# Spreads and Ovoids of the Split Cayley Hexagon

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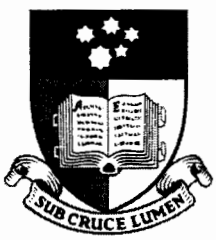


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# Contents

# Abstract

The aim in this thesis has been to investigate translation spreads and ovoids in the split Cayley hexagon $H(q)$, extending on the initial investigations carried out in the paper [BTVM98]. After introducing the necessary background material, using the coordinatization of $H(q)$ as described in [DeSVM93] and [VM98], tools are developed that subsequently find use here and it is believed will continue to be beneficial. Thence the investigation begins in earnest.

Drawing on the analogy between $H(q)$ and the generalized quadrangles $W(q)$ and its dual $Q(4,q)$, results and characterizations are found that are comparable to those given in [BTVM98] for the case of translation spreads and ovoids of the generalized quadrangles. In particular, these spreads and ovoids are shown to be precisely those whose stabilizers in a certain automorphism group of $H(q)$ have order $q^3$, and consequently, the functions appearing in their coordinate representations are linear operators over the prime field.

This thesis also contains classifications of translation spreads and ovoids of $H(q)$ subject to various additional constraints. In particular, it classifies all ovoids translation with respect to a point, spreads translation with respect to a line that have kernel equal to all of $GF(q)$, and spreads that are translation with respect to two disjoint flags. In addition, the nonexistence when $q \neq 3^h$ of ovoids translation with respect to a flag is proved.

Escaping the above-mentioned analogy, and instead making a direct connection with the generalized quadrangles, the "projection along reguli" approach of [BTVM98] for obtaining ovoids of $Q(4,q)$ from locally hermitian spreads of $H(q)$ is generalized to point locally hermitian spreads, and therefore spreads translation with respect to a flag when $q \neq 3^h$. Finally, the geometrical notion of semiclassical for a spread of $H(q)$ is shown to be equivalent to the algebraic one of having all of $GF(q)$ as kernel.