CROSSHOLE RESISTIVITY AND ACOUSTIC VELOCITY IMAGING: 2.5-D HELMHOLTZ EQUATION MODELING AND INVERSION

by

Zhou Bing

B.Sc., M.Sc. (China)

Department of Geology and Geophysics

The University of Adelaide

SA 5005, Australia

Submitted in fulfillment of the requirements for
the degree of Doctor of Philosophy

September 1998

Contents

Abstract V

Statement VII

Acknowledgments VIII

Chapter 1 Introduction 1

1.1 Crosshole Resistivity Imaging 1

1.2 Crosshole Seismic Imaging 3

1.3 2.5-D Approximation 5

1.4 Objectives of The Study and Outline of Thesis 7

Chapter 2 2.5-D Helmholtz Equation 9

2.1 2.5-D Helmholtz Equation 9

2.2 DC Electric Potential Equation 11

2.3 Acoustic Wave Equation 13

2.4 2.5-D Green's Functions 16

2.5 Principle of Reciprocity 19

2.6 The Finite Element Method (FEM) 20

(1) Galerkin's Solution 21

(2) Variational Principle 22

Chapter 3 2.5-D DC Resistivity Modeling 25

3.1 Mixed-Boundary Condition 26

3.2 The FEM Formulation 28

(1) Inner Element Matrix 29

(2) Boundary Element Matrix 35

3.3 Inverse Fourier Transform 36

3.4 Implementation of The Method 39

3.5 Examples of Modeling 43

(1) Accuracy of The Method 43

(2) Resistivity Profiling 46

(3) Crosshole Applied Potential Surveying 46

(4) Monitoring of Underground Water Injection
From Artificial Aquifer Storage 48

Chapter 4 2.5-D Acoustic Wave Modeling in The Frequency-Domain 52

4.1 2.5-D Absorbing Boundary Condition 52
(1) Clayton-Engquist ABC 53
(2) General Form of The 2.5-D ABC 54
4.2 Boundary Condition for The Evanescent Field 57
4.3 Composite Boundary-Valued Solution 58
(1) The FEM Formulation 59
(2) Inner Element Matrix 60
(3) Boundary Element Matrix 62
(4) Numerical Modeling Results 64
4.4 A Damping Method 77
(1) The FEM Formulation 78
(2) Choice of The Damping Term 80
(3) Numerical Modeling Results 81

Chapter 5 Extension to 2.5-D Elastic Wave Modeling in The Frequency-Domain 90

5.1 2.5-D Elastic Wave Equation 91
5.2 The FEM Formulation 95
5.3 Boundary Conditions 98
(1) A Viscous Boundary Condition 99
(2) Paraxial Approximation 100
(3) The Modified Higdon Formula 101

Chapter 6 Inversion Algorithms 104

6.1 Objective Functions 104
6.2 Tikhonov Regularization Solutions 108
(1) General Iterative Solution 109
(2) Iteratively Linearized Solution 110
(3) Conjugate Gradient Solution (CGS) 113
(4) Local-Search Quadratic Approximation CGS 114

6.3 The Smoothest Model Solution (Occam's Inversion) 120

6.4 Generalized Subspace Solution 122

Chapter 7 Computation of The Fréchet and Second Derivatives 128

7.1 Basic Definitions 129

(1) Bilinear Operator 129

(2) Fréchet and Second Derivatives 130

7.2 Common Algorithms 131

7.3 Explicit Expressions for DC Resistivity 134

7.4 Explicit Expressions for 2.5-D Acoustic Waves 139

7.5 Numerical Algorithms 142

7.6 Synthetic Examples 148

(1) Patterns in a Homogeneous Background 148

(2) Patterns for Imhomogeneous Models 152

Chapter 8 Synthetic Study on Crosshole DC Resistivity Imaging 157

8.1 Crosshole Surveying Configurations 157

8.2 Sensitivity Variation of Crosshole Measurements 162

8.3 Numerical Experiments of Crosshole Imaging 170

(1) Pole-Pole Array 172

(2) Pole-Bipole Array 173

(3) Bipole-Pole Array 176

(4) Bipole-Bipole Array 178

8.4 Numerical Experiments for Resistivity Profiling 183

8.5 Physical Model Imaging Experiments 188

Chapter 9 Numerical Simulations for Crosshole Acoustic Velocity Imaging with Spectral Data 191

9.1 Full-waveform Inversion 191

9.2 2.5-D Generalized Diffraction Tomography 195

9.3 Imaging with a Known Source Wavelet 199

(1) Using Real and Imaginary components 201

(2) Using Hartley-Spectral Data 204

(3) Using Amplitude and Phase Spectra 208

9.3 Imaging with an Unknown Source Wavelet 214

(1) Normalized Spectral Data....214
(2) Numerical Imaging Experiments....217

Chapter 10 Conclusions and Future Research....222

10.1 Forward Modeling....222
10.2 Imaging Algorithms....224
10.3 Numerical Imaging Results....225
10.4 Future Research....227

Appendix A The Hartley Transform....228

Appendix B Computation of The Boundary Integral....231

Appendix C Semi-Analytic Acoustic Solution for Two Semi-infinite Media in Contact....234

Appendix D A Constant-Block Approximation in The Computation of The Fréchet and Second Derivatives for Resistivity Imaging....237

Bibliography....239

Abstract

Crosshole resistivity and seismic velocity imaging (diffraction tomography or waveform inversion with the acoustic approximation) are two very useful techniques in geophysical exploration, because they may be employed to map conductivity or velocity inhomogeneities (geological anomalies) between boreholes. Such imaging techniques, when considered in mathematical terms, both reduce to 2.5-D Helmholtz equation modeling and inversion. This thesis presents (1) new finite element solutions for two specified forms of this kind of equation, which define 2.5-D resistivity and 2.5-D acoustic wave problems, as well as an extension to 2.5-D elastic waves solution; (2) iterative algorithms for tackling inversion problems, including Tikhonov regularisation, the smoothest model solution and the subspace solution, in which the generalised algorithms and their inter-relations are formulated, and new approximations for the Fréchet and second derivatives are developed; (3) numerical and physical model experiments for crosshole resistivity imaging with different electrode arrays (or configurations) and synthetic simulations for crosshole acoustic velocity imaging with different full-waveform spectral data (real and imaginary components, amplitude and phase, and the Hartley spectra).

The modeling examples for resistivity and acoustic wavefield show that the presented methods are effective and flexible to compute the 3-D physical response for 2-D arbitrary media. Furthermore, it is shown that the accuracy of the modeling mainly depends upon the wavenumber range and the number of wavenumber samples, both of which can be evaluated in terms of the typical shape of the transformed potential in the resistivity case and the critical values of the wavenumber of the media around the seismic source in the acoustic wave case, respectively. The constant-point approximation based on the explicit expressions for the Fréchet and second derivatives has been shown to be a robust and efficient algorithm for inversion. The numerical and physical imaging experiments show that some specified three- and four-electrode arrays are more suitable for crosshole resistivity imaging than the pole-pole array. For acoustic imaging with spectral data, the real and imaginary components, the Fourier amplitude

and the Hartley spectra are suitable for full-waveform inversion with a known source wavelet, but the phase data do not permit satisfactory object field reconstruction. It is shown in the thesis that the normalised Fourier/Hartley spectral data can be used for imaging when the seismic source wavelet is unknown.