A NEW HILBERT TIME WARPING PRINCIPLE FOR PATTERN MATCHING

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ABSTRACT

In many pattern matching tasks, an important preliminary operation is a transformation of the time or space scale to compensate for object distance or position, or for irrelevant variations, such as the tempo of speech.

A new algorithm based on the phase of the analytic signal representation of the signal patterns has been developed. For example, if two sinewaves are represented in analytic signal form $S_A(t) = s(t) + j\phi(t)$, and $\phi$, the Hilbert transform of $s(t)$, is plotted against $s(t)$, then the resultant figure is a circle. Similar properties pertain to other time functions subjected to time compression or expansion, which may itself be slowly time varying. Each function in the set to be compared requires only one warping to transform it to a function of phase. The phases of the signals are compared with a common phase scale, and signal samples corresponding to matching phases are mapped to a new time series, in the order of the phase sequence of the common phase scale. Each phase point of the common phase scale is associated with a corresponding signal sample. The new time series is defined as the Hilbert warped (HW) version of the original time series.

The most significant and accurate application of this method is to time warp the sampled version of an analogue signal, where the sampling instants have been slowly time varying and there is no significant amplitude distortion. The analysis of the above class of signals has been illustrated in this thesis, and the application is extended to practical signals such as biomedical signals and speech parameter contours, where significant amplitude distortion has been observed.
The performance of the new algorithm is found to be superior for warping speech parameter contours due to computational efficiency and accurate representation of the warping neighbourhood, when compared with the conventional dynamic time warping algorithm (DTW), where computational efficiency can only be improved by a priori information on the warping neighbourhood, and by the use of preconditioning through the identification of signal endpoints or fixed points.