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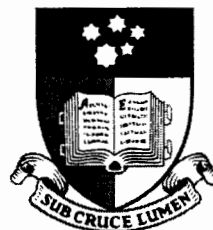
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# Spreads and Ovoids of the Split Cayley Hexagon

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# Abstract

The aim in this thesis has been to investigate translation spreads and ovoids in the split Cayley hexagon  $H(q)$ , extending on the initial investigations carried out in the paper [BTVM98]. After introducing the necessary background material, using the coordinatization of  $H(q)$  as described in [DeSVM93] and [VM98], tools are developed that subsequently find use here and it is believed will continue to be beneficial. Thence the investigation begins in earnest.

Drawing on the analogy between  $H(q)$  and the generalized quadrangles  $W(q)$  and its dual  $Q(4, q)$ , results and characterizations are found that are comparable to those given in [BTVM98] for the case of translation spreads and ovoids of the generalized quadrangles. In particular, these spreads and ovoids are shown to be precisely those whose stabilizers in a certain automorphism group of  $H(q)$  have order  $q^3$ , and consequently, the functions appearing in their coordinate representations are linear operators over the prime field.

This thesis also contains classifications of translation spreads and ovoids of  $H(q)$  subject to various additional constraints. In particular, it classifies all ovoids translation with respect to a point, spreads translation with respect to a line that have kernel equal to all of  $GF(q)$ , and spreads that are translation with respect to two disjoint flags. In addition, the nonexistence when  $q \neq 3^h$  of ovoids translation with respect to a flag is proved.

Escaping the above-mentioned analogy, and instead making a direct connection with the generalized quadrangles, the “projection along reguli” approach of [BTVM98] for obtaining ovoids of  $Q(4, q)$  from locally hermitian spreads of  $H(q)$  is generalized to point locally hermitian spreads, and therefore spreads translation with respect to a flag when  $q \neq 3^h$ . Finally, the geometrical notion of semiclassical for a spread of  $H(q)$  is shown to be equivalent to the algebraic one of having all of  $GF(q)$  as kernel.