\Gamma,\alpha(-)}^\dagger(\mathbf{0})$.

In order to evaluate these, let us first write the sum over all intermediate states as

$$\sum_n |n\rangle \langle n| = \sum_\nu \int \frac{d\mathbf{p}_n}{(2\pi)^3} |\mathbf{p}_n, \nu\rangle \langle \mathbf{p}_n, \nu| , \quad (3.9)$$

where ν refers to the spin-flavour quantum numbers. Let us concentrate on $q_i^{(2)}(x)$ and choose the target rest frame where $p^+ = M$ (It is convenient in this chapter to choose lightcone coordinates where the factor of $\sqrt{2}$ arising on page 13 is incorporated into the spatial coordinates).

We obtain

$$q_i^{(2)}(x) = \frac{M}{(2\pi)^3} \sum_{\nu,\alpha} \int d\mathbf{p}_n \delta(M[1-x] - p_n^+) |\langle \mathbf{p}_n, \nu | \hat{\Psi}_{\Gamma,\alpha(+)}(\mathbf{0}) | 0, \mu \rangle|^2 . \quad (3.10)$$

Here the spin-flavour dependence μ of the quark wavefunctions is explicitly shown.

Let us make use of equation 3.5 and equation 3.6 by converting to the coordinate representation:

$$q_i^{(2)}(x) = \frac{M}{(2\pi)^3} \sum_{\nu, \alpha} \int d\mathbf{p}_n \delta(M(1-x) - p_n^+) \quad (3.11)$$

$$| \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{y}_1 d\mathbf{y}_2 d\mathbf{y}_3 \langle \mathbf{p}_n; \nu | \mathbf{x}_1, \mathbf{x}_2; \nu \rangle$$

$$\langle \mathbf{x}_1, \mathbf{x}_2; \nu | \hat{\Psi}_{\Gamma, \alpha(+)}(\mathbf{0}) | \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3; \mu \rangle \cdot \langle \mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3; \mu | \mathbf{0}; \mu; s \rangle |^2 ,$$

where we have made use of the fact that the operator $\hat{\Psi}_{\Gamma, \alpha(+)}$ acting on a three particle state produces a two particle state.

The operator $\hat{\Psi}_{\Gamma, \alpha(+)}(\mathbf{0})$ may act on any one of the three quarks at positions y_1, y_2 , or y_3 . Let us always relabel so that it acts on the quark at y_3 . Similarly, we shall always reorder y_1 and y_2 to be able to write (note that the spatial part of the wavefunction is symmetric)

$$\langle \mathbf{x}_1, \mathbf{x}_2, | \mathbf{y}_1, \mathbf{y}_2 \rangle = \delta(\mathbf{x}_1 - \mathbf{y}_1) \delta(\mathbf{x}_2 - \mathbf{y}_2) . \quad (3.12)$$

We also need to decide on the action of the projection operators contained in $\hat{\Psi}_{\Gamma, \alpha(+)}(\mathbf{0})$. They act on the wavefunction of the quark at y_3 . The projection operators in spin and flavour space act on the third quark component of μ . It is clear that the sum over ν is a complete set and may therefore be taken out.

We obtain

$$q_i^{(2)}(x) = \frac{M}{(2\pi)^3} \sum_{\alpha} \langle \mu | P_{\Gamma, \alpha(+)} | \mu \rangle \cdot \int d\mathbf{p}_n \delta(M[1-x] - p_n^+) \quad (3.13)$$

$$\cdot | \int d\mathbf{x}_1 d\mathbf{x}_2 \Psi^\dagger(\mathbf{x}_1, \mathbf{x}_2; \mathbf{p}_n) \Psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{0}_\Gamma; \mathbf{0}) |^2 ,$$

where $P_{\Gamma, \alpha(+)}$ is the spin flavour projection contained in $\hat{\Psi}_{\Gamma, \alpha(+)}$ and $\mathbf{0}_\Gamma$ implies that the projection $\gamma^- \gamma^+$ contained in Γ (see equation 2.49) acts on the third quark. Equation 3.13 is general. In order to go further we need to choose a specific nucleon wavefunction, in

this case the Peierls-Yoccoz projected wavefunctions. Before we do this we digress to make a few comments about this projection.

3.3 The Peierls-Yoccoz Procedure

In order to be translationally invariant the wavefunction of a particle with momentum $\mathbf{k}$ made up of N constituents should be of the form

$$\Psi(\mathbf{x}_1, \dots, \mathbf{x}_N) = e^{i\mathbf{k} \cdot \mathbf{x}_{CM}} \Psi(\mathbf{y}_1, \dots, \mathbf{y}_{N-1}) \quad , \quad (3.14)$$

where

$$\mathbf{x}_{CM} = \frac{1}{N} \sum_i \mathbf{x}_i \quad (3.15)$$

and the coordinates $\mathbf{y}_i$ are internal coordinates, independent of the centre of mass variable $\mathbf{x}_{CM}$. It is clear that the operator that translates this wavefunction by a distance $\mathbf{a}$ is given by $e^{i\hat{\mathbf{p}} \cdot \mathbf{a}}$, where $\hat{\mathbf{p}}$ is the centre of mass momentum operator - $i\nabla_{\mathbf{x}_{CM}}$.

Consider now the N -particle wavefunction that is the solution to a Hamiltonian with potential (at rest) centred at $\mathbf{R}$:

$$\Psi(\mathbf{x}_1 - \mathbf{R}, \mathbf{x}_2 - \mathbf{R}, \dots, \mathbf{x}_N - \mathbf{R}) \quad . \quad (3.16)$$

This wavefunction is not of the form 3.14 with $\mathbf{k} = \mathbf{0}$. Not only is its internal wavefunction dependent on $\mathbf{x}_{CM}$, it also has an unphysical dependence on an artificial variable $\mathbf{R}$. This of course reflects the fact that we are solving for the wavefunctions in a potential, where the position of the potential is independent of the positions of the particles.

It was shown by Peierls and Yoccoz [PY57] that we may construct a modified wavefunction by Fourier transforming with respect to the extraneous variable $\mathbf{R}$:

$$\int d\mathbf{R} e^{i\mathbf{k} \cdot \mathbf{R}} \Psi(\mathbf{x}_1 - \mathbf{R}, \mathbf{x}_2 - \mathbf{R}, \dots, \mathbf{x}_N - \mathbf{R}) \quad , \quad (3.17)$$

In a sense we have picked out of the original wavefunction, constructed out of superpositions of plane waves in the variable $\mathbf{R}$, that component with momentum $\mathbf{k}$. If the original wavefunction was normalized, the modified one will not be, so we need to insert a normalization by hand:

$$\Psi_{P.Y.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}) = \frac{1}{\phi_N^{P.Y.}(\mathbf{k})} \int d\mathbf{R} e^{i\mathbf{k} \cdot \mathbf{R}} \Psi(\mathbf{x}_1 - \mathbf{R}, \mathbf{x}_2 - \mathbf{R}, \dots, \mathbf{x}_N - \mathbf{R}) \quad . \quad (3.18)$$

In order to make the physics of this wavefunction more explicit, let us shift the integration by $\mathbf{x}_1$ and define

$$\mathbf{y}_i = \mathbf{x}_i - \mathbf{x}_1 \quad i = 2, \dots, N \quad . \quad (3.19)$$

We obtain

$$\Psi_{P.Y.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}) = \frac{1}{\phi_N^{P.Y.}(\mathbf{k})} \int d\mathbf{R} e^{i\mathbf{k} \cdot (\mathbf{R} + \mathbf{x}_1)} \Psi(-\mathbf{R}, \mathbf{y}_2 - \mathbf{R}, \dots, \mathbf{y}_N - \mathbf{R}) \quad . \quad (3.20)$$

We shall define $\phi_N^{P.Y.}(\mathbf{k})$ to be given by the normalization

$$\int d\mathbf{x}_1, \dots, d\mathbf{x}_N \Psi_{P.Y.}^\dagger(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}') \Psi_{P.Y.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}) = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \quad , \quad (3.21)$$

i.e.

$$|\phi_N^{P.Y.}(\mathbf{k})|^2 = \int d\mathbf{R} e^{-i\mathbf{k} \cdot \mathbf{R}} \langle \mathbf{x}_1 - \mathbf{R}, \mathbf{x}_2 - \mathbf{R}, \dots, \mathbf{x}_N - \mathbf{R} | \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N \rangle \quad . \quad (3.22)$$

Finally note that

$$\mathbf{x}_1 = \mathbf{x}_{CM} - \frac{1}{N} \sum_{i=2}^N \mathbf{y}_i \quad , \quad (3.23)$$

hence

$$\Psi_{P.Y.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}) = \frac{e^{i\mathbf{k} \cdot \mathbf{x}_{CM}}}{\phi_N^{P.Y.}(\mathbf{k})} \int d\mathbf{R} e^{i\mathbf{k} \cdot (\mathbf{R} - \frac{1}{N} \sum_{i=2}^N \mathbf{y}_i)} \Psi(-\mathbf{R}, \mathbf{y}_2 - \mathbf{R}, \dots, \mathbf{y}_N - \mathbf{R}) \quad . \quad (3.24)$$

This is of the form $e^{i\mathbf{k} \cdot \mathbf{x}_{CM}} \Psi(\text{internal variables}; \mathbf{k})$, i.e. it is a momentum eigenstate, with eigenvalue $\mathbf{k}$, albeit with an unphysical dependence on the momentum in the internal wavefunction.

Even this may be removed by the use of the Peierls-Thouless projection [PT62]. Let us define a new state $\Psi_{P.T.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k})$ by a further projection

$$\Psi_{P.T.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}) = \frac{1}{(2\pi)^3 \phi_N^{P.T.}(\mathbf{k})} \int d\mathbf{k}' e^{i(\mathbf{k}-\mathbf{k}') \cdot \mathbf{x}_{CM}} f(\mathbf{k}, \mathbf{k}') \Psi_{P.Y.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}') \quad . \quad (3.25)$$

(Here the normalization of $\Psi_{P.Y.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k})$ is subsumed into $\phi_N^{P.T.}(\mathbf{k})$). If we choose $f(\mathbf{k}, \mathbf{k}') = 1$, the $\mathbf{k}'$ integration gives a $\delta(\mathbf{R} - \mathbf{x}_{CM})$, and so the Peierls-Thouless wavefunction becomes

$$\Psi_{P.T.}(\mathbf{x}_1, \dots, \mathbf{x}_N; \mathbf{k}) = \frac{1}{\phi_N^{P.T.}(\mathbf{k})} e^{i\mathbf{k} \cdot \mathbf{x}_{CM}} \Psi(\mathbf{x}_1 - \mathbf{x}_{CM}, \mathbf{x}_2 - \mathbf{x}_{CM}, \dots, \mathbf{x}_N - \mathbf{x}_{CM}) \quad , \quad (3.26)$$

where

$$|\phi_N^{P.T.}(\mathbf{k})|^2 = \langle \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{N-1}, g(\{\mathbf{x}_1, \dots, \mathbf{x}_{N-1}\}) | \mathbf{x}_1, \mathbf{x}_2, \dots, g(\{\mathbf{x}_1, \dots, \mathbf{x}_{N-1}\}) \rangle \quad . \quad (3.27)$$

This wavefunction is indeed of the form of equation 3.14 (g is a function of all but the last coordinate). It is physically very attractive - the centre of the potential is defined by the positions of the objects moving within the potential, as one might expect. Unfortunately, this is also its weakness. The mixing of the quark coordinates makes this wavefunction extremely difficult to manipulate. For this reason we shall use primarily the Peierls-Yoccoz wavefunction. It would however be nice to generalize this work to the Peierls-Thouless case.

Before we make use of the Peierls-Yoccoz wavefunction we should make one final comment. Both the Peierls-Yoccoz and Peierls-Thouless procedures are essentially non-relativistic. In fact, for a relativistic system the internal wavefunction *should* be dependent on the momentum because it is Lorentz contracted (there would also be a Wigner rotation to take into account). Furthermore, in an interacting theory like QCD there are other degrees of freedom (i.e. gluons) that influence the centre of energy of the system. We may therefore expect our approximation to be a reasonable one if the hadron state has low momentum. For the initial state this is certainly the case (it is at rest), and we shall assume it to be approximately true for the intermediate state also (as we shall see shortly it is a reasonable approximation for x less than about 0.7).

3.4 The Peierls-Yoccoz quark distributions

Let us now specialize to the Peierls-Yoccoz projected MIT bag wavefunctions of equation 3.18. We obtain

$$\begin{aligned} \int d\mathbf{x}_1 d\mathbf{x}_2 \Psi^\dagger(\mathbf{x}_1, \mathbf{x}_2; \mathbf{p}_n) \Psi(\mathbf{x}_1, \mathbf{x}_2, \mathbf{0}_\Gamma; \mathbf{0}) = \\ \frac{1}{\phi_2(\mathbf{p}_n)\phi_3(\mathbf{0})} \int d\mathbf{R}_1 d\mathbf{R}_2 d\mathbf{x}_1 d\mathbf{x}_2 e^{-i\mathbf{p}_n \cdot \mathbf{R}_1} \Psi^\dagger(\mathbf{x}_1 - \mathbf{R}_1) \cdot \Psi(\mathbf{x}_1 - \mathbf{R}_2) \\ \Psi^\dagger(\mathbf{x}_2 - \mathbf{R}_1) \cdot \Psi(\mathbf{x}_2 - \mathbf{R}_2) \Psi(-\mathbf{R}_2) \end{aligned} \quad (3.28)$$

Here $\Psi(\mathbf{r})$ is the MIT bag wavefunction. Changing variables to

$$\begin{aligned} \mathbf{x} &= -\mathbf{R}_2 \\ \mathbf{x}'_1 &= \mathbf{x}_1 + \mathbf{x} \\ \mathbf{x}'_2 &= \mathbf{x}_2 + \mathbf{x} \\ \text{and } \mathbf{R}' &= \mathbf{x} + \mathbf{R}_1 \end{aligned} \quad , \quad (3.29)$$

defining

$$\frac{\phi_2(\mathbf{p}_n)}{\phi_3(\mathbf{0})} \int d\mathbf{x} e^{i\mathbf{p}_n \cdot \mathbf{x}} \Psi_{\Gamma(+)}(\mathbf{x}) \equiv \frac{\phi_2(\mathbf{p}_n)}{\phi_3(\mathbf{0})} \tilde{\Psi}_{\Gamma(+)}(\mathbf{p}_n) , \quad (3.30)$$

equation 3.13 becomes

$$q_i^{(2)}(x) = \frac{M}{(2\pi)^3} \sum_{\alpha} \langle \mu | P_{\Gamma\alpha(+)} | \mu \rangle \cdot \int d\mathbf{p}_n \frac{|\phi_2(\mathbf{p}_n)|^2}{|\phi_3(\mathbf{0})|^2} \delta(M[1-x] - p_n^+) |\tilde{\Psi}_{\Gamma(+)}(\mathbf{p}_n)|^2 . \quad (3.31)$$

It is easiest to do this integration by choosing the magnitude of $\mathbf{p}_n$ and its transverse components $\mathbf{p}_n^\perp$ as integration variables, as well as using the δ -function to do the $\mathbf{p}_n^\perp$ integration. One obtains

$$\int d\mathbf{p}_n \delta(M[1-x] - p_n^+) = 2\pi \int_{\left| \frac{M^2(1-x)^2 - M_n^2}{2M(1-x)} \right|}^{\infty} p_n dp_n , \quad (3.32)$$

where

$$p_\perp^2 = 2M(1-x) \sqrt{M_n^2 + p_n^2} - M^2(1-x)^2 - M_n^2 \quad (3.33)$$

is understood (here M_n is the mass of the intermediate 2 particle state).

Equation 3.31 therefore becomes

$$q_i^{(2)}(x) = \frac{M}{(2\pi)^2} \sum_{\alpha} \langle \mu | P_{\Gamma,\alpha(+)} | \mu \rangle \cdot \int_{\left| \frac{M^2(1-x)^2 - M_n^2}{2M(1-x)} \right|}^{\infty} p_n dp_n \frac{|\phi_2(\mathbf{p}_n)|^2}{|\phi_3(\mathbf{0})|^2} |\Psi_{\Gamma}(\mathbf{p}_n)|^2 . \quad (3.34)$$

It is easy to see from this integral when one might expect the non-relativistic approximation for the intermediate state to be a reasonable one. The main contribution to the integral comes from the region of small p_n so the non-relativistic approximation should be valid as long as the lower limit of the p_n integration is less than M_n - this is true for x less than about 0.7 for the intermediate state masses that we will use (more about the choice of masses later).

The evaluation of the normalizations $\phi_2(\mathbf{p}_n)$ and $\phi_3(\mathbf{0})$, as well as $\tilde{\Psi}_{\Gamma}(\mathbf{p}_n)$, is straightforward but tedious. Using the MIT bag wavefunctions with lowest energy

$$\Psi(\mathbf{x}) = N \begin{pmatrix} j_0(\frac{\Omega|\mathbf{x}|}{R}) \\ i\boldsymbol{\sigma} \cdot \hat{\mathbf{x}} j_1(\frac{\Omega|\mathbf{x}|}{R}) \end{pmatrix} \Theta(R - |\mathbf{x}|) \quad (3.35)$$

where R is the bag radius,

$$N^2 = \frac{1}{4\pi R^3} \frac{\Omega^4}{\Omega^2 - \sin^2 \Omega} \quad (3.36)$$

and Ω is the lowest energy solution of

$$\begin{aligned} j_0(\Omega) &= j_1(\Omega) \\ \text{i.e. } \Omega &= 2.04 \quad , \end{aligned} \quad (3.37)$$

one obtains that

$$|\phi_n(\mathbf{k})|^2 = \int d\mathbf{x} e^{-i\mathbf{k} \cdot \mathbf{x}} \left(\int d\mathbf{y} \Psi^\dagger(\mathbf{y} - \mathbf{x}) \Psi(\mathbf{y}) \right)^n, \quad (3.38)$$

which gives

$$|\phi_2(\mathbf{p}_n)|^2 = \frac{4\pi R^3}{u(\Omega^2 - \sin^2 \Omega)^2} \int_0^\Omega \frac{dv}{v} \sin \frac{2uv}{\Omega} T^2(v) \quad (3.39)$$

and

$$|\phi_3(0)|^2 = \frac{4\pi R^3}{u(\Omega^2 - \sin^2 \Omega)^3} \int_0^\Omega \frac{dv}{v} T^3(v) \quad , \quad (3.40)$$

where

$$\begin{aligned} T(v) &= \left(\Omega - \frac{\sin^2 \Omega}{\Omega} - v \right) \sin 2v - \left(\frac{1}{2} + \frac{\sin 2\Omega}{2\Omega} \right) \cos 2v \\ &+ \frac{1}{2} + \frac{\sin 2\Omega}{2\Omega} - \frac{\sin^2 \Omega}{\Omega^2} v^2 \end{aligned} \quad (3.41)$$

and we have made the substitutions

$$v = \frac{|\mathbf{x}|\Omega}{2R}, \quad u = |\mathbf{p}_n|R \quad (3.42)$$

Let us now become explicit about the projection Γ that was introduced in equation 2.49.

Let us define

$$\Psi_{+m}^{\uparrow\downarrow}(\mathbf{x}) = \frac{1}{2}\gamma^-\gamma^+ \cdot \frac{1}{2}(1 \pm \gamma^5) \cdot \Psi_m(\mathbf{x}) \quad (3.43)$$

so that

$$\Psi^\dagger(\xi^-)\Gamma\Psi(0) = 4 \sum_m \left(\Psi_{+m}^{\uparrow\downarrow}(\xi^-)\Psi_{+m}^\dagger(0) - \Psi_{+m}^{\uparrow\downarrow}(\xi^-)\Psi_{+m}^\dagger(0) \right) \quad (3.44)$$

Here $\uparrow\downarrow$ refers to the two helicity projections $\frac{1}{2}(1 \pm \gamma^5)$, $+$ signifies the projection onto good components $\frac{1}{2}\gamma^-\gamma^+$ and $m = \pm\frac{1}{2}$ indicates the orientation of the z component of total angular momentum of the quarks, i.e. $\Psi_m(\mathbf{x})$ are eigenfunctions of $\hat{j}_z$. $\Psi_{+m}^{\uparrow\downarrow}(\mathbf{x})$ however are not eigenstates of $\hat{j}_z$, but of helicity; i.e. spin-dependent deep inelastic scattering probes the helicity structure (not the spin structure) of the quark distributions in a target with definite spin.

The reader may well ask why we go to all the trouble to quantize $\Psi(\mathbf{x})$ in a basis diagonal in j_z , only to project out components of these into another basis. We do this because the helicity eigenfunctions themselves cannot possibly be solutions of the Dirac equation with the MIT bag boundary conditions. The physical reason for this is closely linked to the chiral symmetry breaking of the MIT Lagrangian - we shall explore some consequences of a (partially) chirally symmetric Lagrangian in Chapter 5. Mathematically it is also easy to see: the non-linear boundary condition of the MIT bag is given by $-\mathbf{r} \cdot \nabla \bar{\Psi}(\mathbf{r})\Psi(\mathbf{r})|_{r=R} = 2B$. If the Ψ 's are eigenstates of helicity this is impossible to fulfill as $\Psi^\dagger(\mathbf{r})(1 \pm \gamma^5)\gamma^0(1 \pm \gamma^5)\Psi(\mathbf{r})$ is zero for all $\mathbf{r}$.

A second reason why a spin-basis (or rather, total angular momentum-basis) is more convenient is that the target has definite spin s . The spin-flavour wavefunction $|\mu\rangle$ is therefore best expressed in terms of j_z , such that the $j_z^{(i)}$ of the three quarks (i) sums to

the total spin s . As we shall see, this distinction between helicity and j_z leads to some non-trivial effects.

We may use equation 3.43 to evaluate $|\Psi_\Gamma(\mathbf{p}_n)|^2$ and obtain

$$|\Psi_m^{\uparrow\downarrow}(\mathbf{p}_n)|^2 = \frac{1}{2} \left\{ f(\mathbf{p}_n) \pm (-1)^{m+\frac{3}{2}} g(\mathbf{p}_n) \right\} \quad (3.45)$$

where $+$ and $-$ refer to the two helicity projections,

$$f(\mathbf{p}_n) = \frac{\pi R^3}{2} \frac{1}{(\Omega^2 - \sin^2 \Omega)} \left\{ s_1^2(u) + 2 \frac{p_n^3}{|\mathbf{p}_n|} s_1(u) s_2(u) + s_2^2(u) \right\} \quad (3.46)$$

and

$$g(\mathbf{p}_n) = \frac{\pi R^3}{2} \frac{1}{(\Omega^2 - \sin^2 \Omega)} \left\{ s_1^2(u) + 2 \frac{p_n^3}{|\mathbf{p}_n|} s_1(u) s_2(u) + \left(1 - 2 \left(\frac{p_n^\perp}{|\mathbf{p}_n|} \right)^2 \right) s_2^2(u) \right\} \quad (3.47)$$

Here u is again $|\mathbf{p}_n|R$,

$$s_1(u) = \frac{1}{u} \left\{ \frac{\sin(u - \Omega)}{u - \Omega} - \frac{\sin(u + \Omega)}{u + \Omega} \right\} \quad (3.48)$$

and

$$s_2(u) = 2j_0(\Omega)j_1(u) - \frac{u}{\Omega} s_1(u) \quad (3.49)$$

Let us now concentrate on a proton target with spin up. Using

$$\begin{aligned} \langle \mu | P_{u, \frac{1}{2}} | \mu \rangle &= \frac{5}{3} \\ \langle \mu | P_{u, -\frac{1}{2}} | \mu \rangle &= \frac{1}{3} \\ \langle \mu | P_{d, \frac{1}{2}} | \mu \rangle &= \frac{1}{3} \\ \langle \mu | P_{d, -\frac{1}{2}} | \mu \rangle &= \frac{2}{3} \end{aligned} \quad (3.50)$$

for the $SU(6)$ $\underline{56}$ -plet spin-flavour wavefunction for the u and d quarks, we obtain for the specific quark distributions corresponding to equation 3.34

$$\begin{aligned}
u_{(2)}^{\uparrow\downarrow}(x) &= F_{(2)}(x) \pm \frac{2}{3}G_{(2)}(x) \\
&\text{and} \\
d_{(2)}^{\uparrow\downarrow}(x) &= \frac{1}{2}F_{(2)}(x) \mp \frac{1}{6}G_{(2)}(x) \quad , \quad (3.51)
\end{aligned}$$

where

$$F_{(2)}(x) = \frac{\rho}{4\pi} \int_{\frac{\rho^2(1-x)^2 - \epsilon^2}{2\rho(1-x)}}^{\infty} du \frac{\int_0^{\Omega} \frac{dv}{v} \sin \frac{2uv}{\Omega} T^2(v)}{\int_0^{\Omega} \frac{dv}{v} T^3(v)} \left\{ s_1^2(u) + 2\frac{u_3}{u} s_1(u) s_2(u) + s_2^2(u) \right\} \quad (3.52)$$

and

$$\begin{aligned}
G_{(2)}(x) &= \frac{\rho}{4\pi} \int_{\frac{\rho^2(1-x)^2 - \epsilon^2}{2\rho(1-x)}}^{\infty} du \frac{\int_0^{\Omega} \frac{dv}{v} \sin \frac{2uv}{\Omega} T^2(v)}{\int_0^{\Omega} \frac{dv}{v} T^3(v)} \\
&\quad \left\{ s_1^2(u) + 2\frac{u_3}{u} s_1(u) s_2(u) + \left[1 - 2\left(\frac{u_{\perp}}{u}\right)^2 \right] s_2^2(u) \right\} \quad , \quad (3.53)
\end{aligned}$$

with $p = MR$, $\epsilon = M_n R$, $u_3 = p_3 R$ and $u_{\perp} = p_{\perp} R$, and we have used the fact that the integral is odd under change of sign of u to simplify the lower limit of the integral.

3.5 Properties of the distribution functions $q_i^{(2)}(x)$

Although we shall shortly present some numerical results of the formalism developed in the last section, let us first make some general observations.

It is relatively straightforward to see where the peak of the distributions will occur. To do this, let us rewrite equation 3.34 as

$$q_i^{(2)}(x) \sim \int_{-\infty}^{\infty} p_n dp_n \Theta(p_n - l(x)) |I(p_n)|^2 \quad , \quad (3.54)$$

where

$$l(x) = \frac{M^2(1-x)^2 - M_n^2}{2M(1-x)} \quad . \quad (3.55)$$

$q_i^{(2)}(x)$ is a maximum when its derivative with respect to x is 0. Noting that

$$\frac{d}{dx}\Theta(p_n - l(x)) = \pm \left(\frac{M}{2} - \frac{M_n^2}{2M(1-x)^2} \right) \delta(p_n - l(x)) \quad (3.56)$$

and using the δ -function to perform the p_n integration we obtain

$$\frac{d}{dx}q_i^{(2)}(x) \sim \pm l(x) |I(l(x))|^2 \left(\frac{M}{2} - \frac{M_n^2}{2M(1-x)^2} \right) , \quad (3.57)$$

which is zero when

$$x = 1 - \frac{M_n}{M} , \quad (3.58)$$

i.e. for intermediate states with mass less than the target mass the distribution peaks in the physical region. In fact, if we neglect any binding energy we would expect $\frac{M_n}{2} = \frac{M}{3}$ which would yield a maximum at $x = \frac{1}{3}$, i.e. all quarks would carry one third of the proton's momentum in the infinite momentum frame. There are, in fact, further observations we can make about the limit in which the binding energy becomes negligible. We expect this to be physically similar to taking the bag radius to infinity. This corresponds, in equations 3.52 and 3.53, to only evaluating the integral for very large u , unless $x = 1 - \frac{M_N}{M}$. When $u \rightarrow \infty$ the integrals for $F_{(2)}(x)$ and $G_{(2)}(x)$ however vanish at least like u^{-5} . Hence the distributions vanish in the limit, except possibly at $x = 1 - \frac{M_N}{M}$.

Let us prove that in fact the distributions cannot vanish at $x = 1 - \frac{M_N}{M}$ itself. We do this by calculating the integral over all x of $F_{(2)}(x)$ and $G_{(2)}(x)$. It is easiest to do this by going back to equation 3.31 and "undoing" the Fourier transforms. The integration over x (from $-\infty$ to ∞ , or equivalently from $-\infty$ to 1) gets rid of the δ -function. The momentum integral give a δ -function which can be used to do one of the integrals of the Fourier transforms, thus giving, for $F_{(2)}(x)$,

$$\int_{-\infty}^1 F_{(2)}(x) dx = \frac{4}{\int \{I_T(\mathbf{x}) + I_B(\mathbf{x})\}^3 d\mathbf{x}} \int d\mathbf{x} (I_T(\mathbf{x}) + I_B(\mathbf{x}))^2$$

$$\int dy \Psi_f^\dagger(y - x) \Psi_f(y) \quad , \quad (3.59)$$

where $I_T(x)$ and $I_B(x)$ are given by

$$I_{T,B}(x) = \int dy \Psi_{T,B}^\dagger(y - x) \Psi_{T,B}(y) \quad . \quad (3.60)$$

Here $\Psi_{T,B}$ refers to the top and bottom parts of the spinors respectively. Ψ_f indicates those parts of Ψ_f that contribute to f (see equation 3.46), and an analogous expression holds for the integral of $G_{(2)}(x)$. It is now relatively straightforward to show that

$$\int_{-\infty}^1 F_{(2)}(x) dx = 1 \quad (3.61)$$

and

$$\begin{aligned} \int_{-\infty}^1 G_{(2)}(x) dx &= \frac{\int \{I_T(x) + I_B(x)\}^2 \{I_T(x) - \frac{1}{3}I_B(x)\} dx}{\int \{I_T(x) + I_B(x)\}^3 dx} \\ &\simeq 0.789 \text{ for } \Omega = 2.04 \quad . \end{aligned} \quad (3.62)$$

This is an important result for two reasons. Firstly, these normalizations are independent of the bag radius R (in fact, they depend only on Ω); hence they provide proof that the distribution cannot vanish at $x = 1 - \frac{M_N}{M}$, even for large R . We therefore have the result that as we increase the bag radius the quark distributions become narrower, tending towards a δ -function at $x = 1 - \frac{M_N}{M}$ in the limit that R goes to infinity, i.e. in the limit of zero binding energy the three quarks carry equal fractions of the nucleons momentum (in the infinite momentum frame).

Secondly, it is clear from the form of $F_{(2)}(x)$ and $G_{(2)}(x)$ that they are non-zero for $x < 0$, the implication being that the integral $\int_0^1 F_{(2)}(x) dx$ does not saturate the normalization requirement. This is of course not unexpected - it reminds us, as we saw in equations 3.7 and 3.8, that we are at the moment only calculating one of the four possible contributions to the structure functions. The total contribution from the other

three terms should be equal to the contribution of $F_{(2)}(x)$ for $x < 0$. We shall calculate them in the next section.

Before we do this however, let us present some numerical results. We show, for various bag radii, the functions $F_{(2)}(x)$ and $G_{(2)}(x)$ in Figures 3.1 and 3.2, and their ratio in Figure 3.3 (The two extreme values $R=0.2$ fm and $R=2$ fm are included to highlight the trend. They are of course not expected to reflect reality). Note that had we taken the wavefunctions $|\mu\rangle$ to be corresponding to helicity rather than spin we would have obtained

$$u_{(2)}^{\uparrow\downarrow}(x) \sim d_{(2)}^{\uparrow\downarrow}(x) \sim F_{(2)}(x) \quad , \quad (3.63)$$

which would correspond to $F_{(2)}(x)/G_{(2)}(x)=1$. The complete description requires the spin dependent structure functions to have a different x -dependence to the spin independent ones, the difference being of the order of 20% at small x for radii around 0.8 Fermi.

The oscillations at large x , especially noticeable for the ratio $\frac{G_{(2)}(x)}{F_{(2)}(x)}$ are presumably due to the sharp boundary imposed by bag surface, i.e. they probably have the same origin as the oscillation observed by Signal and Thomas in their two dimensional version of the model [ST87a].

For the mass of the intermediate state we have used $M_N = \frac{3}{4}M$ in Figures 3.1, 3.2 and 3.3. This estimate is based on the bag virial theorem which tells us that the bag boundary on average carries the same energy as each of the quarks confined by the bag. This is at best an approximation - after all, in QCD the two quark state has net colour and should therefore be infinitely heavy. We shall justify it by noting that the same processes that give the coloured states large mass are also responsible for the fragmentation of the target and are therefore part of the final state interactions. These are part of the nonperturbative processes that are summed over in inclusive interactions and should therefore not be included in the above perturbative treatment. Equivalently one might argue that the time scale of the forward virtual compton scattering is much shorter than the typical time scales involved in the re-adjustment of the bag boundary.

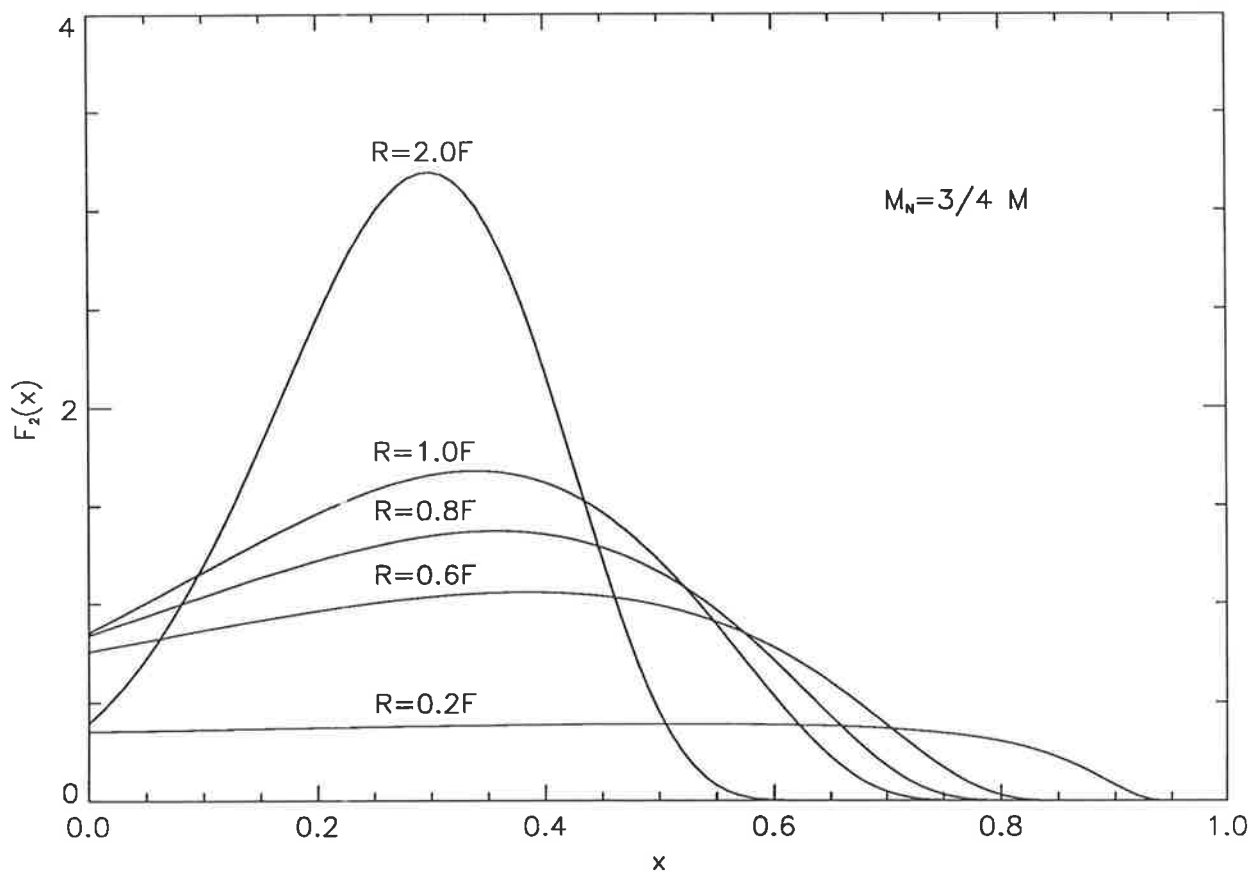


Figure 3.1: $F_2(x)$ as a function of bag radius

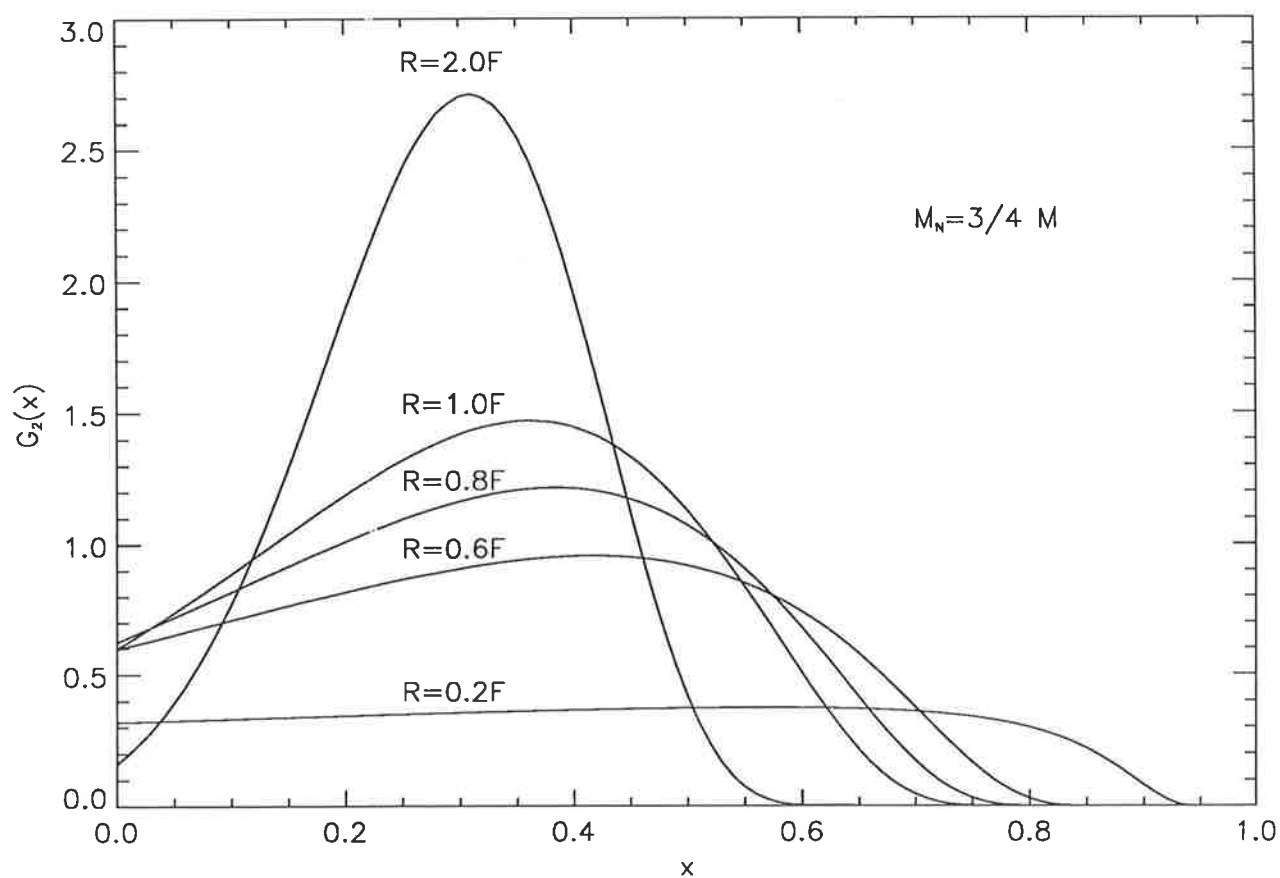


Figure 3.2: $G_2(x)$ as a function of bag radius

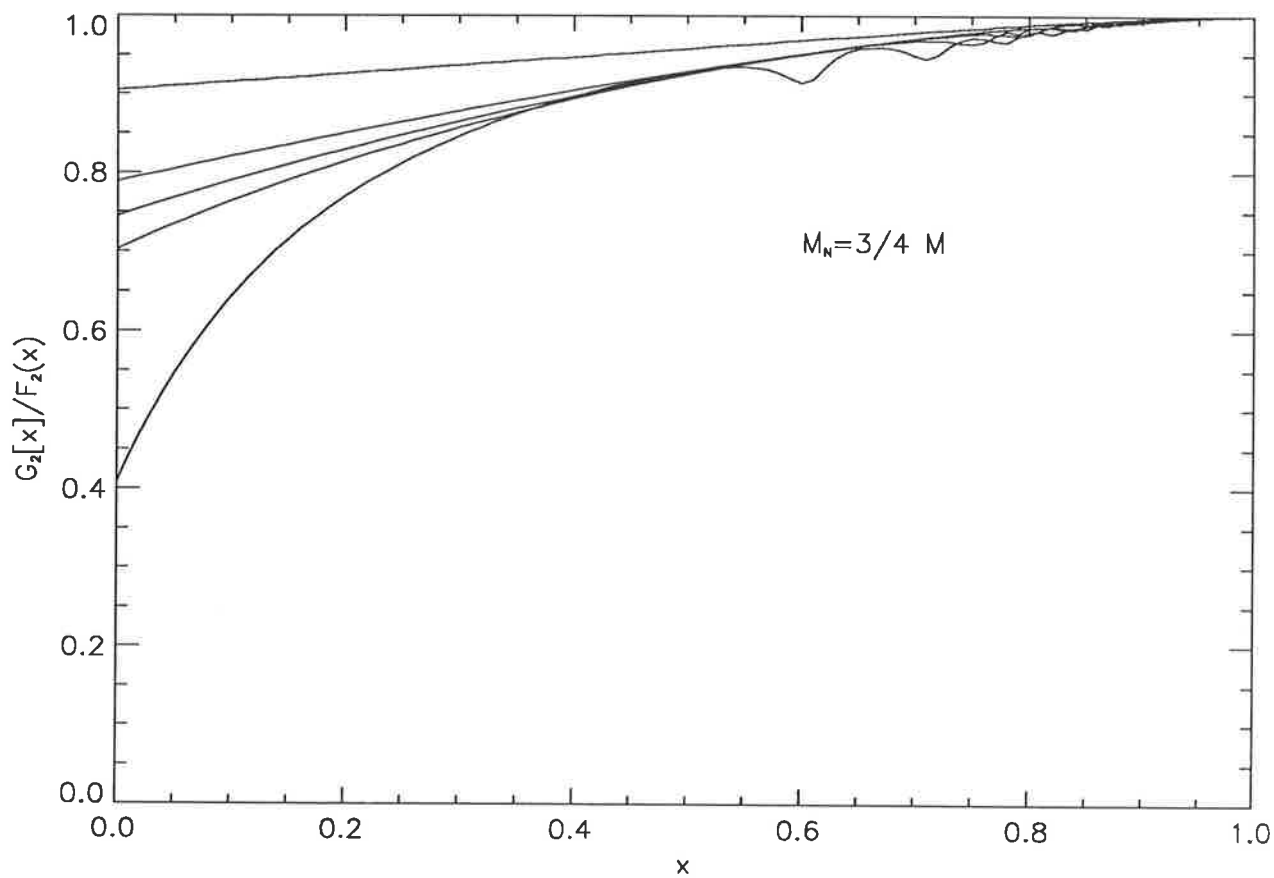


Figure 3.3: $\frac{G_2(x)}{F_2(x)}$ as a function of bag radius R . The 5 curves correspond to (from the top) $R=0.2, 0.6, 0.8, 1.0$ and $2.0 F$

3.6 $q_i^{(4)}(x)$ and $\bar{q}_i^{(4)}(x)$

In the work presented in this thesis we shall only consider initial states that contain no antiquarks, and so $\bar{q}_i^{(2)}(x)$ does not give a contribution. For $q_i^{(4)}(x)$ and $\bar{q}_i^{(4)}(x)$ we may follow a similar procedure as in the previous sections and obtain

$$q_{i(-)}^{(4)}(x) = \frac{M}{(2\pi)^3} \sum_{\alpha} \langle \mu | P_{\Gamma(-), \alpha} | \mu \rangle \int d\mathbf{p}_n \delta(M[1-x] - p_n^+) \frac{|\phi_4(\mathbf{p}_n)|^2}{|\phi_3(\mathbf{0})|^2} |\Psi_{\Gamma(-)}(\mathbf{p})|^2 \quad (3.64)$$

and

$$q_{i(+)}^{(4)}(x) = \frac{M}{(2\pi)^3} \sum_{\alpha} \langle \mu | P_{\Gamma(+), \alpha} | \mu \rangle \int d\mathbf{p}_n \delta(M[1-x] - p_n^+) \frac{|\phi_4(\mathbf{p}_n)|^2}{|\phi_3(\mathbf{0})|^2} |\Psi_{\Gamma(+)}^{\dagger}(\mathbf{p})|^2 \quad (3.65)$$

(see equation 3.31). Here the antiquark spinor needed for $\Psi_{\Gamma(-)}(\mathbf{p}_n)$ is given by

$$\Psi_{\Gamma(-)}(\mathbf{x}) = N \begin{pmatrix} -i\sigma \cdot \hat{\mathbf{r}} j_1(\frac{\Omega r}{R}) \\ j_0(\frac{\Omega r}{R}) \end{pmatrix} \Theta(R-r) \quad (3.66)$$

Performing the same manipulation as before we find that

$$|\Psi_{\Gamma(-)}^{\dagger}(\mathbf{p}_n)|^2 = \frac{1}{2} \left\{ f(-\mathbf{p}_n) \pm (-1)^{m+\frac{3}{2}} g(-\mathbf{p}_n) \right\} \quad (3.67)$$

where f and g are defined in equations 3.46 and 3.47 respectively. Indeed, the same result is true for $q_{i(+)}^{(4)}(x)$ because

$$|\Psi_{\Gamma(+)}^{\dagger}(\mathbf{p}_n)|^2 = |\Psi_{\Gamma(+)}(-\mathbf{p}_n)|^2 \quad (3.68)$$

Let us suppose now that the quark or antiquark is inserted into the n th excited state. If there are no quarks (or antiquarks) in this state to start off with then the spin-flavour matrix elements appearing in equations 3.64 and 3.65 are the same and so their contributions cancel in 3.51.

If one inserted the (anti)quark into the groundstate this argument is not valid because of Pauli blocking. There are 12 possible ways to insert coloured antiquarks into the

groundstate, while there are only 9 possible ways to insert quarks because 3 states are already occupied.

In fact, the various spin-flavour matrix elements for the quarks are

$$\begin{aligned}
\langle \mu | P_{u,+\frac{1}{2}} | \mu \rangle &= \frac{4}{3} \\
\langle \mu | P_{u,-\frac{1}{2}} | \mu \rangle &= \frac{8}{3} \\
\langle \mu | P_{d,+\frac{1}{2}} | \mu \rangle &= \frac{8}{3} \\
\langle \mu | P_{d,-\frac{1}{2}} | \mu \rangle &= \frac{7}{3} \quad ,
\end{aligned} \tag{3.69}$$

using the SU(6) wavefunction. We therefore get the following contributions from the four particle intermediate states:

$$\begin{aligned}
u_{(4)}^{\uparrow\downarrow}(x) &= d_{(4)}^{\uparrow\downarrow}(x) = 3F_{(4)}(x) \\
&\text{and} \\
\bar{u}_{(4)}^{\uparrow\downarrow}(x) &= 2F_{(4)}(x) \mp \frac{2}{3}G_{(4)}(x) \\
\bar{d}_{(4)}^{\uparrow\downarrow}(x) &= \frac{5}{2}F_{(4)}(x) \pm \frac{1}{6}G_{(4)}(x) \quad .
\end{aligned} \tag{3.70}$$

We plot in Figures 3.4 and 3.5 the contribution equivalent to Figures 3.1 and 3.2. Again we have chosen the intermediate state quark masses to be given by the virial theorem, i.e. $M_N = \frac{5}{4}M$.

3.7 Normalization

The quark distributions obey important normalization restrictions. The number of valence up (down) quarks is 2 (1). In our notation this means

$$\begin{aligned}
\frac{1}{2} \int_0^1 dx \{ u_{(2)}(x) + u_{(4)}(x) - \bar{u}_{(4)}(x) \} &= \int_0^1 dx \{ d_{(2)}(x) + d_{(4)}(x) - \bar{d}_{(4)}(x) \} \\
&= \int_0^1 dx \{ F_{(2)}(x) + F_{(4)}(x) \} \quad .
\end{aligned} \tag{3.71}$$

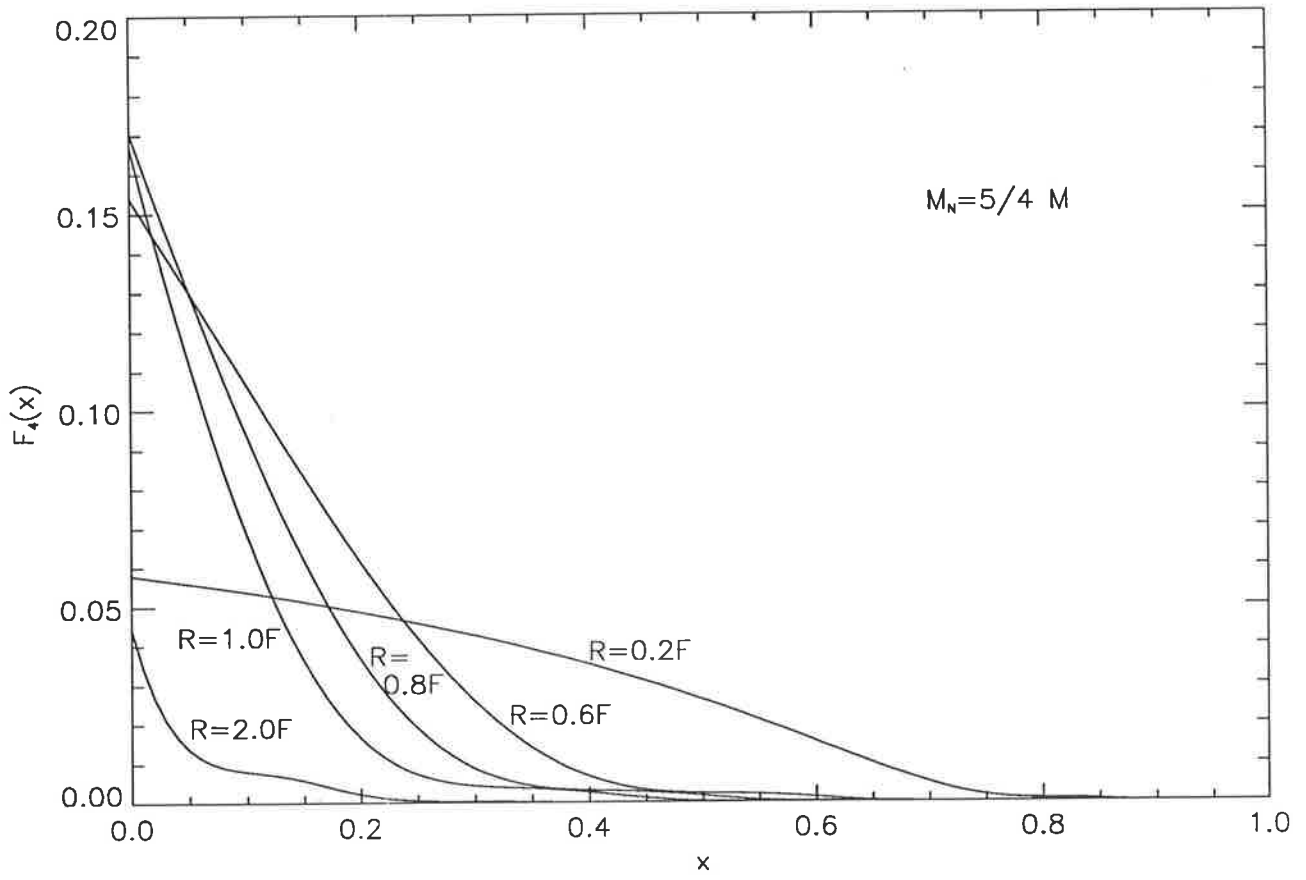


Figure 3.4: $F_4(x)$ as a function of bag radius

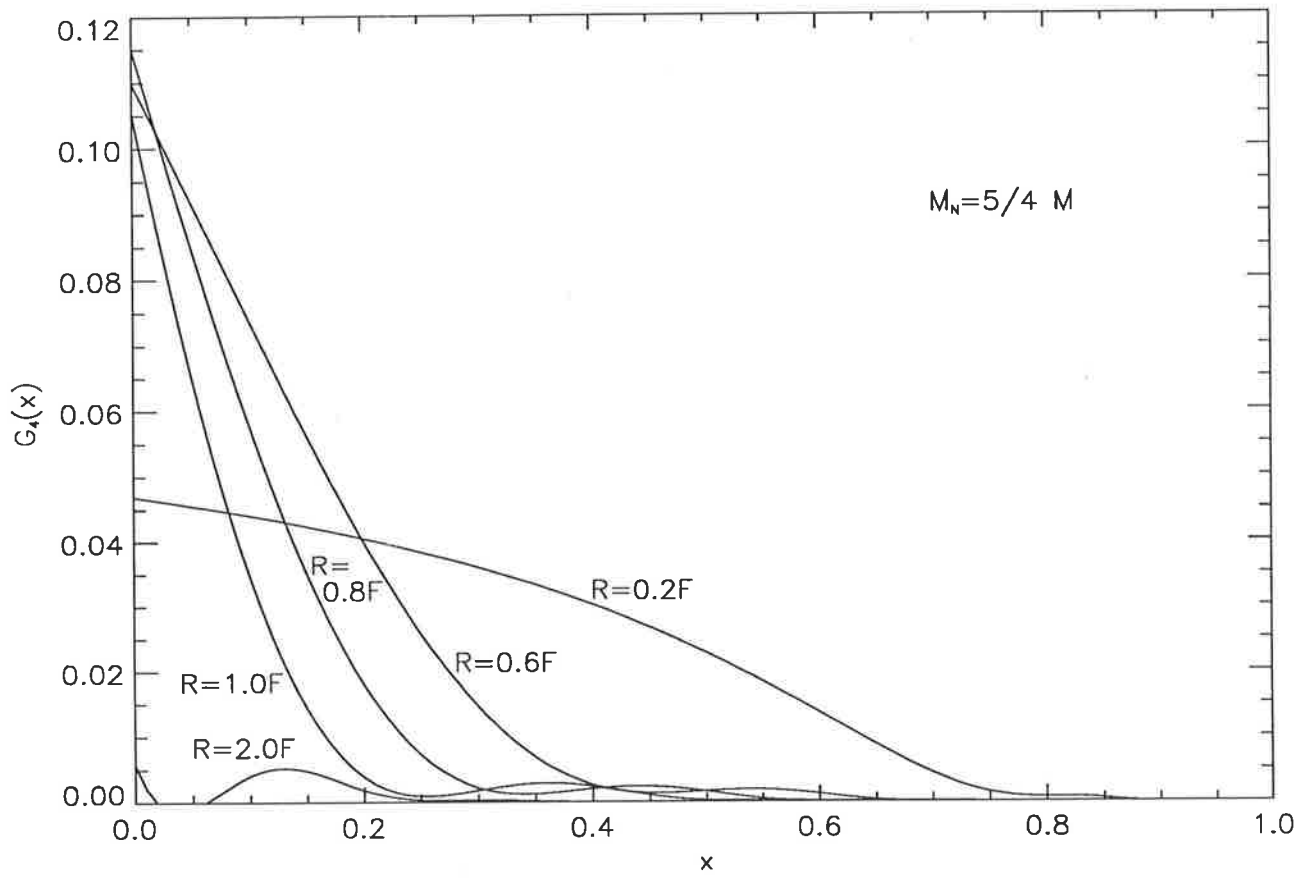


Figure 3.5: $G_4(x)$ as a function of bag radius

Table 3.1: The normalizations of $F_{(2)}$ and $F_{(4)}$

R (fm)	$\int_0^1 dx F_{(2)}(x)$	$\int_0^1 dx F_{(4)}(x)$	$\int_0^1 dx \{F_{(2)}(x) + F_{(4)}(x)\}$
0.2	0.32	0.026	0.35
0.6	0.65	0.029	0.68
0.8	0.75	0.023	0.77
1.0	0.82	0.018	0.84
2.0	0.98	0.003	0.98

The numerical values for the various bag radii are tabulated in Table 3.1.

We see that for reasonable nucleon radii of $\sim 0.8F$ the normalization consists of about 75% from the two quark intermediate state and 2% from the four quark (anti-quark) states. About 23% is missing.

It is difficult to isolate the origin of this normalization problem, in fact, it is not even clear that it is a problem. One could, for example, adopt the attitude that our choice for the masses of the intermediate states was only a rough estimate of the true value given by a consistent solvable interacting field theory.

On the other hand one might argue that there are additional contributions from intermediate 2 quark states that are not in the groundstate. A quick calculation will indeed reveal that the Peierls-Yoccoz wavefunctions (3.24) are not orthogonal so that one might expect contributions from the overlap of the groundstate quarks in the initial state and the intermediate diquark in, say, the first excited state.

This argument is however erroneous and it is easy to show why. The normalization of the n -particle state is given by 3.21

$$\langle n, 0, \mathbf{k}' | n, 0, \mathbf{k} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') \quad (3.72)$$

(The 0 signifies groundstate quantum numbers).

Let us insert a complete set of n particle states

$$1 = \int \frac{d\mathbf{p}}{(2\pi)^3} \sum_{\alpha} c_{\alpha} |n, \alpha, \mathbf{p}\rangle \langle n, \alpha, \mathbf{p}| \quad , \quad (3.73)$$

where c_{α} is the strength of the contribution from the state with quantum numbers α .

We obtain

$$(2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') = c_0 (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') + \int \frac{d\mathbf{p}}{(2\pi)^3} \sum_{\alpha \neq 0} c_{\alpha} \langle n, 0, \mathbf{k}' | n, \alpha, \mathbf{p} \rangle \cdot \langle n, \alpha, \mathbf{p} | n, 0, \mathbf{k} \rangle \quad . \quad (3.74)$$

We see that our previous choice

$$\begin{aligned} c_{\alpha} &= 1 \quad \alpha = 0 \\ &= 0 \quad \alpha \neq 0 \end{aligned} \quad (3.75)$$

is a perfectly valid one, although of course not the only one. (Note however that, for example, the choice $c_{\alpha} = 1$ for all α is not a valid one.)

This highlights a problem with the Peierls-Yoccoz wavefunctions. Although the original MIT bag wavefunctions were a complete set of orthonormal eigenstates of a known Hamiltonian, the projection has created a set of states that are not orthonormal and we do not now have a Hamiltonian that would allow us to re-orthogonalize them. *Our choice of intermediate state therefore emerges as a choice based on physical prejudice, nothing more, that has its origin in the parton model.* i.e. we have arranged things in such a way that the incoming probe interacts only with one quark, leaving the other two in their groundstate.

If we did not have this prejudice we would have been free to construct a different intermediate state, still satisfying equation 3.74, that would reflect the changed physics. We would then need to compare our results to data to determine whether our physical ideas were valid. As we shall see in Chapter 6, our original choice 3.75 is able to fit the data very well. We take this as evidence that 3.75 has the some basis in reality.

3.8 Improvements of the model

Let us now investigate the modifications one expects for the structure functions if we include some known physical effects that we have neglected up to now. We shall look at two: one gluon exchange and the inclusion of a pion cloud.

3.8.1 One gluon exchange

If the spin-flavour wavefunction of the baryons were indeed that predicted by SU(6) symmetry, all the groundstate (i.e. the octet and decuplet) baryons would be degenerate in mass. It is of course well recognized that the strange quark mass breaks the symmetry, leading to a larger mass for all strange baryons when compared to the non-strange ones. This however does not explain the mass splitting between baryons that contain only non-strange quarks, i.e. the Δ and the nucleon.

One possible explanation of the $\Delta - N$ mass difference is one gluon exchange. In QED the magnetic part of 1 photon exchange leads to the so-called hyperfine splitting between, for example, a hydrogen atom in a spin 1 state and a spin 0 state and is proportional to $\langle \sigma_1 \cdot \sigma_2 \rangle$. This process has a direct analogy in QCD, of course with some changes in detail associated with the change from U(1) to SU(3) [DGG75]. The two quark interaction remains proportional to $\langle \sigma_1 \cdot \sigma_2 \rangle$. The effect has been calculated with the MIT bag wavefunction by DeGrand et al. [DJJK75]. The mass splitting also has a consequence on the spin structure functions. This has been worked out, in a simplified version of the model that is presented here, by Close and Thomas [CT88]. (In fact, reference [CT88] assumes that the spin dependent quark distributions are of the form of equation 3.63 rather than equation 3.61.) The essential point is that the mass of the intermediate diquark state will be dependent on whether it is in its singlet or triplet spin state.

Let us make these ideas more quantitative by writing the nucleon and delta masses as

$$M_{N,\Delta} = M^{(0)} + \sum_{i \neq j} \langle B | \sigma_i \cdot \sigma_j | B \rangle \Delta M \quad , \quad (3.76)$$

where B stands for the $J^P = \frac{1}{2}^+$ nucleon state or the $J^P = \frac{3}{2}^+$ delta states. Using

$$\langle B | \sum_{i \neq j} \sigma_i \cdot \sigma_j | B \rangle = \frac{1}{2} \langle B | (\sigma_1 + \sigma_2 + \sigma_3)^2 - \sigma_1^2 - \sigma_2^2 - \sigma_3^2 | B \rangle \quad , \quad (3.77)$$

we obtain

$$M_{\Delta} - M_N = \frac{3}{2} \Delta M \quad (3.78)$$

for the $\Delta - N$ mass difference. This sets the scale of the mass of the intermediate states $|D\rangle$, which are given by

$$\begin{aligned} M_2 &= M_2^{(0)} + \langle D | \sigma_1 \cdot \sigma_2 | D \rangle \Delta M \\ \text{i.e. } M_2(s=1) &= M_2^{(0)} + \frac{1}{4} \Delta M \\ \text{and } M_2(s=0) &= M_2^{(0)} - \frac{3}{4} \Delta M \end{aligned} \quad (3.79)$$

(As before we assume quark number 3 is the one that has interacted with the photon. We shall also totally ignore, for simplicity, the four quark intermediate states in our discussion.) The only unknown that remains is the "unsplit" diquark mass $M_2^{(0)}$. If we use, as in the previous section, the result of applying the virial theorem for the MIT bag, we obtain

$$\begin{aligned} M_2(s=1) &\simeq 800 + 50 = 850 \text{ MeV} \\ M_2(s=0) &\simeq 800 - 150 = 650 \text{ MeV} \end{aligned} \quad (3.80)$$

We shall take these masses as an approximate guide to estimate the effect of the chromomagnetic interaction spin structure functions. It should however be remembered that we have made the assumptions that all of the $N - \Delta$ mass splitting is attributable

to the one-gluon exchange mechanism, we have assumed $\underline{56}$ -plet SU(6) wavefunctions and of course the physics is that of the MIT bag model. The true situation may be somewhat different - for example, in the cloudy bag model pionic corrections change all these assumptions [TTM80,TMT82,TTM81].

Finally, using the SU(6) spin-flavour wavefunction for the proton

$$|\mu\rangle = \frac{1}{\sqrt{2}} \{ |\mu, s=0\rangle + |\mu, s=1\rangle \} \quad , \quad (3.81)$$

where $s=0$ and $s=1$ refer to the mixed antisymmetric and mixed symmetric parts of the wavefunction respectively, we obtain the following matrix elements analogous to equation 3.50:

$$\begin{aligned} \langle \mu, s=0 | P_{u,+\frac{1}{2}} | \mu, s=0 \rangle &= \frac{3}{2} & \langle \mu, s=1 | P_{u,+\frac{1}{2}} | \mu, s=1 \rangle &= \frac{1}{6} \\ \langle \mu, s=0 | P_{u,-\frac{1}{2}} | \mu, s=0 \rangle &= 0 & \langle \mu, s=1 | P_{u,-\frac{1}{2}} | \mu, s=1 \rangle &= \frac{1}{3} \\ \langle \mu, s=0 | P_{d,+\frac{1}{2}} | \mu, s=0 \rangle &= 0 & \langle \mu, s=1 | P_{d,+\frac{1}{2}} | \mu, s=1 \rangle &= \frac{1}{3} \\ \langle \mu, s=0 | P_{d,-\frac{1}{2}} | \mu, s=0 \rangle &= 0 & \langle \mu, s=1 | P_{d,-\frac{1}{2}} | \mu, s=1 \rangle &= \frac{2}{3} \end{aligned} \quad (3.82)$$

This gives the following quark distributions:

$$\begin{aligned} u_{(2)}^{\uparrow\downarrow}(x) &:= \left(\frac{3}{4}F_s(x) + \frac{1}{4}F_v(x) \right) \pm \frac{2}{3} \left(\frac{9}{8}G_s(x) - \frac{1}{8}G_v(x) \right) \\ d_{(2)}^{\uparrow\downarrow}(x) &:= \frac{1}{2}F_v(x) \mp \frac{1}{6}G_v(x) \quad , \end{aligned} \quad (3.83)$$

where the subscripts s and v refer to the relevant intermediate state masses to be used in equations 3.52 and 3.53.

We note that the distribution of u-quarks is dominated by the scalar (i.e. lower mass) intermediate state while the d-quark is entirely given by the vector (i.e. higher mass) intermediate state. From the discussion in the previous section it follows that the u-

quark distribution peaks at a larger x than the d-quark's, leading to a vanishing $\frac{d(x)}{u(x)}$ ratio as $x \rightarrow 1$. This is indeed what is observed experimentally [CKAL89].

In order to be quantitative we have plotted the ratio

$$\frac{d_2(x)}{u_2(x)} = \frac{2F_v(x)}{3F_s(x) + F_v(x)} \quad (3.84)$$

in Figure 3.6 for $M_{(2)}^0 = 700, 800$ and 900 MeV. (Actual comparison with data is postponed until Chapter 6 as we have not yet taken into account QCD corrections.)

The above ratio implies that the ratio of F_2 for the neutron and proton is given by

$$\frac{F_2^n(x)}{F_2^p(x)} = \frac{3F_v(x) + F_s(x)}{2F_v(x) + 4F_s(x)} \rightarrow \frac{1}{4} \text{ as } x \rightarrow 1 \quad (3.85)$$

This is shown in Figure 3.7.

This result has been anticipated for some time [Clo79, page 302-305] on the basis of exactly the sort of physics that has been included in the model presented here.

We are of course also able to calculate the ratio of the spin dependent quark distributions. It is given by

$$\frac{\Delta d_2(x)}{\Delta u_2(x)} = \frac{-2G_v(x)}{9G_s(x) - G_v(x)} \quad (3.86)$$

which is plotted in Figure 3.8, yielding the ratio of spin structure functions

$$\frac{g_1^n(x)}{g_1^p(x)} = \frac{3G_s(x) - 3G_v(x)}{12G_s(x) - 2G_v(x)} \rightarrow \frac{1}{4} \text{ as } x \rightarrow 1 \quad (3.87)$$

(see Figure 3.9).

Note that the neutron structure function is not zero as it would be if the wavefunction were the original $\underline{5}$ -plet wavefunction (i.e. $G_s(x) = G_v(x)$). Finally, we plot the asymmetries

$$A(x) = \frac{2xg_1(x)}{F_2(x)} \quad (3.88)$$

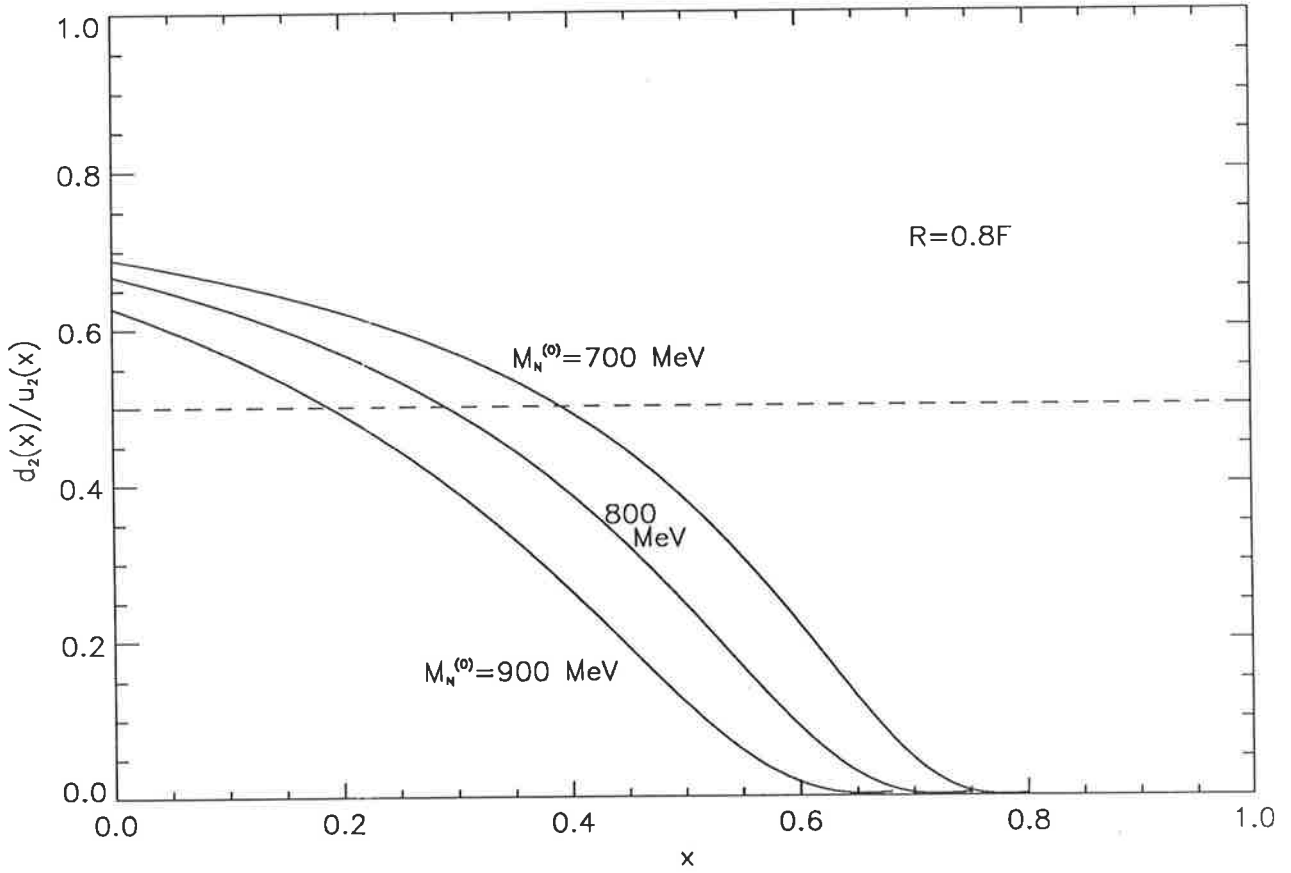


Figure 3.6: The ratio of down to up quarks as a function of the unsplit intermediate diquark state mass $M_N^{(0)}$. The dashed line shows the ratio if the hyperfine splitting is neglected.

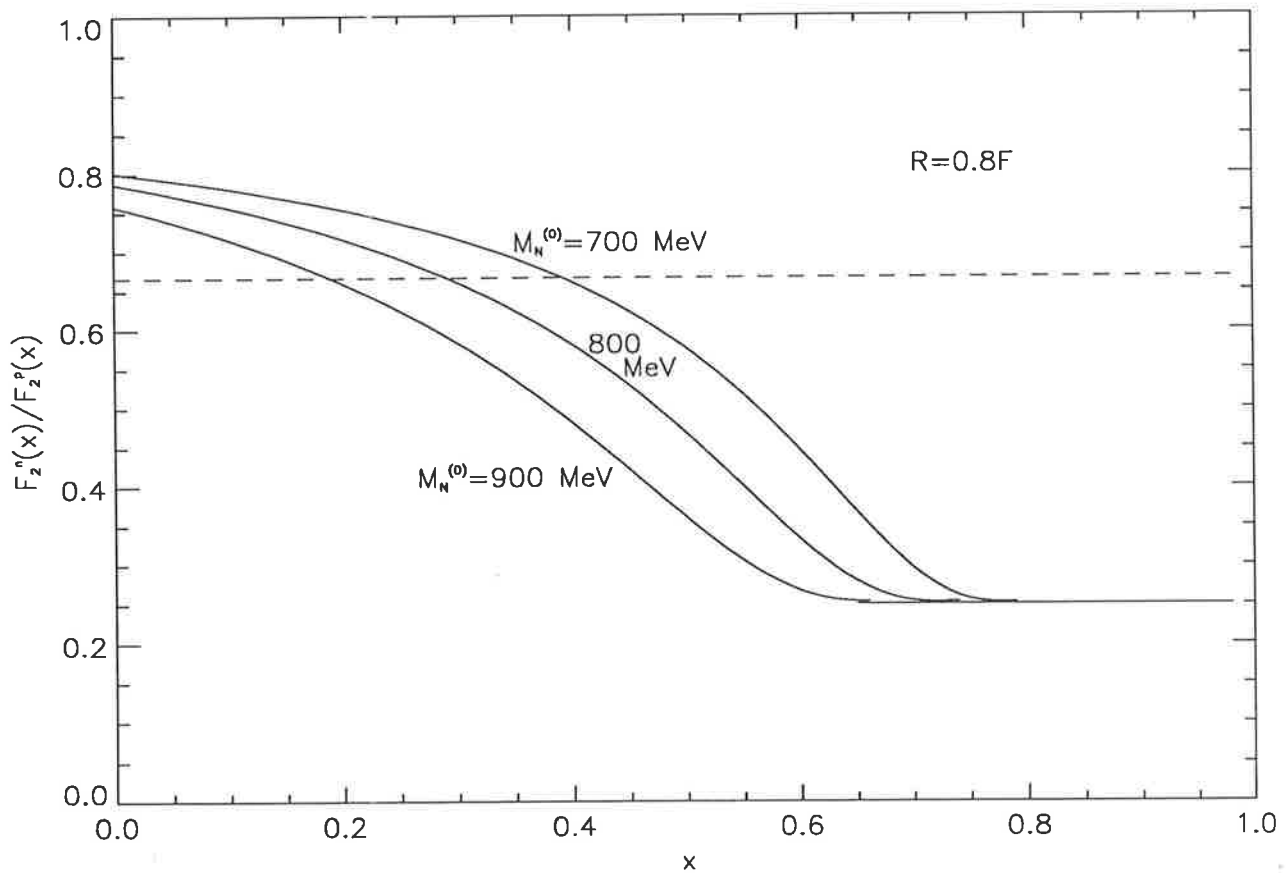


Figure 3.7: The ratio of neutron to proton F_2 's as a function of the unsplit intermediate diquark state mass $M_N^{(0)}$. The dashed line shows the ratio if the hyperfine splitting is neglected.

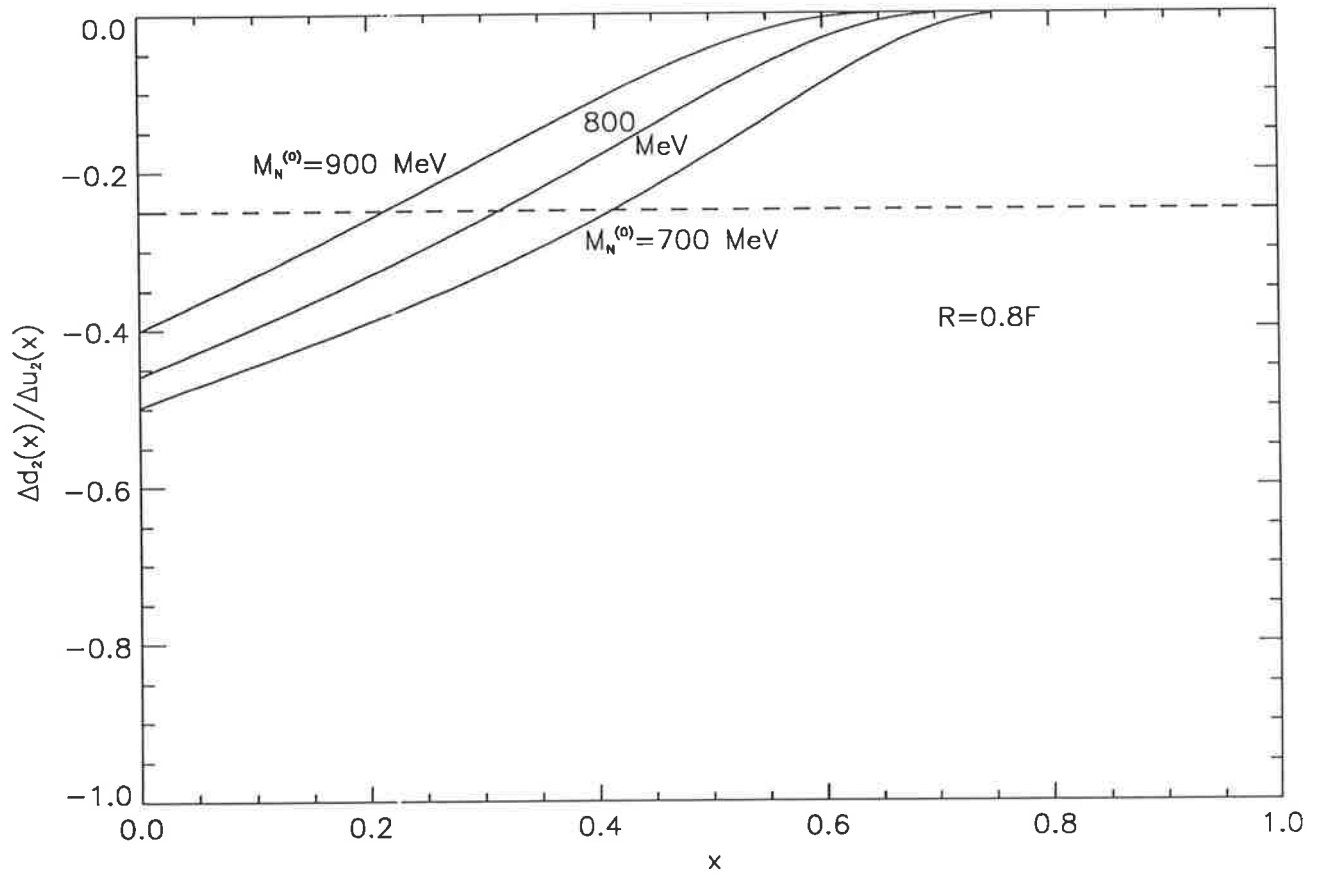


Figure 3.8: The ratio of $\Delta d_2(x)$ to $\Delta u_2(x)$ quarks as a function of the unsplit intermediate diquark state mass $M_N^{(0)}$. The dashed line shows the ratio if the hyperfine splitting is neglected.

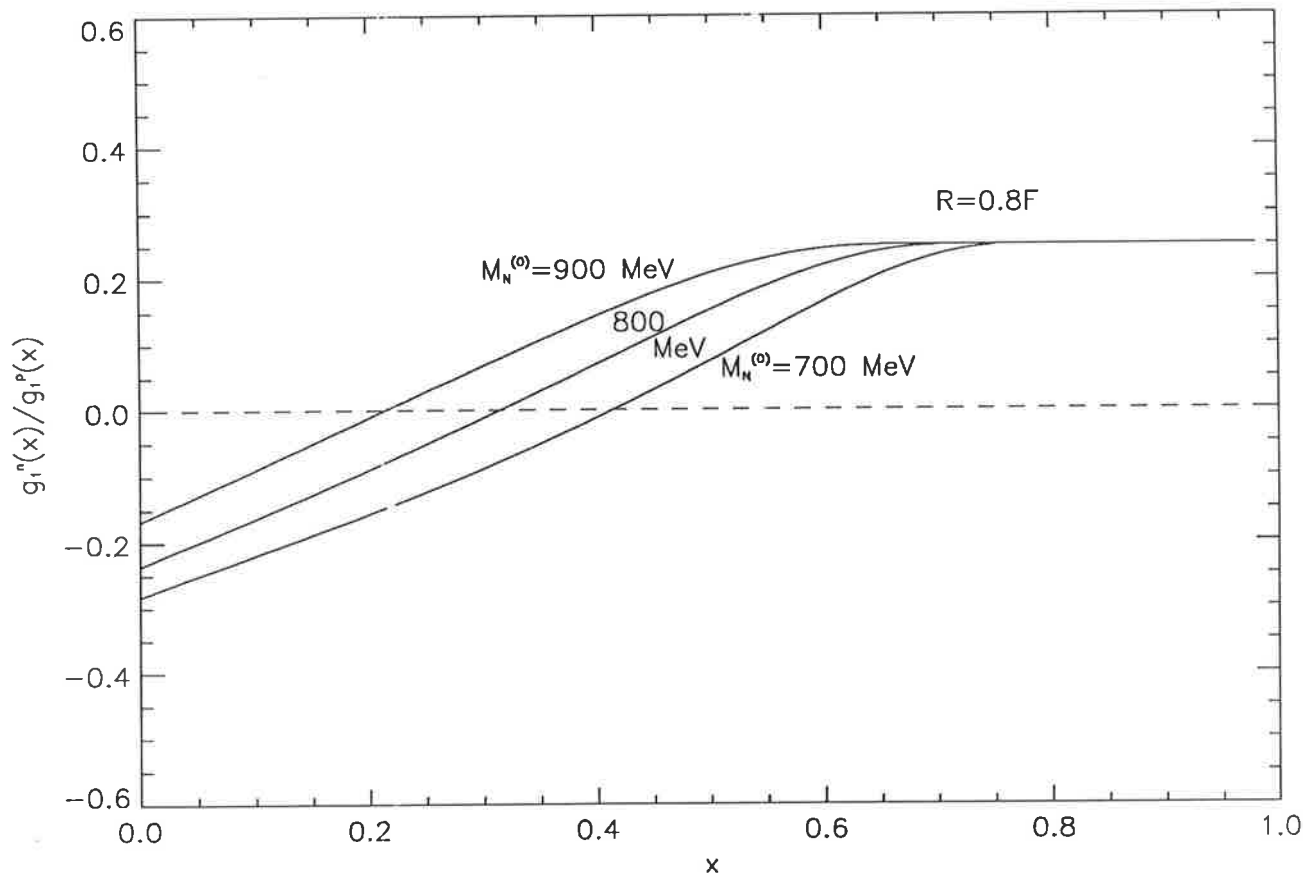


Figure 3.9: The ratio of neutron to proton g_1 's as a function of the unsplit intermediate diquark state mass $M_N^{(0)}$. The dashed line shows the ratio if the hyperfine splitting is neglected.

for both the proton and neutron, given respectively by

$$\begin{aligned} A^p(x) &= \frac{6G_s(x) - G_v(x)}{6F_s(x) + 3F_v(x)} \rightarrow 1 \quad \text{as } x \rightarrow 1 \\ \text{and } A^n(x) &= \frac{G_s(x) - G_v(x)}{F_s(x) + 3F_v(x)} \rightarrow 1 \quad \text{as } x \rightarrow 1 \end{aligned} \quad (3.89)$$

in Figures 3.10 and 3.11. Again, the limit of $A^p(x)$ as x tends towards 1 is in accordance with the experimental findings [EMC88, EMC89, A*76, A*78, B*80, B*83]. (We have cut the curves off at large x as here they become dominated by roundoff errors in the integration.)

3.8.2 Pions

Let us also investigate the modifications of the structure functions that one might expect from the pion cloud around the nucleus. The processes that scale in the Björken limit are shown in Figure 3.12.

It was shown by Berger et al (in the context of the EMC effect) [BCW84] that in the scaling limit one may write the sum of the processes shown in Figure 3.12 and the bare nucleon contribution as

$$q(x) = \{1 - P_r(\pi)\} q_N(x) + \int_x^1 \frac{dy}{y} f_\pi(y) q_\pi\left(\frac{x}{y}\right) + \int_0^{1-x} \frac{dy}{1-y} f_\pi(y) q_N\left(\frac{x}{1-y}\right) \quad (3.90)$$

Here $f_\pi(y)$ is the probability to find a pion with momentum fraction (in the infinite momentum frame) y , $P_r(\pi)$ is the probability that the nucleon consists of a bare nucleon plus a pion, i.e.

$$P_r(\pi) = \int_0^1 dy f_\pi(y), \quad (3.91)$$

and $q_\pi(z)$ and $q_N(z)$ are the quark distributions inside the pion and nucleon respectively.

A process which is not shown, but which also contributes, is that corresponding to $p \rightarrow \Delta\pi$. We shall investigate this effect in Chapter 5 in the context of the cloudy bag

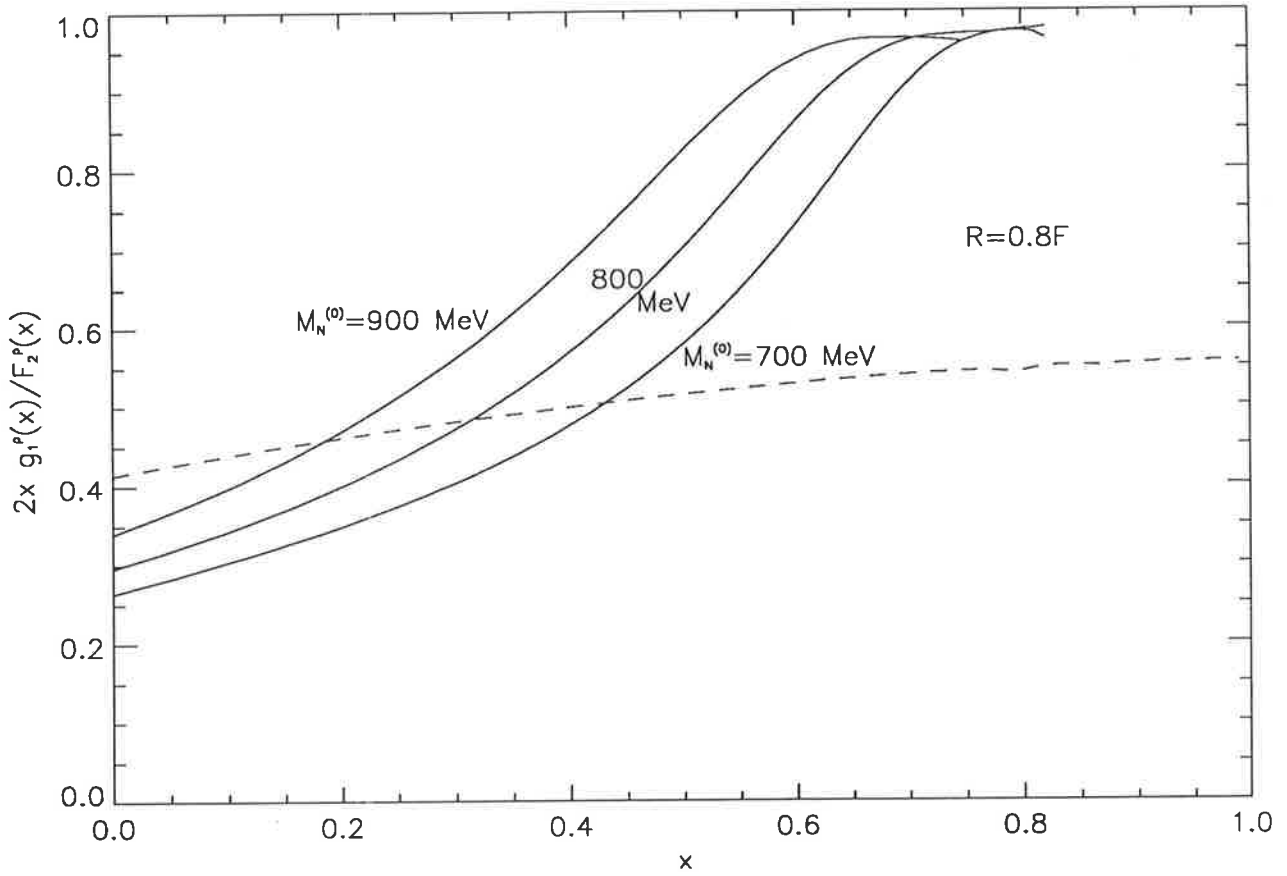


Figure 3.10: The proton asymmetry as a function of the unsplit intermediate diquark state mass $M_N^{(0)}$. The dashed line indicates the case without the one gluon exchange.

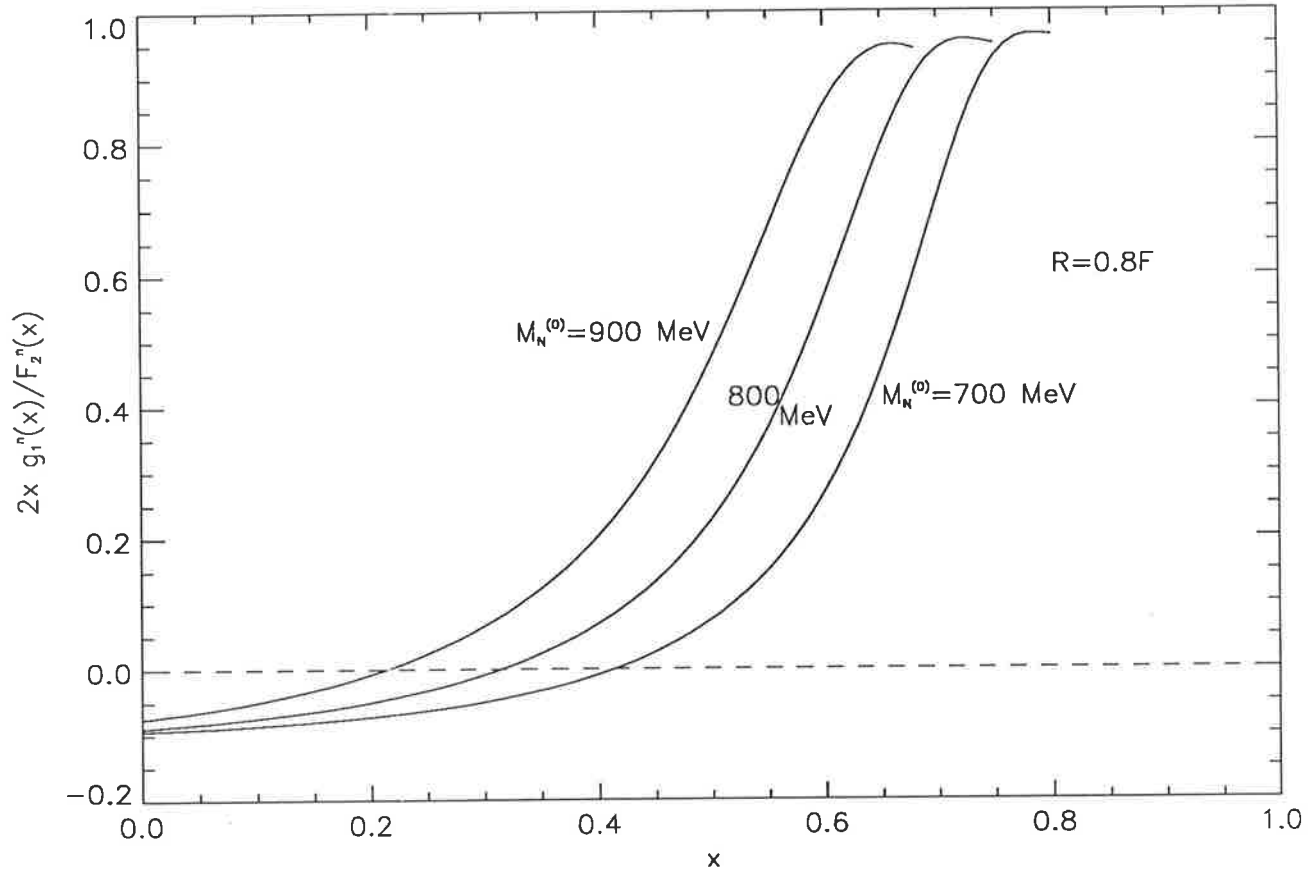


Figure 3.11: The neutron asymmetry as a function of the unsplit intermediate diquark state mass $M_N^{(0)}$. The dashed line indicates the case without the one gluon exchange.

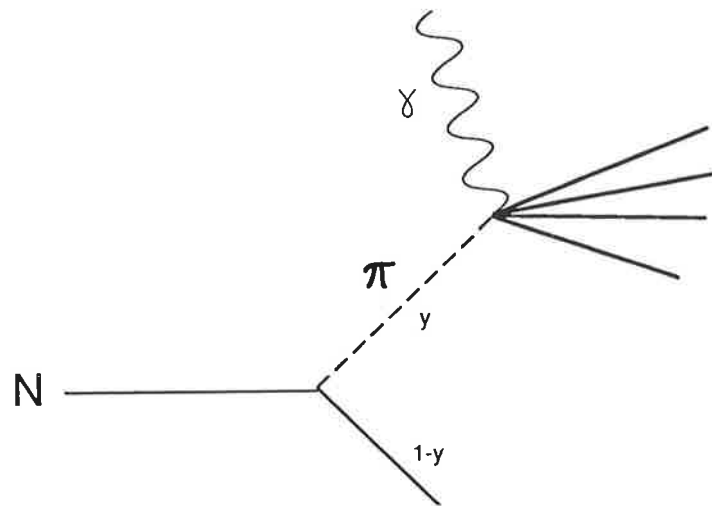
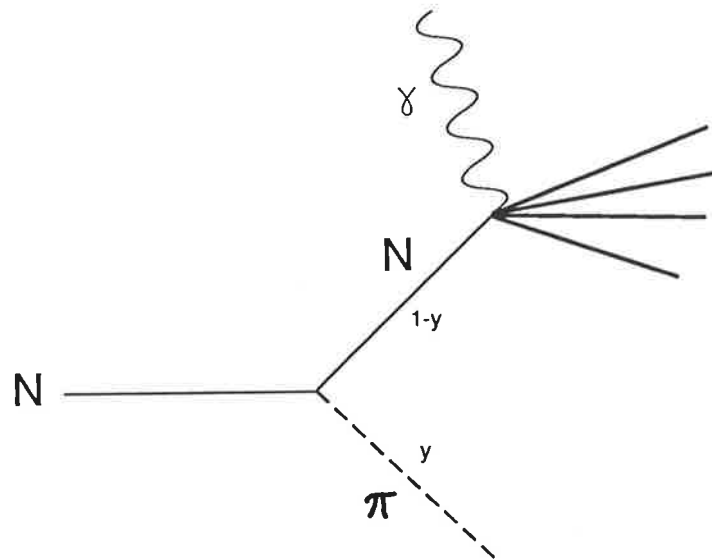


Figure 3.12: Pionic corrections to the scattering cross-section

model. Let us further concentrate only on the spin independent distribution. The term involving the pion structure function $q_\pi(z)$ only contributes to the sea [Sul72,ST87b] so that we are left with only the first and third terms on the right hand side of equation 3.90.

The pion distribution can be written [Sul72,ST87b] as

$$f_\pi(y) = \frac{3g_{\pi NN}^2 y}{16\pi^2} \int_{t_{MIN}}^{\infty} dt \frac{t}{(t + m_\pi^2)^2} |F(t)|^2 . \quad (3.92)$$

Here t is the negative of the square of the four momentum of the pion, $F(t)$ is the form factor governing the πN vertex and $g_{\pi NN}^2$ is the strength of the πN coupling. t_{MIN} is the minimum value of t allowed, which corresponds to forward scattering:

$$t_{MIN} = \frac{M^2 y^2}{1 - y} . \quad (3.93)$$

It is relatively straightforward to derive equation 3.92 and it would lead us too far astray to do so here. The factor $(t + m_\pi^2)^{-2}$ is the square of the pion propagator and t originates with the trace arising from the πN vertex. The factor 3 is due to isospin.

The form factor $F(t)$ arises due to the finite extent of the nucleon and is calculable in models such as the one presented in Chapter 5. Here we shall simply use a phenomenological one which provides a guide to what to expect in a more rigorous treatment. In fact we use

$$F(t) = \frac{\Lambda^2 - m_\pi^2}{\Lambda^2 + t} , \quad (3.94)$$

which gives

$$f_\pi(y) = \frac{3g_{\pi NN}^2 y}{16\pi^2} \left\{ \frac{\Lambda^2 + m_\pi^2}{\Lambda^2 - m_\pi^2} \log \left\{ \frac{\Lambda^2 + t_{MIN}}{m_\pi^2 + t_{MIN}} \right\} - \frac{\Lambda^2}{\Lambda^2 + t_{MIN}} - \frac{m_\pi^2}{m_\pi^2 + t_{MIN}} \right\} . \quad (3.95)$$

In Figure 3.13 we show the valence quark distributions as a function of the cut-off Λ . The corresponding probabilities to observe a pion are 0.31 ($\Lambda = 700 MeV$), 0.40 ($\Lambda = 800 MeV$) and 0.49 ($\Lambda = 900 MeV$).

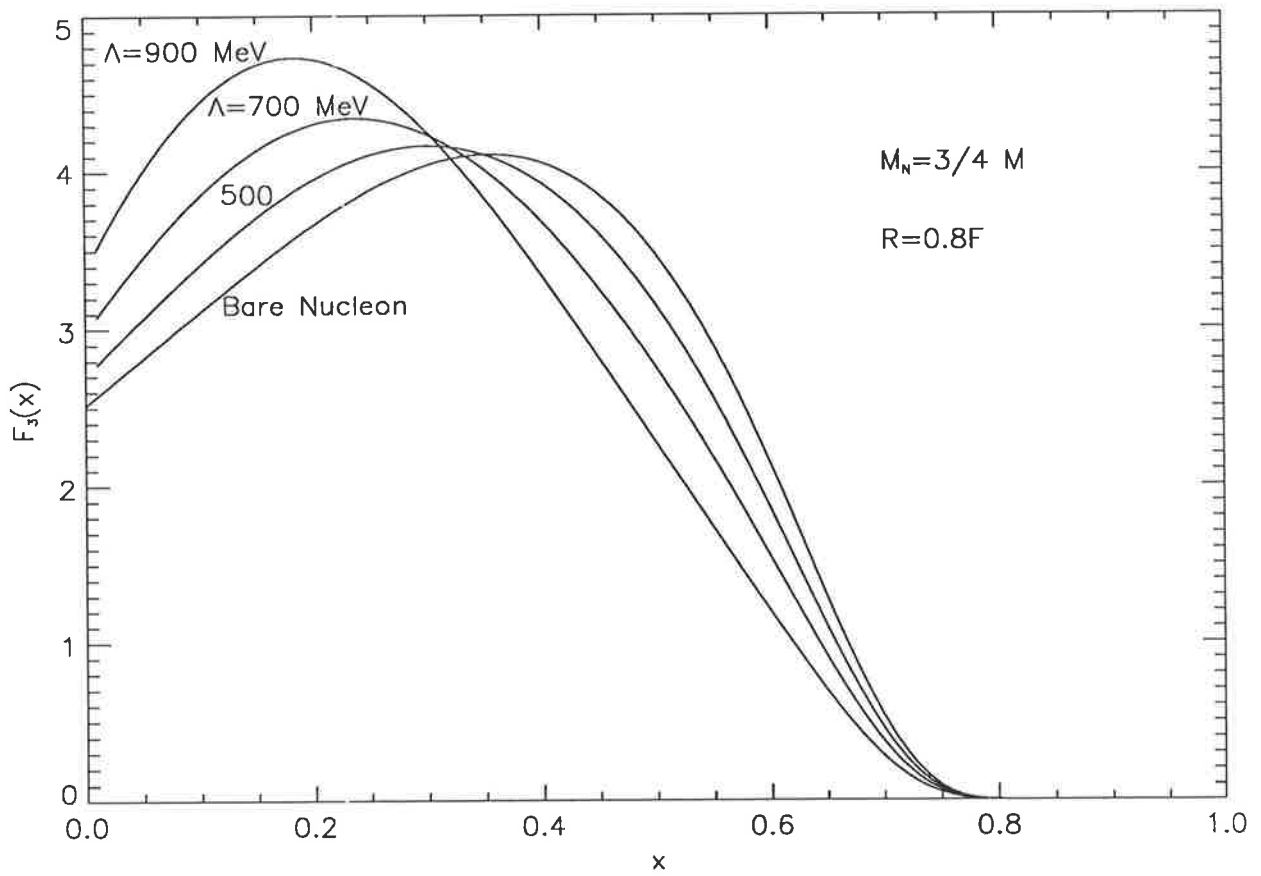


Figure 3.13: The effect of pionic corrections on the valence distribution.

As the form factor becomes harder more pions are produced, causing a corresponding decrease in the average momentum carried by the quarks in the proton, an effect clearly visible in Figure 3.13. The overall normalization of $q(x)$ remains the same of course as we are only considering the valence contribution.

3.9 Conclusion

We have presented in this chapter a detailed model for the quark distributions, both spin dependent and spin independent, that will enable us determine the moments of the matrix elements $\mathcal{A}_{(i,n)}$ that are needed in order to determine the structure functions measured by the experimentalists.

First we developed the model for the bare MIT bag, corrected for centre of mass motion by the Peierls-Yoccoz procedure. We found that in the bag model one does not expect the spin dependent quark distributions to have the same shape, especially at small x , as the spin independent ones.

We also found that in the bag model one expects additional connected contributions to the current correlation functions that arise from the photon probe inserting a quark or an antiquark into the bag. These contributions produce an unavoidable, if small, addition to the valence distributions that looks like that expected from a sea component.

We went on to include chromomagnetic hyperfine splitting which, not unexpectedly, made nontrivial changes - especially to quantities like the d/u ratio. Finally, we looked at the kind of changes that are expected for the valence distribution if one includes the pion cloud.

Chapter 4

QCD evolution of the spin structure functions

4.1 Introduction

In the previous chapter the matrix elements of various twist two operators have been calculated within some specific models. Some of the problems previously associated with these calculations, such as the support above $x = 1$, have been removed, while some others, notably the normalization problem that has been discussed in Section 3.7 remain. In the light of what is now known about the origins of the parton model in the theory of the strong interaction (which shall be taken to be SU(3) colour) it would however not make a great deal of sense to compare the results to data, even if we had a theoretically consistent model. Experimental evidence that something more than the naive parton model is required is shown in Figure 4.1 where the first few moments of the structure function $F_3(x, Q^2)$ are plotted as a function of the four-momentum transfer Q^2 . $F_3(x, Q^2)$ probes the valence quarks inside the nucleon, and so, for example, the fraction of the momentum of the proton carried by the valence quarks is essentially given by the second moment of $F_3(x, Q^2)$. In Figure 4.1 this moment decreases with increasing Q^2 , while the matrix elements of the previous chapter were Q^2 independent. As the momentum carried

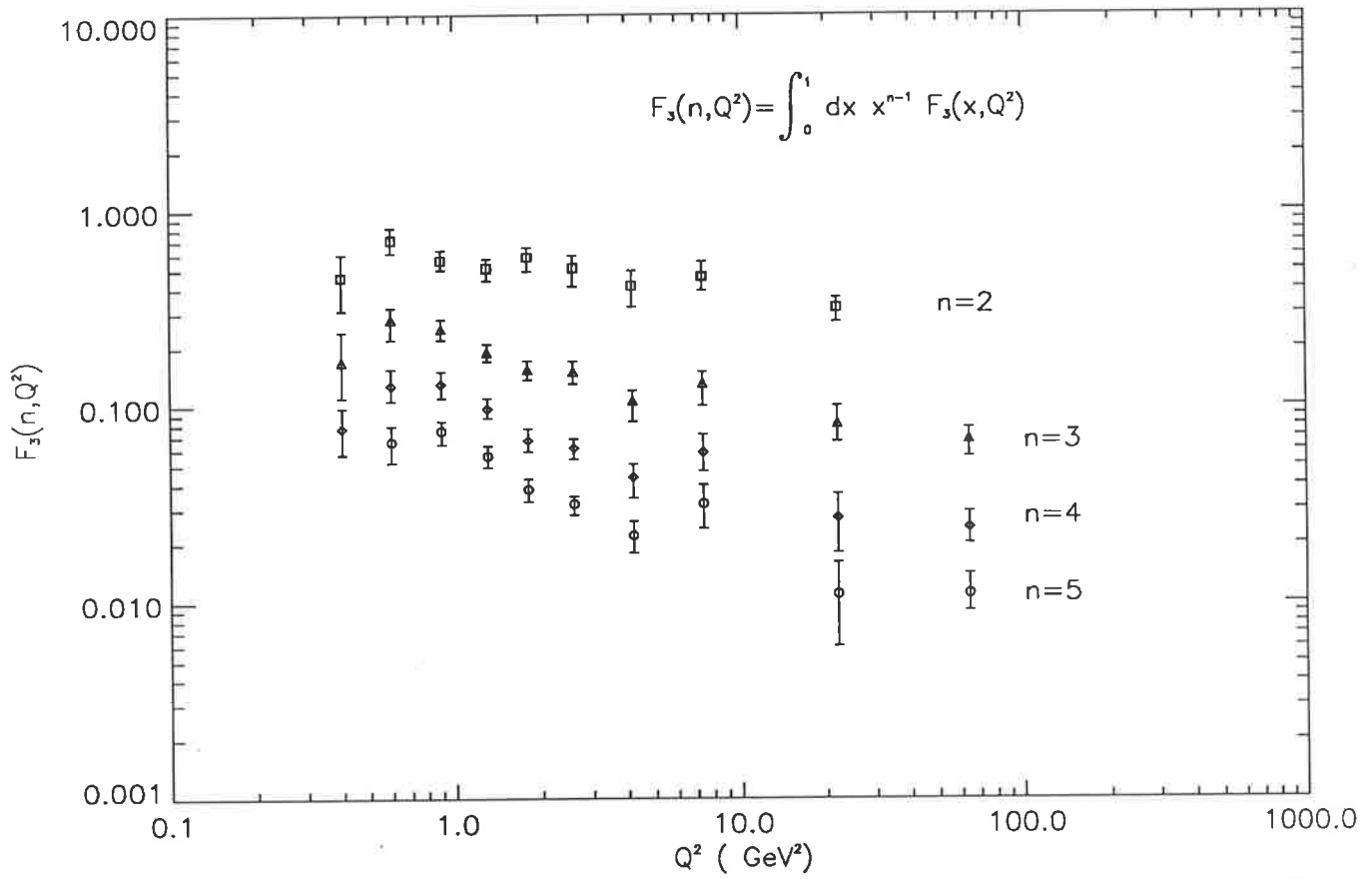


Figure 4.1: The first few moments of $F_3(x, Q^2)$. The data is taken from Bosetti et al [B*78].

by the nucleon must be carried by its constituents we can only conclude that the nucleon must consist of more than three valence quarks.

The origin of these extra constituents is well understood within QCD. In any interacting field theory the structure of a target depends on the scale at which it is being probed. As one decreases this scale, by increasing the four-momentum transfer, more structure becomes apparent. In the case of QCD this structure consists of radiated gluons and $q\bar{q}$ pairs. These carry momentum, thus decreasing the momentum carried by the bare quark. The number of gluons and $q\bar{q}$ pairs increases with Q^2 , providing an explanation of the phenomenon depicted in Figure 4.1.

One is therefore faced with the problem that the structure of the nucleon as probed in deep inelastic scattering is a phenomenon involving both non-perturbative (i.e. the localization of the quarks inside some region) and perturbative (i.e. the corrections to the quark-photon vertex mentioned above) effects. Jaffe and Ross [JR80] have suggested that the physics of both of these regions can be taken into account by postulating that bag model predictions, such as those in Chapter 3 should be interpreted to be relevant at a scale where the proton consists of only three quarks. The scale is dependent on the value of the strong coupling constant and must be determined from the data. Having provided this (non-perturbative) input one must then use perturbation theory in order to relate this prediction to the scale at which the experiments are actually performed.

It should be remembered that the matrix elements that we have calculated are twist 2 matrix elements. The next higher contribution is of twist 4 and is smaller than the leading term by a factor of the order of M^2/Q^2 (M is the proton mass). At the values of Q^2 at which the experiments are done this gives only a small contribution. At the scale of the model however this contribution may indeed be large. The implication is that the model calculations should not be compared to data even at the scale at which they are applicable. Luckily the Q^2 evolutions of operators of different twist do not affect each other so that calculating the twist 2 piece is sufficient, all higher twists having evolved

away by the time Q^2 approaches a few GeV^2 .

It is the purpose of this chapter to calculate these perturbative corrections to the spin dependent structure functions g_1 . There exists a particularly useful formalism that makes these corrections easy to calculate, that of the renormalization group. In Section 4.2 we review this formalism. The results are most readily expressed in terms of the moments of the quark and glue distributions. The reconstruction of distributions from their moments is a long standing mathematical problem. We shall present a novel method of doing this in Section 4.3. At small x (≤ 0.01) the method fails and we shall discuss ways of overcoming this in Section 4.4. Section 4.5 is devoted to exhibiting numerical results for a particular input distribution and we shall conclude in Section 4.6.

4.2 The renormalization group

This section is devoted to a short review of the renormalization group analysis, as applied to spin dependent deep inelastic scattering. Because of the extent of the subject only results relevant to the argument that follows are presented (see for example [Alt82,Bur80,Mut87,SP53,GL54,Low71,Pet79], from which most of the material presented in this section originates). We start with the expression for the moments of the structure function g_1 in terms of the coefficient functions $C^{(i,n)}(Q^2)$ (see equation 2.33):

$$g_1(n, Q^2) \equiv \int_0^1 x^{n-1} g_1(x, Q^2) dx \sim \sum_i \langle P | \mathcal{O}_{(i,n)} | P \rangle \tilde{C}_1^{(i,n)}(q^2) \quad (4.1)$$

Note that in equation 4.1 and the rest of this chapter $\tilde{C}_1^{(i,n)}(q^2)$ shall refer only to the twist two piece of the coefficient functions discussed in Chapter 2. Virtually the same equation could of course be written down for the higher twists, with appropriate factors of $\frac{1}{Q^2}$ out the front. From now on we shall for clarity suppress the superscript i and we shall assume that the operator does not mix under renormalization, i.e. it is multiplicatively renormalized. This corresponds to the case where the operator in question is a flavour

non-singlet operator. We shall quote the results for the singlet at the end.

The possibility to make use of the renormalization group to describe the Q^2 evolution of 4.1 rests on two observations:

- 1) If calculated in a quantum field theory, the operators and wavefunctions appearing in 4.1 need to be renormalized. Even if the original theory had no momentum scale dependence (i.e. if there were only zero mass particles in the theory, which we shall assume) this introduces a momentum scale μ . This is trivially true in calculation schemes in which one introduces a cut-off μ to regularize any integrals that appear. If one uses dimensional regularization the scale still appears through the requirement that the coupling constant remain dimensionless even in $D \neq 4$ dimensions.
- 2) Physical quantities, such as $W_{\mu\nu}$ and g_1 , should not have any dependence on the scale μ . Of course quantities that have not been renormalized should also not depend on this scale.

Let us consider the matrix element $\langle \mathbf{p} | \mathcal{O}_{(n)} | \mathbf{p} \rangle$. Let us define the renormalization of the operator $\mathcal{O}_{(n)}$ by

$$\mathcal{O}_{(n)}^u = Z_{\mathcal{O}_n} \mathcal{O}_{(n)} \quad , \quad (4.2)$$

where $\mathcal{O}_{(n)}^u$ is the unrenormalized operator - i.e. it is independent of μ . Hence

$$\frac{d}{d \log \mu^2} \mathcal{O}_{(n)}^u = \frac{d}{d \log \mu^2} (Z_{\mathcal{O}_n} \mathcal{O}_{(n)}) + Z_{\mathcal{O}_n} \frac{d}{d \log \mu^2} \mathcal{O}_{(n)} = 0 \quad , \quad (4.3)$$

i.e.

$$\left(\frac{d}{d \log \mu^2} + \gamma_{\mathcal{O}_n} \right) \mathcal{O}_{(n)} = 0 \quad , \quad (4.4)$$

where $\gamma_{\mathcal{O}_n}$ is the anomalous dimension associated with the operator $\mathcal{O}_{(n)}$:

$$\gamma_{\mathcal{O}_n} = \frac{1}{Z_{\mathcal{O}_n}} \frac{d}{d \log \mu^2} (Z_{\mathcal{O}_n}) \quad . \quad (4.5)$$

A similar equation holds for any wavefunction

$$(\frac{d}{d \log \mu^2} + \gamma_\psi)\psi = 0 \quad , \quad (4.6)$$

with

$$\gamma_\psi = \frac{1}{Z_\psi} \frac{d}{d \log \mu^2} (Z_\psi) \quad . \quad (4.7)$$

We can therefore write the μ independence of the matrix element $\langle \mathbf{p} | \mathcal{O}_{(n)} | \mathbf{p} \rangle$ as

$$(\frac{d}{d \log \mu^2} + \gamma_{\mathcal{O}_n} + \gamma_{|\mathbf{p}\rangle} + \gamma_{\langle \mathbf{p}|}) \langle \mathbf{p} | \mathcal{O}_{(n)} | \mathbf{p} \rangle = 0 \quad (4.8)$$

and the independence of g_1 on μ gives, using equation 4.1,

$$(\frac{d}{d \log \mu^2} + 2\gamma_j + \gamma_{|\mathbf{p}\rangle} + \gamma_{\langle \mathbf{p}|}) \mathcal{C}^{(n)}(Q^2, \mu^2, g_\mu) \langle \mathbf{p} | \mathcal{O}_{(n)} | \mathbf{p} \rangle = 0 \quad . \quad (4.9)$$

Here γ_j is the anomalous dimension corresponding to the electromagnetic current appearing in the current correlation function while $\gamma_{|\mathbf{p}\rangle}$ and $\gamma_{\langle \mathbf{p}|}$ are the anomalous dimensions corresponding to the wavefunctions. g_μ is the renormalized coupling constant at the scale μ .

Combining 4.8 and 4.9 we get

$$(\frac{d}{d \log \mu^2} + 2\gamma_j(g_\mu) - \gamma_{\mathcal{O}_n}(g_\mu)) \mathcal{C}^{(n)}(Q^2, \mu^2, g_\mu) = 0 \quad , \quad (4.10)$$

or, equivalently,

$$(\frac{d}{d g_\mu} + \frac{2\gamma_j(g_\mu) - \gamma_{\mathcal{O}_n}(g_\mu)}{\beta(g_\mu)}) \mathcal{C}^{(n)}(Q^2, \mu^2, g_\mu) = 0 \quad , \quad (4.11)$$

where

$$\beta(g_\mu) = \frac{\partial g_\mu}{\partial \log \mu^2} \quad . \quad (4.12)$$

We note that in writing down equation 4.8 we have implicitly assumed that both the matrix element as well as the coefficient function are separately gauge independent

(their product must of course always be gauge independent). If this were not the case, further terms (derivatives with respect to the gauge parameter) would have to be added to equation 4.8. Similarly, had we not assumed massless QCD, derivatives with respect to the mass would appear.

Equation 4.11 has the solution

$$C^{(n)}(Q^2, \mu^2, g_\mu) = C^{(n)}(Q^2, \mu_0^2, g_{\mu_0}) \text{Exp} \left[\int_{g_{\mu_0}}^{g_\mu} dg \frac{2\gamma_j(g) - \gamma_{\mathcal{O}_n}(g)}{\beta(g)} \right] . \quad (4.13)$$

Because $C^{(n)}$ is dimensionless, its dependence on Q^2 and μ^2 can only be of the form Q^2/μ^2 [CHM72]. We may therefore write equation 4.13 as

$$C^{(n)}(Q^2/\mu^2, g_\mu^2) = C^{(n)}(1, g_{Q^2}) \text{Exp} \left[\int_{g_{\mu^2}}^{g_{\mu^2}} dg \frac{2\gamma_j(g) - \gamma_{\mathcal{O}_n}(g)}{\beta(g)} \right] , \quad (4.14)$$

where we have set $\mu_0 = Q$. Finally, we point out that if the current j is conserved its anomalous dimension vanishes, thus giving the well known equation governing the Q^2 evolution of the flavour non-singlet:

$$C^{(n)}(Q^2/\mu^2, g_\mu^2) = C^{(n)}(1, g_{Q^2}) \text{Exp} \left[\int_{g_{\mu^2}}^{g_{Q^2}} dg \frac{\gamma_{\mathcal{O}_n}(g)}{\beta(g)} \right] . \quad (4.15)$$

For the singlet one obtains similar expressions, except that because of the mixing of various operators the equations become matrix equations (This has the consequence that some care must be taken when evaluating the expression analogous to equation 4.15 because in general the matrices $\gamma(g)$ and $\gamma(g')$ don't commute). We obtain, instead of equation 4.10,

$$\frac{d}{d \log Q^2} \begin{pmatrix} C_q^{(n)}(Q^2/\mu^2, g_\mu) \\ C_g^{(n)}(Q^2/\mu^2, g_\mu) \end{pmatrix} = \frac{\alpha(Q^2)}{2\pi} \begin{pmatrix} \gamma_{qq}(n) & \gamma_{qg}(n) \\ \gamma_{gq}(n) & \gamma_{gg}(n) \end{pmatrix} \begin{pmatrix} C_q^{(n)}(Q^2/\mu^2, g_\mu) \\ C_g^{(n)}(Q^2/\mu^2, g_\mu) \end{pmatrix} , \quad (4.16)$$

i.e. we get mixing between the renormalization of the quark and gluon operator, the coefficient functions thereof being $C_q^{(n)}$ and $C_g^{(n)}$ respectively.

In perturbation theory the Q^2 evolution of the left hand side of equation 4.15 is evaluated by expanding the right hand side in orders of the running coupling constant:

$$\begin{aligned} \mathcal{C}^{(n)}(1, g_Q) &= C_{0,n} + C_{1,n}g_Q^2 + O(g_Q^4) \\ \gamma_n(g) &= \gamma_{0,n}g^2 + \gamma_{1,n}g^4 + O(g^6) \\ \beta(g) &= -\beta_0g^3 - \beta_1g^5 + O(g^7) \end{aligned} \quad (4.17)$$

The expansion coefficients appearing in equation 4.17 were first calculated up to first order for the unpolarized case by Gross & Wilczek, Politzer and others [GW73,GW74, Pol74,GP74] and later, up to second order, by Floratos,Ross and Sachrajda etc [FRS77, FRS78,FRS79c,FRS79a,FRS79b], Jones [Jon74], Calvo [Cal77], Gonzalez-Arroyo [GLY79b, GLY79a,GL80]and others [RGP77,BBDM78,ZWB74,HS77,AEM78].

For polarized scattering the calculations to leading order have been done by Ahmed and Ross, Ito and Sasaki [AR75,AG76,Ito75,Sas75]. At next to leading order the coefficients $\mathcal{C}^{(n)}$ as well as γ_1 have been calculated by Kodaira et al.[KMM*79,KMSU79,Kod80] For general n the anomalous dimensions have not been calculated beyond leading order.

Equation 4.15 has the solution, to leading order,

$$\mathcal{C}^{(n)}(Q^2/\mu^2, g_\mu) = C_{0,n} \left(\frac{g_Q}{g_\mu} \right)^{-\gamma_{0,n}/\beta_0} \quad (4.18)$$

(note that we have now dropped the subscript $\mathcal{O}$ on the anomalous dimension to conform with the standard notation in the literature), while for the singlet we get the following solution, at leading order, to equation 4.16:

$$\begin{aligned} \mathcal{C}_q^{(n)}(Q^2) &= \left[u_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \mathcal{C}_q^{(n)}(\mu^2) \\ &+ \left[-v_3(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_3(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \mathcal{C}_g^{(n)}(\mu^2) \\ \mathcal{C}_g^{(n)}(Q^2) &= \left[-v_4(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_4(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \mathcal{C}_q^{(n)}(\mu^2) \\ &+ \left[w_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \mathcal{C}_g^{(n)}(\mu^2) \end{aligned} \quad (4.19)$$

where

$$\begin{aligned}
w_{1,2}(n) &= \frac{1}{2} \left(1 \mp \frac{\gamma_{\pm}(n)}{R(n)} \right) \\
w_3(n) &= \gamma_{qg}/R(n) \\
w_4(n) &= \gamma_{gq}/R(n) \\
t_{\pm}(n) &= \frac{1}{4\beta_0} (-\gamma_{\pm}(n) \pm R(n))
\end{aligned} \tag{4.20}$$

and

$$\begin{aligned}
\gamma_{\pm}(n) &= \gamma_{qg} \pm \gamma_{gg} \\
R(n) &= \sqrt{\gamma_{\pm}^2(n) + 4\gamma_{qg}\gamma_{gq}} .
\end{aligned} \tag{4.21}$$

To first order, for three colours and f flavours the spin dependent anomalous dimensions are given by (note that the anomalous dimensions appearing in the expression for the non-singlet (equation 4.18) is just given by $\gamma_{qq}(n)$)

$$\gamma_{qq}(n) = \frac{4}{3} \left(\frac{3}{2} + \frac{1}{n(n+1)} - \frac{2}{n} - 2 \sum_{j=1}^{n-1} \frac{1}{j} \right) \tag{4.22}$$

$$\gamma_{qg}(n) = f \left(\frac{n-1}{n(n+1)} \right) \tag{4.23}$$

$$\gamma_{gq}(n) = \frac{4}{3} \left(\frac{n+2}{n(n+1)} \right) \tag{4.24}$$

$$\gamma_{gg}(n) = 3 \left(\frac{11}{6} - \frac{f}{9} + \frac{2}{n} - \frac{4}{n+1} - 2 \sum_{j=1}^{n-1} \frac{1}{j} \right) , \tag{4.25}$$

while

$$\beta_0 = \frac{33 - 2f}{12} . \tag{4.26}$$

This completes our review of the QCD evolution of the moments of the coefficient functions.

4.3 The reconstruction of the distributions

The developments in the previous section, culminating in equations 4.18 and 4.19, give direct predictions only for the moments of the distribution functions. This is somewhat

unfortunate as the experimentally measured quantities are the actual distributions themselves which typically vary in accuracy as a function of x and are only measured down to some finite small x , with a subsequent extrapolation to $x = 0$ [EMC88,EMC89,A*76,A*78,B*80,B*83]. Hence the extraction of the moments is subject to some uncertainty and there may be considerable correlation between the errors on different moments.

In the case of unpolarized scattering, the reconstruction of the distributions has been performed in a variety of ways. Originally, at first order, Gross and others [Gro74,Par73,DGP74] reconstructed the non-singlet quark distribution using asymptotic expansions of the anomalous dimensions and the inverse Mellin transform technique. Other methods have relied on parameterizations of the structure functions which have then been evolved by numerically solving[CP78] the integro-differential Altarelli - Parisi[AP77] equations, by using the method of Laguerre polynomials [FP82,Ram85] or by using particular parameterizations of the quark and glue distributions [Bur77,BG78,SD89]. In this section we shall extend the method of Gonzalez-Arroyo et al. [GLY79b,GLY79a,GL80] to the spin dependent case. Because the anomalous dimensions have not been calculated at second order we shall restrict ourselves to leading order only. As is the case in [GLY79b,GLY79a,GL80], we expect the generalization to second order to be straightforward.

To illustrate the method we take the case of the spin dependent non-singlet (which, because of parity invariance, is identical to that of the spin independent non-singlet) and then give the straightforward generalization to the singlet. Furthermore, we shall now multiply the coefficient functions by the relevant matrix elements so that from now on we shall talk about the QCD evolution of the quark distributions themselves (this is of course entirely equivalent to talking about the QCD evolution of the coefficient functions).

We suppose that we have a non-singlet quark distribution

$$\Delta q_{NS}(x, Q^2) = q_{NS}^\uparrow(x, Q^2) - q_{NS}^\downarrow(x, Q^2) \quad (4.27)$$

whose n^{th} moment

$$\Delta q_{NS}(n, Q^2) = \int_0^1 x^{n-1} \Delta q_{NS}(x, Q^2) dx \quad (4.28)$$

obeys the evolution equation 4.18, i.e.

$$\Delta q_{NS}(n, Q^2) = \left[\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{-\gamma_{qq}(n)/(2\beta_0)} \Delta q_{NS}(n, \mu^2) . \quad (4.29)$$

Here $q_{NS}^{1(l)}(x, Q^2)$ is the distribution of quarks with helicity equal (opposite) to that of the parent hadron.

In order to reconstruct the distribution $\Delta q_{NS}(x, Q^2)$ from $\Delta q_{NS}(n, Q^2)$ we write

$$\Delta q_{NS}(x, Q^2) = \int_0^1 \Delta(y-x) \Delta q_{NS}(y, Q^2) dy , \quad (4.30)$$

where $\Delta(y-x)$ must have the property

$$f(x) = \int_0^1 \Delta(y-x) f(y) dy . \quad (4.31)$$

A sufficient, but not necessary, choice for $\Delta(y-x)$ is the Dirac δ - function. We however use functions related to the Bernstein Polynomials

$$\begin{aligned} \Delta(y-x) &= \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{(N-k)!k!} y^k (1-y)^{N-k} \\ &= \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l y^{k+l}}{l! (N-k-l)!} . \end{aligned} \quad (4.32)$$

By integrating over y with a testfunction y^c , or otherwise [LY73], it can readily be shown that $\Delta(y-x)$ exhibits the behaviour of a δ - function in the range $0 \leq y \leq 1$ (It need not be symmetric in x and y for our purposes, nor does it have to behave like a δ -function for y outside this interval). Using equations 4.32 and 4.29 in 4.30, we obtain

$$\begin{aligned} \Delta q_{NS}(x, Q^2) &= \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l}{l! (N-k-l)!} \\ &\quad \int_0^1 y^{k+l} \left[\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{-\gamma_{qq}(l+k+1)/(2\beta_0)} \Delta q_{NS}(y, \mu^2) dy . \end{aligned} \quad (4.33)$$

As has been pointed out in [GLY79b, GLY79a, GL80], to sum equation 4.33 numerically is dangerous because of the oscillatory nature due to the appearance of the $(-1)^l$. Because of the form of $\gamma_{qq}(l+k+1)$ it is not possible to do the sum in equation 4.33 analytically. However, if one uses the expansion

$$r^{-\gamma_{qq}(n)/(2\beta_o)} = \sum_{i,j} \frac{c(i,j)}{(p+n)^{i+\rho} [\log a(n+p)]^j} \quad (4.34)$$

it is in fact possible to perform the sum analytically for each individual term in 4.34. (a and p are arbitrary constants; later on we shall use $a = e^{C-\frac{3}{4}}$ and $p = 2$, C being Euler's constant. ρ is dependent on the particular expansion and is a function of r , r being the ratio of the two coupling constants in eq. 4.33) We then find that

$$\lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l}{l!(N-k-l)!} \frac{y^{k+l}}{(p+k+l)^{i+\rho}} = \frac{\theta(y-x)}{\Gamma(i+\rho)} \frac{(x}{y})^{p-1} (\log \frac{y}{x})^{i+\rho-1}, \quad (4.35)$$

and

$$\begin{aligned} \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l}{l!(N-k-l)!} \frac{y^{k+l}}{(p+k+l)^{i+\rho} [\log a(p+k+l)]^j} = \\ \frac{\theta(y-x)}{\Gamma(i+\rho)} \frac{(x}{y})^{p-1} (\log \frac{y}{x})^{i+\rho-1} \int_0^\infty ds \frac{s^{j-1} a^{-s} (\log \frac{y}{x})^s}{\Gamma(i+\rho+s)}. \end{aligned} \quad (4.36)$$

Hence we obtain

$$\Delta q_{NS}(x, Q^2) = \int_x^1 \frac{dy}{y} b_{qq}(x/y, r) \Delta q_{NS}(y, \mu^2), \quad (4.37)$$

with

$$b_{qq}(x/y, r) = \left(\frac{x}{y}\right)^{p-1} \sum_i \log^{(i+\rho-1)} \frac{y}{x} \left[\frac{c_{qq}(i, 0)}{\Gamma(i+\rho)} + \sum_{j \neq 0} \frac{c_{qq}(i, j)}{\Gamma(j)} \int_0^\infty ds \frac{s^{j-1} a^{-s} (\log \frac{y}{x})^s}{\Gamma(i+\rho+s)} \right]. \quad (4.38)$$

By direct re-evaluation of the moments of eq. 4.37 it is easy to check that eq. 4.29 is satisfied. For the particular case of the non-singlet shown here the expression simplifies

considerably because the coefficients $c_{qq}(i, j)$ are 0 for $j \neq 0$. The details of the QCD evolution are contained in the b 's and are independent of the input distributions $\Delta q_{NS}(y, \mu^2)$. This makes the method fast as one only needs to evaluate the b 's once for a suitable set of $\frac{x}{y}$'s and r 's (This is of particular use for the singlet where the coefficients $c_{qq}(i, j)$ are not 0 for $j \neq 0$.)

Finally, we get for the singlets the following expressions analogous to 4.37 :

$$\Delta \Sigma(x, Q^2) = \int_x^1 \frac{dy}{y} [b_{qq}(x/y, r) \Delta \Sigma(y, \mu^2) + b_{qg}(x/y, r) \Delta g(y, \mu^2)] \quad (4.39)$$

and

$$\Delta g(x, Q^2) = \int_x^1 \frac{dy}{y} [b_{gq}(x/y, r) \Delta \Sigma(y, \mu^2) + b_{gg}(x/y, r) \Delta g(y, \mu^2)] \quad (4.40)$$

where $b_{qq}(x/y, r)$, $b_{gq}(x/y, r)$ and $b_{gg}(x/y, r)$ are equivalent to $b_{qq}(x/y, r)$ given in eq. 4.38 with $c_{qq}(i, j)$ replaced by $c_{qgk}(i, j)$, $c_{gqk}(i, j)$ and $c_{ggk}(i, j)$ respectively (k should be summed over). Here $k = \pm$ distinguishes the two sets of expansions necessary for each series of coefficients corresponding to the two distinct powers t_+ and t_- appearing in 4.19.

Choosing $p = 2$, and using

$$\sum_{j=1}^{n-1} \frac{1}{j} = C + \Psi(n) = C + \log n - \frac{1}{2n} - \frac{1}{12n^2} + \dots \quad (4.41)$$

one arrives at the following expansions

$$\begin{aligned} r^{-\gamma_{qq}(n)/(2\beta_0)} &= (az)^{\frac{16}{27}u} \left[1 - \frac{8u}{9} \frac{1}{z} + \left(\frac{-76u}{81} + \frac{32u^2}{81} \right) \frac{1}{z^2} + O(z)^{-3} \right] \\ w_1(n)r^{t_+(n)} &= (az)^{\frac{4}{3}u} \left[\frac{1}{(\log az)^2} \left(\frac{9}{25z^2} \right) + O(z, \log az)^{-3} \right] \\ w_2(n)r^{t_-(n)} &= (az)^{\frac{16}{27}u} \left[1 - \frac{8u}{9z} + \left(\frac{-76u}{81} + \frac{32u^2}{81} \right) \frac{1}{z^2} - \frac{1}{\log az} \frac{4u}{15z^2} \right. \\ &\quad \left. - \frac{1}{(\log az)^2} \frac{9}{25z^2} + O(z, \log az)^{-3} \right] \\ w_3(n)r^{t_+(n)} &= (az)^{\frac{4}{3}u} \left[\frac{1}{(\log az)} \left(-\frac{9}{10z} + \frac{9u}{5z^2} \right) - \frac{1}{(\log az)^2} \frac{27}{20z^2} + O(z, \log az)^{-3} \right] \end{aligned}$$

$$\begin{aligned}
w_3(n)r^{t-(n)} &= (az)^{\frac{16}{27}u} \left[\left(\frac{1}{(\log az)} \right) \left(\frac{9}{10} \frac{1}{z} + \frac{-4u}{5z^2} \right) + \left(\frac{1}{(\log az)^2} \right) \left(\frac{+27}{20z^2} \right) \right. \\
&\quad \left. + O(z, \log az)^{-3} \right] \\
w_4(n)r^{t+(n)} &= (az)^{\frac{4}{3}u} \left[\left(\frac{1}{(\log az)} \right) \left(\frac{-2}{5} \frac{1}{z} + \left(\frac{-6}{5} + \frac{4u}{5} \right) \frac{1}{z^2} \right) + \left(\frac{1}{(\log az)^2} \right) \left(\frac{-3}{5z^2} \right) \right. \\
&\quad \left. + O(z, \log az)^{-3} \right] \\
w_4(n)r^{t-(n)} &= (az)^{\frac{16}{27}u} \left[\left(\frac{1}{(\log az)} \right) \left(\frac{2}{5} \frac{1}{z} + \left(\frac{6}{5} + \frac{-16u}{45} \right) \frac{1}{z^2} \right) + \left(\frac{1}{(\log az)^2} \right) \left(\frac{3}{5z^2} \right) \right. \\
&\quad \left. + O(z, \log az)^{-3} \right] \\
w_2(n)r^{t+(n)} &= (az)^{\frac{4}{3}u} \left[1 - \frac{2u}{z} + \left(\frac{-37u}{9} + 2u^2 \right) \frac{1}{z^2} + \frac{1}{\log az} \frac{4u}{15z^2} \right. \\
&\quad \left. - \frac{1}{(\log az)^2} \frac{9}{25z^2} + O(z, \log az)^{-3} \right] \\
w_1(n)r^{t-(n)} &= (az)^{\frac{16}{27}u} \left[\frac{1}{(\log az)^2} \left(\frac{9}{25z^2} \right) + O(z, \log az)^{-3} \right] . \tag{4.42}
\end{aligned}$$

Here $a = e^{\mathbf{C} - \frac{3}{4}}$, $z = n + 2$ and $u = \log r = \log \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)$. Terms of higher order, as well as the algorithm that enables their calculation, are listed in the Appendix. The rate of convergence of equations 4.42 is a function of $\log r$; more terms need to be kept if the evolution is over a greater range of Q^2 . For the value of r considered in the next section terms up to order $(z \log az)^{-6}$ are more than sufficient. Because the expansions 4.34 are in powers of $\frac{1}{n+p}$ and $\frac{1}{\log(n+p)}$ the series 4.42 converges more rapidly for increasing n . Hence the method is most accurate at large x , and indeed the number of terms that are to be kept is primarily determined by the precision that is required as x approaches 0.

4.4 The First Moment Problem

For the polarized singlet, the expansions appearing in equation 4.42 do not converge for the first moment of both the quark and glue distributions. There are several ways to overcome this. Because all higher moments converge arbitrarily well one might choose to just add a δ -function at $x = 0$, this being the only function that influences the first moment exclusively. Alternatively, one may add to the expansions 4.42 a term like z^{-c} , with c being some large number. Again only the first moment is affected significantly.

Because of the arbitrariness in the choice of c this should be seen purely as a check that the distributions for non-zero x are not sensitive to the problem with the singlet's first moment. No significance should be attached to the predictions in the region that is affected by the term z^{-c} . When we come to use the formalism of this chapter for a physical situation this is the method that we will use because it turns out to be the simplest.

It is also possible to derive the asymptotic behaviour of the spin dependent structure functions for $x \sim 0$ (in a manner similar to that used by Martin [Mar79] for the spin averaged case) provided that the input distribution can be expanded in the following manner:

$$f(x, \mu^2) = \sum_{\beta} \omega_{\beta}(\mu^2) x^{\beta} \quad , \quad (4.43)$$

where β can be non-integer (for β less than -1 the distribution however has an undefined first moment). It is the purpose of this section to derive the analytic expressions for the spin-dependent case.

Let us write

$$\Delta\Sigma(x, \mu^2) = \sum_{\sigma} \omega_{\sigma}(\mu^2) x^{\sigma} \quad (4.44)$$

and

$$\Delta g(x, \mu^2) = \sum_{\gamma} \omega_{\gamma}(\mu^2) x^{\gamma} \quad . \quad (4.45)$$

For clarity we shall concentrate on one term of the above expressions - we shall see shortly that the dominant contribution will come from the term with the lowest σ or γ . Let us further concentrate on that part of $\Delta\Sigma(x, Q^2)$ that originates in $\Delta\Sigma(x, \mu^2)$ (see equation 4.19), i.e.

$$\Delta\Sigma(n, Q^2) = \left[w_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta\Sigma(n, \mu^2) \quad . \quad (4.46)$$

The result for the other terms will be shown at the end.

It is well known that a distribution can be reconstructed from its moments by performing the inverse Mellin Transform:

$$\Delta\Sigma(x, Q^2) = \frac{1}{2\pi i} \int_{N_0-i\infty}^{N_0+i\infty} dn x^{-n} \Delta\Sigma(n, Q^2) \quad , \quad (4.47)$$

where N_0 is arbitrary, as long as it is to the right of all singularities of $\Delta\Sigma(n, Q^2)$ [Doe50, page 262].

Using 4.44 we find

$$\Delta\Sigma(x, Q^2) = \frac{1}{2\pi i} \int_{N_0-i\infty}^{N_0+i\infty} dn \frac{x^{-n}}{n+\sigma} \frac{1}{2} \{r^{t_+(n)} + r^{t_-(n)} - \frac{\gamma_-}{R} (r^{t_+(n)} - r^{t_-(n)})\} \quad , \quad (4.48)$$

where the notation of Section 4.2 has been adopted. Using [GR80, eq.8.362 1]

$$\sum_{j=1}^{n-1} \frac{1}{j} = 1 - \frac{1}{n} - \frac{1}{n+1} + \sum_{k=2}^{\infty} \frac{n}{k(n+k)} \quad (4.49)$$

we see that the integrand has singularities at $n = -\sigma, 0, -1, -2, \dots$, all but the first arising from the anomalous dimensions 4.25.

If the integrand goes to zero sufficiently fast for large $|n|$ we may close the contour and thus equation 4.48 becomes a sum over residues. That this condition holds may readily be seen from the explicit form of the anomalous dimensions (equation 4.25). We shall need the following result to evaluate the residues, which may be proved by induction:

$$\lim_{x \rightarrow 0} \frac{d^k}{dn^k} x^{-n} f(n) = x^{-n} [(\log \frac{1}{x})^k f(n) + O(\log \frac{1}{x})^{k-1}] \quad . \quad (4.50)$$

Defining $r^{-1/4\beta_0} \equiv e^t$, we may expand

$$\frac{1}{2} (r^{t_+(n)} + r^{t_-(n)}) = e^{t\gamma_+} \sum_{i=0}^{\infty} \frac{t^{2i} R^{2i}}{(2i)!} \quad (4.51)$$

and

$$\frac{\gamma_-}{2R} (r^{t_+(n)} - r^{t_-(n)}) = \gamma_- e^{t\gamma_+} \sum_{i=0}^{\infty} \frac{t^{2i+1} R^{2i+1}}{(2i+1)!} \quad . \quad (4.52)$$

We see that the general form of the contribution of the singularity at $-\alpha$ becomes

$$\frac{1}{2\pi i} \oint dn x^{-n} \frac{1}{(n+\alpha)^q} \mathcal{F}(n) e^{\frac{\kappa}{n+\alpha}} \quad , \quad (4.53)$$

where $\mathcal{F}(n)$ is finite and does not vanish at $n=-\alpha$. Using equation 4.50 we obtain the contribution of this pole near $x \sim 0$ as

$$\frac{1}{\sqrt{4\pi}} \left(\sqrt{\frac{\log \frac{1}{x}}{\kappa}} \right)^{q-1} \frac{x^\alpha \mathcal{F}(-\alpha) e^{2\sqrt{\kappa \log \frac{1}{x}}}}{(\kappa \log \frac{1}{x})^{1/4}} , \quad (4.54)$$

where we have used

$$\lim_{z \rightarrow \infty} I_\nu(z) = \frac{e^z}{\sqrt{2\pi z}} \left(1 + O\left(\frac{1}{z}\right) \right) . \quad (4.55)$$

Equation 4.54 has two very useful properties: 1) Independently of q or c , the dominant contribution near $x \sim 0$ is that of the smallest α ; i.e. the rightmost pole dominates. 2) For any given α (which fixes c) the term with the largest power q dominates near $x \sim 0$.

This makes the rest of the analysis straightforward. One must distinguish the three cases of the lowest power σ being less than, equal to or greater than 0.

If σ is less than zero its pole dominates and we find

$$\Delta\Sigma(x, Q^2) \sim x^\sigma . \quad (4.56)$$

For $\sigma \geq 0$, the dominant term takes the form

$$\Delta\Sigma(x, Q^2) \sim \frac{\exp\left[\left(\sqrt{\frac{40}{3}} + \sqrt{\frac{32-f}{5}}\right)\sqrt{\frac{\log \frac{1}{r} \log \frac{1}{x}}{\beta_0}}\right]}{\left(\log \frac{1}{x}\right)^{\frac{3}{4} - \frac{1}{2}\delta_{0,\sigma}}} . \quad (4.57)$$

Note that as Q^2 increases, r decreases and hence $\Delta\Sigma(x, Q^2)$ becomes more divergent at $x \sim 0$.

Equivalent calculations for the other three terms contributing to the singlet yield expressions which have the same x behaviour as that above. Finally, for the non-singlet the behaviour near $x \sim 0$ is

$$\Delta q_{NS}(x, Q^2) \sim \begin{cases} x^\sigma & \sigma < 0 \\ \frac{\exp\left[32\sqrt{\frac{\log \frac{1}{r} \log \frac{1}{x}}{12\beta_0}}\right]}{\left(\log \frac{1}{x}\right)^{\frac{3}{4} - \frac{1}{2}\delta_{0,\sigma}}} & \sigma \geq 0 \end{cases} . \quad (4.58)$$

The primary deficiency of the above analysis is its limited range of applicability. The approximation 4.55 is valid for large z , corresponding to small x . The x dependence is however logarithmic, thus implying a slow convergence (for example for the non-singlet and for $r = 0.5$, corresponding to evolution from, say, $Q^2 = 1 \text{ GeV}^2$ to $Q^2 = 25 \text{ GeV}^2$, the approximation 4.55 gives about 75 % of the true value of the Bessel function $I_1(z)$ for $x \sim 0.007$). The approximation gets better if the range of evolution is increased. For the purpose of the work presented in this thesis, which is primarily concerned with evolving structure functions up to $Q^2 \sim 10 \text{ GeV}^2$, we shall prefer to use the method outlined at the start of this section on page 84.

Before presenting numerical results one more comment should be made. It is well known that Regge theory may be used to predict the small x behaviour of these spin structure functions [Hei73]. Its predictions are independent of Q^2 while the above analysis yields asymptotic behaviour that is Q^2 dependent, at least for $\sigma \geq 0$. This is not necessarily a contradiction (indeed, for $\sigma < 0$ there is none) because it should be remembered that the only term that has been calculated is the twist-two, leading logarithmic order one. It may well be that higher order corrections change the small x behaviour.

4.5 Numerical Results

This section is devoted to exhibiting some numerical results that will give a guide to the accuracy of the procedure developed in this chapter. For definiteness we shall use input distributions of the form

$$\begin{aligned}\Delta q_\Sigma(x, \mu^2) &= \Delta \Sigma(x, \mu^2) = 0.6 \frac{105}{16} x^{\frac{1}{2}} (1-x)^2 \\ \Delta q_3(x, \mu^2) &= 1.25 \frac{105}{16} x^{\frac{1}{2}} (1-x)^2 \\ \Delta g(x, \mu^2) &= 0\end{aligned}\tag{4.59}$$

The results corresponding to input distributions that are physically motivated will be presented in Chapter 6.

Table 4.1: Accuracy of evolution of the non-singlet $\Delta q_3(x, r)$ as a function of r , with $N_{max}=9$. The entries are the reconstructed moments divided by the exact moments as calculated from equation 4.29.

n	r=0.1	r=0.3	r=0.5	r=0.7
1	0.952	0.985	0.993	0.997
2	0.997	0.999	1.000	1.000
3	1.000	1.000	1.000	1.000
4	1.000	1.000	1.000	1.000

Table 4.2: Accuracy of evolution of the singlet $\Delta\Sigma(x, r)$ as a function of r , with $N_{max}=9$ and $c=16$. The entries are the approximated moments divided by the exact moments for the terms proportional to $\Delta\Sigma(x, 1)$ (I) and $\Delta g(x, 1)$ (II). Note that the first moment of the term originating from $\Delta g(x, 1)$ is 0.

n	r=0.1		r=0.3		r=0.5		r=0.7	
	I	II	I	II	I	II	I	II
1	1.000	-	1.000	-	1.000	-	1.000	-
2	0.995	0.936	1.000	0.968	1.000	0.971	1.000	0.971
3	1.000	0.996	1.000	0.999	1.000	0.999	1.000	0.999
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

In Figure 4.2 and Table 4.1 we show the result of evolving the nonsinglet over various ranges of Q^2 , the corresponding information for the both the quark and glue singlets being shown in Figures 4.3, 4.4 and Tables 4.2, 4.3. $N_{max}=9$ terms in the expansions 4.42 have been kept, and the convergence of the first moment of the singlet is guaranteed by the addition of a term proportional z^{-c} , where $c=16$.

The area under the non-singlet curves is constant - a consequence of massless QCD, where the probability for a gluon to create a $q\bar{q}$ pair is independent of flavour. The area under the quark singlet curves is also constant because massless QCD conserves helicity. The area under the gluon curves however increases - the virtual photon probe resolves an

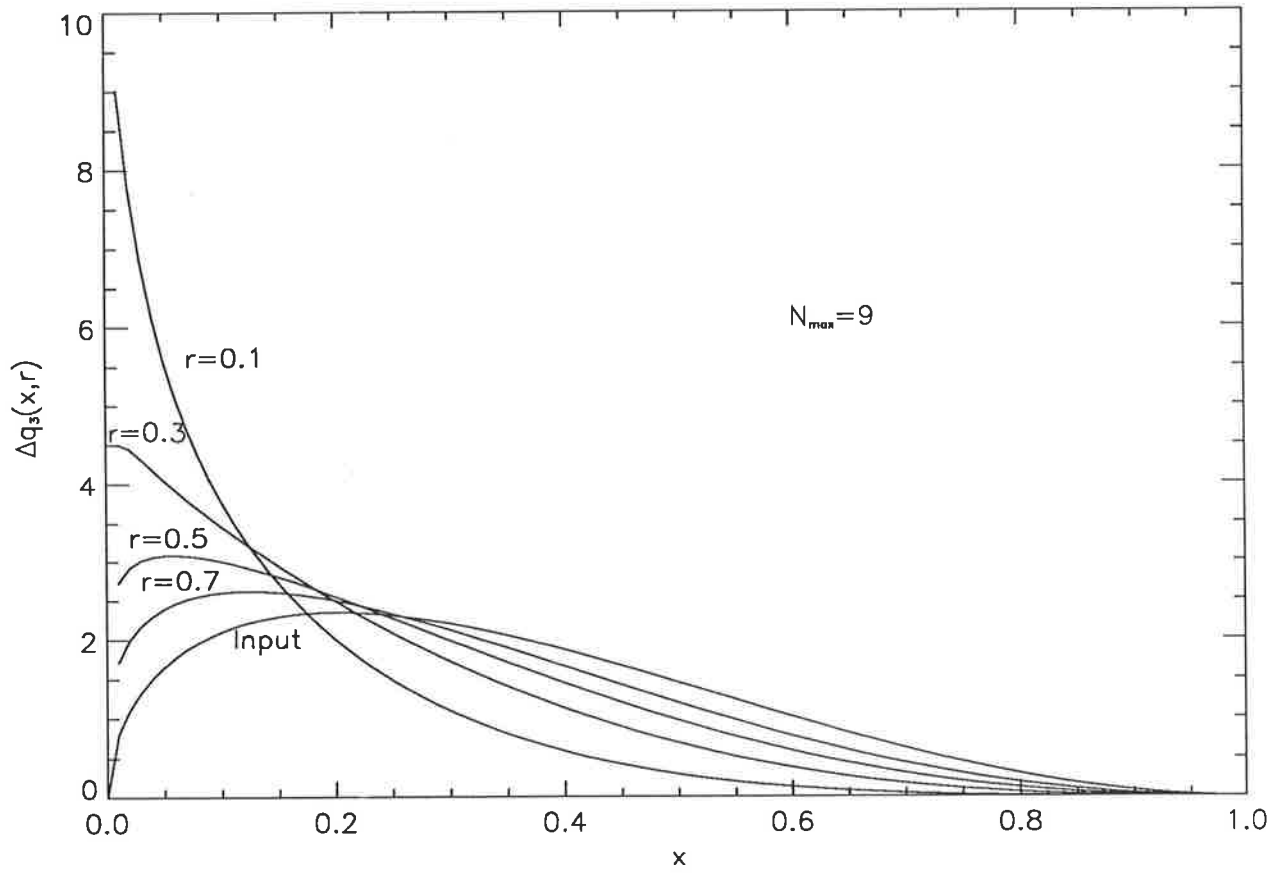
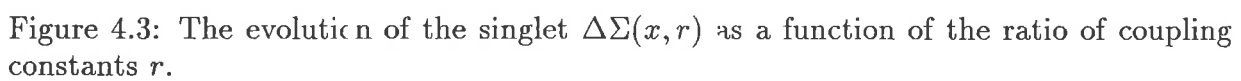


Figure 4.2: The evolution of the non-singlet $\Delta q_3(x, r)$ as a function of the ratio of coupling constants r .



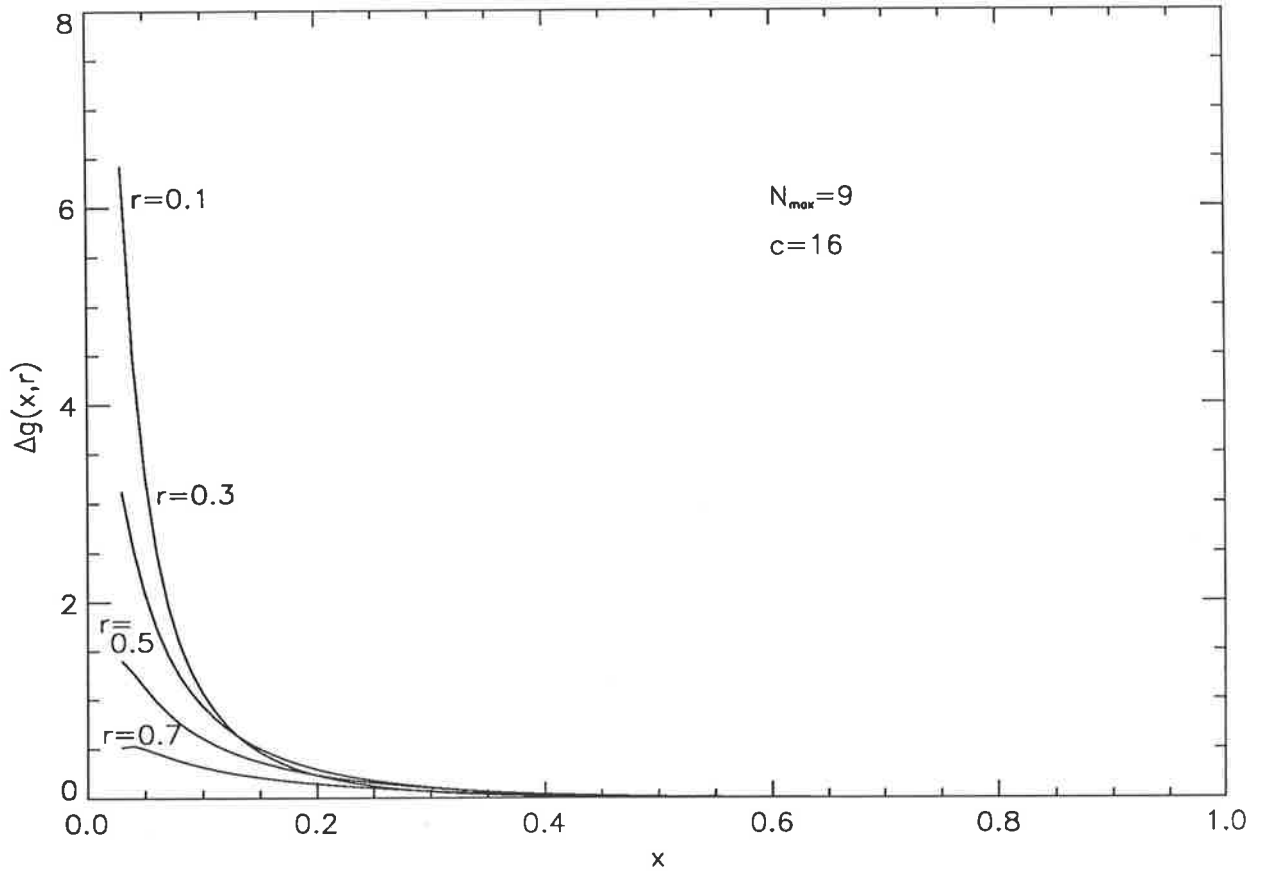


Figure 4.4: The evolution of the singlet $\Delta g(x, r)$ as a function of the ratio of coupling constants r .

Table 4.3: Accuracy of evolution of the singlet $\Delta g(x, r)$ as a function of r , with $N_{max}=9$ and $c=16$. The entries are the approximated moments 4.29 divided by the exact moments for the terms proportional to $\Delta \Sigma(x, 1)$ (I) and $\Delta g(x, 1)$ (II).

n	r=0.1		r=0.3		r=0.5		r=0.7	
	I	II	I	II	I	II	I	II
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.969	1.038	0.941	0.985	0.947	0.992	0.952	0.997
3	0.985	0.995	0.990	0.997	0.993	0.999	0.995	1.000
4	0.996	0.998	0.998	0.999	0.999	1.000	0.999	1.000

increasing number of gluons as its momentum transfer increases, even though there were no gluons present at the starting scale. The increase with Q^2 in the number of partons implies that each parton must carry a smaller fraction of the hadron's momentum. This is clearly visible in the tendency for all the curves to peak at smaller x as r is decreased.

In order to obtain an idea how many terms it might be necessary to keep in 4.29, we show in Figure 4.5 and Table 4.4 the effect of keeping only a limited number N_{max} of terms for the non-singlet. The equivalent curves for the singlet are shown in Figures 4.6, 4.7 and Tables 4.5, 4.6. (Note that calculating $N_{max} = 14$ for the singlet is computationally not feasible) All curves are calculated using $r = 0.1$. If r is decreased the series converges more slowly. Most of the work presented in this thesis will in fact involve larger r , so that the case presented here is in fact a 'worst case'.

$N_{max} = 4$ is the number of terms kept by Gonzalez-Arroyo et al. [GLY80] in their evaluation of the spin-independent structure functions. For the ratio of r used it is a good approximation down to $x \sim 0.1 - 0.2$. Keeping 9 terms allows sensible predictions down to $x \sim 0.02 - 0.04$. This is the number of terms that we shall keep in all the calculations that follow.

The final thing that needs to be done is to check the validity of adjusting the first moment by hand. In Figures 4.8, 4.9 and Tables 4.7, 4.8 we show curves calculated

Table 4.4: Accuracy of evolution of the non-singlet $\Delta q_3(x, r = 0.1)$ as a function of N_{max} . The entries are the reconstructed moments divided by the exact moments as calculated from equation 4.29.

n	$N_{max}=4$	$N_{max}=9$	$N_{max}=14$
1	0.782	0.952	0.980
2	0.950	0.997	1.000
3	0.984	1.000	1.000
4	0.994	1.000	1.000

Table 4.5: The convergence of the series for $\Delta\Sigma(x, r = 0.1)$ as a function of N_{max} , with $c=16$. Note that the first moment of the part originating from $\Delta g(x, 1)$ is 0.

n	$N_{max}=4$		$N_{max}=9$	
	I	II	I	II
1	1.000	-	1.000	-
2	0.930	0.843	0.995	0.936
3	0.978	0.944	1.000	0.996
4	0.992	0.975	1.000	1.000

Table 4.6: The convergence of the series for $\Delta g(x, r = 0.1)$ as a function of N_{max} , with $c=16$.

n	$N_{max}=4$		$N_{max}=9$	
	I	II	I	II
1	1.000	1.000	1.000	1.000
2	0.649	0.721	0.969	1.038
3	0.754	0.796	0.985	0.995
4	0.862	0.895	0.996	0.998

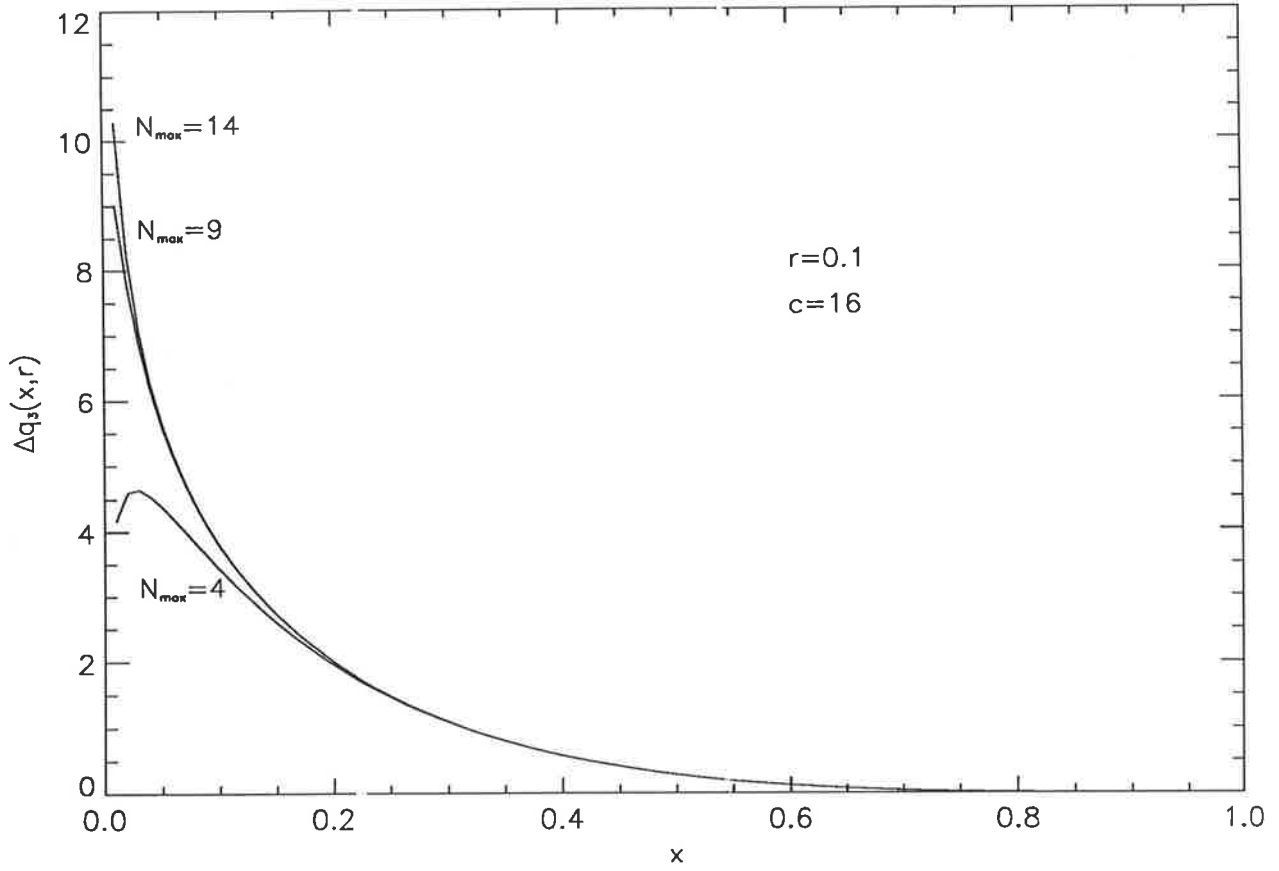


Figure 4.5: The convergence of the series for $\Delta q_3(x, r = 0.1)$ as a function of N_{max} . The input distribution is that of the non-singlet $\Delta q_3(x, 1)$.

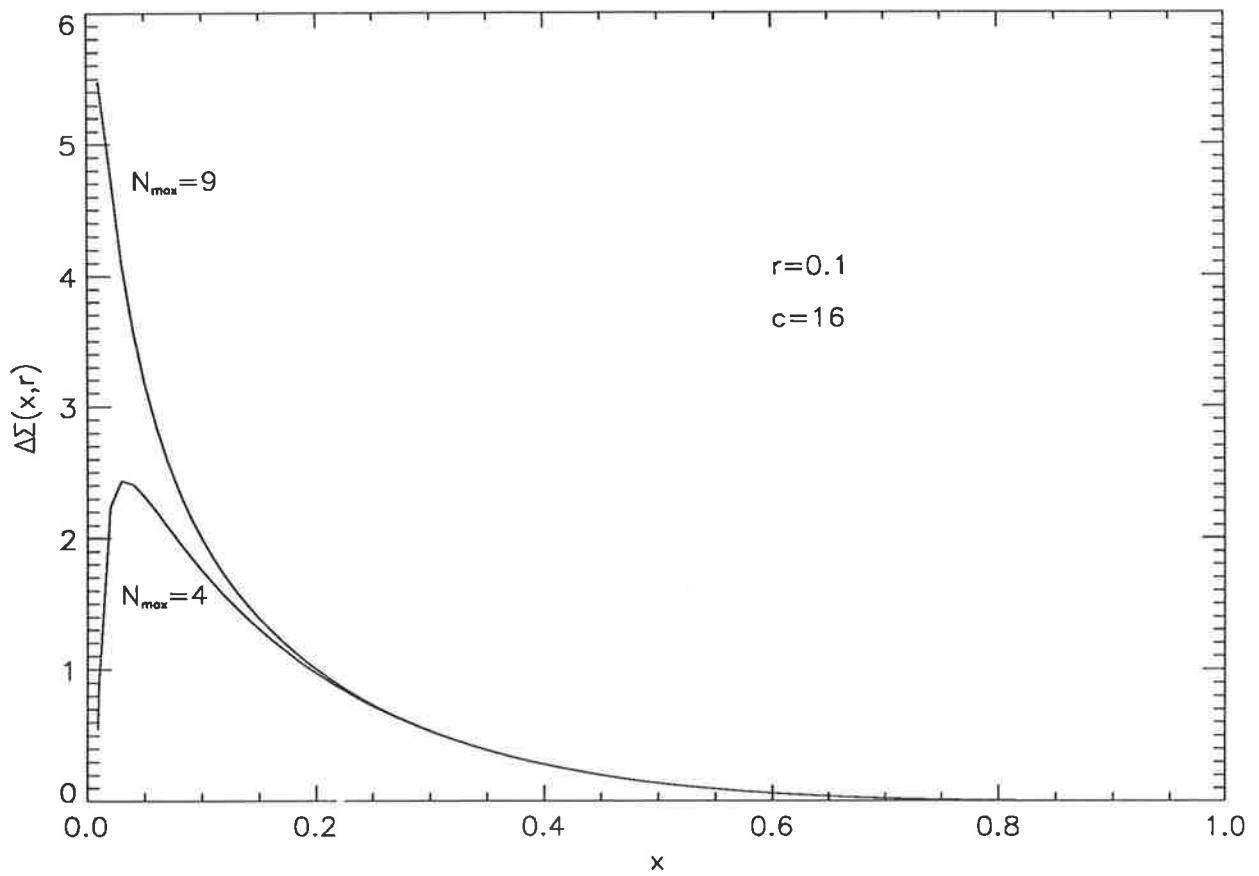


Figure 4.6: The convergence of the series for $\Delta\Sigma(x, r = 0.1)$ as a function of N_{max} .

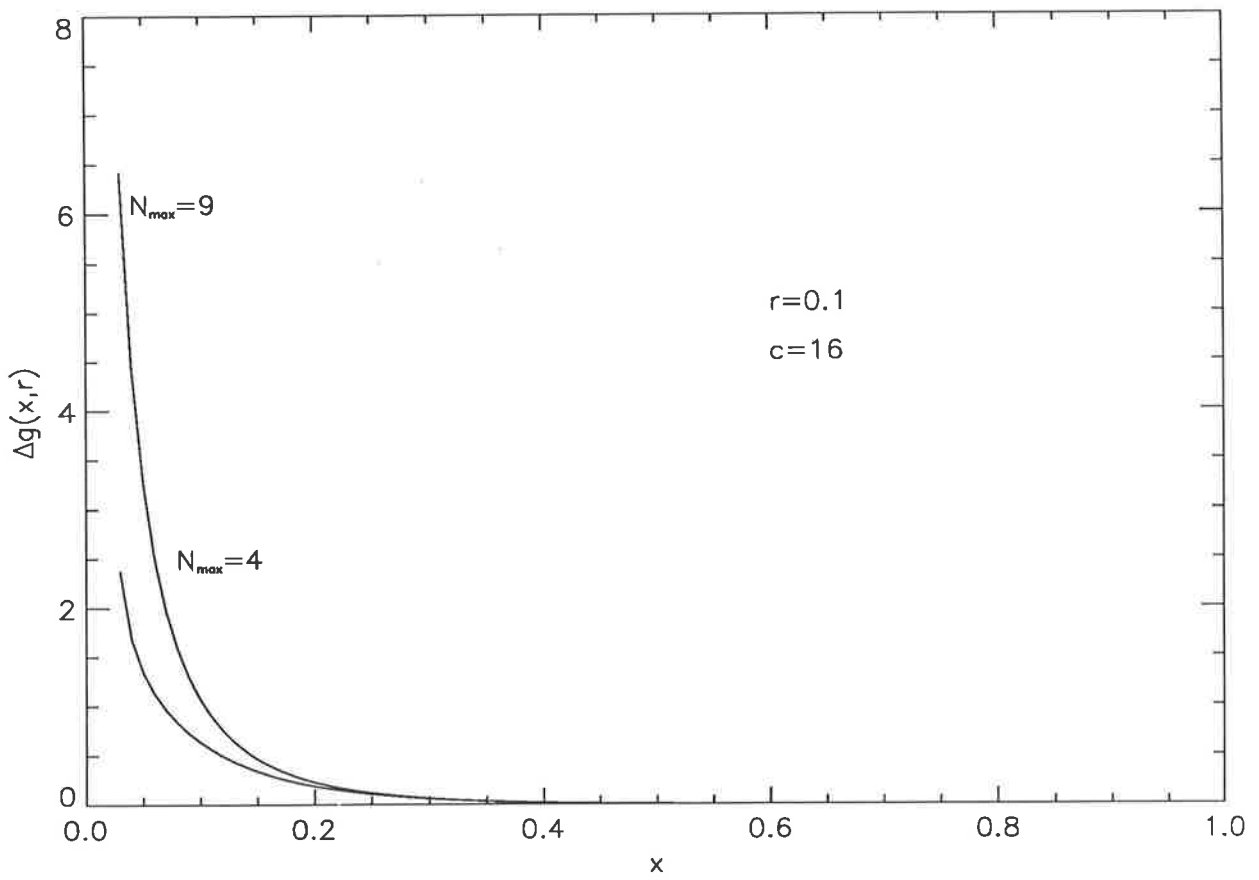


Figure 4.7: The convergence of the series for $\Delta g(x, r = 0.1)$ as a function of $N_{\max}$.

Table 4.7: The moments corresponding to $\Delta\Sigma(x, r = 0.1)$ as a function of c . Note that the first moment of the part originating from $\Delta g(x, 1)$ is 0.

n	No term		c=10		c=13		c=16	
	I	II	I	II	I	II	I	II
1	1.093	-	1.000	-	1.000	-	1.000	-
2	0.998	1.010	0.986	0.596	0.993	0.835	0.995	0.936
3	1.000	1.001	0.998	0.892	0.999	0.977	1.000	0.996
4	1.000	1.000	1.000	0.969	1.000	0.996	1.000	1.000

Table 4.8: The moments corresponding to $\Delta g(x, r = 0.1)$ as a function of c . The entries are the approximated moments divided by the exact moments for the terms proportional to $\Delta\Sigma(x, 1)$ (I) and $\Delta g(x, 1)$ (II).

n	No term		c=10		c=13		c=16	
	I	II	I	II	I	II	I	II
1	0.287	0.257	1.000	1.000	1.000	1.000	1.000	1.000
2	0.872	0.861	1.419	1.854	1.102	1.280	0.969	1.038
3	0.978	0.977	1.134	1.351	1.011	1.058	0.985	0.995
4	0.995	0.996	1.040	1.123	1.001	1.011	0.996	0.998

for the case $r = 0.1$, the convergence of the first moment having been assured with the addition of a term z^{-c} , with $c=10, 13, 16$. The curve with no added term is also shown. We see that the addition of something like z^{-16} leaves the region above $x \sim 0.05$ and hence the higher moments ($n > 1$) unaffected. This is the value of c that we shall use from now on, so we should take care not to read too much into the results below $x \sim 0.05$. (Note that when one considers the actual structure functions, rather than the quark and gluon distributions themselves, the problem becomes numerically less important because the nonsinglet evolution does not suffer from the problem with the first moment.)

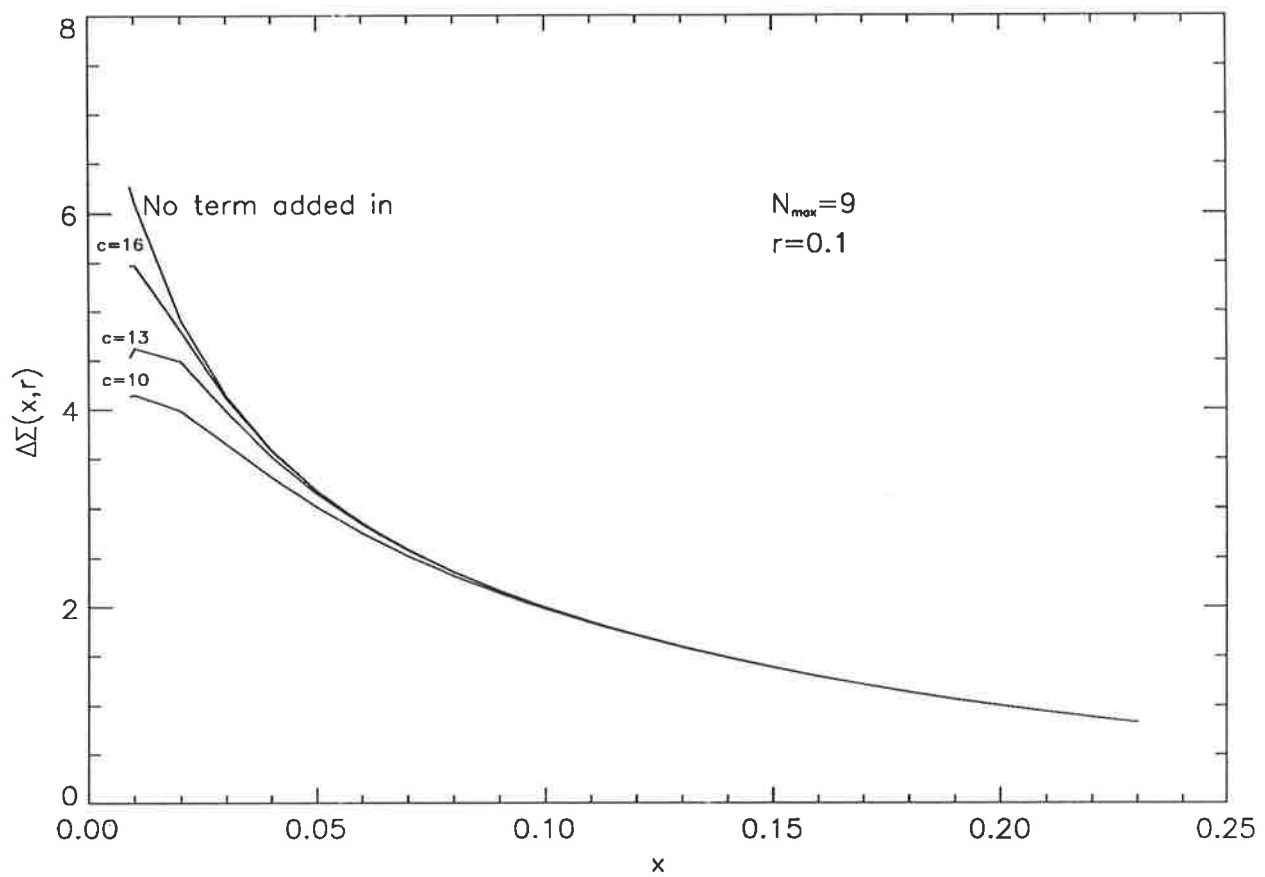


Figure 4.8: The effect of the term z^{-c} on the evolution of the quark singlet $\Delta\Sigma(x, 1)$.

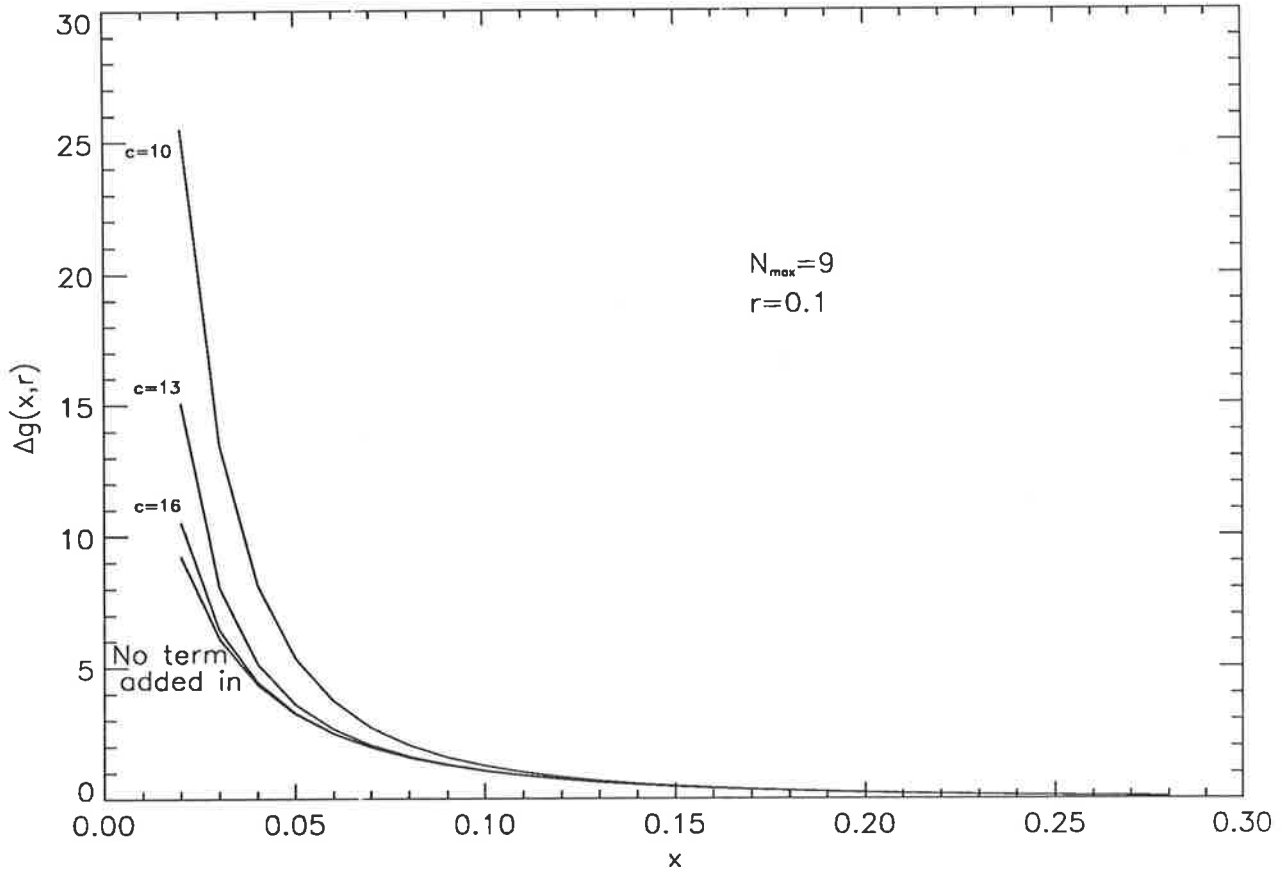


Figure 4.9: The effect of the term z^{-c} on the evolution of the glue singlet $\Delta g(x, 1)$.

4.6 Conclusion

In this chapter we have presented the details of a formalism that enables us to predict the behaviour of the spin dependent quark and glue distributions at any Q^2 , given that they are known at some reference scale μ^2 . The QCD evolution has been worked out to leading twist and leading logarithmic order, although the formalism can easily be extended to higher order. We have also presented the analytic form of the distributions near $x \sim 0$ which will be useful for predictions of distributions at very high Q^2 . Finally, we have examined the accuracy of the numerical results for a test case.



Chapter 5

The spin structure function in the cloudy bag model

5.1 Introduction

In Chapter 2 we derived the expression for the moments of the spin structure function of a nucleon polarized in the positive z direction (see equation 2.55). In particular for the first moment, using the result of Chapter 4 that the first moment doesn't evolve with Q^2 , we obtained that

$$g_1^{n,p} = \int_0^1 g_1^{n,p}(x) dx = \langle n, p | \sum_i c_i \bar{\Psi}(0) \lambda_i \gamma^z \gamma^5 \Psi(0) | n, p \rangle, \quad (5.1)$$

where the states are normalized as

$$\langle \vec{p} | \vec{p}' \rangle = (2\pi)^3 \delta(\vec{p} - \vec{p}') \quad (5.2)$$

and the coefficients c_i were given in equation 2.47.

We have not included higher order QCD corrections here. Furthermore, we shall not include possible contributions from the anomaly [AR88,ET88] (see Section 6.2 for a discussion of the role of the anomaly). We note that the matrix element appearing on the right side of equation 5.1 is translationally invariant and we can therefore replace it by

$$g_1^{n,p} = \langle n, p | \int d^4x e^{iq \cdot x} \sum_i c_i \bar{\Psi}(x) \lambda_i \gamma^z \gamma^5 \Psi(x) | n, p \rangle |_{q=0} . \quad (5.3)$$

(Implicit in 5.3 is a normalization of the states differing from 5.2 by a four dimensional volume factor.) This is a matrix element corresponding to zero momentum transfer and for the case of the nucleon being at rest can therefore be evaluated in any model that describes the structure of a static nucleon. Note that the introduction of intermediate states needed in Chapter 3 to ensure vanishing support for $x > 1$ is not necessary here, i.e. the expression 5.3 does not suffer the same normalization problem encountered in Section 3.7.

It is the purpose of this chapter to evaluate equation 5.3 in the cloudy bag model. In Section 5.2 we give a review of the relevant ingredients of the cloudy bag model, the main result being the modified wavefunction of the nucleon. This will be used in Section 5.3 in order to evaluate g_1^p and g_1^n . In Section 5.4 centre of mass corrections are discussed.

Before we turn to the cloudy bag model, let us discuss its relevance. In order to do this it is instructive to link equation 5.3 to a matrix element that can actually be measured. Because the neutron and proton carry equal strangeness and are isospin partners, only the component proportional to λ_3 will contribute differently to g_1^p and g_1^n . The difference of g_1^p and g_1^n can therefore be written as

$$\begin{aligned} g_1^p - g_1^n &= \frac{1}{12} \int d^4x e^{iq \cdot x} \langle p | \bar{\Psi}(x) [\lambda_3, \lambda_+] \gamma^z \gamma^5 \Psi(x) | n \rangle |_{q=0} \\ &= \frac{1}{6} \int d^4x e^{iq \cdot x} \langle p | \bar{\Psi}(x) \lambda_+ \gamma^z \gamma^5 \Psi(x) | n \rangle |_{q=0} , \end{aligned} \quad (5.4)$$

where we have used the commutation relations of the SU(3) flavour matrices. The right hand side of equation 5.4 is the Gamow-Teller part of the matrix element governing the weak decay of the neutron, which at zero four momentum transfer is the axial charge $|g_a|$. We therefore obtain the Bjørken Sum Rule [Bjo66]

$$g_1^p - g_1^n = \frac{1}{6} \left| \frac{g_a}{g_v} \right| . \quad (5.5)$$

(g_v is inserted here because the weak decay in fact also involves the Cabibbo angle, which cancels in the ratio.) Although of course equations 5.3 and 5.5 contain the same physics, it is perhaps better to discuss the motivation for the work described in this chapter by using the latter because the value of $|\frac{g_a}{g_v}|$ has been derived previously in many models [CJTT74,TTM81,Tho84]

It is well known [Clo79] that $|\frac{g_a}{g_v}|$ is $\frac{5}{3}$ for the 56-plet wavefunctions of SU(6) if one assumes that the quarks in the nucleon are non-relativistic. This should be compared to the experimental value of 1.259 ± 0.004 [Par88]. The discrepancy is explained by the observation that the nucleon has a size of about 1 fm and therefore its constituents must have momenta of about $\frac{\hbar}{1fm} \sim 0.2$ GeV/c, i.e. the constituents are highly relativistic. This implies that the nucleon spin is not only carried by the spin of the quarks, as assumed in the non-relativistic quark model, but also by the angular momentum corresponding to their transverse momenta. One would therefore expect the real spin carried by the quarks to be somewhat less than that of the proton. This idea was made more quantitative in the MIT bag model which gives $|\frac{g_a}{g_v}| = 1.09$ [CJTT74]. (The statements in this paragraph are somewhat inexact. More precisely, the wavefunction of a confined quark is neither an eigenstate of angular momentum nor spin but of the total angular momentum. One therefore only expects that $\sum \langle S_z \rangle + \langle L_z \rangle = \frac{1}{2}$, where the sum is over all constituents of the proton. $|\frac{g_a}{g_v}|$ is only sensitive to the expectation value $\langle S_z \rangle$.)

The MIT bag model however has a well known deficiency which is potentially very relevant to its spin structure - its non-conservation of helicity. This problem arises because the bag surface is a scalar object. A quark must have the same total angular momentum before and after "colliding" with the boundary and hence the expectation value of its helicity must change. This is not a property exhibited by massless QCD so that we may suspect that at least in this respect the MIT bag does not quite reflect reality.

Because of this we are naturally led to consider a model that explicitly incorporates chiral symmetry, e.g. the cloudy bag model. Of course one could also use other models

Before we turn to the technical details of the cloudy bag model, let us remind the reader once more about the reason for using this (or other) models in structure function calculations in the first place. We use them as effective theories designed to mimic certain aspects of QCD. For example, in Chapter 3, we used the MIT bag to describe the (suspected) confining property of QCD. Similarly, in this chapter, we couple the MIT bag to a pion field as a phenomenologically successful way of implementing the chiral symmetry of massless QCD. The model's pointlike coupling of the pion to the nucleon is an approximation to the more complex, presently not calculable, coupling between the two that must take place in reality. In a similar manner, the model approximates the pion as a point particle. Clearly this approximation must fail if we consider processes in which the sub-structure of the pion would be expected to play a role. We are thinking here particularly about the contribution, already mentioned in Section 3.8.2, where the incoming photon interacts with the pion itself. Processes such as this would be calculable, for example, in a convolution model (see equation 3.90 and [Sul72,ST87b]). Indeed it would be absolutely necessary to calculate them for the spin independent structure functions in order to obtain the correct sum rules. For the spin dependent structure functions which we calculate in this chapter these terms are however not necessary as they must cancel because the pion is a spin-0 object; the standard SU(6) spin-flavour wavefunction for the pion is antisymmetric in spin and hence the probability to observe a quark with a particular flavour must be the same for both spin states of the quark, leading to a vanishing contribution to g_1 . We therefore concentrate in the rest of this chapter on the terms in which the photon interacts with the baryon itself.

with the same property, which in fact was done by others (with similar results) after this work was completed [BEK88,DEF88,HM88,HM89,Wak89].

The physical mechanism enabling helicity conservation in the cloudy bag model is the possibility of angular momentum being carried by the pions. This produces a well defined mixing of the SU(6) MIT bag wavefunction of the nucleon in terms of bare nucleons, deltas and pions. For the structure functions of the nucleons one therefore expects two effects: the average spin carried by the quarks will be smaller than in the MIT bag (in fact, $|\frac{g_a}{g_v}| \sim 1.00$ in the CBM) but moreover the mixed wavefunction suggests that the neutron structure function may become non-zero (Note that it is the spin - isospin structure of the 56-plet SU(6) wavefunction that yields a vanishing g_1^n .)

(continued opposite)

5.2 The Cloudy Bag Model

The Lagrangian of the cloudy bag model is given by [TTM80,TTM81,TMT82,Tho84]

$$\begin{aligned} \mathcal{L}(x) = & \left[\sum_{a=1}^3 \frac{i}{2} \bar{q}_a(x) \vec{\partial} q_a(x) - B \right] \theta_v + \frac{1}{2} \left[(D_\mu \phi(x)) \cdot (D^\mu \phi(x)) - m_{0\pi}^2 \phi(x) \cdot \phi(x) \right] \\ & - \frac{1}{2} \sum_{a=1}^3 \bar{q}_a(x) \epsilon x p(i\tau \cdot \phi(x) \gamma_5 f_{0\pi}) q_a(x) \Delta_s . \end{aligned} \quad (5.6)$$

Here B is the bag pressure, θ_v is a step function which is 1 inside the cavity, ϕ is the pion field and $q_a(x)$ is the quark field with colour a, Δ_s is a surface delta function, $m_{0\pi}$ is the (bare) pion mass, τ is a Pauli matrix in isospin space and $f_{0\pi}$ is related to the pion decay constant through

$$f_{0\pi} = f_\pi \frac{m_\pi^2}{m_{0\pi}^2} . \quad (5.7)$$

The first and second terms correspond to non-interacting quarks confined in a cavity and free pions respectively while the third term is responsible for the chiral symmetry

restoration in the cloudy bag model. (Actually equation 5.6 is not quite chirally symmetric because of the non-zero pion mass introduced in the second term. This is, of course, intentional and a reflection of PCAC.)

The energy momentum Tensor for the cloudy bag model is given by

$$T^{\mu\nu}(x) = \frac{\partial \mathcal{L}(x)}{\partial(\partial_\mu q)}(\partial^\nu q) - g^{\mu\nu} \mathcal{L}(x) \quad , \quad (5.8)$$

which yields a Hamiltonian

$$\tilde{H} = \int dx T^{00}(x) \quad . \quad (5.9)$$

We shall only work in the space of non-strange baryons, in which case we have

$$H = H_0 + H_I \quad , \quad (5.10)$$

where

$$H_0 = m_{0N} N^\dagger N + m_{0\Delta} \Delta^\dagger \Delta + \int dk \, w_{0k} \, \mathbf{a}_k^\dagger \mathbf{a}_k \quad . \quad (5.11)$$

Here N, Δ ($N^\dagger, \Delta^\dagger$) are bare baryon annihilation (creation) operators, m_{0N} and $m_{0\Delta}$ are the bare baryon masses, and $\mathbf{a}_k$ ($\mathbf{a}_k^\dagger$) annihilates (creates) a pion with momentum $\mathbf{k}$ and energy w_{0k} .

H_I is [TMT82]

$$H_I = \int dk (\mathbf{V}_{0k} \cdot \mathbf{a}_k + \mathbf{V}_{0k}^\dagger \cdot \mathbf{a}_k^\dagger) \quad , \quad (5.12)$$

where

$$\mathbf{V}_{0k} = \sum_{\alpha\beta} \alpha^\dagger v_{0k}^{\alpha\beta} \beta, \quad \alpha, \beta \in \{N, \Delta\} \quad (5.13)$$

and

$$v_k^{\alpha\beta} = \frac{if_0^{\alpha\beta}}{m_\pi} \frac{\sqrt{4\pi} u(kR)}{\sqrt{2w_{0k}} \sqrt{(2\pi)^3}} (\mathbf{S}^{\alpha\beta} \cdot \mathbf{k}) \mathbf{T}^{\alpha\beta} \quad (5.14)$$

$$\mathbf{S}^{\alpha\beta} = \sum_m C(S_\beta 1 \rightarrow S_\alpha | s_\beta m \rightarrow s_\alpha) \hat{\mathbf{s}}_m^* \cdot \sqrt{3} \quad (5.15)$$

$$\mathbf{T}^{\alpha\beta} = \sum_n C(T_\beta 1 \rightarrow T_\alpha | t_\beta n \rightarrow t_\alpha) \hat{\mathbf{t}}_n^* \cdot \sqrt{3} \quad (5.16)$$

$$\begin{aligned} f_0^{N\vee} &= f_0^{\Delta\Delta} = \frac{5}{4\sqrt{2}} f_0^{N\Delta} = \frac{5}{2\sqrt{2}} f_0^{\Delta N} \\ &= \frac{5}{18\sqrt{4\pi}} \frac{m_\pi}{f_{0\pi}} \frac{\Omega}{\Omega - 1} \end{aligned} \quad (5.17)$$

$\hat{\mathbf{s}}_m^*$ and $\hat{\mathbf{t}}_n^*$ are spherical spin and isospin vectors respectively, the C 's are Clebsch-Gordan coefficients, $u(kR)$ is the form factor for the cloudy bag ($u(kR) = j_0(kR) + j_2(kR)$) and Ω/R is the lowest energy eigenvalue of the MIT bag.

The bare and dressed nucleon states are defined through

$$H_0 |N_{H_0}\rangle = m_{0N} |N_{H_0}\rangle \quad (5.18)$$

and

$$H |N_H\rangle = m_N |N_H\rangle \quad (5.19)$$

respectively, m_N being the dressed nucleon's mass (we shall neglect the kinetic energy of the baryons). Similar expressions hold for the Δ .

Our aim is to express all the relevant physics of the cloudy bag model in the wavefunction of the nucleon. To this end we shall define

$$|N_H\rangle = |N_H^0\rangle + |N_H^I\rangle, \quad (5.20)$$

with

$$|N_H^0\rangle = (|N_{H_0}\rangle \langle N_{H_0}|) |N_H\rangle \equiv \sqrt{Z_2^N} |N_{H_0}\rangle \quad (5.21)$$

and

$$|N_H^I\rangle = (1 - |N_{H_0}\rangle \langle N_{H_0}|) |N_H\rangle \equiv \Lambda |N_H\rangle. \quad (5.22)$$

Using 5.18 and 5.19 we then obtain

$$H_0 (|N_H^0\rangle + |N_H^I\rangle) + H_I |N_H\rangle = m_N (|N_H^0\rangle + |N_H^I\rangle), \quad (5.23)$$

i.e.

$$(m_N - H_0)|N_H^I\rangle = (H_0 - m_N)|N_H^0\rangle + H_I|N_H\rangle \quad . \quad (5.24)$$

Noting that $|N_H^I\rangle$ is an eigenstate of Λ while $|N_H^0\rangle$ is annihilated by it, and that Λ commutes with H_0 , one then only needs to operate with Λ on both sides of 5.24 to obtain

$$|N_H\rangle = \sqrt{Z_2^N}|N_{H_0}\rangle + \frac{1}{m_N - H_0}\Lambda H_I|N_H\rangle \quad , \quad (5.25)$$

i.e.

$$|N_H\rangle = \sqrt{Z_2^N} \left(1 - \frac{1}{m_N - H_0}\Lambda H_I\right)^{-1} |N_{H_0}\rangle \quad . \quad (5.26)$$

Assuming that the π -baryon coupling constants are small we shall keep only the terms up to first order in H_I (i.e. we shall keep only the terms with 1 pion):

$$|N_H\rangle \sim \sqrt{Z_2^N} \left(1 + \frac{1}{m_N - H_0}\Lambda H_I\right) |N_{H_0}\rangle \quad (5.27)$$

Finally then we get

$$|N_H\rangle \simeq \sqrt{Z_2^N} \left(1 + \frac{1}{m_N - H_0}H_I\right) |N_{H_0}\rangle \quad , \quad (5.28)$$

where we have made use of the fact that $\langle N_0 | H_I | N_0 \rangle = 0$.

5.3 g_1^p and g_1^n in the cloudy bag model

We are now in a position to evaluate the typical matrix element diagonal in spin and isospin $\langle N_H | \mathcal{O} | N_H \rangle$. By using equation 5.28 as well as equations 5.12 to 5.17, noting that

$$\langle 0 | a_{\mathbf{k}}^i a_{\mathbf{k}'}^{\dagger j} | 0 \rangle = \delta_{ij} \delta(\mathbf{k} - \mathbf{k}') \quad , \quad (5.29)$$

(i and j being isospin indices) and inserting appropriate intermediate states, we obtain straightforwardly

$$\langle N_H | \mathcal{O} | N_H \rangle = Z_2 \{ \langle N_{H_0} | \mathcal{O} | N_{H_0} \rangle + M_\pi \} \quad , \quad (5.30)$$

where

$$M_\pi = \frac{9}{4\pi^2 m_\pi^2} \sum_{\alpha, \beta} f_0^{N_0 \beta} f_0^{N_0 \alpha} \int \frac{d\mathbf{k} u^2(kR) k_z^2}{w_{0k}(m_{N_0} - m_\alpha - w_{0k})(m_{N_0} - m_\beta - w_{0k})} \cdot \mathcal{T}(\alpha, \beta, N_0) \mathcal{S}(\alpha, \beta, N_0) \langle \alpha, t_\alpha, S_d | \mathcal{O} | \beta, t_\beta, S_\beta \rangle \quad . \quad (5.31)$$

Here

$$\mathcal{T}(\alpha, \beta, N_0) = \sum_n C(T_\beta 1 \rightarrow T_{N_0} | t_\beta m \rightarrow t_{N_0}) C(T_\alpha 1 \rightarrow T_{N_0} | t_\alpha m \rightarrow t_{N_0}) \quad (5.32)$$

and

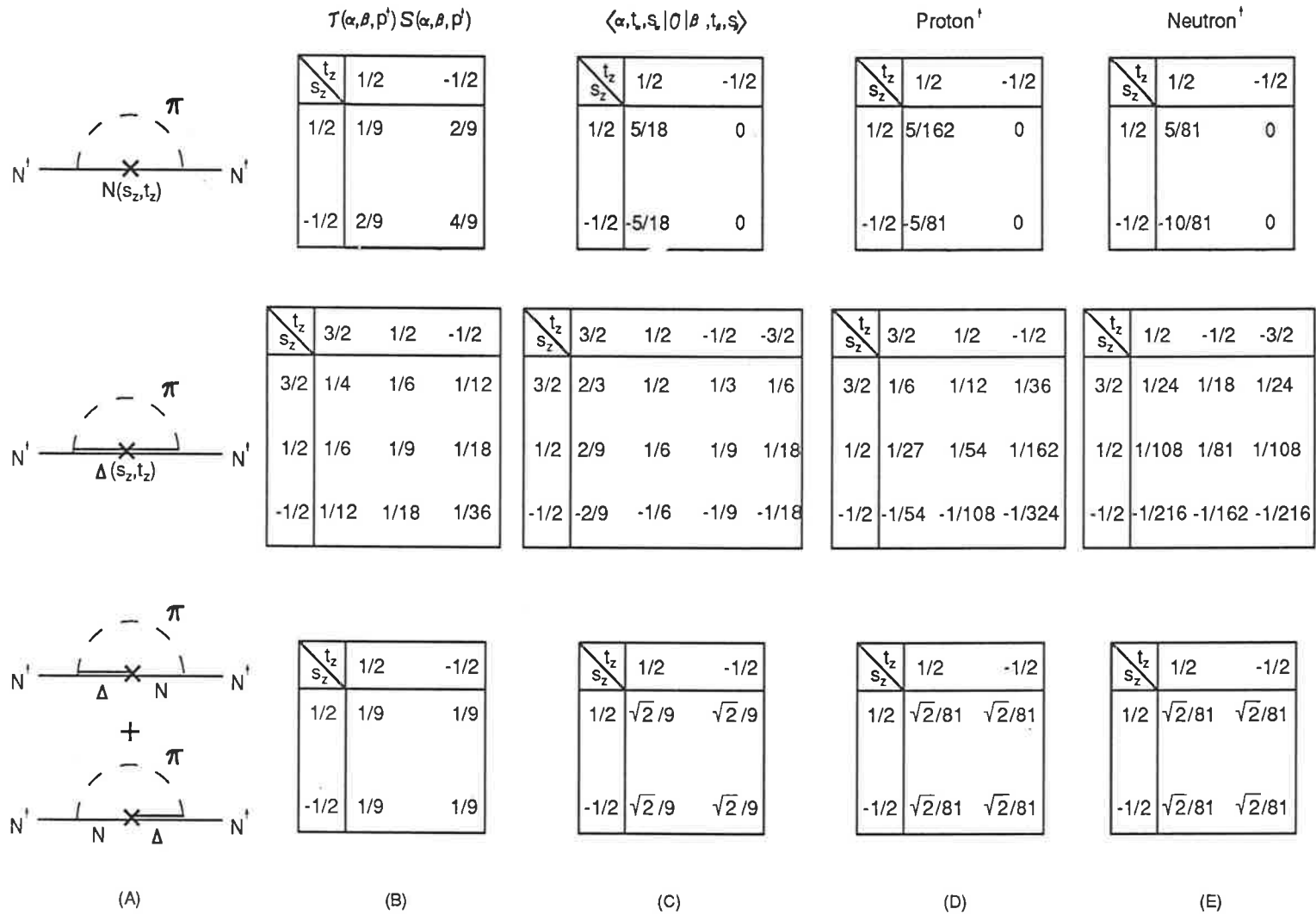
$$\mathcal{S}(\alpha, \beta, N_0) = \sum_n C(S_\beta 1 \rightarrow S_{N_0} | s_\beta n \rightarrow s_{N_0}) C(S_\alpha 1 \rightarrow S_{N_0} | s_\alpha n \rightarrow s_{N_0}) \quad . \quad (5.33)$$

Note that because the matrix element is diagonal in spin and isospin that $t_\alpha = t_\beta \equiv t$ and $s_\alpha = s_\beta \equiv s$. We see that we have three distinct terms (apart from the term not containing pions): α and β may be both nucleons, both deltas or one nucleon and one delta (see Column (A) of Figure 5.3). The product of the 4 Clebsch Gordon coefficients for these terms is shown in the Column(B) of Figure 5.3 for the case of N_0 being a proton with spin up.

To go further, we need to evaluate $\langle \alpha, t_\alpha, s_\alpha | \mathcal{O} | \beta, t_\beta, s_\beta \rangle$ within a model. For the first moment of the structure functions the operator $\mathcal{O}$ is

$$\int d^4x \sum_i c_i \bar{\Psi}(x) \lambda_i \gamma^z \gamma^5 \Psi(x) \quad . \quad (5.34)$$

Let us suppose that the baryon wavefunctions are given by products of MIT bag wavefunctions (see equation 3.35)

Figure 5.1: Coefficients needed for the calculation of g_i in the cloudy bag model

$$\Psi(\mathbf{x}) = N \begin{pmatrix} j_0(\frac{\Omega}{R}|\mathbf{x}|) \\ i\sigma \cdot \mathbf{x} j_1(\frac{\Omega}{R}|\mathbf{x}|) \end{pmatrix} e^{-i\frac{\Omega}{R}t} \Theta(R - |\mathbf{x}|) , \quad (5.35)$$

with the spin-flavour structure of the bare baryons being just those of the groundstate $\underline{56}$ -plet of $SU(6)$. The normalization N is given in equation 3.36. We obtain

$$\langle \alpha, t_\alpha, s_\alpha | \mathcal{O} | \beta, t_\beta, s_\beta \rangle = N^2 \int d\mathbf{x} \langle \nu_\alpha, t_\alpha, s_\alpha | \sum_i c_i \lambda_i \sigma_3 \left\{ j_0^2(\frac{\Omega}{R}|\mathbf{x}|) - \frac{1}{3} j_1^2(\frac{\Omega}{R}|\mathbf{x}|) \right\} | \nu_\alpha, t_\alpha, s_\alpha \rangle, \quad (5.36)$$

where the bra and ket refer now to the spin-flavour wavefunctions alone. Performing the integral, one obtains

$$\langle \alpha, t_\alpha, s_\alpha | \mathcal{O} | \beta, t_\beta, s_\beta \rangle = \frac{\Omega}{3(\Omega - 1)} \langle \alpha, t_\alpha, s_\alpha | \sum_i c_i \lambda_i \sigma_3 | \beta, t_\beta, s_\beta \rangle . \quad (5.37)$$

$\frac{\Omega}{3(\Omega-1)}$ is the famous factor that reduces $|\frac{g_a}{g_v}|$ from $\frac{5}{3}$ to 1.09 in the MIT bag model. The non-zero spin flavour matrix elements are shown in Column (C) of Figure 5.3, the product between it and the Clebsch-Gordon coefficients being evaluated in Column (D). Until evaluating the integral in equation 5.31 we can of course not compare the strengths of the three separate terms, however it is clear that the contribution from the nucleon intermediate state is that of a proton spinning in the opposite direction of the original proton, while the most important contribution of the Δ intermediate state is the $\Delta^{++}(s_z = +\frac{3}{2})$. All the interference terms contribute with equal strength. Summing the contributions, we get

$$g_1^p = \frac{1}{6} \frac{\Omega}{3(\Omega - 1)} \left\{ \frac{5}{3} Z_2 - \frac{5}{27} P_{N\pi} + \frac{50}{27} P_{\Delta\pi} + \frac{4}{9} P_{N\Delta\pi} \right\} , \quad (5.38)$$

where

$$P_{N\pi} = \frac{3Z_2}{\pi m_\pi^2} (f_0^{NN})^2 \int_0^\infty \frac{dk k^4 u^2(kR)}{(w_{0k})^3}$$

Table 5.1: Probabilities and coupling constants as a function of radius in the cloudy bag model

R(fermi)	Z	$P_{N\pi}$	$P_{\Delta\pi}$	$P_{N\Delta\pi}$	f_r^2	g_r^2
0.8	0.61	0.25	0.14	0.35	0.0780	0.25
0.9	0.68	0.21	0.11	0.28	0.0770	0.24
1.0	0.74	0.17	0.08	0.22	0.0750	0.23
1.1	0.79	0.14	0.06	0.18	0.0735	0.22

$$P_{\Delta\pi} = \frac{3Z_2}{\pi m_\pi^2} (f_0^{N\Delta})^2 \int_0^\infty \frac{dk k^4 u^2(kR)}{w_{0k}(m_N - m_\Delta - w_{0k})^2} \quad (5.39)$$

$$P_{N\Delta\pi} = \frac{4\sqrt{2}Z_2}{\pi m_\pi^2} (f_0^{NN} f_0^{N\Delta}) \int_0^\infty \frac{dk k^4 u^2(kR)}{w_{0k}^2(-m_N + m_\Delta + w_{0k})} \quad (5.40)$$

$P_{N\pi}$ and $P_{\Delta\pi}$ are the probabilities of the proton being made up of a bare pion and a bare nucleon or Δ respectively. The normalization of $P_{N\Delta\pi}$ is arbitrary and is chosen here according to the convention established in equations 5.17. The bare bag probability is given by $Z_2 = 1 - P_{N\pi} - P_{\Delta\pi}$.

The integrals corresponding to these probabilities can be evaluated numerically and are listed as a function of the bag radius in Table 5.1, where we have also shown the values of the renormalized coupling constants taken from the paper by Théberge et al[TMT82]. The bare bag probability is of the order of 70%, causing a corresponding decrease in the value of g_1^p when compared with the MIT bag value. (Table 5.2) Approximately half of the decrease is recovered from the graphs with pionic corrections to the vertex, mostly from the contribution of the $\Delta^{++}(s_z = +\frac{3}{2})$, leading to the values for g_1^p listed in Table 5.3. These should be compared to the experimental values listed in Table 5.4

It is straightforward to derive the corresponding value of the neutron structure function. The isospin Clebsch-Gordon coefficients occurring in $T(\alpha, \beta, N_0)$ 5.33 are related to those with the isospin flipped by [Edm60, page 46-47]

$$C(T_\alpha, T_\beta, T|t_\alpha, t_\beta, t) = (-1)^{T_\alpha + T_\beta + T - 2t} C(T_\alpha, T_\beta, T|-t_\alpha, -t_\beta, -t) \quad (5.41)$$

Table 5.2: MIT bag results for g_1 of the proton and neutron and the axial coupling constant

g_1^P	$\frac{1}{6} \cdot \frac{5}{3} \frac{\Omega}{3(\Omega-1)} = 0.182$
g_1^n	0
$\left \frac{g_a}{g_v} \right $	$\frac{5}{3} \frac{\Omega}{3(\Omega-1)} = 1.090$

Table 5.3: CBM results for g_1 of the proton and neutron and the axial coupling constant

	R=0.8	R=0.9	R=1.0	R=1.1
g_1^p	0.151	0.155	0.158	0.161
g_1^n	-0.013	-0.011	-0.009	-0.008
$\left \frac{g_a}{g_v} \right $	0.98	1.00	1.00	1.02

Table 5.4: Experimental values for g_1^p [EMC89](adjusted to infinite Q^2) and $\left| \frac{g_a}{g_v} \right|$ [Par88]. g_1^n has been derived using the Björken Sum Rule 5.5.

g_1^P	0.138 ± 0.011 (stat.) ± 0.016 (sys.)
g_1^n	-0.084 ± 0.010 (stat.) ± 0.015 (sys.)
$\left \frac{g_a}{g_v} \right $	1.259 ± 0.004

Hence all we need to do is to flip the isospin in Column (B) of Table 5.3 and introduce an additional sign change for the interference terms. The resulting magnitude of the individual terms is shown in Column (E) of Figure 5.3, yielding

$$g_1^n = \frac{1}{6} \frac{\Omega}{3(\Omega - 1)} \left\{ \frac{-10}{27} P_{N\pi} + \frac{25}{27} P_{N\Delta} - \frac{4}{9} P_{N\Delta\pi} \right\} . \quad (5.42)$$

Note that in equation 5.42 there is no "bare neutron" term, corresponding to the well known result that the 56-plet SU(6) wavefunction for the neutron yields a vanishing structure function. The only non-vanishing contributions come from the terms where the probe interacts with either a proton or a delta. It should also be noted that equations 5.38 and 5.42 obey the Bjørken Sum rule, i.e.

$$g_1^p - g_1^n = \frac{1}{6} \frac{\Omega}{3(\Omega - 1)} \left\{ \frac{5}{3} Z + \frac{5}{27} P_{N\pi} + \frac{25}{27} P_{\Delta\pi} + \frac{8}{9} P_{N\Delta\pi} \right\} = \frac{1}{6} \left| \frac{g_a}{g_v} \right| . \quad (5.43)$$

Numerically, the Δ and interference terms roughly cancel for the neutron so that the main contribution is that of the $p\pi^-$ graph, with the proton spinning in the opposite direction to the original neutron. Numerical values for both g_1^n and $|\frac{g_a}{g_v}|$ corresponding to various radii, are given in Table 5.3.

By looking at the results of the cloudy bag model for g_1^p one might be persuaded to argue that the inclusion of pionic effects explains the discrepancy between the MIT result and the experimental value at the 1 standard deviation level. This would however be misleading as we note from Table 5.3 that the value of $|\frac{g_a}{g_v}|$ for the cloudy bag model is in less favourable agreement with the experimental value than that of the MIT value (Table 5.2). For example, scaling the values of $g_1^{p,n}$ by a factor that ensures agreement of $|\frac{g_a}{g_v}|$, gives $g_1^p \approx 0.19 - 0.20$ for the cloudy bag model, which, although closer than the (rescaled) MIT bag value of ≈ 0.21 , is still about two standard deviations from the experimental value.

5.4 Centre of mass corrections

A well known deficiency with bag models that indeed affects the calculated value of $\left| \frac{g_a}{g_v} \right|$ is that they describe quarks moving independently in a static potential. This results in a spurious centre of mass motion; i.e. the nucleon is not in an eigenstate of momentum. Corrections to hadronic observables due to this problem are typically 10-20%, and therefore should be of importance when calculating g_1^N . Let us therefore try to include these (see, for example, [Won86] and references therein for other methods that correct for this).

We can do this most easily by using the results of Chapter 3. Indeed, using the wavefunction of equation 3.18

$$|p\rangle = \frac{1}{\phi_N(p)} \int d\mathbf{x} e^{i\mathbf{p}\cdot\mathbf{x}} \Psi(\mathbf{x}_1 - \mathbf{x}, \mathbf{x}_2 - \mathbf{x}, \dots, \mathbf{x}_N - \mathbf{x}) \quad , \quad (5.44)$$

we easily obtain the corresponding expression to equation 5.36 for the Peierls-Yoccoz projected wavefunction:

$$\langle \alpha, t_\alpha, s_\alpha | \mathcal{O} | \beta, t_\beta, s_\beta \rangle = \langle \nu_\alpha, t_\alpha, s_\alpha | \frac{1}{|\phi_3(\mathbf{0})|^2} \int d\mathbf{a} d\mathbf{x} \langle 0_2, \mathbf{a} | 0_2, \mathbf{0} \rangle \quad (5.45)$$

$$\bar{\Psi}(\mathbf{x} - \mathbf{a}) \sum_i c_i \lambda_i \gamma^z \gamma^s \Psi(\mathbf{x}) | \nu_\beta, t_\beta, s_\beta \rangle \quad , \quad (5.46)$$

where $\langle 0_2, \mathbf{a} | 0_2, \mathbf{0} \rangle$ is the two particle overlap

$$\langle 0_2, \mathbf{a} | 0_2, \mathbf{0} \rangle = (\langle 0_1, \mathbf{a} | 0_1, \mathbf{0} \rangle)^2 \quad (5.47)$$

with $\langle 0_1, \mathbf{a} | 0_1, \mathbf{0} \rangle$ defined in equation 3.60. The $\mathbf{x}$ integral in equation 5.46 becomes, after substitution of the MIT bag wavefunctions,

$$\int d\mathbf{x} \left\{ j_0\left(\frac{\Omega}{R}|\mathbf{x} - \mathbf{a}|\right) j_0\left(\frac{\Omega}{R}|\mathbf{x}|\right) \sigma_3 + \sigma \cdot \widehat{\mathbf{x} - \mathbf{a}} \sigma_3 \sigma \cdot \hat{\mathbf{x}} j_1\left(\frac{\Omega}{R}|\mathbf{x} - \mathbf{a}|\right) j_1\left(\frac{\Omega}{R}|\mathbf{x}|\right) \right\} \quad (5.48)$$

$$\Theta(R - |\mathbf{x}|) \Theta(R - |\mathbf{x} - \mathbf{a}|) \quad .$$

Now

$$\begin{aligned} \sigma \cdot \widehat{\mathbf{x} - \mathbf{a}} \sigma_3 \sigma \cdot \hat{\mathbf{x}} &= i(\hat{\mathbf{x}} \times \widehat{\mathbf{x} - \mathbf{a}})_3 + \hat{x}_3 \sigma \cdot \widehat{\mathbf{x} - \mathbf{a}} - \sigma_3 \widehat{\mathbf{x} - \mathbf{a}} \cdot \hat{\mathbf{x}} \\ &\quad + (\widehat{\mathbf{x} - \mathbf{a}})_3 \sigma \cdot \hat{\mathbf{x}} . \end{aligned} \quad (5.49)$$

Any terms odd in $\mathbf{x}_1, \mathbf{x}_2$, or $\mathbf{x}_3$ don't contribute to the integral, so the only contribution comes from

$$\sigma_3 \left\{ \frac{2\mathbf{x}_3^2}{|\mathbf{x}||\mathbf{x} - \mathbf{a}|} - \frac{|\mathbf{x}|}{|\mathbf{x} - \mathbf{a}|} \right\} , \quad (5.50)$$

which gives the same contribution as $\frac{-\sigma_3|\mathbf{x}|}{3|\mathbf{x} - \mathbf{a}|}$. In the notation of equation 3.60 we get

$$\begin{aligned} \langle \alpha, t_\alpha, s_\alpha | \mathcal{O} | \beta, t_\beta, s_\beta \rangle &= \frac{\int [I_T(\mathbf{a}) + I_B(\mathbf{a})]^2 [I_T(\mathbf{a}) - \frac{1}{3}I_B(\mathbf{a})] d\mathbf{a}}{\int [I_T(\mathbf{a}) + I_B(\mathbf{a})]^3 d\mathbf{a}} \\ &\quad \langle \mu_\alpha, t_\alpha, s_\alpha | \sum_i c_i \lambda_i \sigma_3 | \mu_\alpha, t_\alpha, s_\alpha \rangle , \end{aligned} \quad (5.51)$$

which is of course nothing else than equation 3.62. Hence we obtain, for the lowest energy state, that

$$\langle \alpha, t_\alpha, s_\alpha | \mathcal{O} | \beta, t_\beta, s_\beta \rangle = \langle \nu_\alpha, t_\alpha, s_\alpha | 0.789 \sum_i c_i \lambda_i \sigma_3 | \nu_\alpha, t_\alpha, s_\alpha \rangle . \quad (5.52)$$

This is to be compared with equation 5.37 for the MIT bag without centre of mass corrections where the coefficient of $\sum c_i \lambda_i \sigma_3$ was $\frac{\Omega}{3(\Omega-1)} = 0.654$. We have therefore found that the inclusion of centre of mass corrections using the Peierls-Yoccoz procedure tends to increase the values of g_1 , and hence $\left| \frac{g_a}{g_v} \right|$, by about 20%. The results are tabulated in Table 5.5.

5.5 Conclusion

Let us summarize the results of this chapter. We have found that the corrections expected from the pion cloud of the nucleon shift some of the strength of g_1^p to g_1^n . The largest contribution to the change in g_1^p is due to the virtual photon interacting with a $s_z =$

Table 5.5: Results for g_1 of the proton and neutron and $\left|\frac{g_a}{g_v}\right|$ for the MIT and cloudy bag models, with centre of mass corrections applied.

	MIT	CBM			
		R=0.8	R=0.9	R=1.0	R=1.1
g_1^p	0.219	0.182	0.187	0.191	0.194
g_1^n	0	-0.016	-0.013	-0.011	-0.010
$\left \frac{g_a}{g_v}\right $	1.32	1.18	1.21	1.21	1.23

$\frac{3}{2} \Delta^{++}$, while a π^- is "in the air". We have also calculated the correction expected when one allows for the spurious centre of mass motion using the Peierls-Yoccoz projection. This yields a value of $\left|\frac{g_a}{g_v}\right|$ for both the MIT and cloudy bag essentially in agreement with experiment. The first moment of the proton structure function g_1^p is reduced by about 20 % in the cloudy bag model when compared with the MIT bag, and disagrees with the experimental value at the one standard deviation level, although it does agree at the two standard deviation level. Finally, the cloudy bag model predicts a sizable neutron structure function, the first moment being about 5-8 % of that of the proton.

Chapter 6

Numerical Results - Comparison with Data

6.1 Introduction

Given the model for the matrix elements $\mathcal{A}_{(n)}$ that we have developed in Chapter 3, and using the evolution formalism in Chapter 4, we now have all the ingredients to perform a comparison with experimental data as well as making predictions for structure functions that have not yet been measured.

This is not the first time an attempt has been made to derive structure functions from bag models. Already before QCD had become established as a theory of the strong interactions, Jaffe [Jaf75] derived spin-independent functions in the cavity approximation to the bag theory while, sometime later, Hughes [Hug77,Bar79] did the same for the spin-dependent ones. Of course none of these papers was in a position to use the framework of perturbative QCD to make meaningful predictions at higher Q^2 . Although this became possible in the mid-seventies when the first and second order calculations of the relevant anomalous dimensions were performed (see Chapter 4 for references), it was not until 1980 that it was actually done. Jaffe and Ross [JR80] argued that the bag model calculation of structure functions corresponds to the calculation of the twist two piece of

the current correlation function at a scale μ where the quarks carry all the momentum of the proton which, when evolved to higher Q^2 , becomes the dominant contribution to the scattering cross-section. They calculated various moments of the non-singlet function $F_3(x)$ and, by comparison with data, found the scale μ to be of the order of 1 GeV.

Since then a considerable amount of work has been done on the subject [BM87,BM88a,BM88b,ST89b], in particular after the advent of the (nuclear) EMC effect [EMC83]. Only one prediction has been made for the QCD improved spin-dependent structure function g_1 [BM89].

The work described in this thesis enables us to make self consistent predictions for spin-dependent and spin-independent structure functions for both the neutron and proton. We shall concentrate on $F_3(x)$ and $g_1(x)$ as well as ratios such as $\frac{d_v(x)}{u_v(x)}$ which are of interest when investigating SU(6) symmetry-breaking effects. (Our model for the matrix elements would of course also allow us to make predictions for $F_2(x)$ and $g_2(x)$. In particular $g_2(x)$ is an interesting structure function as it is expected that some higher twist effects will become important here [Jaf90]. However as we have not worked out the QCD evolution for these two in Chapter 4 we shall not consider them here.)

As in previous work, the scale μ is a priori undetermined. It is usually fixed by comparing a low moment of the predicted structure function with data [JR80,GR77]. This can be improved upon considerably here - we shall fix the scale μ by comparing the shape of the entire distribution with experiment. Having fixed the scale we are then free to evaluate all other quantities that may be of interest.

Before we do this we need to decide what to do about the normalization of the distributions. As has been outlined in Chapter 3, the component of the distribution functions with the two quark intermediate state does not, nor should it, saturate the normalization. Unfortunately, this is still the case after adding in the four quark (antiquark) intermediate states. We shall assume in this chapter that in a fundamental description of the non-perturbative physics involved we would be able to derive the intermediate state

masses, leading to a saturation of the normalization requirement.

Our procedure will therefore be as follows: We shall choose parameters (bag radius, diquark state mass) for the two quark component and shall then add in a term, corresponding to the four quark component, to fix the normalizations. For simplicity we shall mimic the behaviour of the four quark term by $(1-x)^7$ and $\frac{1}{\sqrt{x}}(1-x)^7$, which we shall refer to as scenario A and B respectively. $(1-x)^7$ is a very good approximation to the actual behaviour of the four quark term shown in Figures 3.4 and 3.5 for radii of 0.6 to 1.0 fermi, while $\frac{1}{\sqrt{x}}(1-x)^7$ (which also has similar behaviour to the four quark term) has the asymptotic behaviour expected of the valence distribution at small x from Regge theory [Hei73, KW71, LPS71].

We shall then fix the scale μ by comparing our prediction for the valence distribution $u_v(x) + d_v(x)$ to four different parameterizations of data (all rescaled to $Q^2 = 10\text{GeV}^2$), as compiled by Charchula et al [CKAL89] :

- 1) Eichten, Hinchliffe, Lane, Quigg - Set I [EHLQ86b, EHLQ86a] (solid curves)
- 2) Martin, Roberts, Stirling - Set I [MRS88] (dotted curves)
- 3) Duke, Owens - Set I [DO84] (dashed curves)
- 4) Diemoz, Ferroni, Longo - Set II [DFLM88] (dot-dashed curves)

We are then free to make predictions for all other distributions. In Section 6.3 we present the results for the bare bag, in Section 6.4 those with the one gluon exchange included and in Section 6.5 we estimate by how much the scale μ will be influenced by pions. In Section 6.6 we make predictions for forthcoming experiments.

To start of with, however, we need to discuss the role of the anomaly in spin dependent scattering. We shall in fact find, in Section 6.2, that regardless of any theoretical questions about its existence, it is likely to be numerically insignificant.

6.2 The anomaly and the first moment of g_1^p

In 1987 Jaffe [Jaf87] proposed that the first moment of g_1^p may vary rapidly with Q^2 because the U(1) anomaly results in a non-zero anomalous dimension for the flavour singlet axial current, beginning at second order.

The experimental results published around the same time [EMC88] indeed showed a significant shortfall for the integral when compared to the predictions of the Ellis-Jaffe sum rule [EJ74]: 0.114 ± 0.012 (stat) ± 0.026 (syst) as compared to 0.189 ± 0.005 . (The latter number includes the QCD corrections to the coefficient functions.) The significance of this experimental result lies in the observation that the experimental value is essentially saturated by the flavour non-singlet contributions to the g_1 , implying that the flavour singlet carries little of the spin of the proton

$$\Delta\Sigma(Q^2) = \sum_f \Delta q_f(Q^2) \sim 0 \quad (6.1)$$

and that the polarization by the strange quarks is negative and large. (The flavour non-singlet contributions can be estimated as they are related via SU(3) symmetry arguments to weak decay matrix elements; see however [Lip88a,Lip88b,Lip89] for a critique)

The validity of the experimental results has been questioned by Close and Roberts [CR88] due to uncertainties in the extrapolation to small x and has subsequently been revised upwards by the EMC Collaboration [EMC89] to 0.126 ± 0.010 (stat) ± 0.015 (syst) (this includes older SLAC data [A*76,B*80,B*83]). The Ellis-Jaffe estimate for the first moment has also been revised since then to take into account the change in the F/D ratio that follows from the revised value for the neutron lifetime [Par88], the new estimate being 0.17 [Clo90] (including the QCD correction) which brings it to being within two standard deviations of the experimental result. As we have seen in Chapter 5 the cloudy bag model predicts essentially the same result.

Even though the difference of the experimental result from the theoretical expectation is now only a two standard deviation effect the question of the role of the anomaly has

remained under much discussion, mainly due to contributions by Efremov and Teryaev [ET88], Altarelli et al. [AR88,AS89] and Carlitz et al. [CCM89]. These authors claim that the anomaly gives rise to an effective contribution to the first moment of g_1 , from perturbatively generated gluons, of the form

$$- \frac{\alpha_s(Q^2)}{2\pi} N_f \Delta g(Q^2) \quad . \quad (6.2)$$

The important point made by Altarelli et al. is that although $\alpha_s(Q^2)$ decreases logarithmically with Q^2 , $\Delta g(Q^2)$ increases logarithmically and hence one gets a finite contribution even at infinite Q^2 . The authors argue that if one identifies a current quark distribution of a given flavour f as

$$\Delta q_f^{cur}(Q^2) = \Delta q_f^{con}(Q^2) - \frac{\alpha_s(Q^2)}{2\pi} \Delta g(Q^2) \quad (6.3)$$

then the EMC result can be reconciled with an essentially vanishing constituent strange quark contribution $\Delta s^{con}(Q^2)$. This proposal by Altarelli et al. has been criticized for several reasons. As we shall show in the next subsection the result is sensitive to the way the infrared divergence that appears is regularized and questions have been raised as to the sensitivity of the box diagram to the kinematical region which should not be calculated in perturbation theory [BQ90,MS90,JM90,BNT90,Man90]. Furthermore, Jaffe and Manohar [JM90] have pointed out that the gluonic contribution should not exist as there is no gauge invariant twist two operator that can contribute in the operator product expansion of the current correlation product.

Under the assumption that the gluonic contribution does indeed exist for light quarks, Glück and Reya [GR89] have been able to estimate its contribution by calculating the amount of perturbatively generated glue present at 10 GeV^2 , assuming that the proton consists of three valence quarks at some lower scale μ^2 . The scale μ^2 is determined by the requirement that all the proton's momentum is carried by the valence quarks at that scale. We shall return to this after discussing the box diagram itself.

6.2.1 The box diagram

The contribution of Figure 6.1(a) is given by [CCM89]

$$\begin{aligned}
A_1(x) &= \frac{N_f \alpha_s(Q^2)}{\pi} \frac{1}{2\pi} \int d^4 k \frac{\delta((p-k)^2 - m^2) \delta((q+k)^2 - m^2)}{(k^2 - m^2 + i\epsilon)^2} \\
&\quad \cdot \frac{1}{4} \epsilon^{ij} \epsilon^{lm} \text{Tr} \{ \gamma_j (\not{k} + m) \gamma_m (\not{q} + \not{k} + m) \gamma_l (\not{k} + m) \gamma_i (\not{p} - \not{k} + m) \} \\
&= -\frac{N_f \alpha_s(Q^2)}{\pi} \frac{1}{2\pi} \int d^4 k \delta((p-k)^2 - m^2) \delta((q+k)^2 - m^2) \\
&\quad \cdot 4 \left\{ \frac{1}{2} + \frac{k_\perp^2 + p \cdot q}{k^2 - m^2} + \frac{p^2 q^2 + 4k_\perp^2 p \cdot q}{2(k^2 - m^2)^2} \right\} , \tag{6.4}
\end{aligned}$$

while Figure 6.1(b) contributes

$$\begin{aligned}
A_2(x) &= -\frac{N_f \alpha_s(Q^2)}{\pi} \frac{1}{2\pi} \int d^4 k \frac{\delta((p-k)^2 - m^2) \delta((q+k)^2 - m^2)}{(k^2 - m^2 + i\epsilon)((q-p+k)^2 - m^2 + i\epsilon)} \\
&\quad \cdot \frac{1}{4} \epsilon^{im} \epsilon^{lj} \text{Tr} \{ \gamma_j (\not{q} - \not{p} + \not{k} + m) \gamma_m (\not{q} + \not{k} + m) \gamma_l (\not{k} + m) \gamma_i (\not{k} - \not{p} + m) \} \\
&= -\frac{N_f \alpha_s(Q^2)}{\pi} \frac{1}{2\pi} \int d^4 k \delta((p-k)^2 - m^2) \delta((q+k)^2 - m^2) 4k_\perp^2 p \cdot q . \tag{6.5}
\end{aligned}$$

Here ϵ^{ij} is the two dimensional antisymmetric tensor and we are working in a frame where both the photon and gluon do not have any transverse momentum, while the gluon polarization is in the transverse direction; m is the quark mass.¹

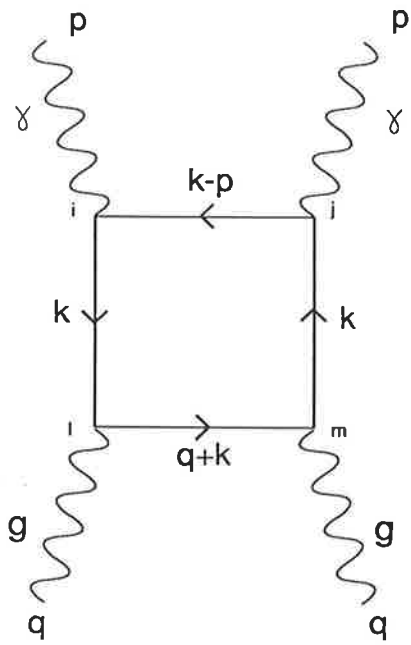
Adding the two terms together and noting that the two δ -functions require

$$\begin{aligned}
k^- &= \frac{p^- - q^-}{2} + \frac{p^- + q^-}{2} a R \\
k^+ &= \frac{p^+ - q^+}{2} - \frac{p^+ + q^+}{2} a R , \tag{6.6}
\end{aligned}$$

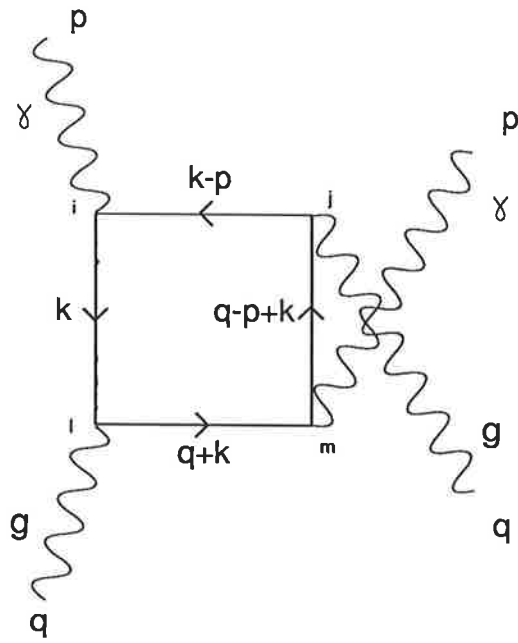
where $a = \pm 1$ and $R = \sqrt{1 - \frac{4(k_\perp^2 + m^2)}{(p+q)^2}}$, and we have converted to lightcone variables. We may do the k^+ and k^- integrations (as well as an angular integration) to obtain

¹This calculation was performed concurrently, but independently, by S. D. Bass and N. N. Nikolaev.

These authors went on to investigate the regulator dependence, resulting in reference [BNT90]; I would like to acknowledge the opportunity to verify the results with them.



(a)



(b)

Figure 6.1: Gluonic contributions to g_1

$$A(x) = -N_f \frac{\alpha(Q^2)}{2\pi} \int_0^\lambda dR \frac{\Delta^2 R^2 + (p \cdot q)^2}{[\Delta^2 R^2 - (p \cdot q)^2]^2} \left\{ (1 - R^2)(p \cdot q W^2 - \Delta^2) - 4m^2 p \cdot q \right\} . \quad (6.7)$$

Here

$$\Delta^2 = (p \cdot q)^2 - p^2 q^2 ,$$

$$\lambda = \sqrt{1 - \frac{4m^2}{W^2}}$$

and

$$W^2 = (p + q)^2 . \quad (6.8)$$

The region of small $k_\perp^2$ corresponds to $R \rightarrow \lambda \sim 1$. Because Δ^2 approaches $(p \cdot q)^2$ in the Björken limit the denominator in equation 6.8 gets small so that $A(x)$ receives a considerable contribution from the low $k_\perp^2$ region. As has been pointed out by Bodwin and Qiu [BQ90] one should not really do perturbation expansions in this region. The precise differentiation of what one should and should not include in the perturbative and non-perturbative parts of the calculation is the source of much current debate [BQ90, BNT90, MS90, Man90].

If one does integrate equation 6.8 over all allowed $k_\perp^2$ one obtains

$$\begin{aligned} A(x) &= \frac{N_f \alpha(Q^2)}{2\pi} \left\{ \frac{p \cdot q}{\Delta} \left(-1 + \frac{W^2}{\Delta^2} p \cdot q \right) \log \frac{p \cdot q - \lambda \Delta}{p \cdot q + \lambda \Delta} \right. \\ &\quad \left. + \lambda \left[\frac{+2(p^2 + p \cdot q)(q^2 + p \cdot q)}{\Delta^2} + \frac{4m^2 \Delta^2}{4m^2 \Delta^2 + p^2 q^2 W^2} \right] \right\} \\ &= \frac{N_f \alpha(Q^2)}{2\pi} \left\{ \frac{(1 - 2x)(1 + 2x\rho^2)}{\delta^3} \log \frac{1 - \lambda\delta}{1 + \lambda\delta} \right. \\ &\quad \left. + \lambda \left[\frac{2(1 - 2x)(1 + 2x\rho^2)}{\delta^2} - \frac{\mu^2 \delta^2}{\rho^2 x(1 - x) + \rho^4 x^2 - \mu^2 \delta^2} \right] \right\} . \quad (6.9) \end{aligned}$$

$$\text{Here } \delta = \frac{\Delta}{Q^2} = (1 + 4\rho^2 x^2)^{\frac{1}{2}} ,$$

$$\lambda = \sqrt{1 - \frac{4\mu^2 x}{1 - x + \rho^2 x}}$$

$$\text{and } \rho = \frac{p}{Q}, \mu = \frac{m}{Q} \quad (6.10)$$

Expression 6.9 is exact. One can read off the limits for $\mu^2 = 0$, $\rho^2 \sim 0$:

$$A_\rho(x) \sim \frac{\alpha(Q^2)N_f}{2\pi} \{1 - 2x\} \left\{ \log(-\rho^2 x^2) + 2 \right\} \quad (6.11)$$

and $\rho^2 = 0$, $\mu^2 \sim 0$

$$A_\mu(x) \sim \frac{\alpha(Q^2)N_f}{2\pi} \left\{ (1 - 2x) \left(\log \frac{\mu^2 x}{1 - x} + 2 \right) + 1 \right\} \quad (6.12)$$

which agree with those quoted by Bodwin and Qiu [BQ90]. Integration of these limits over $0 \leq x \leq 1$ gives

$$\int_0^1 A_\rho(x) dx = \frac{-\alpha(Q^2)N_f}{2\pi} \quad (6.13)$$

and

$$\int_0^1 A_\mu(x) dx = 0 \quad (6.14)$$

which are the results of Carlitz et al. [CCM89]. As has been pointed out in [BQ90], these results should be treated with a great deal of caution. The explicit dependence on the regulators ρ and μ only drops out because the integral from 0 to 1 of $1-2x$ vanishes. We see however, from 6.10, that for non-zero μ the physical region of x does not extend all the way to 1, so that at least for the case $\rho^2 = 0$, $\mu^2 \sim 0$ the cancellation between the small x and large x region will not be exact. Because a large part of the integral of A_μ comes from the extreme regions of x equation 6.14 is misleading. It would be nice to evaluate the integral of equation 6.9 exactly to see this explicitly, this has however not been done yet.

6.2.2 Phenomenology of the gluon contribution

Notwithstanding the problems mentioned above, let us now look at the proposal that if the gluonic contribution exists it would solve the problem of the missing spin of the proton.

Let us first concentrate on the first moment. From equation 4.19 we can read off that to the leading order the quark singlet contribution is independent of Q^2 while the glue component is given by

$$\Delta g_{\text{LO}}(Q^2) = \frac{4}{9} \left(\frac{\alpha(\mu^2)}{\alpha(Q^2)} - 1 \right) \Delta \Sigma(\mu^2) \quad , \quad (6.15)$$

where μ^2 is a scale low enough so that there is no glue present. The singlet contribution to $g_1^p(Q^2)$ then becomes

$$g_{1 \text{ SINGLET}}^p(Q^2) = \frac{1}{9} \left(1 - \frac{2}{3\pi} [\alpha(\mu^2) - \alpha(Q^2)] \right) \Delta \Sigma(\mu^2) \quad (6.16)$$

to leading order. At next to leading order we have the corresponding expressions

$$\begin{aligned} \Delta \Sigma_{\text{NLO}}(Q^2) &= \Delta \Sigma(\mu^2) \\ \Delta g_{\text{NLO}}(Q^2) &= \frac{2\pi}{3\alpha(Q^2)} \left[1 - \exp \left\{ \frac{2}{3\pi} [\alpha(Q^2) - \alpha(\mu^2)] \right\} \right] \Delta \Sigma(\mu^2) \end{aligned} \quad (6.17)$$

and hence

$$\begin{aligned} g_{1 \text{ SINGLET}}^p(Q^2) &= \frac{1}{9} \left[\left(1 - \frac{\alpha(Q^2)}{\pi} \right) \Delta \Sigma_{\text{NLO}}(Q^2) - \frac{3\alpha(Q^2)}{2\pi} \Delta g_{\text{NLO}}(Q^2) \right] \\ &= \frac{1}{9} \left[\frac{-\alpha(Q^2)}{\pi} + \exp \left\{ \frac{2}{3\pi} [\alpha(Q^2) - \alpha(\mu^2)] \right\} \right] \Delta \Sigma(\mu^2) \quad . \end{aligned} \quad (6.18)$$

(Here we have used the leading order expression for $\alpha(Q^2)$ rather than the next to leading order one in order to avoid having to redefine the QCD variable Λ_{QCD} .) Both the leading order and next to leading order predictions are shown as a function of μ in Figure 6.2, using $\Delta \Sigma(\mu^2) = 0.6$ [GR89].

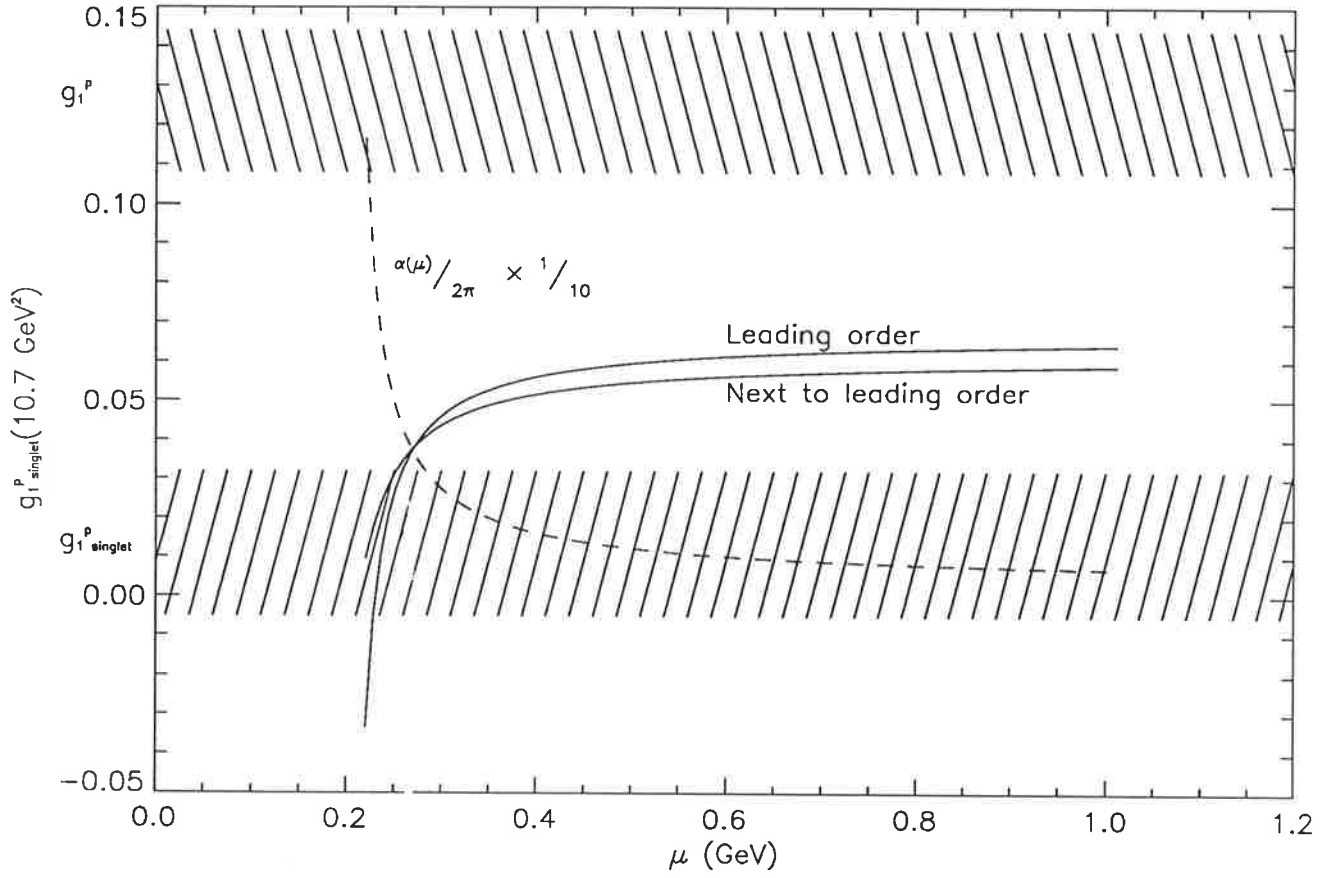


Figure 6.2: The first moment of the singlet contribution to g_1^p . The experimental values and their uncertainties for the singlet contribution as well as the total value of g_1^p are indicated by the dashed areas. Also shown is the behaviour of the running coupling constant. Λ is assumed to be 0.2 GeV

We see that in order for the gluonic contribution to be large enough so that the singlet contribution falls within the one standard deviation limits one needs to assume that the scale μ is extremely close to Λ_{QCD} . If this is indeed the case, the near-vanishing of the singlet contribution can only be described as accidental. A small change of μ in this region would produce a large change in the contribution of the singlet to g_1^p . One should of course be wary of trusting the leading order or even next to leading order prediction in this region in any case. On the other hand, a scale of $\mu \sim 1$ GeV will clearly not do very much for the sum rule (if $\mu = 1$ GeV the anomalous term gives a correction of 0.003 to the sum rule).

As we shall see shortly in a quantitative manner (see also [STL90a]), a scale μ so close to Λ_{QCD} implies that the original valence quarks are producing a great deal of gluon bremsstrahlung when viewed at 10 GeV. As already examined in Chapter 4 this implies that each quark and gluon carries relatively little of the momentum of the hadron and so the quark distributions are shifted quite dramatically towards small x . It is very difficult, if not impossible, to reconcile this with both the EMC spin data and the data for other structure functions like $F_3(x)$. In other words, a very small scale μ may indeed be able to fit the experimentally observed first moment of g_1 but one then has great difficulty fitting the shape of the distribution at small x .

We must therefore conclude, apart from any theoretical difficulties with anomalous gluon contribution, that numerically it does not seem to play an important role if our Ansatz (i. e. a model of the proton at the scale μ consisting of only valence quarks and no gluons) is correct. We shall therefore neglect it.

6.3 Bare Bag results

To start off with we shall set the intermediate diquark state mass to be $M_2 = \frac{3}{4}M$ and shall assume no pionic corrections. In Figure 6.3 we show the best fits to the valence distribution as a function of bag radius R and the ratio of coupling constants r . Also

included are the original (i.e. unevolved) curves at the scale μ which is implied by the latter. The labels A and B refer to the two scenarios of page 119.

The radius of best fit is about 0.6 fermi. Any larger radius tends to decrease the large x region while enhancing the medium x region, thus leading to a larger (negative) slope at $x \sim 0.4$. Radii of up to around 0.8 fermi also give acceptable fits.

The ratio of coupling constants implies, given the scale of the data as $Q^2 = 10 \text{ GeV}^2$, that $\alpha_s(Q^2 = \mu^2) = 1.9$ or $\mu = 0.29 \text{ GeV}$ (using $\Lambda_{QCD} = 0.2 \text{ GeV}$). This is a very low scale, although not quite as low as that advocated by Glück and Reya [GR77]. Note that these authors point out that the perturbative expansion is in the variable $\frac{\alpha_s}{2\pi}$, which in our case would be around $\frac{1}{3}$. Nevertheless, if the bag model indeed corresponds to physics at such a low scale one really needs to evaluate at least the second order correction to obtain some guide as to how much to trust the bag model predictions. (Some comparisons of leading and next to leading order calculations have recently been carried out by Glück, Reya and Vogt [GRV89]. They find surprisingly little difference between the two.)

The predictions for $\sigma_1^p(x, Q^2 = 10 \text{ GeV}^2)$ obtained by using this scale of μ is shown in Figure 6.4, together with the data from the EMC collaboration [EMC88, EMC89]. Although the data have large error bars, the fits are not really very good. At small x the predictions fall significantly above the data, which is where the EMC collaboration collected most of their data. Glancing back at the predictions for the valence distributions in Figure 6.3 one sees that here too they exceed the data at small x .

It might seem that an obvious way of justifying this discrepancy is to resort to the argument that the bag model should not be expected to yield valid results for processes that occur over distance of the order of the bag radius, which may indeed be the case for small x (see the argument in Section 2.3).

Although this is certainly true, it is unlikely that it is the explanation for the excess strength of the cross sections at small x because the phenomenon is relatively independent of the behaviour at small x of the input distribution (i.e. the distribution at the scale μ)

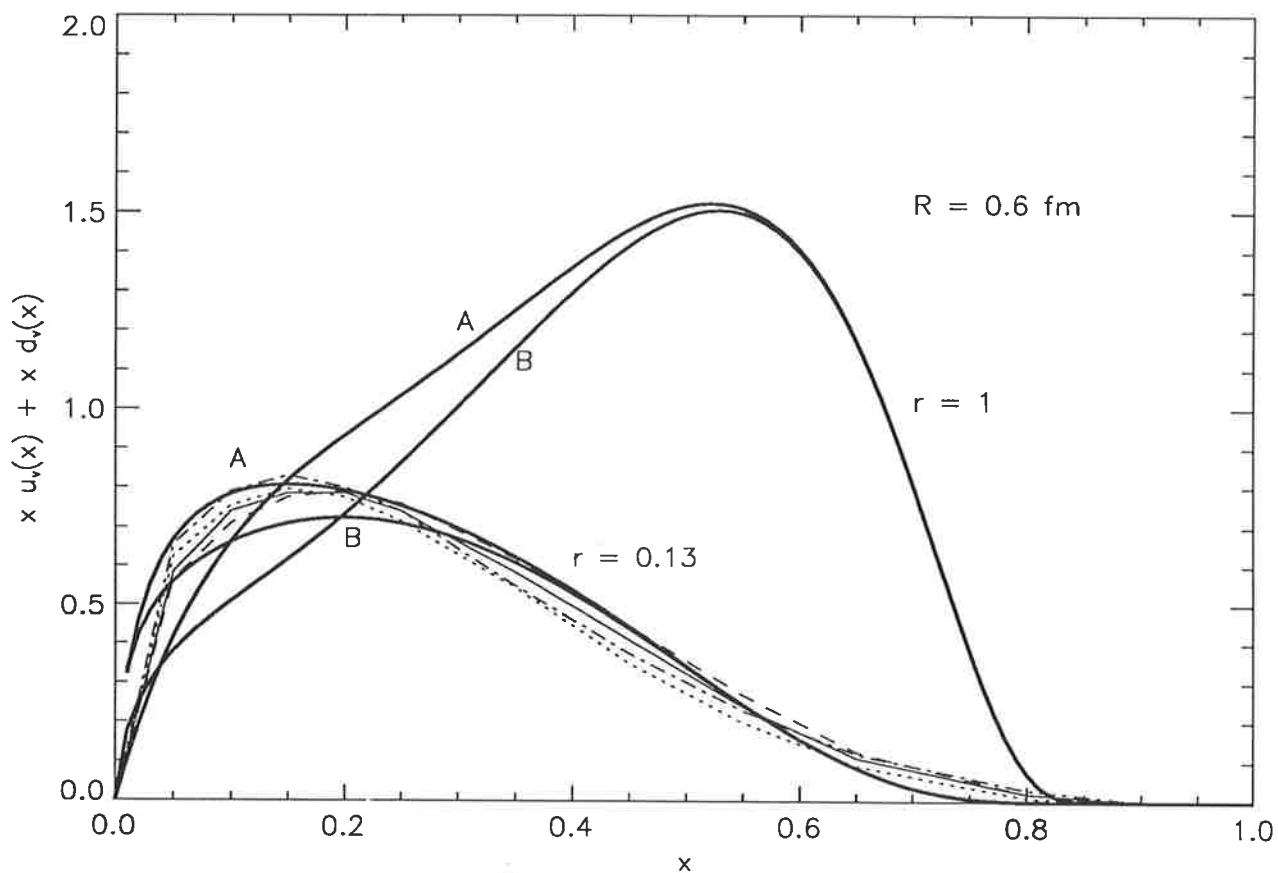


Figure 6.3: The valence distribution for the bare bag. The thick curves correspond to the theoretical results at both μ (~ 0.29 GeV) and a scale where the strong coupling constant is 13% of the coupling constant at μ . The thin curves are parameterizations of the data as explained in the text. The average value of x is $\langle x \rangle \sim 0.12$

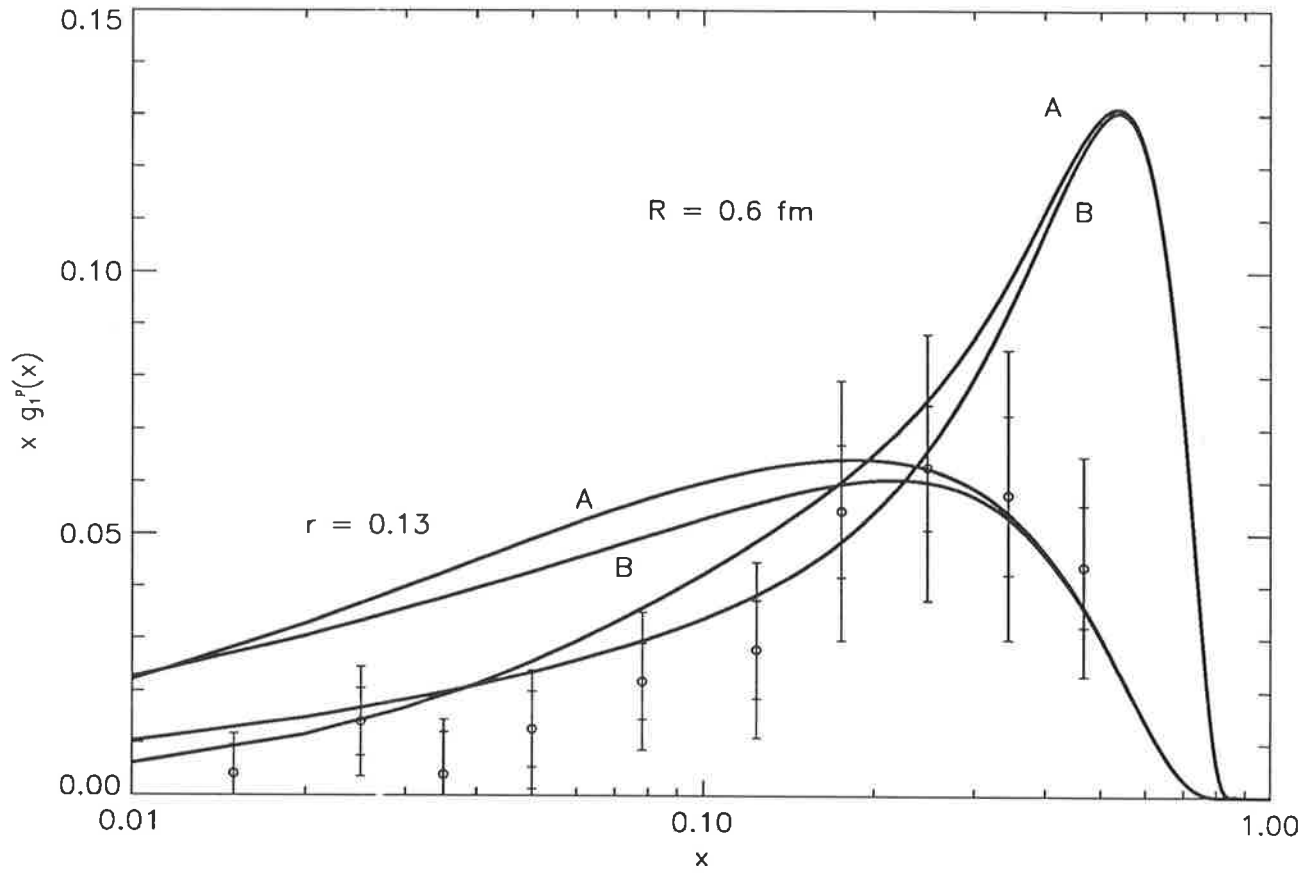


Figure 6.4: $g_1^p(x, 10\text{GeV}^2)$ corresponding to $\mu = 0.29 \text{ GeV}$ and $R = 0.6 \text{ fm}$. The data is that of the EMC collaboration.

[STL90a]. A significant part of the small x behaviour of the distribution at $Q^2 = 10\text{GeV}^2$ originates from contributions at larger x at the scale μ^2 .

A more natural explanation is that the scale μ is too low. The choice of μ that was necessary to fit the valence distribution implies that at the higher scale a significant portion of the spin structure function is provided by the radiatively generated sea, which is concentrated at low x . However, choosing a larger value of μ , and hence r , destroys the nice agreement for the valence distribution. We are therefore led to the conclusion that the bare bag model does not really give a satisfactory fit to both the valence distribution and g_1^p . Note in addition that the ratio of $\frac{d_v(x, Q^2)}{u_v(x, Q^2)}$ is predicted to be $\frac{1}{2}$, independent of Q^2 and x - also in disagreement with data. Let us therefore proceed to the model that takes into account the one-gluon exchange effects.

6.4 Bare bag with one gluon exchange

We shall include one gluon exchange along the lines developed in Section 3.8.1 and shall again allow the "unsplit" diquark mass $M_2^{(0)}$, as defined in equation 3.79, to vary around 800 MeV. As was the case in the previous section the best fits to the valence quark distribution are obtained for rather low values of μ (Figure 6.5) and a bag radius of 0.6 fm.

The diquark masses used are $M_2 = 500$ MeV and 750 MeV for the scalar and vector contributions respectively (i.e. $M_2^{(0)} = 700$ MeV). The ratio of coupling constants implies a bare bag scale of about 0.26 GeV.

In Figure 6.6 we show the corresponding ratio $\frac{u_v(x, Q^2)}{d_v(x, Q^2)}$ plotted against the parameterizations of the data.

Given the large discrepancies between the various parameterizations of the data the fits are very good except perhaps for $x \geq 0.7$. As has been pointed out in Section 3.5 at large x the unphysically sharp boundary to the bag effects the distributions so that we should not worry too much about this discrepancy.

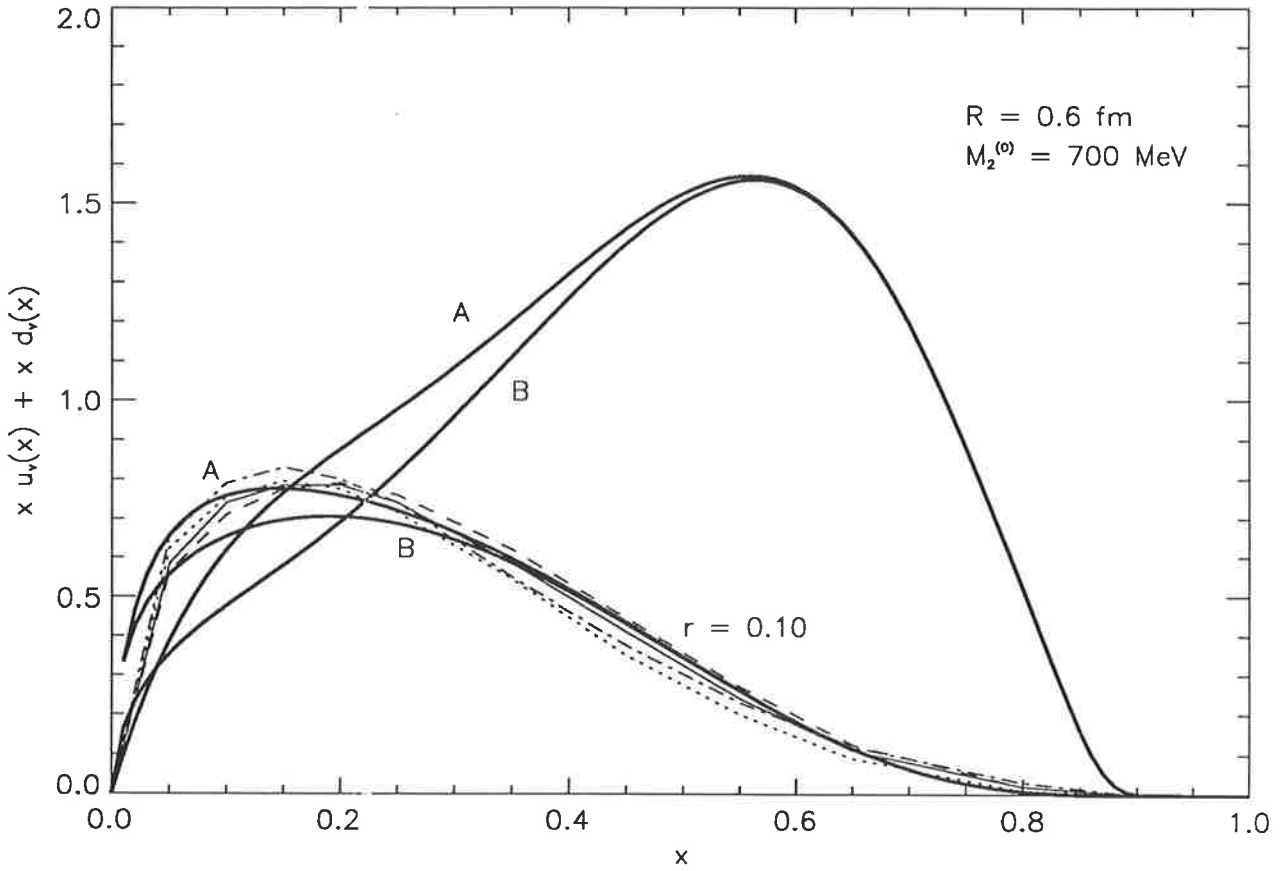


Figure 6.5: The valence distribution for the bare bag, with one gluon exchange included. $M_2^{(0)}$ is 700 MeV. The thick curves correspond to the theoretical results at both μ (~ 0.26 GeV) and a scale where the strong coupling constant is 10% of the coupling constant at μ .

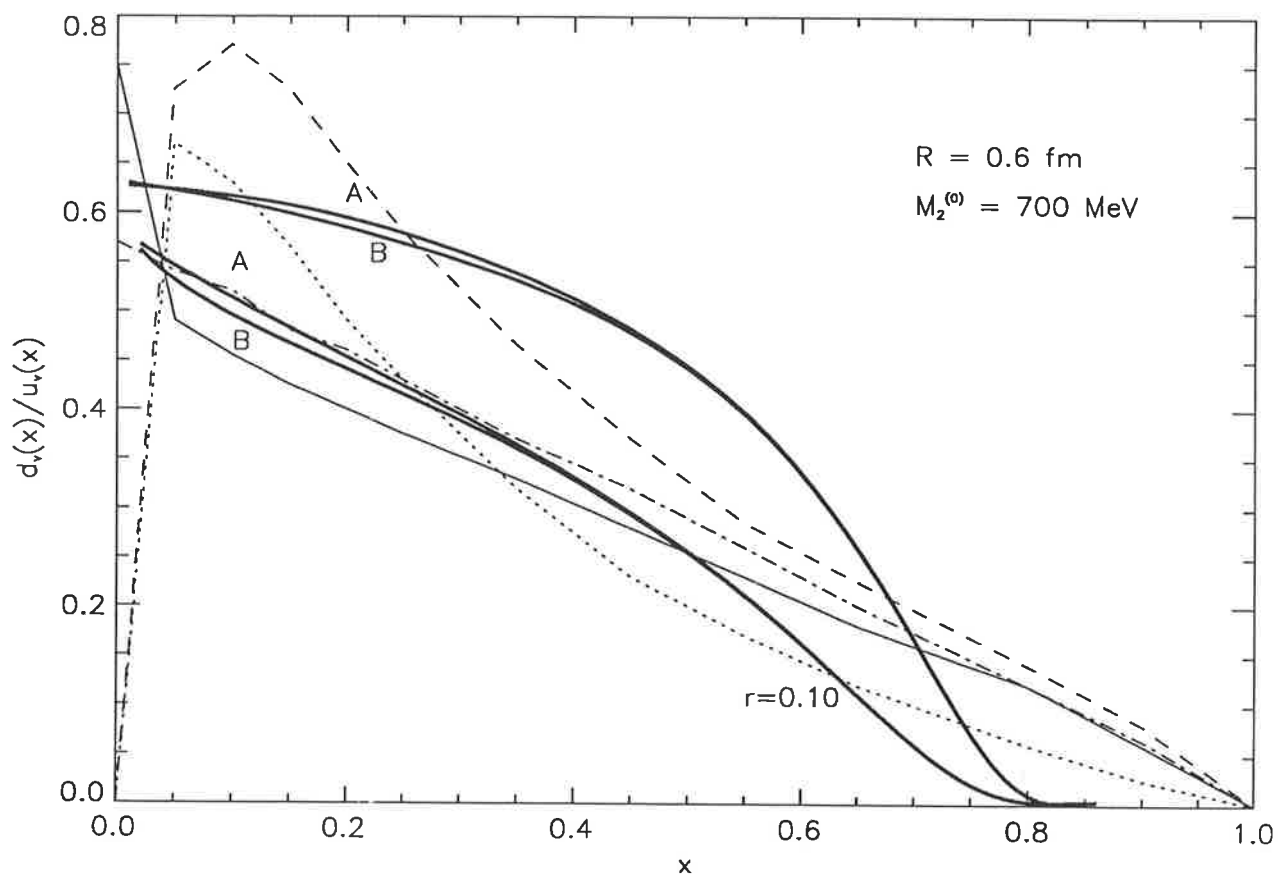


Figure 6.6: The ratio of valence distributions of the down and up quarks corresponding to Figure 6.5.

More worrying is the small scale μ , again leading to an overestimate of the valence distribution and $g_1^p(x)$ (Figure 6.7) at small x .

It also effects the neutron structure functions $g_1^n(x)$, shown in Figure 6.8. In an SU(6) symmetric model $g_1^n(x)$ is zero everywhere. The glue splitting causes it to become finite, although the first moment still vanishes. (The QCD evolution also induces a small value of $g_1^n(x, Q^2)$, even if $g_1^n(x, \mu^2) = 0$, because the singlet and non-singlet parts evolve differently. The effect is typically $\frac{1}{3}$ of the size of the effect considered here.) The large peak in $g_1^n(x, \mu^2)$ at large x as well as the value of μ causes the region where $g_1^n(x, Q^2 = 10\text{GeV}^2)$ is negative to move to rather small x . It will be interesting to compare this to data. A significantly larger value for the intercept would indicate that the bag model scale should not be as low as we have made it here.

Given this problem with the small value of μ it is of course tempting to ask how much worse the fits would become if we tried to increase it while adjusting the mass $M_2^{(0)}$. In Figures 6.9, 6.10, 6.11 and 6.12 we show corresponding curves for $r = 0.3$, which corresponds to $\mu = 0.46$ GeV.

The diquark masses in this case are 750MeV (scalar) and 950MeV (vector) (i.e. $M_2^{(0)} = 900$ MeV). Although the fit to the valence distribution is somewhat worse than before it is still in reasonable agreement. The larger diquark masses have increased the magnitude of the slope of the $\frac{d_v(x)}{u_v(x)}$ ratio, again leading to some discrepancy with the data. The distribution $g_1^p(x, Q^2)$ has not been effected greatly, however note that the intercept of $g_1^n(x, Q^2)$ has now moved to $x \sim 0.1$. A slightly better fit to $g_1^p(x, Q^2)$, not shown here, is obtained for a bag radius of 0.8 fm [SST].

Overall the fits with the larger value of r are probably still acceptable. Note however that the larger diquark-mass has shifted the peaks of the distributions to smaller x , thus increasing the contribution of the four quark intermediate states. This has led to a greater difference between the set of curves A and B, i.e. those with the four quark state parameterized by $(1-x)^7$ and $\frac{1}{\sqrt{x}}(1-x)^7$. If these parameters indeed reflect physical

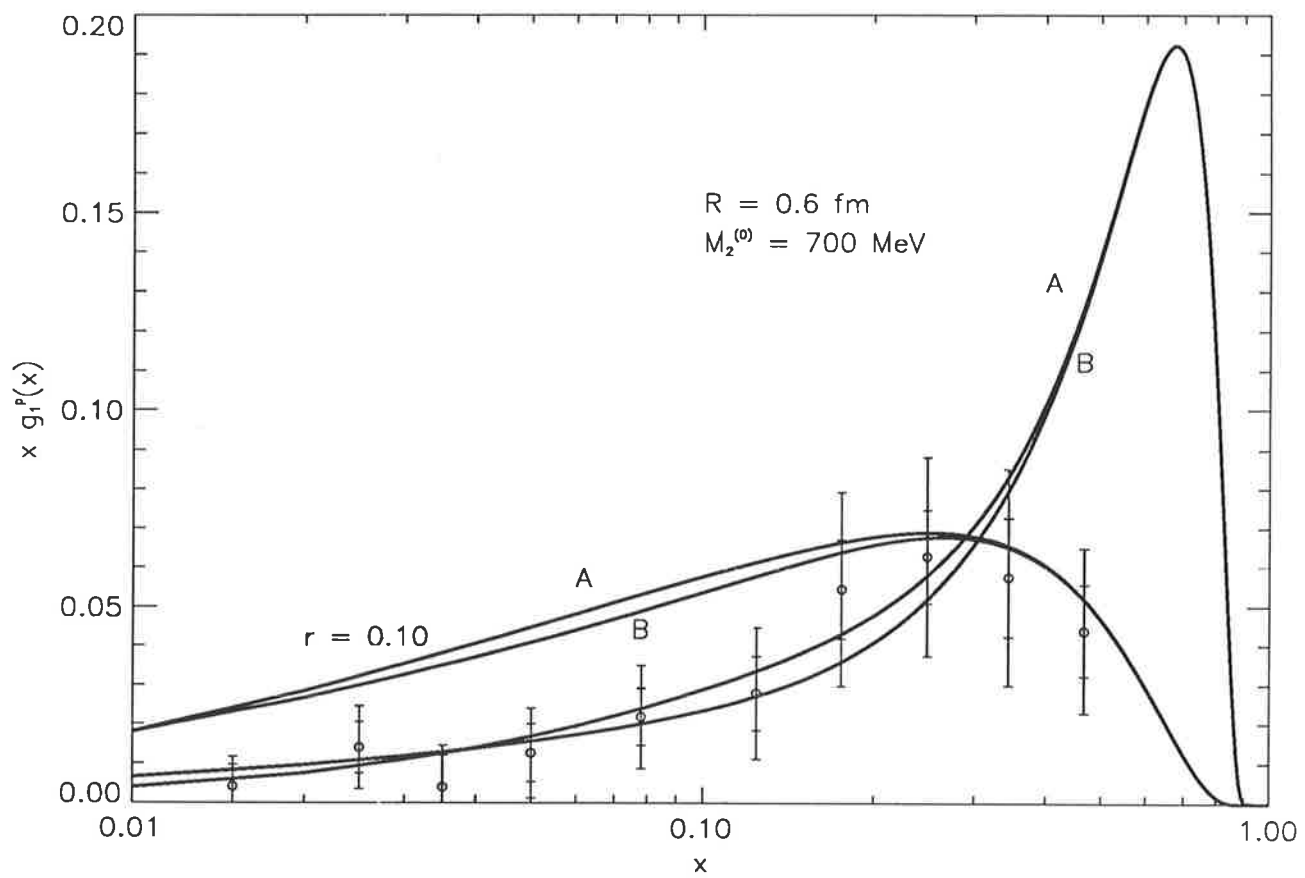


Figure 6.7: $g_1^p(x)$ corresponding to Figure 6.5.

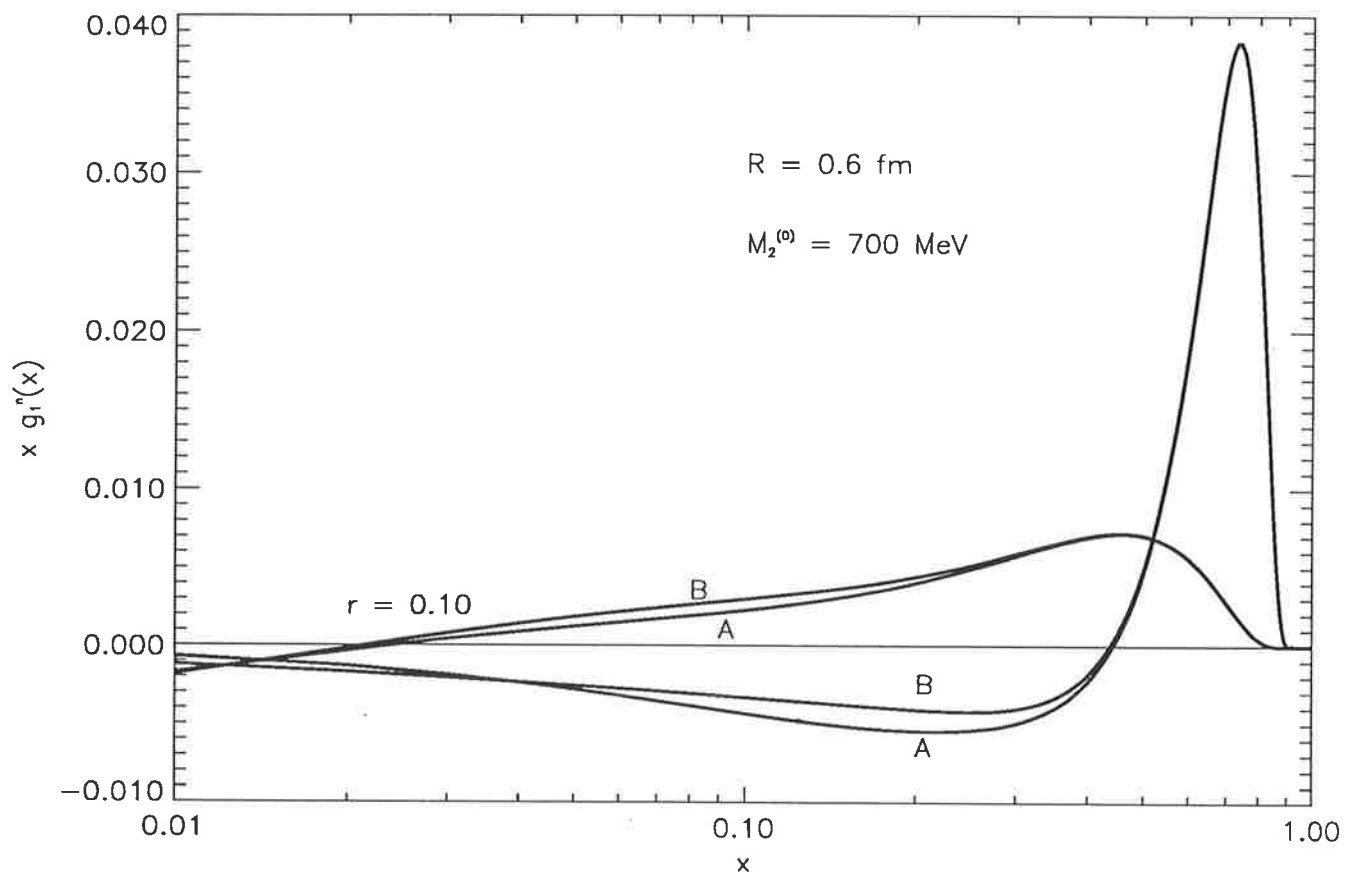


Figure 6.8: $g_1^n(x)$ corresponding to Figure 6.5.

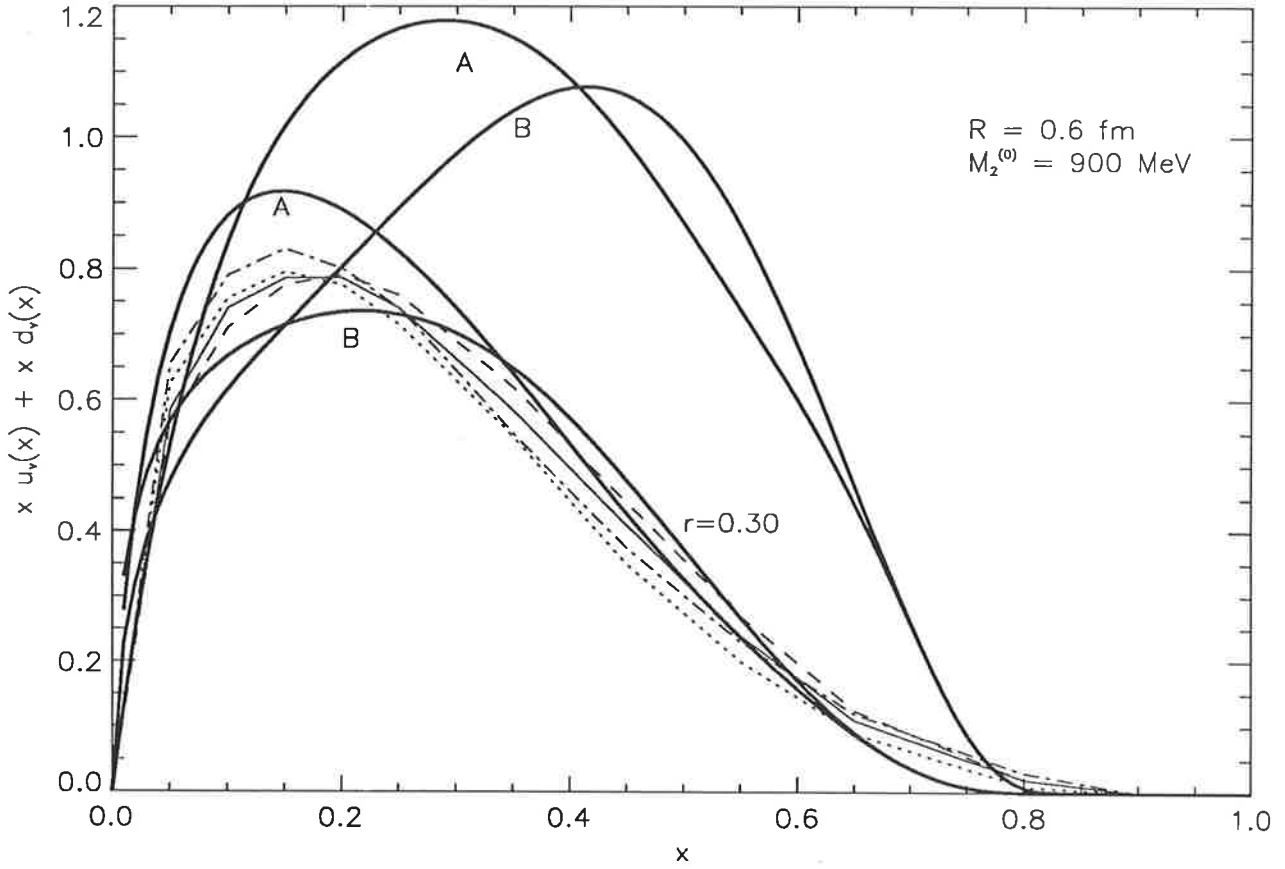


Figure 6.9: The valence distribution for the bare bag, with one gluon exchange included. The scalar and vector diquark masses are 750 MeV and 950 MeV respectively. The thick curves correspond to the theoretical results at both μ (~ 0.46 GeV) and a scale where the strong coupling constant is 30% of the coupling constant at μ .

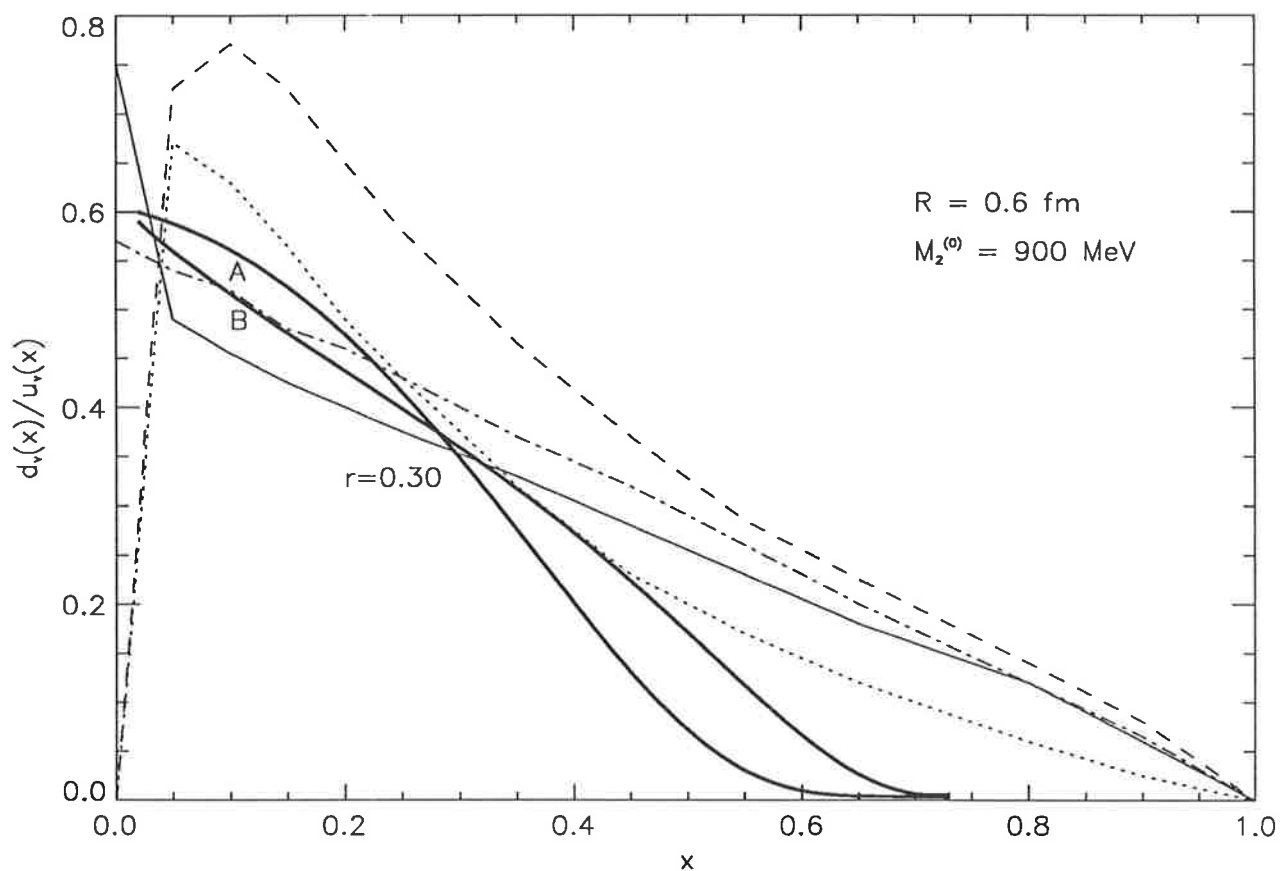


Figure 6.10: The ratio of valence distributions of the down and up quarks corresponding to Figure 6.9.

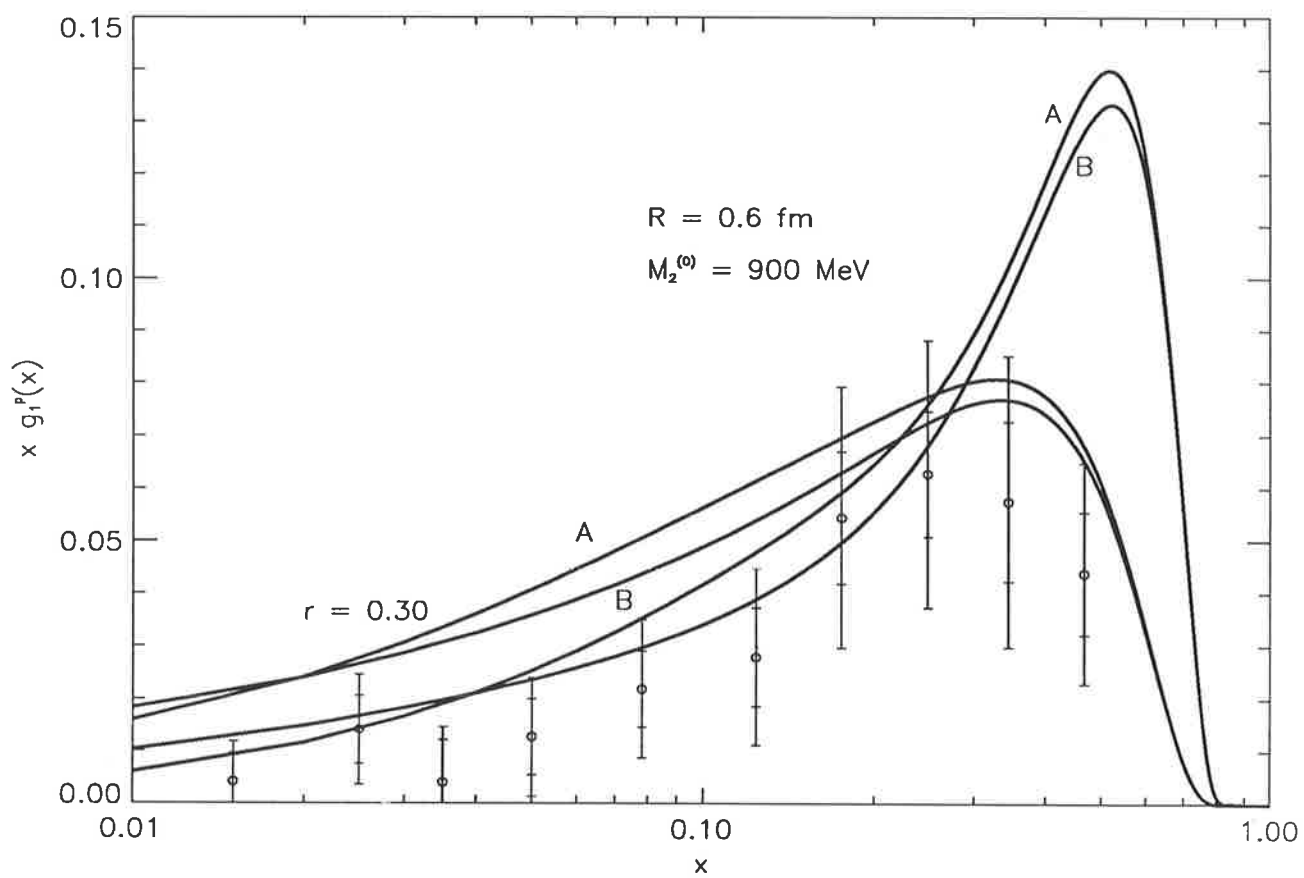


Figure 6.11: $g_1^p(x)$ corresponding to Figure 6.9.

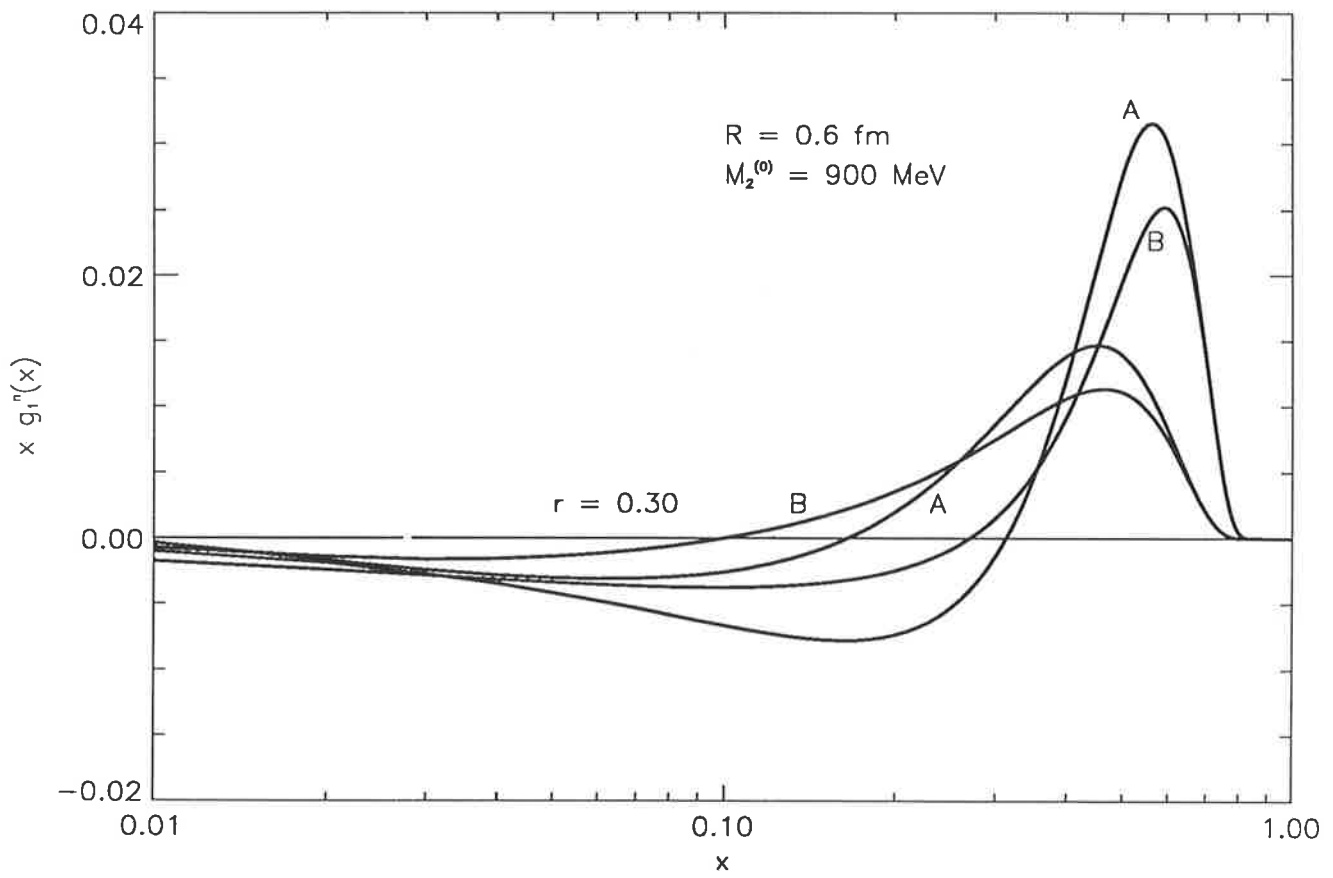


Figure 6.12: $g_1^n(x)$ corresponding to Figure 6.9.

reality then it is imperative to obtain a more satisfactory solution to the normalization problem. If the parameters corresponding to $r=0.1$ are closer to the truth the importance of the four quark state is less and the solution to the problem becomes less urgent, at least from a practical point of view.

6.5 Pions

Finally we shall estimate the effect of the pion cloud. As we have discussed in Section 3.8.2 pionic corrections are expected to shift the distributions to smaller x , thus having a similar effect to decreasing the value of the bag scale μ . We thus might expect that pionic corrections might allow us to use a larger value of μ . Until the spin-isospin structure of these corrections has been determined (and of course non-valence distributions receive sea contributions from the virtual photon scattering on the pion itself) we should restrict ourselves to examining the isospin independent valence contribution $u(x, Q^2) + d(x, Q^2)$. In fact we shall take the two fits of the last section, with $M_2^{(0)} = 700$ and 900 MeV, and shall examine by how much we are able to increase the scale μ .

QCD evolution and the pion cloud have a similar effect on the valence distribution in that both decrease the average momentum of the valence quarks, so it is not a surprise that we are able to fit the data for wide range of pion form factors (i.e. a wide range of Λ , defined in Section 3.8.2.) The probability of finding the nucleon as a bare nucleon plus pions increases with Λ and so the required range of evolution can be expected to decrease. For example, if the input distribution of the last section with $M_2 = 550$ and 750 MeV (scalar and vector masses respectively) is used we can obtain fits with $r = 0.15$, $\Lambda = 700$ MeV (i.e. $\mu = 0.30$ GeV and $\text{Pr}(\pi)=0.31$, see Figure 6.13) or $r = 0.18$, $\Lambda = 900$ MeV (i.e. $\mu = 0.33$ GeV and $\text{Pr}(\pi)=0.49$, see Figure 6.14).

Alternatively, for the case where $M_2 = 750$ and 950 MeV we obtain $r = 0.35$, $\Lambda = 700$ MeV (i.e. $\mu = 0.53$ GeV, see Figure 6.15) and $r = 0.40$, $\Lambda = 900$ MeV (i.e. $\mu = 0.60$ GeV, see Figure 6.16).

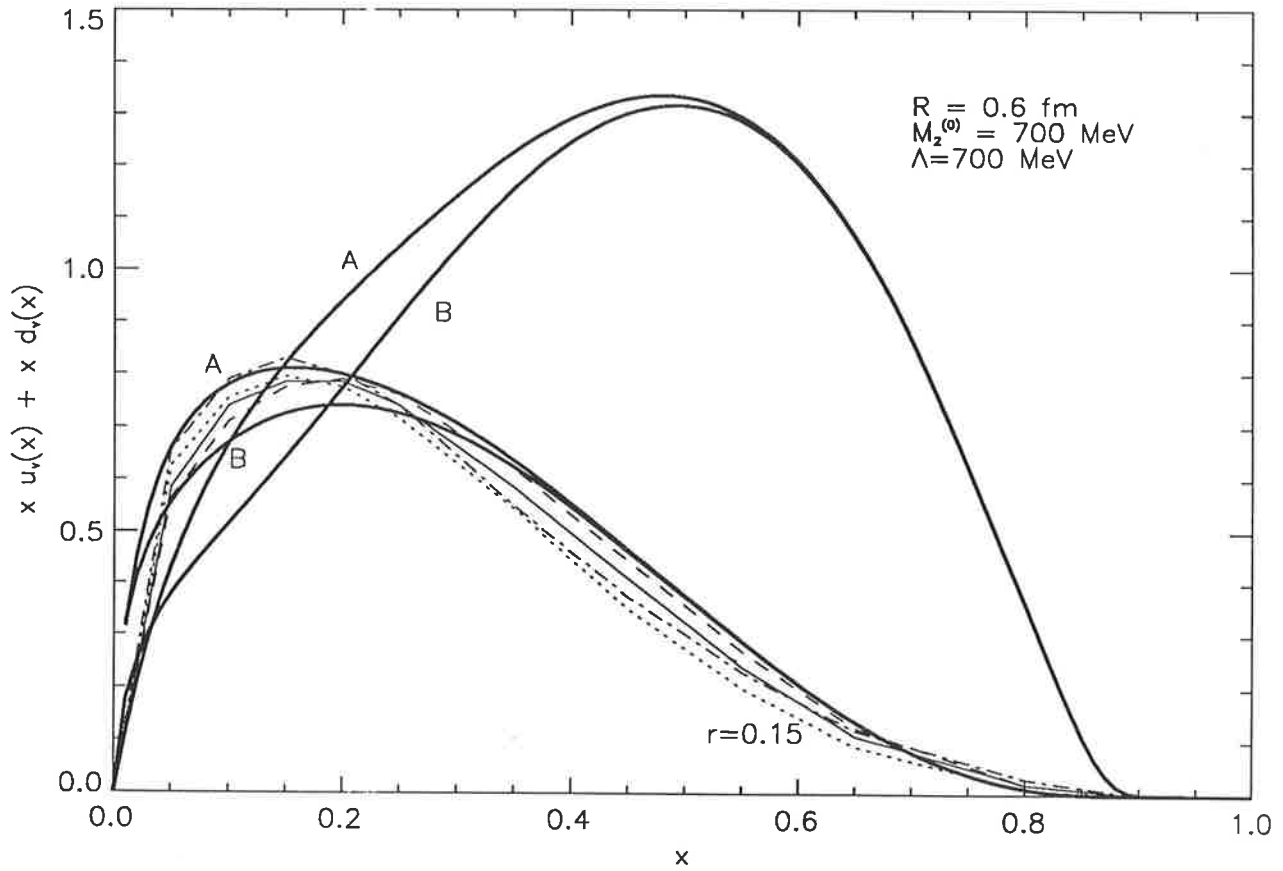


Figure 6.13: The valence distribution for the bag with pionic corrections and one gluon exchange included. The scalar and vector diquark masses are 550 MeV and 750 MeV respectively. Λ is 700 MeV, leading to $r = 0.15$ and $\mu = 0.30$ GeV

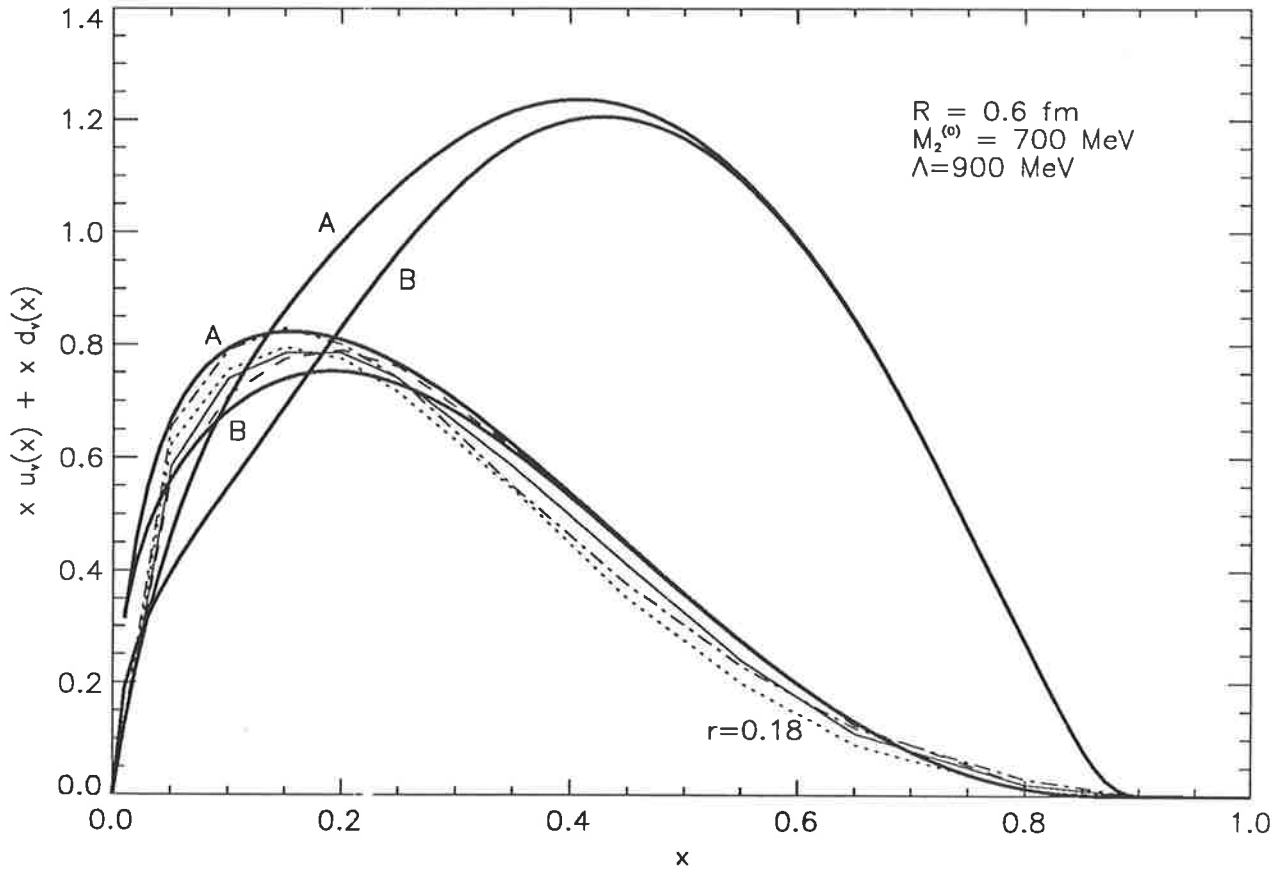


Figure 6.14: The valence distribution for the bag with pionic corrections and one gluon exchange included. The scalar and vector diquark masses are 550 MeV and 750 MeV respectively. Λ is 900 MeV, leading to $r = 0.18$ and $\mu = 0.33$ GeV

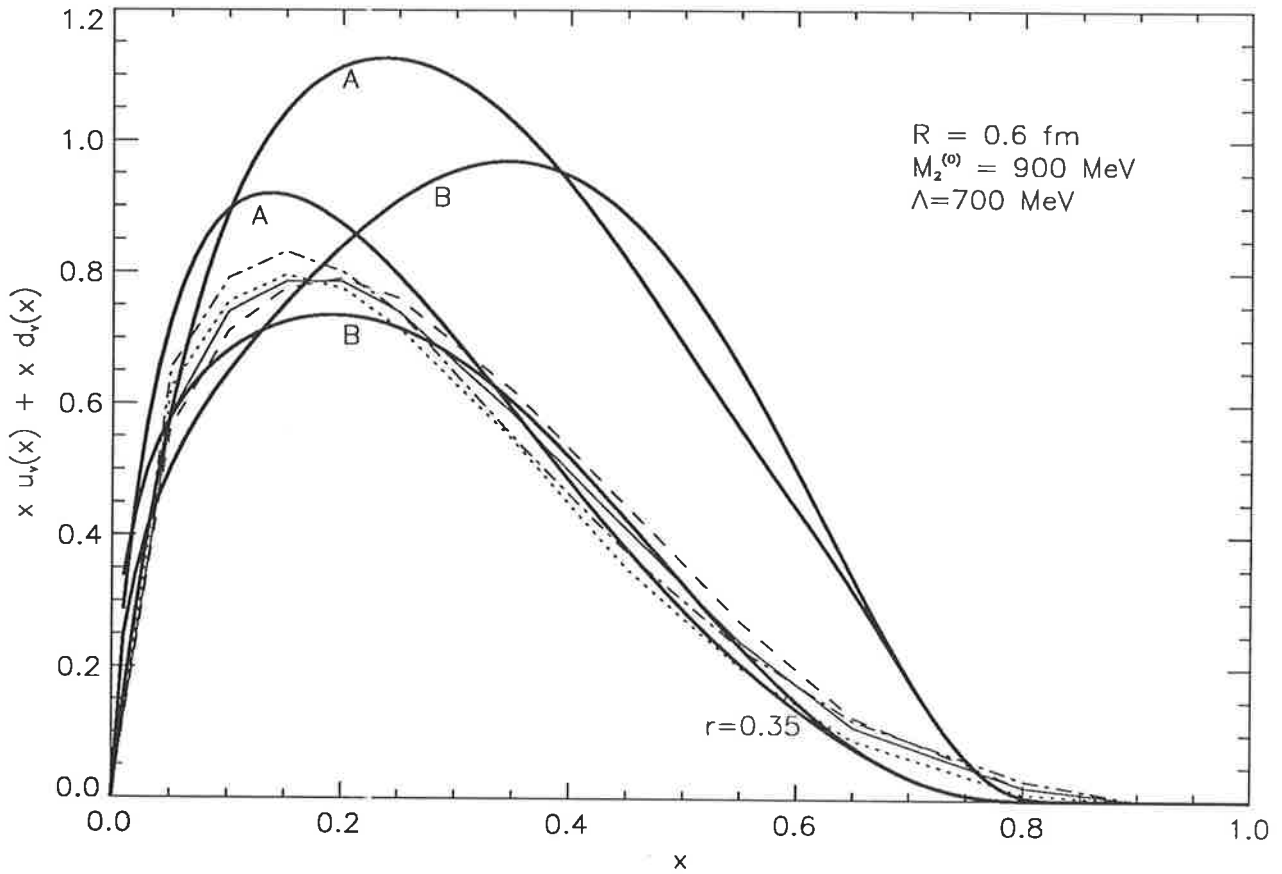


Figure 6.15: The valence distribution for the bag with pionic corrections and one gluon exchange included. The scalar and vector diquark masses are 750 MeV and 950 MeV respectively. Λ is 700 MeV, leading to $r = 0.35$ and $\mu = 0.53$ GeV

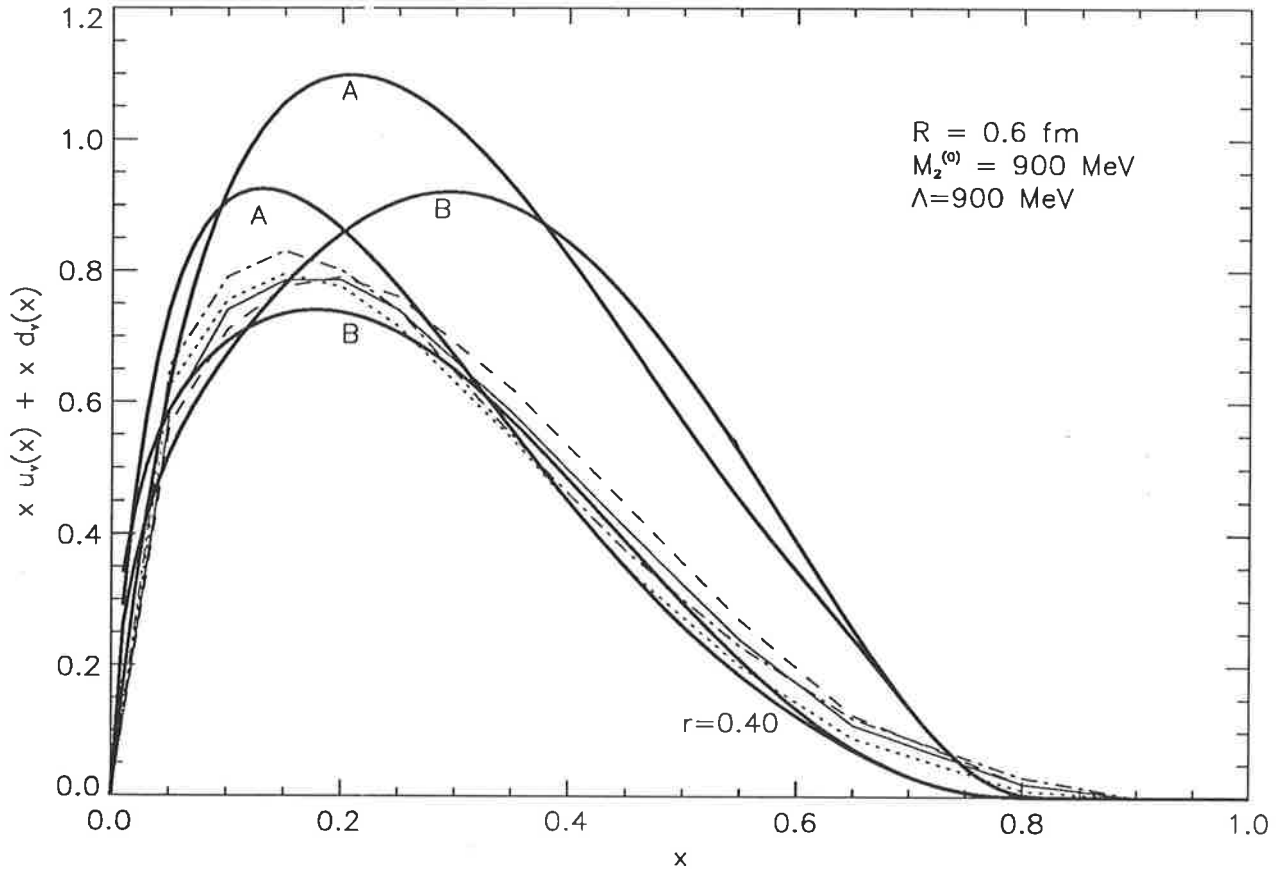


Figure 6.16: The valence distribution for the bag with pionic corrections and one gluon exchange included. The scalar and vector diquark masses are 750 MeV and 950 MeV respectively. Λ is 900 MeV, leading to $r = 0.40$ and $\mu = 0.60 \text{ GeV}$

6.6 Predictions

One of the purposes of model calculations such as the one presented in this thesis is to make predictions for future experiments, which we shall do in this section. Because of the surprising EMC results [EMC88,EMC89] several proposals for new measurements of the nucleons' spin dependent structure functions have been made:

- 1) The group at SLAC [A*89] propose to measure the neutron structure function $g_1^n(x)$ for $0.04 < x < 0.6$ and $1 < Q^2 < 6 \text{ GeV}^2$.
- 2) The SMC [SMC89] plan to run another experiment, this time for both the neutron's and proton's $g_1(x)$ in the range $0.01 < x < 0.7$ and $2.3 < Q^2 < 19.7 \text{ GeV}^2$.
- 3) At HERA the HERMES collaboration [HER88,HER89] has proposed to measure the neutron's and proton's $g_1^n(x)$ and, for the first time, $g_2(x)$. (The SMC proposal also calls for a preliminary measurement of this.) The kinematic ranges are $0.02 < x < 0.8$ and $1 < Q^2 < 20 \text{ GeV}^2$.

The Q^2 ranges of these experiments are essentially identical to the EMC's range, whose data we compared with in the previous sections. (Note that the EMC's data, with an average $Q^2=10.7 \text{ GeV}^2$, was collected over a range $3.5 < Q^2 < 29.5 \text{ GeV}^2$. The range of Q^2 for the SLAC measurement is somewhat lower than this but already in the range where the strong coupling constant is varying slowly; e.g. $r=0.1$ for $\langle Q^2 \rangle = 10 \text{ GeV}^2$ becomes $r=0.12$ for $\langle Q^2 \rangle = 4 \text{ GeV}^2$.) Because of this overlap in the ranges of Q^2 , predictions for the three proposed experiments can essentially be found in the previous sections. Here we shall merely exhibit the results for higher energies - $Q^2 \sim 10^2 \text{ GeV}^2$ and 10^3 GeV^2 .

For definiteness we shall concentrate on the results with one gluon exchange included but shall neglect the pions (essentially because the full spin-isospin structure has not yet been calculated for these.) As we have found in the previous section, model scales μ between 0.26 GeV and 0.46 GeV were acceptable. We shall choose the lower scale because the fits at $Q^2 \sim 10 \text{ GeV}^2$ were slightly better, but one should keep in mind that

this scale will increase when pionic corrections are included. Furthermore we include only the curves corresponding to scenario A because, as we saw in the previous sections, the two scenarios give similar results for the parameters we have chosen here.

In Figure 6.17 we show the expected valence distribution, in Figure 6.18 the d/u ratio, in Figure 6.19 the proton structure function g_1^p and in Figure 6.20 g_1^n .

We see that there is very little difference between the results for the three values of Q^2 . This reflects the fact that for values of the momentum transfer much above a few GeV^2 the coupling constant is changing only very slowly. Note that for g_1^n the x -intercept is pushed to very low values of x because of the prominent peak at large x of the primordial distribution.

6.7 Conclusion

The results presented in this chapter have been encouraging in the sense that despite the crudity of the model we are able to fit the experimental data for the valence distributions, the $\frac{d}{u}$ ratio and, at least qualitatively, the spin structure function g_1^p with the one set of parameters. The question that remains to be solved is the scale μ at which the bag model applies. We have found that we are able to do so for a large part of the x distribution for any scale between 0.25 and 0.6 GeV , depending on the particular model under consideration. The small x behaviour favours a scale that is towards the top of this range.

Perhaps the cleanest way to determine the model scale is a comparison with data for the, as yet unmeasured, neutron structure function $g_1^n(x, Q^2)$. We found the point at which this structure function changes sign to be highly dependent on the ratio of coupling constant r . If the scale is indeed very low ($\mu \sim 0.25 \text{ GeV}$) the structure function only seems to change sign around $x \sim 0.01$ while for higher scales ($\mu \sim 0.6 \text{ GeV}$) the intercept occurs near $x \sim 0.2$.

Pionic corrections affect the scale in that they take away momentum from the valence

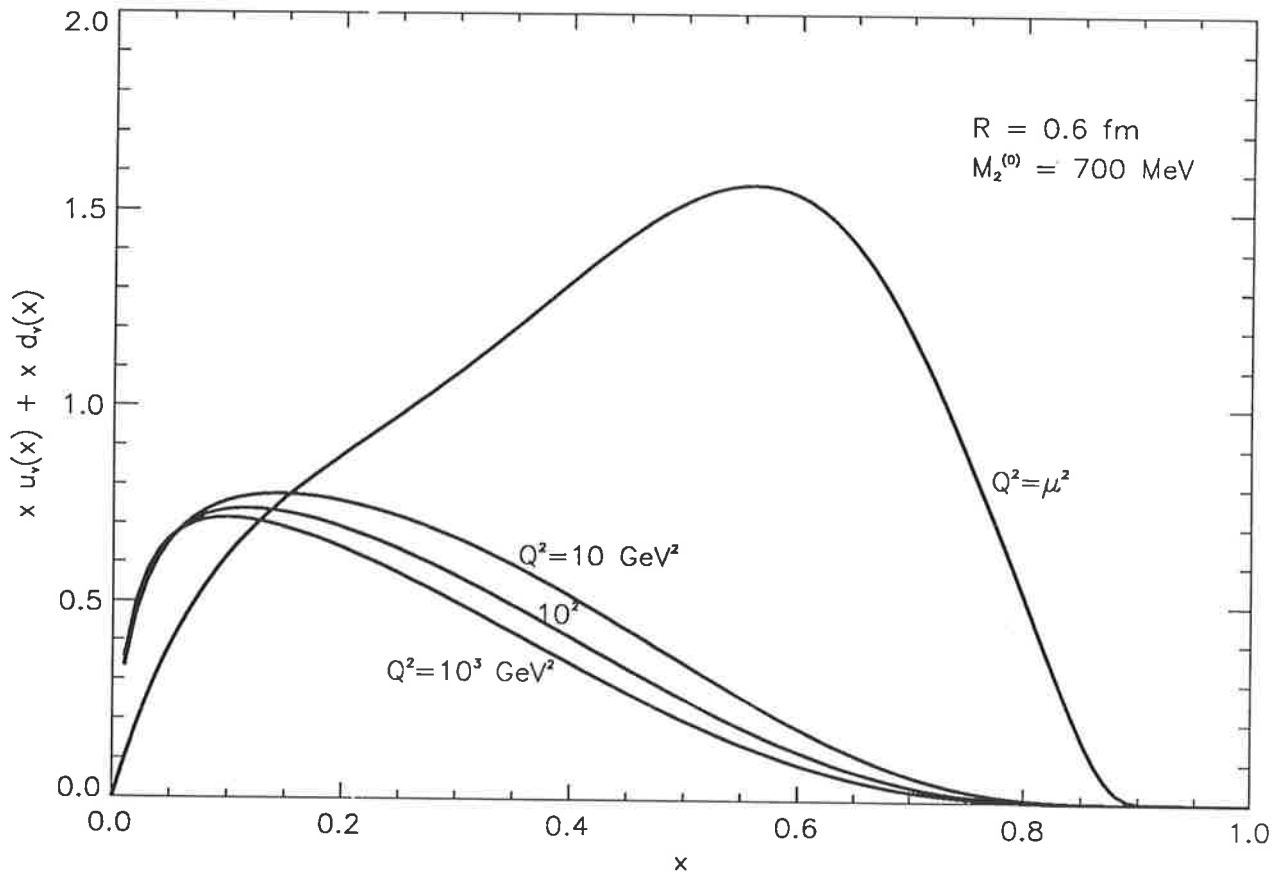


Figure 6.17: The valence distribution for the bag with one gluon exchange included. The scalar and vector diquark masses are 550 MeV and 750 MeV respectively.

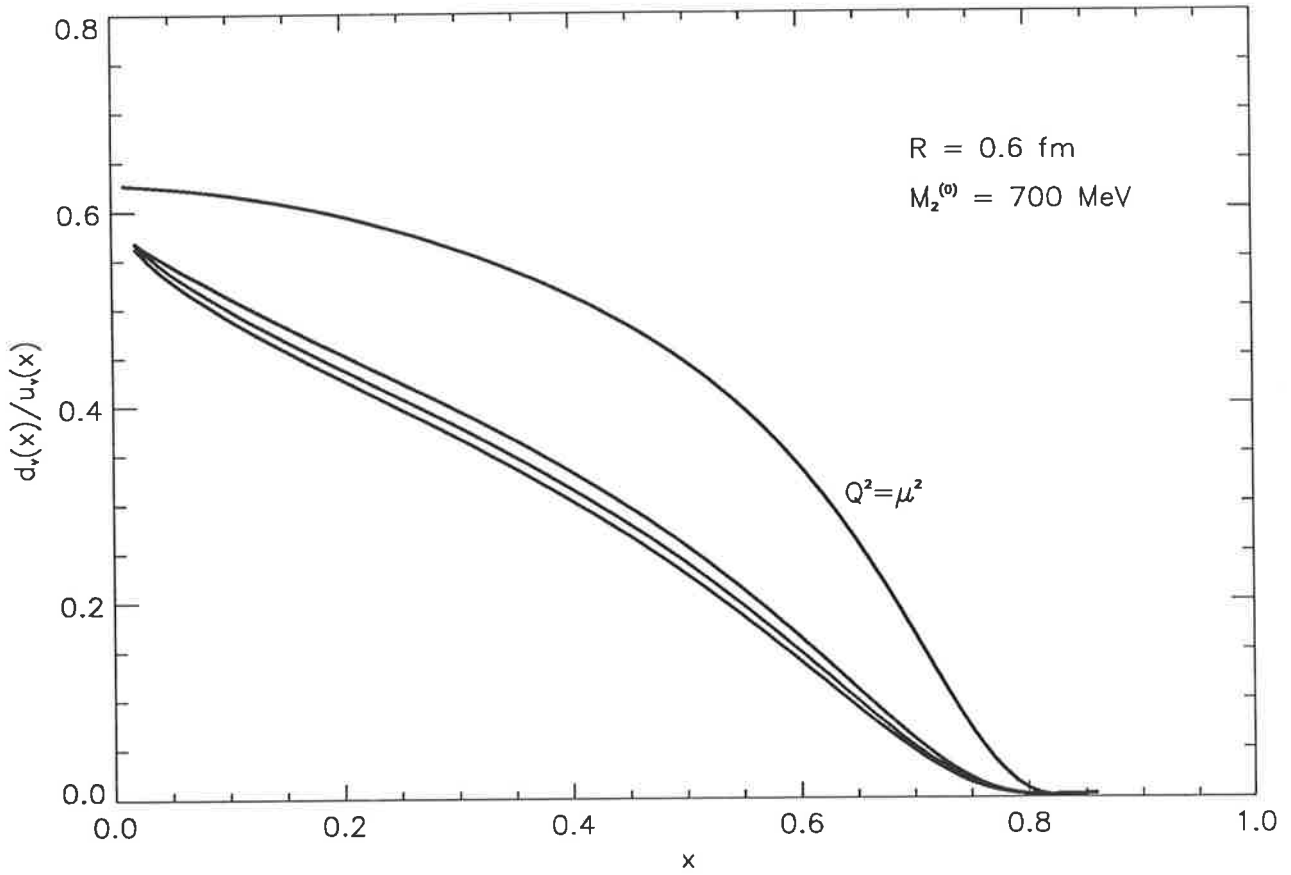


Figure 6.18: The d/u ratio for the bag with one gluon exchange included. The three curves correspond, from the bottom to the top, to $Q^2=10$, 100 and 1000 GeV^2 respectively.

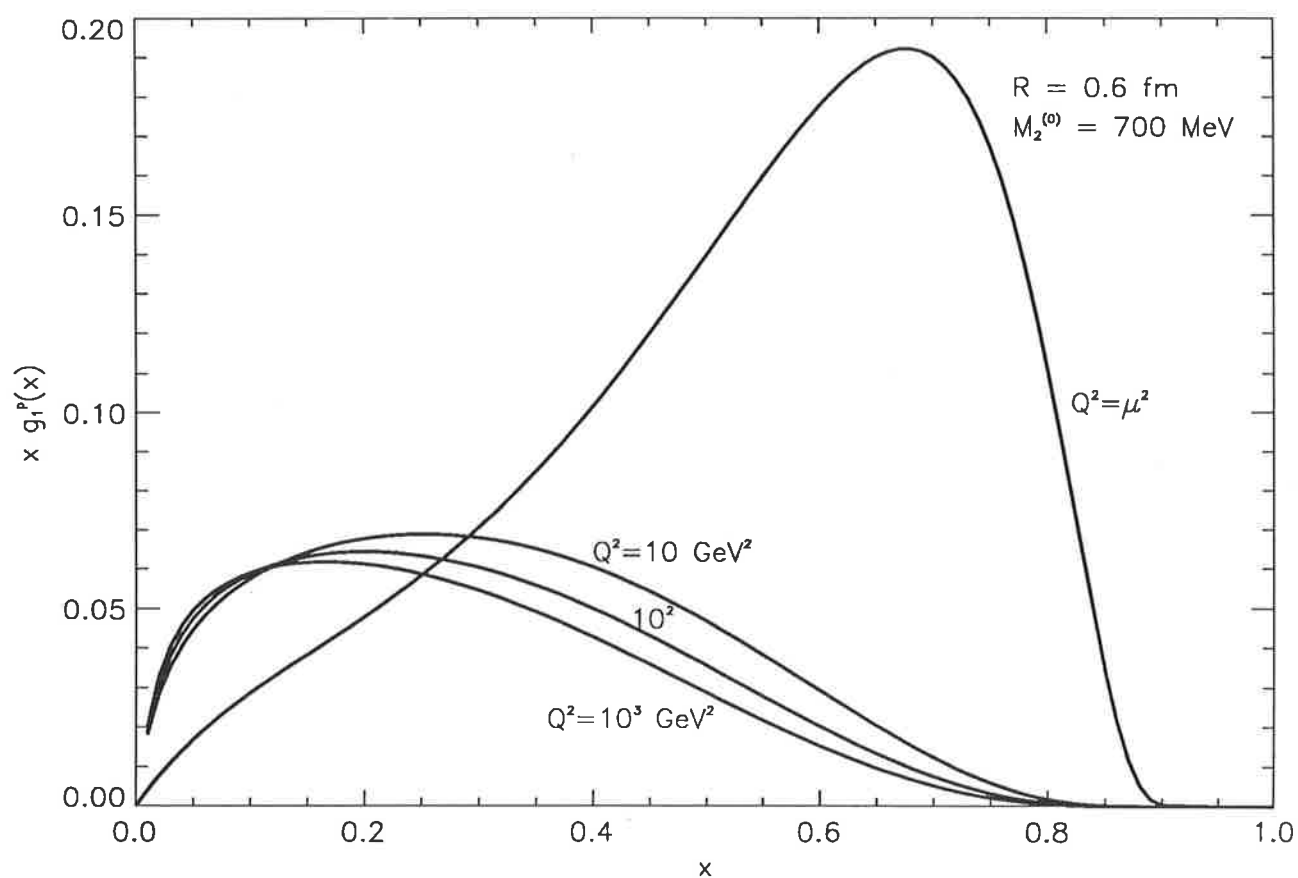


Figure 6.19: $g_1^p(x)$ for the bag with one gluon exchange included.

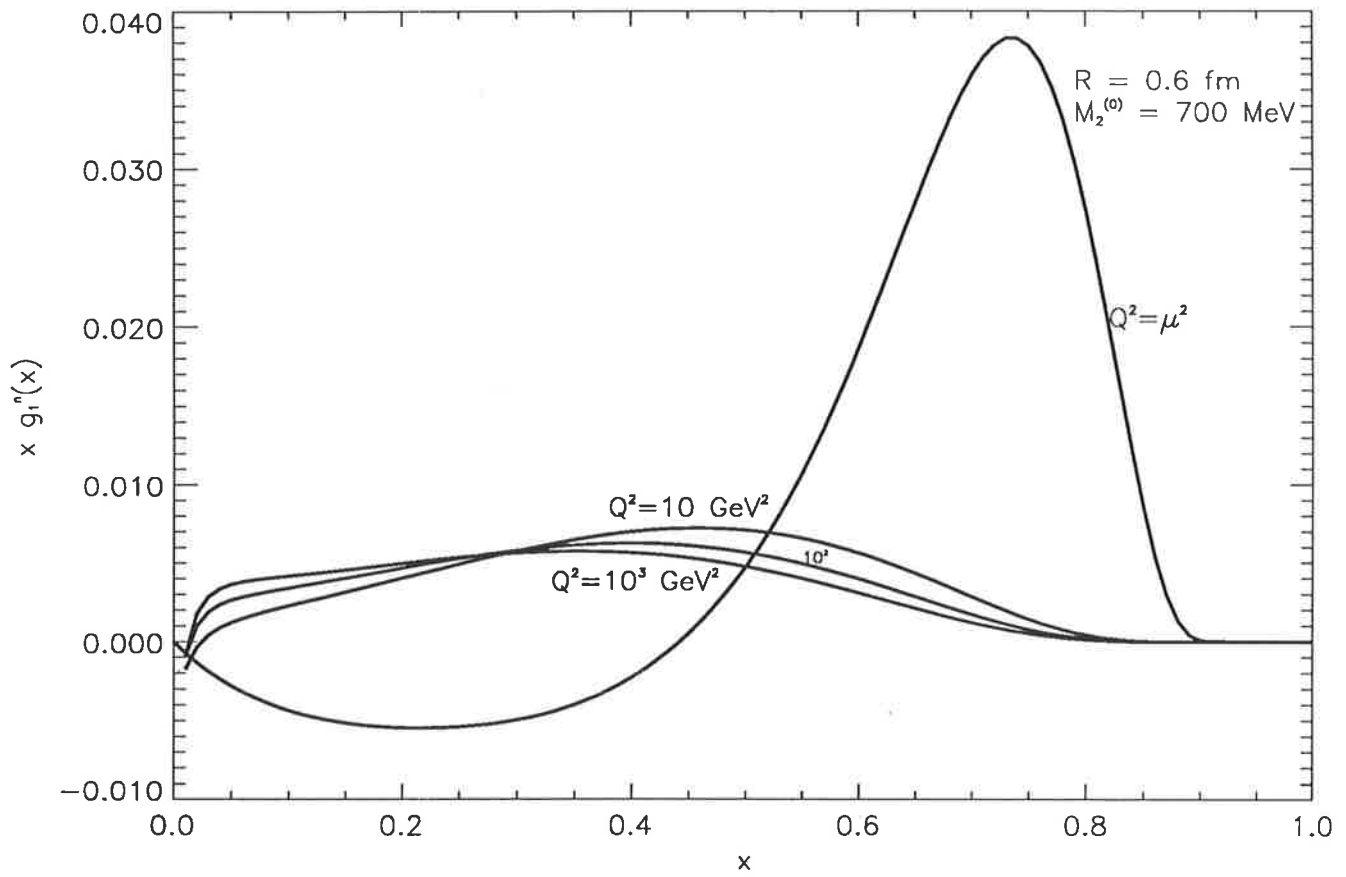


Figure 6.20: $q_1^n(x)$ for the bag with one gluon exchange included.

quarks. At present we are only able to give precise predictions for the corrections to the isospin singlet valence distribution. The full spin-isospin structure of these corrections could be worked out along the lines of the formalism presented in Chapter 5, but this has as yet not been done.

Perhaps the most pressing problem however is to obtain the correct normalizations within the model. If the intermediate state masses are small this is not so important from a phenomenological point of view, because then most of the normalization is carried by the contributions with the two quark intermediate states. Nevertheless, it is of course of interest to solve it in any case from a theoretical point of view.

Chapter 7

Conclusion & Outlook

It is time for us to summarize what has been achieved and outline what remains to be done. Our aim has been to obtain a comprehensive description of deep inelastic scattering processes by using our present knowledge of the low energy structure of hadrons. Particular attention has been paid to the spin structure of the theory.

Operationally the input provided by the low energy physics is contained in our choice of the quark wavefunctions. These are essentially those of the MIT bag, however with some important modifications necessitated by the inevitable deficiencies of a model like this.

One of these deficiencies is that the nucleon wavefunction, if it is taken to be just the product of three quark wavefunctions, is not a momentum eigenstate and is translationally not invariant. This problem has been dealt with by using a non-relativistic method, the Peierls-Yoccoz procedure, to project out a momentum eigenstate. Inevitably this modification introduces some extra problems: Apart from being non-relativistic (which is, as we have shown, probably a reasonable approximation - at least for x less than about 0.7), the projected internal wavefunction contains an unphysical momentum dependence. (Paradoxically we would, due to Lorentz contraction, expect momentum dependence if we were able to obtain a relativistic wavefunction; the internal momentum dependence in our formulation clearly has nothing to do with this. An additional conse-

quence of a description using an interacting relativistic field theory would be a shift of some of the momentum carried by the valence quarks to the dynamical bag boundary.) As we have seen, in principle the momentum dependence can be eliminated by the use of the Peierls-Thouless projection. It would be interesting to compare predictions of the latter with those presented here in order to be able to gauge the effect of the unphysical momentum dependence.

The quark distributions presented in this thesis are not expressed in terms of forward scattering matrix elements as is usually done, but rather in terms of the modulus square of the matrix element of a quark field operator between a three quark initial state and a two (or four) quark (antiquark) intermediate state - multiplied by a delta function. It is because of this delta function, which is extracted before any approximations are made, that we are able to achieve vanishing support above $x = 1$ - a definite improvement over previous model calculations.

Extracting the delta function by the insertion of a complete set of intermediate states has another important advantage: It enables us to describe, in a simple and straightforward manner, the effects of one gluon exchange. We are therefore able to make meaningful predictions for the d/u ratio as well as the proton and neutron asymmetries.

A further refinement has been the inclusion of a pion cloud. We calculated the corrections to the first moment of $g_1^p(x)$ and $g_1^n(x)$ within the cloudy bag model and found that the first moment of the proton structure function is decreased by about 15 % when compared to the MIT bag value, while that of the neutron acquires a non-zero value. As well as this we investigated the modification to the actual distributions themselves when one allow the nucleon to radiate a pion, although only the isoscalar case has been considered. A complete calculation, using the full spin-isospin structure of the cloudy bag model, would be a natural next step.

In principle one should also calculate the contribution from the virtual photon scattering of the quarks in the pion itself (this has in fact been done to some extent in the

literature). We feel however that at the present time this is perhaps not so important because the contribution is expected to be concentrated at small x . In this region the distributions we have calculated are somewhat uncertain due to the normalization problem discussed in Chapter 3. It would seem that this problem should be addressed first so that one has an internally consistent formulation in a simple model, to which one may then add such refinements as $\gamma\pi$ scattering contributions.

The *raison d'être* for the model predictions is of course the possibility of a comparison to data. This is enabled through the QCD evolution formalism of Chapter 4, where we have extended a formalism developed for the spin independent case to the spin dependent one. We found that as far as the model calculations went there is good agreement with data except perhaps for rather small x . Even this disagreement may disappear if the rigorous inclusion of the pion cloud enables the model scale μ to be increased to around 0.5 - 1 GeV.

Two extensions to the evolution formalism that should be undertaken spring to mind. In the light of the attempts currently underway to measure the structure functions $g_{1,2}$ of the proton and neutron it is imperative to perform a complete calculation of the anomalous dimensions of the spin dependent operators to next to leading logarithmic order. As was the case for the spin independent calculation, the formalism of Chapter 4 can presumably be straightforwardly extended once this calculation is performed.

One should also extend the evolution calculations to the twist three operators needed for the structure function g_2 . Again there should be no problem in doing this along the lines of Chapter 4. When this is done attention should be paid to the calculation of the twist three operators within the model of Chapter 3. (The twist two piece of g_2 is essentially already calculated as it is merely given by an integral of g_1).

Finally, the work presented in this thesis can easily be extended to the calculation of the spin independent structure function F_2 as well as those measured in neutrino scattering. It is hoped that this will be undertaken in the near future to provide a

complete confrontation of the predictions of a low energy model of hadron structure with deep inelastic scattering data.

Appendix A

Appendix

In this appendix we list the essential definitions for evaluating coefficients for the expansions 4.42 up to arbitrary order with the use of the algebraic manipulation program *MATHEMATICA*. An example of their use as well as the coefficients up to sixth order are also listed.

Definitions

(* The anomalous dimensions of equation 4.25 for three colours. The variable log is defined as $\text{Log}[a z]$, where a is defined after equation 4.42 *)

gam1/: gam1[z_, log_] := 4/3*(3/2 + 1/(z - 1) + 1/(z - 2) - 2*EulerGamma - 2*phi[z, log])

gam2/: gam2[z_] := 3*(z - 3)/(z - 2)/(z - 1)

gam3/: gam3[z_] := 4/3 * z/(z - 2)/(z - 1)

gam4/: gam4[z_, log_] := 3*(3/2 + 4/(z - 2) - 2/(z - 1) - 2*EulerGamma - 2*phi[z, log])

(* The definition of phi - up to order z^{-20} *)

phi/: phi[z_, log_] := log - EulerGamma + 3/4 + sum[z]

sum/: sum[z_] := -1/(2*) - Sum[BernoulliB[2*n]/(2*n*z^(2*n)), n, 1, 10]

(* The various definitions of equations 4.20 and 4.21 *)

```
w1/: w1[z_, log_] := 1/2*(1 - gamminus[z, log]/root[z, log])
w2/: w2[z_, log_] := 1/2*(1 + gamminus[z, log]/root[z, log])
w3/: w3[z_, log_] := gam2[z]/root[z, log]
w4/: w4[z_, log_] := gam3[z]/root[z, log]
gamminus/: gamminus[z_, log_] := gam1[z, log] - gam4[z, log]
gamplus/: gamplus[z_, log_] := gam1[z, log] + gam4[z, log]
tplus/: tplus[z_, log_] := -1/9*(gamplus[z, log] + root[z, log])
tminus/: tminus[z_, log_] := -1/9*(gamplus[z, log] - root[z, log])
root/: root[z_, log_] := Abs[Sqrt[gamminus[z, log]^2 + 4*gam2[z]*gam3[z]]]
```

An example

In[1]:= <<def.m (* load definitions *)

In[2]:= nonsinglet[z,log]=ExpandAll[Exp[-2/9 u gam1[z,log]]] (* define nonsinglet *)

$$Out[2] = \exp \left[\frac{16 \log u}{27} + \frac{349222u}{22275z^{20}} - \frac{175468u}{96957z^{18}} + \frac{3617u}{13770z^{16}} - \frac{4u}{81z^{14}} + \frac{1382u}{110565z^{12}} - \frac{4u}{891z^{10}} + \frac{u}{405z^8} - \frac{4u}{1701z^6} + \frac{2u}{405z^4} - \frac{4u}{81z^2} - \frac{8u}{27z} - \frac{8u}{27(-2+z)} - \frac{8u}{27(-1+z)} \right]$$

In[3]:= %/Exp[16 log u/27] (* divide by term that blows up at infinity *)

$$Out[3] = \exp \left[\frac{349222u}{22275z^{20}} - \frac{175468u}{96957z^{18}} + \frac{3617u}{13770z^{16}} - \frac{4u}{81z^{14}} + \frac{1382u}{110565z^{12}} - \frac{4u}{891z^{10}} + \frac{u}{405z^8} - \frac{4u}{1701z^6} + \frac{2u}{405z^4} - \frac{4u}{81z^2} - \frac{8u}{27z} - \frac{8u}{27(-2+z)} - \frac{8u}{27(-1+z)} \right]$$

In[4]:= Series[%,log,Infinity,3,z,Infinity,3] (* obtain series *)

$$Out[4] = 1 - \frac{8u}{9z} + \frac{-\frac{76u}{81} + \frac{32u^2}{81}}{z^2} + \frac{-\frac{40u}{27} + \frac{608u^2}{729} - \frac{256u^3}{2187}}{z^3} + O\left(\frac{1}{z}\right)^4$$

Coefficients

$$c_{NS}[0,0] = 1$$

$$c_{NS}[1,0] = -\frac{8u}{9}$$

$$c_{NS}[2,0] = -\frac{76u}{81} + \frac{32u^2}{81}$$

$$c_{NS}[3,0] = -\frac{40u}{27} + \frac{608u^2}{729} - \frac{256u^3}{2187}$$

$$c_{NS}[4,0] = -\frac{1078u}{405} + \frac{11528u^2}{6561} - \frac{2432u^3}{6561} + \frac{512u^4}{19683}$$

$$c_{NS}[5,0] = -\frac{136u}{27} + \frac{41072u^2}{10935} - \frac{57664u^3}{59049} + \frac{19456u^4}{177147} - \frac{4096u^5}{885735}$$

$$c_{NS}[6,0] = \frac{-16636u}{1701} + \frac{264808u^2}{32805} - \frac{19329568u^3}{7971615} + \frac{184576u^4}{531441} - \frac{38912u^5}{1594323} + \frac{16384u^6}{23914845}$$

$$\begin{aligned}
c_{qq+}[2, 2] &= \frac{9}{25} \\
c_{qq+}[3, 2] &= \frac{27}{25} - \frac{18u}{25} \\
c_{qq+}[3, 3] &= \frac{27}{25} \\
c_{qq+}[4, 2] &= \frac{9}{5} - \frac{91u}{25} + \frac{18u^2}{25} \\
c_{qq+}[4, 3] &= \frac{1581}{250} - \frac{258u}{125} \\
c_{qq+}[4, 4] &= \frac{5103}{2500} \\
c_{qq+}[5, 2] &= \frac{27}{25} - \frac{57u}{5} + \frac{128u^2}{25} - \frac{12u^3}{25} \\
c_{qq+}[5, 3] &= \frac{5571}{250} - \frac{2064u}{125} + \frac{246u^2}{125} \\
c_{qq+}[5, 4] &= \frac{11772}{625} - \frac{4563u}{1250} \\
c_{qq+}[5, 5] &= \frac{3159}{1250} \\
c_{qq+}[6, 2] &= -\frac{27}{5} - \frac{6709u}{250} + \frac{10009u^2}{450} - \frac{22u^3}{5} + \frac{6u^4}{25} \\
c_{qq+}[6, 3] &= \frac{29199}{500} - \frac{59117u}{750} + \frac{7493u^2}{375} - \frac{156u^3}{125} \\
c_{qq+}[6, 4] &= \frac{201111}{2000} - \frac{105591u}{2500} + \frac{4039u^2}{1250} \\
c_{qq+}[6, 5] &= \frac{444447}{12500} - \frac{12177u}{3125} \\
c_{qq+}[6, 6] &= \frac{41553}{50000}
\end{aligned}$$

$$\begin{aligned}
c_{qq-}[0,0] &= 1 \\
c_{qq-}[1,0] &= \frac{-8u}{9} \\
c_{qq-}[2,0] &= -\frac{76u}{81} + \frac{32u^2}{81} \\
c_{qq-}[2,1] &= \frac{-4u}{15} \\
c_{qq-}[2,2] &= -\frac{9}{25} \\
c_{qq-}[3,0] &= -\frac{40u}{27} + \frac{608u^2}{729} - \frac{256u^3}{2187} \\
c_{qq-}[3,1] &= \frac{-4u}{5} + \frac{32u^2}{135} \\
c_{qq-}[3,2] &= -\frac{27}{25} - \frac{2u}{25} \\
c_{qq-}[3,3] &= -\frac{27}{25} \\
c_{qq-}[4,0] &= -\frac{1078u}{405} + \frac{11528u^2}{6561} - \frac{2432u^3}{6561} + \frac{512u^4}{19683} \\
c_{qq-}[4,1] &= \frac{-4u}{3} + \frac{1168u^2}{1215} - \frac{128u^3}{1215} \\
c_{qq-}[4,2] &= -\frac{9}{5} - \frac{47u}{45} + \frac{56u^2}{225} \\
c_{qq-}[4,3] &= -\frac{1581}{250} + \frac{69u}{125} \\
c_{qq-}[4,4] &= -\frac{5103}{2500} \\
c_{qq-}[5,0] &= -\frac{136u}{27} + \frac{41072u^2}{10935} - \frac{57664u^3}{59049} + \frac{19456u^4}{177147} - \frac{4096u^5}{885735} \\
c_{qq-}[5,1] &= \frac{-4u}{5} + \frac{944u^2}{405} - \frac{5888u^3}{10935} + \frac{1024u^4}{32805} \\
c_{qq-}[5,2] &= -\frac{27}{25} - \frac{383u}{75} + \frac{1312u^2}{675} - \frac{896u^3}{6075} \\
c_{qq-}[5,3] &= -\frac{5571}{250} + \frac{64u}{25} + \frac{16u^2}{375} \\
c_{qq-}[5,4] &= -\frac{11772}{625} - \frac{2223u}{1250} \\
c_{qq-}[5,5] &= -\frac{3159}{1250} \\
c_{qq-}[6,0] &= -\frac{16636u}{1701} + \frac{264808u^2}{32805} - \frac{19329568u^3}{7971615} + \frac{184576u^4}{531441} - \frac{38912u^5}{1594323} + \frac{16384u^6}{23914845} \\
c_{qq-}[6,1] &= 4u + \frac{23432u^2}{6075} - \frac{163616u^3}{98415} + \frac{18944u^4}{98415} - \frac{2048u^5}{295245} \\
c_{qq-}[6,2] &= \frac{27}{5} - \frac{36949u}{2250} + \frac{51988u^2}{6075} - \frac{7424u^3}{6075} + \frac{2816u^4}{54675}
\end{aligned}$$

$$\begin{aligned}
c_{qq-}[6, 3] &= -\frac{29199}{500} + \frac{19517u}{4500} + \frac{1744u^2}{1125} - \frac{448u^3}{3375} \\
c_{qq-}[6, 4] &= -\frac{201111}{2000} + \frac{39747u}{2500} - \frac{358u^2}{625} \\
c_{qq-}[6, 5] &= -\frac{444447}{12500} + \frac{41013u}{12500} \\
c_{qq-}[6, 6] &= -\frac{41553}{50000}
\end{aligned}$$

$$\begin{aligned}
c_{qg+}[1, 1] &= -\frac{9}{10} \\
c_{qg+}[2, 1] &= \frac{9u}{5} \\
c_{qg+}[2, 2] &= -\frac{27}{20} \\
c_{qg+}[3, 1] &= \frac{9}{5} + \frac{37u}{10} - \frac{9u^2}{5} \\
c_{qg+}[3, 2] &= -\frac{771}{200} + \frac{123u}{50} \\
c_{qg+}[3, 3] &= -\frac{1377}{1000} \\
c_{qg+}[4, 1] &= \frac{27}{5} + \frac{24u}{5} - \frac{37u^2}{5} + \frac{6u^3}{5} \\
c_{qg+}[4, 2] &= -\frac{171}{25} + \frac{627u}{50} - \frac{111u^2}{50} \\
c_{qg+}[4, 3] &= -\frac{9621}{1000} + \frac{1017u}{500} \\
c_{qg+}[4, 4] &= -\frac{243}{2000} \\
c_{qg+}[5, 1] &= \frac{63}{5} - \frac{21u}{100} - \frac{749u^2}{36} + \frac{37u^3}{5} - \frac{3u^4}{5} \\
c_{qg+}[5, 2] &= -\frac{2097}{400} + \frac{4969u}{120} - \frac{983u^2}{60} + \frac{33u^3}{25} \\
c_{qg+}[5, 3] &= -\frac{156553}{4000} + \frac{20687u}{1000} - \frac{673u^2}{500} \\
c_{qg+}[5, 4] &= -\frac{178929}{20000} - \frac{5589u}{5000} \\
c_{qg+}[5, 5] &= \frac{349191}{100000} \\
c_{qg+}[6, 1] &= 27 - 27u - \frac{6737u^2}{150} + \frac{533u^3}{18} - \frac{74u^4}{15} + \frac{6u^5}{25} \\
c_{qg+}[6, 2] &= \frac{1701}{100} + \frac{10373u}{100} - \frac{15069u^2}{200} + \frac{962u^3}{75} - \frac{29u^4}{50} \\
c_{qg+}[6, 3] &= -\frac{230871}{2000} + \frac{234531u}{2000} - \frac{4841u^2}{250} + \frac{23u^3}{50} \\
c_{qg+}[6, 4] &= -\frac{3131613}{40000} + \frac{14589u}{2500} + \frac{11673u^2}{5000} \\
c_{qg+}[6, 5] &= \frac{998163}{50000} - \frac{433431u}{50000} \\
c_{qg+}[6, 6] &= \frac{391473}{40000}
\end{aligned}$$

$$\begin{aligned}
c_{qg-}[1,1] &= \frac{9}{10} \\
c_{qg-}[2,1] &= \frac{-4u}{5} \\
c_{qg-}[2,2] &= \frac{27}{20} \\
c_{qg-}[3,1] &= -\frac{9}{5} - \frac{38u}{45} + \frac{16u^2}{45} \\
c_{qg-}[3,2] &= \frac{771}{200} - \frac{36u}{25} \\
c_{qg-}[3,3] &= \frac{1377}{1000} \\
c_{qg-}[4,1] &= -\frac{27}{5} + \frac{4u}{15} + \frac{304u^2}{405} - \frac{128u^3}{1215} \\
c_{qg-}[4,2] &= \frac{171}{25} - \frac{406u}{75} + \frac{56u^2}{75} \\
c_{qg-}[4,3] &= \frac{9621}{1000} - \frac{243u}{125} \\
c_{qg-}[4,4] &= \frac{243}{2000} \\
c_{qg-}[5,1] &= -\frac{63}{5} + \frac{307u}{75} + \frac{3172u^2}{3645} - \frac{1216u^3}{3645} + \frac{256u^4}{10935} \\
c_{qg-}[5,2] &= \frac{2097}{400} - \frac{16763u}{1350} + \frac{2372u^2}{675} - \frac{512u^3}{2025} \\
c_{qg-}[5,3] &= \frac{156553}{4000} - \frac{703u}{50} + \frac{152u^2}{125} \\
c_{qg-}[5,4] &= \frac{178929}{20000} - \frac{918u}{625} \\
c_{qg-}[5,5] &= -\frac{349191}{100000} \\
c_{qg-}[6,1] &= -27 + \frac{72u}{5} - \frac{1544u^2}{6075} - \frac{4384u^3}{6561} + \frac{9728u^4}{98415} - \frac{2048u^5}{492075} \\
c_{qg-}[6,2] &= -\frac{1701}{100} - \frac{164u}{9} + \frac{4034u^2}{405} - \frac{26176u^3}{18225} + \frac{128u^4}{2025} \\
c_{qg-}[6,3] &= \frac{230871}{2000} - \frac{88909u}{1500} + \frac{2152u^2}{225} - \frac{64u^3}{135} \\
c_{qg-}[6,4] &= \frac{3131613}{40000} - \frac{25779u}{1250} + \frac{876u^2}{625} \\
c_{qg-}[6,5] &= -\frac{998163}{50000} + \frac{17739u}{12500} \\
c_{qg-}[6,6] &= -\frac{391473}{40000}
\end{aligned}$$

$$\begin{aligned}
c_{gq+}[1, 1] &= -\frac{2}{5} \\
c_{gq+}[2, 1] &= -\frac{6}{5} + \frac{4u}{5} \\
c_{gq+}[2, 2] &= -\frac{3}{5} \\
c_{gq+}[3, 1] &= -\frac{14}{5} + \frac{182u}{45} - \frac{4u^2}{5} \\
c_{gq+}[3, 2] &= -\frac{527}{150} + \frac{82u}{75} \\
c_{gq+}[3, 3] &= -\frac{153}{250} \\
c_{gq+}[4, 1] &= -6 + \frac{214u}{15} - \frac{256u^2}{45} + \frac{8u^3}{15} \\
c_{gq+}[4, 2] &= -\frac{679}{50} + \frac{664u}{75} - \frac{74u^2}{75} \\
c_{gq+}[4, 3] &= -\frac{764}{125} + \frac{113u}{125} \\
c_{gq+}[4, 4] &= -\frac{27}{500} \\
c_{gq+}[5, 1] &= -\frac{62}{5} + \frac{3203u}{75} - \frac{10657u^2}{405} + \frac{44u^3}{9} - \frac{4u^4}{15} \\
c_{gq+}[5, 2] &= -\frac{4307}{100} + \frac{60701u}{1350} - \frac{6913u^2}{675} + \frac{44u^3}{75} \\
c_{gq+}[5, 3] &= -\frac{321577}{9000} + \frac{26789u}{2250} - \frac{673u^2}{1125} \\
c_{gq+}[5, 4] &= -\frac{20691}{5000} - \frac{621u}{1250} \\
c_{gq+}[5, 5] &= \frac{38799}{25000} \\
c_{gq+}[6, 1] &= -\frac{126}{5} + \frac{2903u}{25} - \frac{22253u^2}{225} + \frac{2254u^3}{81} - \frac{404u^4}{135} + \frac{8u^5}{75} \\
c_{gq+}[6, 2] &= -\frac{2433}{20} + \frac{27149u}{150} - \frac{5779u^2}{90} + \frac{5036u^3}{675} - \frac{58u^4}{225} \\
c_{gq+}[6, 3] &= -\frac{158497}{1000} + \frac{26351u}{300} - \frac{11701u^2}{1125} + \frac{46u^3}{225} \\
c_{gq+}[6, 4] &= -\frac{472103}{10000} + \frac{1379u}{1250} + \frac{1297u^2}{1250} \\
c_{gq+}[6, 5] &= \frac{338211}{25000} - \frac{48159u}{12500} \\
c_{gq+}[6, 6] &= \frac{43497}{10000}
\end{aligned}$$

$$\begin{aligned}
c_{gq-}[1, 1] &= \frac{2}{5} \\
c_{gq-}[2, 1] &= \frac{6}{5} - \frac{16u}{45} \\
c_{gq-}[2, 2] &= \frac{3}{5} \\
c_{gq-}[3, 1] &= \frac{14}{5} - \frac{584u}{405} + \frac{64u^2}{405} \\
c_{gq-}[3, 2] &= \frac{527}{150} - \frac{16u}{25} \\
c_{gq-}[3, 3] &= \frac{153}{250} \\
c_{gq-}[4, 1] &= 6 - \frac{568u}{135} + \frac{2944u^2}{3645} - \frac{512u^3}{10935} \\
c_{gq-}[4, 2] &= \frac{679}{50} - \frac{584u}{135} + \frac{224u^2}{675} \\
c_{gq-}[4, 3] &= \frac{764}{125} - \frac{108u}{125} \\
c_{gq-}[4, 4] &= \frac{27}{500} \\
c_{gq-}[5, 1] &= \frac{62}{5} - \frac{7292u}{675} + \frac{92176u^2}{32805} - \frac{9472u^3}{32805} + \frac{1024u^4}{98415} \\
c_{gq-}[5, 2] &= \frac{4307}{100} - \frac{112366u}{6075} + \frac{15536u^2}{6075} - \frac{2048u^3}{18225} \\
c_{gq-}[5, 3] &= \frac{321577}{9000} - \frac{9946u}{1125} + \frac{608u^2}{1125} \\
c_{gq-}[5, 4] &= \frac{20691}{5000} - \frac{408u}{625} \\
c_{gq-}[5, 5] &= -\frac{38799}{25000} \\
c_{gq-}[6, 1] &= \frac{126}{5} - \frac{5852u}{225} + \frac{151568u^2}{18225} - \frac{343424u^3}{295245} + \frac{13312u^4}{177147} - \frac{8192u^5}{4428675} \\
c_{gq-}[6, 2] &= \frac{2433}{20} - \frac{42922u}{675} + \frac{220504u^2}{18225} - \frac{6400u^3}{6561} + \frac{512u^4}{18225} \\
c_{gq-}[6, 3] &= \frac{158497}{1000} - \frac{178423u}{3375} + \frac{59456u^2}{10125} - \frac{256u^3}{1215} \\
c_{gq-}[6, 4] &= \frac{472103}{10000} - \frac{20858u}{1875} + \frac{1168u^2}{1875} \\
c_{gq-}[6, 5] &= -\frac{338211}{25000} + \frac{1971u}{3125} \\
c_{gq-}[6, 6] &= -\frac{43497}{10000}
\end{aligned}$$

$$\begin{aligned}
c_{gg+}[0,0] &= 1 \\
c_{gg+}[1,0] &= -2u \\
c_{gg+}[2,0] &= -\frac{37u}{9} + 2u^2 \\
c_{gg+}[2,1] &= \frac{4u}{15} \\
c_{gg+}[2,2] &= -\frac{9}{25} \\
c_{gg+}[3,0] &= -\frac{28u}{3} + \frac{74u^2}{9} - \frac{4u^3}{3} \\
c_{gg+}[3,1] &= \frac{4u}{5} - \frac{8u^2}{15} \\
c_{gg+}[3,2] &= -\frac{27}{25} + \frac{28u}{25} \\
c_{gg+}[3,3] &= -\frac{27}{25} \\
c_{gg+}[4,0] &= -\frac{1799u}{90} + \frac{4393u^2}{162} - \frac{74u^3}{9} + \frac{2u^4}{3} \\
c_{gg+}[4,1] &= \frac{4u}{3} - \frac{364u^2}{135} + \frac{8u^3}{15} \\
c_{gg+}[4,2] &= -\frac{9}{5} + \frac{1346u}{225} - \frac{334u^2}{225} \\
c_{gg+}[4,3] &= -\frac{1581}{250} + \frac{321u}{125} \\
c_{gg+}[4,4] &= -\frac{5103}{2500} \\
c_{gg+}[5,0] &= -\frac{124u}{3} + \frac{10577u^2}{135} - \frac{2881u^3}{81} + \frac{148u^4}{27} - \frac{4u^5}{15} \\
c_{gg+}[5,1] &= \frac{4u}{5} - \frac{76u^2}{9} + \frac{512u^3}{135} - \frac{16u^4}{45} \\
c_{gg+}[5,2] &= -\frac{27}{25} + \frac{1474u}{75} - \frac{2528u^2}{225} + \frac{272u^3}{225} \\
c_{gg+}[5,3] &= -\frac{5571}{250} + \frac{7936u}{375} - \frac{1076u^2}{375} \\
c_{gg+}[5,4] &= -\frac{11772}{625} + \frac{2574u}{625} \\
c_{gg+}[5,5] &= -\frac{3159}{1250} \\
c_{gg+}[6,0] &= -\frac{15877u}{189} + \frac{168803u^2}{810} - \frac{2805899u^3}{21870} + \frac{2377u^4}{81} - \frac{74u^5}{27} + \frac{4u^6}{45} \\
c_{gg+}[6,1] &= -\frac{13418u^2}{675} + \frac{20018u^3}{1215} - \frac{88u^4}{27} + \frac{8u^5}{45} \\
c_{gg+}[6,2] &= \frac{27}{5} + \frac{54523u}{1125} - \frac{41663u^2}{810} + \frac{23896u^3}{2025} - \frac{158u^4}{225}
\end{aligned}$$

$$\begin{aligned}
c_{gg+}[6,3] &= -\frac{29199}{500} + \frac{1399291u}{13500} - \frac{102634u^2}{3375} + \frac{20714u^3}{10125} \\
c_{gg+}[6,4] &= -\frac{201111}{2000} + \frac{30513u}{625} - \frac{4941u^2}{1250} \\
c_{gg+}[6,5] &= -\frac{44447}{12500} + \frac{50247u}{12500} \\
c_{gg+}[6,6] &= -\frac{41553}{50000}
\end{aligned}$$

$$\begin{aligned}
c_{gg-}[2, 2] &= \frac{9}{25} \\
c_{gg-}[3, 2] &= \frac{27}{25} - \frac{8u}{25} \\
c_{gg-}[3, 3] &= \frac{27}{25} \\
c_{gg-}[4, 2] &= \frac{9}{5} - \frac{292u}{225} + \frac{32u^2}{225} \\
c_{gg-}[4, 3] &= \frac{1581}{250} - \frac{132u}{125} \\
c_{gg-}[4, 4] &= \frac{5103}{2500} \\
c_{gg-}[5, 2] &= \frac{27}{25} - \frac{236u}{75} + \frac{1472u^2}{2025} - \frac{256u^3}{6075} \\
c_{gg-}[5, 3] &= \frac{5571}{250} - \frac{2704u}{375} + \frac{64u^2}{125} \\
c_{gg-}[5, 4] &= \frac{11772}{625} - \frac{1404u}{625} \\
c_{gg-}[5, 5] &= \frac{3159}{1250} \\
c_{gg-}[6, 2] &= -\frac{27}{5} - \frac{5858u}{1125} + \frac{40904u^2}{18225} - \frac{4736u^3}{18225} + \frac{512u^4}{54675} \\
c_{gg-}[6, 3] &= \frac{29199}{500} - \frac{98434u}{3375} + \frac{13504u^2}{3375} - \frac{1664u^3}{10125} \\
c_{gg-}[6, 4] &= \frac{201111}{2000} - \frac{14052u}{625} + \frac{752u^2}{625} \\
c_{gg-}[6, 5] &= \frac{444447}{12500} - \frac{10638u}{3125} \\
c_{gg-}[6, 6] &= \frac{41553}{50000}
\end{aligned}$$

Bibliography

- [A*76] M. J. Alguard et al. Deep-Inelastic Scattering of Polarized Electrons by Polarized Protons. *Phys. Rev. Lett.*, **37**:1261–1265, 1976.
- [A*78] M. J. Alguard et al. Deep-Inelastic e-p Asymmetry Measurements and Comparison with the Bjørken Sum Rule and Models of Proton Spin Structure. *Phys. Rev. Lett.*, **41**:70–73, 1978.
- [A*89] R. Arnold et al. A proposal to measure the neutron spin dependent structure function. October 1989. SLAC Proposal E-142.
- [AEM78] Altarelli, R Ellis, and G. Martinelli. Leptoproduction and Drell-Yan Processes beyond the Leading Approximation in Chromodynamics. *Nucl. Phys. B*, **143**:521–545, 1978.
- [AG76] M. A. Ahmed and G. G. Ross. Polarized Lepton-Hadron Scattering in Asymptotically Free Gauge Theories. *Nucl. Phys. B*, **111**:441–460, 1976.
- [Alt82] G. Altarelli Partons in Quantum Chromodynamics. *Phys. Rep.*, **81**:1–129, 1982.
- [AP77] G. Altarelli and G. Parisi. Asymptotic Freedom in Parton Language. *Nucl. Phys. B*, **126**:298–318, 1977.
- [AR75] M. A. Ahmed and G. G. Ross. Spin-Dependent Deep Inelastic Electron Scattering in an Asymptotically Free Gauge Theory. *Phys. Lett. B*, **56**:385–390, 1975.

- [AR88] G. Altarelli and G. G. Ross. The Anomalous Gluon Contribution to Polarized Leptoproduction. *Phys. Lett. B*, **212**:391, 1988.
- [AS89] G. Altarelli and W. J. Stirling. Phenomenology of the Anomalous Gluon Contribution to Polarized Leptoproduction. *Particle World*, **1**:40, 1989.
- [B*69a] E. D. Bloom et al. High-energy inelastic e-p scattering at 6° and 10° . *Phys. Rev. Lett.*, **23**:930–934, 1969.
- [B*69b] M. Breidenbach et al. Observed behaviour of highly inelastic electron-proton scattering. *Phys. Rev. Lett.*, **23**:935–939, 1969.
- [B*78] P. C. Bosetti et al. Analysis of nucleon structure functions in CERN bubble chamber neutrino experiments. *Nucl. Phys. B*, **142**:1–28, 1978.
- [B*80] G. Baum et al. Measurement of Asymmetry in Spin Dependent e - p Resonance Region Scattering. *Phys. Rev. Lett.*, **45**:2000–2003, 1980.
- [B*83] G. Baum et al. New Measurement of Deep-Inelastic e-p Asymmetries. *Phys. Rev. Lett.*, **51**:1135–1138, 1983.
- [Bar79] Jan A. Bartelski. Comment on the G_2 structure function in the MIT bag model. *Phys. Rev. D*, **20**:1229–1231, 1979.
- [BBDM78] William A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta. Deep-inelastic scattering beyond the leading order in asymptotically free gauge theories. *Phys. Rev. D*, **18**:3998–4017, 1978.
- [BCD*75] William A. Bardeen, M. S. Chanowitz, S. D. Drell, M. Weinstein, and T. -M. Yan. Heavy quarks and strong binding: A field theory of hadron structure. *Phys. Rev. D*, **11**:1094–1136, 1975.
- [BCW84] F. I. Berger, F. Coester, and R. B. Wiringa. Pion density in nuclei and deep-inelastic lepton scattering. *Phys. Rev. D*, **29**:398–411, 1984.

- [BEK88] Stanley J. Brodsky, John Ellis, and Marek Karliner. Chiral symmetry and the spin of the proton. *Phys. Lett. B*, **206**:309–315, 1988.
- [Bel65] J. S. Bell. Difficulties of relativistic $U(6)$. In A. Zichichi, editor, *Recent developments in particle physics, International Summer School, Erice*, pages 138–175, 1965.
- [Bel74] J. S. Bell. The Melosh Transformation and the Pryce-Tani-Foldy-Wouthuysen Transformation. *Acta Physica Austriaca*, Supp.8:395–445, 1974.
- [BG78] A. J. Buras and K. J. F. Gaemers. Simple Parametrizations of Parton Distributions with Q^2 Dependence given by Asymptotic Freedom. *Nucl. Phys. B*, **132**:249–267, 1978.
- [BH74] J. S. Bell and A. J. G. Hey. A theoretical argument for something like the second Melosh transformation. *Phys. Lett. B*, **51**:365–366, 1974.
- [Bjo66] J. D. Bjørken. Applications of the Chiral $U(6) \otimes U(6)$ Algebra of Current Densities. *Phys. Rev.*, **148**:1467–1478, 1966.
- [Bjo69] J. D. Bjørken. Asymptotic Sum Rules at Infinite Momentum. *Phys. Rev.*, **179**:1547–1553, 1969.
- [BKS*70] F. Buccella, H. Kleinert, C. A. Savoy, E. Celeghini, and E. Sorace. A mixing operator for the $SU_3 \times SU_3$ chiral algebra. *Il Nuovo Cimento A*, **69**:133–149, 1970.
- [BM87] Charles J. Benesh and Gerald A. Miller. Deep-Inelastic Structure Functions in the MIT bag model. *Phys. Rev. D*, **36**:1344–1349, 1987.
- [BM88a] C. Benesh and G. A. Miller. Valence Quark Distribution in the Soliton Bag Model. *Phys. Lett. B*, **215**:381–386, 1988.

- [BM88b] Charles J. Benesh and Gerald A. Miller. Deep-inelastic scattering in a modified bag model. *Phys. Rev. D*, **38**:48–55, 1988.
- [BM89] C. J. Benesh and G. A. Miller. The Spin Structure of the Proton. *Phys. Lett. B*, **222**:476–480, 1989.
- [BNP72] F. Buccella, F. Nicolò, and A. Pugliese. A representation of $SU_3 \otimes SU_3$ chiral algebra for baryons. *Il Nuovo Cimento A*, **12**:640–650, 1972.
- [BNT90] S. D. Bass, N. N. Nikolaev, and A. W. Thomas. Gluons and the Proton Spin. March 1990. Adelaide University preprint ADP-133-T80.
- [BP69] J. D. Björken and E. A. Paschos. Inelastic Electron-Proton and γ -Proton Scattering and the Structure of the Nucleon. *Phys. Rev.*, **185**:1975–1982, 1969.
- [BP71] Richard A. Brandt and Giuliana Preparata. Operator product expansion near the light cone. *Nucl. Phys. B*, **27**:541–567, 1971.
- [BQ90] Geoffrey T. Bodwin and Jianwei Qiu. Gluonic contribution to g_1 and its relationship to the spin-dependent parton distributions. *Phys. Rev. D*, **41**:2755–2766, 1990.
- [BR79] G. E. Brown and M. Rho. The little bag. *Phys. Lett. B*, **82**:177, 1979.
- [Bra69] Richard A. Brandt. Light-cone commutator and high-energy lepton-hadron inelastic scattering. *Phys. Rev. Lett.*, **23**:1260–1263, 1969.
- [Bur77] A. J. Buras. Asymptotic Freedom Effects in ν and $\bar{\nu}$ Deep Inelastic Scattering. *Nucl. Phys. B*, **125**:125–144, 1977.
- [Bur80] A. Buras. Asymptotic freedom in deep inelastic processes in the leading order and beyond. *Rev. Mod. Phys.*, **52**:199–276, 1980.

- [C*75] C. Chang et al. Observed deviations from scale invariance in high-energy muon scattering. *Phys. Rev. Lett.*, **35**:901–904, 1975.
- [Cal77] Miguel Calvo. Higher-order effects of asymptotically free field theories. *Phys. Rev. D*, **15**:730–754, 1977.
- [CCM89] R. D. Carlitz, J. C. Collins, and A. H. Mueller. The Role of the Axial Anomaly in Measuring Spin-Dependent Parton Distributions. *Phys. Lett. B*, **214**:229–236, 1989.
- [CHJR83] N. S. Craigie, K. Hidaka, M. Jacob, and F. M. Renard. Spin Physics at Short Distances. *Phys. Rep.*, **99**:71–236, 1983.
- [CHM72] N. Christ, B. Hasslacher, and A. H. Mueller. Light-cone behaviour of perturbation theory. *Phys. Rev. D*, **6**:3543–3562, 1972.
- [CJJ*74] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf. New extended model of hadrons. *Phys. Rev. D*, **9**:3471–3495, 1974.
- [CJJT74] A. Chodos, R. L. Jaffe, K. Johnson, and C. B. Thorn. Baryon structure in the bag theory. *Phys. Rev. D*, **10**:2599–2604, 1974.
- [CK77] R. Carlitz and J. Kaur. Chiral Substructure of the Nucleon. *Phys. Rev. Lett.*, **38**:673–676, 1977.
- [CKAL89] K. Charchula, M. Krawczyk, H. Abramowicz, and A. Levy. *Some topics in ep scattering at HERA - Parton distributions in the nucleon*. Technical Report, ZEUS, 1989. Note 89-121.
- [Clo74] F. E. Close. On the transformation between current and constituent quarks and consequences for polarized electroproduction structure functions. *Nucl. Phys. B*, **80**:269–298, 1974.
- [Clo79] F. E. Close. *An Introduction to Quarks and Partons*. Academic Press, 1979.

- [Clo90] F. E. Close. Polarized Strange Quarks in the Proton and the Validity of Quantum Chromodynamics. *Phys. Rev. Lett.*, **64**:361–363, 1990.
- [CP78] N. Cabibbo and R. Petronzio. Two-Stage Model of Hadron Structure: Parton Distributions and their Q^2 Dependence. *Nucl. Phys. B*, **137**:395–412, 1978.
- [CR88] F. E. Close and R. G. Roberts. The spin content of the proton. *Phys. Rev. Lett.*, **60**:1471–1474, 1988.
- [Cre83] Michael Creutz. *Quarks, Gluons and Lattices*. Cambridge University Press, 1983.
- [CT75] A. Chodos and C. B. Thorn. Chiral invariance in a bag theory. *Phys. Rev. D*, **12**:2733–2743, 1975.
- [CT88] F. E. Close and A. W. Thomas. The Spin and Flavor Dependence of Parton Distribution Functions. *Phys. Lett. B*, **212**:227, 1988.
- [DEF88] H. Dreiner, J. Ellis, and R. A. Flores. The Spin of the Proton in a Hybrid Chiral Bag Model. December 1988. CERN-TH.5251/88.
- [DFLM88] M. Diemoz, F. Ferroni, E. Longo, and G. Martinelli. Parton densities from deep inelastic scattering to hadronic processes at super collider energies. *Zeit. Phys. C*, **39**:21–37, 1988.
- [DGG75] A. De Rújula, Howard Georgi, and S. L. Glashow. Hadron masses in a gauge theory. *Phys. Rev. D*, **12**:147–162, 1975.
- [DGP74] A. De Rújula, H. Georgi, and H. Politzer. Breakdown of scaling in neutrino and electron scattering. *Phys. Rev. D*, **10**:2141–2146, 1974.
- [DJJK75] T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis. Masses and other parameters of the light hadrons. *Phys. Rev. D*, **12**:2060–2076, 1975.

- [DO84] D. W. Duke and J. T. Owens. Q^2 -dependent parametrizations of parton distribution functions. *Phys. Rev. D*, **30**:49–54, 1984.
- [Doe50] Gustav Doesch. *Handbuch der Laplace-Transformations*. Volume 1, Verlag Birkhäuser Basel, 1950.
- [Edm60] A. R. Edmonds. *Angular Momentum in Quantum Mechanics*. Princeton University Press, 1960.
- [EHLQ86a] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg. Erratum: Supercollider physics [Rev. Mod. Phys. **56**, 579,(1984)]. *Rev. Mod. Phys.*, **58**:1065–1072, 1986.
- [EHLQ86b] E. Eichten, I. Hinchliffe, K. Lane, and C. Quigg. Supercollider physics. *Rev. Mod. Phys.*, **56**:579–708, 1986.
- [EJ74] John Ellis and Robert Jaffe. Sum rule for deep-inelastic electroproduction from polarized protons. *Phys. Rev. D*, **9**:1444–1446, 1974.
- [EMC83] EMC Collaboration. The Ratio of the Nucleon Structure Function F_2^N for Iron and Deuterium. *Phys. Lett. B*, **123**:275–278, 1983.
- [EMC88] EMC Collaboration. A Measure of the Spin Asymmetry and Determination of the Structure Function g_1 in Deep Inelastic Muon-Proton Scattering. *Phys. Lett. B*, **206**:364–370, 1988.
- [EMC89] EMC Collaboration. An Investigation of the Spin Structure of the Proton in Deep Inelastic Scattering of Polarised Muons on Polarised Protons. 1989. Cern report CERN-EP/89-73.
- [ET88] A. V. Efremov and O. V. Terjaev. . 1988. Dubna preprint E2-88-287.
- [Fey69] R. P. Feynman. Very high-energy collisions of hadrons. *Phys. Rev. Lett.*, **23**:1415–1417, 1969.

- [Fey70] R. P. Feynman. *High Energy Collisions*, page 237. Gordon and Breach, 1970.
- [Fey72] R. P. Feynman. *Photon-hadron Interactions*. The Benjamin Cummings Publishing Company, 1972.
- [FG72] H. Fritzsch and M. Gell-Mann. Current Algebra: Quarks and what else? . In J. D. Jackson and A. Roberts, editors, *Proceedings of the 16th International Conference on High Energy Physics, Fermilab*, pages 135–165, 1972.
- [FGL73] Harald Fritzsch, M. Gell-Mann, and H. Leutwyler. Advantages of the colour octet gluon picture . *Phys. Lett. B*, **47**:365–368, 1973.
- [FH68] David Faiman and Archibald W. Hendry. Harmonic-Oscillator Model for Baryons. *Phys. Rev.*, **173**:1720–1729, 1968.
- [FH69] David Faiman and Archibald W. Hendry. Harmonic-Oscillator Model for Baryons. *Phys. Rev.*, **180**:1609–1610, 1969.
- [FP67] L. D. Faddeev and V. N. Popov. Feynman diagrams for the Yang-Mills field. *Phys. Lett. B*, **25**:29–30, 1967.
- [FP82] W. Furmanski and R. Petronzio. A Method of Analyzing the Scaling Violations of Inclusive Spectra in Hard Processes. *Nucl. Phys. B*, **195**:237–261, 1982.
- [FRS77] E. G. Floratos, D. A. Ross, and C. T. Sachrajda. Higher-Order Effects in Asymptotically Free Gauge Theories: The Anomalous Dimensions of Wilson Operators. *Nucl. Phys. B*, **129**:66–88, 1977.
- [FRS78] E. G. Floratos, D. A. Ross, and C. T. Sachrajda. Erratum. *Nucl. Phys. B*, **139**:545, 1978.

- [FRS79a] E. G. Floratos, D. A. Ross, and C. T. Sachrajda. Detailed Description of the Violations of Bjørken Scaling in QCD (Including Higher-order Corrections). *Phys. Lett. B*, **80**:269–274, 1979.
- [FRS79b] E. G. Floratos, D. A. Ross, and C. T. Sachrajda. Erratum. *Phys. Lett. B*, **87**:403, 1979.
- [FRS79c] E. G. Floratos, D. A. Ross, and C. T. Sachrajda. Higher-Order Effects in Asymptotically Free Gauge Theories (II) Flavour singlet Wilson operators and coefficient functions. *Nucl. Phys. B*, **152**:493–520, 1979.
- [Gel61] M. Gell-Mann. The eightfold way: A theory of strong interaction symmetry. 1961. Caltech Preprint CTSL-20.
- [Gel64] M. Gell-Mann. A schematic model of baryons and mesons. *Phys. Lett.*, **8**:214–215, 1964.
- [GL54] M. Gell-Mann and F. E. Low. Quantum electrodynamics at small distances. *Phys. Rev.*, **95**:1300–1312, 1954.
- [GL80] A. Gonzalez-Arroyo and C. Lopez. Second-order Contributions to the Structure Functions in Deep Inelastic Scattering (III) The singlet case. *Nucl. Phys. B*, **166**:429, 1980.
- [GLY79a] A. Gonzalez-Arroyo, C. Lopez, and F. J. Ynduráin. Second-order Contributions to the Structure Functions in Deep Inelastic Scattering (II) Comparison with experiment for the non-singlet contributions to e,μ -nucleon scattering. *Nucl. Phys. B*, **159**:512–527, 1979.
- [GLY79b] A. Gonzalez-Arroyo, C. Lopez, and F. J. Ynduráin. Second-order Contributions to the Structure Functions in Deep Inelastic Scattering (I) Theoretical calculations. *Nucl. Phys. B*, **153**:161–186, 1979.

- [GLY80] A. Gonzalez-Arroyo, C. Lopez, and F. J. Ynduráin. Testing QCD Predictions beyond the Leading Order for Non-Singlet Structure Functions in Neutrino Scattering. *Nucl. Phys. B*, **174**:474–490, 1980.
- [GN64] M. Gell-Mann and Y. Ne'eman. *The eightfold way*. Benjamin, New York, 1964.
- [GP74] Howard Georgi and H. David Politzer. Electroproduction scaling in an asymptotically free theory of strong interactions. *Phys. Rev. D*, **9**:416–420, 1974.
- [GR77] M. Glück and E. Reya. Dynamical Determination of Parton and Gluon Distributions in Quantum Chromodynamics. *Nucl. Phys. B*, **130**:76–92, 1977.
- [GR80] I. S. Gradshteyn and I. M. Ryzhik. *Table of Integrals, Series and Products*. Academic Press, Inc., 1980.
- [GR89] M. Glück and E. Reya. Polarized Leptoproduction and Dynamically Generated Spin-dependent Parton Distributions. *Zeit. Phys. C*, **43**:679, 1989.
- [Gro74] D. Gross. How to Test Scaling in Asymptotically Free Theories. *Phys. Rev. Lett.*, **32**:1071–1073, 1974.
- [GRV89] M. Glück, E. Reya, and A. Vogt. Radiatively Generated Parton Distributions for High Energy Collisions. 1989. DO-TH 89/20.
- [GW73] D. Gross and F. Wilczek. Asymptotically Free Gauge Theories. I. *Phys. Rev. D*, **8**:3633–3652, 1973.
- [GW74] David J. Gross and Frank Wilczek. Asymptotically free gauge theories. II. *Phys. Rev. D*, **9**:980–993, 1974.
- [Hei73] R. L. Heimann. Spin Dependent High Frequency Inelastic Electron Scattering and Helicity Flip Couplings. *Nucl. Phys. B*, **64**:429–463, 1973.

- [HER88] HERMES Collaboration. Measurement of the deep inelastic spin dependent structure functions of the proton and neutron at HERA. 1988. Submitted to HERA Physics Review Committee for consideration March 1988.
- [HER89] HERMES Collaboration. A proposal to measure deep inelastic spin dependent structure functions of the nucleon at HERA. 1989. Draft 9-11-89.
- [Hey74] A. J. G. Hey. *Polarization Effects in Electron Proton Scattering -Form Factors, Scaling, Quarks and $SU(6)_W$* . Daresbury Lecture Note Series 13, Daresbury Laboratory, 1974.
- [HK78] Peter Hasenfratz and Julius Kuti. The quark bag model. *Phys. Rep.*, **40**:75-179, 1978.
- [HM72] A. J. G. Hey and J. E. Mandula. Light-Cone Analysis of Spin - Dependent Deep Inelastic Electron Scattering. *Phys. Rev. D*, **5**:2610-2614, 1972.
- [HM88] H. Høgaasen and F. Myhrer. The Spin Content of the Proton in the Chiral Bag. *Phys. Lett. B*, **214**:123-126, 1988.
- [HM89] H. Høgaasen and F. Myhrer. The second EMC Effect, Semileptonic Baryon Decays and the Chiral Bag. 1989.
- [HN65] M. Y. Han and Y. Nambu. Three-Triplet model with double $SU(3)$ symmetry. *Phys. Rev.*, **139**:B1006-B1010, 1965.
- [HS77] I. Hinchliffe and C. H. Llewellyn Smith. Detailed Treatment of Scaling Violations in Asymptotically Free Gauge Theories. *Nucl. Phys. B*, **128**:93-130, 1977.
- [Hug77] R. J. Hughes. Structure functions of spin-dependent deep-inelastic scattering in the cavity approximation to the MIT bag model. *Phys. Rev. D*, **16**:622-629, 1977.

- [IK78] Nathan Isgur and Gabriel Karl. P-wave baryons in the quark model . *Phys. Rev. D*, **18**:4187–4205, 1978.
- [IK79a] Nathan Isgur and Gabriel Karl. Ground-state baryons in a quark model with hyperfine interactions. *Phys. Rev. D*, **20**:1191–1194, 1979.
- [IK79b] Nathan Isgur and Gabriel Karl. Positive-parity excited baryons in a quark model with hyperfine interactions. *Phys. Rev. D*, **19**:2653–2677, 1979.
- [IM75] T. Inoue and T. Maskawa. Bag Theory with the Dirichlet Boundary Conditions and Spontaneous Symmetry breakdown. *Prog. Theor. Phys.*, **54**:1833–1847, 1975.
- [Ito75] Hiroaki Ito. Polarized Electroproduction in an Asymptotically Free Gauge Theory. *Prog. Theor. Phys.*, **54**:555–562, 1975.
- [Jaf75] R. L. Jaffe. Deep-inelastic structure functions in an approximation to the bag theory. *Phys. Rev. D*, **11**:1953–1968, 1975.
- [Jaf83] R. L. Jaffe. Parton Distribution Functions for Twist 4. *Nucl. Phys. B*, **229**:205–230, 1983.
- [Jaf85] R. L. Jaffe. Deep Inelastic Scattering with Application to Nuclear Targets. In M. B. Johnson and A. Pickleseimer, editors, *Relativistic Dynamics and Quark-Nuclear Physics*, pages 1–82, Wiley New York, 1985.
- [Jaf87] R. L. Jaffe. The Axial Anomaly and the Sum Rules for Spin Dependent Electroproduction. *Phys. Lett. B*, **193**:101–104, 1987.
- [Jaf90] R. L. Jaffe. g_2 - The Nucleon's Other Spin-Dependent Structure Function. *Comments Nucl. Part. Phys.*, **19**:239–257, 1990.

- [JJ90] R. L. Jaffe and Xiangdong Ji. Studies of the transverse spin-dependent structure function $g_2(x, Q^2)$. 1990. preprint CTP 1848 Submitted to Physical Review D.
- [JM90] R. L. Jaffe and A. V. Manohar. The g_1 problem: Deep Inelastic electron scattering and the spin of the proton. *Nucl. Phys. B*, **337**:509–546, 1990.
- [Jon74] D. R. T. Jones. Two-Loop Diagrams in Yang-Mills Theory. *Nucl. Phys. B*, **75**:531–538, 1974.
- [JR80] R. L. Jaffe and G. G. Ross. Normalizing the renormalization group analysis of deep inelastic scattering. *Phys. Lett. B*, **93**:313–317, 1980.
- [KMM*79] J. Kodaira, S. Matsuda, T. Muta, T. Uematsu, and K. Sasaki. Quantum chromodynamic effects in polarized electroproduction. *Phys. Rev. D*, **20**:627–629, 1979.
- [KMSU79] J. Kodaira, S. Matsuda, K. Sasaki, and T. Uematsu. QCD higher order effects in spin-dependent deep inelastic electroproduction. *Nucl. Phys. B*, **159**:99–124, 1979.
- [Kod80] J. Kodaira. QCD higher-order effects in polarized electroproduction. *Nucl. Phys. B*, **165**:129–140, 1980.
- [KW71] Julius Kuti and Victor F. Weisskopf. Inelastic Lepton-Nucleon Scattering and Lepton Pair Production in the Relativistic Quark-Parton Model. *Phys. Rev. D*, **4**:318–3439, 1971.
- [Lee81] T. D. Lee. *Particle physics and introduction to field theory*. Harwood Academic Publishers, 1981.
- [Lig58] M. J. Lighthill. *Introduction to Fourier Analysis and Generalized Functions*. University Press, Cambridge, 1958.

- [Lip88a] Harry J. Lipkin. Flavor symmetry and the spin of the proton. 1988. Argonne preprint ANL-HEP-CP-88-46, to appear in the Proceedings of the Workshop on Future Polarization at Fermilab, June 1988.
- [Lip88b] Harry J. Lipkin. Flavour Symmetry, EMC results and the spin content of the proton. *Phys. Lett. B*, **214**:429–433, 1988.
- [Lip89] Harry J. Lipkin. Hyperon decays, EMC results and the spin content of the proton. *Phys. Lett. B*, **230**:135–140, 1989.
- [LOPR74] A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal. Prediction of the $SU(3) \otimes SU(3)$ configuration mixing as a relativistic effect in the naive quark model. *Phys. Rev. D*, **9**:2636–2645, 1974.
- [LOPR75] A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal. Chiral configuration mixing and deep-inelastic scattering of polarized leptons by polarized nucleons. *Phys. Rev. D*, **11**:680–687, 1975.
- [LOPR77] A. Le Yaouanc, L. Oliver, O. Pène, and J. C. Raynal. Combined effects of internal quark motion and $SU(6)$ breaking on the properties of the baryon ground state. *Phys. Rev. D*, **15**:844–853, 1977.
- [Low71] J. H. Lowenstein. Differential Vertex Operations in Lagrangian Field Theory. *Communications of Mathematical Physics*, **24**:1–21, 1971.
- [LP83] Elliot Leader and Enrico Predazzi. *An Introduction to Gauge Theories and the 'New Physics'*. Cambridge University Press, 1983.
- [LPS71] P. V. Landshoff, J. C. Polkinghorne, and R. D. Short. A Non Perturbative Parton Model of Current Interactions. *Nucl. Phys. B*, **28**:225–239, 1971.
- [LY73] C. López and F. Ynduráin. *Padé approximants and their applications*, pages 217–240. Academic Press, New York, 1973.

- [Man90] Lech Mankiewicz. On the hard part of the gamma-gluon cross section in deep inelastic scattering off polarized target. 1990. CAMK-213/90, submitted to *Phys. Rev. D*.
- [Mar79] Francois Martin. Results on nucleon structure functions in quantum chromodynamics. *Phys. Rev. D*, **19**:1382–1397, 1979.
- [Mel74] H. J. Melosh. Quarks: Currents and constituents. *Phys. Rev. D*, **9**:1095–1112, 1974.
- [MP78] William Marciano and Heinz Pagels. Quantum Chromodynamics. *Phys. Rep.*, **36**:137–276, 1978.
- [MRS88] A. Martin, R. G. Roberts, and W. J. Stirling. Structure-function analysis and Ψ , jet W , and Z production: Determining the gluon distribution. *Phys. Rev. D*, **37**:1161–1173, 1988.
- [MS90] Lech Mankiewicz and Andreas Schäfer. The Infrared-Stability of the Anomalous Perturbative Gluon Contribution to the Nucleon Spin. *Phys. Lett. B*, **242**:455–460, 1990.
- [Mut87] T. Muta. *Foundations of Quantum Chromodynamics*. World Scientific, 1987.
- [Nee61] Y. Ne’eman. Derivation of strong interactions from a gauge invariance. *Nucl. Phys.*, **26**:222–229, 1961.
- [Neg89] J. W. Negele. QCD and hadrons on a lattice. In , to be published in Plenum (N. Y.), Aug. 1989. Lectures presented at the Cargese Institute on Hadrons and Hadronic Matter.
- [NSVZ80] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov. Operator expansion in Quantum Chromodynamics beyond perturbation theory. *Nucl. Phys. B*, **174**:378 – 396, 1980.

- [Obe74] Fritz Oberhettinger. *Tables of Mellin Transforms*. Springer-Verlag, 1974.
- [Pan68] W. K. H. Panofsky. Low q^2 electrodynamics, elastic and inelastic electron (and muon) scattering. In J. Prentki and J. Steinberger, editors, *Proceedings of the 14th International Conference on High Energy Physics, Vienna*, pages 23–39, 1968.
- [Par73] G. Parisi. Experimental Limits on the Values of Anomalous Dimensions. *Phys. Lett. B*, **43**:207–208, 1973.
- [Par88] Particle Data Group. Review of particle properties. *Phys. Lett. B*, **204**:1 – 486, 1988.
- [Pet79] A. Peterman. Renormalization Group and the Deep Structure of the Proton. *Phys. Rep.*, **53**:157–248, 1979.
- [Pol73] H. David Politzer. Reliable perturbative results for strong interactions? *Phys. Rev. Lett.*, **30**:1346–1349, 1973.
- [Pol74] H. David Politzer. Asymptotic Freedom: An Approach to Strong Interactions. *Phys. Rep.*, **14**:129–180, 1974.
- [PT62] R. E. Peierls and D. J. Thouless. Variational Approach to Collective Motion. *Nucl. Phys.*, **38**:154–176, 1962.
- [PY57] R. E. Peierls and J. Yoccoz. The Collective Model of Nuclear Motion. *Proc. Phys. Soc.*, **A70**:381–387, 1957.
- [Ram85] Gordon P. Ramsey. The Laguerre Method for Solving Integro-Differential Equations. *J. Comp. Phys.*, **60**:97–118, 1985.
- [RGP77] A. De Rújula, Howard Georgi, and H. David Politzer. Demythification of Electroproduction Local Duality and Precocious Scaling. *Ann. of Phys.*, **103**:315–353, 1977.

- [Sas75] Ken Sasaki. Polarized Electroduction in Asymptotically Free Gauge Theories. *Prog. Theor. Phys.*, **54**:1816–1827, 1975.
- [SD89] X. Song and J. Du. Spin-dependent Structure Functions of the Nucleon and their QCD Evolutions. *Phys. Rev. D*, **40**:2177–2183, 1989.
- [SMC89] SMC Collaboration. Measurement of the spin-dependent structure functions of the neutron and proton. December 1989. CERN/SPSC 88-47.
- [SP53] E. C. G. Stueckelberg and A. Peterman. La normalisation des constantes dans la theorie des quanta. *Helv. Phys. Acta*, **26**:499–520, 1953.
- [SST] A. W. Schreiber, A. I. Signal, and A. W. Thomas. A model calculation of deep inelastic structure functions. In preparation.
- [ST87a] A. Signal and A. W. Thomas. Deep Inelastic Scattering from a Momentum Conserving Two-Dimensional Cavity. 1987. ADP-87-54/T47.
- [ST87b] A. I. Signal and A. W. Thomas. Possible Strength of the Non-Perturbative Strange Sea of the Nucleon. *Phys. Lett. B*, **191**:205–208, 1987.
- [ST88a] A. W. Schreiber and A. W. Thomas. Spin dependent structure functions in the cloudy bag model. *Phys. Lett. B*, **215**:141–144, 1988.
- [ST88b] A. I. Signal and A. W. Thomas. The Structure Function of the Nucleon. *Phys. Lett. B*, **211**:481–484, 1988.
- [ST89a] A. W. Schreiber and A. W. Thomas. Corrections to the Ellis-Jaffe sum rule. *Nucl. Phys. A*, **497**:287c–288c, 1989.
- [ST89b] A. I. Signal and A. W. Thomas. Calculation of quark distribution functions using bag-model wave functions. *Phys. Rev. D*, **40**:2832, 1989.

- [STL90a] A. W. Schreiber, A. W. Thomas, and J. T. Londergan. On the QCD Evolution of the Spin Structure Function of the Proton. *Phys. Lett. B*, **237**:120, 1990.
- [STL90b] A. W. Schreiber, A. W. Thomas, and J. T. Londergan. QCD Evolution of the Spin Structure Functions of the Neutron and Proton. 1990. ADP-90-137/T84, accepted for publication in *Phys. Rev. D*.
- [Sul72] J. D. Sullivan. One-Pion Exchange and Deep-Inelastic Electron-Nucleon Scattering. *Phys. Rev. D*, **5**:1732–1737, 1972.
- [SVZ79] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov. QCD and Resonance Physics. Theoretical Foundations. *Nucl. Phys. B*, **147**:385–447, 1979.
- [t H71] G. 't Hooft. Renormalization of massless Yang-Mills fields. *Nucl. Phys. B*, **33**:173–199, 1971.
- [Tho84] A. W. Thomas. Chiral Symmetry and the Bag Model: A New Starting Point for Nuclear Physics. *Advances in Nucl. Phys.*, **13**:1–137, 1984.
- [TMSG89] A. W. Thomas, A. Michels, A. W. Schreiber, and P. A. M. Guichon. A new approach to nuclear structure functions. *Phys. Lett. B*, **233**:43–47, 1989.
- [TMT82] S. Th  berge, G. A. Miller, and A. W. Thomas. The cloudy bag model. IV. Pionic corrections to nucleon properties. *Can. J. Phys.*, **60**:59–72, 1982.
- [TTM80] S. Th  berge, A. W. Thomas, and Gerald A. Miller. Pionic corrections to the MIT bag model: The (3,3) resonance. *Phys. Rev. D*, **22**:2838–2852, 1980.
- [TTM81] A. W. Thomas, S. Th  berge, and Gerald A. Miller. Cloudy bag model of the nucleon. *Phys. Rev. D*, **24**:216–229, 1981.
- [W*75] Y. Watanabe et al. Test of scale invariance in ratios of muon scattering cross sections at 150 and 56 GeV. *Phys. Rev. Lett.*, **35**:898–901, 1975.

- [Wak89] M. Wakamatsu. The Spin Content of the Proton in a Chiral Quark Model. *Phys. Lett. B*, **232**:251–256, 1989.
- [Wei60] S. Weinberg. High-Energy Behaviour in Quantum Field Theory. *Phys. Rev.*, **118**:838–849, 1960.
- [Wil64] Kenneth G. Wilson. *On Products of Quantum Field Operators at Short Distances*. Technical Report, Cornell Report, 1964.
- [Wil69] Kenneth G. Wilson. Non-Lagrangian Models of Current Algebra. *Phys. Rev.*, **179**:1499 – 1512, 1969.
- [Wil74] Kenneth G. Wilson. Confinement of Quarks. *Phys. Rev. D*, **10**:2445–2459, 1974.
- [Won86] C. W. Wong. An overview of quarks in nuclei. *Phys. Rep.*, **136**:1–102, 1986.
- [WW77] S. Wandzura and F. Wilczek. Sum rules for spin-dependent electroproduction - test of relativistic constituent quarks. *Phys. Lett. B*, **72**:195–198, 1977.
- [Ynd83] F. J. Ynduráin. *Quantum Chromodynamics*. Springer-Verlag, 1983.
- [Zim71] Wolfhart Zimmermann. Local Operator Products and Renormalization in Quantum Field Theory. In S. Deser, M. Grisaru, and H. Pendleton, editors, *Lectures on Elementary Particles and Quantum Field Theory, Proc. 1970 Brandeis Summer Institute in Theor. Phys.*, page 396, MIT Press, 1971.
- [Zim73] Wolfhart Zimmermann. Normal Products and the Short Distance Expansion in the Perturbation Theory of Renormalizable Interactions. *Ann. Phys.*, **77**:570–601, 1973.
- [ZWB74] A. Zee, F. Wilczek, and S. B. Treiman. Scaling deviations for neutrino reactions in asymptotically free field theories. *Phys. Rev. D*, **10**:2881–2891, 1974.

[Zwe64a] G. Zweig. . 1964. Cern Preprint 8419/Th412.

[Zwe64b] G. Zweig. . 1964. Cern Preprint 8182/Th401.

ON THE QCD EVOLUTION OF THE SPIN STRUCTURE FUNCTION OF THE PROTON

A.W. SCHREIBER, A.W. THOMAS

Department of Physics and Mathematical Physics, The University of Adelaide, G.P.O. Box 498, Adelaide, SA 5001, Australia

and

J.T. LONDERGAN

Department of Physics and Nuclear Theory Center, Indiana University, Bloomington, IN 47408, USA

Received 13 September 1989; revised manuscript received 25 November 1989

We examine the proposal that the value of the first moment of the spin structure function of the proton $g_1^p(x, Q^2)$ found by the EMC is explained through QCD evolution, taking into consideration the effect of the anomaly. We conclude that this is unlikely to be the complete explanation as we find that the accurate prediction of the first moment implies a distribution $g_1^p(x, Q^2)$ that does not agree with the data. It does not seem to be possible to fit both the shape of $g_1^p(x, Q^2)$ and its first moment simultaneously in such a picture.

The unexpectedly low value for the integral of the proton spin structure function [1]

$$\int_0^1 g_1^p(x, Q^2 \sim 10 \text{ GeV}^2) dx = 0.126 \pm 0.010 \text{ (stat.)} \pm 0.015 \text{ (sys.)}$$

has prompted a reappraisal of our understanding of polarised deep inelastic lepton scattering. In particular, under the assumption of SU(3) flavour symmetry for the axial charges of the baryon octet, the measured value of $g_1^p(Q^2)$ seems to imply that the total spin carried by the quarks at $Q_0^2 = 10 \text{ GeV}^2$ is $0.120 \pm 0.094 \pm 0.138$. This is in contrast to the nonrelativistic parton model expectation of 1, (i.e. the quarks carry all the spin) or, relativistically, about 0.7 [2].

Attention has focused recently on the corrections to the parton results expected from QCD. The logarithmic scaling variations familiar from unpolarised deep inelastic scattering due to gluon bremsstrahlung are also encountered for the polarised structure functions and have been worked out by Kodaira et al. [3]. They, however, yield only $\alpha(Q_0^2)$ corrections to the parton results and, since $\alpha(Q_0^2) = 0.25$, do not come close to explaining the EMC result.

More recently it has been pointed out [4] that the anomaly [5] produces an effective correction to the flavour singlet contribution of $g_1^p(Q^2)$ of the form

$$\Delta\Sigma'(Q^2) \rightarrow \Delta\Sigma(Q^2) - f \frac{\alpha(Q^2)}{2\pi} \Delta g(Q^2). \quad (1)$$

Here f is the number of active flavours, and $\Delta\Sigma(Q^2) = \sum_j \Delta q_j(Q^2)$, where $\Delta q_j(Q^2)$ is the difference of the first moments of the quark distributions, with helicity equal to and with helicity opposite to that of the proton (for a quark of flavour j). $\Delta g(Q^2)$ is the analogous quantity for the gluons. The important point is that although

$\alpha(Q^2)$ falls with increasing Q^2 , the first moment $\Delta g(Q^2)$ increases with Q^2 . Consequently, the anomaly produces a correction to $\Delta g_1^p(Q^2)$ that approaches a nonzero constant as $Q^2 \rightarrow \infty$ in the leading log approximation, and may possibly be large enough to yield the required experimental result $\Delta \Sigma'(Q_0^2) \sim 0$ ^{#1}.

This idea has recently been taken up by Glück and Reya [7]. Using the experimentally determined proportion of the proton's (plus component of) momentum carried by the valence quarks at momentum Q_0^2 [8] they have determined through (first order) QCD evolution the scale μ at which the valence quarks carry all of the proton's momentum [9]. They find the coupling constant

$$\alpha(Q^2) = \frac{4\pi}{\beta_0 \log(Q^2/\Lambda^2)}$$

at this scale to be $\alpha(\mu^2) \sim 3.5$, corresponding to $\mu \sim 0.24$ GeV if one uses $\Lambda_{\text{QCD}} = 0.2$ GeV and assumes 3 flavours. They have identified this scale as one where the naive parton model is expected to hold and hence $\Delta \Sigma'(\mu^2) = \Delta \Sigma(\mu^2)$, i.e. there is no net helicity carried by the glue here. Using the value of $\Delta q_u(\mu^2) + \Delta q_d(\mu^2)$, derived from the experimental values of F and D for the axial charges of the baryon octet (under the assumption of SU(3) symmetry), they obtain through second order QCD evolution that the contribution of $\Delta \Sigma'(Q_0^2)$ to $g_1^p(Q_0^2)$ is $g_{1,s}^p(Q_0^2) \sim 0.03$, in acceptable agreement with the EMC result.

In this letter we would like to examine this proposal in more detail. We first study the sensitivity of $g_{1,s}^p(Q^2)$ to the quark model scale μ and then turn to the behaviour of the higher moments of $g_1^p(x, Q^2)$ by investigating the evolution of typical parameterisations of quark distributions. As in ref. [9], we neglect the possibility that higher order corrections may in fact change the value of $\alpha(Q_0^2)/\alpha(\mu^2)$ ($\approx \frac{1}{4}$ in the above analysis) significantly. In the light of previous investigations of second order effects, summarised for example in ref. [10], we consider it likely that higher order corrections could be important. Nevertheless, our aim in this letter is to investigate the consequences of the ideas presented in refs. [7,9].

Following ref. [7], at leading order

$$\Delta \Sigma_{\text{LO}}(Q_0^2) = \Delta \Sigma(\mu^2), \quad \Delta g_{\text{LO}}(Q_0^2) = \frac{4}{9} \left(\frac{\alpha(\mu^2)}{\alpha(Q_0^2)} - 1 \right) \Delta \Sigma(\mu^2), \quad (2a,b)$$

so that

$$g_{1,s}^p(Q_0^2) = \frac{1}{9} \left(1 - \frac{2}{3\pi} [\alpha(\mu^2) - \alpha(Q_0^2)] \right) \Delta \Sigma(\mu^2), \quad (2c)$$

while the next to leading order expressions become

$$\Delta \Sigma_{\text{NLO}}(Q_0^2) = \Delta \Sigma(\mu^2), \quad \Delta g_{\text{NLO}}(Q_0^2) = \frac{2\pi}{3\alpha(Q_0^2)} \left[1 - \exp \left(+ \frac{2[\alpha(Q_0^2) - \alpha(\mu^2)]}{3\pi} \right) \right] \Delta \Sigma(\mu^2), \quad (3a)$$

yielding

$$\begin{aligned} g_{1,s}^p(Q_0^2) &= \frac{1}{9} \left[\left(1 - \frac{\alpha(Q_0^2)}{\pi} \right) \Delta \Sigma_{\text{HO}}(Q_0^2) - \frac{3\alpha(Q_0^2)}{2\pi} \Delta g_{\text{HO}}(Q_0^2) \right] \\ &= \frac{1}{9} \left[- \frac{\alpha(Q_0^2)}{\pi} + \exp \left(\frac{2}{3\pi} [\alpha(Q_0^2) - \alpha(\mu^2)] \right) \right] \Delta \Sigma(\mu^2). \end{aligned} \quad (3c)$$

We note that the leading order prediction diverges as $\mu \rightarrow \Lambda$ because of the appearance of $\alpha(\mu^2)$. The next to leading order expression does not diverge, but still has a rapid dependence on μ near $\mu \sim \Lambda$, with the limit

^{#1} For a view to the contrary, see ref. [6].

$$g_{1,s}^p(Q_0^2) \xrightarrow{\mu \rightarrow A} \frac{-\alpha(Q_0^2)}{9\pi} \Delta\Sigma(\mu^2).$$

The limit $\mu \rightarrow A$ is not physically meaningful in the sense that the perturbation expansion does not exist in this region. However, the strong dependence on μ persists into the perturbative region. We show this in fig. 1 under the assumption that $A_{\text{QCD}} = 0.2$ GeV for both the leading and next to leading order calculations. We see that by changing the quark model scale from $\mu = 0.24$ GeV to 0.22 GeV the leading order prediction actually changes sign while the next to leading order result is greatly reduced. Seen from this perspective the vanishing of the singlet contribution at Q_0^2 must be described as accidental. Conversely, if the above is correct, the experimental measurement of $g_1^p(Q_0^2)$ is an extraordinarily sensitive probe of μ and it may therefore be of interest to measure this quantity in a nuclear environment, in order to test for the dynamical rescaling [11] often mentioned in relation to the "old" EMC effect [12].

Most of the interest in the EMC result has focussed on the integrated spin structure function, i.e. the first moment of $g_1^p(x, Q_0^2)$. This is understandable, considering the striking result that seemingly no spin is carried by the quarks. This not only contradicts the naive parton model result that all the spin is carried by the quarks, but is also in significant contradiction to the Ellis–Jaffe sum rule [13].

However, some doubt has been cast on the validity of the experimental value of the first moment of $g_1^p(x, Q_0^2)$ because of the uncertain behaviour at small x [14] (extraction of the first moment involves an integral over $0 \leq x \leq 1$, whereas only the region $x \geq 0.01$ was measured by the EMC group). In fact it was found by Close and Roberts that by making slightly different assumptions to those of the EMC group about the small x behaviour it was possible to restore agreement between experiment and the Ellis–Jaffe sum rule.

Moreover, QCD evolution not only predicts the variation of the first moment of $g_1^p(x, Q^2)$ with Q^2 , but that of all other moments as well. As knowledge of all moments of a distribution is equivalent to a knowledge of the distribution itself, any prediction of the latter provides a more stringent test of the theory than the prediction of just the first moment. This, as well as the problem of the small x region, has led us to pursue this line further. Unfortunately, whereas one has information on the first moment of $g_1^p(x, \mu^2)$ from the weak axial charges [15], one does not have similar information for the other moments. Hence we resort to reasonable parameterisations of the distribution, the details of which we turn to after outlining our method for the evolution of the structure function.

The solution to the evolution of the n th moment of the nonsinglet quark distribution for three flavours is given by [10]

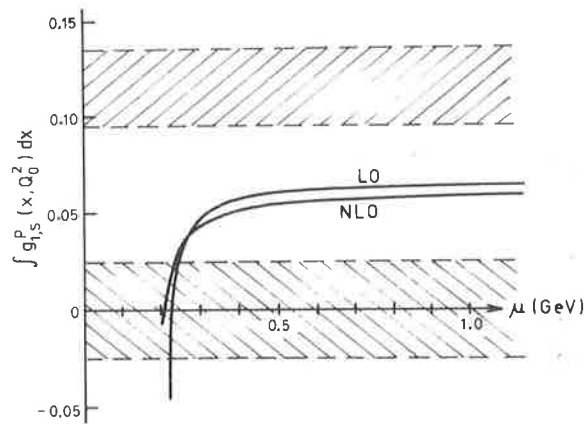


Fig. 1. Leading order (LO) and next to leading order (NLO) behaviour of the first moment of $g_{1,s}^p(x, Q_0^2)$ as a function of the quark model scale μ . Also shown are the experimental values of $g_{1,s}^p(Q_0^2)$ (hatched) and of $g_1^p(Q_0^2)$ (cross-hatched). The value of μ implied by ref. [7] is 0.24 GeV. A is taken to be 0.2 GeV.

$$\Delta M_n^{\text{NS}}(Q^2) = \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-(2/9)\Delta A_{qq}(n)} \Delta M_n^{\text{NS}}(\mu^2), \quad (4a)$$

while for the singlet one has

$$\begin{aligned} \Delta \Sigma_n(Q^2) = & \left[\omega_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + \omega_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta \Sigma_n(\mu^2) \\ & + \left[-\omega_3(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + \omega_3(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta g_n(\mu^2), \end{aligned} \quad (4b)$$

$$\begin{aligned} \Delta g_n(Q^2) = & \left[-\omega_4(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + \omega_4(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta \Sigma_n(\mu^2) \\ & + \left[\omega_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + \omega_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta g_n(\mu^2), \end{aligned} \quad (4c)$$

where

$$\omega_{1,2}(n) = \frac{1}{2} \left(1 \mp \frac{A_-(n)}{R(n)} \right), \quad \omega_3(n) = \frac{\Delta A_{qg}(n)}{R(n)}, \quad \omega_4(n) = \frac{\Delta A_{gq}(n)}{R(n)}, \quad t_{\pm}(n) = \frac{1}{9} [-A_+(n) \pm R(n)], \quad (5)$$

$$A_{\pm}(n) = \Delta A_{qq}(n) \pm \Delta A_{gg}(n), \quad R(n) = \sqrt{A_-^2(n) + 4\Delta A_{qg}(n)\Delta A_{gq}(n)}.$$

Here $\Delta A_{qq}(n)$, $\Delta A_{qg}(n)$, $\Delta A_{gq}(n)$, $\Delta A_{gg}(n)$ refer to the entries in the anomalous dimension matrix for polarised electron scattering, listed for example in ref. [16]^{#2}. In order to extract the distributions from the above equations we have extended the method of Gonzales-Arroyo et al. [17], developed by them for the spin averaged distributions, to the spin dependent distributions. The method is based on the idea that we can write any distribution $f(x)$, defined over the interval $0 \leq x \leq 1$, as $f(x) = \int_0^1 \mathcal{A}(x-y)f(y) dy$, where $\mathcal{A}(x-y)$ is chosen so that it behaves like a δ function for $0 \leq x \leq 1$. One then expands $\mathcal{A}(x-y)$ in a power series in y , so that $f(x) = \sum_n a_n f_{n+1}$, where f_n is the n th moment of $f(x)$. Using the nonsinglet as an illustration, we obtain

$$\begin{aligned} \Delta M^{\text{NS}}(x, Q^2) &= \sum_n a_n \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-(2/9)\Delta A_{qq}(n+1)} \Delta M_{n+1}^{\text{NS}}(\mu^2) \\ &= \int_0^1 dy \left\{ \sum_n a_n \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-(2/9)\Delta A_{qq}(n+1)} y^n \right\} \Delta M^{\text{NS}}(y, \mu^2). \end{aligned} \quad (6)$$

Thus, provided one can calculate the quantity in brackets in eq. (6), it is possible to calculate the distribution at any Q^2 . Gonzales-Arroyo et al. have shown that this is possible analytically if the function $\mathcal{A}(x-y)$ is constructed from Bernstein polynomials and provided that one can make the following expansion:

$$\left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{-(2/9)\Delta A_{qq}(n+1)} = \sum_{\alpha, \beta \geq 0} \frac{c(\alpha, \beta)}{n^\alpha \log n^\beta}. \quad (7)$$

Numerical details of this expansion and its convergence are rather lengthy, in particular for the singlet, and will be presented elsewhere [18].

We now turn to the choice of input quark distributions. As we have pointed out there is no direct experimental input that we can use. We therefore parameterize the valence quark distribution by $Nx^a(1+bx)(1-x)^c$, where

^{#2} Note, however, that our ΔA_{qg} is $4T(R)$ times that of ref. [16].

a and c are chosen such that, after evolution from μ to Q_0 , they approximate the experimental distributions of $F_3(x, Q_0^2)$. We note that a and c are similar to those given, for example, in ref. [10] for $Q^2 = 1.8 \text{ GeV}^2$. We fix b by the requirement [7,9] that the three valence quarks carry all of the nucleon momentum at the scale μ^2 , that is

$$q_1(\mu^2)/q_2(\mu^2) = 3,$$

where $q_i(\mu^2)$ is the i th moment of the unpolarised singlet distribution $q(x, \mu^2)$. For simplicity we have approximated the shape of the quark distribution to be spin-flavour independent, so that N is fixed by

$$\Delta q_3(\mu^2) = 1.25, \quad \Delta q_0(\mu^2) = \Delta q_8(\mu^2) = 0.6$$

for the various flavour combinations [7]. More realistic calculations, using input distributions from the bag model [19], do not significantly alter our results. We show the predictions for $g_1^p(x, Q_0^2)$ corresponding to several parameterisations of the quark distribution in fig. 2.

One sees that the various parameterisations give similar results; however, they all disagree significantly with the data in that they greatly overestimate the value of $xg_1(x)$ at small x and perhaps underestimate it at larger x . There may be several reasons for this. An obvious possibility is that our quark model parameterisations are inadequate. If no restrictions are made, it is always possible to find a distribution at μ^2 that will produce an evolved distribution at Q_0^2 that matches the data. We believe this to be unlikely as one can see in fig. 2 that $xg_1^p(x, Q^2)$ does not vary greatly for our set of parameterisations, and so one would have to depart rather dramatically from the naive expectations we have used. A more likely explanation is that the value chosen for $L_0 = \alpha(\mu^2)/\alpha(Q_0^2)$ is too large, i.e. that the scale $\mu \sim 0.24 \text{ GeV}$ is too low. This hypothesis is supported by the observation that as a given quark distribution is evolved in Q^2 its peak tends to move to lower x . (The change in the momentum being accounted for by the gluon bremsstrahlung.) The large change in $\alpha(Q^2)$ considered in fig. 2, which guarantees the agreement with the experimental first moment, has meant that the evolved quark and gluon distributions are peaked at $x \sim 0$ causing the overestimation at small x . In fig. 3 we show the same curves evolved from $\alpha(\mu^2) = 0.5$, corresponding to $\mu = 0.8 \text{ GeV}$. In this case the peak in the distributions is at larger x than in fig. 2, and hence a much better fit is obtained for the shape. We note, however, that now the overall normalisation is too large, as was to be expected for this value of μ (see fig. 1).

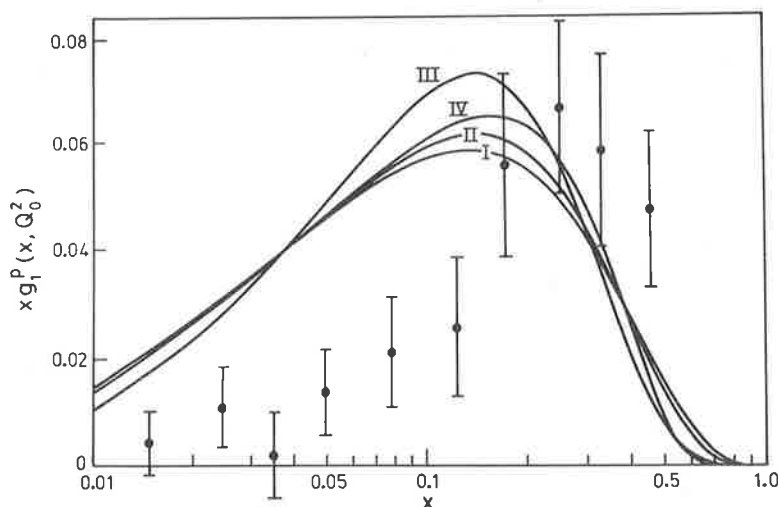


Fig. 2. Evolved distributions $xg_1(x, Q_0^2)$ corresponding to the parameterisation $x^a(1+bx)(1-x)^c$ for $q(x, \mu^2)$ with: (I) $a=0.5, b=0.0, c=2$, (II) $a=0.5, b=4.33, c=3$, (III) $a=0.5, b=-17, c=5$, (IV) MIT bag results. $\alpha(\mu^2)$ is that of ref. [7].

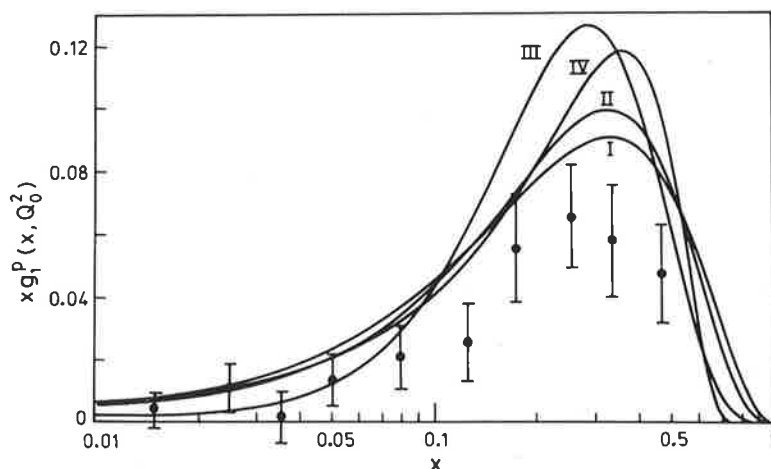


Fig. 3. Same as for fig. 2, with $\alpha(\mu^2) = 0.5$, corresponding to $\mu = 0.8$ GeV.

In summary, we have examined in this letter a recent proposal explaining the vanishing of the first moment of $g_{1,s}^p(x, Q_0^2)$ by using perturbative QCD (taking into account the anomaly) to evolve from a scale μ at which the valence quarks carry all the proton's momentum. We have found, using reasonable parameterisations of the quark distribution at the scale μ , that one can fit the first moment of $g_1^p(x, Q_0^2)$ at the expense of the shape of the distribution, or vice versa. It is, however, not possible to fit both simultaneously. Hence, to reproduce the experimental data, new ingredients have to be introduced. It has been proposed, for example, that a large strange quark component is compatible with the data [20]. A strange quark sea, as well as a glue contribution [21], present at the scale μ would evolve to give an additional contribution to the distribution at the scale Q_0 . Furthermore, a glue and sea component, or for that matter pions [2,22], present at the scale μ would invalidate the argument leading to $\alpha(Q_0^2)/\alpha(\mu^2) \sim \frac{1}{14}$ [9] as they would presumably carry some of the proton's momentum. Investigation of the effect of some of these ideas on the shape of distribution $g_1^p(x, Q^2)$ are currently under way.

This work was supported by the Australian Research Council. One of the authors (J.T.L.) was supported in part by the US National Science Foundation under contract NSF-PHY88-805640. A.W.S. acknowledges the support of Adelaide University through an Adelaide University Postgraduate award.

References

- [1] EM Collab., J. Ashman et al., Phys. Lett. B 206 (1988) 364; CERN preprint CERN-EP/89-73;
M.L. Alguard et al., Phys. Rev. Lett. 37 (1976) 1261; 41 (1978) 70;
G. Baum et al., Phys. Rev. Lett. 51 (1983) 1135; 45 (1980) 2000.
- [2] A.W. Schreiber and A.W. Thomas, Phys. Lett. B 215 (1988) 141.
- [3] J.K. Kodaira et al., Phys. Rev. D 20 (1979) 627;
J. Kodaira et al., Nucl. Phys. B 159 (1979) 99;
J. Kodaira, Nucl. Phys. B 165 (1980) 129.
- [4] G. Altarelli and G.G. Ross, Phys. Lett. B 212 (1988) 391;
R.D. Carlitz et al., Phys. Lett. B 214 (1988) 229;
G. Altarelli and W.J. Stirling, CERN preprint TH. 5249/88;
A.V. Efremov and O.V. Terjaev, Dubna preprint E2-88-287 (1988);
J. Kunz et al., Phys. Lett. B 222 (1989) 481;
R. Jaffe, Phys. Lett. B 193 (1987) 101.

- [5] S.L. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cimento A 51 (1969) 47.
- [6] R.L. Jaffe and A.V. Manohar, MIT preprint CTP 1706;
G. Bodwin and J. Qui, Argonne preprint ANL-HEP-PR 89/83;
S. Forte, Saclay preprints SPHT/89/018, SPHT/89/106.
- [7] M. Glück and E. Reya, University of Dortmund report DO-TH 88/22.
- [8] M. Diemoz et al., Z. Phys. C 39 (1988) 21.
- [9] M. Glück et al., Z. Phys. C 41 (1989) 667.
- [10] A.J. Buras, Rev. Mod. Phys. 52 (1980) 199.
- [11] F. Close et al., Phys. Lett. B 142 (1984) 202.
- [12] J.J. Aubert et al., Phys. Lett. B 123 (1983) 275.
- [13] J. Ellis and R.L. Jaffe, Phys. Rev. D 9 (1974) 1444; D 10 (1974) 1669 (E).
- [14] F. Close and R. Roberts, Phys. Rev. Lett. 60 (1988) 1471.
- [15] Particle Data Group, G.P. Yost et al., Review of particle properties, Phys. Lett. B 204 (1988) 1.
- [16] G. Altarelli and G. Parisi, Nucl. Phys. B 126 (1977) 298.
- [17] A. Gonzales-Arroyo et al. Nucl. Phys. B 153 (1979) 161; B 159 (1979) 512; B 166 (1980) 429.
- [18] A.W. Schreiber et al., in preparation.
- [19] T. Signal and A.W. Thomas, Phys. Lett. B 211 (1988) 481;
A. Schreiber and A.W. Thomas, in preparation.
- [20] M. Glück and E. Reya, Z. Phys. C 39 (1988) 569.
- [21] F. Close and A.W. Thomas, Phys. Lett. B 212 (1988) 227;
F. Myhrer and A.W. Thomas, Phys. Rev. D 38 (1988) 1633.
- [22] H. Høgaasen and F. Myhrer, Phys. Lett. B 214 (1988) 123.

ABSTRACT

**QCD EVOLUTION OF THE SPIN STRUCTURE
FUNCTIONS OF THE NEUTRON AND PROTON**

A. W. SCHREIBER* AND A. W. THOMAS

*Dept. of Physics and Mathematical Physics, University
of Adelaide, Adelaide 5001, South Australia*

and

J. T. LONDERGAN†

Department of Physics and Nuclear Theory Center, Indiana University, Bloomington IN 47408

Models of the long distance behaviour of QCD can be used to make predictions for the twist two piece of the current correlation function appearing in deep inelastic scattering. The connection between the model predictions, valid at a four momentum transfer squared Q^2 of the order of $1(\text{GeV}/c)^2$, and the data, collected at much higher Q^2 , is provided by perturbative QCD. We present here the details of a method, previously used for spin averaged lepton scattering, applied to the spin dependent case. We then use the method to make predictions for the spin structure functions g_1^{proton} and g_1^{neutron} .

* Address after September 1990: NIKHEF-K, P. O. Box 41882, 1009 DB Amsterdam, The Netherlands

† Research supported in part by the U. S. National Science Foundation under contract NSF-PHY88-805640.

1. Introduction

For some years now QCD has been the leading candidate for a theory of the strong interaction. Perturbative QCD (PQCD) has provided us with a unifying framework for studying all high energy scattering phenomena involving hadrons and, because of its property of asymptotic freedom, has given us an a posteriori justification of the popular parton model.¹

At finite Q^2 moreover PQCD predicts logarithmic deviations from exact scaling. These are usually derived using the renormalization group² which makes predictions for the n^{th} moment of the structure functions, or equivalently (at least at leading order) the quark and gluon distributions, of the form

$$q(n, Q^2) = A_n C_n(Q^2/\mu^2, g_c) + \text{higher twists} \quad (1)$$

where

$$C_n(Q^2/\mu^2, g_c) = C_n(1, \bar{g}) e^{-\int_{\bar{g}}^{\bar{g}} dg \gamma_n(g)/\beta(g)} \quad (2)$$

Here A_n is a matrix element which is dependent on the quark distribution under consideration and the C_n 's are coefficient functions which give rise to the Q^2 dependence of the moments $q(n, Q^2)$. $\bar{g}$ is the running coupling constant while g_c is the coupling constant at the reference scale μ . γ_n and β are the usual anomalous dimensions and the Callan-Symanzik functions respectively.¹

In perturbation theory one typically expands the C_n , γ_n and β up to some power of the coupling constant and evaluates them to this order. This was done for the unpolarized case by Gross & Wilczek, Politzer and others up to first order^{3,4} and later, up to second order, by Floratos, Ross and Sachrajda etc.^{5,6,7}

For polarized scattering the calculations to leading order have been done by Ahmed and Ross, Ito and Sasaki⁸. At next to leading order the coefficients C_n as well as γ_1 have been calculated by Kodaira et al.⁹ For general n the anomalous dimensions have not been calculated beyond leading order.

The works mentioned above give direct predictions only for the moments of the distribution functions. This is somewhat unfortunate as the experimentally measured quantities are the actual distributions themselves which typically vary in accuracy as a function of x and are only measured down to some finite small x , with a subsequent extrapolation to $x=0$ ^{10,11}. Hence the extraction of the moments is subject to some uncertainty and there may be considerable correlation between the errors on different moments. Moreover, in the case of polarized scattering a great deal of attention has recently been given to the QCD

evolution of the first moment of the spin structure function of the proton¹², while relatively little attention has been paid to the consistent evolution of the other moments¹³. As we have previously pointed out¹⁴ consideration of the latter in fact makes it unlikely that the small value of $\int_0^1 g_1^p(x, Q^2 = 10 \text{ GeV}^2) dx$ measured by the EMC group can be reconciled with the picture that at low energies the proton consists exclusively of three non-strange valence quarks¹⁵.

In the case of unpolarized scattering, the evolution of the relevant distributions has been calculated in a variety of ways. Originally, at first order, Gross and others¹⁶ reconstructed the non-singlet quark distribution using asymptotic expansions of the anomalous dimensions and the inverse Mellin transform technique. Other methods have relied on parameterizations of the structure functions which have then been evolved by numerically solving¹⁷ the integro-differential Altarelli - Parisi¹⁸ equations, by using the method of Laguerre polynomials¹⁹ or by using particular parameterizations of the quark and glue distributions²⁰. In this paper we shall use the method of Gonzales-Arroyo et al., the details of which can be found in a series of papers, starting with the second order evolution of the unpolarized non-singlet⁶ and leading to the more complicated case of the singlet⁷.

In section II we present their method applied to the polarized case. Because the anomalous dimensions for the polarized singlet have not been calculated at second order we shall restrict ourselves to leading order only. As is the case in refs (6,7), we expect the generalization to second order to be straightforward. Because of its speed the method described in Section II is particularly useful for comparing model predictions of structure functions, which apply typically at some low scale $Q^2 = \mu^2$, to data at higher Q^2 (in our case $Q^2 = 10 \text{ GeV}^2$). This is because the model dependent scale μ is a priori unknown and has to be obtained by fitting some characteristic prediction of the model to data. In ref. (21), for example, the scale of a model where the quarks carry all the momentum of the proton is extracted from the known momentum fraction that they carry at $Q^2 = 10 \text{ GeV}^2$. In section III we shall present the results for a particular model where we use the shape of the spin independent valence distribution to determine μ , enabling us to make predictions for $g_1(x, Q^2)$ of the proton and neutron.

2. The reconstruction via the Bernstein Polynomials

We describe here the method of refs. (6,7), applied to the spin dependent quark and glue distributions. To illustrate the method we take the case of the spin dependent non-singlet (which, because of parity invariance, is identical to that of the spin independent non-singlet) and then give the straightforward generalization to the singlet. The main result of this paper is the calculation of sets of coefficients, given in eq. (23) and in the appendix. The reader familiar with method may want to proceed directly to these results.

We suppose that we have a non-singlet quark distribution

$$\Delta q_{NS}(x, Q^2) = q_{NS}^1(x, Q^2) - q_{NS}^2(x, Q^2) \quad (3)$$

whose n^{th} moment

$$\Delta q_{NS}(n, Q^2) = \int_0^1 x^{n-1} \Delta q_{NS}(x, Q^2) dx \quad (4)$$

obeys the evolution equation ((1)). To first order this yields

$$\Delta q_{NS}(n, Q^2) = \left[\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{-A_{qq}(n)/(2\beta_0)} \Delta q_{NS}(n, \mu^2) \quad (5)$$

Here $q_{NS}^{I(1)}(x, Q^2)$ is the distribution of quarks with helicity equal (opposite) to that of the parent hadron, $\alpha(Q^2)$ is the strong coupling constant at the scale Q^2 , $A_{qq}(n)$ is the (lowest order) anomalous dimension arising from the polarized quark-quark splitting function and β_0 is the lowest order coefficient in the expansion of the Callan - Symanzik function and is given by, for SU(3) colour,

$$\beta_0 = \frac{33 - 2f}{12} \quad (6)$$

where f is the number of flavours. In order to reconstruct the distribution $\Delta q_{NS}(x, Q^2)$ from $\Delta q_{NS}(n, Q^2)$ we write

$$\Delta q_{NS}(x, Q^2) = \int_0^1 \Delta(y - x) \Delta q_{NS}(y, Q^2) dy \quad (7)$$

where $\Delta(y - x)$ must have the property

$$f(x) = \int_0^1 \Delta(y - x) f(y) dy \quad (8)$$

A sufficient, but not necessary, choice for $\Delta(y - x)$ is the Dirac δ -function. We however use functions related to the Bernstein Polynomials

$$\begin{aligned} \Delta(y - x) &= \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{(N-k)!k!} y^k (1-y)^{N-k} \\ &= \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l y^{k+l}}{l! (N-k-l)!} \end{aligned} \quad (9)$$

By integrating over y with a testfunction y^c , or otherwise²², it can readily be shown that $\Delta(y - x)$ exhibits the behaviour of a δ -function in the range $0 \leq y \leq 1$ (It need not be symmetric in x and y for our purposes, nor does it have to behave like a δ -function for y outside this interval). Using eqs. (9) and (5) in (7), we obtain

$$\Delta q_{NS}(x, Q^2) = \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l}{l! (N-k-l)!} \int_0^1 y^{k+l} \left[\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right]^{-A_{qq}(l+k+1)/(2\beta_0)} \Delta q_{NS}(y, \mu^2) dy. \quad (10)$$

As has been pointed out in ref (6,7), to sum equation (10) numerically is dangerous because of the oscillatory nature due to the appearance of the $(-1)^l$. Because of the form of $A_{qq}(l+k+1)$ it is not possible to do the sum in equation (10) analytically. However, if one uses the expansion

$$r^{-A_{qq}(n)/(2\beta_0)} = \sum_{i,j} \frac{c(i,j)}{(p+n)^i [\log a(n+p)]^j} \quad (11)$$

it is in fact possible to perform the sum analytically for each individual term in (11). (a and p are arbitrary constants; later on we shall use $a = e^{C-3/4}$ and $p=2$, C being Euler's constant. r is the ratio of the two coupling constants in eq. (10)) We then find that

$$\lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l}{l!(N-k-l)!} \frac{y^{k+l}}{(p+k+l)^i} = \frac{\theta(y-x)}{\Gamma(i)} \left(\frac{x}{y}\right)^{p-1} \left(\log \frac{y}{x}\right)^{i-1}, \quad (12)$$

and

$$\begin{aligned} \lim_{\substack{N, k \rightarrow \infty \\ k/N \rightarrow x}} \frac{(N+1)!}{k!} \sum_{l=0}^{N-k} \frac{(-1)^l}{l!(N-k-l)!} \frac{y^{k+l}}{(p+k+l)^i [\log a(p+k+l)]^j} \\ = \frac{\theta(y-x)}{\Gamma(i)} \left(\frac{x}{y}\right)^{p-1} \left(\log \frac{y}{x}\right)^{i-1} \int_0^\infty ds \frac{s^{j-1} a^{-s} (\log \frac{y}{x})^s}{\Gamma(i+s)}. \end{aligned} \quad (13)$$

Hence we obtain

$$\Delta q_{NS}(x, Q^2) = \int \frac{dy}{y} b_{qq}(x/y, r) \Delta q_{NS}(y, \mu^2), \quad (14)$$

with

$$b_{qq}(x/y, r) = \left(\frac{x}{y}\right)^{p-1} \sum_i \log^{(i-1)} \frac{y}{x} \left[\frac{c_{qq}(i, 0)}{\Gamma(i)} + \sum_{j \neq 0} \frac{c_{qq}(i, j)}{\Gamma(j)} \int_0^\infty ds \frac{s^{j-1} a^{-s} (\log \frac{y}{x})^s}{\Gamma(i+s)} \right] \quad (15)$$

At this stage several remarks should be made. (i) In the derivation of (14) and (15) one interchanges a limit and an integration. Because of the singular nature of the Δ -function the validity of this is not clear. Perhaps the easiest way to check that the result is indeed correct is by direct re-evaluation of the moments of eq.(14) and checking that they indeed satisfy eq.(5). (ii) For the particular case of the non-singlet shown here the expression simplifies considerably because the coefficients $c_{qq}(i, j)$ are 0 for $j \neq 0$. (iii) The details of the QCD evolution are contained in the b's and are independent of the input distributions $\Delta q_{NS}(y, \mu^2)$. This makes the method fast as one only needs to evaluate the b's once for a suitable set of $\frac{x}{y}$'s and r's (This is of particular use for the singlet where the coefficients $c_{qq}(i, j)$ are not 0 for $j \neq 0$.)

In the case of the singlet distributions the differential equation governing the evolution of its moments is given by

$$\frac{d}{d \log Q^2} \begin{pmatrix} \Delta \Sigma(n, Q^2) \\ \Delta g(n, Q^2) \end{pmatrix} = \frac{\alpha(Q^2)}{2\pi} \begin{pmatrix} A_{qq}(n) & A_{qg}(n) \\ A_{gq}(n) & A_{gg}(n) \end{pmatrix} \begin{pmatrix} \Delta \Sigma(n, Q^2) \\ \Delta g(n, Q^2) \end{pmatrix}. \quad (16)$$

Here $\Delta g(n, Q^2)$ are the moments of the glue distributions analogous to eqs.(3) and (4). The A's are

proportional to the anomalous dimensions and for three flavours are given by

$$\begin{aligned} A_{qq}(n) &= \frac{4}{3} \left(\frac{3}{2} + \frac{1}{n(n+1)} - \frac{2}{n} - 2 \sum_{j=1}^{n-1} \frac{1}{j} \right) \\ A_{qg}(n) &= 3 \left(\frac{n-1}{n(n+1)} \right) \\ A_{gq}(n) &= \frac{4}{3} \left(\frac{n+2}{n(n+1)} \right) \\ A_{gg}(n) &= 3 \left(\frac{3}{2} + \frac{2}{n} - \frac{4}{n+1} - 2 \sum_{j=1}^{n-1} \frac{1}{j} \right) \end{aligned} \quad (17)$$

Equation (16) has the solution

$$\begin{aligned} \Delta \Sigma_n(Q^2) &= \left[w_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta \Sigma_n(\mu^2) \\ &+ \left[-w_3(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_3(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta g_n(\mu^2) \\ \Delta g_n(Q^2) &= \left[-w_4(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_4(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta \Sigma_n(\mu^2) \\ &+ \left[w_2(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_+(n)} + w_1(n) \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)^{t_-(n)} \right] \Delta g_n(\mu^2) \end{aligned} \quad (18)$$

where

$$\begin{aligned} w_{1,2}(n) &= \frac{1}{2} \left(1 \mp \frac{A_-(n)}{R(n)} \right) \\ w_3(n) &= \Delta A_{qg}(n)/R(n) \\ w_4(n) &= \Delta A_{gq}(n)/R(n) \\ t_{\pm}(n) &= \frac{1}{9} (-A_+(n) \pm R(n)) \end{aligned} \quad (19)$$

and

$$\begin{aligned} A_{\pm}(n) &= \Delta A_{qq}(n) \pm \Delta A_{gg}(n) \\ R(n) &= \sqrt{A_-^2(n) + 4 \Delta A_{qg}(n) \Delta A_{gq}(n)} \end{aligned} \quad (20)$$

Finally, we get for the singlets the following expressions analogous to (14):

$$\Delta\Sigma(x, Q^2) = \int_x^1 \frac{dy}{y} [b_{qq}(x/y, r) \Delta\Sigma(y, \mu^2) + b_{gg}(x/y, r) \Delta g(y, \mu^2)] \quad (21)$$

and

$$\Delta g(x, Q^2) = \int_x^1 \frac{dy}{y} [b_{gq}(x/y, r) \Delta\Sigma(y, \mu^2) + b_{gg}(x/y, r) \Delta g(y, \mu^2)] \quad (22)$$

where $b_{qg}(x/y, r)$, $b_{gq}(x/y, r)$ and $b_{gg}(x/y, r)$ are equivalent to $b_{qq}(x/y, r)$ given in eq (15) with $c_{qqk}(i, j)$ replaced by $c_{qgk}(i, j)$, $c_{gqk}(i, j)$ and $c_{ggk}(i, j)$ respectively. Here $k = \pm$ distinguishes the two sets of expansions necessary for each series of coefficients corresponding to the two distinct powers t_+ and t_- appearing in (18).

Choosing $p=2$, and using

$$\begin{aligned} \sum_{j=1}^{n-1} \frac{1}{j} &= C + \Psi(n) \\ &= C + \log n - \frac{1}{2n} - \frac{1}{12n^2} + \dots \end{aligned}$$

one arrives at the following expansions

$$\begin{aligned} r^{-A_{\text{eff}}(n)/(2\beta_0)} &= (az)^{\frac{16}{27}n} \left[1 - \frac{8u}{9} \frac{1}{z} + \left(\frac{-76u}{81} + \frac{32u^2}{81} \right) \frac{1}{z^2} + O(z)^{-3} \right] \\ w_1(n)r^{t_+(n)} &= (az)^{\frac{4}{3}n} \left[\frac{1}{(\log az)^2} \left(\frac{9}{25z^2} \right) + O(z, \log az)^{-3} \right] \\ w_2(n)r^{t_-(n)} &= (az)^{\frac{16}{27}n} \left[1 - \frac{8u}{9} \frac{1}{z} + \left(\frac{-76u}{81} + \frac{32u^2}{81} \right) \frac{1}{z^2} - \frac{1}{\log az} \frac{4u}{15z^2} - \frac{1}{(\log az)^2} \frac{9}{25z^2} + O(z, \log az)^{-3} \right] \\ w_3(n)r^{t_+(n)} &= (az)^{\frac{4}{3}n} \left[\frac{1}{(\log az)} \left(-\frac{9}{10z} + \frac{9u}{5z^2} \right) - \frac{1}{(\log az)^2} \frac{27}{20z^2} + O(z, \log az)^{-3} \right] \\ w_3(n)r^{t_-(n)} &= (az)^{\frac{16}{27}n} \left[\left(\frac{1}{(\log az)} \right) \left(\frac{9}{10} \frac{1}{z} + \frac{-4u}{5z^2} \right) + \left(\frac{1}{(\log az)^2} \right) \left(\frac{+27}{20z^2} \right) + O(z, \log az)^{-3} \right] \\ w_4(n)r^{t_+(n)} &= (az)^{\frac{4}{3}n} \left[\left(\frac{1}{(\log az)} \right) \left(\frac{-2}{5} \frac{1}{z} + \left(\frac{-6}{5} + \frac{4u}{5} \right) \frac{1}{z^2} \right) + \left(\frac{1}{(\log az)^2} \right) \left(\frac{-3}{5z^2} \right) + O(z, \log az)^{-3} \right] \\ w_4(n)r^{t_-(n)} &= (az)^{\frac{16}{27}n} \left[\left(\frac{1}{(\log az)} \right) \left(\frac{2}{5} \frac{1}{z} + \left(\frac{6}{5} + \frac{-16u}{45} \right) \frac{1}{z^2} \right) + \left(\frac{1}{(\log az)^2} \right) \left(\frac{3}{5z^2} \right) + O(z, \log az)^{-3} \right] \\ w_2(n)r^{t_+(n)} &= (az)^{\frac{4}{3}n} \left[1 - \frac{2u}{z} + \left(\frac{-37u}{9} + 2u^2 \right) \frac{1}{z^2} + \frac{1}{\log az} \frac{4u}{15z^2} - \frac{1}{(\log az)^2} \frac{9}{25z^2} + O(z, \log az)^{-3} \right] \\ w_1(n)r^{t_-(n)} &= (az)^{\frac{16}{27}n} \left[\frac{1}{(\log az)^2} \left(\frac{9}{25z^2} \right) + O(z, \log az)^{-3} \right] \end{aligned} \quad (23)$$

Here $a = e^{C-\frac{3}{4}}$, $z=n+2$ and $u = \log r = \log \left(\frac{\alpha(Q^2)}{\alpha(\mu^2)} \right)$. Terms of higher order are listed in the appendix.

The rate of convergence of eq (23) is a function of $\log r$; more terms need to be kept if the evolution is over a greater range of Q^2 . For the value of r considered in the next section terms up to order $(z \log az)^{-6}$ are more than sufficient. Because the expansions (11) are in powers of $\frac{1}{n+p}$ and $\frac{1}{\log(n+p)}$ the accuracy of the series (23) increases rapidly as n increases. Hence the method is most accurate at large x , and indeed the number of terms that are to be kept is primarily determined by the precision that is required as x approaches 0.

Finally, for the polarized singlet only, it turns out that the expansions do not converge for $n=1$. There are several ways to overcome this. Because all higher moments converge arbitrarily well one might choose to just add a δ -function at $x=0$, this being the only function that influences the first moment exclusively. Alternatively, and this is the course taken by us, one may add to the expansions (23) a term like z^{-c} , with c being some large number. Again only the first moment is affected significantly. Because of the arbitrariness in the choice of c this should be seen purely as a check that the distributions for non-zero x are not sensitive to the problem with the singlet's first moment. No significance should be attached to the predictions in the region that is effected by the term z^{-c} . We will examine the effect of this term in the next section, after briefly outlining a model that we shall use to illustrate the method that has been developed in this section.

3. Results

We now wish to apply the procedure developed in the previous section to a realistic problem. We shall start with a model prediction for the proton and neutron spin dependent quark distributions at some, in principle unknown, scale μ^2 . This scale is determined, within the model, by evolving the model prediction for the twist two piece of the non-singlet spin-independent valence distribution $xq_v(x, \mu^2) = xu_v(x, \mu^2) + xd_v(x, \mu^2)$, and comparing the resultant $xq_v(x, Q^2)$ with parameterizations of the data at $Q^2 = 10 \text{ GeV}^2$. This enables us to make predictions for $g_1^p(x, Q^2 = 10 \text{ GeV}^2)$ and $g_1^n(x, Q^2 = 10 \text{ GeV}^2)$.

The model we use should describe the low energy properties of the nucleons, such as g_A , the magnetic moment, the $N-\Delta$ mass splitting, etc. For this reason we shall use MIT bag wavefunctions for the nucleons, with the centre of mass momentum projected out through the use of the Peierls-Yoccoz procedure²⁴. We shall assume that $\Delta s(x, \mu^2) = \Delta g(x, \mu^2) = 0$.

$\Delta u(x, \mu^2)$ and $\Delta d(x, \mu^2)$ are given by

$$\Delta q_f(x, \mu^2) = 2M \sum_m \int \frac{d^3 p_2}{(2\pi)^3} \delta(M - xM - p_2^+) \left(|\langle p_2 | b_{m,f} \Psi_m^\dagger + (0) | p = 0 \rangle|^2 - |\langle p_2 | b_{m,f} \Psi_m^\dagger + (0) | p = 0 \rangle|^2 \right) \quad (24)$$

Here $|p = 0\rangle$ is the Peierls-Yoccoz projected wavefunction of the nucleon at rest while $\langle p_2 |$ is the wavefunction of the two quark intermediate state with momentum p_2 . The destruction operator for a quark of flavour f and total angular momentum m is $b_{m,f}$ and Ψ_m refers to the corresponding spinor. The $+$ indicates the projection $\frac{1}{2}\gamma^-\gamma^+$ and $\uparrow\downarrow$ refer to the two helicity projections (Note that while Ψ_m is an eigenstate of the total angular momentum operator, $\Psi_m^{\uparrow\downarrow}$ are not). Further details of the model can be found in refs (25).

The N - Δ mass splitting is incorporated into the model by adjusting the diquark mass appearing in equation (24) (through $p_2^+ = (p_2^2 + m_2^2)^{\frac{1}{2}} + p_2$), depending on whether the state is in the mixed symmetric or mixed antisymmetric part of the spin-flavour wavefunction²⁶. We emphasize that the model is not entirely satisfactory as the Peierls-Yoccoz procedure does not produce an energy eigenstate, nor does it eliminate the centre of mass momentum variable from the internal wavefunction of the nucleon. Furthermore, it has been pointed out that connected contributions to the twist-two piece of the scattering process also arise from four quark intermediate states²⁵, which are not considered in (24). As a consequence of these deficiencies the first moment of the spin independent quark distributions (summed over flavours) is not guaranteed to be precisely three, nor is the Björken sum rule satisfied. We shall overcome these problems phenomenologically by simply adding in a term of the form $x^{-1/2}(1-x)^7$ to the quark distributions in order to mimic the four quark term. We note that this does not affect g_1^p and g_1^n above $x=0.3$. Note however that the vanishing of the support above $x=1$ is guaranteed because of the δ -function in eq.(24) which ensures energy-momentum conservation explicitly. For a bag of radius $R=0.8$ fermi the model predicts the following normalizations at the scale μ :

$$\begin{aligned} \int_0^1 u^1(x, \mu^2) dx &= 1.53 \\ \int_0^1 u^1(x, \mu^2) dx &= 0.47 \\ \int_0^1 d^1(x, \mu^2) dx &= 0.37 \\ \int_0^1 d^1(x, \mu^2) dx &= 0.63 \end{aligned} \quad (25)$$

giving the value of g_A corresponding to the Peierls-Yoccoz projected MIT bag model

$$g_A = \int_0^1 \Delta q_3(x, \mu^2) dx = 1.32 \quad (26)$$

and, for the first moments of the nucleon structure functions

$$\begin{aligned} \int_0^1 g_1^{p(n)}(x, \mu^2) dx &= \int_0^1 \left[\pm \frac{1}{12} \Delta q_3(x, \mu^2) + \frac{1}{36} \Delta q_8(x, \mu^2) + \frac{1}{9} \Delta \Sigma(x, \mu^2) \right] dx \\ &= 0.22 (0.00) \end{aligned} \quad (27)$$

These are the same values as those expected with wavefunctions that are completely $SU(6)$ symmetric. Clearly this is only an approximation to reality. For example, we have shown previously that a pion cloud reduces the integral of $g_1^p(x, Q^2)$ by about 15 %²⁷. It should be noted however that the distributions themselves are different from the $SU(6)$ symmetric ones, in particular the neutron structure function is not zero everywhere. For the purpose of this paper we shall assume that $R=0.8$ Fermi, $m_2^{scalar(vector)} = 750 (950) MeV$ and $M=938 MeV$. The difference between the two diquark masses is fixed because of the N - Δ mass difference²⁶ while their absolute value has been treated as an adjustable parameter.

We now turn to the QCD evolution of the above model. In Figure 1 we show the evolved function $x q_v(x, Q^2)$ and compare it with parameterized data at $10 GeV^{22}$. As r decreases, i.e. as Q^2/μ^2 increases, the momentum carried by the initial valence quarks is decreased and generates the rise of the quark and gluon sea contributions at low x . A value of $r \approx 0.3$ (corresponding to $\mu \approx 0.5 GeV$ and $\alpha_s/2\pi = 0.13$ if $\Lambda_{QCD} = 0.2 GeV$) gives a reasonable approximation to the data. We emphasize that using a different low energy model, in particular one including a pion cloud, will change this value of r . (In fact, because the pions

tend to carry some of the proton's momentum, the distributions will be peaked at smaller x and so the fitted value of r should increase).

Before we turn to the predictions this enables us to make for the spin dependent structure functions we need to make a comment about the effect of the anomaly. Given the scale μ obtained above, one finds that the first moment of the glue distribution at $Q^2 = 10\text{GeV}^2$ is about 0.8. It has been proposed that the anomaly produces an additional contribution to the first moment of the singlet of the form^{12,13}

$$-\frac{1}{9} \frac{f\alpha(Q^2)}{2\pi} \Delta g \quad (n = 1, Q^2)$$

The form of the corresponding effect of the anomaly on higher moments of the singlet is not clear as they depend on the regularization adopted. Whatever this form, however, with the above value of Δg ($n = 1, Q^2$) it only gives a contribution of order 0.01 to the sum rule. We shall therefore neglect it.

In Figure 2 we show the evolved quark singlet and glue distributions at $Q^2 = 10\text{GeV}^2$, as well as the variation expected at low x due to the arbitrariness of the term inserted to reproduce the first moment. The region $x \geq 0.1$ is essentially unaffected by this term and hence reliable. This is also evident from the moments of the distributions, shown in Table I.

In Figure 3 we show $x g_1^p(x, Q^2 = 10\text{GeV}^2)$ corresponding to the quark distribution in Figure 2, as well as the EMC data for $x g_1^p(x, Q^2 = 10\text{GeV}^2)$. It is immediately evident that although the shape of the prediction for $x g_1^p(x, Q^2)$ matches the data, the 1st moment is not reproduced for the ratio of coupling constants considered here. A reduction of this ratio, translating into a smaller scale μ^2 , decreases the first moment (if one includes the proposed effect of the anomaly).

Finally we turn to the prediction for $x g_1^n(x, Q^2)$ (Figure 4). The non-zero value of this structure function has two origins: The glue splitting breaks the SU(6) symmetry of the wavefunction and the singlet and nonsinglet parts of the structure function evolve differently, giving rise to a non-zero neutron structure function at all $\mu^2 \neq Q^2$, irrespective of whether the original wavefunction was SU(6) symmetric.

4. Conclusions

We have presented a useful scheme for implementing the QCD evolution of spin dependent quark and glue distributions. The method is computationally fast because the details of the QCD evolution are independent of the input distributions. It is therefore particularly useful in fitting model predictions to data. We have applied the procedure to a model of the nucleon that includes the one gluon exchange responsible for the $N - \Delta$ mass splitting in a phenomenological way. This, as well as the QCD evolution itself, induces a non-zero neutron structure function $g_1^n(x, Q^2)$.

APPENDIX

We list below the higher order coefficients occurring in eq ((23)). Terms up to order $(z \log az)^{-9}$ may be obtained from the authors.

$$\begin{aligned}
 c_{qq+}[2, 2] &= \frac{9}{25} \\
 c_{qq+}[3, 2] &= \frac{27}{25} - \frac{18u}{25} \\
 c_{qq+}[3, 3] &= \frac{27}{25} \\
 c_{qq+}[4, 2] &= \frac{9}{5} - \frac{91u}{25} + \frac{18u^2}{25} \\
 c_{qq+}[4, 3] &= \frac{1581}{250} - \frac{258u}{125} \\
 c_{qq+}[4, 4] &= \frac{5103}{2500} \\
 c_{qq+}[5, 2] &= \frac{27}{25} - \frac{57u}{5} + \frac{128u^2}{25} - \frac{12u^3}{25} \\
 c_{qq+}[5, 3] &= \frac{5571}{250} - \frac{2064u}{125} + \frac{246u^2}{125} \\
 c_{qq+}[5, 4] &= \frac{11772}{625} - \frac{4563u}{1250} \\
 c_{qq+}[5, 5] &= \frac{3159}{1250} \\
 c_{qq+}[6, 2] &= -\frac{27}{5} - \frac{6709u}{250} + \frac{10009u^2}{450} - \frac{22u^3}{5} + \frac{6u^4}{25} \\
 c_{qq+}[6, 3] &= \frac{29199}{500} - \frac{59117u}{750} + \frac{7493u^2}{375} - \frac{156u^3}{125} \\
 c_{qq+}[6, 4] &= \frac{201111}{2000} - \frac{105591u}{2500} + \frac{4039u^2}{1250} \\
 c_{qq+}[6, 5] &= \frac{444447}{12500} - \frac{12177u}{3125} \\
 c_{qq+}[6, 6] &= \frac{41553}{50000}
 \end{aligned}$$

$$\begin{aligned}
 c_{qq-}[0, 0] &= 1 & c_{qq-}[1, 0] &= \frac{-8u}{9} & c_{qq-}[2, 0] &= -\frac{76u}{81} + \frac{32u^2}{81} \\
 c_{qq-}[2, 1] &= \frac{-4u}{15} \\
 c_{qq-}[2, 2] &= -\frac{9}{25} \\
 c_{qq-}[3, 0] &= -\frac{40u}{27} + \frac{608u^2}{729} - \frac{256u^3}{2187} \\
 c_{qq-}[3, 1] &= \frac{-4u}{5} + \frac{32u^2}{135} \\
 c_{qq-}[3, 2] &= -\frac{27}{25} - \frac{2u}{25} \\
 c_{qq-}[3, 3] &= -\frac{27}{25} \\
 c_{qq-}[4, 0] &= -\frac{1078u}{405} + \frac{11528u^2}{6561} - \frac{2432u^3}{6561} + \frac{512u^4}{19683} \\
 c_{qq-}[4, 1] &= \frac{-4u}{3} + \frac{1168u^2}{1215} - \frac{128u^3}{1215} \\
 c_{qq-}[4, 2] &= -\frac{9}{5} - \frac{47u}{45} + \frac{56u^2}{225} \\
 c_{qq-}[4, 3] &= -\frac{1581}{250} + \frac{69u}{125} \\
 c_{qq-}[4, 4] &= -\frac{5103}{2500} \\
 c_{qq-}[5, 0] &= -\frac{136u}{27} + \frac{41072u^2}{10935} - \frac{57664u^3}{59049} + \frac{19456u^4}{177147} - \frac{4096u^5}{885735} \\
 c_{qq-}[5, 1] &= \frac{-4u}{5} + \frac{944u^2}{405} - \frac{5888u^3}{10935} + \frac{1024u^4}{32805} \\
 c_{qq-}[5, 2] &= -\frac{27}{25} - \frac{383u}{75} + \frac{1312u^2}{675} - \frac{896u^3}{6075} \\
 c_{qq-}[5, 3] &= -\frac{5571}{250} + \frac{64u}{25} + \frac{16u^2}{375} \\
 c_{qq-}[5, 4] &= -\frac{11772}{625} + \frac{2223u}{1250} \\
 c_{qq-}[5, 5] &= -\frac{3159}{1250} \\
 c_{qq-}[6, 0] &= -\frac{16636u}{1701} + \frac{264808u^2}{32805} - \frac{19329568u^3}{7971615} + \frac{184576u^4}{531441} - \frac{38912u^5}{1594323} + \frac{16384u^6}{23914845} \\
 c_{qq-}[6, 1] &= 4u + \frac{23432u^2}{6075} - \frac{163616u^3}{98415} + \frac{18944u^4}{98415} - \frac{2048u^5}{295245} \\
 c_{qq-}[6, 2] &= \frac{27}{5} - \frac{36949u}{2250} + \frac{51988u^2}{6075} - \frac{7424u^3}{6075} + \frac{2816u^4}{54675} \\
 c_{qq-}[6, 3] &= -\frac{29199}{500} + \frac{19517u}{4500} + \frac{1744u^2}{1125} - \frac{448u^3}{3375} \\
 c_{qq-}[6, 4] &= -\frac{201111}{2000} + \frac{39747u}{2500} - \frac{358u^2}{625} \\
 c_{qq-}[6, 5] &= -\frac{444447}{12500} + \frac{41013u}{12500} \\
 c_{qq-}[6, 6] &= -\frac{41553}{50000}
 \end{aligned}$$

$$\begin{aligned}
c_{gg}+[1,1] &= -\frac{9}{10} \\
c_{gg}+[2,1] &= \frac{9u}{5} \\
c_{gg}+[2,2] &= -\frac{27}{20} \\
c_{gg}+[3,1] &= \frac{9}{5} + \frac{37u}{10} - \frac{9u^2}{5} \\
c_{gg}+[3,2] &= -\frac{771}{200} + \frac{123u}{50} \\
c_{gg}+[3,3] &= -\frac{1377}{1000} \\
c_{gg}+[4,1] &= \frac{27}{5} + \frac{24u}{5} - \frac{37u^2}{5} + \frac{6u^3}{5} \\
c_{gg}+[4,2] &= -\frac{171}{25} + \frac{627u}{50} - \frac{111u^2}{50} \\
c_{gg}+[4,3] &= -\frac{9621}{1000} + \frac{1017u}{500} \\
c_{gg}+[4,4] &= -\frac{243}{2000} \\
c_{gg}+[5,1] &= \frac{63}{5} - \frac{21u}{100} - \frac{749u^2}{36} + \frac{37u^3}{5} - \frac{3u^4}{5} \\
c_{gg}+[5,2] &= -\frac{2097}{400} + \frac{4969u}{120} - \frac{983u^2}{60} + \frac{33u^3}{25} \\
c_{gg}+[5,3] &= -\frac{156553}{4000} + \frac{20687u}{1000} - \frac{673u^2}{500} \\
c_{gg}+[5,4] &= -\frac{178929}{20000} - \frac{5589u}{5000} \\
c_{gg}+[5,5] &= \frac{349191}{100000} \\
c_{gg}+[6,1] &= 27 - 27u - \frac{6737u^2}{150} + \frac{533u^3}{18} - \frac{74u^4}{15} + \frac{6u^5}{25} \\
c_{gg}+[6,2] &= \frac{1701}{100} + \frac{10373u}{100} - \frac{15069u^2}{200} + \frac{962u^3}{75} - \frac{29u^4}{50} \\
c_{gg}+[6,3] &= -\frac{230871}{2000} + \frac{234531u}{2000} - \frac{4841u^2}{250} + \frac{23u^3}{50} \\
c_{gg}+[6,4] &= -\frac{3131613}{40000} + \frac{14589u}{2500} + \frac{11673u^2}{5000} \\
c_{gg}+[6,5] &= \frac{998163}{50000} - \frac{433431u}{50000} \\
c_{gg}+[6,6] &= \frac{391473}{40000}
\end{aligned}$$

$$\begin{aligned}
c_{gg}-[1,1] &= \frac{9}{10} \\
c_{gg}-[2,1] &= \frac{-4u}{5} \\
c_{gg}-[2,2] &= \frac{27}{20} \\
c_{gg}-[3,1] &= -\frac{9}{5} - \frac{38u}{45} + \frac{16u^2}{45} \\
c_{gg}-[3,2] &= \frac{771}{200} - \frac{36u}{25} \\
c_{gg}-[3,3] &= \frac{1377}{1000} \\
c_{gg}-[4,1] &= -\frac{27}{5} + \frac{4u}{15} + \frac{304u^2}{405} - \frac{128u^3}{1215} \\
c_{gg}-[4,2] &= \frac{171}{25} - \frac{406u}{75} + \frac{56u^2}{75} \\
c_{gg}-[4,3] &= \frac{9621}{1000} - \frac{243u}{125} \\
c_{gg}-[4,4] &= \frac{243}{2000} \\
c_{gg}-[5,1] &= -\frac{63}{5} + \frac{307u}{75} + \frac{3172u^2}{3645} - \frac{1216u^3}{3645} + \frac{256u^4}{10935} \\
c_{gg}-[5,2] &= \frac{2097}{400} - \frac{16763u}{1350} + \frac{2372u^2}{675} - \frac{512u^3}{2025} \\
c_{gg}-[5,3] &= \frac{156553}{4000} - \frac{703u}{50} + \frac{152u^2}{125} \\
c_{gg}-[5,4] &= \frac{178929}{20000} - \frac{918u}{625} \\
c_{gg}-[5,5] &= -\frac{349191}{100000} \\
c_{gg}-[6,1] &= -27 + \frac{72u}{5} - \frac{1544u^2}{6075} - \frac{4384u^3}{6561} + \frac{9728u^4}{98415} - \frac{2048u^5}{492075} \\
c_{gg}-[6,2] &= -\frac{1701}{100} - \frac{164u}{9} + \frac{4034u^2}{405} - \frac{26176u^3}{18225} + \frac{128u^4}{2025} \\
c_{gg}-[6,3] &= \frac{230871}{2000} - \frac{88909u}{1500} + \frac{2152u^2}{225} - \frac{64u^3}{135} \\
c_{gg}-[6,4] &= \frac{3131613}{40000} - \frac{25779u}{1250} + \frac{876u^2}{625} \\
c_{gg}-[6,5] &= -\frac{998163}{50000} + \frac{17739u}{12500} \\
c_{gg}-[6,6] &= -\frac{391473}{40000}
\end{aligned}$$

$$\begin{aligned}
c_{gq}+[1,1] &= -\frac{2}{5} \\
c_{gq}+[2,1] &= -\frac{6}{5} + \frac{4u}{5} \\
c_{gq}+[2,2] &= -\frac{3}{5} \\
c_{gq}+[3,1] &= -\frac{14}{5} + \frac{182u}{45} - \frac{4u^2}{5} \\
c_{gq}+[3,2] &= -\frac{527}{150} + \frac{82u}{75} \\
c_{gq}+[3,3] &= -\frac{153}{250} \\
c_{gq}+[4,1] &= -6 + \frac{214u}{15} - \frac{256u^2}{45} + \frac{8u^3}{15} \\
c_{gq}+[4,2] &= -\frac{679}{50} + \frac{664u}{75} - \frac{74u^2}{75} \\
c_{gq}+[4,3] &= -\frac{764}{125} + \frac{113u}{125} \\
c_{gq}+[4,4] &= -\frac{27}{500} \\
c_{gq}+[5,1] &= -\frac{62}{5} + \frac{3203u}{75} - \frac{10657u^2}{405} + \frac{44u^3}{9} - \frac{4u^4}{15} \\
c_{gq}+[5,2] &= -\frac{4307}{100} + \frac{60701u}{1350} - \frac{6913u^2}{675} + \frac{44u^3}{75} \\
c_{gq}+[5,3] &= -\frac{321577}{9000} + \frac{26789u}{2250} - \frac{673u^2}{1125} \\
c_{gq}+[5,4] &= -\frac{20691}{5000} - \frac{621u}{1250} \\
c_{gq}+[5,5] &= \frac{38799}{25000} \\
c_{gq}+[6,1] &= -\frac{126}{5} + \frac{2903u}{25} - \frac{22253u^2}{225} + \frac{2254u^3}{81} - \frac{404u^4}{135} + \frac{8u^5}{75} \\
c_{gq}+[6,2] &= -\frac{2433}{20} + \frac{27149u}{150} - \frac{5779u^2}{90} + \frac{5036u^3}{675} - \frac{58u^4}{225} \\
c_{gq}+[6,3] &= -\frac{158497}{1000} + \frac{26351u}{300} - \frac{11701u^2}{1125} + \frac{46u^3}{225} \\
c_{gq}+[6,4] &= -\frac{472103}{10000} + \frac{1379u}{1250} + \frac{1297u^2}{1250} \\
c_{gq}+[6,5] &= \frac{338211}{25000} - \frac{48159u}{12500} \\
c_{gq}+[6,6] &= \frac{43497}{10000}
\end{aligned}$$

$$\begin{aligned}
c_{gq}-[1,1] &= \frac{2}{5} \\
c_{gq}-[2,1] &= \frac{6}{5} - \frac{16u}{45} \\
c_{gq}-[2,2] &= \frac{3}{5} \\
c_{gq}-[3,1] &= \frac{14}{5} - \frac{584u}{405} + \frac{64u^2}{405} \\
c_{gq}-[3,2] &= \frac{527}{150} - \frac{16u}{25} \\
c_{gq}-[3,3] &= \frac{153}{250} \\
c_{gq}-[4,1] &= -6 - \frac{568u}{135} + \frac{2944u^2}{3645} - \frac{512u^3}{10935} \\
c_{gq}-[4,2] &= \frac{679}{50} - \frac{584u}{135} + \frac{224u^2}{675} \\
c_{gq}-[4,3] &= \frac{764}{125} - \frac{108u}{125} \\
c_{gq}-[4,4] &= \frac{27}{500} \\
c_{gq}-[5,1] &= \frac{62}{5} - \frac{7292u}{675} + \frac{92176u^2}{32805} - \frac{9472u^3}{32805} + \frac{1024u^4}{98415} \\
c_{gq}-[5,2] &= \frac{4307}{100} - \frac{112366u}{6075} + \frac{15536u^2}{6075} - \frac{2048u^3}{18225} \\
c_{gq}-[5,3] &= \frac{321577}{9000} - \frac{9946u}{1125} + \frac{608u^2}{1125} \\
c_{gq}-[5,4] &= \frac{20691}{5000} - \frac{408u}{625} \\
c_{gq}-[5,5] &= -\frac{38799}{25000} \\
c_{gq}-[6,1] &= \frac{126}{5} - \frac{5852u}{225} + \frac{151568u^2}{18225} - \frac{343424u^3}{295245} + \frac{13312u^4}{177147} - \frac{8192u^5}{4428675} \\
c_{gq}-[6,2] &= \frac{2433}{20} - \frac{42922u}{675} + \frac{220504u^2}{18225} - \frac{6400u^3}{6561} + \frac{512u^4}{18225} \\
c_{gq}-[6,3] &= \frac{158497}{1000} - \frac{178423u}{3375} + \frac{59456u^2}{10125} - \frac{256u^3}{1215} \\
c_{gq}-[6,4] &= \frac{472103}{10000} - \frac{20858u}{1875} + \frac{1168u^2}{1875} \\
c_{gq}-[6,5] &= -\frac{338211}{25000} + \frac{1971u}{3125} \\
c_{gq}-[6,6] &= -\frac{43497}{10000}
\end{aligned}$$

$$\begin{aligned}
c_{gg+}[0,0] &= 1 & c_{gg+}[1,0] &= -2u & c_{gg+}[2,0] &= -\frac{37u}{9} + 2u^2 \\
c_{gg+}[2,1] &= \frac{4u}{15} \\
c_{gg+}[2,2] &= -\frac{9}{25} \\
c_{gg+}[3,0] &= -\frac{28u}{3} + \frac{74u^2}{9} - \frac{4u^3}{3} \\
c_{gg+}[3,1] &= \frac{4u}{5} - \frac{8u^2}{15} \\
c_{gg+}[3,2] &= -\frac{27}{25} + \frac{28u}{25} \\
c_{gg+}[3,3] &= -\frac{27}{25} \\
c_{gg+}[4,0] &= -\frac{1799u}{90} + \frac{4393u^2}{162} - \frac{74u^3}{9} + \frac{2u^4}{3} \\
c_{gg+}[4,1] &= \frac{4u}{3} - \frac{364u^2}{135} + \frac{8u^3}{15} \\
c_{gg+}[4,2] &= -\frac{9}{5} + \frac{1346u}{225} - \frac{334u^2}{225} \\
c_{gg+}[4,3] &= -\frac{1581}{250} + \frac{321u}{125} \\
c_{gg+}[4,4] &= -\frac{5103}{2500} \\
c_{gg+}[5,0] &= -\frac{124u}{3} + \frac{10577u^2}{135} - \frac{2881u^3}{81} + \frac{148u^4}{27} - \frac{4u^5}{15} \\
c_{gg+}[5,1] &= \frac{4u}{5} - \frac{76u^2}{9} + \frac{512u^3}{135} - \frac{16u^4}{45} \\
c_{gg+}[5,2] &= -\frac{27}{25} + \frac{1474u}{75} - \frac{2528u^2}{225} + \frac{272u^3}{225} \\
c_{gg+}[5,3] &= -\frac{5571}{250} + \frac{7936u}{375} - \frac{1076u^2}{375} \\
c_{gg+}[5,4] &= -\frac{11772}{625} + \frac{2574u}{625} \\
c_{gg+}[5,5] &= -\frac{3159}{1250} \\
c_{gg+}[6,0] &= -\frac{15877u}{189} + \frac{168803u^2}{810} - \frac{2805899u^3}{21870} + \frac{2377u^4}{81} - \frac{74u^5}{27} + \frac{4u^6}{45} \\
c_{gg+}[6,1] &= -4u - \frac{13418u^2}{675} + \frac{20018u^3}{1215} - \frac{88u^4}{27} + \frac{8u^5}{45} \\
c_{gg+}[6,2] &= \frac{27}{5} + \frac{54523u}{1125} - \frac{41663u^2}{810} + \frac{23896u^3}{2025} - \frac{158u^4}{225} \\
c_{gg+}[6,3] &= -\frac{29199}{500} + \frac{1399291u}{13500} - \frac{102634u^2}{3375} + \frac{20714u^3}{10125} \\
c_{gg+}[6,4] &= -\frac{201111}{2000} + \frac{30513u}{625} - \frac{4941u^2}{1250} \\
c_{gg+}[6,5] &= -\frac{444447}{12500} + \frac{50247u}{12500} \\
c_{gg+}[6,6] &= -\frac{41553}{50000}
\end{aligned}$$

$$\begin{aligned}
c_{gg-}[2,2] &= \frac{9}{25} \\
c_{gg-}[3,2] &= \frac{27}{25} - \frac{8u}{25} \\
c_{gg-}[3,3] &= \frac{27}{25} \\
c_{gg-}[4,2] &= \frac{9}{5} - \frac{292u}{225} + \frac{32u^2}{225} \\
c_{gg-}[4,3] &= \frac{1581}{250} - \frac{132u}{125} \\
c_{gg-}[4,4] &= \frac{5103}{2500} \\
c_{gg-}[5,2] &= \frac{27}{25} - \frac{236u}{75} + \frac{1472u^2}{2025} - \frac{256u^3}{6075} \\
c_{gg-}[5,3] &= \frac{5571}{250} - \frac{2704u}{375} + \frac{64u^2}{125} \\
c_{gg-}[5,4] &= \frac{11772}{625} - \frac{1404u}{625} \\
c_{gg-}[5,5] &= \frac{3159}{1250} \\
c_{gg-}[6,2] &= -\frac{27}{5} - \frac{5858u}{1125} + \frac{40904u^2}{18225} - \frac{4736u^3}{18225} + \frac{512u^4}{54675} \\
c_{gg-}[6,3] &= \frac{29199}{500} - \frac{98434u}{3375} + \frac{13504u^2}{3375} - \frac{1664u^3}{10125} \\
c_{gg-}[6,4] &= \frac{201111}{2000} - \frac{14052u}{625} + \frac{752u^2}{625} \\
c_{gg-}[6,5] &= \frac{444447}{12500} - \frac{10638u}{3125} \\
c_{gg-}[6,6] &= \frac{41553}{50000}
\end{aligned}$$

References

1. G. Altarelli, Phys.Rep. **81** (1982) 1; A. Buras, Rev. of Mod. Phys. **52** (1980)199; T.Muta, Foundations of Quantum Chromodynamics, World Scientific 1987
2. E. C. G. Stueckelberg and A. Peterman, Helv. Phys. Acta **26** (1953) 499; M. Gell-Mann and F. E. Low, Phys. Rev. **95**(1954)1300
3. D. Gross and F. Wilczek, Phys. Rev. **D8** (1973) 3633, **D9** (1974) 980
4. H. D. I. Politzer, Phys. Rep. **14** (1974)129; H. Georgi and H. D. I. Politzer, **D9** (1974) 416
5. E. Floratos, D. Ross and C. Sachrajda, Nucl. Phys. **B129** (1977) 66; **E:B139** (1978) 545; **B152** (1979) 493; Phys. Lett. **B56** (1979) 269; D.R.T. Jones, Nuc. Phys. **B75** (1974) 531; M. Calvo, Phys. Rev. **D15** (1977) 730; A. de Rújula, H. Georgi and H. Politzer Ann. of Phys. **103** (1977) 315; W. Bardeen, A. Buras, D. Duke and T. Muta, Phys. Rev. **D18** (1978) 3998; A.Zee, F. Wilczek and S. Treiman, Phys. Rev. **D10** (1974) 2881; I. Hinchliffe and C. Llewellyn Smith, Nuc. Phys. **B128** (1977) 93; G. Altarelli, R. Ellis and G. Martinelli, Nuc. Phys. **B143** (1978) 521
6. A. Gonzales-Arroyo *et al.*, Nucl. Phys. **B153** (1979) 161; *ibid.*, **B159** (1979) 51
7. A. Gonzales-Arroyo *et al.*, Nucl. Phys. **B166** (1980) 429.
8. M. Ahmed and G. Ross, Phys. Lett. **B56** (1975) 385; Nuc. Phys. **B111** (1976) 441; H. Ito, Prog. Theor. Phys. **54** (1975) 555; K. Sasaki, Prog. Theor. Phys. **54** (1975) 1816
9. J. Kodaira *et al.*, Phys. Rev. **D20** (1979) 627; Nucl. Phys. **B159** (1979) 99; J. Kodaira, Nucl. Phys. **B165** (1980) 129.
10. J. Ashman *et al.*, Phys. Lett. **B206** (1988) 364; Cern report CERN-EP/89-73, June 1989
11. M. J. Alguard *et al.*, Phys. Rev. Lett. **37**(1976)1261; **41**(1978) 70; G. Baum *et al.*, Phys. Rev. Lett. **45**(1980)2000; **51**(1983)1135
12. G. Altarelli and G. G. Ross, Phys. Lett. **B212** (1988) 391; R. D. Carlitz *et al.*, Phys. Lett. **B214** (1988) 229; A. V. Efremov and O. V. Terjaev, Dubna preprint E2-88-287 (1988); R. L. Jaffe, Phys. Lett. **B193**(1987)101; R. L. Jaffe and A. V. Manohar, M. I. T. preprint CTP 1706 (1989); G. Bodwin and J. Qui, Argonne preprint ANL-HEP-PR 89/83; S. Forte, Phys. Lett. **B224**(1989) 189; Saclay preprint SPHT/89-106(1989), to be published in Nuc. Phys. B;

13. G. Altarelli and W. J. Stirling, Particle World **1**(1989)40; M. Glück, E. Reya and W. Vogelsang, Nuc. Phys. **B329**(1990)347
14. A. W. Schreiber, A. W. Thomas, and J. T. Londergan, Phys. Lett. **B237**(1990)120
15. M. Glück and E. Reya, Z. Phys. **C43**(1989)679
16. D. Gross, Phys. Rev. Lett. **32** (1974) 1071; G. Parisi, Phys. Lett. **B43** (1973)207; A. de Rújula, H. Georgi and H. Politzer, Phys. Rev. **D10** (1974) 2141
17. N. Cabibbo and R. Petronzio, Nuc. Phys. **B137** (1978) 395
18. G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298.
19. W. Furmanski and R. Petronzio, Nuc. Phys. **B195** (1982)237; G. P. Ramsey, J. Comp. Phys. **60** (1985)97
20. A. Buras, Nuc. Phys. **B125** (1977)125; A. Buras and K. Gaemers, Nuc. Phys. **B132** (1978)249, X. Song and J. Du, Phys. Rev. **D40** (1989)2177
21. M. Glück *et al.*, Zeits. Phys. **C41** (1989) 667.
22. C. López and F. Ynduráin, in *Padé approximants and their applications*, ed. P. Graves-Morris, Academic Press, New York, 1973.
23. K. Charchula *et al.*, "Some topics in *ep* scattering at HERA - Parton distributions in the nucleon", ZEUS - Note 89-121
24. R. E. Peierls and J. Yoccoz, Proc. Phys. Soc. **A70**(1957)381
25. A. I. Signal and A. W. Thomas, Phys. Lett. **B211**(1988)481; Phys. Rev. **D40**(1989)2832; A. W. Thomas in Lectures presented at the Cargèse Institute on Hadrons and Hadronic Matter, ADP-89-130/T78 (Aug. 1989), to be published in Plenum (N.Y.); A.W. Schreiber, A.I.Signal and A.W.Thomas, in preparation
26. F. E. Close and A. W. Thomas, Phys. Lett. **B212**(1988)227
27. A. W. Schreiber and A. W. Thomas, Phys. Lett. **215**(1988)141

Table 1.

Moments of quark nonsinglet, singlet and glue distributions. All moments are in percent of expected ones. The singlet moments correspond to those obtained from eq.(23) with the first moment readjusted by a term z^{-10} (see text). The numbers in brackets do not have this additional term.

Moment	$\Delta q_{NS}(n, Q^2)$	$\Delta \Sigma(n, Q^2)$	$\Delta g(n, Q^2)$
1	0.93	1.00 (0.99)	1.00 (0.40)
2	0.99	0.99 (1.00)	1.00 (0.84)
3	1.00	1.00 (1.00)	0.99 (0.95)
4	1.00	1.00 (1.00)	0.99 (0.98)

Figure Captions

1. $xF_3(x, Q^2)$ at the model scale μ^2 and at $10GeV^2$ (solid lines). The dashed and dotted lines correspond to the Duke-Owens and Martin- Roberts-Stirling parameterizations of $xu_v(x, 10GeV^2) + xd_v(x, 10GeV^2)$ respectively (Figures I.1b and I.2b of ref.23)
2. The quark singlet (a) and glue (b) distributions at μ^2 and $10GeV^2$ (Note that $\Delta g(x, \mu^2) = 0$). The solid and dashed lines refer to the addition of terms proportional to z^{-16} and z^{-10} respectively to ensure the convergence of the first moment (see Text). The dot-dashed line does not have this term added in.
3. The proton structure function $g_1^p(x, Q^2)$ both at the model scale μ^2 and at $10GeV^2$. The data is taken from ref. 10
4. The predicted neutron structure function $g_1^n(x, Q^2)$ both at the model scale μ^2 and at $10GeV^2$.

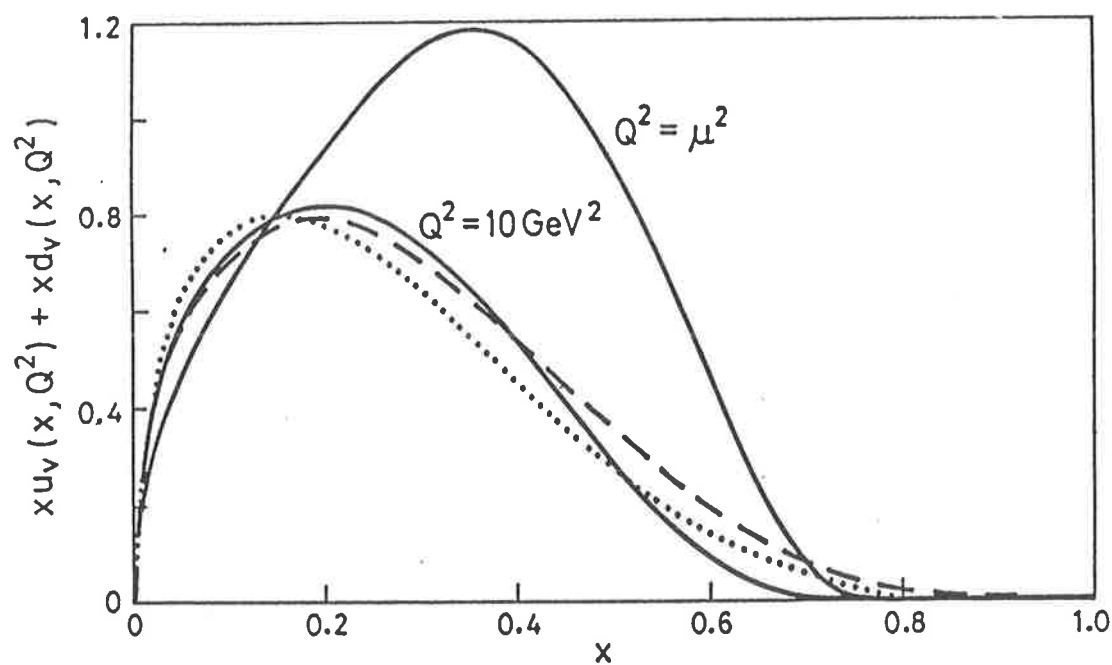
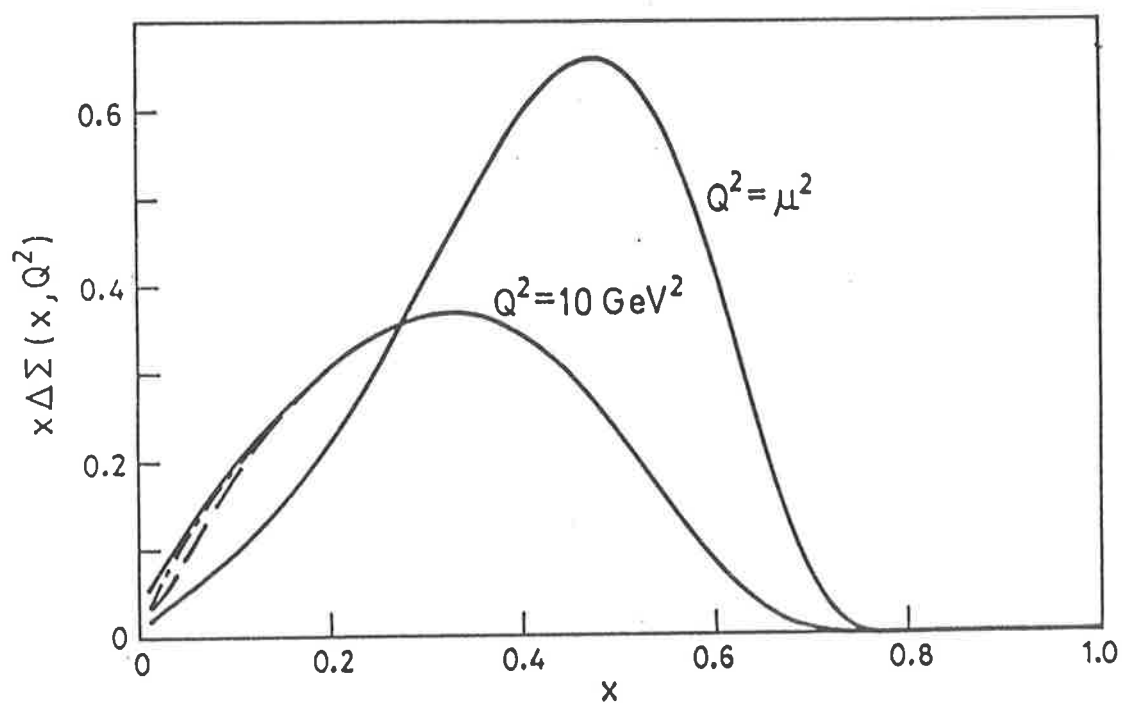


FIGURE 1



FIGURE

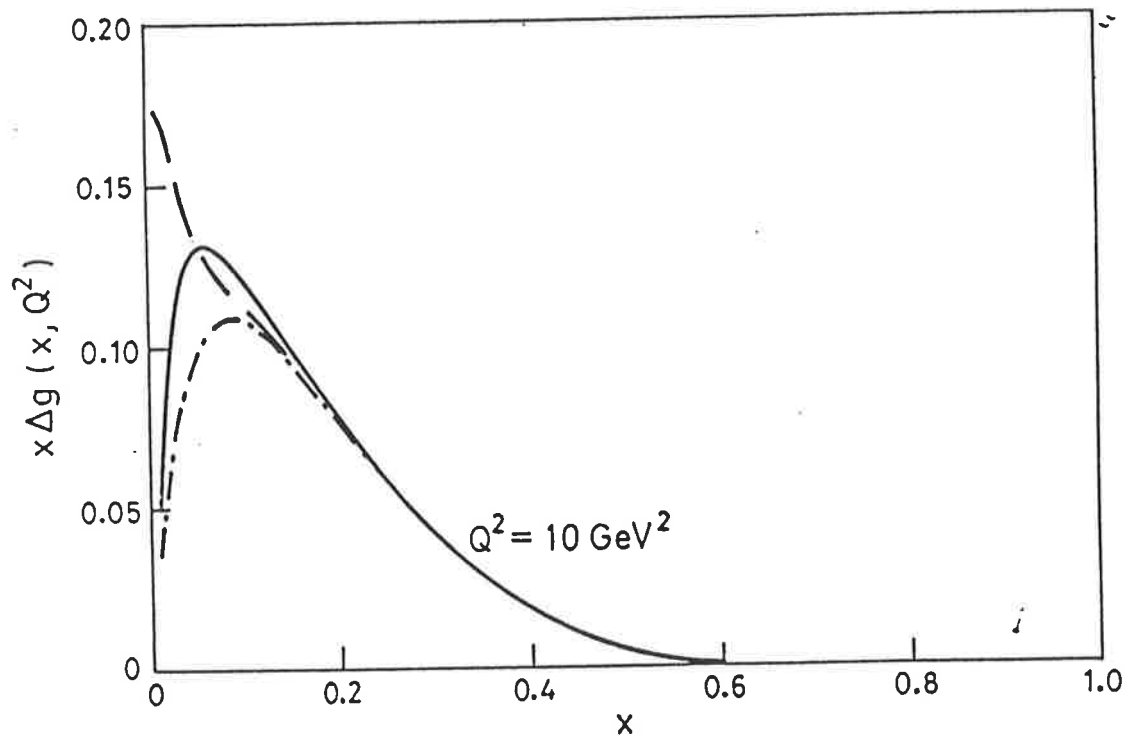


FIGURE 2b

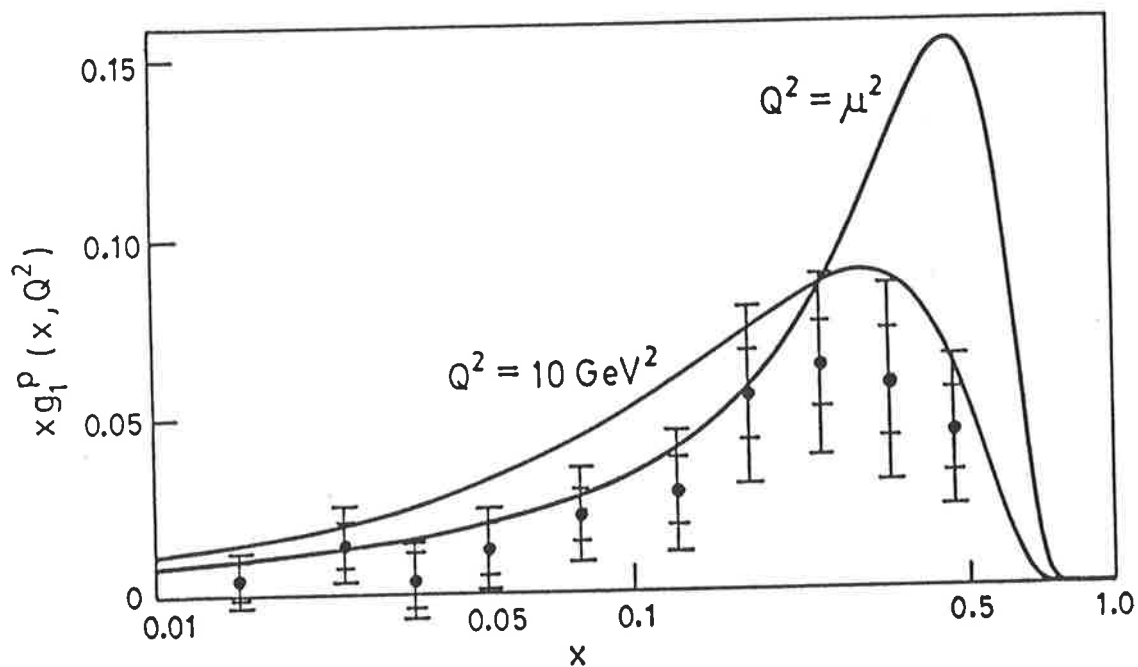
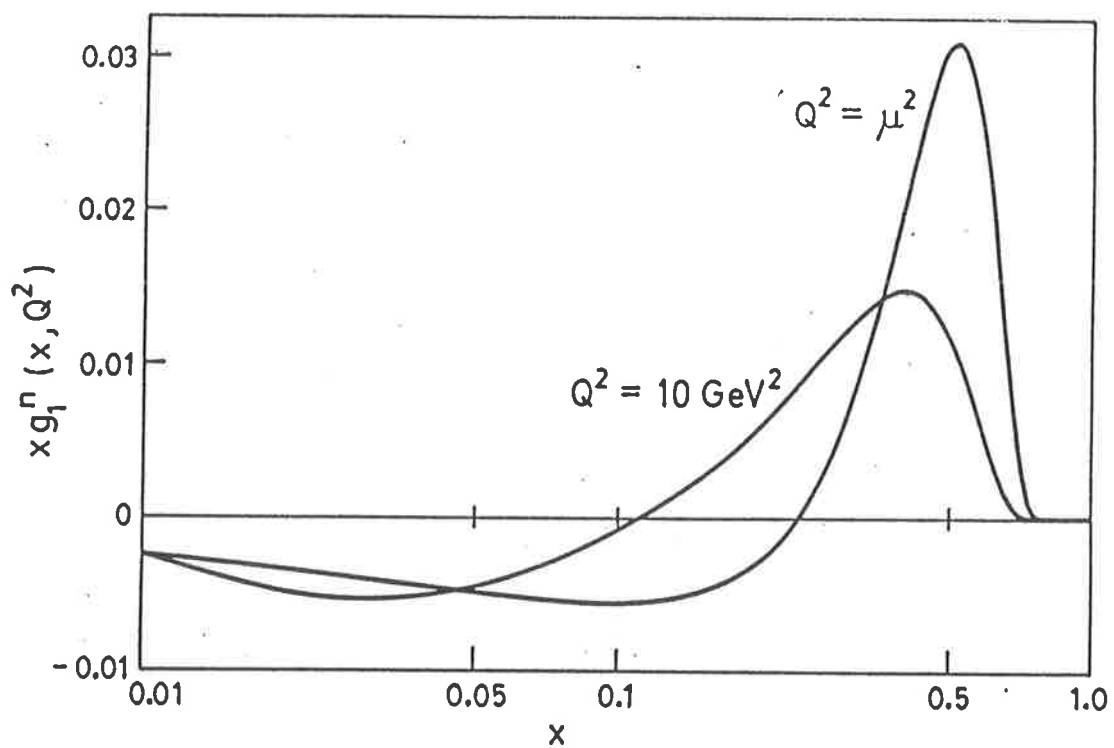


FIGURE 3



FIGURE

A NEW APPROACH TO NUCLEAR STRUCTURE FUNCTIONS

A.W. THOMAS, A. MICHELS, A.W. SCHREIBER

Department of Physics and Mathematical Physics, University of Adelaide, PO Box 498, Adelaide, S.A. 5001, Australia

and

P.A.M. GUICHON

D Ph N, CEN Saclay, Orme des Merisiers, F-91191 Gif-sur Yvette, France

Received 18 August 1989

We calculate the structure function of an MIT bag model nucleon bound in self-consistent scalar and vector mean fields corresponding to nuclear matter at an appropriate density. This approach enables us to investigate a number of features of the usual binding correction for the EMC effect. When Fermi motion is included the calculation reproduces the main features of the data.

Over the past six years enormous interest has been generated by the discovery of a nuclear dependence of the deep-inelastic structure functions [1–3]. Clearly, one would like to rule out explanations based on standard nuclear physics before ascribing the effect to some exotic phenomenon [4]. For this reason it is crucial to understand the conventional approach which includes corrections from binding and Fermi motion [5–7]. While that approach has produced promising results it has also been plagued by two problems. First, the nucleon structure function may vary significantly off-mass-shell, and this dependence is unknown [6]. Second, the approach necessarily involves an impulse approximation whose validity has never been tested.

In order to make progress towards resolving these questions one needs two ingredients. One of these is a method of relating quark models to structure functions. The other is a nuclear model in which quarks figure explicitly. Fortunately there has been significant progress in both of these areas quite recently. In particular, one of us has presented a model [8]^{#1} of nuclear matter in which the nucleons consist of quarks

confined in the MIT bag [10]. Furthermore, Signal and Thomas have recently shown how to overcome the problem of bad support in the calculation of the nucleon structure functions from a quark model (like the MIT bag) [11,12]. By applying this method to the model of nuclear structure presented in refs. [8,9], we are able to throw some light on the earlier calculations of the EMC effect. (For an initial study of such effects in the one-dimensional bag we refer to the work of Signal and Thomas [12].)

Let us begin by briefly recalling the key features of the Guichon model of nuclear matter. The interactions between the bags were replaced by self-consistent, effective scalar and vector (“ σ ” and “ ω ”) potentials in the mean field approximation. By adjusting the σ and ω coupling constants it was possible to obtain nuclear saturation at the correct energy and density. Saturation was a natural consequence of the effect of these fields on the internal structure of the nucleons. In this case it is crucial that the quarks, not the nucleons, move relativistically. As a bonus, the model also gave the correct value of the nuclear incompressibility. All the necessary formulae may be found in ref. [8] – see also ref. [13] where the effects of nucleon Fermi motion are shown to be minor. In particular, the wave functions are still very like those of the free bag model, but with an eigenfrequency

^{#1} The first study of the effect of the σ -field on nucleon structure was by Wong [9], but there was no reference to nuclear saturation.

which must be self-consistently determined in the medium.

As a first step towards calculating the EMC ratio of nuclear structure functions we begin with the simpler problem of a nucleon at rest in nuclear matter at a certain density (We denote a bound nucleon bag of momentum p as $|\tilde{N}p\rangle$, while $|Np\rangle$ is a free nucleon of the same momentum) The twist-2 quark distribution at a scale, μ^2 , appropriate to the model (viewed as a classical approximation to QCD equations of motion) is [14,15]

$$\begin{aligned} \tilde{q}^{(2)}(x, \mu^2) \\ = \frac{m}{2\pi} \int_{-\infty}^{\infty} dz e^{-imxz} \langle \tilde{N}0 | \psi_+^\dagger(\xi^-) \psi_+(0) | \tilde{N}0 \rangle_c \end{aligned} \quad (1)$$

Here ξ^- is $(z, 00-z)$, ψ_+ is the projection $\frac{1}{2}(1+\alpha^3)\psi$, c denotes a connected matrix element, there is an implicit spin average and x is the Bjorken variable defined in units of m (the free nucleon mass) In order to evaluate eq (1) we insert a complete set of states $|\tilde{n}p\rangle$ between $\psi^\dagger$ and ψ and use translational invariance to remove the z -dependence from $\psi^\dagger$ In the case of a free nucleon target this ensures that the quark distribution has the correct support [1 e $q^{(2)}(x, \mu^2)=0$ for $x \geq 1$ in the free case] Effectively one builds in energy-momentum conservation before making any model dependent approximations

For a nucleon bound in scalar and vector mean fields (V_s and V_0 respectively) the twist-2 quark distribution becomes

$$\begin{aligned} \tilde{q}^{(2)}(x, \mu^2) \\ = \frac{m}{(2\pi)^3} \sum_n \int dp \delta[(\tilde{E}_N - mx) - \tilde{p}_n^+] \\ \times |\langle \tilde{n}p | \psi_+(0) | \tilde{N}0 \rangle|^2 \end{aligned} \quad (2)$$

Here m is the free nucleon mass, $\tilde{E}_N$ is the energy of the bound, target nucleon at rest and $\tilde{p}_n^+$ is the energy plus the z -component of the momentum of the intermediate state (again in the medium) In the presence of V_s and V_0 a nucleon consisting of three massless quarks in the 1s state has an effective mass $\tilde{m}$ given by

$$\tilde{m} = m + 3 \left(\frac{(\chi^2 + R^2 V_s^2)^{1/2}}{R} - \frac{2.043}{R} \right), \quad (3)$$

so that

$$\tilde{E}_N \simeq \tilde{m}_N + 3V_0 \quad (4)$$

The quark spatial frequency χ (2.043 in the free case) is determined by solving the Dirac equation subject to the usual linear boundary condition For the dominant, two-quark intermediate state [a $(1s)^2$ configuration] the effective mass is

$$\tilde{m}_2 = m_2 + 2 \left(\frac{(\chi^2 + R^2 V_s^2)^{1/2}}{R} - \frac{2.043}{R} \right), \quad (5)$$

and hence

$$\tilde{p}_n^+ = (\tilde{m}_2^2 + p^2)^{1/2} + 2V_0 + p_z. \quad (6)$$

By choosing the Peierls-Yoccoz method [16] to construct approximate momentum eigenstates from the bag model wave functions one can evaluate eq. (2) and compare the result with the free twist-2 quark distribution $q^{(2)}(x, \mu^2)$ In passing we observe that the scale μ^2 need not be the same for the free and the bound case Any difference would lead to an additional EMC correction of the same mathematical form as "dynamical rescaling" [17] On the other hand, the admittedly naive physical arguments used to justify dynamical rescaling relied on the change in confinement size for quarks in a nucleus In the present model the nucleon size changes by no more than 1%, and thus one might expect a change in μ which was no bigger than this In what follows we ignore any change in μ^2 for the free and bound case

For the purpose of illustration we choose a bag radius of 1 fm and consider the nucleon bound in nuclear matter of density one third of nuclear matter density ($\frac{1}{3}\rho_0$) The corresponding scalar and vector potentials felt by the quarks are respectively -38 MeV and $+15$ MeV We choose the mass of the diquark state (m_2) to be 650 MeV A more sophisticated treatment of spin and flavor [18] would allow for the mass splitting of the $S=0$ and $S=1$ states by gluon exchange, but that is unnecessary for the present purpose In the same spirit we do not adjust the mass of the $3q-\bar{q}$ intermediate state ($n=4$) to give the additional 15% normalisation of the valence quark distribution – rather we add a phenomenological term

proportional to $(1-x)^7$ (This has no effect for x beyond about 0.3)

In a real nucleus the nucleons are not at rest but undergo Fermi motion. Furthermore, the EMC ratio is quite sensitive to the shape of the input quark distribution. While the quark distribution calculated in the bag model is qualitatively acceptable, it does not yet provide a good representation of twist-2 data at an appropriate scale [18,19] (As well at the $S=0$ and $S=1$ mass splitting already mentioned, we should also include the effect of configuration mixing [20]). Therefore, in order to reduce the model dependence as much as possible, we take the phenomenological quark distributions, for a free nucleon, from Buras and Gaemers [21] (at their reference scale Q_0^2) and multiply by the calculated ratio $\tilde{q}^{(2)}(x, \mu^2)/q^{(2)}(x, \mu^2)$. From this quark distribution, which incorporates binding and off-shell corrections, we can calculate a modified nucleon structure function – e.g. $\tilde{F}_{2N}(x, Q^2)$. The nuclear structure function is then obtained by conventional Fermi averaging, that is

$$F_{2A}(x, Q^2) = \int_x^1 dy y f(y) \tilde{F}_{2N}(x/y, Q^2) \quad (7)$$

For $f(y)$ we use the harmonic oscillator distribution of Dunne and Thomas corresponding to ^{56}Fe . Of course, since we are calculating the binding correction explicitly we replaced $m - |\epsilon_i|$ by m in their formulas. We also choose $\hbar\omega$ to be 18 MeV, which gives the same $\langle p^2 \rangle$ for Fe as a Woods-Saxon potential. (The factor of y in eq. (7) is the flux factor omitted in many early calculations.)

After all these preliminaries we are ready to show the results. The change of $q^{(2)}$ to $\tilde{q}^{(2)}$ involves two effects. One is truly a structural effect resulting from the change in the quark wave functions (i.e. $|\tilde{N}0\rangle \neq |N0\rangle$ and $|\tilde{n}p\rangle \neq |np\rangle$). This can be investigated by arbitrarily switching off the effects of V_s and V_0 in eqs. (2)–(6). On the other hand, the kinematic effects due to V_0 and V_s [through the δ -function in eq. (2)] are potentially large. Moreover, unlike the usual binding approximation we see that this approach includes an automatic correction to the impulse approximation. It is illustrated in fig. 1. While a conventional calculation of binding corrections would include the interaction energy of the nucleon with the mean field, it omits the interaction of the

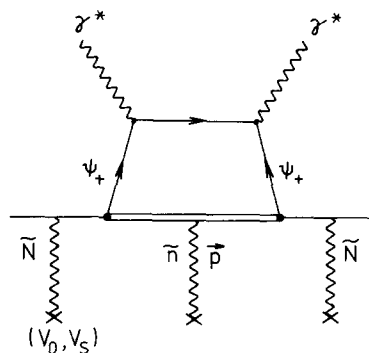


Fig. 1 Schematic illustration of the calculation of the structure function of a nucleon ($|\tilde{N}\rangle$) bound by the scalar and vector potentials V_s and V_0 .

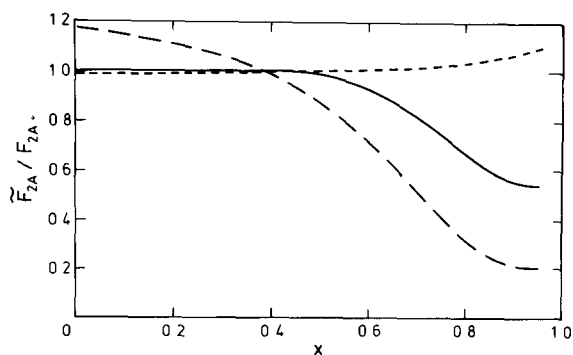


Fig. 2 Illustration of the effect on the Fermi-averaged structure function of (a) changing just the quark wave functions (short dash), (b) also including the kinematic effect of V_s and V_0 (solid), (c) arbitrarily dropping V_s and V_0 in the intermediate state (long dash).

intermediate di-quark state with the same field. Since this debris is relatively slow there is no reason to ignore its interaction with the rest of the nucleus.

Fig. 2 illustrates the effects we just described. In each case we plot the ratio of the Fermi-averaged, medium modified structure function to the Fermi-averaged free structure function. The short-dashed curve, which is close to unity for all x , shows the minor effect of the change in the wave functions of the bound quarks. The solid curve also includes the kinematic effect of V_s and V_0 . Although we tried to find an approximation by which $\tilde{q}^{(2)}$ could be replaced by $q^{(2)}$ at a shifted argument (as in x -rescaling) this did not prove possible. Lastly, the long-dashed curve in

fig 2 illustrates the result of arbitrarily dropping the potentials in the intermediate state. Clearly this approximation, which corresponds to the usual impulse approximation, is very poor.

Although the dip in the solid curve in fig 2 occurs at too large a value of x it clearly resembles the EMC effect. One is therefore tempted to compare with the experimental data for F_{2A}/F_{2D} . This requires a model for F_{2D} which we do not have. Instead we ignore binding in the deuteron and Fermi average the free nucleon structure function as in ref [6] – with $\hbar\omega = 7$ MeV (The fact that we cannot yet estimate the effect of binding in this model for deuterium is what led us to choose $\frac{1}{3}\rho_0$ rather than $\frac{1}{2}\rho_0$ which would probably be more appropriate to Fe.) The resulting EMC ratio is compared to data in fig 3 (dashed curve). The solid curve shows the effect of evolving both F_{2A} and F_{2D} to the same Q^2 as the data (10 GeV^2) before taking the ratio. (Recall that a quark model necessarily represents nucleon structure at poor resolution, or $\mu^2 \approx 0.8 \text{ GeV}^2$.) This clearly is a big effect, and the resulting theoretical calculation bears a striking similarity to the data. It will certainly encourage us to remove some of the approximations made here. For example, instead of a single density ($\frac{1}{3}\rho_0$) we could

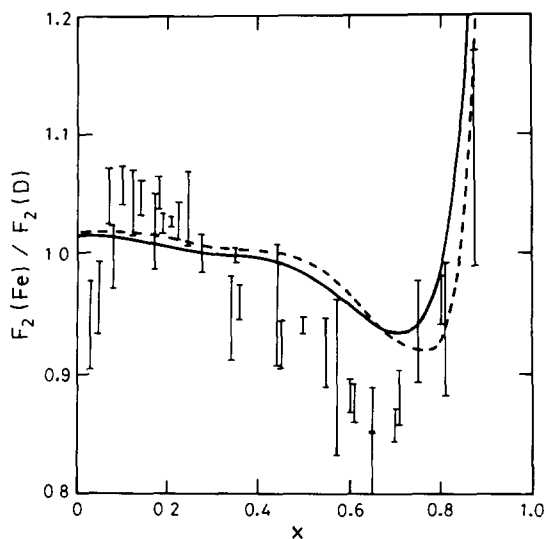


Fig 3 Comparison with data [1–3] of our estimate of the ratio of the Fe to D structure functions at the reference scale of Buras and Gaemers [21] (dashed line) and after evolution to Q^2 of order 10 GeV^2 (solid line)

perform the calculation in the local density approximation. In this way one could also get an estimate of F_{2D} .

In summary, we have presented for the first time a calculation of a nuclear structure function based upon an explicit quark model of nuclear matter. The results have led us to question the accuracy of conventional estimates of the nuclear binding correction (Alternatively we might say that off-mass-shell corrections can be quite large.) It is encouraging that this crude calculation reproduces the qualitative behaviour of the EMC data. This will undoubtedly lead to more sophisticated work along the same lines.

We have benefitted particularly from discussions with R P Bickerstaff and A I Signal. One of us (A W T) would like to acknowledge the hospitality of the DPhN at Saclay where part of the work was completed. This work was supported by the Australian Research Council.

References

- [1] European Muon Collab, J Ashman et al, Phys Lett B 206 (1988) 364
- [2] S Dasu et al (SLAC), Phys Rev Lett 61 (1988) 1061
- [3] BCDMS Collab A C Benvenuti et al, preprint CERN EP/89-06
- [4] R P Bickerstaff and A W Thomas, preprint ADP-89-121/T68, J Phys G, to appear, L L Frankfurt and M I Strikman, Phys Rep 160 (1988) 235, and references therein
- [5] S V Akulinichev, S A Kulagin and G M Vagradoy, JETP Lett 42 (1985) 127, S V Akulinichev and S Shlomo, Phys Rev C 33 (1986) 1551
- [6] G V Dunne and A W Thomas, Nucl Phys A 455 (1986) 701
- [7] G L Li, K F Liu and G E Brown, Phys Lett B 213 (1988) 531
- [8] P A M Guichon, Phys Lett B 200 (1988) 235
- [9] C W Wong, Nucl Phys A 435 (1985) 669
- [10] A Chodos et al, Phys Rev D 10 (1974) 2599
- [11] A I Signal and A W Thomas, Phys Lett B 211 (1988) 481
- [12] A I Signal and A W Thomas, Adelaide preprint ADP-89-118/T65
- [13] P A M Guichon, in Proc Seillac Workshop (CEN, Saclay, 1988)
- [14] H D Politzer, Nucl Phys B 172 (1980) 349, R K Ellis, W Furmanski and R Petronzio, Nucl Phys B 212 (1983) 29

- [15] R L Jaffe, Nucl Phys B 229 (1983) 205
- [16] R E Peierls and J Yoccoz, Proc Phys Soc (London) A 70 (1957) 381
- [17] F E Close et al, Phys Rev D 31 (1985) 1004
- [18] F E Close and A W Thomas, Phys Lett B 212 (1988) 227
- [19] A W Thomas, preprint ADP-88-106/T64, in Proc 1st European Workshop on Hadronic Physics in the 1990's with GeV Electrons (Seillac, 1988), to appear
- [20] S Bass and A W Thomas, contributed paper H7 to Few body XII, TRIUMF report TRI-898-1
- [21] A Buras and K Gaemers, Nucl Phys B 132 (1978) 249

SPIN DEPENDENT STRUCTURE FUNCTIONS IN THE CLOUDY BAG MODEL

A.W. SCHREIBER AND A.W. THOMAS

*Department of Physics and Mathematical Physics, University of Adelaide,
North Terrace, Adelaide, South Australia 5000, Australia*

Received 17 May 1988

We derive expressions for the integrals of the spin dependent structure functions $g_1(x)$ for the proton and the neutron in the context of the cloudy bag model. We find that the neutron contributes 5–10% to the Bjorken sum rule, while there is a corresponding decrease for the proton's contribution. It is difficult to reconcile these results with those reported in a recent experiment.

The EMC group has recently reported a measurement of the spin structure function $g_1(x)$ of a polarised proton [1]. Having measured this quantity for a large range of x ($0.01 < x < 0.7$) they extrapolate to $x=0$ and 1 and hence claim to be able to give a result for the integrated structure function $\int_0^1 g_1^p(x) dx$. The Bjorken sum rule relates this quantity and the corresponding one for the neutron:

$$\int_0^1 [g_1^p(x) - g_1^n(x)] dx = \frac{1}{6} |g_A/g_V|. \quad (1)$$

Using this sum rule and the experimental value of the ratio of axial vector to vector coupling constants (g_A/g_V), the EMC group derives a value for the integrated neutron structure function. These results are summarised in the first column of table 1. Although some doubt has been cast on the validity of their analysis (in particular the extrapolation to small $x^{(2)}$), it certainly suggests that the integrated neutron asymmetry may not be zero.

In the Bjorken limit the spin-dependent structure function $g_1(x)$ can be expressed in terms of spin-dependent quark distributions:

$$g_1^N(x) = \frac{1}{2} \sum_f Q_f^2 [q_f^+(x) - q_f^-(x)]. \quad (2)$$

Here $q_f^{(\pm)}(x)$ is the number density for quarks of flavour f (charge Q_f) at momentum fraction x , and with spin parallel (anti-parallel) to that of the nucleon

($N=n$ or p). Using current algebra one can show that the integrated value of $g_1^N(x)$ is given by the matrix element

$$\int_0^1 dx g_1^N(x) = \frac{1}{2} \langle N \uparrow | \bar{\Psi}(0) Q^2 \gamma_5 \Psi(0) | N \uparrow \rangle. \quad (3)$$

It is a remarkable fact that whereas the left-hand side involves very high energy scattering, the right-hand side (RHS) involves a low energy matrix element. This is the basis of our calculation.

The Bjorken sum rule (eq. (1)) is easily obtained from eq. (3) (using $Q^2 = \frac{2}{3}B + \frac{1}{6}Y + \frac{1}{3}I_3$ for a non-charmed quark). For quarks with equal mass, the RHS of eq. (3) is proportional to the spin-flavour matrix element $\frac{1}{2} \langle N \uparrow | \sum_f Q_f^2 \sigma_{z_f} | N \uparrow \rangle$. In the simplest case, where the nucleon is in the 56-plet of SU(6) and the quarks are non-relativistic [3], the constant of proportionality is unity (and g_A/g_V is $\frac{5}{3}$).

It was realised early on that the discrepancy between this value of $|g_A/g_V|$ and the experimental value is largely due to the fact that the confinement of the quarks to volumes of the order of fm^3 endows them with high velocities and that therefore the non-relativistic picture is inadequate [4]. For example, if one simulates the confinement mechanism by the use of the MIT bag [5], one obtains $|g_A/g_V| = 1.09$ – in much better agreement with experiment, especially if one applies centre-of-mass corrections which tend to increase this value by about 15–20%. The corresponding values for the nucleon asymmetries are

Table 1
Expectations for spin dependent quantities in the non relativistic quark, the MIT bag and the cloudy bag models. For the cloudy bag model the renormalised coupling constants of ref. [12] were used.

Experimental	Non relativistic quark model	MIT bag model	Naive CBM		Cloudy bag model normalised to $g_A/g_V = 1.254$	
			$R = 0.8 \text{ fm}$	$R = 1.1 \text{ fm}$	$R = 0.8 \text{ fm}$	$R = 1.1 \text{ fm}$
$ g_A/g_V $	1.254 ± 0.006	$\frac{5}{3} = 1.67$	0.98	1.02	1.254	1.254
$\int g_1^p(x) dx$	$0.114 \pm 0.012 \pm 0.026$	$\frac{2}{18} = 0.28$	0.151	0.161	0.193	0.198
$\int g_1^n(x) dx$	$-0.077 \pm 0.012 \pm 0.026$	0	-0.013	-0.008	-0.017	-0.010

listed in the third column of table 1. We note that although the value of the axial coupling constant is now predicted very well, the neutron asymmetry is still zero because the MIT bag uses the same spin-isospin structure for the wavefunction as the original non-relativistic quark model.

It is however well known that, in contrast to QCD, the MIT bag model badly violates chiral symmetry [6]. In particular, the reflection of a quark at the MIT bag surface does not result in a reversal of its spin and hence its helicity is not conserved. Since $g_1(x)$ is intimately related to the quark spins (see eq. (2)) one might expect that there would be significant corrections to the predictions of the MIT model for these structure functions.

The cloudy bag model does conserve the axial vector current [7] by including an extra term in the lagrangian. Physically this requires the inclusion of pions in the theory. Put simply, the change of the quark spin upon reflection at the bag boundary is compensated by the emission of a pion with unit angular momentum. This has the immediate consequence that the quark spins in the nucleon need not add up to $\pm \frac{1}{2}$ anymore – hence the spin structure revealed by an electromagnetic probe is changed.

The pions essentially play the role of a quark-antiquark sea, which has been previously incorporated into quark models of the nucleon [3,8]. We note that in the cloudy bag model (as used in this paper) the “sea” is unpolarised. The important effect of the sea is to yield an effective, well-defined mixing of the SU(6) wavefunction of the nucleon in terms of bare nucleons, deltas and pions. The effect has similar consequences to the phenomenological representation mixing used by Bucella and others [9]^{#1} to describe the spin structure of the nucleon. Sea polarisation effects, as well as the spin carried by the gluons, are not described but may of course give additional contributions [11].

The diagrams relevant for the calculation of $\int g_1(x) dx$ and $|g_A/g_V|$ are shown in fig. 1. We use the same formalism and notation as in ref. [12] and find the following expression for the axial vector coupling constant:

^{#1} See also ref. [10] which has extensive references.

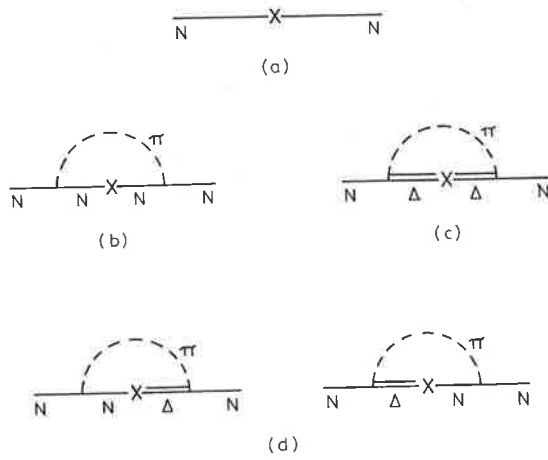


Fig. 1. Diagrams contributing to $|g_A/g_V|$ and hence to the sum rule for g_1 in the cloudy bag model. (The cross represents the action of the axial current operator.)

$$|g_A/g_V| = [\Omega/3(\Omega-1)] \times \left(\frac{5}{3}Z + \frac{5}{27}P_{N\pi} + \frac{25}{27}P_{\Delta\pi} + \frac{8}{9}P_{N\Delta\pi} \right), \quad (4)$$

where Z is the bare bag probability (see fig. 1a), $P_{N\pi}$ and $P_{\Delta\pi}$ are the probabilities corresponding to figs. 1b and 1c, and $P_{N\Delta\pi}$ describes the interference term depicted in fig. 1d^{#2}. These terms are explicitly written down in ref. [12]. Numerically Z is about 0.6–0.7 while $P_{N\pi}$, $P_{\Delta\pi}$ and $P_{N\Delta\pi}$ are ~ 0.2 , 0.1 and 0.3, respectively. We note here that the spatial information (which reduced g_A/g_V from the naive SU(6) value of $\frac{5}{3}$ is contained purely in the term $\Omega/3(\Omega-1)$ (Ω is the ground state solution to $j_0(\Omega) = j_1(\Omega)$, as usual), while the expression in the bracket is given (apart from integrations over the pion momentum) by the spin-isospin content of the wavefunction. One can therefore immediately write down the corresponding expressions for the (integrated) proton and

neutron structure functions as calculated from the spin-isospin matrix element $\langle \tilde{N} | Q^2 \sigma_z | \tilde{N} \rangle$ ($|\tilde{N}\rangle$ is the cloudy bag wavefunction for the nucleon):

$$\int dx g_1^p(x) = \frac{1}{6} [\Omega/3(\Omega-1)] \times \left(\frac{5}{3}Z - \frac{5}{27}P_{\Delta\pi} + \frac{50}{27}P_{\Delta\pi} + \frac{4}{9}P_{N\Delta\pi} \right), \quad (5)$$

$$\int dx g_1^n(x) = \frac{1}{6} [\Omega/3(\Omega-1)] \times \left(-\frac{10}{27}P_{N\pi} + \frac{25}{27}P_{\Delta\pi} - \frac{4}{9}P_{N\Delta\pi} \right). \quad (6)$$

The spatial factor has to be identical to that of $|g_A/g_V|$ so that the Bjorken sum rule is satisfied.

As expected, the contributions involving a pion now give rise to a non-zero neutron asymmetry. The numerical values corresponding to eqs. (4)–(6) are shown in the fourth column of table 1 for two values of the bag radius, the only free parameter in the cloudy bag model. The numbers are not particularly sensitive to the bag radius.

The decrease in the proton asymmetry from the MIT value is largely due to the bare bag probability, Z , being of the order of 60%. Approximately half of the decrease is recovered from the graphs with pionic corrections to the vertex, mostly from the contribution of the Δ^{++} with $s_z = \frac{3}{2}$ (fig. 1c). In the case of the neutron, the contributions of figs. 1c and 1d roughly cancel because they appear with opposite sign in eq. (6). The asymmetry is therefore given essentially by graphs involving only the nucleon and the pion.

It is not wise to compare the results in column 4 of table 1 (and column 3 for that matter) directly to experiment. The well-known problem of the MIT and cloudy bag models, as presented here, is that the nucleon wavefunction is not translationally invariant. Corrections to $|g_A/g_V|$ arising from this defect have been estimated to be of the order of 15–20% [7], yielding a value in excellent agreement with experiment in both models. To compare our results with experiment, we shall adopt the view here that in principle the cloudy bag model – with corrections – gives the experimental value of $|g_A/g_V|$. This amounts to normalising the results in column 4 of table 1 to the experimental value of $|g_A/g_V|$. The result is given in the last column of table 1.

We see that the result for the integrated proton structure function is in disagreement with the exper-

^{#2} This equation for $|g_A/g_V|$ is identical to that derived in ref. [7] using the Goldberger-Treiman relation. Here we use the MIT value for $\Omega/(\Omega-1)$ in order to highlight the similarities and differences to the MIT model. In ref. [7] (eq. (2.13)) this was written in terms of the pion decay constant and the coupling constant $f_{NN\pi}$. Hence there is a small difference in the numerical result for $|g_A/g_V|$ in column 4 of table 1 with respect to the value derived in ref. [7].

imental value given by the EMC group at the 2σ -level. The predicted contribution of the neutron to the Bjorken sum rule is 5–10%, compared to $37 \pm 14\%$ according to the EMC group.

Ellis and Jaffe [13], as well as Sehgal [14], have used existing data on $|g_A/g_V|$ for the nucleons and hyperons to extract estimates for these integrated structure functions. To do this, they assume an unpolarised strange quark sea as well as SU(3) symmetry so that they can relate the quark distribution in the hyperons to those in the nucleons. Their values (0.200 ± 0.006 for the proton and -0.009 ± 0.006 for the neutron, using the modern value for F/D [15]) are similar to those obtained by us.

Our results are also in rough agreement with those of Carlitz and Kaur [11] (0.21 and -0.003 for the integrated proton and neutron structure functions, respectively). They consider, phenomenologically, a similar dilution of the spin of the valence quarks, however in their case this is taken to be due to the interactions with a gluon sea. They assume that the quark sea gives a negligible contribution with respect to this.

Finally, we note that Jaffe [15] has pointed out that as a consequence of the U(1) axial anomaly there may be substantial QCD corrections beginning at the two-loop level. He finds lower bounds for the proton and neutron integrated structure functions of 0.113 and -0.078 , respectively (using $\alpha_s = 0.27$). These results overlap with those reported by the EMC group.

In summary, we have shown that in the cloudy bag

model one expects a non-zero contribution from the neutron to the Bjorken sum rule. This contribution amounts to 5–10% depending on the bag radius. This result is in agreement with previous phenomenological analyses, but differs from the value reported by the EMC group. It would be natural to extend this work to calculate the x -dependence of $g_1^n(x)$.

This work was supported by the University of Adelaide and the Australian Research Grant Scheme.

References

- [1] J. Ashman et al., Phys. Lett. B, 206 (1988) 364.
- [2] F.E. Close and R.G. Roberts, preprint RAL-88-007.
- [3] J. Kuti and V.F. Weisskopf, Phys. Rev. D 4 (1971) 3418.
- [4] P.N. Bogolioubov, Ann. Inst. Henri Poincaré 8 (1968) 163.
- [5] A. Chodos, R.L. Jaffe, K. Johnson and C.B. Thorn, Phys. Rev. D 10 (1974) 2599.
- [6] E.g. A.W. Thomas, Adv. Nucl. Phys. 13 (1984) 1.
- [7] A.W. Thomas, S. Th  berge and G.A. Miller, Phys. Rev. D 24 (1981) 216.
- [8] G.W. Look and E. Fischbach, Phys. Rev. D 16 (1977) 211.
- [9] F. Bucella, M. DeMaria and M. Lusignoli, Nucl. Phys. B 6 (1968) 430.
- [10] F.E. Close, Nucl. Phys. B 80 (1974) 269.
- [11] R. Carlitz and J. Kaur, Phys. Rev. Lett. 38 (1977) 673; J. Kaur, Nucl. Phys. B 128 (1977) 219; F.E. Close and D. Sivers, Phys. Rev. Lett. 39 (1977) 1116.
- [12] S. Th  berge, G.A. Miller and A.W. Thomas, Can. J. Phys. 60 (1982) 59.
- [13] J. Ellis and R. Jaffe, Phys. Rev. D 9 (1974) 1444.
- [14] L.M. Sehgal, Phys. Rev. D 10 (1974) 1663.
- [15] R.L. Jaffe, Phys. Lett. B 193 (1987) 101.

CORRECTIONS TO THE ELLIS-JAFFE SUM RULE

A.W. Schreiber & A.W. Thomas
Physics & Mathematical Physics Department
University of Adelaide
North Terrace, Adelaide, South Australia, 5000

We report on two contributions to the neutron spin structure function: a pionic effect as well as a correction due to gluon exchange.

The Bjorken sum rule relates the spin structure functions of the proton and neutron to the ratio of axial to vector coupling constants of the nucleons:

$$\int_0^1 \left[g_1^p(x) - g_1^n(x) \right] dx = \left| \frac{g_A}{g_V} \right| \quad (1)$$

As the latter is well known, the recent determination by the European Muon collaboration¹ of the proton's contribution enables one to deduce that of the neutron : $-0.077 \pm 0.012 \pm 0.026$.

The Bjorken sum rule is remarkable in that it implies that, although the structure functions themselves are high energy quantities (measured in deep inelastic scattering experiments such as that of the EMC), their integrals can be related to low energy matrix elements:

$$\int_0^1 g_1^N(x) dx \propto \left\langle N \left| \bar{\psi}(0) \gamma^2 \gamma^z \gamma^5 \psi(0) \right| N \right\rangle \quad (2)$$

Low energy models of nucleon structure can therefore predict these.

While not as rigorous as the Bjorken sum-rule, the Ellis-Jaffe sum rule², based on SU(3) symmetry, suggests that essentially all of the strength is carried by the proton - hence the surprise at the EMC result. This result can be readily understood in a simple model like the MIT bag model. It gives identically 0 for the neutron matrix element using eqn. (2), because the nucleon wavefunction is usually taken to be in the 56-plet of SU(6). However, the MIT bag breaks the chiral symmetry exhibited by QCD. It may

This work was supported by the Australian Research Council and the University of Adelaide.

therefore not be wise to trust its predictions here, as the chiral symmetry of the model is intimately connected to its spin structure. The cloudy bag model does incorporate chiral symmetry by changing the SU(6) character of the wavefunction. Evaluating the matrix element (2) for this model³, we find that it does indeed predict a non-vanishing contribution for the neutron:

$$\int_0^1 g_1^n(x) dx \propto -\frac{10}{27} P_{N\pi} + \frac{25}{27} P_{\Delta\pi} - \frac{4}{9} N_{\Delta\pi}, \quad (3)$$

Here $P_{N\pi}$ ($P_{\Delta\pi}$) are the probabilities of the nucleon being a nucleon (delta) and a pion, and $P_{N\Delta\pi}$ is an interference term. Numerically the neutron's contribution amounts to (depending on the bag radius) 13 - 22% of the value found by the EMC.

SU(6) results of the MIT bag model are also changed by one gluon exchange⁴. In particular, the spin structure observed by the virtual photon is modified by the possibility of the quarks having non-zero angular momentum in the intermediate state. At the tree level, this effect can be shown to be given by⁵

$$\int_0^1 g_1^N(x) dx < N \left| \sum_{i \neq j} \sigma_z(j) Q^2(i) \right| N >, \quad (4)$$

where i and j label the quarks exchanging the gluon (i is the quark interacting with the virtual photon). This matrix element gives a contribution of 24% of the EMC result for the integral.

Hence, roughly 37 - 46% of the neutron's contribution to the Bjorken sum rule as determined by EMC can be explained by these two effects. Taking into consideration the large uncertainty in the experimental value⁶ this may in fact be all there is.

REFERENCES

- 1) J. Ashman et al. Phys. Lett. B206 (1988) 364.
- 2) Ellis and R. Jaffe, Phys. Rev. D9 (1974) 1444.
- 3) A.W. Schreiber and A.W. Thomas. submitted to Phys. Lett. B.
- 4) H. Högaasen and F. Myhrer, Phys. Rev. D37 (1988) 1950.
- 5) F. Myhrer and A.W. Thomas, Phys. Rev. D, rapid comm. (to appear, 1988).
- 6) F.E. Close and R.G. Roberts, Phys. Rev. Lett. 60 (1988) 1471.